TRANSACTIONS

OF THE

AMERICAN SOCIETY

OF

MECHANICAL ENGINEERS

1935

VOLUME 57

PUBLISHED BY
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
29 WEST THIRTY-NINTH STREET, NEW YORK, N. Y.

Copyright, 1936, by
The American Society of Mechanical Engineers

Printed in the United States of America by
The Mack Printing Co., Easton, Pa.

~~Engr~~ Engin. Lib.
Engin.
Dept
2-28-36

Foreword

VOLUME 57 of the Transactions of The American Society of Mechanical Engineers contains the individual papers published during 1935 under the sponsorship of the Society's professional divisions and technical committees, including the contributions of the Applied Mechanics Division issued originally in a quarterly known as *Journal of Applied Mechanics*, and the 1935 Society Records and Index. The technical papers and reports that make up this volume represent the Society's annual contribution to the permanent record of mechanical-engineering achievement. Most of these papers and reports were presented at meetings of the Society and of its professional divisions and local sections, and were published in monthly issues, eight being distributed as the Transactions of The American Society of Mechanical Engineers and four as the *Journal of Applied Mechanics*. Other papers published by the Society but not included in this volume may be located by means of the Index on page RI-121 at the end of the volume.

Because of the fact that material of which this volume is composed was originally issued periodically as Transactions, *Journal of Applied Mechanics*, and Society Records, three sets of page numbers will be found. Numbers without letter symbols are those of the eight issues of the Transactions, those with letter symbol A preceding the number refer to the pages of the *Journal of Applied Mechanics*, which follow, and those with letter symbol RI to the Society Records section, which concludes the volume.

All sections of the Transactions are bound together at the end of the year for the convenience of libraries and of engineers who wish all of the papers in permanent form. Copies of the bound Transactions will be found in depositories located in selected engineering, university, and public libraries throughout the world. A complete list of these depositories will be found on pages RI-117 to RI-120 of the Society Records and Index. Copies of the bound Transactions have also been set aside for sale.

The Society Records of the A.S.M.E. for 1935, which form a part of these Transactions, are the permanent records of the Society's activities for the year. They include lists of committees, annual reports of the Council and all Society Committees, and memorial notices of deceased members. The Index provides a means of locating special A.S.M.E. publications, and articles in *Mechanical Engineering*, as well as those in the Transactions.

THE PUBLICATIONS COMMITTEE

The Elastic Properties of Steel at High Temperatures[1]

By GUY VERSÉ,[2] ANN ARBOR, MICH.

This paper deals with the determination of the modulus of elasticity and of the modulus of rigidity of steel at elevated temperatures up to 500 C.

Both a static and a dynamic method were used and their results are compared. Whereas previously most of the static determinations of these moduli have been made under increasing load, in the tests described in this paper they were determined under decreasing load, according to the method proposed by F. L. Everett. The tests show that within certain limits the results obtained by this method are practically independent of the rate of loading. This appears to be due to a strain-hardening effect occurring upon loading and detected upon unloading.

The comparison between the static and the dynamic tests shows that the values of E, modulus of elasticity, and G, modulus of rigidity, determined statically under decreasing load are close to the values obtained dynamically. However they are somewhat lower for temperatures above 400 C.

The values obtained for E and G by static tests under increasing load are not reliable owing to the difficulty of discriminating between the elastic and the plastic deformation.

The values determined dynamically are the most reliable.

The variation of Poisson's ratio with temperature is also presented, the values of μ being obtained by calculation from the values found for E and G.

THE need for more research relating to the determination of the elastic properties of steel at high temperatures has been pointed out in recent years by many investigators and designers of power-plant equipment.

The question, however, has been given attention for a considerable time. As early as 1877, Pisati investigated Young's modulus for iron up to 300 C, and expressed his results by means of a third-degree empirical formula which shows a gradual decrease of the modulus as the temperature rises.

The existence of non-elastic or permanent deformations, of increasing importance with rising temperature, was soon discovered and many investigators have attempted to eliminate the time effect and determine the modulus of elasticity by measuring the so-called instantaneous strain.

Further, when the phenomenon of the continuous creep of metals at high temperatures was discovered, the attention of the investigators became focused on the determination of creep characteristics and it appeared to many that it was impossible to arrive at any definite value for the modulus at high temperatures.

In researches undertaken at the Westinghouse Research Laboratories in 1923, S. Timoshenko found that if the material be previously loaded and an appreciable strain-hardening produced the rate of creep occurring under a load smaller than this initial load was considerably reduced. This takes place to a sufficient extent in some cases, to permit the determination of the elastic deformation with reasonable accuracy. It will be shown that this phenomenon can be explained on the basis of the ordinary creep curves.

F. L. Everett,[3] in his determinations of the modulus of rigidity of a medium-carbon steel up to 500 C, used the original scheme of determining the modulus on unloading rather than on loading.

Because of observations by Honegger,[4] of the Brown, Boveri & Co., Switzerland, it was felt that an investigation of the possibilities and limitations of this scheme would be of value. Dynamic determinations of the moduli have been made also and their results compared with those obtained statically.

Testing Equipment and Test Procedure

(1) *Static Tensile Tests.* A sketch of the apparatus used for the static tensile tests is presented in Fig. 1. Its principle can

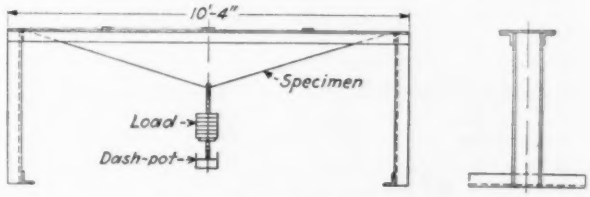


FIG. 1 TENSILE TEST APPARATUS

readily be seen, namely, the specimen is in the form of a wire rigidly fixed at both ends and loaded at the middle. The deflections of the middle-point under various loads are measured by a cathetometer. The heating of the specimen is produced by an electric current passing through it and its temperature is determined by measuring its electrical resistance.

A number of considerations lead to the adoption of such a scheme:

(*a*) The system is one of great simplicity involving no delicate measuring device

(*b*) No part of the measuring instruments is subjected to high temperature

(*c*) Measurement of an elongation of 1.5×10^{-6} is obtained by using an instrument with no greater sensitivity than a cathetometer. This is due to the fact that small

---

[1] This paper is a portion of a dissertation presented for the degree of Doctor of Science in the University of Michigan. The experimental program was conducted in the Applied Mechanics Laboratory under the direction of Prof. S. Timoshenko.

[2] Associated with Sté. Ame. des Ateliers de Construction Mécanique de Tirlemont, Belgium. Dr. Versé was graduated from the University of Brussels, Belgium, in 1931, with the degree of Ingenieur Civil des Mines. He devoted two years to graduate studies at the University of Michigan as holder of a fellowship of the Commission for Relief in Belgium Educational Foundation, Inc.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., December 4 to 8, 1933, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "Strength of Materials Subjected to Shear at High Temperatures," by F. L. Everett, Trans. A.S.M.E., vol. 53 (1931), paper APM-53-10.

[4] "The Modulus of Elasticity of Steel at High Temperatures," by E. Honegger, *Brown Boveri Review*, vol. 19, no. 5 (1932), p. 1.

elongations of the wire result in appreciable deflections of the middle-point

(*d*) The method of heating is very simple and the specimen reaches the required stable temperature in a short time.[5]

The electric current was supplied by a large battery set kept on charge during the entire test. In order to prevent air currents from producing temperature variations and to cut down the radiation, asbestos tubing was placed around the wire.

The stress and the strain were easily calculated from geometrical considerations, admitting that the shape of the loaded

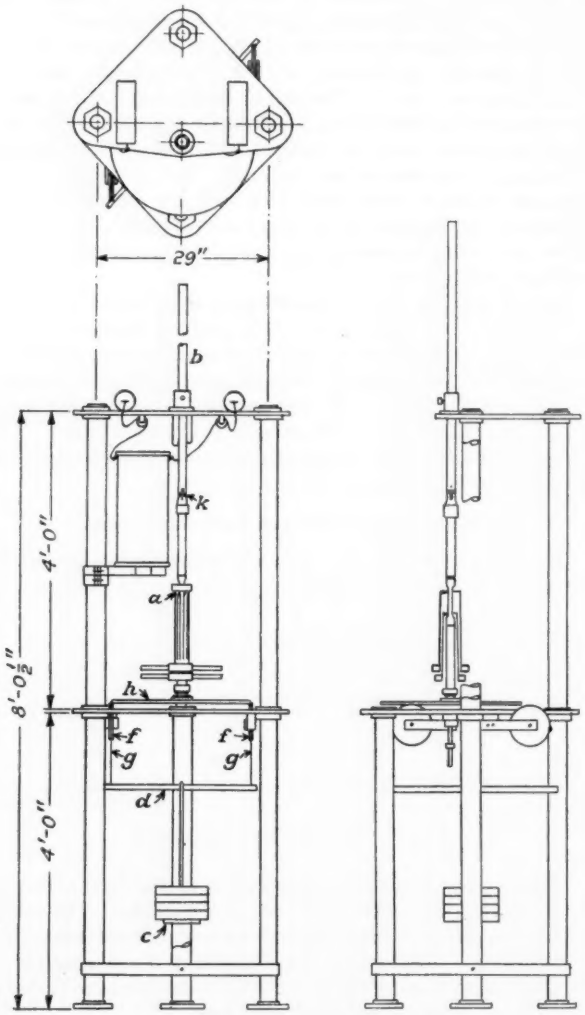


FIG. 2 TORSION TEST APPARATUS

wire is composed of two straight lines forming an angle at the loading point. A relatively large initial sag, at least one-tenth of the span, was thus resorted to.

The load was applied and removed by steps at the following rates:

about 2500 lb per sq in. every 3 min
about 2500 lb per sq in. every 5 min
about 2500 lb per sq in. every 10 min

[5] From the experiments of H. L. Dodge, *Physical Review*, vol. 2, series 2, p. 431, in which the heating was produced alternately by an electric current and by external source, it has been proved that heating by electric current has no effect other than that caused by the accompanying temperature rise.

The measurements of the deflection were taken 10 sec after the application of the load and at the end of the loading period, that is, 3 minutes, 5 minutes, or 10 minutes after the application of the load, depending on the rate of loading.

(2) *Static Torsion Tests.* The apparatus used in making the static torsion tests was designed by F. L. Everett for his researches on the strength of materials submitted to shear at high temperatures. The essential features of this machine are shown in Fig. 2. A description of it will be found in a paper presented by Dr. Everett.[3]

The load was applied and removed by steps at the following rates:

1575 lb per sq in. every 2 min
1575 lb per sq in. every 5 min
1575 lb per sq in. every 10 min

Measurements of the shear strain were taken 10 seconds after the application of the load and at the end of the loading period.

(3) *Dynamic Tensile Tests.* In the dynamic tests, the values of E and G were arrived at by determining the period of

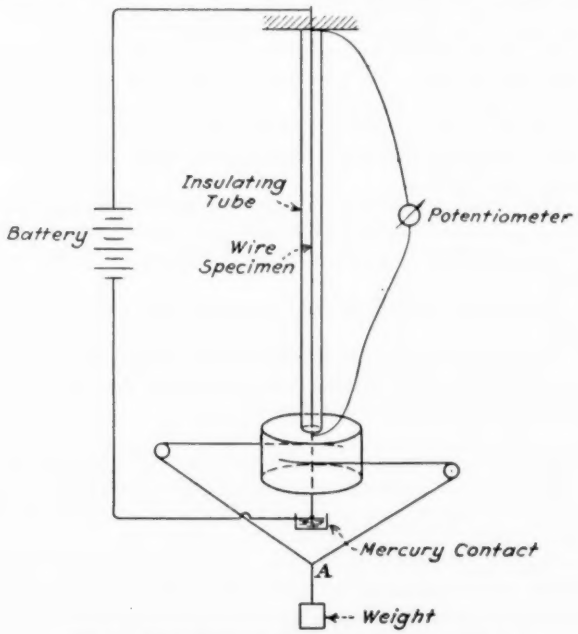


FIG. 3 APPARATUS TO MEASURE G DYNAMICALLY

free longitudinal and free torsional vibration of a wire. Since the stresses were kept very low it is considered that the amount of plastic deformation taking place in each cycle was negligible. Thus the moduli of elasticity and of rigidity were calculated from the observed periods of free vibration by the well-known formulas based on Hooke's law of elasticity.

For the dynamic tensile tests the same apparatus as for the static tension tests, and shown in Fig. 1, was used except that the horizontal plate of the damper was replaced by vertical fins. The wire, loaded at the middle, is given a first vertical impulse and the free vibrations so produced are recorded on a drum rotating at a known constant speed. The period of free vibration can thus be measured on the record.

The method of heating and of measuring the temperature are the same as for the static tensile tests.

(4) *Dynamic Torsion Tests.* The principle used for the determination of the period of free torsional vibration involves the apparatus shown diagrammatically in Fig. 3 and is as follows: A couple is applied at the lower end of a wire hanging vertically and

held fast at the upper end. This couple is suddenly removed by burning the strings at A and free torsional vibrations result. The period of these vibrations is measured by a stop-watch.

The specimen is heated by passing electric current through it. Its temperature is determined by measuring its electrical resistance. The temperature difference between the top and the bottom of the specimen is minimized by the use of a thin copper

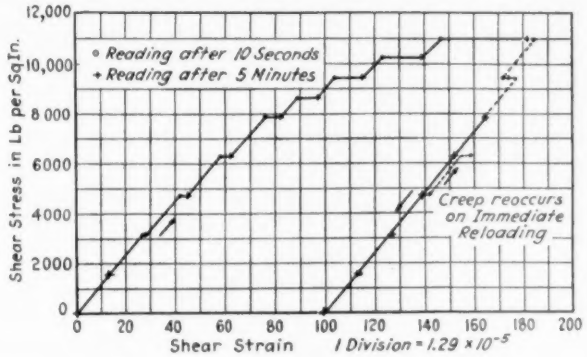


FIG. 4 TORSION TEST AT 450 C (842 F)
(Load increased every 5 minutes. Rate of unloading: 1575 lb per sq in. every 5 minutes.)

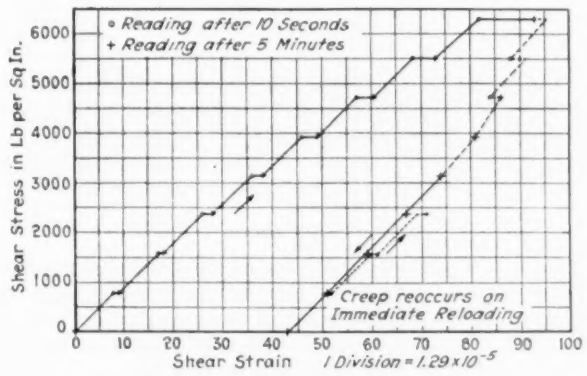


FIG. 5 TORSION TEST AT 500 C (932 F)
(Rate of loading and unloading: 1575 lb per sq in. every 5 minutes.)

sheet inside of a glass tube surrounding the wire with asbestos blocking the openings of the tube. Besides, there is an averaging effect along the wire in both the measurements of the temperature and of G.

MATERIAL TESTED

For the static and dynamic tensile tests and the dynamic torsion tests, the specimens were in the shape of a wire 0.0307 in. in diameter and made of carbon steel of the composition:

Carbon	0.43 per cent
Manganese	0.86 per cent
Sulphur	0.038 per cent
Phosphorus	0.018 per cent
Silicon	0.135 per cent

The wire for these tests was annealed before testing.

The static torsion tests were made on tubular carbon-steel specimens of the following composition:

Carbon	0.34 per cent
Manganese	0.80 per cent
Sulphur	0.03 per cent
Phosphorus	0.02 per cent
Silicon	0.10 per cent

The specimens for the static torsion tests were fully annealed by heating to 900 C before testing.

RESULTS

(1) *Static Tests.* The tension tests as well as the torsion tests show that in determining the moduli of elasticity and of rigidity, it is of decided advantage to operate under decreasing rather than under increasing load.

The diagrams, Figs. 4 and 5, for shear at 450 and 500 C are characteristic also of those obtained for tension. It seems that a strain-hardening effect occurring on loading and detected on unloading tends to reduce considerably the amount of plastic deformation taking place in unit time under a certain stress.

It was found also that, within the range of loading rates previously mentioned, the values obtained on unloading for the moduli were independent of the rate of loading and unloading.

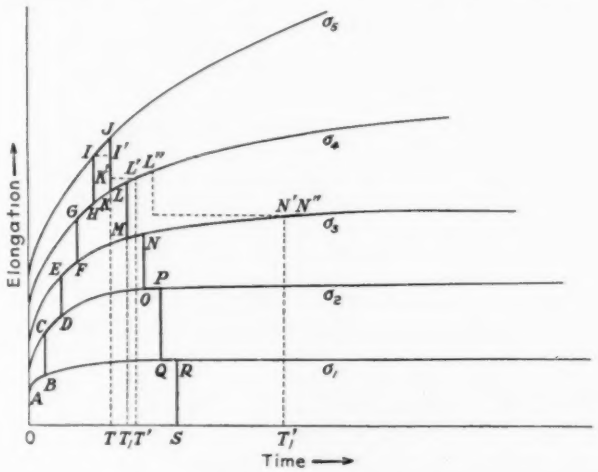


FIG. 6 TIME-ELONGATION CREEP CURVES

This results in a more reliable determination of the moduli and it is felt that the values obtained under decreasing load are more nearly the physical constants for the material.

The strain-hardening effect is not sufficient, however, to permit an accurate determination, for medium carbon steels, of the moduli at temperatures above 500 C. It is felt that for such temperatures it is better to resort to dynamic tests.

This strain-hardening effect can be explained on the basis of the well-known deformation-time creep curves. It can be seen from Fig. 6 that the step-loading and unloading process used in our experiments can be represented, as far as creep rates are concerned, by the line 0 *A B C D E F G H I J K L M N O P Q R S*. It can be noticed that, under such a stress as σ_2, the rate of creep CD on loading is much larger than the rate of creep OP on unloading. The restriction "as far as creep rates are concerned" has been made because the line $0AB$—QRS does not give the actual amount of elongation of the specimen after each step. It is easily seen that upon unloading from the stress σ_5 to the stress σ_4 the contraction will not recover the plastic deformation $I'J$ undergone from I to J. Now, even considering the creep rates only, the line $0AB$—QRS is not accurate. Various investigators of the question of creep, Dr. Everett[3] for instance, have shown that the rate of creep under a definite stress is much more a function of the total amount of deformation than of the time. In other words, the total amount of deformation after unloading from σ_5 to σ_4 will not be TK but TK', and the rate at which creep will take place under σ_4 will not be given by KL but by $L'L''$ which is smaller. At the second step of unloading, σ_4 to σ_3, the decrease of creep rate is even larger being from MN to $N'N''$.

TABLE 1 VALUES OF E AND G DETERMINED STATICALLY

Temp, deg C	Modulus of elasticity E, lb per sq in. (millions)	Temp, deg C	Modulus of rigidity G, lb per sq in. (millions)
25	29.9	25	11.53
220	27.5	300	11.1
300	26.9	350	10.7
400	24.5	400	10.0
460	23.0	450	9.15
...	...	500	7.9

TABLE 2 VALUES OF E AND G DETERMINED DYNAMICALLY

Temp, deg C	Modulus of elasticity E, lb per sq in. (millions)	Temp, deg C	Modulus of rigidity G, lb per sq in. (millions)
25	30.2	25	11.55
220	28.4	290	10.9
310	26.9	387	10.3
395	25.7	430	9.73
462	24.2	500	8.5

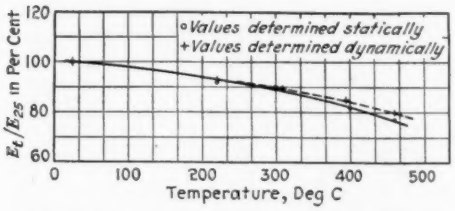


FIG. 7 VARIATION OF E WITH TEMPERATURE

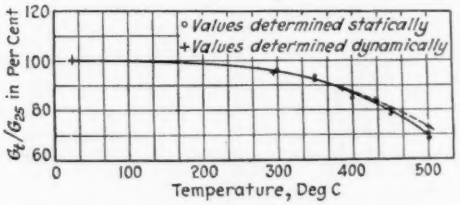


FIG. 8 VARIATION OF G WITH TEMPERATURE

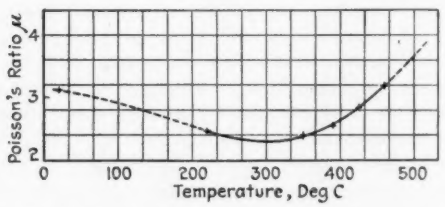


FIG. 9 VARIATION OF μ WITH TEMPERATURE

The values of E and G, determined statically, are given in Table 1.

(2) *Dynamic Tests.* The results of the dynamic tests proved to be more consistent and accurate than those of the static tests, especially at temperatures above 400 C. The cause of this is believed to lie in the fact that the stresses having been kept small, the amount of plastic deformation taking place in each cycle was negligible.

The values of E and G determined dynamically are given in Table 2.

The values of the ratio E_t/E_{25C} as obtained from both static and dynamic tests are plotted against the temperature in Fig. 7. It appears that, for temperatures above 350 C, the values of E_t/E_{25C} when determined statically are somewhat lower than when determined dynamically. The values of the ratio G_t/G_{25C} as obtained from both static and dynamic tests are plotted against temperature in Fig. 8. Again it appears that the values of the static ratio are somewhat lower than the values of the dynamic ratio for temperatures above 400 C.

(3) *Variation of Poisson's Ratio With Temperature.* The theory of elasticity shows that Poisson's ratio μ is related to the modulus of elasticity E and the modulus of rigidity G as follows:

$$G = \frac{E}{2(1 + \mu)}$$

or

$$\mu = \frac{E}{2G} - 1$$

From the values of E and G determined dynamically, the values of μ at various temperatures have been calculated. Although it is fully realized that this method of calculation of μ cannot give very accurate results it is felt that it is interesting to present the results of these calculations (see Fig. 9). It is realized also that these findings are not in agreement with the results obtained by Carrington.[6] The numerical values are presented in Table 3.

TABLE 3 VARIATION OF POISSON'S RATIO WITH TEMPERATURE

Temp, deg C	Modulus of elasticity E, lb per sq in. (millions)	Modulus of ridigity G, lb per sq in. (millions)	Poisson's ratio μ
25	30.2	11.55	0.307
220	28.4	11.4	0.246
300	27.2	11.05	0.23
350	26.3	10.6	0.24
390	25.8	10.23	0.255
425	25.1	9.77	0.283
460	24.2	9.18	0.318

CONCLUSIONS

The comparison between the dynamic and the static tests shows that the values of E and G determined statically under decreasing load are practically independent of the rate of loading and are close to the values obtained dynamically. However, they are somewhat lower for temperatures above 400 C.

The values obtained for E and G by static tests under increasing load are not reliable, owing to the difficulty of discriminating between the plastic and the elastic deformation.

It is, therefore, the author's opinion that for the determination of the moduli at high temperatures dynamic tests should always be resorted to. For the application of these results to the calculation of static constructions, such methods as the one presented by K. Baumann[7] should be used in order to take into account the plastic deformation.

Additional experiments are necessary to reach a definite conclusion as to the variation of Poisson's ratio with temperature.

ACKNOWLEDGMENTS

The research work reported in this paper was carried out under the direction of Prof. S. Timoshenko. To him the author wishes to express his indebtedness for his invaluable help, advice, and inspiration. Dr. F. L. Everett's and Prof. A. E. White's willingness to give the author the benefit of their own experience on the subject is also greatly appreciated.

[6] *Engineering*, vol. 117 (1924), p. 69.

[7] Ibid., vol. 130 (1930), p. 597.

The Calculation of the Dispersion of Flue Dust and Cinders From Chimneys

By HUBER O. CROFT,[1] IOWA CITY, IOWA

The dispersion of flue dust from a chimney depends primarily upon: (a) the velocity of the flue gas in the chimney; (b) the size and weight of the particle; (c) the height of the chimney; (d) the wind velocity; and (e) the aerodynamic resistance of the particle. The value of the last-mentioned quantity is usually the most uncertain of these variables and is usually obtained from Stokes's equation. Comparisons of different equations for the fluid resistance to falling particles are made in the following article and a new equation is suggested. Using this latter equation and assuming values of the other variables mentioned, a method is demonstrated for calculating the annual dust loading at different distances from a chimney.

MUCH has been written concerning the separation of flue dust from the flue-gas stream by mechanical, hydraulic, or electrical methods, but little has been written about the disposal of that dust and ash which actually reaches the chimney. It is the purpose of this paper to examine the aerodynamics and mechanics of dust-carrying streams and by making certain assumptions, predict approximately what dust particles will be carried up and discharged by a chimney, and further, at what distance from the chimney these particles will be deposited on the earth.

Resistance of Falling Bodies

Heretofore, it has been customary to determine the frontal resistance of falling bodies from Stokes's equation or Allen's equation. In this paper, a new equation is suggested which is the result of the study of numerous falling-body experiments both in liquids and gases.

After the resistance relation has been determined, the time of fall of a particle from a given height may be predicted. Hence, knowing the height of fall and the time of fall, and assuming a wind velocity, the distance a particle can be carried horizontally may be calculated.

Considering a particle falling in still air, the force tending to cause the particle to fall is that of gravity minus the buoyancy of the air. This is opposed by the frontal resistance of the particle, or air friction due to motion. The net force of gravity F can be expressed:

$$F = \frac{\pi d^3}{6} g (\delta - \rho) \quad [1]$$

where F is the net force of gravity, poundals; d is the diameter of the particle, ft; g is the acceleration of gravity, ft per sec per sec; δ is the unit weight of the particle, lb per cu ft; and ρ is the unit weight of the air in lb per cu ft.

The resistance P of the particle to motion due to air friction can be defined by the equation

$$P = \frac{f}{2} \pi \left(\frac{d}{2}\right)^2 \rho V^2 \quad [2]$$

where P is the air resistance due to motion, poundals; f is the resistance coefficient, a pure number; d is the diameter of the particle, ft; ρ is unit weight of the gas, lb per cu ft; and V is the velocity of the particle, fps.

A falling particle will decelerate rapidly until such a velocity is reached that the net force pulling the particle downward will be just equal to the air resistance to motion of the particle; in which case a constant and maximum downward velocity is obtained. This condition can be expressed mathematically by

$$F = \frac{\pi d^3}{6} g (\delta - \rho) = P = \frac{f}{2} \pi \left(\frac{d}{2}\right)^2 \rho V^2 \quad [3]$$

$$\text{or } f = \frac{4 d g (\delta - \rho)}{3 \rho V^2} \quad [4]$$

where f is the friction coefficient when the maximum velocity is reached.

This maximum, "free-fall," velocity is known as the "end velocity" or "terminal velocity." At such a condition the frontal resistance of the particle is just equal to the force of gravity.

If conditions were reversed, and the atmosphere were to move upward with a speed equal to the end velocity of a particle, the particle would then be supported at a constant elevation by the air velocity.

This end velocity, then, is also the minimum velocity of an upward, vertical, wind stream supporting a given particle. A greater wind velocity than the terminal velocity would carry a given particle upward, because the upward force due to frontal resistance is greater than the force of gravity acting downward upon the particle.

The resistance coefficient, as defined by Equation [2] for regularly shaped bodies, such as spheres, disks, etc., has been studied by many physicists and engineers.

The variation of the resistance coefficient is usually most conveniently demonstrated by plotting this coefficient f as ordinate and Reynolds' number, $\frac{d V \rho}{\mu} = R$, as abscissa, where μ is the absolute viscosity, lb per ft-sec, and the other symbols are the same as already indicated.

These relations between the resistance coefficients and Reynolds' numbers R are shown in Fig. 1 together with paths of equations expressing certain well-known resistance laws.

---

[1] Head of department of mechanical engineering at the University of Iowa. Mem. A.S.M.E. Professor Croft was graduated from the University of Colorado in 1918 and entered the Air Service at Post Field, Okla. From 1919 to 1921, he was employed by Swift and Company and Durbin Van Law, in Denver. From 1921 to 1927 he was a member of the faculty of the University of Illinois, where he obtained his M.S. degree. In 1927 he became a member of the faculty of Stanford University, and in 1929 was appointed to his present position. Professor Croft has been employed by the City of Saint Louis, the Public Service Company of Northern Illinois, and the Murray Iron Works. He is a member of the A.S.M.E. Power Test Code Committee No. 21.

Contributed by the Fuels Division and presented at the Semi-Annual Meeting, Denver, Colo., June 25 to 28, 1934, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

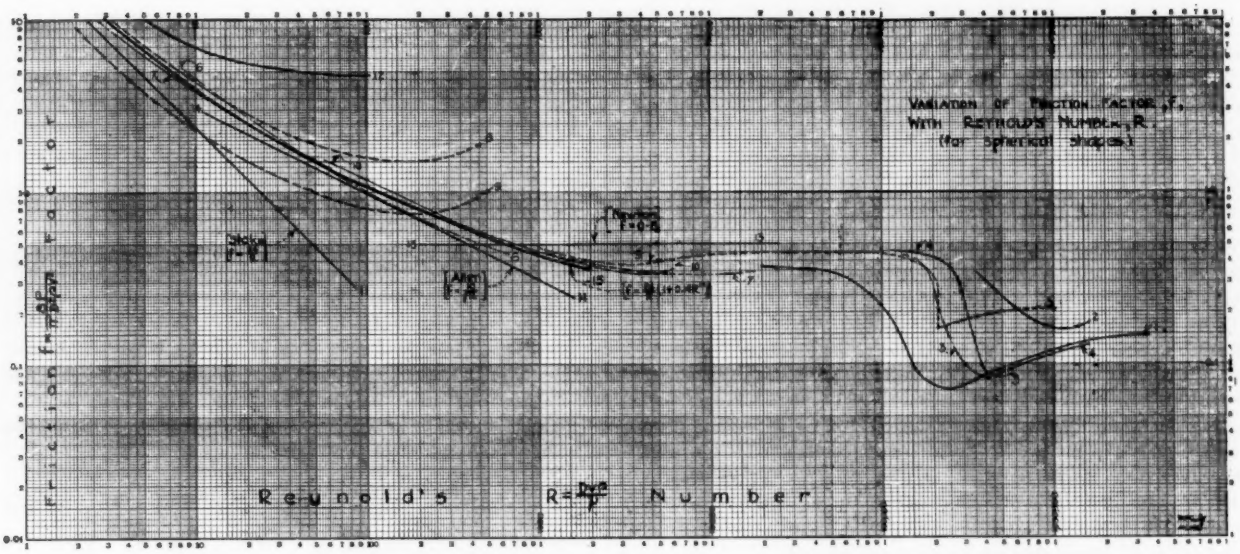


FIG. 1 VARIATION OF RESISTANCE COEFFICIENT f WITH REYNOLDS' NUMBER R FOR SPHERICAL SHAPES

(Curves 1, 3, and 4 are from various wind-tunnel experiments for spheres up to 20-cm diam; curve 2 is for free-falling experiments of spheres from 20 cm to 30 cm diam made of wax, rubber, and wood (3); curve 5 shows the results of experiments by Wieselberger; curve 6 shows the results of falling steel spheres by Liebster (4); curve 7 is for spheres; curves 8 and 9 are for cinders and fly-ash by Rosin and Kayser (5); and 10 gives Allen's results (2). Numbers in parentheses refer to similarly numbered references at the end of the paper.)

The graphs include results obtained from a large number of experimental determinations made with a great variety of particles varying from the resistance of a sphere 20 cm diam, fixed in a wind tunnel, to free-falling rubber spheres about 20 cm diam, and very small spheres in liquids, and cinders in air.

Graphs are also shown for the well-known equations for the gravity separation (sedimentation) of very finely divided particles in liquids. For example, from Stokes's law (1)[1]

$$P = 3\pi \mu d V \quad [5]$$

$$f = \frac{8 P}{\pi d^2 \rho V^2} = \frac{24 \mu}{d \rho V} = \frac{24}{R} \quad [6]$$

From Oseens' equation (1)

$$P = 3 \pi d V \left(1 + \frac{3}{16} R\right) \quad [7]$$

$$f = \frac{8 P}{\pi d^2 \rho V^2} = \frac{24}{R}\left(1 + \frac{3}{16} R\right) \quad [8]$$

From Allen's equation (2)

$$f = \frac{10}{\sqrt{R}} \quad [9]$$

From Newton's equation

$$f = 0.5 \quad [10]$$

From Fig. 1 it is seen that apparently there is a distinct connection between resistance determinations on fixed spheres, and the gravity settling of very small particles in liquids as expressed by Equations [5], [7], and [9].

It is further seen that a "critical region" is apparently reached when $R = 100{,}000 \pm$ similar to that in the flow of fluids in pipes when $R = 2000 \pm$ for the pipe condition.

The usual practise in work of this nature for obtaining the friction factor f has been to use different expressions for f for different ranges of R; for example, Kirkup (6) uses Stokes's Equation [6] (curve 11, Fig. 1) as a basis for $R < 8$; Allen's Equation [9] (curve 14, Fig. 1) from $R = 8$ to 450; and Newton's Equation [10] (curve 13, Fig. 1) for $R > 450$.

It is the author's opinion that a single Equation [11] (curve 15, Fig. 1) can be used up to $R = 2000$ without great error. This curve, represented by the heavy line in Fig. 1, has been so drawn that it approaches the curve of Stokes's equation at small values of R, since Stokes's equation is confirmed experimentally for conditions at small values of R.

The equation representing these average conditions is

$$f = \frac{24}{R}(1 + 0.14 R^{0.7}) \quad [11][2]$$

From Fig. 1 it would seem that this equation would give engineering accuracy up to values of $R = 2000$.

Equation [11] and Stokes's Equation [6] result in approximately the same value for f and end velocity up to about $R = 1.0$ (com-

[1] Numbers in parentheses refer to similarly numbered references at the end of the paper.

[2] Schiller and Naumann (7) from less general data express this relation for $R < 800$ as: $f = \frac{24}{R}(1 + 0.15 R^{0.678})$.

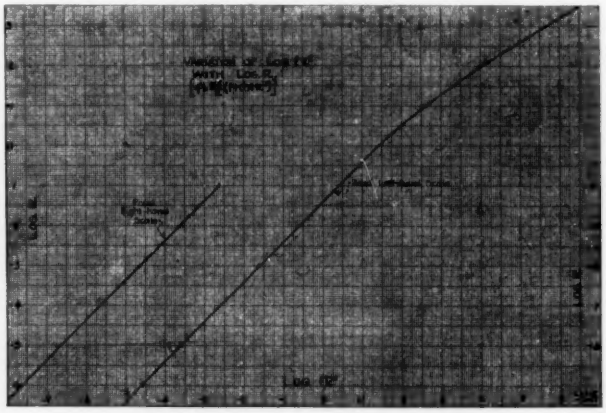


FIG. 2 VARIATION OF LOG fR^2 WITH LOG R

(Value of f as obtained from Equation [11].)

TABLE 1 DATA AND METHOD FOR FINDING END VELOCITY

1	Approximate Mesh	30		100		200		325		1250		2500							
2	Dia. in Microns	590		149		74		44		10		5		3		1		0·2	
3	Density of Ash δ – lb/cu ft.	80		105		120		120		120		120		120		120		120	
4	Flue-Gas Temp. Deg. Fahr.	300	600	300	600	300	600	300	600	300	600	300	600	300	600	300	600	300	600
5	$fR^2 = \frac{4}{3}g\frac{(\delta-\rho)}{\rho\nu^2}D^3$	5850	2230	124	47·2	17·4	6·63	3·61	1·38	·0427	·0163	·00534	·00204	·00115	·00044	·0000427	·0000163	$\cdot 34\times10^{-6}$	$\cdot 13\times10^{-6}$
6	Log. fR^2	3.767	3.48	2.093	1.674	1.241	822	·538	·139	-1.37	-1.788	-2.27	-2.69	-2·94	-3.35	-4.37	-4.787	-6.467	-6.885
7	Log. R	1.89	1.6	0·57	0.20	-1.81	-1.41	-1.16	-2·73	-3·25	-4.8	-4·34	-5·92	-5·68	-5·25	-6·23	-7·81	-8.15	-9·75
8	R	77·6	39·8	3·72	1·585	0.645	0.257	0·145	·0537	.00178	·00063	.000219	·000089	$\cdot 478\times10^{-6}$	$\cdot 178\times10^{-6}$	1.7×10^{-6}	$\cdot 645\times10^{-6}$	1.41×10^{-6}	$\cdot 562\times10^{-6}$
9	$V=\frac{R\nu}{D}$ (flue-gas)	11·4	11·3	2·17	1.77	·756	·578	.286	.204	.0155	.0105	·0038	·0028	.00138	.000989	.000148	.000107	$6\cdot12\times10^{-6}$	$4\cdot63\times10^{-6}$
10	$V=\frac{R\nu}{D}$ (Air)	9.62		2·4		0·9		·332		.0188		·0048		.00165		.000192		8.17×10^{-6}	

Values used: For Flue-Gas – At 300° Fahr., ρ=.0523 lb/ft³; μ = .0000149 lb/ft sec.; ν=.000285 ft²/sec.
" " " 600° " " " .0373 " " " .0000204 " " " .000547 " "
For Air – " " 70° " " " .0740 " " " .0000120 " " " .000162 " "

parable to a diameter of 35 microns for this work). For greater values of R, however, Stokes's and Allen's equations result in lower values of f and, hence, greater end velocities. For example, when $R = 400$ (with a 1000-micron particle), the end velocity by Equation [11] is approximately 19.6 fps, while the end velocity from Stokes's Equation [6] is 196 fps for this same diameter.

The relations between f and R for values above $R = 2000$ are rather indefinite because apparently a critical range has been reached in which unstable flow conditions are obtained.

The relations obtained from Fig. 1 between f and R can be transposed to be of great help in the computations involved. From Equation [3], representing the terminal velocity condition

$$\frac{\pi d^3}{6} g\,(\delta - \rho) = \frac{f}{2}\pi\left(\frac{d}{2}\right)^2 \rho\, V^2 \quad \ldots\ldots\ldots\ldots [12]$$

If both sides of the above equation are multiplied by ρ^2/ν^2, where ν is the kinematic viscosity, we have

$$\frac{4 g d^3}{3\nu^2}\left(\frac{\delta-\rho}{\rho}\right) = f R^2 \quad \ldots\ldots\ldots\ldots [13]$$

Equation [13] is useful because the problem is: With a particle of a given diameter and density falling in a gas of known density and viscosity, what is the terminal velocity? Since the factors in the left-hand side of Equation [13] are known, the value of fR^2 can be calculated, and, by referring to Fig. 2, the proper value of R can be determined from the above value of fR^2 and since

$$V = \frac{R\mu}{d\rho} \quad \ldots\ldots\ldots\ldots [14]$$

the desired end velocity can be calculated.

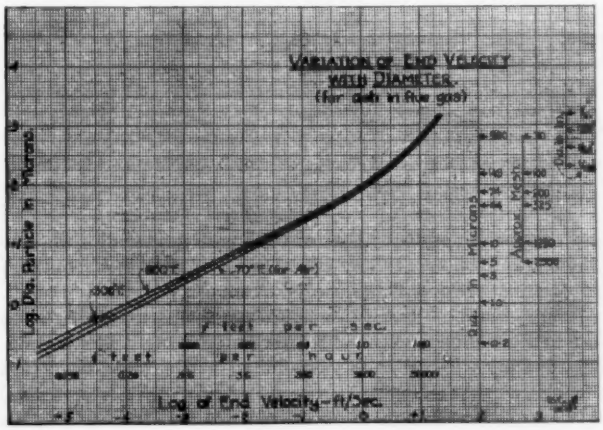


FIG. 3 VARIATION OF END VELOCITY OF ASH IN FLUE GAS WITH THE DIAMETER OF THE PARTICLE

Values for end velocities of different sized particles in flue gas, calculated by this method, can be obtained directly from Fig. 3, which was plotted from the results obtained from the data of Table 1.

The maximum sized particle which can be carried upward by a given flue-gas velocity may also be determined from Fig. 3, since the end velocity determined from this graph is also the flue-gas velocity required to support the particle at a fixed elevation.

Any gas velocity greater than the end velocity will tend to carry the particle upward and out of the chimney; conversely the particle will fall for any gas velocity less than the end velocity.

TIME OF FALL OF ASH PARTICLE WHEN DISCHARGED BY CHIMNEY

In order to calculate the time required for an ash particle to fall vertically from the height as discharged by the chimney, certain assumptions are made, namely:

(*a*) The upward velocity of the particle in the chimney may be considered $(U - V)$, where U is the flue-gas velocity, fps, and V is the end velocity of the particle, fps.

(*b*) The upward force of the flue gas becomes negligible when the top of the chimney has been reached.

(*c*) The temperature and the pressure of the surrounding air is constant at 70 F and 14.7 lb per sq in., respectively, for the height of the chimney.

(*d*) The ash particle is similar to a sphere in shape and the diameter of this sphere is the clear distance between the meshes of a screen when meshes are indicated. This assumption is warranted by the results of Rosin and Kayser (5) (curves 8 and 9, Fig. 1) where this approximation is utilized.

The method of making the calculation is best demonstrated by Table 1.

Referring to Table 1, item 1 is the approximate screen mesh corresponding to the diameter of the particle in microns ($1\ \mu = 3.28 \times 10^{-6}$ ft, or $800\ \mu = \frac{1}{32}$ in.). Item 3, the particle density, is varied according to the size (8). Item 5 is calculated from Equation [12]. Item 6 is the logarithm of item 5. Entering Fig. 2 at the value of item 6, item 7 is determined. Item 9, calculated from Equation [14], is then the desired terminal velocity of the ash particle for the condition given in the footnote of Table 1. Item 10 is the terminal velocity of the same particle in air at 70 F and 14.7 lb per sq in. The values obtained from Table 1 are shown plotted for convenience in Fig. 3.

In Table 2 are shown the remaining calculations required for determining the time of descent for the ash particle. This table has been calculated for a flue-gas temperature of 300 F. A flue-gas temperature of 600 F alters the final result less than 1 per cent from that of the same gas at 300 F.

TABLE 2 TIME OF FALL FOR DIFFERENT STACK VELOCITIES AND HEIGHTS

Stack Vel. f.p.s.		Approximate Mesh / Dia. in Microns / Stack Ht.–ft.	30 / 590				100 / 149				200 / 74				325 / 44				1250 / 10				2500 / 5				3				10			
			50	100	150	200	50	100	150	200	50	100	150	200	50	100	150	200	50	100	150	200	50	100	150	200	50	100	150	200	50	100	150	200
10	a	Net Vel.–f/s	0	0	0	0	7.83				9.24				9.71				9.8				←—				←—				←—			
	b	t – Sec.					.245				.287				.303				.313				←—				←—				←—			
	c	Max. Ht.–ft.					.96				1.326				1.47				1.56				←—				←—				←—			
	d	Total Fall–ft.					51	101	151	201	51	101	151	201	51	101	151	201	52	102	152	202	52	102	152	202	52	102	152	202	52	102	152	202
	e	Time of Fall–Sec.					21.2	42	63	83	56.6	112	169	223	155	304	455	606	2750	5400	8040	10600	11100	21900	32600	43500	31500	61500	91800	2040	4480	8820	13150	17500
20	a	Net Vel.–f/s.	8.6				17.8				19.24				19.7				19.8				←—				←—				←—			
	b	t – Sec.	.27				.556				.597				.616				.626				←—				←—				←—			
	c	Max. Ht.–ft.	1.16				4.95				5.74				6.06				6.26				←—				←—				←—			
	d	Total Fall–ft.	51	101	151	201	55	105	155	205	56	106	156	206	56	106	156	206	56	106	156	206	256	106	156	206	56	106	156	206	56	106	156	206
	e	Time of fall–Sec.	5.3	10.5	15.6	20.9	23	44	64	85	629	117	173	228	169	319	470	618	2990	5640	8250	1000	12800	22900	33800	44500	34400	64200	94800	2080	4890	9220	13550	17900
30	a	Net Vel. f/s.	18.6				27.8				29.24				29.7				29.8				←—				←—				←—			
	b	t – Sec.	.58				.868				.908				.928				.939				←—				←—				←—			
	c	Max. Ht.–ft.	5.4				12.1				13.3				13.8				14.1				←—				←—				←—			
	d	Total Fall–ft.	55	105	155	205	62	112	162	212	63	113	163	213	64	114	164	214	64	114	164	214	64	114	164	214	64	114	164	214	64	114	164	214
	e	Time of Fall–Sec.	5.72	10.9	16.1	21.3	25.9	46.6	67.2	88.2	70.3	126	181	237	192	343	493	642	3410	6060	8700	11400	13800	24400	35500	46300	38900	69200	99600	2120	5530	9900	14250	18600
40	a	Net Vel.–f/s.	28.6				37.8				39.24				39.7				39.8				←—				←—				←—			
	b	t – Sec.	.89				1.18				1.22				1.24				1.252				←—				←—				←—			
	c	Max. Ht.– ft.	12.7				22.3				23.9				24.6				25.04				←—				←—				←—			
	d	Total Fall– ft.	63	113	163	213	72	122	172	222	74	124	174	224	75	125	175	225	75	125	175	225	75	125	175	225	75	125	175	225	75	125	175	225
	e	Time of Fall–Sec.	8.48	11.7	16.9	22.1	30.1	50.8	71.4	92.4	82	137	193	248	224	376	527	672	3990	6660	9300	1900	16200	27000	37800	48600	45500	75600	1770	2270	6810	10540	19200	19200

Note: Underlined Values in Items "e" = Minutes Instead of Seconds.

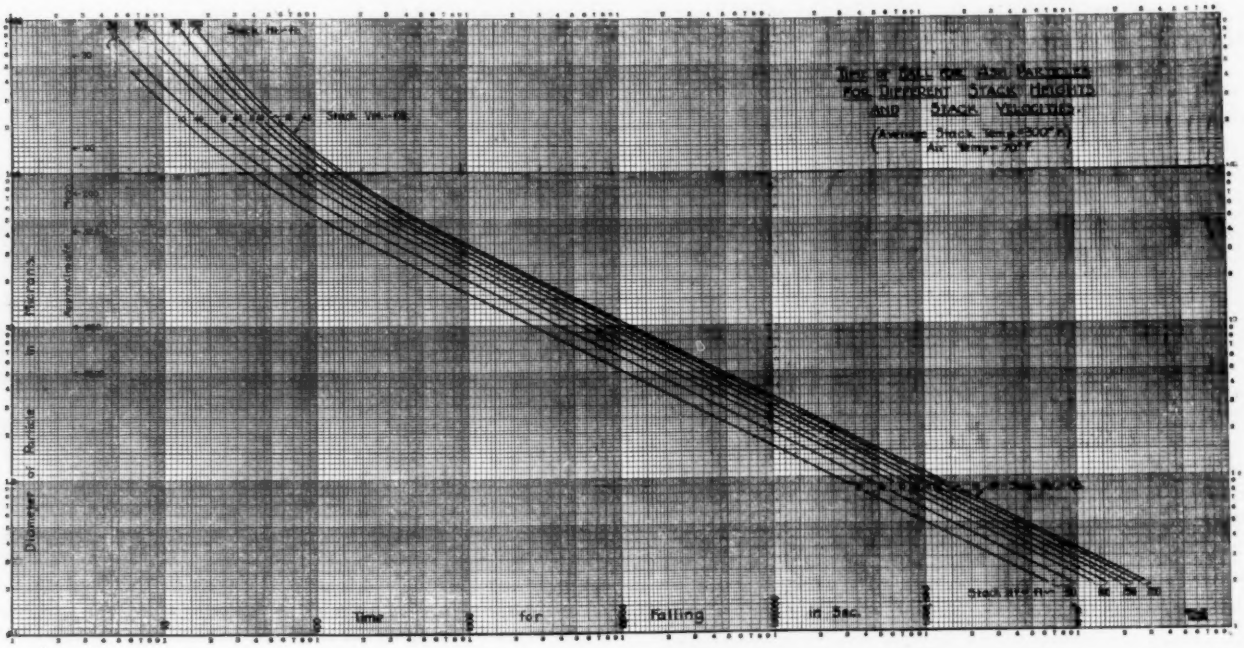


FIG. 4 TIME OF FALL FOR ASH PARTICLES FOR DIFFERENT STACK HEIGHTS AND STACK VELOCITIES
(Avg stack temp = 300 F. Air temp = 70 F.)

Chimney velocities of 10, 20, 30, and 40 ft per sec and chimney heights of 50, 100, 150, and 200 ft have been chosen.

Referring to Table 2, item *a* is the net velocity of the particle up the chimney, that is, the flue-gas velocity indicated minus the terminal velocity of the particle in the flue gas (item 9, Table 1).

Item *b* represents the time required for the particle propelled from the chimney to reach the maximum height above the chimney or when the vertical velocity of the particle becomes zero. This is obtained from the equation of falling bodies: $V = -V' + 32t$.

When $V = 0$, V' = the velocity of the ejected particle, item *a*, and t, item *b*, is the required time in seconds. This is an approximation, neglecting the resistance of the particle which introduces a negligible error in the time of fall.

Item *c* is the maximum height above the chimney to which the particle rises and is obtained by multiplying the average velocity of the particle by the time in seconds. Thus, $h = \frac{(V + V')}{2} t$, where $V = 0$ at maximum height. Items *a*, *b*, and *c* are assumed to be constant for particles smaller than 10 microns and at the values given for 10 microns without the introduction of appreciable error.

Item *d*, the total height through which the particle falls, is the sum of the chimney height plus item *c*. This method of calculation does not consider the time required to accelerate the falling particle up to the end velocity, but the error involved is negligible.

Item *e*, the time required for the particle to fall to earth, is the total height, item *d* divided by the end velocity in the air (item 10, Table 1).

The results given in Table 2 are shown plotted in Fig. 4.

For convenience, Fig. 5 illustrates the distance to which different sized particles are carried for different wind velocities: If the chimney is 100 ft high, the chimney velocity is 20 ft per sec, the average flue-gas temperature is 300 F, and the outside air temperature is 70 F. Other data used are the same as in Table 1.

The distance a particle is carried is determined by multiplying the wind velocity in ft per sec by the falling time (obtained from Fig. 4) in seconds for the proper particle size and chimney height.

Application of the Results

A practical problem will now be solved by the use of the foregoing plotted data, showing the methods of approach.

The following data assumed are in no way to be considered an accurate representation of the performances of the different types of firing, or dust-separating equipment. These data are used merely to demonstrate a method of attack.

The assumed data are: Chimney height = 100 ft; chimney velocity = 20 ft per sec; chimney-gas temperature = 300 F; average wind velocity = 5 mph or 10 mph; size of unit = 50,000 kw; lb of coal per kwhr = 1.7; carry-over for stoker = 5 per cent of the coal fired; and carry-over for pulverized fuel = 6 per cent of the coal fired.

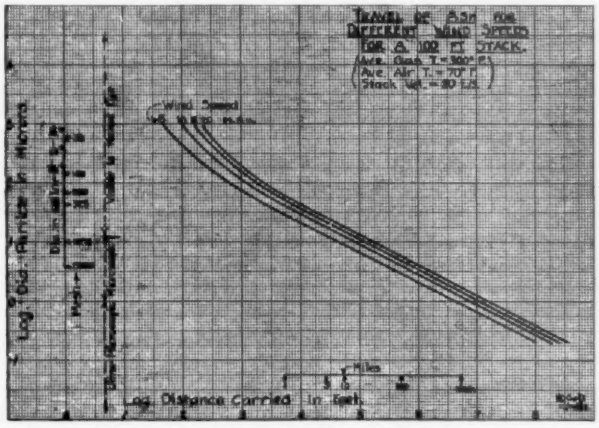


Fig. 5 Travel of Ash for Different Wind Speeds for a 100-Ft Stack

(Avg gas temp = 300 F. Avg air temp = 70 F. Stack velocity = 20 fps.)

The assumption is also made that all of this carry-over will be discharged by the chimney in order to calculate the worst condition of loading, although some of the larger sizes would probably be deposited in the breeching and chimney, especially if no induced-draft fan is used and low breeching velocities exist.

The assumed percentage by weight of the various sizes of the carry-over and the assumed efficiency of the separator for each size as well as the resulting calculated yearly loading at different distances are given in Table 3.

The distance that the particle is carried was obtained from Fig. 5.

The dust loading per square foot per year given in Table 3 was obtained by using the distance carried for the largest particle as a radius with the chimney at the center and assuming an equal distribution of the dust for that size over the area of the circle formed.

The distribution of the dust for the next larger size was determined by using the distance carried for this sized particle as a radius of a circle with the chimney at the center, subtracting the area covered by the next larger size, and dividing the total weight of ash for that size by the annular area so formed.

TABLE 3 DISTRIBUTION AND YEARLY DUST LOADING BY CARRYING OVER FROM A 100-FT CHIMNEY

Particle Diameter		Distance Carried by Wind—in ft.		Sep. Eff. %	Stoker—Carry Over = 5% of Coal						Pul. Coal—Carry Over = 6% of Coal					
					Carry Over		Dust Loading in lb. per sq ft per yr.				Carry Over		Dust Loading in lb per sq ft per yr.			
							5 M.p.h.		10 M.p.h.				5 M.p.h		10 M.p.h.	
Mesh	Microns	5 M.p.h.	10 M.p.h.		%	Lb/hr	No Sep.	with Sep.	No Sep.	with Sep.	%	Lb/hr	No Sep.	with Sep.	No Sep.	with Sep.
20	832	56	112	99.9	1.0	42.5	37.6	.038	9.4	.009	0	0	0	0	0	0
50	294	144	288	98.0	4.0	170	26.9	0.54	6.73	.135	1.0	51	8.08	.162	2.02	.040
70	208	220	440	93.0	5.0	213	20.1	1.49	5.57	.390	1.0	51	4.82	.337	1.32	.092
140	104	560	1120	81.0	15.0	228	12.6	2.94	3.12	.580	3.0	153	8.45	1.6	2.05	.39
270	53	1410	2820	70.0	25.0	1325	6.34	2.07	1.73	.518	5.0	255	1.34	.401	.331	.0994
	30	5000	10000	61.0	20.0	1133	0.431	0.168	.108	.0421	25	1275	.486	.190	.0122	.0476
	20	11,200	22,000	35.0	15.0	637	.0056	.0025	.0039	.0063	35	1785	.0157	.0098	.0039	.0175
1250	10	44,700	88,400	42.0	15.0	637	.003	.0017	.0007	.0004	30	1530	.007	.0041	.001	.00058

50,000 kw. Unit; 1.7 lb coal/kw-hr. Chimney Velocity = 20 ft/sec.

It should be noted that doubling the chimney height has approximately the same effect as doubling the wind velocity. Therefore, in Table 3 (for a 100-ft chimney), the column headed "10 mph" indicates what might be expected with a 5-mph wind with a 200-ft chimney.

In considering the yearly dust loading indicated by Table 3, it is interesting to note that the yearly rate of erosion is approximately 0.03 lb of earth per sq ft (9).

Conclusions

(1) An equation for the resistance of particles is derived (Equation [11]) which is useful over the entire range of particle sizes usually found in cinders and fly-ash.

(2) A method is demonstrated for calculating the distance an ash particle of a given size may be carried from the top of a chimney with a given wind velocity.

(3) By making certain assumptions, a method of calculating the yearly dust loading at different distances from a chimney is advanced.

References

(1) "Handbook Exp. Physik," Wien-Harms, vol. 4, p. 341.
(2) *Philosophical Magazine*, vol. 50, pp. 324 and 519.
(3) Technical Report No. 185, N.A.C.A.
(4) "Handbook Exp. Physik," Wien-Harms, vol. 4, p. 2.
(5) *Zeit. V.D.I.*, 1931, vol. 75, p. 854.
(6) *Fuel*, 1931, vol. 10, p. 196.
(7) *Zeit. V.D.I.*, 1933, vol. 77, p. 319.
(8) *Combustion*, June, 1933, p. 35.
(9) "Hydrology," by D. W. Mead, McGraw-Hill, p. 324.

Discussion

T. A. Marsh.[4] The Department of Smoke Abatement of the City of Chicago has given much attention to atmospheric conditions, not only in the matter of visible smoke, but also of fly-ash and dust, over a period of years and has thereby been able to draw conclusions and to submit some very definite facts.

It has been our observation that many dust-fall determinations have been unreliable and often misleading, due sometimes to the method of making these observations, and at other times to such factors as increasing or decreasing amounts of coal burned within the district during the period of observation. It must, of course, be recognized that not all of the dust fall is chargeable to fuel burning although dust samples indicate that fuel is the major source.

During 1934, the Department of Smoke Abatement, under Civil Works Administration Project No. 1504 which pertains to

[4] Member, Advisory Board of Engineers, Chicago Department of Smoke Abatement, and Central Division Engineer, Iron Fireman Mfg. Company. Mem. A.S.M.E.

smoke observation and abatement, had allocated to it 175 men, some of whom are engineers of national reputation and of wide experience in this work. The writer is authorized, by the Department, to present to the Society any of the facts or figures from the report compiled by these men.

One of the significant figures of the report is that the tonnage of coal burned in the Chicago district has doubled between 1911 and 1933, yet the amount of smoke in 1933 was only 95 per cent of that of 1911, and the density of the smoke only 49 per cent.

Over this period of time, the changes in smoke produced by classifications are as follows:

Railroads	93 per cent decrease
Power plants	15 per cent decrease
Metallurgical furnaces	61 per cent decrease
Manufacturing plants	1 per cent decrease
Boats	2 per cent decrease
Apartment buildings	1620 per cent increase
Domestic (residences)	900 per cent increase

It is evident that we have mastered the situation in the larger plants, but in the meantime the great increase in the number of apartment buildings and residences has so increased the tonnage of coal burned that these two classes of chimneys have become responsible for a very large proportion of our smoke.

An analysis shows the sources of Chicago smoke to be as follows, in order of importance:

Apartments and large heating plants	43.0 per cent
Power plants	25.4 per cent
Domestic	20.4 per cent
Manufacturing	5.1 per cent
Railroad locomotives	2.8 per cent
Metallurgical and special processes	2.4 per cent
Boats	0.9 per cent

We have therefore logically centered our attention on the apartment buildings and heating plants.

Until the development of small automatic stokers for this field we had no reliable and economically sound weapon with which to attack this increasing source of half the smoke. The small stoker offers a definite solution. Four years ago we therefore wrote into our ordinance that all new plants of 1200 sq ft of steam radiation and over must use automatic firing of some kind, i.e., stokers, oil, or gas. At the present time there are approximately 6000 stokers installed in Chicago, each serving 1200 sq ft of radiating surface or more.

The dust fall in Chicago in tons per square mile per month for every year 1926 to 1932, inclusive, is as follows: 326.41, 390.28, 384.81, 355.02, 323.52, 268.52, and 230.33.

We have thus, due to our active campaign, not only reduced the volume and density of smoke in the City of Chicago, but have reduced the dust fall from a high figure of 390.28 in 1927, to 230.33 in 1932, the reduction being 41 per cent. These figures must, however, be considered in the light of the decrease of industrial coal burned in 1933 as compared to 1928. We are encouraged by the improvement, and, inasmuch as apartment buildings are very rapidly installing stokers purely for economic reasons, we feel that within a few years we shall make a measurable reduction in smoke from that particular group which is the worst offender. The improvements in stoker designs as now being made will further decrease the fly-ash.

The current research work and reports of the Department of Smoke Abatement of the City of Chicago are available to all who are interested in this work and our department will be glad to cooperate to the maximum with other cities or engineers toward improvement of atmospheric conditions.

There has never been any question as to whether stokers abated smoke. There has been a question, however, as to whether stokers actually decrease dust and fly-ash from chimneys. Our surveys indicate that stokers do decrease the dust and fly-ash emission because of decreased tonnage and maintenance of a uniform fuel bed.

Fuel and air distribution are the vital factors in fly-ash emission. Stokers with improper distribution and improper air regulation increase fly-ash. During the past three years sufficient advancement has been made in the art to make as much as a 75 per cent reduction in fly-ash from grates.

Cooperation Between Industrial and Public-Utility Companies in Generating Steam and Electricity

By H. DRAKE HARKINS,[1] WILMINGTON, DEL.

PUBLIC-UTILITY systems in the United States (those having an annual output exceeding 100,000,000 kwhr each during 1932) have an installed generating capacity of 30,700,000 kw, 22,000,000 being steam and 8,700,000 being hydro (1).[2]

Manufacturing plants in 1929 had an installed generating capacity of 7,800,000 kw divided among 17,270 generators averaging about 450 kw each (2). Mines and quarries had 740,000 kw (3) installed in electric generators, bringing the total in industrial plants to 8,540,000 kw. The installed capacity of industrial plants is, therefore, approximately 27.8 per cent of the installed capacity of the larger public-utility systems.

Office buildings, hotels, hospitals, schools and colleges, and the like are not tabulated as manufacturers in the census, but for the purpose of this paper they could be included as industrial plants if the figures were available. Therefore, it is probable that the 27.8 percentage of installed capacity is approximately correct even though the very small utilities are not included.

The generating capacity actually installed in industrial plants is only that which management (unfamiliar with power problems) has seen fit to install with economic conditions as they now are and in the face of strong public-utility sales pressure.

Glenn B. Warren (4) states: "An estimate recently made showed that if all the industrial back-pressure and bleeder turbines manufactured in each of the years 1927 to 1929, inclusive, had been built for 1200 lb initial pressure instead of the 200 to 400 lb pressure for which they were built and arranged to supply the same heat, these turbines would have had an aggregate capacity over and above what they did have equivalent to about one-fifth of all the condensing turbines manufactured in the respective years. Similarly, an extension of this investigation indicated that if all these turbines which were installed to supply heating steam had been of the mercury-vapor-process type and, as before, arranged to supply the same demand for heat, these installations would have had an aggregate capacity over and above what they did have equivalent to almost one-half of that of all the condensing turbines manufactured in the respective years."

The author recently reviewed a proposed electrolytic process which involved the distillation of large quantities of mercury. The mercury vapor could generate twice as much electricity, 12,000 kw, as the electrolytic process required. The industry had no use for the surplus electricity and it could not be credited to the operation. In this instance, the absence of a market for surplus electrical energy hindered industrial progress and technical development.

Table 1, by A. R. Smith (5), shows how an industrial steam flow of 400,000 lb per hr at 200 lb pressure could generate no surplus energy, yet 42,000 kw were obtainable through present technology.

TABLE 1 STEAM FLOW TO INDUSTRIAL—400,000 LB PER HR

Initial steam conditions[a]	Entering evaporator[a]	Exhaust-steam conditions[a]	Electric power
200 lb, 525 F	No evaporator	200 lb, 525 F	None
400 lb, 750 F	No evaporator	200 lb, 625 F	5,970 kw
600 lb, 750 F	400 lb, 670 F	200 lb, sat.	5,150 kw
1200 lb, 750 F	400 lb, 518 F	200 lb, sat.	13,750 kw
2400 lb, 1000 F	400 lb, 618 F	200 lb, sat.	24,050 kw
125 lb mercury	28 in. hg vacuum to 400 lb steam	200 lb, 625 F	42,200 kw

[a] Pressures in lb per sq in. gage.

Therefore, it may be concluded that the present capacity installed in industrial plants is by no means a measure of what might justifiably be installed if an outlet were given for surplus energy, if full economic advantage were taken of technical development as shown in Table 1, if industrial management were educated in the economics of power generation, and if the utilities would accept the industrial plant as a possible source of cheap energy. It is obvious that these new conditions would result in a large increase in industrial capacity, mostly available for generating surplus energy. With industrial capacity now 28 per cent of utility capacity, it is equally obvious that any large increase in industrial capacity is of such magnitude as to satisfy the nation's demand curve for some considerable future period and leads us to the startling conclusion that the larger part of future electrical generating capacity should very probably be installed in or near industrial plants rather than in condensing-steam or hydro central stations. The superior efficiency of industrial "back-pressure" generation and its economic justification are well known to engineers and are shown elsewhere in this paper.

The public-utility systems are generally interconnected into a few great regional power pools, giving them the advantage of emergency supply, diversity, and reduction in spare equipment. Very few of the industrial generating plants are so interconnected either with each other or with the regional system. Many of the industrial plants generate electrical energy and utilize the exhaust heat from the turbine resulting

[1] Industrial Engineer, E. I. du Pont de Nemours & Co., Inc. Mem. A.S.M.E.

[2] Numbers in parentheses refer to bibliography at end of paper.

Contributed by the Power Division and presented at the Annual Meeting, New York, N. Y., December 4 to 8, 1933, of The American Society of Mechanical Engineers.

in a production of electrical energy with much higher thermal efficiency (80 per cent) than can be obtained in utility steam stations (25 to 30 per cent). The seasonal heating load of many industrial plants enables them to produce a surplus of electrical energy above the plant requirement and at very low cost during the peak of the heating season, which is approximately coincident with the load peak on the utility systems.

Alfred W. Fox works out a specific case (6) for providing the top 20,000 kw of an actual public-utility load-duration curve which contained 867,000 kwhr. If a new central station is provided, the annual costs are $262,000 for a peak load steam plant, $229,000 for a base-load steam plant, $255,000 for a hydraulic peak-load plant, and $257,000 for a steam-accumulator plant. An industrial plant having or needing the boiler capacity can provide the 20,000 kw in generating capacity and the energy at an annual cost of $171,000. This is an annual saving of $58,000. All costs are total, including fixed and operating charges. Mr. Fox concludes, "the figures used above may be open to question. I believe they are sufficiently accurate for a discussion of this nature and are moreover so variable as to require special attention and estimate on each particular problem. When times become normal, the peak-load problem of the public utility will again assume its previous importance, and I believe that solving the problem in conjunction with one of surplus power can mean a great saving to the utility and a source of additional profit to the industrial plant."

Many industrial power engineers believe the general refusal of the utilities to buy their available surplus energy is unsound national economics, unfair competition, and shortsighted policy. Guy B. Randall (7) also champions this opinion. They feel that such purchase or some interchange or other cooperative agreement will economically justify the installation of a great many more industrial power plants with a reduction in their own and the nation's power bills. These industrial engineers point out that most hydro stations and a great many utility steam stations are necessarily located remotely from the load center, requiring expensive, elaborate, and hazardous transmission systems which contribute to the result that the delivered cost of electricity to the ultimate consumer is from two to twenty times the cost at the generating station. (This spread should not irritate industrial plant engineers who pay two-dollar freight on twenty-cent coal, but should be accepted as economic fact.) A great many of the industrial plants are very near other consumers where industrial-plant surplus energy, lower in generating cost, could be delivered with small transmission loss to their neighbors if the utility transmission system were opened to the industrials.

The advantage of interconnection and other advantages of interchange with industrial plants have been ably pointed out by B. F. Wood (8). W. F. Ryan, a thorough student of this subject, outlines the advantage of cooperation and other industrial opportunities in his paper (9). W. S. Monroe has emphasized the importance of by-product power generation and pointed out the necessity for cooperation between utility and industrial companies (10).

Utility companies generally refuse this industrial-plant energy and other cooperative schemes for the following reasons (11):

(1) Possibility of having to face the charge of discrimination in cases where the utility chooses to make a working arrangement with one industrial and to deny it to another.

(2) Hazards to service presented by a number of small, isolated plants whose engineering and operating standard may be below the utility grade.

(3) Complications of supplying process steam non-electrical service to industrials.

(4) Financial hazards and disagreements potential in joint ownership and operation of utility and industrial plant equipment.

(5) Danger of increasing industrial business to an unsafe volume in proportion to total business.

(6) Objection to committing utility funds to investment in power facilities in industrial plants of uncertain business continuity.

(7) Possibility that an industrial plant with excess capacity will attempt to sell this directly to users at a higher profit than the interchange rate permits.

(8) Variations in efficiency of industrial plants delivering by-product electricity as a function of blast or other process heat usage.

(9) Inflexibility of such by-product plants as compared with utility generating stations.

But at least one utility executive (12) recognized the importance of power interchange and its benefit to the whole community, pointing out that it is the industrial plant's payroll which supports the utility and it is the utility's service which improves living conditions for the industrial's workmen. Cooperation between the two, he concluded, has a broad economic and sociological foundation.

Much of the difficulty in reaching a cooperative agreement for the mutual sale or interchange of electrical energy between utility and industrial companies lies in the misunderstanding of the value of a unit of energy. The value varies widely if it be "firm or dump," with the time of the day and year, and with the installed capacity of each party at the instant of generation. W. B. Skinkle has admirably and thoroughly expounded the fundamental principles in his papers on this subject (13, 14, 15). (These three papers are recommended to every engineer contemplating or negotiating a cooperative power contract.) Especially pertinent to the subject of this paper are Figs. 1 and 2 reproduced from Mr. Skinkle's discussion (13). Fig. 1 illustrates how the large generating units of a public-utility system increase capacity in steps, making the capacity growth very unlike the more uniform growth of the load curve. Fig. 2 illustrates how industrials and utilities may both delay investment and avoid installations of capacity far beyond requirement by a mutual and cooperative exchange of power. These curves refer principally to large-capacity industrial installations. It would be interesting to see the result if some utility system endeavored to provide for the gradual growth of its load by an equally gradual utilization of surplus power from industrial plants. The inspiration for such an attempt must come from the management or engineering groups of the utility company. At present, contact between utility and industrial is usually made by the power-sales division of the utility which, naturally enough, is only interested in selling energy and is not interested in the economic or engineering aspects of the power problem as a whole.

We hear much, especially from political sources, of the undeveloped hydro energy available in this country. Undeveloped hydro becomes more and more remote and it has already been pointed out that transmission costs now exceed energy-conversion cost. The public has heard very little in this country concerning the use of this great quantity of cheap industrial surplus electrical energy which is now available in the very heart of our consuming districts.

The power utility is given a monopoly on the assumption that it will serve the community more efficiently than any other means yet devised. Therefore, it is obligated to investigate sources and secure cheap energy, either by purchase or genera-

tion, and to distribute energy cheaply and efficiently. It is not unreasonable to ask these companies to purchase surplus industrial energy, if such purchase will benefit the community by fostering local industries and reducing the utility cost. Certainly this conception of the utility does not justify the intense and extravagant effort sometimes made to prevent industrial generation. Many individual utility companies are, in fact, only distributors, owning no active generating equipment and buying from affiliates. These distributors may well be asked to purchase industrial energy at a price commensurate with that they now pay.

This whole proposition is naturally so involved, technically and economically, that we can find a solution only through study and experiment made with greatest honesty and caution. So involved is each problem that the smoke-screen of confusion is ever at the disposal of partizans. So technical is each problem that all but the engineer are lost. Industrial managers are confused alike by equipment and power salesmen. This is a task for the most capable industrial and utility engineers.

While cooperation between utilities and industrial plants has no existence on a national scale, there are some isolated

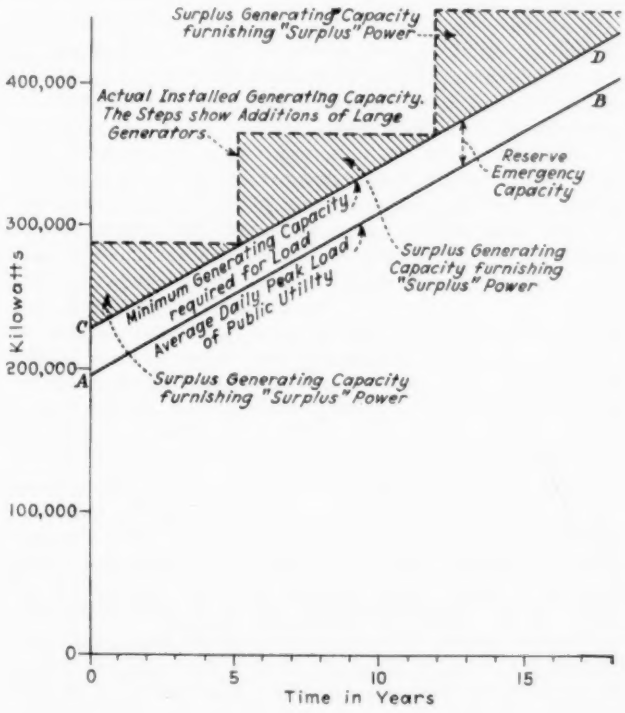


FIG. 1

instances, and it is the object of this paper to examine such cases as exist, review the contractual and physical arrangements, and evaluate the success, in the hope that this study may assist progress in cooperation between utility systems and industrial plants.

EXAMPLES OF COOPERATION ABROAD

In England, the Electricity Act of 1926 created a governing board to integrate public-utility generating systems. This Act provides for current to be fed into the "grid" from industrial plants and gives power to the Electricity Board to purchase surplus electricity in such cases. A British correspondent (16) stated in April, 1933, that at that moment there was no actual case from which he could obtain any particulars but gave as a reason that it was only recently that the grid has been put into actual operation and it is not yet complete. He believes that industrial cooperative schemes have been left over until the distribution of supply is more completely organized and the unification of frequencies achieved.

A prominent American engineer (17) states:

> The Belgium state has organized a corporation on which manufacturers, users, engineers, and the government are represented in the directorate to connect up all of the power plants, both utility and industrial, in the eastern half of Belgium. This corporation is known as the Union Générale Belge d'Électricité and has been running now for about five years. The arrangements have been most satisfactory, I understand, to all users and manufacturers, and the profits, while they have not been large, have been substantial and of sufficient amount to insure the continuation of the corporation. . . .
>
> The western part of Belgium is handled by Les Centrales des Flandres with headquarters at Ghent, while another public-service company operates Brussels and the surrounding territory. It is intended that in the end the whole of Belgium will be tied-in in one generating and operating company, but this will probably be sometime in the future.

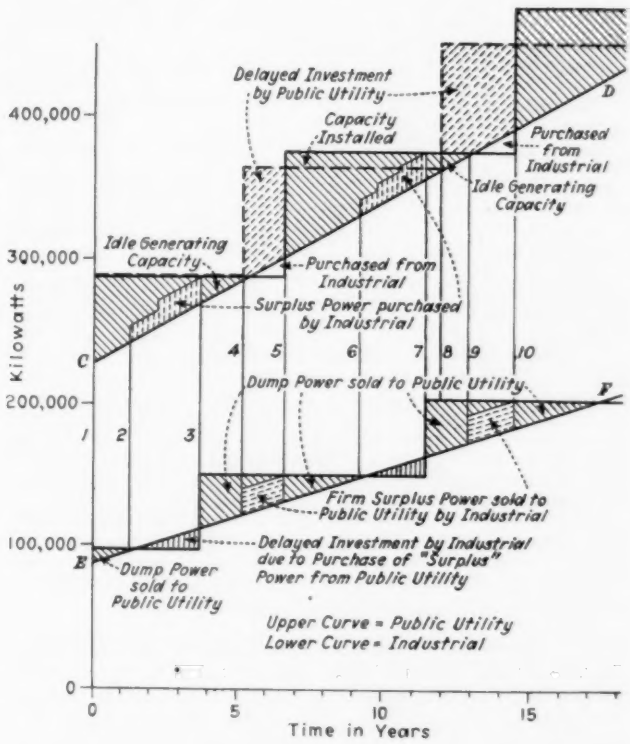


FIG. 2

While not of national importance, a regional cooperative arrangement of great interest and significance was consummated at Sydney, Nova Scotia. Here is a case where the power corporation and several industrial companies and a municipal lighting plant cooperated, with the approval of the Board of Public Utilities, with a result that has been satisfactory as regards service, economy, and earnings. Steel plant and mine operations are scheduled in such a way that the power corporation can provide necessary power at a minimum of cost. The effect of a proper rate schedule on power demand and usage at coal mines and steel plant has been extraordinary and has resulted in substantial economies for these industrial concerns. Refuse coals and coke breeze are used for generating electricity and domestic current usage tends to increase due to

reduced rates. K. H. Marsh (18) has provided data for 1929 in Table 2. He says:

TABLE 2 STATISTICS OF POWER GENERATION AT SYDNEY, N. S., 1929

Generating station	Kw installed	Frequency, cycles	Annual output, kwhr, in millions	Fuel
Steel Co., station No. 1	11,500	60	45	Blast-furnace gas, coke breeze, and coal
D. Coal Co., station No. 2	7,000	25	41	Slack coal, stoker-fired
D. Coal Co., station No. 3	4,000	25	15	Slack coal, stoker-fired
S. Coal Co., station No. 4	2,400	60	11	Slack coal, hand-fired
Utility Co., station No. 5	1,000	60	4	Slack coal, hand-fired
Municipal, station No. 6	400	60	1	Slack coal, hand-fired
Total.....			120	

The above plants [Table 2] are listed in order of efficiency. With more normal industrial activity all stations were overloaded at times.

Stations No. 2 and No. 3 (25 cycle) were tied together electrically.

Stations No. 1 and No. 5 exchanged stand-by service.

Stations No. 4 and No. 6 assisted each other at times.

Now in 1930, a power corporation was formed and put into operation a modern 7500-kw steam-electric generating plant, which was interconnected with Stations Nos. 1, 2, 3, 4, 5, and 6. A 25–60-cycle reversible frequency changer was installed, and a program of colliery electrification was completed. The power corporation leased Stations Nos. 1, 2, and 3. Stations Nos. 4, 5, and 6 were closed down.

The power corporation put into effect a schedule of rates approved by the Board of Public Utilities, and operates its very efficient modern station on base load, while Stations Nos. 1, 2, and 3 operate to suit the load from steel plant, coal mines, utility company, municipal lighting, as well as lighting for several small mining villages.

There are other instances in Canada where large industrial plants provide steam-electric stand-by service to promote good-will in communities where small hydro utilities could ill afford to provide their own steam stand-by. And the hydroelectric utility assists the industrial steam plants at certain times. There are cases where hydro installations with unused capacity sell electrical energy to industrials for use in steam boilers at little more than fuel cost, but, of course, on short-term contracts.

Examples in the United States—Parallel Operation

The simplest case of cooperation between utility and industrial in the generation of electrical energy occurs when the industrial secures part of its demand from the utility and operates its own generating equipment in parallel with the utility but does not feed back. So many installations of this kind exist in the United States that it may be called common practise, although a few utilities have earned the ill will of their customers by refusing to cooperate to this extent. This attitude is untenable, as can be proved by the entire success of the numerous existing installations. The utility in one large Eastern city states that it has twelve customers operating in parallel, the largest taking 6500 kw from the power company and the smallest 100 kw. All have contracts of three years' initial duration which continue from year to year with a 60-day cancelation clause. In none of these cases is there any pump-back of power into the utility lines and the power purchased is taken in each case under a standard form of contract.

There is an unusual case of parallel operation in an Eastern seaboard city where an existing industrial plant which generated all its own steam and electricity had an opportunity to sell steam to a new industrial located on adjoining property. The first industrial had sufficient installed capacity to supply its neighbor with both high- and low-pressure steam. The low-pressure steam could be extracted from one of the turbo-generators, but this extracted steam would only generate part of the neighbor's requirement. The local utility might possibly have blocked this sale of electricity because of existing laws, but generously cooperated and sold electrical energy to the first industrial for resale to the neighbor in accordance with the demand. All of the three parties profited. The utility gained by selling some energy and also succeeded in running its service lines into the plant of the first industrial which had never before been connected with the utility. The first industrial profited by the sale of steam and electricity without the necessity of increasing its installed capacity to meet the new load. The newly located industrial obtained steam and electricity at a cost lower than could have been obtained by building its own power plant and/or purchasing electricity from the utility. This arrangement has been in operation several years and has been entirely satisfactory to all of the three parties.

Many additional installations for parallel operation would be made economically attractive were it not for the additional financial burden imposed by the necessity for the industrial company to install spare generating equipment or pay a burdensome demand charge because of the periods when its one generator would be out of service. In a few instances, cooperatively minded utility companies have waived the demand charge on the grounds that the unusual demand was caused by an unavoidable breakdown. Modern central-station generating equipment has an availability better than 90 per cent and industrial turbo-generators without condensers probably have a higher availability. Industrial plants having normal week-end and annual inventory shut-downs can depend on emergency shut-downs occurring less frequently than yearly. Nevertheless, an industrial power engineer must conservatively include the maximum possible annual demand charge when preparing his project for the installation of a single turbo-generator and this prevents the authorization of many.

Utilities defend this high demand charge by maintaining that the business is unattractive and that generating and transmission capacity must be provided for the ultimate unavoidable breakdown. This contention is, undoubtedly, sound when considering a single installation or an isolated industrial at the end of a long transmission line, but if industrials which are located within a reasonable distance of each other were permitted to feed back their surplus energy into the utility system, the statistical effect, diversity, load factor, etc., would probably result in placing no additional burden on the utility's generating equipment by infrequent individual breakdowns and would require but very little additional transmission capacity.

An example of prohibitive demand charge existed in an industrial plant which generated all its energy but had a public-utility connection of some 300 kw demand for use in starting a cold plant after a total shut-down. The demand charge included some energy which was completely used. After two years' experience, the industrial secured a handsome return on an investment which was made for the installation of a 300-kw gasoline set. The service connection from the utility was discontinued.

A large utility which has numerous customers in parallel operation but none feeding back states that its company policy is an exceedingly broad one in dealing with special conditions and if, at a future time, conditions might arise wherein it would be advisable and profitable to both the customer and

the company to draw up a form of contract wherein any power would flow both ways, it is sure the company would give this serious consideration.

Industrials Which Sell Energy to Utilities

This subject is so shrouded in secrecy that no quantitative data can be secured or given. The issue of a magazine (19) devoted to this subject lists the following:

(1) Northern Paper Mills at Green Bay, Wis., developed two hydro sites with capacity over its requirement for sale to the Wisconsin-Michigan Power Co.

(2) Newton Falls Paper Company sells surplus hydro to Northern New York Utilities.

(3) The Wauregan Mills, Wauregan, Conn., sells surplus hydro up to 1000 kw and purchases power deficiency up to 300 kw.

(4) North American Cement Corporation, Security, Md., generates from waste heat and sells to the Potomac-Edison Co.

(5) The Colorado Portland Cement Company, Boettcher, Col., sells surplus electricity generated from waste heat to the local utility.

(6) The Manitowoc Plant of Medusa Portland Cement Company interchanges power with the Wisconsin Public Service Company, using waste heat to generate surplus energy.

(7) A woodworking plant in New Hampshire uses refuse to generate surplus energy for sale to the Twin States Gas and Electric Co.

(8) The Commerce Royalty and Mining Company, Cardin, Okla., operates a Diesel plant which supplies 18 lead and zinc mines, also the Northeast Oklahoma Railway in the city of Miami.

(9) The Sapphire Cotton Mills, Brevord, N. C., sells energy over the week-end to the local utility.

(10) The Fairbanks-Morse test floor sells surplus energy to the local utility.

(11) The University of Michigan sells surplus energy to the Detroit Edison Company during the heating season and purchases in summer.

(12) In Pittsfield, Mass., the electric company buys surplus energy from a large textile and dye works and sells stand-by.

(13) The Twin City Plant of the Ford Motor Company operates a hydro station which was built under the government requirement that all available hydro be generated. The surplus energy is sold to Northern States Power Company. The government gets free power for lock operation.

An interesting aspect of the last case quoted is the governmental requirement that surplus available energy be diverted into useful channels.

An industrial plant in Virginia generates all its own power and sells power to the utility for use in two near-by industrial plants during times of utility power failure. This cooperation by the industrial plant prevented the utility from losing two customers who would otherwise have built their own plants to secure continuity of service.

The Eli Lilly Company at its Indianapolis Laboratories purchases steam from the Indianapolis Power and Light Company, generates electricity with this steam, and sells the electricity to the power company. The electrical demands of the Eli Lilly Company are satisfied from the lines of the power company (20).

The Crocker-Burbank Company, of Fitchburg, Mass., sells surplus by-product power to the local utility (21).

A large motor company believed in 1931 that its power-plant loads had dropped to the point where it would be advantageous to purchase some current on week-ends and holidays and, accordingly, entered into an interchange agreement with the local electric company. This contract carries no demand charge and calls for an interchange of energy at a straight energy charge. Neither party guarantees to furnish the other power at any particular time and each party is the sole judge of its ability to serve. While this rate is classed as an interchange rate, the electric company has never permitted the motor company to deliver any power to it under the terms of the contract and in effect it is a straight energy contract without any demand charge. The contract may be classed as a failure, and one of the industrial company's officials states, "It has been my experience that there is not sufficient genuine cooperation between the utility and the industrial to result in the most advantageous mutual arrangements. The industrial plant is too often viewed as a competitor of the utility which the utility must endeavor to eliminate. This attitude is sure to interfere with entering into those contractual relations which would contribute most to the benefit of both parties."

For some years prior to the construction of its new central station, an Eastern utility bought energy from a large industrial which was so located on the utility's system that it could supply energy to great advantage during peak loads and when transmission troubles occurred. This was a straight sale of firm power subject to the call of the utility's dispatcher. This is an example of the utility's using available industrial capacity as exhibited in Fig. 2.

A utility has an interconnection with a large industrial company for the exchange of electric power. The interconnection consists of two banks of transformers rated at 30,000 kva each. Both banks are owned by the utility but the industrial is billed for the rental of the second bank. There is no formal contract covering the exchange of power. A letter, accepted by both parties, fixes the charge per kilowatthour of net exchange. There is, ordinarily, no such net exchange. During the reconstruction of the industrial's power house, a considerable supply of energy went to the industrial at a fixed price. On occasion, during tests, etc., the industrial has returned power to the utility but at a special rate much lower than fixed in the letter. Both parties stand ready to pick up loads in the event of an emergency and such an emergency has occurred two or three times. The utility states that the interconnection has been most useful. The arrangement may be terminated at any time by mutual agreement.

An industrial in an Eastern city has an installed capacity of 1250 kw with a day load steadily over 1000 kw and with occasional 1300-kw peaks. The plant is connected with the utility, and more than ten years ago made a cooperative agreement for exchange at $3\frac{1}{2}$ cents per kwhr and without demand charge. The exchange of power was nearly even and considered fair by both concerns. This agreement survived one change in ownership of the utility. After a larger holding company had taken over the local utility, it made many different propositions for supplying all the industrial's energy, but none was found acceptable. Two years ago the utility reduced the exchange rate to $1\frac{1}{2}$ cents and stopped taking energy from the industrial. One year ago the utility abrogated the exchange rate and put the industrial under a standard service classification. This increased the industrial's rate to 5.3 cents per kwhr. The industrial complains because its demand charge is figured from the maximum setting of its transformer switch. This, it feels, is unfair, and if not changed will force the industrial to build its own stand-by. Its own service had always been dependable. This switch must be set well above the actual demand because when set close to the

demand the industrial lost its load several times while running in parallel with the utility. The industrial's engineer states: "During the winter months when exhaust steam is needed, operation is such that the utility cannot expect to compete, but during the summer months when our loads are so high that we waste exhaust steam it is just too bad that the power company cannot compete with such methods."

Large-Scale Operations

Although utility companies have been reluctant to make cooperative agreements with smaller industrial plants, several large-scale operations exist in the United States where advantage has been taken of industrial waste heat and fuel or of industrial demand for large quantities of steam; thus giving opportunity for the cheap generation of electrical energy.

The Deepwater Station, located on the New Jersey side of the Delaware River some 30 miles below Philadelphia, is an outstanding example and has been adequately described in the technical press (22). This 1200-lb central station houses two condensing turbo-generators, each owned by a different utility, and also contains one turbo-generator which exhausts to evaporators. The evaporator vapor is delivered in quantity up to 400,000 lb per hr to an adjacent industrial plant. This is strictly a cooperative venture, although the industrial has no capital invested in the station. The industrial was about to build its own high-pressure plant when the utility contemplated a new station in the neighborhood, and one plant was built instead of two in the expectation of realizing the benefits which naturally accrue to a single large-scale operation. The station has been in service approximately three years and the entire operation has been successful. The contractual success is almost entirely due to the spirit of cooperation with which each party has settled unforeseen contingencies as they arose. Although the contract covered many pages and took upward of six months' careful negotiation to complete, it, nevertheless, did not foresee all possibilities, and if a spirit of cooperation had not existed among all contracting parties, ultimate failure might have occurred. Advantages to the utility have been a larger-scale operation and a high boiler load factor with consequent economies. The industrial has secured the benefit of skilled and efficient power-plant operation without burdening management with the responsibility. The disadvantages to the industrial were: First, the much longer construction period with consequent delay in achieving the economy of high-pressure operation. Second, there is not the flexibility in meeting changes in conditions which would exist if the industrial could operate entirely within its own organization and through its own procedure. The author is not in a position to state the disadvantages to the utility.

In 1929 the growth of load on a Southern utility system had reached the point where it was apparent that additional power supply facilities would be necessary. The ideal location for additional capacity was at the eastern terminus of the system which, as it so happened, was the location of an oil refinery. The refinery was planning extensions. A large generating station (technically not a utility) was built adjacent to the refinery. This station planned to supply the refinery with a maximum of 840,000 lb of steam per hr and 14,000 kw. The oil company could have built its own power plant and generated the electrical requirement with about 400 lb boiler pressure. By building for 625 lb pressure, a large surplus of electricity could be generated and is sold to the local utility. The refinery supplies several kinds of liquid refinery waste to be used as fuel in the power plant. Petroleum coke is delivered in railroad cars. Both parties had many vital interests to protect. The oil company was anxious to get the very best possible price for its non-merchantable but burnable waste fuels and to assure itself a continuity of service. The steam company needed guarantees for its investment and assurance of continued business, a very definite relation between the price at which it had to sell steam and electricity and the cost of fuel, and the ability to convert its plant to a normal type of condensing station should the contract with the oil company ever be terminated. As the situation developed, it became apparent that this contract could only be made on some mutual basis. Since the plant went into operation, an additional boiler has been installed to permit the sale of additional steam up to a total of about 1,000,000 lb per hr. The company which operates the plant is not a utility. It confines all its operations to its own property and its customers come to it with their pipe lines and electrical connections. It is so located that it is in a position to serve other customers at equitable rates, preferably those with needs for both steam and electricity. At present the entire requirements of the oil company and of one utility are well cared for, and the surplus energy is utilized by another utility.

A technical publication (22) states that the Iowa Railway and Light Corporation and the Quaker Oats Company at Cedar Rapids have an arrangement whereby the utility generates steam for the industrial and uses the industrial's waste fuel, oat hulls.

The same publication (22) states that the Rochester Gas and Electric Company supplies steam to several industrial plants and also heating steam to the business section and has 12,000 kw installed in non-condensing units.

Regional energy coordination in the Chicago district developed a unique cooperative arrangement between steel plants and electrical and gas utilities as described by A. H. Dyckerhoff (23). A very careful, extensive, and thorough survey of steel mills under all load conditions indicated that a saving of from $0.43 to $1.36 per ton of steel could be effected by selling coke-oven gas and purchasing electricity. This startling conclusion displaced the previously held opinion that steel mills should use all their available heat for metallurgical and power-generation purposes. An exchange of energy between steel plants and gas utilities has been worked out in the region of Chicago, linking six basic steel plants with electrical and gas utilities, resulting in purchasing electrical energy, selling coke-oven gas, and using blast-furnace gas for heating rather than for power generation.

The General Electric Company is building a mercury-vapor plant at Schenectady (5). Additional steam supply was needed and the New York Power and Light Corporation desired additional electric generating capacity, it being agreed that a steam plant erected at or near the Schenectady works could and would economically supply both. The General Electric Company constructed the plant on its own property, placed it strategically for steam distribution, and is leasing it to the power corporation which, in turn, operates and maintains the plant. The steam delivered to the Schenectady works at 225 lb pressure will amount to 630,000 lb per hr. The industrial provided the capital and constructed the plant, and the utility leases the plant on the basis of the fixed charges on the actual investment as a yearly rental. The utility admits that a plant having a capacity of 26,000 kw has for it a certain value per kilowatt of capacity, and the fixed charges on such investment are credited to the industrial. This amount is less than the rental, because the actual cost of the plant provided not only for the production of 26,000 kw, but 650,000 lb of steam also. The utility also allows a credit for each kilowatthour produced by the plant, all of which

goes into its system. This credit is based on the prevailing costs of producing power with modern plants supplying its system, taking into account the fact that the utility must always accept the power produced according to the steam demand, regardless of the hour of the day or the season of the year. The utility sells all the power that the industrial demands at scheduled rates, regardless of this particular agreement; and the excess cost covering the rental, operation, and maintenance and the fuel over and above the two credits allowed the industrial is charged to the industrial as its cost of steam. The agreement between the two companies covering operation is in effect for five years; thereafter continuing from year to year unless terminated by either party on one year's written notice.

Conclusion

Modern technical developments in higher steam pressures and mercury-vapor cycles make available more and more surplus electrical energy from industrial steam flow, and this technical development favors the generation of electricity in connection with industrial processes.

The steam flow to industrial plants is of such magnitude and the industrial plants are so located that the larger part of future electrical generating capacity should very probably be installed in or near these industries rather than in new condensing steam or hydro central stations.

Cooperation between public utilities and industrial plants in the generation of steam and electrical energy will reduce generating cost and the investment in generating equipment, thus ultimately reducing the nation's power bill and resulting in the still wider dissemination and use of electrical energy, which is the basis of our industrial civilization.

An unsound economic condition in power generation is maintained by the utilities' competitive sales campaign.

Industrial management must be educated in the apparent paradox that small-scale "back-pressure" power generation is more efficient than the "quantity production" in condensing central stations.

A spirit of cooperation must be fostered by public-utility executives and engineers if industrial engineering antagonism is to be avoided and public relations improved.

A detailed survey of American conditions will probably prove, and the success of the Union Générale Belge d'Electricité indicates, that a program of cooperation will be successful.

BIBLIOGRAPHY

1 Supplement to *Electrical World*, May 6, 1933.
2 Fifteenth Census of U. S., Manufacturers, 1929, vol. 1, p. 112.
3 Fifteenth Census of U. S., Mines and Quarries, 1929, vol. 1, p. 16.
4 Discussion of "Higher Steam Pressures and Temperatures," Winter Convention, A.I.E.E., January, 1933.
5 "Coordinated Production of Industrial Steam and Utility Power," by A. R. Smith, *General Electric Review*, July, 1933.
6 "Surplus Power—Availability, Cost, and Economic Disposal," by Alfred W. Fox, *Power Plant Engineering*, July, 1933, p. 312.
7 "The Pooling of Power," by Guy B. Randall, *Power Plant Engineering*, June 1, 1932.
8 "Engineering Aspects of Interchange of Power With Industrial Plants," by B. F. Wood, Trans. A.S.M.E., 1931, FSP-53-26b.
9 "Combined Heat and Power Supply in Industrial Plants," by W. F. Ryan, Trans. A.S.M.E., 1931, FSP-53-26a.
10 "Tendencies in Future Power Station Developments," an interview with W. S. Monroe, *Power Plant Engineering*, January, 1933.
11 *Electrical World*, May 30, 1931, p. 1005.
12 "Industrial Power Interchange," an interview with T. O. Kennedy, vice-president and general manager, Ohio Public Service Co., Cleveland, Ohio, *Electrical World*, March 17, 1928.
13 "Discussion of Papers on Interconnection Between Public Utility and Large Industrials and the Interchange of Power Between the Two Systems," by W. B. Skinkle, engineer, Pittsburgh District Power Committee, Subsidiary Companies of U. S. Steel Corp., *Iron and Steel Eng.*, May, 1931.
14 "Cost Sheets and Their Relation to Engineering Economics," by W. B. Skinkle. Engineers' Society of Western Pennsylvania.
15 "Purchasing Public Utility Power for Industrial Use," by W. B. Skinkle. Engineers' Society of Western Pennsylvania.
16 F. W. Gardner, C.E., Turbine Department, C. A. Parsons & Co., Ltd., Newcastle-on-Tyne, 6.
17 Geo. A. Orrok, consulting engineer, New York, N. Y.
18 K. H. Marsh, C.E., Dominion Steel & Coal Corp., Ltd., Sydney, Nova Scotia.
19 *Power*, May 27, 1930.
20 "Plant and Utility Exchange Steam and Power," *Heating, Piping, and Air Conditioning*, May, 1932.
21 "Crocker-Burbank Central Steam Plant Extension," *Heat Engineering* (Foster-Wheeler Corp.), May, 1933.
22 "Utilities That Supply Steam," *Power*, May 27, 1930.
23 Series of four articles, "Regional Energy Coordination," by A. H. Dyckerhoff, *Electrical World*, Feb. 27, Mar. 5, 12, and 19, 1932.

Discussion

J. H. Cather[3] wrote: As Mr. Harkins points out, extracting more electric power from steam required for industrial purposes would seem to be a field with considerable future possibilities.

The largest steam-electric power plant of the Eastman Kodak Co. has a minimum summer steam load which averages about 66 per cent of the maximum winter load. Careful investigations by the engineers of the local utility and by this company's engineers were made recently regarding the possibilities of extracting a constant amount of surplus power from the minimum steam load.

A new 125-lb mercury plant and also an alternate 1400-lb steam plant were designed—these plants to supply approximately 300,000 lb of steam per hr to present 260-lb steam turbines.

The maximum the utility could afford to pay for this firm power was naturally the cost at which it could produce this power in a new high-pressure condensing plant of its own. The final report, written in July, 1932, commenting on this investigation, concludes "that the installation cost or the price to be paid per kilowatthour must change before the scheme would be attractive." It seems probable that future developments will make installations such as these investigated economical for conditions such as ours. Mercury boilers for 125 lb will probably reach a more commercial basis after experience with the two plants now going into operation.

The generating plant of the Eastman Kodak Co. has, for a number of years, operated with no unusual difficulties on an almost exact steam-electric balance, using non-condensing turbines and engines. This is made possible by the sale of surplus or "dump" power to the local utility in the colder portions of the year and the purchase from it of part of the required power at other times.

A. G. Christie[4] wrote: The idea of interchanging steam and electric power between utilities and industries seems to be making headway very slowly in this country. Mr. Harkins points out a number of reasons why utilities generally refuse to purchase the surplus power of industries. He might have added another reason; the desire of the utilities to dominate and monopolize all power generation and distribution. The Government yardstick for electrical rates announced by the Tennessee Valley Authority may cause utilities to look more

[3] Engineer in Charge of Power, Kodak Park Plant of Eastman Kodak Co., Rochester, N. Y. Mem. A.S.M.E.

[4] Professor of Mechanical Engineering, The Johns Hopkins University, Baltimore, Md. Mem. A.S.M.E.

carefully into every possible source of cheap power in order to enable them to lower rates, and this may lead to further interchange agreements.

We may possibly be forced to copy the Belgian idea with a regional committee under Government authority to coordinate all sources of heat and electrical energy for joint benefits to all concerned. This is not at all unlikely under the present socialistic trends of our Government.

Mr. Harkins comes close to the facts when he places the blame for lack of cooperation upon the influence of the power-sales departments of the utilities in determining policies toward the industries. Were these matters left to the production and distribution engineers, agreements could be effected much more readily.

The examples quoted by Mr. Harkins are helpful as indicating the nature of agreements now in effect. There seems to be no standardization of plan. Possibly a little more study on the part of the utility executives would lead to a more systematic approach to the problem. As I pointed out in my paper[5] before the Industrial Power Group last year, there must exist a spirit of mutual confidence on the part of both utility and industrial plant for any interchange of power to be mutually satisfactory. As long as one attempts to take advantage of the other, no lasting or satisfactory agreement can be consummated.

Much can be gained from more whole-hearted attempts to find a basis for interchange. This timely paper will add encouragement to efforts in this direction.

E. D. Dreyfus[6] wrote: Mr. Harkins has set forth many interesting statements and references that demand careful thought. Economy should always be the byword with the engineer in the matter of investments as well as with materials and forces. "Material economy" may be encompassed with a reasonable degree of definiteness, while "investment economy" is too frequently influenced by many intangible and often unforeseen factors such that the ultimate results for the long pull may fall considerably short of expectations. Fortunately, in an advancing engineering and developmental business period, errors in "economic" judgment are sometimes overshadowed by the profits of associated operations and by surrounding improvements. During a stationary period or a business depression, such errors, as are all too well known from our recent lesson, are likely to prove fatal. With this premise it might be well to view seriously what may be made of a utility-industrial interconnection and coordination of power-producing facilities to mutual advantage. As interestingly brought out by Mr. Harkins, there is apparent saving under such a program, particularly where by-product power of one form or another may be involved. Therefore, both utility and industry owe it to the community in which they operate to make the most of the opportunity to improve the region's economic position.

Many good examples have been cited as evidence of the fact that interchanges of steam and electric power have been successful in some localities. The question naturally follows: Why has the practise not been more universal in the past? Possibly we all may agree on one factor as responsible for the limited progress made in this direction and that is the element of *risk* involved.

Every planner is duty-bound to safeguard adequately the capital entrusted to his care; figuratively, a factor of safety must be introduced in relatively the same conservative way as is done in dealing with fiber stresses. No one would dare approach the elastic limit in the latter case, but this may be done in the case of an investment unless restraint is placed upon those sponsoring the development. The engineer has gone a long way in mastering physical problems and it is now incumbent upon him to look to the financial side with the same degree of perspicacity and conservatism—an economic error being as monumental as a structural failure. By so doing, the stigma that the banker placed on the engineers decades ago to the effect that financial backing for their projects would only be forthcoming provided the development would still pay with the actual cost assumed to be twice the estimate and the earning power reduced by one-half, will no longer be justified. Therefore, the approach to the problem of joint utility-industrial power interchange must be made with such foresight as may be commanded.

Those of us who have attended the meetings of this Society during the past quarter of a century have witnessed some startling changes; the coming and going of the gas engine; the low-pressure-turbine innovation and its comparative dying-out; and the like; and have listened with deep interest to discussions about the change in "style" of power-plant equipment over the years. This may sound fanciful but is nevertheless real as borne out by the mute evidence in the machinery graveyards in many quarters. Furthermore, there is no guarantee today that we have reached anything like a stationary position. Some new discovery in the matter of the heat cycle; chemical process using the "cold" in place of the "heated" state; substitution of electricity for fuels in metallurgical reactions and treatment; the possibility of a different kind of heating and ventilating system; and other industrial applications may arise to supersede the type we now know, thus destroying the economic balance of an inter-industry development which would have been profitable under the present circumstances. While it is not fear that should rule, but in ordinary investment as in ordinary dangers, "discretion is the better part of valor."

If we look further into the fundamental differences in the general character of the utility and the industry, we find that the utility has been declared by law to be "affected by the public interest" and therefore has been brought under the police powers of the state and closely regulated. The industrial establishment is still pretty much of a free lance although temporarily NRA laws now exert some slight restraining influence.

With the utility, the earning power is limited and turnover of capital is slow. The rates charged for service embrace only a very moderate depreciation accrual on the basis of virtually continuous life. On the other hand, the industry is only limited in its profit realization by whatever competition it experiences. In good times the industrial may earn sufficient surplus after a fair yield on its investment to write off the cost of the property in a comparatively few years. Consequently, the accrual policies of utility and an industrial plant necessarily vary widely.

I sometimes wonder when this subject of utility-industrial power interchange is discussed whether it is fully borne in mind that, as far as the utility is concerned, it is a 100 per cent business-producing investment, whereas with the industry the power-generating facilities represent only a small fraction of their investment in total plant for manufacturing purposes. Consequently, any economic error that might be made in either case would be of a magnitude with the utility many times that of the industry.

Another feature deserving close attention is that changes in the points of delivery to or from the power system may have a disturbing effect upon the investment economy as a whole if the use of existing facilities is modified to an appreciable extent through changes of manufacturing methods or processes, and thereby more or less power for interchange.

[5] "Surplus Power From Industrial Plants," by A. G. Christie. Mechanical Engineering, November, 1932, pp. 771 to 774.

[6] Engineer, Pittsburgh, Pa. Mem. A.S.M.E.

It is also to be remembered that the utility must pass on to its customers the benefits of the savings realizable through the interchange of power supply under discussion. While profit may be the incentive to the industry, duty must be largely the impelling reason for the utility to enter an arrangement of this kind. It may be safely stated that the utility has been giving serious consideration to the interconnection possibility, but in weighing all factors it has been forced to the conclusion that undue risk must be reasonably avoided.

The industry should by right be entitled to an outlet for surplus by-product power it may be able to generate and the utility should receive and market the power provided it does not, by so doing, impose any disadvantages upon its dependent customers and upon its owners.

In the limited time for discussion I will attempt to outline very briefly (as one individual way of attack) one plan that might stimulate proper utilization of industry's waste power. To view the problem adequately with the object of establishing equitable relationships, a set of basic conditions are briefly presented, as follows:

(1) It is presupposed that normally where no by-product power obtains, the utility may supply the power needs of the industry and thus minimize investment expenditures for the district.

(2) Where by-product power of one kind or another is available, the utility would install the necessary prime mover, cooperating with the engineers of the industry.

(3) The industry would finance the installation and the utility would refund the cost on a basis of credits according to the power developed—the rate to be based upon equivalent cost to the utility for like power, taking into account the question of necessary reserve to be allowed for and reasonable compensation for the intermediary service provided.

(4) Title would pass to the utility after credits had equaled the installation costs and the utility would be the sole agency for the power supply of the district and therefore in position to establish the lowest power cost for the district. Proper terms, minimum operation, and other factors must be prescribed to insure achievement of the plan. In event the industry underwent a radical change in its manufacturing method, the loss to the utility would be a minimum as it would come into possession of the generating equipment and recover partly on their outlay made through credits against revenues.

In the foregoing, I have merely tried to stress that there must be a mutual financial responsibility in the plan—with risk factor duly weighted—and further to give some indication of how such an idea may be carried out in practise. There have been similar methods employed in somewhat like situations heretofore which have been worked satisfactorily and I am confident they may be successfully applied to interchange problems.

Finally, I wish to add the observation that the careful reasoning on these investment matters will constitute as progressive a move as designing new types and installations that may not have real economic merit when all factors—time, business, and manufacturing variables, and specific operating characteristics—have been completely tested. The A.S.M.E. is doing a splendid piece of work in the contributions it has encouraged and published, which are intended to provide an understanding (which should aid in the solution) of our economic problems.

A. H. Dyckerhoff[7] wrote: Beyond doubt, there is a great desirability of such cooperation between utilities and industrials under proper economic conditions and I feel safe in saying that the utilities are more desirous of cooperating than *most* industrials, notwithstanding the different view expressed in the paper under discussion. In this statement I am guided by my work for an industrial and for a group of larger utilities having close contacts with numerous large industrials. I cannot agree with the author of the paper when he states that "the power-sales division of the utility is only interested in selling energy and is not interested in the economic or engineering aspects of the power problem as a whole" or that "sometimes extravagant effort is made to prevent industrial generation," or that "an unsound economic condition in power generation is maintained by the utilities' competitive sales campaign."

Mr. Harkins shows two diagrams evolved by W. B. Skinkle, suggesting coordinated increase of generating capacity by the utility and the industrial for the purpose of avoiding generating capacity beyond requirements. This procedure is theoretically interesting, but is applicable in very few instances, because of the vast difference of capacity of generators required by utilities and industrials.

Another stumbling block in the road of cooperation is the very frequent extravagant idea expressed by engineers of industrials as to the value of dump or surplus electric energy available by their plants. It is not kept sufficiently clearly in mind that a utility is obliged to give service at all times on demand. Therefore, it must have sufficient generating capacity at its disposal and under its control to meet such requirements immediately. For this reason any dump energy to be delivered by the industrial can have fuel value only as determined by the most efficient power plant of the utility. If we assume, for instance, that an energy of 13,500 Btu *net* is required per kilowatthour by large generating units at, say, 40 per cent capacity factor, and the cost of coal burned is 1.7 cents per therm, the value of the dump energy is but about 0.23 cent per kwhr. From this value certain deductions must be made for losses and fixed charges for investment, if the lines of the utility are not available within the immediate proximity of the industrial plant. If proper allowance is made for such items and if consideration is given to the approximate price of 0.4 to 0.5 cent per kwhr, which is expected by some industrial plant engineers, it is evident that such schemes are bound to collapse if the absorption of surplus electric energy of the industrial by the utility is the sole purpose of the interconnection of the utility and the industrial. However, if the industrial will purchase at least a fairly substantial block of electric energy from the utility, and wishes to feed back into the utility system surplus energy in proper proportion to the energy purchased at the approximate fuel value of the utility power station, a good economical foundation for cooperation would be established.

Likewise, difficulties arise frequently when it is intended to tie private plants having small capacity to large utility systems for lack of proper regulation, a low power factor of the industrial plant, proper capacity of circuit breakers, etc. To overcome these difficulties additional expenses must be incurred militating somewhat against the unidirectional flow of surplus energy from industrial to utility.

It should be kept in mind that, while the regulation by the Commerce or Railroad Commission applies mainly to the sale of electric energy at definite rates without discrimination, the utility must grant similar conditions of service to *all* industrial plants. Since these naturally show considerable different economic set-ups in the generation of energy, it is obvious that variations in set-up stand somewhat in the way of effecting the exchange of electric power between utilities and industrials.

Since several instances of cooperation abroad have been cited in the paper, it should be mentioned that overall utility practise in this country is somewhat further advanced than that abroad and also that the sale of energy per capita is considerably

[7] Engineer, Chicago, Ill.

higher. As a result, the electric rates are relatively lower than in Europe if we consider that the cost of equipment is 40 to 70 per cent higher in this country. The conclusion is that the natural flow of energy is from utility to industrial. It should also be kept in mind that the European and foreign plants, some of which have been cited in the paper, are relatively small and in many cases are possible because many European utilities are not subject to the rulings of commissions and are freer in negotiating power contracts as dictated by individual conditions.

Dual-purpose utility power plants occupy a distinctive position in the vast field of producers of energy in various forms and will be built wherever the economic and local conditions warrant. It cannot be construed as lack of cooperation on the part of utilities if they find themselves unable in times like the present to respond in every instance to a demand for steam and power in a certain locality. Unfortunately, little definite practical information of such dual-purpose power plants have been made available.

Lastly, it should be mentioned that energy in various gaseous forms must be considered in the interchange or buying and selling of energy. Long-distance natural-gas lines, replacement of rich gas by lean gas, use of mixed gases, availability of refinery gas, low off-peak gas rates, adjustable off-peak electric rates, all play important rôles in this adjustment, which has been going on during the last five to ten years. Judging by conditions in the Chicago district, the reproach made by the author is wholly unjustified. Such projects cannot be forced through as a general policy; each project must be considered individually and must stand on its own merits.

Alfred W. Fox[8] wrote: Mr. Harkins has presented an interesting résumé, pursuing his analysis to logical conclusions with which I am in full agreement.

Unfortunately, some of the public utilities maintain an "all or nothing" attitude, that is, they do their utmost to prevent the installation of a private plant, but if it is built, they will refuse to cooperate in an interchange of power and will not furnish stand-by service except at relatively exorbitant rates. Obviously, if the industry's low-pressure steam requirements are greatly in excess of the electrical, it is deprived of an additional source of profit from the sale of surplus energy.

In all fairness to the utility, however, such surplus energy, available only in small quantities and at odd moments, is worth only the saving in generating cost at the least efficient central station which happens to be on duty at the moment. The figure, consequently, would not be the same in the different utility systems and would vary in any one system with the load, but it may be taken roughly as 3.5 to 4.5 mills per kwhr.

Assuming that the steam and electric generating plant is already installed, the cost of producing such surplus energy is made up of the fixed charges on and the operation of the necessary substation and switching equipment to tie into the system. The additional fuel required may be neglected in most cases, since the advantage, for process purposes, of superheated steam from a reducing valve over dry saturated steam from a properly proportioned back-pressure or bleeder turbine is negligible in the average industrial plant. This cost must be deducted from the 3.5 to 4.5 mills given above to determine the true value to the utility or the net profit to the industry. It is apparent that unless large quantities of surplus energy are available there will not be a great amount of profit for either party.

The point which I should like to emphasize is that proposed industrial plants in a position to supply surplus energy could, at relatively slight expense, be converted to give firm power either under direct utility supervision and operation or subject to the call of the load dispatcher. Energy in this form is worth much more than 1 or 2 mills per kwhr; it is worth that plus the capacity value of the peak load which it is able to carry. This should be self-evident but a simple illustration may be of interest.

Table 1 of the paper presents interesting data, showing that an industry requiring up to 400,000 lb per hr of 200-lb steam can generate from 0 to 42,000 kw, depending on the type of plant selected. If the industry requires up to 5000 kw and if the steam and electric demands are fairly coincident, it might install a 400-lb 750-F boiler plant and back-pressure turbine which would take care of its own needs.

If, however, a market for surplus power existed, at slight additional expense, say, $7 or $8 per kw, a 1200-lb 750 F plant could be installed and would supply 13,750 kw, or 8750 kw additional. Assume that the local utility company has an installed capacity of 100,000 kw and is carrying a peak load of 93,000 kw. A year from now the load is expected to increase up to or beyond capacity and a new unit or plant will have to be added to the system. The cost of a new plant would probably be not less than $80 per kw, but the cost of adding capacity to the industrial plant should be about $40 per kw. With 15 per cent fixed charges, a net capacity value of $5.35 per kw or $46,875 should be credited to the industrial plant.

This materially enhances the value of the surplus energy. It may be argued that the industrial plant's maximum steam demand of 400,000 lb per hr might not coincide with the utility's peak-load periods and that only 2000 or 3000 kw might be available. But what is to prevent the generation of 400,000 lb of steam per hour during these relatively few hours per year and even venting the exhaust steam to the atmosphere if the installation of a low-pressure turbine could not be justified? If the plant were called on as often as 100 days a year with 50 hr of actual use, the waste would be less than $10,000.

It is in this respect of providing peak-load capacity that stations like Deepwater, Baton Rouge, and Schenectady are distinguished from the numerous smaller ones which furnish merely surplus energy. Undoubtedly many existing industrial power plants could provide the utilities with stand-by service which will be of value when loads again pick up.

As Mr. Harkins points out, industrial executives must be educated in the natural advantages for economic electrical generation possessed by their plants, and a spirit of cooperation must be developed by both parties to insure success. Perhaps many of these projects will never be developed until both parties are brought together by a disinterested third person—an engineer with the ability and patience to work out the many complex details which must arise.

C. F. Hirshfeld[9] wrote: When one discusses a paper on so controversial a subject as that treated by Mr. Harkins, there is always the danger that one may be accused of a partizan viewpoint. For this reason I think it well to start with certain remarks of a personal character.

I am employed by a well-known light and power company and for that reason may be assumed to have the utility viewpoint.

But the utility with which I am connected is widely known as an open-minded and progressive one and has, in fact, not only studied interconnection with industrials for many years but is actually operating with several such interconnections at the present time.

Further, as early as 1929 I presented at a national meeting of

[8] Consulting Engineer, Utility Consumers' Service, New York, N. Y. Jun. A.S.M.E.

[9] Chief of Research, Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

light and power company executives a rather elaborate and comprehensive study of the possibilities, advantages, and limitations of such interconnections and have been identified with the discussion of the subject more or less ever since.

And finally, as a private consultant, I have handled, for both power companies and industrials, studies and negotiations in this field. I believe you will agree that, in spite of my utility connection, I should be able to view this problem in a manner at least approximately non-partizan.

I want to start my discussion by complimenting Mr. Harkins upon the very broad and impartial way in which he has treated the subject. He has endeavored to point out honestly and fearlessly both the good and the bad. He has criticized equally the mental inhibitions and ineptitudes of both parties to such agreements. He has, on the whole, endeavored to give an elder statesman's treatment of the subject and I believe he has succeeded very well indeed. Moreover, while doing this he has collected in one place and in easily readable form a mass of very valuable information.

But it must be noted that his treatment rests fundamentally on certain statistical facts, such as the relative capacities of central-station and industrial plants for the country as a whole, the increased electrical output obtainable with improved equipment in industrial plants, and the like. This is a valuable but also, on occasion, a dangerous method of analysis as is well illustrated in the field of by-product central steam heating.

For example, it has been shown many times by this method that power companies could do a very profitable central-heating business by generating electricity in non-condensing equipment and distributing the exhaust steam for heating purposes. And yet, those companies having the largest central-heating installations have resorted almost entirely to the direct generation of heating steam instead of the use of combination plants. And it should be noted that in some cases these companies actually started the business with combination plants. There is no overall and simple explanation of this phenomenon. Noncoincidence of heating and power loads and different characteristics of the two loads form parts of the explanation. The occurrence of electrical peaks at times of the year in which steam loads are close to minimum values is another part. The cheaper distribution system obtainable in a large area when high-pressure feeders are used in preference to low-pressure feeders is another. In fact, the problem is so complex that each case must be studied separately and on its individual characteristics.

The same sort of thing is true with respect to interconnection of industrial and central-station properties. There is no doubt that there are cases in which such interconnection is highly advantageous to both parties, but it would be wrong to argue from these that this must necessarily be true in all cases or in the great majority of cases.

The author correctly puts his finger on one of the great difficulties met in attempting to negotiate such interconnections when he refers to the misunderstanding of the value of a unit of energy. He might properly go further and state that this is only one of many difficulties which may be summed up either under the head of difference in viewpoint or difference in character of business. After all, each industrial establishment is run for the purposes of itself and not to supply by-product energy. In general, the by-product is not of sufficient value to justify its production in the absence of need for the principal product. And therein lies the great stumbling block in the way of many apparently possible interconnections.

The paper refers to the necessity of conducting negotiations through or with the full knowledge of the engineers of both parties in interest. He undoubtedly has in mind, though possibly unconsciously, the engineering staff of an industrial of the type he is connected with. However, if any such extent of cooperation as he envisages is to be brought about it will involve dealings with industrials of much smaller magnitude. Here one meets, in general, executives who know little or nothing about steam and power problems and operating engineers who have their reputations and their jobs to protect. The combination is well-nigh hopeless and such experience in this field as I am familiar with indicates that this combination probably presents more difficulties than any other single phase of this complicated problem.

The paper under discussion lays great stress upon the economic advantages of these interconnections from the national viewpoint. I think it correct to assume that the refinement of all arts and businesses which comes with age and with competition will automatically lead to more and more of these arrangements. But in every case they must be based upon business advantage to each of the parties to the agreement and not upon any altruistic motive such as the conserving of the national store of fuel or the improvement of national business.

In closing, I would like to draw attention to one aspect of this matter that the paper does not touch upon and which I have never seen referred to elsewhere in this connection. When a company undertakes to render public service it incurs certain legal obligations. Having placed itself in the position of a public utility it has, for instance, certain obligations with respect to the supply of that service to the public. I am an engineer and not a lawyer, but I suspect that if the interconnection of industrial plants to public-utility systems goes very far some industrial executive who has not had adequate legal advice will wake up some day to discover that his power plant is legally part of a public-utility system and must be run to the extent that public service dictates rather than as his private business demands. I know of at least one case that is somewhat similar to this and in which the court decided against the well-meaning industrial.

J. C. Hobbs[10] wrote: The total cost of producing service is less when industry and utility cooperate than when they operate independently. The saving in operating expense is then available for distribution between them in the proper ratio. If cooperation does not exist this saving is not accomplished and a preventable waste occurs.

A simple rule which has proved successful in individual inter-company cases is:

(1) Determine what action would be taken if the properties were unified. This is what is done in every case of detailed equipment design and it should be done in inter-department and inter-company problems. Utility engineers do not adopt motor drives for main auxiliaries because of the savings shown when figured on the basis of their standard rate schedules but rather on the actual station cost. Power-station design also incorporates the same heat-balance problems which the present subject covers.

(2) Determine an equitable distribution of the benefits derived from combination operation and adjust the costs to each accordingly.

In other words, many of our problems today are not engineering or financial problems but questions of policy which should be solved on a sound basis so as to take full advantage of the opportunities.

Unfortunately, the elements of such problems are not always as clear and as easily recognized as are the almost exact quanti-

[10] Superintendent of Power, Diamond Alkali Co., Painesville, Ohio. Mem. A.S.M.E.

ties found in steam and electrical calculations. Nevertheless, they represent dollars and should be so equated.

E. C. HUTCHINSON[11] wrote: Mr. Harkins has brought together and correlated an extremely interesting series of thoughts, facts, and figures appropriate to the title subject. Through the paper is a number of pertinent statements, any one of which may be developed easily into a small book. For this reason the paper is exceedingly stimulating to any one who will wish to give it serious thought.

There is no doubt that a predominant policy among public-utility companies heretofore has been to discourage the installation of industrial power by one means or another. There are, of course, some notable exceptions to this statement, but in the main it has been correct.

I make this statement in the past tense because I believe that there is rapidly coming into the minds of utility executives the newer concept that the economics of any power-service requirement for industry or elsewhere must ultimately prevail. Therefore, it is unwise for any utility to take a position in which its policies are assailable and subject to destruction by the pure logic of the circumstances.

Those utilities which have cooperated with industrial companies by exchanging power services, and even by extending steam services from utility to industry, are, to the best of my knowledge, well satisfied with the relationships they have developed. Some express frankly their satisfaction with the results and decry the attitude of other utilities which persist in the old viewpoint. There are, for instance, utilities which, while not denying the economic superiority of the Diesel engine under certain services, still flatly refuse to use one or have anything to do with oil engines, because they have been the instruments through which some of their most difficult competition has been developed. There are likewise other utilities which have hospitably taken the Diesel engine into the family circle because it is a willing and efficient worker and may as well be doing it for the utility as for any one else.

In discussing the cost of transmitting electricity, the author mentions that it costs from two to twenty times as much to distribute electricity as it does to generate it at the station. He then makes an interesting and not ordinarily thought-of comparison between the cost of producing a ton of coal and transporting it to the point of consumption. While the paper states a case in which the distribution cost was ten times the cost at the mine, it should be pointed out that there are many cases where it is fifteen times, or even more.

One is also reminded of the occasion in Germany during the World Power Conference of 1930 in which the American Ambassador was strenuously taken to task by Samuel Insull for a statement made in a speech criticizing the electrical industry for the large spread between the cost of producing electricity and the cost to consumer.

It will be well indeed to give constructive publicity to the thought that transmission costs for electricity are, after all, no greater proportionately than are numerous other distribution costs. Distribution costs will not increase if they are not economically justified. Moreover, they will be sure to decrease in the face of more efficient power generation in industrial plants.

Thus is seen definitely today the trend away from the blind purchase of utility power and the entrance into the question of "shall we purchase" or "shall we make" is a higher type of intelligence and a better grade of engineering than in any previous time. This, I believe, is going to have the effect of limiting the expansion of utility business, generally speaking, to those fields in which their service is economically superior to anything else. There are many such places and there are tremendous prospects for the growth of utilities. Such growth will, in my opinion, be enhanced and become effective in proportion to the willingness of the utilities to accept properly qualified isolated industrial plants into their network upon an equitable and mutually satisfactory plan for cooperative development and the interchange of steam and electric service.

[11] New York, N. Y. Mem. A.S.M.E.

K. M. IRWIN[12] discussed the paper as follows: Mr. Harkins mentions the cooperation between the company by which I am employed and industrial concerns. I personally have spent much time with Mr. Harkins on one cooperative arrangement which is working out to the satisfaction of both parties. I am assuming, therefore, that this paper excludes "present company" and I want to assure him that my remarks do also.

The impression given by the statements and data on the first two pages is that the capacity and energy that might be obtained by increasing the pressure in industrial plants and expanding the steam through back-pressure turbines is usable and of value to utility companies and that it is only their "shortsighted policy" and "unfair competitive method" that prevent its purchase. The first statement of the conclusion also presupposes that the capacity obtained is firm and that the energy has a definite value.

In our experience, it has been difficult to find an industrial development where the excess capacity would be firm on the load curve. A definition of firm capacity is "capacity which will be, beyond question of doubt, available for use when needed." If the capacity is not firm, the utility cannot pay money for it. We are sorry that "many industrial power engineers believe that the general refusal of the utilities" to pay money for something that is of no value to them "is unsound national economics." I fear they mix "national economics" with what is best for their own particular firm. The surplus energy is worth, to the utility, something less than it would cost the utility to make it itself, less the transmission losses. In these days of interconnections between neighboring large companies, surplus power is, broadly speaking, a commodity of which there is a surplus and every fifth row should be plowed in.

It should not be forgotten that in taking surplus energy, great care must be exercised not to use up transmission and distribution capacity which is needed for the normal business, as savings on surplus power can hardly be large enough to carry additional investments.

A great deal of time and money has been spent by our company in making studies of concrete cases where it at first appeared possible to build a plant to supply a particular consumer or groups of two or three consumers electrical energy and steam. To date, only two cases have worked out to a solution which would be mutually profitable to both parties and would also safeguard them against the contingencies of load variations. One of these cases is the Deepwater arrangement, and the installation of the other has been delayed due to business conditions. This type of arrangement ties up a relatively large amount of capital to the load stability of one or possibly two or three customers.

Industrial plants have undergone radical changes in their steam and power requirements in the past. With business fluctuations, they may shut down for periods of time, change their products, change their process of manufacture, eliminate or add processes, or even go out of business. Tariff changes have

[12] Assistant to Vice-President, Philadelphia Electric Co., Philadelphia, Pa. Mem. A.S.M.E.

caused radical changes in the paper and sugar businesses. We know of a large oil refinery that uses practically no steam while others have a very large steam use. One cannot help but wonder whether some of those using large quantities of steam would not be tempted to change their process if they were not burdened with investments in steam-generating facilities. The loss of the load or a radical change in the steam or electrical requirements of a customer supplied from a special plant may leave idle a large amount of investment, while the normal consumers which are connected to the main distribution system render only a relatively small amount of distribution investment idle when their loads are lost.

These considerations make it appear necessary that a contract be at a rate and for a term which will amortize the power- and steam-generating equipment during the life of the contract, A ten-year contract may cause the rate to be too high to be attractive and it is difficult to persuade industrials to sign fifteen-year contracts. When, due to load growth, it is necessary to increase the installation during the life of a contract, the problem of amortizing the increased investment is present, especially if it is necessary to make the installation during the later years of the contract. Retirements and replacements which will probably occur during the contract must also be provided for.

The economical advantage of this type of plant lies in the ability of the utility company to supply reserve for the generating equipment and to take the excess production or to supply the deficit above the steam demand.

Adding back-pressure turbines in the central supply company's main generating stations is, of course, a more attractive proposition, as the buildings and boiler equipment, etc., are useful for the general business of the company and even the back-pressure turbine unit may be utilized by the installation of a low-pressure unit or by exhausting into existing low-pressure equipment. The opportunity of locating customers within economical steam distribution radius of the main generating stations is generally limited. The feedwater problem, if the condensate is not returned, is also serious as the installation of evaporators takes up pressure head which could be used for power generation.

The form of the corporate set-up and type of rate schedule also offer complications. Considerations as to whether the individual plant should be a part of the public service company or a separate company must be made and this decision has a direct bearing on the form of rate schedule. With a single customer served from the plant, methods of charges are relatively easy of solution but, with two or more customers each having a different ratio of steam to electric requirements, it becomes more difficult to distribute both the fixed and production costs equitably.

The Deepwater station arrangement, referred to in the paper, is interesting in some of the results that have been experienced. I believe the parties to this agreement have faith in the fairness of each other. They know that the differences that have arisen are honest divergences of interpretation of intent and have settled them on a basis of equity rather than trying to take an advantage of literal interpretations of the contract. The satisfaction to the utility companies in this contract is not dependent on receiving either firm capacity or surplus energy from the back-pressure turbine, although it was expected when the contract was signed that both firm capacity and surplus energy would be available. The operation at this plant is an example of the way industrial loads and load characteristics vary and demonstrates why utility engineers are hesitant in considering the power from back-pressure turbines as firm capacity or believing that steam and electrical peaks coincide. At the time the arrangements were entered into, the industrial's engineers thought that the steam and electrical requirements varied together, that the steam used was such that excess energy would always be available, that there would be excess capacity available in the back-pressure turbine which the utility company could count on as firm capacity, and that the industrial would only have to purchase additional power when the back-pressure turbine was out of service.

Now, let's look at the record! It covers the years 1931, 1932, and the first eight months of 1933.

The maximum monthly kilowatthour use was 65 per cent greater than the minimum monthly kilowatthour use
The maximum monthly steam use was 86 per cent greater than the minimum monthly steam use
The kilowatthour required per 1000 lb of steam required varied from 34.6 to 44.7.

The maximum number of kilowatthours that it was necessary to purchase from the power companies' machines to make up for the lack of power generated by the back-pressure turbine was, in one month, 20 per cent of the entire industrial's requirements. In the month of minimum requirements, it was necessary to purchase but $^{24}/_{100}$ of one per cent.

The maximum amount of surplus power from the back-pressure turbine which the power companies could purchase was 10.65 per cent of the total generated. There were five months when the back-pressure turbine produced no surplus that could be purchased. In one month, it was necessary to purchase 10 per cent of the power required by the industrial from the power companies due to shortage in the steam requirements and, during the same month, an amount of power equivalent to 5 per cent of the industrial's consumption was sold back to the power companies. All of the above data were for months in which the back-pressure turbine was in operation throughout the month.

I believe these figures clearly show the unsynchronized use of steam and electrical energy in a large industrial establishment.

James F. Muir[13] wrote: In his review of the many papers, which have been published on the interchange of steam and electric power, Mr. Harkins stresses, in a somewhat unique manner, the necessity of cooperation between industries and power utilities.

The basis for interchange must, of course, be mutual profit, and a spirit of cooperation must necessarily be a function of this objective. If the parties to any prospective interchange plan do not cooperate, there is then no common ground for negotiations or agreement.

In a problem so involved in technical and economic considerations it is but natural that differences of opinion, misunderstandings, and possible disagreements should arise. But when we consider that these same conditions are an every-day occurrence among all kinds of business and among industrials in the same business, it should be a matter of satisfaction that some progress has been made, as evidenced by the author's list of interchange arrangements already in service.

For the purpose of analysis and discussion, it becomes necessary to establish the extent to which the interchange plan can be adopted and to determine the respective responsibilities of the utilities and the industrials in the promotion of this plan.

As a matter of general interest and to give a picture of the entire industrial situation, the author might have included data showing the industrial plants which purchase power from central power systems. While most of these utility customers

[13] Power Engineer, The West Penn Electric Company, Pittsburgh, Pa. Mem. A.S.M.E.

are not in a position to interchange, this information would give an indication and a measure of the extent to which industrials depend on the central-station system for power service.

It is common knowledge that a very large percentage of central-station power is generated for industrial purposes. In fact, in the system I am associated with, more than 80 per cent of the total output is delivered to industrial plants.

In this connection Mr. Harkins discloses that in the $8^1/_2$ million kilowatts of industrial capacity, the average size of generators is 450 kw. Furthermore, the paper states, in effect, that the interconnection of a great many small units presents a hazard and reduces the quality of service to all customers. It is reasonable to expect that this conclusion has been arrived at after a careful analysis of conditions. It must be agreed, therefore, that the average plant is too small to warrant interchange considerations. In any case, our investigations indicate that these average plants have little or no surplus power available and the installation of additional equipment for this purpose cannot be justified.

In considering, then, the plants which are qualified to be included in a study of interchange arrangements, and which are not a part of the central power system, it becomes a vital part of this study to sort out or separate the favorable and unfavorable possibilities.

Plants with large power loads and relatively small process- and heating-steam requirements may be considered as the steam-condensing group. This group, which includes glass, coal and coke, iron and steel, metal products, etc., represents a very large percentage of the plants above the average size. With the greater portion of their power needs generated by condensing units, the advantage of the interchange is of small consequence. Although power from these plants could be delivered by relatively short transmission systems to local customers, this advantage would be offset, to some extent, by the fact that the industry must be served first. The outside customer is of more or less secondary importance. In this group, however, there are a few plants where waste heat and fuel are available. In such cases the installation of interchange capacity might be warranted, but the number of plants in this category represents an insignificant part of the industrial field.

There now remains the group of plants with large steam demands and relatively small power requirements. These types of plants constitute a field for the development of possible interchange arrangements. As a result of investigations in the system in which I am employed, the number of plants having large steam demands and relatively small power requirements represents only one-half of one per cent of all industrial plants in the territory. This territory covers an area of 20,000 square miles and is typical of a highly industrialized region.

In summarizing the situation as outlined above, it is quite evident that the field of interchange is very small indeed, probably consisting of a few of the industrial plants with large steam and small power requirements.

Having arrived at this conclusion by investigation and a consideration of the many factors involved and encountered in industries of almost every description, it would seem appropriate that some further comments be added with respect to the attitude of the industrials in connection with the supply of power.

In the past (and it is likely to apply in the future) industrial concerns have been more interested in investments which would show a high return in manufacturing equipment. While the savings in the power-plant department by high pressures and mercury cycles may show some advantages, these savings would be insignificant compared with the profits on industrial-plant manufacturing equipment investments of the same magnitude. This statement of investment policy is not merely an assumption, it comes directly from both small and large industrial interests. These concerns are not interested in complicating their normal manufacturing activities by power developments beyond their local requirements. They purchase electricity from the central power system because it is profitable to do so and because these purchase agreements can be discontinued should the arrangement prove to be unsatisfactory.

Industrial concerns are specialists in the manufacture of commercial products. In fact they must specialize and concentrate on the improvement of their methods of production and the quality of their goods. These activities result in an almost continual change from year to year, and in some cases, month to month, in processes, in the demand for goods, in equipment of new types and kinds, in the character of business, and in the use and amount of their power and steam demands. In fact, changes are so rapid that the average life of industrial plants is astonishingly short. It is less than a decade.

In operations surrounded by such uncertainties it is only logical that the attitude of industrials should be one that points definitely toward a release from the responsibilities of the technical problems and the highly specialized operations of the power business.

It is a known fact that with but few exceptions industrial power plants are not equipped with steam and electric generators which can compete in performance and production costs with those in service in central power systems. In order to meet this competitive situation and make the interchange plan effective and profitable, new high-pressure steam and electric generating equipment would have to be provided. From our record of 10,000 personal contacts each year with industrials it can be stated with confidence that the vast majority are not interested in the installation of new high-pressure equipment for interchange.

Mr. Harkins points out that the larger part of future electrical generating capacity should very probably be installed in or near industrial plants rather than in condensing-steam and hydro central stations.

The economic advantage of stations conveniently located in industrial centers for the supply of steam as well as power is thoroughly appreciated by the power companies. In fact, in the summer of 1929 our company completed studies and was prepared to undertake an installation of this kind. These plans were upset by the depression and its duration has altered temporarily the power-supply situation. However, as soon as normal business conditions are established, we are fully committed to a policy of active promotion of such schemes as are warranted by sound economic conclusions. Indeed, the major part of my activities are centered around investigations into these possibilities.

Although the development of such projects is governed by the demand for process steam, the turbine equipment in these stations cannot be confined solely to the back-pressure unit as sponsored by the author. Seasonal variations and wide fluctuations in industrial demands would, in most of these installations, result in very inefficient loading of turbines of the back-pressure type.

From the author's comments it is not clear whether he favors the development of these projects by the utilities or by the industrials. It would seem a reasonable assumption that this type of power enterprise is an activity which should, and probably will, be undertaken by the utility. The power companies are in a position to provide high-efficiency condensing base-load plants of capacities considerably in excess of local steam and power loads. Extraction steam would be used for process, and surplus power would be delivered to the system. On account of the larger capacity and the high economic advantage

of base-load operation, the unit cost of production would be reduced to a minimum. A plant of this type would not be affected by changes in manufacturing processes and it would have the same reliability as any modern super-power station.

Let us hope that the few plants of this type already in service will furnish the incentive for more active cooperation between central power companies and industrials in an expansion of this most recent and interesting plan for the development of power facilities.

Regarding the attitude of the utilities with respect to the purchase or interchage of power from industrials, I can, of course, give only an expression of the policy of the company by whom I am employed. It can be stated definitely that we are interested in obtaining adequate and reliable power at any place, provided the cost is in line with incremental value of similar blocks of power from existing plants already in operation in the system.

This discussion has attempted to point out as a result of experience and contact with a great variety of industries, that possible interchange arrangements are more or less insignificant when considered in terms of the industrial field as a whole. However, regardless of the number of possibilities, it is important that the power utilities exercise all possible interest and cooperate to the fullest extent. This, I am sure, is the honest intent of all progressive central power systems.

GEO. A. ORROK[14] wrote: In preparing a discussion of this paper I found that I would greatly exceed the ten minutes allowed me so I prepared an article which appeared in *Power Plant Engineering*, December, 1933, where I have given facts and figures concerning the most interesting interconnection at present in existence, that of the Union Générale Belge d'Electricité.

In the years preceding 1925, Mr. Fernand Courtoy, of Brussels, worked up a plan for the coordination and interconnection of the power supplies of Belgium. This proposal was soon followed by the appointment of a commission whose report was embodied in the law of March 10, 1925, establishing the Union Générale Belge d'Electricité. This corporation grouped together all the large producers and users of power from all sources in the eastern half of Belgium. The size of the plants connected was limited by the requirement that they should take at least from 500 kw to 1500 kw of demand. These plants were connected by high-tension lines into one system managed by the load dispatcher at Liége, and by 1929 an aggregate of about 500,000 kw had joined the system. The higher-cost plants had been shut down or relegated to peak-load operation. The net savings for 1928 operation were more than 4,000,000 francs and the profits to members were of the order of 2 centimes per kwhr for the 1928 operation. In 1929 the profits on a much larger business were about $1^1/_4$ centimes per kwhr. The 1930 business approached 1,000,000,000 kwhr, with profits of about the same figure.

Since the preparation of the article referred to, I have received a letter from Mr. F. Bochkoltz, director of the Union Générale Belge d'Electricité, who tells me that the 150,000-volt transmission line connecting the provinces of Hainaut and Liége has been completed and the paralleling of the entire system is now in operation. The generating capacity is substantially the same as in 1932, while the returns paid to the affiliates have amounted in each year, 1931, 1932, and 1933, to one centime per kilowatthour. He corrects one statement which I made in *Power Plant Engineering* by saying that up to date no government representative has been appointed to the board of directors, although the possibility is provided for in the law of 1927.

J. A. POWELL[15] wrote: I am thoroughly in accord with the broad idea of interchange of steam and electric power between industrials and public utilities, having studied this subject for several years.

Such projects, however, should only be executed after a very thorough study has been made, taking into account not only the cost of the steam and electric generation, but also the long-time economy and availability of such generation from industrial plants, the changing energy demand on the utility system, the method of joint plant operation, and the local conditions peculiar to that particular project. For such studies a thoroughly competent engineering organization should be called in, who, with the cooperation of the industrial and the utility, can ascertain all facts and work out a most economical solution for approval.

I have been associated with a number of such investigations during the last four years. In general, the industrial and the utility have fully cooperated in all studies. These studies revealed the necessity of a thorough examination of the conditions for each individual project. Not only should the present steam and power consumption of the industrial be checked, but possible improvements in operation should be investigated, as these may materially affect the steam demand and thereby the result of the study.

In several studies, it was found that an interchange of power between the industrial and the utility was not as favorable as a superficial study seemed to indicate; in other cases, such interchange of power was found definitely economical and feasible. In general, it may be said that an industrial plant must be of a certain magnitude and must be doing a stable business to make it attractive for the utility to connect this plant to its system for power generation.

In one study made, the steam demand of the industrial was approximately 500,000 lb per hr, with a yearly steam requirement of 3.5 billion pounds. The thoroughness of this study, which was completed about two years ago, may be judged from the fact that complete estimates were made for eight arrangements, including installation and operating costs for a plant using steam at 1400 lb, 600 lb, and a mercury boiler. The smallest mercury unit which the manufacturer was willing to install was for 20,000 kw, thereby limiting the benefit of the mercury cycle to the largest industrial plants.

The result of this study was gratifying in that it showed that a plant built for the supply of steam to the industrial and electrical energy to the utility was economical, feasible, and beneficial to both parties. Due to the slump in business conditions which reduced the steam requirements of the industrial and also the energy requirements of the utility, the postponement of the actual installation was necessary. In this case, as in several others, plans are complete and the actual purchase of equipment and installation of units can be made without delay when business conditions warrants such action.

A number of engineers and executives in this country are fully aware of the possibilities of such interchange of steam and electric power and that the studies made will prevent them from the pitfalls of overenthusiasm in a new field, and lead to quick action based on a careful analysis of the facts when conditions permit.

Many joint power projects have unquestionably been delayed, due to cheap fuel and dump hydro power experienced during

[14] Consulting Engineer, Orrok, Myers, and Shoudy, New York, N. Y. Mem. A.S.M.E.

[15] Vice-President, W. S. Barstow & Co., Reading Pa. Mem. A.S.M.E.

the last few years, and difficulty in obtaining new capital for such projects.

GUY B. RANDALL[16] said: There are so many interesting statements in this paper by Mr. Harkins that it will naturally develop a lot of interesting discussion. But his seven conclusions, well summarized at the end of the paper, must appeal to the vast majority of engineers who have followed this development as being quite sound.

In the body of his paper, as well as in conclusion No. 2, Mr. Harkins, in my opinion, has perhaps unintentionally left an erroneous impression when he makes his brief references to hydroplants. As I have suggested in at least two previous articles dealing with this subject, there can hardly be any serious competition between steam power and water power if our national power development is carried out in a rational way. One has only to visualize the inevitable national, and even international, "power pool" to realize fully the fundamental truth of this statement. A water-power plant not only can be very simply and quickly placed in service and likewise stopped but by drawing on water storage can develop enormous peak-load capacities. Water-power plants can also be arranged in many cases to pump water for storage, thus consuming certain steam-power and water-power surpluses. It is, among other considerations, for reasons such as this that water-power rights will, in the future, prove highly valuable in a unique way. So steam-power development, either on a condensing or a by-product basis, does not operate to make water power useless but rather tends to modify and to enhance the importance of its ultimate economic application. I say this, and yet in the same breath admit I am a "steam" man. But when I talk about a water-power plant I do it "with my fingers crossed." Because, after all, a hydroplant is nothing but an elementary steam plant in disguise. In it the boiler feed pump is simply reversed and magnified to the dimensions of the main prime mover, a hydraulic turbine, while the sun and the ocean act as the heat source and the boiler, respectively. So if anything can be regarded as certain it is that we should not be faced with a problem of steam power versus hydro power but rather with the problem of the proper coordination of our three great steam-power sources, namely, (1) condensing steam, (2) by-product steam, and (3) "hydro steam."

Mr. Harkins follows his water-power statement with a second point which is tied into conclusions Nos. 3, 4, 5, 6, and 7. I do not question these conclusions. But I do think that the paragraph beginning at the bottom of page 12 may reasonably be examined from a slightly different angle. I refer to the general subject of the rights and responsibilities of a utility power company in dealing with an industrial plant as to surplus power and power interchange. The key to the solution of this problem jointly confronting the industries and the utilities is probably to be found in a careful review of the essential character of the utility power companies and their proper relations to the communities they serve. This involves consideration of their obligations no less than their privileges. They are privileged to exercise the right of eminent domain, to occupy streets, roads, and other public places in a monopolistic manner for the conveyance of their product, and to charge such prices as will net them a reasonable or fair return on their investment. Their chief collateral responsibilities are to exercise reasonable prudence in management, to avoid interfering unduly with the use of public places or highways, and to see that service is adequate to meet the ever-changing needs and standards of the communities they serve in this quasi-public relationship. They are in no sense granted any monopoly on the making of electricity. Any one may do that. Nor on the selling of it. Any one may do that. Their monopoly lies in the rather exclusive use of public property for conveying and distributing electricity. That is, they are permitted to install private "powerways" on and under public highways. Now if I have a block of electricity to sell and ten squares down the street another party has a block to buy I, as an ordinary citizen or company, cannot make the sale unless I make delivery over a private transmission line running on private property. That is, I must follow this method, which is usually prohibitively expensive, unless I can prevail upon the power company to do the conveying. In this event, I sell the electricity to the power company, which in turn allows it to flow over their wires to the buyer's premises where they then re-sell it to the buyer. Suppose that as a surplus-power producer I finally induce the utility to make me a price for my surplus. This price, of course, would be discouragingly small as compared to what the customer pays. As an alternative I might then suggest to the utility that they simply place three independent wires on their existing structures and connect me with my customer. At one stroke I have cleared away a wilderness of entanglements and complications. The basis on which the deal may be negotiated now assumes relatively simple proportions. I can make money, the power company can make money, the customer can make money, and the community is better off. This is the common-carrier idea reduced to its simplest terms where electricity is the commodity. The full solution, of course, involves another step which I have, for almost exactly ten years, labeled and described as a "power pool." This great power pool is coming inevitably. Many members here in this meeting can easily recall when it was quite common for railroads to refuse freight originating on other lines. But such a custom could not long prevail because it was so obviously contrary to the public interest. The same reasoning applies to the power problem. Pooling is natural. The only question really open to discussion is how best it may be done. We can recognize it and arrange for an orderly development. Or we can refuse to recognize it but nevertheless will get it through a course of zig-zag progress which will serve mainly to increase both the expense and the time for a given development. So perhaps lexicographers, as well as engineers, should add to waterway, highway, railway, and airway, an equally portentous development logically called "powerway," because, like its four consorts, it will be essentially a common carrier.

In the foregoing I have emphasized in a little different way some of the features of the problem which Mr. Harkins has ably dealt with. These are features which are important, not only for engineers in general but for much of the public to understand. Such understanding will, perhaps, keep us from losing a sense of proportion—a thing which has occurred too often in the past. Mr. Harkins, perhaps, recognized this fact but in one paper cannot include everything. As another industry interchanging power with a utility system, I might add The Champion Coated Paper Company to the list he discusses. This company, in its Hamilton plant alone, generates well over one hundred million kilowatthours per year, sells about ten per cent, and buys about one per cent. The interchange is on "when, as, and if" basis chiefly because of the utility's insistence. The utility's selling price is three times the industrial's. Probably neither party is satisfied with the arrangement though relations are certainly friendly. The satisfactory solution of this case and all other cases must, in the final analysis, depend upon some sort of public agreement as to the principles involved. This in turn calls for a very considerable amount of free and open discussion. The ends of logic, justice, and econ-

[16] Superintendent of Power, Champion Coated Paper Company, Hamilton, Ohio. Mem. A.S.M.E.

omy are not well served by silence, secrecy, or equivocation. The existence of a north pole and a south pole implies an equator. Let us all hope we find it without resort to a complicated legal instrument. Yet some kind of a code will doubtless be necessary for a good many reasons. But it must be remembered the present by-product or surplus-power situation is not exactly something new under the sun. It is rather the old, old story of a natural and gradual technological advance which may ultimately exert enormous pressure—like water freezing in a granite rock—simply because it is constrained.

One peculiar result of this constraint lies in connection with the maximum-demand and emergency-service policy followed by most of the utilities. The set-up is such that if I want to install one thousand kilowatts of capacity to generate some power, the utilities indirectly force me to install another thousand kilowatts of capacity for emergency reserve. I say this because in every case I have known much about it has been cheaper to install used or second-hand equipment in a first-class manner to supply emergency service than to buy it from the utility. I have been a little surprised at some cases where engineers have recommended purchasing emergency service when the used-equipment solution would pay 20 to 100 per cent yearly dividends. In this connection used equipment—or old equipment—is not to be taken lightly. A keen engineer can often accomplish wonders with it for service with a 5- or 10-per cent use factor. And, believe it or not, some of the old equipment is better for standby service than some of the new. But most utilities seem to feel that in any event this reserve equipment is an industrial plant's own ghost. I am not so sure. I often wonder if, in the natural course of future events, the utilities may not some night awaken in a cold sweat to find it haunting them.

Whatever the material facts may be there is little ground for belief that industrial plants, as surplus-power producers, will receive unprejudiced consideration of their claims by the utilities except as they arrange to press them diligently. If any one is inclined to doubt this statement, let him investigate the known facts of electric-appliance retailing and the laws that have been passed—or are pending—in many states to govern it. Much information is available in this connection from the various state retail associations. But this very situation offers a unique opportunity to the utilities to demonstrate that they do realize and do assume the responsibilities as well as the privileges conferred through their public-granted monopoly for conveying and distributing electricity. If they do make use of this opportunity they will not only have an added reason for further development but will rise above that level of thought and action that now seems to justify applying to some of them that old but logical and classical injunction, "There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

In closing, I feel compelled to mention once more[17] the moot point as to the necessity for evaporators. All who have read this paper must have noticed that in Table 1 there is a curious recession in the kilowatts of power available when the increasing steam pressure reaches 650 lb. This simply means that Mr. Smith thinks evaporators are not required at 450 lb but are required at 650 lb. There is an abundance of actual operating evidence that they are not needed up to approximately 750 lb, inclusive. The recognized authorities on feedwater and boiler-water control also seem to agree that they are not needed for 1300-lb operation. Operation of the plant with which I am connected has been notably successful and free from trouble of any kind without evaporators and with a make-up of 35 per cent of hard water for 650-lb operation. I think the secret—if indeed there is any secret as some seem to think—is that water-treatment plants must be controlled by the hour rather than by the week. With evaporators or without evaporators first-class control is economically necessary because it pays handsome dividends to those who exercise it.

[17] See Trans. A.S.M.E., 1931, paper FSP-53-26a, p. 346.

C. Richard Soderberg[18] wrote: It is a curious fact that, although the fundamentals of the problem of interchanging steam and electric power have been well known for a long time, it is only during recent years that it has received due attention. Of the available possibilities of materially improving the efficiency of power generation, the generation of by-product power offers by far the greatest possibilities, wherever there is a demand for steam for heating purposes. It will eventually transform a large percentage of our industry from power consumers into efficient power-generating stations. The author has ably brought out the inherent facts of this problem.

It is in this field that the development toward higher initial steam conditions is bound to come into its own. The principal objection to high pressure, namely, the wetness in the exhaust of condensing turbines, is removed when the expansion is interrupted at the range of pressure generally used in industrial heating application. In this connection, I should like to call attention to the next to the last item in Table 1 where steam is expanded from 2400 lb per sq in., 1000 F, to 200 lb per sq in. with a by-product power generation of 24,050 kw for an exhaust steam flow of 400,000 lb per hr. This is presumably obtained on the assumption of no extraction for feed heating. Regenerative feed heating is of particular importance in this connection, however. When this device is brought into the picture, it will be found that the by-product power available comes close to the value obtained by the mercury process. Considering, for example, the more practical case of 2400 lb per sq in., 900 F, exhausting to 200 lb per sq in., and extracting for feedheating in three stages, including at the exhaust pressure, it is possible to generate about 35,000 kw for a steam flow to the process line of 400,000 lb per hr.

W. B. Skinkle.[19] There is so much to be said on the possibilities resulting from the interchange of power facilities as between two industrial companies or between the industrial and the utility that it is difficult to determine what should be left out in order to bring the discussion within the limit of the time allowed.

This paper by Mr. Harkins very effectively handles a problem to which the writer has given a great deal of study during the past fifteen years. He should be complimented on the brief yet able way in which he has treated this subject which is capable of an almost infinite number of variations.

The author makes one statement in his paper as follows: "if industrial management were educated in the economics of power generation, etc.," to which I should like to make a considerable enlargement. The writer believes this statement should read: "If utility and industrial management, operators, and engineers were educated in the general subject of industrial economics, etc."

The inability of each group to visualize and recognize problems faced by the other group is one of the major stumbling blocks in the conclusion of sound interchange agreements.

Fundamental Economics. The two large oversights occurring in both groups are a failure to recognize and properly evaluate

[18] Manager, Turbine Apparatus Division, Westinghouse Elec. & Mfg. Co., South Philadelphia Works, Philadelphia, Pa. Mem. A.S.M.E.

[19] Secretary and Engineer, Pittsburgh District Power Committee, United States Steel Corp., Pittsburgh, Pa.

d e "time factor" in production, and second, a failure to realize that this mythical thing called "cost" is subject to considerable analysis and variation. There very decidedly *is* such a thing as a "constant" cost which is independent of the production and there is also such a thing as an "increment" or "out-of-pocket" cost which is dependent on the production and which varies widely in accordance with a constantly changing set of operating conditions.

It has been gratifying to note how this present industrial depression is educating all classes of both utility and industrial men into some of these basic facts in industrial economics.

Attention is called to one point illustrated by Fig. 2 of Mr. Harkins' paper (see page 13) which it is believed is generally overlooked.

Without considerable study and analysis of the subject, most

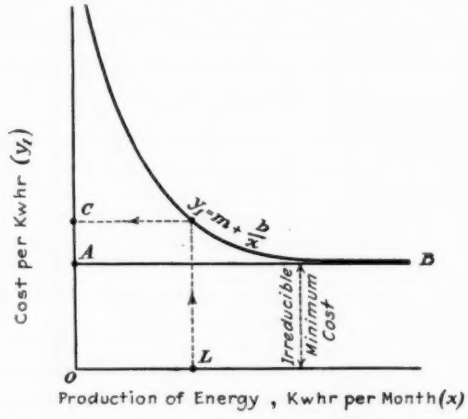


FIG. 3 TYPICAL UNIT-COST PRODUCTION CURVE

people assume that the great value of interchange results from the recovery of industrial by-product heat and the conversion of this heat into power. Without the all-important "time factor," energy recovered from industrial by-product heat, while not absolutely valueless, has in fact only a very small value as it must compete with the "increment" or "out-of-pocket" fuel cost of large utility equipment capable of putting a kwhr on the station bus bar at an expenditure of between three-fourths of a pound and one pound of coal.

It is quite remarkable to note from a close study of Fig. 2 what a small and almost insignificant quantity of "firm" power will be exchanged between the two systems.

The area under the line "*CD*" represents the quantity of energy developed by the utility whereas only the small vertically sectioned areas between the vertical lines 2 to 3 and 6 to 7 represent the help from industrial sources that is actually required by the utility company. The situation with the industrial is similar, where the diagonal dotted cross-sectioned areas between the vertical lines 4 and 5, and 9 and 10 represent the utility help in the form of "firm" power actually required by the industrial.

It appears, therefore, that the chief advantage of interchange arises from the exchange of emergency power and the ability of the two systems to delay large investments in power equipment which, under independent development would have to be completely installed before any part of its output would be required and would then have to stand idle accumulating a high cost in fixed charges until the load grows to a point where its operation becomes necessary.

The addition of generating equipment on a major utility system can easily cost as much as $6,000,000 which if delayed for one year means the avoidance of approximately $780,000 expense in fixed charges. A large industrial plant may easily invest $2,000,000 in addition to power facilities on which the fixed charges would amount to $260,000 a year. With utility support, the industrial plant could probably delay investments much longer than would be possible in the case of a utility plant.

During the past ten years I have had occasion to develop the economies involved in several interchange problems. It is my opinion that a sufficient number of interesting points were developed to make a review of some of these problems particularly opportune at the present time.

FUNDAMENTAL ECONOMICS

In order to clarify the points presented it will be necessary to make a short review of some of the fundamentals of industrial economics which have already been presented in other papers but which it is believed will stand repetition.

Fig. 3 is a typical curve of unit costs plotted against production and illustrates how the unit cost decreases with increased production until it approaches an irreducible minimum.

It can be shown that such a typical curve answers the general definition of a hyperbola of the form of

$$y_1 = m + b/x$$

If the total operating cost for any given rate of production is wanted, the units produced *OL* would be multiplied by the

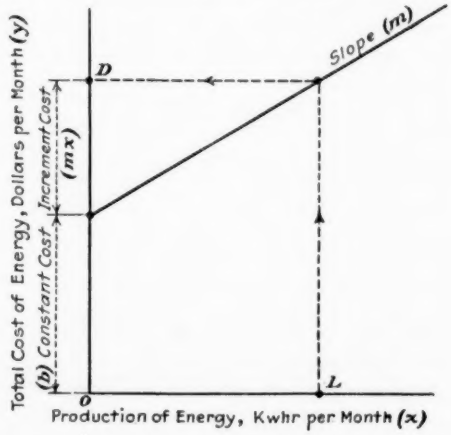


FIG. 4 TYPICAL TOTAL COST-PRODUCTION CURVE

unit cost *OC*. It can be shown also that the curve of Fig. 3 can be expanded into a total cost curve like Fig. 4, which is a typical curve of total cost plotted against production.

In this figure the total cost is shown to be made up of two parts: A constant cost b, which is independent of the production plus an increment cost mx, which varies directly as the production.

The truth and practical application of this conception of costs have been proved many times in the writer's investigation of industrial economies. An interesting side light to the foregoing is shown in Fig. 5.

Variations in efficiency of the units in question change only the slope of the increment-cost line so that economies resulting from increased efficiency are dependent on a large production for satisfactory results, whereas changes in the constant cost of operating a given unit hold throughout the entire range of production. This principle has been of great value during the present periods of low production in pointing out those elements of cost on which attention and effort should be concentrated.

As an actual illustration of how closely industrial costs follow this principle, Fig. 6 is offered for inspection.

In Fig. 6(*a*) actual "book costs" for every month of three consecutive years are plotted for a boiler house of about 3500 hp of rated capacity. The variations from the cost lines shown on the graph are accounted for by changes in the cost of coal and also by the fact that repairs and "distributive costs" shown on the lower line are charged "in total" against the production of the month in which the expense is incurred. This means that although many items of repair and similar costs gradually accumulate over long periods of time, the production during the month in which the money is actually spent gets "soaked" with the entire burden. In Fig. 6(*b*), the repair costs and similar items were "funded." A given constant plus an increment was charged against each month, so that months of large production carried a greater amount of these costs than months of smaller production. The fuel costs were then charged at the weighted average cost of the fuel.

This particular installation usually operated as a "make-up" unit to supply any deficiency in the by-product heat. During periods of low operation by-product heat was not available and the coal boilers were called upon for heavy production whereas at other times with a plentiful supply of by-product heat the coal station floated on the line and only picked up the load for short periods of deficiency in the supply of by-product heat.

The result is an unusually wide variation in load, this being something over 600 per cent. The points on the cost line show how closely actual costs follow this principle when the unavoidable variations in costs of raw material and some of the peculiarities in accounting practises are removed.

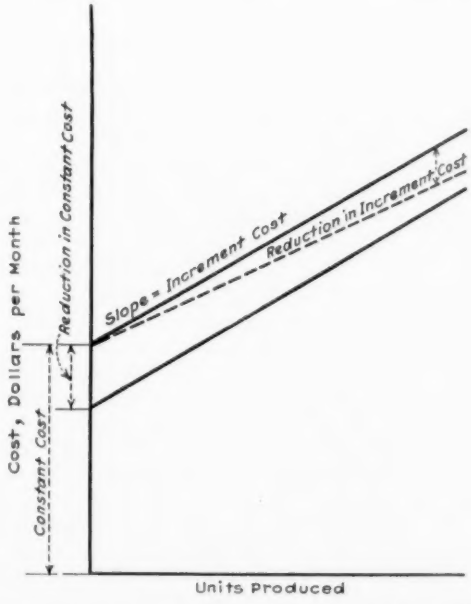
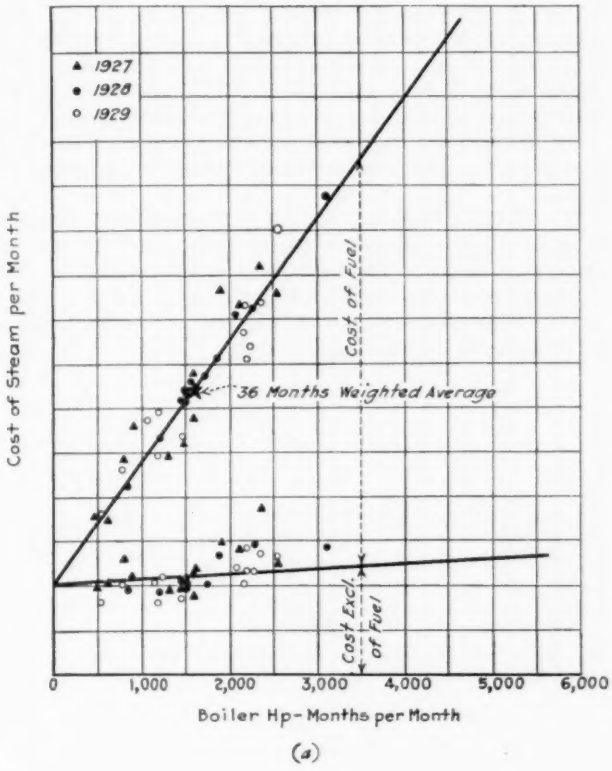


FIG. 5 COMPARISON OF INCREMENT COST AND CONSTANT COST

Many similar analyses of steam costs have been made with equally uniform results. This particular example was used because of the extremely wide variation in the steam production. Before leaving this analysis your attention is called again to the irregularities in the book costs of steam as shown in Fig. 6(*a*). The same irregularities occur in the turbine room costs when taken by themselves. Usually when the



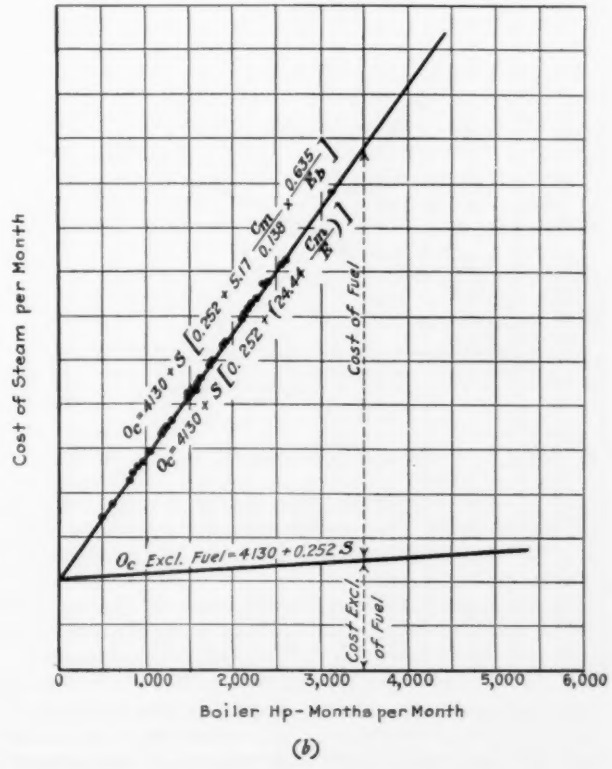


FIG. 6 COSTS OF PRODUCING STEAM FOR 1927, 1928, AND 1929
[(*a*) Book Cost. (*b*) Corrected Cost Based on 63.5 Per Cent Boiler Efficiency and Fuel at 13.8 Cents per Million Btu.]

turbine is shut down for repairs the boilers are "down" and also undergoing repairs with the result that these irregularities are cumulative and electric costs look something like those shown in Fig. 7 where the total cost of 33 consecutive months of operation is plotted against the production of the unit.

Fig. 8 shows the analysis of costs on this unit. The difference in unit cost between the points 2 and 5 is nearly 240 per cent.

Figs. 7 and 8 have been shown to illustrate the difficulties in attempting to formulate economically sound interchange agreements based on the book costs as reflected in the cost sheets of the average industrial plant.

With this background of the fundamental economics, let us see what can be done in an actual case.

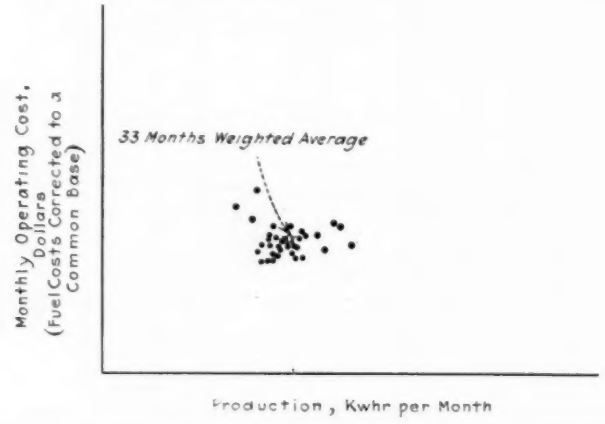


FIG. 7 TURBO-GENERATOR STATION COSTS PLOTTED AGAINST PRODUCTION

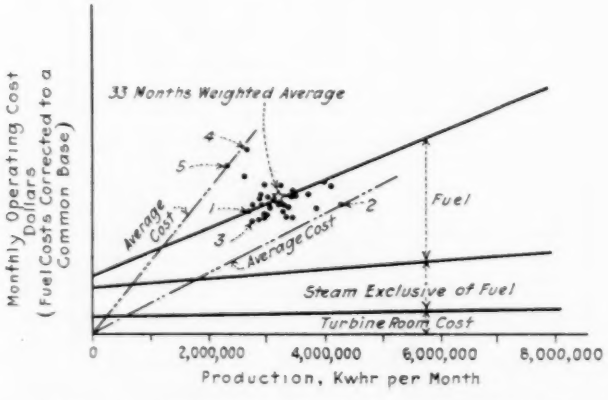


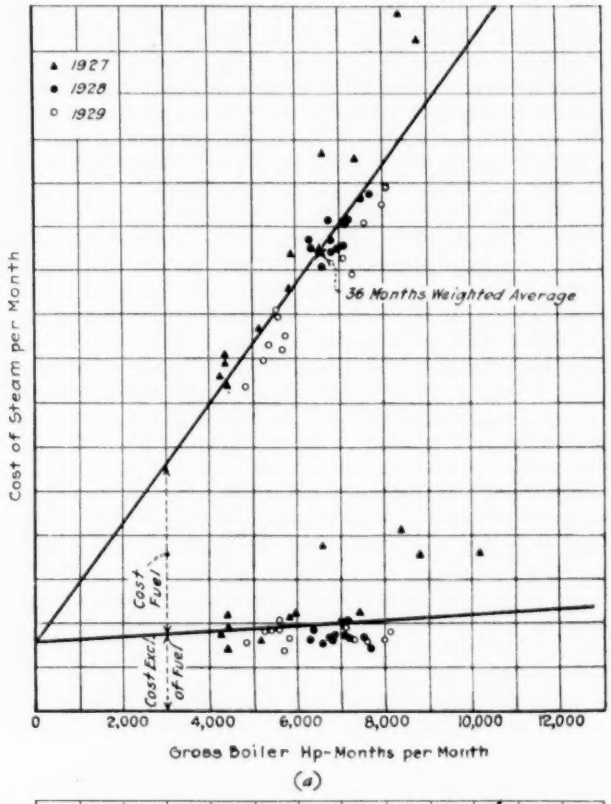
FIG. 8 VARIATIONS IN POWER PLANT COST SHEETS

Case 1 Steam Interchange. Two companies called company "C" and company "A" operate adjoining properties. Company "C" has a large modern boiler house and company "A" a 30-year-old boiler house.

The economies possible are the elimination of the "constant cost" of operating boiler house "A" and the supply of the joint steam from boiler house "C." The coal-fired boilers of company "C" are "make up" boilers and are required to float on the line when large quantities of by-product heat are available.

The analysis of the company "C" steam costs from coal are shown in Fig. 9 with (*a*) the book costs and (*b*) the corrected costs.

In similar manner the steam costs of company "A" are shown in Fig. 10. There was, however, a correction that had to be



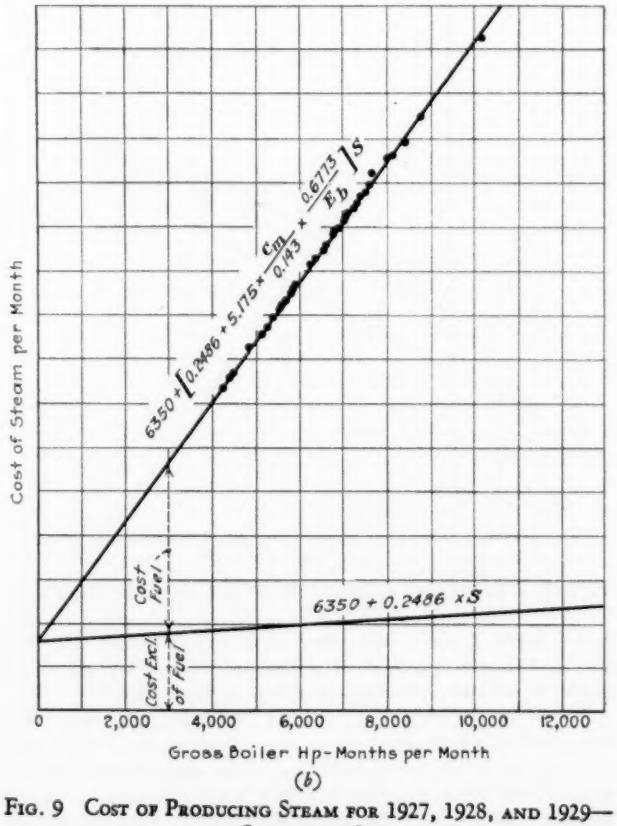


FIG. 9 COST OF PRODUCING STEAM FOR 1927, 1928, AND 1929—COMPANY "C"

[(*a*) Book Cost. (*b*) Corrected Cost Based on 67.73 Per Cent Boiler Efficiency and Fuel at 14.3 Cents per Million Btu.]

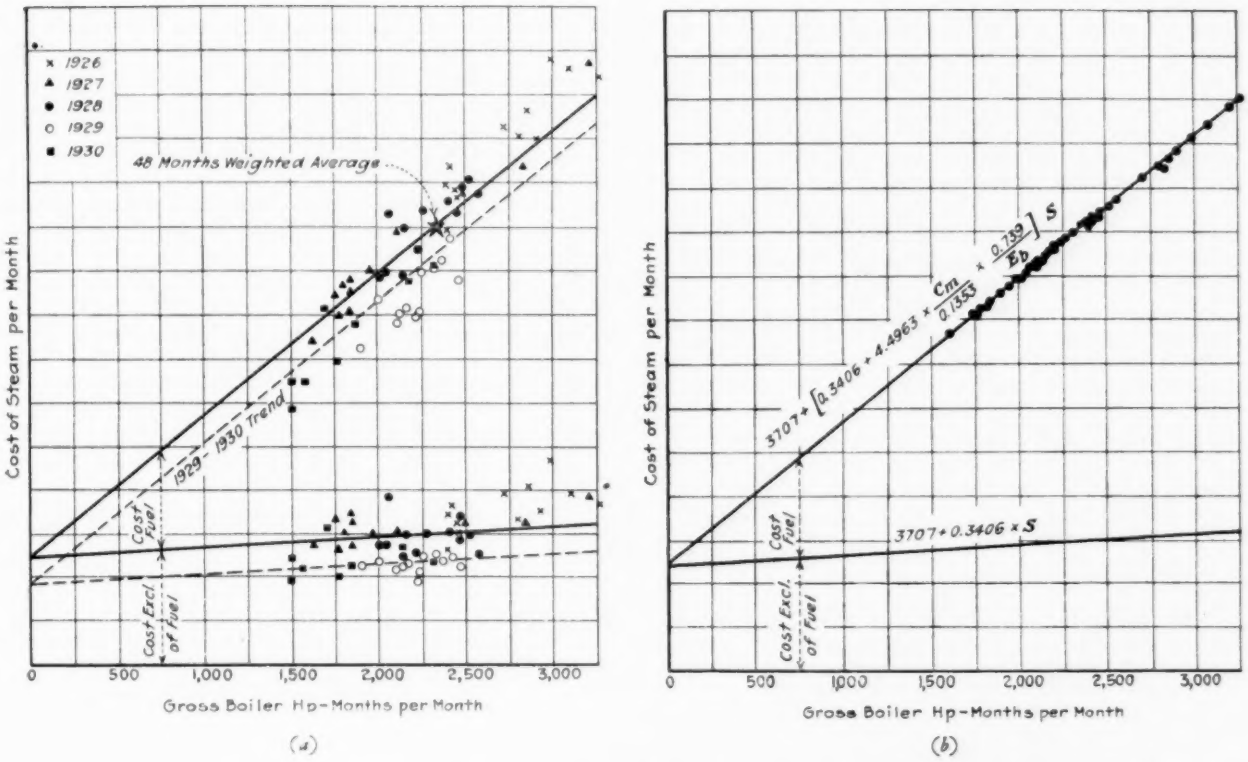


FIG. 10 COSTS OF PRODUCING STEAM FOR 1926, 1927, 1928, AND 1929—COMPANY "A"

[(*a*) Book Cost. (*b*) Corrected Cost Based on 73.9 Per Cent Boiler Efficiency and Fuel at 13.53 Cents per Million Btu.]

made in these costs. If you will notice the costs, exclusive of fuel for 1929 and 1930 are materially lower than similar costs for other years. This shows clearly the results of efforts on the part of the operators of company "A" to reduce their costs. The basis of intercharge between these two companies was therefore made using a constant boiler-house cost of $2800 a month for company "A" instead of $3707 as was indicated by the five-year analysis.

If the total cost of both boiler houses were to be plotted together, the graph would look like Fig. 11(*a*), in which the load and cost graph of company "A" is superimposed on the load and cost graph of company "C."

Changes in either the cost of fuel or in the efficiency of operation of either boiler plant will cause the increment cost lines to change their slope using their points of origin as centers. The interchange agreement is now faced with the problem of dividing a positive but always changing cost reduction equally between the participating parties.

The manner in which this was accomplished is shown in Fig. 11(*b*) which is an enlargement of the upper portion of Fig. 11(*a*). The lines *A* and *B* of Fig. 11(*a*) form the coordinate on which Fig. 11(*b*) is built.

It was assumed that inasmuch as both companies purchased coal from the same mine the fuel cost C_m would be the same for both boiler houses.

It was also assumed that inasmuch as the average efficiency of boiler house "A" over a four-year period had been maintained at 73.9 per cent it was fair to assume that such a figure should continue to be maintained and should, therefore, be used in estimating the probable costs after the boilers were shut down.

The assumption of a constant efficiency enables the cost equation of company "A" to be simplified to the form shown on the upper line of Fig. 11(*b*).

The increment cost of company "C" is shown on the lower line. If these two costs are then added together and divided by two the result will be a selling price that no matter what change in fuel cost or what change in efficiency takes place, the cost reduction resulting from joint operation will be divided equally between the two participating companies. This equation is shown along the central dotted line in Fig. 11(*b*).

Case 2 Interchange of Electric Energy. Interchange of electric energy between utilities and industrials is possible mainly when the industrial has at his disposal large quantities of by-product fuel available in such form that it is not possible or economical to store it or to dispose of it in the form of fuel or in any other manner.

Problems of this kind open up a vast field of special conditions which may radically change from one hour to the next. The difficulties in drawing up agreements which are sufficiently flexible to meet these ever-changing conditions are very great unless the men conducting the negotiations are well versed in the field of cost analysis and industrial economics.

The most essential feature for the successful operation of such agreements is that both parties must benefit financially from *all* parts of its operation.

Industrials generally fail to recognize that there are several kinds of kilowatthours which it may receive from or deliver to a utility, and that each of these different kinds of energy has radically different values to the receiver and radically different costs to the producer. Until this fact is recognized and admitted by the industrial, interchange negotiations will simply become a case of "horse trading" whereby the utility

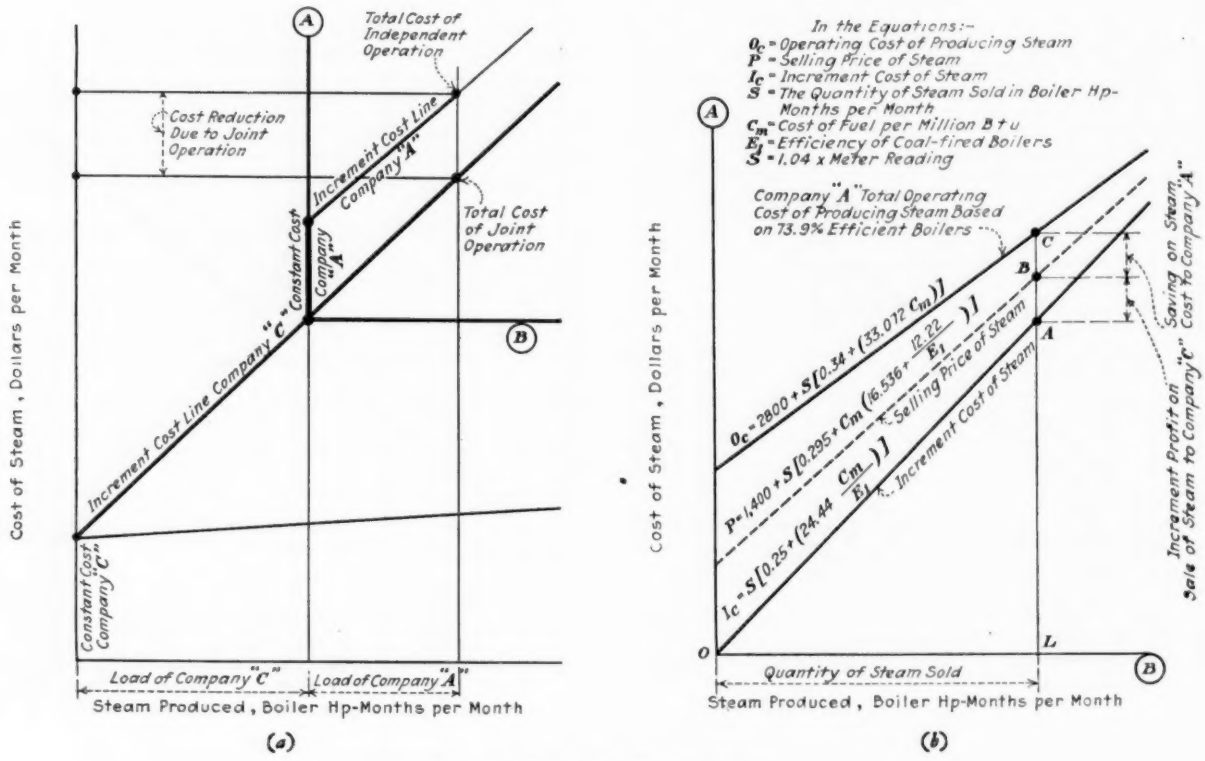


FIG. 11 LOAD-COST RELATIONSHIPS

[(*a*) Total Costs of Individual and Joint Operation. (*b*) Saving and Profit in Joint Operation.]

representatives will "hope" to effect an agreement out of which they can make some money regardless of the benefits accruing to the customer, and the industrial has hopes along exactly similar lines.

Needless to say agreements built on such a foundation will be terminated as soon as the party who is losing money recognizes the fact and can bring the agreement to an end.

Interchanged energy can be of four different kinds, namely:

(1) Firm power sold under regular contracts and available at any time up to the limits of the equipment or contract.

(2) Surplus power available by reason of surplus generating capacity idle and awaiting growth of the system on which it is installed.

(3) Dump power available from surplus by-product heat in the industrial system, which it is not practicable or economical to store or hold in reserve.

(4) Emergency power available from either system to the other on short notice in order to meet sudden, unforeseen difficulties.

Different values should be assigned to each of these, and the number of these values is doubled depending on whether or not the exchange is made during "on-peak" or "off-peak" hours of the utility.

Peak firm power should carry every item of cost that can be charged to it. This includes fixed charges, sales and executive costs, all operating costs, and losses.

The utilities recognize the value of firm power sales secured in "off-peak" hours by placing a low energy charge of 4, 5, or 6 mills on all power sold at load factors above from 25 per cent to 40 per cent. Some utilities go further by removing power factor penalty clauses during off-peak hours or by allowing the customer to create demands much higher than their "on-peak" demands, during off-peak hours without the increased bill that these high demands would normally require.

Surplus power costs should omit many items that enter into the cost of firm power. Investments in equipment are already made and the complete organization is already set up and operating. Additional power under these conditions can be produced for very little over the increment fuel cost.

Dump power from the industrial to the utility is of very low value. The utility must keep equipment operating and prepared to pick up the load carried by the dump power, the instant there is a failure in the industrial supply of by-product fuel. The maximum value that can legitimately be assigned to dump power is, therefore, the lowest "increment" or "out-of-pocket" cost of power to the utility, plus line losses to the point where the two systems are connected. The geographical location of the industrial with relation to the utility generating station is therefore of importance. The price of this kind of power should be based on consideration of these facts and should be protected by suitable coal-cost clauses.

Emergency power values are subject to considerable negotiation as it is almost impossible to place values on this type of service. The costs of these various types of power can be seen readily by referring to Fig. 12.

"On-peak" firm power costs are shown by the vertical height from the base to the line *GH;* "off-peak" firm power by the vertical height between the lines *AB* and *GH;* and surplus power costs by the slope of the line *GH* and should vary from approximately $1\frac{1}{2}$ mills per kwhr with very low cost fuel, to $3\frac{1}{2}$ mills or even 4 mills in small stations using higher priced fuel.

Dump Power. In the case of dump power made from surplus by-product fuel which must be either converted into electric energy in an already operating station or discharged into the

atmosphere, the cost of this conversion is very small. By referring to Fig. 12 it will be seen that there is no change in the fixed charges and that all the constant costs are already being absorbed. If no other use can be found immediately for the surplus fuel its value to the industrial is zero. The only part of the costs which do increase is a very small part of the repairs, due to the increased load, a little of the water-treating costs for additional steam and a little more water pumping for the condenser. This increase in costs is represented by the slope of the line *EF* and in actual money would be approximately $^{1}/_{10}$ of one mill per kwhr.

Analysis of an Interchange Agreement. On the cost basis set forth, an interchange agreement was worked out several years ago. This had for its fundamental concept an equal division of any economies that might result from the interchange between the two parties to the agreement.

Surplus and emergency power were worked out according to a schedule shown in Fig. 13.

In this figure the vertical ordinate is "total cost in dollars" and the horizontal ordinate is "kwhr of energy."

The full lines show cost of power on the station bus bars to the industrial and the dotted lines show the cost of energy in various conditions according to the terms of the interchange rates. The entire figure is similar to the final steam diagram shown in Fig. 11(*b*).

The lower line *OA* represents the utility increment cost of surplus power and includes increment station costs, line losses

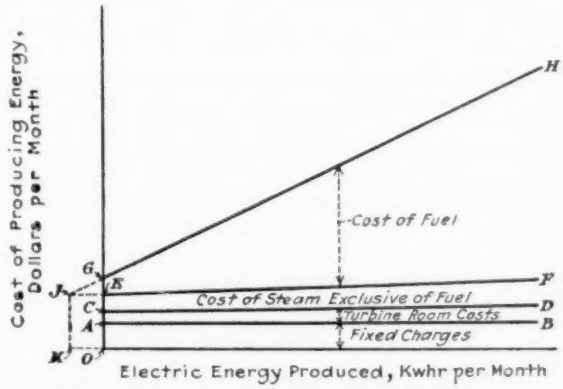


FIG. 12 ELEMENTS OF ELECTRIC POWER COSTS

to the interchange point, and some taxes that would result from the power sales.

The next line *ONB* is the interchange rate based on 60-cycle, 22,000-volt energy above which are two lines showing the unavoidable conversion losses necessary to make the energy available at 25 cycles, 6600 volts.

The vertical line *L* to *R* at approximately $3^{1}/_{2}$ million kwhr per month is the estimated point of average operation. It will be noticed that at this point the utility increment costs would be equivalent to the vertical heights *LM*. The utility selling price would be *LN* and the utility profit would be *MN*. The unavoidable losses would be *NP* and the industrial saving would be *PQ*. The last assumes that the industrial has available by-product fuel which it dissipates rather than make the immediate investment necessary to convert this fuel into electric energy.

The general surplus power rate was fixed at 4 mills per kwhr as indicated by the light line originating at *O*, the continuation of which forms the right end of the line *OB*.

In order to handle emergency power sales and "on-peak" surplus power sales the original slope of this line from the origin at *O* is made very steep. The slope and positions of the lines *JN* and *KP* were placed so as to make as nearly as possible an equal division of the resulting economies.

This could not be accomplished through all loads. It is evident from a glance at the figure that the utility cost line *OA* will cross the industrial cost line *EF* and this crossing point is the limit to which cooperation can be carried without involving an economic loss.

The *EF* cost line is used because the industrial already had available sufficient by-product fuel to produce the required energy. The fuel cost for the proposed unit was, therefore, considered to be zero.

The equal division was further complicated by the unavoidable conversion losses between the two systems which, as the figure shows, further reduced the point of profitable cooperation between the two companies.

Where the steep slope originating at *O* intersected the line

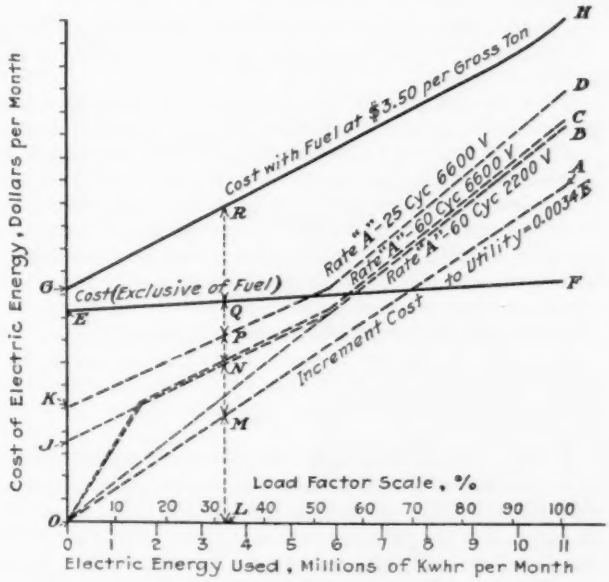


FIG. 13 SURPLUS AND EMERGENCY POWER SCHEDULE

JN the rate slope flattened out until a load was reached that made the average sale price of 4 mills. From this point on the surplus power rate remained at 4 mills.

Surplus power agreements could be made and canceled by either party at any time it proved acceptable to the other party. If, however, any agreement was made for a long period, it was subject to cancelation on eighteen months' notice. This period of time being agreed upon as sufficient to enable the purchasing party to either build his own power plant extension or make other provision to secure the power. The industrial power supply was always protected by reason of the fact that he could at any time make application to the utility for a a supply of power under the regular wholesale tariffs. The utility could not refuse the supply under these last conditions. Dump power from the industrial to the utility required only 24 hours' notice of the amount to be dumped in order to make it acceptable.

Variations of 50 per cent above and below the estimated amounts were allowed.

The purchaser was required to pay for at least 50 per cent of the contracted power for any given day and pay emergency rates for all over 150 per cent of the daily contracts both in demand and energy.

Engineers of both the utility and the industrial plants after careful study of the provisions of the agreement believed that it was sufficiently flexible to permit unrestricted interchange and yet be profitable to both parties to the agreement.

There is one peculiar point in this agreement, however, to which attention should be called.

Usually when two companies agree to do business together the greater the volume of business the greater the profits. A glance at Fig. 13 shows that the agreement is decidedly contrary to this.

It will be noticed that at about $5^1/_2$ million kwhr per month the industrial profit becomes zero and at all loads between $1^1/_2$ million and $5^1/_2$ million kwhr the utility profit decreases.

This seems to indicate that the right to interchange and to secure emergency service is a valuable asset even if no use of the service is made. It also tends to justify the much debated contention of the utility companies on their "ready-to-serve" charge.

The Legal Aspect. There is one other very serious obstacle to interchange to which attention should be called, and that is the legal aspect.

When the interchange agreement just described was completed and in a form satisfactory to both engineers and attorneys it was placed before the Public Service Commission for examination and suggestion.

The Commission's engineers and rate men were very much interested and pleased with the manner in which the project had been handled. The proposed rates were closely examined and the unanimous opinion was expressed that so far as they could see there was nothing in either rate or agreement that was discriminatory or detrimental to the public interest, but right there the matter stopped. When asked what steps should be taken to get the Commission's formal approval before large capital investments were made it was found that there were no steps.

It was explained that the Commission could act only in case of a complaint. If such a complaint were made the Commission would hold a hearing on the case and then, if nothing detrimental to the public interest was found and the complainant failed to show discrimination, the Commission would issue its approval.

In the meantime a large investment, made in good faith by both parties to the agreement, would be seriously jeopardized.

It is believed that serious thought should be given by legislative bodies to this phase of the subject if industrial development along economic lines is to be encouraged.

Neither an industrial nor a utility would care to make investments ranging anywhere from $50,000 to $1,000,000 for equipment which they would be liable to find useless on their hands because of unfavorable decisions that could in no manner be foreseen.

In Conclusion. Agreements of the interchange type are of necessity subordinate to the main business objects of the participants and operate in what might be termed the edges or fringes of the field of industrial economics. Efforts to make economies in this field therefore require a broad knowledge of the elements which go to build up costs and an equally broad knowledge of economics. Any changes in operating methods must be carefully examined to avoid interference with the main business of the participants or with effective operations. It is hoped that some of the chief problems that will be encountered in this field and methods for attacking their solution have been pointed out.

Author's Closure

It is very gratifying to review the thoughtful and thorough discussions presented by so many capable engineers. It is unfortunate that the discussions indicate that we are still far from a meeting of minds which will result in a united cooperative effort along the line indicated in my paper. Evidently, individual experience leads to very different conclusions. Professor Christie agrees with my condemnation of the utilities' sales campaign as forcefully as Mr. Dyckerhoff praises their cooperative attitude.

Mr. Dreyfus, Mr. Dyckerhoff, Mr. Hirshfeld, Mr. Irwin, Mr. Muir, and Mr. Powell all emphasize the possible disadvantages of cooperating with *individual* industrial plants. The reasons advanced are beyond dispute; a plant may suspend operations, change its load, its load factor, or its process. Many plants do all these things. Yet utilities find it profitable to sell electrical energy to them. The same diversity factor, statistical effect, and business stability can be obtained by *numerous* cooperative or purchase agreements. The study of the individual case is no study of the whole problem. I urge regional surveys and experimental published purchase rates by utilities as the only proper approach.

Mr. Dyckerhoff, Mr. Muir, and Mr. Orrok point out the commercial and technical difficulties existing by reason of the small capacity and great number of industrial plants. Each of the great majority of consumers has a one-kw demand and a minimum monthly bill of one dollar. With this in mind, it is difficult to believe that the economic and technical problems cannot be solved if we attempt to include in our systems industrial generating stations averaging 450 kw. The careful engineer will be content to say that the technique is not yet developed.

Mr. Hirshfeld presents impressive evidence against the superficial attractiveness of by-product electricity when he gives reasons why central-heating plants do not usually employ back-pressure turbines. I think the picture would be more complete and less conclusive if he had added these three: old franchises do not always permit electrical generation; the unsolved problem of allocating cost between exhaust steam and by-product electricity may bring the utility into an uncomfortable position before rate-supervising commissions; and only very recently has the art of feedwater treatment permitted the construction of extremely efficient high-pressure central-heating plants.

In the first part of his discussion, Mr. Irwin proves that surplus industrial energy is of no value to the utility. In the latter part he complains because he gets none from Deepwater Station. He gets very little because the contract price is so low that the industrial company has every reason to invent new uses for this energy and has no incentive for selling. Mr. Irwin's statistics are correct but do not represent the actual steam and electric loads of the industrial plant. The industrial company operates its own boilers and turbines from time to time to secure the lowest possible power cost. The terms of the Deepwater contract and fluctuations in oil and coal prices occasionally make such operation profitable. It is conceded that steam and electric loads are not synchronized. But I think Mr. Irwin will admit that the stability of the industrial load throughout the depression was better than that of his system load. It leads one to believe that supplying steam and electricity to an industrial plant does not make the utility business more hazardous.

It is gratifying to hear from Messrs. Cather, Irwin, and Powell that new cooperative arrangements of some magnitude may be expected with an improvement in business. Such progress should encourage us all to real cooperative effort in reducing both utility and industrial-power cost. Mr. Hobbs shows us the true cooperative method of attack.

A New Method of Investigating Performance of Bearing Metals

By JOHN R. CONNELLY,[1] BETHLEHEM, PA.

Any method to be used to investigate the performance of bearing metals should reproduce wear under service conditions and in addition it should show variation in wear with unit pressure. Various methods have been proposed in the past, one of which involves an accelerated test using an abrasive between two surfaces rubbing across each other, and another utilizes two tangent cylinders rotating relatively to each other and making contact along their common element. The first method does not reproduce wear under service conditions, while the second results in a constant contact area so that in order to test a metal completely many runs must be made at various unit pressures. The new method described in this paper reproduces wear under service conditions and gives variation in rate of wear with unit pressure.

THE essential elements of the new test are a specimen of bearing metal with one side machined to a plane surface and a steel cylinder rotating in a bath of lubricant (see Fig. 1). A constant force holds the specimen of bearing metal with its plane side tangent to the cylinder.

As the cylinder rotates, a depression is worn in the specimen of bearing metal (see Fig. 2). With a constant force holding the surfaces together, this increase in contact area gives a decrease in unit pressure and the test continues until there is no measurable increase in the depth of the depression.

The characteristics of the new test are:

(*a*) The conditions existing in an actual bearing are reproduced as to lubrication, metals used, physical proportions, and wear. Wear in the presence of a lubricant results from high unit pressures

(*b*) The validity of the test data is independent of any assumption that data on abrasion are directly convertible to data on wear

(*c*) The feature of varying area of contact gives rate of wear for a wide range of unit pressures. An investigator using a test involving a constant area of specimen must necessarily make a great number of runs, each at a different unit pressure

(*d*) The data obtained are directly usable for design purposes.

---

[1] Instructor, Dept. of Mechanical Engineering, Lehigh University. Jun. A.S.M.E. Mr. Connelly was graduated in 1927 with the degree of B.S. in mechanical engineering from the University of Illinois and received the degree of M.S. from the same institution in 1929. During 1927 and 1928 he was with the Illinois Power & Light Corporation in the capacity of junior mechanical engineer and in 1929 as special research assistant for the Engineering Experiment Station at the University of Illinois. In the fall of 1929 Mr. Connelly joined the staff of Lehigh University and received the degree of M.A. in 1934.

Contributed by the Iron and Steel Division and presented at the Annual Meeting, New York, N. Y., December 4 to 8, 1933, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

ANALYSIS OF A MACHINE BEARING

No matter how careful and painstaking the manufacture and assembly of a sliding-contact bearing may be, it is practically impossible to have the mating surfaces absolutely aligned so that the load shall be evenly distributed over the surface intended to carry it. As a result, the actual surface in contact is reduced so that high unit pressures exist at points on the surface.

The first stage in the life of a sliding-contact bearing is a period during which the surface in contact wears away until a "fit" is obtained. That is, as the bearing metal wears, the area of surface in contact gradually increases. This permits the load to spread itself over a greater area with a reduction in unit pressure. Wear continues until the maximum unit pressure existing in the bearing is insufficient to rupture the oil film. Such a condition completes the first stage. The extent of the bearing area in actual contact and the corresponding unit pressure when wear ceases vary with the temperature and properties of the lubricant.

The second stage in the life of a sliding-contact bearing consists of relative motion between surfaces separated by a lubricant. If this condition persists indefinitely, the only necessary

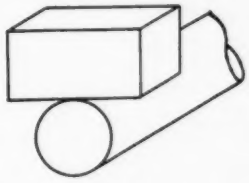


FIG. 1

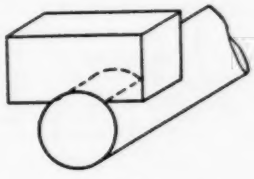


FIG. 2

quality of a bearing metal would be an ability to attract and retain lubricant on its surface. However, all bearings stop occasionally with consequent disturbance of lubrication equilibrium and restarting results in wear.

PRESENT DESIGN OF BEARINGS

In this paper the term bearing metal shall apply to the part on which the major portion of wear is to take place.

At present the design of bearings considers (*a*) mechanical strength of both journal and bearing, (*b*) heat dissipation, and (*c*) wear.

An ideal bearing metal should (*a*) have sufficient mechanical strength to support the load without plastic deformation, (*b*) retain a lubricant on its surface, and (*c*) have a maximum resistance to wear without causing undue wear on the mating metal.

At present, the consideration of wear is met by using a combination of unit working pressure and lubricant that past experience has shown will not give excessive wear during continuous operation. Very little information is available on rate of wear during running-in or restarting.

APPLICATION OF NEW METHOD

The new method will furnish data that will make possible a far more accurate consideration of wear. This will be accom-

plished by giving values of ultimate bearing pressure for a lubricated bearing, and relative rate of wear during running-in and restarting.

The depth of the worn volume and the revolutions of the cylinder are measured periodically. There may be computed from these readings the total travel of a point on the surface of the cylinder and the volume worn away and corresponding unit pressure. The equations to accomplish this are derived in the Appendix.

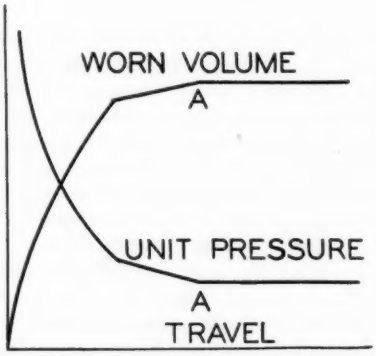
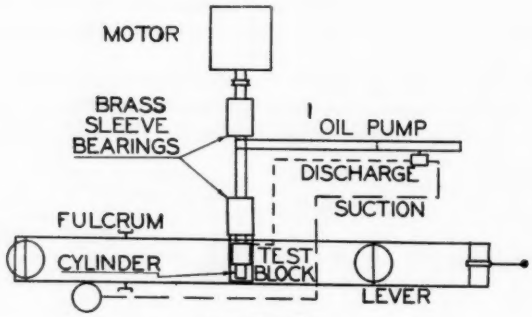
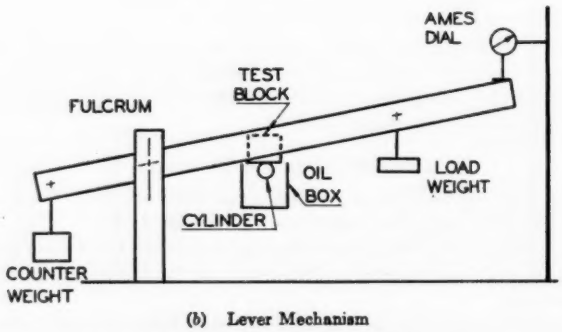


FIG. 3 TYPICAL CURVES OF WORN VOLUME AND UNIT PRESSURE VS. TRAVEL



(a) Plan View



(b) Lever Mechanism

FIG. 4 EXPERIMENTAL TESTING MACHINE

The curves of volume worn away and unit pressure against linear feet of travel are shown in Fig. 3. Wear ceases at *A* and any further running is to insure accuracy. To aid in comparing the relative performance of various metals, the test information may be simplified to one curve. This curve is obtained by plotting rate of wear against unit bearing pressure. To obtain the values of rate of wear we must measure worn volume per unit of travel for a variety of unit pressures. This work will be materially shortened if a mathematical expression can be found that will represent the data. The worn volume per unit of travel may then be obtained by taking the first derivative of the expression of worn volume with respect to travel or *d volume/d travel.*

The curve showing rate of wear for a given metal will furnish information as to the behavior of that metal during running-in and restarting. The unit pressure at which wear ceases will be the ultimate pressure for the conditions of the test. The test information so obtained is of course applicable only to the same metals, lubricant, temperatures, ratio of length to diameter, and rubbing speed. With this more definite knowledge at his command, the designer can select the most desirable combination of metals, ubricant, and physical proportions.

DESCRIPTION OF EXPERIMENTAL TESTING MACHINE

The mechanism, shown diagrammatically in Fig. 4, consists essentially of a rigid lever supported at the fulcrum and carrying the test block which in turn bears on the cylinder. This arrangement permits motion of the block of bearing metal only in a circle about the fulcrum as a center. The angle moved through is so small that for all practical purposes the block of bearing metal may be said to have straight-line motion.

In designing and constructing the lever care was exercised to meet the several limitations imposed. The fulcrum of the lever

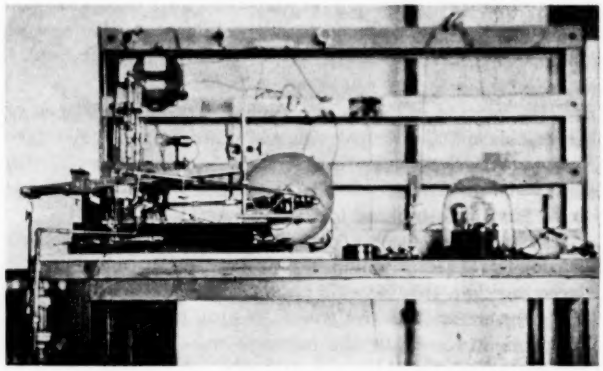


FIG. 5 COMPLETED EXPERIMENTAL TESTING MACHINE

was placed in a horizontal plane tangent to the top of the test cylinder so that the moment of the friction force would not be a disturbing element. A fulcrum consisting of hardened steel points working in conical recesses was employed so that the lever would not move in any way except about its own fulcrum. A counterbalance was provided to prevent any pressure between the test pieces due to the weight of any of the parts. A calibrated weight was hung from steel points on the lever to supply the definite known pressure between the cylinder and the block of bearing metal. The block of bearing metal was clamped securely to a holder plate which is in turn bolted to the lever with provision for adjustment in any direction between the holder plate and the lever. Fig. 5 shows a photograph of the machine as set up.

The lever serves the further purpose of magnifying the depth of worn volume, thereby improving the accuracy of reading. Various methods have been used to measure the movement of the lever as wear progressed. A 0.0001-in. Ames dial gage was found valuable for studying the early stages when the lever movement is relatively fast. For a complete test, however, a 0.0001-in. micrometer screw in conjunction with a telephone head set or a grid-glow tube is a desirable combination.

The cylinder as used in this experimental machine consists of a length of $^1/_4$-in. steel rod rigidly supported by two brass sleeve bearings mounted on the same heavy cast-iron base used for the

lever fulcrum support. One end of the steel rod was connected through a flexible coupling to a 1/4-hp, 1750-rpm, ball-bearing motor. The other end of the steel rod, which constitutes the test cylinder or test journal proper, was prepared by grinding to as near a true right circular cylinder as is easily possible. The maximum variation in diameter at any cross-section was of the order of 0.0001 in.

To lubricate the test pieces a small metal box kept full of oil at a constant level was so placed that the test journal, which enters it through a felt packing, and the bearing block are completely flooded with oil. A gear pump was provided to recirculate the oil continually.

The temperature of the lubricant supplied to the bearing is measured by means of a copper-constantan thermocouple installed in a glass tube filled with oil. The tube rests in the box of lubricant; the glass tube electrically insulates the thermocouple from its metal surroundings.

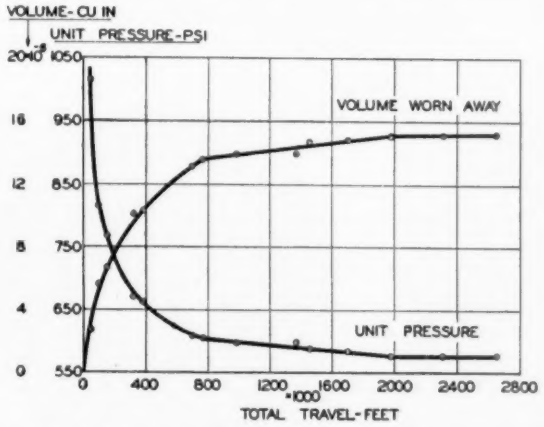


FIG. 6(a) TEST CURVES OF WORN VOLUME AND UNIT PRESSURE VS. FEET OF TRAVEL
(Meta IB; Oil A.)

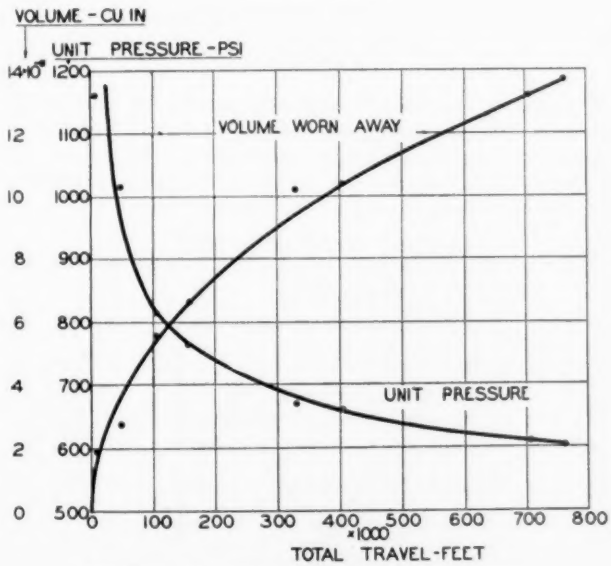


FIG. 6(b) PORTION OF TEST CURVES ENLARGED
[Curves of Fig. 6(a) from 0 to 800,000 ft.]

The temperature of the block of bearing metal is measured by means of copper and a constantan wire, each peened into opposite faces of the block near the test surface. In order to reduce to a minimum the dissipation of heat from the block, asbestos paper was placed between the block and its support.

The rpm of the cylinder is obtained by measuring the slip of the motor, using a mark on the shaft and a neon lamp.

ILLUSTRATIVE TEST

The following example will illustrate the use of the results obtained by this new method of testing.

A specimen of bearing metal is clamped in the holder described and adjusted tangent to the test journal with the aid of Prussian blue. It is not absolutely necessary that the bearing metal be exactly tangent to the cylinder provided it is approximately so. The projected area is measured at the completion of a run and if the bearing metal were not quite tangent at the beginning the projected area of the worn volume would be a trapezoid rather than a rectangle.

The time is recorded as the machine is started. At present no attempt is made to read the lever deflection at starting. When the load weight is first applied, conditions will be very unstable, probably due to a slight amount of plastic deformation. About 30 seconds after starting it is possible to obtain readings of lever deflections and these are recorded throughout the test. To furnish an idea of the length of a run it may be stated that the total time varies from two weeks to a month of continuous operation. During this period a point on the surface of the cylinder travels a distance of the order of 3,000,000 ft. After the test is complete the lever deflection readings are corrected to agree with the projected area of worn volume obtained by measurement. Other readings taken are rpm of cylinder, temperature of bearing metal, temperature of lubricant, temperature of the room, and power consumed by the driving motor.

The tabular values of data and results appear in the Appendix. Curves of total feet of travel as abscissas and unit pressure and total volume worn away as ordinates are shown in Fig. 6(*a*). The general shape of the curves shows quite definitely that a discontinuity occurs at about 800,000 ft of travel. Beyond this point wear takes place at a greatly reduced rate. After about 2,000,000 ft of travel, wear seems to cease altogether even though the run was continued to about 2,700,000 ft of travel. The ready location of these different phases of wear is an advantage of this type of test.

The curve of worn volume from 800,000 ft of travel on to 1,800,000 ft of travel seems to be a straight line. The part of the volume curve from 0 to 800,000 ft of travel is drawn to an enlarged scale on Fig. 6(*b*). After some experimentation with a log curve, a parabola, and an ellipse, the ellipse was selected as the curve most closely representing the data. The ellipse selected was of the form

$$y^2 = B^2/A^2(2Ax - x^2)$$

where y = volume worn away in cu in.
x = travel in feet
B = the semi minor axis = 0.00001840 in.
A = the semi major axis = 2,280,000 ft travel

Substituting

$$volume^2 = \frac{0.0000184^2}{2{,}280{,}000^2}\,(2 \times 2{,}280{,}000 \times travel - travel^2)$$

As outlined earlier in this paper the slope of this volume curve at any point represents rate of wear. The first derivative of the ellipse is

$$\frac{dy}{dx} = \frac{B^2}{A^2}\left(\frac{A - x}{y}\right)$$

or

$$\frac{d(volume)}{d(travel)} = \frac{0.00001840^2}{2{,}280{,}000^2}\left(\frac{2{,}280{,}000 - travel}{volume}\right)$$

This derived curve, evaluated for ten increments of travel from 100,000 ft of travel on, gives Fig. 7.

This curve of $d(volume)/d(travel)$ plotted against the corresponding unit pressures graphically portrays the relation between rate of wear and bearing pressure for this particular metal under the conditions of the test.

The point at which wear ceases is simply determined by the pressure curve Fig. 6(*a*) dropping to a value where an oil film may be maintained. This value may be appropriately called

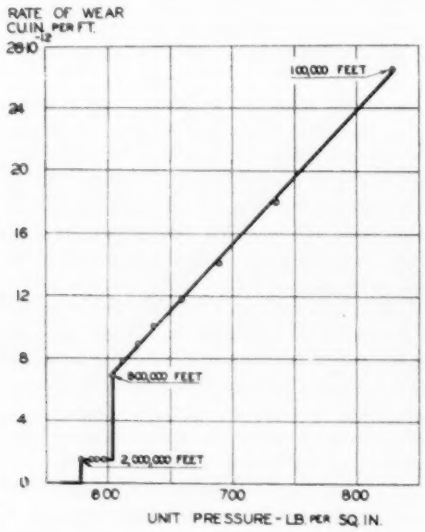


FIG. 7 RATE OF WEAR VS. UNIT PRESSURE
[Derived from curves of Fig. 6(*a*); see Table 2, Appendix.]

the ultimate bearing pressure and used for design with a factor of safety.

The pressure at which an oil film may be maintained seems to be affected by a number of variables among which are: the lubricant, the temperature of the lubricant, and the ratio L/d. The bearing metal is subject to at least two known variations: that of composition and that of crystal size which is dependent upon conditions of casting and cooling.

Fig. 8 shows two photomicrographs of the bearing-metal surface; (*a*) before the test and (*b*) after. The metal surface after the test was completed shows definitely that the crystal structure is that of a bearing in service. It is anticipated that the wear qualities of a metal will be influenced by the size and distribution of the supporting crystal.

GENERAL USE OF TEST

There are several problems connected with the use of bearing metals concerning which much needed information may be obtained by the test described:

(*a*) The allowable working pressure can be determined by the ultimate bearing pressure in conjunction with a factor of safety

(*b*) The amount of wear during starting of an actual bearing will be indicated by the relative position of the rate-of-wear curve

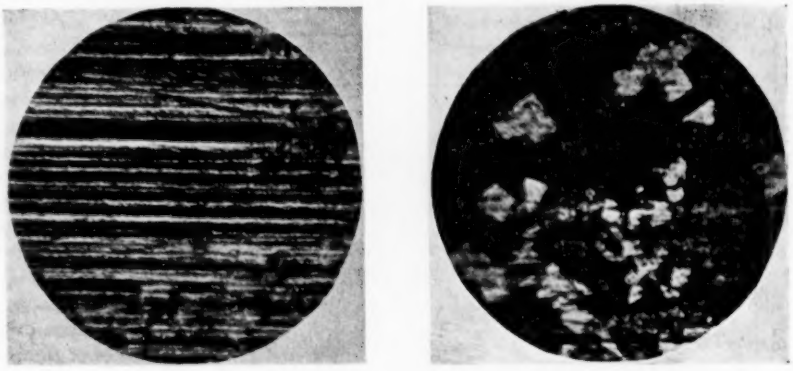
(*c*) Interrelation of the variables, for example the optimum L/d ratio, may vary with both the lubricant and the bearing metal.

The intended type of service for a given bearing will determine the relative emphasis to be placed on items *a* and *b* during design.

In order to obtain the data on which to base these selections, each one of the variables must be investigated separately.

The field of investigation into the performance of bearing metals, to which this method may be applied particularly, is broadly divided into three phases:

1. Using a certain bearing metal as a standard of comparison, the wear-reducing properties of various lubricants may be determined, together with the effect of variation in viscosity of a given lubricant
2. Using a certain lubricant as a standard of comparison, the effect of composition, crystal size, and physical proportions may be investigated
3. Using the representative lubricants and the representative metals, the interrelation of the variables may be determined.



(*a*) Before Testing (*b*) After Testing

FIG. 8 PHOTOMICROGRAPHS OF BEARING-METAL SURFACE

Appendix

The mathematical relationships (*a*) between the variable of projected area and readings taken during a test, and (*b*) between volume worn away and readings taken during a test are derived as follows. The use of these expressions considerably reduces the labor of working up results from data obtained.

Let P_x = any position of the face of the block
n = CD (Fig. 9), the diametral travel of P_x from its original tangent position

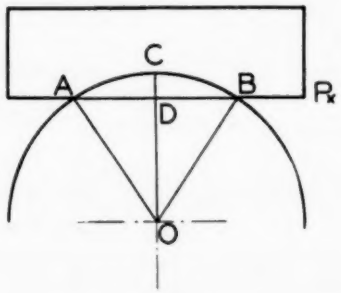


FIG. 9

w = AB (Fig. 9) the width of worn volume
L = the dimension of worn volume along the axis of the cylinder
d = the diameter of the cylinder
v = the volume worn away.

(*a*) Find w in terms of n and d.

$$w = 2AD = 2(nd - n^2)^{1/2} \quad [1]$$

Rearranging

$$w = 2d\left[\frac{n}{d} - \left(\frac{n}{d}\right)^2\right]^{1/2} = 2dK_1 \quad [2]$$

where

$$K_1 = \left[\frac{n}{d} - \left(\frac{n}{d}\right)^2\right]^{1/2} \quad [3]$$

and the projected area

$$wL = 2LdK_1 \quad [4]$$

(*b*) Find v in terms of n and d.

$$v = Lx \text{ (the area of the segment } ABCD) \quad [5]$$

Substituting

$$v = Ld^2\left\{{}^1/_4 \text{ arc sin } 2\left[\frac{n}{d} - \left(\frac{n}{d}\right)^2\right]^{1/2} - \left({}^1/_2 - \frac{n}{d}\right)\left[\frac{n}{d} - \left(\frac{n}{d}\right)^2\right]^{1/2}\right\} \quad [6]$$

and letting

$$K_2 = {}^1/_4 \text{ arc sin } 2\left[\frac{n}{d} - \left(\frac{n}{d}\right)^2\right]^{1/2} - \left({}^1/_2 - \frac{n}{d}\right)\left[\frac{n}{d} - \left(\frac{n}{d}\right)^2\right]^{\frac{1}{2}} \quad [7]$$

the worn volume

$$v = Ld^2K_2 \quad [8]$$

The work of calculating the results of any test is very simple, requiring only a graph of K_1 and K_2 plotted against the ratio of n/d for values of n/d from 0 to 0.5. Such a set of curves is shown in Fig. 10. The curve for K_1 is the arc of a circle thus giving a check on the accuracy of those computations.

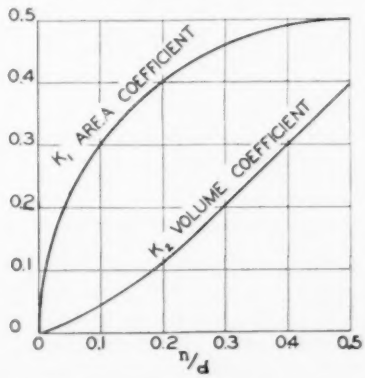


FIG. 10 AREA AND VOLUME COEFFICIENTS

Starting with the test data consisting of values of n for various times during the test, the values of K_1 and K_2 may be read off the chart and substituted in the appropriate equations. The resulting values of area and worn volume may then be used as indicated in the paper.

TABLE 2 DATA FOR RATE OF WEAR CURVE, FIG. 7

Travel, ft	Volume, cu in. × 10^3	Unit pres., lb/in.^{2}	Slope, in.^{3}/ft × 10^{12}
100,000	0.539	830	26.4
200,000	0.753	735	18.0
300,000	0.913	690	14.1
400,000	1.040	660	11.8
500,000	1.145	638	10.1
600,000	1.244	625	08.8
700,000	1.325	612	07.8
800,000	1.395	602	06.9

TABLE 1 DATA AND RESULT SHEET

Test of B metal

Average rpm = 1759; L = 0.375 in.; d = 0.492 in.; Bearing load 10.78 lb

A Time	B Deflection × 10^3	C n × 10^4	D (n/d) × 10^4	E K_1 × 100	F w × 100	G wL × 100	H p, lb/in.^{2}	J P, 1000 (lb/ft^2)	M Travel, 1000 ft	N K_2 × 10^5	P v, in.^{3} × 10^4
0.017	0.25	0.73	1.49	1.220	1.20	0.45	2390	344.0	0.23	0.28	0.25
0.033	0.40	1.18	2.40	1.575	1.55	0.58	1855	267.0	0.46	0.53	0.48
0.100	0.70	2.08	4.24	2.075	2.04	0.76	1410	203.0	1.38	1.16	1.05
0.167	0.90	2.65	5.39	2.325	2.29	0.86	1250	180.0	2.30	1.64	1.49
0.283	1.05	3.09	6.29	2.485	2.46	0.92	1170	168.0	3.91	2.00	1.81
0.533	1.10	3.24	6.60	2.515	2.48	0.93	1160	167.0	7.35	2.10	1.90
3.500	1.40	4.12	8.36	2.865	2.82	1.06	1015	146.0	48.3	3.00	2.71
7.633	2.25	6.62	13.40	3.655	3.51	1.32	816	118.0	105.5	6.20	5.61
11.416	2.45	7.22	14.70	3.825	3.76	1.41	764	110.0	158.0	7.36	6.67
24.066	3.20	9.42	19.20	4.360	4.29	1.61	669	96.2	331.0	11.20	10.20
29.10	3.25	9.51	19.50	4.410	4.34	1.63	660	95.0	403.0	11.50	10.40
51.13	3.85	11.30	23.00	4.790	4.71	1.77	610	86.6	705.0	14.64	13.20
55.40	3.95	11.60	23.60	4.850	4.77	1.79	602	86.6	764.0	15.10	13.70
71.08	4.00	11.70	23.80	4.890	4.81	1.80	597	86.0	980.0	15.50	14.00
96.11	4.00	11.70	23.80	4.890	4.81	1.80	597	86.0	1330.0	15.50	14.00
105.5	4.15	12.20	24.80	4.980	4.89	1.83	588	84.6	1458.0	16.40	14.80
120.3	4.20	12.30	25.00	5.010	4.94	1.85	582	83.7	1695.0	16.50	14.90
143.5	4.25	12.50	25.40	5.050	4.98	1.87	575	82.6	1980.0	16.90	15.30
168.2	4.25	12.50	25.40	5.050	4.98	1.87	575	82.6	2320.0	16.90	15.30
192.3	4.25	12.50	25.40	5.050	4.98	1.87	575	82.6	2655.0	16.90	15.30

EXPLANATION OF ITEMS IN TABLE 1

Col.	Explanation
A	Time elapsed
B	Deflection measured at end of lever
C	Col. B/3.41: 3.41 being the constant for lever used
D	Col. C/0.492
E	Read from chart of K_1 vs. (n/d), Fig. 10
F	$w = 2LdK_1$
G	wL
H	10.78 lb/Col. G
J	Col. H × 144
M	$\frac{0.492 \times 1759 \times 60 \text{ (hr elapsed)}}{12}$ Since the rpm of the motor was constant within the limits of experimental error the total travel of a point on the surface of the test cylinder is directly proportional to the time elapsed so that "M" follows directly from Col. A
N	Read from chart of K_2 vs. (n/d)
P	$v = Ld^2K_2$

Classification of Drying, Including Graphical Analysis of Air Drying as Developed Abroad

By A. WEISSELBERG,[1] JERSEY CITY, N. J., CHAS. W. THOMAS,[2] NEW YORK, N. Y., AND T. R. OLIVE,[3] NEW YORK, N. Y.

In order that the art of drying, or in other words the process of removing liquid from a mixture, may become a field of true engineering endeavor, it must be freed from the existing confusion with regard to classification and terms, and the analytical approach to the problem must be facilitated.

In this paper the authors classify the liquids as to kind, nature, and state, enumerate the means available for their removal, and discuss briefly the limitations of the various drying methods with regard to the material being processed and the results obtained.

A graphical method of analyzing air drying, the most widely used drying process, is described by the authors, and by means of this analysis, terms which are now confusing are defined and factors governing the performance of drying equipment are explained.

DRYING is usually defined as the partial or complete removal of evaporable liquids from a mixture of the liquid with solids, other liquids, or gases. To be more specific, the definition should include the words "by thermal means," i.e., by supplying or removing heat. Thus we differentiate it from the other category of moisture removal, wherein this end is accomplished by mechanical means and is commonly designated as dehydration. Desiccation has been suggested as an appropriate technical term that would embrace both methods of moisture removal.

[1] Consulting Mechanical Engineer. Jun. A.S.M.E. Mr. Weisselberg received his engineering diploma from the Technical University of Vienna, Austria. He came to the United States in 1923 and, after a diversified practise in various engineering fields, established himself in 1930 as a consulting engineer. He is the author of a number of articles published in recent years on the subject of air-conditioning and drying and has modified and introduced in this country the Mollier water-air mixture chart reproduced in this paper.

[2] Consulting Engineer. Mem. A.S.M.E. Mr. Thomas was graduated from the Stevens Institute of Technology with the degree of mechanical engineer in 1884 and after graduation became assistant superintendent of the Dixon Pencil Works, Jersey City, N. J. He has had a wide experience in the development, design, and construction of machinery for inventors and has lectured in machine design at the Newark Evening Technical School and at Columbia University. Mr. Thomas retired recently from the staff of the university as professor of machine design and has entered into consulting practise, specializing in the dehydration of food products. He is chairman of the Drying Committee of the Process Division, A.S.M.E.

[3] Associate Editor, *Chemical and Metallurgical Engineering*. Jun. A.S.M.E. Mr. Olive was graduated in 1923 with the degree of A.B. from Harvard University, where he specialized in the field of engineering sciences, with special reference to mechanical and chemical engineering. After graduation he spent a number of years in the employ of several chemical companies, including the National Aniline & Chemical Co. and the DuPont Rayon Co. Since 1927 he has been a member of the editorial staff of *Chemical & Metallurgical Engineering*, of which he is now associate editor. He is the author of numerous articles on various phases of chemical engineering, co-author of the chapter on process control in Perry's Chemical Engineers' Handbook, and secretary of the Process Industries Division, A.S.M.E.

Contributed by the Process Industries Division and presented at the Metropolitan Section Meeting, New York, N. Y., October 18, 1932, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Kind, Nature, and State of Liquid

As to liquids, we distinguish between two groups: (*a*) pure liquids, and (*b*) solutions. Pure liquids are classified as (*a*1) water and (*a*2) liquids other than water, commonly known as solvents. Liquids which undergo a change in state by chemical action do not belong here, even though the term "drying" is applied to the process which accomplishes this change in state. Oils which harden by oxidation exemplify such liquids, and the use of the term "drying" in this case is altogether out of place.

Solutions are either (*b*1) crystalloidal or (*b*2) colloidal. The former are readily diffusible and are also characterized by the fact that, at the same temperature, their vapor pressures are lower than those of the pure liquids of which they are composed. Colloidal solutions, on the other hand, diffuse with difficulty, but their vapor pressures are hardly affected by the presence of the colloids. The difference in these properties will, obviously, influence the removal of moisture, as will later be seen.

It is assumed that when we dry or dehydrate, the state of the material from which the liquid is removed does not change, or, in other words, we deal with the removal of free moisture as distinguished from the moisture of crystallization and that of decrepitation. The last is really free moisture mechanically entrapped within certain crystals. Its removal precludes the retention of the crystal form and the crackling noise by which the removal is accompanied warns of the approaching danger of complete disintegration into powder form into which the crystals fall when the moisture of crystallization is driven out. The removal of the moisture of crystallization is called calcination and is beyond the scope of this paper.

The free moisture with which we are concerned may be found present in one of the following four states: surface moisture, capillary moisture, cellular moisture, and as vapor.

Means of Removing Moisture

Evidently, removal of moisture by mechanical means is confined to extraction of moisture without changing the state of the moisture, since any change of state implies that heat must be supplied or removed. This limits dehydration to surface and capillary moisture, while moisture in any of the four states mentioned may be removed by drying. Thus arises the question as to when one method should be used in preference to the other as far as the removal of surface or capillary moisture is concerned. Aside from the fact that the removal of large percentages of moisture is always accomplished with greater efficiency by dehydration, the choice of the method may also be governed by the desired result. Thus, if the liquid carries salts in solution, the presence of which is undesirable in the final product, dehydration to the lowest possible moisture content must be resorted to. On the other hand, just the opposite result may be desired, since, for instance, the salts in solution may have a nutritive value, in which case preference will be given to drying.

Thus, either method may be used alone, or both in combination, depending upon the end to be accomplished. A knowledge of both methods is required, therefore, when the problem of moisture removal is to be dealt with.

Dehydration is accomplished by subjecting the mixture to pressure and providing a resistance for the retention of one of the ingredients. In this operation we differentiate between (*a*) pressure applied directly, compressing the mixture to a smaller volume, as in pressing or squeezing, and (*b*) pressure applied indirectly, without compressing the mixture to a smaller volume,

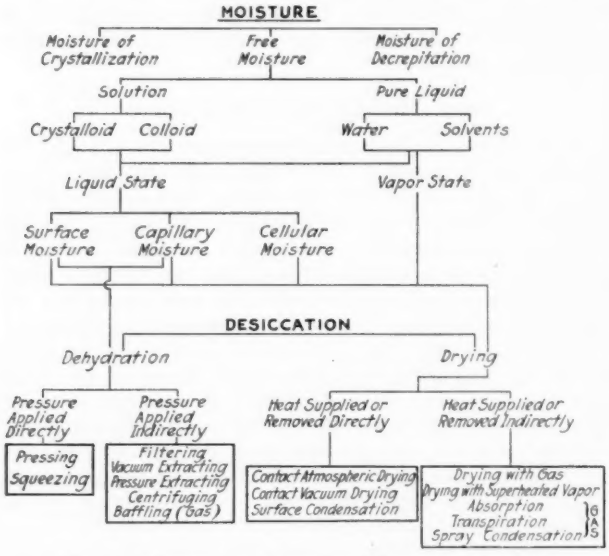


FIG. 1 REMOVAL OF MOISTURE FROM AIR OR GAS

as in filtering, vacuum extracting, centrifuging, pressure extracting, and baffling, the latter applying to gases.

As already stated, drying deals with the removal of moisture by thermal means. Heat is supplied or removed in order to change the liquid from one state to another, thus accomplishing separation. If the moisture is in vapor state, as is the case in drying a gas, the vapor must be reduced to the liquid state by removing the latent heat of vaporization. The reverse is true if the original state of the moisture was liquid.

With respect to the manner in which the heat is applied, we distinguish between (*a*) heat supplied or removed directly by a medium other than the one carrying the moisture away (contact drying), and (*b*) heat supplied or removed indirectly by the same medium which carries the moisture away (air drying).

With method (*a*) of applying heat, if the moisture must be removed at low temperature, or quickly, or without exposure to the oxidizing action of the atmosphere, vacuum drying will be resorted to as distinguished from atmospheric drying in which the vapors are discharged into the surrounding atmosphere. In this category there also belongs the drying of a gas by condensing the vapors by passing the gas over cooled surfaces.

With method (*b*) of applying heat the use of air as a drying medium is the most common practise. If the product to be dried is subject to oxidation, an inert gas or products of combustion will be used instead of air. The ability of superheated steam to evaporate moisture also falls into this group. On the face of it, its use in connection with drying at high temperatures (above 212 F) looks rather advantageous from the point of view of thermal efficiency. Actually, the efficiency would not be higher than with air drying at sufficiently high temperatures and saturation, inasmuch as, with the approach of the boiling temperature, the heat consumed in heating the air becomes negligible. On the other hand, the power consumption would be materially greater with superheated-steam drying, since a greater volume would have to be moved per pound of moisture removed than with air drying. This, together with other difficulties which this practise involves, makes the use of superheated steam for drying of little practical value, except when other conditions call for it.

The removal of vapor moisture from air or gas by indirect removal of heat may be accomplished by absorption through chemical reagents or by adsorption through adsorbent materials, by transpiration through hygroscopic fibrous material, or by precipitation through spray condensation. Fig. 1 shows this at-

TABLE 1 FACTORS WHICH INFLUENCE DRYING CONDITIONS AND THEIR LIMITATIONS

Factors	Limitations			
	Physical	Chemical	Biological	Market Requirements
Final moisture content.	Optimum for maximum strength. Not too low if soaking ability be not destroyed. Low if grindability is a factor.	Not too low to avoid discoloration. Low if by reabsorption of moisture product will melt.	Below hygroscopic point if growth of bacteria be prevented.	Low to lower freight cost. Not lower than regain at average air condition. ¼ of hygroscopic point if powdered or granular, as these tend to pack in storage. Very low if bitter aromatics must be driven out.
Temperature.	Low if strength is affected. Low if there is danger of melting. Low if change in shape is objectionable.	Low if danger of disintegration. Low if danger of decomposing acids. Low if danger of chemical discoloration. Low if coagulation of albumen is to be avoided to retain solubility. Low if sublimation of valuable volatiles be prevented.	Low if vitamin A and C should be retained. High if germs or bacteria be destroyed.	High to improve taste by production of desirable aromas.
Drying time.	Short if soaking ability be high. Long if there is danger of distortion, case - hardening, cracking.	Short if the damaging effect of high temperatures as listed above is to be offset.	Short if damaging effect of high temperature on vitamins be offset. Short to prevent growth of bacteria if present. Short to conserve aromatic oils if present.	Long if desired uniformity is not otherwise obtainable.
Drying medium.	Cleaned if impurities in drying medium damage the surface of the dried product.	Non-oxidizing if there is danger of oxidation.	Non-oxidizing to prevent promotion of destruction of vitamins A and C. Cleansed and filtered if danger of contamination from bacteria in air.	.....
Handling.	Careful to prevent objectionable physical damage. Without tension to prevent objectionable lowering of strength by not allowing for shrinkage.	Preventing contact with materials that would cause objectionable chemical contamination.	.....	Careful if appearance is stressed.

tempted classification in chart form. Obviously it cannot claim perfection and criticism is invited.

It appears desirable to carry the classification further to the factors which influence the drying conditions with respect to certain limiting requirements. These factors are: (1) final moisture content, (2) temperature, (3) drying time, (4) drying medium, and (5) the way the product is being handled in the course of moisture removal. The limitations which may have to be considered in connection with these factors may be grouped as follows: (*a*) physical limitations, (*b*) chemical limitations, (*c*) biological limitations, and (*d*) marketing-requirements limitations. This classification will cover almost any condition that may have to be considered as shown in Table 1, for which completeness cannot be claimed.

Definitions of Terms

It is of greater importance, however, to discuss the definitions of the principal terms used in connection with drying, about some of which there seems to exist considerable confusion.[4] These are as follows:

Moisture Content. The desirability of expressing the moisture content in per cent by weight of the dry material is often still not fully realized. Moisture contents are still very often given on a wet basis and sometimes the basis is not given at all. Expressed on a dry basis a change in percentage tells directly the change in moisture by weight per pound of dry material. It may be well, however, to use a shorter expression such as "dry-basis moisture" instead of "moisture content on a dry basis."

Capacity. If M designates the quantity of dry material expressed in any unit of measure (lb, yd, etc.) that the drying equipment holds at one time, and τ the time the material must remain in the drier to dry to the required moisture content, then M/τ is the "production capacity" per unit length of time. M is the holding capacity, and τ the drying time. For any one drier, the holding capacity is the same, but the drying time will vary in accordance with a number of factors.

Drying Time. If w represents the drying rate in pound of moisture removed per square foot of total drying area of material A per hour and Y_1, Y_2, the initial and final "dry-basis moisture" of the material, respectively, then the drying time τ equals $\frac{M\ (Y_1 - Y_2)}{wA}$; i.e., it is inversely proportional to the drying rate. As long as the wet product still has surface moisture, one may expect that the rate of removal of moisture will not differ from that of the evaporation of a film of liquid. The drying rate accordingly will be equal to the rate of evaporation. After the surface moisture is gone, however, the capillary moisture will begin to diffuse to the surface and two different cases become possible.

In the first, the possible diffusion rate per unit area is at first greater than the rate of evaporation for the same area. Then we can assume that the moisture will overflow the end of the capillaries at the surface and keep the surface partially or totally wet, depending upon the nature of the surface and the nature of the liquid. The drying rate will remain the same, or drop, according to the extent of wet-surface reduction. This will continue until the diffusion rate per unit area of total surface becomes equal to the drying rate or, in other words, until the diffusion rate per unit area of wet surface equals the evaporation rate. Sherwood[5] calls the period during which the rate of diffusion exceeds the rate of evaporation the zone of unsaturated surface drying, assuming that during this period the amount of wet surface gradually becomes less, which does not necessarily hold true for all cases. When the rate of diffusion falls below the rate of evaporation for the same unit area, the diffusion rate alone controls the rate of drying.

As a second case the diffusion rate per unit evaporation area is lower than the evaporation rate. The surface will then dry out unless the drying conditions are changed in such a manner as to make the evaporation rate equal to the diffusion rate. In other words, the drying rate must be equal to or lower than the diffusion rate.

Suppose that the drying conditions are so maintained that the evaporation rate per unit of wet area is greater than the diffusion rate for the same area. Then the surface will dry and the plane of evaporation will retreat somewhat from the surface. With the same drying conditions the rate of evaporation in this plane will be lower because of the much increased resistance to heat transfer through the dry layer. On the other hand, the rate of diffusion for this plane is greater than for the surface. Hence, the plane of evaporation would not have to retreat very far before subsequent stages of equilibrium are reached. The result, however, will be the undesirable condition of a dried surface enveloping the wet interior and a considerable drop in drying rate. In general, then, depending upon the initial and final moisture content, the average drying rate and, in turn, the drying time will be a function of either the evaporation rate or the diffusion rate, or both.

Evaporation. Evaporation is essentially a heat-consuming process. In terms of sensible heat transfer, the rate of evaporation in pounds of moisture per square foot per hour is expressed by the general equation $\frac{U(t_a - t_l)}{h_v - h_l}$ in which U stands for the overall heat-transfer coefficient in Btu per square foot per hour per degree F, $(t_a - t_l)$ for the temperature difference between heating medium and liquid to be evaporated, and $(h_v - h_l)$ for the difference in heat content between vapor and liquid, i.e., the latent heat of vaporization.

We may also express the rate of evaporation in terms of vapor heat transfer, by the equation $\frac{L(p_l - p_v)}{h_v - h_l}$ in which L, the vapor, heat-transfer coefficient, stands for Btu in the vapor diffused per inch of mercury per square foot per hour and p_l and p_v the vapor pressure of the liquid at temperature t_l and the partial pressure of the vapor, respectively. Substituting K for $\frac{L}{h_v - h_l}$, the evaporation rate can also be set equal to $K(p_l - p_v)$, K being the mass transfer coefficient in pounds per square foot per hour per inch of mercury pressure difference.

Another way of expressing the rate of evaporation in case the heat for evaporation is supplied only by the air was introduced by Lewis[6] under the assumption that $U(t_a - t_l) = r(X_s - X_a)k'$, r being the latent heat of vaporization at wet-bulb temperature, X_s and X_a, respectively, the absolute humidities at saturation and of the air, and k', a coefficient of vapor diffusion in pounds of moisture per square foot per hour per unit absolute humidity difference. On this assumption $k' = U/s$, s being the specific heat of the humid air (defined by Grosvenor as humid heat) since $(X_s - X_a)r = s(t_a - t_l)$. However Schmidt[7] has shown that $k' = \frac{U}{s} \times \frac{\kappa}{\alpha}$, wherein κ stands for the mass diffusivity constant and α for the thermal diffusivity. For evaporation of water

[4] "Fundamentals of Drying Methods," by C. W. Thomas and A. Weisselberg, presented at the 1933 Annual Meeting of the A.S.M.E. This paper covered definitions of terms employed in classifying drying methods and equipment.

[5] "The Drying of Solids," by T. K. Sherwood, *Ind. Eng. Chem.*, vol. 21, 1929, p. 12 and p. 976, also vol. 22, 1930, p. 132.

[6] "Evaporation of a Liquid Into a Gas," by W. K. Lewis, Trans. A.S.M.E., vol. 44, 1922, p. 325.

[7] *Chemische Fabrik*, vol. 29, 1929, p. 527.

into air κ/α was determined at 1.09, which is close enough to unity to make the Lewis formula usable for practical purposes. We see, therefore, that we may express the rate of evaporation in four different ways. Values for all four coefficients, U, K, L, and k', are available from a great deal of experimental data. Results with values for still air check fairly well. For moving air, the values given as functions of velocity and turbulence lead in many instances to quite different answers.

If the heat for evaporation is derived only from the air the liquid assumes the wet-bulb temperature t_w. The rate of evaporation is therefore proportional to the temperature difference $(t_a - t_w)$ termed the wet-bulb depression. According to the foregoing it is also proportional to the partial pressure difference $(p_w - p_a)$, p_w being the vapor pressure at saturation corresponding to the wet-bulb temperature. The term drying potential for $(p_w - p_a)$ is quite appropriate because it signifies the drying potentiality of the air. In contact drying with the heat for evaporation supplied from an outside source, the heat may be supplied at such a rate that t_l exceeds the boiling point of the liquid, i.e., the temperature at which the vapor pressure is equal to the total pressure p of the air above the liquid. In this case, the evaporation rate depends on the overall heat-transfer rate from heating medium to liquid and the available temperature drop. However, if the temperature of the liquid is lower than the boiling point the rate of evaporation will be controlled by the rate of diffusion of the vapor into the surrounding medium at the existing pressure drop $(p_l - p_v)$. Unless the rate of heat flow from heating medium to liquid is checked to suit the rate of diffusion, the temperature of the liquid will rise to a level at which equilibrium exists.

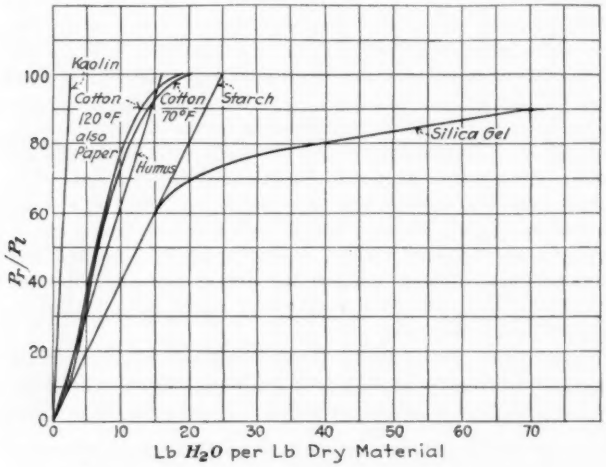


FIG. 2 REGAIN CURVES FOR VARIOUS MATERIALS

In connection with evaporation, there still remains to be discussed the hygroscopic moisture and its evaporation. In the beginning it was mentioned that the vapor pressure of crystalloidal solutions is lower than the vapor pressure of the pure liquid at the same temperature. As an extreme example, a saturated soda-ash solution boils at 572 F. This corresponds to a pressure of 1227 lb per sq in. Most organic materials contain, in their cells, both crystalloidal and colloidal solutions. As the moisture content decreases, the concentration of these solutions increases, especially that of the crystalloid solution, since, as stated, it diffuses readily. With the increase in concentration a point is reached when the vapor pressure becomes sensibly affected. This is called the hygroscopic point. If p_r denotes the reduced pressure and p_l the pressure of the pure liquid at the same temperature, then p_r/p_l will decrease as the moisture content decreases. This relation for various materials is shown in Fig. 2. Substituting the reduced pressure, p_r for p_l, in the evaporation formula, $(p_r - p_v)$ must be positive if evaporation is to take place. If p_r, corresponding to a certain moisture content, is equal to p_v, no more moisture can evaporate. Hence each moisture content below the hygroscopic point is associated with a drying potential equal to the reduction in pressure $(p_l - p_r)$, which is the same as saying that the equilibrium moisture content is a function of the drying potential or of the wet-bulb depression of the drying air. Hygroscopic moisture-content curves (regain curves), which have been published for various substances, show the moisture content plotted against the relative humidity of the air at a certain temperature. From the foregoing, it appears more logical, although admittedly less convenient, to plot the hygroscopic moisture content against the wet-bulb depression or the drying potential and present the relation independently of the temperature. Even those inorganic materials which themselves do not go into crystalloidal solution with the liquid they contain may be hygroscopic if they contain salts as impurities which will go into solution with the liquid. Common salt, for instance, is only slightly hygroscopic, but it contains magnesium salts which make it very hygroscopic. It is for this reason that, in order to obtain a non-hygroscopic salt, the mother liquor should be removed by dehydration as far as possible.

As the air picks up moisture, its drying potential decreases. The average rate of evaporation is equal to the logarithmic mean of the initial and final rates of evaporation, and is found directly by using the logarithmic mean for $(t_a - t_w)$, $(p_w - p_v)$, and $(X_w - X_a)$, respectively, in the given formulas for evaporation.

Diffusion. Compared with evaporation, the mechanism of diffusion is much more complicated, especially when no outside force is applied, the difference of concentration of liquid being the only driving force that causes the liquid to flow. This force diminishes and changes with the location. Like all problems of this nature in which the rate of flow of a quantitative substance changes with the time and location, the relation is expressed by a differential equation of the second order. In the case of liquid diffusion in a body, it is assumed with sufficiently close approximation that the rate of propagation of concentration of liquid K, in pounds per foot per hour per unit of concentration, is constant. Then the flow μ of the substance in the direction x is equal to $-\kappa' \dfrac{d\sigma}{dx}$, σ denoting the concentration. The change of the concentration with the time will be proportional to change in flow along X, or $\dfrac{d\sigma}{d\vartheta} = \dfrac{d\mu}{dx} = \dfrac{\kappa' d^2\sigma}{dx^2}$. The integration of even this equation, let alone those in which we would have to consider diffusion in three directions and with variable κ', is extremely difficult to solve. Of interest to practical engineers are only the curves and tables worked out by physicists[8] of which the latest and most complete ones are those by Prof. Albert B. Newman.[9] With their help the results of laboratory tests can be interpreted for practical purposes, and where coefficients are known, the change of moisture content with the time for bodies of certain geometrical shape may be determined under varying conditions of initial moisture distribution. Although there are admittedly practical drying problems to which this information may be applied directly, it must be borne in mind that the solutions are based on constant conditions, such as constant temperature of the material and constant flow, and constant temperature and

[8] Williamson and Adams, *Phys. Rev.*, vol. 14, 1919, p. 39; Lederer, *Zeitschrift für Angew. Chem.*, 1924, pp. 370–750; Sherwood, *Ind. Eng. Chem.*, vol. 21, 1929, p. 12; Peck, *Phys. Rev.*, 1930, p. 35; Newman, *Cooper Union Bulletin*, no. 5, 1932.

[9] *Cooper Union Bulletin*, no. 5, 1932.

humidity of the air. Not only are these seldom encountered, but we should actually change them in order that we may improve on nature, so to speak.

The question arises whether we should let the difference of concentration of liquid be the only force to drive the moisture out, and whether we cannot apply an outside source of energy to speed up diffusion. How this is accomplished in contact drying does not require further explanation. But from it we may recall the fact that the moisture diffuses in the direction of vapor-pressure drop, and where there is vapor-pressure drop there must be temperature drop. Hence, in the case of air drying, if we do not want to depend upon "natural diffusion," we must maintain a temperature drop from the center of the body to the surface. We may do this continuously or intermittently, in both cases utilizing the heat of the air for this purpose. In the latter case, we will have alternating periods during one of which the heat input, $U(t_a - t_l)$, will be greater than the heat in the steam evaporated, $L(p_l - p_v)$, while during the other it will be equal or lower. On the other hand, during the period of drying, which would otherwise be controlled by the diffusion, we may so adjust conditions by changing U, which it should be recalled is a function of air velocity and turbulence, that $U(t_a - t_l)$ will always be somewhat greater than $L(p_l - p_v)$, the difference being utilized to cause "forced diffusion" at a rate corresponding to $\frac{L(p_l - p_v)}{p}$. Obviously, because of the change that occurs in the rate of forced diffusion with reduced moisture content, $U(t_a - t_l)$ and $L(p_l - p_v)$ must also be changed to suit. The calculations for the proper determination of the variables are rather cumbersome. A graphical solution suggested by Prof. E. Schmidt[10] quite simplifies the problem.

In connection with diffusion there is also to be considered the effect of the capillaries and the cells upon it. Undoubtedly, if the surface of evaporation is at right angles to the capillaries, the flow of moisture to the surface will be assisted by the capillary force. As soon as the moisture surrounding the cells becomes less, the cell moisture will diffuse into the capillaries, whence the capillary force will assist it to the surface. No reference is known of investigations which take these questions into account. The fact that the moisture in the cells has crystalloids and colloids in solution, only adds to the complication. From what we said in the beginning, one may anticipate that in the case of a crystalloidal solution, the entire solution will diffuse to the surface, where the salts will be left behind after the moisture has evapo-

[10] Föppl Festschrift, Springer Verlag, Berlin, 1924.

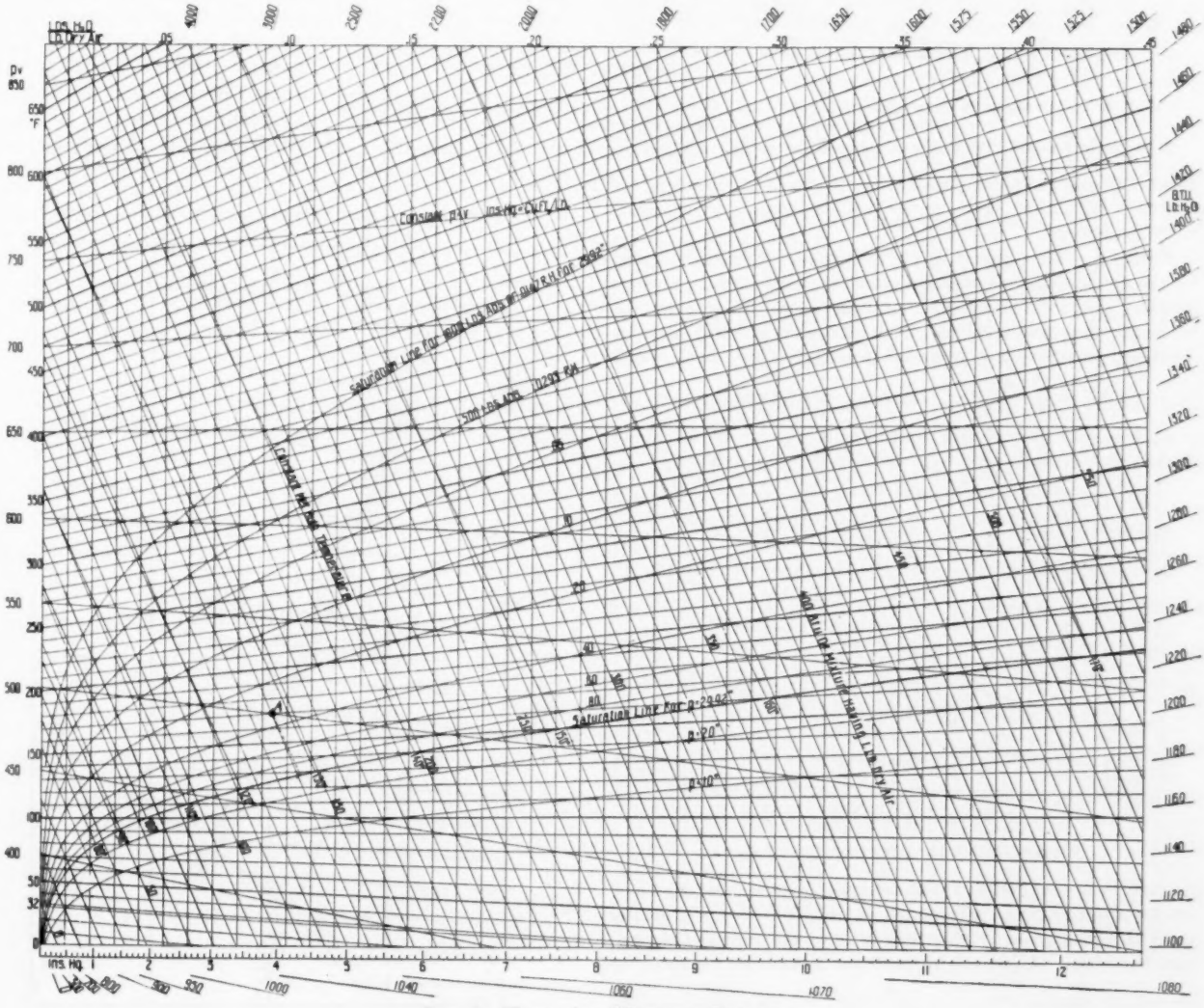


FIG. 3 WATER-AIR MIXTURE CHART

(Based on: lb mol H_2O/lb mol air = 0.622; C_p air at 100 F = 0.2420; C_p air at 700 F = 0.2460; C_p H_2O obtained from Keenan steam tables.)

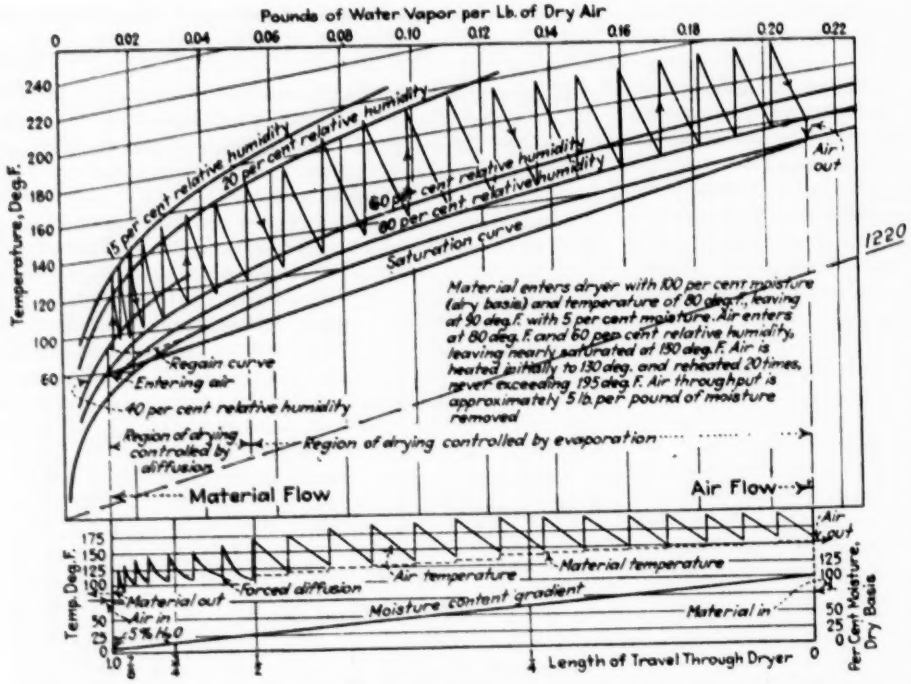


FIG. 4 PERFORMANCE OF A COUNTERFLOW REHEATING AIR DRIER SHOWN ON A SKELETON MOLLIER WATER-AIR MIXTURE CHART

(Air and material temperatures and moisture gradient of material are shown in lower diagram.)

rated. In the case of a colloidal solution in the cells, only the moisture will diffuse, leaving the colloids behind on the inside walls of the cells.

Efficiency. The amount of air that passes through the drier for the evaporation of a certain quantity of moisture is a direct function of the difference in humidities X_i and X_o, respectively, of the air going into and coming out of the drier. In a continuous drier, the rate of flow expressed in pounds of dry air per hour, d_f, is equal to $\frac{w}{X_o - X_i}$. The power P_f required to move this quantity of air through the drier is only part of the air power needed in an efficient drier employing recirculation. The

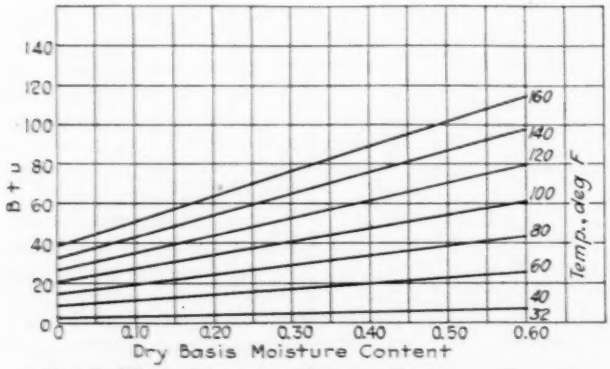


FIG. 5 WATER-MATERIAL MIXTURE CHART FOR $C_p = 3$

balance P_r is required to recirculate the air at a rate d_r, which can be approximately determined as follows: If q is the total thermal transmission in Btu per hour and t, the average temperature rise of the air passing through the heaters at a considered air velocity, then $n = \frac{q}{d_f\left(1 + \frac{X_o - X_i}{2}\right) s\,\Delta t}$, the number of times d_f must be circulated per hour and $d_r = d_f\left(1 + \frac{X_o - X_i}{2}\right) n/\rho$, the rate at which the air is recirculated in cubic feet per hour, if ρ is the average density of the air in the drier. The temperature rise Δt of the air passing through the heater in each section of the drier must correspond to the temperature drop from evaporation in that section of the drier. The efficiency of the drier expressed in terms of air horsepower, therefore, will depend a great deal upon the design of the drier with respect to the distribution of heating surface and the resistance to flow of air to and from the heaters. At the expense of increased air horsepower a continuous drier employing reheating may attain the same thermal efficiency as a similar drier of superior design. For the purpose of comparison of performance, therefore, one should know both heat and power consumption per pound of moisture removed. The investigation

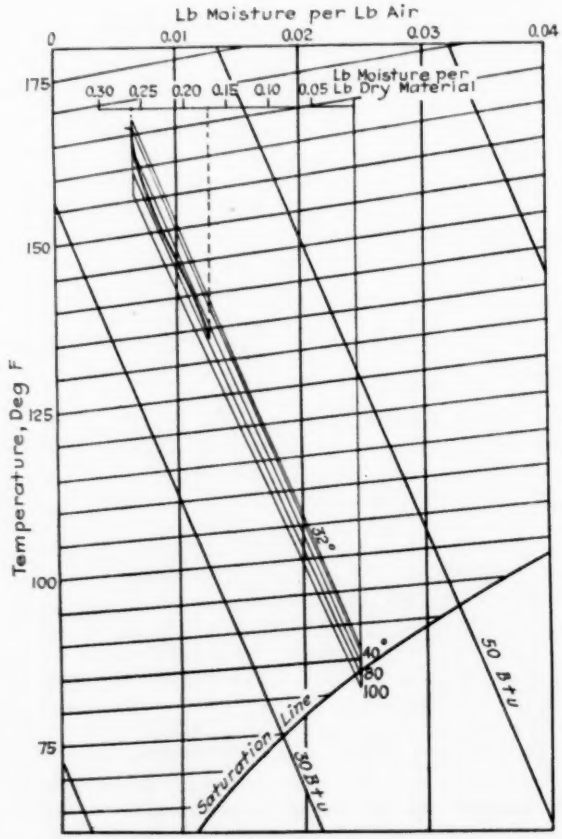


FIG. 6 WATER-MATERIAL MIXTURE CHART ($C_p = 3$) SUPERIMPOSED ON WATER-AIR MIXTURE CHART

should also include a heat balance in which the heat input appears in the following items:

(1) Heat to evaporate the moisture at the temperature at which it enters the drier
(2) Heat to raise the temperature of the evaporated moisture from the temperature at which the moisture enters the drier to the temperature of the exit air
(3) Heat to raise the temperature of the air entering the drier to the exit temperature, including its initial vapor content but not including the moisture added in the course of the drying process
(4) Heat carried from the drier with the material (including final moisture content) above the temperature at which it enters the drier
(5) Heat loss due to radiation and convection from the inclosure.

Graphical Analysis of Drying Process

Graphical presentation facilitates the analysis of a process. For drying, two principal charts are needed, one a liquid-gas mixture chart in which the change of condition of the air or gas may be shown, the other a liquid-material mixture chart for presenting the change of condition of material. For each kind of liquid and gas, and liquid and material, a different chart has to be drawn. A chart for water and air is shown in Fig. 3 and is based on Professor Mollier's suggestion to use heat content and moisture content as coordinates and set these coordinates at an angle. A most useful chart is thus obtained. Referring to it and assuming an atmospheric pressure condition of 29.92 in., point *A* indicates a dry-bulb temperature of 170 F (left end scale), a wet-bulb temperature of 128 F (constant wet-bulb line), a partial pressure of 3.9 in. Hg (bottom scale), a dew-point temperature of 124 F (intersection of vertical with saturation line), a humidity of 0.0935 (top scale), a percentage humidity of $\frac{0.0935}{0.428} = 0.2185$ (ratio of moisture content to saturated moisture content on top scale), a relative humidity of $\frac{3.9}{12.2} = 0.32$ (ratio of partial pressure to saturated pressure at same temperature, on bottom scale), a heat content of 140 Btu, a specific volume per lb of mixture of $\frac{500}{29.92} = 16.7 \left(\frac{\text{constant } pv}{29.92}\right)$. The marginal scale at top, bottom, and right-hand side points toward the origin (0 F) and the slope of the connecting lines indicate Btu per lb of moisture added per pound of dry air if the air changes in a direction parallel to this slope. Thus, for instance, in Fig. 4, the line connecting the initial and final state of the air is parallel to the 1220 Btu line, indicating the Btu consumed per pound of moisture removed per pound dry air. All the above readings may be made relative to any other pressure within the range of the chart. Furthermore, adiabatic compression (vertical) may be followed on the chart if the compression ratio to the 1.4 power is taken from tables or calculated.[11] Fig. 5 shows a water-pulp mixture chart. The specific heat for the pulp is 0.3. Hirsch[12] suggests superimposing the one chart upon the other as shown in Fig. 6. For a drying process of a simple nature this may be practical, but when reheating stages with changing quantities of air are used, the procedure becomes too involved to be practical.

[11] The application of this chart to the graphical treatment of the so-called "Oxford drying process" was presented in a paper by A. Weisselberg at a meeting of the Metropolitan Section of the A.S.M.E., April 18, 1933.

[12] "Die Trokentechnik," by Hirsch, Springer Verlag, 1932.

Ten Years of Stoker Development at Hudson Avenue

By JOHN M. DRISCOLL[1] and W. H. SPERR,[2] BROOKLYN, N. Y.

Hudson Avenue Station is a stoker-fired generating station of 770,000-kw rated capacity. The station was started in 1922 with 14-retort underfeed stokers of 355 sq ft projected grate and ashpit area and completed in 1932 with an installation of 15-retort stokers of 694 sq ft projected grate and ashpit area. Development of the rated station capacity requires that the latter installation of stokers burn coal at the rate of about 65 lb per sq ft per hr, using eastern semi-bituminous coal of about 14,000 Btu per lb as fired. On test, coal was burned at the rate of 75 lb per sq ft per hr for 48 consecutive hours, at a steam-generating-unit efficiency of 77 per cent.

The development of the various stoker units, of which there are five different types, is described and illustrated in this paper. This station was the location of the development by the Westinghouse Company of the first "link-grate" stoker, which was successfully applied here in a 15-retort design having a projected grate and ashpit area of 524 sq ft.

The 1932 installation of American Engineering Company's Taylor stokers is equipped with manual zoned-air control, which divides the air supplied to the grate section into 69 separately controlled zones. These stokers are the longest single-ended underfeed stokers on record, measuring 26 ft 7 in. from front furnace wall to the rear of ashpit.

The various steps in the development of the underfeed stokers to obtain the desired capacities are described, and an apparent limitation in capacity of a purely underfeed stoker of any considerable length is discussed.

This paper discusses also the unburned-gas losses and the cinder losses in the heat balance of a boiler-stoker test. The cinder loss attains considerable magnitude in stoker operation at high-burning rates, and loss data are given for a number of stoker tests at coal-burning rates up to 75 lb per sq ft per hr.

The extent of improvement in efficiency in stoker operation over the period of ten years due to stoker design and associated equipment is discussed from the standpoint of the installations in this particular station, efficiency data being given in curve form for the various stoker installations.

It is the hope of the authors in presenting this paper that it will serve steam-plant designers as a basis of facts on efficiency and capacity possibilities of large stoker-fired installations in the present state of the art.

A companion paper by Messrs. Hardie and Cooper, of the Brooklyn Edison Company, entitled "The Test Performance of Hudson Avenue's Most Recent Steam Generating Units"[3] presents test data and describes the test procedure and methods.

WHEN the design of Hudson Avenue Station was initiated in 1922, underfeed stokers were the outstanding means available for burning a high-grade bituminous coal used by a steam-generating station located in the Middle Atlantic states. Subsequent to the original installation of twelve stokers at Hudson Avenue when the later additions of capacity were being provided, pulverized-fuel equipment was seriously considered because by then it had been developed so that its reliability was no longer questioned. However, after considering such factors as layout, operation, and investment costs, stokers were chosen consistently as the better method of fuel firing at Hudson Avenue.

Because of the large increase in size of each successive unit, it was necessary on each installation to utilize stokers which were longer than any in use at the time of their selection. The results of this design, from the standpoint of station investment cost and operating economy, have been very satisfactory.

Stoker Development

The station structure was built in two stages, the original building providing for three 50,000-kw turbines and three rows of boilers with stokers which went into operation in May, June, and October, 1924. The first two units were engineered and installed at the same time. The designers and operators, in order to obtain comparative operating data between two of the stokers which were on a parity on the basis of a comparison of bids, recommended the purchase of two Combustion Engineering Company's Frederick stokers, and six Westinghouse stokers. The Combustion stokers are 14-retort, 31-tuyère units with a projected grate area of 380 sq ft and a length of 15 ft 6 in. while the Westinghouse stokers are 14-retort, 27-tuyère units with a projected grate area of 355 sq ft and a projected length of 14 ft 5 in. including ashpit. The third unit was engineered during the installation of the first two, and the stokers selected were four Frederick units of the same size and design as the two purchased for No. 1 unit. The first three turbine and boiler installations were all of the same size, and the design data required a coal-burning rate of more than 50 lb per sq ft per hr to carry full station turbine capacity with eleven of twelve boilers in service. The coal-burning-rate guarantees on the first three units are tabulated in Table 1.

Since the station was designed with exceptionally high stacks,

[1] Plant Equipment Engineer, Brooklyn Edison Co., Inc. Jun. A.S.M.E. Mr. Driscoll received the degree of B.S. in mechanical engineering in 1925 from Brown University. He was employed by the Brooklyn Edison Co., Inc., in 1925, in the mechanical engineering department and has been connected with the mechanical design work of Hudson Avenue Station since that time. Since 1929 he has been plant equipment engineer.

[2] Assistant Plant Equipment Engineer, Brooklyn Edison Co., Inc. Mr. Sperr received the degree of M.E. from Stevens Institute of Technology in 1925. He was employed by the Brooklyn Edison Co., Inc., in 1925, as a cadet engineer. Since 1927 he has been with the plant-equipment bureau and in 1930 was made assistant plant equipment engineer.

Contributed by the Fuels Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until April 10, 1935, for publication in a later issue of Transactions.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] See A.S.M.E. Transactions, November, 1934, paper FSP-56-15.

TABLE 1

Unit	Combustion rate, lb per sq ft per hr — Continuous	Four hours	Two hours
No. 1 Frederick	51.6	..	62.5
Nos. 1 and 2 Westinghouse	57.0	..	73.0
No. 3 Frederick	55.2	71.0	..

it was hoped to attain the required boiler outputs on natural draft, and consequently no induced-draft fans were installed for this first section. Unfortunately, the draft proved insufficient to carry the stokers much above 45 lb per sq ft per hr so that their capacity to meet the guaranteed coal-burning rates has never been tested.

Fig. 1 shows a cross-section of the Combustion stoker, and Fig. 2 shows a cross-section of the Westinghouse stoker unit as originally installed. During the first few years of their operation some changes were made from the construction shown on the drawings. The Westinghouse stoker was operated with the lower agitator grates in a stationary position, and the agitator operating mechanism removed. On the Combustion stokers the single grinder roll was rearranged to turn toward the lower grate and the retort bottoms were rearranged to use a shallower type of secondary ram which produced less disturbance and seemed to reduce clinkering troubles.

The efficiency, as determined on test of Westinghouse stokered boiler No. 14 (the fourth boiler in No. 1 row), is shown at the left of Fig. 3, plotted against boiler output in millions of Btu per hour.

In 1924, load growth was still continuing at a rate which indicated the need for capacity above that of the first three units. Engineering studies made at the time indicated the advisability of raising the operating pressure of the station from 265 lb per sq in. to 400 lb per sq in. for improved economy of operation. When this was done it was also decided to install an 80,000-kw turbine, to take care of the increasing load demand, also to reduce the cost per kilowatt of capacity of the ultimate station.

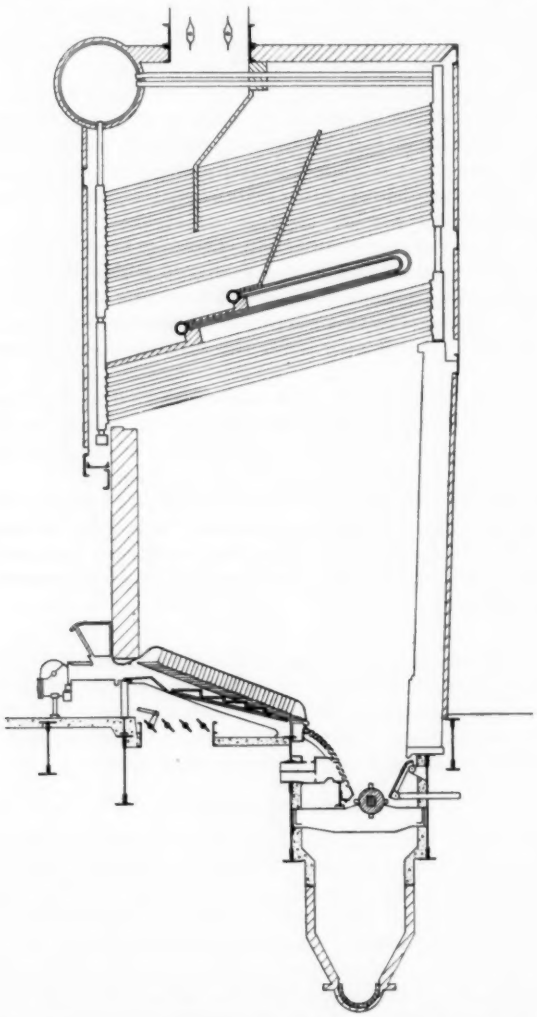


FIG. 1 COMBUSTION ENGINEERING COMPANY'S FREDERICK STOKER, UNITS NOS. 1 AND 3

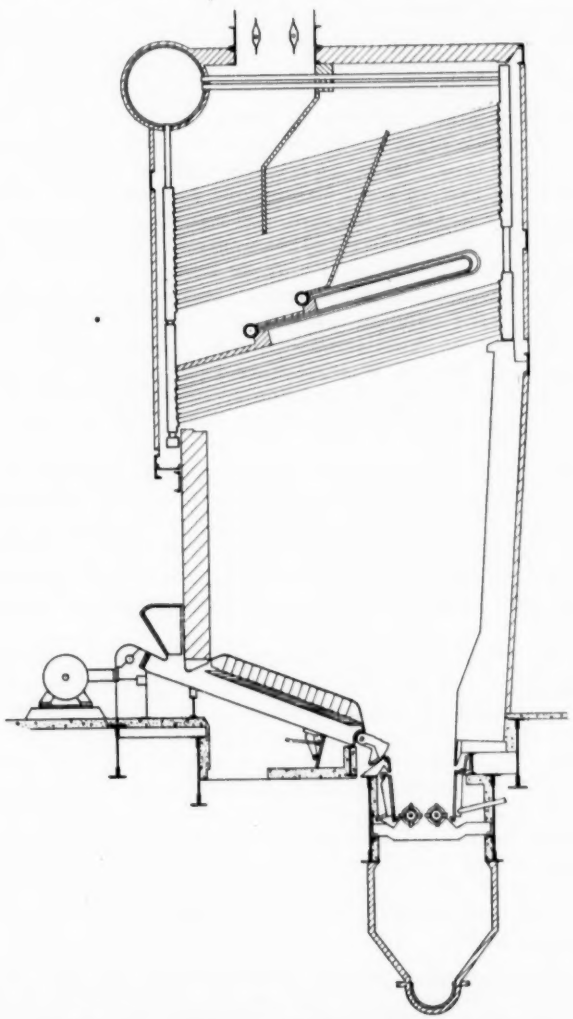


FIG. 2 WESTINGHOUSE STOKER UNITS NOS. 1 AND 2

The stoker specifications issued for unit No. 4 called for a 14-retort stoker with a length not to exceed 20 ft 6 in. and capable of burning 32,000 lb of coal per hr continuously. In other words the specification called for a continuous coal-burning rate of not less than 63 lb per sq ft per hr. It should be noted in connection with the coal quantities just mentioned that at the time of unit No. 4 design work, neither the company's nor the manufacturer's engineers appreciated the existence of a rather sharp falling off in stoker efficiency at these high outputs. Hence the required coal per hour in actual operation was even higher than that specified to obtain the desired output.

The No. 4 stokers were to operate with preheated air at a tem-

perature of 300 to 450 F. Only one of the four bidders had built a stoker of the maximum length allowed. Two of them offered propositions covering stokers of the maximum length permitted. The two others did not take advantage of the full length, but offered propositions on shorter stokers, which were, however, longer than any that these two manufacturers had previously built. As a result of these negotiations, Westinghouse stokers were purchased, 39 tuyères long, having a projected length of 17 ft 4 in. and a projected grate and pit area of 427 sq ft. To burn a total of 32,000 lb of coal per hr on each stoker required a burning rate of 75 lb per sq ft per hr.

The first No. 4 stoker was initially operated in September, 1926. As shown in Fig. 4 it was designed to underfeed practically its entire length. The windbox was divided into four compartments so that different air pressures could be carried under sections of the stoker from head to foot, with an additional zone for the agitator grate. Severe burning of the tuyères occurred immediately above the plates dividing the windbox along the length of the stoker, and to overcome this the division plates were lowered to give transfer area from section to section immediately beneath the tuyères.

Numerous changes were made in the design of the retort bottom. The original bottom, as indicated in Fig. 4, had three secondary rams, the lower one acting at a different angle to the tuyère row, from the others. After several days of running, a heavy clinker formation would develop at the head end due to the lack of upward motion of the secondary rams. High ratings could not be maintained because of excessive blowing of the fuel bed and lack of burning on the clinker-blanketed upper tuyères.

The next major change made was to use a straight-line retort bottom, in which the three secondary rams acted in line along a retort bottom inclined at an angle to the tuyère row. This retort

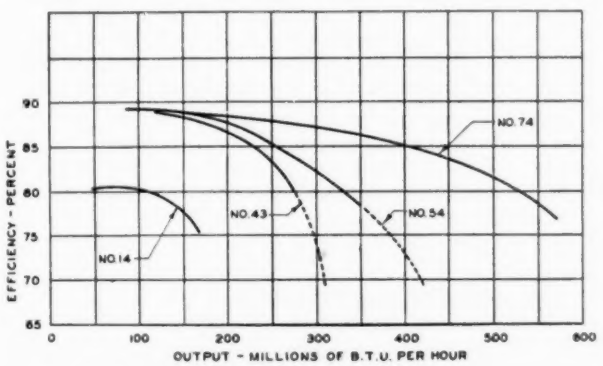


FIG. 3 CURVES SHOWING OVERALL BOILER EFFICIENCIES

produced better overall results but made the fuel bed heavy and inactive at the head in spite of many minor adjustments. Clinker on the tuyère rows, while not giving trouble at low ratings, still interfered with the obtaining of high coal-burning rates. With the thought that a greater air-entry area into the upper section of the stoker would increase the combustion rate at this point, the next step was to install longer tuyères at the head end which had double the air-passage area of the standard tuyère. This change seemed to have no appreciable beneficial effect on the obtaining of high ratings. A third major change consisted in raising the retort bottom along its entire length to shallow the retort and obtain greater agitation and looseness of the fuel bed. Again this third attempt failed to eliminate clinker on the tuyère row at the upper end. In order to get still more upward motion of coal out of the retorts at the upper end, the fourth major change consisted of making the upper secondary move upward toward the tuyères as well as down the retort and lift coal on its top surface into the fuel bed. This change eliminated, very effectively, clinker immediately above the special upper secondary ram but in the section of fuel bed immediately following, tuyère clinker was experienced.

The final change made was to install telescoping rams as shown in the lower view of Fig. 4, which combined the effective motion down the retort of the straight-line design, and a uniform upward motion of green fuel toward the tuyères. On test, this design was capable of maintaining a uniform fuel bed over its entire

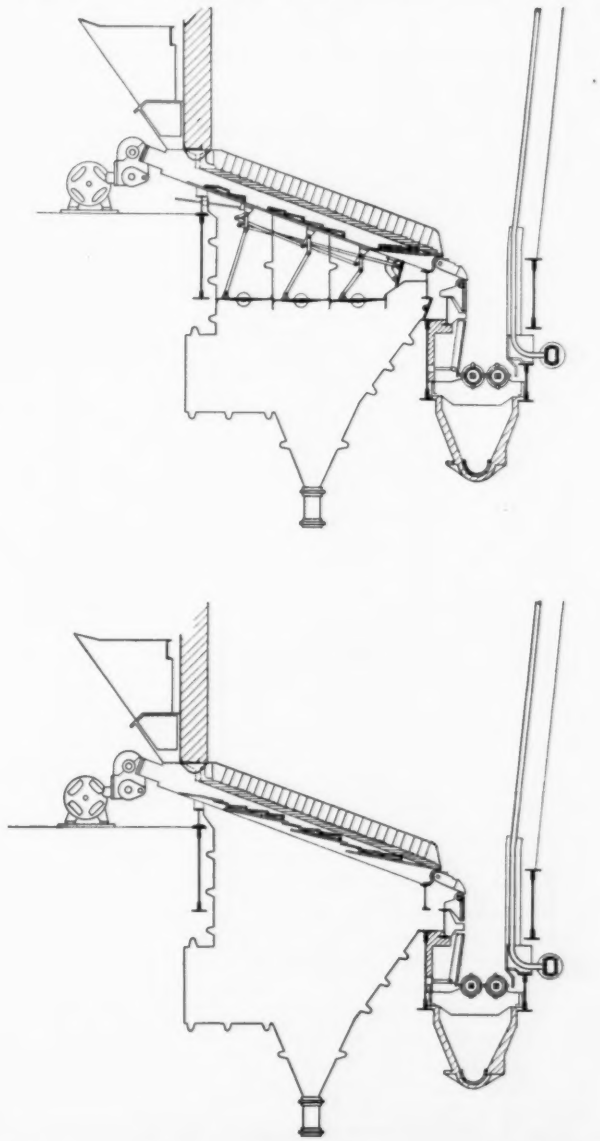


FIG. 4 WESTINGHOUSE STOKER UNIT NO. 4. UPPER VIEW, ORIGINAL INSTALLATION; LOWER VIEW, LATER UNDERFEED ARRANGEMENT

length at coal-burning rates up to approximately 53 lb per sq ft per hr for 24-hr periods. Rates as high as 60 to 65 lb per sq ft per hr could be maintained for 4-hr periods. At this burning rate, however, the fuel bed tended to become sensitive and was likely to blow off in any section. A point had been reached where there was an apparent limitation on continuous capacity, after every reasonable possibility had been investigated in the development of the underfeeding retort.

While this development work was going on for unit No. 4, plans were in progress for further addition to the generating capacity of the station, calling for four additional boilers and stokers to serve a 110,000-kw turbine known as unit No. 5. The development of the thin, water-cooled furnace wall made it possible to provide one extra retort in the width of each stoker within the same overall limits of furnace dimension. A return was made to

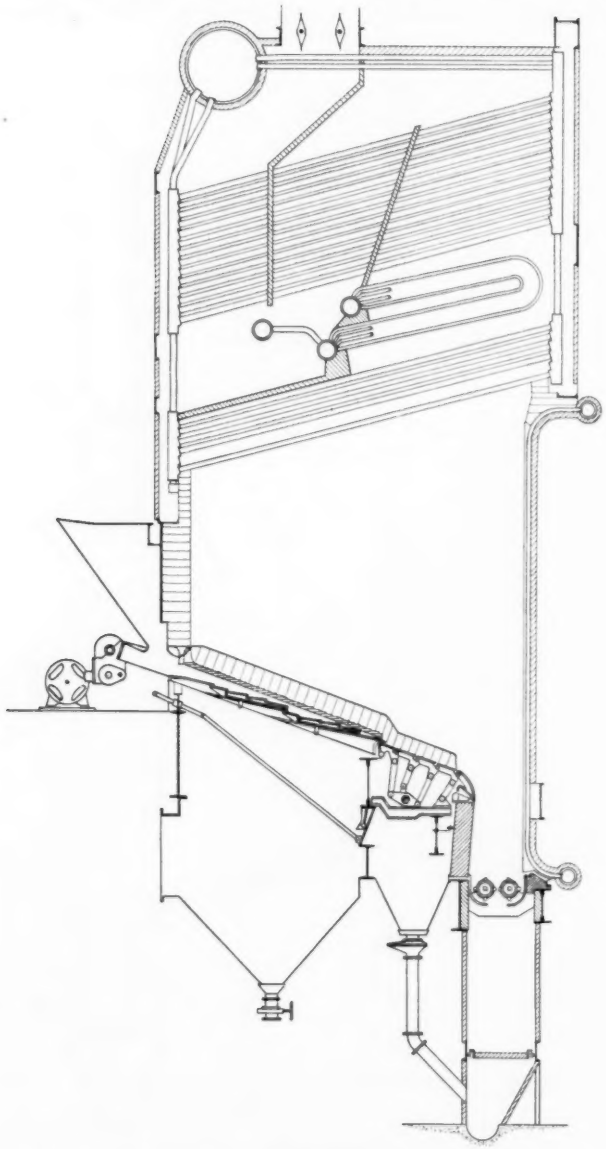


FIG. 5 WESTINGHOUSE LINK-GRATE STOKER UNIT NO. 5

cold-air operation, because economizers offered an effective substitute for air heaters in attaining economy and because space limitations practically prohibited the installation of the necessary duct work for preheated air on two adjacent units. It has been found, too, that the common-duct, preheated-air system, which had been necessary due to lack of space for a unit system, severely handicapped stoker- and boiler-maintenance work because of high temperatures resulting from leaky dampers.

For No. 5 unit, 15-retort Westinghouse stokers were purchased, having a projected length from inside of front furnace wall to rear of ashpit of 19 ft 8$^3/_4$ in. and a projected stoker and ashpit area of 524 sq ft. The proposal was based on a stoker designed along the lines of No. 4 unit stokers, but of 45-tuyères length instead of 39. The guarantees were somewhat more conservative than those for unit No. 4, based on capacity per sq ft, and were as follows:

Continuous, 24-hr	58 lb per sq ft per hr
Four hours	65 lb per sq ft per hr
Two hours	73 lb per sq ft per hr

Before development of the drawings for this installation had proceeded very far, the Westinghouse Company had developed the idea of their "link-grate" stoker. They proposed this design for installation on this contract, making the statement that they believed this new development had possibilities greater than the conventional underfeed stoker. It was agreed that the installation should proceed on this basis. A sealed-ashpit design was incorporated in the No. 5 unit installation and subsequent installations, in order to eliminate the downward flow of air and

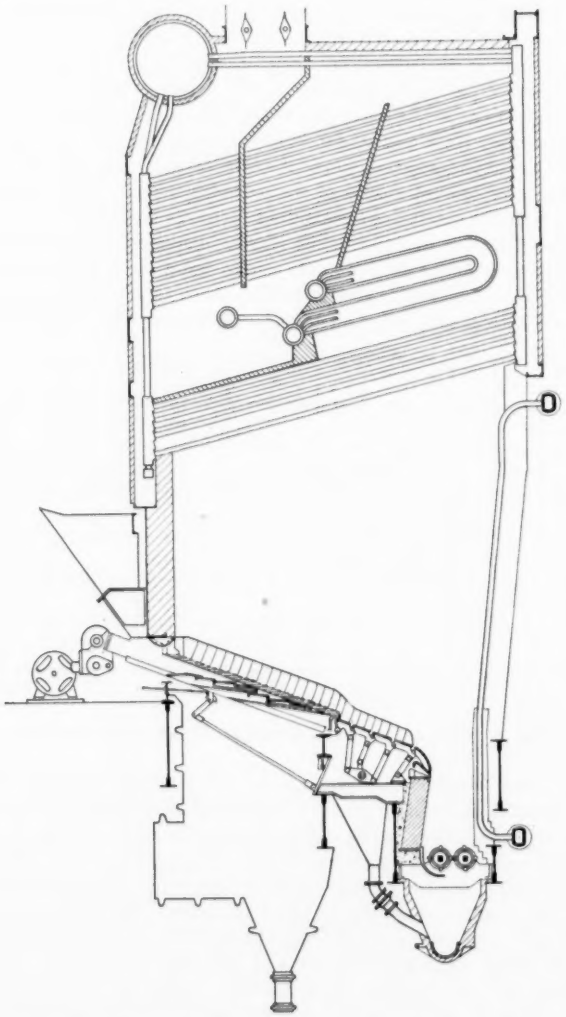


FIG. 6 WESTINGHOUSE LINK-GRATE STOKER UNIT NO. 4

gas into the ashpit which had often been experienced on the unit No. 4 stokers and had resulted in high maintenance costs for the ashpit and lower stoker parts.

The link-grate stoker, of which unit No. 5 at Hudson Avenue was the first installation, has been described a number of times

in the technical press.[4] Fig. 5 is a drawing of this stoker. Briefly the link-grate section consists of a six-foot-long overfeed section made up longitudinally of five bar elements which are linked together by a driving mechanism which gives the bars an undulating or wave-like motion which breaks down and distributes the fuel as it is received from the underfeed section and propels it toward the ashpit.

This installation was successful from the first, and practically no development work and operating experience were necessary in order to meet and surpass the capacity guarantees. Results of efficiency tests on No. 54 boiler and stoker are shown in Fig. 3. Data up to coal-burning rates of 62 lb per sq ft per hr (350×10^6 Btu per hr output) are for 24-hr tests, while the tests at burning rates of 76 and 89 lb per sq ft per hr are for durations of 3 hr and 2 hr, respectively.

Within a short time of the operation of unit No. 5, it was necessary to start engineering-design work on the next unit, No. 6. Because of the very satisfactory performance of the boilers and stokers for unit No. 5, and the need for speed in installation, unit No. 6 was made essentially a duplicate of unit No. 5 in these equipments. Unit No. 6 went into operation in 1930 with the same highly satisfactory results obtained on No. 5. In the entire station experience, these two rows of stokers have been outstanding for developing rated capacities without a long period of experimental work.

Subsequent to the successful operation of unit No. 5 link-grate stokers, the Westinghouse Company decided to install a link-grate section for the unit No. 4 stokers as a means of approaching their capacity guarantees. Before doing this work, the Westinghouse Company submitted revised capacity guarantees for the unit No. 4 stokers, which made them consistent in "performance per square foot," with those submitted for unit No. 5. These revised guarantees, although somewhat of a reduction, were accepted and the work proceeded and was completed in 1930. The revised capacity guarantees were easily met. Provision was also made at this time for sealing the ashpit and introducing air under pressures beneath the clinker-grinder rolls, insuring a definite upward flow of air through the clinker pit. Fig. 6 is a drawing of the link-grate stoker unit No. 4.

Specifications were issued in 1930 calling for stokers for units Nos. 7 and 8 to complete the station. Since two 160,000-kw turbines had been selected, it was necessary to obtain steam-generating capacity to supply their requirements. Originally it was thought that a boiler-house extension with two boilers, in addition to the eight then contemplated within the existing house, might be necessary for this purpose. Station-layout drawings were sent out with the specification indicating the available floor area and column arrangement in the building and the stoker space limitations. In defining the capacity requirements, it was indicated that four stoker units when operated together should have a combined instantaneous output rate, in steam, which at no time during a cycle of operation of five hours' duration per night for six consecutive nights should fall below a rate equivalent to 2,080,000,000 Btu per hr. Expressed as an average, this corresponds to a steam output of 520,000,000 Btu per hr per stoker unit. (455,000 lb of steam per hr.) This output was sufficient to supply the required steam from eight boilers inside the existing building. Stoker width was limited to 15 retorts by existing building columns carrying the coal bunkers.

Very serious consideration was given at this time to the use of double-ended stokers, because of the advantages in control of a shorter fuel bed. The problems of providing coal feed to an additional aisle, in particular the space requirements of equipment which would have been required to feed reliably from the existing bunkers, could not be solved by any reasonable layout, so that the double-end design was abandoned reluctantly and the specification was issued for single-ended stokers. Proposals were received as given in Table 2.

TABLE 2

	Stoker area, sq ft	Combustion rate, lb per sq ft per hr		
		24 hr	5 hr	2 hr
American Engineering Co.	694	65.5	73	82
Combustion Engineering Co.	630	60	64.3	75
Westinghouse Co.	578	57.8	69.5	73.7

The Westinghouse Company also offered a stoker with an area of 625 sq ft, but with the same guaranteed total heat output per unit as for the 578-sq ft stoker; that is, the unit combustion rates were correspondingly reduced. While the Brooklyn Edison Company's experience with units Nos. 5 and 6 had indicated these Westinghouse guarantees to be conservative, there was still considerable margin between the total maximum output guaranteed and the desired output from the eight units. The Combustion Company's guarantee was also not high enough to obtain the desired steam output from the eight stokers. American Engineering stokers were therefore purchased on the basis of their guarantee to perform in accordance with the preceding conditions, which met the specification and station-capacity requirements. This guarantee was the only one which offered to produce the required steam without going outside the limits of the existing building.

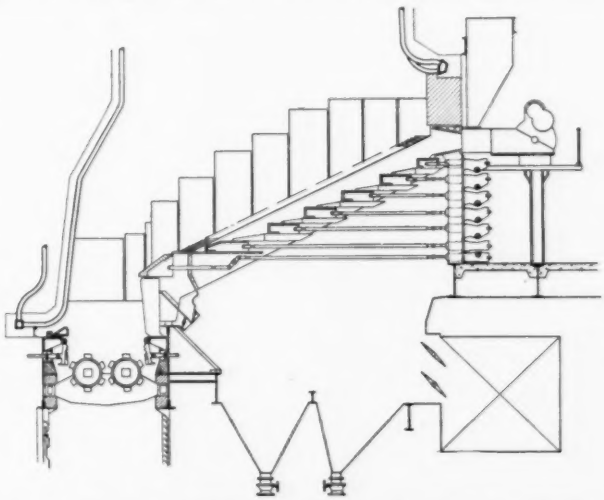


FIG. 7 TAYLOR STOKER UNIT NO. 7, ORIGINAL INSTALLATION

The stokers installed were 69 tuyères long, an extension of a design previously installed at the Delray Power House No. 3 of the Detroit Edison Company which were 57 tuyères long. The retort contained six horizontally acting secondary pushers, in addition to a reciprocating extension-grate section at the lower end. Each pusher and extension grate had provision for individual adjustment of stroke external to the stoker while the stoker is in operation. Five pushers were originally of the square-nosed type, 8 in. high. The arrangement of the stoker originally installed is shown in Fig. 7.

Early operation disclosed lack of ignition at the head and a tendency for the fuel to blow off the grates as coal-burning rates of 45 to 50 lb per sq ft per hr were approached. The first move was to restrict the air supply to the front-wall air boxes and also to restrict the air to the upper tuyères at the head end of the stoker by installing baffles underneath the tuyères. Constant experiment was carried on to determine the best combination of

[4] "The Westinghouse Link Grate Stoker," *Blast Furnace and Steel Plant*, Dec., 1929.
"Underfeed Stoker Has Overfeed Rear Section," *Power Plant Engineering*, Nov. 15, 1929.

TABLE 3

Unit No.	1	3	4		5	7
Stoker No.	14	34	43		54	74
Manufacturer	Westinghouse	Combustion (Frederick)	Westinghouse		Westinghouse	American Engr. (Taylor)
Year installed	1924	1924	1926	1929	1928	1932
Width, retorts	14	14	14	14	15	15
Projected area, including ashpit, sq ft.	355	380	427	427	524	694
Projected length, including ashpit...	14′5″	15′6″	17′4″	17′4″	19′6″	26′7″
Projected length, underfeed section..	8′10″	9′8″	13′3″	9′4″	11′2″	18′6″
Projected length, overfeed section...	1′7″	1′4″	1′7″	5′0″	5′0″	2′6″
Projected width, clinker pit	4′0″	4′6″	2′6″	3′0″	3′4″	5′7″
Boiler surface, sq ft	19,650	19,650	22,920		23,760	24,450
Economizer surface, sq ft	0	0	0		14,960	22,400
Air-heater surface, sq ft	0	0	43,290		0	0
Furnace vol., cu ft	7500	7500	7750		8500	14,000
Furnace volume per sq ft grate area..	21.1	19.8	18.0		16.2	20.2
Type of furnace wall	Refractory	Refractory	Spaced water tube		Bailey block water wall	Fin tube water wall
Air temperature	Cold	Cold	300–400 F		Cold	Cold

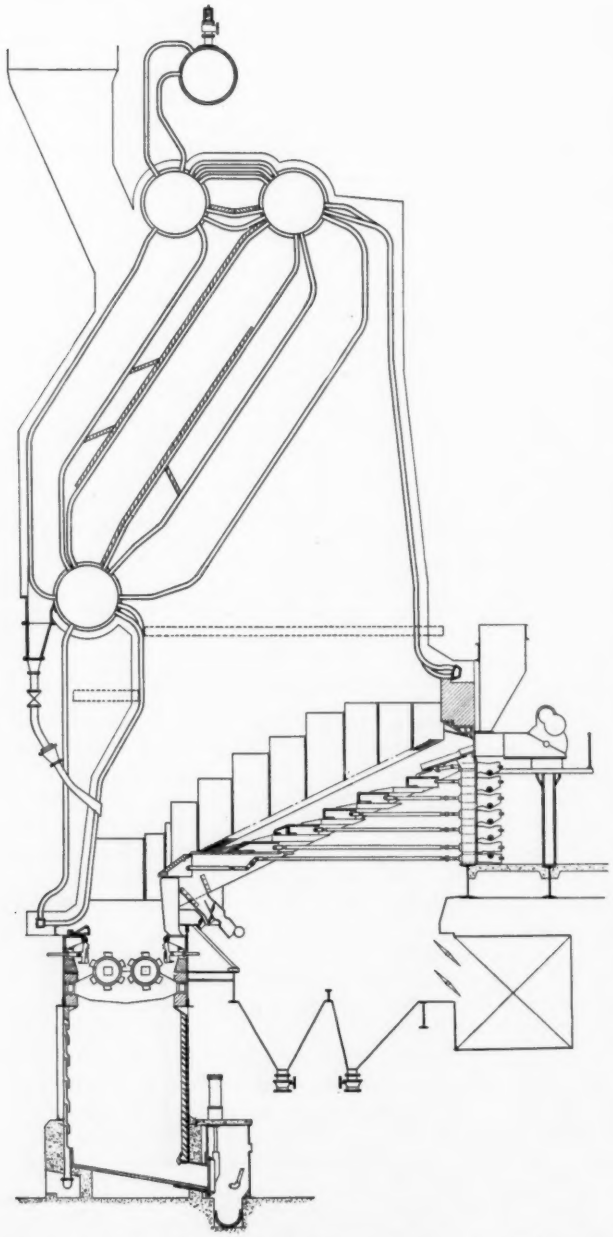


FIG. 8 TAYLOR STOKER UNIT NO. 7, REVISED ARRANGEMENT

secondary-ram strokes. Also the upper secondary ram was removed and replaced by an inclined dead plate. This effectively deepened the retort at this location, but the increased depth was below the tuyère line so that the fuel-bed thickness above the tuyère line was not increased. Then the ram-box caps, which originally had an extremely flat slope from the ram, were replaced by others, the roof slope of which could be adjusted to give an upward flare of as much as $7^1/_2$ in. from the ram out to the furnace and thus give greater depth of coal over the tuyères. With these two latter changes, some difficulty was experienced with packing of coal on the bottom of the retort at this location, which effectively shallowed the retort and also produced a disturbance to the underfeeding action. The upper dead plate was replaced experimentally with an inclined flat plate capable of reciprocating motion, with some improvement.

Other experimental changes were made on the other secondary pushers, by pulling them toward the front of the stoker, thus effectively deepening the retort; and also on other occasions by adding sections having inclined faces to the pusher noses, which effectively shallowed the retort. At the end of about a year's work, the various possibilities had been quite thoroughly investigated and the retort profile determined which gave the best results. This comprised an inclined sliding plate for the No. 1 secondary ram, which was later modified to give the retort a rounded bottom in order to reduce friction of the coal in the retort. The ram-box caps were set at a height giving a rise of about 6 in. Pushers Nos. 2, 3, and 4 were of the square-nose type, while Nos. 5 and 6 had inclined faces, with slopes approaching that of the retort. No. 6 pusher was finally installed with an inclined stepped face. With this stoker, fairly reliable operation at coal-burning rates of about 60 lb per sq ft per hr for periods of 4 to 5 hr could be obtained, with somewhat longer runs and higher ratings sometimes attainable. The experimental work described was conducted chiefly on one stoker, and during this time the operation of the stokers was entirely reliable and without excessive maintenance, the only limitation being on the maximum capacity as described.

Since the American Engineering Company had done considerable work with zoned-air control on other installations,[5] they then stated their desire to make the application to one of these stokers as their next step toward meeting the capacity guarantees. This equipment was designed and installed first on No. 74 stoker. Each row of tuyères, including the two rows of half-tuyères at the side, was divided into four equal zones and a control damper and metering venturi box and gage were provided for each zone, making 64 zones in all. The extension-grate section was divided into five zones across the width of the stoker with control damper and gage for each. The air-control apparatus arrangement under each tuyère row is illustrated in Fig. 9.

After a short period of experimental operation, this stoker

[5] "Zone Air Control for Stokers Made Automatic," F. J. Chatel, *Power*, Jan. 26, 1932.

"Zoned Air Control for Underfeed Stokers Increases Boiler Capacity," Griswold and Brown, *Power*, June 23, 1931.

"Stoker Developments at the Detroit Edison Company's Delray Power House No. 3," P. W. Thompson and F. J. Chatel, A.S.M.E. Trans., 1933, paper FSP-55-11.

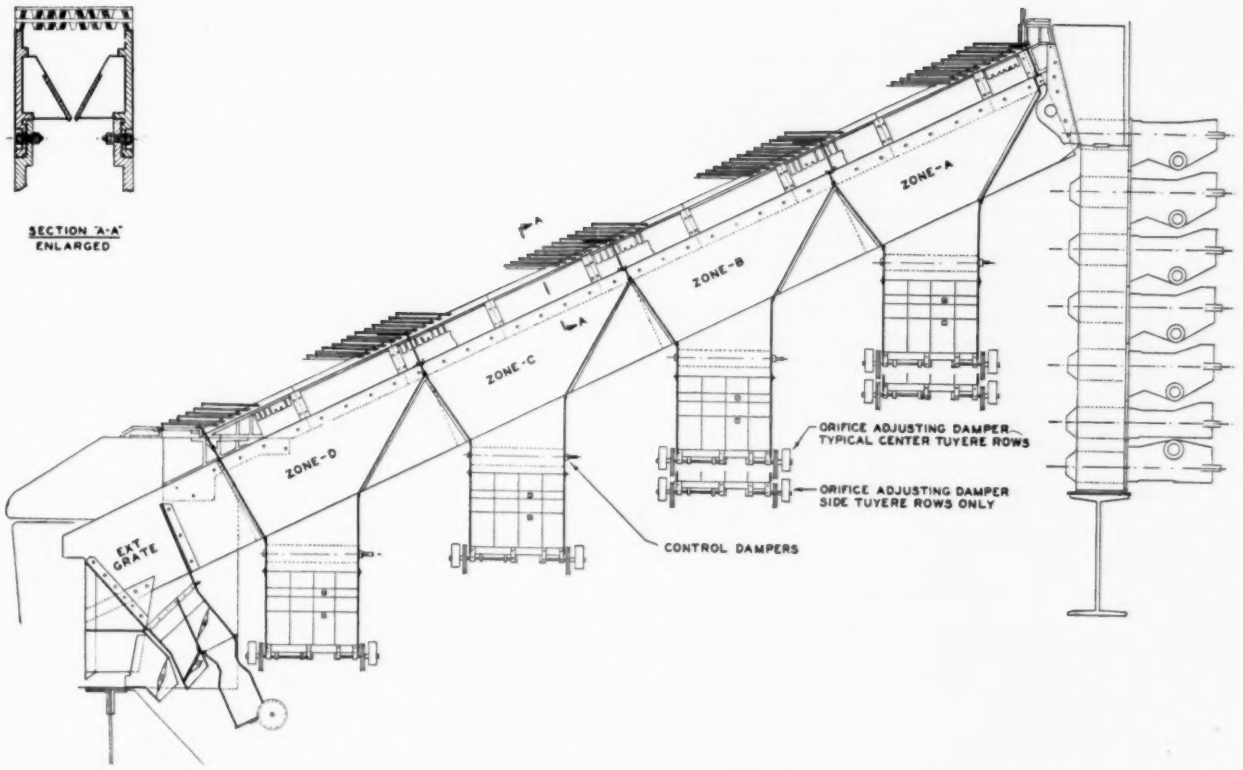


FIG. 9 STOKER AIR-CONTROL EQUIPMENT

proved itself capable of meeting the capacity guarantees and of exceeding them by a substantial margin. During the final tests on this stoker, for which water and coal were weighed, it was operated for a period of 48 consecutive hours at an average coal-burning rate of 75 lb per sq ft per hr at an average efficiency of steam-generating equipment of 77 per cent. This was at an output of approximately 500,000 lb of steam per hr.

While this point has not been mentioned in the descriptions of the individual stokers, there has been complete adoption at the Hudson Avenue Station of the so-called "thin tuyère." Thus the unit No. 4 Westinghouse stoker with link grate, having a rated underfeed section of 27 tuyères, now has actually 48 tuyères, and the newest Taylor stokers, originally 69-tuyère stokers, now actually have 100 tuyères. This type of tuyère was, in these cases, adopted by the manufacturers, and the use continued by the station operators. While they do not show any marked improvement in the manipulation of the stoker, they lessen clinkering difficulties and burning troubles, and lower maintenance costs.

DISCUSSION OF RESULTS

In considering the stoker installations, the history of which is presented here, it has been pointed out that capacity requirements made necessary the use of the most advanced developments which the stoker art afforded, both as to length of stokers and in capacity per square foot. The following points seem to the authors to warrant mentioning in a discussion of the results of these installations:

(1) As will be seen from Table 4, the general type of coal in use at Hudson Avenue has been fairly constant in character. A heating value of 14,000 Btu per lb, as fired, moisture content of 4 per cent, volatile matter of 17 to 19 per cent, and ash of 7 to 9 per cent would serve to represent this coal. Comparisons of coal-burning rates per square foot are based on coal as fired.

TABLE 4 COMPARISON OF REPRESENTATIVE COALS—STOKER TESTS

Stoker No.	14	34	43	54	74
Btu as fired	13,650	13,800	14,000	13,650	14,000
Btu dry	14,200	14,400	14,500	14,300	14,500
Moisture, per cent	4.0	4.1	3.3	4.6	3.3
Proximate analysis, dry basis					
Volatile, per cent	18.0	19.0	18.2	19.4	17.8
Fixed carbon, per cent	73.3	73.0	74.5	72.4	74.7
Ash, per cent	8.7	8.0	7.3	8.2	7.5
Ultimate analysis					
Carbon, per cent	81.0	80.8	82.3	80.7	82.1
Hydrogen, per cent	4.3	4.3	4.2	4.6	4.4
Oxygen, per cent	3.4	4.0	3.9	3.9	3.6
Nitrogen, per cent	1.3	1.3	1.9	1.3	1.3
Sulphur, per cent	1.3	1.6	0.9	1.3	1.1
Ash, per cent	8.7	8.0	7.3	8.2	7.5

While these values might have been expressed on a dry-coal basis, no standard practice in this respect seems to have been adopted, and there would still remain the variable of heating value when comparing with other installations. The importance of the heating value in comparing capacities has been pointed out in the paper by Messrs. Foresman and Mosshart of the Westinghouse Company, on "Stokers and Furnaces for New England Fuels,"[6] presented before the Boston Section of the A.S.M.E., May 15, 1929.

(2) There is some variation in the furnace volume per square foot of stoker area, which will be noted by reference to Table 3. There are also differences in the amount of "cold"-water-cooled surface exposed in the furnace. The most radical difference from the preceding units is presented by units Nos. 7 and 8, with almost completely water-cooled furnace, and an entirely different shape of furnace and disposition of furnace height with respect to the stoker.

(3) From a review of Fig. 10 it will be noted that the difference in steam-generating efficiency between the 1924 installation

[6] A.S.M.E. Trans., 1931, paper FSP-53-1.

and those made since 1926 is about 8 per cent. Of this difference, 7 per cent increase is accomplished by the reduction of dry-gas loss. This reduction is due chiefly to the installation of heat-recovery equipment beyond the boiler surface. A portion of it, representing about one-half of one per cent in the heat balance, was accomplished by a reduction of excess air. The use of water-cooled furnace walls made practical the use of higher furnace temperatures of combustion, and the improved methods of fuel and air control, particularly in the extension-grate zone, facilitated the accomplishment of the reduction of excess air. The introduction of the sealed ashpit probably assisted in the reduction of excess air at low ratings.

The combustible in refuse loss has been reduced from about 1.5 or 1.7 per cent in the heat balance to a loss of generally less

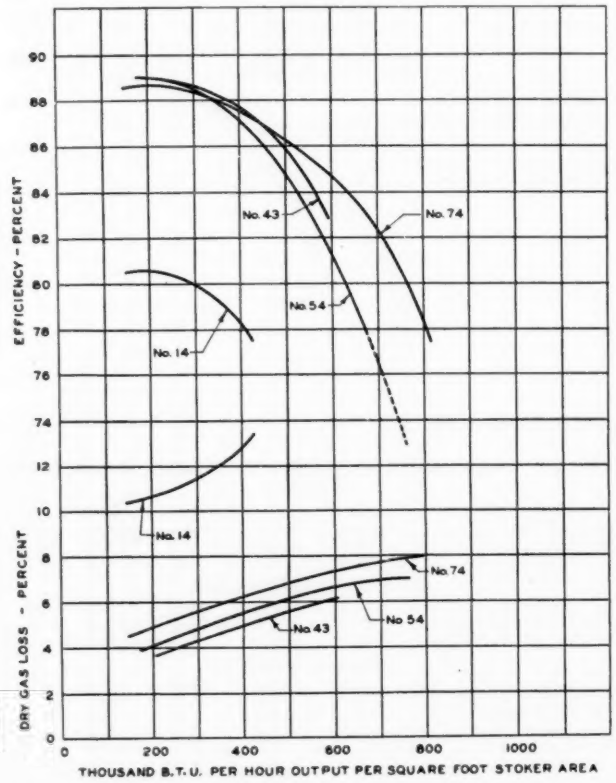


FIG. 10 DIAGRAM SHOWING STEAM-GENERATING EFFICIENCIES AND DRY-GAS LOSS FOR VARIOUS STOKERS

than 0.5 per cent. Credit for this is due mainly to the improvements in the extension-grate section and to the sealed ashpit, which latter permits more complete burning out of combustible in the ash particularly at higher ratings when "popcorning" of fuel into the more quiet zone of the pit takes place on all stokers.

(4) As operating capacities were increased to burning rates of 50 lb per sq ft per hr and above, the standard boiler test showed increasingly large unaccounted-for losses in the heat balance. Consideration of this question indicated two probable identities for these unmeasured losses; first, the fly cinder and coke carried off the fuel bed with the gas, and second, unburned gases or products of partial combustion, such as hydrogen, hydrocarbons, and carbon monoxide. Carbon monoxide is mentioned because of indications that the usual Orsat analysis apparatus did not give as high an indication of this gas as was shown to be warranted by other check methods. The Brooklyn Edison Company has given considerable attention to the development of cinder-sampling methods for determining the cinder losses and to laboratory methods of gas analysis from samples taken in the field for the more complete determination of losses due to unburned or partially burned combustible gases. This investigational work was reviewed by W. F. Davidson of this company in a paper entitled "Reduction of Unaccounted-For Losses in Boiler Tests," presented at the Chicago Meeting of the A.S.M.E. in 1931. Appli-

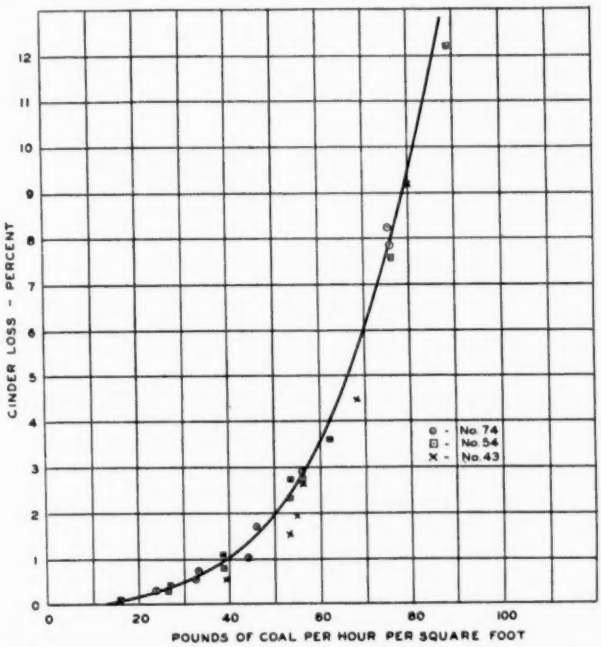


FIG. 11 CINDER LOSS PLOTTED FOR THREE STOKER UNITS

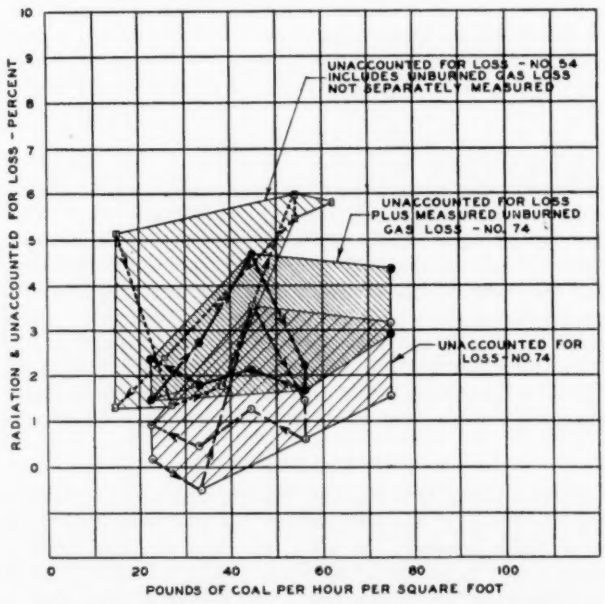


FIG. 12 DIAGRAM SHOWING UNACCOUNTED-FOR LOSSES

cation of these methods of test and analysis has resulted in the obtaining of extremely consistent results regarding cinder-loss values, and a gratifying reduction in the residual values of "radiation and unaccounted-for losses" obtained on test. A discussion of the methods and apparatus used on the 1934 tests is given in a companion paper by Messrs. Hardie and Cooper of the Brooklyn Edison Company research bureau, entitled "The Test Perform-

ance of Hudson Avenue's Most Recent Steam-Generating Units."[3]

(5) The increase in cinder loss at the higher ratings which has been referred to is a reflection of the growing instability of the fuel bed as burning rates are increased. This condition, resulting from inequalities in the thickness and porosity of the fuel bed, has also been discussed by Messrs. Foresman and Mosshart with reference to the duration of time for which any rating may be carried.

The cinder losses for three of the stoker units most recently tested, namely, Nos. 43, 54, and 74, are plotted in Fig. 11. These losses were calculated from the gas flow after determining the cinder loading of the gas in the uptake by the sampling method as described in the paper by Hardie and Cooper. In this connection, it should be pointed out that the cinder loss for No. 74 stoker was measured under conditions of operation which provided for the return to the ashpit of that portion of the cinder which was trapped in the hoppers of the boiler at the bottom of the third boiler pass. These hoppers and the return piping system are indicated in Fig. 8. This method of operation assisted in raising the efficiency of No. 74 stoker on test by reclaiming heat which otherwise would have appeared in the cinder loss. The low value of combustible in refuse loss indicates that the cinder returned to the pit was quite completely burned.

TABLE 5 OPERATING DATA FOR HUDSON AVENUE STATION, 1925–1933

(Based on service hours per year)

Year / Row	1925	1926	1927	1928	1929	1930	1931	1932	1933
1—Average steaming rate—1000 lb per hr per boiler									
11–14	69.3	76.7	75.3	64.5	56.0	63.0	52.3	.....	.....
21–24			78.1	64.4	58.4	64.1	54.9	.....	.....
31–34			72.9	61.9	53.9	58.0	46.6	.....	.....
41–44	....	123.7	142.8	144.8	138.1	132.0	126.2	103.2	.....
51–54	....	.....	.....	166.8	181.8	180.7	162.4	134.0	131.7
61–64	....	.....	.....	.....	.....	160.8	165.8	135.3	127.3
71–74	....	.....	.....	.....	.....	.....	.....	179.2	197.0
81–84	....	.....	.....	.....	.....	.....	.....	172.6	186.9
2—Average coal-burning rate, lb coal per sq ft per hr									
11–14	20.3	20.9	21.0	17.9	15.7	17.3	14.7	.....	.....
21–24			21.8	18.6	16.7	17.9	15.7	.....	.....
31–34			19.4	16.6	14.5	15.5	12.9	.....	.....
41–44	....	27.8	31.9	33.0	31.8	30.7	27.1	22.4	.....
51–54	....	.....	.....	32.1	34.8	33.4	29.4	24.5	24.0
61–64	....	.....	.....	.....	.....	29.7	31.1	25.1	24.0
71–74	....	.....	.....	.....	.....	.....	.....	24.9	27.2
81–84	....	.....	.....	.....	.....	.....	.....	23.3	25.2
3—Average steam-generating efficiency—per cent									
11–14	72.7	79.0	78.0	77.5	75.5	75.7	74.9	.....	.....
21–24			80.2	77.1	76.7	77.2	76.0	.....	.....
31–34			78.9	78.0	76.0	75.4	72.9	.....	.....
41–44	....	79.9	81.1	79.8	76.1	75.5	82.3	83.3	.....
51–54	....	.....	.....	83.4	82.7	84.2	86.4	86.3	85.9
61–64	....	.....	.....	.....	.....	85.2	85.5	86.5	85.3
71–74	....	.....	.....	.....	.....	.....	.....	85.3	85.0
81–84	....	.....	.....	.....	.....	.....	.....	87.5	87.3
4—Stoker-maintenance cost—dollars per net ton									
11–14	0.141	0.143	0.128	0.121	0.108	0.174	0.108	.....	.....
21–24	0.119	0.094	0.065	0.086	0.085	0.089	0.099	.....	.....
31–34	0.138	0.170	0.173	0.174	0.232	0.124	0.132	.....	.....
41–44	....	0.086	0.054	0.144	0.210	0.203	0.090	0.173	.....
51–54	....	.....	.....	0.025	0.043	0.075	0.071	0.063	0.043
61–64	....	.....	.....	.....	.....	0.031	0.052	0.044	0.034
71–74	....	.....	.....	.....	.....	.....	.....	0.036	0.080
81–84	....	.....	.....	.....	.....	.....	.....	0.034	0.068

(6) In comparing the efficiency results of stokers Nos. 54 and 74, it will be noted that while the cinder-loss points, as determined at the boiler outlet, lie close to a common curve and the dry-gas loss for No. 74 is higher than for No. 54, the efficiency for the No. 74 unit is higher at the same coal-burning rate. The reason for the difference in efficiency is of interest and lies in the greater "unaccounted-for" loss in the No. 54 test. Although the unburned-gas losses were not measured on the No. 54 test, it is believed that these largely account for the difference. The "unaccounted-for" losses, which include radiation and all test errors, are shown in Fig. 12. A comparison is also given with the sum of "unaccounted-for" and unburned-gas losses for No. 74, since the unburned-gas loss for No. 54 is included in the "unaccounted-for" classification. This comparison shows the No. 54 losses somewhat higher. All of the test points shown on this illustration are for 24-hour tests with weighed coal and water. Cinder loss and other measured losses were obtained by similar procedures in each case. The reduction in unburned-gas and unaccounted-for losses in the No. 74 test may be attributed partly to the air-control equipment and also to better furnace conditions, namely, greater furnace volume and what is undoubtedly a superior arrangement of furnace for stoker firing, namely, a long gas path from the head of the stoker to the boiler-heating surface, as contrasted with the comparatively short gas path for No. 54.

(7) It is interesting to note that for unit No. 4, which was installed as an underfeed stoker, with a projected length including ashpit of 17 ft 6 in., after a long period of experimenting and numerous changes which developed it to its best operating condition as a purely underfeed stoker before the installation of the link grate, a limitation on stable operation for long periods was reached at a burning rate of about 55 lb per sq ft per hr. This stoker operated with preheated air.

For unit No. 7, an underfeed stoker of different manufacture with a projected length of 26 ft 7½ in. including ashpit, after a long period of development which brought it to its best operating condition as a purely underfeed stoker, without zoned-air control apparatus, a limitation on stable operation for long periods was reached at about the same value of 55 lb per sq ft per hr.

It is known that this capacity limitation does not apply to extremely short stokers, probably because of the better opportunity for operating supervision and the quicker response of the fuel bed to control measures. The exact length at which this capacity limitation becomes effective is not known from experience at Hudson Avenue; it is probably considerably shorter than the approximately 13-ft-long underfeed section of the original unit No. 4 installation.

(8) The use of the link-grate overfeed section at the lower end of an underfeed stoker had the effect of raising the capacity limitation compared with the purely underfeed stoker. With a six-foot section of link-grate construction substituted for the usual tuyère row and retort construction, the maximum, continuous, overall coal-burning rate on the combined underfeed and overfeed area of the stokers described was raised to approximately 65 lb per sq ft per hr. An explanation of this increase in coal-burning rate may be in the following: The pure underfeed section is shortened, thereby facilitating its control and raising the combustion-rate limitation on that section. As the coked fuel passes on to the link-grate section it is broken down and spread out to form a very porous fuel bed, through which a windbox pressure lower than that needed for the underfeed section will pass the required combustion air. Thus, in the event of the formation of a weak spot in the fuel bed, the potentialities for the blowing out of the local section of the fuel bed, due to a sudden conversion from windbox static pressure to velocity pressure, are reduced.

(9) The use of zoned-air control equipment brings back to the long stoker the control element formerly lost by comparison with the short stoker. The provision of metering boxes and

gages to indicate the flow conditions enables the operator to interpret the condition of the fuel bed at various points. The adjacent damper control provides the means to control the flow to correct any irregular condition indicated by the metering gages. The effectiveness of this control may be judged from the fact that during the two consecutive 24-hour test runs on No. 74 stoker at a coal-burning rate of about 75 lb per sq ft per hr, the air control alone was manipulated, the strokes of the six secondary pushers were not changed, gear-box ratio was not changed, and stoker speed was adjusted on an average of only once in two hours, by an almost negligible percentage.

(10) It is interesting to note that the apparent limitation of the plain underfeed stoker, due to lack of control of fuel and air, has been met by two manufacturers by two entirely different developments, opening up further possibilities in the efficient burning of coal for heat and power. This method of increasing capacity by the more effective use of existing floor areas and more intensive use of equipment results in a lower cost per kilowatt of installed capacity, and is particularly adapted to the peak requirement which is characteristic of electrical central-station service.

(11) A summary of the results of nine years of stoker operation, from 1925 to 1933, inclusive, giving average boiler steam-output rate, average coal-burning rate, average boiler efficiency, and average stoker-maintenance costs is given in Table 5. The relatively low average coal-burning rates result from the low station loads during the early morning hours. The completion of the link-grate installation on No. 4 unit in 1930 is reflected in the improved steam-generating efficiency and stoker maintenance figures in the years following.

(12) As to the future possibilities for higher coal-burning rates per square foot of stoker area (it being an author's prerogative to make a few predictions), the chief loss of magnitude which remains to be attacked by future development is the cinder loss. It would seem that to decrease this loss, stoker grates and coal-feeding mechanisms will have to be changed radically from their present, design to one which will produce a much higher ratio of air-admitting area to total grate area and which will distil the volatile, and coke the fuel rapidly within a very short section of the total stoker length.

In closing, the authors wish to state that in presenting this paper it is their intention to make available information on the performance and capacity abilities and limitations of long stokers when burning eastern bituminous coal; the stokers discussed representing, in the present state of the art, the most advanced developments in stoker length and coal-burning capacity.

The authors also wish to compliment the manufacturers on the persistence displayed by them in finally overcoming the many problems with which they had to cope and on the successful completion of their contracts and meeting of their guarantees.

Use of Current Meters for Precise Measurement of Flow

By FLOYD A. NAGLER[1]

In this paper the author deals with the art of flow determination by current meter, compares the accuracy of this method with that of other means of water measurement, and draws attention to a number of precautions which should be observed in order to minimize error. He compares also the characteristics of cup and screw meters, discusses meter ratings, the effects of turbulence and angularity of flow, describes the channel sections best suited to accurate gaging, and covers in some detail the technique of flow determination under varying conditions in the field.

Precision Obtainable Only With Proper Use of Reliable Meter

THE CURRENT METER is an indicator of the velocity at only one point in the cross-section of a stream; it does not measure flow directly. Flow may be obtained, however, with any reliable current meter more or less precisely depending upon the intelligence displayed in the use of this instrument in scanning the cross-section of the stream and the consideration given to the variations and disturbances in flow. Its use in measuring flow is analogous to the technique required in using the pitot tube or any other velocity-measuring instrument. Thus, perfect instruments do not insure precise measurements. The proper use of the current meter in integrating the velocities is as essential to a precise measurement of flow as the reliability of the current meter itself in indicating the correct forward component of velocity along its axis.

Types of Meters

Current meters in common use at the present time are generally classified, with respect to the type of their rotating element, as either cup or screw meters.

The rotating element of a cup meter turns about a vertical axis. This type is being used extensively by the United States Geological Survey who have adopted the improved Price meter (see Fig. 1) as the most dependable current meter for use in the measurement of the flow of American rivers.

The propellers of screw meters rotate about a horizontal axis. Meters of this type are available in many designs, the most common of which are the Fteley, Haskell, Hoff, and Ott meters. The Haskell meter (Fig. 2) has been employed in measuring the discharge of some of the larger American rivers, the new Hoff meter (Fig. 3) has been used extensively in some of the western states, while Ott meters of several different designs (Figs. 4 and 5) are occasionally used in making the more precise measurements of flow in volved in laboratory work and hydroelectric development

Meter Ratings

The characteristics of these meters as indicated by their ratings in still water are well known:

(*a*) Meters are available which with good care maintain their rating in an entirely satisfactory manner for velocities greater than one foot per second.[2]



Fig. 1 Improved Price Current Meter

(*b*) In contrast to the screw meter, the cup meter does not register properly near the water surface or the walls of the channel.[3]

(*c*) Like all instruments of precision, meters of the same type differ in their rated characteristics. They also give different results with different types of support. Hence, in rating, meters should always be supported in the manner in which they are used in the field.[4]

---

[1] Deceased. Dr. Nagler was graduated from Michigan State College in 1914 with the degree of B.S. In 1915 he received the degree of M.S.E. and in 1917 that of Ph.D. from the University of Michigan. From July, 1909, to September, 1910, he was electric meterman with the Commonwealth Power Co. and from October, 1914, to June, 1917, he was a graduate student in hydraulic engineering at the University of Michigan. From 1917 to 1920 Dr. Nagler was associated with Robert E. Horton, Consulting Engineer, Albany, N. Y., and in 1920 went to the State University of Iowa as assistant professor. In 1922 he became associate professor of mechanics and hydraulics and in 1928 he was appointed professor of hydraulic engineering at the State University of Iowa. Dr. Nagler held this position until his death, November 10, 1933.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., December 4 to 8, 1933, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "The Rating and Use of Current Meters," by C. Rohwer, Tech. Bull. No. 3, Colorado Agric. College, p. 126.

[3] "Accuracy of Stream Measurements," by E. C. Murphy, Water Supply and Irrigation Paper No. 56, 1901, U. S. Geological Survey.

"Behavior of Cup Meters Under Conditions Not Covered by Standard Ratings," by F. C. Scobey, *Jour. of Agric. Research*, May 25, 1914, pp. 77–83.

"The Rating and Use of Current Meters," by C. Rohwer, Tech. Bull. No. 3, Colorado Agric. College, p. 126.

[4] "Accuracy of Stream Measurements," by E. C. Murphy, Water Supply and Irrigation Paper No. 56, 1901, No. 95, 1903, U. S. Geological Survey.

"Meter Ratings With Various Suspension Arrangements" and "The Rating and Use of Current Meters," by C. Rohwer, Tech. Bull. No. 3. Colorado Agric. College, pp. 127 and 128.

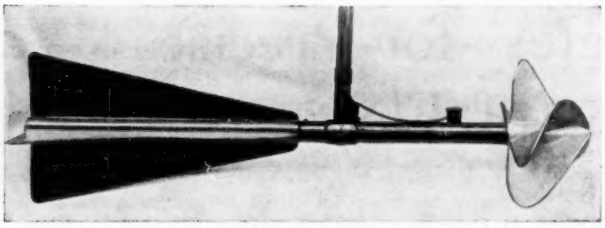


FIG. 2 HASKELL CURRENT METER

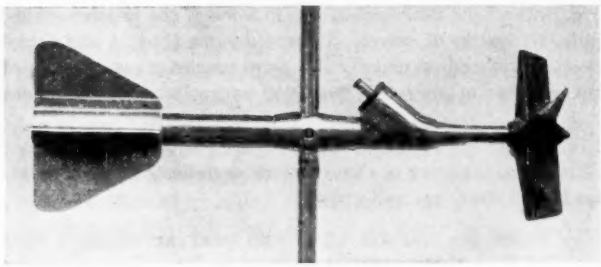


FIG. 3 HOFF CURRENT METER

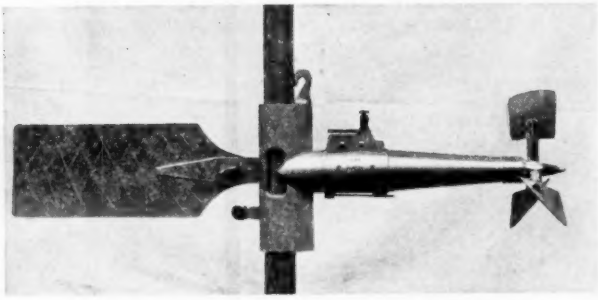


FIG. 4 OTT CURRENT METER—TYPE VIB

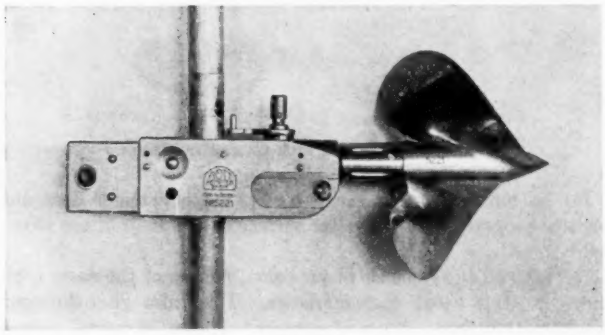


FIG. 5 OTT CURRENT METER—TYPE V

(*d*) Ratings at tangent stations have generally been considered to be more reliable than ratings made at rotary stations. However, there is generally good agreement between the results obtained at tangent and rotary stations, although a small difference in rotation of the meter has been observed at some rotary stations.[5]

STILL-WATER RATING VS. PERFORMANCE IN FLOWING WATER

The streamline filaments encountered by a current meter when towed through quiet water are obviously quite different from the more or less perturbed movements of water that strike the rotating element of a meter when held stationary in a flowing stream where parallel filaments of flow seldom are seen and cross currents, vortices, and eddies are frequently encountered.

A few attempts have been made to discover the difference between still-water ratings and performance in flowing water by towing the meter with and against a steady current.[6] At ordinary velocities the still-water rating has given no different result from that obtained in mildly flowing water. However, to date, these tests have been made only in well-behaved water with only a mild current, hence the only conclusion that may safely be drawn is that still-water ratings should give satisfactory indication of meter performance when ideal conditions of flow are encountered. Towing experiments of this type in streams with higher velocity and greater turbulence are needed to check the results obtained by analytical methods.

EFFECT OF TURBULENCE

When a current meter operates in a turbulent stream, the action of the water upon the propeller differs from the action of the streamline flow encountered in rating, in that

(*a*) The velocity of successive portions of the stream coming in contact with the propeller may vary rapidly in magnitude

(*b*) The distribution of the filaments of velocity over the plane of the propeller may not be uniform

(*c*) The water frequently strikes the rotating element at an oblique angle

(*d*) The obliquity of the current coming in contact with the propeller may vary rapidly in direction.

ERROR DUE TO INERTIA OF ROTATING ELEMENT

The error in registration due to failure of the meter to respond to rapid changes in water velocity has been studied both by oscillating the meters in an upstream and downstream direction while being towed in still water and also by producing similar oscillations while measuring the velocity of flowing water. Under the influence of rapid variations in the velocity of the flowing water, experiments have indicated that all types of meters give satisfactory performance unless the velocity is exceedingly low, or unless upstream velocities are encountered. The meters with light aluminum or rubber propellers give no better performance in this respect than those with rotating elements made of brass or copper.[7] In this connection it is interesting to note that the Robinson cup anemometer over-registers in gusty winds of variable speed.[8]

EFFECT OF OBLIQUE CURRENTS

The effect of oblique currents on various types of meters has been studied by towing meters in still water with the meter axis inclined at various horizontal and vertical angles, by holding the

[5] Discussion by Charles M. Allen, Proc. Engrg. Soc. of Western Pa., vol. 30, 1914–1915, pp. 401–402.

"The Rating and Use of Current Meters," by C. Rohwer, Tech. Bull. No. 3, Colorado Agric. College, pp. 128–130.

[6] "Über die Umlaufbewegung der hydrometrischen Flügel," by Hajos, 1902.

"The Rating and Use of Current Meters," by C. Rohwer, Tech. Bull. No. 3, Colorado Agric. College, pp. 56 and 57.

[7] "Characteristics of Cup and Screw Meters," by B. F. Groat, Trans. A.S.C.E., vol. 76, 1913, pp. 819–840 and 852–870.

"Chemi-Hydrometry and Its Application to the Precise Testing of Hydroelectric Generators," by B. F. Groat, Trans. A.S.C.E., vol. 80, 1916, pp. 1231–1271.

"Effect of Turbulence on the Registration of Current Meters," by David L. Yarnell and Floyd A. Nagler, Trans. A.S.C.E., vol. 95, 1931, pp. 780–784.

[8] "The Effect of Inertia on the Motion of an Anemometer Cup Wheel Exposed to a Wind of Variable Speed," by G. Grimminger, Bull. Amer. Meteorological Soc., June-July, 1933, pp. 161–164.

meters stationary in a steady current with the meter axis turned at various angles to the direction of flow, and by oscillating the meter laterally or vertically while towing it in still water.[9,10,11] The results of all such experiments are in fair agreement, and apparently the effect of oblique currents upon the registration of various types of meters can be determined with satisfactory precision. The following significant facts have been determined from these tests:

(*a*) Screw meters under-register the forward component of oblique lateral currents while cup meters over-register. However, at small angles some older types of screw meters have been known to over-register slightly, while some newer types have practically perfect characteristics.

(*b*) Oblique vertical currents are also under-registered by screw meters, while cup meters may slightly under- or over-register the forward component of these same currents, depending somewhat upon the type of cup meter and whether the current is upward or downward.

(*c*) The interference of the meter frame is such that the registration of the forward component of an oblique current varies slightly, depending upon whether the current approaches the meter from right or left and from above or below.

(*d*) All meters show a slight variation in the percentage of over- or under-registration of the forward component of oblique currents of a given angle depending upon its velocity.

(*e*) The Haskell and Hoff meters under-register ordinary oblique currents to a greater extent than the Price meter over-registers these same currents.

(*f*) The best characteristics are shown by the Ott meter. If the obliquity does not exceed 20 deg its effect upon the registration of the better designs is negligible.

Other Effects of Turbulence

Very few published data are available upon the effect of non-uniform velocity distribution over the plane of the propeller and rapid changes in direction and obliquity of the stream filaments. Although there are records of many flow measurements by current meter compared with other standard methods, the results cannot be considered conclusive since the technique of using the meter and the other standard method may have entirely obliterated the deviation in meter characteristics due to turbulence.

Perhaps the Iowa tests,[12] in which various meters were held at fixed points in violently turbulent streams, are most significant in that the various meters arranged themselves in exactly the same order in degree of over- and under-registration as that determined by test of these same meters in smoothly oblique flow. This fact tends to support the present practice of precise measurements in turbulent water where the amount of absolute error in measuring the forward component of velocity is determined by measuring this velocity with two meters of different under- or over-rating characteristics. The difference in registration applied to the "oblique characteristic curve" indicates the absolute error of the meter giving the better performance. Hence, this average obliquity is assumed to represent a rather obscure measure of the degree of turbulence.

Use of Current Meter

As a metering instrument, all that is expected of the ideal current meter is that it register the true forward component of velocity of only that filament of the stream encountering the propeller. Whether this instrument or any other instrument that indicates only the velocity at a single point in the cross-section of a stream can be used to obtain a precise measurement of the flow of a stream, raises a question which is entirely independent of the considerations that have thus far been given to meter characteristics. The current meter can be used to scan the irregularities in velocity across the section of a stream, but the precise measurement of flow demands that this scanning be thoroughly done and that there be no doubt as to the exact cross-section of flow.

Ideal Section

In addition to a section free from turbulence so that the meter performance will be beyond question, the ideal location for a precise current-meter measurement should (*a*) insure reliable scanning by the maintenance of steady flow, with the minimum amount of irregular variations in velocity throughout the section; (*b*) furthermore, in order to accurately measure the area of the cross-section, the metering section should be fixed and regular. It is ridiculous to expect to obtain a precise measurement of flow where the bed of the stream is rocky and irregular or soft and movable. Unfortunately, ideal sections seldom exist, and the degree of precision attained in a measurement can always be gaged by the extent to which the ideal section is realized.

Steady Flow

Water generally moves in pulsations and waves. Flow cannot be measured precisely with current meters when the pulsations are large in magnitude, or when the rate of discharge is changing rapidly. Increase in the number of meters and period of observation may reduce errors in certain cases of unsteady flow.

Expanding cross-sections must be avoided. Due to the steadying influence of contracting channels, sections located where flow is accelerating often give better measuring conditions than channels with a uniform cross-section.

Excellent metering conditions were encountered in a vertical section through the nappe of water discharging at a depth of eleven feet over the Keokuk spillway.[13] Meter readings could be duplicated to the tenth of a second at any given point. The accelerating water was absolutely steady, and lacked the pulsations in velocity commonly encountered in most measurements. Converging irrigation flumes have also been found by Rohwer to furnish the best conditions for current-meter measurements.[14]

---

[9] "On the Current Meter, Etc.," by F. P. Stearns, Trans. A.S.C.E., vol. 12, 1883, pp. 301–338.

"An Investigation of the Use and Rating of the Current Meter," by C. P. Rumpf, *Engineering News*, vol. 71, 1914, pp. 1083–1084.

"The Measurement of the Velocity of Flowing Water," by L. F. Moody, Proc. Eng. Soc. of Western Pa., vol. 30, 1914–1915, pp. 320–322.

"Behavior of Cup Meters Under Conditions Not Covered by Standard Ratings," by F. C. Scobey, *Jour. Agric. Research*, May 25, 1914, pp. 77–83.

"The Rating and Use of Current Meters," by C. Rohwer, Tech. Bull. No. 3, Colorado Agric. College, pp. 35–51.

[10] Discussion by E. H. Brown and Floyd A. Nagler, Proc. Eng. Soc. of Western Pa., vol. 30, 1914–1915, pp. 415–424.

"Effect of Turbulence on the Registration of Current Meters," by David L. Yarnell and Floyd A. Nagler, Trans. A.S.C.E., vol. 95, 1931, pp. 784–791.

[11] "Characteristics of Cup and Screw Meters," by B. F. Groat, Trans. A.S.C.E., vol. 86, 1913, pp. 819–840 and 852–870.

"Pitot Tube Formulas, Facts and Fallacies," by B. F. Groat, Proc. Eng. Soc. of Western Pa., vol. 30, 1914–1915, pp. 351–366.

"Chemi-Hydrometry, Etc.," by B. F. Groat, Trans. A.S.C.E., vol. 80, 1916, pp. 1231–1271.

[12] "Effect of Turbulence on the Registration of Current Meters," by David L. Yarnell and Floyd A. Nagler, Trans. A.S.C.E., vol. 95, 1931, pp. 770–780.

[13] "Experiments on Discharge Over Spillways and Models, Keokuk Dam," by Floyd A. Nagler and A. Davis, Trans. A.S.C.E., vol. 94, 1930, pp. 777–844.

[14] "The Rating and Use of Current Meters," by C. Rohwer, Tech. Bull. No. 3, Colorado Agric. College, pp. 120–122.

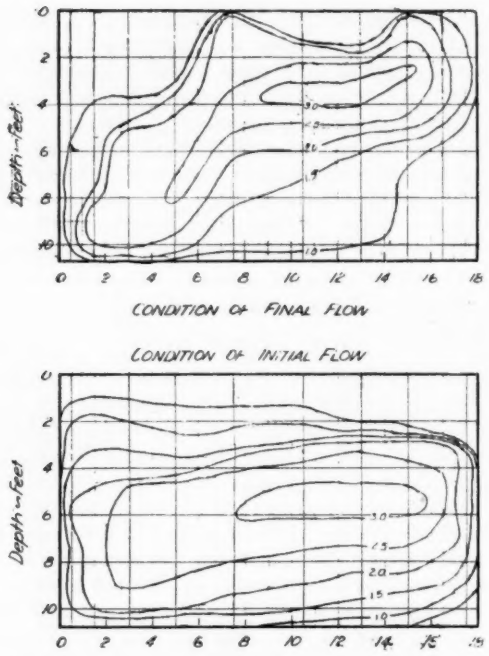


FIG. 6 VELOCITIES IN OPEN WHEEL PIT, 10 FT UPSTREAM FROM TURBINE
(Velocity contours in ft per sec; discharge = 344 cfs.)

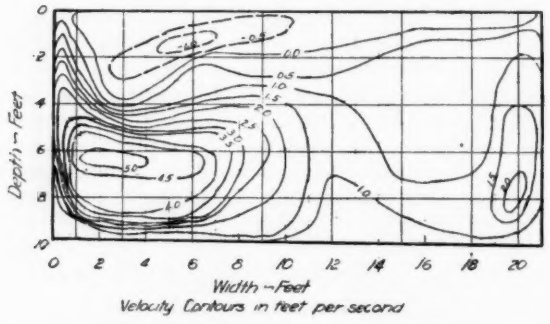


FIG. 7 VELOCITIES IN TAILRACE, 10 FT DOWNSTREAM FROM OUTLET OF ELBOW DRAFT TUBE
(Velocity contours in ft per sec; discharge = 330 cfs.)

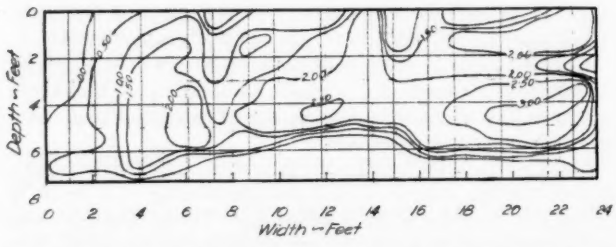


FIG. 8 DISCHARGE MEASUREMENT BY CURRENT METER AT SECTION 2 FT DOWNSTREAM FROM TRASH RACKS WITH INTERFERENCE BY SUPPORTING BEAMS AND POSTS
(Velocity contours in ft per sec; discharge = 241 cfs.)

Unstable conditions of flow must also be avoided. In an otherwise good metering section a complete turnover in velocity distribution has been experienced during a measurement. Either distribution shown in Fig. 6 could be secured, depending upon whether the condition was approached with increasing or decreasing flow. Such changes in flow distribution, if near the water turbine, are not without effect upon the turbine efficiency.

TYPE OF SECTION GENERALLY ENCOUNTERED IN WATER-POWER DEVELOPMENTS

Unfortunately, ideal current-meter sections are seldom provided in the construction of water-power plants. In this country, current meters have been used mainly to measure the discharge of low-head plants with open-flume settings where the current meter has generally offered the only means of more or less precise measurement. Draft-tube design has not yet reached the plane where flow conditions in short tailraces are at all adequate for the precise measurement of flow. The distortion of contours in Fig. 7 is only a mild indication of the actual disturbances in the tailrace.

Generally, the flow is metered upstream from the water tur-

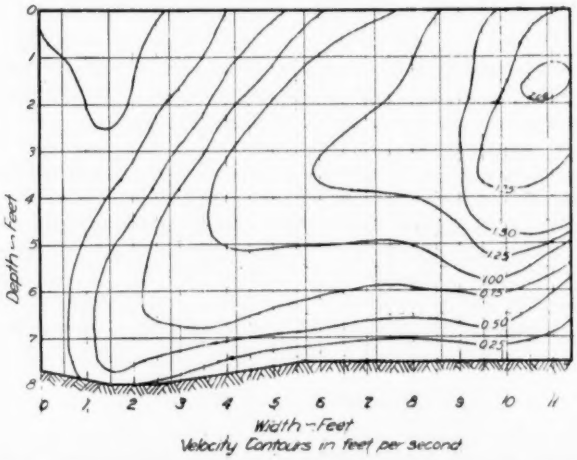


FIG. 9 DISCHARGE MEASUREMENT BY CURRENT METER IN INTAKE FLUME DOWNSTREAM FROM TRASH RACKS
(Velocity contours in ft per sec; discharge = 72.8 cfs.)

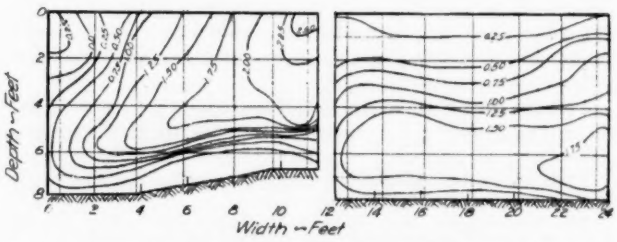


FIG. 10 DISCHARGE MEASUREMENT BY CURRENT METER AT SECTION 7 FT DOWNSTREAM FROM TRASH RACKS
(Velocity contours in ft per sec; discharge = 190.5 cfs.)

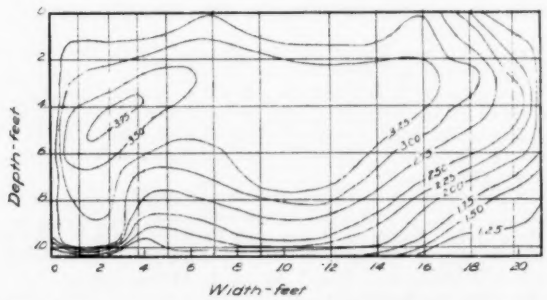


FIG. 11 DISCHARGE MEASUREMENT BY CURRENT METER AT SECTION IN APPROACH FLUME TO VACUUM SETTING
(Velocity contours in ft per sec; discharge = 597.3 cfs.)

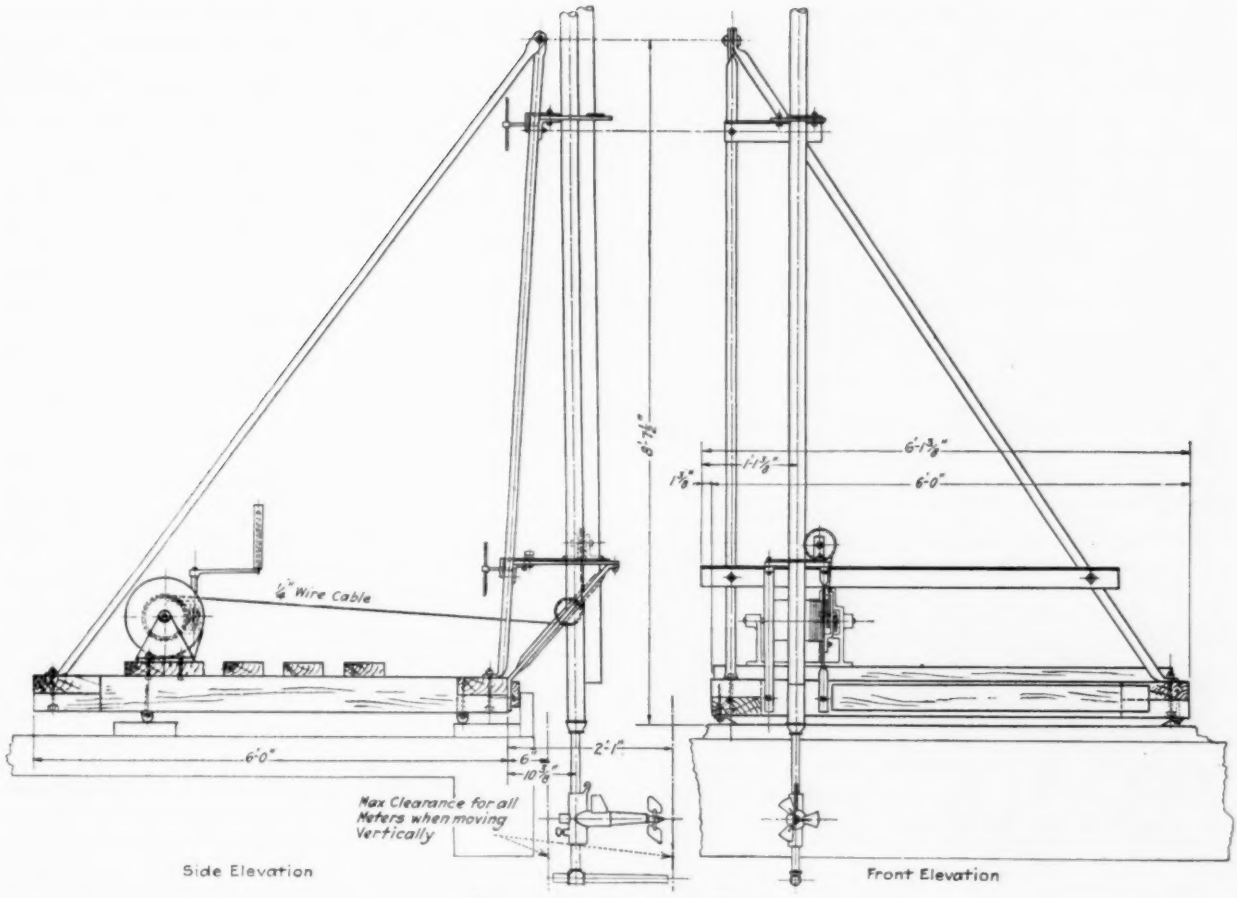


FIG. 12 CURRENT-METER HOIST

bine in the headrace or intake flume. The opening at the head gate or stop log groove has usually been chosen as the best available section, although dead water and turbulence caused by contractions around curtain walls, pier noses, trash racks and their supports often introduce complications into the metering section. Sometimes these may be eliminated at slight expense by the construction of false guide walls. As many meters as desired may be mounted on a frame using the slots for guides to lower and hold the meters in the gaging section. If available, a gaging section upstream in the headrace generally is preferred to metering in the power house or too near the unit itself.

Fig. 8 shows the type of velocity distortion obtained when operating downstream from and adjacent to interfering posts and rack supports. It is generally impractical to scan the velocities in a section of this type with great accuracy. Sections located a greater distance downstream from the trash racks give better, although not ideal, metering conditions as shown in Figs. 9 and 10. The distribution shown in Fig. 11 was almost ideal.

SUSPENSION OF METERS

If the variation in velocities across the metering section is to be determined by measuring the velocities at a limited number of points, the place at which the meter is operated must be exactly known. Thus, suspension of the meter by cable is in general eliminated as a feasible procedure in precise measurements of flow. Usually suspension from rods will insure the required accuracy in location, although it may be necessary to support the rods from a rigid frame, by stay wires, or a specially designed hoist similar to that shown in Fig. 12.

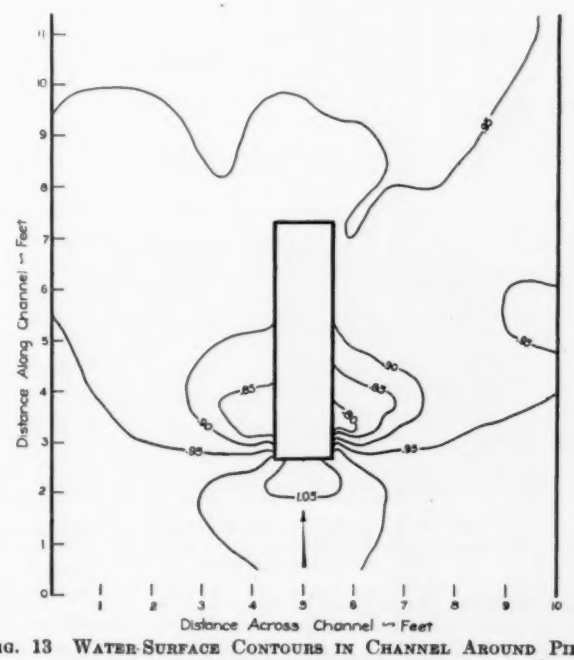
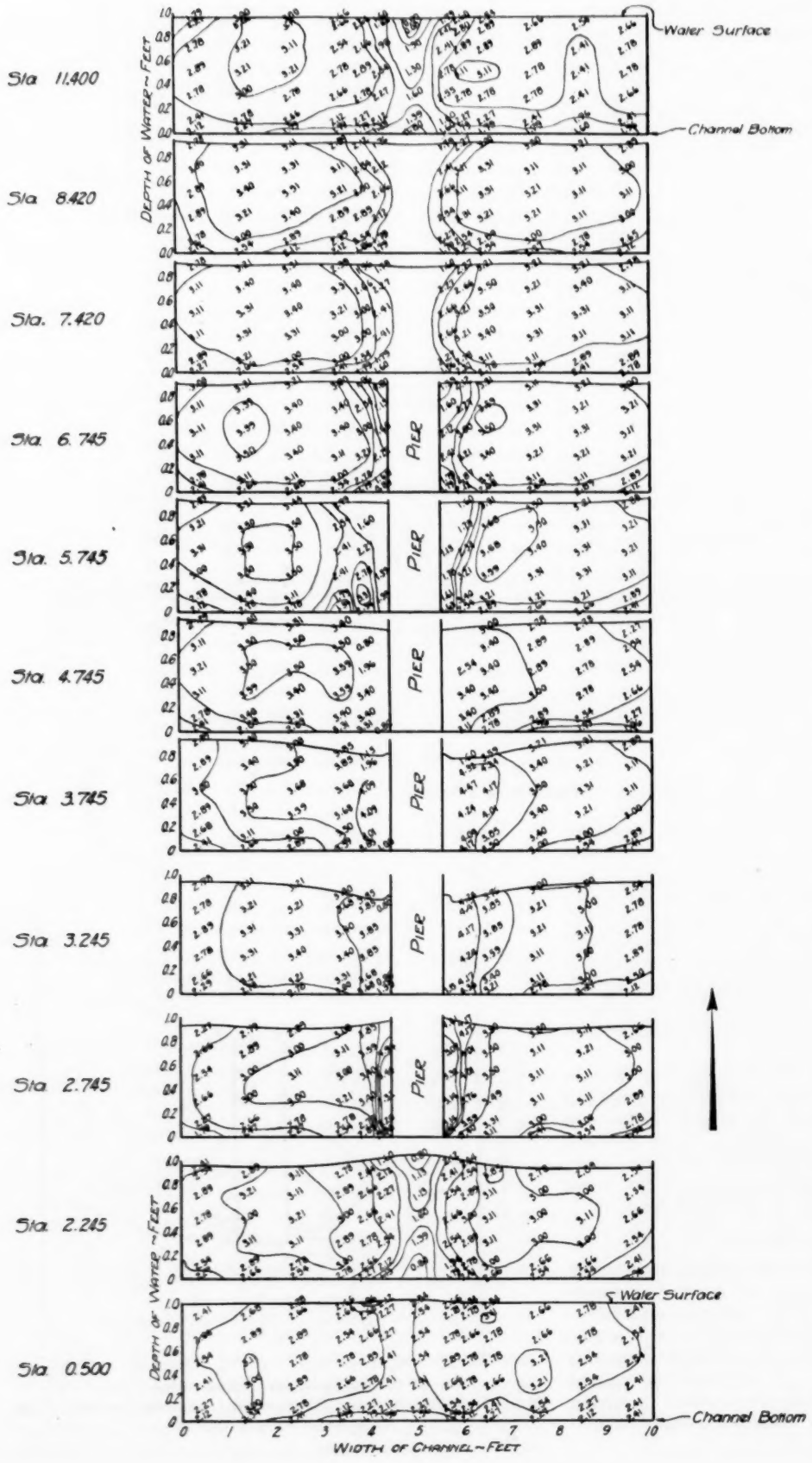


FIG. 13 WATER-SURFACE CONTOURS IN CHANNEL AROUND PIER
(Test No 2; discharge 24.1 cfs.)



When rigidly supported by a rod, meters have generally been found to rotate faster than with cable suspension. Hence, they should be rated with this same type of support.

When steady oblique currents of high velocity are encountered, it may be advisable to allow the meter to swing in line with the filaments of flow making an accurate measurement of the angle, rather than attempting to operate the meter cross-wise with the current.

The obstruction offered by the supporting rods or frame may tend to retard or sometimes even accelerate the rotation of the meter, depending upon the location of the meter with respect to the obstruction. A picture of the alteration in current velocities adjacent to a 14-in. obstruction in a 10-foot channel is shown in Figs. 13 and 14. Current velocities are decreased immediately ahead of the obstruction, but not for a great distance, and there is considerable acceleration of the filaments of flow curving around the obstruction. The possible effect of objects near the meter is so questionable that meters should always be rated in the manner used in the field.

Number and Distribution of Measuring Points

One may compute by formula the number of points required to adequately scan the cross-section of a stream,[13] but consideration must always be given to the configuration of the velocity diagram. Measurements using only one point located at $^2/_{10}$ depth, two points in a vertical at $^2/_{10}$ and $^8/_{10}$ depth or three points in a vertical at $^2/_{10}$, $^6/_{10}$, and $^8/_{10}$ depth are not

[13] Swiss standard for water measurements gives $n = 4.3$ to $7.65 \times \sqrt{A}$, where A is the area in sq ft and n the number of measuring points.

Fig. 14 (Left) Velocity Distribution in Channel Around Pier—Test No. 2

(Single pier in center of channel; width = 1.17 ft, length = 4.67 ft; square nose and tail with nose at sta. 2.63 and tail at 7.30; discharge past pier measured by sharp-crested weir = 24.1 cfs; contours plotted at 0.5 ft per sec interval.)

adequate for precise measurements. In order to reproduce a curved velocity surface by observations at a limited number of points, measurements must be made at a sufficient number of points to define the velocity surface throughout the entire cross-section.

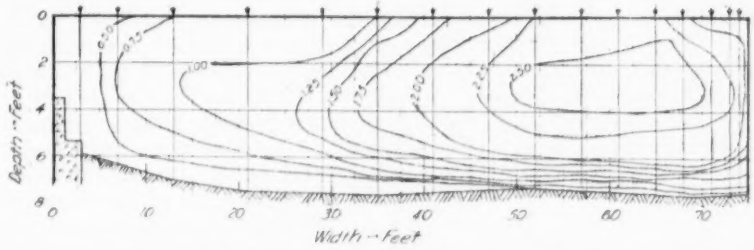


FIG. 15 DISCHARGE MEASUREMENT BY CURRENT METER AT SECTION 5 FT AHEAD OF TRASH RACKS
(Velocity contours in ft per sec; discharge = 807 cfs.)

Hence, measuring points should be distributed so that they are more numerous where the curvature in the velocity surface is greatest. Thus, a closer spacing of the verticals and points on the vertical is generally required as one approaches the periphery of the cross-section. An irregular distribution of vertical traverses to meet a distorted condition of flow is shown in Fig. 15. In some instances it has been necessary to use a shrouded meter similar to the Ott meter shown in Fig. 16, or build a special guard around the meter to protect the propeller against injury in sections adjacent to the walls or bottom. The guard shown in Fig. 17 caused a two per cent retardation of the meter, although no portion of the frame was within an inch of the propeller.

The number of verticals is generally reduced to a minimum, consistent with accuracy. Reducing the number of verticals to one-half has been found to double the error, and further reduction to one-quarter has tripled the error. In any event, a preliminary exploration of the section is advisable to fix the meter traverses in their proper locations and to determine the number of points required for each traverse. Care should be exercised that there is no unbalance between the vertical and horizontal spacing of the measuring points.

Small meters are preferred for use in small measuring sections. The United States Engineer Department while engaged in model research at the Iowa Institute of Hydraulic Research developed the tiny meter shown in Fig. 18. The friction in this meter has been reduced to the extent that a "straight line" velocity relation is obtained for current velocities as low as one-tenth of a foot per second.

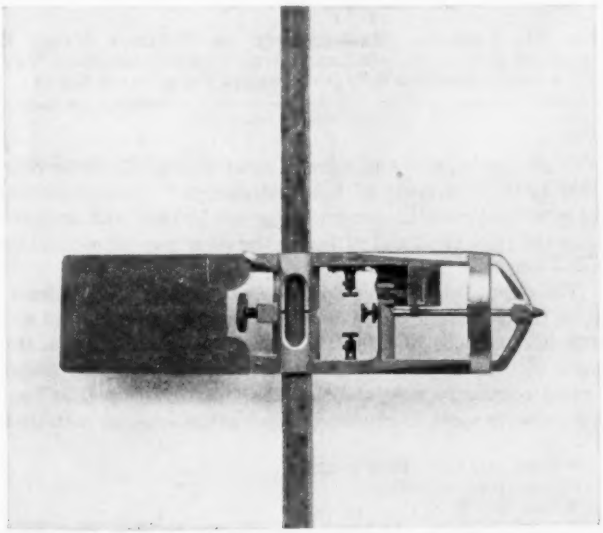


FIG. 16 SMALL OTT CURRENT METER—TYPE X—WITH CIRCULAR GUARD

NUMBER OF METERS

The simultaneous use of one meter for each vertical traverse is generally the maximum practical desideratum. If only one or a few meters are available, the measurement consumes more time and for precision requires steadier flow. Where conditions are stable, one meter may be permanently installed at a single point as an index to general flow conditions, but the record of an index meter does not constitute a precise measurement of flow.

The mean velocity in a vertical traverse is generally determined by plotting the velocity readings at the specific measuring points. However, the average velocity is sometimes obtained by moving the meter at a constant speed through the vertical—thus obtaining a single "integrated" reading—or by both moving and stopping for a fixed period at predetermined equally spaced points, thus obtaining a single "semi-integrated" reading as well as velocity measurements at definite points from which the vertical velocity curve may be drawn. Mr. Strieff has suggested the use of the Tchebycheff formula for determining the proper loca-

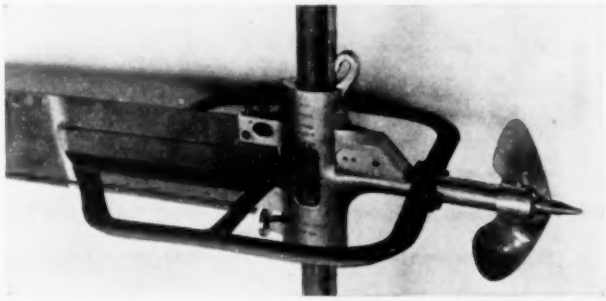


FIG. 17 OTT CURRENT METER—TYPE IVA—WITH SPECIAL METAL GUARD

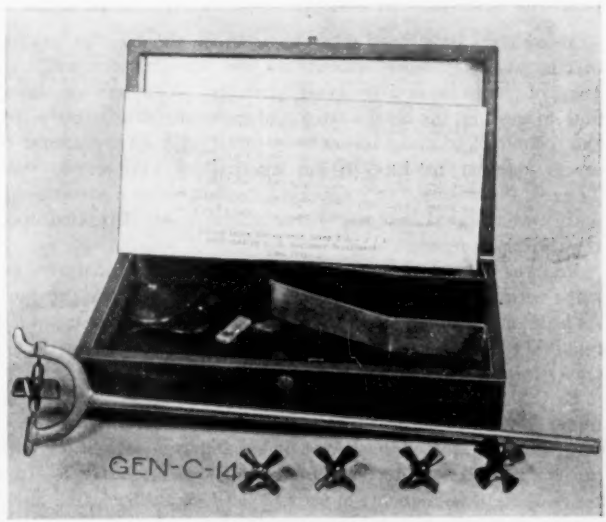


FIG. 18 SMALL METERS FOR LABORATORY SERVICE
(Developed in 1931 at Iowa Institute of Hydraulic Research by United States Engineer Dept.)

tion of any number of points in the vertical, the arithmetic average of which will give the true mean vertical velocity.[16] All of these methods are designed primarily to save labor and time. They are not, however, fool proof against gross error, and to be used with confidence the measuring section must be a good one.

Measuring Turbulent Flow

The existence of turbulence greatly reduces the reliability of current-meter measurements. If the turbulence is mild, the better Ott meters may be trustworthy alone, although the cup meters are not. However, as already mentioned, the use of two types of meters with different registration characteristics is advisable, since the differences between their registration may be used as an index to the amount of the correction to be applied to the more accurate of the two meters.

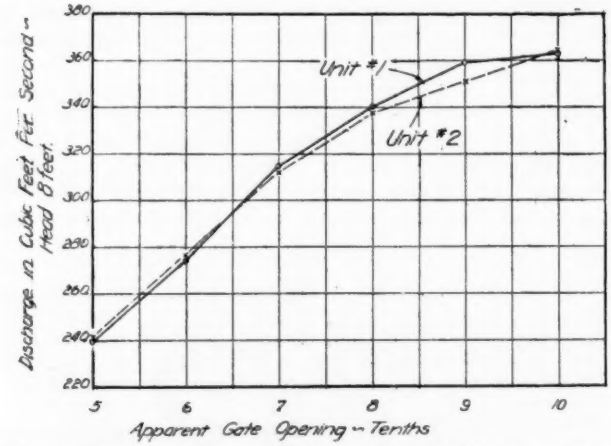


Fig. 19 Discharge Measurements by Current Meter for Two Similar 74-In. Turbines
(Separate settings; metering sections reduced to head of 8 ft.)

Comparative Tests by Current Meter and Other Methods

Many tests have been made to establish the error or validity of current-meter measurements by the simultaneous measurement of the same flow by other methods. Although significant and interesting, the results are not always conclusive, since either the technique of using the current meter may be questioned or errors may be involved in the standard of comparison itself. Thus F. P. Stearns,[17] in comparing current-meter measurements with a weir, found that the degree of agreement depended upon the use of the meter.

Perhaps only those who have obtained good comparative results with current meters have ventured to publish the fact. At least, engineering literature contains many verifications of the reliability of current-meter measurements. The following extracts from a few may be worthy of note.

E. C. Murphy[18] concluded that the Price meter can be used under ideal conditions to measure discharge within 1 or 2 per cent, but bad use of the meter under abnormal conditions produced departures of 40 per cent from the weir discharge. In a series of 50 measurements in the Cornell Laboratory Canal, measurements by Haskell and Price meters showed a maximum variation of 4.8 per cent, but twenty of the tests showed differences of less than 1 per cent.

W. B. Gregory[19] obtained a maximum variation of 2 per cent and an average variation of 0.1 per cent when compared with a pitot tube in measuring pump discharge.

The current-meter measurements made at Massena by B. F. Groat[20] agreed with the hydrochemical gagings with an error of less than 1 per cent. He concluded that results of a "remarkably high degree of precision can be obtained, even under very unfavorable hydraulic conditions, with meters of properly related types in conjunction with an intelligent statistical study of their performance."

Robert E. Horton[21] states that the current meter has been a much abused instrument, and under suitable conditions it may quite certainly give results accurate within less than 1 per cent.

In 1914, A. Strieff made measurements of flow with Ott current meters at the Croton, Michigan, power house all of which agreed with hydrochemical gagings within 1.3 per cent.

F. C. Scobey measured discharge through wood-stave pipe[22] simultaneously by color, weir and current meter, with differences varying from zero to 6 per cent.

D. L. Yarnell made 102 comparative discharge measurements with pitot tube, weir and current meter during the period 1922–1924 at the University of Iowa Laboratory.[23] From those tests he concluded that the current meter can be used with less error than the pitot tube, and with care the error may be reduced below 2 per cent.

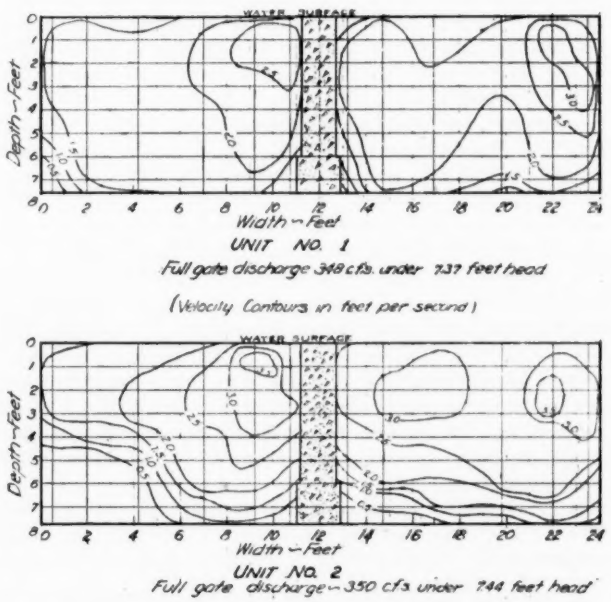


Fig. 20 Discharge Measurements by Current Meter in Separate Intake Flumes Leading to Two Similar 74-In. Turbines at Sections 6 Ft Downstream From Trash Racks
(Velocity contours in ft per sec; both sections give identical discharge of 362.5 cfs under head of 8 ft.)

The recent comparative flow measurements of Carl Rohwer[24] show the magnitude of the errors that might be anticipated with different methods of using the meter. It is unfortunate that most of the comparisons are based upon flow over an uncalibrated contracted weir and that the "multipoint method" was not properly applied. However, his conclusions agree with those

[16] Trans. A.S.C.E., vol. 95, 1931, pp. 859 and 860.
[17] Ibid., 1883, pp. 301–338.
[18] Water Supply and Irrigation Papers 47, 64, U. S. Geological Survey; also Trans. A.S.C.E., vol. 1902, pp. 370–391.
[19] Trans. A.S.C.E., vol. 28, pp. 745–769.
[20] Trans. A.S.C.E., 1916, p. 1233.
[21] Ibid., 1916, p. 1283.
[22] Bull. 376, U. S. Dept. of Agric.
[23] "Comparison of Discharge Measurements by Weir, Pitot Tube, and Current Meter" (unpublished thesis).
[24] Tech. Bull. No. 3, Colorado Agric. College.

from other investigations, that reasonably accurate measurements can be made by the use of the current meter.

Many European tests have confirmed the reliability of flow measurements by current meter mostly under ideal conditions. Perhaps the experiments of O. Kirschmer and B. Esterer[25] are the most convincing. All measurements with the Ott current meter checked reliable volumetric measurements within a maximum error of 1 per cent.

Fig. 19 shows the results of flow measurements made by current meter on two similar 74-inch water turbines in different wheel pits, requiring the use of individual metering sections. The velocity contours for full gate at the two sections are shown in Fig. 20. The close agreement, which could hardly be improved upon by any other method of water measurement, cannot be the result of an accident.

Precision in Water Measurements

Perhaps the word "precise" as applied to water measurements is merely a relative term. As applied to a current-meter measurement, the measure of precision no doubt depends upon the accuracy obtainable with other methods, all of which must ultimately be gaged by the measurement of the actual volume or weight of water concerned. However, there are no scales and but few measuring basins large enough to hold the discharge commonly measured with meters.

Even in the laboratory, errors of 1 and 2 per cent are commonly made by inexperienced engineers in so simple an experiment as checking a gravimetric measurement against a volumetric test. It is not surprising, therefore, to encounter greater errors than these in the inexperienced application of the more complicated methods of measurement of larger flows in the field, where the current meter has sometimes been selected as the last alternative where no other method can be applied.

Under ideal conditions, the best current meters can be used to secure precise measurements of flow, dependable within 2 per cent, and if used intelligently in the hands of experts, the deviation from the true value should not exceed 1 per cent. If conditions are not ideal, the degree of precision with current meters may still be better than with any other known method, and the accuracy of the result will depend upon the extent to which the metering conditions depart from the ideal.

[25] *Zeit. V.D.I.*, 1930, vol. 74, no. 44, p. 1499.

Report on Oil-Engine Power Cost for 1933[1]

THIS report presents information on the production cost of oil-engine power plants. Production cost in the meaning of this report is defined as consisting of the following items: fuel cost; lubrication cost; cost of attendance and superintendence; cost of supplies and miscellaneous; cost of engine and plant repairs.

The report includes information from 156 oil-engine generating plants, containing 398 engines, totaling 216,010.5 rated bhp. The total net output for the 156 plants in this report amounted to 259,209,519 kwhr. The coverage of this report as compared to that of previous reports is shown by the following tabulation:

Year of report.	1929	1930	1931	1932	1933
Number of plants	36	94	119	140	156
No. of engines...	107	283	330	377	398
Total rated bhp.	68,775	161,583	190,768	213,910.5	216,010.5
Total output, net kwhr.	134,766,761	309,369,930	333,066,644	282,466,690	259,209,519

The engines listed in the report are full-Diesel, vertical type and direct-connected to generators, unless otherwise noted in Table III. All Diesel plants listed are located in the United States.

Plant Numbers. The system used in former reports of designating plants by numbers has been retained. Numbers identifying plants previously reported correspond to the same plants in this report.

Period Covered. All but two of the plants of this report submitted data for a period of exactly 12 months each. Of the two exceptions, one submitted for a period of 9.3 months; the other for a period of 7.2 months.

Bases for Costs and Performances. Unit costs referred to in this report were calculated on the basis of net kilowatt-hours. The net kilowatt-hour output is found by subtracting the power used for plant auxiliaries and station lights from the total gross output of the plant.

Figures given for power output per gallon of fuel oil and of lubricating oil were calculated on the basis of the gross output of the individual units and plants.

Formulas defining running-engine-capacity factor, running-plant-capacity factor, annual-plant-load factor, and plant-service factor are as follows:

$$\text{Running-engine-capacity factor, per cent} = \frac{\text{Engine output in gross kwhr} \times 100}{\text{Kw rating} \times \text{number of hours operated}}$$

$$\text{Running-plant-capacity factor, per cent} = \frac{\text{Plant output in gross kwhr} \times 100}{\text{Total rated kwhr of individual units}}$$

$$\text{Annual-plant-load factor, per cent} = \frac{\text{Plant output in gross kwhr} \times 100}{\text{Peak load in kw} \times \text{number of hours in period}}$$

$$\text{Plant-service factor, per cent} = \frac{\text{Total rated kwhr of individual units} \times 100}{\text{Total installed kw} \times \text{number of hours in period}}$$

The expression "rated kwhr" refers to the kilowatt rating of an engine-generator set multiplied by the number of hours operated. For example, if a unit having a rating of 200 kw was operated 4000 hours, the rated kwhr equals 800,000, no matter what the actual output may have been. Thus the denominator of the expression for "Running-plant-capacity factor" and likewise the numerator for the "Plant-service factor" are arrived at by totaling the rated kilowatt hours of all plant units. In this report, the kilowatt rating of an engine-generator set is considered equal to: Rated bhp $\times$ 0.746 $\times$ 0.9.

In the strict sense of its definition, the annual-plant-load factor cannot be correctly applied to data covering any period other than one year. However, the committee extended the application of this to plants operated 8760 hr. plus or minus 2 per cent, using the actual number of hours in the denominator in each case.

The formula for "Plant-service factor" requires further explanation for special cases. The expression is an index of the actual number of hours of operation as compared with the total number of hours installed for operation. Therefore, when some units have been installed for longer periods than have others in the same plant, account must be taken of the fact in the calculation. For example, assume that a plant is reported for a twelve-month period, during which time one unit rated at 200 kw, installed before the start of the period, was operated 5000 hours. Six months after the start of the period, a unit rated at 300 kw was installed, and subsequently operated 2500 hours. The plant-service factor in per cent is therefore:

$$\frac{200 \times 5000 + 300 \times 2500}{200 \times 8760 + 300 \times 4380} \times 100$$

Since the 300-kw unit was not installed during the entire 8760 hours, but only for 4380 hours, this adjustment must be made.

Fuel and Lubricating-Oil Data. The lubricating-oil economies of 120 plants generating 95 per cent or more of their outputs by means of full-Diesel units are shown graphically in Fig. 1, in which the kwhr output per gallon of lubricating oil is plotted against running plant capacity factor. (Five of the full-Diesel plants did not report unit hours of operation, and running-capacity factor could not be calculated for these. One full-Diesel plant operated only a nominal length of time and did not require any lubricating oil. The amount of lubricating oil for one full-Diesel plant was not available.) Fuel-oil economies of 122 full-Diesel plants are shown graphically in Fig. 2, in which the gross kwhr output per gallon of fuel oil is likewise plotted against the running-plant-capacity factor. (The same five plants which did not report units hours of operation are not included.) The values plotted in Fig. 2 are not corrected for the heat content of the fuel or for altitude. The lubricating-oil and fuel-oil economies of 24 plants generating more than 5 per cent of their output by means of semi-Diesel units are shown in Figs. 3 and 4, respectively. (Five plants generating more than 5 per cent of output by semi-Diesel units did not report unit hours of operation, and running-capacity factors could not be calculated for these.)

The type of the plant was judged to be that of the engines generating 95 per cent or more of the gross output. The following types of plants are illustrated in Figs. 1 to 4, inclusive:

Diesel, four-stroke cycle, air injection

Diesel, four-stroke cycle, mechanical injection

Diesel, two-stroke cycle, air injection

Diesel, two-stroke cycle, mechanical injection, separate scavenging

---

[1] Submitted by the Subcommittee on Oil-Engine Power Cost, Oil and Gas Power Division, A.S.M.E.; H. C. Major, chairman, Committee of Public Utilities, Municipal Bldg., Rockville Center, L. I., N. Y.

Presented at the Seventh National Oil and Gas Power Meeting, State College, Pa., June 20–23, 1934, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

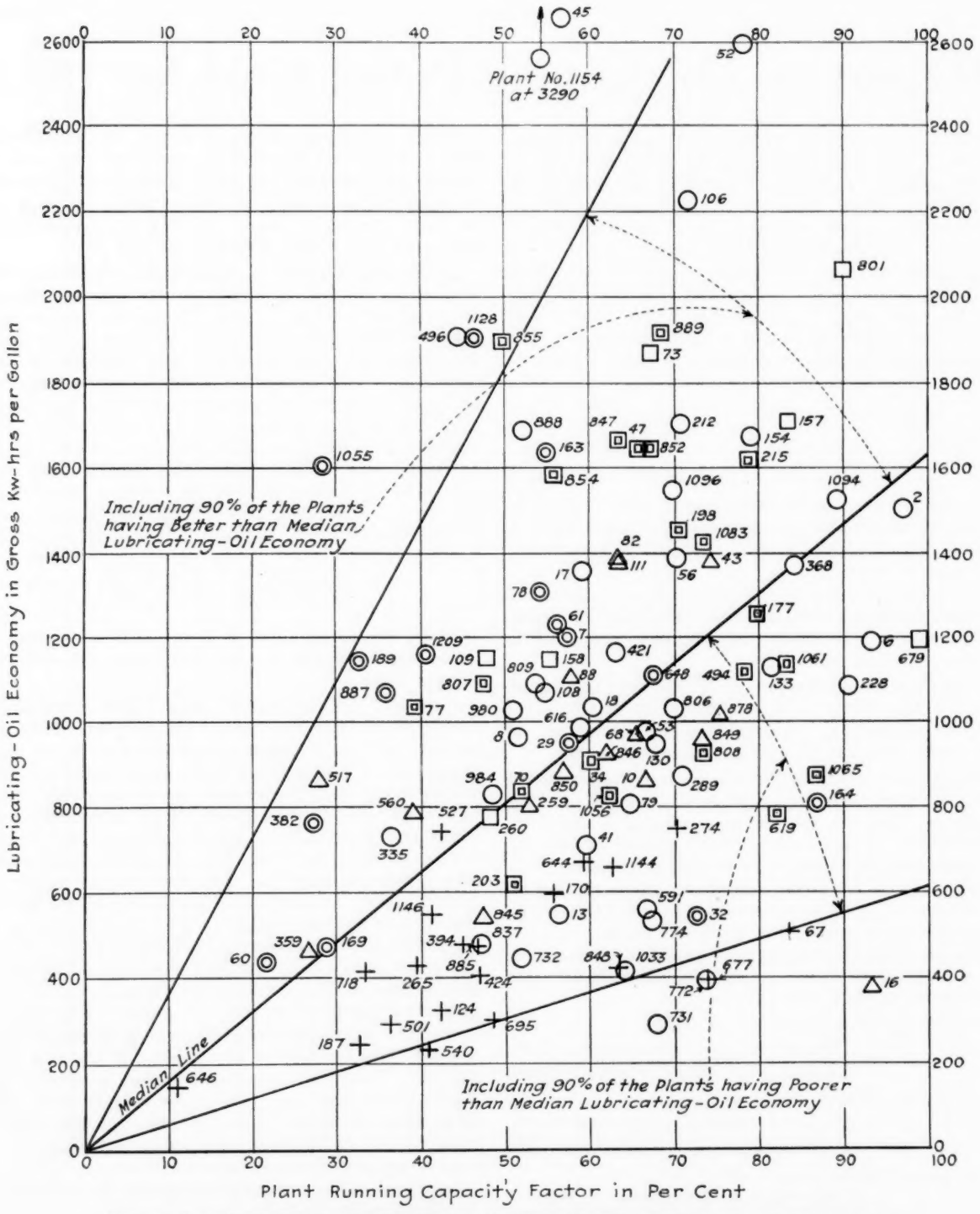


FIG. 1. LUBRICATING OIL ECONOMIES OF 120 FULL DIESEL PLANTS, NOT INCLUDING 5 FULL DIESEL PLANTS, RUNNING CAPACITY FACTORS FOR WHICH ARE NOT AVAILABLE, NOR ONE SUCH PLANT WHICH RAN A NOMINAL NUMBER OF HOURS AND REQUIRED NO NEW LUBRICATING OIL, NOR ONE FOR WHICH LUBRICATING OIL GALLONAGE IS NOT AVAILABLE

(For Key to Plant Symbols see Fig. 2)

Diesel, two-stroke cycle, mechanical injection, crankcase scavenging

Mixed-type full Diesel

Mixed Diesel and semi-Diesel.

A median line was drawn on each chart, the number of points above the line being equal to the number below. High and low boundary lines were drawn to include all but 10 per cent of the plants on the high and low sides, respectively.

Unless it is otherwise noted in Table III plants received fuel oil in tank cars.

All fuel- and lubricating-oil costs include costs for the handling of oil from the cars to the tanks.

Cooling Water. The various water-cooling systems are indicated by the following symbols:

System A—Raw water going to waste after one pass

System B—Raw water recirculated after passing over cooling tower or spray pond

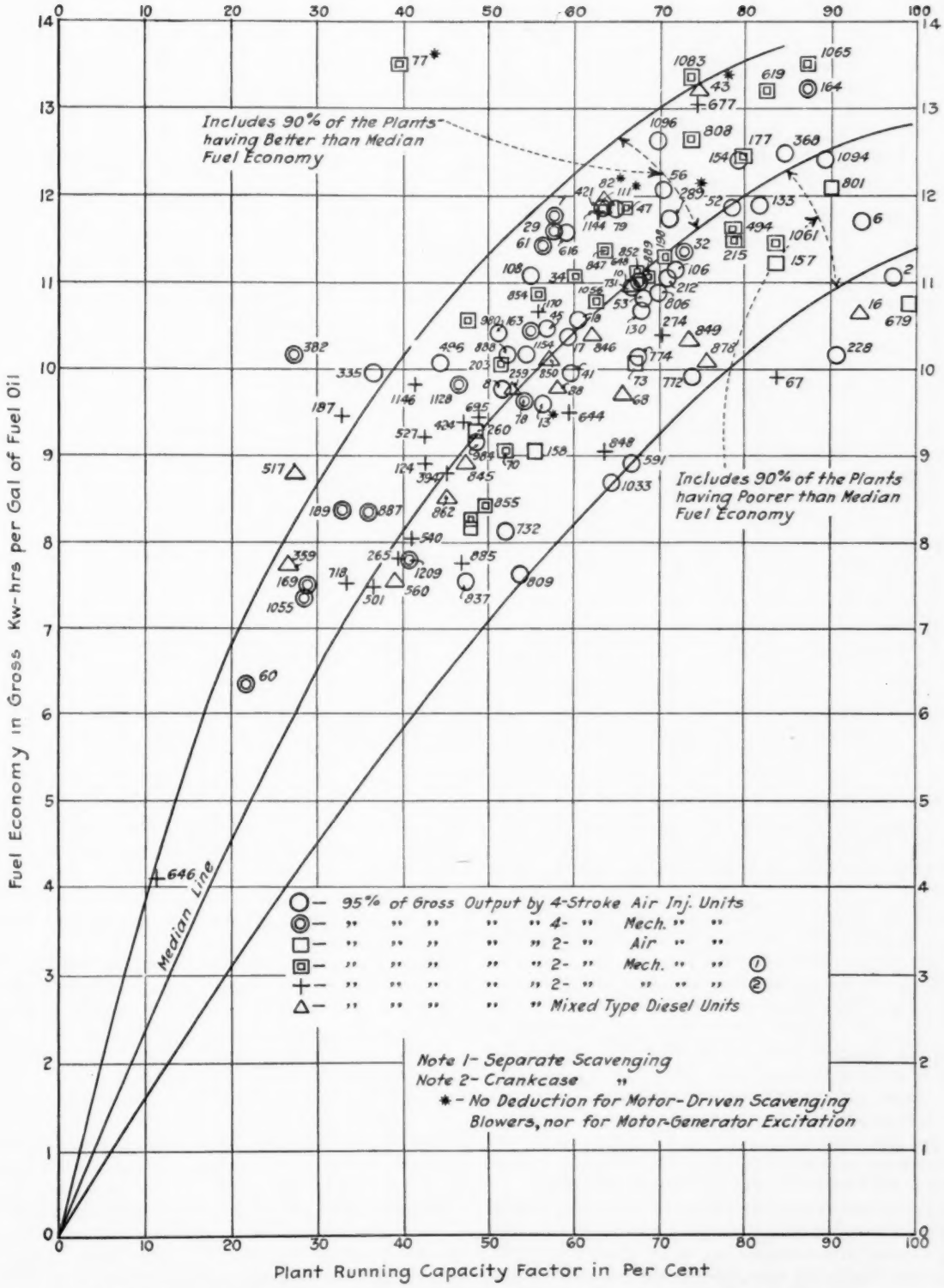


FIG. 2. FUEL ECONOMIES OF 122 FULL DIESEL PLANTS, NOT INCLUDING 5 FULL DIESEL PLANTS, RUNNING CAPACITY FACTORS FOR WHICH ARE NOT AVAILABLE
(Not Corrected for Heat Content of Fuel nor for Altitude)

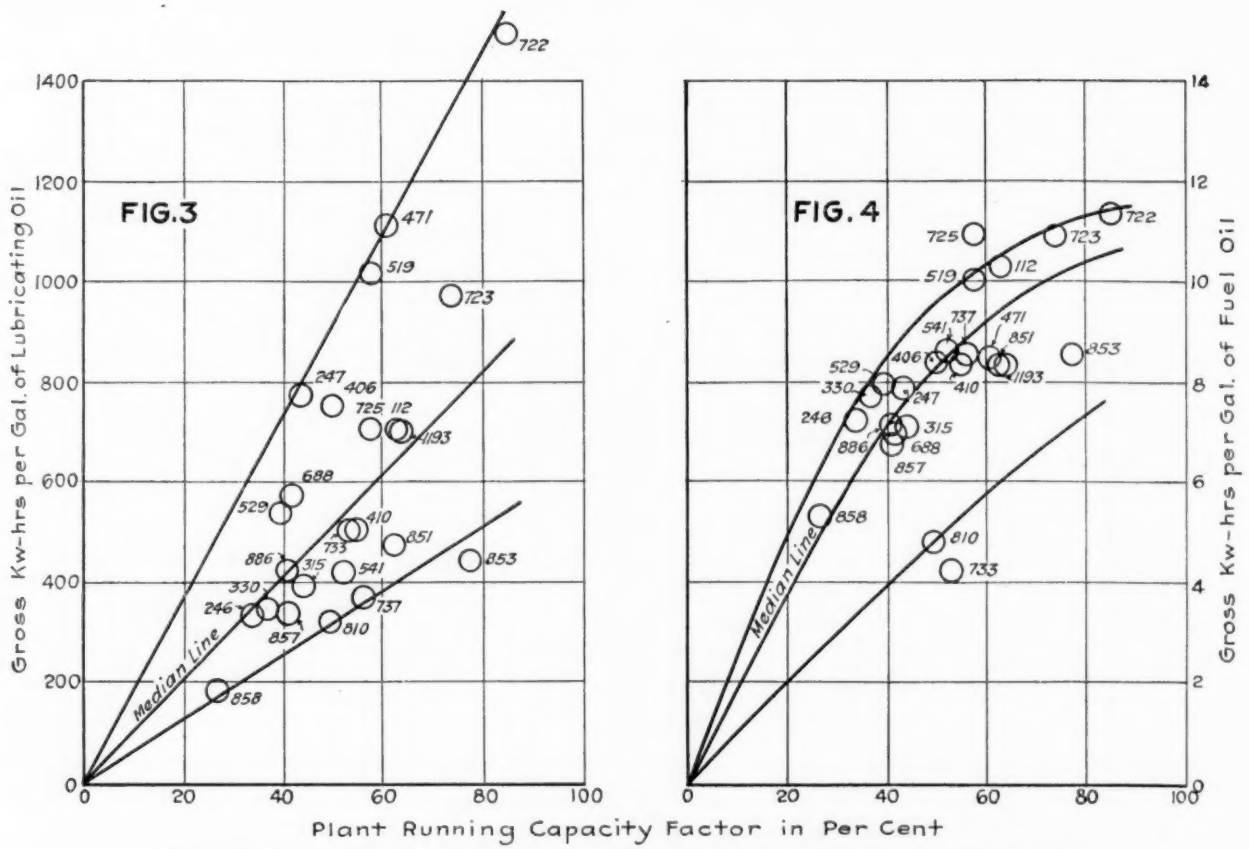
System C—Soft water continuously recirculated, cooled by raw water going to waste after one pass through heat exchanger

System D—Soft water continuously recirculated, cooled by raw water also recirculated after cooling by cooling tower or spray pond

System E—Engine circulating water cooled by radiator and fan

System F—Any of the foregoing systems with engine circulating water treated (added as a suffix)

Enforced Shutdowns. The term "enforced shutdown" is defined as any stoppage caused by actual or imminent engine trouble. The duration of an enforced shutdown is the time elapsing from the shutdown of the engine to the time at which the engine is again ready for service. A



FIGS. 3 AND 4. FUEL AND LUBRICATING OIL ECONOMIES OF 24 PLANTS GENERATING MORE THAN 5 PER CENT OF OUTPUT BY SEMI-DIESEL UNITS, NOT INCLUDING 5 SUCH PLANTS, RUNNING CAPACITY FACTORS FOR WHICH ARE NOT AVAILABLE

prearranged shutdown for maintenance work is not considered an enforced shutdown. Regular maintenance time where listed is the total time put in on regular maintenance work whether or not the units were actually needed during the time.

Peak Loads. The peak loads presented in the report are the highest average loads sustained for 15 minutes, unless otherwise stated in Table III.

Repair Costs. All costs for repairs whether to engines only or to all other plant equipment, listed in Table I, include the cost of materials delivered at the plant in question and the cost of any extra labor employed for the purpose of making these repairs. Unless otherwise noted, however, these costs do not include any charges for work done by regular attendants. Correspondingly, unless otherwise noted, the costs given in Table I for attendance and superintendence have not been subject to deduction because of repair work done by the regular attendance crews.

Attendance and Superintendence. Table III presents data on the number of shifts per year, the length of shifts and the number of attendants per shift, where these data are available. Table III also presents the net kwhr output per man hour of attendance for those plants operated by full-time attendance and also for those plants having part-time attendance, provided only one man out of at least four works part time.

Supplies and Miscellaneous. Supplies in the meaning of this report include those items used in the power-generating plant which are consumed in the operating process such as: waste, packing, wipers, gage glasses, gaskets, bolts, screws, nails, dynamo and motor brushes, cans for containing rags and waste, transformer oil and hand oil cans. The term "miscellaneous" as used in this report refers to such items as expenditures for lighting, heating, cleaning systems, fire-protection systems, janitor's supplies, ice water, meals and carfare, stationery, telephone and toilet service and care of streets, yards and sidings.

Type of load. The terms used for type of load are defined as follows:

Complete Power—Plant run regularly alone when needed, without assistance from any base or peak-load service

Base Load—Plant run at substantially full load whenever its capacity can be used; usually supplemented by a peak-load service. When full or nearly full capacity cannot be used, plant is shut down

Peak Load—Plant run only when load exceeds capacity of regular source of power

Standby—Plant run only when regular source of power is interrupted.

For this investigation, the committee requested information also on the type of power supplemented by base load, peak load, and standby plants. Information obtained in accordance with this request is presented in Table I.

Total Production Costs. Total production costs for 154 plants reporting for one year each (plus or minus one month) are shown graphically on logarithmic coordinates in Fig. 5. In this chart total production cost in mills per net kwhr is plotted against specific output, or the output in net kwhr per year per kw of installed capacity.

Calculations. The committee was assisted in its work by Messrs. Robert T. Brown and F. E. Bunbery, Jr.

SUBCOMMITTEE ON OIL-ENGINE POWER COST

H. C. Major, Chairman	Howard McCurdy
M. J. Reed, Secretary	A. B. Morgan
L. R. Ford	L. H. Morrison
W. G. G. Godron	Lee Schneitter
K. M. Irwin	P. H. Schweitzer
Edgar J. Kates	H. C. Thuerk
H. C. Lenfest	C. A. Trimmer

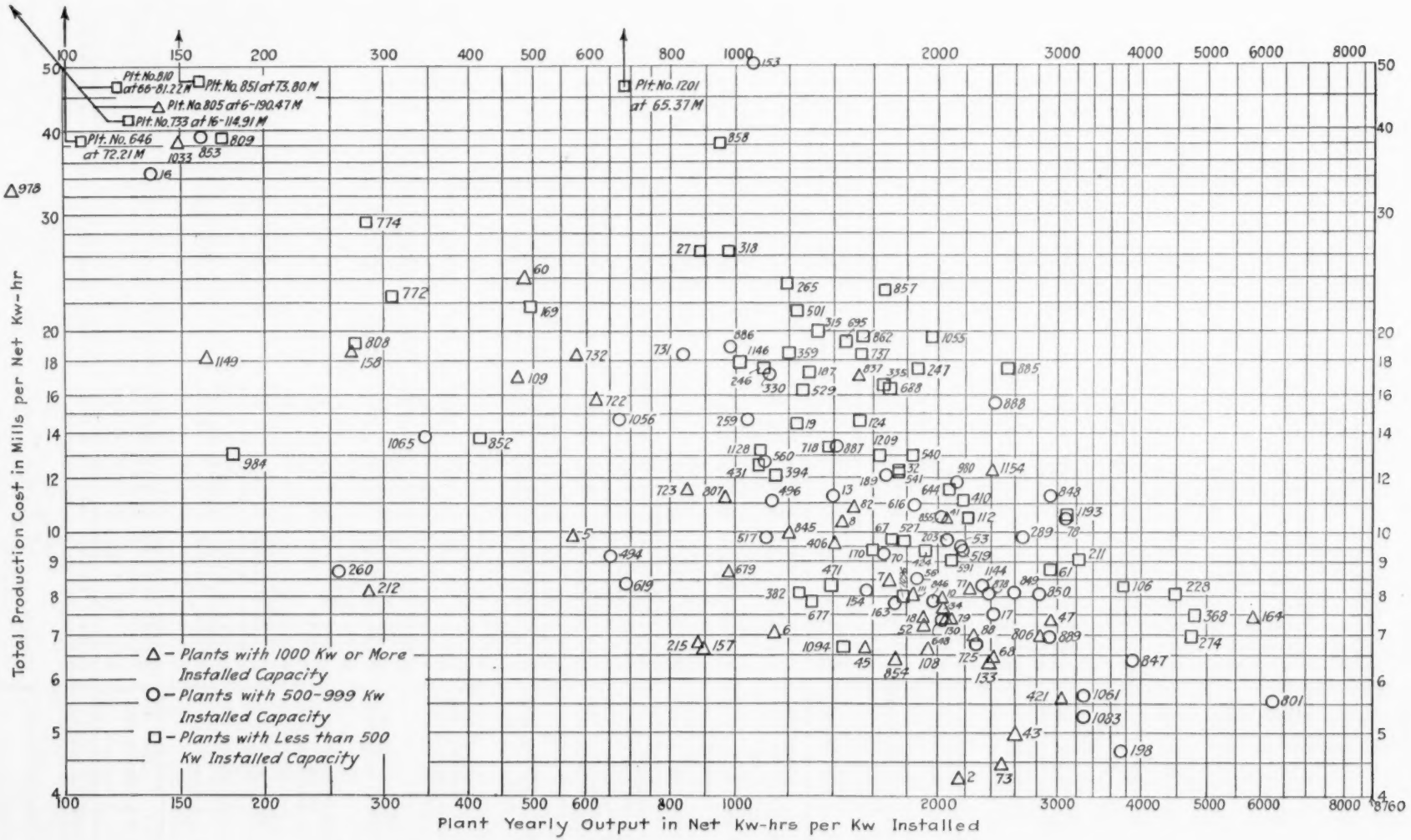


FIG. 5. TOTAL PRODUCTION COSTS FOR ALL PLANTS REPORTED, WITH THE EXCEPTION OF 2 WHICH OPERATED FOR PERIODS OF LESS THAN 11 MONTHS—MILLS PER NET KW HR.

TABLE I—INFORMATION ON PRODUCTION COST (Page 1)

Plant Number	Character of Plant (See Notes)	Type of Load (See Notes)	Number of Engines	Total Installed B.H.P.	Total Installed K.V.A.	Total Plant Hours Operated in Reported Period	Number of Months in Reported Period	Total Gross Output—K.W. Hrs.	Determination of Gross K.W. Hrs. (See Notes)	Total Net Output—K.W. Hrs.	Determination of Net K.W. Hrs. (See Notes)	Percent of Gross K.W. Hrs. for Plant Purposes	Annual Plant Load Factor (See Text)	Running Plant Capacity Factor (See Text)	Plant Service Factor (See Text)	Average Cost of Fuel Oil—Cents per Gallon	Average Cost of Lubricating Oil—Cents per Gallon	Costs per Net K.W. Hr.—Mills: Fuel Oil Cost	Lubricating Oil Cost	Attendance Cost Including Superintendence	Cost of Supplies and Misc., including Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Combined Cost of All Repairs, Supplies and Miscellaneous	Total Production Cost	Plant Number
43	U	C	9	15,640	14,850	8,760	12	30,795,736	M	27,434,541	A	10.9	55.1	74.3	45.1	3.39	28.8	2.88	0.23	0.77	0.09	0.75	0.23	1.07	4.95	43
82	M-W	C	7	8,960	8,102	8,760	12	9,995,550	M	9,014,095	M	9.8	34.0	63.2	30.0	4.24	58.5	3.94	0.47	2.74[a]	0.87	1.37[a]	1.45[a]	3.69	10.84	82
52	M	C	5	7,445	6,380	8,760	12	9,949,800	M	9,530,892	M	4.2	37.6	78.3	29.0	4.26	58.6	3.75	0.24	2.43	0.21	0.48	0.09	0.78	7.20	52
73	M-W	C	4	5,000	4,500	8,760	12	8,921,600	M	8,379,100	M	6.1	44.3	67.2	45.1	1.60	56.0	1.69	0.32	2.03	0.08	0.33	0.02	0.43	4.47	73
45	M	C	5	4,250	3,625	8,760	12	4,984,325	M	4,471,550	M	10.3	35.6	56.9	35.1	3.10	37.2	3.30	0.16	2.56	0.07	0.39	0.15	0.61	6.63	45
111	M	C	4	3,675	3,160	8,760	12	5,135,900	M	4,543,778	M	11.5	39.1	63.2	37.6	3.45	49.8	3.29	0.41	3.27	0.44	0.25	0.35	1.04	8.01	111
130	M	C	4	3,600	3,150	8,760	12	5,238,400	M	4,919,360	M	6.1	43.7	67.8	36.5	4.27	61.4	4.27	0.69	1.50	0.43	0.37	0.10	0.90	7.36	130
157	U	V-T	2	3,520	3,000	5,840	12	2,162,100	M	2,122,944	M	1.8	10.7	83.6	12.5	4.48	52.6	4.07	0.31	1.56	0.32	0.32	0.02	0.66	6.60	157
2	M-W	B&P-T	3	3,450	3,000	2,266	12	5,071,720	M	4,980,440	M	1.8	24.6	97.3	25.7	1.92	21.5	1.77	0.15	1.82	0.11	0.36	0.04	0.51	4.24	2
164	I	B-Z	4	3,360	4,000	8,760	12	13,784,200	M	13,289,000	E	3.6	....	87.0	80.1	4.01	22.5	3.15	0.29	1.32	0.24	1.80	0.60	2.64	7.41	164
77	U	C-Z[b]	1	3,000	2,812	8,760	12	5,790,900	M	4,511,158	A	22.1	38.9	39.3	83.5	4.91	38.9	4.67	0.48	1.87	0.35	0.83	0.01	1.19	8.21	77
109	M	S-H	3	2,925	2,450	8,760	12	959,960	M	931,425	M	2.9	9.5	47.9	11.7	5.95	81.8	7.51	0.73	6.85	0.90	0.80	0.10	1.80	16.89	109
7	M	C	5	2,730	2,625	8,760	12	3,359,900	M	3,114,320	M	7.3	41.6	57.5	36.4	3.80	30.0	3.48	0.27	3.04	0.35	0.72	0.58	1.65	8.44	7
732	U	P-T	5	2,520	2,050	8,760	12	1,222,560	M	988,290	M	19.2	10.0	52.0	15.8	2.62	45.0	3.99	1.25	8.32	1.55	2.82	0.35	4.72	18.28	732
978	U	S-	3	2,520	2,100	.....	12	184,300	M	142,039	M	22.9	1.9	....	....	3.97	49.6	4.00	0.47	18.09	4.04	5.74		9.78	32.34	978
679	M	P-H	2	2,500	2,130	2,264	12	1,677,900	M	1,639,420	M	2.3	10.4	99.2	11.5	5.28	44.0	5.03	0.38	1.74	0.84	0.70	0	1.54	8.69	679
60	I	C	3	2,400	2,025	8,760	12	939,700	M	777,920	E	17.2	20.6	21.6	30.8	5.16	55.1	9.83	1.52	11.34	0.25	0.96		1.21	23.90	60
1149	U	S-	2	2,400	2,091	.....	12	290,760	M	262,329	M	9.7	2.0	....	....	3.90	55.0	3.68	0.21	11.44	2.00	0.81		2.81	18.14	1149
41	M-W	C	3	2,370	1,800	8,760	12	3,329,790	M	3,279,790	E	1.5	31.7	59.7	40.0	4.37	46.1	4.45	0.66	4.44	0.39	0.17	0.39	0.95	10.50	41
723	U	S-T	4	2,370	2,010	2,242	12	1,411,510	M	1,349,830	M	4.4	10.2	73.8	13.7	4.41	49.7	4.24	0.53	4.59	0.72	1.11	0.28	2.11	11.47	723
34	M-W	C-Z[b]	3	2,250	1,920	8,760	12	3,149,100	M	3,101,023	M	1.5	44.9	60.2	39.5	4.31	26.7	3.95	0.30	2.15	0.13	1.14		1.27	7.67	34
79	M	C	5	2,170	1,858	8,760	12	3,230,400	M	3,065,966	A	5.1	41.0	64.8	39.1	3.23	39.9	2.88	0.52	3.12	0.43	0.39	0	0.82	7.34	79
5	U	S-	3	1,160	.....	.....	12	918,180	M	834,039	M	9.2	7.0	....	....	4.02	47.6	4.59	0.54	3.65	0.45	0.65		1.10	9.89	5
6	U	C	4	2,100	.....	.....	12	1,652,500	M	1,616,773	M	2.2	12.7	93.5	14.3	4.34	50.2	3.79	0.43	1.67	0.32	0.83		1.15	7.04	6
68	M-W	C	4	2,060	1,744	8,760	12	3,516,800	M	3,347,210	M	4.8	59.9	65.4	44.4	3.73	46.2	4.02	0.50	1.29	0.13	0.46	0.04	0.63	6.44	68
133	I	B-Z	2	2,000	1,376[d]	3,561	12	3,224,550	M	3,216,935	E	0.3	26.8	81.6	33.6	4.81	37.9	4.05	0.34	1.49	0.07[c]	0.37	0	0.44	6.32	133
806	U	B-T	3	2,000	1,758	8,760	12	3,981,520	M	3,814,300	M	4.2	49.9	69.9	48.4	2.10	66.6	2.01	0.68	2.24	1.00	0.22	0.81	2.03	6.95	806
29	I	C	3	1,980	1,689	1,800	9.3	926,340	M	886,688	M	4.3	....	57.6	17.6	4.75	67.4	4.27	0.74	6.39	0.27	0	0	0.27	11.67	29
215	U	S-H&T	2	1,960	1,690	8,760	12	1,194,900	M	1,163,860	E	2.6	11.4	78.9	13.1	4.49	50.7	4.02	0.32	1.44	0.55	0.46	0	1.01	6.79	215
807	U	P-T	4	1,885	1,572	8,760	12	1,416,913	M	1,223,713	M	13.6	32.3	47.5	26.9	3.56	80.5	3.90	0.86	5.06	0.88	0	0.44	1.32	11.15	807
108	M-W	C	3	1,875	1,601	8,760	12	2,548,000	M	2,440,000	M	4.3	36.9	54.9	42.1	2.31	52.0	2.18	0.51	2.94	0.61	0.23	0.10	0.94	6.57	108
837	U	C	5	1,875	1,625	8,760	12	2,064,771	M	1,920,591	M	7.0	39.3	47.1	39.8	2.51	40.4	3.57	0.90	6.01	0.51	5.15	0.98	6.64	17.12	837
1033	U	P&S-T	4	1,865	1,300	...	12	206,550	M	186,791	..	9.5	2.8	64.1	2.9	4.17	30.2	5.31	0.80	21.35	3.76	3.17	3.87	10.80	38.26	1033
421	M-W	B-G	2	1,800	1,576	8,760	12	3,802,600	M	3,689,473	E	3.0	....	63.1	....	2.32	53.0	2.02	0.47	2.85	0.16	0.07	0	0.23	5.57	421
845	U	C	3	1,770	1,500	8,760	12	1,489,775	M	1,434,575	M	3.7	36.2	47.3	30.2	3.93	38.6	4.57	0.74	2.97	0.44	1.26	0	1.70	9.98	845
854	U	B-T	3	1,770	1,500	8,760	12	2,177,480	M	2,064,467	M	5.2	28.3	55.8	37.5	3.98	36.5	3.87	0.24	1.76[e]	0.32	0.15	0.09	0.56	6.43	854
158	U	P&S-T	1	1,760	1,500	2,920	12	343,172	M	317,690	M	7.5	3.9	55.4	6.0	6.06	59.3	7.24	0.56	5.35	4.90	0.07[f]	0.39	5.36	18.51	158
212	W	S-T	2	1,750	1,342	8,760	12	337,200	M	336,189	E	0.3	6.8	70.8	4.6	3.60	49.0	3.27	0.29	2.17	0.30	2.07	0	2.37	8.10	212
722	U	S-T	4	1,705	1,450	8,760	12	774,780	M	709,180	M	8.5	8.0	84.9	9.1	4.53	77.2	4.36	0.56	7.98	1.25	1.26	0.32	2.83	15.73	722
805	U	S-Z	2	1,680	1,400	8,760	12	7,870	M	6,585	M	16.3	0.2	48.1	0.2	2.76	....	4.00	0	182.48	2.64	0	1.36	4.00	190.47	805
18	U	C	3	1,650	.....	.....	12	2,205,070	M	2,112,805	M	4.1	26.8	60.4	37.6	4.22	49.9	4.16	0.50	1.74	0.31	0.72		1.03	7.44	18
88	M-W	C	4	1,620	1,318	8,760	12	2,532,910	M	2,470,900	M	2.5	42.8	57.9	45.9	3.04	52.1	3.18	0.48	2.84	0.36	0.10	0	0.46	6.96	88
8	M-W	C	3	1,560	1,350	8,760	12	1,554,430	M	1,511,411	E	2.8	....	51.6	32.8	3.44	40.0	3.64	0.43	4.84	0.26	1.07	0.08	1.41	10.32	8
406	M	C	4	1,560	1,300	8,760	12	1,589,240	M	1,482,043	E	6.8	42.7	50.0	34.6	3.51	58.4	4.48	0.83	3.05	0.71	0.24	0.28	1.23	9.59	406
10	W	C	2	1,500	1,250	6,199	12	2,080,220	M	2,057,220	E	1.1	54.0	66.6	35.4	1.82	21.0	1.68	0.25	3.72[g]	0.47	0.74	1.07	2.28	7.92	10
47	U	C-T	1	1,500	1,250	8,760	12	3,021,340	M	2,967,310	M	1.8	32.9	65.9	52.0	6.01	45.1	5.15	0.28	1.18	0.49	0.26	0	0.75	7.36	47
1154	M	C	2	1,500	1,500	8,760	12	2,455,900	M	2,433,000	A	0.9	37.4	54.3	51.2	5.51	71.3	5.47	0.22	6.16	0.23	0.12	0.05	0.40	12.25	1154
289	M	C	4	1,465	1,232	8,760	12	2,747,030	M	2,625,010	E	4.5	38.3	71.0	44.9	4.30	52.0	3.83	0.62	4.29	0.33	0.54	0.26	1.13	9.87	289
496	M	C	3	1,450	1,240	8,760	12	1,201,175	M	1,108,150	M	7.7	21.1	44.4	31.7	4.08	64.8	4.39	0.37	4.98	0.89	0.44	0	1.33	11.07	496
850	U	B-T	2	1,440	1,200	8,760	12	2,844,460	M	2,736,620	M	3.8	33.2	56.9	59.0	4.69	39.0	4.81	0.46	1.83	0.30	0.61	0.01	0.92	8.02	850

TABLE I—INFORMATION ON PRODUCTION COST (Page 2)

TABLE I—INFORMATION ON PRODUCTION COST (Page 2)

70	M-W	C	3	1,420	1,190	8,760	12	1,791,883	M	1,594,598	M	11.0	41.7	52.0	41.2	3.16	45.0	3.92	0.61	3.20	0.24	0.08	1.21	1.53	9.26	70
849	U	P-T&Z	2	1,410	1,200	8,760	12	2,516,180	M	2,463,870	M	2.1	30.3	73.2	41.5	5.01	35.2	4.94	0.38	1.98	0.12	0.64	0	0.76	8.06	849
154	U	C	3	1,400	.....	.....	12	1,513,170	M	1,483,854	M	1.9	30.3	79.2	23.2	3.87	52.8	3.18	0.33	2.81	0.31	1.48		1.79	8.11	154
163	I	..	3	1,400	1,311	8,760	12	1,771,725	M	1,636,205	E	7.7	20.2	55.0	39.1	3.44	26.3	3.58	0.17	1.24	0.74	2.07		2.81	7.80	163
198	U	P-T	1	1,400	1,392	5,976	12	3,666,420	M	3,514,032	A	4.2	41.9	70.6	63.1	3.52	15.3	3.26	0.11	0.77	0.11	0.42		0.53	4.67	198
517	M	C	2	1,375	1,250	8,760	12	1,122,560	M	1,034,979	E	7.8	34.2	27.7	50.1	3.65	60.6	4.51	0.76	4.19	0.30	0.07	0.01	0.38	9 84	517
888	U	C	3	1,375	1,200	8,760	12	2,326,490	M	2,252,600	M	3.2	31.6	52.1	55.2	4.69	51.9	4.76	0.32	5.45	2.02	2.52	0.68	5.22	15.75	888
725	U	B-T	4	1,310	1,107	8,760	12	2,071,200	M	2,013,550	M	2.8	37.5	57.8	46.5	3.59	55.1	3.38	0.80	2.19	0.32	0.04	0.02	0.38	6.75	725
886	U	C	4	1,305	1,090	8,760	12	913,325	M	865,747	M	5.2	34.0	40.7	29.3	4.74	50.6	7.00	1.26	5.88	1.71	2.76	0.13	4.60	18.74	886
13	M-W	C	3	1,225	1,000	8,760	12	1,300,700	M	1,153,244	M	11.3	39.6	56.4	32.0	4.72	41.9	5.55	0.86	3.35	0.70	0.37	0.38	1.45	11.21	13
16	U	B-T	3	1,200	1,012	282	12	113,136	M	109,596	M	3.2	1.6	93.3	1.7	3.23	50.0	3.13	1.35	11.95	6.25	8.83	2.89	17.97	34.40	16
17	U	C	2	1,200	.....	.....	12	1,997,800	M	1,946,839	M	2.6	26.2	59.2	47.8	4.23	50.5	4.19	0.38	1.69	0.43	0.81		1.24	7.50	17
846	U	P-T	2	1,200	1,000	8,760	12	1,668,900	M	1,599,147	M	4.1	25.1	61.8	38.2	4.09	39.8	4.10	0.45	2.67[e]	0.31	0.34	0.01	0.66	7.88	846
889	U	C-T	1	1,200	900	8,760	12	2,461,720	M	2,365,854	M	3.9	36.5	68.5	50.9	5.20	36.6	4.89	0.20	1.06	0.18	0.62	0	0.80	6.95	889
980	M	C	2	1,200	1,000	8,760	12	1,757,700	M	1,724,300	A	1.9	39.4	51.0	48.8	4.35	43.3	4.26	0.43	6.79	0.15	0.19	0.04	0.38	11.86	980
1083	U	B-T&Z	1	1,200	1,042	.....	12	2,695,050	M	2,652,550	M	1.6	38.5	73.5	51.9	4.00	47.3	3.04	0.34	1.18	0.25	0.43	0.01	0.69	5.25	1083
330	M	C	4	1,165	890	8,760	12	944,470	M	876,218	E	7.2	30.8	37.1	37.2	5.55	53.1	7.75	1.66	6.28	0.62	0.84	0	1.46	17.15	330
878	U	-T	3	1,160	977	8,760	12	2,258,880	M	1,860,122	M	17.7	36.8	75.3	44.0	3.24	50.4	3.89	0.60	1.80	0.53	0.69	0.54	1.75	8.04	878
616	M	C	3	1,150	982	8,760	12	1,581,800	E	1,430,450	M	9.6	36.1	59.0	39.6	4.40	52.9	4.20	0.59	4.75	0.45	0.73	0.23	1.41	10.96	616
887	U	C	3	1,140	938	8,760	12	1,162,920	M	1,086,909	M	6.5	39.1	36.0	48.2	4.53	47.5	5.82	0.48	5.46	1.52	0.06	0.04	1.62	13.38	887
78	I	C	2	1,060	1,000	8,760	12	2,335,124	M	2,239,317	M	4.1	31.9	54.2	69.1	5.06	48.1	5.48	0.38	3.53	0.20	0.67	0.16	1.03	10.42	78
519	M-W	C	3	1,060	890	8,760	12	1,668,238	M	1,563,450	E	6.2	48.8	57.9	46.2	3.40	42.4	3.62	0.45	2.40	0.17	2.57	0.13	2.87	9.34	519
203	M	C	2	1,050	894	8,760	12	1,586,400	M	1,460,200	M	8.0	40.3	51.1	50.4	4.16	51.3	4.49	0.90	4.11	0.17	0.02	0	0.19	9.69	203
259	M-W	C	3	1,050	870	8,760	12	810,365	M	734,445	M	9.3	44.1	52.8	24.9	5.18	57.2	5.84	0.78	7.03	0.37	0.57	0.05	0.99	14.64	259
847	U	B-T	1	1,050	900	8,760	12	2,815,400	M	2,745,800	M	2.5	45.9	63.4	71.9	4.28	38.2	3.86	0.24	1.63	0.27	0 39	0.03	0.69	6.42	847
1144	M-W	C	3	1,050	873	8,760	12	1,730,300	M	1,648,088	A	4.7	38.3	62.9	44.6	4.83	49.9	4.29	0.79	2.17	....	....	....	1.10	8.35	1144
56	I	C	2	1,000	876	2,880	12	1,351,199	M	1,251,011	E	7.4	19.3	70.3	32.7	4.96	45.0	4.43	0.35	3.27	0.30	0.14		0.44	8.49	56
801	U	B-Z	1	1,000	937	8,760	12	4,428,200	M	4,206,500	M	5.0	72.2	90.0	83.7	3.99	61.8	3.48	0.32	0.88	0.20	0.57	0.09	0.86	5.54	801
1061	U	B-Z&T	1	980	975	.....	12	2,256,400	M	2,155,929	A	4.4	36.8	83.5	46.9	3.46	26.4	3.16	0.24	0.76	0.16	1.32		1.48	5.64	1061
53	U	B-T	3	900	758	8,760	12	1,368,900	M	1,314,144	E	4.0	31.2	66.7	38.8	2.49	63.6	2.36	0.68	4.06	0.48	1.91	0.01	2.40	9.50	53
177	U	B-T	1	900	774	5,256	7.2	1,635,670	M	1,510,370	M	7.7	....	79.9	64.4	3.81	47.0	3.31	0.40	1.96	0.27	0.23	0.12	0.62	6.29	177
260	M	S&P-H	1	900	750	.....	12	180,000	M	156,190	M	13.2	3.6	48.3	7.0	3.95	60.0	4.92	0.89	2.56	0	0.35		0.35	8.72	260
560	M-W	C	3	900	746	8,760	12	709,290	M	670,266	E	5.5	25.3	39.0	34.3	2.98	55.0	4.15	0.74	6.74	0.45	0.71	0	1.16	12.79	560
848	U	P-T	2	900	750	8,760	12	1,880,400	M	1,777,476	M	5.4	41.3	63.5	56.0	5.19	41.6	6.07	1 04	2.42	0.20	1.51	0.05	1.76	11.29	848
1065	U	B-T	1	900	774	420	12	221,120	M	209,620	M	5.2	4.6	87.0	4.8	4.13	50.2	3.22	0.61	2.69	2.54	2.26	2.54	7.34	13.86	1065
189	M	C	3	850	730	8,760	12	1,010,820	M	956,138	M	5.4	38.5	32.8	61.6	3.61	75.6	4.57	0.70	6.24	0.34	0.21		0.55	12.06	189
153	U	S-	1	840	700	.....	12	81,080	M	60,200	M	25.7	1.7	....	....	4.00	66.7	4.70	1.66	25.76	6.48	11.43		17.91	50.03	153
494	U	B-T	1	840	700	.....	12	376,760	M	369,381	M	2.0	7.8	78.4	9.7	4.20	50 2	3.69	0.46	2.22	0.71	0.79	1.28	2.78	9.15	494
619	U	B-T	1	840	700	.....	12	402,770	M	390,760	M	3.0	8.2	82.3	9.9	4.28	50.1	3.34	0.66	2.59	1.01	0.52	0.28	1.81	8.40	619
855	U	P-T	1	840	700	.....	12	1,266,600	M	1,145,142	M	9.6	30.7	49.8	51.4	3.90	39.2	5.12	0.23	3.22[e]	0.44	1.46	0.05	1.95	10.52	855
1056	U	P-Z&T	1	840	700	.....	12	425,890	M	379,021	A	11.0	8.1	62.3	13 8	3.50	21.2	3.65	0.29	4.96	2.10	3.76		5.86	14 76	1056
731	U	P-T	1	825	750	8,760	12	478,260	M	462,470	M	3.3	9.1	68.0	14.5	5.29	48.0	5.05	1.70	8.59	1.31	0.52	1.05	2.88	18 22	731
853	U	S-T	3	810	675	2,920	12	100,000	M	87,586	M	12.4	2.1	77.4	2.7	4.36	49.1	5.82	1 27	24.48	4.95	1.70	0.94	7.59	39.16	853
648	U	B-T	2	800	666	5,000	12	1,125,370	M	1,095,200	M	2.7	32.1	67.6	35.4	3.90	46.0	3.64	0.43	2.84	0.28	0.16	0.05	0.49	7 40	648
112	M-W	C	3	740	620	8,750	12	1,193,300	M	1,104,807	M	7.5	34.1	63.1	43.4	3.78	45.1	3.94	0.69	5.28	0.25	0.31	0.05	0.61	10 52	112
1128	M	C	3	725	.....	8,760	12	619,650	M	531,546	E	14.2	27.2	46.4	31.4	4.90	50.2	5.81	0.31	6.69	0.36	0	0	0.38	13 19	1128
67	P	C	2	720	500	7,512	12	832,900	M	832,067	E	0.1	45.3	83.8	23.4	5.34	43.6	5.40	0.86	2.24	0.34	0.76	0.17	1.27	9 77	67
170	I	C	2	720	600	3,399	12	814,242	M	780,252	E	4.8	19.8	55.8	34.5	4.95	44.1	4.84	0.77	2.42	0.32	1.03		1.35	9.38	170
394	M	C	3	720	590	8,760	12	562,900	M	558,059	M	0.9	37.9	45.0	29.6	3.13	36.9	3.59	0.78	7.06	0.17	0.37	0.12	0.66	12.09	394
527	M	C	2	720	600	8,760	12	914,300	M	864,300	E	5.5	31.6	42.4	50.9	3.23	60.7	3.71	0.87	3.82	0.91	0.34	0.06	1.31	9 71	527
718	U	C	2	720	600	8,760	12	729,970	M	658,898	M	9.7	41.7	33.5	51.3	3.76	42.5	5.53	1.13	5.52	0.25	1.15	0.10	1.50	13.68	718
19	U	C	3	700	.....	.....	12	629,550	M	579,880	M	7.9	38.8	....	....	4.08	52.3	5.61	1.31	5.18	0.69	1.68		2 37	14.47	19
274	I	C	2	700	582	.....	12	2,305,000	M	2,236,000	M	3.0	61.9	70.2	79.7	4.10	46.0	4.07	0.63	1.68	0.15	0.43		0.58	6.96	274
368	I	B-H&Z	2	700	590	5,982	12	2,292,000	M	2,250,009	E	1.8	55.4	84.4	65.8	5.90	63.0	4.81	0.47	1.91	0.01	0.26	0.04	0.31	7.50	368
851	U	P-T	2	660	550	2,920	12	91,800	M	68,859	M	25.0	2.1	62.6	3.8	4.34	38.3	6.93	1.07	46.00	6.78	10.13	2.89	19.80	73.80	851
810	U	S-T	3	650	545	4,380	12	31,900	M	28,810	M	9.7	1.6	49.3	1.7	3.20	63.6	7.45	2.19	64.31	1.92	0	5.35	7.27	81.22	810
1146	M	C	3	630	511	8,760	12	495,000	M	429,120	M	13.3	28.2	41.3	32.4	4.38	51.5	5.15	1.08	7.98	3.45	0.22	0.02	3.69	17.90	1146
529	M	C	4	620	490	8,760	12	558,084	M	524,025	E	6.1	35.4	39.4	38.9	4.00	31.4	5.34	0.62	7.45	2.01	0.60	0.19	2.80	16.21	529
359	M	C	2	615	510	8,760	12	510,000	M	495,640	M	2.8	34.3	26.5	53.2	4.49	43.7	5.97	0.96	7.02	1.14	0.31	2.98	4.43	18.38	359
431	M-W	C	3	610	500	8,760	12	451,900	M	446,060	E	1.3	29.5	....	....	2.50	38.8	2.84	1.01	7.62	0.28	0.66	0.26	1.20	12.67	431
32	I	C	1	600	572	8,760	12	726,050	M	706,050	E	2.7	18.4	72.8	28.3	6.00	66.9	5.43	1.26	5.52	0.08	0.05	0	0.13	12.34	32
61	I	C	2	600	550	8,760	12	1,259,600	M	1,188,400	E	5.7	32.7	56.2	63.4	4.98	60.9	4.62	0 52	2.85	0.30	0.50	0	0.80	8.79	61
106	U	C	1	600	.....	.....	12	1,570,920	M	1,522,216	M	3.1	39.0	71.8	62.0	4.31	51.4	3.99	0.24	2.73	0.52	0.76		1.28	8.24	106
228	M-W	C	2	600	500	8,760	12	1,898,000	M	1,814,780	E	4.3	57.8	90 7	59.4	4.00	44.9	4.12	0.43	3.34	0.02	0.13	0	0.15	8.04	228
644	M-W	C	2	600	496	8,760	12	913,700	M	840,958	A	7.9	39.4	59.5	43 5	5.28	63.9	6.03	1.03	3.42	0.85	0.18	0.06	1.08	11.57	644
809	U	S-T	1	600	500	2,555	12	76,300	M	69,433	M	9.0	2.2	53.6	4.0	4.40	71.0	6.35	0.72	23.51	1.57	0	6.92	8.49	39.06	809
1094	U	B-T&Z	1	600	513	.....	12	596,800	M	583,920	M	2.2	16.6	89.4	18.9	4.18	50.1	3.43	0.34	1.39	0.60	0.71	0.24	1.55	6.71	1094
1096	U	B-T	1	600	513	2,606	12	733,520	M	716,299	M	2.3	20.4	69.8	29.8	4.20	49.9	3.40	0.33	2.40	0.62	1.06	0.16	1.84	7.97	1096
410	M	C	3	590	495	8,760	12	922,470	M	867,657	M	5.9	40.5	55.0	48.4	3.07	50.0	3.91	1.05	5.46	0.45	0.23		0.68	11.10	410
862	U	C	4	588	495	8,760	12	653,190	M	609,080	M	6.8	33.3	44.9	42.1	3.44	....	4.33	1.00	9.18	0.81	4.08	0.40	5.29	19.80	862

TABLE I—INFORMATION ON PRODUCTION COST (Page 3)

Plant Number	Character of Plant (See Notes)	Type of Load (See Notes)	Number of Engines	Total Installed B.H.P.	Total Installed K.V.A.	Total Plant Hours Operated in Reported Period	Number of Months in Reported Period	Total Gross Output—K.W. Hrs.	Determination of Gross K.W. Hrs. (See Notes)	Total Net Output—K.W. Hrs.	Determination of Net K.W. Hrs. (See Notes)	Percent of Gross K.W. Hrs. for Plant Purposes	Annual Plant Load Factor (See Text)	Running Plant Capacity Factor (See Text)	Plant Service Factor (See Text)	Average Cost of Fuel Oil—Cents per Gallon	Average Cost of Lubricating Oil—Cents per Gallon	Costs per net K.W Hr.—Mills: Fuel Oil Cost	Lubricating Oil Cost	Attendance Cost Including Superintendence	Cost of Supplies and Misc., including Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Combined Cost of All Repairs, Supplies and Miscellaneous	Total Production Cost	Plant Number
808	U	S-T	1	560	470	1,460	12	106,700	M	102,432	M	4.0	2.9	73.5	4.4	5.28	69.9	4.35	0.78	8.16	0.82	0	5.19	6.01	19.31	808
852	U	S-T	1	560	460	.....	12	174,660	M	156,631	M	10.3	6.2	67.3	7.9	5.78	34.0	5.80	0.23	1.25	5.45	0.73	0.41	6.59	13.87	852
246	M	C	3	540	430	8,760	12	412,525	M	397,000	E	3.8	29.4	33.9	38.4	5.34	57.8	7.65	1.80	7.83	0.27	0.10	0.08	0.45	17.73	246
540	M-W	C	2	540	.....	8,760	12	700,500	M	666,616	A	4.8	28.6	41.0	53.7	3.55	46.6	4.64	2.08	5.46	0.74	0.06	0.06	0.86	13.04	540
541	M-W	C	3	540	451	8,760	12	650,900	M	636,000	M	2.3	37.2	52.2	39.3	3.92	50.9	4.65	1.24	5.04	0.59	0.34	0.40	1.33	12.26	541
211	U	C	1	510	438	.....	12	1,151,190	M	1,098,749	M	4.5	39.8	....	....	4.24	51.3	3.60	0.81	2.16	0.68	1.82		2.50	9.06	211
772	U	B-T	2	500	513	780	12	107,572	M	103,495	M	3.8	3.8	73.8	5.0	4.36	51.3	4.57	1.35	6.74	5.32	3.27	1.19	9.78	22.44	772
424	M	C	2	480	400	8,159	12	665,600	M	621,800	E	6.6	34.5	47.0	50.2	3.00	49.9	3.43	1.32	4.39	0.24	0	0	0.24	9.38	424
501	M	C	3	480	380	8,760	12	417,049	M	397,167	M	4.8	35.3	36.5	40.4	4.30	51.3	6.02	1.83	11.64	0.31	1.44	0.23	1.98	21.47	501
124	M	C	3	470	385	8,760	12	515,200	E	480,890	M	6.7	39.2	42.4	43.9	4.60	42.0	5.55	1.38	7.48	0.21	0.17	0	0.38	14.79	124
315	M	C	3	460	380	8,760	12	418,900	M	408,995	M	2.4	23.7	44.2	35.0	4.70	49.1	6.79	1.28	8.80	2.29	0.71	0.14	3.14	20.01	315
471	I	B-T&Z	2	450	313	2,331	12	428,110	M	422,283	E	1.4	22.2	60.8	26.6	3.37	64.4	4.03	0.59	1.99	0.75	0.92	0	1.67	8.28	471
695	M	C	3	450	390	8,760	12	456,140	M	440,760	M	3.4	31.6	48.8	35.3	4.81	36.0	5.26	1.24	9.39	0.67	2.90	0	3.57	19.46	695
885	U	C	3	435	360	8,758	12	778,200	M	738,709	M	5.0	38.1	46.7	65.2	3.59	40.4	4.87	0.88	6.94	2.34	2.35	0.27	4.96	17.65	885
335	M-W	C	2	430	369	8,760	12	491,320	M	479,720	M	2.3	36.9	36.5	53.2	4.72	54.2	4.85	0.76	9.08	0.94	0.87	0.04	1.85	16.54	335
1209	M-W	C	2	430	375	8,760	12	515,490	M	471,690	M	8.5	38.2	40.7	50.2	4.42	55.4	6.18	0.52	6.23	0.06	0.02	0	0.08	13.01	1209
247	M-W	C	3	390	315	8,760	12	490,040	M	486,820	M	0.7	24.9	43.4	49.1	5.50	34.0	7.03	0.44	9.74	0.28	0.18	0	0.46	17.67	247
187	M-W	C	2	350	281	8,760	12	315,879	M	301,279	M	4.6	25.8	32.9	46.7	3.82	47.0	4.23	2.00	10.64	0.43	0.15	0.05	0.64	17.51	187
688	M	C	3	350	278	8,760	12	412,000	M	400,000	E	2.9	29.4	41.8	47.8	4.82	38.5	7.14	0.69	6.86	0.97	0.41	0.19	1.57	16.26	688
774	U	B-T	2	345	270	500	12	66,822	M	65,645	M	1.8	7.6	67.4	4.9	4.59	50.4	4.61	0.96	7.28	3.31	12.98	0.06	16.35	29.20	774
857	U	C	2	330	240	8,760	12	412,136	M	369,378	M	10.5	42.8	40.8	51.9	3.25	52.0	5.36	1.71	7.39	1.79	6.12	0.70	8.60	23.07	857
265	M	C	3	320	249	8,760	12	262,100	M	257,000	M	2.0	26.0	39.6	35.2	7.35	59.0	9.59	1.41	11.67	0.19	0.70	0.14	1.03	23.70	265
1055	M-W	C-Z	1	320	265	8,760	12	521,127	M	421,352	E	19.2	33.1	28.4	97.5	4.32	60.6	7.27	0.47	11.29	0.54	0	0.05	0.59	19.62	1055
27	U	C	2	315	.....	.....	12	203,800	M	186,510	M	8.5	31.0	....	....	4.14	40.0	7.29	2.13	10.88	0.77	5.16		5.93	26.23	27
737	U	C	4	305	250	8,760	12	409,926	M	314,646	M	23.2	42.5	56.3	40.6	3.96	48.7	6.02	1.73	8.09	0.94	1.14	0.44	2.52	18.36	737
382	I	C	2	300	.....	6,630	12	254,000	M	251,413	E	1.0	....	27.2	52.6	3.64	37.5	3.63	0.50	3.73	0.19	0.02	0	0.21	8.07	382
858	U	C	2	270	215	8,760	12	211,600	M	172,400	M	18.5	39.6	26.7	49.6	3.12	46.4	7.17	3.13	16.40	3.81	6.85	1.00	11.66	38.36	858
169	I	C	1	240	200	1,809	12	84,090	M	79,570	E	5.4	6.0	28.8	20.7	4.50	50.8	6.34	1.13	4.52	0.98	8.74		9.72	21.71	169
677	I	C	1	240	200	.....	12	235,000	E	210,000	M	10.6	19.9	74.1	22.5	4.61	41.7	3.95	1.19	1.43	0.36	0.95	0	1.31	7.88	677
591	U	B-T	1	225	200	8,760	12	329,143	M	317,143	M	3.6	23.5	66.8	37.3	3.43	50.1	3.99	0.93	3.60	0.16	0.23	0.21	0.60	9.12	591
984	U	S	1	225	219	2,160	12	29,000	M	26,851	E	7.4	1.8	48.5	4.5	4.29	45.7	5.07	0.60	5.59	0.75	0.82	0.19	1.76	13.02	984
318	M	C	4	212½	143	8,760	12	144,230	M	139,730	E	2.9	26.6	....	....	3.69	53.2	6.81	2.37	13.43	2.00	0.34	1.20	3.54	26.15	318
1193	M-W	C	1	200	170	8,760	12	424,400	M	408,307	A	3.8	42.1	64.0	56.5	5.00	42.9	6.24	0.64	2.74	.....	0.26	....	0.97	10.59	1193
733	U	S-T	2	175	135	68	12	2,135	M	1,892	M	11.4	0.2	53.2	0.4	6.03	50.0	16.00	3.17	38.34	20.20	0	37.20	57.40	114.91	733
646	M	S-H	1	120	90	8,736	12	8,990	M	8,074	M	10.2	....	11.1	11.4	5.39	55.5	14.62	4.34	21.92	12.87	9.91	8.55	31.33	72.21	646
1201	M	C	2	100	78	8,760	12	58,550	M	45,350	E	22.6	29.1	....	....	6.36	65.4	15.52	6.22	39.85	3.60	0.18	0	3.78	65.37	1201

NOTES—Character of Plant
- U—Private Power Company
- P—Private Pumping Company
- M—Municipal Power Plant
- W—Municipal Pumping Plant
- I—Industrial Power Plant
- B—City Building Power Plant

Determination of Output
- M—Metered
- E—Estimated from Gross (or Net)
- A—Metered Plant Consumption Subtracted from Gross (or Added to Net)

Type of Load
- C—Complete Power
- B—Base Load
- S—Stand by
- P—Peak Load
- V—Type Varied during Period
  - —T—Supplemented by Transmission Line
  - —H—Supplemented by Hydro-Electric Units
  - —Z—Supplemented by Steam Units
  - —G—Supplemented by Gas Engines

Lettered Notes
- *a*—Attendance Cost includes no Repair Labor; Repair Costs include All Repair Labor.
- *b*—Steam Plant used in Emergencies Only.
- *c*—No Cost included for Water, which goes to House Supply.
- *d*—K.W. Rating; Direct Current.
- *e*—No Deduction made for Attendance of Ice Plant.
- *f*—Does not include $1,885 for Cast Iron Piston Heads to Replace Steel; Change made for Operating Reasons and Charged to Capital Account.
- *g*—No Deduction made for Attendance of Water Pumping Plant.

TABLE II—COMPARATIVE COSTS—1929, 1930, 1931, 1932 AND 1933 REPORTS (Page 1)

Plant No.	Year	Total B.H.P Installed	Total Net Output K.W. Hrs.	Annual Plant Load Factor	Running Plant Capacity Factor	Plant Service Factor	Aver. Cost Fuel Oil—Cents per Gallon	Costs per Net K.W. Hr.—Mills								Year	Plant No.
								Fuel Cost	Lub. Oil Cost	Atten. Cost, Incl. Superint.	Cost Supplies & Misc., Incl. Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Total All Supply Repair & Misc. Costs	Total Production Cost		
43	1929	11,540	22,591,027	.....	69.9	....	4.25	3.82	0.69	0.60	.....	.....	.....	1.04	6.15	1929	43
	1930	11,540	26,221,176	46.1	70.4	57.8	4.37	3.95	0.53	0.94	0.21	0.67	0.05	0.93	6.35	1930	
	1931	11,540	29,849,848	52.5	73.2	65.7	3.78	3.51	0.47	0.70	0.28	0.63	0.29	1.20	5.88	1931	
	1932	15,640	28,374,009	57.1	83.3	46.0	3.48	3.23	0.31	0.86	0.30	0.56	0.14	1.00	5.40	1932	
	1933	15,640	27,434,541	55.1	74.3	45.1	3.39	2.88	0.23	0.77	0.09	0.75	0.23	1.07	4.95	1933	
82	1930	5,660	7,958,851	31.3	65.0	35.9	5.71	5.22	0.52	3.73	1.38	1.91	1.08	4.37	13.86	1930	82
	1931	5,660	8,778,880	33.2	69.0	39.1	5.13	4.93	0.45	2.94	0.92	1.96	1.22	4.10	12.42	1931	
	1932	8,960	9,065,633	33.0	60.3	40.5	4.48	4.52	0.63	2.81	0.47	1.13	1.57	3.17	11.13	1932	
	1933	8,960	9,014,095	34.0	63.2	30.0	4.24	3.94	0.47	2.74	0.87	1.37	1.45	3.69	10.84	1933	
52	1929	3,330	5,722,032	31.1	65.3	45.2	5.58	5.38	0.55	3.83	0.88	0.74	0.40	2.02	11.78	1929	52
	1930	4,580	7,257,703	33.5	75.2	45.4	5.70	5.30	0.38	2.83	0.74	0.56	0.48	1.78	10.29	1930	
	1931	4,580	8,417,102	33.7	78.6	40.4	4.83	4.46	0.30	2.55	0.63	0.49	0.38	1.50	8.81	1931	
	1932	7,445	9,110,000	36.3	78.1	43.8	4.41	3.97	0.27	2.35	0.52	0.35	0.14	1.01	7.60	1932	
	1933	7,445	9,530,892	37.6	78.3	29.0	4.26	3.75	0.24	2.43	0.21	0.48	0.09	0.78	7.20	1933	
73	1930	5,000	8,016,000	37.9	64.0	44.4	1.30	1.34	0.31	2.62	0.01	0.10	0.01	0.12	4.39	1930	73
	1931	5,000	8,341,500	41.9	66.3	45.1	0.95	0.93	0.26	1.93	0.13	0.22	0.05	0.40	3.52	1931	
	1932	5,000	8,030,200	42.0	66.4	43.4	1.22	1.26	0.29	2.01	0.16	0.16	0.02	0.34	3.90	1932	
	1933	5,000	8,379,100	44.3	67.2	45.1	1.60	1.69	0.32	2.03	0.08	0.33	0.02	0.43	4.47	1933	
45	1929	4,250	7,442,350	36.8	72.5	43.6	5.23	5.04	0.09	1.65	0.06	....	....	0.92	7.70	1929	45
	1930	4,250	6,908,500	34.9	69.5	42.4	5.29	5.18	0.09	1.77	0.04	1.01	0.10	1.15	8.20	1930	
	1931	4,250	5,537,000	46.4	66.5	36.6	4.79	4.89	0.09	2.04	0.07	1.28	0.55	1.90	8.92	1931	
	1932	4,250	5,015,575	42.3	62.1	35.8	3.87	4.07	0.11	2.05	0.05	1.33	0.32	1.70	7.93	1932	
	1933	4,250	4,471,550	35.6	56.9	35.1	3.10	3.30	0.16	2.56	0.07	0.39	0.15	0.61	6.63	1933	
111	1932	2,475	4,105,401	37.6	....	....	3.23	3.03	0.49	3.88	0.31	0.24	0.48	1.03	8.43	1932	111
	1933	3,675	4,543,778	39.1	63.2	37.6	3.45	3.29	0.41	3.27	0.44	0.25	0.35	1.04	8.01	1933	
130	1930	3,600	4,484,700	41.8	54.1	50.7	4.72	4.59	0.89	1.36	0.77	0.82	0.17	1.76	8.60	1930	130
	1931	3,600	4,168,750	42.4	67.0	31.4	4.19	3.93	0.56	1.58	0.71	1.96	0.18	2.85	8.92	1931	
	1932	3,600	4,281,230	44.7	68.6	31.3	4.02	3.83	0 66	1.65	0.47	0.68	0.19	1.34	7.48	1932	
	1933	3,600	4,919,360	43.7	67.8	36.5	4.27	4.27	0.69	1.50	0.43	0.37	0.10	0.90	7.36	1933	
157	1932	3,520	4,378,509	19.8	82.3	26.0	4.16	3.88	0.24	0.80	0.24	0.58	0.05	0.87	5.79	1932	157
	1933	3,520	2,122,944	10.7	83.6	12.5	4.48	4.07	0.31	1.56	0.32	0.32	0.02	0.66	6.60	1933	
2	1929	3,450	16,479,500	83.2	94.4	84.5	2.71	2.62	0.33	1.40	0.13	....	....	0.47	4.82	1929	2
	1930	3,450	16,077,095	78.8	94.3	77.1	2.18	1.98	0.45	1.14	0.09	1.56	0.09	1.74	5.31	1930	
	1931	3,450	17,745,325	86.5	97.8	89.8	2.11	2.00	0.26	1.09	0.08	1.16	0.09	1.33	4.68	1931	
	1932a	3,450	8,839,511	....	97.8	91.2	2.33	2.22	0.24	1.03	0.10	1.11	0.08	1.29	4.78	1932a	
	1933	3,450	4,980,440	24.6	97.3	25.7	1.92	1.77	0.15	1.82	0.11	0.36	0.04	0.51	4.24	1933	
164	1930	3,360	14,709,450	81.4	90.7	72.7	3.66	2.91	0.31	1.38	0.16	1.12	0.19	1.47	6.07	1930	164
	1931	3,360	11,037,481	59.5	76.3	76.9	3.34	2.66	0.28	1.73	0.35	1.52	0.30	2.17	6.84	1931	
	1932	3,360	12,838,281	....	80.0	84.9	2.59	2.12	0.27	1.28	0.55	1.37	0.46	2.38	6.05	1932	
	1933	3,360	13,289,000	....	87.0	80.1	4.01	3.15	0.29	1.32	0.24	1.80	0.60	2.64	7.41	1933	
77	1932	3,000	3,978,113	34.9	42.8	68.9	4.52	4.25	0.54	2.10	0.48	....	....	0.97	7.86	1932	77
	1933	3,000	4,511,158	38.9	39.3	83.5	4.91	4.67	0.48	1.87	0.35	0.83	0.01	1.19	8.21	1933	
109	1930b	1,800	885,399	....	48.1	16.3	5.53	6.33	1.26	3.78	0.88	0.60	0.11	1.59	12.96	1930b	109
	1931	2,925	1,858,860	....	54.1	30.3	5.60	6.29	1.25	2.32	0.27	0.80	0	1.07	10.93	1931	
	1932	2,925	959,606	8.9	49.2	11.5	5.69	6.85	0.64	5.01	0.24	0.75	0.10	1.09	13.59	1932	
	1933	2,925	931,425	9.5	47.9	11.7	5.95	7.51	0.73	6.85	0.90	0.80	0.10	1.80	16.89	1933	
7	1929	1,830	3,231,938	36.0	64.8	53.2	5.77	5.31	0.36	3.06	1.34	....	....	3.98	12.71	1929	7
	1930	2,730	3,211,821	38.0	61.8	53.1	4.79	4.52	0.35	3.17	1.19	1.57	0.31	3.07	11.11	1930	
	1931	2,730	3,088,885	43.9	57.3	36.4	3.43	3.08	0.39	3.79	0.68	1.28	0.51	2.47	9.73	1931	
	1932	2,730	3,105,742	42.5	57.9	36.3	3.27	2.78	0.25	3.29	0.42	0.96	0.39	1.77	8.09	1932	
	1933	2,730	3,114,320	41.6	57.5	36.4	3.80	3.48	0.27	3.04	0.35	0.72	0.58	1.65	8.44	1933	
732	1930	2,520	4,848,480	37.4	59.7	61.1	3.65	3.73	0.91	1.61	....	....	....	2.08	8.33	1930	732
	1931	2,520	2,632,385	25.0	49.1	44.1	2.82	3.68	1.14	2.19	0.59	1.33	0.77	2.69	9.70	1931	
	1932	2,520	3,497,637	27.5	50.4	47.4	3.17	3.27	0.73	2.09	0.57	2.48	0.37	3.42	9.51	1932	
	1933	2,520	988,290	10.0	52.0	15.8	2.62	3.99	1.25	8.32	1.55	2.82	0.35	4.72	18.28	1933	
978	1931	2,520	2,247,295	....	....	....	3.33	3.11	0.47	1.23	0.42	....	....	0.71	5.52	1931	978
	1932	2,520	2,223,852	14.4	87.6	17.5	3.57	3.11	0.27	1.41	0.35	....	....	0.74	5.53	1932	
	1933	2,520	142,039	1.9	....	....	3.97	4.00	0.47	18.09	4.04	....	....	9.78	32.34	1933	
60	1930	2,400	1,658,900	24.1	37.0	33.7	4.62	5.54	0.94	4.81	....	....	....	3.24	14.53	1930	60
	1931	2,400	No reply to inquiries for 1931.													1931	
	1932	2,400	1,015,200	13.7	24.8	32.3	4.69	7.48	0.73	5.22	0.60	....	....	2.76	16.19	1932	
	1933	2,400	777,920	20.6	21.6	30.8	5.16	9.83	1.52	11.34	0.25	....	....	1.21	23.90	1933	
1149	1932c	2,400	592,781	....	....	....	3.68	3.19	0.22	1.99	0.57	....	....	0.60	6.00	1932c	1149
	1933	2,400	262,329	2.0	....	....	3.90	3.68	0.21	11.44	2.00	....	....	2.81	18.14	1933	
41	1929	2,370	3,798,810	39.6	....	....	5.51	5.43	0.48	4.11	0.24	0.22	0.07	0.53	10.55	1929	41
	1930	2,370	4,068,290	39.2	59.0	44.2	5.19	5.09	0.48	4.08	0.37	0.63	0.15	1.15	10.80	1930	
	1931	2,370	4,092,730	39.4	67.3	44.2	4.57	4.48	0.51	3.85	0.25	0.24	0.13	0.62	9.46	1931	
	1932	2,370	3,698,090	35.6	65.5	41.0	4.10	3.86	0.62	4.05	0.23	0.26	0.08	0.57	9.10	1932	
	1933	2,370	3,279,790	31.7	59.7	40.0	4.37	4.45	0.66	4.44	0.39	0.17	0.39	0.95	10.50	1933	
723	1930	1,320	1,270,520	19.7	68.0	26.3	5.62	6.46	1.15	4.01	0.39	2.37	0.97	3.73	15.35	1930	723
	1931	2,370	1,556,510	....	73.3	....	4.21	4.23	0.95	3.56	0.53	0.86	0.70	2.09	10.83	1931	
	1932	2,370	1,371,480	10.9	67.2	15.4	4.31	4.13	0.74	3.71	0.76	....	....	2.55	11.13	1932	
	1933	2,370	1,349,830	10.2	73.8	13.7	4.41	4.24	0.53	4.59	0.72	1.11	0.28	2.11	11.47	1933	
79	1930	2,170	3,184,025	42.6	67.3	38.8	3.33	3.12	0.46	4.13	0.08	0.49	0.31	0.88	8.59	1930	79
	1931	2,170	3,204,390	38.6	66.8	39.6	2.62	2.38	0.30	3.31	0.13	0.20	0.13	0.46	6.45	1931	
	1932	2,170	3,280,357	39.3	66.4	40.6	2.49	2.21	0.45	3.53	0.11	0.23	0	0.34	6.53	1932	
	1933	2,170	3,065,966	41.0	64.8	39.1	3.23	2.88	0.52	3.12	0.43	0.39	0	0.82	7.34	1933	
5	1929	2,160	319,120	3.7	....	....	4.15	5.45	1.77	11.61	3.56	....	....	17.99	36.82	1929	5
	1930	2,160	1,013,269	8.1	....	....	4.04	4.86	1.25	5.56	1.14	....	....	8.17	19.84	1930	
	1931	2,160	1,789,653	10.0	....	....	3.33	3.86	0.80	2.15	0.79	....	....	2.18	8.99	1931	
	1932	2,160	2,597,100	21.5	94.4	22.6	3.54	3.90	0.65	1.31	0.53	....	....	1.39	7.25	1932	
	1933	2,160	834,039	7.0	....	....	4.02	4.59	0.54	3.65	0.45	....	....	1.10	9.89	1933	

TABLE II—COMPARATIVE COSTS—1929, 1930, 1931, 1932 AND 1933 REPORTS (Page 2)

Plant No.	Year	Total Installed B.H.P.	Total Net Output K.W. Hrs.	Annual Plant Load Factor	Running Plant Capacity Factor	Plant Service Factor	Aver. Cost Fuel Oil—Cents per Gallon	Costs per Net K. W. Hr.—Mills: Fuel Cost	Lub. Oil Cost	Atten. Cost, Incl. Superint.	Cost Supplies & Misc., Incl. Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Total All Supply Repair & Misc. Costs	Total Production Cost	Year	Plant No.
6	1929	2,100	1,318,898	10.0	....	....	4.70	4.17	0 72	2.67	0.47	....	....	1.61	9.17	1929	6
	1930	2,100	1,576,234	18.8	....	....	4.51	3.93	0.57	2.07	0.40	....	....	2.03	8.60	1930	
	1931	2,100	1,612,957	11.8	....	....	4.12	3.53	0.62	1.89	0.31	....	....	1.13	7.17	1931	
	1932	2,100	1,720,413	11.7	97.4	14.5	3.94	3.40	0.45	1.64	0.25	....	....	1.05	6.54	1932	
	1933	2,100	1,616,773	12.7	93.5	14.3	4.34	3.79	0.43	1.67	0.32	....	....	1.15	7.04	1933	
68	1931	2,060	3,336,860	59.4	67.5	42.7	3.37	3.30	0.44	1.41	0.11	0.31	0.37	0.79	5.94	1931	68
	1932	2,060	3,038,210	54.8	63.9	40.9	3.60	3.60	0.42	1.54	0.18	0.43	0.08	0.69	6.25	1932	
	1933	2,060	3,347,210	59.9	65.4	44.4	3.73	4.02	0.50	1.29	0.13	0.46	0.04	0.63	6.44	1933	
215	1930*d*	1,960	1,372,941	....	78.5	17.5	5.40	4.68	0.58	1.12	0.33	0.01	0.10	0.44	6.82	1930*d*	215
	1931	1,960	1,568,700	13.1	89.9	15.5	3.97	3.45	0.21	1.40	0.38	0.53	0.08	0.99	6.05	1931	
	1932	1,960	1,163,370	9.8	84.2	12.3	5.14	3.84	0.29	1.48	0.55	0.07	0	0.62	6.23	1932	
	1933	1,960	1,163,860	11.4	78.9	13.1	4.49	4.02	0.32	1.44	0.55	0.46	0	1.01	6.79	1933	
837	1930	1,875	2,016,465	33.2	49.0	38.8	3.07	3.80	0.68	7.39	0.66	3.32	1.12	5.10	16.97	1930	837
	1931	1,875	2,101,374	35.9	51.1	40.2	2.72	3.43	0.65	6.01	0.49	3.36	0.92	4.77	14.86	1931	
	1932	1,875	2,057,699	37.9	48.9	41.2	2.49	3.29	0.80	5.89	0.48	6.63	0.90	8.01	17.99	1932	
	1933	1,875	1,920,591	39.3	47.1	39.8	2.51	3.57	0.90	6.01	0.51	5.15	0.98	6.64	17.12	1933	
1033	1932	1,865	480,770	6.2	64.8	7.1	3.42	3.73	0.58	10.33	4.05	6.44	1.18	11.67	26.31	1932	1033
	1933	1.865	186,791	2.8	64.1	2.9	4.17	5.31	0.80	21.35	3.76	3.17	3.87	10.80	38.26	1933	
421	1932	800	2,485,908	57.1	69.6	80.2	2.20	2.04	0.18	4.23	0	0	0	0	6.45	1932	421
	1933	1,800	3,689,473	....	63.1	....	2.32	2.02	0.47	2.85	0.16	0.07	0	0.23	5.57	1933	
158	1932	1,760	512,498	6.0	32.7	17.0	5.87	8.69	1.06	3.00	2.62	1.09	0.77	4.48	17.23	1932	158
	1933	1,760	317,690	3.9	55.4	6.0	6.06	7 24	0.56	5.35	4.90	0.07	0.39	5.36	18.51	1933	
212	1931	1,750	228,700	3.6	69.6	3.2	4.43	4.13	0.51	2.21	0.28	0.09	0	0.37	7.22	1931	212
	1932	1,750	217,147	4.4	71.8	2.9	3.60	3.22	0.30	2.14	0.29	6.40	0	6.69	12.35	1932	
	1933	1,750	336,189	6.8	70.8	4.6	3.60	3.27	0.29	2.17	0.30	2.07	0	2.37	8.10	1933	
722	1930	1,705	796,450	9.3	82.1	11.2	5.61	5.97	0.69	7.99	1.49	1.19	0.90	3.58	18.23	1930	722
	1931	1,705	826,340	10.1	72.8	13.2	4.26	4.63	0.56	8.48	1.03	0.14	0.44	1.61	15.28	1931	
	1932	1,705	896,220	10.0	62.4	15 7	4.68	5.07	0.63	6.53	0.94	....	....	2.55	14.78	1932	
	1933	1,705	709,180	8.0	84.9	9.1	4.53	4.36	0.56	7.98	1.25	1.26	0.32	2.83	15.73	1933	
18	1929	1,650	2,351,123	26.4	....	....	4.71	4.52	0.41	2.58	0.38	....	....	2.30	9.81	1929	18
	1930	1,650	2,406,690	27.2	....	....	4.49	4.51	0.61	2.08	0.21	....	....	4.80	12.00	1930	
	1931	1,650	2,406,609	26.0	....	....	3.94	3.93	0.58	1.98	0.41	....	....	2.24	8.73	1931	
	1932	1,650	2,327,071	26.3	62.4	40.0	3.74	3.78	0.51	1.72	0.34	....	....	1.44	7.45	1932	
	1933	1,650	2,112,805	26.8	60.4	37.6	4.22	4.16	0.50	1.74	0.31	....	....	1.03	7.44	1933	
88	1932	1,620	2,497,042	41.2	58.6	....	2.31	2.39	0.47	2.86	0.36	0.10	0	0.46	6.18	1932	88
	1933	1,620	2,470,900	42.8	57.9	45.9	3.04	3.18	0.48	2.84	0.36	0.10	0	0.46	6.96	1933	
8	1930	1,560	2,049,000	34.2	57.0	39.1	3.37	3.40	0.58	4.68	0.70	0.27	0	0.97	9.63	1930	8
	1931	1,560	1,876,410	33.4	58.7	35.3	2.81	2.92	0.41	3.61	....	....	....	0.54	7.48	1931	
	1932	1,560	1,627,980	31.4	55.0	32.7	3.04	3.26	0.54	4.85	....	....	....	0.62	9.27	1932	
	1933	1,560	1,511,411	....	51.6	32.8	3.44	3.64	0.43	4.84	0.26	.07	0.08	1.41	10.32	1933	
10	1929	1,500	3,154,085	91.2	74.7	48.9	2.97	3.00	0 97	3.28	0.19	....	....	0.46	7.71	1929	10
	1930	1,500	3,058,452	....	72.1	48.5	2.42	2.33	0.78	3.09	0.07	0.78	0.42	1.27	7.47	1930	
	1931	1,500	3,000,700	78.5	69.9	49.0	2.45	2.27	0.66	5.38	0.13	0.84	0.52	1.49	9.80	1931	
	1932	1,500	Intermittent	Operat	ion in 1	932: Re	cords In	complet	e.							1932	
	1933	1,500	2,057,220	54.0	66.6	35.4	1.82	1.68	0.25	3.72	0.47	0.74	1.07	2.28	7.92	1933	
1154	1932	1,500	2,044,267	36.3	46.5	50.5	5.84	6.07	0.70	7.04	1.17	0.07	0.39	1.63	15.44	1932	1154
	1933	1,500	2,433,000	37.4	54.3	51.2	5.51	5.47	0.22	6.16	0.23	0.12	0.05	0.40	12.25	1933	
289	1931	1,465	2,523,740	38.4	71.3	42.7	4.15	3.67	0.51	4.71	0.32	....	....	1.44	10.33	1931	289
	1932	1,465	2,528,017	36.5	71.1	43.9	4.17	3.78	0.43	4.60	0.32	0.12	1.13	1.57	10.38	1932	
	1933	1,465	2,625,010	38.3	71.0	44.9	4.30	3.83	0.62	4.29	0.33	0.54	0.26	1.13	9.87	1933	
496	1932	1,450	1,101,905	21.1	45.5	31.1	4.02	4.32	0.34	5.48	0.52	0.51	0.42	1.45	11.59	1932	496
	1933	1,450	1,108,150	21.1	44.4	31.7	4.08	4.39	0.37	4.98	0.89	0.44	0	1.33	11.07	1933	
70	1930	1,420	1,980,400	....	....	....	3.68	3.81	0.44	3.84	0.13	0.12	0.05	0.30	8.39	1930	70
	1931	1,420	1,868,725	41.7	62.0	39.4	2.83	3.25	0.46	3.48	0.32	0.85	0.38	1.55	8.74	1931	
	1932	1,420	1,675,057	38 4	54 3	41.5	3.03	3.71	0.53	4.25	0.27	0.53	0	0.80	9.29	1932	
	1933	1,420	1,594,598	41.7	52.0	41.2	3 16	3.92	0.61	3.20	0.24	0.08	1.21	1.53	9.26	1933	
154	1930	1,400	1,571,594	26.8	....	....	4.48	3.79	0.32	3.95	0.35	....	....	0.71	8.77	1930	154
	1931	1,400	1,608,939	29.3	....	....	3.59	2.94	0.22	4.22	0.44	....	....	1.58	8.96	1931	
	1932	1,400	1,521,613	29.4	81.9	23.0	3.47	2.81	0.29	4.18	0.33	....	....	0.81	8.09	1932	
	1933	1,400	1,483,854	30.3	79 2	23.2	3.87	3.18	0.33	2.81	0.31	....	....	1.79	8.11	1933	
163	1930	1,400	5,125,835	60.4	71.4	82.1	2.44	2.13	0.21	1.12	0.29	....	....	3.44	6.90	1930	163
	1931	1,400	3,793,300	43.6	76.3	63.7	1.92	1.75	0.16	1.60	0.21	....	....	2.16	5.67	1931	
	1932	1,400	127,335	1.7	50.4	3.4	0.97	1.21	0.36	7.30	0.36	5.53	0.58	6.47	15.34	1932	
	1933	1,400	1,636,205	20.2	55.0	39 1	3.44	3.58	0.17	1.24	0.74	....	....	2.81	7.80	1933	
198	1932*e*	1,400	2,089,869	....	83.6	56.7	3.63	3.31	0.34	0.42	0.06	0.12	0.05	0.23	4.30	1932*e*	198
	1933	1,400	3,514,032	41.9	70.6	63.1	3.52	3.26	0.11	0.77	0.11	....	....	0.53	4.67	1933	
517	1932	1,375	1,050,715	30.7	27.9	50.1	3.74	4.48	0.61	4.84	0.25	0.05	0.40	0.70	10.63	1932	517
	1933	1,375	1,034,979	34.2	27.7	50.1	3.65	4.51	0.76	4.19	0.30	0.07	0.01	0.38	9.84	1933	
888	1932	1,375	2,168,250	31.0	49.8	55.4	4.02	4.21	0.35	5.19	1.43	1.35	0.76	3.54	13.29	1932	888
	1933	1,375	2,252,600	31.6	52.1	55.2	4.69	4.76	0.32	5.45	2.02	2.52	0.68	5.22	15.75	1933	
725	1930	1,310	1,976,000	....	....	....	4.40	4.39	0.84	2.58	0.38	0.46	0.20	1.04	8.85	1930	725
	1931	1,310	2,640,671	....	....	....	3.35	3.16	0.58	1.78	0.40	0.21	0.15	0.76	6.28	1931	
	1932	1,310	3,324,653	59.8	74.4	63.8	3.62	3.43	0.81	1.70	0.18	0.24	0.02	0.44	6.38	1932	
	1933	1,310	2,013,550	37.5	57.8	46.5	3.59	3.38	0.80	2.19	0.32	0.04	0.02	0.38	6.75	1933	
886	1932	1,305	889,868	28.5	39.3	31.4	4.74	7.00	1.39	5.20	1.68	0.89	0.15	2.72	16.31	1932	886
	1933	1,305	865,747	34.0	40.7	29.3	4.74	7.00	1.26	5.88	1.71	2.76	0.13	4.60	18.74	1933	

TABLE II—COMPARATIVE COSTS—1929, 1930, 1931, 1932 AND 1933 REPORTS (Page 3)

Plant No.	Year	Total Installed B.H.P.	Total Net Output K.W. Hrs.	Annual Plant Load Factor	Running Plant Capacity Factor	Plant Service Factor	Aver. Cost Fuel Oil—Cents per Gallon	Costs per Net K. W. Hr.—Mills								Year	Plant No.
								Fuel Cost	Lub. Oil Cost	Atten. Cost, Incl. Superint.	Cost Supplies & Misc., Incl. Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Total All Supply Repair & Misc. Costs	Total Production Cost		
13	1929	1,225	1,169,971	33.8	55.9	34.1	6.00	7.44	0.57	4.74	1.00	0.89	0.77	2.66	15.41	1929	13
	1930	1,225	1,131,661	32.7	59.2	31.2	5.69	6.60	0.65	4.88	0.54	0.39	0.08	1.01	13.14	1930	
	1931	1,225	1,259,958	38.4	58.3	34.1	5.41	6.29	0.73	4.69	0.24	0.31	0.05	0.60	12.31	1931	
	1932	1,225	1,176,048	36.0	57.4	32.4	5.13	5.94	0.68	4.50	0.48	0.59	0.37	1.44	12.56	1932	
	1933	1,225	1,153,244	39.6	56.4	32.0	4.72	5.55	0.86	3.35	0.70	0.37	0.38	1.45	11.21	1933	
16	1932	1,200	379,610	5.5	98.2	5.6	3.69	3.43	0.54	3.82	2.04	0.65	0.87	3.56	11.35	1932	16
	1933	1,200	109,596	1.6	93.3	1.7	3.23	3.13	1.35	11.95	6.25	8.83	2.89	17.97	34.40	1933	
17	1929	1,200	1,611,876	20.7	....	....	4.52	4.69	0.53	2.64	0.67	....	....	2.11	9.97	1929	17
	1930	1,200	2,148,910	27.5	....	....	4.45	4.23	0.50	2.06	0.36	....	....	1.96	8.75	1930	
	1931	1,200	2,207,300	27.2	....	....	3.97	3.83	0.40	1.82	0.42	....	....	1.89	7.94	1931	
	1932	1,200	2,064,882	24.1	58.3	51.4	3.71	3.67	0.38	1.66	0.38	....	....	1.36	7.07	1932	
	1933	1,200	1,946,839	26.2	59.2	47.8	4.23	4.19	0.38	1.69	0.43	....	....	1.24	7.50	1933	
980	1931	1,200	2,143,170	38.5	....	....	5.32	5.06	0.46	4.39	0.44	0.56	0	1.00	10.91	1931	980
	1932	1,200	No Reply to Inquiries for 1932.													1932	
	1933	1,200	1,724,300	39.4	51.0	48.8	4.35	4.26	0.43	6.79	0.15	0.19	.04	0.38	11.86	1933	
1083	1932	1,200	2,144,290	24.9	70.5	43.8	3.58	2.81	0.41	1.04	0.26	0.15	0.08	0.49	4.75	1932	1083
	1933	1,200	2,652,550	38.5	73.5	51.9	4.00	3.04	0.34	1.18	0.25	0.43	0.01	0.69	5.25	1933	
878	1932	1,160	1,878,245	32.3	74.6	43.9	2.71	3.44	0.70	2.39	0.63	0.50	0.60	1.73	8.26	1932	878
	1933	1,160	1,860,122	36.8	75.3	44.0	3.24	3.89	0.60	1.80	0.53	0.69	0.54	1.75	8.04	1933	
616	1930	1,150	1,553,500	37.2	....	....	5.10	4.83	0.85	4.81	....	....	....	0.39	10.88	1930	616
	1931	1,150	1,609,550	38.8	....	....	4.50	4.35	0.76	4.32	....	2.48	....	2.79	12.22	1931	
	1932	1,150	1,565,000	39.0	56.5	44.7	4.51	4.31	0.64	4.04	1.06	1.97	0	3.03	12.02	1932	
	1933	1,150	1,430,450	36.1	59.0	39.6	4.40	4.20	0.59	4.75	0.45	0.73	0.23	1.41	10.96	1933	
887	1932	1,140	1,102,053	42.5	45.9	38.7	4.16	5.26	0.99	5.25	1.74	0.06	0.09	1.89	13.39	1932	887
	1933	1,140	1,086,909	39.1	36.0	48.2	4.53	5.82	0.48	5.46	1.52	0.06	0.04	1.62	13.38	1933	
78	1928/30f	1,060	4,867,305	....	55.5	88.7	5.08	4.92	0.82	2.79	0.08	0.61	0.13	0.82	9.35	1928/'0f	78
	1931	1,060	1,906,347	31.4	46.5	69.2	5.07	5.88	1.13	4.69	0.23	0.15	0.19	0.57	12.27	1931	
	1932g	1,060	1,507,615	....	44.4	59.8	5.15	6.23	0.38	4.50	0.49	....	....	1.00	12.11	1932g	
	1933	1,060	2,239,317	31.9	54.2	69.1	5.06	5.48	0.38	3.53	0.20	0.67	0.16	1.03	10.42	1933	
519	1932	1,060	1,561,470	47.2	....	....	5.42	4.58	0.70	3.08	....	....	....	3.75	12.11	1932	519
	1933	1,060	1,563,450	48.8	57.9	46.2	3.40	3.62	0.45	2.40	0.17	2.57	0.13	2.87	9.34	1933	
203	1932h	1,050	1,464,600	42.0	55.7	45.0	3.37	3.81	0.95	4.56	0.72	0	0	0.72	10.04	1932h	203
	1933	1,050	1,460,200	40.3	51.1	50.4	4.16	4.49	0.90	4.11	0.17	0.02	0	0.19	9.69	1933	
259	1932	1,050	725,222	41.4	49.5	26.1	5.08	5.77	0.91	7.30	0.43	0.02	0	0.45	14.43	1932	259
	1933	1,050	734,445	44.1	52.8	24.9	5.18	5.84	0.78	7.03	0.37	0.57	0.05	0.99	14.64	1933	
1144	1932	1,050	1,534,790	36.4	62.6	40.9	4.87	4.12	0.77	2.03	....	....	....	0.88	7.80	1932	1144
	1933	1,050	1,648,088	38.3	62.9	44.6	4.83	4.29	0.79	2.17	....	....	....	1.10	8.35	1933	
801	1930	1,000	4,110,052	74.3	74.6	94.9	3.82	4.19	0.43	1.25	0.79	0.89	0.17	1.85	7.72	1930	801
	1931	1,000	4,616,205	78.0	84.9	95.8	3.31	3.28	0.30	1.04	1.20	0.28	0.02	1.50	6.12	1931	
	1932	1,000	4,524,387	78.7	85.2	96.2	3.20	3.36	0.27	1.04	0.14	0.30	0.06	0.50	5.17	1932	
	1933	1,000	4,206,500	72.2	90.0	83.7	3.99	3.48	0.32	0.88	0.20	0.57	0.09	0.86	5.54	1933	
1061	1930	980	1,405,473	....	96.7	49.4	4.32	3.22	0.56	0.68	0.10	....	....	0.55	5.01	1930	1061
	1931	980	1,974,459	33.4	81.4	43.7	3.98	3.59	0.56	0.92	0.18	0.63	0.10	0.91	5.98	1931	
	1932	980	2,974,884	50.1	94.5	56.4	3.25	2.95	0.36	0.61	0.14	0.59	0.25	0.98	4.90	1932	
	1933	980	2,155,929	36.8	83.5	46.9	3.46	3.16	0.24	0.76	0.16	....	....	1.48	5.64	1933	
53	1930	900	2,300,000	50.4	70.6	63.7	3.26	3.68	0.73	2.12	0.43	1.76	0.05	2.24	8.77	1930	53
	1931	900	2,355,470	52.8	71.0	65.2	2.65	3.06	0.53	2.00	0.42	2.94	0.05	3.41	9.00	1931	
	1932	900	1,975,660	44.2	69.0	56.2	2.55	2.90	0.41	2.21	0.41	3.79	0.02	4.22	9.74	1932	
	1933	900	1,314,144	31.2	66.7	38.8	2.49	2.36	0.68	4.06	0.48	1.91	0.01	2.40	9.50	1933	
260	1931	900	38,740	1.5	50.6	3.0	5.00	11.17	0.65	6.24	4.44	0	0	4.44	22.50	1931	260
	1932	900	68,389	1.7	47.0	3.6	3.39	5.34	1.42	5.26	0	0.18	0	0.18	12.20	1932	
	1933	900	156,190	3.6	48.3	7.0	3.95	4.92	0.89	2.56	0	....	....	0.35	8.72	1933	
1065	1932	900	363,000	8.7	83.3	8.6	3.81	3.22	0.66	1.65	1.82	1.04	1.25	4.11	9.64	1932	1065
	1933	900	209,620	4.6	87.0	4.8	4.13	3.22	0.61	2.69	2.54	2.26	2.54	7.34	13.86	1933	
189	1930	850	800,000	32.4	27.4	59.5	4.10	5.22	0.93	8.63	1.03	....	....	2.53	17.31	1930	189
	1931	850	951,426	38.4	31.5	64.0	3.68	4.40	0.36	5.99	0.78	....	....	2.44	13.19	1931	
	1932	850	998,200	41.4	32.8	64.1	2.76	3.38	0.82	5.86	0.34	....	....	1.46	11.52	1932	
	1933	850	956,138	38.5	32.8	61.6	3.61	4.57	0.70	6.24	0.34	....	....	0.55	12.06	1933	
153	1930	840	49,340	2.2	....	....	4.08	7.70	1.01	44.30	11.92	....	....	24.43	77.44	1930	153
	1931	840	557,470	12.1	....	....	3.41	3.46	0.61	4.71	0.90	....	....	1.83	10.61	1931	
	1932	840	929,460	19.7	82.7	23.6	3.55	3.18	0.53	2.65	0.78	....	....	1.24	7.60	1932	
	1933	840	60,200	1.7	....	....	4.00	4.70	1.66	25.76	6.48	....	....	17.91	50.03	1933	
494	1931	840	915,865	21.4	76.8	24.7	3.56	3.09	0.34	1.59	0.70	0.61	0.28	1.59	6.61	1931	494
	1932	840	482,220	11.3	74.5	13.4	3.51	3.38	0.54	1.88	0.87	0.60	1.22	2.69	8.49	1932	
	1933	840	369,381	7.8	78.4	9.7	4.20	3.69	0.46	2.22	0.71	0.79	1.28	2.78	9.15	1933	
560	1932	900	719,930	26.9	38.2	36.2	2.76	4.57	0.74	6.79	0.32	2.26	0.19	2.77	14.87	1932	560
	1933	900	670,266	25.3	39.0	34.3	2.98	4.15	0.74	6.74	0.45	0.71	0	1.16	12.79	1933	
619	1931	840	828,880	18.1	74.1	23.9	3.72	3.21	0.65	1.66	0.85	1.70	0	2.55	8.07	1931	619
	1932	840	575,200	12.2	76.5	15.5	3.53	3.51	0.37	1.96	1.06	1.55	0.32	2.93	8.77	1932	
	1933	840	390,760	8.2	82.3	9.9	4.28	3.34	0.66	2.59	1.01	0.52	0.28	1.81	8.40	1933	
1056	1930	840	2,122,057	46.5	92.5	50.2	4.10	4.13	0.56	1.61	0.15	....	....	0.37	6.67	1930	1056
	1931	840	1,906,182	41.5	76.2	55.0	3.84	3.70	0.51	1.56	0.50	0.66	0.28	1.44	7.21	1931	
	1932	840	1,655,114	34.0	76.2	47.4	3.12	3.09	0.23	1.80	0.62	0.42	0.24	1.28	6.40	1932	
	1933	840	379,021	8.1	62.3	13.8	3.50	3.65	0.29	4.96	2.10	....	....	5.86	14.76	1933	
731	1930	825	454,041	9.2	49.2	18.7	5.75	5.96	1.84	9.43	0.37	1.56	0	1.93	19.16	1930	731
	1931	825	592,895	11.8	61.2	20.8	4.84	4.91	1.00	6.58	0.23	0.59	0	0.82	13.31	1931	
	1932	825	554,765	11.1	68.2	17.7	4.00	3.70	1.17	7.50	1.20	1.63	0.09	2.92	15.29	1932	
	1933	825	462,470	9.1	68.0	14.5	5.29	5.05	1.70	8.59	1.31	0.52	1.05	2.88	18.22	1933	

TABLE II—COMPARATIVE COSTS—1929, 1930, 1931, 1932 AND 1933 REPORTS (Page 4)

Plant No.	Year	Total Installed B.H.P.	Total Net Output K.W. Hrs.	Annual Plant Load Factor	Running Plant Capacity Factor	Plant Service Factor	Aver. Cost Fuel Oil—Cents per Gallon	Costs per Net K. W. Hr.—Mills: Fuel Cost	Lub. Oil Cost	Atten. Cost, Incl. Superint.	Cost Supplies & Misc., Incl. Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Total All Supply Repair & Misc. Costs	Total Production Cost	Year	Plant No.
648	1931	800	1,180,517	32.1	80.9	31.7	3.53	3.23	0.38	2.97	0.76	0.02	0.06	0.84	7.42	1931	648
	1932	800	1,068,900	31.3	70.2	33.2	3.23	3.12	0.39	3.13	0.40	0.46	0.11	0.97	7.61	1932	
	1933	800	1,095,200	32.1	67.6	35.4	3.90	3.64	0.43	2.84	0.28	0.16	0.05	0.49	7.40	1933	
112	1930	740	1,165,300	29.7	68.1	42.2	4.43	5.19	0.83	5.46	0.19	0.54	0.12	0.85	12.33	1930	112
	1931	740	1,195,128	34.8	71.3	41.8	3.42	3.47	0.74	5.38	0.18	0.35	0.25	0.78	10.37	1931	
	1932	740	1,443,130	41.9	75.8	46.7	3.52	3.57	0.64	4.19	0.23	0.06	0.04	0.33	8.73	1932	
	1933	740	1,104,807	34.1	63.1	43.4	3.78	3.94	0.69	5.28	0.25	0.31	0.05	0.61	10.52	1933	
67	1929	720	1,163,600	64.0	79.1	33.7	5.94	6.03	0.55	1.60	0.21	0.04	0	0.25	8.43	1929	67
	1930	720	1,098,900	58.4	82.9	30.3	4.97	4.73	0.71	1.70	0.23	1.03	0.02	1.28	8.42	1930	
	1931	720	891,200	48.5	82.3	25.6	4.58	4.38	0.87	2.09	0.24	1.05	0.10	1.39	8.73	1931	
	1932	720	810,190	44.0	81.5	23.4	4.47	4.39	0.94	2.30	0.30	1.04	0.12	1.46	9.09	1932	
	1933	720	832,067	45.3	83.8	23.4	5.34	5.40	0.86	2.24	0.34	0.76	0.17	1.27	9.77	1933	
170	1930	720	860,110	19.5	64.7	32.7	6.51	7.28	0.89	2.50	0.12	1.30	0	1.42	12.09	1930	170
	1931	720	835,219	22.4	64.5	31.6	6.20	6.78	0.78	2.19	0.20	2.64	0	2.84	12.59	1931	
	1932	720	675,376	17.4	58.2	28.5	5.53	6.01	0.84	2.71	0.22	....	....	1.63	11.19	1932	
	1933	720	780,252	19.8	55.8	34.5	4.95	4.84	0.77	2.42	0.32	....	....	1.35	9.38	1933	
527	1930*d*	720	673,300	....	37.0	56.2	3.87	5.02	1.59	3.97	0.25	0.04	0.04	0.33	10.91	1930*d*	527
	1931	720	933,200	31.9	41.4	60.5	2.83	3.35	0.82	3.48	0.29	0.13	0.01	0.43	8.08	1931	
	1932	720	967,000	31.1	39.6	59.2	3.10	3.52	0.81	5.08	0.24	0.25	0.03	0.52	9.93	1932	
	1933	720	864,300	31.6	42.4	50.9	3.23	3.71	0.87	3.82	0.91	0.34	0.06	1.31	9.71	1933	
718	1930*d*	720	604,312	....	36.9	51.5	4.27	6.05	1.33	6.50	0.83	1.70	0.58	3.11	16.99	1930*d*	718
	1931	720	691,477	38.6	34.9	51.4	3.45	4.66	1.33	5.04	0.68	6.28	0.14	7.10	18.13	1931	
	1932	720	644,337	51.0	33.0	51.2	3.61	4.98	1.02	5.39	0.64	1.05	0.77	2.46	13.85	1932	
	1933	720	658,898	41.7	33.5	51.3	3.76	5.53	1.13	5.52	0.25	1.15	0.10	1.50	13.68	1933	
19	1929	700	603,260	29.8	....	....	4.50	6.32	1.40	5.40	1.07	....	....	2.24	15.36	1929	19
	1930	700	710,280	34.9	....	....	4.13	5.72	1.26	4.96	1.07	....	....	4.05	15.99	1930	
	1931	700	694,060	34.8	....	....	3.73	5.27	1.47	5.15	0.80	....	....	5.59	17.48	1931	
	1932	700	612,980	36.7	48.5	33.1	3.63	4.87	1.35	4.96	1.01	....	....	1.68	12.86	1932	
	1933	700	579,880	38.8	....	....	4.08	5.61	1.31	5.18	0.69	....	....	2.37	14.47	1933	
529	1932	620	507,836	34.3	38.4	38.6	3.60	5.28	0.93	6.21	1.92	0.21	0.20	2.33	14.75	1932	529
	1933	620	524,025	35.4	39.4	38.9	4.00	5.34	0.62	7.45	2.01	0.60	0.19	2.80	16.21	1933	
32	1932	600	402,600	10.5	77.9	15.0	5.45	5.09	0.61	10.95	0.17	0.29	0	0.46	17.11	1932	32
	1933	600	706,050	18.4	72.8	28.3	6.00	5.43	1.26	5.52	0.08	0.05	0	0.13	12.34	1933	
61	1930	600	1,539,900	....	52.3	79.6	6.39	5.31	0.59	5.53	0.33	0.61	0.06	1.00	12.43	1930	61
	1931	600	1,555,200	38.7	68.4	64.7	4.20	3.46	0.36	5.37	0.11	1.07	0	1.18	10.37	1931	
	1932	600	1,380,330	37.9	64.3	60.7	4.63	3.97	0.38	5.76	0.04	1.02	0.03	1.09	11.20	1932	
	1933	600	1,188,400	32.7	56.2	63.4	4.98	4.62	0.52	2.85	0.30	0.50	0	0.80	8.79	1933	
106	1929	600	750,419	....	....	....	4.66	4.75	0.66	2.66	0.61	....	....	2.62	10.69	1929	106
	1930	600	1,435,434	35.6	....	....	4.49	4.18	0.27	3.09	0.65	....	....	2.52	10.06	1930	
	1931	600	1,504,859	28.1	....	....	3.97	3.71	0.24	3.05	0.68	....	....	1.76	8.76	1931	
	1932	600	1,358,285	31.5	71.4	55.8	3.77	3.57	0.27	3.07	0.74	....	....	2.07	8.98	1932	
	1933	600	1,522,216	39.0	71.8	62.0	4.31	3.99	0.24	2.73	0.52	....	....	1.28	8.24	1933	
1094	1931	600	678,530	19.5	78.5	25.3	3.84	3.43	0.27	2.65	0.91	1.80	0.37	3.08	9.43	1931	1094
	1932	600	572,600	17.2	78.0	22.4	3.79	3.78	0.35	1.52	1.24	0.37	0.50	2.11	7.76	1932	
	1933	600	583,920	16.6	89.4	18.9	4.18	3.43	0.34	1.39	0.60	0.71	0.24	1.55	6.71	1933	
1096	1931	600	726,486	20.8	74.0	29.3	3.68	3.13	0.50	4.46	0.76	0.48	0.41	1.65	9.74	1931	1096
	1932	600	708,626	19.7	69.9	29.3	3.88	3.39	0.50	2.09	0.65	1.45	0.27	2.37	8.35	1932	
	1933	600	716,299	20.4	69.8	29.8	4.20	3.40	0.33	2.40	0.62	1.06	0.16	1.84	7.97	1933	
862	1930	588	627,820	25.3	47.7	39.7	4.43	4.62	0.83	8.84	0.42	1.63	1.88	3.93	18.22	1930	862
	1931	588	640,360	25.7	46.9	41.7	3.27	3.81	0.70	10.77	0.40	1.24	0.97	2.61	17.89	1931	
	1932	588	609,820	31.2	47.9	38.6	3.14	3.87	0.75	8.69	0.47	1.71	0.48	2.66	15.97	1932	
	1933	588	609,080	33.3	44.9	42.1	3.44	4.33	1.00	9.18	0.81	4.08	0.40	5.29	19.80	1933	
246	1930	540	364,500	28.8	....	....	6.47	11.42	2.69	9.80	0.69	0.14	0	0.83	24.74	1930	246
	1931	540	407,085	33.6	38.0	35.4	6.29	9.15	2.19	8.78	0.41	0.35	0	0.76	20.88	1931	
	1932	540	412,636	30.7	35.1	38.5	6.17	9.05	1.86	8.66	0.28	0.10	0	0.38	19.95	1932	
	1933	540	397,000	29.4	33.9	38.4	5.34	7.65	1.80	7.83	0.27	0.10	0.08	0.45	17.73	1933	
540	1931	540	713,333	32.9	....	....	3.30	4.10	1.72	5.52	0.35	0.38	0.06	0.79	12.13	1931	540
	1932	540	633,751	28.2	40.9	51.3	3.36	4.46	1.91	6.22	0.69	0.19	0	0.88	13.47	1932	
	1933	540	666,616	28.6	41.0	53.7	3.55	4.64	2.08	5.46	0.74	0.06	0.06	0.86	13.04	1933	
772	1931	500	222,100	9.5	74.5	10.7	3.72	3.67	0.68	9.55	3.41	3.67	0.56	7.64	21.54	1931	772
	1932	500	142,730	6.2	73.7	7.1	3.72	3.90	0.64	4.90	4.15	0.57	0.46	5.18	14.62	1932	
	1933	500	103,495	3.8	73.8	5.0	4.36	4.57	1.35	6.74	5.32	3.27	1.19	9.78	22.44	1933	
644	1932	600	846,916	40.4	59.8	43.1	4.04	4.53	0.98	4.02	0.75	0.04	0.06	0.85	10.38	1932	644
	1933	600	840,958	39.4	59.5	43.5	5.28	6.03	1.03	3.42	0.85	0.18	0.06	1.08	11.57	1933	
424	1932	480	586,935	36.1	41.8	50.9	3.00	3.58	1.34	4.37	0.34	0.14	0	0.48	9.77	1932	424
	1933	480	621,800	34.5	47.0	50.2	3.00	3.43	1.32	4.39	0.24	0	0	0.24	9.38	1933	
501	1930	480	403,022	32.5	40.6	37.5	5.50	7.58	1.80	10.58	0.40	0.14	0.31	0.85	20.81	1930	501
	1931	480	452,970	37.3	39.3	42.6	3.93	5.53	1.60	8.50	0.49	0.62	0.33	1.44	17.07	1931	
	1932	480	421,162	33.4	37.9	41.7	4.71	6.85	1.94	8.37	0.40	1.04	0.39	1.83	18.99	1932	
	1933	480	397,167	35.3	36.5	40.4	4.30	6.02	1.83	11.64	0.31	1.44	0.23	1.98	21.47	1933	
695	1932	450	423,414	29.2	47.6	34.6	4.59	5.13	0.77	9.63	0.62	0	0	0.62	16.15	1932	695
	1933	450	440,760	31.6	48.8	35.3	4.81	5.26	1.24	9.39	0.67	2.90	0	3.57	19.46	1933	
335	1932	430	490,130	38.9	37.1	53.4	3.11	3.20	0.71	7.15	1.26	0.03	0.03	1.32	12.38	1932	335
	1933	430	479,720	36.9	36.5	53.2	4.72	4.85	0.76	9.08	0.94	0.87	0.04	1.85	16.54	1933	
247	1930	390	428,290	28.9	....	....	6.00	8.12	1.10	10.79	0.11	0	0	0.11	20.12	1930	247
	1931	390	480,204	25.1	44.6	47.1	4.62	6.03	1.01	9.62	0.05	0.40	0.26	0.71	17.37	1931	
	1932	390	485,760	24.7	54.2	39.2	4.55	5.88	0.81	9.76	0.07	0.12	0.05	0.24	16.69	1932	
	1933	390	486,820	24.9	43.4	49.1	5.50	7.03	0.44	9.74	0.28	0.18	0	0.46	17.67	1933	
152	1932	360	Net Generation Negative in 1933.														

TABLE II—COMPARATIVE COSTS—1929, 1930, 1931, 1932 AND 1933 REPORTS (Page 5)

Plant No.	Year	Total Installed B.H.P.	Total Net Output K.W. Hrs.	Annual Plant Load Factor	Running Plant Capacity Factor	Plant Service Factor	Aver. Cost Fuel Oil—Cents per Gallon	Costs per Net K. W. Hr.—Mills								Year	Plant No.
								Fuel Cost	Lub. Oil Cost	Atten. Cost, Incl. Superint.	Cost Supplies & Misc., Incl. Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Total All Supply Repair & Misc. Costs	Total Production Cost		
187	1932*i*	350	297,000	....	....	....	4.15	5.02	2.18	11.84	0.82	0.10	0.03	0.95	19.99	1932*i*	187
	1933	350	301,279	25.8	32.9	46.7	3.82	4.23	2.00	10.64	0.43	0.15	0.05	0.64	17.51	1933	
688	1931	350	437,000	....	....	....	5.00	6.69	0.65	6.86	0.51	0.17	0.69	1.37	15.57	1931	688
	1932	350	432,500	29.3	42.6	49.8	3.93	5.32	0.45	8.32	0.12	0.25	0.12	0.49	14.58	1932	
	1933	350	400,000	29.4	41.8	47.8	4.82	7.14	0.69	6.86	0.97	0.41	0.19	1.57	16.26	1933	
774	1931	345	109,735	7.2	66.2	8.4	3.79	4.74	0.71	9.97	3.03	6.09	1.07	10.19	25.61	1931	774
	1932	345	77,122	5.1	72.6	5.5	3.58	3.31	1.03	6.57	3.52	1.36	2.97	7.85	18.76	1932	
	1933	345	65,645	7.6	67.4	4.9	4.59	4.61	0.96	7.28	3.31	12.98	0.06	16.35	29.20	1933	
857	1930	330	307,163	44.3	....	....	3.39	5.91	2.06	11.50	1.75	1.22	0.84	3.81	23.28	1930	857
	1931	330	No Reply to Inquiries for 1931.													1931	
	1932	330	379,380	....	42.8	51.3	2.80	4.30	1.57	6.92	1.82	3.97	0.84	6.63	19.42	1932	
	1933	330	369,378	42.8	40.8	51.9	3.25	5.36	1.71	7.39	1.79	6.12	0.70	8.60	23.07	1933	
265	1930	180	184,000	....	....	....	9.14	13.48	1.76	13.58	1.10	0.71	0	1.81	30.63	1930	265
	1931	320	216,000	....	....	37.0	7.79	11.81	1.83	11.57	0.32	0.46	0.58	1.36	26.57	1931	
	1932	320	220,000	25.4	37.9	34.8	7.30	11.00	1.86	11.37	0.16	0.11	0.34	0.61	24.84	1932	
	1933	320	257,000	26.0	39.6	35.2	7.35	9.59	1.41	11.67	0.19	0.70	0.14	1.03	23.70	1933	
1055	1932*k*	320	204,875	....	29.6	100.0	4.50	5.79	1.49	8.30	0.46	0	0	0.46	16.04	1932*k*	1055
	1933	320	421,352	33.1	28.4	97.5	4.32	7.27	0.47	11.29	0.54	0	0.05	0.59	19.62	1933	
27	1930	315	190,375	30.1	....	....	4.03	9.00	2.42	12.68	0.77	....	....	6.68	30.78	1930	27
	1931	315	208,890	30.8	....	....	4.06	7.72	1.52	10.58	0.95	....	....	7.29	27.16	1931	
	1932	315	194,105	32.0	28.8	39.4	3.63	7.12	3.05	11.06	0.92	....	....	13.05	34.28	1932	
	1933	315	186,510	31.0	....	....	4.14	7.29	2.13	10.88	0.77	....	....	5.93	26.23	1933	
382	1931*l*	300	86,887	....	20.5	37.4	3.35	3.62	0.82	4.47	0.43	0	0	0.43	9.34	1931*l*	382
	1932	300	99,300	....	21.2	28.4	3.48	4.76	0.66	5.08	0.42	0	0	0.42	10.92	1932	
	1933	300	251,413	....	27.2	52.6	3.64	3.63	0.50	3.73	0.19	0.02	0	0.21	8.07	1933	
858	1930	180	123,538	23.0	....	....	3.41	6.87	3.26	31.90	4.02	4.66	2.08	10.76	52.79	1930	858
	1931	180	No Reply to Inquiries for 1931.													1931	
	1932	270	175,200	....	26.7	49.7	2.82	6.46	2.78	17.45	3.31	1.54	0.43	5.28	31.97	1932	
	1933	270	172,400	39.6	26.7	49.6	3.12	7.17	3.13	16.40	3.81	6.85	1.00	11.66	38.36	1933	
169	1930	240	151,850	11.3	41.5	27.1	5.98	6.23	0.94	2.56	0.55	0.51	0	1.06	10.79	1930	169
	1931	240	184,360	13.7	42.5	31.9	5.56	6.23	0.89	2.11	0.87	4.39	0	5.26	14.49	1931	
	1932	240	77,310	5.8	27.7	20.9	5.04	6.52	1.25	3.84	0.85	....	....	14.77	26.38	1932	
	1933	240	79,570	6.0	28.8	20.7	4.50	6.34	1.13	4.52	0.98	....	....	9.72	21.71	1933	
984	1932	225	27,290	1.6	47.1	4.6	4.50	6.43	0.57	5.22	1.10	0	0	1.10	13.32	1932	984
	1933	225	26,851	1.8	48.5	4.5	4.29	5.07	0.60	5.59	0.75	0.82	0.19	1.76	13.02	1933	
318	1932	212.5	138,700	29.9	....	....	4.10	6.81	1.89	15.14	2.78	0.23	0.25	3.26	27.10	1932	318
	1933	212.5	139,730	26.6	....	....	3.69	6.81	2.37	13.43	2.00	0.34	1.20	3.54	26.15	1933	
733	1930	175	14,800	1.6	65.0	1.6	4.82	5.54	2.91	8.92	2.91	0	0	2.91	20.28	1930	733
	1931	175	3,293	0.3	62.5	0.5	5.78	10.93	0.91	10.93	9.72	0	0	9.72	32.49	1931	
	1932	175	1,788	0.2	62.1	0.3	3.00	7.92	1.96	18.18	34.30	0.56	20.43	55.29	83.35	1932	
	1933	175	1,892	0.2	53.2	0.4	6.03	16.00	3.17	38.34	20.20	0	37.20	57.40	114.91	1933	
646	1930	120	15,893	3.8	13.7	7.0	7.00	12.65	3.15	147.25	11.26	0	0	11.26	174.31	1930	646
	1931	120	21,035	5.3	16.1	20.4	7.00	16.70	2.57	16.97	3.28	0	0	3.28	39.52	1931	
	1932	120	5,832	1.6	13.4	7.6	5.37	18.82	5.70	27.14	0	0	0	0	51.66	1932	
	1933	120	8,074	....	11.1	11.4	5.39	14.62	4.34	21.92	12.87	9.91	8.55	31.33	72.21	1933	

Plant 54 did not operate in 1933. Purchased Power of Dump Power Contract.
Following Plants were used for Standby Power only in 1933: 735-736-738.
" " Data for 1933 was not received: 132-451-626-863.
" " did not submit sufficient Data for 1933: 161-532-1129-83.
" " made no reply to inquiries for 1933: 3-14-91-92-93-95-96-98-101-102-103-105-143-160-193-333-411-516-720-908-1016-1091.

NOTES:

a— 8.1 months
b—13 months
c— 6 months
d—10 months
e— 6.7 months
f—36 months
g—11 months
h—12.1 months
i—12.8 months
k— 5 months
l— 8 months

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 1)

Plant Number	Engine Data												Lubrication					Fuel			Gross Output—K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Gross K.W. Hrs. per Gallon of Fuel Oil
	Engine Designation	Engine Cycle	Injection System (Notes)	Scavenging System (Notes)	Trunk Piston or Crosshead?	Rated B.H.P.	Equivalent K.W.—90% Generating Efficiency	Number of Cylinders	Cylinder Dimensions Bore x Stroke—Inches	Rated R.P.M.	Generator Rating—K.V.A.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Total Gallons of New Lubricating Oil Used	Gals. of New Lub. Oil for Cylinder Lub. Only	Gals. of Unfit Lubricating Oil Discarded	Rated H.P. Hours per Gal. of New Lubricating Oil	Lubricating Oil Treatment (See Notes)	Fuel Oil Used—Gallons	Nature of Fuel Oil Used (See Notes)	Is Fuel Centrifuged?			
43	1	4	A	..	TP	500	336	3	22 x29½	150	375	1915	93	155	19	.....	300	S	2,162	20.1°API	No	21,456	138	9.92
	2	4	A	..	TP	500	336	3	22 x29½	150	375	1916	157	183	35	.....	429	&	3,232	75.5 SSF @ 77F	No	34,720	190	10.74
	3	4	A	..	TP	520	349	4	19 x24½	200	500	1919	255	171	63	.....	776	F	7,406	1.1%S; 0.02% Ash	No	68,710	402	9.28
	4	4	A	..	TP	520	349	4	19 x24½	200	500	1920	254	236	64	.....	560		7,114	0.05% BS&W	No	65,930	279	9.27
	5	4	A	..	CHa	1,000	671	6	20½x36	150	925	1922	465	429	196	.....	1,083		19,514	19.3% Asphaltic Residue	No	197,810	461	10.13
	6	4	A	..	CHa	1,000	671	6	20½x36	150	925	1925	132	366	81	.....	361		6,975	after ½ hour @ 400C	No	57,210	156	8.20
	7	2	A	P	CH	3,750	2,517	6	30 x42	124	3,750	1928	5,250	7,144	2,553	.....	2,755		826,012	1.91% hard asphalt	No	10,024,400	1,403	12.13
	8	2	A	B	CH	3,750	2,517	6	30 x42	124	3,750	1929	2,430	3,831	1,224	.....	2,376	CC	359,831		No	4,650,700b	1,213b	12.92b
	9	2	M	B	CH	4,100	2,753	6	30 x42	124	3,750	1932	7,803	9,737	3,347	.....	3,285		1,094,781		No	15,674,800b	1,610b	14.31b
	Plant					15,640	10,499				14,850			22,252	7,582	.....	2,778		2,327,027			30,795,736b	1,383b	13.23b
82	1	4	A	..	TP	360	242	4	16½x21	225	312	1921	808	86	81	.....	3,380		11,992	33.7°API	No	113,000	1,313	9.42
	2	2	A	P	CH	750	504	4	17 x27	180	740	1923	1,346	276	227	.....	3,657		39,826	40.5 SSU @ 100F	No	336,030	1,217	8.44
	3	2	A	P	CH	750	504	4	17 x27	180	740	1924	1,146	216	194	.....	3,977		31,237	0.38% S; 0.06% CC	No	284,300	1,316	9.10
	4	2	A	P	CH	1,150	772	6	17 x27	180	1,000	1927	2,699	645	543	.....	4,810	BC	102,266	0.004% Ash	No	995,130	1,542	9.73
	5	2	A	P	CH	1,150	772	6	17 x27	180	1,000	1927	2,696	1,384	584	.....	2,239		119,717	BS&W—Trace	No	1,287,390	930	10.75
	6	2	A	P	CH	1,500	1,007	8	17 x27	180	1,350	1929	2,739	1,556	723	.....	2,639		172,891		No	1,899,600	1,220	10.98
	7	2	M	B	TP	3,300	2,216	10	19½x27	240	2,960	1932	3,362	3,019	2,473	.....	3,674		359,547		No	5,080,100b	1,683b	14.12b
	Plant					8,960	6,017				8,102			7,182	4,825	1,952	3,282		837,476			9,995,550b	1,391b	11.93b
52	1	4	A	..	CHa	830	557	4	23 x32	164	700	1928	3,508	610	329	0	4,774		118,998	25.6° API	No	1,371,500	2,250	11.52
	2	4	A	..	CHa	1,250	840	6	23 x32	164	1,060	1928	4,379	970	598	0	5,645	BC	243,557	0.6% S; 0.3% CC	No	2,880,600	2,970	11.82
	3	4	A	..	CHa	1,250	840	6	23 x32	164	1,060	1928	2,360	698	322	0	4,225	&	128,720	0.6% BS&W; Ash—Trace	No	1,500,700	2,150	11.65
	4	4	A	..	CHa	1,250	840	6	23 x32	164	1,060	1930	2,900	730	397	0	4,965	S	163,760		No	1,909,400	2,615	11.65
	5	4	A	..	CHa	2,865	1,924	8	29 x48	120	2,500	1933d	1,380	823	395	0	4,805		182,651		No	2,287,600	2,780	12.51
	Plant					7,445	5,001				6,380			3,831	2,041	0	4,935		837,686			9,949,800	2,595	11.87
73	1	2	A	P	CH	1,250	840	5	20½x26	180	1,125	1924	2,247	766	175	0	3,667		125,364		No	1,272,500	1,661	10.15
	2	2	A	P	CH	1,250	840	5	20½x26	180	1,125	1926	2,616	927	225	0	3,528	CC	147,877	10-14° API	No	1,516,300	1,636	10.25
	3	2	A	P	CH	1,250	840	5	20½x26	180	1,125	1928	5,181	1,549	363	0	4,181	&	280,072		No	2,805,800	1,811	10.02
	4	2	A	P	CH	1,250	840	5	20½x26	180	1,125	1928	5,765	1,525	367	0	4,725	F	332,239		No	3,327,000	2,182	10.01
	Plant					5,000	3,360				4,500			4,767	1,130	0	4,145		885,552			8,921,600	1,872	10.07
45	1	4	A	..	TP	600	403	6	16½x24	200	500	1923	49	24	.....	.....	1,225	BC,	696	21.2° API; 65 SSU @ 122F;	No	9,850	411	14.13
	2	4	A	..	TP	600	403	6	16½x24	200	500	1923	148	44	.....	.....	2,018	C&F	2,682	7.74% CC; 1.47% S	No	24,475	556	9.12
	3	4	A	..	CHa	750	504	4	23 x32	150	625	1923	2,548	460	405	.....	4,154		60,058	2.5% BS&W	No	581,600	1,262	9.69
	4	4	A	..	CHa	1,150	772	6	23 x32	150	1,000	1926	4,677	682	572	.....	7,880	BC	195,146		No	2,101,650	3,080	10.77
	5	4	A	..	CHa	1,150	772	6	23 x32	150	1,000	1926	4,907	665	553	.....	8,485	&C	216,707		No	2,266,750	3,408	10.47
	Plant					4,250	2,854				3,625			1,875	.....	.....	6,960		475,289			4,984,325	2,658	10.46
111	1	4	A	..	TP	675	453	6	17 x24	225	580	1928	4,251	.....	.....	.....	.....		.....	19.4° API	No	1,163,200	.....	.....
	2	4	A	..	TP	900	604	8	17 x24	225	770	1928	2,921	.....	.....	.....	.....	CC	.....		No	1,165,100	.....	.....
	3	4	A	..	TP	900	604	8	17 x24	225	770	1928	3,026	.....	.....	.....	.....	&	.....		No	1,167,600	.....	.....
	4	4	M	..	TP	1,200	806	6	20 x24	277	1,040	1932	3,225	.....	.....	.....	.....	S	.....		No	1,640,000	.....	.....
	Plant					3,675	2,467				3,160			3,700	800	50	3,270		433,188			5,135,900h	1,385h	11.83h
130	1	4	A	..	TP	600	403	6	16½x23	225	513	1924	5,816	1,694	220	0	2,058		145,990	16°—30° API	No	1,592,800	940	10.91
	2	4	A	..	TP	600	403	8	17 x24	200	513	1926	4,571	1,150	210	0	2,383	BC	111,322	0.9% S	No	1,197,200	1,041	10.75
	3	4	A	..	TP	1,200	806	6	20 x24	300	1,062	1929	2,068	1,115	198	0	2,225	&	108,700		No	1,153,100	1,033	10.61
	4	4	A	..	TP	1,200	806	6	20 x24	300	1,062	1930	2,321	1,571	253	0	1,773	CC	124,930		No	1,295,300	825	10.37
	Plant					3,600	2,418				3,150			5,530	881	0	2,078		490,942			5,238,400	947	10.67
157	1	2	A	P	CH	1,760	1,182	8	17 x23	257	1,500	1930	1,109	.....	.....	.....	.....	S	98,122	24°—26° API	No	1,068,500	.....	10.89
	2	2	A	P	CH	1,760	1,182	8	17 x23	257	1,500	1930	1,079	.....	.....	.....	.....	&F	94,834		No	1,093,600	.....	11.53
	Plant					3,520	2,364				3,000			1,263	495	.....	3,049		192,956			2,162,100	1,711	11.21
2	1	4	A	..	TP	1,150	772	6	23 x29½	166⅔	1,000	1925	2,266	.....	.....	0	.....	CC	155,699	27° API; 40 SSU @ 100 F	No	1,690,240	.....	10.85
	2	4	A	..	TP	1,150	772	6	23 x29½	166⅔	1,000	1925	2,251	.....	.....	0	.....	&S	151,270	0.5% S; 2.0% CC;	No	1,690,240	.....	11.17
	3	4	A	..	TP	1,150	772	6	23 x29½	166⅔	1,000	1925	2,236	.....	.....	0	.....		151,598	0.04% Ash; Tank Wagon	No	1,691,240	.....	11.15
	Plant					3,450	2,316				3,000			3,367	1,687	0	2,306		458,567	Deliv.		5,071,720	1,506	11.06
164	1	4	M	..	TP	840	564	6	19½x24	214	1,000	1926	7,244	.....	.....	.....	.....		.....	28° API; 42 SSU @ 100 F;	No	3,502,300	.....	.....
	2	4	M	..	TP	840	564	6	19½x24	214	1,000	1926	7,217	.....	.....	.....	.....	BC	.....	0.05% S; 0.028% CC;	No	3,436,300	.....	.....
	3	4	M	..	TP	840	564	6	19½x24	214	1,000	1926	7,574	.....	.....	.....	.....		.....	0.001% Ash; BS&W—nil	No	3,802,200	.....	.....
	4	4	M	..	TP	840	564	6	19½x24	214	1,000	1926	6,041	.....	.....	.....	.....		.....	By Refinery Pipe Line	No	3,043,400	.....	.....
	Plant					3,360	2,256				4,000			17,069	.....	800	1,382		1,043,710			13,784,200	808	13.21
77	1	2	M	B	TP	3,000	2,014	10	19½x27	240	2,812	1931	7,316	5,575	5,103	0	3,936	CC&F	428,744	25.2° API; 50 SSU @ 100 F; 0.4% S; 3.25% CC; 0.01% Ash; 0.75% BS&W	Yes	5,790,900b	1,038b	13.50b
109	1	2	A	P	CH	675	453	4	16½x22	240	500	1927	2,258	344	167	0	4,431		.....		No	454,580	1,321	.....
	2	2	A	P	TP	1,125	755	8	14½x21	257	975	1929	635	212	171	0	3,370	CC	.....	24°—26° API; 0.5% S	No	231,650	1,093	.....
	3	2	A	P	TPa	1,125	755	8	14½x21	257	975	1931	666	279	172	0	2,685		.....		No	273,730	981	.....
	Plant					2,925	1,963				2,450			835	510	0	3,578		117,502			959,960	1,150	8.17
7	1	4	M	..	TP	350	235	6	11½x16	277	375	1925	1,890	.....	.....	.....	.....		.....	36°—40° API;	Yes	263,900	.....	.....
	2	4	M	..	TP	350	235	6	11½x16	277	375	1925	1,560	.....	.....	.....	.....		.....	44.4 SSU @ 100 F;	Yes	210,600	.....	.....
	3	4	M	..	TP	350	235	6	11½x16	277	375	1925	1,226	.....	.....	.....	.....	CC	.....	0.23% S; 0.016% CC;	Yes	169,000	.....	.....
	4	4	M	..	TP	750	504	6	16 x22	240	750	1927	3,747	.....	.....	.....	.....		.....	0.012% Ash; 1.3% BS&W	Yes	940,800	.....	.....
	5	4	M	..	TP	930	624	6	18 x22	257	750	1930	4,584	.....	.....	.....	.....		.....		Yes	1,775,600	.....	.....
	Plant					2,730	1,833				2,625			2,805	.....	420	3,104		284,995			3,359,900	1,197	11.77
732	1	4	A	..	TP	250	168	4	13¾x17½	257	200	1916	0	.....	.....	.....	.....		.....	.....	No	.....	.....	.....
	2	4	A	..	TP	250	168	4	13¾x17½	257	200	1916	635	.....	.....	.....	.....	CC	.....	.....	No	.....	.....	.....
	3	4	A	..	TP	520	349	4	19½x24½	200	400	1921	1,230	.....	.....	.....	.....	&F	.....	.....	No	.....	.....	.....
	4	4	A	..	TP	750	504	4	23 x32	150	625	1923	1,600	.....	.....	.....	.....		.....	.....	No	.....	.....	.....
	5	4	A	..	TP	750	504	4	23 x32	150	625	1926	2,000	.....	.....	.....	.....		.....	.....	No	.....	.....	.....
	Plant					2,520	1,693				2,050			2,740	.....	.....	1,276		150,757			1,222,560	446	8.11
978	1	2	M	P	TP	840	564	6	16 x20	257	700	1931	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	2	2	M	P	TP	840	564	6	16 x20	257	700	1931	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	3	2	M	P	TP	840	564	6	16 x20	257	700	1931	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	Plant					2,520	1,692				2,100			135	.....	.....	.....		14,295			184,300	1,365	12.89
679	1	2	A	P	CH	1,250	840	5	20½x26	180	1,065	1925	1,053	707	66	.....	1,862	BC	80,521	28°—30° API	No	877,650	1,241	10.90
	2	2	A	P	CH	1,250	840	5	20½x26	180	1,065	1925	964	696	38	.....	1,732	&S	75,776	75 SSU @ 100 F	No	800,250	1,150	10.56
	Plant					2,500	1,680				2,130			1,403	104	.....	1,797		156,297			1,677,900	1,196	10.74

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 1, Continued)

Plant Number	Loading: Running Engine Capacity Factor (See Text)	Loading: Running Plant Capacity Factor (See Text)	Loading: Peak Load During Reported Period—Gross K.W.	Loading: B.M.E.P. at Rated B.H.P.—Lbs. per Sq. In.	Loading: B.M.E.P. at Peak Load—90% Generating Efficiency	Piston Cooling (See Notes)	Are Air Filters Used?	Type Cooling System (See Text)	Average Temperature Incoming Cooling Water—Degs. F.	Average Temperature Outgoing Cooling Water—Degs. F.	Purpose for which Jacket Water Heat is Utilized (See Notes)	Purpose for which Exhaust Heat is Utilized (See Notes)	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Material	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Extra Labor	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Material	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Extra Labor	Maintenance and Repairs: Total Engine Maintenance in Dollars per Rated B.H.P. per Year	Maintenance and Repairs: Major Engine Parts Renewed During Reported Period (See Note gg)	Maintenance and Repairs: No. of Enforced Engine Shutdowns	Maintenance and Repairs: Total Duration of Enforced Engine Shutdowns—Hours	Maintenance and Repairs: Total Engine Maintenance Time Not Inc. in Enforced Shutdown Time—Hours	Attendance: No. of Shifts in Period	Attendance: No. of Hours per Shift (See Note ff)	Attendance: No. of Attendants per Shift	Output per Man-Hour—Net K.W. Hrs.	Plant Altitude—Feet Above Sea Level	Plant Number
43	68.7		300	78.4	70.0	W	No						104	587	0	0	0.69		0	0	101							43
	65.8		300	78.4	70.0	W	No												0	0	170							
	77.2		400	74.0	84.8	W	No						2	397	0	0	0.38		0	0	99							
	74.4		400	74.0	84.8	W	No								0	0			0	0	98							
	63.4		740	73.9	81.5	W	No						130	323	0	0	0.23		0	0	123							
	64.6		740	73.9	81.5	W	No								0	0			0	0	36							
	75.9		3,000	67.2	80.1	W	Yes								0	0		Cyl.Liners; 1-Cyl.Head	0	0	796	317½	8	2				
	76.1		3,000	67.2	80.1	W	Yes						— 18,921 —		0	0	1.63	1-Liner	0	0	486	317½	8	3				
	73.0		3,000	73.5	80.1	W	Yes								0	0		Fuel V.; Starting V.	0	0	892	317½	8	2				
		74.3	6,400c					B-F	90	130	N	N										317½	8	1	1,350	2,500		
82	57.8		260	72.7	78.1	A	No						44	420	0	0	1.29	8-Piston Rings	1	5½	91							82
	49.5		400	67.3	53.4	W	Yes						64	490	0	0	0.74	(See gg1)	0	0	0							
	49.2		410	67.3	54.8	W	Yes						26	363	0	0	0.52	1-Piston Cooling Tube	0	0	0							
	47.8		745	68.8	66.4	W	Yes						731	1,178	0	0	1.66	(See gg2)	0	0	1,104	313	8	2				
	61.8		755	68.8	67.3	W	Yes						961	1,819	0	0	2.42	(See gg3)	1	2½	1,393	313	8	2½				
	68.9		1,060	67.3	70.8	W	Yes						988	2,182	410	540	2.75	(See gg4)	1	210	2,568	313	8	4½				
	68.2		2,220	67.6	67.8	O	Yes						355	1,759	0	0	0.64	(See gg5)	4	34	16	208	8	1				
		63.2	3,360					B-F	75	115	N	N					1.38					156	8	1	354	23		
52	70.1		560	75.3	75.7	W	Noe						54	410g	0	15g	0.58	Exh.V. Seat & Sp.	1	2	81							52
	78.3		860	75.7	77.5	W	Noe						106	818g	176	187g	1.03	(See gg6)	2	873f	188	313	8	2				
	75.7		860	75.7	77.5	W	Noe						409	839g	0	32g	1.02	(See gg7)	1	1	176	313	8	2				
	78.4		850	75.7	76.6	W	Noe						101	683g	0	0	0.63	Exh. Valve Springs	0	0	157	313	8	3				
	86.1		2,280	74.5	88.3	W	Noe						28	697g	0	0	0.25	None	0	0	248	312	8	1				
		78.3	3,020					B	90	115	N	B/W					0.61					52	8	1	466	30		
73	67.4		900	62.5	67.0	W	Yes						1,750	0	0	0	1.40	1-Set Cylinder Liners;	0	0	.....	.....	7	....				73
	69.0		900	62.5	67.0	W	Yes						300	0	0	0	0.24	2-Main Bearings;	0	0	.....	.....	6	....				
	64.5		900	62.5	67.0	W	Yes						285	0	0	0	0.23	Piston Rings;	0	0	.....	.....	6	....				
	68.7		900	62.5	67.0	W	Yes						420	0	0	0	0.34	Scavenging Valves	0	0	.....	.....	6	....				
		67.2	2,300					B-F	80	120	F	B					0.55					.....	6	....	.....	500		
45	50.0		275	77.2	52.6	A	Noe								0	0		None	0	0	16							45
	41.1		400	77.2	76.6	A	Noe								0	0		None	0	0	18							
	45.4		350	74.5	51.7	W	Noe						410	398	622	27	0.41	Cylinder Head	0	0	127	313	8	2				
	58.2		700	76.1	69.0	W	Noe								241	51		Piston Rod	0	0	381	313	8	4				
	59.9		700	76.1	69.0	W	Noe								0	0		None	0	0	288	313	8	2				
		56.9	1,600					B	85	100	N	N										313	8	1	198	50		
111	60.4		460	72.6	73.7	A	Yes						— 1,132 —				0.31	Piston Rings & Gaskets	0	0	50							111
	66.1		600	72.6	72.1	A	Yes												0	0	45	313	8	2				
	63.8		600	72.6	72.1	A	Yes												1	1	29	313	8	2				
	63.1		800	75.8	75.3	O	No												1	2	28	313	8	2				
		63.2	1,500					C	95	120	N	B										313	8	1	259	200		
130	67.9		450	73.8	82.4	A	Yes						512	10	0	0	0.87	Rocker Arms; A.C.; A.V.	1	8	112	.....						130
	65.0		430	72.7	77.6	A	Yes						122	0	0	0	0.20	Pump Plungers	5	40	130	.....	8	....				
	69.1		960	70.1	83.5	O	Yes						381	18	0	0	0.33	Pu. Pl.; Se.; A. C. R. W. P.	2	72	68	.....	8	....				
	69.2		1,010	70.1	87.8	O	Yes						622	130	0	0	0.63	Main B.; Se.; A. C.;	5	120	124	.....	8	....				
		67.8	1,370					A	60	110	N	L					0.50	A. C. R.				.....	8	....	.....	750		
157	81.5		1,250	64.9	68.6	O	Yes						689	0	0	0	0.20	Comp.Valves & Spgs.;	1	1	.....	.....						157
	85.8		1,300	64.9	71.4	O	Yes							0	0	0		Scavenging Valves;	0	0	.....	365	8	1				
		83.6	2,300					A/Ci	34/85	100—120	N	B/F						Rings				365	8	1	364	1,016		
2	96.6		785	74.3	75.6	W	No								0	0												
	97.3		785	74.3	75.6	W	No						888	916	0	0	0.52	Injection; Inlet &	1	2½	.....	.....	8	....				2
	98.0		785	74.3	75.6	W	No								0	0		Exhaust Valve parts;	1	2½	.....	.....	8	....				
		97.3	2,355					A	63	110	N	L						Pipe; Misc.	2	3½	.....	.....	12	....	.....	735		
164	85.7		538j	72.4	....	O	No															234	8	3				
	84.4		567j	72.4	....	O	No						— 23,94 —		—	—	7.13	.....	.....	.....	.....	234	8	3				164
	89.0		554j	72.4	....	O	No												.....	.....	.....	234	8	3				
	89.3		542j	72.4	....	O	No											1-Con. Rod; 1-Cyl.;	.....	.....	.....	234	8	3				
		87.0	2,083j					D	110	130	N	N						1-Piston; ½-Base				156	8	3	507	30		
77	39.3	39.3	1,700	61.4	51.8	O	Yes	D-F	96	120	N	N	1,965	1,800	0	0	1.26	(See gg8)	2	548	896	305	8	3	264	4,000	77	
																						305	8a	2				
109	44.4		430	57.4	54.5	O	No								0	0			0	0	240							109
	48.3		700	59.4	55.1	O	Yes						620	123	0	0	0.25	Pistons	1	5	435	365	8	1½				
	54.4		700	59.4	55.1	O	Yes								0	0			0	0	136	365	8	1				
		47.9	1,150					B	90	110	N	N										365	8	1	91	4,300		
7	59.4		225	100.3	96.0	A	Yes								0	0		Spray Valve Bushings;	0	0	9							7
	57.5		225	100.3	96.0	A	Yes								0	0		Fuel Pump Bodies;	0	0	149							
	58.7		225	100.3	96.0	A	Yes						540	320	0	0	0.82	Fuel Pump Plungers	0	0	401							
	49.8		400	93.2	74.0	A	Yes								1,120	261		1-H.; 1-L.; 1-P. & R.; R	1	99	80	365	8	2				
	62.1		580	85.2	79.2	A	Yes								0	0			0	0	63	365	8	2				
		57.5	922					D‡	60	120	N	N						Liner Gaskets				365	8	2	178	1,230		
732	.....		.....		74.1	....	A	Yes							0	0	0		.....	.....	.....							732
	.....		.....		74.1	....	A	Yes							0	0	0		.....	.....	.....							
	.....		.....		70.4	....	W	Yes					2,787		0	0	0	1.11	1-Cyl. Head	.....	.....	.....	313	8	2				
	.....		.....		74.5	....	W	Yes							0	0	0		.....	.....	.....	313	8	3				
	.....		.....		74.5	....	W	Yes							0	0	0		.....	.....	.....	313	8	2				
		52.0	1,400					B	.....	.....	N	N										313	8	1	49	850		
978	.....		.....		53.6	....	O	....											.....	.....	.....	.....	.....	.....	.....	.....		978
	.....		.....		53.6	....	O	....											.....	.....	.....	.....	.....	.....	.....	.....		
	.....		.....		53.6	....	O	....											.....	.....	.....	.....	.....	.....	.....	.....		
	.....	....	1,100					.....	.....	.....	.....	.....													22	.....		
679	99.3		975	62.5	72.6	W	No						740	408	0	0	0.46	Pistons	0	0	104							679
	98.8		925	62.5	68.9	W	No								0	0			0	0	56	191	8	1¼				
		99.2	1,850					A	70l	130	N	N										83	8	2	.....	603		

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 2)

Plant Number	Engine Data												Lubrication					Fuel			Gross Output—K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Gross K.W. Hrs. per Gallon of Fuel Oil
	Engine Designation	Engine Cycle	Injection System (Notes)	Scavenging System (Notes)	Trunk Piston or Crosshead?	Rated B.H.P.	Equivalent K.W.—90% Generating Efficiency	Number of Cylinders	Cylinder Dimensions Bore x Stroke—Inches	Rated R.P.M.	Generator Rating—K.V.A.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Total Gallons of New Lubricating Oil Used	Gals. of New Lub. Oil for Cylinder Lub. Only	Gals. of Unfit Lubricating Oil Discarded	Rated H.P. Hours per Gal. of New Lubricating Oil	Lubricating Oil Treatment (See Notes)	Fuel Oil Used—Gallons	Nature of Fuel Oil Used (See Notes)	Is Fuel Centrifuged?			
60	1	4	M	.	TP	800	537	8	19½x22	200	675	1924	2,145	.....	.....	.....	.....	BC	.....	.....	No	223,000	.....	.....
	2	4	M	.	TP	800	537	8	19½x22	200	675	1924	2,824	.....	.....	.....	.....		.....	.....	No	356,200	.....	.....
	3	4	M	.	TP	800	537	8	19½x22	200	675	1924	3,122	.....	.....	.....	.....		.....	.....	No	360,500	.....	.....
	Plant					2,400	1,611				2,025			2,143	.....	887	3,020		148,009			939,700	438	6.35
1149	1	2	M	P	TP	1,200	806	8	16 x20	257	1,050	1932	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	2	2	M	P	TP	1,200	806	8	16 x20	257	1,041	1932	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	Plant					2,400	1,612				2,091			100	.....	.....	.....		24,750			290,760	2,908	11.75
41	1	4	A	..	TP	520	349	3	22 x29½	164	400	1922	3,520	.....	.....	.....	.....	CC	70,900	20° API	No	673,990	.....	9.50
	2	4	A	..	TP	700	470	4	22 x29½	164	500	1923	3,073	.....	.....	.....	.....	S,	75,990	47 SSU @ 100 F	No	786,200	.....	10.34
	3	4	A	..	TP	1,150	772	6	23 x29½	164	900	1928	3,761	.....	.....	.....	.....	&F	187,126	1.3% S; 0.15% CC	No	1,869,600	.....	9.99
	Plant					2,370	1,591				1,800			4,689	930	.....	1,770		334,016			3,329,790	710	9.97
723	1	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1925	199	42	.....	.....	1,421	BC&F	3,320	32°—36° API	No	32,030	763	9.65
	2	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1925	341	68	.....	.....	1,504	BC&F	4,845		No	46,480	684	9.59
	3	2	M	P	TP	720	484	6	16 x20	257	600	1927	1,205	615	.....	.....	1,411	BC	41,175		No	406,200	660	9.87
	4	2	M	P	TP	1,050	705	7	16 x20	257	910	1931	1,733	725	.....	.....	2,510	BC	80,285		No	926,800	1,278	11.54
	Plant					2,370	1,593				2,010			1,450	.....	.....	1,965		129,625			1,411,510	973	10.89
34	1	2	M	P	TP	750	504	5	16 x20	257	640	1932	3,382	.....	.....	.....	.....	BC	93,500	32.5° API	No	1,026,760	.....	10.98
	2	2	M	P	TP	750	504	5	16 x20	257	640	1932	3,501	.....	.....	.....	.....		94,100		No	1,050,750	.....	11.17
	3	2	M	P	TP	750	504	5	16 x20	257	640	1932	3,494	.....	.....	.....	.....		96,853		No	1,071,590	.....	11.06
	Plant					2,250	1,512				1,920			3,462	.....	.....	2,246		284,453			3,149,100	910	11.07
79	1	4	A	..	TP	225	151	3	16 x24	164	198	1911	1,191	360	15	.....	744		11,505	24° API; 166 SSU @ 100° F	No	108,300	283	8.88
	2	4	A	..	TP	225	151	3	16 x24	164	198	1912	86	23	2	.....	841		694	0.45% S; 4.3% CC	No			
	3	4	A	..	TP	600	403	6	16½x23½	225	512	1929	5,024	772	202	100	3,905	S	105,545	Trace—Ash; BS&W—Nil	No	1,274,400	1,650	12.07
	4	4	A	..	TP	520	349	4	19 x26	200	438	1923	2,559	1,078	161	165	1,234		53,852		No	612,500	569	11.36
	5	4	A	..	TP	600	403	6	16½x23½	225	512	1925	4,658	1,768	190	165	1,580		100,931		No	1,235,200	699	12.23
	Plant					2,170	1,457				1,858			4,001	570	.....	1,856		272,527			3,230,400	807	11.85
5	1	2	M	P	TP	720	484	6	16 x20	257	.....	1926	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	2	2	M	P	TP	720	484	6	16 x20	257	.....	1927	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	.	2	M	P	TP	720	484	6	16 x20	257	.....	1927	.....	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	Plant					2,160	1,452				.....			946	.....	.....	.....		95,288			918,180	971	9.64
6	1	2	M	C	TP	300 *m*	202	6	14 x17	257	.....	1926	101	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	2	4	A	..	TP	600	403	6	17 x25	200	.....	1926	1,373	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	3	4	A	..	TP	600	403	6	17 x25	200	.....	1927	1,431	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	4	4	A	..	TP	600	403	6	17 x25	200	.....	1928	1,529	.....	.....	.....	.....		.....	.....	.....	.....	.....	.....
	Plant					2,100	1,411				.....			1,388	.....	.....	1,895		141,204			1,652,500	1,191	11.70
68	1	2	M	C	TP	240	161	4	14 x17	257	200	1925	2,765	.....	.....	.....	.....		.....		No	236,900	.....	.....
	2	2	M	C	TP	360	242	6	14 x17	257	300	1925	4,050	.....	.....	.....	.....	BC	.....	30°—34° API	No	512,600	.....	.....
	3	2	M	P	TP	560	376	4	16 x20	257	470	1928	4,673	.....	.....	.....	.....		.....		No	1,239,400	.....	.....
	4	2	M	P	TP	900	604	6	16 x20	257	774	1930	3,630	.....	.....	.....	.....		.....		No	1,527,900	.....	.....
	Plant					2,060	1,383				1,744			3,622	.....	97	2,209		361,270			3,516,800	971	9.73
133	1	4	A	..	TP	1,000	671	8	17½x24½	225	688°	1930	2,967	1,438	912	366	2,061	CC	134,400	24°API; 75 SSU @ 122F;	No	1,598,730	1,111	11.88
	2	4	A	..	TP	1,000	671	8	17½x24½	225	688°	1930	2,918	1,422	896	366	2,050		136,573	1.25% S; 0.5% CC;	No	1,625,820	1,142	11.89
	Plant					2,000	1,342				1,376°			2,860	1,808	732	2,057		270,973	0.05% Ash; 1.0% BS&W		3,224,550	1,127	11.89
																				Delivered by Tank Wagon				
806	1	4	A	..	TP	300	202	3	16½x24	200	240	1916	1,429	95	.....	.....	4,513	BC	23,187	24.9° API; 0.32% S	Yes	232,550	2,448	10.03
	2	4	A	..	TP	500	336	4	18½x28½	164	475	1921	3,744	422	.....	.....	4,436	&C	85,653	4.9% BS&W	Yes	906,670	2,149	10.59
	3	4	A	..	TP	1,200	806	6	20 x24	300	1,043	1930	5,151	3,351	.....	.....	1,845		257,111	Partly Delivered in Tank	Yes	2,842,300	848	11.05
	Plant					2,000	1,344				1,758			3,868	.....	.....	2,193		365,951	Trucks		3,981,520	1,029	10.88
29	1	4	M	..	TP	660	443	6	16 x20	300	563	1932	1,206	306	.....	.....	2,600	CC	26,403	28°—32° B; 36 SSU @ 100F;	No	299,200	978	11.32
	2	4	M	..	TP	660	443	6	16 x20	300	563	1932	1,226	352	.....	.....	2,298	&S	26,146	0.6% S; 0.035% CC;	No	318,700	905	12.18
	3	4	M	..	TP	660	443	6	16 x20	300	563	1932	1,196	318	.....	.....	2,480		27,207	Trace—Ash; BS&W—Nil;	No	308,440	970	11.33
	Plant					1,980	1,329				1,689			976	.....	.....	2,454		79,756	Delivered by Tank Ship		926,340	949	11.60
215	1	2	M	P	TP	980	658	7	16 x20	257	845	1930	1,740	.....	.....	.....	.....	BC	.....	33.5° API; 0.22% S	No	917,800	.....	.....
	2	2	M	P	TP	980	658	7	16 x20	257	845	1930	561	.....	.....	.....	.....		.....	0.3% Ash	No	277,100	.....	.....
	Plant					1,960	1,316				1,690			738	.....	.....	3,055		104,059			1,194,900	1,619	11.48
807	1	4	A	..	TP	500	336	5	16½x24	200	425	1916	187	50	.....	.....	1,870		3,691		No	41,720	834	11.30
	2	4	A	..	TP	365	245	4	16½x21	225	312	1921	204	60	.....	.....	1,241	BC	2,057	28°—30° API	No	24,040	401	11.69
	3	4	A	..	TP	180	121	4	11½x15	276	135	1925	0	0	.....	.....		&C	0	S < 0.5%	No	0		
	4	2	M	P	TP	840	564	6	16 x20	257	700	1929	5,090	1,194	.....	.....	3,581		128,595		No	1,351,153	1,132	10.51
	Plant					1,885	1,266				1,572			1,304	.....	.....	3,408		134,343			1,416,913	1,087	10.55
108	1	4	A	..	TP	600	403	6	17 x24	200	513	1927	2,527	.....	.....	.....	.....	CC	.....	24°—26° B; 75 SSU @ 100 F	Yes	546,100	.....	.....
	2	4	A	..	TP	600	403	6	17 x24	200	513	1928	3,331	.....	.....	.....	.....	&	.....	CC < 0.5%	Yes	698,800	.....	.....
	3	4	A	..	TP	675	453	6	17 x24	225	575	1930	5,027	.....	.....	.....	.....	BC	.....	S < 0.5%	Yes	1,303,100	.....	.....
	Plant					1,875	1,259				1,601			2,385	.....	300	2,895		229,970			2,548,000	1,070	11.08
837	1	4	A	..	CHa	750	504	4	23 x32	150	625	1922	7,500	2,831	.....	.....	1,987		229,798	.....	No	1,763,770	623	7.68
	2	4	A	..	TP	450	302	6	16 x24	164	413	1918	824		.....	.....				.....	No			
	3	4	A	..	TP	225	151	3	16 x24	164	200	1904	1,030	1,460	.....	.....	609	F	43,737	.....	No	301,001	206	6.88
	4	4	A	..	TP	225	151	3	16 x24	164	200	1904	711		.....	.....				.....	No			
	5	4	A	..	TP	225	151	3	16 x24	164	187	1906	562		.....	.....				.....	No			
	Plant					1,875	1,259				1,625			4,291	.....	.....	1,517		273,535			2,064,771	481	7.55
1033	1	4	A	..	TP	500	336	4	18½x28½	164	350	.....	9	10	5	.....	450		.....		No	1,820	182	.....
	2	4	A	..	TP	500	336	4	18½x28½	164	350	.....	339	157	123	.....	1,080	S	.....	22°+API	No	71,040	452	.....
	3	4	A	..	TP	500	336	4	18½x28½	164	350	.....	605	328	199	.....	922	&F	.....		No	132,720	405	.....
	4	4	A	..	TP	365	245	4	16½x21	225	250	.....	9	2	2	.....	1,643		.....	26°+API	No	970	485	.....
	Plant					1,865	1,253				1,300			497	329	.....	965		23,802			206,550	416	8.68
421	1	4	A	..	TP	800	537	8	17 x24	200	681	1930	5,428	1,535	260	.....	2,828	CC	.....	26°—28° API	No	1,808,300	1,177	.....
	2	4	A	..	TP	1,000	671	8	17½x25	225	895	1933	4,644	1,728	1,228	.....	2,686		.....	1.0% S	No	1,994,300	1,153	.....
	Plant					1,800	1,208				1,576			3,263	1,488	500	2,753		320,675			3,802,600	1,164	11.85
845	1	2	M	C	TP	360	242	6	14 x17	257	300	1927	4,384	.....	.....	.....	.....		.....	30°—36° B	No	.....	.....	.....
	2	2	M	C	TP	360	242	6	14 x17	257	300	1927	5,858	.....	.....	.....	.....	BC	.....	S < 1.0%	No	.....	.....	.....
	3	2	M	P	TP	1,050	705	7	16 x20	257	900	1931	955	.....	.....	.....	.....		.....	Ash < 0.05%	No	.....	.....	.....
	Plant					1,770	1,189				1,500			2,748	.....	.....	1,706		166,853			1,489,775	542	8.93

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 2, Continued)

Plant Number	Loading: Running Engine Capacity Factor (See Text)	Loading: Running Plant Capacity Factor (See Text)	Loading: Peak Load During Reported Period—Gross K.W.	Loading: B.M.E.P. at Rated B.H.P.—Lbs. per Sq. In.	Loading: B.M.E.P. at Peak Load—90% Generating Efficiency	Loading: Piston Cooling (See Notes)	Loading: Are Air Filters Used?	Type Cooling System (See Text)	Average Temperature Incoming Cooling Water—Degs. F.	Average Temperature Outgoing Cooling Water—Degs. F.	Purpose for which Jacket Water Heat is Utilized (See Notes)	Purpose for which Exhaust Heat is Utilized (See Notes)	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Material	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Extra Labor	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Material	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Extra Labor	Maintenance and Repairs: Total Engine Maintenance in Dollars per Rated B.H.P. per Year	Maintenance and Repairs: Major Engine Parts Renewed During Reported Period (See Note gg)	Maintenance and Repairs: No. of Enforced Engine Shutdowns	Maintenance and Repairs: Total Duration of Enforced Engine Shutdowns—Hours	Maintenance and Repairs: Total Engine Maintenance Time Not Inc. in Enforced Shutdown Time—Hours	Attendance: No. of Shifts in Period	Attendance: No. of Hours per Shift (See Note ff)	Attendance: No. of Attendants per Shift	Output per Man-Hour—Net K.W. Hrs.	Plant Altitude—Feet Above Sea Level	Plant Number
60	19.4		520	60.3	58.4	A	No						.....	.....	.....	.....	.....	Piston Rings	1	.....	.....	365	7	2			60
	23.5		520	60.3	58.4	A	No						.....	.....	.....	.....	.....	None	0	0	.....	365	7	1			
	21.5		520	60.3	58.4	A	No						.....	.....	.....	.....	.....	None	0	0	.....	365	7	1			
		21.6	520					D	90	130	N	N										365	3	1	69	4,000	
1149	.....		.....	57.5	....	O	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		1149
	.....		.....	57.5	....	O	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		
	....	1,650						.....	.....	.....	.....	.....					.....								37	.....	
41	54.8		400	74.6	85.6	W	No																				41
	54.4		500	75.3	80.1	W	No						568	0	0	0	0.24	P. Pin Bs.; Cp. P. Rs.; P. C. Tube Parts; Fuel	....	.....	.....	365	8	2			
	64.3		900	75.5	88.0	W	No							0	0	0		Oil Pu. Parts; H.P. Air C. Coil	....	.....	.....	365	8	3			
		59.7	1,200					A	60	110	N	B										365	8	2	160	20	
723	79.7		200	29.4	29.1	A	No								0	0		.....	0	0	7						723
	67.5		200	29.4	29.1	A	No								0	0		.....	0	0	11	339	8	2			
	69.7		460	46.0	43.8	O	Yes						—1,501—		0	0	0.63	Pu. Push Rods & Bush. Pu. Push Rod & Bush.	0	0	90	339	8	1			
	75.9		720	57.5	53.7	O	Yes								0	0		1-Fuel Pump Nozzle	1	19	94	339	8	1			
		73.8	1,580					C	96	110	N	N										78	8	1	118	1,200	
34	60.2		520	57.5	59.2	O	Yes											1-Telescope Tube & Pocket; 2-Comp. Relief V.; Springs & Disks for Comp. Valves	1	8	.....	313	8	1			34
	59.5		520	57.5	59.2	O	Yes												0	0	.....	341	8b	1			
	60.8		520	57.5	59.2	O	Yes												0	0	.....	72	8	1	275	770	
		60.2	800					B	.....	.....	L	B															
79	56.2		110	75.0	54.6	A	No						0	0	0	0	0	.....	0	0	.....						79
			100	75.0	49.6	A	No						0	0	0	0	0	.....	0	0	.....						
	63.0		400	73.8	73.2	A	No						350	0	0	0	0.58	Bearings	0	0	.....						
	68.6		325	74.0	69.0	W	No						150	0	0	0	0.29	Rings	0	0	.....	365	8	3			
	65.8		400	73.8	73.2	A	No						689	0	0	0	1.15	Cylinders	0	0	.....	365	8	2			
		64.8	900					B	85	96	N	N					0.55					365	8	2	150	360	
5	.....		.....	46.0	....	O	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		5
	.....		.....	46.0	....	O	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		
	.....		.....	46.0	....	O	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		
	....	1,500						.....	.....	.....	.....	.....					.....								152	.....	
6	.....		.....	29.4	....	A	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		6
	.....		.....	69.8	....	A	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		
	.....		.....	69.8	....	A	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		
	.....		.....	69.8	....	A	...						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....		
		93.5	1,490					.....	.....	.....	.....	.....					.....								.....	.....	
68	53.2		120	35.3	26.3	A	No											1-Cyl. Head; Scavenging Pump Valves; Rings, Fuel Pumps	0	.....	.....						68
	52.3		200	35.3	29.2	A	No						——	—1,55	——	——	0.76		2	.....	.....	312	8	1n			
	70.5		320	53.6	45.6	O	Yes												0	.....	.....	312	8	1n			
	69.7		600	57.5	57.1	O	Yes												1	.....	.....	312	8	1n			
		65.4	670					B-F	90	110	N	N										156	8	1n	.....	832	
133	80.2		688	74.6	76.5	A	Yes						253	344	0	0	0.60	P. Rs.; Cp. Rs.; C. Coil	2	7	146						133
	83.0		688	74.6	76.5	A	Yes						253	344	0	0	0.60		1	3½	140	297	7⅓	2n			
		81.6	1,376					A	75	120	W	N					0.60					297	4⅓	1n	.....	10	
806	80.6		.....	77.2	....	A	Yes						25	0	0	0	0.02	Valve Springs & Gaskets	0	0	322	365	8	1			806
																						313	8	3			
	72.1		270	75.9	61.1	A	Yes						38	0	0	0	0.02	Valve Springs & Gaskets	0	0	493	313	8a	2			
	68.5		630	69.6	54.8	O	Yes						763	0	0	0	0.64	Valve Springs; Gaskets; 1-Piston; Oil C. Pipes	20	532	361						
		69.9	910					A	60	80	N	B/F					0.41					313	8	1	166	1,850	
29	56.0		425	72.2	69.3	A	Yes						0	0	0	0	0	None	0	0	0						29
	58.7		450	72.2	73.4	A	Yes						0	0	0	0	0	None	0	0	0						
	58.2		460	72.2	74.9	A	Yes						0	0	0	0	0	None	0	0	0						
		57.6	980					B	70	110	N	N										225	8	3½	154	20	
215	80.2		600c	53.6	48.9	O	Yes						191	75	0	0	0.14	Piston Rings; General Inspection	0	0	168	312	8	⅓			215
	75.1		600c	53.6	48.9	O	Yes						191	75	0	0	0.14		0	0	168	312	8a	⅓			
		78.9	1,200c					B	99	104	N	N					0.14					156	8	⅓	.....	2,150	
807	66.4		.....	77.1	....	A	Yes						0	0	0	0	0	None	0	0	0						807
	48.1		150	73.7	45.1	A	Yes						0	0	0	0	0	None	0	0	0	365	6	2			
	0		0	79.4	....	A	Yes						0	0	0	0	0	None	0	0	0	365	6	1			
	47.1		500	53.6	47.5	O	Yes						0	0	0	0	0	None	0	0	0	365	6	1			
		47.5	500					C	70	100	N	N					0					365	6	1	112	2,950	
108	53.6		420	72.6	75.7	A	No											1-P. Pin; 6-Exh. V. & S	0	0	175						108
	52.1		410	72.6	73.9	A	No						565	0	0	0	0.30	1-Vert.T. B. & Gov Shft	0	0	175						
	57.2		475	72.9	76.1	A	No							0	0	0		6-Exh. Valves & Seats; 1-Fuel Pump Body	1	4	171	354	8c	1			
		54.9	790					C	100	130	N	L										50	8	1	209	850	
837	46.6		.....	74.5	....	W	No											(See gg0)									837
			.....	75.0	....	A	No											3-Cyl. Heads; 1-Liner	....	.....	.....						
			.....	75.0	....	A	No						2,859	3,164	3,402	473	5.28	None	....	.....	.....						
	50.5		.....	75.0	....	A	No											None	....	.....	.....	365	8	2p			
			.....	75.0	....	A	No											1-Cyl. Head	....	.....	.....	365	8	2p			
		47.1	600					A	80	126	N	F										365	8	2p	110	5	
1033	60.2		280c	76.0	63.3	A	No								0	0		3-Exh. Manifolds	0	0	.....						1033
	62.4		310c	76.0	70.1	A	No								0	0		(See gg10)	0	0	.....						
	65.3		300c	76.0	67.9	A	No						592		0	0	0.32	1-Exh. M. Gov. Shft.	0	0	....	.....	8	1n			
	44.0		150c	73.5	45.0	A	No								0	0		None [Oil Guard]	0	0	.....	.....	8	3n			
		64.1	850c					B	.....	.....	N	N										.....	8	2n	.....	3,800	
421	62.0		.....	72.7	....		Yes						216	24	0	0	0.30	Piston Rings	0	0	120	365	8	4			421
	64.0		.....	73.2	....		Yes						0	0	0	0	0		0	0	0	365	8	2			
		63.1	.....					A	70	110	N	N					0.13					365	8	2	158	620	
845	.....		200	35.3	29.2	A	No												....	.....	.....	.....	8	1			845
	.....		200	35.3	29.2	A	No						——	—1,800	——	——	1.02	Piston Rings	....	.....	.....	.....	8	1			
	.....		470	57.5	38.3	O	No												....	.....	.....	.....	8	1			
		47.3	470					B-F	70	90	N	N											8	1	.....	1,050	

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 3)

Plant Number	Engine Designation	Engine Cycle	Injection System (Notes)	Scavenging System (Notes)	Trunk Piston or Crosshead?	Rated B.H.P.	Equivalent K.W.—90% Generating Efficiency	Number of Cylinders	Cylinder Dimensions Bore x Stroke—Inches	Rated R.P.M.	Generator Rating—K.V.A.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Total Gallons of New Lubricating Oil Used	Gals. of New Lub. Oil for Cylinder Lub. Only	Gals. of Unfit Lubricating Oil Discarded	Rated H.P. Hours per Gal. of New Lubricating Oil	Lubricating Oil Treatment (See Notes)	Fuel Oil Used—Gallons	Nature of Fuel Oil Used (See Notes)	Is Fuel Centrifuged?	Gross Output—K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Gross K.W. Hrs. per Gallon of Fuel Oil
854	1	2	M	C	TP	360	242	6	14 x17	257	300	1928	132	.....	.....	.....	.....	BC	.....	30°—36° B	No	.....	.....	.....
	2	2	M	C	TP	360	242	6	14 x17	257	300	1928	355	.....	.....	.....	.....		.....	S < 1.0%; Ash < 0.05%	No	.....	.....	.....
	3	2	M	P	TP	1,050	705	7	16 x20	257	900	1931	5,372	.....	.....	.....	.....		.....	BS&W < 0.3%	No	.....	.....	.....
	Plant					1,770	1,189				1,500			1,377	.....	.....	4,222		200,464			2,177,480	1,582	10 86
158	1	2	A	P	CH	1,760	1,182	8	17 x23	257	1,500	1930	524	300	100	.....	3,074	S&F	37,969	18°—22°API; 0° F Pour Pt.	No	343,172	1,144	9 04
212	1	4	A	..	TP	875	588	8	17½x24	200	671	1929	398	100	65	29	3,483	S	14,998	33° B; 42 SSU @ 210 F	No	168,200	1,682	11 21
	2	4	A	..	TP	875	588	8	17½x24	200	671	1929	412	98	57	28	3,679	&F	15,508	0.08% S;	No	169,000	1,725	10 89
	Plant					1,750	1,176				1,342			198	122	57	3,379		30,506	Delivered by Tank Truck		337,200	1,702	11 05
722	1	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1923	188	55	.....	.....	684		2,241		No	.....	.....	.....
	2	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1923	191	43	.....	.....	1,332	BC	3,560	32°—36° API	No	.....	.....	.....
	3	2	M	P	TP	840	564	6	16 x20	257	700	1929	1,348	345	.....	.....	3,282		52,727		No	.....	.....	.....
	4	4	A	..	TP	365	245	4	16½x21	225	330	1916	362	76	.....	.....	1,739		9,670		No	.....	.....	.....
	Plant					1,705	1,145				1,450			519	.....	.....	2,619		68,198			774,780	1,493	11 36
805	1	2	M	P	TP	840	564	6	16 x20	257	700	1928	16	0	0	.....	.....	BC	519	28°—30° API	No	4,230	.....	8 15
	2	2	M	P	TP	840	564	6	16 x20	257	700	1929	13	0	0	.....	.....	&C	435	S < 0.5%	No	3,640	.....	8 37
	Plant					1,680	1,128				1,400			0	0	.....	.....		954			7,870	.....	8 25
18	1	2	M	C	CH	450	302	5	15½x16	275	.....	1922	465	.....	.....	.....	.....		.....		.....	.....	.....	.....
	2	4	A	..	TP	600	403	6	17 x25	200	.....	1926	4,763	.....	.....	.....	.....		.....		.....	.....	.....	.....
	3	4	A	..	TP	600	403	6	17 x25	200	.....	1928	3,955	.....	.....	.....	.....		.....		.....	.....	.....	.....
	Plant					1,650	1,108				.....			2,129	.....	.....	2,552		208,390			2,205,070	1,035	10 57
88	1	4	A	..	TP	285	191	3	16½x24	200	240	1920	4,894	.....	.....	.....	.....	CC	53,790	32°—36° API;	No	555,650	.....	10 32
	2	4	A	..	TP	285	191	3	16½x24	200	240	1920	2,983	.....	.....	.....	.....	&S	35,270	42 SSU @ 100 F;	No	332,160	.....	9 42
	3	4	A	..	TP	300	202	3	16½x24	200	188	1925	3,767	.....	.....	.....	.....	&F	46,530	0.35% S; 0.01% Ash	No	477,800	.....	10 27
	4	2	A	P	TP	750	504	5	14½x21	257	650	1932	4,188	.....	.....	.....	.....		123,028		No	1,167,300	.....	9 49
	Plant					1,620	1,088				1,318			2,289	1,338	300	2,846		258,618			2,532,910	1,106	9 79
8	1	4	A	..	TP	400	269	4	16½x24	200	350	1921	4,497	.....	.....	.....	.....	CC&F	.....		No	463,890	.....	.....
	2	4	A	..	TP	560	376	3	22½x22½	225	500	1922	82	.....	.....	.....	.....	CC	.....	26°—34° API	No	12,350	.....	.....
	3	4	A	..	TP	600	403	6	17 x25	200	500	1926	4,399	.....	.....	.....	.....	CC	.....		No	1,078,190	.....	.....
	Plant					1,560	1,048				1,350			1,609	.....	.....	2,787		159,418			1,554,430	965	9 75
406	1	2	M	P	TP	600	403	4	16 x20	257	500	1930	2,104	264	.....	0	4,781	BC	.....	32°—36° API; 1.0% S	No	512,600	1,941	.....
	2	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1923	3,592	660	.....	0	1,632	C	.....	0.25% Ash; 0.25% BS&W	No	343,000	520	.....
	3	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1922	3,336	672	.....	0	1,488	C	.....		No	313,230	466	.....
	4	2	M	C	TP	360	242	6	14 x17	257	300	1925	3,872	516	.....	0	2,701	C	.....		No	420,410	814	.....
	Plant					1,560	1,049				1,300			2,112	.....	0	2,241		189,211			1,589,240	753	8 40
10	1	4	A	..	TP	750	504	4	22 x29½	166⅔	625	1924	5,648	2,118	816	0	2,000	CC	174,723	27° API; 40 SSU @ 100 F;	No	1,923,300	908	11 01
	2	2	A	P	CHa	750	504	4	17 x27	176	625	1925	551	294	169	0	1,406		14,583	0.5% S; 1.5% CC;	No	156,920	534	10 76
	Plant					1,500	1,008				1,250		6,199	2,412	985	0	1,928		189,306	0.004% Ash	No	2,080,220	862	10 99
47	1	2	M	P	TP	1,500	1,007	5	19½x27	240	1,250	1932	4,550	1,833	.....	.....	3,722	BC&S	254,353	Delivered by Tank Wagon 30°—36° B; S < 1.0% Ash <0.05%; BS&W<0.3%	No	3,021,340	1,647	11 87
1154	1	4	A	..	TP	750	504	6	17½x24½	225	750	1931	4,524	.....	.....	0	.....	BC	121,861	32°—34° API	No	1,255,900	.....	10 30
	2	4	A	..	TP	750	504	6	17½x24½	225	750	1931	4,445	.....	.....	0	.....	&S	119,912		No	1,200,000	.....	10 01
	Plant					1,500	1,008				1,500			746	706	0	9,020		241,773			2,455,900	3,290	10 16
289	1	4	A	..	TP	250	168	4	13¾x17½	257	210	1920	5,478	651	297	0	2,103		56,160		No	644,970	991	11 48
	2	4	A	..	TP	250	168	4	13¾x17½	257	210	1920	5,462	908	316	0	1,503	BC	55,872	26°—30° API	No	640,470	706	11 46
	3	4	A	..	TP	365	245	4	16½x21	225	300	1915	4,529	891	279	0	1,856		62,803		No	743,790	834	11 84
	4	4	A	..	TP	600	403	6	16½x23	225	512	1927	2,291	697	234	0	1,972		59,254		No	717,800	1,029	12 11
	Plant					1,465	984				1,232			3,147	1,126	0	1,830		234,089			2,747,030A	873A	11 73A
496	1	4	A	..	TP	400	269	4	16½x24½	200	340	1925	3,674	154	117	0	9,540	CC	41,693	32°—36° API	No	437,625	2,830	10 50
	2	4	A	..	TP	300	201	3	16½x24½	200	250	1925	4,601	341	117	0	4,047	BC	40,463	S < 1.0%	No	390,550	1,145	9 65
	3	4	A	..	TP	750	504	6	17½x24½	225	650	1928	1,569	134	121	0	8,780	S&C	37,095		No	373,000	2,783	10 06
	Plant					1,450	974				1,240			629	355	0	6,400		119,251			1,201,175	1,910	10 07
850	1	4	A	..	TP	600	403	6	17 x25	200	500	1926	8,214	.....	.....	.....	.....	BC	.....	30°—36° B; S — 1.0%	No	.....	.....	.....
	2	2	M	P	TP	840	564	6	16 x20	257	700	1928	2,996	.....	.....	.....	.....		.....	Ash <0.05%; BS&W<0.3%	No	.....	.....	.....
	Plant					1,440	967				1,200			3,220	.....	.....	2,311		280,723			2,844,460	884	10 13
70	1	2	M	P	TP	560	376	4	16 x20	257	470	1928	4,408	.....	.....	.....	.....	CC	.....		No	864,783	.....	.....
	2	2	M	P	TP	560	376	4	16 x20	257	470	1928	4,487	.....	.....	.....	.....	S&F	.....	32°—36° API	No	869,250	.....	.....
	3	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1923	496	.....	.....	.....	.....		.....		No	57,850	.....	.....
	Plant					1,420	954				1,190			2,150	.....	55	2,385		198,000			1,791,883	834	9 05
849	1	2	M	C	TP	360	242	6	14 x17	257	300	1928	3,473	.....	.....	.....	.....	BC	.....	30°—36° B; S < 1.0%	No	.....	.....	.....
	2	2	M	P	TP	1,050	705	7	16 x20	257	900	1931	3,684	.....	.....	.....	.....		.....	Ash <0.05%; BS&W<0.3%	No	.....	.....	.....
	Plant					1,410	947				1,200			2,618	.....	.....	1,955		243,220			2,516,180	961	10 34
154	1	4	A	..	TP	600	403	6	17 x24	200	.....	1927	1,688	.....	.....	.....	.....		.....		.....	.....	.....	.....
	2	4	A	..	TP	400	269	4	17 x24	200	.....	1926	2,921	.....	.....	.....	.....		.....		.....	.....	.....	.....
	3	4	A	..	TP	400	269	4	17 x24	200	.....	1925	1,659	.....	.....	.....	.....		.....		.....	.....	.....	.....
	Plant					1,400	941				.....			904	.....	.....	3,150		121,977			1,513,170	1,674	12 41
163	1	4	M	..	TP	400	269	4	17½x22	225	343	1924	1,225	120	.....	0	4,083		12,045	37.7° API; 45 SSU @ 100 F	No	123,600	1,030	10 26
	2	4	M	..	TP	400	269	4	17½x22	225	343	1927	4,601	440	.....	120	4,180	BC	62,837	0.2% S; 0.002% Ash	No	653,700	1,485	10 40
	3	4	M	..	TP	600	403	6	17½x22	225	625	1929	4,110	522	.....	120	4,724		95,041	BS&W—Nil. Delivered	No	994,425	1,904	10 46
	4	4	A	..										1,082	.....	240	4,434		169,923	by Refinery Pipe Line		1,771,725	1,635	10 42
	Plant					1,400	941				1,311													
198	1	2	M	P	TP	1,400	941	8	16 x20	300	1,392	1932	5,525	2,517	.....	0	3,073	BC	324,674	(35.4°API; 41.3 SSU @ 100F; Ash-Nil; 0.29%S; 0.01%CC; BS&W—Trace	No	3,666,420	1,456	11 28
517	1	4	A	..	TP	625	420	5	17½x25	225	625	1930	4,237	816	270	0	3,245	CC	64,340	28°—30° API; 0.5% S	No	542,830	665	8 44
	2	4	M	..	TP	750	504	6	17 x25	225	625	1932	4,523	482	274	0	7,040		63,400	0.1% CC	No	579,730	1,203	9 14
	Plant					1,375	924				1,250			1,298	544	0	4,654		127,740			1,122,560	865	8 79
888	1	4	A	..	TP	300	202	3	17 x24	200	250	1925	1,686	.....	.....	.....	.....	CC&F	.....	33.9° B; 0.338% S;	No	158,650	.....	.....
	2	4	A	..	TP	400	269	4	17 x24	200	375	1925	3,520	.....	.....	.....	.....	CC&F	.....	CC and BS&W—Nil	No	495,900	.....	.....
	3	4	A	..	TP	675	453	6	17 x24	225	575	1931	7,017	.....	.....	.....	.....	CC	.....	Fuel by Barge	No	1,671,940	.....	.....
	Plant					1,375	924				1,200			1,378	.....	.....	4,825		228,830			2,326,490	1,687	10 16

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 3, Continued)

Plant Number	Loading: Running Engine Capacity Factor (See Text)	Loading: Running Plant Capacity Factor (See Text)	Loading: Peak Load During Reported Period—Gross K.W.	Loading: B.M.E.P. at Rated B.H.P.—Lbs per Sq. In.	Loading: B.M.E.P. at Peak Load—90% Generating Efficiency	Loading: Piston Cooling (See Notes)	Are Air Filters Used?	Type Cooling System (See Text)	Average Temperature Incoming Cooling Water—Degs. F.	Average Temperature Outgoing Cooling Water—Degs. F.	Purpose for which Jacket Water Heat is Utilized (See Notes)	Purpose for which Exhaust Heat is Utilized (See Notes)	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Material	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Extra Labor	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Material	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Extra Labor	Maintenance and Repairs: Total Engine Maintenance in Dollars per Rated B.H.P. per Year	Maintenance and Repairs: Major Engine Parts Renewed During Reported Period (See Note gg)	Maintenance and Repairs: No. of Enforced Engine Shutdowns	Maintenance and Repairs: Total Duration of Enforced Engine Shutdowns—Hours	Maintenance and Repairs: Total Engine Maintenance Time Not Inc. in Enforced Shutdown Time—Hours	Attendance: No. of Shifts in Period	Attendance: No. of Hours per Shift (See Note f)	Attendance: No. of Attendants per Shift	Output per Man-Hour—Net K.W. Hrs.	Plant Altitude—Feet Above Sea Level	Plant Number
854	.....		200	35.3	29.2	A	No						306				0.17					312	8	1q			854
	.....		200	35.3	29.2	A	No															312	8	1q			
	.....		700	57.5	57.1	O	No															312	8	1q			
		55.8	880					.....	70	80	N	N										159	8	1q	236	900	
158	55.4	55.4	1,000	64.9	54.9	O	Yes	B-Fr	56r/90	100r/	N	B/F										365	8	1	109	1,612	158
								D-F		120			23s	0	0	0	0.01s	Comp. Valves & Springs	3	1	.....						
212	71.9		570	75.1	72.8	A	No						697	0	0	0											212
	69.7		560	75.1	71.5	A	No							0	0	0	0.40	1-Cyl.; 1-Liner; Bearings for Fuel Pump; Gaskets	0	0	.....	365	8	2n			
		70.8	570					A-F	53	125	N	N										365	8	1n	.....	585	
722	.....		120	29.4	26.3	A	No								0	0											722
	.....		180	29.4	26.2	A	No								0	0	0.53	Piston Rings; Fuel Oil Pumps; Air Valves	0	0	0						
	.....		600	53.6	57.0	O	Yes						896		0	0			0	0	9	339	8	2			
	.....		2.0	73.8	66.3	A	No								0	0			0	0	243	339	8	1			
		84.9	1,100					C	95	109	N	N							0	0	33	339	8	1			
																						78	8	1	62	1,250	
805	46.9		380	53.6	36.1	O	Yes						0	0	0	0	0	None	0	0	0	365	10	1			805
	49.6		380	53.6	36.1	O	Yes						0	0	0	0	0	None	0	0	0	365	7	1			
		48.1	380					C	65	90	N	N										365	7	1	1	2,533	
18	.....		.....	44.3	....	A	....																				18
	.....		.....	69.8	....	A	....																				
	.....		.....	69.8	....	A	....																				
		60.4	940					.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	212	.....	
88	59.4		200	73.3	76.8	A	No																				88
	58.3		200	73.3	76.8	A	No						237	0	0	0	0.15	Piston Rings; Valves; Packing; 1-Comp. Bearing; Gaskets	0	0	.....	355	8	3			
	62.8		200	77.1	76.8	A	No							0	0	0			0	0	.....	355	8	2			
	55.3		520	64.3	66.4	O	Yes							0	0	0			0	0	.....	355	8	2			
		57.9	675					A	65	100	N	N										70	8	1	121	821	
8	38.3		.....	77.2	....	A	No						1,330	0	0	0	3.33	2-P.; 1-H.; R Cam Shft.	0	0	.....	.....	8	3			8
	40.1		.....	73.4	....	O	No						0	0	0	0	0	None [Grs.	0	0	.....	.....	8	1			
	60.8		.....	69.8	....	A	No						280	0	0	0	0.47	Rings	0	0	.....	.....	8	1			
		51.6	.....					B-F	75	110	B	N					1.03					.....	8	1	.....	1,100	
406	60.5		400	57.5	57.2	O	Yes						13	0	0	0	0.02	No Major Parts	0	0	56						406
	47.3		155	29.4	22.7	A	No						112	0	0	0	0.37	Rings	0	0	56						
	46.5		155	29.4	22.7	A	No						121	0	0	0	0.40	Rings	0	0	56	365	8	1			
	44.9		185	35.3	27.1	A	No						116	0	0	0	0.32	Rings	0	0	56	365	8	1			
		50.0	425					D	80	120	N	N					0.23					365	8	1	169	1,310	
10	67.6		440	79.5	69.4	W	No						811	168	0	0	1.31	Injection, Inlet & Exh. Valve Parts; Piston Rings; Misc.	3	4½	.....						10
	56.5		440	68.8	60.1	W	Yes						159	395	0	0	0.74		0	0	.....	274	8b	2n			
		66.6	440					A	72	110	N	L					1.02					273	8	2n	.....	21	
47	65.9	65.9	1,050	61.4	64.0	O	Yes	B	70	90	N	N	775				0.52					.....	8c	1 n	.....	3,400	47
1154	55.1		.....	74.7	....	A	Yes						291	0	0	0	0.19	Piston Rings; Gaskets; Springs	1	½	16	315	8	3			1154
	53.6		.....	74.7	....	A	Yes							0	0	0			0	0	16						
		54.3	750					A	44	110	N	B										315	8b	2	107	1,450	
289	70.1		.....	74.1	....	A	Yes								0	0											289
	69.8		.....	74.1	....	A	Yes						1,413	0	0	0	0.97	1-Piston; 1-Cyl. Head; 1-Set of Timing Gears; 1-Main Thrust Bearing	0	0	120						
	67.0		.....	73.8	....	A	Yes							0	0	0			0	0	168						
	77.8		.....	73.8	....	A	Yes							0	0	0			0	0	192	365	8	2			
		71.0	820					A	50	105	F	B										365	8	3			
																						365	8	2	128	1,670	
496	44.3		280	75.4	78.8	A	Yes						0	0	0	0	0		0	0	.....						496
	42.2		220	75.4	82.6	A	Yes						0	326	0	0	1.09		0	0	.....	365	10	2			
	47.2		520	74.4	76.8	A	Yes						105	55	0	0	0.21	H.P. Cp. Coil; Cp. V.	0	0	.....	365	10	1			
		44.4	650					D	100	122	F	N					0.34					365	10	1	76	1,200	
850	.....		.....	69.8	....	A	No						1,676				1.16					.....					850
	.....		.....	53.6	....	O	No															.....					
		56.9	980					A	55	70	N	B										.....	8c	1	.....	2,385	
70	52.2		360	53.6	51.4	O	Yes						0	0	0	0	0	None	3	1	.....						70
	51.5		340	53.6	48.5	O	Yes						126	0	0	0	0.23	1-Cyl. Head	0	0	.....	365	8	2			
	57.8		170	29.4	24.7	A	No						0	0	0	0	0	None	0	0	.....	365	8	1			
		52.0	490					A	.....	80	N	N					0.09					365	8	1	137	1,330	
849	.....		220	35.3	32.1	A	No						1,564				1.11					.....					849
	.....		730	57.5	59.5	O	No															.....					
		73.2	950					A	55	70	N	N										.....	8c	1n	.....	3,200	
154	.....		.....	72.7	....	A	....																				154
	.....		.....	72.7	....	A	....																				
	.....		.....	72.7	....	A	....																				
		79.2	570					.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	142	.....	
163	37.5		270	64.7	64.9	A	No						.....	.....	.....	.....	.....	Piston Rings	.....	.....	.....	184	a	1n			163
	52.8		290	64.7	69.7	A	No						.....	.....	.....	.....	.....		.....	.....	.....	300	....	1n			
	60.0		440	64.7	70.6	A	No						.....	.....	.....	.....	.....	Piston Rings	.....	.....	.....	185	....	1n			
		55.0	1,000					B	53	76	N	N					.....					542	....	3n	.....	1,020	
198	70.6	70.6	1,000	57.5	61.1	O	Yes	D	107	113	N	N	.....	.....	.....	.....	.....	1-Scavng. Air Piston; 2) Scav. Cyl. Heads; All at Mfgr's Expense	3	202	28	348	8	2n			198
																						348	8	1n	.....	1,109	
																						51	8	1n			
517	30.5		375	73.2	65.4	A	Yes						65	0	0	0	0.10	(See gg 11)	0	0	215						517
	25.4		375	77.6	57.8	A	Yes						10	0	0	0	0.01	4-Spray Valve Springs	0	0	120	353	8b	1	118	1,100	
		27.7	375					D-F	85	115	N	B					0.06					36	8	1			
888	46.6		160u	72.7	57.6	A	No								0	0		Rb. 3 Cr. Pin B.; 9-P.R.	0	0	416u						888
	52.4		260u	72.7	70.3	A	No						4,605	1,075	0	0	4.13	(See gg 12)	1	144	584u	365	8	2			
	52.6		460u	72.7	73.2	A	No								0	0		(See gg 13)	2	241	1,047u	365	8	2			
		52.1	840					A	54	.....	N	N										365	8	2	129	10	

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 4)

Plant Number	Engine Data: Engine Designation	Engine Cycle	Injection System (Notes)	Scavenging System (Notes)	Trunk Piston or Crosshead?	Rated B.H.P.	Equivalent K.W.—90% Generating Efficiency	Number of Cylinders	Cylinder Dimensions Bore x Stroke—Inches	Rated R.P.M.	Generator Rating—K.V.A.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Lubrication: Total Gallons of New Lubricating Oil Used	Gals. of New Lub. Oil for Cylinder Lub. Only	Gals. of Unfit Lubricating Oil Discarded	Rated H.P. Hours per Gal. of New Lubricating Oil	Lubricating Oil Treatment (See Notes)	Fuel: Fuel Oil Used—Gallons	Nature of Fuel Oil Used (See Notes)	Is Fuel Centrifuged?	Gross Output—K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Gross K.W. Hrs. per Gallon of Fuel Oil
725	1	4	A	..	TP	600	403	6	17 x25	200	500	1926	5,682	.....	1,103	.....	.....	BC	.....		No	.....	.....	.....
	2	4	A	..	TP	400	269	4	17 x25	200	337	1925	3,575	.....	476	.....	.....	S&	.....	32°—36° API; 0.5% S	No	.....	.....	.....
	3	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1924	2,500	.....	330	.....	.....	C	.....		No	.....	.....	.....
	4	2	M	C	TP	110 *m*	74	2	14 x17½	257	100	1923	0	.....	0	.....	.....		.....		No	.....	.....	.....
	Plant					1,310	880				1,107			2,941	1,909	100	1,815		189,733			2,071,200	704	10.92
886	1	2	M	C	TP	120	81	2	14 x17	257	85	1927	2,018	.....	.....	.....	.....	BC,	.....		No	70,662	.....	.....
	2	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1925	2,540	.....	.....	.....	.....	S	.....		No	177,461	.....	.....
	3	2	M	C	TP	360	242	6	14 x17	257	300	1929	2,199	.....	.....	.....	.....	&F	.....		No	222,478	.....	.....
	4	4	A	..	TP	625	420	5	17½x25	225	535	1931	2,881	.....	.....	.....	.....		.....		No	442,724	.....	.....
	Plant					1,305	877				1,090			2,150	.....	.....	1,555		127,740			913,325	424	7.15
13	1	4	A	..	TP	275	185	4	13½x19½	257	219	1923	6,509	1,177	871	306	1,520	CC	75,450	30° API; 0.28% S;	No	756,500	643	10.02
	2	4	A	..	TP	200	134	3	13½x19½	257	156	1923	2,660	451	339	386	1,180	&S	19,140	0.02% CC	No	210,700	467	11.00
	3	4	A	..	TP	750	504	4	23 x29½	164	625	1925	1,480	745	396	55	1,490		40,970	Ash and BS&W—Nil	No	333,500	448	8.14
	Plant					1,225	823				1,000			2,373	1,606	747	1,446		135,560			1,300,700A	548A	9.59A
16	1	4	A	..	TP	600	403	6	17 x25	200	512	1927	213	.....	.....	.....	.....	BC	.....		No	83,760	.....	.....
	2	2	M	C	TP	240	161	4	14 x17	257	200	1925	87	.....	.....	.....	.....	&	.....	28°—30° API	No	11,536	.....	.....
	3	2	M	C	TP	360	242	6	14 x17	257	300	1925	89	.....	.....	.....	.....	CC	.....		No	17,840	.....	.....
	Plant					1,200	806				1,012			296	.....	50	611		10,619			113,136	382	10.65
17	1	4	A	..	TP	600	403	6	17 x25	200	.....	1926	4,274	.....	.....	.....	.....		.....			.....	.....	.....
	2	4	A	..	TP	600	403	6	17 x25	200	.....	1926	4,095	.....	.....	.....	.....		.....			.....	.....	.....
	Plant					1,200	806				.....			1,472	.....	.....	3,411		192,906			1,997,800	1,357	10.36
846	1	2	M	C	TP	360	242	6	14 x17	257	300	1927	5,006	.....	.....	.....	.....	BC	.....	30°—36° B; 1.0% S;	No	.....	.....	.....
	2	2	M	P	TP	840	564	6	16 x20	257	700	1929	2,641	.....	.....	.....	.....		.....	Ash <0.05%; BS&W<0.3%	No	.....	.....	.....
	Plant					1,200	806				1,000			1,802	.....	.....	2,231		160,448			1,668,900	927	10.40
889	1	2	M	P	TP	1,200	806	8	16 x20	257	900	1931	4,460	1,283	.....	.....	4,165	BC	222,613	30°—36°B; S< 1.0%; Ash <0.05%; BS&W<0.3%	No	2,461,720	1,918	11.06
980	1	4	A	..	TP	750	504	6	17½x24½	225	625	1929	3,674	.....	462	0	.....		.....	25.4° API; 0.5% S;	No	.....	.....	.....
	2	4	A	..	TP	450	302	4	17 x24½	225	375	1927	5,271	.....	540	0	.....	CC	.....	1.14% CC; 0.002% Ash;	No	.....	.....	.....
	Plant					1,200	806				1,000			1,713	1,002	0	2,993		168,650	0.1% BS&W		1,757,700	1,026	10.41
1083	1	2	M	P	TP	1,200	806	8	16 x20	257	1,042	1931	4,548	1,890	.....	170	2,886	BC	201,872	26°—30° API	No	2,695,050	1,426	13.35
330	1	4	A	.	TP	625‡	420	5	17½x25	225	450	1930	2,435	.....	.....	.....	.....		.....		No	429,350	.....	.....
	2	2	M	C	TP	240	161	4	14 x17	257	200	1928	4,745	.....	.....	.....	.....	BC	.....	32°—36° API	No	215,350	.....	.....
	3	2	M	C	TP	100 *m*	67	2	14 x17	257	70	1917	2,555	.....	.....	.....	.....		.....		No	91,250	.....	.....
	4	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1919	4,380	.....	.....	.....	.....		.....		No	208,520	.....	.....
	Plant					1,165	782				890			2,747	.....	.....	1,380		122,255			944,470	344	7.72
878	1	2	M	P	TP	560	376	4	16 x20	257	470	1929	4,960	.....	.....	.....	.....		.....	28°—30° API; Not over	No	1,368,600	.....	.....
	2	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1919	69	.....	.....	.....	.....	BC	.....	220 SSU @ 100 F;	No	6,280	.....	.....
	3	4	A	..	TP	400	400	4	17 x25	200	337	1924	4,201	.....	.....	.....	.....		.....	S <0.5%; BS&W<1.0%	No	884,000	.....	.....
	Plant					1,160	779				977			2,221	.....	.....	2,013		223,376			2,258,880	1,017	10.11
616	1	4	A	..	TP	300	202	3	16½x23	200	250	1923	5,774	857	.....	0	2,021	CC	66,947		No	812,380	948	12.13
	2	2	M	C	TP	100 *m*	67	2	14 x17	257	75	1920	100	25	.....	0	400	&F	650	20°—22° API	No	5,000u	200	7.69
	3	4	A	..	TP	750	504	4	23 x32	150	657	1925	2,994	720	.....	0	3,119		68,850		No	764,420	1,062	11.10
	Plant					1,150	773				982			1,602	.....	0	2,489		136,447			1,581,800	987	11.59
887	1	2	M	C	TP	150 *m*	101	3	14 x17	257	125	1922	0	.....	.....	.....	.....	BC	.....		No	0	.....	.....
	2	2	A	P	CH	440	295	4	15 x20	225	375	1930	58	.....	.....	.....	.....	&C	.....		No	8,050	.....	.....
	3	4	M	..	TP	550	369	5	17 x24	225	438	1928	8,702	.....	.....	.....	.....		.....		No	1,154,870	.....	.....
	Plant					1,140	765				938			1,090	.....	.....	4,411		139,521			1,162,920	1,067	8.33
78	1	4	M	..	TP	400	269	4	17 x24	225	375	1928	6,691	831	45	162	3,220	CC	87,787	30.9° API	No	710,200	855	8.09
	2	4	M	..	TP	660	443	6	17 x24	240	625	1930	5,676	953	48	268	3,930		154,912		No	1,624,924	1,705	10.48
	Plant					1,060	712				1,000			1,784	93	430	3,595		242,699			2,335,124	1,308	9.62
519	1	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1925	1,900	.....	.....	0	.....	S	.....		No	.....	.....	.....
	2	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1922	807	.....	.....	0	.....	C&F	.....	32°—36° API	No	.....	.....	.....
	3	2	M	P	TP	560	376	4	16 x20	257	470	1929	6,552	.....	.....	0	.....		.....		No	.....	.....	.....
	Plant					1,060	712				890			1,644	.....	0	2,609		166,500			1,668,238	1,014	10.02
203	1	2	M	P	TP	525	352	5	14 x17	300	447	1931	4,260	.....	.....	0	.....	CC	.....		No	790,700	.....	.....
	2	2	M	P	TP	525	352	5	14 x17	300	447	1931	4,548	.....	.....	0	.....		.....		No	795,700	.....	.....
	Plant					1,050	704				894			2,560	.....	0	1,806		157,693			1,586,400	619	10.06
259	1	2	M	C	TP	210	141	3	14 x17	300	170	1931	6,498	646	.....	.....	2,111	BC	45,390		No	523,376	810	11.52
	2	2	M	C	TP	360	242	6	14 x17	257	300	1926	1,367	190	.....	.....	2,590	&F	20,220		No	148,949	784	7.37
	3	2	M	P	TP	480	322	4	16 x20	257	400	1928	895	170	.....	.....	2,526		17,130		No	138,040	812	8.06
	Plant					1,050	705				870			1,006	.....	.....	2,272		82,740			810,365	805	9.79
847	1	2	M	P	TP	1,050	705	7	16 x20	257	900	1931	6,303	1,694	.....	.....	3,905	BC	248,040	30°—36° B; S <1.0% Ash <0.05%; BS&W<0.3%	No	2,815,400	1,662	11.34
1144	1	2	M	C	TP	210	141	3	14 x17	300	167	1931	5,320	800	.....	0	1,396	BC	37,907		No	440,100	550	11.60
	2	2	M	C	TP	420	282	6	14 x17	300	353	1931	3,564	894	.....	0	1,674	C&F	54,233	28°—32° B	No	644,500	721	11.88
	3	2	M	C	TP	420	282	6	14 x17	300	353	1931	3,533	933	.....	0	1,590		54,425		No	645,700	692	11.86
	Plant					1,050	705				873			2,627	.....	0	1,559		146,565			1,730,300	659	11.81
56	1	4	A	..	TP	500	336	5	17 x24½	200	438	1926	2,872	478	436	0	3,004	CC	57,326	28°—32° API; 50 SSU @	No	668,900	1,398	11.66
	2	4	A	..	TP	500	336	5	17 x24½	200	438	1929	2,853	493	452	0	2,893		54,422	100 F; 0.01% S; 0.01% CC;	No	682,299	1,383	12.53
	Plant					1,000	672				876			971	888	0	2,945		111,748	0.0008%Ash; BS&W—Trace		1,351,199	1,391	12.08
801	1	2	A	P	CH	1,000	671	4	20½x26	180	937	1927	7,329	2,150	.....	0	3,409	BC&S	366,030	22°—26° API; 1.0% S	Yes	4,428,200	2,060	12.10
1061	1	2	M	P	TP	980	658	7	16 x20	257	975	1930	4,104	1,989	.....	60	2,020	BC&S	196,725	35.2° API; 41 SSU @ 100F 0.282% S; 0.01% CC; Ash—Nil; BS&W—Trace	No	2,256,400	1,135	11.48
53	1	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1923	118	51	.....	5	463	S	.....		No	13,350	262	.....
	2	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1925	318	153	.....	5	623	&C	.....	23.5° API	No	48,610	317	.....
	3	4	A	.	TP	400	269	4	16½x23	225	338	1925v	7,350	1,194	334	60	2,460		.....		No	1,306,940	1,092	.....
	Plant					900	605				758			1,398	.....	70	2,185		125,025			1,368,900	979	10.95

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 4, Continued)

Plant Number	Loading: Running Engine Capacity Factor (See Text)	Loading: Running Plant Capacity Factor (See Text)	Loading: Peak Load During Reported Period—Gross K.W.	Loading: B.M.E.P. at Rated B.H.P.—Lbs. per Sq. In.	Loading: B.M.E.P. at Peak Load—90% Generating Efficiency	Piston Cooling (See Notes)	Are Air Filters Used	Type Cooling System (See Text)	Average Temperature Incoming Cooling Water—Degs. F.	Average Temperature Outgoing Cooling Water—Degs. F.	Purpose for which Jacket Water Heat is Utilized (See Notes)	Purpose for which Exhaust Heat is Utilized (See Notes)	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Material	Cost of Engine Regular Upkeep—Dollars: Extra Labor	Cost of Repairs for Engine Accidents—Dollars: Material	Cost of Repairs for Engine Accidents—Dollars: Extra Labor	Total Engine Maintenance in Dollars per Rated B.H.P. per Year	Major Engine Parts Renewed During Reported Period (See Note gg)	No. of Enforced Engine Shutdowns	Total Duration of Enforced Engine Shutdowns—Hours	Total Engine Maintenance Time Not Inc. in Enforced Shutdown Time—Hours	Attendance: No. of Shifts in Period	Attendance: No. of Hours per Shift (See Note ff)	Attendance: No. of Attendants per Shift	Output per Man-Hour—Net K.W. Hrs.	Plant Altitude—Feet Above Sea Level	Plant Number
725	.....		325	69.8	56.3	A	No													1	65	0						725
	.....		205	69.8	53.2	A	No								80				Spray Valve Parts	0	0	0	313	8	1			
	.....		109	29.4	23.9	A	No										0.06			1	5	0	313	8	1			
	.....		0	31.5	....	A	No													0	0	0	313	8	1			
		57.8	630					D-F	90	110	F	N										156	8	1	230	572	
886	43.3		69	35.3	30.1	A	No													0	0	0						886
	52.1		138	29.4	30.3	A	No								2,386					0	0	120	313	8	1			
	41.9		193	35.3	28.2	A	No										1.83			0	0	600	313	8	1			
	36.6		307	73.2	53.5	A	No													0	0	0	313	8	1			
		40.7	307					D	.....	.....	N	N											156	8	1	99	2,970	
13	62.8		180	75.8	73.8	A	No						101	0	0	0	0.37	Piston Rings & Piston Pin Bearings	0	0	60	340	8	1			13
	59.1		140	73.5	76.8	A	No						103	0	0	0	0.52		0	0	35	340	8	1			
	44.7		375	73.8	54.9	W	No						225	0	0	0	0.30		0	0	120	340	8	1			
		56.4	375					A	80	115	N	L					0.35					75	8	1	132	70	
16	97.7		420	69.8	72.8	A	Yes						0	0	420	140	0.93	Exh. Manifold Failure	1	316	0						16
	82.4		160	35.3	35.1	A	Yes						0	0	312	95	1.70	Piston Rod Failure	1	288	0						
	82.8		240	35.3	35.0	A	Yes						0	0	0	0	0	None	0	0	0	.....	8	1n			
		93.3	810					B-F	90	110	N	N					0.81					.....	8	1n	.....	1,100	
17	.....		.....	69.8	....	A	....																				17
	.....		.....	69.8	....	A	....																				
		59.2	870					.....	.....	.....	.....	.....					.....								189	.....	
846	.....		200	35.3	29.2	A	No								536												846
	.....		560	53.6	53.2	O	No										0.45										
		61.8	760					B	70	90	N	N											8c	1q	.....	2,300	
889	68.5	68.5	770	57.5	54.9	O	No	A	55	80	N	N			1,465		1.22						8c	1n	.....	4,100	889
980	.....		510	74.6	75.5	A	No						197	0	0	0	0.26	Piston Rings	0	0	.....						980
	.....		320	71.2	75.5	A	No						132	0	0	0	0.29		0	0	.....	307	8b	2			
		51.0	510					A	70	110	N	N					0.27					187	8	2	97	15	
1083	73.5	73.5	800	57.5	57.1	O	Yes	B-F	90	110	N	F	800	330	0	0	0.94	Spray V. P. and Sle.; 1-Cracked Cyl.	0	0	336	...	8c	1n	.....	1,100	1083
330	42.0		300	73.2	52.2	A	Yes											None	0	0	.....						330
	28.2		170	35.3	37.3	A	No						553	184	0	0	0.63	Piston Rings & Gaskets	0	0	.....	313	8	1½			
	53.3		50	29.4	21.9	A	No								0	0		Piston Rings & Gaskets	0	0	.....	313	8	1			
	35.5		120	29.4	26.3	A	No								0	0		Piston Rings & Gaskets	0	0	.....	313	8	1			
		37.1	350					B	50	95	N	N										156	8	1	87	4,551	
878	73.4		400	53.6	57.0	O	Yes											5 Piston Rings	6	.....	.....	288	10	1			878
	68.0		120	29.4	26.3	A	No								1,273		1.10	None	0	0	.....	288	10	2			
	78.2		275	69.8	71.4	A	No											12 P. R.; 1 C. B.	2	.....	.....	288	10	1			
		75.3	700					D-F	85	110	N	N										77	8b	1	139	2,163	
616	69.7		200	77.2	76.4	A	Noe												0	0	.....						616
	74.6		50	29.4	21.9	A	No						911	64	0	0	0.91	Piston Rings	0	0	.....	365	8	3			
	50.7		500	74.5	74.0	W	Noe							64	0	0			0	0	.....	365	8	1			
		59.0	500					B	90	120	N	F										365	8	1	98	15	
887	.....		0	29.4	....	A	No												0	0	0	313	8	1			887
	47.1		180	54.7	33.4	W	Yes								63				0	0	15	313	8	1			
	36.0		340	71.1	65.5	A	Yes										0.06		2	2	56	313	8	1			
		36.0	340					D-F	.....	.....	N	N										156	8	1	124	4,082	
78	39.5		.....	64.6	....	A	Yes						124	444	0	0	1.42		0	0	.....	.....	.....	.....			78
	64.6		.....	66.7	....	A	Yes						295	628	0	0	1.40		1	2	.....	.....	.....	.....			
		54.2	835					A	72	95	N	N					1.41					365	6c	1	256	172	
519	.....		135	29.4	29.5	A	No						380	0	0	0	1.90	Rings and Bearings	0	0	320						519
	.....		200	29.4	29.1	A	No						227	0	0	0	0.76	Rings	1	¼	120	365	10	2n			
	.....		390	53.6	55.5	O	Yes						3,418	0	0	0	6.10	4 Pistons; 4 Cyls. Bored; Main Bearings; Rings	0	0	690	365	10	1½n			
		57.9	390					A	70	80	N	N					3.80					365	10	1n	.....	500	
203	52.7		425	52.9	63.9	O	Yes						0	27	0	0	0.03	None	0	0	168	365	8	1			203
	49.7		425	52.9	63.9	O	Yes						0		0	0		None	0	0	120	365	8	1			
		51.1	450					D-F	103	107	N	N										365	8	1	167	1,100	
259	57.1		130	35.3	32.6	A	No						0	0	0	0	0	1-Piston; 1-Cyl. Head; Bearings, Gaskets; etc.	....	.....	29						259
	45.0		175	35.3	25.5	A	No						14	11	297	16	0.94		....	.....	338	365	8	1			
	47.9		200	46.0	28.6	O	No						82	0	0	0	0.17		....	.....	139	365	8	1			
		52.8	210					C	100	110	N	N					0.40					365	8	1	84	3,467	
847	63.4	63.4	700	57.5	57.1	O	No	B	70	90	N	N			1,077		1.03		.....	.....	.....	.....	8c	1	.....	2,000	847
1144	58.7		125	35.3	31.3	A	No						.....	.....	.....	.....	.....		0	0	20	308	8	1			1144
	64.1		250	35.3	31.3	A	No						.....	.....	.....	.....	.....		0	0	30	309	8	1			
	64.8		250	35.3	31.3	A	No						.....	.....	.....	.....	.....	1-Cylinder	1	2	28	308	8	1			
		62.9	515					C	100	120	N	N										200	8	1	183	800	
56	69.4		400	71.2	84.8	A	Yes						.....	.....	.....	.....	.....	1-Connecting Rod Brg.	0	0	336						56
	71.2		400	71.2	84.8	A	Yes						.....	.....	.....	.....	.....	Rings & Cooling Coil	0	0	336	180	8	1			
		70.3	800					A	60	105	N	L						2-Rings				180	8	1	435	850	
801	90.0	90.0	700	62.5	65.2	W	Yes	A	.....	.....	N	N	2,062	348	0	0	2.41	Cyl. Liners; Oil Filter; Piston Rods; Piston Heads; Rings	18	141	582	313	8	1½	420	591	801
																						313	8a	1			
																						156	8	1			
1061	83.5	83.5	700	53.6	57.0	O	No	D	101	106	N	N	.....	.....	.....	.....	.....	Only Minor Parts; Incl. Cam Shaft Previously Installed	3	27½	166	348	8	2n	.....	.....	1061
																						348	8	1n			
																						51	8	1n			
53	84.5		160c	29.4	35.1	A	No						0	0	0	0	0	None	0	0	0	321	8	1			53
	75.7		200c	29.4	29.1	A	No						14	0	0	0	0.05	None	1	2	12	321	8	1			
	66.1		240c	73.8	65.9	A	No						1,933	567	0	0	6.25	Rebab. Main Bearings	13	46	1,364	322	8	1			
		66.7	500c					B-F	77	110	N	N					2.79					131	8	1	150	1,650	

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 5)

Plant Number	Engine Data												Lubrication					Fuel			Gross Output—K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Gross K.W. Hrs. per Gallon of Fuel Oil
	Engine Designation	Engine Cycle	Injection System (Notes)	Scavenging System (Notes)	Trunk Piston or Crosshead?	Rated B.H.P.	Equivalent K.W.—90% Generating Efficiency	Number of Cylinders	Cylinder Dimensions Bore x Stroke—Inches	Rated R.P.M.	Generator Rating—K.V.A.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Total Gallons of New Lubricating Oil Used	Gals. of New Lub. Oil for Cylinder Lub. Only	Gals. of Unfit Lubricating Oil Discarded	Rated H.P. Hours per Gal. of New Lubricating Oil	Lubricating Oil Treatment (See Notes)	Fuel Oil Used—Gallons	Nature of Fue (Oil Used (See Notes)	Is Fuel Centrifuged?			
177	1	2	M	P	TP	900	605	6	16 x20	257	774	1933e	3,384	1,300	.....	200	2,342	CC	131,336	27°—30° API	No	1,635,670	1,258	12.45
260	1	2	A	P	TP	900	605	6	14½x21	257	750	1928	617	232	181	.....	2,393	CC	19,399	27.8° API; 0.51% S; Ash—Trace; 0.6% BS&W; Tank Wagon Delivery	Yes	180,000	776	9.28
560	1	2	M	C	CH	200	134	4	12½x13½	327	169	1924	5,419	852	.....	0	2,592	CC	.....		No	.....	.....	.....
	2	2	M	C	CH	200	134	4	12½x13½	327	169	1924	5,621		.....	0			.....	32°—36° API; 0.07% S	No	.....	.....	.....
	3	4	A	..	TP	500	336	5	17 x24	200	408	1929	1,003	48	.....	0	10,448		.....	Delivered Partly by Tank Truck	No	.....	.....	.....
	Plant					900	604				746			900	.....	0	3,011		93,537			709,290	788	7.88
848	1	2	M	C	CH	450	302	6	15½x16	277	375	1931	5,021	.....	.....	.....	.....	S	.....	30°—36° B; S<1.0%	No	.....	.....	.....
	2	2	M	C	CH	450	302	6	15½x16	277	375	1931	4,776	.....	.....	.....	.....		.....	Ash <0.05%; BS&W<0.3%	No	.....	.....	.....
	Plant					900	604				750			4,431	.....	.....	995		207,919			1,880,400	425	9.05
1065	1	2	M	P	TP	900	605	6	16 x20	257	774	1931	420	253	.....	50	1,493	CC	16,350	28°—30°API; 0°F Pour Test	No	221,120	874	13.52
189	1	4	M	..	TP	550	369	5	17 x24	240	470	1929	8,086	808	303	0	5,500	S&	.....		No	960,540	1,187	.....
	2	4	M	..	TP	200z	134	2	17 x24	200	170	1922	674	75	75	0	1,798	C	.....	28°—30° B	No	50,280	670	.....
	3	4	M	..	TP	100z	67	1	17 x24	200	90	1915	0	0	0	0			.....		No	0		
	Plant					850	570				730			883	378	0	5,190		120,968			1,010,820	1,144	8.36
153	1	2	M	P	TP	840	564	6	16 x20	257	700	1929	.....	150	.....	.....	.....	.....	7,070	.....	.....	81,080	541	11.47
494	1	2	M	P	TP	840	564	6	16 x20	257	700	1930	852	337	337	50	2,123	CC	32,440	28°—30° API	No	376,760	1,117	11.61
619	1	2	M	P	TP	840	564	6	16 x20	257	700	1929	868	513	.....	70	1,421	CC	30,500	28°—30° API	No	402,770	785	13.20
855	1	2	M	P	TP	840	564	6	16 x20	257	700	1929	4,502	667	.....	.....	5,670	Cent.	150,531	30°—36° B; S < 1.0%; Ash <0.05%; BS&W<0.3%	No	1,266,600	1,899	8.41
1056	1	2	M	P	TP	840	564	6	16 x20	257	700	1929	1,212	515	.....	0	1,980	BC	39,467	35.1° API; 41 SSU @ 100F; 0.3% S; 0.01% CC; Ash—Nil; BS&W—Trace	No	425,890	826	10.79
731	1	4	A	..	TP	825	554	4	23 x32	164	750	1926	1,269	1,636	.....	.....	640	BC, S &F	44,190	32° API; 0.5% S	No	478,260	292	10.82
853	1	2	M	C	TP	150 m	101	3	14 x17	257	125	1925	14	.....	.....	.....	.....	S	.....	30°—36° B; S < 1.0%;	No	.....	.....	.....
	2	2	M	C	TP	300 m	202	6	14 x17	257	250	1925	255	.....	.....	.....	.....	&F	.....	Ash < 0.05%;	No	.....	.....	.....
	3	2	M	C	TP	360	242	6	14 x17	257	300	1926	316	.....	.....	.....	.....		.....	BS&W< 0.3%	No	.....	.....	.....
	Plant					810	545				675			226	.....	.....	851		11,687			100,000	443	8.85
648	1	4	M	..	TP	400	269	4	17 x24	225	333	1928	2,900	451	.....	.....	2,575	S	45,600	26°—30° API	No	503,600	1,121	11.08
	2	4	M	..	TP	400	269	4	17 x24	225	333	1928	3,308	562	.....	.....	2,354	&C	56,750		No	619,770	1,102	10.92
	Plant					800	538				666			1,013	.....	.....	2,450		102,350			1,125,370	1,110	10.99
112	1	2	M	C	TP	200 m	134	4	14 x17	257	170	1925	2,021	260	.....	.....	1,553	BC	20,739	32°—36° API	No	176,700	679	8.52
	2	2	M	C	TP	240	161	4	14 x17	257	200	1927	7,328	907	.....	.....	1,939	&S	62,377		No	726,400	801	11.64
	3	2	M	C	TP	300 m	202	6	14 x17	257	250	1925	2,178	525	.....	.....	1,244		32,716		No	290,200	553	8.87
	Plant					740	497				620			1,692	.....	.....	1,665		115,832			1,193,300	705	10.30
1128	1	4	M	..	TP	125	84	6	8 x10½	400	.....	1932	3,670	.....	.....	0	.....	BC	16,077	36° API; 36-49 SSU @ 60F	No	149,430	.....	9.29
	2	4	M	..	TP	300	202	4	13½x17½	327	.....	1932	2,425	.....	78	0	.....		21,417	0.19—0.21% S;	No	213,590	.....	9.97
	3	4	M	..	TP	300	202	4	13½x17½	327	.....	1932	2,681	.....	78	0	.....		25,677	BS&W—Trace	No	256,630	.....	10.00
	Plant					725	488				.....			325	.....	0	6,120		63,171			619,650	1,906	9.81
67	1	2	M	C	TP	360	242	6	14 x17	257	250	1925	2,487	1,034	.....	100	867	C	51,892	23° B; 52 SSU @ 100F;	No	512,820	495	9.89
	2	2	M	C	TP	360	242	6	14 x17	257	250	1925	1,624	610	.....	50	960	&F	32,291	0.005% S; 0.037% CC	No	320,080	525	9.92
	Plant					720	484				500			1,644	.....	150	901		84,183			832,900	507	9.90
170	1	2	M	C	TP	360	242	6	14 x17	257	300	1929	2,954	684	.....	98	1,554	S,C	.....	30.1° API; 34 SSU @ 100F; 0.8% S; 0.015% CC;	No	.....	.....	.....
	2	2	M	C	TP	360	242	6	14 x17	257	300	1929	3,089	675	.....	94	1,647	&F	.....	Ash and BS&W—Nil	No	.....	.....	.....
	Plant					720	484				600			1,359	.....	192	1,600		76,309	(also Some 24.6° API) Delivered by Tank Truck		814,242	509	10.67
394	1	2	M	C	TP	120	81	2	14 x17	257	90	.....	2,603	148	.....	.....	2,110	F	10,004		No	79,040	534	7.90
	2	2	M	C	TP	240	161	4	14 x17	257	200	.....	5,525	832	.....	.....	1,595		48,519		No	426,850	513	8.79
	3	2	M	C	TP	360	242	6	14 x17	257	300	.....	632	194	.....	.....	1,170		5,460		No	57,010	294	10.44
	Plant					720	484				590			1,174	.....	.....	1,590		63,983			562,900	480	8.80
527	1	2	M	C	TP	360	242	6	14 x17	257	300	1930	2,244	.....	.....	0	.....	S,C	.....	½—32°—36° API; 38 SSU @ 100F; ½—24°—26° API; 50 SSU @ 100F;	No	254,000	.....	.....
	2	2	M	C	TP	360	242	6	14 x17	257	300	1930	6,679	.....	.....	0	.....	&F	.....		No	660,300	.....	.....
	Plant					720	484				600			1,232	.....	0	2,607		99,360	All—S<0.33%;CC<0.33%; Ash—Trace; BS&W<0.25%		914,300	742	9.20
718	1	2	M	C	TP	360	242	6	14 x17	257	300	1926	3,930	850	.....	.....	1,664	S,C	.....	32°—36° API;				
	2	2	M	C	TP	360	242	6	14 x17	257	300	1926	5,061	900	.....	.....	2,024	&F	.....	36 SSU @ 100F; 0.7% S; 0.01% CC; 0.001% Ash;	No	313,990	369	.....
	Plant					720	484				600			1,750	.....	.....	1,850		96,838		No	415,980	462	.....
																				BS&W—Trace		729,970	417	7.54
19	1	2	M	C	TP	100 m	67	2	14 x17	257	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	2	2	M	C	CHa	240	161	4	12½x13½	325	.....	1927	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	3	2	M	C	CHa	360	242	4	15½x16	275	.....	1927	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	Plant					700	470				.....			1,452	.....	.....	.....		79,850			629,550	434	7.89
274	1	2	M	C	TP	350	235	5	14 x17	300	291	1932	6,984	.....	.....	.....	.....	F	.....	32°—36° B; 0.25% S;	Yes	.....	.....	.....
	2	2	M	C	TP	350	235	5	14 x17	300	291	1932	6,984	.....	.....	.....	.....		.....	0.5% BS&W	Yes	.....	.....	.....
	Plant					700	470				582			3,085	.....	.....	1,584		221,733			2,305,000	747	10.39
368	1	4	A	..	TP	400	269	4	16½x23	225	340	1925	5,918	974	.....	.....	2,430	CC,S	107,725	26.2° API; 54 SSU @ 100F;	No	1,366,000	1,402	12.67
	2	4	A	..	TP	300	202	3	16½x23	225	250	1929	5,586	696	.....	.....	2,405	&F	75,526	0.648% S; 0.026% CC;	No	926,000	1,330	12.26
	Plant					700	471				590			1,670	.....	.....	2,420		183,251	Ash—Nil; BS&W—Trace		2,292,000	1,372	12.52
851	1	2	M	C	TP	300 m	202	6	14 x17	257	250	1926	219	.....	.....	.....	.....	S	.....	30°—36° B; S < 1.0%;	No	.....	.....	.....
	2	2	M	C	TP	360	242	6	14 x17	257	300	1926	424	.....	.....	.....	.....	&F	.....	Ash <0.05%; BS&W<0.3%	No	.....	.....	.....
	Plant					660	444				550			193	.....	.....	1,130		10,987			91,800	475	8.35

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 5, Continued)

Plant Number	Loading: Running Engine Capacity Factor (See Text)	Loading: Running Plant Capacity Factor (See Text)	Loading: Peak Load During Reported Period—Gross K.W.	Loading: B.M.E.P. at Rated B.H.P.—Lbs. per Sq. In.	Loading: B.M.E.P. at Peak Load—90% Generating Efficiency	Piston Cooling (See Notes)	Are Air Filters Used	Type Cooling System (See Text)	Average Temperature Incoming Cooling Water—Degs. F.	Average Temperature Outgoing Cooling Water—Degs. F.	Purpose for which Jacket Water Heat is Utilized (See Notes)	Purpose for which Exhaust Heat is Utilized (See Notes)	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Material	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Extra Labor	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Material	Maintenance and Repairs: Cost of Repairs for Engine Accidents—Dollars: Extra Labor	Maintenance and Repairs: Total Engine Maintenance in Dollars per Rated B.H.P. per Year	Maintenance and Repairs: Major Engine Parts Renewed During Reported Period (See Note gg)	Maintenance and Repairs: No. of Enforced Engine Shutdowns	Maintenance and Repairs: Total Duration of Enforced Engine Shutdowns—Hours	Maintenance and Repairs: Total Engine Maintenance Time Not Inc. in Enforced Shutdown Time—Hours	Attendance: No. of Shifts in Period	Attendance: No. of Hours per Shift (See Note ff)	Attendance: No. of Attendants per Shift	Attendance: Output per Man-Hour—Net K.W. Hrs.	Plant Altitude—Feet Above Sea Level	Plant Number
177	79.9	79.9	600	57.5	57.0	O	Yes	B-F	90	110	N	F	246	100	0	0	0.63	Spray V. Pl. & Casings	3	6	44	.....	.....	.....	.....	1,100	177
260	48.3	48.3	575	64.4	61.2	O	No	A	46	120	N	N	.....	.....	.....	.....	.....	Rings, Fuel Valves, etc.	0	0	.....	.....	.....	.....	.....	4,850	260
560	.....		120	37.2	33.3	A	No						413	61	0	0	1.19	Piston Rings	2	6	47	.....	8	1n			560
	.....		120	37.2	33.3	A	No								0	0		Piston Rings	5	15	49	.....	8	2n			
	.....		320	72.6	69.2	A	No						0	0	0	0	0	None	0	0	2	.....	8	1n			
		39.0	320					A	75	120	N	N										.....	8	1n	.....	35	
848	.....		260	36.7	31.6	O	No						2,689				2.99	.....	.....	.....	.....	.....	.....	.....			848
	.....		260	36.7	31.6	O	No											.....	.....	.....	.....	.....	.....	.....			
		63.5	520					A	60	70	N	N										.....	8c	1	.....	3,900	
1065	87.0	87.0	550	57.5	52.3	O	Yes	B-F	90	110	N	N	300	173	0	0	0.53	Spray V. Pl. and Sle.	0	0	240	.....	.....	1n	.....	1,100	1065
189	32.2		300	66.7	54.2	A	Yes						.....	.....	.....	.....	.....		0	0	70						189
	55.7		125	72.6	67.7	A	No						.....	.....	.....	.....	.....	No Major Parts	0	0	0	365	10	1			
	0		0	72.6	....	A	No						.....	.....	.....	.....	.....		0	0	0	365	10	1			
		32.8	300					D	95	120	N	N										365	4	1	109	1,850	
153	.....	.....	540	53.6	51.3	O	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	14	.....	153
494	78.4	78.4	550	53.6	52.3	O	Yes	B-F	90	110	N	N	241	50	0	0	0.35	No Major Parts	0	0	25	.....	8	1n	.....	1,100	494
619	82.3	82.3	560	53.6	53.2	O	Yes	B-F	90	110	N	N	100	102	0	0	0.24	Master Fuel Pump	0	0	10	.....	8a	1n	.....	1,100	619
855	49.8	49.8	470	53.6	44.7	O	Yes	.....	75	90	N	N	1,666				1.98	.....	.....	.....	.....	.....	8c	1q	.....	.....	855
1056	62.3	62.3	600	53.6	56.9	O	Yes	D	105	.....	N	N	.....	.....	.....	.....	.....	1-Set of Main Bearings; 1-Cam Shaft (at Mfgr's Expense)	2	681	154	348 348 51	8 8 8	2n 1n 1n	.....	1,200	1056
731	68.0	68.0	600	74.9	81.1	W	Yes	D	.....	.....	N	N	241	0	0	0	0.29	(See gg 14)	1	1	1	313 313	8a 8a	1 2	31	865	731
853	.....		.....	29.4	....	A	No						149				0.18	.....	.....	.....	.....						853
	.....		.....	29.4	....	A	No											.....	.....	.....	.....						
	.....		.....	35.3	....	A	No											.....	.....	.....	.....						
		77.4	550					B	70	90	N	N										365	8	1	30	2,070	
648	64.9		200	64.6	48.0	A	Yes						50	25	0	0	0.19	No Major Parts	1	1,440	0						648
	69.6		200	64.6	48.0	A	Yes						50	50	0	0	0.25		.....	.....	.....	.....	.....	1n			
		67.6	400					B-F	90	110	N	B					0.22					.....	.....	1n	.....	1,100	
112	65.3		130	29.4	28.5	A	No																				112
	61.6		145	35.3	31.8	A	No						342	0	0	0	0.46	1-Main Bearing; Piston Rings; Gaskets	0	0	20	350	10	1			
	66.0		190	29.4	27.7	A	No							0	0	0			1	6	24	365	4	1			
		63.1	400					D-F	110	125	N	N		0	0	0			0	0	30	350	10	1			
																						29	10	1	126	845	
1128	48.5		90	78.1	83.7	A	Yes						0	0	0	0	0	None	0	0	.....						1128
	43.6		220	75.2	81.9	A	Yes						0	0	0	0	0	None	0	0	.....	365	4	1			
	47.4		210	75.2	78.1	A	Yes						0	0	0	0	0	None	0	0	.....	365	10	1			
		46.4	260					D	57/80	105	N	B					0					365	10	1	61	1,000	
67	85.2		210	35.3	30.6	A	No						544	92	0	0	0.88	1-Piston; 8-Bearings Rebabbitted	....	.....	.....	313	8	1n			67
	81.5		210	35.3	30.6	A	No								0	0			....	.....	.....	313	8	1n			
		83.8	210					A	58	120	N	N										313	8	1n	.....	860	
170	.....		.....	35.3	....	A	No						.....	.....	.....	.....	.....	1-Seized Piston	3	19	.....	.....	10	2n			170
	.....		.....	35.3	....	A	No						.....	.....	.....	.....	.....	1-Seized Piston	4	18½		.....	.....	1n			
		55.8	470					B-F	.....	85/100	N	N					.....								.....	50	
394	37.5		65	35.3	28.3	A	No						15	0	0	0	0.13		0	0	8						394
	48.0		140	35.3	30.7	A	No						190	0	0	0	0.79	2-Conn. Rod B; 1M.B.	0	0	36	365	10	1½			
	37.3		170	35.3	24.8	A	No						0	0	0	0	0		0	0	0	365	10	1			
		45.0	170					A	75	110	N	N					0.28					365	4	1	53	2,650	
527	46.8		260	35.3	37.9	A	Yes											12-D.S. Rings; 24 Inj. Nozzle Tips	1	1/20	90	.....	.....	.....			527
	40.8		260	35.3	37.9	A	Yes						293	0	0	0						.....	.....	.....			
		42.4	330					D-F	110	116	N	N					0.41	1-Lubricator	1	1/12	30	351 42	8b 8	1 1	99	750	
718	33.0		200	35.3	29.2	A	No						760				1.06	(See gg 15)	2	.....	.....						718
	34.0		200	35.3	29.2	A	No												3	.....	.....	313	8b	1			
		33.5	200					B	.....	.....	N	N										156	8	1	75	1,200	
19	.....		.....	29.4	....	A	....						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	19
	.....		.....	45.0	....	A	....						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
	.....		.....	44.3	....	A	....						.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
	....		185					.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	55	.....	
274	.....		215	35.3	32.3	A	Yes						.....	.....	.....	.....	.....	Conn. Rod Bearings; Gaskets	1	12	0	.....	8	1			274
	.....		215	35.3	32.3	A	Yes						.....	.....	.....	.....	.....		2	18	0	.....	8	1			
		70.2	425					B-F	95	102	F	N										.....	8	1	.....	700	
368	85.8		272	73.8	74.6	A	No						384	0	0	0	0.96	1-Intercooler Tube	2	16	48	231	11a	1			368
	82.0		200	73.8	73.0	A	No						204	0	0	0	0.68	Blowout; 1-Piston	0	0	45	192	12	1			
		84.4	472					B	60	110	N	N					0.84					63	8b	1	342	1,400	
851	.....		.....	29.4	....	A	No						698				1.06	.....	.....	.....	.....	.....	.....	.....			851
	.....		.....	35.3	....	A	No											.....	.....	.....	.....						
		62.6	490					B	70	90	N	N										365	8	1	24	2,500	

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 6)

Plant Number	Engine Data: Engine Designation	Engine Cycle	Injection System (Notes)	Scavenging System (Notes)	Trunk Piston or Crosshead?	Rated B.H.P.	Equivalent K.W.—90% Generating Efficiency	Number of Cylinders	Cylinder Dimensions Bore x Stroke—Inches	Rated R.P.M.	Generator Rating—K.V.A.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Lubrication: Total Gallons of New Lubricating Oil Used	Gals. of New Lub. Oil for Cylinder Lub. Only	Gals. of Unfit Lubricating Oil Discarded	Rated H.P. Hours per Gal. of New Lubricating Oil	Lubricating Oil Treatment (See Notes)	Fuel: Fuel Oil Used—Gallons	Nature of Fuel Oil Used (See Notes)	Is Fuel Centrifuged?	Gross Output—K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Gross K.W. Hrs. per Gallon of Fuel Oil
810	1	2	M	C	TP	150 *m*	101	3	14 x17	257	125	1920	19	10	.....	.....	285		231		No	570	57	2.47
	2	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1919	110	25	.....	.....	880	None	1,856	28°—30° API; S < 0.5%	No	6,600	264	3.56
	3	2	M	C	TP	300 *m*	202	6	14 x17	257	250	1922	238	64	.....	.....	1,116		4,607		No	24,730	386	5.37
	Plant					650	437				545			99	.....	.....	972		6,694			31,900	322	4.77
1146	1	2	M	C	TP	140	94	2	14 x17	300	111	1931	4,212	.....	.....	.....	.....	BC,	17,500	28°—36° API	No	162,900	.....	9.32
	2	2	M	C	TP	210	141	3	14 x17	300	170	1931	1,116	.....	.....	.....	.....	S	10,145	0.5% S	No	100,400	.....	9.94
	3	2	M	C	TP	280	188	4	14 x17	300	230	1931	3,432	.....	.....	.....	.....	&F	22,860		No	231,700	.....	10.10
	Plant					630	423				511			902	.....	5	1,980		50,505			495,000	549	9.81
529	1	2	M	C	TP	100 *m*	67	2	14 x17	257	75	1922	1,230	108	.....	0	1,139	BC	4,893		No	.....	.....	.....
	2	2	M	C	TP	100 *m*	67	2	14 x17	257	75	1922	1,326	102	.....	0	1,300	BC	4,965	32°—36° API	No	.....	.....	.....
	3	2	M	C	TP	120	81	2	14 x17	257	90	1927	4,272	309	.....	0	1,659	BC&F	17,985		No	.....	.....	.....
	4	2	M	P	TP	300	201	6	10½x12½	360	250	1931	4,482	517	.....	0	2,600	BC	42,075		No	.....	.....	.....
	Plant					620	416				490			1,036	.....	0	2,038		69,918			558,084	539	7.98
359	1	2	M	C	TP	240	161	4	14 x17	257	200	1929	3,119	439	439	.....	1,704	F	20,666	.....	No	138,800	316	6.72
	2	4	M	..	TP	375	252	6	12 x15½	360	310	1932	5,641	654	654	.....	3,234		45,263	.....	No	371,200	567	8.20
	Plant					615	413				510			1,093	1,093	.....	2,619		65,929			510,000	467	7.74
431	1	2	M	C	TP	100 *m*	67	2	14 x17	257	75	1922	.....	.....	.....	.....	.....		.....		No	.....	.....	.....
	2	2	M	C	TP	150 *m*	101	3	14 x17	257	125	1923	.....	.....	.....	.....	.....	F	.....	34° API	No	.....	.....	.....
	3	2	M	C	TP	360	242	6	14 x17	257	300	1930	.....	.....	.....	.....	.....		.....		No	.....	.....	.....
	Plant					610	410				500			1,160	.....	.....	.....		50,735			451,900	389	8.91
32	1	4	M	..	TP	600*y*	403	5	17½x22	234*z*	572	1928	2,472	1,333	.....	200	1,113	S&F	63,867	36° API; CC and Ash—Trace; BS&W—Nil	No	726,050	544	11.36
61	1	4	M	..	TP	300	202	3	17½x22	225	275	1928	5,533	.....	.....	.....	.....	CC	.....	30° API; 40 SSU @ 100 F;	No	651,300	.....	.....
	2	4	M	..	TP	300	202	3	17½x22	225	275	1928	5,599	.....	.....	.....	.....	&F	.....	0.3% S; 0.15% CC;	No	608,300	.....	.....
	Plant					600	404				550			1,022	.....	75	3,268		110,379	0.25% BS&W; Ash—Trace		1,259,600	1,231	11.41
106	1	4	A	..	TP	600	403	6	17 x25	200	.....	1929	5,432	706	.....	.....	4,616	.....	141,092	.....	.....	1,570,920	2,225	11.13
228	1	4	A	..	TP	300	202	3	17 x25	200	250	1925	5,198	.....	.....	.....	.....	S	.....	24°—26° API;	No	.....	.....	.....
	2	4	A	..	TP	300	202	3	17 x25	200	250	1925	5,198	.....	.....	.....	.....	&F	.....	150 SSU @ 100F; 1.0%CC;	No	.....	.....	.....
	Plant					600	404				500			1,750	360	.....	1,782		187,000	1.0% BS&W; S—Trace		1,898,000	1,084	10.15
644	1	2	M	C	TP	240	161	4	14 x17	257	196	1931	7,217	945	945	0	1,832	BC	69,970	32°—36° B; 0.5% S;	No	.....	.....	.....
	2	2	M	C	TP	360	242	6	14 x17	257	300	1931	1,543	413	413	0	1,344		26,200	0.5% BS&W	No	.....	.....	.....
	Plant					600	403				496			1,358	1,358	0	1,683		96,170			913,700	672	9.50
809	1	4	A	..	TP	600	403	6	16½x24	200	500	1923	353	70	.....	.....	3,026	S&F	10,016	28°—30° API; S <0.5%	No	76,300	1,090	7.62
1094	1	4	A	..	TP	600	403	6	16½x23	225	513	1928	1,658	391	.....	70	2,542	S&C	47,857	.....	No	596,800	1,526	12.47
1096	1	4	A	..	TP	600	403	6	16½x23	225	513	1928	2,606	475	.....	70	3,290	S&C	58,000	.....	No	733,520	1,544	12.64
410	1	2	M	C	TP	150 *m*	101	3	14 x17	257	125	1923	3,918	.....	.....	.....	.....		.....		No	198,200	.....	.....
	2	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1923	7,850	.....	.....	.....	.....	C	.....		No	608,830	.....	.....
	3	2	M	C	TP	240	161	4	14 x17	257	200	1928	1,421	.....	.....	.....	.....		.....		No	115,440	.....	.....
	Plant					590	396				495			1,829	.....	.....	1,366		110,292			922,470	504	8.36
862	1	2	M	C	TP	75 *m*	50	2	12 x15	300	60	1920	445	.....	.....	.....	.....		.....		No	.....	.....	.....
	2	2	M	C	TP	75 *m*	50	2	12 x15	300	60	1922	985	.....	.....	.....	.....	CC	.....	33.1° API; 0.6% S;	No	.....	.....	.....
	3	2	M	C	TP	180	121	3	14 x17	257	150	1925	4,345	648	.....	70	1,207	&F	.....	Trace—BS&W	No	.....	.....	.....
	4	4	A	..	TP	258	173	3	16½x24½	180	225	1918*aa*	4,950	752	433	169	1,698		.....		No	.....	.....	.....
	Plant					588	394				495			.....	.....	.....	.....		76,756			653,190	.....	8.51
808	1	2	M	P	TP	560	376	4	16 x20	257	470	1930	386	115	.....	0	1,880	BC	8,434	28°—30° API; S <0.5%; Delivered Partly in Tank Trucks	No	106,700	928	12.65
852	1	2	M	P	TP	560	376	4	16 x20	257	460	1928	690	106	.....	.....	3,644	BC	15,720	30°—36° B; S <1.0%; Ash <0.05%; BS&W <0.3%	No	174,660	1,647	11.11
246	1	2	M	C	TP	100 *m*	67	2	14 x17	257	60	1918	2,350	.....	.....	0	.....	C,	.....	36°—40° API; Delivered	No	.....	.....	.....
	2	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1922	4,435	.....	.....	0	.....	S,	.....	Partly by Tank Truck	No	.....	.....	.....
	3	2	M	C	TP	240	161	4	14 x17	257	200	1929	2,875	.....	.....	0	.....	&F	.....		No	.....	.....	.....
	Plant					540	362				430			1,239	.....	0	1,461		56,880			412,525	333	7.25
540	1	2	M	C	TP	360	242	6	14 x17	257	.....	1930	5,361	.....	.....	30	.....	F	.....	32°—36° API	No	.....	.....	.....
	2	2	M	C	TP	180	121	3	14 x17	257	.....	1926	3,399	.....	.....	20	.....		.....	0.5% S	No	.....	.....	.....
	Plant					540	363				.....			2,965	.....	50	858		87,120			700,500	236	8.04
541	1	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1922	2,834	582	.....	.....	974	BC	26,132		No	206,500	355	7.90
	2	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1922	2,712	542	.....	.....	1,001	&S	25,076	36° API	No	192,700	356	7.68
	3	2	M	C	TP	140	94	2	14 x17	300	111	1931	5,354	423	.....	.....	1,772		24,145		No	251,700	595	10.42
	Plant					540	362				451			547	.....	.....	1,202		75,353			650,900	421	8.64
211	1	4	A	..	TP	510	342	6	14 x21	277	438	1932	.....	1,741	.....	.....	.....	.....	93,221	.....	.....	1,151,100	662	12.35
772	1	4	A	..	TP	250	168	4	13¾x17½	257	250	1916	440	138	.....	15	797	CC	5,530	28°—30° API	No	54,770	397	9.91
	2	4	A	..	TP	250	168	4	13¾x17½	257	263	1916	428	135	.....	15	793		5,330	0°F Pour Test	No	52,802	391	9.91
	Plant					500	336				513			273	.....	30	795		10,860			107,572	394	9.91
424	1	2	M	C	TP	240	161	4	14 x17	257	200	1929	4,175	799	.....	35	1,254	F	33,071	Delivered Partly by	No	309,325	387	9.35
	2	2	M	C	TP	240	161	4	14 x17	257	200	1929	4,606	844	.....	35	1,310		37,996	Tank Truck	No	356,275	422	9.38
	Plant					480	322				400			1,643	.....	70	1,282		71,067				405	9.37
501	1	2	M	C	TP	120	81	2	14 x17	257	90	1925	2,521	289	.....	40	1,046	CC,	11,170	.....	No	63,845	221	5.72
	2	2	M	C	TP	120	81	2	14 x17	257	90	1925	3,533	513	.....	40	826	S	15,259	.....	No	90,604	177	5.94
	3	2	M	C	TP	240	161	4	14 x17	257	200	1929	4,054	612	.....	30	1,590	&F	29,152	.....	No	262,600	429	9.01
	Plant					480	323				380			1,414	.....	110	1,201		55,581			417,049	295	7.50
124	1	2	M	C	TP	80	54	2	12 x15	300	65	1927	1,181	177	.....	0	534	BC,	4,675		No	.....	.....	.....
	2	2	M	C	TP	180	121	3	14 x17	257	150	1930	4,489	743	.....	0	1,087	S	28,408	30° API; 1.91% S	No	.....	.....	.....
	3	2	M	C	TP	210	141	3	14 x17	300	170	1931	4,329	664	.....	0	1,368	&F	24,790		No	.....	.....	.....
	Plant					470	316				385			1,584	.....	0	1,143		57,873			515,200	325	8.90

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 6, Continued)

Plant Number	Loading: Running Engine Capacity Factor (See Text)	Loading: Running Plant Capacity Factor (See Text)	Loading: Peak Load During Reported Period—Gross K.W.	Loading: B.M.E.P. at Rated B.H.P.—Lbs. per Sq. In.	Loading: B.M.E.P. at Peak Load—90% Generating Efficiency	Loading: Piston Cooling (See Notes)	Loading: Are Air Filters Used?	Type Cooling System (See Text)	Average Temperature Incoming Cooling Water—Degs. F.	Average Temperature Outgoing Cooling Water—Degs. F.	Purpose for which Jacket Water Heat is Utilized (See Notes)	Purpose for which Exhaust Heat is Utilized (See Notes)	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Material	Cost of Engine Regular Upkeep—Dollars: Extra Labor	Cost of Repairs for Engine Accidents—Dollars: Material	Cost of Repairs for Engine Accidents—Dollars: Extra Labor	Total Engine Maintenance in Dollars per Rated B.H.P. per Year	Major Engine Parts Renewed During Reported Period (See Note gg)	No. of Enforced Engine Shutdowns	Total Duration of Enforced Engine Shutdowns—Hours	Total Engine Maintenance Time Not Inc. in Enforced Shutdown Time—Hours	Attendance: No. of Shifts in Period	Attendance: No. of Hours per Shift (See Note ff)	Attendance: No. of Attendants per Shift	Attendance: Output per Man-Hour—Net K.W. Hrs.	Plant Altitude—Feet Above Sea Level	Plant Number
810	29.7		50	29.4	14.6	A	No						0	0	0	0	0	None	0	0	0						810
	44.8		80	29.4	17.6	A	No						0	0	0	0	0	None	0	0	0						
	51.4		125	29.4	18.2	A	No						0	0	0	0	0	None	0	0	0						
		49.3	225					B-F	90	115	N	N										.....	.....	1	.....	1,500	
1146	41.1		70	35.3	26.2	A	Yes						28	0	0	0	0.20	Bearing Rebabb.; Gask.	0	0	32	348	10	1			1146
	63.8		140	35.3	35.1	A	Yes						32	0	0	0	0.15	Bearing Rebabb.; Gask.	0	0	24	348	10	1			
	35.9		200	35.3	37.7	A	Yes						35	0	0	0	0.12	Bearing Rebabb.; Gask.	0	0	64	348	10	1			
		41.3	200					B	100	112	N	B					0.15					52	10	1	39	800	
529	.....		55	29.4	24.1	A	No						0	0	0	0	0	None	0	0	0						529
	.....		55	29.4	24.1	A	No						0	0	0	0	0	None	0	0	0	365	6	1½			
	.....		65	35.3	28.3	A	No						6	5	0	0	0.10	None	0	0	40	365	6	1			
	.....		180	50.8	45.5	O	Yes						24	20	240	22	1.02	1-Cyl. and Piston	1	504er	96	365	6	1			
		39.4	180					D	95	115	B	B					0.51					365	6	1	53	700	
359	27.6		160	35.3	35.1	A	Yes						77	74	0	0			0	0	8	365	10	1			359
	26.1		170	78.4	52.9	A	Yes								0	0	0.25	1-New Oil Line—Improved Design	5	45	214	365	10	1			
		26.5	170					D-F	.....	115	N	N										365	8	1	48	1,600	
431	.....		.....		29.4	....	A	No					16	0	0	0	0.16	Spray Valve Tips; Gask.	0	0	16	358	8	1			431
	.....		.....		29.4	....	A	No					220	0	0	0	1.47	(See gg 16)	2	150	30	358	8	1			
	.....		.....		35.3	....	A	No					58	0	0	0	0.16	1-Inj. Rock. Arm; Gask.	0	0	72	358	8	1			
		....	175					B-F	80	110	N	N					0.48					21	8	1	51	212	
32	72.8	72.8	450	74.6	83.4	A	No	A	95	110	N	B	28	6	0	0	0.06	No Major Parts	0	0	72	365	12a	1	81	9,200	32
61	58.3		220	64.7	70.5	A	No						38	180	0	0	0.73	No Major Parts	0	0	336	365	8	½			61
	53.8		220	64.7	70.5	A	No						11	172	120	73	1.25	2-Sets Crank Bearings	2	75	408	365	8	½			
		56.2	440					B	85	115	T	N					0.99					365	8	½	.....	620	
106	71.8	71.8	460	69.8	79.7	A	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	151	.....	106
228	.....		200	69.8	69.1	A	No						150	40	0	0	0.63	Governor Parts	2	20	40	365	8	4n			228
	.....		210	69.8	72.6	A	No						50	0	0	0	0.17	.....	0	0	40	365	8	2n			
		90.7	375					A	63	100	N	N					0.40					365	8	2n	.....	272	
644	.....		152	35.3	33.3	A	No						148	0	0	0	0.25	Water Piping; Nozzles	0	0	.....	313	10	1½n			644
	.....		250	35.3	36.5	A	No							0	0	0		and Tips; Gaskets	0	0	.....	313	10a	1n			
		59.5	265					C	90	130	N	N										156	12	1n	.....	579	
809	53.6	53.6	400	77.1	76.5	A	No	B-F	80	108	N	N	0	0	0	0	0	None	0	0	0	.....	.....	1	.....	1,900	809
1094	89.4	89.4	410	73.8	75.1	A	No	B-F	90	110	N	B	301	111	0	0	0.69	Main Shaft T. B.	0	0	240	.....	.....	.....	.....	1,100	1094
1096	69.8	69.8	410	73.8	75.1	A	Yes	B-F	90	110	N	N	560	202	0	0	1.27	.....	0	0	236	....	8b	1n	.....	1,100	1096
410	50.1		75	29.4	21.8	A	No						.....	.....	.....	.....	.....	.....	0	0	.....						410
	57.4		100	29.4	21.8	A	No						.....	.....	.....	.....	.....	2-Main Bearings	0	0	.....	365	8	1			
	50.1		115	35.3	25.1	A	No						.....	.....	.....	.....	.....	.....	0	0	.....	365	8	1			
		55.0	260					D	100	105	N	N	.....	.....	.....	.....	.....	.....				365	8	1	99	1,850	
862	.....		35	29.1	20.4	A	No						———	2,489	———			None	0	0	24						862
	.....		35	29.1	20.4	A	No											None	0	0	24	348	8	1			
	.....		95	35.3	27.7	A	No										4.23	1-Conn. Rod Bearing	2	—524—		348	8	1			
	.....		180	72.2	75.1	A	No											New Rings & Bearings	2	—1,128—		348	8	1			
		44.9	224					B	90	110	N	N										51	8	1	70	221	
808	73.5	73.5	425	53.6	60.6	O	Yes	B-F	65	75	F	N	0	0	0	0	0	None	0	0	0	.....	.....	1	.....	1,900	808
852	67.3	67.3	320	53.6	45.6	O	No	A	60	80	N	N	———	114	———		0.20	.....	.....	.....	.....	.....	.....	1n	.....	3,556	852
246	.....		58	29.4	25.4	A	No																				246
	.....		95	29.4	20.8	A	No						23	15	0	0	0.07	Gaskets and Misc.	0	0	15						
	.....		120	35.3	26.3	A	No								0	0		Small Parts	0	0	28	365	10	1			
		33.9	160					B	60/80	110	B	N										365	10	1			
																						365	4	1	45	5,000	
540	.....		260	35.3	37.9	A	Yes						13	27	0	0	0.07	Pist. Rings, Gask., etc.	0	0	14	365	8	1			540
	.....		110	35.3	32.1	A	Yes								0	0		Pist. Rings, Gask., etc.	0	0	10	365	8	1			
		41.0	280					D-F	.....	100	N	N										365	8	1	76	525	
541	54.4		120	29.4	26.3	A	No						215	0	0	0	0.40	No Major Parts	0	0	200						541
	53.0		120	29.4	26.3	A	No							0	0	0		1-Piston	0	0	200	365	5	1			
	50.0		90	35.3	33.8	A	No							0	0	0		1-Conn. Rod Bearing	0	0	72	365	9½	1			
		52.2	200					B	110	130	N	N										365	9½	1	73	780	
211	.....	.....	330	75.2	72.6	A	No	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	132	.....	211
772	74.1		160	74.1	70.6	A	Yes						201	137	0	0	0.68	No Major Parts	0	0	168						772
	73.5		160	74.1	70.6	A	Yes								0	0			0	0	144	.....	8	1n			
		73.8	320					B-F	90	110	N	N										.....	8	1n	.....	1,100	
424	46.0		160	35.3	35.1	A	No						0	0	0	0	0	None	1	4	192	350	11a	1			424
																						365	2	1			
	48.0	47.0	160	35.3	35.1	A	No	B	75	100	N	N	0	0	0	0	0	None	0	0	192	30	11	1	71	225	
501	31.3		220 40	35.3	17.4	A	No								0	0		Piston Rings	0	0	32						501
	31.6		65	35.3	28.3	A	No						545	28	0	0	1.19	P. & R.; Reboring	0	0	96	365	5/10	1			
	40.2		135	35.3	29.6	A	No								0	0		1-Cyl. and Piston	1	1/6	10	365	10	1			
		36.5	135					B-F	90	110	N	N										365	9	1	.....	1,106	
124	.....		45	31.1	25.9	A	Yes						40	0	0	0	0.50	4-Special Rings	.....	.....	.....						124
	.....		100	35.3	29.2	A	Yes						32	0	0	0	0.18	12-Std. Rings	.....	.....	.....	365	8	1			
	.....		120	35.3	30.0	A	Yes						9	0	0	0	0.04	6-Std. Rings	.....	.....	.....	365	8	1			
		42.4	150					B	70	90	N	N					0.17					365	8	1	55	4,550	

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 7)

Plant Number	Engine Designation	Engine Cycle	Injection System (Notes)	Scavenging System (Notes)	Trunk Piston or Crosshead?	Rated B.H.P.	Equivalent K.W.—90% Generating Efficiency	Number of Cylinders	Cylinder Dimensions Bore x Stroke—Inches	Rated R.P.M.	Generator Rating—K.V.A.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Total Gallons of New Lubricating Oil Used	Gals. of New Lub. Oil for Cylinder Lub. Only	Gals. of Unfit Lubricating Oil Discarded	Rated H.P. Hours per Gal. of New Lubricating Oil	Lubricating Oil Treatment (See Notes)	Fuel Oil Used—Gallons	Nature of Fuel Oil Used (See Notes)	Is Fuel Centrifuged?	Gross Output—K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Gross K.W. Hrs. per Gallon of Fuel Oil
315	1	2	M	C	TP	240	161	4	14 x17	257	200	1929	3,320	413	.....	.....	1,928	F	30,750	32°—36° API	No	240,200	582	7.81
	2	2	M	C	TP	120	81	2	14 x17	257	90	1926	4,466	486	.....	.....	1,102	F	23,600		No	156,900	323	6.65
	3	2	M	C	TP	100 *m*	67	2	14 x17	257	90	1917	793	166	.....	.....	478	None	4,680		No	21,800	131	4.66
	Plant					460	309				380			1,065	.....	.....	1,325		59,030			418,900	393	7.10
471	1	2	M	C	TP	150 *m*	101	2	14½x16	327	94	1924	2,330	.....	.....	.....	.....	BC	.....	35° API	No	.....	.....	.....
	2	2	M	C	TP	300 *m*	202	4	14½x16	327	219	1927	2,331	.....	.....	.....	.....		.....		No	.....	.....	.....
	Plant					450	303				313			385	.....	.....	2,725		50,406			428,110	1,112	8.50
695	1	2	M	C	TP	165	111	3	12 x15	360	150	1931	2,536	.....	.....	0	.....	BC	.....	28°—32° API	No	.....	.....	.....
	2	2	M	C	TP	165	111	3	12 x15	360	150	1931	2,609	.....	.....	0	.....	&F	.....		No	.....	.....	.....
	3	2	M	C	TP	120	81	2	14 x17	257	90	1931	4,536	.....	.....	0	.....		.....		No	.....	.....	.....
	Plant					450	303				390			1,513	.....	0	921		48,230			456,140	302	9.45
885	1	2	M	C	TP	180	121	3	14 x17	257	150	1930	6,672	.....	.....	.....	.....	BC,	.....		No	.....	.....	.....
	2	2	M	C	TP	180	121	3	14 x17	257	150	1926	6,670	.....	.....	.....	.....	S	.....		No	.....	.....	.....
	3	2	M	C	TP	75 *m*	50	2	12 x15	300	60	1923	1,068	.....	.....	.....	.....	&F	.....		No	.....	.....	.....
	Plant					435	292				360			1,626	.....	.....	1,526		99,991			778,200	479	7.78
335	1	4	A	..	TP	250	168	4	13¾x17½	257	219	1924	6,090	442	332	110	3,444	F	33,477	28°—30° API	No	342,910	776	10.23
	2	4	A	..	TP	180	121	4	11½x15	300	150	1920	2,670	229	188	41	2,098	&S	15,788	0.5% S	No	148,410	648	9.40
	Plant					430	289				369			671	520	151	2,985		49,265			491,320	732	9.97
1209	1	4	M	..	TP	215	144	3	14½x18	300	188	1930	4,426	.....	.....	.....	.....	BC	34,170	38°—40° API	No	262,700	.....	7.69
	2	4	M	..	TP	215	144	3	14½x18	300	188	1930	4,344	.....	.....	.....	.....	&F	31,844		No	252,790	.....	7.93
	Plant					430	288				376			444	14	.....	4,250		66,014			515,490	1,160	7.80
247	1	2	M	C	TP	240	161	4	14 x17	257	200	1928	4,200	.....	.....	0	.....	S,	.....	34° B	No	258,090	.....	.....
	2	2	M	C	TP	100 *m*	67	2	14 x17	257	75	1920	6,624	.....	.....	0	.....	C	.....		No	227,150	.....	.....
	3	2	M	C	TP	50 *m*	34	1	14 x17	257	40	1915	200	.....	.....	0	.....	&F	.....		No	4,800	.....	.....
	Plant					390	262				315			632	.....	0	2,658		62,109			490,040	775	7.89
187	1	2	M	C	TP	140	94	2	14 x17	300	111	1931	5,840	.....	.....	.....	.....	CC	.....	32°—36° API	No	.....	.....	.....
	2	2	M	C	TP	210	141	3	14 x17	300	170	1931	2,920	.....	.....	.....	.....		.....		No	.....	.....	.....
	Plant					350	235				281			1,285	.....	.....	1,113		33,355			315,879	246	9.47
688	1	2	M	C	TP	50 *m*	34	1	14 x17	257	38	1923	3,156	.....	.....	.....	.....	CC,	.....	36° API	No	.....	.....	.....
	2	2	M	C	TP	120	81	2	14 x17	257	90	1926	5,925	.....	.....	.....	.....	S	.....		No	.....	.....	.....
	3	2	M	C	TP	180	121	3	14 x17	257	150	1927	3,334	.....	.....	.....	.....	&F	.....		No	.....	.....	.....
	Plant					350	236				278			720	.....	100	2,040		59,200			412,000	572	6.96
774	1	4	A	..	TP	165	111	4	11½x15	277	135	1917	452	60	.....	5	1,243	CC	3,200	28°—30° API	No	32,600	543	10.18
	2	4	A	..	TP	180	121	4	11½x15	300	135	1920	406	65	.....	10	1,124		3,400	0°F Pour Test	No	34,222	526	10.06
	Plant					345	232				270			125	.....	15	1,181		6,600			66,822	535	10.13
857	1	2	M	C	TP	180	121	3	14 x17	257	150	1929	5,322	.....	.....	.....	.....	S,C	.....	29° B	No	287,600	.....	.....
	2	2	M	C	TP	150 *m*	101	3	14 x17	257	90	1927	3,618	.....	.....	.....	.....	&F	.....	1.0% S	No	124,536	.....	.....
	Plant					330	222				240			1,216	.....	.....	1,234		60,936			412,136	339	6.76
265	1	2	M	C	TP	60	40	1	14 x17	257	48	1927	2,190	.....	.....	.....	.....	F	.....	36°—40° API	No	.....	.....	.....
	2	2	M	C	TP	120	81	2	14 x17	257	90	1927	3,285	.....	.....	.....	.....		.....		No	.....	.....	.....
	3	2	M	C	TP	140	94	2	14 x17	300	111	1931	3,285	.....	.....	.....	.....		.....		No	.....	.....	.....
	Plant					320	215				249			613	.....	250	1,610		33,505			262,100	428	7.82
1055	1	4	M	..	TP	320	215	4	15 x18½	300	265	1932	8,535	325	3	0	8,400	BC	70,807	16°—18° API	No	521,127	1,603	7.36
27	1	2	M	C	TP	75 *m*	50	2	12 x15	300	.....	1926	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	2	2	M	C	CHa	240	161	4	12½x13½	325	.....	1926	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	Plant					315	211				.....			992	.....	.....	.....		32,820			203,800	205	6.21
737	1	2	M	C	TP	100 *m*	67	2	14 x17	257	75	1929 *ee*	5,642	502	.....	0	1,124	S&	23,532	.....	No	221,788	442	9.43
	2	2	M	C	TP	75 *m*	50	2	12 x15	300	60	1926 *ee*	5,237	380	.....	0	1,034		18,598	.....	No	144,102	379	7.75
	3	2	M	C	TP	50 *m*	34	1	14 x17	257	40	1918	2,406	229	.....	0	525	F	5,347	.....	No	42,776	187	8.00
	4	2	M	C	TP	80 *m*	54	2	12½x13½	300	75 *bb*	1927	101	7	.....	0	1,154		423	.....	No	1,260	180	2.98
	Plant					305	205				250			1,118	.....	0	971		47,900			409,926	367	8.56
382	1	4	M	..	TP	150	101	4	9½x14	300	140	1927	6,630	.....	.....	.....	.....	BC	.....	36°—40° API; 1.0% S;	No	168,500	.....	.....
	2	4	M	..	TP	150	101	4	9½x14	300	135 *o*	1927	2,623	.....	.....	.....	.....		.....	Ash—Nil; CC—Trace	No	85,500	.....	.....
	Plant					300	202				.....			333	.....	.....	4,170		25,026			254,000	763	10.14
858	1	2	M	C	TP	150 *m*	101	3	14 x17	257	125	1929 *ee*	4,071	.....	.....	.....	.....	S,	.....	29° B	No	112,200	.....	.....
	2	2	M	C	TP	120	81	2	14 x17	257	90	1929	4,693	.....	.....	.....	.....	C&F	.....	1.0% S	No	99,400	.....	.....
	Plant					270	182				215			1,161	.....	.....	1,011		39,671			211,600	182	5.33
169	1	4	M	..	TP	240	161	6	10½x13½	327	200	1928	1,809	177	.....	0	2,452	CC	11,207	31° API; 37 SSU @ 100F; 0.61% S; 0.06% CC; Ash—Nil; BS&W—Trace	No	84,090	475	7.51
677	1	2	M	C	TP	240	161	4	14 x17	257	200	1928	1,971	600	.....	.....	788	Cent.	18,000	32°—36° API	No	235,000	392	13.05
591	1	4	A	..	TP	225	151	3	18½x24	150	200	1921	3,265	587	367	180	1,251	None	36,934	32°—36° API; 0.5% S	No	329,143	561	8.91
984	1	4	A	..	TP	225	151	6	9½x13½	400	219	1928	396	35	.....	35	2,545	F	3,170	Delivered by Tank Wagon	No	29,000	828	9.15
318	1	2	M	C	CHa	100	67	2	12½x13½	327	70 *bb*	1922 *dd*	.....	.....	.....	0	.....	S	.....	37° API	No	29,900	.....	.....
	2	2	M	C	TP	50 *m*	34	1	14 x17	257	35 *bb*	1920	.....	.....	.....	0	.....		.....		No	74,000	.....	.....
	3	2	M	C	TP	37½ *m*	25	1	12 x15	300	23 *bb*	1924	.....	.....	.....	0	.....		.....		No	25,830	.....	.....
	4	2	M	C	TP	25 *m*	17	1	10 x13	325	15 *bb*	1918	.....	.....	.....	0	.....		.....		No	14,500	.....	.....
	Plant					212½	143				143			624	.....	0	.....		25,815			144,230	231	5.59
1193	1	2	M	C	TP	200 *m*	134	4	14 x17	257	170	1924	4,944	606	.....	.....	1,630	S&F	50,968	26°—30° B; 0.5% S; 0.1% CC; 0.1% BS&W	No	424,400	700	8.32
733	1	2	M	C	CHa	100	67	2	12½x13½	327	75	1922	35	7	0	0	500	None	.....			1,130	.....	.....
	2	2	M	C	TP	75 *m*	50	2	12 x15	300	60	1914	33	5	0	0	495	F	.....	Delivered by Tank Wagon	No	1,005	.....	.....
	Plant					175	117				135			12	0	0	498		502		No	2,135	502	4.26
646	1	2	M	C	TP	120	81	2	14 x17	257	90	1927	998	63	.....	.....	1,900	BC&F	2,192	38°—40° API	No	8,990	143	4.10
1201	1	2	M	C	TP	x25 *m*	17	1	10 x12	325	25 *bb*	1930 *ee*	.....	.....	.....	.....	.....	F	.....	28°—38° API; Delivered Partly by Tank Wagon	No	18,135	.....	.....
	2	2	M	C	TP	75 *m*	50	1	14 x17	300	53	1932 *ee*	.....	.....	.....	.....	.....		.....		No	40,415	.....	.....
	Plant					100	67				78			431	.....	.....	.....		11,056			58,550	136	5.30

TABLE III—ENGINE DETAILS AND OPERATING INFORMATION (Page 7, Continued)

Plant Number	Loading: Running Engine Capacity Factor (See Text)	Loading: Running Plant Capacity Factor (See Text)	Loading: Peak Load During Reported Period—Gross K.W.	Loading: B.M.E.P. at Rated B.H.P.—Lbs. per Sq. In.	Loading: B.M.E.P. at Peak Load—90% Generating Efficiency	Loading: Piston Cooling (See Notes)	Loading: Are Air Filters Used?	Type Cooling System (See Text)	Average Temperature Incoming Cooling Water—Degs. F.	Average Temperature Outgoing Cooling Water—Degs. F.	Purpose for which Jacket Water Heat is Utilized (See Notes)	Purpose for which Exhaust Heat is Utilized (See Notes)	Maintenance and Repairs: Cost of Engine Regular Upkeep—Dollars: Material	Cost of Engine Regular Upkeep—Dollars: Extra Labor	Cost of Repairs for Engine Accidents—Dollars: Material	Cost of Repairs for Engine Accidents—Dollars: Extra Labor	Total Engine Maintenance in Dollars per Rated B.H.P. per Year	Major Engine Parts Renewed During Reported Period (See Note gg)	No. of Enforced Engine Shutdowns	Total Duration of Enforced Engine Shutdowns—Hours	Total Engine Maintenance Time Not Inc. in Enforced Shutdown Time—Hours	Attendance: No. of Shifts in Period	Attendance: No. of Hours per Shift (See Note ff)	Attendance: No. of Attendants per Shift	Output per Man-Hour—Net K.W. Hrs.	Plant Altitude—Feet Above Sea Level	Plant Number
115	44.9		165	35.3	36.2	A	No																					315
	43.4		84	35.3	36.6	A	No						292	0	0	0	0.63	Valves; Springs; Injection Nozzles, etc.	0	0	14						
	41.0		67	29.4	29.4	A	No							0	0	0			0	0	10						
		44.2	202					A	50	100	N	N									0	365	12	1½			
																						365	12	1	.....	2,565	
471	.....		65	35.6	22.9	A	Yes																				471
	.....		155	35.6	27.3	A	Yes						279	52	36	21	0.86	Pistons; Bearings; Piston Pins and Bushings; etc.	2	3½	15						
		60.8	220					A	85	130	P	N							2	2½	31	259	9	1	181	.....	
695	.....		105	35.6	33.7	A	Yes						125				0.76	Piston Rings & Gaskets	.....	.....	.....						695
	.....		105	35.6	33.7	A	Yes						125				0.76	Piston Rings & Gaskets	.....	.....	.....	365	8	1			
	.....		72	35.3	31.4	A	Yes						1,029				8.58		.....	.....	.....	365	8	1			
		48.8	165					A	60	100	.....	B					2.84	(See gg 17)				365	8	1	50	300	
885	.....		95	35.3	27.7	A	No																				885
	.....		95	35.3	27.7	A	No						1,738				4.00		10	120	600	313	8	1			
	.....		43	29.1	25.0	A	No												4	48	672	313	8	1			
		46.7	233					D	75/95	95/120	N	N							0	0	200	313	8	1			
335	33.5		152	74.1	67.1	A	No															156	8	1	84	1,464	
	45.9		115	73.0	69.4	A	No						129	0	290	0	1.68	1-Lub.Oil P; Exh.&A.V.	1	240	0	365	7	1			335
		36.5	152					A	55	110	S	B	0	0	0	0	0		0	0	0	339	7	1			
																	0.98					339	8	1			
209	41.2		154	63.7	68.1	A	No															352	2	1			
	40.4		147	63.7	65.0	A	No						7	0	0	0	0.02	8-Spray Valve Check bodies; 24-Spray Valve Strainers; Gaskets	0	0	.....	52	8	1			1209
		40.7	154					D	70	110	N	N							0	0	.....	365	4	1			
247	38.2		158	35.3	34.6	A	No															365	10	1	54	1,290	
	51.2		75	29.4	32.9	A	No						88	0	0	0	0.23	Replaced Snap Rings with Double Seal Rings	0	0	12	365	8	1			247
	70.6		25	29.4	21.9	A	No							0	0	0			0	0	12	365	8	1			
		43.4	225					A	55	120	N	N							0	0	0	365	8	1			
																						365	8	1	56	5,000	
187	.....		110	35.3	41.3	A	No						17	0	0	0	0.12	Reconditioned Spray Nozzles	0	0	43	355	10a	1			187
	.....		140	35.3	35.0	A	No						29	0	0	0	0.14		0	0	21	361	4	1			
		32.9	140					B-F	.....	105	N	N					0.13					25	11	1	34	560	
688	.....		.....	29.4	....	A	No																				688
	.....		.....	35.3	....	A	No						125	0	40	0	0.47	1-Piston Pin and Bushing	0	0	12						
	.....		.....	35.3	....	A	No							0		0			1	10	30	365	8	1			
		41.8	160					D-F	.....	.....	N	N							0	0	35	365	8	1			
																						365	8	1	46	870	
774	65.0		100	72.6	65.4	A	No																				774
	69.6		100	73.1	60.4	A	No						0	0	0	0	0	None	0	0	0						
		67.4	100					B-F	90	110	N	N	0	0	653	200	4.74	1-Cyl. Head and Piston	1	700	0						
																	2.47					.....	8	1a	.....	1,100	
857	44.7		.....	35.3	....	A	No																				857
	34.1	40.8	110	29.4	....	A	No	B	.....	.....	L	N	2,261				6.85	Cylinders and Pistons; Crank Pin Bearings	.....	.....	.....	313	8b	1			
																						156	8	1	42	4,010	
265	.....		30	35.3	26.3	A	No																				265
	.....		65	35.3	28.5	A	No						80	0	100	0	0.56	1-Cyl. Head; Rings; Gaskets; etc.	.....	.....	.....	365	8	1			
	.....		100	35.3	37.6	A	No							0		0			.....	.....	.....	365	8	1			
		39.6	115					B-F	90	110	N	N										365	8	1	29	894	
1055	28.4	28.4	180	64.6	54.1	A	Yes	D	125	140	N	N	0	0	0	0	0	None	.....	.....	225	313	8b	1a	.....	1,053	1055
																						156	8	1a			
27	.....		.....	29.1	....	A	....																				27
				45.0	....	A	....																				
		....	75														.....								23	.....	
737	58.7		70	29.4	30.7	A	No																				737
	55.0		55	29.1	32.0	A	No						22	338	0	0	1.18	Piston Rings	....	525						
	52.3		30	29.4	25.9	A	No								0	0			....	504							
	23.1		50	33.8	31.3	A	No								0	0			....	314		365	10	1a			
		56.3	110					B	70	110	N	N							....	10		365	6.8	1a			
																						365	6.8	1a	.....	1,300	
382	25.2		.....	99.7	....	A	Yes																				382
	32.2		.....	99.7	....	A	Yes						4	0	0	0	0.01	Discharge Valve Repairs	0	0	.....						
		27.2	.....					A	.....	.....	M	N							0	0	.....						
																						275	8	1a	.....	875	
858	27.3		.....	35.3	....	A	No																				858
													—1,181—		0	0	4.37	1-Cyl. and Piston	.....	.....	.....						
	26.1	26.7	61	29.4	....	A	No	B	.....	.....	L	N							.....	.....	.....	313	8b	1	20	4,509	
																						156	8	1			
169	28.8	28.8	160	77.6	77.1	A	Yes	B	.....	125	B	N	.....	.....	.....	.....	.....	No Major Parts	0	0	.....	213	8½	1a	.....	20	169
677	74.1	74.1	135	35.3	29.6	A	No	B	70	110	N	N	149	50	0	0	0.83	Piston Rings	2	2	98	105	8	1a	.....	730	677
591	66.8	66.8	160	63.0	66.8	A	No	B	90	130	N	N	52	20	0	0	0.32	P. R. & Spray V. Parts	0	0	.....	117	9.7	1a			
984	48.5	48.5	180	77.6	92.5	A	Yes	D	60	.....	N	N	15	7	0	0	0.10	No Major Parts	0	0	.....	312	8c	1a	.....	578	591
	.....		62	37.2	34.4	A	No															66	....	1a	.....	190	984
	.....		33	29.4	28.8	A	No						18	12	0	0	0.30		0	0	36						318
	.....		21	29.1	24.5	A	No						4	5	0	0	0.18		0	0	10½						
	.....		10	29.8	17.5	A	No						3	6	0	0	0.23		0	0	15	365	10	1			
		....	62					A	50	140	B	N	0	0	0	0	0		0	0	8	365	6	½			
																	0.23					365	8	½	.....	1,275	
1193	64.0	64.0	115	29.4	25.2	A	No	A	55	110	N	N	63	46	0	0	0.54	1-Crank B; Minor Parts	0	0	62	351	8b	1a	.....	.....	1193
																						42	8	1a			
733	48.2		66	37.2	36.7	A	No						0	0	0	0	0	None	0	0	0						733
	60.9		54	29.1	31.4	A	No						0	0	0	0	0	None	0	0	0						
		53.2	120					A	.....	.....	N	N										.....	.....	1	.....	850	
646	11.1	11.1	.....	35.3	....	A	No	D-F	.....	120	....	B	80				0.67	(See gg 18)	1	6	.....	365	12a	1a	.....	600	646
1201	.....		10	32.3	19.0	A	No																				1201
	.....		23	37.8	17.4	A	No						8	0	0	0	0.08	None	0	0	.....	365	12	1a			
		....	23					B	65	108	N	N										365	12	1a	.....	3,400	

NOTES OF TABLE III

gg—Abbreviations—(A—Air); (B—Bearings); (C—Cooler); (Cp—Compressor); (F—Fuel); (H—Head); (L—Liner); (M—Manifold); (P—Piston); (Pu—Pump); (Pl—Plunger); (R—Rings); (Rb—Rebabbitt); (S—Seats); (Sc—Silencer Coils); (T—Thrust); (V—Valves); (W—Water).

*gg*1—6-Piston Rings; 2-Cooling Tubes; 10-lbs. Babbitt; 12-L. P. Valve Blades.

*gg*2—1-Cyl. Liner; 1-Cyl. Head Gasket; 2-Long Studs (Piston cooling); 2-Head Bearings; 55 lbs. Babbitt; 2-H.P. Valve Screws, 2-H.P. Valve Plates, 1-H.P. Cooler.

*gg*3—1-Cyl. Liner; 2-Crank Bearings; 5-Head Bearings; 1-H.P. Cooler (rebuilt); 1-Piston Cooling Tube; 1-H.P. Drain Valve; 3-H.P. Valve Screws; 3-H.P. Valve Plates; 10-Cyl. Head Gaskets; 9-Cyl. Liner Gaskets; 12-Piston Scraper Rings; 24-Piston Snap Rings; 12-Piston Double Seal Rings.

*gg*4—1-H.P. Cooler; 1-Main Bearing; 4-H.P. Valves (Suction); 4-H.P. Valves (Discharge); 6-Cyl. Head Gasket; 6-Cyl. Liner Gasket; 16-Piston Scraper Rings; 32-Piston Snap Rings; 16-Piston Double Seal Rings.

*gg*5—1-Fuel Pump Plunger and Bushing; 2-Fuel Pump Springs; 1-Set Governor Balls and Separator.

*gg*6—Governor Spring Assem.; Exh. Valves; Piston Rings.

*gg*7—Crank Pin Bearing Bolts; Exh. Valve and Springs.

*gg*8—1-Liner; 1-Piston Pin and Bearing; 1-Set Governor Bevel Gears (10-Con. Rod Bearings, material gratis by manufacturer).

*gg*9—4-Cyl. Heads; 1-Piston; 2-Piston Rods; 2-Con. Rods.

*gg*10—1-Crk. Pin Bolt; Gov. Drive Gear; 5-SKF Bearings; 3 Rings.

*gg*11—1-Inj. Valve; 1-Receiver Hd.; 1-Comp. Piston; 1-Fuel Pump Plunger.

*gg*12—Rebabbitt 4 Cr. Pin Brs.; 1-Piston Pin Br.; 1-Cr. Pin; Fuel Pump Drive.

*gg*13—Rebabbitt Cr. Pin Brs. 10 Times; 17-Rings; 1-Comp. Liner and Rings.

*gg*14—Starting Head for 1 Cyl.; 1-Rocker Arm Bushing; 1-Speed Control Shaft.

*gg*15—1-Wrist Pin; 1-Cyl. with Piston Rings; 2-Crank Pin Bearings; Bushings; 2-Wrist Pin Bearings; 1-Wrist Pin; 1-Crank Pin Bearing.

*gg*16—1-Piston; 3-Wrist Pin; 2-W. P. Bushings; Gasket.

*gg*17—1-Cyl. and Piston; 3-W. Pins and Bushings; 1-Conn. Rod Bearings; Piston Rings; Gaskets.

*gg*18—4-Piston Rings; 4-Water Chamber Gaskets; 24-Cyl. Head Gaskets.

LETTERED NOTES

Injection System:
A—Air
M—Mechanical

Scavenging System (2 Stroke Cycle Only):
C—Crank Case Compression
P—Attached Pump or Blower
B—Independently Driven Blower

Lubricating Oil Treatment:
CC—Continuous Centrifuging
BC—Batch Centrifuging
Cent—Centrifuging (Kind Not Stated)
C—Chemically
F—Filtering
S—Settling

Nature of Fuel:
API—American Petroleum Institute
B—Baume
SSU—Seconds Saybolt Universal
SSF—Seconds Saybolt Furol
S—Sulphur
CC—Carbon by Conradson Method
BS&W—Bottom Sediment and Water

Piston Cooling:
A—Air
W—Water
O—Oil

Heat Utilization:
N—Not Utilized
B—To Heat Building
F—To Heat Fuel Oil
L—To Heat Lubricating Oil
T—Thawing Ice Cans
P—To Process
M—Boiler Feed Makeup
W—Hot Water Supply
S—To Swimming Pool

a—Cylinders Closed off from Crankcase
b—No Deduction Made for Motor-Driven Scavenging Blowers
c—60 Minute Peak Load
d—Before beginning of Reported Period (March 1st)
e—Injection Air Compressor Air is Filtered
f—Waited 870 Hours for Parts; This Time Included
g—Entire Labor Cost
h—No Deduction Made for Motor-Generator Excitation
i—Type A in Summer; Type B in Winter
j—Average for 24 Hours
k—Secondary Water also Soft
l—Recirculated; Temperature of Source
m—Semi-Diesel Engine
n—Part Time on Diesel Plant
o—K.W.; Direct Current
p—Changed under NRA; but Man-Hours Unaffected
q—Ice Plant attended also by Reported Staff
r—Summer-Winter Conditions
s—Does Not Include $1,885 for Cast Iron Piston Heads Replacing Steel; Made for Operating Reasons and charged to Capital Account
t—Arranged for Supercharging; Not Supercharged in Practice
u—Estimated
v—Installed in Plant No. 53 in 1926
w—Installed 25th Day of 5th Month of Period
x—Horizontal Engine
y—Supercharged to Give Sea Level Rating at 9,200 feet Altitude
z—Sold for 257 RPM; Operated at 234 RPM
aa—Date of first Installation; Installed in Plant 862 in 1928
bb—Belt Driven Generator
cc—Date Installed in Reported Plant; First installed Date Unknown
dd—Date of First Installation; Date Installed in Reported Plant Unknown
ee—Waited Two Weeks for Parts; This Time Included
ff—(a) indicates 2 similar shifts.
(b) 3 similar shifts; (c) 4 similar shifts.

MSP-57-1

A New Basis for the Rating of Roller-Chain Drives

By G. M. BARTLETT,[1] WEST LAFAYETTE, IND.

Reference tables giving horsepower ratings for roller transmission chains are commonly based upon certain allowable working pressures per square inch of projected bearing area at the chain joints. The allowable working load is assumed to increase in proportion to the projected area of the pins and to decrease with an increase in chain velocity and an increase in the "centrifugal pull" of the chain.

An analysis of the bending action of the links at both driving and driven wheels and the work of friction between pins and bushings yields an algebraic expression for the heat generated per minute, or for the rate of wear in the chain. This is proportional to the rate of elongation of the chain; and this rate of elongation is found to be directly proportional to the horsepower transmitted and inversely proportional to the pitch, the length of the bushing, the number of links in the chain, and the product of the number of teeth in the two sprockets (very nearly). A set of tables is developed based upon chain velocity and the number of teeth in both sprockets. The horsepowers taken from these tables are such as will produce approximately the same rate of elongation in all drives whose design and operation conform to average conditions. Similar tables are computed on the basis of the rpm of the smaller sprocket instead of the chain velocity.

These tables also allow for the extra pull on the chain due to centrifugal force, and for the chain lengths and center distances necessary to keep the rate of elongation within the limits allowed.

An examination of these tables shows that, with certain combinations of sprocket teeth and chain lengths, loads twice as great as those usually given in tables can be transmitted. And these figures seem to be supported by actual practice.

The subject of limiting speeds for sprockets is treated and three sets of limiting conditions are investigated, namely: (*a*) the effect of a single impact between chain rollers and sprocket teeth, (*b*) the energy of impact per tooth per minute, and (*c*) the effect of centrifugal action. A table is compiled giving maximum chain velocities and sprocket speeds, based upon these investigations, for various chains and various numbers of teeth in the sprockets. This table shows the conditions under which certain chains can be operated at velocities above 3000 fpm and other conditions under which the same chains should not be operated at more than 240 fpm.

IN DESIGNING a chain drive it is customary to refer to tables and charts to determine the allowable load carried by a given chain at a given speed, and, after making due allowances for special conditions as dictated by one's judgment, a selection of a suitable chain model is made.

By means of one chart frequently used, the chain is selected on the basis of the horsepower to be transmitted and the revolutions per minute of the smaller sprocket. By means of certain tables in common use, the chain is selected on the basis of chain velocity and either chain pull or horsepower. In another table, the revolutions per minute of the smaller sprocket and its number of teeth are the given data.

While such tables are easy to use and economical to print, it has long been realized that unless all other conditions are satisfactory the tabulated ratings are likely to be erroneous. It is the purpose of this paper to present the results of a mathematical investigation of the conditions that cause chain and sprocket wear, and also to present evidence of the possibility as well as the desirability of compiling a set of tables for the proper rating of roller chains on the basis of all factors that affect the satisfactory performance of the drive.

The number of teeth in the smaller sprocket affects the rapidity of wear of the chain, the uniformity of chain velocity, the efficiency of the drive, and the endurance limit of the chain.

The rapidity of chain elongation is affected by the chain pull, the horsepower, the sprocket speed, the length of the chain bushing, the chain length, the product of the number of teeth in the two sprockets, and the character of the lubrication; but it is independent of the pin diameter unless the unit pressure on the pin is excessive.

The rapidity of sprocket-tooth wear due to roller impact is affected by the pitch, the weight of the chain, the size of the rollers, the revolutions per minute, the material and heat-treatment of the sprockets, the design of the tooth form, and the character of the lubrication.

The centrifugal force of the chain is affected by the weight of the chain and its velocity.

The efficiency of a chain drive is affected by the number of teeth, the chain pull, the pin diameter, the chain weight, the revolutions per minute, the character of the lubrication, and the tangential distance along the tight strand of the chain.

The principal consideration in the design of a chain drive is long service for both chain and sprockets. But such considerations as compactness, quiet action, light weight, or low initial cost may sometimes become paramount. In the present investigation it is assumed that the end to be attained is a certain

[1] Professor of Machine Design, Purdue University, and Consulting Engineer, Diamond Chain & Manufacturing Co., Indianapolis, Ind. Mem. A.S.M.E. Mr. Bartlett was graduated from Amherst College with the degree of B.S. in 1901 and received the M.A. degree in 1905. He was instructor in mathematics and drawing at Case School of Applied Science from 1901 to 1903, and instructor in mechanism and drawing, University of Michigan, from 1903 to 1910. From 1910 until 1920 was chief engineer of the Diamond Chain & Manufacturing Co., for which concern he has been consulting engineer since 1920. He has been professor of machine design at Purdue University since 1927. He is the author of several textbooks and articles in the technical press, and the inventor of many machines and tools.

Contributed by the Machine Shop Practice Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication in a later issue of the Transactions.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

number of hours of service under average conditions before the chain has elongated to such an extent that it is no longer satisfactory.

The first procedure will be the development of formulas from which maximum chain velocities and maximum sprocket speeds may be calculated for various standard chains and for various numbers of sprocket teeth. Having tabulated the results obtained from these formulas, other formulas will be developed to express the rate of chain wear and the allowable horsepower of a chain based upon pitch, pin diameter, bushing length, chain length, number of sprocket teeth, and sprocket speed. From these it will be possible to construct a set of tables for use in the selection of a chain expected to operate for a reasonable number of hours before being worn out.

Cause of Sprocket Wear[2]

The wear of sprocket teeth is mainly due to roller impact. While engagement between a chain and sprocket is taking place, each link, AB, as shown in Fig. 1, turns about the pin A with an

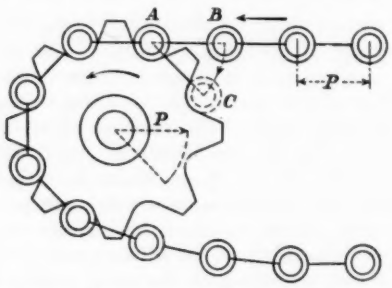


Fig. 1 Engagement Between a Chain and Sprocket; Analysis of Impact Conditions

angular speed equal to that of the sprocket. Hence the linear speed of the roller B along the arc BC, relative to the sprocket, is equal to the linear speed of a point on the sprocket at a distance P from the center. That is, the roller strikes the sprocket with a speed equal to $2\pi Pn/12$ fpm, or $\pi Pn/360$ fps, where P is the pitch and n is the revolutions per minute of the sprocket. The energy of one impact is $wv^2/2g$ ft-lb, in which w is the weight of the impinging body in pounds, v the speed of the roller along the arc BC in feet per second, and g the acceleration of gravity.

It is uncertain how much of the mass of the chain plays a part in the impact, but we may reasonably assume that it is proportional to the weight of a link and nearly equal to it.

Substituting $\pi Pn/360$ for v, and 32.2 for g, the energy of impact in foot-pounds is

$$\frac{wv^2}{2g} = \frac{wP^2n^2}{845617} \quad \ldots\ldots [1]$$

where w is the mean weight of one chain link.

Examination of this expression shows that, in the case of any given chain, the destructive action due to impact between the chain and the sprocket is proportional to the square of the revolutions per minute, and is independent of both the chain velocity and the number of teeth in the sprocket.

The destructive action is also proportional to the weight of the chain link and to the square of the pitch. Hence, if sprocket wear is to be reduced, the revolutions per minute, the pitch, and the chain weight should be as low as is consistent with strength and pin bearing area.

The question might arise as to whether the chain tension F does not affect the amount of the impact. Undoubtedly it does, but to only a slight extent as is deduced from the fact that under conditions of high sprocket speed the wear on the sprocket teeth seems to be as rapid under a light load as under a heavy load. Such as it is, this effect is proportional to F and, very nearly, inversely proportional to the distance from the point of impact to the nearest seated roller. This distance is the pitch P. The expression for the energy in foot-pounds of one impact would then be

$$\frac{P^2n^2}{845617}\left(w + \frac{KF}{P}\right) \quad \ldots\ldots [2]$$

in which K is a very small constant, undetermined.

Effects of Roller Impact

There are four detrimental tendencies which result from excessive roller impact, namely, the breaking of rollers, noise, heating, and the wear of sprocket teeth. Roller breakage has been nearly eliminated by the use of better materials and proper heat treatment. Noise caused by impact has been reduced by the introduction of new designs of sprocket teeth,[3] and can be further reduced by better lubrication and the use of short-pitch multiple-width chains. Heating is caused by wear and is always accompanied by a loss of power. Sprocket-tooth wear is the result of a continuous succession of blows administered by the rollers. This wear may be reduced in the case of the smaller sprockets by case-hardening. But assuming that reasonable attention is given to all other matters the most important consideration is that of keeping the impact per square inch of roller area within practical limits determined by experience. And this means the determination of the maximum sprocket speed to be used with a chain of a given pitch, weight, and roller size.

Maximum Allowable Sprocket Speed

The allowable amount of impact between a roller and a sprocket is proportional to the projected area A of the roller, which is the product of its length by its diameter.

The impact in foot-pounds per square inch of projected roller area is

$$\frac{P^2n^2}{845617A}\left(w + \frac{KF}{P}\right) \quad \ldots\ldots [3]$$

Experience shows that for satisfactory service the energy of impact should not exceed, say, 0.362 ft-lb per sq in. of projected roller area. Hence, putting [3] equal to 0.362 and solving for n, the maximum value of the sprocket speed in revolutions per minute is

$$n_{max} = \sqrt{\frac{845617 \times 0.362 \times A}{P^2\left(w + \frac{KF}{P}\right)}} = \frac{554}{P}\sqrt{\frac{A}{w + \frac{KF}{P}}}$$

where w is the mean weight of one link.

If we disregard the effect of the chain pull F, the constant K being very small, and substitute $W_FP/12$ for w, where W_F is the weight per foot of chain, the maximum value of n will be

$$n_{max} = \frac{1920}{P}\sqrt{\frac{A}{W_FP}} \quad \ldots\ldots [4]$$

By means of this formula a table of sprocket speeds may be made, showing the maximum number of revolutions per minute at which the smaller sprocket should run for each standard

[2] "Roller Chain Drives in Theory and Practice," by G. M. Bartlett, *Product Engineering*, July, 1931, p. 299.

[3] The British tooth form introduced by Hans Renold; the Diamond tooth form designed by the author; and the American tooth form adopted by the A.S.A.

chain when the pitch, the roller diameter, the width, and the weight per foot are known. Substituting $12V/NP$ for n in formula [4] and solving for the maximum value of V, the velocity of the chain in feet per minute, we have

$$V_{max} = 160\ N \sqrt{\frac{A}{W_F P}} \quad \text{[5]}$$

Maximum Revolutions per Minute Based Upon Energy of Impact per Tooth per Minute. Multiplying the energy of one roller impact in Equation [1] by the number of impacts per tooth per minute, and dividing by A, the total energy of impact in foot-pounds per minute per tooth per square inch of projected roller area is

$$\frac{wP^2n^3}{845617\,A}$$

Substituting $W_F P/12$ for w, this becomes

$$\frac{W_F P^2 n^3}{12 \times 845617 \times A}$$

Let K_1 be the maximum value of $W_F P^2 n^3/A$. Then the maximum value of n will be

$$n_{max} = \sqrt[3]{\frac{AK_1}{W_F P^3}} = \frac{K}{P}\sqrt[3]{\frac{A}{W_F}}$$

in which K is $\sqrt[3]{K_1}$.

If K is taken as 2000, the maximum value of n will be

$$n_{max} = \frac{2000}{P}\sqrt{\frac{A}{W_F}} \quad \text{[6]}$$

This is based upon an assumed maximum impact in foot-pounds per tooth per minute, with only moderate lubrication.

Maximum Chain Velocity. To find the maximum chain velocity based upon the rate of sprocket wear per tooth per minute due to roller impact, substitute $12\,V/NP$ for n in formula [6] and solve for V. Then

$$V = 166\ N \sqrt[3]{\frac{A}{W_F}} \quad \text{[7]}$$

This means that, in so far as the rate of tooth wear is concerned, the permissible chain velocity is proportional to the number of teeth in the smaller sprocket for any given chain.

Allowable Chain Velocity as Affected by Centrifugal Force. The formula here developed will be based upon the theory that the maximum allowable centrifugal force for one chain link is proportional to the projected area of the roller.

A body, such as a chain link, rotating about an axis at a speed of n revolutions per minute will exert a centrifugal pull directed radially and equal in pounds to

$$F_R = 0.0000284\ W_L r n^2$$

where F_R = centrifugal force of one chain link, lb
r = virtual radius of sprocket = $PN/2\pi$ in.
W_L = weight of one link = $W_F P/12$, lb
W_F = weight per ft of chain, lb
P = pitch of chain, in.
N = number of teeth in sprocket
n = rpm of sprocket = $6V/\pi r$
V = chain velocity, fpm

Substituting

$PN/2\pi$ for r, $W_F P/12$ for W_L, and $6V/\pi r$ for n

$$F_R = \frac{W_F V^2}{18454\,N}$$

According to this theory the maximum permissible value of this quantity bears a constant ratio to the value of the projected roller area A.

Max $W_F V^2/18454\ N = K_1 A$ where K_1 is a constant, and A is the product of the roller diameter times the chain width. Then

$$\max V = K\sqrt{\frac{NA}{W_F}}$$

This applies to multiple-width chains as well as to single chains.

Assuming 1634 fpm as the maximum velocity of a standard $^1/_2$-in.-pitch chain over a 17-tooth sprocket, the value of the constant K will be 793, and

$$\max V = 793\sqrt{\frac{AN}{W_F}} \quad \text{[8]}$$

where N is the number of teeth in the smaller sprocket.

Maximum Sprocket Speed Based Upon the Effect of Centrifugal Force. Substituting $nNP/12$ in place of V in the centrifugal formula, and solving for n, we have a formula for the maximum revolutions per minute based upon maximum centrifugal action,

$$\max \text{rpm} = \frac{9516}{P}\sqrt{\frac{A}{NW_F}} \quad \text{[9]}$$

Table 1 gives the maximum chain velocities and maximum sprocket speeds for standard roller chains and for various numbers of teeth in the sprockets. They are the minimum calculated results from formulas [4] to [9], inclusive.

Chain Elongation or Stretch

This is caused by wear between the pins and bushings which increases the size of the holes and reduces the size of the pins. This causes an increase in the pitch distance between the roller centers of each pin link, but does not affect the pitch of the roller links.

In Fig. 3*a*, the forward roller B of a pin link BC has just seated itself between the teeth of the driving sprocket. The pin link now begins to turn about the center of the forward pin B until the next roller C has seated itself. The angle of bend is $360/N_1$. During this turn, the pressure between the pin and bushing is equal to the whole load on the chain; but there is no wear between the bushing and the roller, nor between the roller and the sprocket. At this instant, the next link, CD, in Fig. 4*a*, which is a roller link, begins to turn about the rear pin C. During this turn the average pressure between the rear pin and its bushing is less than the load on the chain, and there is definite wear between the bushing and the roller.

As the same links leave the driven sprocket, Figs. 3*b* and 4*b*, the reverse conditions hold. It is evident that if the driving sprocket is smaller than the driven sprocket the forward pin of the pin link will wear more rapidly than the rear pin, and if the driving sprocket is the larger of the two, the rear pin will wear more rapidly.

Maximum Elongation. The stretching that can take place in a chain, before it becomes unfit for further use, varies according to the number of teeth on the larger sprocket, and the character of the drive. In some cases, chains are not discarded until they have stretched to the extent of 3 per cent or more of their original length, and in others they are considered unfit for use before they have stretched half that amount. As the number of teeth increases, a chain with 3 per cent elongation acts at higher points on the teeth, until when the number of teeth has reached

TABLE 1 MAXIMUM REVOLUTIONS PER MINUTE AND MAXIMUM CHAIN VELOCITIES FOR AMERICAN STANDARD ROLLER-CHAINS

Chain No.	Pitch	Roll diam	No. of teeth in smaller sprocket															
			7	9	11	13	14	15	16	17	20	24	28	32	37	48	54	62
35N	3/8	0.200	3043[a]	3043	3043	3043	2935	2835	2744	2662	2455	2240	2073	1940	1805	1585	1495	1395
			667[b]	857	1048	1238	1284	1330	1373	1414	1524	1680	1817	1940	2088	2377	2520	2700
41	1/2	0.306	2654	2654	2654	2654	2654	2654	2580	2500	2306	2106	1950	1823	1695	1488	1407	1310
			772	993	1213	1434	1545	1655	1718	1770	1920	2100	2275	2428	2612	2973	3195	3380
40	1/2	5/16	2517	2517	2517	2517	2517	2460	2381	2310	2130	1945	1800	1685	1567	1375	1298	1210
			732	942	1150	1360	1465	1534	1585	1634	1770	1940	2100	2240	2411	2748	2910	3120
50	5/8	0.400	1900	1900	1900	1900	1900	1900	1885	1830	1688	1542	1426	1335	1241	1089	1028	958
			692	890	1089	1287	1384	1480	1570	1620	1760	1925	2080	2222	2390	2722	2885	3095
60	3/4	15/32	1500	1500	1500	1500	1500	1500	1500	1500	1392	1270	1176	1100	1023	898	848	791
			657	844	1032	1220	1313	1407	1500	1584	1740	1905	2060	2200	2368	2697	2860	3061
80	1	5/8	940	940	940	940	940	940	940	940	940	940	885	828	771	677	638	594
			549	705	862	1019	1097	1173	1254	1330	1567	1880	2032	2195	2360	2690	2850	3055
100	1 1/4	3/4	645	645	645	645	645	645	645	645	645	645	645	639	593	522	492	459
			470	604	738	872	940	1010	1075	1145	1347	1612	1880	2131	2292	2611	2768	2968
120	1 1/2	7/8	520	520	520	520	520	520	520	520	520	520	520	520	515	453	427	398
			455	585	715	845	910	975	1040	1104	1300	1560	1820	2080	2385	2715	2878	3085
140	1 3/4	1	370	370	370	370	370	370	370	370	370	370	370	370	370	370	364	339
			378	486	594	702	756	810	864	917	1080	1296	1510	1728	1996	2590	2851	3060
160	2	1 1/8	325	325	325	325	325	325	325	325	325	325	325	325	325	325	301	281
			379	487	595	704	758	813	867	922	1083	1300	1516	1734	2003	2600	2708	2905
200	2 1/2	1 9/16	240	240	240	240	240	240	240	240	240	240	240	240	240	240	240	229
			350	450	550	650	700	750	800	850	1000	1200	1400	1600	1850	2400	2700	2955

NOTE: [a] Upper figures are maximum revolutions per minute. [b] Lower figures are maximum chain velocities.

about 60 the roller centers are acting as far out as the addendum circle of the sprocket.

FORCES ACTING ON THE LINKS

In Fig. 2, the sprocket is driving counter-clockwise and the tight strand of the chain is moving in the direction XBA. The roller B has just seated itself, and at this instant, its pressure upon the tooth is zero. The first active roller on the sprocket is A, upon which there are three forces acting, namely, AX which is equal to the tension F in the chain, ZA which is normal to the tooth curve at the point of contact with the roller, and YA which acts along the center line of the link AD. By drawing XZ through X parallel to AD, we have the force diagram AXZ in which XZ is the tension in the link AD, and ZA is the pressure between the roller and the tooth. The angle XAZ is the pressure angle. During the time that the link AB has been turning about the pin A the tension in the link has not changed.

If AB is a pin link, the forward pin A supports a bearing pressure equal to AX during the entire bending motion of AB about A.

If AB is a roller link, the pin is attached to the pin link AD and hence, during the bending, the pin pressure has decreased from AX to XZ, and the average pressure can be taken as about one half the sum of AX and XZ.

If N_1 is the number of teeth; ϕ_1 the pressure angle; and F the tension AX in the straight portion of the chain, then the pressure in pounds on the first tooth at A is, from the law of sines,

$$AZ = \frac{F \sin \frac{360}{N_1}}{\sin\left(\frac{360}{N_1} + \phi_1\right)} \quad \ldots\ldots\ldots\ldots [10]$$

The tension in pounds in the link AD is

$$F' = XZ = \frac{F \sin \phi_1}{\sin\left(\frac{360}{N_1} + \phi_1\right)} \quad \ldots\ldots\ldots [11]$$

The average pin pressure on the *rear* pin of a pin link during the flexing of a link at the driver is, in pounds, very nearly

$$\frac{F}{4}\left[1 + \frac{\sin \phi_1 + 2 \sin\left(\frac{180}{N_1} + \phi_1\right)}{\sin\left(\frac{360}{N_1} + \phi_1\right)}\right] \quad \ldots\ldots [12]$$

By laying out the force triangle successively for each link, the tooth pressures and link tensions can be determined for any link.

No significant rolling action takes place between the chain rollers and the sprocket teeth. Each bushing, however, turns within its roller an amount equal to $360/N_1$ every time it passes around a sprocket with N_1 teeth. The forward bushing on a roller link turns in the roller during approach and the rear bushing turns during departure. Thus the hole in the roller

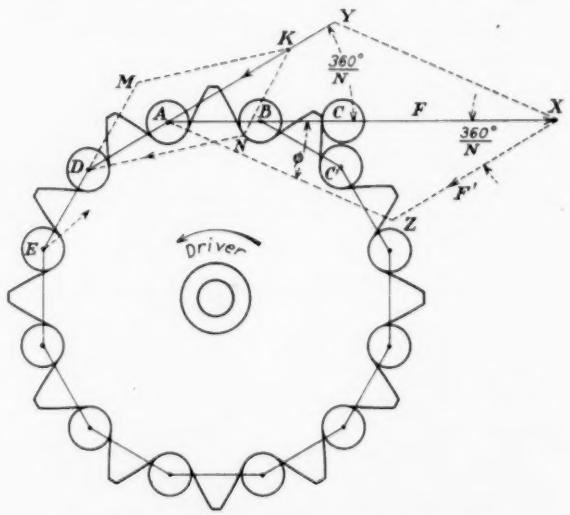


FIG. 2 FORCES ACTING ON THE LINKS

receives uniform wear in all parts throughout the life of the chain. The rate at which the rollers turn over their bushings in turns per minute is theoretically equal to $n_1/N_1 + n_2/N_2$, where n_1 and n_2 are the sprocket speeds, and N_1 and N_2 the numbers of teeth on the sprockets.

Rate of Wear Between Pins and Bushings. As chain elongation or "stretch" is caused by wear between the pins and the bushings in which they turn, and since a reduction of this elongation means a longer period of service for the chain, a careful study of what takes place when a chain approaches and leaves each sprocket is important.

In Fig. 3 are shown a driving sprocket at a, and a driven sprocket at b, and a chain in which a link on each sprocket is about to flex. Fig. 4 represents the same sprockets and chain as in Fig. 3, but advanced one pitch.

In Fig. 3a, the link AB is a roller link and BC is a pin link.

B will be called the forward pin and C the rear pin of the pin link. While the driving sprocket is turning through $1/N_1$ revolutions, C moves to the position B and seats itself between the next pair of teeth. In doing so, the link BC turns through an angle of $360/N_1$ with respect to AB, and the pin B turns within its bushing the same amount. During this rotation, the bearing pressure between pin and bushing has been equal to the working load F of the chain.

If d is the pin diameter, F the chain pull in pounds, and f the coefficient of friction, one bend of a pin link at the driver produces a frictional loss, in foot-pounds, of

$$\frac{\pi dfF}{12\,N_1} \qquad [13]$$

During this action, the roller has not turned between the teeth of the sprocket, nor has the bushing turned within its roller. This pin comes into action on the driving sprocket $n_1N_1/2L$ times per minute, where L is the total number of links in the chain, and n_1 is the revolutions per minute of the driving sprocket. Hence, the frictional loss in foot-pounds per minute on each forward pin over the driver is

$$\frac{\pi dfFn_1}{24\,L} \qquad [14]$$

Now this same pin B which plays a part in the bending of a pin link over the driver, plays its part in the bending of a roller link when leaving the driven sprocket, Fig. 3*b*. In this case the pressure between pin and bushing at B is F lb at the end of the motion; but at the beginning of the turn it is less than F lb as given in formula [11]. The average pressure in pounds during the turn is very nearly as in formula [12], as follows:

$$\text{Av. pressure} = \frac{F}{4}\left[1 + \frac{\sin\phi_2 + 2\sin\left(\frac{180}{N_2} + \phi_2\right)}{\sin\left(\frac{360}{N_2} + \phi_2\right)}\right] \qquad [12]$$

where ϕ_2 is the pressure angle corresponding to N_2 teeth. Multiplying by $\frac{\pi dfn_2}{24\,L}$ as in [14], the frictional loss in foot-pounds per minute on each forward pin over the driven sprocket is

$$\frac{\pi dfFn_2}{96\,L}\left[1 + \frac{\sin\phi_2 + 2\sin\left(\frac{180}{N_2} + \phi_2\right)}{\sin\left(\frac{360}{N_2} + \phi_2\right)}\right] \qquad [15]$$

Assuming the average pressure angle during the life of the chain to be (26 deg — 92 deg/N_2),[4] substituting this for ϕ_2, and adding formulas [14] and [15], we have for the frictional loss of each forward pin over both sprockets, in foot-pounds per minute

$$\frac{\pi dfFn_1}{96\,L}\left[4 + \frac{N_1}{N_2}\left(1 + \frac{\sin\left(26 - \frac{92}{N_2}\right) + 2\sin\left(26 + \frac{88}{N_2}\right)}{\sin\left(26 + \frac{268}{N_2}\right)}\right)\right] \qquad [16]$$

By a similar process each rear pin of a pin link can be shown to undergo a rate of wear, in foot-pounds per minute over both sprockets, equal to

$$\frac{\pi dfFn_2}{96\,L}\left[4 + \frac{N_2}{N_1}\left(1 + \frac{\sin\left(26 - \frac{92}{N_1}\right) + 2\sin\left(26 + \frac{88}{N_1}\right)}{\sin\left(26 + \frac{268}{N_1}\right)}\right)\right] \qquad [17]$$

in which n_2 can be replaced by N_1n_1/N_2, since $n_2/n_1 = N_1/N_2$.

The projected area of the pin is dB, where B is the effective length of the bushing. If the sum of Equations [16] and [17] is divided by 2, we shall have the average foot-pounds per minute per pin lost in friction. If this is then divided by dB the result

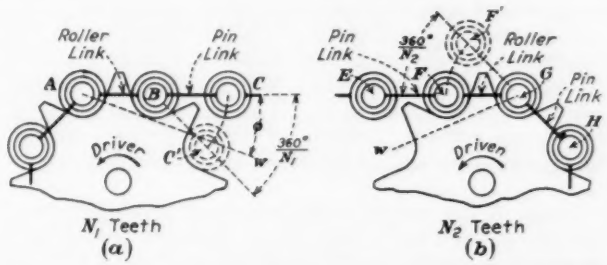


FIG. 3 ANALYSIS OF WEARING CONDITIONS BETWEEN ROLLERS, BUSHINGS, AND PINS

(*a*, Driving sprocket, *b* driven sprocket, and a chain in which a link on each sprocket is about to flex.)

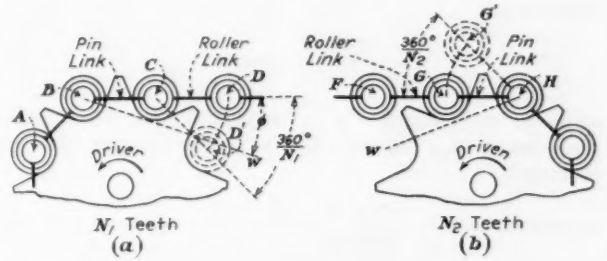


FIG. 4 THIS REPRESENTS THE SAME SPROCKET AND CHAIN AS IN FIG. 3 BUT ADVANCED ONE PITCH

will be the number of foot-pounds per minute per square inch of pin area lost in friction, namely,

$$\frac{\pi fFn_1}{192\,LB}\Big([16] + N_1/N_2\,[17]\Big) \qquad [18]$$

where [16] and [17] represent the parenthetic portion of formulas [16] and [17], respectively.

For purposes of study, we must assume that the chain length bears a constant relationship to the size of the sprockets. As the minimum number of pitches in a practical drive is about equal to the sum of the teeth in the two wheels, we may put $N_1 + N_2$ in place of L in formula [18]. Also, since the expression for the horsepower of a chain drive is $H = \frac{N_1PFn_1}{396000}$, we can substitute $\frac{396000\,H}{N_1P}$ for Fn_1. Then the work of friction in foot-pounds per minute per square inch of projected pin area is

$$\frac{6480\,fH}{BP}(M) \qquad [19]$$

where (M) is equal to the expression

[4] See A.S.A. Standard, B29a—1930.

$$\left[\frac{5+\sin\left(26-\frac{92}{N_1}\right)+2\sin\left(26+\frac{88}{N_1}\right)}{N_1(N_1+N_2)\sin\left(26+\frac{268}{N_1}\right)}+\frac{5+\sin\left(26-\frac{92}{N_2}\right)+2\sin\left(26+\frac{88}{N_2}\right)}{N_2(N_1+N_2)\sin\left(26+\frac{268}{N_2}\right)}\right]$$

TABLE 2 VALUES OF M AND $7.35/N_1N_2$

Teeth on larger sprocket	Teeth on smaller sprocket: 12	16	20	30	60
20	0.02915[a]	0.02220	0.01792	.....	.....
	0.03062[b]	0.02293	0.01836	.....	.....
30	0.01953	0.01492	0.01209	0.00819	.....
	0.02043	0.01533	0.01224	0.00817	.....
40	0.01465	0.01121	0.00910	0.00619	.....
	0.01533	0.01147	0.00918	0.00612	.....
60	0.00974	0.00747	0.00607	0.00414	0.00212
	0.01021	0.00767	0.00612	0.00408	0.00204
100	0.00582	0.00447	0.00363	0.00248	0.00128
	0.00612	0.00459	0.00367	0.00245	0.00122

[a] Upper figures are values of M. [b] Lower figures are values of $7.35/N_1N_2$.

Values of this quantity, M, for various combinations of N_1 and N_2 are found to be very close to those obtained from the simple expression $7.35/N_1N_2$. In the body of Table 2, the upper figures are values of M and the lower figures are values of $7.35/N_1N_2$. It is evident that without serious error we can put this shorter expression in place of M and simplify formula [19] thus:

Work of friction in foot-pounds per minute per square inch of pin area is

$$\frac{47628fH}{BPN_1N_2}\text{ (nearly)}\qquad[20]$$

where the chain length $= N_1 + N_2$ pitches. But for any other chain length, L_p, the work lost in friction between pins and bushings is

$$\frac{47628fH}{BPN_1N_2}\times\frac{N_1+N_2}{L_p}\qquad[21]$$

where L_p is the actual chain length in pitches.

Assuming that chain elongation is proportional to the unit wear between pins and bushings, and that unit wear is proportional to the work of friction per square inch of projected pin area, the value of the expression in any one of formulas [18], [19], [20], or [21] is proportional to the rate of elongation of the chain due to wear between pins and bushings.

An inspection of formulas [18], [20], and [21] shows that the rapidity of chain elongation varies directly as the chain tension, the sprocket speed, the coefficient of friction, and the horsepower. It varies inversely as the chain length, the bushing length, the pitch, and, very nearly, the product of the numbers of teeth in the two sprockets, assuming that $L = N_1 + N_2$. It is independent of the pin diameter providing that the diameter is sufficient for strength and for proper bearing area. It also appears that if N_1 and N_2 are interchanged in the formula, the value of the expression is not changed, and so we can consider N_1 as the number of teeth on the smaller sprocket regardless of whether it is the driving or the driven wheel.

Maximum Horsepower. In order to derive a formula for use in the selection of a chain for a given horsepower, sprocket speed, and number of teeth on driving and driven wheels, let it be assumed tentatively that the chain length L will be equal to the minimum of $N_1 + N_2$ links. In the average drive this will be somewhat greater, but the difference will be on the side of safety.

Let it be further assumed that the maximum rate of wear (expressed in terms of the work of friction) should not exceed, say, 44 ft-lb per min per sq in. of projected pin area, and that the coefficient of friction f is 0.04. Then, 44 will be the maximum value of

$$\frac{47628\times 0.04\times H}{BPN_1N_2}$$

Whence, max hp $= 0.023\ BPN_1N_2$ .....[22]

It should be noted that the constant 0.023 is arbitrary and can be increased or decreased depending upon what may be considered a reasonable rate of wear.

For a chain whose length is other than $N_1 + N_2$ links, the horsepower determined from [22] should be multiplied by $L_p/N_1 + N_2$, where L_p is the number of links or pitches in the chain.

Maximum Unit Bearing Pressure on Pins. From formula [22] it might appear that the amount of power that a given chain drive can transmit satisfactorily depends only upon the pitch, the bushing length, the number of links in the chain, and the product of the number of teeth in the two sprockets. This would mean that the horsepower is independent of the pin diameter and of the chain velocity. If, however, the chain velocity is too low, the working pull on the chain may become very high and produce an excessive bearing pressure on the pins. The pressure per square inch of projected pin area should not exceed 5000 to 6000 lb for standard roller transmission chains.

The unit pressure on a pin is equal to $\frac{33000H}{VBd}$ where Bd is the projected pin area, and if this is put equal to 5000 lb as a maximum, we have

$$\text{max hp} = 0.152VBd.\qquad[23]$$

It now appears that the maximum horsepower of a drive whose chain contains $N_1 + N_2$ links is the lesser of the two values obtained from formulas [22] and [23].

Centrifugal Pull in Chain. The extra pull in the chain in pounds due to centrifugal force is

$$F_c = \frac{W_FV^2}{115920}\qquad[24]$$

Where W_F is the weight in pounds per foot of chain, and V is the chain velocity in feet per minute. This formula is derived in the following way. The well-known expression for the tension induced in a belt by centrifugal force is Wv^2/g lb; where W is the weight of a strip of belt 1-ft long and 1 sq in. cross-section, v is the belt velocity in feet per second, and g is the force of gravity. If V is the velocity in feet per minute,

$$v^2 = \left(\frac{V}{60}\right)^2 \text{ and } F_c = \frac{W_FV^2}{3600\times 32.2} = \frac{W_FV^2}{115920}$$

This extra chain pull imposes an added horsepower load equivalent to:

$$\frac{VF_c}{33000} = \frac{V}{33000}\times\frac{W_FV^2}{115920}$$

Hence, $$\text{centrifugal hp} = \frac{W_FV^3}{3{,}825{,}360{,}000}\qquad[25]$$

This amount of power is not actually added to that required by the chain; but its effect on the rate of wear of the chain is the same as if that amount of extra power actually were being transmitted. The only additional demand upon the source of power

is that required to overcome the extra friction due to such an increase in the load; and this is comparatively small.

If formulas [22] and [23] are to express the maximum power that can be transmitted by a given drive without producing excessive wear or excessive pin pressure, this centrifugal horsepower must be deducted from that given by these formulas. We now have, for a drive using a chain containing $N_1 + N_2$ links, the lesser of the values found from the two formulas:

$$\text{max hp} = 0.023BPN_1N_2 - \frac{W_F V^3}{3{,}825{,}360{,}000} \quad \ldots\ldots [26]$$

$$\text{max hp} = 0.152VBd - \frac{W_F V^3}{3{,}825{,}360{,}000} \quad \ldots\ldots [27]$$

Horsepower Tables Based on Chain Velocity and the Product $N_1 \times N_2$

The horsepower values yielded by the two formulas, [26] and [27], are equal when $V = \dfrac{0.1513PN_1N_2}{d}$.

Table 3 shows horsepower values for various values of N_1N_2 and various chain velocities for 1-in.-pitch standard roller chain

$$\delta = 0.6P\left(\frac{1}{N_1^2} + \frac{1}{N_2^2}\right)$$

And if the tangential distance is an exact multiple of the pitch, the elastic stretch is a minimum and equal, in inches, to

$$\delta = D_1 \sin\theta - D_2 \sin\frac{N_1\theta}{N_2}$$

where $\sin\theta = \sqrt{1 - \dfrac{N_1}{\pi D_1}}$, and D_1 and D_2 are the pitch diameters of the smaller and larger sprockets, respectively.

A part of this stretch may be absorbed by the films of lubricant between the pins and bushings if the working load is not too great, and a part is taken up by an elastic stretch in the chain itself. This induces stresses in the chain over and above those due to the working load, and these stresses are inversely proportional to the length of the chain span and directly proportional to the elastic stretch. Hence, a short center distance should not be combined with a low number of teeth on the sprockets. A safe rule for the minimum center distance is 20 pitches. This insures that the minimum span of the chain from

TABLE 3 HORSEPOWERS FOR STANDARD ROLLER CHAIN NUMBER 80[a]

$N_1 \times N_2$	Chain velocity in fpm														
	48	60	78	97	121	155	194	242	310	388	485	620	775	1200	1600
100	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.00	1.99	1.96	1.91	1.81	1.27	..
125	2.01	2.51	2.51	2.51	2.51	2.51	2.51	2.51	2.50	2.49	2.46	2.41	2.31	1.77	..
160	2.01	2.51	3.22	3.22	3.22	3.22	3.22	3.22	3.21	3.20	3.17	3.12	3.02	2.48	1.48
200	2.01	2.51	3.22	4.02	4.02	4.02	4.02	4.02	4.01	4.00	3.97	3.92	3.82	3.28	2.20
250	2.01	2.51	3.22	4.02	5.03	5.03	5.03	5.03	5.02	5.01	4.98	4.93	4.83	4.29	3.29
320	2.01	2.51	3.22	4.02	5.03	6.44	6.44	6.44	6.43	6.42	6.39	6.34	6.24	5.70	4.70
400	2.01	2.51	3.22	4.02	5.03	6.44	8.04	8.04	8.03	8.02	7.99	7.94	7.84	7.30	6.30
500	2.01	2.51	3.22	4.02	5.03	6.44	8.04	10.0	10.0	10.0	10.0	9.95	9.85	9.31	8.31
640	2.01	2.51	3.22	4.02	5.03	6.44	8.04	10.0	12.8	12.8	12.8	12.8	12.7	12.1	11.1
800	2.01	2.51	3.22	4.02	5.03	6.44	8.04	10.0	12.8	16.0	16.0	16.0	15.9	15.3	14.3
1000	2.01	2.51	3.22	4.02	5.03	6.44	8.04	10.0	12.8	16.0	20.0	20.0	19.9	19.4	18.4
1280	2.01	2.51	3.22	4.02	5.03	6.44	8.04	10.0	12.8	16.0	20.0	25.6	25.5	25.0	24.0
1600	2.01	2.51	3.22	4.02	5.03	6.44	8.04	10.0	12.8	16.0	20.0	25.6	31.9	31.4	30.4

[a] P = 1 in., B = 0.875 in., d = 0.312 in., roller diameter = 0.625 in., and W_F = 1.63 lb.

No. 80. Where the chain length is other than $N_1 + N_2$ pitches, the tabulated horsepowers *above* the heavy line should be multiplied by the actual chain length in pitches and divided by $N_1 + N_2$, but they should not be multiplied by more than 1.5 nor should they in any case exceed those shown in the same column below the heavy line. For chain velocities below 100 fpm, the horsepower is proportional to the velocity.

Values of N_1N_2 lying between 400 and 1000 will probably comprise 80 per cent or more of all chain drives in use. Any value of N_1N_2, such as 720, may stand for a number of combinations, such as 8×90; 15×48; 18×40; etc.

Not all of these possible combinations will make satisfactory drives. For example, 10 teeth combined with a chain velocity of 1000 fpm or 16 teeth combined with a velocity of 1500 fpm would not be good practice because these velocities are beyond those given as the maximum in Table 1.

Elastic Stretch in the Chain. Due to the polygon effect in sprocket action, the angular speed ratio between the two wheels tends to vary between a maximum and a minimum during the passage of each tooth. But due to the inertia of the wheels and their attached masses this variation is resisted, and the result is that in order to compensate for the polygon action there must be a rapid succession of changes in the length of that portion of the chain which spans the distance between the two sprockets. This elastic stretch is greatest when the tangential distance between the sprockets is an odd multiple of half the pitch; in which case it is very nearly equal, in inches, to

the driving to the driven sprocket will generally be between 17 and 20 pitches.

Horsepower Tables Based on Sprocket Speed

In most cases of chain drive calculations, the known quantity is the revolutions per minute of the smaller sprocket rather than the chain velocity; and so a more convenient and informative table for reference would be one in which the sprocket speeds are read at the top of the table and various combinations of teeth are listed in the left-hand column, as shown in Table 4 which is calculated for the same chain (No. 80) as in Table 3.

Tables of this sort should be much more extensive than Table 4, but their greater usefulness offsets the disadvantage of cumbersomeness.

For the computation of such tables, the necessary data are the pitch P, the bushing length B, the pin diameter d, and the weight per foot W_F. Then, putting V equal to $PN_1n_1/12$ in formulas [26] and [27], we have for the horsepower the lesser of the two values,

$$\text{hp} = 0.023PBN_1N_2 - \frac{W_F P^3 N_1^3 n_1^3}{6{,}609{,}081{,}600{,}000} \quad \ldots\ldots [28]$$

$$\text{hp} = 0.0127PBdN_1n_1 - \frac{W_F P^3 N_1^3 n_1^3}{6{,}609{,}081{,}600{,}000} \quad \ldots\ldots [29]$$

This assumes a chain length of $N_1 + N_2$. But a more reasonable chain length is one that will make the center distance be-

TABLE 4 HORSEPOWERS FOR STANDARD ROLLER-CHAIN NUMBER 80[a]

N_1	N_2	L_P	Cent. dist. (pitches)	Speed of the smaller sprocket, rpm: 162	183	202	225	252	281	313	348	386	433	487	547	940
15	15	56	20.50	8.42	8.42	8.41	8.41	8.41	8.40	8.39	8.38	8.37	8.32	8.32	8.28	7.72
15	18	58	20.74	8.42	9.52	9.51	9.51	9.51	9.50	9.39	9.38	9.37	9.32	9.32	9.28	8.72
15	21	60	20.97	8.42	9.52	10.5	10.5	10.5	10.5	10.5	10.5	10.5	10.4	10.4	10.4	9.81
15	25	62	20.93	8.42	9.52	10.5	11.7	11.7	11.7	11.7	11.7	11.7	11.6	11.6	11.6	11.0
15	30	66	21.61	8.42	9.52	10.5	11.7	13.1	13.1	13.1	13.1	13.1	13.0	13.0	13.0	12.4
15	36	68	20.97	8.42	9.52	10.5	11.7	13.1	14.6	14.6	14.6	14.6	14.5	14.5	14.5	13.9
15	43	72	21.01	8.42	9.52	10.5	11.7	13.1	14.6	16.2	16.2	16.2	16.2	16.2	16.1	15.6
15	51	78	21.73	8.42	9.52	10.5	11.7	13.1	14.6	16.2	18.1	18.1	18.0	18.0	18.0	17.4
15	60	84	22.06	8.42	9.52	10.5	11.7	13.1	14.6	16.2	18.1	20.8	20.8	20.8	20.7	20.2
15	72	90	21.28	8.42	9.52	10.5	11.7	13.1	14.6	16.2	18.1	20.8	22.4	22.4	22.3	21.8
15	86	102	22.91	8.42	9.52	10.5	11.7	13.1	14.6	16.2	18.1	20.8	22.4	25.2	25.2	24.6
15	102	116	24.82	8.42	9.52	10.5	11.7	13.1	14.6	16.2	18.1	20.8	22.4	25.2	28.3	30.1
19	19	60	20.50	10.8	11.6	11.6	11.6	11.6	11.6	11.6	11.5	11.5	11.4	11.4	11.3	10.2
19	21	62	20.99	10.8	12.1	12.4	12.4	12.4	12.4	12.4	12.4	12.3	12.3	12.2	12.1	11.0
19	25	64	20.97	10.8	12.1	13.3	13.9	13.9	13.9	13.9	13.9	13.8	13.8	13.7	13.7	12.5
19	30	68	21.67	10.8	12.1	13.3	14.8	15.7	15.7	15.7	15.7	15.7	15.6	15.6	15.5	14.4
19	36	70	21.07	10.8	12.1	13.3	14.8	16.6	17.7	17.7	17.7	17.7	17.6	17.6	17.5	16.4
19	43	74	21.15	10.8	12.1	13.3	14.8	16.6	18.5	19.9	19.9	19.8	19.8	19.7	19.7	18.5
19	51	80	21.89	10.8	12.1	13.3	14.8	16.6	18.5	20.6	22.2	22.1	22.1	22.0	22.0	20.8
19	60	86	22.27	10.8	12.1	13.3	14.8	16.6	18.5	20.6	22.9	24.9	24.8	24.8	24.7	23.6
19	72	92	21.57	10.8	12.1	13.3	14.8	16.6	18.5	20.6	22.9	25.3	27.8	27.8	27.7	26.6
19	86	104	23.26	10.8	12.1	13.3	14.8	16.6	18.5	20.6	22.9	25.3	28.4	31.4	31.3	30.2
19	102	120	26.38	10.8	12.1	13.3	14.8	16.6	18.5	20.6	22.9	25.3	28.4	31.4	35.7	37.6

[a] P = 1 in., B = 0.875, d = 0.312, W_F = 1.63 lb., and maximum speed = 940 rpm.

tween the sprockets approximately 20 pitches. This will be accomplished if the chain length is made equal to 38 + 0.6 $(N_1 + N_2)$ pitches for all values of $N_1 + N_2$ less than 95, but equal to $N_1 + N_2$ when the sum exceeds 95.

Formula [28], then, can be used when $N_1 + N_2$ is over 95, but for values of 95 and less, the first term of [28] must be multiplied by the quotient of 38 + 0.6$(N_1 + N_2)$ divided by $N_1 + N_2$ and the formula will then be

$$\text{hp} = 0.0138\, PBN_1N_2\left(\frac{63.33}{N_1 + N_2} + 1\right) - \frac{W_F P^3 N_1^3 n_1^2}{6{,}609{,}081{,}600{,}000} \quad \ldots\ldots [30]$$

Table 4 gives horsepower values for drives using 15 teeth on the smaller wheel, and also for drives using 19 teeth. Values of N_2 are chosen for convenience in an approximate geometric series. Values for the revolutions per minute may be arbitrarily chosen, but in the first half of this table they were computed from the formula

$$n_1 = \frac{1.0866\, N_2}{d}\left(\frac{63.33}{N_1 + N_2} + 1\right)$$

for the several values of N_2, so that the horsepower found from formula [30], for any given value of N_2, would be equal to that found from formula [29] using the corresponding value of n_1. This determines the location of the heavy lines, below which are the horsepower values based upon a maximum unit pin pressure of 5000 lb and calculated from [29]. All values *above* the heavy line are calculated from formula [30], except where $N_1 + N_2$ is greater than 95, in which case, formula [28] is used. For convenience in this case the same values of revolutions per minute have been tabulated for 19 teeth as for 17 teeth.

In tables previously published it has been customary to reduce the allowable pin pressure as the speeds increase. It can be shown that the actual rubbing distance in feet per minute between any pin and bushing acting over both sprockets is

$$\frac{\pi d n_1 (N_1 + N_2)}{12\, N_2 L_P}$$

This is the expression for the average rubbing velocity. It gives 3.06 fpm for No. 80 chain with 10 and 10 teeth at 940 rpm; and it gives 0.96 fpm for the same chain with 16 and 100 teeth at 940 rpm. As we have numerous examples of punch-press pressures ranging from 5000 to 7000 lb at more than 100 fpm rubbing speed, it would seem that there is no theory supporting a gradual reduction of unit bearing pressures in chains where the maximum rubbing speed is as low as 3 or 4 fpm, and where the periods of rest are relatively much greater than in punch presses.

Radiation Intensities and Heat Transfer by Radiation in Boiler Furnaces

By HUBER O. CROFT[1] and C. F. SCHMARJE,[2] IOWA CITY, IOWA

This paper describes the development of an absorption calorimeter with a fused-quartz window and discusses its use in connection with a series of boiler tests. The results of these tests are compared with the Hudson-Orrok formula and the Wohlenberg method of computing radiation heat transfer.

A tentative empirical equation is given for computing radiation intensities, taking into account the dirtiness of the cold surfaces. Tentative formulas are also given for computing the average heat transfer by radiation in boiler furnaces.

THERE have been several methods suggested for the computation of the radiant-heat exchange occurring in boiler furnaces. Of these, probably the best known are the Hudson-Orrok formula, and the Wohlenberg method.

The purpose of this investigation is, first, to secure experimental data which will serve as a basis for comparing the results obtained by the two methods of computation mentioned above, and, second, to arrive at a simple empirical equation to fit these data.

The method consisted of determining, experimentally, the intensities of radiation at various locations inside a boiler furnace during a series of boiler tests.[3] For this purpose there has been developed a quartz-window radiation calorimeter.

Description of the Radiation Calorimeter and Its Calibration

Description of the Calorimeter. The principle of operation of the radiation calorimeter developed for this investigation is to absorb in a stream of water, the radiant energy passing through a fused-quartz window. The energy absorbed by the water is indicated by the increase in the temperature of a measured quantity of flow.

The calorimeter is shown in Fig. 1. The pyralin-walled, water chamber, *A*, is surrounded by an air chamber, except at the front end, which is closed by means of the fused-quartz window *W*, held in place by an aluminum tube *T* against a bakelite partition *D*. This partition in addition to being the support for the inner assembly, consisting of the absorption chamber and its surrounding air chamber (shown in detail in Fig. 1-*A*), is also the boundary between the front cooling chamber *F* and the rear cooling chamber *R*. Into the rear of the outer copper shell *S* is screwed an iron water-cooled pipe *P*, through which are led the water tubes to the absorption and cooling chambers, *A*, *F*, and *R*, together with the thermocouple leads.

The thermocouples for the main chamber are made of No. 22 copper and constantan wires with the junctions formed by soldering the two wires together. These couples are insulated with a thin layer of pyralin and sealed into the inlet and outlet pyralin water tubes of the absorption-chamber *A*. These couples were calibrated in air, and in flowing water before installation, and the calibration was checked after assembly in the tubes.

The thermocouple leads, insulated with silk and two coats of varnish, are carried out through a glass tube to the rear of the supporting pipe, *P*.

Operation of the Calorimeter. In operation, the three chambers, *A*, *F*, and *R* were supplied with water from a constant level tank. The flow through the absorption chamber was regulated to about 30 to 35 lb per hr, which should give turbulent flow about the outlet thermocouple.

The electromotive forces of the couples were determined against a third copper-constantan junction placed in a thermos jar of ice water, by means of a Leeds and Northrup precision potentiometer; the electromotive forces were determined for both the inlet and the outlet thermocouple about 15 separate times in the course of observations of approximately six minutes duration, readings being made to 0.005 millivolt, corresponding to a temperature interval of 0.2 F.

The water discharged from the absorption chamber was collected in a calibrated glass flask, the time of collection being noted with a stopwatch. The probable error in the water-rate determinations is less than one-half of 1 per cent.

Calibration of the Calorimeter. The calibration of the calorimeter consisted in comparing the radiation received by the calorimeter from an electrically heated carbofrax plate, with that received by a special 'black-body" absorber in the same location.

The black-body absorber was similar to those which have been used by H. C. Hottel and J. D. Keller,[4] and by E. Schmidt.[5] It consists of a cylindrical cavity *C*, Fig. 2, with blackened walls which are surrounded by a water chamber *W*. Pyralin-insulated, copper-constantan thermocouples are sealed in the inlet and outlet tubes of the water chamber. The outer walls of the water chamber, *W*, are thermally insulated with 85 per cent magnesia. Water-cooled shields *F*, and *O*, protect the absorption chamber

---

[1] Professor and Head of Department of Mechanical Engineering, University of Iowa, Iowa City, Iowa. Mem. A.S.M.E. Professor Croft received the B.S. degree from the University of Colorado in 1918, and M.S. from the University of Illinois in 1925. He served on the faculties of the University of Illinois and Stanford before assuming his present responsibilities. Professor Croft represented the Society at a joint meeting with the Mexican Society of Engineers in Mexico during the summer of 1934.

[2] Graduate Student in Mechanical Engineering, University of Iowa. Jun. Mem. A.S.M.E. Mr. Schmarje received the degree of B.S. in M.E. from the University of Iowa in 1932. Following his graduation he was research assistant in mechanical engineering, engaged in work relating to the utilization of Iowa coals. He received the M.S. degree in 1934. At present he is pursuing post-graduate work in an experimental investigation of heat transfer by radiation in boiler furnaces.

[3] These tests were conducted in cooperation with the Committee on Utilization of Iowa Coal. A report of these investigations will be made in the near future.

Contributed by the Fuels Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication in a later issue of Transactions.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] "Effects of Reradiation on Heat Transmission in Furnaces and Through Openings," by H. C. Hottel and J. D. Keller, Trans. A.S.M.E., vol. 55, 1933, paper IS-55-6-39.

[5] "Die Wärmestrahlung von Wasser und Eis, von bereiften und benetzten Oberflächen," by E. Schmidt, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 5, 1934, p. 1.

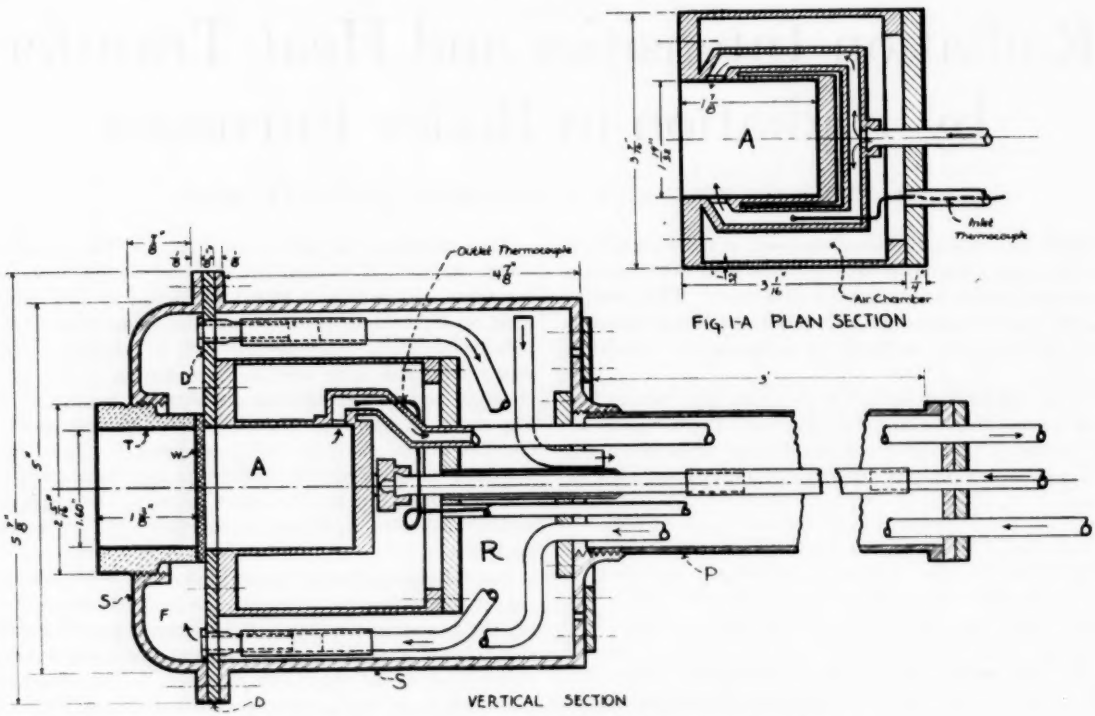


FIG. 1 DETAILS OF THE QUARTZ-WINDOW RADIATION CALORIMETER

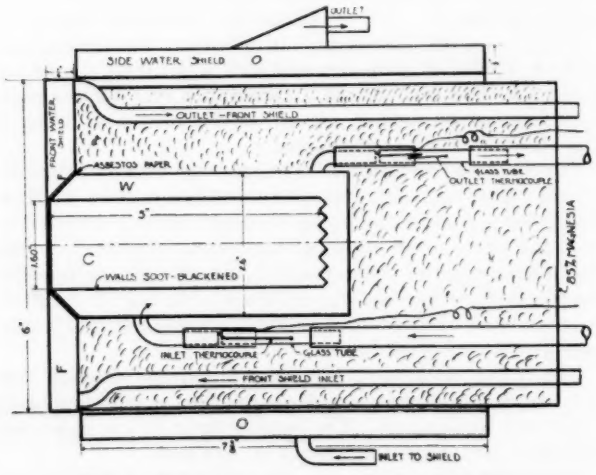
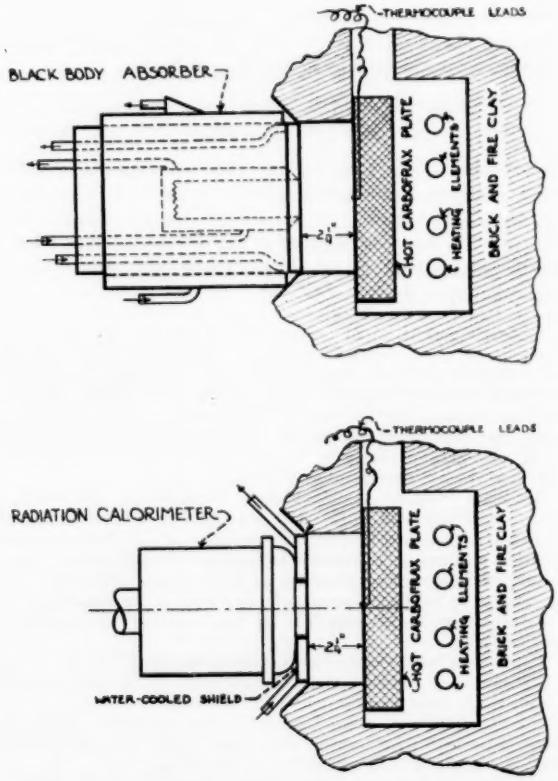


FIG. 2 DETAILS OF THE BLACK-BODY ABSORBER



FIGS. 3 AND 4 VERTICAL SECTIONS THROUGH CALIBRATION ARRANGEMENTS

from radiation at the front and sides, and define the opening of C, which is of the same size as the opening of the quartz-window calorimeter.

The calibration arrangements are shown in Figs. 3 and 4. In Fig. 3, the absorption of the black body was determined for a series of different temperatures of the heated plate, as indicated by the chromel-alumel thermocouple imbedded in the front surface of the hot-plate. Then in another series of tests (Fig. 4), the absorption of the radiation calorimeter with the quartz-window was determined for the corresponding hot-plate temperatures. A special water-cooled shield was used with the quartz-window instrument in order to duplicate the shape of the black-body absorber.

The calibration data are shown in Fig. 5. In these graphs the energy absorbed by the quartz-window calorimeter, and by the special black-body absorber, have been plotted against the radiation intensity at the surface of the hot plate. The radiation

intensity has been computed by means of the Stefan-Boltzmann law, which may be expressed as

$$I = 0.172 \times 0.92[(T_f/100)^4 - (T_s/100)^4] \quad [1]$$

in which I is the radiation intensity in Btu per sq ft per hr; 0.172 is the radiation constant; 0.92 is the emissivity of the hot plate as determined experimentally in an earlier investigation;[6] T_f is the absolute temperature of the surface of the hot plate as measured by the chromel-alumel thermocouple; and T_s is the absolute temperature of the receiver.

It will be noted in Fig. 5 that the energy absorbed by the black-body absorber is directly proportional to the radiation intensity I. Also, the energy absorbed by the quartz-window instrument is apparently proportional to the radiation intensity I.

The ratio between the energy absorbed by the quartz-window instrument and the energy which would be absorbed by the special black-body absorber is 0.464 from Fig. 5. The calibration equation for the quartz-window instrument is

$$I = M/0.464 = 2.155M \quad [2]$$

in which I is the radiation intensity in Btu per sq ft per hr; and M is the measured energy-absorption rate of the calorimeter in Btu per sq ft per hr. The radiation intensity, I, which has been studied in this investigation, is the rate of radiant-energy absorption in Btu per sq ft per hr, which would be experienced by a perfectly black surface at 90 F.

It was the object of the calibration tests to have the two arrangements of Figs. 3 and 4 such that it would not be necessary to determine the exact temperature of the front surface of the hot plate, but to use the temperature as indicated by the chromel-alumel thermocouple as a guide to show when the true surface temperatures were identical in the two separate arrangements.

Then, for a given hot-plate temperature, the energy absorbed by the quartz-window calorimeter could be compared directly with that received by a black-body absorber of similar size for the same radiation intensity. Thus the calibration does not depend upon the accurate determination of the temperature at the exact boundary of the hot-plate, nor upon geometrical "form factors." It depends, rather, almost wholly upon how nearly the so-called black-body absorber approximates a true black body (which absorbs all incident radiation, reflecting none)

[6] "The Construction and Calibration of an Instrument for the Measurement of Radiant Energy in Boiler Furnaces," by L. P. Meade, Thesis, State University of Iowa, 1934.

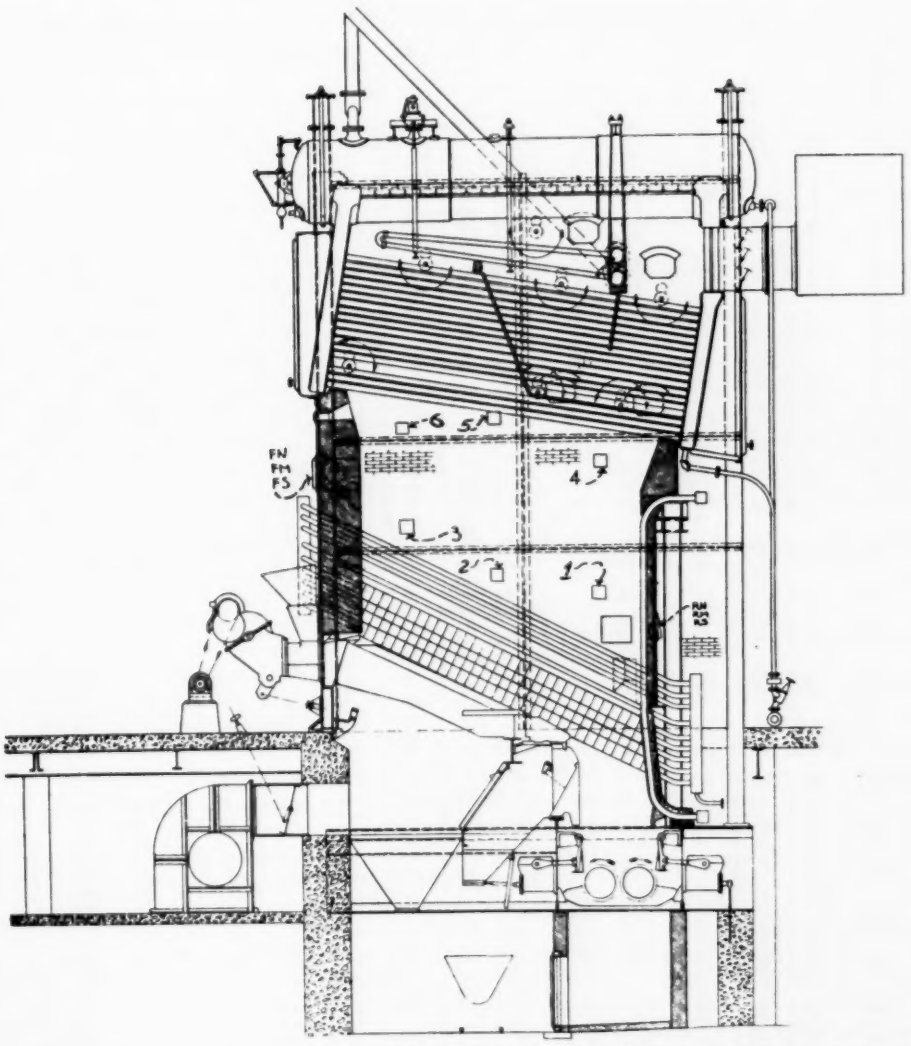


FIG. 6 CROSS-SECTION OF BOILER, STOKER, AND SETTING, SHOWING THE LOCATION OF THE OBSERVATION DOORS

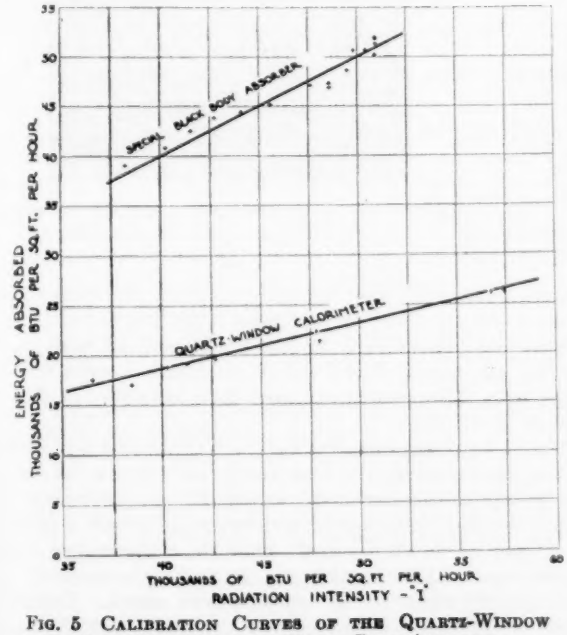


FIG. 5 CALIBRATION CURVES OF THE QUARTZ-WINDOW CALORIMETER AND THE BLACK-BODY ABSORBER

TABLE 1 ACTUAL RADIATION INTENSITIES,[a] I^b

Door No.[c] / Test No.	1	2	3	4	6	RS	RM	RN	FM	FN	Average
53	30.5	49.6	34.0	28.5	..	..	..	..	..	..	36.1
	34.8	..	..	..	..	..	..	..	..	..	
54	36.9	49.9	46.9	30.6	..	..	..	..	..	..	41.0
	37.1	55.2	..	41.2	..	..	..	..	..	..	
	19.5	..	..	..	..	..	..	..	..	..	
	18.5	..	..	..	..	..	..	..	..	..	
55	36.6	49.2	50.0	44.4	..	41.4	42.5	51.9	..	..	44.1
						..	46.8	33.2	..	..	
S2	33.8	45.8	48.8	23.3	..	34.0	38.8	37.2	27.4	..	37.1
						..	41.2	..	..	..	
						..	50.6	..	..	..	
S3	41.6	70.1	52.1	43.6	50.1	40.0	57.5	47.5	70.5	..	49.5
						..	60.0	33.2	..	..	
						..	..	30.4	..	..	
56	57.8	79.0	65.9	62.4	..	35.7	48.1	32.6	75.9	..	62.0
	58.9	78.3	..	..	..	51.6	62.6	..	76.0	..	
						..	66.0	..	77.5	..	
57	33.5	89.0	76.1	49.3	..	51.5	65.9	47.5	74.0	..	64.9
	72.0	88.0	77.6	63.5	..	..	..	..	70.1	..	
	48.0	..	..	..	..						
58	48.4	77.6	84.6	55.2	..	69.7	78.1	57.8	81.0	67.8	64.9
						52.2	72.6	43.0	56.5	79.0	
59	86.9	62.5	72.9	88.8	..	71.6	70.1	80.6	..	..	75.6
						70.5	97.0	68.9	..	..	
60	52.0	75.0	60.6	40.2	..	76.0	53.0	64.0	65.4	67.6	65.9
	69.7	79.7	76.2	66.4	..	61.7	73.3	61.0	72.8	72.7	
S5	87.6	107.0	77.6	71.5	..	40.6	60.0	43.6	..	..	62.6
						..	61.1	67.5	..	..	
						..	60.4	..	..	..	
S6	57.7	79.1	65.9	52.4	..	54.3	51.6	40.3	80.3	47.3	63.4
	..	86.1	84.5	33.7	..	46.5	55.5	39.3	48.1	80.3	
	..	83.3	..	..	..	..	..	..	97.0	83.0	
S7	67.0	105.3	71.2	72.0	..	62.1	89.1	83.1	..	..	81.9
	..	103.5	76.0	..	..	88.4	90.2	75.4	..	..	
69	46.5	77.8	79.6	58.3	..	52.1	62.8	49.4	..	..	61.2
	41.4	75.9	90.7	59.1	..	54.6	59.1	44.6	..	..	
						57.7	63.3	53.5	..	..	
70	76.6	89.4	81.8	79.7	..	63.1	84.4	73.2	..	..	65.2
	49.4	86.0	72.1	58.8	..	57.1	62.5	71.0	..	..	
						48.9	..	52.2	..	..	
						..	..	32.1	..	..	
S8	66.0	80.6	77.3	70.5	..	51.3	56.0	44.1	..	..	68.9
	73.0	65.4	79.4	78.9	..	52.9	54.1	38.2	..	..	
						..	64.9	..	..	..	
S9	23.9	64.3	48.1	31.2	..	38.5	49.6	55.8	..	..	45.5
	31.2	55.2	59.3	38.8	..	46.6	51.0	35.6	..	..	
	25.8	55.8	64.2	42.7	..	37.6	47.2	..	..	..	
S10	29.3	78.6	60.4	45.3	..	52.1	56.5	43.1	..	..	51.0
	29.3	62.6	74.0	63.9	..	44.5	53.0	44.2	..	..	
	40.8	50.9	..	54.9	..	44.1	47.6	46.2	..	..	
S11	16.4	42.0	38.6	19.3	..	36.3	50.1	38.9	..	..	34.6
	27.0	39.0	36.6	29.0	..	32.1	31.8	30.4	..	..	
	20.5	53.2	47.1	23.6	..	36.2	49.2	28.1	..	..	
S12	24.3	41.0	48.1	21.9	30.9	43.4	42.9	35.2	..	..	40.3
	24.0	55.7	50.6	26.5	35.8	44.4	50.6	44.9	..	..	
	21.9	58.6	49.2	26.2	32.0	38.4	64.4	59.8	..	..	
S13	36.6	53.6	37.7	45.5	33.7	31.5	50.6	37.4	..	..	42.7
	40.5	59.0	43.1	39.1	31.3	39.4	52.7	33.6	..	..	
						34.3	54.3	47.5	..	..	
S17	20.8	39.4	39.4	29.0	29.9	30.4	38.6	22.5	37.8	25.0	32.3
	19.4	43.9	51.5	23.9	23.4	20.2	41.7	..	42.8	34.4	
S18	25.3	51.1	56.6	24.1	34.3	28.9	39.4	23.9	42.3	35.4	35.8
	30.7	43.1	39.1	24.1	31.2	32.1	41.3	33.2	43.1	37.6	
71	24.5	38.6	34.1	25.4	20.9	25.0	28.1	26.1	30.3	24.2	27.9
	24.8	33.3	24.0	19.7	17.9	38.3	32.6	24.7	34.6	30.7	
72	27.3	48.0	31.3	26.8	22.6	22.6	29.5	26.2	32.7	28.7	34.1
	29.6	86.8	40.3	40.5	31.0	25.3	41.6	25.6	34.3	32.3	
78	27.6	42.4	29.9	34.3	21.9	27.3	40.5	26.5	31.0	34.2	35.4
	37.6	63.2	32.6	30.8	31.6	31.6	41.9	28.2	45.2	37.2	
									56.5	..	
79	30.2	45.6	33.5	40.8	22.3	46.0	54.4	39.1	46.2	39.4	38.2
	19.9	57.4	35.7	24.5	29.3	33.4	45.3	37.0	44.4	34.1	
80	12.6	54.0	30.6	20.1	41.9	49.2	61.5	46.4	61.1	24.2	44 2
	44.5	67.0	44.8	43.8	45.1	57.1	53.2	43.4	44.4	34.1	

[a] All intensities expressed in thousands of Btu per sq ft per hr.
[b] $I = 2.155\ M$, see Equation [2].
[c] For location of various doors see Figs. 6 and 13.

It has been assumed in the foregoing discussion, that the emissivity of the black-body absorber (Fig. 2) is 1.00, i.e., that the absorber is a true black body. Actually, the emissivity might be slightly less than unity. The emissivity of the inner, acetylene-soot-covered surfaces is about 0.945.[7] But the net emissivity of the cavity is considerably greater than the individual emissivities of the surfaces which form its walls, because the absorbing surfaces form an inclosure, and since the amount of incident radiation which is not completely absorbed at the blackened surface which it strikes inside the cavity will be more likely to be absorbed at another part of the blackened surface than to escape through the front opening. It is reasonable to suppose that the resultant emissivity is about 0.98.

If it were assumed that the emissivity of the special absorber is 0.98, the calibration equation for the quartz-window instrument would be changed by 2 per cent. Since the resulting correction for emissivity is only 2 per cent or less, it will not be used at this time.

[7] "Surface Heat Transmission," by R. H. Heilman, Trans. A.S.M.E., vol. 51, 1929, p. 289, paper FSP-51-41.

Application of Radiation Calorimeter to Boiler Tests

Description of Boiler and Furnace. The tests herein described were conducted on a steam-generation unit equipped with an underfeed stoker, located in the heating plant of the State University of Iowa. The unit tested (Fig. 6) has a 6000-sq-ft, straight-tube boiler, equipped with an extended-tube superheater. The rear water wall, consisting of twenty-one $3^1/_4$-in. bare tubes, spaced 6 in. on centers, provides 91 sq ft of cold surface. The side walls, with four $3^1/_4$-in. bare slag-drip tubes, and five $3^1/_4$-in. armored side-wall tubes on each side, have a total of 95 sq ft of cold surface; the top of the furnace has 150 sq ft of cold surface. All the surfaces have been computed as projected areas. The total cold surface in the furnace is 336 sq ft. The total wall area of the furnace is 835 sq ft, so the "fraction cold" of the furnace is 336/835, or 0.402.

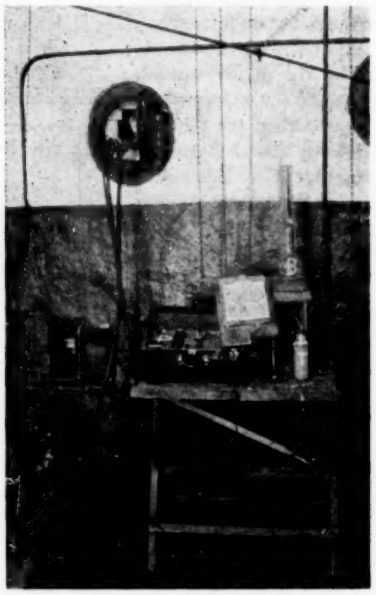


Fig. 7 View of Radiation Calorimeter in Door No. 1, With Accessory Equipment

Radiation Observations. Radiation observations were made through ten special observation doors, during a series of 8-hr boiler tests. The locations of these doors are shown in Figs. 6 and 13.

During an observation, the radiation calorimeter was placed in the observation door as shown in Fig. 7. The water rate in the absorption chamber of the calorimeter was determined by noting the time required to fill a calibrated glass flask *B* (Fig. 7) to a mark on its narrow neck, while the electromotive forces of the inlet and outlet thermocouples were determined continuously by means of the potentiometer shown. During a single observation, lasting about six minutes, about 15 sets of

temperature determinations were made, of which, the average values were used in determining the temperature increase of the water.

The actual radiation intensities as measured and corrected by Equation [2], are shown in Table 1. The average value of the radiation intensity for each boiler test, shown in Table 1, is the arithmetic average of all the observations made in all the doors. For this paper, no attempt has been made to determine a weighted average; the authors believe that the system of averaging here used is best for the purposes of this paper, because the radiation intensity was not constant at the various observation points throughout an entire test.

Methods of Computing Heat Absorption

Computations of Radiation by the Hudson-Orrok Formula. There are shown in Table 2 the average heat-transfer rates by radiation, as computed by means of the Hudson-Orrok formula.[8] These heat-transfer rates are plotted in Fig. 9. The quantity U in this table is the heat released in the furnace per lb of fuel burned. U also enters into the calculation of radiation by the Wohlenberg method, and is computed in Table 3, in which it is shown as the calorific heating value of the fuel, in Btu per lb, minus the losses due to (*a*) incomplete combustion of carbon to carbon-dioxide, (*b*) combustible in the refuse, and (*c*) evaporation of moisture.

Computations of Radiation by the Wohlenberg Method. In the computations of furnace heat balances by the Wohlenberg method,[9,10,11] as shown in Table 3, some shortcuts have been taken, the most important one being in evaluating the solid angles which enter into the computations of the radiation coefficients. The "fraction cold" for the furnace used in these tests was found to be 0.402. This corresponds to the Type *B* cubical furnace mentioned by Wohlenberg and Lindseth, which has one wall and the top cold. It has been assumed that the furnace in question is approximately represented, in so far as our purposes are concerned, by the Type *B* furnace, even though its shape differs from that of a cube. Hence the values for the various solid angles have been taken for those quoted for the Type *B* cubical furnace.

The radiation from the carbon dioxide and water vapor present in the burning gases is a function of the temperature, and of the product of the percentage concentration and the thickness of the radiating gas; this product may be expressed as $c = ps/328$, where p is the percentage by volume of the carbon dioxide or water vapor, and s is the thickness of the gas column in feet.[10] For a given temperature, the radiation from these gases increases with the factor c, until $c = 0.15$. Any further increase of c above 0.15 produces no increase in the radiation intensity from the gases at constant temperature.

In all the boiler tests of this investigation, the factor c was greater than 0.15 for both the carbon dioxide and water vapor; hence, the radiation from these gases could be plotted as a function of the temperature alone.

The radiation term in the heat balance for this furnace, then, may be plotted as a function of the "mean flame temperature" as shown in Fig. 8. From this graph, the total value of the radiation term may be determined for any assumed mean flame temperature.

TABLE 2 VARIOUS FACTORS USED IN COMPUTING RADIATION BY THE HUDSON-ORROK FORMULA

Test No.	A	C_r	μ	U	X_h
53	8.83	16.88	0.427	9209	67100
54	9.32	13.05	0.447	9639	56300
55	8.65	12.83	0.466	9383	56100
S2	10.24	11.69	0.430	10185	51200
S3	10.22	15.61	0.400	9209	57500
56	8.85	12.32	0.474	8375	49000
57	8.42	14.45	0.456	8475	55900
58	9.51	16.80	0.409	8874	61000
59	8.35	17.36	0.437	8790	66600
60	9.06	13.33	0.449	8514	51000
S5	8.92	14.62	0.416	9888	60100
S6	10.45	10.59	0.443	10385	48700
S7	9.50	13.90	0.433	9894	59600
S8	9.70	14.38	0.424	9821	59900
S9	9.77	12.35	0.430	9997	53000
S10	9.49	14.75	0.426	9708	60900
S11	10.82	10.13	0.440	10408	46300
S12	10.48	12.40	0.422	10316	54000
S13	8.75	15.03	0.452	9296	63200
S17	9.94	14.88	0.443	9674	63700
S18	10.16	15.83	0.448	10203	72400
69	9.11	13.55	0.446	9965	60100
70	7.65	15.83	0.469	10572	78500
71	9.55	11.98	0.394	8704	41100
72	9.92	12.86	0.431	8772	48600
78	8.65	14.42	0.452	8995	58600
79	9.40	12.30	0.450	9539	52900
80	8.99	15.90	0.447	9364	66500

$\mu = \dfrac{1}{1 + \dfrac{A\sqrt{C_r}}{27}}$ = fraction of energy released which is transferred to the cold surfaces in the furnace by radiation.

A = lb of air per lb of fuel.

U = heat release per lb of coal.

$X_h = C_r U \left[\dfrac{1}{1 + \dfrac{A\sqrt{C_r}}{27}}\right]$ = heat transfer rate by radiation, Btu per sq ft per hr.

C_r = fuel burned, lb per hr per sq ft of water-cooled surface exposed to radiation.

The solution of the heat-balance equation consists of finding the mean flame temperature, T_m; for this temperature, the energy released in the furnace per hr (called GU in Table 3) is equal to the sum of the three quantities, (*a*) the radiation term, Q_r, (*b*) the heat transferred by convection to the surfaces exposed to radiation (except for that radiant surface in the aperture through which the gases leave the furnace), and (*c*) the sensible heat of the products of combustion leaving the furnace at the mean flame temperature (represented in Table 3 by $\Sigma G_b \Delta h_b$). It was usually possible to select the correct mean flame temperature after a choice of three or four values.

The radiation-heat-absorption rates as computed by the Wohlenberg method are shown in Fig. 8. It is interesting to compare the results calculated from the same data using the two different methods.

Discussion of Test Results

Effect of Dirty Surfaces. In Fig. 10, the measured average radiation intensities are plotted against the energy release rate in the furnace. The measured radiation intensity, according to Fig. 10, varies considerably with constant energy release rate. This effect was noticed early in the tests, when it was found that the variation was due to the difference in dirtiness of the water-cooled surfaces in the furnace.

Attempts were made to estimate, in each of the tests, the fraction of the radiant surface which was covered with slag or ash. The covering on the tubes was found to consist mostly of patches of powdery ash, between $^1/_8$ in. and $1^1/_8$ in. in thickness. At times, after the boiler had been operated at high ratings, the slag-drip tubes of the side walls were covered with slag. In Table 4 are listed the values of the furnace dirtiness D, which is the estimated fraction of the total water-cooled surface in the furnace that is covered with slag or ash. Fig. 11 shows a portion of the rear water wall, which the authors estimate to be about 0.4 covered with ash, i.e., $D = 0.4$.

[8] "Radiation in Boiler Furnaces," by Geo. A. Orrok, Trans. A.S.M.E., vol. 47, 1925, pp. 1148–1155.

[9] "Radiation in the Pulverized-Fuel Furnace," by W. J. Wohlenberg and D. G. Morrow, Trans. A.S.M.E., vol. 47, 1925, p. 127.

[10] "The Influence of Radiation in Coal-Fired Furnaces on Boiler Surface Requirements and a Simplified Method for Its Calculation," by W. J. Wohlenberg and E. L. Lindseth, Trans. A.S.M.E., vol. 48, 1926, p. 848.

[11] Complete details of the computations and boiler test data are given in a thesis, "Radiation in Steam Boiler Furnaces," by C. F. Schmarje, which is on file in the library of the State University of Iowa.

TABLE 3 SUMMARY OF CALCULATIONS BY THE WOHLENBERG METHOD

Boiler Test No.	53			54			55			S-2			S-3			56			57			58			59			60			S-5			56			S-7			S-8		
T_a °F	2450			2350			2375			2315			2405			2155			2343			2330			2423			2195			2438			2268			2380			2365		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Product of Combustion																																										
CO_2	1.995	616	1229	2.010	585	1217	2.02	593	1199	2.209	575	1270	2.269	602	1364	1.922	524	1010	1.916	552	1059	1.99	579	1152	1.935	607	1175	1.958	538	1055	2.155	612	1319	2.285	539	1280	2.190	595	1305	2.186	590	1290
O_2	0.299	531	159	0.349	499	184	0.22	506	111	0.468	488	227	0.414	515	213	0.644	483	279	0.436	462	201	0.486	491	238	0.346	521	128	0.543	446	242	0.174	526	91	0.492	470	231	0.319	506	161	0.38	502	191
N_2+CO	7.082	577	4060	7.38	546	4030	6.87	554	3805	8.05	535	4310	8.065	562	4520	7.15	486	3478	6.836	513	3520	7.486	540	4040	6.796	547	3823	7.36	498	3665	7.16	572	4100	8.27	520	4300	7.57	554	4200	7.57	550	4265
Refuse	0.147	565	83	0.112	531	62	0.154	539	83	0.190	521	57	0.124	547	68	0.172	474	81	0.180	500	90	0.093	525	49	0.085	532	32	0.164	486	80	0.102	587	52	0.094	507	48	0.125	540	67	0.187	535	100
H_2O	0.488	1167	581	0.501	1105	576	0.485	1120	544	0.527	1083	572	0.525	1139	598	0.711	985	700	0.644	1039	671	0.696	1043	761	0.636	1151	731	0.720	1009	726	0.717	1140	831	0.653	1054	690	0.675	1123	758	0.682	1114	770
$\Sigma G_p a h_p \div G$			6112			6069			5742			6436			6743			5548			5541			6240			5709			5766			6348			6799			6791			6616
G Pounds of Fuel per Hour	5646			4884			4309			3801			5209			4114			4486			5650			5831			4481			4918			5560			4674			4827		
H Heating Value of Fuel – Btu/Lb.	9930			10340			9960			10925			10870			9850			9890			9847			9920			9405			11010			11223			10893			10710		
Losses (from H) Btu/Lb.																																										
(a) Due To Unconsumed Carbon (CO)		80			82			10			25			4			0			0			154			366			0			246			43			47			28		
(b) Due To Combustible In Refuse		95			68			36			187			580			106			206			56			66			101			90			76			172			101		
(c) Due To Evaporation of Moisture		546			571			531			538			576			780			709			743			698			790			786			719			740			760		
L Total Losses Btu/Lb.	721			721			577			750			960			886			915			953			1130			891			1122			838			959			889		
U Heat Released (H-L) Btu/Lb.		9209			9619			9383			10185			9910			8964			8975			8894			8790			8514			9888			10385			9934			9821		
$\Sigma G_p a h_p$ Btu/Hr.	34,650,000			24,600,000			24,730,000			25,100,000			35,210,000			22,850,000			24,860,000			35,250,000			34,450,000			25,840,000			31,450,000			25,315,000			30,360,000			31,900,000		
Z_c Btu/Hr.	685,000			574,000			585,000			564,000			664,000			436,000			576,000			615,000			660,000			802,000			660,000			490,000			610,000			595,000		
Q_R Btu/Hr.	16,780,000			14,680,000			15,600,000			14,010,000			15,770,000			11,130,000			12,660,000			14,280,000			16,160,000			11,800,000			16,500,000			13,120,000			15,260,000			14,960,000		
$\Sigma G_p a h_p + Z_c + Q_R$ Btu/Hr.	52,115,000			41,856,000			40,473,000			39,674,000			51,644,000			34,416,000			38,036,000			50,145,000			51,270,000			38,140,000			48,610,000			38,925,000			46,230,000			47,455,000		
G·U Heat Released Per Hour Btu/Hr.	52,130,000			41,800,000			40,500,000			39,670,000			51,600,000			34,455,000			38,019,000			51,138,000			51,250,000			38,150,000			48,630,000			34,930,000			46,220,000			47,406,000		
ERR $GU \div$ Volume $= GU/2160$ Btu/Ft³/Hr.	24,100			19,320			18,720			18,350			23,800			16,720			18,500			24,300			24,900			18,550			22,450			17,200			21,460			21,900		
Heat-Transfer Rate By Radiation: $X_R = Q_R/\text{Area} = Q_R/336$ Btu/Ft²/Hr.	49,900			43,600			45,100			41,700			46,900			33,100			37,700			42,700			48,100			35,100			49,100			39,000			45,400			44,500		

NOTE: Col. C = (Col. B) × (Col. A)
Col. A = Lbs. of Product per Pound of Fuel.

Col. "B" = Heat Capacity of Product above 60°F [for water, above 212°F] per pound.

Boiler Test No.	S-9			S-10			S-11			S-12			S-13			S-17			S-18			69			71			72			78			79			80								
T_a °F	2330			2378			2230			2325			2393			2360			2413			2395			2155			2170			2522			2280			2390								
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C						
Product of Combustion																																													
CO_2	2.200	580	1278	2.147	595	1278	2.317	549	1272	2.300	577	1325	2.045	598	1230	2.114	588	1243	2.240	605	1356	2.045	600	1223	1.961	825	1030	1.970	562	1108	2.000	576	1152	2.155	563	1203	2.08	597	1242						
O_2	0.450	490	199	0.366	506	185	0.586	457	268	0.526	469	257	0.345	511	142	0.426	500	213	0.460	515	239	0.480	512	215	0.688	452	297	0.734	472	348	0.397	487	194	0.436	474	206	0.434	510	222						
N_2+CO	7.775	540	4200	7.515	553	4190	8.545	509	4350	8.277	537	4442	8.935	558	3870	7.886	548	4320	8.220	544	4428	7.247	551	4085	7.447	485	3736	7.986	522	4165	7.09	514	3800	7.628	524	4010	7.726	557	4045						
Refuse	0.116	525	61	0.119	539	64	0.066	496	33	0.078	523	41	0.154	544	84	0.093	534	49	0.152	550	73	0.117	544	63	0.170	471	80	0.184	509	80	0.13	522	68	0.121	510	62	0.115	543	61						
H_2O	0.628	1095	686	0.706	1123	791	0.660	1031	680	0.660	1090	719	0.675	1132	766	0.68	1111	755	0.68	1140	775	0.731	1135	827	0.68	981	669	0.68	1059	720	0.68	1087	738	0.68	1061	722	0.68	1130	769						
$\Sigma G_p a h_p \div G$			6419			6506			6403			6784			6119			6980			7075			6413			5824			6421			5952			6203			6359						
G Pounds of Fuel per Hour	4151			4959			3905			4165			5054			4918			5312			4958			4032			4522			4851			4132			5340								
H Heating Value of Fuel – Btu/Lb.	10890			10670			11210			11180			10250			10700			11130			10853			9600			9580			9840			10340			10230								
Losses (from H) Btu/Lb.																																													
(a) Due To Unconsumed Carbon (CO)		81			62			0			9			0			208			0			7			0			0			36			12			19								
(b) Due To Combustible In Refuse		125			125			78			131			214			73			182			76			151			63			73			44			102								
(c) Due To Evaporation of Moisture		691			775			724			724			740			745			745			801			745			745			745			745			745								
L Total Losses Btu/Lb.	893			962			802			864			954			1026			927			885			896			808			654			801			846								
U Heat Released (H-L) Btu/Lb.		9997			9708			10408			10316			9296			9674			10203			9965			8704			8772			8995			9534			9364								
$\Sigma G_p a h_p$ Btu/Hr.	24,650,000			32,370,000			22,480,000			28,260,000			30,900,000			32,900,000			37,600,000			29,240,000			23,460,000			24,180,000			28,860,000			25,620,000			33,900,000								
Z_c Btu/Hr.	370,000			615,000			470,000			550,000			600,000			610,000			620,000			600,000			460,000			470,000			570,000			520,000			620,000								
Q_R Btu/Hr.	14,230,000			15,220,000			12,420,000			14,190,000			15,300,000			14,850,000			15,950,000			15,580,000			11,250,000			11,360,000			14,130,000			13,350,000			15,460,000								
$\Sigma G_p a h_p + Z_c + Q_R$ Btu/Hr.	41,490,000			48,105,000			35,370,000			43,000,000			47,000,000			48,360,000			54,170,000			45,390,000			35,045,000			37,980,000			43,560,000			39,490,000			49,980,000								
G·U Heat Released Per Hour Btu/Hr.	41,500,000			48,100,000			35,400,000			43,000,000			47,000,000			48,400,000			54,200,000			45,420,000			35,045,000			37,913,000			43,610,000			39,420,000			50,000,000								
ERR $GU \div$ Volume $= GU/2160$ Btu/Ft³/Hr.	19,200			22,200			16,400			19,900			21,800			22,400			25,100			21,000			16,200			17,600			20,200			18,250			23,100								
Heat-Transfer Rate By Radiation: $X_R = Q_R/\text{Area} = Q_R/336$ Btu/Ft²/Hr.	42,500			45,300			37,000			42,200			46,100			44,200			47,500			46,300			33,500			33,800			42,000			39,700			46,000								

In Fig. 10 curves of equal dirtiness, D, have been drawn for the furnace clean ($D = 0$), slightly dirty ($D = 0.1$), moderately dirty ($D = 0.2$), and very dirty ($D = 0.4$). Since there was a similarity between the constant dirtiness curves and the Orrok curve of Fig. 9, and also an increase in intensity with D, at a given energy-release rate, a tentative formula (based upon the Hudson-Orrok formula) for calculating the average radiation intensity has been derived.

The proposed formula is as follows:

$$I = C_r U \left[\frac{0.5 + 1.7\,D}{1 + \dfrac{A\sqrt{C_r}}{27}} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

in which I is the average radiation intensity at the furnace walls in Btu per sq ft per hr; U is the energy release per lb of fuel burned as found by subtracting from the heating value of

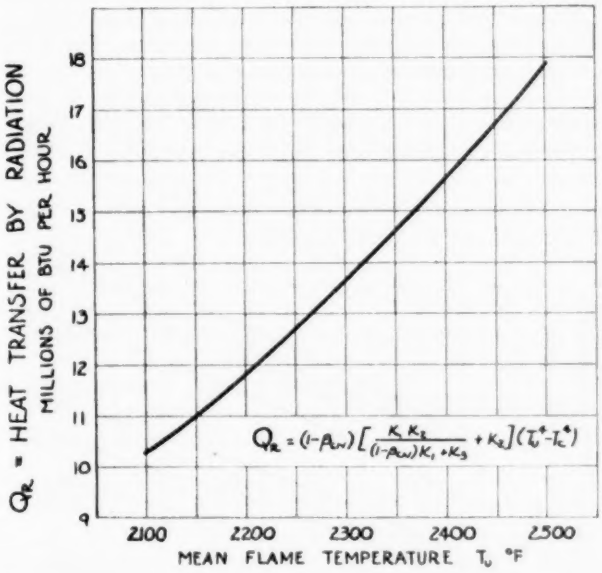


FIG. 8 HEAT TRANSFERRED BY RADIATION AS COMPUTED BY THE WOHLENBERG METHOD

the fuel the losses due to unconsumed carbon, evaporation of moisture, and combustible in the refuse; A is the lb of air entering the furnace per lb of fuel burned; C_r is the fuel burned,

TABLE 4 DIRTINESS FACTORS, D

Test No.	D	Test No.	D
S3	0.1	70	0.4
S4	0.1	S8	0.4
S5	0.1	S9	0.2
S2	0.1	S10	0.2
S3	0.1	S11	0.1
S6	0.4	S12	0.1
S7	0.4	S13	0.1
S8	0.4	S17	0.0
S9	0.4	S18	0.0
60	0.4	71	0.1
S5	0.4	72	0.1
S6	0.4	78	0.1
S7	0.4	79	0.1
69	0.4	80	0.15

lb per hr per sq ft of projected cold surface exposed to radiation; and D is the fraction of the cold surface in the furnace which is covered with slag or ash.

It is to be noted that Equation [3], here tentatively proposed, gives "radiation intensity," as contrasted to "heat-transfer rate" by radiation. The two quantities may be very nearly equal for a clean furnace, but for a dirty furnace the

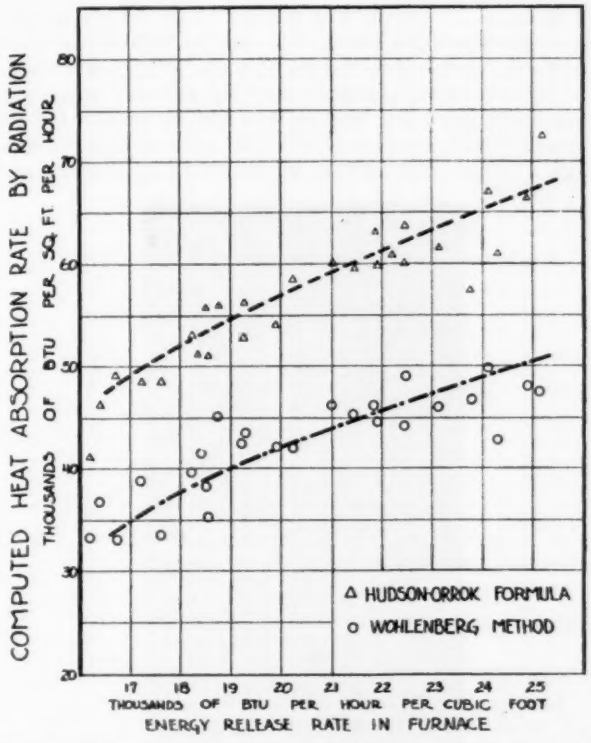


FIG. 9 RADIANT-HEAT-ABSORPTION RATES COMPUTED BY THE HUDSON-ORROK FORMULA AND THE WOHLENBERG METHOD

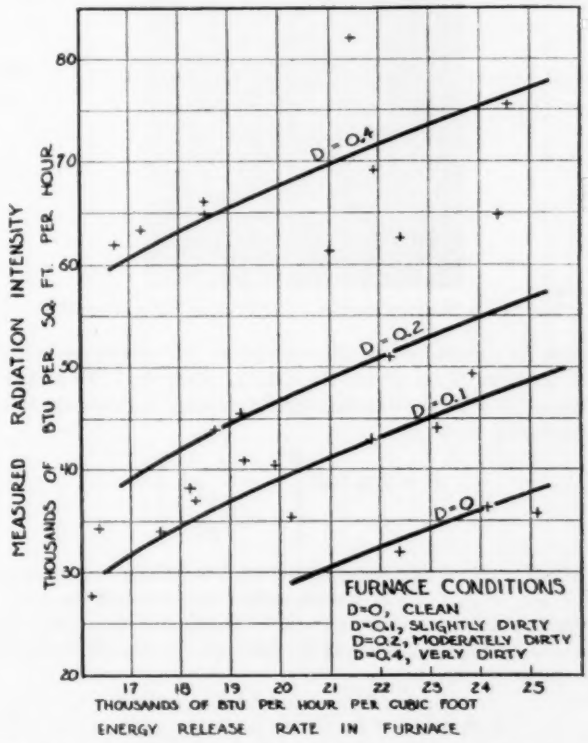


FIG. 10 RADIATION INTENSITIES, I, DETERMINED DURING THE BOILER TESTS

average radiation intensity is probably much greater than the average heat-transfer rate by radiation.

Heat-Transfer Rates by Radiation. Having found an equation

for radiation intensities, it is possible to predict an approximate equation for the average heat-transfer rates by radiation. A perfectly black, cold surface placed in the furnace would absorb heat by radiation at a rate equal to the radiation intensity I, given by Equation [3]. But the cold surfaces employed in the furnace probably absorb only about 95 per cent of the radiation which a perfectly black surface would at the same temperature.[9]



FIG. 11 A PORTION OF THE REAR WATER WALL (D = 0.4)

Hence, for a clean furnace, the average heat absorption by radiation would be 95 per cent of the radiation intensity. For a clean furnace (D = 0), the equation for the average heat-transfer rate by radiation would be

$$X = 0.95\, C_r U \left[\frac{0.5}{1 + \frac{A\sqrt{C_r}}{27}} \right] \quad \ldots\ldots\ldots\ldots [4]$$

Those parts of the water-cooled surfaces which are covered with ash or slag would absorb only about one-half as much radiant energy as a clean surface.[12] Taking this into account, the average heat-transfer rate by radiation for a partially dirty furnace would then be

$$X = 0.95(1 - 0.5D) C_r U \left[\frac{0.5 + 1.7D}{1 + \frac{A\sqrt{C_r}}{27}} \right] \quad \ldots\ldots\ldots [5]$$

[12] This conclusion is based upon a study of data given in the paper, "Heat Absorption in Water-Cooled Furnaces," by W. L. DeBaufre, Trans. A.S.M.E., vol. 53, 1931, paper FSP-53-19a. Figs. 3 and 4 of this paper especially would indicate that 0.5 is about the correct value.

where X is the average heat-transfer rate by radiation to the water-cooled surfaces in the furnace in Btu per sq ft per hr, and the other symbols are the same as in Equation [3]. This may be closely represented, when the value of D is between 0 and 0.5, by the formula,

$$X = C_r U \left[\frac{0.48 + 1.1D}{1 + \frac{A\sqrt{C_r}}{27}} \right] \quad \ldots\ldots\ldots\ldots [6]$$

which the authors propose (tentatively) to replace the Hudson-Orrok formula. For values of D greater than 0.5, Equation [5] should be used.

The constants in the numerator of this new equation depend upon the relationship between heat absorption and radiation intensity, with varying degrees of furnace dirtiness. An experimental investigation is now under way to study this relationship.

Hudson-Orrok Formula and Wohlenberg Method Compared to Results Based Upon Measured Radiation Intensities. In Fig. 12, the heat-transfer rates by radiation, as computed by the new formula, Equation [6], are shown in comparison to those corresponding to the Hudson-Orrok formula and the Wohlenberg method. The single curves for the Hudson-Orrok formula

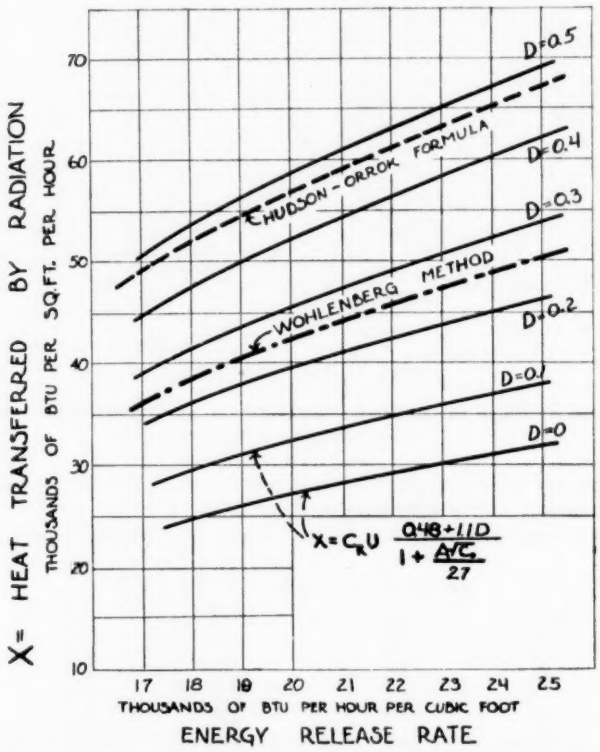


FIG. 12 COMPARISON OF HEAT TRANSFER BY RADIATION FROM THE HUDSON-ORROK FORMULA AND THE WOHLENBERG METHOD WITH THE HEAT TRANSFER BASED UPON MEASURED RADIATION INTENSITIES

and the Wohlenberg method are justified, because the computed data, for these boiler tests, all cluster closely about these two curves, as shown in Fig. 9.

It is found, for the boiler tests represented here, that for a given energy release rate, the Hudson-Orrok formula results in higher heat-transfer rates than does the Wohlenberg method. The proposed formula, which represents actual heat-transfer

rates by radiation, gives values at constant energy release depending upon the dirtiness of the furnace. The Hudson-Orrok formula is in agreement with the proposed formula, Equation [6], for these tests, when the furnace is fairly dirty. The Wohlenberg method gives results which are apparently correct for a comparatively clean furnace.

Distribution of Radiation. The typical distribution of radiation intensities about the furnace is shown in Fig. 13 for three different tests of approximately equal energy-release rates.

During one of these tests, the furnace was clean, having been in operation only two days following a thorough cleaning; in the other tests the fractions of the radiant surface dirty were approximately 0.2 and 0.4.

Summary

Conclusions. The development of a fused-quartz-window absorption calorimeter for determining radiation intensities in boiler furnaces is described. This instrument has been used during 28 boiler tests and the results compared with the Hudson-Orrok formula and the Wohlenberg method of computation of radiation heat transfer.

The results show that, for these tests, the Hudson-Orrok formula gives results which are correct for a furnace with a considerable part of the water-cooled surfaces covered with ash or slag. The Wohlenberg method of computation shows correct results, for these tests, when the water-cooled surfaces are comparatively clean.

A tentative empirical equation, Equation [3], is given for computing radiation intensities, taking into account the dirtiness of the cold surfaces; tentative formulas, Equations [5] and [6], are given for computing the average heat transfer by radiation in boiler furnaces.

There is a variation in radiation intensity from point to point at the walls of the furnace, as indicated by Fig. 13.

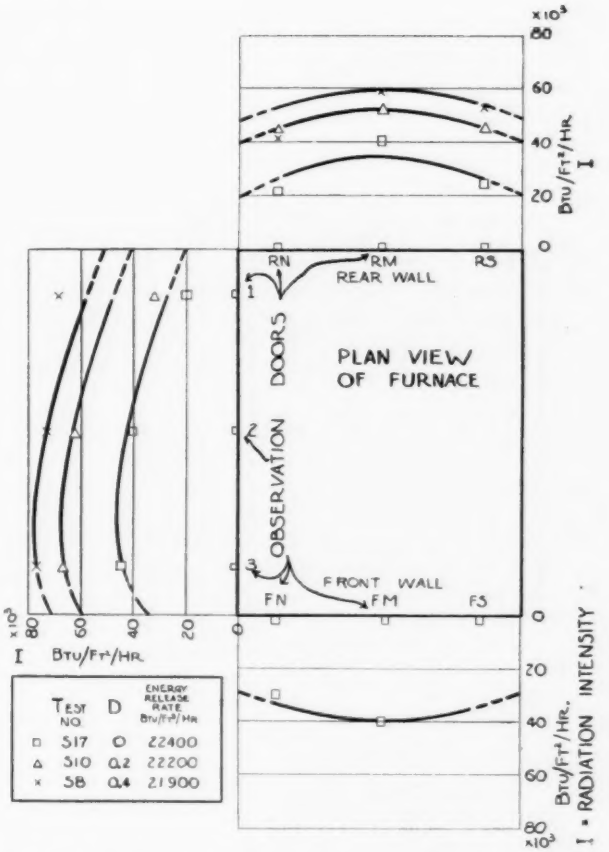


Fig. 13 Distribution of Radiation Intensities About the Boiler Furnace

The Leakage of Steam Through Labyrinth Seals

By ADOLF EGLI,[1] PHILADELPHIA, PA.

This paper gives a rational theoretical treatment of the labyrinth problem, based on the actual flow characteristics typical for a sharp-edged orifice. The general relations between leakage, number of throttlings, and pressure distribution are given in graphical form on a chart intended for use in practical turbine design.

It is also shown how the effect of kinetic energy being carried from one throttling into the next can be considered rationally. The numerical constants left open by the theory are derived experimentally with the aid of a static labyrinth leakage-testing device.

INTRODUCTION

With ever-increasing operating pressures, leakage loss of steam through unavoidable seals (shaft packings, dummy pistons, etc.) becomes more and more detrimental to the efficiency of modern steam turbines. Also, it has become common practice to improve the efficiency of turbine stages by introducing leakage seals which reduce the amount of steam uselessly by-passing each row of blades. Inasmuch as the labyrinth-type seal is still the most common, an accurate method for predicting the leakage through it is desirable. It is also important to know accurately the pressure distribution in the labyrinth when calculating the thrust of certain types of dummy pistons. In this paper a refined, yet sufficiently simple method, is developed for predicting the performance of labyrinths.

DEVELOPMENT OF A RATIONAL THEORY FOR THE EXPANSION OF COMPRESSIBLE FLUIDS THROUGH LABYRINTHS

1 Expansion Through a Single Sharp-Edged Throttling

A schematic illustration of two-dimensional flow through a single sharp-edged throttling is given in Fig. 1. The steam (or gas) expands from the pressure p_0 to p_1 when passing through the slot, forming at the same time some kind of a jet with a minimum area A_m. We are not particularly interested in knowing this minimum area. In some other section, A_1, which may be located somewhere near (before or after) the minimum section, the pressure in the steam jet will be equal to the chamber pressure, p_1. If A_1 is known and adiabatic change of state of the steam, for instance, is assumed, it is possible to calculate the weight of the theoretical flow passing through the throttling by applying Saint Venant's equation. This equation, neglecting the approach velocity w_0 of the steam, is given as

$$G_{th} = A_1 \sqrt{\left(2g \frac{k}{k-1}\left[\beta^{2/k} - \beta^{1+1/k}\right]\frac{p_0}{v_0}\right)} \quad \ldots\ldots [1]$$

where G_{th} = theoretical flow, lb per sec; g = acceleration of gravity, 32.17 ft per sec per sec; k = exponent of adiabatic expansion (1.3 for superheated steam); β = the pressure ratio p_1/p_0; p_0 = abs pressure before throttling, lb per sq ft; p_1 = abs pressure after throttling, lb per sq ft; v_0 = specific volume before throttling, cu ft per lb; and A_1 = jet area, sq ft.

Introducing the flow coefficient

$$\alpha = A_1/A \quad \ldots\ldots [2]$$

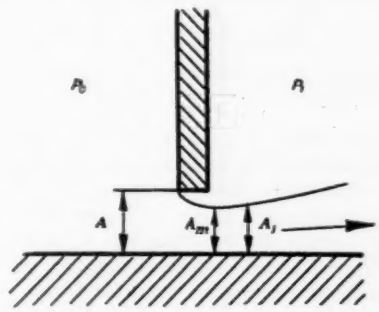


Fig. 1 Expansion Through a Single Sharp-Edged Throttling

and abbreviating the dimensionless expression

$$\psi_{th} = \sqrt{\left(\frac{2k}{k-1}\left[\beta^{2/k} - \beta^{1+1/k}\right]\right)} \quad \ldots\ldots [3]$$

we finally write for the theoretical flow

$$G_{th} = A\alpha\psi_{th}\sqrt{\left(g\frac{p_0}{v_0}\right)} \quad \ldots\ldots [4]$$

where A is the area of the throttling opening (Fig. 1). For superheated steam ($k = 1.3$), ψ_{th} was calculated and is plotted in Fig. 2 as a function of the pressure ratio β. It has the well-known nozzle characteristics, reaching a maximum value of 0.667 at the critical pressure ratio $\beta_c = p_c/p_0 = 0.546$.

The flow coefficient, α, of a sharp-edged opening, when plotted against Reynolds' number, gives a curve such as shown in Fig. 3. For Reynolds' numbers higher than the critical value R_{cr} the coefficient α is constant, i.e., independent of the velocity of the steam. For sharp-edged round orifices, the critical value of $R = wd/\nu$ is about 10^3, where w = velocity of the jet discharging from the orifice, ft per sec; d = diameter of the orifice opening, (clearance of throttling); and ν = kinematic viscosity of the fluid in ft² per sec. Although results of tests on the flow coefficient

---

[1] Research Engineer, Westinghouse Electric & Manufacturing Co. Mr. Egli received his technical education at the Federal Technical University in Zurich, Switzerland, where he was graduated as Diplom Mechanical Engineer under Professor Stodola in 1929. After graduation he was engaged as assistant to Professor Stodola and Professor Eichelberg and as instructor of thermodynamics and heat-engine design. From 1930 to 1931 he was employed as experimenting engineer with the Terry Steam Turbine Company, Hartford, Conn. Since 1931, Mr. Egli has been connected with the Westinghouse Company at South Philadelphia as research engineer on turbine efficiency and design.

Contributed by the Power Division for presentation at the semi-annual meeting, Cincinnati, Ohio, June 19 to 21, 1935, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of a narrow slot as a function of Reynolds' number are not available, we may assume that R_c is in the order of 10^3. In a steam labyrinth, R is as high as 10^4 and above. We can assume that for velocities far enough below the acoustic velocity (about 500 ft per sec or less for superheated steam) α is independent of the velocity. In this range (that is, for throttling pressure ratios β = 0.8 to 1.0) we should expect Equation [4] to give the correct flow.

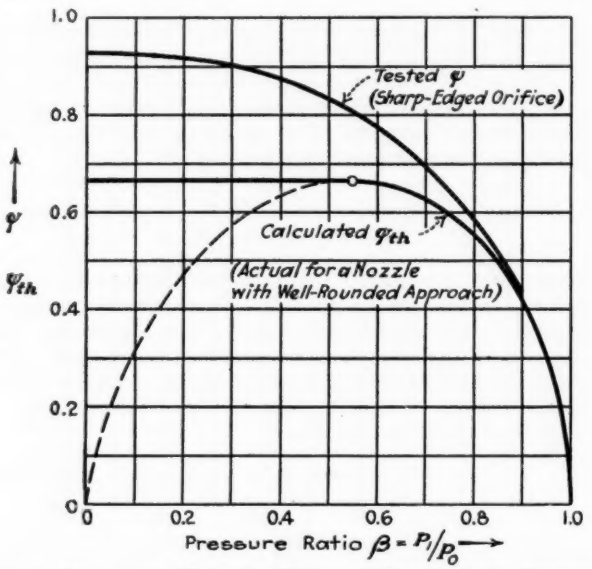


FIG. 2 EXPANSION FUNCTION ψ OF A SHARP-EDGED ORIFICE OPERATING WITH SUPERHEATED STEAM

The last throttlings in a labyrinth very often work with velocities comparable with, or as high as, the acoustic velocity (β less than 0.8). Under these conditions, adiabatic change of state and the constance of α can no longer be assumed. The actual flow G through the throttling will differ from the theoretical value G_{th}, calculated from Equation [4].

There is complete information available on the flow through sharp-edged round orifices which can be applied to this problem. Assuming, for simplicity, a constant-flow coefficient, Schiller[2] calculated from flow tests made with VDI orifices, the function ψ as defined by the equation

$$G = A\alpha\psi\sqrt{\left(g\frac{p_0}{v_0}\right)} \quad \ldots\ldots [5]$$

where G = the actual measured flow (see Table 1). The values in Table 1 are used in plotting the curve ψ versus β shown in Fig. 2. A comparison of the (theoretical) ψ_{th} curve with the (tested) ψ curve reveals the fact that for pressure ratios of β = 0.8 and above, the theoretical flow checks rather closely with the actual. As the pressure ratio decreases below 0.8, ψ rises increasingly above ψ_{th}. It is remarkable that no critical pressure ratio exists for the actual flow through a sharp-edged opening.

TABLE 1 EXPANSION FUNCTION ψ OF A SHARP-EDGED ORIFICE AND OF A WELL-ROUNDED OPENING

Pressure ratio β	Sharp-edged orifice ψ	Theoretical ψ_{th}	Well-rounded opening ψ
1.00	0.000	0.000	0.000
0.95	...	0.307	0.307
0.90	0.432	0.421	0.421
0.85	...	0.499	0.499
0.80	0.588	0.555	0.555
0.70	0.698	0.629	0.629
0.60	0.778	0.663	0.663
0.546	...	0.667	0.667
0.50	0.835	0.664	0.667
0.40	0.877	0.635	0.667
0.30	0.905	0.575	0.667
0.20	0.921	0.475	0.667
0.10	0.928	0.322	0.667
0.00	0.929	0.00	0.667

[2] "Überkritische Entspannung kompressibler Flüssigkeiten," by W. Schiller, *Forschung auf dem Gebiete des Ingenieurwesens*, 1933, p. 128.

2 EXPANSION THROUGH A SERIES OF THROTTLINGS

General Thermodynamic Relations. A group of throttlings as used in steam-turbine labyrinths is shown diagrammatically in Fig. 4.[3] The arrangement shall be isolated perfectly so that no heat can be exchanged with the surroundings. As the steam flows through the labyrinth, a pressure drop occurs across each throttling. After each throttling, a small part of the kinetic energy of the steam jet will be reconverted into pressure energy, a second part will be destroyed and transferred into heat, and the remaining kinetic energy will enter the following throttling. The velocity w_ν of the steam jet issuing from the νth throttling is obtained from the energy equation

$$(w_\nu/223.7)^2 = i_0 - i_\nu \quad \ldots\ldots [6]$$

where w_ν = velocity of steam jet issuing from the νth throttling, ft per sec; i_0 = total heat of steam before entering the labyrinth, Btu per lb; and i_ν = total heat of steam issuing from ν^{th} throttling, Btu per lb.

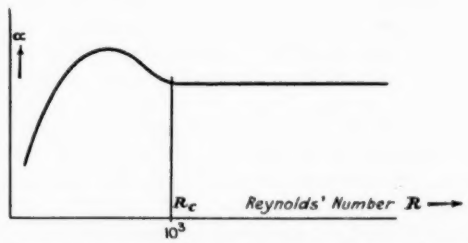


FIG. 3 FLOW COEFFICIENT α OF A SHARP-EDGED ORIFICE AS A FUNCTION OF REYNOLDS' NUMBER

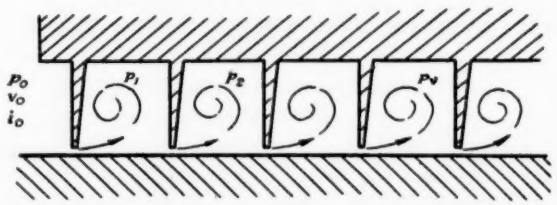


FIG. 4 EXPANSION THROUGH A SERIES OF LABYRINTH THROTTLINGS

Inasmuch as Equation [6] is general, nothing has to be said about the nature of the flow preceding the νth labyrinth. The condition of continuity for the νth throttling, however, may be written as

$$Gv_\nu = \alpha A_\nu w_\nu \quad \ldots\ldots [7]$$

where G = the flow, lb per sec; A_ν = the leakage area of the νth throttling, sq ft; v_ν = the specific volume of steam leaving the νth throttling, cu ft per lb; and α = the flow coefficient.

For the great majority of the throttlings of the labyrinth, the pressure ratio β is higher than 0.8 and α is practically constant. Then, in a labyrinth with a constant leakage area A, the expression

$$W_\nu/v_\nu = G/\alpha A \quad \ldots\ldots [7a]$$

is a constant. Combining Equation [7a] with Equation [6]

[3] A series of instructive illustrations of flow through labyrinths produced with water is published in *Escher Wyss News*, January-February, 1934.

$$\text{Constant} = G/\alpha A = (223.7/v_\nu)\sqrt{(i_0 - i_\nu)} \quad [8]$$

which represents a definite relation between v_ν and i_ν. For a given initial total heat i_0, Equation [8] determines a series of Fanno curves[4] $G/\alpha A$ = a constant on the total-heat-entropy diagram as shown in Fig. 5. In a labyrinth of constant-leakage area A, all points of the condition in the steam jet issuing

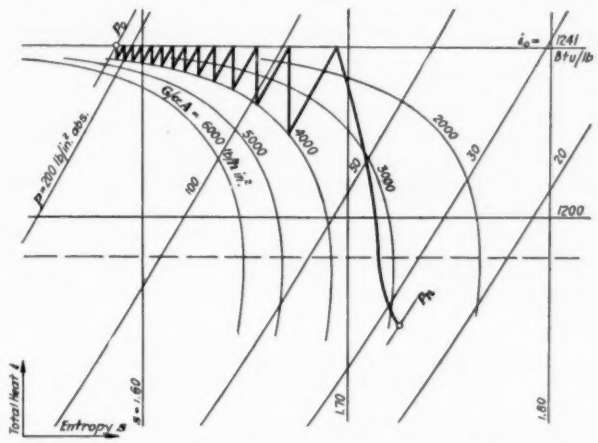


FIG. 5 TOTAL HEAT-ENTROPY DIAGRAM WITH FANNO CURVES
(The heavy zig-zag line is the condition curve for an ideal labyrinth.)

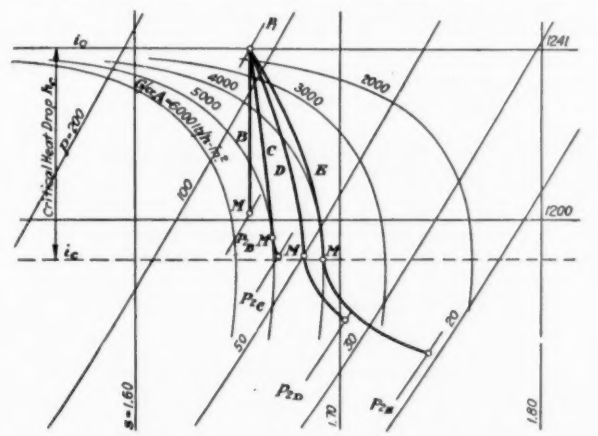


FIG. 6 CONDITION CURVES FOR THE EXPANSION OF STEAM THROUGH A SHARP-EDGED THROTTLING

from each throttling lie on a Fanno curve,[5] as long as α is constant. In an ideal labyrinth with completely destroyed kinetic energy after adiabatic expansion in each throttling, the condition of the steam will follow the zig-zag line of Fig. 5. In the last throttling the pressure ratio β often is considerably smaller than 0.8, and the conditions appear as observed previously.

[4] "Steam and Gas Turbines," by A. Stodola, McGraw-Hill, 1927, sixth edition, p. 61. The curves are named in honor of Fanno, who first used this relationship.

[5] A total-heat-entropy diagram with drawn-in Fanno curves, such as shown in Fig. 6, is helpful when studying the expansion through all kinds of channels. It shows at once that the flow in a passage with constant cross-section never exceeds the critical velocity $w_c = 223.7\sqrt{(h_c)}$, because a flow with decreasing entropy is impossible. The condition of the steam when expanding through a sharp-edged opening may be reproduced by curves like B, C, D, or E, Fig. 6, for decreasing pressure ratios, β. The velocity in the minimum section of the jet (points M, Fig. 6) cannot exceed the critical. This, however, does not mean that when the acoustic velocity in the narrowest section is reached, the flow has to be independent of the back pressure. Because of the change of contraction of the jet, the flow can still increase when the back pressure is lowered.

The curve for the conditions in the last throttling will be similar to the one in Fig. 5.

Derivation of a Practical Method of Calculating the Performance of Ideal Labyrinths With Constant-Leakage Area. In view of the complicated relation between pressure ratio and flow in each throttling, a graphical method is the most practical for computing the conditions in the whole labyrinth.

The equation for leakage through an ideal labyrinth (a labyrinth in which the kinetic energy of the steam jet is completely destroyed and reconverted into heat after each throttling) is similar to the equation for the single throttling

$$G = A\alpha\varphi\sqrt{\left(g\frac{p_0}{v_0}\right)} \quad [9]$$

where G = the leakage, lb per sec; A = leakage area of the single throttling, sq ft; α = flow coefficient, assumed to be constant for all throttlings; p_0 = abs pressure before the labyrinth, lb per sq ft; v_0 = specific volume before the labyrinth, cu ft per lb; p_n = pressure after the labyrinth, lb per sq ft; φ is a function of the labyrinth pressure ratio p_n/p_0 and the number n of throttlings.

For the labyrinth with one throttling, ($n = 1$) φ is identical with ψ in Fig. 2. The calculation of φ is a matter of mathematical analysis, the result of which is represented graphically in Figs. 7 and 7*a*.

The φ curves were obtained as follows:

In a labyrinth with two throttlings, the relation for the first is

$$G_1 = A\alpha\varphi_1\sqrt{\left(g\frac{p_0}{v_0}\right)} \quad [10]$$

and for the second throttling

$$G_2 = A\alpha\psi\sqrt{\left(g\frac{p_1}{v_1}\right)} \quad [11]$$

Since $G_1 = G_2$

$$\varphi_1\sqrt{\left(\frac{p_0}{v_0}\right)} = \psi\sqrt{\left(\frac{p_1}{v_1}\right)} \quad [12]$$

The pressure and volume of the steam on a constant total-heat line are, with sufficiently close approximation, related in the form

$$p_0v_0 = p_1v_1 \quad [13]$$

Thus, we can write Equation [12] as

$$\psi = \left(\frac{1}{p_1/p_0}\right)\varphi_{n=1} \quad [14]$$

Inasmuch as ψ versus $\beta = p_2/p_1$, and φ_1 versus p_1/p_0 (in this case identical with ψ versus β) are given graphically in Fig. 2, Equation [14] must also be solved graphically. The result is the curve $\varphi_{n=2}$ of Fig. 7*a*.

For the labyrinth with three throttlings, the following equation is obtained similar to Equation [14]:

$$\psi = \left(\frac{1}{p_2/p_0}\right)\varphi_{n=2} \quad [15]$$

from which $\varphi_{n=3}$ of Fig. 7*a* is calculated. In this manner we can proceed up to any number of throttlings. For a large number of throttlings, however, this method would be too cumbersome and inaccurate since errors are cumulative. For a labyrinth with $n + 1$ throttlings, Equation [14] can be generalized as

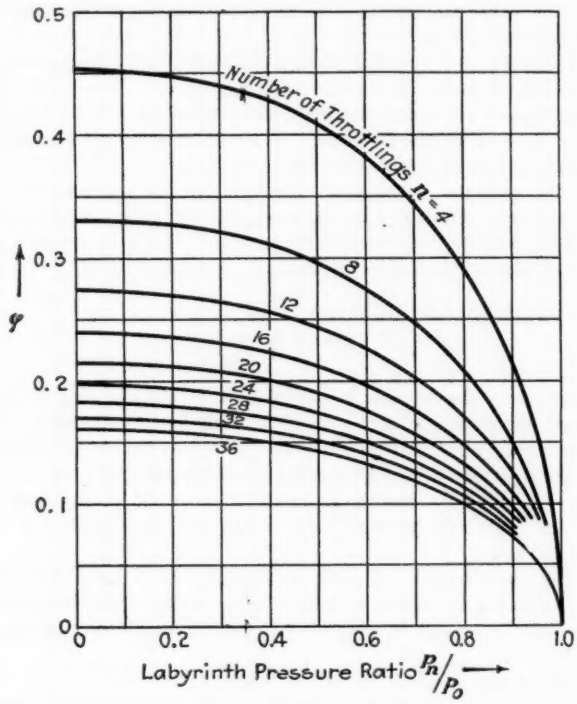


FIG. 7 LEAKAGE FUNCTION φ FOR LABYRINTHS WITH FOUR AND MORE THROTTLINGS

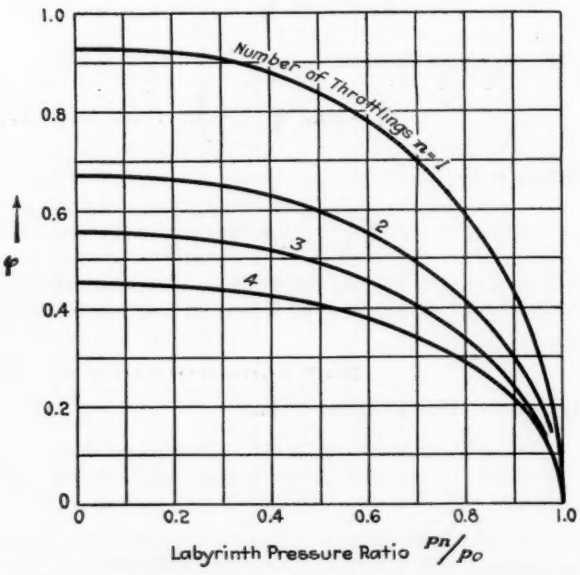


FIG. 7a LEAKAGE FUNCTION φ FOR LABYRINTHS WITH ONE TO FOUR THROTTLINGS

$$\psi\beta = p_{n+1}/p_0 = \left(\frac{1}{p_n/p_0}\right)\varphi_{n,p_n/p_0} \quad \ldots\ldots [16]$$

which, when φ_n for p_n/p_0 is known, can again be solved graphically for φ_{n+1}

$$\varphi_{n+1,p_{n+1}/p_0} = \varphi_{n,p_n/p_0} \quad \ldots\ldots [17]$$

If the number of $n + 1$ throttlings is sufficiently large, the throttling pressure ratio β in the first n throttlings is greater than 0.8 and it is only in the last $(n + 1^{th})$ throttling that β is smaller than 0.8. For the first n throttlings, therefore, an explicit formula, based on the theoretical Equation [4], can be developed. The formula, as derived in the appendix, takes the form

$$\varphi_n = \sqrt{\left[\frac{1 - (p_n/p_0)^2}{n + \log_e (p_0/p_n)}\right]} \quad \ldots\ldots [18]$$

and is identical with the expansion term in Martin's[6] labyrinth formula.

The solution with Equations [16], [17], and [18] was used in calculating the φ curves for $n = 8$ to 36 of Fig. 7.

It is now possible to predict the leakage through any ideal labyrinth by applying Equation [9] and utilizing Figs. 7 and 7a, once the flow coefficient α has been determined by actual leakage measurement.

The pressure distribution in the labyrinth is obtained from Fig. 7 by simply intersecting the curves n = constant with the horizontal line φ = constant corresponding to the operating conditions.

APPLICATION OF THE THEORY ON THE TYPES OF LABYRINTHS USED IN PRACTICE

From the point of view of flow, there are two types of labyrinths used in practical turbine design; (*a*) a so-called "straight-through" type as shown in Fig. 8, and (*b*) the staggered type schematically shown in Fig. 9.

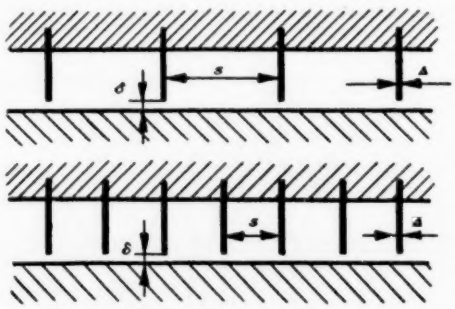


FIG. 8 STRAIGHT-THROUGH LABYRINTHS

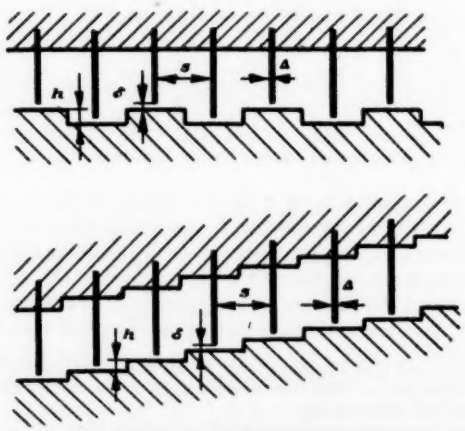


FIG. 9 STAGGERED-TYPE LABYRINTHS

The staggered-type labyrinth obviously gives a better seal than the straight-through type, but it is more costly and involves a more difficult machine assembly. By making the steps h (Fig. 9) sufficiently high, it is possible to have practically all

[6] "Steam Leakage in Dummies of the Ljungstrom Type," by H. M. Martin, *Engineering*, Jan. 3, 1919, pp. 1, 2, and 3.

of the kinetic energy of the steam jet destroyed before the steam enters the following throttling. The theory of the ideal labyrinth can, therefore, be applied directly to this case. Leakage and pressure distribution are obtained with Equation [9] and Figs. 7 and 7*a* as already described.

The straight-through-type labyrinth is frequently used in turbine design since it is easy and cheap to manufacture and has many advantages from an assembly standpoint. It is not as tight, however, as the staggered-type labyrinth because it permits a considerable percentage of kinetic energy to be carried from one throttling into the next. This results in conditions represented by the heavy zig-zag line in Fig. 10. The zig-zag dashed line in Fig. 10 represents the conditions in an ideal labyrinth (with the same leakage area A and the same flow coefficient α) through which the same amount of steam is leaking. The Fanno curve is the same for both labyrinths. These two curves, representing the two conditions, demonstrate clearly that the straight-through-type labyrinth needs more strips for the same leakage than is required in the ideal labyrinth. Apparently, we

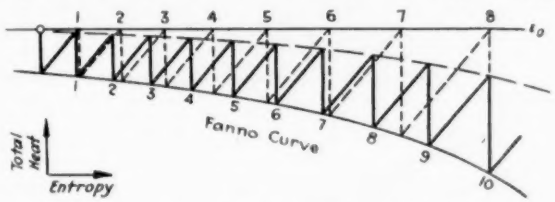


FIG. 10 CONDITION CURVE OF A STRAIGHT-THROUGH LABYRINTH

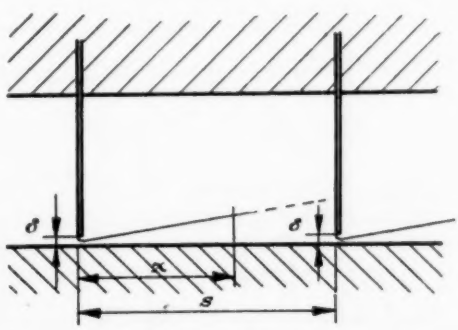


FIG. 11 BROADENING OF A STEAM JET ISSUING FROM A THROTTLING

can calculate the leakage through such a straight-through labyrinth by again utilizing Figs. 7 and 7*a* and applying Equation [9], but, instead of taking n as the number of strips, a smaller number of strips n' has to be used which will be equivalent to the number used in the ideal labyrinth. Neglecting the term $\log_e(p_0/p_n)$, it is seen that the flow through a labyrinth is approximately proportional to $\sqrt{(1/n)}$. Therefore, for the leakage through a straight-through labyrinth

$$G = A\alpha\varphi\gamma\sqrt{\left(g\frac{p_0}{v_0}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [9a]$$

where $\gamma = \sqrt{(n/n')}$, and n' = the number of strips in the equivalent ideal labyrinth. It is assumed that α is constant and of the same magnitude as determined from tests with staggered labyrinths. (Strictly, α depends on the percentage of kinetic energy carried through each throttling, i.e., on γ itself. For practical reasons of simplicity it is assumed, nevertheless, that α is constant and we take account of the change in leakage by varying only γ.) Values of γ are obtained by comparing Equation [9a] with the leakage determined from tests. As the jet issuing from the throttling broadens out proportionally with the distance x (Fig. 11), the percentage of kinetic energy available for expansion through the following throttling must be expected to be primarily a function of the ratio δ/s, where δ is the strip clearance and s is the pitch of the strips (see Fig. 8).

TEST RESULTS

Experiments have been conducted with various types of labyrinths in a static-testing device built especially for this purpose. A schematic drawing of the test arrangement and test

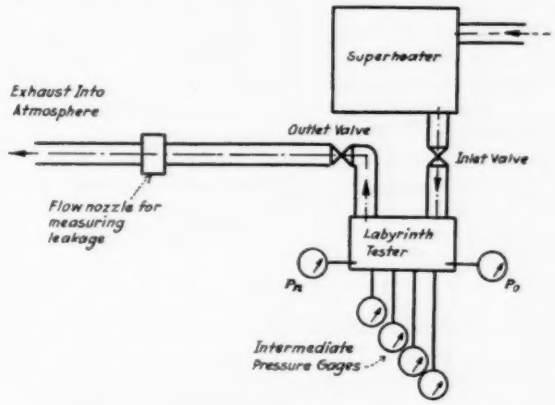


FIG. 12 ARRANGEMENT FOR THE STATIC TESTING OF LABYRINTHS

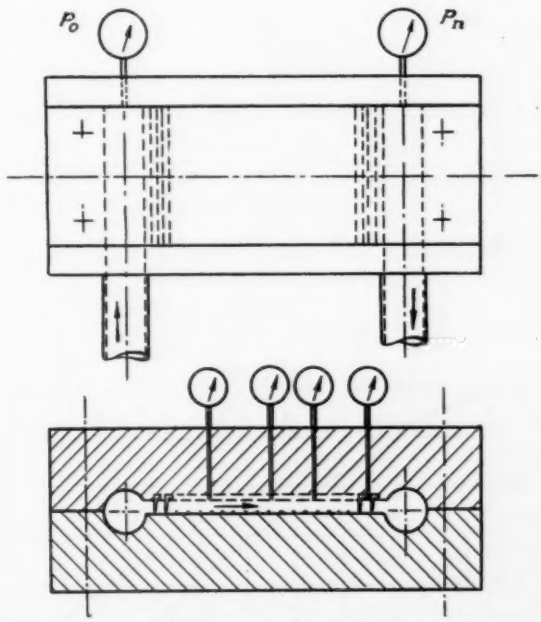


FIG. 13 TEST BLOCK USED IN THE STATIC TESTING OF LABYRINTHS

block is shown in Figs. 12 and 13. When testing, both the inlet and the back pressure were varied over a wide range, and the temperature was adjusted so that the whole labyrinth always operated with superheated steam.

Staggered-Type Labyrinth. In Fig. 14, test curves $\alpha\varphi$ are shown as a function of p_n/p_0 for a well-staggered labyrinth. Fig. 15 demonstrates the agreement between theoretical and actual pressure distribution. By dividing the values of $\alpha\varphi$ obtained from the test curves in Fig. 14 with φ from Fig. 7, the flow coefficient α is determined, which in turn is plotted in Fig. 18 as a function of the ratio δ/Δ = clearance/strip thickness (Friedrich's tests, compared on the basis of the same theory, give for

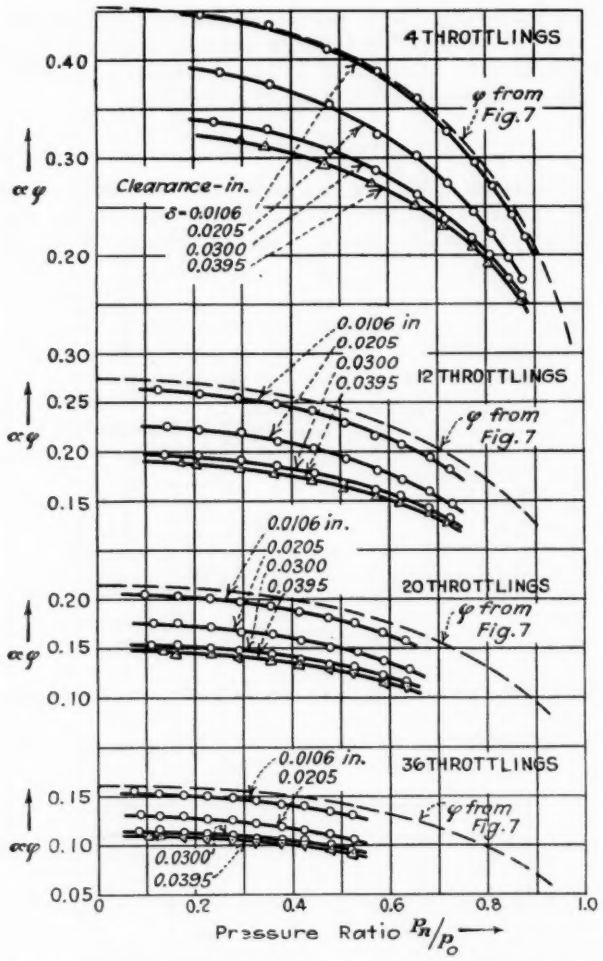


FIG. 14 RESULTS OF STATIC TESTS OF STAGGERED-TYPE LABYRINTH (Thickness of strips Δ = 0.010 in.)

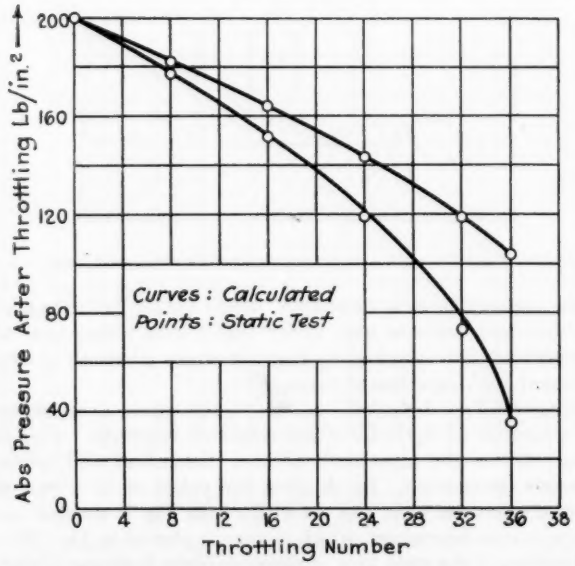


FIG. 15 PRESSURE DISTRIBUTION IN A STAGGERED-TYPE LABYRINTH

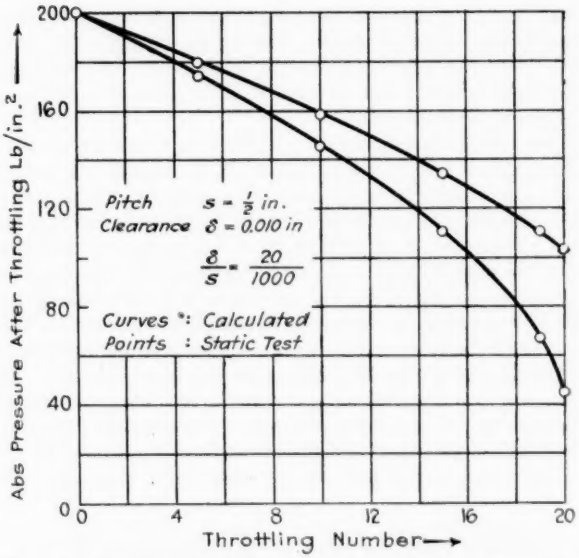


FIG. 16 PRESSURE DISTRIBUTION IN A STRAIGHT-THROUGH LABYRINTH WITH 0.5-IN. PITCH AND 0.010-IN. CLEARANCE

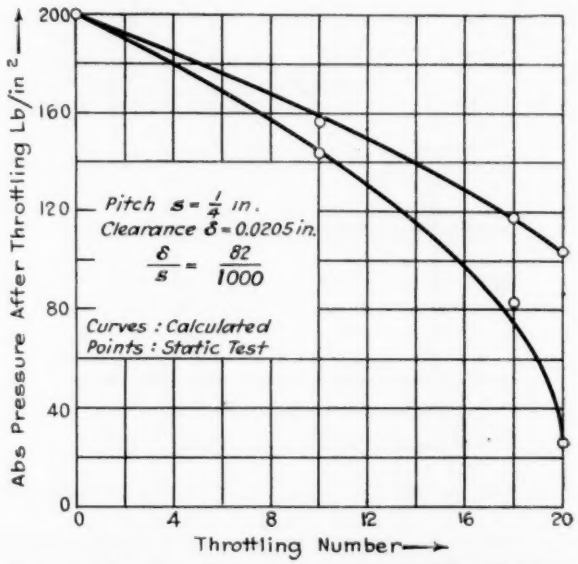


FIG. 17 PRESSURE DISTRIBUTION IN A STRAIGHT-THROUGH LABYRINTH WITH 0.25-IN. PITCH AND 0.0205-IN. CLEARANCE

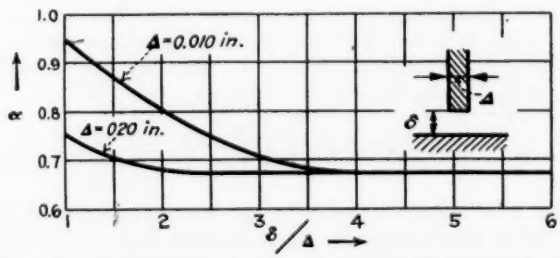


FIG. 18 FLOW COEFFICIENT α FOR LABYRINTHS WITH SHARP-EDGED STRIPS DETERMINED FROM TESTS

the range of $\delta/\Delta = 1.3$ to 2.3, an average value of $\alpha = 0.71$ which checks the average in the same range of δ/Δ in Fig. 18).[7]

[7] "Untersuchungen ueber das Verhalten der Schaufelspaltdichtungen in Gegenlauf-Dampfturbinen," by H. Friedrich, *Mitt. Forsch. Anst. G. H. H.*, Oct., 1933.

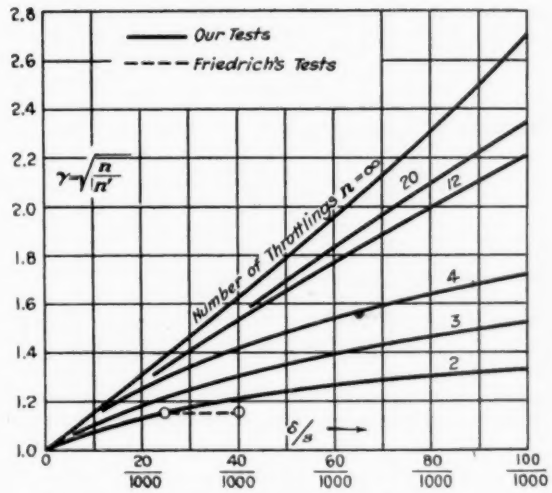
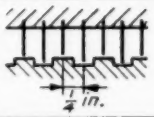
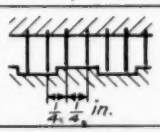
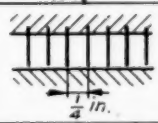
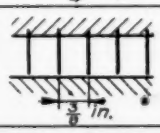
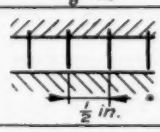


FIG. 19 CARRY-OVER CORRECTION FACTOR γ FOR STRAIGHT-THROUGH LABYRINTHS
(Refer to footnote 7 for Friedrich's tests.)

TABLE 2- LABYRINTHS WITH CONSTANT LEAKAGES

These Labyrinths (with 0.010-in. Thick Sharp-edged Strips) have the Same Leakage

Type of Labyrinth	Clearance - Inches 0.010		0.020		0.030	
	Number of Strips n	Total Length Inches	Number of Strips n	Total Length Inches	Number of Strips n	Total Length Inches
	4	1.0	10	2.5	18	4.5
	5	1.25	18	4.5	34	85
	8	2.0	54	13.5	183	45 7
	5	1.9	35	13.1	97	36.4
	5	2.5	25	12.5	64	32.0

Straight-Through Type Labyrinths. During tests of this type labyrinth, the pitch s and the clearance δ were both varied. The pressure distribution obtained with Fig. 7 checks the test results very closely as can be seen from a study of Figs. 16 and 17. The carry-over correction factor γ was calculated upon the assumption that α is the same as found in the tests of the staggered-type labyrinths. It is found that γ is a function of the ratio δ/s = clearance/pitch, only, and is given in Fig. 19. When evaluating these tests it was considered that a factor $\gamma = 1$ always applies to the first throttling of a group, and it was necessary to solve graphically the equation

$$\varphi_{n-1,\,p_2/p_1} = \frac{1}{\gamma_\infty}\,\frac{1}{p_1/p_0}\,\varphi_{1\,p_1/p_0}$$

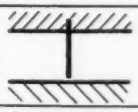
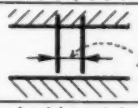
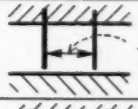
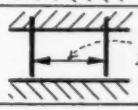
Friedrich's findings[7] with straight-through-type labyrinths of two and three throttlings are somewhat contradictory to our test results. The discrepancy can be explained probably when we consider that Friedrich tested clearances of as little as 0.006 to 0.010 in. It seems possible that for such narrow clearances, the boundary layer which develops along the smooth surface across the strip prevents kinetic energy from being carried over from one throttling to the other. Our tests were made with clearances of 0.015 to 0.040 in., which is about what occurs in axial-flow turbines built in this country.

CONCLUSION

The leakage through labyrinth seals with constant-leakage area is calculated by applying Equation [9a], in which α depends on the type of sealing strip used. For sharp-edged strips, α is given in Fig. 18.

TABLE 3-COMPARISON OF SINGLE-STRIP AND DOUBLE-STRIP SEALS
(Thickness of Sharp-Edged Strip Δ = 0.010 In.)

Clearances of One Column Give the Same Leakage

Type of Seal	Clearance - Inches					
	0.010	0.015	0.020	0.030	0.040	0.050
	0.012	0.017	0.022	0.031	0.041	0.050
	0.013	0.018	0.023	0.033	0.043	0.053
	0.014	0.020	0.025	0.035	0.045	0.055

The value of γ is equal to unity in well-staggered labyrinths. In straight-through labyrinths, γ depends on the ratio δ/s (clearance/pitch) and the number of throttlings in the labyrinth group (Fig. 19).

The expansion function φ is given graphically in Figs. 7 and 7a.

The pressure distribution in any labyrinth with constant leakage area is obtained from Figs. 7 and 7a by intersecting with a line φ = constant.

In Tables 2 and 3, examples are calculated to demonstrate the effectiveness of various types of labyrinths. Particular attention should be given to the double-strip seals in Table 3 which are often used in sealing shrouded reaction stages. For clearances greater than 0.020 in. the single-strip seal is practically as tight as the double strip. The second strip is only effective if a step is provided to prevent the steam jet from shooting directly into the second throttling.

Appendix

The derivation of an explicit formula for the leakage through an ideal labyrinth when the throttling-pressure ratio is greater than 0.8 is as follows:

The velocity of the steam jet after having expanded adiabatically from p_1 to p_2 (Fig. 20) is obtained by integrating the energy equation

$$d\left(\frac{w^2}{2g}\right) = -vdp \quad \ldots\ldots [19]$$

with

$$p^{1/k}v = \text{constant} \quad \ldots\ldots [20]$$

as the condition for the adiabatic change of state we get

$$\frac{w^2}{2g} = p_1v_1 \frac{k}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{1-1/k}\right] \quad \ldots\ldots [21]$$

We write

$$\Delta p = p_2 - p_1$$

and develop the series

$$\left(\frac{p_2}{p_1}\right)^{1-1/k} = \left(1 + \frac{\Delta p}{p_1}\right)^{1-1/k}$$
$$= 1 + \frac{k-1}{k}\left(\frac{\Delta p}{p_1}\right) - \frac{k-1}{2k^2}\left(\frac{\Delta p}{p_1}\right)^2 + \ldots\ldots$$

which when introduced in Equation [21] gives

$$\frac{w^2}{2g} = -p_1v_1\left[\frac{\Delta p}{p_1} - \frac{1}{2k}\left(\frac{\Delta p}{p_1}\right)^2 + \ldots\ldots\right]$$

Since this formula is used only as long as p_2/p_1 is larger than 0.8 or $\Delta p/p_1$ is less than 0.2, the term with $(\Delta p/p_1)^2$ can be neglected so that

$$\frac{w^2}{2g} = -vdp \quad \ldots\ldots [22]$$

The equation of continuity for the jet is

$$\frac{G}{\alpha A} = \frac{w}{v'_2} \quad \ldots\ldots [23]$$

where v'_2 is the specific volume at the end of the adiabatic expansion and is obtained from

$$v'_2 = \left(\frac{p_1}{p_2}\right)^{1/k} v_1$$

or in a series of $(\Delta p/p_1)$,

$$v'_2 = v_1\left[1 - \frac{1}{k}\left(\frac{\Delta p}{p_1}\right) + \frac{k+1}{2k^2}\left(\frac{\Delta p}{p_1}\right)^2 + \ldots\ldots\right]$$

We can neglect again the term with $(\Delta p/p_1)^2$ so that

$$v'_2 = v_1\left[1 - \frac{1}{k}\left(\frac{\Delta p}{p_1}\right)\right] \quad \ldots\ldots [24]$$

Combining Equations [22], [23], and [24] we obtain

$$\left(\frac{G}{\alpha A}\right)^2 = \frac{-2g\Delta p}{v_1\left[1 - \frac{2}{k}\left(\frac{\Delta p}{p_1}\right) + \ldots\ldots\right]} \quad \ldots\ldots [25]$$

All initial-state points to the various throttlings lie on a curve of constant total heat, along which the relation

$$p_1v_1 = p_0v_0 = \text{constant} \quad \ldots\ldots [26]$$

holds sufficiently well for gases and steam. Thus we have

$$\frac{1}{v} = \frac{p_1}{p_0v_0}$$

and combined with Equation [25]

$$\left(\frac{G}{\alpha A}\right)^2 = -\frac{2gp_1\Delta p}{p_0v_0\left(1 - \frac{2}{k}\frac{\Delta p}{p_1}\right)} \quad \ldots\ldots [27]$$

We now write this equation in the form

$$\left(\frac{G}{\alpha A}\right)^2 \frac{1}{\Delta x} - \left(\frac{G}{\alpha A}\right)^2 \frac{2}{k}\frac{1}{p_1}\left(\frac{\Delta p}{\Delta x}\right) = -\frac{2g}{p_0v_0}p_1\left(\frac{\Delta p}{\Delta x}\right)$$

which can be integrated if the number of throttlings is large enough so that $\Delta p/\Delta x$ can be replaced by dp/dx.

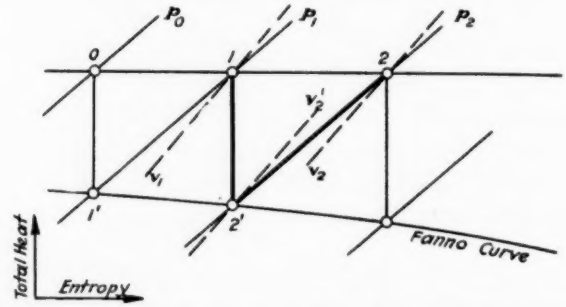


FIG. 20 CONDITION CURVE FOR AN IDEAL LABYRINTH THROTTLING

We write

$$\frac{1}{\Delta x}\left(\frac{G}{\alpha A}\right)^2 \int_{x_0}^{x_n} dx - \left(\frac{G}{\alpha A}\right)^2 \frac{2}{k}\int_{p_0}^{p_n}\frac{dp}{p_1} = -\frac{2g}{p_0v_0}\int_{p_0}^{p_n} p_1dp$$

and, noting that $(x_n - x_0)/\Delta x = n$ = the number of throttlings, we obtain as the leakage formula

$$G = A\alpha\varphi\sqrt{\left(g\frac{p_0}{v_0}\right)} \quad \ldots\ldots [28]$$

in which

$$\varphi = \sqrt{\left(\frac{1 - (p_n/p_0)^2}{n + \frac{2}{k}\log_e(p_0/p_n)}\right)}$$

where p_n = pressure after n^{th} throttling, and $k = 1.3$ for superheated steam.

It has been found by comparing φ from Equation [28] with the thermodynamically strict graphical solution, using the Fanno curve, that for small numbers of strips, a more correct numerical result is obtained if for steam $k = 2$. For large numbers of throttlings n, the logarithmic term is of little importance. We, therefore, write for any number of throttlings, the final formula for the leakage as

$$G = A\alpha\varphi\sqrt{\left(g\frac{p_0}{v_0}\right)} \quad \ldots\ldots [28a]$$

in which

$$\varphi = \sqrt{\left(\frac{1 - (p_n/p_0)^2}{n + \log_e(p_0/p_n)}\right)}$$

This formula can with sufficient accuracy also be used for air and other gases.

The Division of Load Among Generating Units for Minimum Cost

By JAMES E. MULLIGAN,[1] CAMBRIDGE, MASS.

The criteria for minimum-input division of load among several generating units which are to be operated together depend upon the forms of the input-output curves of the units. When the curves are continuous and uninflected the greatest input-curve slope at which any unit operates should be kept a minimum. When the curves are discontinuous or inflected a comparison of two curves of total input against total output will determine the load at which a change should be made from one loading definitely prescribed by the slopes of the input curves, to another definite distribution. When a varying load is to be supplied with part of the units operating at constant load, minimum-input loading may be determined by means of curves of average input against average output.

DURING the whole history of power generation an enormous amount of effort has been applied to the problem of increasing the efficiencies of individual generating units. The fruitfulness of this endeavor is well known. The margin of possible improvement in present types of apparatus has, however, become so small that it is at least exceedingly unlikely that the striking improvements of recent years can be duplicated.

In view of the limitations on further increase of unit efficiencies and of a current hesitancy to increase capital expenditures in order to decrease future operating costs, it is not surprising that attention has recently been turned to the increasing of system efficiency by a better distribution of the load among existing units. The improvement obtainable may be large, a case having been reported where an effort to operate a hydro plant with optimum load distribution resulted in an increase in output of approximately eight per cent.[2]

Although the development of methods for the optimum distribution of load has been altogether from the viewpoint of the operation of power-generating apparatus, the principles involved have no such limited application. It should be recognized that the same principles may be used to obtain a given output with minimum input to several productive agencies of any kind, whether the agencies are turbo-generators, generating stations, machines, or factories.

The first publication of the basic principles affecting optimum distribution of a constant load between two generating units, except for the simplest case where the input curves are all straight lines, was by F. H. Rogers in 1924.[3] The treatment was extended by Rogers and Moody during the following year.[4] Since that time there have been published many papers dealing principally with the practical application of the methods of Rogers and Moody.

It is the purpose here to expand the theory underlying load division by a demonstration of the criteria for distribution where more than two units are involved, where the curves of input plotted against output are discontinuous or inflected, and where part of the units operate at constant load as the total load changes.

Any method for distributing load for maximum economy will require a knowledge of the relationship between input and output for the units concerned, with each of the inputs and each of the outputs expressed in common units. Thus hydroelectric generating units in the same station, taking water from the same forebay and delivering energy to the same bus, can be represented by their turbine discharge curves. The input of a distant generating station might be plotted against the output at the receiving end of its transmission line. In some cases dollars of cost will be the only common measure of input. Much study has been given to the problem of determining such input curves.[4,5]

When the amount of generating capacity which shall be operated to supply an expected load has been decided upon there still remain two questions: (1) Which of a number of available generating units shall be operated to supply the required capacity? (2) How shall the system load be divided among the units which are to be operated?

In general the first question will be answered by giving preference to the generating units of higher efficiency. With increasing load the units will be started in order of decreasing efficiency. In some simpler cases a comparison of individual input-output curves will indicate the order in which the units should be started. When the comparison of individual curves becomes inadequate, curves of total input versus total output for various combinations of units, with most efficient distribution of load among the units in each case, may be plotted together to give an envelope curve of minimum input. The envelope curve will determine the particular units which will give least input at each load. Such curves have been published. We are here directing attention to the second question of the preceding paragraph; that is, to the division of load among a number of units all of which are to be operated together. The no-load inputs to all of them must then be supplied and load division among them depends only on the relative increments of input above the no-load values.

---

[1] Instructor in Electrical Engineering, Massachusetts Institute of Technology. Jun. A.S.M.E. Mr. Mulligan was graduated from the University of Maine with the degree of B.S. in 1925. Since that time he has been assistant and instructor in the department of electrical engineering at the Massachusetts Institute of Technology. He received the degree of S.M. from the Massachusetts Institute of Technology in 1933. This paper is taken from a part of a thesis submitted for that degree and prepared under the direction of Dr. V. Bush. He is also the author of a paper on power-system switching.

[2] "Automatic Operator for Economy Control," by S. Logan Kerr, A.I.E.E. Trans., vol. 50, March, 1931, p. 133.

Contributed by the Power Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication in a later issue of Transactions.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "Acceptance Tests of Hydroelectric Plants," by F. H. Rogers, A.I.E.E. Trans., vol. 43, 1924, p. 568.

[4] "Interrelation of Design and Operation of Hydraulic Turbines," F. H. Rogers and L. F. Moody, *Engineers and Engineering*, vol. 42, 1925, p. 174.

[5] "Incremental Loading of Generating Stations," by M. J. Steinberg and T. H. Smith, *Electrical Engineering*, vol. 53, 1933, no. 3, p. 432 and no. 4, p. 571.

Input Curves Continuous and Uninflected

The great majority of input-output curves are either made up of straight lines or have an upward concavity which increases with the load. In Fig. 1, I_1, I_2, etc. are curves of this kind, representing input plotted against output for any number of generating units which are to be operated together. The curves are continuous, although the slopes of some of them are discontinuous. I'_1, I'_2, etc. are the first derivatives of the input curves, that is, they simply indicate the slope at all values of output of I_1, I_2, etc., respectively.

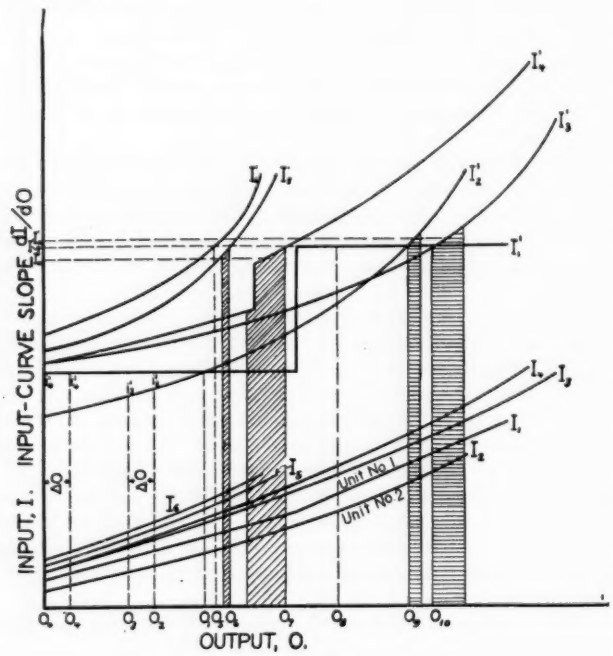


Fig. 1

It will be proved that for units with this type of input-output curve the load should always be so apportioned that the greatest value of input-curve slope at which any unit operates is kept a minimum. This criterion will be examined for three cases; for low values of slope which apply to only one unit, for intermediate values where there may be any number of units, and for the highest values applying only to the unit with greatest slope.

Any loads in the range from no load to O_1 should be applied to Unit No. 2 because the slope of I_2 is less than that of any other in this range. Suppose a load of O_2, which is all being supplied by No. 2, is redistributed by applying any fraction of it to any of the other units. For instance, suppose the load on No. 2 is reduced by the amount ΔO from O_2 to O_3 and the load on No. 1 is increased by the same amount from O_3 to O_4. The decrease in input to No. 2, $\frac{dI_2}{dO}\Delta O = \Delta I_2$, is shown by the area $O_3i'_3i'_2O_2$, with a width equal to the change in output and a height equal to the slope of the input curve. The increase in input to No. 1, $\frac{dI_1}{dO}\Delta O$, is shown by the area $O_3i'_3i'_4O_4$, of the same width but of greater height than $O_3i'_3i'_2O_2$. Since

$$\frac{dI_2}{dO} < \frac{dI_1}{dO}, \text{ then } \frac{dI_2}{dO}\Delta O < \frac{dI_1}{dO}\Delta O, \text{ and } \Delta I_2 < \Delta I_1.$$

That is, the decrease in input to No. 2 is less than the increase in input to No. 1, and the redistribution of load has resulted in a net increase in input at constant output. For best efficiency all the load should be supplied by the unit whose slope is least at the given load.

Total loads greater than O_1 cannot be applied to a single unit so that it will be loaded to a point where its slope is less than that of any other, but the load should still be so distributed that the greatest slope at which any unit operates is as small as possible. This will lead to operation with load so divided that all units operate at equal values of slope. Any load in the range of equal slopes,

$$O = O_6 + O_6 + O_7 + O_8 + O_9 + O_{10}$$

should be so distributed that the operating points on all the input curves have the same slope, I', as is now to be proved. Suppose any other distribution of load whatever be used. Then in order that the total output still be O, some of the outputs will be less with smaller values of slope and some greater with larger values of slope than with the equal-derivatives distribution. Let the load of N of the units be less by a total of $\Delta_1 O$ and that of M of the units be greater, with the loads of any number of the units remaining unchanged. With O constant the load on the M units must be greater by a total of $\Delta_1 O$.

The decrease in input to the N units is shown in Fig. 1 by the diagonally cross-hatched areas of average height I'_N and the increase to the M units by the horizontally cross-hatched areas of average height I'_M. Since the derivatives either remain constant or increase with increased load, $I' < I'_M$ and $I'_N < I'$ and therefore $I'_N < I'_M$. The sum of the widths of the N areas representing load decrease is equal to the sum of the widths of the M areas representing load increase, each sum being $\Delta_1 O$. The total decrease in input is then $I'_N\ \Delta_1 O$ and the total increase $I'_M\ \Delta_1 O$. Since $I'_N < I'_M$, $I'_N \Delta_1 O < I_M \Delta_1 O$ and the net input has again been increased by any change from operation at minimum slope.

When increase in load at equal slopes results in any unit reaching its maximum output, equal-slopes operation of all units will have to be abandoned and minimum input for higher loads will result from leaving the output of that unit constant at its maximum value and continuing to load the others at equal slopes. The load on the fully loaded unit will either have to be left constant or be reduced. If it is reduced, some of its load will have to be transferred to other units with greater input-curve slope, and the change will result in a net increase in input. This will hold until the unit of highest input-curve slope reaches full load.

To summarize: For input-output (input as ordinates against output as abscissas) curves which are continuous and which have no decrease of slope with increasing load, a given output will be supplied with minimum input if it is so divided among the units chosen to operate that the greatest slope at which any one of them operates is kept a minimum. At points of discontinuity in the slopes of their input curves, units should operate at constant load until load change on other units gives equality of slopes beyond the discontinuity.

Discontinuous Input Curves

Although the great majority of input-output curves are of the kind just discussed, some of them, notably those of some turbines whose valves open successively as load increases, are discontinuous or show a decreasing slope at the valve points. The criterion of equal slopes alone is inadequate with curves of these kinds.

In Fig. 2, are shown the input-output curves of two identical turbines which have an abrupt increase in input at load at O_1 where a by-pass valve is opened. The criterion of equal slopes, which means equal loads on the identical units used in this example, still holds with both units operating above, or both below, the

discontinuity. The derivative curve, however, gives no indication of the amount of increase in input at the valve point. For the range of loads between twice O_1 and the maximum load of one unit, curves of total input for each of the two conditions may be used to determine whether one unit should be operated at O_1 with the other taking the remainder of the load, or whether both units should be operated above O_1 at equal loads. The load above which it becomes economical to supply the increased input caused by opening the second by-pass valve rather than to go further up the increasing slope toward full load can be determined by the intersection of the two curves of Fig. 3. Curve BB' gives total input with one unit operating at load O_1 (Fig. 2) with its by-pass valve closed, and is AA' (Fig. 2) with O_1 added to its abscissas and I_1 to its ordinates. CC' is a curve of total input with equal loading obtained by multiplying by two the abscissas and ordinates of AA'. Evidently best operation will be that along the envelope curve of minimum input BDC', so for total loads less than O_2 one unit should operate at a load of O_1 with its by-pass valve closed, and for greater total loads the unit loads should be equal.

Although the discussion of discontinuous curves was simplified by the use of only two similar curves with a single discontinuity, the procedure is the same for any number of units, similar or not.

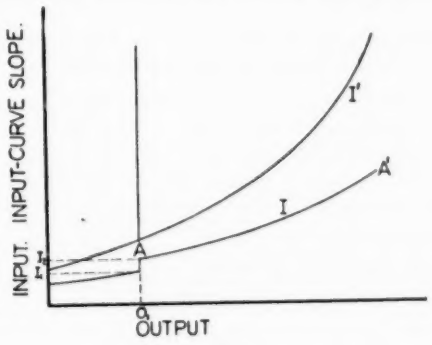


Fig. 2

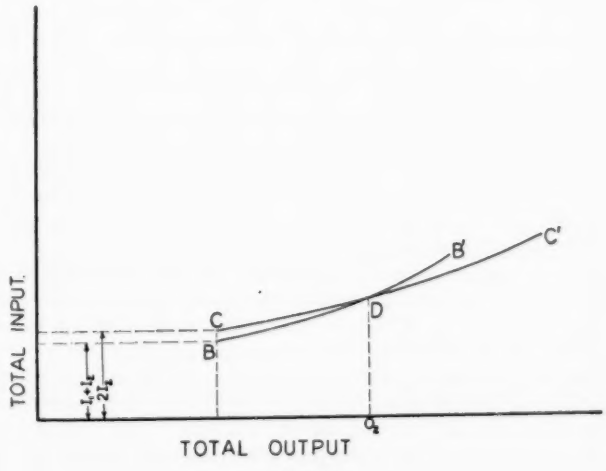


Fig. 3

The region in which equal-slopes operation must be abandoned will be determined by the comparison of curves of total input for the two methods of operation.

The use of curves of total input versus total output, as thus described, to determine the load at which operation should be changed from one distribution definitely prescribed by the derivative curves to another definite distribution, should not be confused with the use of input curves alone for minimum-input-load division. It is possible to determine optimum distribution between two units as the locus of the minima of curves of total input versus output of one of the two units, with total output as a parameter. This latter method requires the drawing of many curves even when only two units are involved, and would rapidly become unwieldy with an increase in the number of generators.

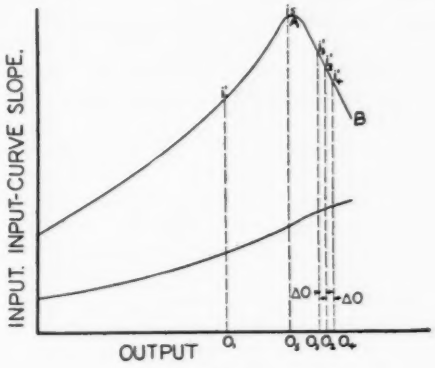


Fig. 4

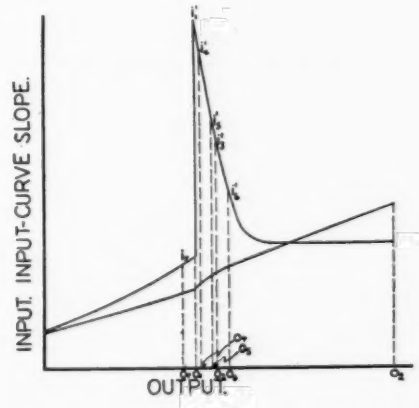


Fig. 5

The method used here requires the determination of only one intersection at each critical point in the loading schedule, regardless of the number of units in operation.

Inflected Input Curves

Another type of input curve is continuous but exhibits points of inflection. Here again, as in the case of discontinuous input curves, there will be loads at which it will be incorrect to operate at equal slopes, or to operate similar units at equal loads. The methods which have been discussed may again be used to determine most economical operation.

Fig. 4 shows an input curve whose first derivative curve has a region AB whose negative slope is greater than that of the positive slope of the curve at lighter loads. Light loads should be divided equally between two similar units of this kind. For instance, for a load of twice O_1 distributed equally the argument is the same as for single curvature; any redistribution by moving the operating point of one unit from i'_1 toward A will sweep off a larger area representing load increase than will be swept off by the moving of the operating point of the other to the left from i'_1, where the derivative curve is lower. On the other hand, a heavier load such as twice O_2 is not supplied at minimum input by operating both units at O_2. Remove ΔO from one unit and apply it to the other so that operation is at O_3 and O_4. The decrease in in-

put, $O_3i'_3i'_2O_2$, is greater than the increase in input, $O_2i'_2i'_4O_4$, and the change has resulted in a net decrease in input. Improvement will continue with further change until one unit is operated at full load. The division between the regions where the units should be operated at equal slopes and that where one should operate at full load is at a load of twice O_5, the load where the area under the derivative curve from i'_5 to full load is equal to the area under the curve from i'_5 an equal distance to the left. For any greater load a change to full load on one unit will result in decreased total input, because the area under the derivative curve from any such point to full load is less than the area from the same point an equal distance toward no load. For a practical determination of O_5 the measurement of areas under the derivative curve may be avoided by comparing two curves of total input, as was done in the case of discontinuous input curves. A curve of total input with equal loads and one of total input with one unit at full load will intersect at a total output of twice O_5.

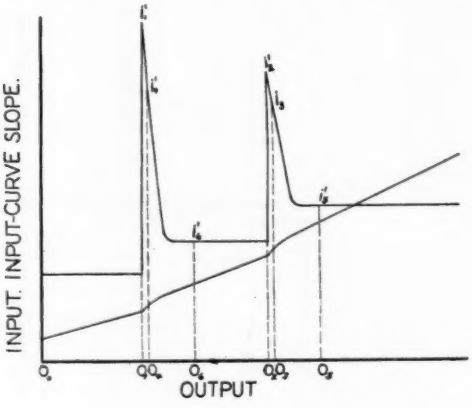


FIG. 6

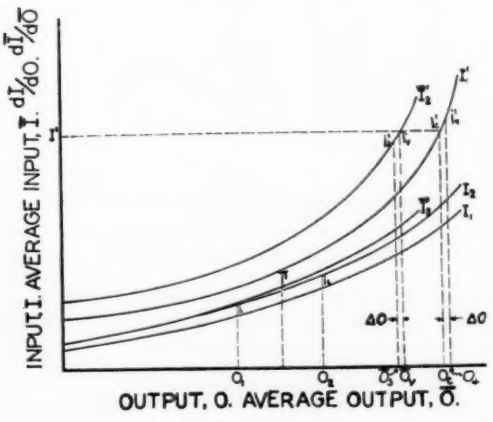


FIG. 7

In Fig. 5 the input curve again has points of inflection, but here the change in curvature is abrupt at a valve point, giving a derivative curve with positive slope greater than negative, and making unnecessary the use of curves of total input in prescribing load division. For loads below twice O_1, two equal units should divide output equally. For loads greater than twice O_1 and less than $O_1 + O_2$, one unit should carry O_1 and the other should carry the remainder. For greater loads one unit should operate at full load. For loads between twice O_1 and $O_1 + O_2$ the reasoning is as follows: Let the units operate at O_1 and O_3. A change of distribution by increasing load on one unit above O_1 will result in an input increase $O_1i'_1i'_4O_4$, which is greater than the decrease in input $O_3i'_3i'_2O_2$ caused by an equal decrease in output of the other unit. In the same way a decrease in output below O_1, with an equal increase above O_3, is shown to result in a net increase in input by the greater height of area $O_3i'_3i'_6O_6$ over $O_1i'_1i'_7O_7$. Therefore, for any load in this range one unit should operate at O_1. For the other two ranges of load the reasoning is similar to that in cases which have been treated previously and need not be repeated.

For curves with more than one point of inflection, curves of total input may again be used to determine critical points in the operating program. In Fig. 6 loads from O_0 to twice O_1 should be distributed equally, and for loads from twice O_1 to $O_1 + O_3$ one unit should operate at O_1 as in the previous case. For a range of loads beyond $O_1 + O_2$ it will be better to supply a smaller increment of input, $O_2i'_2i'_5O_5$, than the larger one, $O_1i'_1i'_4O_4$, so with increasing load one unit should remain for a time at O_1. Because of the increasing trend of curvature, the more heavily loaded unit may reach a load such as O_5, where a decrease in its load to O_3 will result in a decrease in input which is greater than the increase produced by an equal increase in input to the other unit beyond O_1. In the figure this point would be reached when $O_2i'_2i'_5O_5 = O_1i'_1i'_6O_6$, $O_6 - O_1$ being equal to $O_5 - O_3$. This load may be determined practically by the comparison of curves of total input for one unit at O_1 and for one unit at O_2.

Distribution of a Varying System Load—Constant-Load Operation of Some Generators

The criteria for least input which have been discussed can be followed exactly only if the loads of all units can be adjusted continuously as the total load changes. The ordinary method of power-system operation, where part of the generating units operate at fixed throttle, and the remainder control frequency by sharing the load changes, is not covered.

Although an assumption that good operation will be obtained by keeping the loads near those which the previous theory would dictate (by, for instance, keeping the load on constant-load units equal to the average load on similar frequency-controlling units) may in some cases give nearly optimum operation, a definite criterion is desirable. Approximations may be far wrong when the load swing on the frequency-controlling units is large. Such a definite criterion may be used in comparing the relative efficiency of different methods of operation, to show the improvement which would result from a more frequent readjustment of the load on constant-load units, or in determining the advantage to be derived by having more units share in frequency control.

A criterion for this case is easily established if it is assumed that a curve of load against time will be a straight line within the range of load change absorbed by the frequency-controlling units. The effect of this assumption with a typical load curve and a typical range of load swing on the frequency-controlling units was investigated and the assumption was found to be justified. In Fig. 7, I_1 is the input curve of the unit or units which are to be operated at constant load while the system load changes by $O_2 - O_1$. The load change is to be absorbed by a frequency-controlling unit or units represented by input curve I_2. If there are several constant-load units their share of the load is distributed in accordance with the principles previously discussed, and I_1 represents total input to them. If there are several frequency-controlling units their controls are so interlocked that they always operate with optimum distribution of load among themselves, so that I_2 may be treated in the same way as the input curve of a single frequency-controlling unit.

Curve $\bar{I}_2$ shows average input plotted against average output for the varying-load units as the load is changed uniformly over an interval, $O_2 - O_1$. The method of determining a point on this curve, $\bar{i}$ for instance, is as follows: Area $O_1i_1i_2O_2$ is measured and

divided by the length of its base, $O_2 - O_1$, to give the ordinate of $\bar{i}$. Then $\bar{i}$ is plotted against average output in the interval and under the assumption of linear load change with time, the average is at the middle of the interval, at $\frac{O_1 + O_2}{2}$. For other points the position of the constant interval, $O_2 - O_1$, on the output scale is changed. $\bar{I}'_2$ is $\frac{d\bar{I}_2}{d\bar{O}}$, the derivative of $\bar{I}_2$, and I'_1 is $\frac{dI_1}{dO}$ the derivative of I_1.

It can be proved by a method similar to those already used that the average input will be minimum when the constant-load units are operated so that the slope of their input curve is equal to the slope of the curve of average input to the frequency-controlling units. A load which is to vary over the range $O_2 - O_1$, for which I_2 was drawn, is distributed so that

$$\frac{dI_1}{dO} = \frac{d\bar{I}_2}{d\bar{O}} = I'$$

with the constant-load units carrying a steady load of O_c and the frequency-controlling units supplying a varying load whose average is $\bar{O}_v$. It may be shown that the total average load $\bar{O} = O_c + \bar{O}_v$ is being supplied with minimum average input by decreasing the average load on the frequency-controlling units by any amount, ΔO, and increasing the load on the constant-load units by ΔO. The average output is unchanged at $\bar{O}$. From the geometry of the figure, the total average input has been increased because the increase in constant input, $O_c i'_c i'_4 O_4$, is greater than the decrease in average input, $\bar{O}_3 i'_3 i'_v \bar{O}_v$.

An increase in the average load on the frequency-controlling units with an equal decrease in load on the constant-load units would evidently give the same result of increased average input at the same average output. The equal-slopes distribution gives best operation.

All other methods of load distribution, equal fractions of rating, equal efficiencies, all units but one at maximum efficiency, etc., will lead to larger inputs except when they accidentally coincide with the methods which have been discussed.

Discussion

The Design of Light-Weight Trains[1]

E. L. Larson.[2] I regret that I cannot agree with Dr. Eksergian's statement, "On the other hand, without any partition whatsoever, the torsional effect is manifested by a couple exerted on the main side frames, ..." in the section entitled, "Torsional Strength Through Step Wells and Openings." His plain meaning is that the roof and underframe-floor lateral construction do not exert a couple "without any partition whatsoever." Actually, they exert a couple under this condition. Equilibrium in the doorway region can be established in two ways: (1) Without rigid bulkheads ("without any partition whatsoever"), by means of (*a*) a couple exerted by the roof and the underframe-floor construction acting as lateral members, and (*b*) a couple exerted by the side frames acting as vertical members; (2) with rigid bulkheads fore and aft of the door openings, by means of either couple (*a*), or (*b*), or both couples. In other words, with rigid bulkheads, one couple is sufficient for equilibrium but without rigid bulkheads, one couple is not enough; both couples are required. The actions in portions of the framing depend upon how these portions function with respect to the couples mentioned. Local torque, in the individual framing members across the door opening, is of minor importance compared with the action of the couples discussed, and, therefore, the local torque will not be referred to after this. For the sake of brevity, an intermediate condition, which might be termed "partially rigid bulkheads," will not be discussed. Theoretically, what we are here calling a rigid bulkhead is at best but a partially rigid bulkhead. For the same reason, the action of the bulkheads in attempting to retain the original form of cross-section (such as trying to keep a rectangular section from becoming that of a parallelogram) will not be referred to.

The accompanying diagram, Fig. 1, and equations show why the two couples, instead of only one couple, are required in the case of the door region when "without any partition whatsoever." The diagram and the equation set forth an approach to a first approximation. For the sake of simplicity, a car with rectangular cross-section is considered. Some tacit assumptions are made as to end conditions. The diagram is essentially that of a box unfolded, with sides separated. Only one source of torsion is considered, namely, uneven jacking at diagonally opposite corners of an articulated unit of a train. In order to avoid complication of force arrows, the total torsional moment transmitted to the unit by the external forces is indicated by T and the semi-circular arrows in the end views. The deflections are not shown.

From the diagram, it is evident that "without any partition whatsoever," equilibrium of moments is established in the various planes as follows:

$$\text{In the end views:} \quad fW\,H + fH\,W = T \text{ or } 2fHW = T$$
$$\text{In plan and bottom view:} \quad fW\,L = fL\,W$$
$$\text{In side elevations:} \quad fH\,L = fL\,H$$

From equilibrium in the end views, it follows that the portion of the total torsional moment resisted by (*a*), the couple exerted by the roof and the underframe-floor construction acting as lateral members, is equal to the portion of the total torsional moment resisted by (*b*), the couple exerted by the side frames acting as vertical members.

Obviously, the approach indicated in the diagram does not apply to open top cars, nor does it apply to certain types of closed-top freight cars, those with so-called flexible roofs. The action of torque perhaps explains some of the creaking of wood-framed passenger cars.

The five equations, at the end of the section referred to in Dr. Eksergian's paper, can be made of service under the conditions for which he evidently intended them ("without any partition whatsoever") by substituting $T/2$ for his T, where T still has the value assigned by him, namely, the total torsional moment transmitted across the door opening.

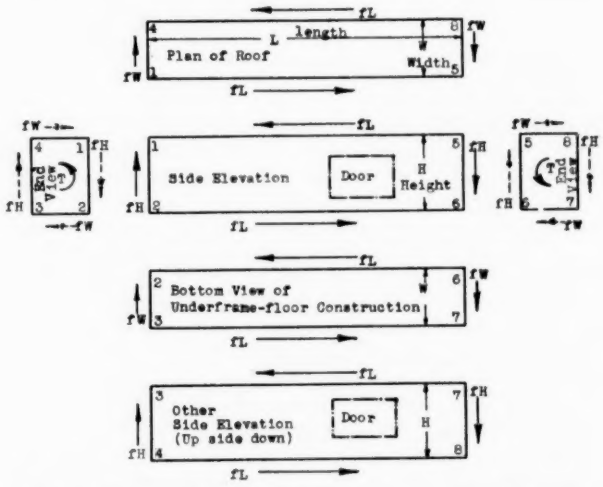


Fig. 1 General Torsional Action in a Car With Hollow Rectangular Cross-Section, Without Rigid Bulkheads Fore and Aft of the Side Door Openings

[All views are *outside* views.
The arrows with long heads refer to external forces or moments, in the plane considered, with reference to the member of the car shown.
The forces shown in the end views by dotted arrows show how equilibrium is established in the end views with reference to the applied torque.
The numerals, 1 to 8 inclusive, refer to the corners in order to assist in comparing the views.
The thicknesses of the members are neglected for simplicity in discussing the general overall actions and because secondary actions are neglected. (Among the secondary actions are restraints, at edges, other than those of direct shear.)]

Dr. Eksergian also states, ". . . ., if the bulkheads are very rigid, a pure torque is applied by the truss to the openings." In this case, probably in most instances, there will be considerable beam action on the part of the members extending across the door openings, whereas the pure torque in these members will be of less importance, and then this will be merely as secondary action.

As a matter of possible interest, reference might be made to "The Torsional Stress of Wings," by C. P. Burgess in the fifteenth annual report of the National Advisory Committee for Aeronautics, 1929, as well as to "The Torsion of Members Having Sections in Common in Aircraft Construction," by G. W. Trayer and H. W. Marsh in the same report.

[1] Published as paper RR-56-4, by R. Eksergian, in the September, 1934, issue of the A.S.M.E. Transactions.

[2] Chicago, Ill. Mem. A.S.M.E.

J. C. FITTERER.[3] In recent years, the automobile and the airplane have seriously challenged the economic existence of the locomotive as a result of the design of internal-combustion engines with a marked reduction of weight per horsepower developed, and the introduction of cheaper and better fuels for these engines. The young science of metallurgy is swinging into a vigorous stride and places at our disposal alloys such as duraluminum and stainless steel, in which desirable properties have been obtained by controlling composition. Very often, substances of three ingredients offer useful compounds whose properties can be plotted on a tri-linear diagram in which each element occupies one of the vertices. Any point, at a vertex, on a side, or within the triangle, locates a substance containing one, two, or all three ingredients. The memorable researches of Prof. Thurston clearly brought this means of graphical analysis into general prominence. Any property under investigation can be noted for a given composition, and after the field has been thoroughly covered, a characteristic topography may be developed, and the compound best suited for a given purpose easily selected.

Stainless steel possesses a triangular array of its own in which steel, chromium, and nickel compose the leading triumvirate. Compounds of these furnish interesting avenues of investigation of stainless steels and enhance our knowledge of their properties such as tensile strength, compressive strength, torsion, flexure, corrosion, weight, etc. (with or without previous heat treatment). Suitable alloys, thus attained, have contributed much to the design of engines of low unit weight, such as found in the *Zephyr*.

Regarding mathematical calculations developed in the paper, the unhesitating use of the process of resolving indeterminateness by setting up an inclusive energy function and differentiating, may be an augury of greater future usefulness in the application of modern methods on the part of the progressive engineer thoroughly grounded in the use of mathematical analysis.

As an addendum or extension to Dr. Eksergian's paper, it might be interesting and practical for some one to investigate the effect that varying widths of track gage would have on the economics of design in equipment of the *Zephyr* type. We have railroad-track gages, varying from the monorail to gages wider than the so-called standard gage. We might include among them the negative gage as a limiting case, clearly indicating the airplane which is not fettered to any ground track whatsoever. If a suitable function could be set up, the orthodox process of differentiation and equating the derivative to zero might lead to some suggestive results applicable to future designs. Any reduction in costly trackage must surely reflect its helpfulness in diminishing capital charges and, perchance, in a further reduction in the cost of operation, all of which may prolong the economic life of the railroads in competition with newer developments arising on all sides. Adaptation to the changing order of events is surely a healthful sign of continued progress.

A. M. WRIGHT.[4] Dr. Eksergian states that train resistance may be divided into two primary components which are (1) that due to rolling and bearing friction, and (2) that due to wind resistance. This is true, but from tests on the actual performance of trains, it is impossible to determine what portion of the total resistance is due to friction, and what portion is due to the air. All tests hitherto made indicate merely that the total resistance can be approximately represented by the equation

$$R = A + BV + CV^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where V is the velocity of the train, and A, B, and C, are constants depending on the dimensions and weight of the train. It is usual to assume that the term CV^2 is due to air resistance alone, and in designing a train of a radically new type, such as the *Zephyr*, it was logical to resort to wind-tunnel tests to determine the value of the constant C. Such tests, however, must be accepted with caution, for several reasons. An important one is the scale effect. It is well known that for the flow around similar bodies to be similar in any two cases, the Reynolds number $\left(\frac{\rho v l}{\mu}\right)$ must be the same in both cases. When testing a small scale model in a wind tunnel, this equality requires the velocity used to be much higher than the velocity of the full-size train. It is found, however, that the drag coefficient changes very little over a wide range of Reynolds' numbers above the critical region, so that the tunnel tests may usually be carried out with reasonably low wind velocities. On the other hand, it seems very necessary to know just how much this small variation in the drag coefficient amounts to, in order that the results obtained with one value of vl may be applied to another. An error in the coefficient C in the train resistance equation may lead to very incorrect results at high speeds. To the writer, it is not obvious that the same scale effect applies to both the conventional and the streamlined train, for the air flow is by no means the same in both cases. Another point which would seem to require careful attention in testing such an elongated body as a train model is that the velocity of the undisturbed air stream should be uniform over the whole length occupied by the train. It would appear that very different results would be obtained with different types of wind tunnels due to variations in velocity along the length of the model. Dr. Eksergian makes no mention of this effect, and it would be interesting to know how this was allowed for in predicting the train resistance of the *Zephyr*.

For the above reasons, Dr. Eksergian's train-resistance formula can be considered only as contingent. The most satisfactory way of determining the train resistance, of course, is by road tests. In a train of the Zephyr type, which will always operate as a unit, resistance values can be obtained very simply from a coasting test. The only apparatus needed is an accurate speedometer. Suppose the power is shut off from the motors at some point on the run, and that the train is allowed to coast down to a low speed or even to a stop, under the retarding influence of the train resistance and the grades encountered. At the instant of shutting off power, the total energy in the train is composed of potential energy due to its elevation H above some datum level, and kinetic energy due to its velocity v. The total energy in the train is then

$$E = WH + \frac{Wv^2}{2g}$$

where g should be taken at about 30 instead of 32.2, to allow for rotational inertia.

Dividing through by W, $\frac{E}{W} = H + \frac{v^2}{2g}$ ..... [2]

In Fig. 2, let the distance traversed from the point of shutting off power be denoted by the abscissas, and the quantity $\left(\frac{H + v^2}{2g}\right)$ feet be denoted by the ordinates. Then if OM is the total value of $\frac{E}{W}$ at the beginning of the test, it is clear that if there were no train resistance, this value would remain constant ever after, as shown by the line MN. Now plotting the profile of the railway on the same diagram, as shown by the line $OABCD$, the distance of the points on this line above the axis OX give the values of H in the above equation, and the vertical distances between $OABCD$

[3] Professor of mathematics, Colorado School of Mines, Golden, Colo.

[4] Electrical engineer, The Reading Company, Philadelphia, Pa.

and the line OM are then equal to $\frac{v^2}{2g}$. From this the velocity may be obtained at any point. The quantity $\frac{E}{W}$ in Equation [2] has the dimension of a length, and may be referred to as the "total head," from analogy with hydraulics. Now actually there is a dissipation of energy due to the train resistance, so that the

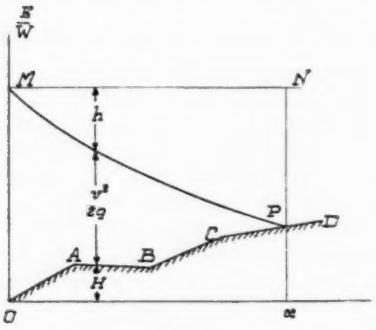


FIG. 2

line MN is lowered to some such position as MP. This line may be obtained by observing the speed at marked points on the route of the test, and plotting the value $\frac{v^2}{2g}$ above the line of the profile $OABCD$ on the diagram. It is then obvious that length NP gives a measure of the work done in overcoming train resistance while the train travels from O to P. Symbolically, if we denote the ordinates of the curve MP by h, and the elevation of the track above the datum line OX by H, we have for the work done against the train resistance

$$\frac{1}{W}\int Rdx = OM - h = OM - \left[\frac{v^2}{2g} + H\right] \ldots\ldots [3]$$

We can thus, by differentiating the curve MP, get a value of the total resistance R at each point, and to this corresponds an observed value of the speed. The train resistance at each speed is therefore completely determined.

Since the performance curves shown in Figs. 1 to 3 in Dr. Eksergian's paper are based on the wind-tunnel tests of train resistance, they also must be regarded as contingent on revision of the train-resistance values.

The author's remarks on the low power requirements of light-weight trains are more significant than appears from the horse-power requirements alone when oil-electric motive power is involved. The limitations of the Diesel engine are the piston speed and the mean effective pressure in the cylinder. The latter of course is a function of the cylinder volume, to the extent that the engine cannot burn more fuel than the air in the cylinder requires for complete combustion. Leaving supercharging out of consideration, we may assume that each pound of fuel requires about 14.6 pounds of free air for its complete combustion, and in that case the horsepower developed by the engine is proportional to the piston area and to the piston speed of the engine. If H is the Btu content of the fuel per pound, and if 1 lb of free air weighs 0.0727 lb, we get for the output of the engine

$$\text{bhp} = \frac{Av}{n}\,\frac{0.0727 \times 778\,H}{14.6 \times 33000}\,K \ldots\ldots [4]$$

where A is the piston area in sq ft, v is the piston speed in ft per min, and n is the number of strokes per cycle. The constant K for modern engines is usually between 0.13 and 0.16, and it may be increased to 0.20 by supercharging. The piston speed v is usually between 1000 and 1800 ft per min. To see the variation in the size and weight of oil-electric sets for railway service, we may use Equation [4] in a hypothetical line of engines, as shown in the writer's Table 1. It is seen that as the engine size increases

TABLE 1

(Output of Diesel Generating Sets—2-Stroke Cycle)

Diam × stroke	Piston speed, ft/min	No. cylinders	Bhp (K = 0.16)	Rpm	Relative generator weight per kw%
4 × 4	1500	6	190	2250	100
5 × 5	1500	6	300	1800	125
6 × 6	1500	6	430	1500	150
8 × 8	1500	6	760	1125	200
10 × 10	1500	6	1200	900	250

the speed of revolution decreases. This has a very adverse effect on the size of the electric generator. It may easily be shown that the output of an electrical machine is proportional to D^2L, where D is the diameter of the frame and L the length. The output is also proportional to the angular velocity of the machine. Assuming the same proportional constant in all cases in the table, the last column shows how the weight of the electric generator per kilowatt increases as the size of the engine increases. The weight per brake horsepower of a large Diesel engine is also greater than that of a small engine, so that the net weight per horsepower of large-cylinder-capacity oil-electric sets is considerably greater than that of small sets. It is therefore desirable to use generating equipment of as small a capacity as possible. The implication of this is that if a service requires a given horsepower per ton of total train weight, as shown in Dr. Eksergian's Fig. 4, the total horsepower required by the train must be as small as we can make it, if a reasonable overall-weight efficiency is to result. The economical use of Diesel-electric power in important passenger service, therefore, compels the railway engineer to use equipment of as low a weight as possible, and it seems that the best field for the Diesel-electric drive in main line service is in light-weight trains of the *Zephyr* type.

An attempt to duplicate the performance of the *Zephyr* with heavy conventional equipment, using oil-electric drive, leads to extravagant sizes and weights of motive power.

AUTHOR'S CLOSURE

Mr. Larson's criticism is constructive and well received because his analysis really points out that in discussing torsional conditions through an opening, it is necessary to differentiate the loading conditions due to centrifugal inertia rolling moments which result in augmented torsional loadings under operating conditions, and extreme jacking loadings with a light car body. In the former, the end couples act as balancing moments for a distributed inertia moment along the car, while in the latter under extreme jacking conditions, with supports at the ends diagonally opposite, we have the condition of equal and opposite balancing couples at the ends of the cars. (Along with the distributed inertia rolling moment we also have distributed lateral forces, equal to the centrifugal loadings, reduced to the floor plane and balanced by lateral reactions at the center pins.) The author's discussion was primarily postulated on the former condition because it represented augmented torsional loadings which would superimpose on the loadings previously discussed in the paper, and based on maximum operating loading conditions. Mr. Larson's analysis pertains strictly only to extreme jacking conditions, and should be classified in a separate analysis since much higher stresses would be permissible. On the basis of the author's postulation of augmented operating loadings, the distributed inertia rolling moments increase the loadings on one main side frame and decrease the loadings on the other main side frame over and above the normal vertical loadings. From the point of view of equilibrium conditions alone, such augmented loadings

can be balanced entirely by vertical supports at the ends of the main side frames and no resultant longitudinal-shear forces at top and bottom of main side frames are needed since no balancing couple is then required for equilibrium of a side frame. If, therefore, reacting longitudinal-shear forces do not act on the roof, transverse-shear forces would likewise not necessarily exist in the roof or floor system. However, the ability to transmit lateral-shear forces to the roof and floor across an opening is very much dependent on the relative rigidity of the bulkheads adjacent to the opening. As an extreme case, without any partitions whatsoever, only the augmented vertical loadings and the corresponding total bending moment are maintained across the openings, such up and down reactions on either side of opening being the resisting torsional moment for the particular section at which the step well is located.

Obviously under the dynamic torsional loading postulated by the author, the torsional shear varies for different sections of the car body in a manner similar to a beam with distributed gravity loadings and balancing supporting reactions at the ends. Now in a car body, openings near the ends are subjected to drastic vertical-shear loadings resulting in high secondary bending stress. For this reason augmented torsional-shear loadings which may induce additional bending stresses become of considerable importance. With a uniform lateral centrifugal loading inducing a corresponding distributed rolling moment, it is seen that the torsional shear would coincide with the vertical-shear loading at any section of the car. For this reason the dynamic roll loading may assume considerable importance for openings toward the ends subjected to heavy vertical-shear loading and therefore to correspondingly large torsional-shear loadings, augmenting the vertical-shear loadings.

On the other hand for openings near the center of the car, though vertical-shear loadings and corresponding dynamic torsional-shear loadings are greatly decreased, ample reinforcement of the roof structure is still of major importance due to secondary bending under operating conditions, and to the extreme jacking conditions, resulting in transverse-shear forces through the roof under light load static conditions.

For the case of extreme jacking, with equal and opposite end couples, Mr. Larson presents an analysis as a proof that lateral-shear forces equal to the vertical-shear forces in side frames, are induced in the roof and floor. His analysis, however, actually postulates rigid bulkheads at the ends of the car, which may only partly exist, and also a uniform distribution of longitudinal shear which may far from exist in an actual car construction. It is, of course, true that with equal and opposite couples applied at the ends, as under the extreme jacking conditions assumed, transverse-shear forces are induced in both roof and floor system in any actual car construction. However, the magnitude and distribution of such forces at different sections of the car body depend upon the distribution and the yield of the bulkheads as well as the cross-framing of the car.

In the *Zephyr* construction, the end cross frames are relatively rigid, so that the assumption of rigid end bulkheads is permissible. For a first approximation, the author is in agreement with Mr. Larson's recommendation that without any adjacent bulkheads at the opening, and with a symmetrical car, the moment resulting from extreme eccentric jacking conditions is resisted equally by a transverse couple for roof-and-floor system and a couple with up-and-down reactions exerted on the side frames.

In the analysis of a highly redundant structure as a car body, the criterion of the equilibrium of a system of reactions acting on several portions of the structure is not sufficient to determine their distribution. The distribution is effected by the relative yields of the various elastic members. It is always possible for non-statical systems to have various combinations of equilibrating systems, where the transfer between parts is effected by the action and reaction between any two portions of the structure. In fact the condition equations of a redundant system are necessarily supplied by the equilibrium conditions for each and every portion of the structure, together with the mutual reactions between the parts. The particular distribution of the equilibrating system, however, is dependent on the relative deformation of the various members due to their elastic properties. On the basis of equal and opposite torsional moments applied at the ends of the car, it is evident that every cross-section of the car must sustain a mutual torsional-shear moment between the parts. However, the distribution of the shearing forces which make up the resultant resisting couple at the section is dependent on the elastic yield of the structure on either side of the opening. Mr. Larson does not specify the elastic properties of his system; only the balancing system of reactions, he assumes, postulates the elastic system considered and, therefore, the effectiveness of end bulkheads in transferring a lateral-shear couple to the roof and floor equal to the vertical-shear couple in the side frames.

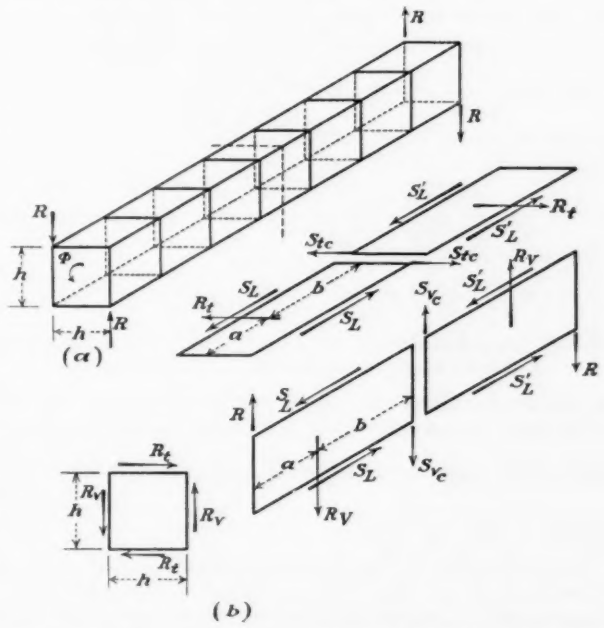


FIG. 3 SYMMETRICAL CAR WITHOUT END BULKHEADS SUBJECTED TO EQUAL AND OPPOSITE END BALANCING COUPLES

There arises the important question as to the effectiveness of partial end bulkheads as compared with rigid bulkheads in transferring transverse shear to a roof-and-floor system in extreme jacking conditions. Without end bulkheads, the following approximate analysis is of interest in showing that the maximum lateral-shear force in a roof-and-floor system may exceed values postulated on the rigidity of end bulkheads.

Case 1. Symmetrical Car Without End Bulkheads Subjected to Equal and Opposite Couples at Ends as in Extreme Jacking Conditions. Referring to Fig. 3*a*, we assume a symmetrical car with end balancing couples $\Phi = Rh$. For simplicity the car cross-section is assumed square with the common dimension, h. Let mn be the mid-section, which sustains a vertical shear S_{v_c} in the side walls, and a transverse shear S_{t_c} in the roof. We will assume the torque transferred to the roof by light bulkheads consisting simply of frame bents at panel points, i.e., posts, carlines, and cross-floor beam. Let the resultant reactions of these cross frames on the side frame be R_t on either side of the mid-section at distance b from the mid-section "a" and a distance a from ends as shown in

Fig. 3*b*. Evidently $S_{ve} = R - R_v$ for the vertical shear at the mid-section.

To obtain the longitudinal shear S_L or S'_L, observing that due to symmetry no bending noment exists at center section, we have

$$S_L h = R_v a + S_v(a + b) = R(a + b) - R_v b$$

now for the equilibrium conditions of the cross-frame bents, we have $R_v h = R_t h$, so that the resultant transverse reaction on the roof equals the vertical reaction on the side frame, and this

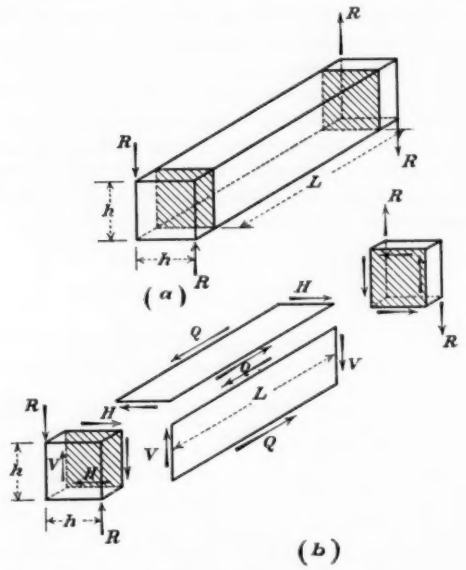


FIG. 4 RIGID BULKHEAD WITH A COUPLE THAT IS BALANCED BY A COUPLE CAUSED BY SIDE-FRAME AND ROOF REACTION

equals the transverse shear in the roof $S_{te} = R_t = R_v$. But for equilibrium of the roof, $R_t b = S_L h$, so that since $R_t = R_v$, then, $R_v b = R(a + b) - R_v b$, and since $S_{te} = R_t = R_v$, $S_{te} = R\left(\frac{a+b}{2b}\right)$ and $S_{ve} = R\left(\frac{b-a}{2b}\right)$.

It is evident that the values of the transverse- and vertical-shear forces depend upon the relative values of a and b which in turn depend upon the relative flexibilities and distribution of the bulkhead systems in general. Thus with rigid heavy end bulkheads, $a = 0$, and $b = L/2$ so that $S_{te} = S_{ve} = R/2$.

If, on the other hand, the bulkheads were so distributed elastically that $a = b$, then $S_{te} = R$ and $S_{ve} = 0$.

It is to be noted that the structure itself is assumed to be a symmetrical plated walled structure with symmetrical distribution of bulkheads. The action of a trussed system is different and is considered later.

Case 2. With Rigid End Bulkheads. As another extreme we will consider the case postulated by Mr. Larson's analysis for rigid end bulkheads. In this case, Fig. 4, the rigid bulkhead has a couple $Rh = \Phi$, which is balanced by a couple $Vh + Hh$, due to side frame and roof reaction. For equilibrium of side frame $VL = Qh$ and for the roof $Qh = HL$, so that $H = V$. This is based on the premise that the rigidity of cross frames is small compared with end bulkhead and the frame and roof are reasonably symmetrical. Since $Hh = Vh$, evidently H which also is the transverse shear for any section of the roof, is $\Phi/2h = R/2$.

Thus, on the premise of rigid end bulkheads, the transverse shear in the roof is reduced to one-half the end reaction R and the transverse shear in the roof equals the vertical shear in the side frame.

Distribution of Torsional Shear in Truss Space Framework. In order to show more clearly the effect of the elasticity of the system on the distribution of the lateral-shear forces, the simple frame structure shown in Fig. 5 has been analyzed. Three cases have been considered: (*a*) with rigid end bulkheads, (*b*) with the forward end bulkhead having a diagonal with the same rigidity as the diagonals in subsequent cross-panel frames, and (*c*) with light diagonals in both forward and intermediate cross frames with a rigid bulkhead at the back end. Evidently the transverse and vertical shear for top and bottom and the side frames correspond to the components of the diagonal tensions in these directions.

(*a*) Truss space framework with rigid end bulkheads. If Φ is the total torque transmitted, Vb is the reacting couple on the rigid bulkhead due to the vertical force V, and Hh is the reacting couple on the rigid bulkhead due to the lateral forces H, then the total torque requires the condition:

$$Hh + Vb = \Phi \text{ (equilibrium of rigid bulkhead)}$$

We may therefore choose either H or V as redundant. In addition we select the tensions in the diagonals of the truss bulkheads

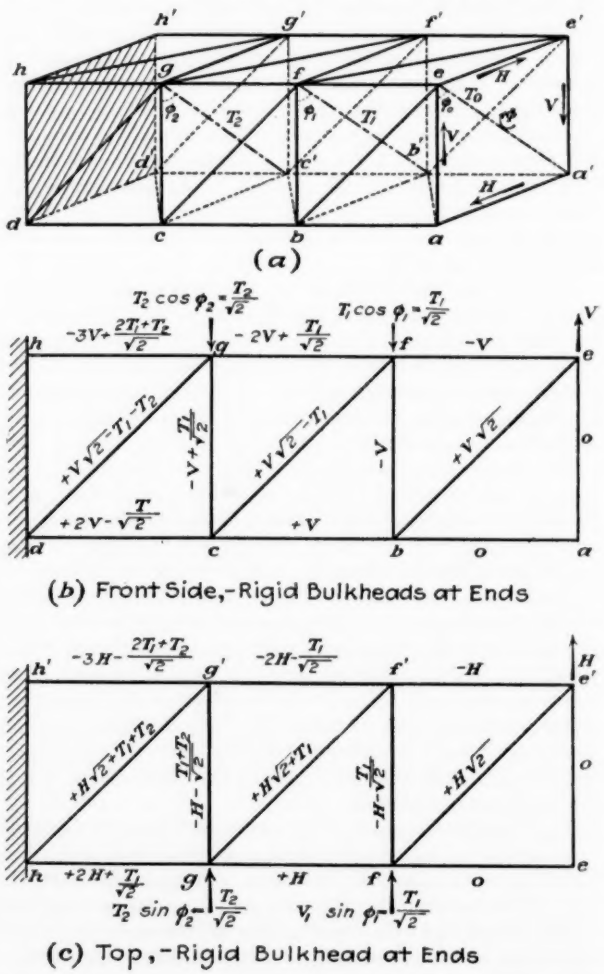


FIG. 5 DISTRIBUTION OF TORSIONAL SHEAR IN TRUSS SPACE FRAMEWORK

as tension redundant reactions. The system is indeterminate to the third degree. The redundants are H, T_1, and T_2. Evidently $V = \frac{\Phi - Hh}{L}$ (lb). Assume the diagonals in the bulkheads make angles ϕ_1 and ϕ_2 with respect to the vertical. Then the loadings

on the left vertical side frame are V, $T_1 \cos \phi_1$ and $T_2 \cos \phi_2$. For the top horizontal side frame, the loadings are H, $T_1 \sin \phi_1$ and $T_2 \sin \phi_2$. In like manner we have a similar loading system for the right and bottom side frames.

Considering now the rear end bulkhead, the reactions of the vertical side frames are

$$V' = V - T_1 \cos \phi_1 - T_2 \cos \phi_2$$

and for top and bottom frame

$$H' = H + T_1 \sin \phi_1 + T_2 \sin \phi_2$$

The couple on the rear bulkhead is

$$\Phi' = V'b + H'h = Vb + Hh = \Phi$$

since $h \sin \phi - b \cos \phi = 0$ at any intermediate bulkhead, thus giving an equilibrium check.

It is to be noted that the distribution for the component couples due to vertical- and horizontal-shear forces are definitely different. In other words they are affected by the relative rigidity of the intermediate truss bulkheads.

For a solution of this system, we have the loading conditions for each side truss. Evidently the total stress in any member has the form

$$T = \alpha\Phi + \beta H + \gamma T_1 + \mu T_2$$

where H, T_1, and T_2 are the redundant forces. Therefore, equating the differential coefficients of the elastic energy $\Sigma \frac{T^2 l}{2EA}$ with respect to each of the redundants to zero, we have

$$\Sigma (\alpha\Phi + \beta H + \gamma T_1 + \mu T_2) \beta \frac{l}{A} = 0$$

$$\Sigma (\alpha\Phi + \beta H + \gamma T_1 + \mu T_2) \gamma \frac{l}{A} = 0$$

$$\Sigma (\alpha\Phi + \beta H + \gamma T_1 + \mu T_2) \mu \frac{l}{A} = 0$$

from which we can solve for H, T_1, and T_2 in terms of the applied moment, and V from the relation $Vb = \Phi - Hh$. On the basis of equal cross-sectional areas for truss members and diagonals in intermediate cross frames, and assuming square panels, i.e., $h = b$, we have

$$\begin{aligned} 187.94H + 74.69T_1 + 25.68T_2 &= 93.97\,\Phi/h \\ 74.69H + 38.23T_1 + 14.16T_2 &= 36.99\,\Phi/h \\ 25.68H + 14.16T_1 + 11.57T_2 &= 12.49\,\Phi/h \end{aligned}$$

hence $H = 0.5149\,\Phi/h \quad T_1 = -0.027\,\Phi/h$

$V = 0.4851\,\Phi/h \quad T_2 = -0.0305\,\Phi/h$

(*b*) With tension diagonal bulkheads of same rigidity as truss frames, considering rigid bulkhead only at rear end. In this case we have no rigid forward bulkhead, the forward bulkhead being replaced by a diagonal brace. The tension T_0 in this diagonal replaces the redundant reaction H in the previous analysis. The redundants are now T_0, T_1, and T_2, the redundant tensions in the diagonals of the cross-frames.

On the basis of equal cross-sections for all members, we have

$$\begin{aligned} 96.88T_0 + 52.81T_1 + 18.16T_2 &= 66.45\,\Phi/h \\ 52.81T_0 + 38.23T_1 + 14.16T_2 &= 36.99\,\Phi/h \\ 18.16T_0 + 14.16T_1 + 11.57T_2 &= 12.49\,\Phi/h \end{aligned}$$

so that

$$T_0 = 0.638\,\Phi/h,\ T_1 = 0.104\,\Phi/h,\ T_2 = -0.050\,\Phi/h$$

(*c*) Assuming flexible bulkheads, with tension members in cross-frames having one quarter the cross-sectional area of main members in truss frames. In this case we arrive at the equations:

$$\begin{aligned} 101.13T_0 + 52.81T_1 + 18.16T_2 &= 66.45\,\Phi/h \\ 52.81T_0 + 42.47T_1 + 14.16T_2 &= 36.99\,\Phi/h \\ 18.16T_0 + 14.16T_1 + 15.81T_2 &= 12.49\,\Phi/h \end{aligned}$$

so that

$$T_0 = 0.577\,\Phi/h,\ T_1 = 0.158\,\Phi/h,\ T_2 = -0.015\,\Phi/h$$

For the conditions in (*a*), the stresses in the side-frame diagonals and stresses in the top- and bottom-frame diagonals are listed in Table 2. The vertical- and transverse-shear components were similarly estimated for conditions (*b*) and (*c*).

TABLE 2 STRESSES IN MEMBERS AND DISTRIBUTION OF SHEAR IN PANELS FOR CONDITION (*a*)

Stresses in side-frame diagonals:

Member	Tension	Vertical Shear Comp.
be	$V\sqrt{2} = 0.686\,\Phi/h$	$V\frac{\sqrt{2}}{\sqrt{2}} = 0.4851\,\Phi/h$
cf	$(V\sqrt{2} - T_1) = 0.713\Phi/h$	$(V\sqrt{2} - T_1)\frac{1}{\sqrt{2}} = 0.5043\,\Phi/h$
dg	$(V\sqrt{2} - T_1 - T_2) = 0.744\,\Phi/h$	$(V\sqrt{2} - T_1 - T_2)\frac{1}{\sqrt{2}} = 0.5258\,\Phi/h$

Stresses in Top- and Bottom-Frame Diagonals:

Member	Tension	Transverse Shear Comp.
fe'	$H\sqrt{2} = 0.728\,\Phi/h$	$H\frac{\sqrt{2}}{\sqrt{2}} = 0.5149\,\Phi/h$
gf'	$(H\sqrt{2} + T_1) = 0.701\,\Phi/h$	$(H\sqrt{2} + T_1)\frac{1}{\sqrt{2}} = 0.4957\,\Phi/h$
hg'	$(H\sqrt{2} + T_1 + T_2) = 0.670\,\Phi/h$	$(H\sqrt{2} + T_1 + T_2)\frac{1}{\sqrt{2}} = 0.4742\Phi/h$

The values of the transverse- and vertical-shear components in bays I, II, and III are given in Table 3.

TABLE 3 VALUES OF TRANSVERSE- AND VERTICAL-SHEAR COMPONENTS IN BAYS I, II, AND III

Bay	Rigid end bulkhead	Bulkhead with diagonal	Bulkheads with light diagonals (A/A)
Transverse shear in the roof:			
I	0.5149 R[a]	0.4514 R	0.4080 R
II	0.4957 R	0.5252 R	0.5200 R
III	0.4742 R	0.4896 R	0.5095 R
Vertical shear in side frame:			
I	0.4851 R	0.5486 R	0.5920 R
II	0.5043 R	0.4748 R	0.4800 R
III	0.5258 R	0.5104 R	0.4905 R

[a] $R = \Phi/h$.

From this we note that as the end bulkheads become more flexible the transverse shear is reduced in the end bay but, even with relative flexible end bulkheads due to the greater distortion at the ends, they are very effective in equalizing the torsional-shear distribution in roof and floor system and side frames.

It is also of interest to note that intermediate bulkheads are not subjected to heavy loadings as indicated by the relative small values of the cross-frame tension diagonals T_1 and T_2 for all three cases.

On the other hand to transmit torsional-shear stresses through openings under operating conditions with heavy inertia rolling moments, relative rigid bulkheads adjacent to the openings are effective in inducing lateral-shear forces in the roof and floor system and thereby providing a better torsional-shear distribution. The relative magnitudes of this distribution are obviously greatly dependent on the relative flexibilities in the transverse and vertical directions through the opening.

Mr. Wright's discussion is also most interesting and valuable. He raises the question of the validity of premising the train-resistance component due to air resistance on the basis of wind-tunnel experiments on models mentioned in the paper. On similarity tests with models and full-scaled trains let P' and P be the total resistance of model and the air-resistance component for a full-size train.

From dimensional analysis,

$P' = k\rho' v'^2 A' f'(R')$ for model

$P = k\rho v^2 A f(R)$ for large-size train

where k is numeric, consistent for units used, ρ is the mass density, v is the velocity, A is the cross-section of train, and

$R = \frac{\rho v L}{\mu}$ the Reynolds number, a non-dimensional parameter.

If we write the drag coefficient in the form $P = D_e A v^2$ then

$D_e' = k\rho' f'(R')$ for the model

$D_e = k\rho f(R)$ for the large-size train.

Since $\rho' = \rho$, approximately, evidently the condition for no scaling up from the model to full-size train, requires equality of the drag coefficients, so that

$$f'(R') = f(R)$$

Let us first assume that $\frac{P}{k\rho v^2 A} = f(R)$ has the same function for both the model in the wind tunnel and for the full-size train. Then the condition for equal drags reduces to $f(R') = f(R)$. Now for the full-size train, even assuming considerable speeding up of the velocity v' in the wind tunnel, $R = \frac{\rho v L}{\mu}$ is still greater than $R' = \frac{\rho v' L'}{\mu'}$ for the model. But in the range of turbulent motion considered which in both cases is considerably above the critical velocity, the function $f(R)$ is practically constant so that $f(R)$ can be assumed practically equal to $f(R')$ of the model, even though $R > R'$.

Actually it would be anticipated that the function of the Reynolds number, $\frac{P}{k\rho v^2 A} = f(R)$ to decrease somewhat with increasing Reynolds numbers. Therefore since R is inherently greater than R', the drag coefficient for the full-size train should be somewhat less than for the model. Actually it has been found that a positive scaling up is necessary.

This would indicate that in scaling up the drag coefficient, the $f'(R)$ is not the same as $f(R)$ for the full-size train, $f(R)$ in general being greater than $f'(R)$ for the model. This would be anticipated inasmuch as the boundary disturbing effect at the rail is different from that in a wind tunnel even with a dividing plane in an image duplication of the model.

Mr. Wright also raises the question that the scale effect in a wind-tunnel experiment may be different for conventional and streamlined trains due to the fact that the air flow is different; also that the scaling may be seriously affected by non-uniformity and disturbances of air flow in a long model in a wind tunnel. This conclusion seems reasonable and further points to the fact that the Reynolds function may be appreciably different for these cases as well.

On the other hand from a practical point of view, on the basis of the Davis formula modified for drag coefficients for air resistance obtained from the Massachusetts Institute of Technology wind-tunnel tests, test results for balance speeds of the *Zephyr* indicate a remarkable consistency. In the author's paper, the drag coefficient was scaled up by 20 per cent in the correction for the air-resistance component in the Davis formula. At 4.3 hp per ton computed balance speeds indicated a top speed of 96 mph. Actual top speeds of the *Zephyr* were somewhat over 100 mph and test results indicated that actually very little scaling up of the drag coefficient was necessary. For these reasons the author is convinced that reasonably satisfactory drag coefficients can be obtained from wind-tunnel tests, particularly for short trains with small modification in drag coefficients.

Mr. Wright suggests simple coasting retardation tests for obtaining train resistance on the basis of accurate speedometer data and a profile of the road. He introduces an ingenious graphical method which has the advantage of materially aiding in the faring of the plotting data by resolving the energy components in terms of heads. The gradient of the curve MP corresponds to the resistance at any point.

From the energy equation

$$d\left[\frac{kW}{2g}v^2 + WH\right] = -Rdx, \text{ where } dH = dx \sin \alpha$$

we have, between any two points,

$$\left[H_2 + \frac{kv_2^2}{2g}\right] - \left[H_1 + \frac{kv_1^2}{2g}\right] = -\frac{1}{W}\int_1^2 Rdx \ldots [5]$$

Instead of plotting the curve,

$$\frac{kv_0^2}{2g} - \left[H + \frac{kv^2}{2g}\right] = \frac{1}{W}\int_0^x Rdx \ldots\ldots [6]$$

and differentiating with respect to x for the train resistance, perhaps a closer approximation would be obtained by directly plotting the velocity curve and below it the profile curve. Then for any two adjacent points 1 and 2 along the curves, we can write either Equation [5] or [6] in the form

$$k\left(\frac{v_1^2 - v_2^2}{2g}\right) - (H_2 - H_1) = \frac{R}{W}(x_2 - x_1)$$

or

$$\frac{k}{g}\left(\frac{v_2 + v_1}{2}\right)(v_1 - v_2) - (H_2 - H_1) = \frac{R}{W}(x_2 - x_1)$$

where the interval $x_2 - x_1$ is taken sufficiently small to assume R constant.

Allowable Working Stresses Under Impact[1]

AUTHOR'S CLOSURE

IN a criticism of the author's paper, E. Dillon Smith states that the "raising factor" of the yield point or ultimate strength under impact cannot always be the same and that it depends on the velocity of impact, the inertia of the beams, and its natural period. The illustration of the idea given by the author at the end of this article will show that in some cases the dynamic force may produce a greater deflection and consequently a greater stress than the static one.

The author fears that Mr. Smith does not make enough distinction between two different things namely, (1) the relation between the dynamical and statical yield point and tensile strength and (2) the relation between the statical and dynamical external

[1]By N. N. Davidenkoff, Head of the Mechanical Department of the Physico-Technical Institute in Leningrad, U.S.S.R. Published with discussion in A.S.M.E. Trans., March, 1934, vol. 56, no. 3, paper APM-56-1.

forces, revealing the same stresses in the specimen. The data given by the author in Tables 1 and 2 do not relate to the method of calculation of dynamical stresses, but only assume that this method of calculation is correct. In cases similar to those cited by Mr. Smith, as when an elastic impact is being produced by a light hammer against a heavy beam, the inertia of the beam cannot be neglected when calculating the stress. If we take into account this inertia and observe that the beam is deflected, not only by the pressure of the hammer, but also by its own inertia forces, then there will be no discrepancy between the deflection and the actual force. The author however, wishes to assure Mr. Smith that the values given in Tables 1 and 2 have been obtained from tests of small specimens in longitudinal tension or compression using sufficiently large hammers. In this case the inertia forces of the specimen can be completely neglected, especially due to the long duration of non-elastic impact and the high natural frequency of the longitudinal oscillations of the specimen. In this manner the measurement of the impact stresses was reduced to the simple measurement of the impact pressure of the hammer.

The question of the influence of the impact rate or rather the rate of increase of the impact stresses on the value of the "raising factors" will now be considered. There is no doubt that this influence exists. Up to now, however, all attempts which have been made to investigate this influence have led either to doubtful results[2] which could be explained by the conditions of the experiment or a negative[3] answer. R. Plank[4] gives some data stating the following dependence of dynamic tensile strength σ on the velocity v.

$$\sigma = \frac{a\,v}{v + b}$$

where a and b are constants. Owing to the peculiarity of these data the author did not dwell on them in his paper.

Mr. Smith objects to the connection of the upper and lower branches of the curve Fig. 8. In admitting this operation for Fig. 7, he must accept it for Fig. 8, which represents a picture of the same physical phenomenon, but only more definitely pronounced. Moreover, some of the values in Fig. 8 tend to approach the connecting curve.

Mr. Smith does not seem to agree with the author as to the difficulty of accurately double differentiating curves. With small-scale diagrams this difficulty will undoubtedly exist and the best proof of this is the discrepancy which exists between diagrams obtained by different investigators[5,6] under similar conditions. In the proper choice of equations for the curves to be differentiated, apart from the tediousness of the process, there is the risk of smoothing out imperceptible deviations which under double differentiation will grow into considerable waves. See for instance Mr. Smith's diagram, Fig. 11.

As to the use of piezo quartz, the author does not understand why Mr. Smith neglects the possibility of safeguarding the quartz crystal from rupture during the experiment by keeping the proper relation between the size of the crystal and specimen.

The author is greatly obliged to Mr. Smith for the detailed account of the principles of the piezo-quartz method on which the author did not dwell in his paper. Mr. Smith, however, suspects that the photographs given in the paper are not those given by a cathode-ray oscillograph. From the explanation given of Fig. 3, Mr. Smith might have seen that the cathode rays get two mutually normal deviations, the latter of which is created by the magnetic field for the special purpose of developing a picture without using a moving film.

Finally it might have been better if the author had given a description of the calibration of the piezo quartz which was done by a simple static loading. This omission may have given rise to the suspicion that a more complicated method was used.

The author is greatly interested in the work of Prof. R. V. Southwell, which shows how careful one must be in respect to the value of the so-called "mean impact force" computed as the quotient of the energy of fracture and the deformation of the specimen. It is obvious that under ordinary test conditions we will always obtain an exaggerated meaning of the impact force.

In connection with Figs. 7 and 8, Messrs. Mason and Stone mention the phenomenon of "blue-heat brittleness" whereas the phenomenon is displayed under impact at higher temperatures namely, above 500 C and the phenomenon shown in Figs. 7 and 8 should be called "cold brittleness," and is of quite a different physical nature.

[2] Proc. Intern. Assoc. of Testing Materials, 1912, no. 9.

[3] Charpy a Cornu Thenard., *Jour. of Iron & Steel Inst.*, vol. 96, 1917, p. 61.

[4] *Zeit. V.D.I.*, vol. 56, 1912, p. 17.

[5] Mitt. K-W. Instit. f. Eisenfor., vol. 7, 1925, p. 81.

[6] *Deutsch. Ver. f. Materialpr.*, 1927, no. 78.

Effects of Side Leakage in 120-Degree Centrally Supported Journal Bearings[1]

MAYO D. HERSEY.[2] The author brings out the point that the side-leakage effect depends not only on the bearing design, but also upon the load carried. Since, however, the side-leakage effect is expressed as a simple ratio, it would seem probable that the situation might be more completely and exactly described by saying that the side-leakage ratio depends also on the dimensionless generalized variable G, more commonly known as ZN/P, and which, in the author's notation, is written as $\mu N/p_0$.

The author has presented two side-leakage factors or ratios, one being the ratio of the frictional resistance of the actual bearing to that of an ideal bearing without side leakage; the other the corresponding ratio of load capacities. The writer's previous statement regarding G has reference only to the friction ratio. It is not clear to him what is meant by saying that the other ratio, the ratio of load capacities, depends on the load. It is suggested, however, that the side-leakage factor for load capacity depends not only on the bearing design, but also on the product of the viscosity μ by the speed N, and might be so represented on a diagram.

Another point which is not yet clear to the writer is how it comes to pass that the Sommerfeld variable $G\left(\frac{a}{\eta}\right)^2$ remains intact in the side-leakage problem, as shown by Fig. 13 in the article under discussion. When this variable was introduced by Sommerfeld in 1904, it resulted from a detailed integration, valid only for the ideal bearing free from side leakage.

If the paper presented by Mr. Needs contains a mathematical proof that this simple result may be derived from Reynolds' equations, as I believe it probably does, it would seem well worth while to segregate this proposition from the rest of the paper. Otherwise, it might be construed that the proposition has been tacitly assumed in the very beginning.

Some remarks might be offered concerning the methods of graphical representation selected by Mr. Needs. It is natural that he should lean toward the viewpoint of the machine de-

[1] Published as paper APM-56-16, by S. J. Needs, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Division of Engineering, Brown University, Providence, R. I. Mem. A.S.M.E.

signer as contrasted with that of the operating or lubrication engineer. Consequently, most of his results are presented in terms of the more intangible factors such as the eccentricity ratio c. A welcome exception is found in Figs. 13 and 13a of the author's paper. Here the Sommerfeld variable has been selected for the independent variable, or abscissa, in the coefficient of friction diagram. Would it be possible to give a similar diagram for the eccentricity ratio, or better still, for the relative film thickness, h_0/η?

Referring to Fig. 10 in the author's paper, it seems paradoxical that the friction ratio decreases as the bearing becomes narrower and side leakage worse. The explanation lies in the fact that the curves shown are for equal eccentricities. But in actual practice, if we compare a collection of bearings, all having the same clearance-diameter ratio, and all being tested under the same operating conditions, that is, at the same value of G, the eccentricities will not be equal. A curve of this type plotted against the length-width ratio would show an increasing frictional resistance ratio as the bearing is made narrower. Such a curve might be constructed by cross-plotting from the author's Figs. 13 and 13a.

In conclusion it might be asked if the author has compared his results with the approximate solutions of Gümbel, Stodola, Boswall, Karelitz, or Duffing?

R. Baudry.[3] The bearing characteristics given by Mr. Needs have been compared with those obtained by a method of analysis[4] based on data previously presented.

In Fig. 1 are plotted curves from data given by Mr. Howarth and Mr. Needs, which show the characteristics of a 120-deg. bearing of infinite width. The characteristics of the bearing have been given in functions of the characteristic number ZN/P, where Z is the viscosity in centipoises, N is the revolutions per minute, and P is the unit pressure in lb per sq in. The curve for n/r gives the relation between the clearance ratio n/r and ZN/P given eccentricity C.

Similar curves are plotted for the coefficient of friction, oil-film thickness, location of the minimum oil-film thickness, with respect to the time of action of the load (ϕ), and the ratio of the maximum to the minimum oil-film thickness, h_m/h_o. These curves reveal the condition of minimum friction and maximum oil-film thickness as shown by Mr. Kingsbury. It is also clearly seen that for a rather large variation of the clearance ratio, there is only a small change in the coefficient of friction and minimum oil-film thickness.

Boswall, in his book, "The Theory of Film Lubrication," has given some data for the leakage factors of a plane bearing having an $\frac{\text{inlet}}{\text{outlet}}$ oil-film ratio of 2 and 3. These data have been plotted as solid curves in Fig. 2.

In his paper Mr. Needs presented data for the location of the point of negative pressure, which for large eccentricities is located between the point of maximum pressure and the outlet edge. This shows that it is not permissible to assume that the negative pressure begins at the point of nearest approach as many authors have done. In the absence of complete data for the location of this point, we have assumed, in finite bearings, that it is located half way between the outlet edge and the point of nearest approach for the infinitely wide bearing. We obtain in this manner the oil-film ratio of $\frac{\text{length}}{\text{width}}$, from which the broken-

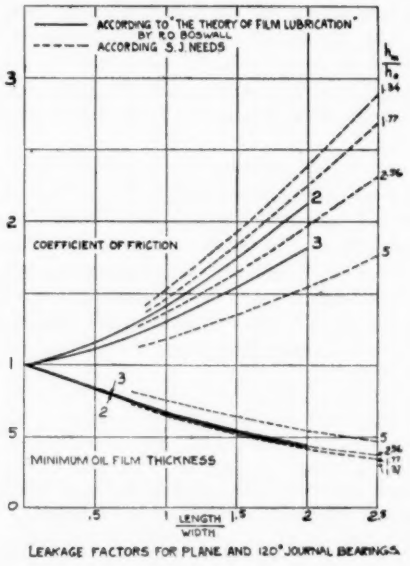


Fig. 2

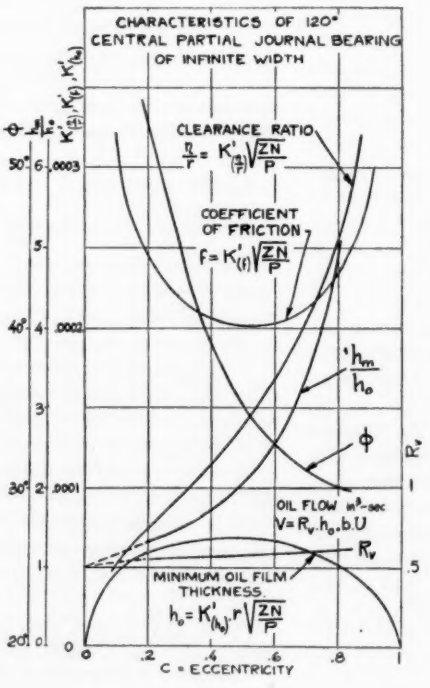


Fig. 1

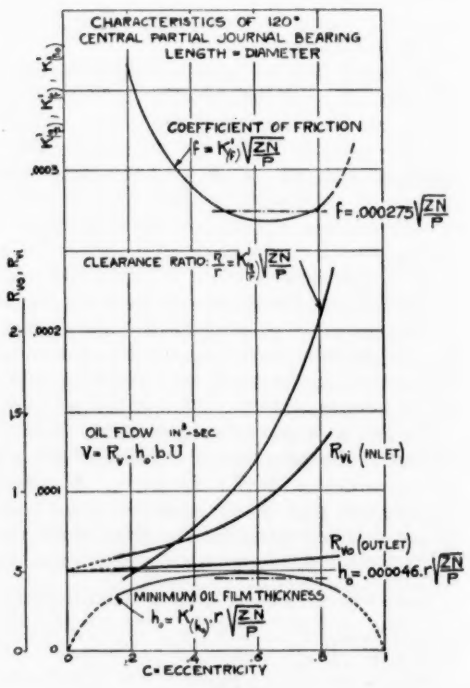


Fig. 3

[3] Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

[4] "Performance of Large Journal Bearings," by R. Baudry and L. M. Tichvinsky, presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of The American Society of Mechanical Engineers.

line curves in Fig. 2 were plotted. These correspond to the different bearing eccentricities calculated by Mr. Needs. It is interesting to note that these curves agree closely with data published by Boswall and interpolated for different values of h_m/h_o.

In Fig. 3 are shown the characteristics of a 120-deg bearing in which the length of the bearing is equal to the diameter of the shaft. It is seen that for a variation of the clearance ratio from less than 0.001 per in. to more than 0.002 per in., the coefficient of friction will vary only plus or minus two per cent of the mean

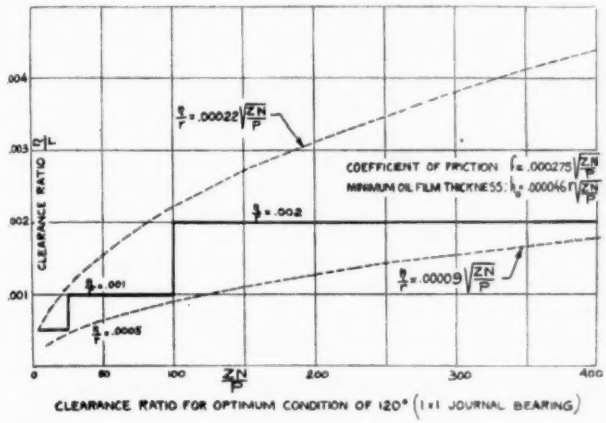


FIG. 4

value shown on the curve. There is also a variation of the same order for the oil-film thickness.

In Fig. 4 is shown the permissible clearance ratio necessary to operate at the optimum condition. It is interesting to note that this corresponds with the clearance used on Westinghouse bearings. For high-speed turbo-generators which operate under values of ZN/P from 100 to 400, a clearance ratio of 0.002 is used. For formerly used slow-speed power bearings, with a 120-deg angle and having a value of ZN/P below 100, a clearance ratio of 0.001 was used. This shows a rather close agreement between theory and practice.

The leakage factors for oil flow published by Boswall also show close agreement with those of Mr. Needs. From this comparison it is seen that the different leakage factors are mainly a function of the $\frac{\text{length}}{\text{width}}$ ratio and h_m/h_o. It seems, as suggested by Mr. Needs, that a single set of curves for leakage factors can be used with a reasonable degree of accuracy for many different types of bearings.

ALBERT KINGSBURY.[5] The paper on "Optimum Conditions in Journal Bearings," to which Mr. Needs refers, contains several assumptions which are known to be not strictly true, but were taken as near enough to the truth to provide for the development of a theory that would be serviceable for practical work.

Among these assumptions are: (1) That the viscosity of the oil during its passage through the film may be taken as a constant value represented by its average value; (2) that the form of the surfaces is definitely known; (3) that the friction per unit width, with given clearance ratio and eccentricity, is the same for narrow bearings as for wide bearings; and (4) that the optimum conditions are obtained with the same eccentricity for both wide and narrow bearings.

The theory is open to improvement in respect to these assumptions. Mr. Needs has investigated the degree of inaccuracy involved in the third and fourth of these assumptions, while still retaining the first two. The results of his work are interesting, and form a valuable extension of the theory.

J. R. CONNELLY.[6] In that part of the paper by Mr. Needs dealing with the selection of dimensions of a proposed bearing, it is implied that wear at starting and stopping in the absence of an oil film is directly proportional to p_0 (the ratio of total load to projected area).

Experimental evidence obtained in the laboratory at Lehigh University using the same journal, lubricant, and bearing metal seems to indicate that variation of the length to diameter ratio results in a variation in the unit pressure at which wear ceases. Also, for certain values of the length to diameter ratio, there seems to be an inverse relation with the unit pressure at which wear ceases.

Should the above condition be found to exist for other metals, journals, and lubricants, then the selection of the dimensions of a bearing must include a consideration of the maximum value of p_0 at which wear ceases.

AUTHOR'S CLOSURE

The statement referred to by Mr. Hersey, that side-leakage factors depend on bearings loads as well as $\frac{\text{length}}{\text{width}}$ ratios, assumes that the speed, viscosity, and clearance ratio remained constant while the load was varied. Fig. 9 of the paper shows the side-leakage factors to vary with the eccentricity for a given $\frac{\text{length}}{\text{width}}$ ratio. Any change in eccentricity causes a change in the side-leakage factor. Hence, Mr. Hersey's suggestion that the side-leakage factors depend on the speed and viscosity as well as the load, gives a much clearer picture of the situation. Another factor affecting eccentricity is the clearance ratio, which must also be included in the group determining the side-leakage factors.

It may be shown mathematically that the Sommerfeld variable remains intact in the side-leakage problem. If in Reynolds' equation,

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\mu U\frac{\partial h}{\partial x} \quad \ldots\ldots [1]$$

we write $h=\eta(1+c\cos\theta)$, $x=a\theta$, $z=a\phi$, and $p=\left[\frac{6\mu Ua}{\eta^2}\right]q$, Equation [1] becomes $\frac{\partial}{\partial\theta}\left[(1+c\cos\theta)^3\frac{\partial q}{\partial\theta}\right]+\frac{\partial}{\partial\phi}\left[(1+c\cos\theta)^3\frac{\partial q}{\partial\phi}\right]=-c\sin\theta$. Any solution of this equation is of the form, $q=f'(\theta,\phi,c)$. Hence, any solution of Equation [1] is of the form,

$$p=\left[\frac{6\mu Ua}{\eta^2}\right]f'\left(\frac{x}{a}\frac{z}{a}c\right)$$

When, for infinite width Equation [1] reduces to

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right)=6\mu U\frac{dh}{dx} \quad \ldots\ldots [2]$$

the solution is of the form,

$$p=\left[\frac{6\mu Ua}{\eta^2}\right]f'\left(\frac{x}{a}c\right)$$

[5] President, Kingsbury Machine Works, Philadelphia, Pa. Mem. A.S.M.E.

[6] Instructor, Department of Mechanical Engineering, Lehigh University, Bethlehem, Pa. Jun. A.S.M.E.

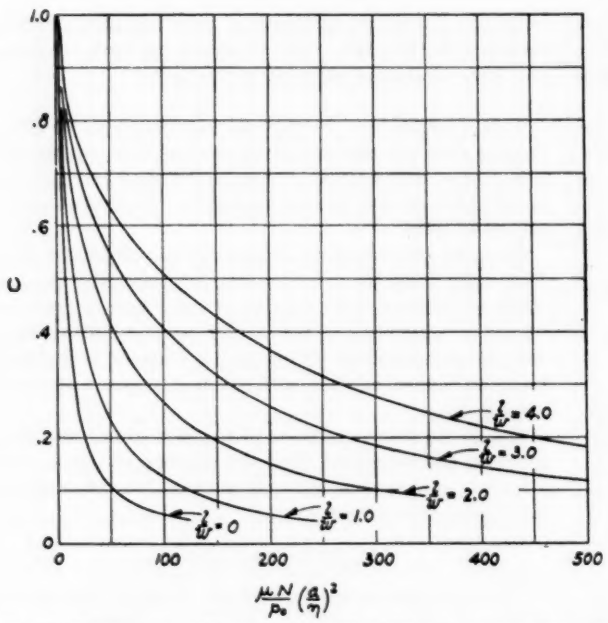


FIG. 5 VARIATION OF ECCENTRICITY WITH VALUES OF THE SOMMERFELD VARIABLE FROM 0 TO 500

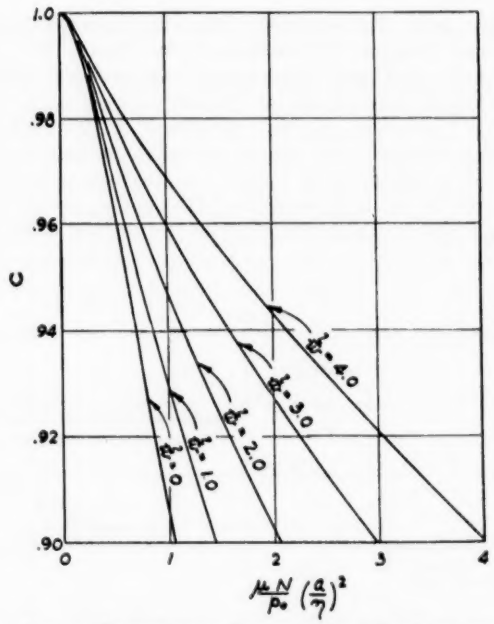


FIG. 6 VARIATION OF ECCENTRICITY WITH VALUES OF THE SOMMERFELD VARIABLE FROM 0 TO 4

As far as the absence of η is concerned f_r and f are of the desired form. The group coefficients of f_r and f_s are identical in the cases of finite and infinite widths, hence, when the Sommerfeld variable is taken as the abscissa, the curve for each $\frac{\text{length}}{\text{width}}$ ratio will be independent of the clearance ratio.

Variation of eccentricity with the Sommerfeld variable is given in Figs. 5 and 6 of this discussion. Since $h_0/\eta = (1 - c)$, the relative film thickness may also be found from these charts.

No comparison of the results of the present paper with the approximate solutions mentioned by Mr. Hersey has been made by the author. Mr. Baudry has made some comparisons of side-leakage factors with data published by Boswall. Comparison with other formulas should prove equally interesting.

Mr. Baudry's curves are of special interest in that they express the thought that the data as presented are not in suitable form for all designers who might care to use them. The results of future research, together with the information now available, will no doubt indicate a more logical and useful method of presentation. Attention should be called to the fact that Mr. Baudry has plotted the values of K' and not the absolute value of the quantities referred to by the curves. This accounts for the peculiar appearance of the h_0 curve, which at first glance appears to indicate that $h_0 = 0$ when $c = 0$; when in fact, when $c = 0$, h_0 = the radial clearance η and the film is of uniform thickness.

It should be pointed out that assumptions (1) and (2) mentioned by Dr. Kingsbury may be included in solutions by the electrical method. He has made investigations of (1), particularly with reference to plane surfaces; and has pointed out that the electrical method may be used for any form of bearing surfaces.

From Mr. Connelly's comments it may be inferred that there is some value of the unit pressure, other than zero, at which there will be no wear at starting and stopping; but he does not state that the experimental evidence on which his remarks are based was obtained with a journal bearing under actual starting and stopping conditions. Experimental results, showing the relationship between wear, unit loading, and length-width ratio in a practical bearing, as the speed is varied from zero to the point where a film is formed, would be of great interest and value.

Attention is called to one or two typographical errors in the paper itself. In Tables 1 and 2, the third column from the left should be headed c and not c, deg. In Fig. 17, the left-hand vertical scale should read 0, 0.2, 0.4, 0.6, 0.8, 1.0.

Exact Construction of the $(\sigma_1 + \sigma_2)$-Network From Photoelastic Observations[1]

E. E. WEIBEL.[2] In its practical application, Dr. Neuber's proposed method of determining the values of principal normal stresses from photoelastic observations does not appear to the writer to permit a saving in time over the existing methods.

It may be compared with Filon's integration method which also makes use of quantities determined graphically and obtains its results by means of a step-by-step process. In Filon's methods, in one of its forms, either principal stress is found directly by numerically adding quantities of the type $(\sigma_1 - \sigma_2)\frac{ds}{dx}$, where dx is the intercept between isoclinics, drawn in the direction of one of the principal stresses, and ds is the increment of length along the stress trajectory which is the path of integration.

In the method proposed by Dr. Neuber, the graphical construction which must be made at each point considered requires the taking into account of six quantities: the directions 1, 5, and 7; the spacings b and c; and the value of $(\sigma_1 - \sigma_2)$.

In a simple curved-beam problem studied recently, about 400 points would have had to be considered for either the Filon or the Neuber method in order to cover satisfactorily that portion of the member which was of interest. It is the large amount of detailed measurement which has caused Filon's method to be described by a number of workers as tedious. The Neuber method does not reduce the amount of measurement and

[1] Published as paper APM-56-17, by H. P. Neuber, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] University of Michigan, Ann Arbor, Mich.

requires in addition a graphical construction at each point considered.

It is the writer's opinion that problems for which principal stresses can be obtained by either of these graphical methods can be solved in a shorter time by means of the membrane-analogy method, using either a soap film or a stretched rubber membrane.

THOMAS H. EVANS.[3] Although it appears to the writer that two other methods for constructing a $(\sigma_1 + \sigma_2)$-network have decided advantages over any yet developed, that proposed by the author seems somewhat simpler than the graphical integration used by Filon and others.

Any such method, no matter how exact theoretically, that requires the plotting of lines obtained with the polariscope, and the construction of others at given gradients to these, is susceptible to graphical error. Although the method under discussion does not require a plot of the stress trajectories, as does that of Filon, it is necessary to trace out isoclinics, which in some parts of a stressed model are extremely difficult to resolve accurately. Assuming, however, that the basic systems upon which the network is constructed are accurate, a great deal of tedious plotting and checking is necessary unless further inaccuracy is to be introduced by sketching in lines from only a small number of calculated points.

The two methods mentioned above, which appeal to the writer as being simpler to apply once the necessary equipment has been assembled, are the membrane analogy, and an optical method. The former was described in an excellent paper by McGivern and Supper[4] while the latter was successfully used by Tesar and Cornu in France. The membrane analogy relies only on the accuracy of boundary stress determinations, photoelastically, to plot accurately a $(\sigma_1 + \sigma_2)$-surface. The optical method of measuring thickness variation in a stressed model (and hence $\sigma_1 + \sigma_2$) by the method of interference-fringes is entirely independent of the previous photoelastic work to determine $(\sigma_1 - \sigma_2)$, and is extremely simple and accurate.

M. S. NOYES.[5] This is the first paper that the writer has seen, on the photoelastic examination of material under stress, which sets up mathematical descriptions of the observed lines of stress. The writer feels that the author has bridged the gap between the photographic record of the stressed model and the older approximate formulas for the apparent stresses.

The usefulness of the theory and method given, however, is exactly in proportion to the ease with which they can be applied to all of the ordinary problems met in design work. The writer wishes, therefore, that the author had lengthened his short paper sufficiently to include in some detail the application of his graphical method to the symmetrical angle plate analyzed in Figs. 8 and 9. It is not readily apparent, for instance, whether the formula, $-100 = \frac{P}{dh}\left(\frac{6l}{h} + 1\right)$, in these figures, is supposed to represent the true stress for $(\sigma_1 + \sigma_2)$ or an apparent one.

A. M. WAHL.[6] In connection with the problem of determining the principal stresses from photoelastic observations, a recently developed lateral extensometer described elsewhere[7] has proved very useful.

This lateral extensometer differs from that used by Coker in that the points are pressed against the specimen with a small but definite pressure and are held in one location while the specimen is being loaded. There is thus no possibility of sidewise motion of the points during the loading of the specimen, and hence no errors are introduced due to slight variations in thickness from point to point. The whole apparatus is balanced and supported by springs so that the specimen may deform slightly without causing slippage of the points (which carry practically no load). The results obtained by using this extensometer were very consistent, successive readings obtained by removing and reclamping the extensometer at the same point seldom differing by more than 50 lb per sq in. for a 0.33 in. thick specimen (bakelite may easily carry values of $\sigma_1 + \sigma_2$ equal to 5000 lb per sq in. within the elastic limit).

The instrument was checked against a theoretical solution (obtained by Howland[8]) for a plate having a hole with a diameter half the width. It was also checked against photoelastic values of σ_1 (σ_2 being zero) near the free edges of some specimens designed to simulate the conditions in press fits. In all these cases the results obtained by the lateral extensometer agreed closely with those obtained by other means.

In most practical cases, we are interested only in the stresses in a comparatively localized region, such as, for example, the minimum section in the case of a tension bar with a hole or along the line of contact between hub and shaft in the case of a press fit. In such cases the instrument has proved particularly useful, since a few measurements made near such points will give a good idea of the stress distribution in a relatively short time.

Dr. Neuber has presented a very ingenious method of determining $\sigma_1 + \sigma_2$, and one which appears to represent a definite improvement over previously used graphical methods. The writer feels, however, that the use of an accurate lateral extensometer will result in a considerable saving of labor as compared to the graphical method, while, at the same time, giving sufficiently accurate results for most practical work.

AUTHOR'S CLOSURE

In the discussion of the paper, the author's graphical method for determining the construction of the $(\sigma_1 + \sigma_2)$-network is compared with other methods used for this purpose. In order to understand both the usefulness of the author's method and its practical application, one must realize that it has two important advantages:

(*a*) It eliminates the necessity of drawing the principal stress lines.

(*b*) In most practical problems it will be sufficient to make use of the particular cases (constructions 1 *a,b,c;* 2 *a,b,c;* 3 *a,b*) in which the application of the method becomes extraordinarily simple. These particular cases will furnish a sufficient number of points for obtaining an excellent conception of the $(\sigma_1 + \sigma_2)$-network, inasmuch as the boundary values are already known.

Further details of the method and its application are outlined in, "Einfuhrung in die Photoelastizitat," by Dr. L. Foppel and the author, a treatise which has not yet been published.

It would be very useful for investigators to compare results

---

[3] Instructor in Engineering Mechanics, Sheffield Scientific School, Yale University, New Haven, Conn.

[4] "A Membrane Analogy Supplementing Photoelasticity," by J. G. McGivern and H. L. Supper, A.S.M.E. Trans., August, 1934, paper APM-56-9.

[5] Bureau of Engineering, Washington, D. C. Assoc-Mem. A.S.M.E.

[6] Westinghouse Research Laboratories, East Pittsburgh, Pa. Assoc-Mem. A.S.M.E.

[7] "Fatigue of Shafts at Fitted Members (With a Related Photoelastic Analysis)," by R. E. Peterson and A. M. Wahl, presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

[8] "On the Stresses in the Neighborhood of a Circular Hole in a Strip Under Tension," by R. C. J. Howland, Proceedings Royal Society of London, January, 1930.

of experimental methods (membrane-analogy or optical methods) with the exact theory of the $(\sigma_1 + \sigma_2)$-network given by the author.

Dynamic Balancing of Rotating Machinery in the Field[1]

T. C. RATHBONE.[2] In referring to the writer's paper[3] on the same subject, the author may leave the impression that heretofore the problem had been carried only to the point of solving for the effect of unbalance at one bearing at a time, using the crude shaft marking for phase-angle determination. This paper[3] not only describes an accurate stroboscopic phase-determining apparatus[4] developed and used in the field some ten years ago for the study of vibration phenomena, but also points out the necessity for taking into account the inter-action of influences from one bearing to the other. It also describes a method of determining at all bearings, "unit vectors," which are produced by a unit of unbalance at each bearing, and then describes a rapid practical method for determining the necessary corrections at both ends of the rotor simultaneously, from the results of two trial runs.

The experiments leading up to this solution were carried out on large turbine rotors at speeds up to 2000 rpm. At first, the complete vibratory motion was recorded. Starting with the rotor in practically perfect balance, known unbalances were inserted, first at one end and then the other, and observations made with stroboscopic vibrometers at both ends simultaneously.

It was found that the resulting vibration for a variety of combinations responded with great fidelity—both in shape, size, and inclination of the ellipse, and in phase. The angles could be read to within 2 or 3 deg. From these data the "unit ellipses" were determined for a unit of unbalance and it was found that the resultant figure for any combination of unbalance weights at both ends could be predicted with almost uncanny accuracy, by superimposing the adjusted unit ellipses. Reversing the procedure, unknown unbalance corrections were determined by building up the unit ellipses to produce synthetically the same figures resulting from the unknown unbalance.

Professor Den Hartog solved the problem analytically for this general case, using the complete data observed. As two independent solutions were possible with the ellipse information, which should check, the observations were then confined to either the vertical or lateral component, and this made possible the graphical solution by unit vectors described in the writer's paper. The method involves reproducing synthetically the initial unbalance vectors by adjusting the unit vectors, and the solution is rapidly determined. Mr. Thearle's vector-operator method applied to these unit vectors gives a more direct and mathematically satisfactory solution, and for this contribution, as well as for his improvements in stroboscopic vibrometers, he is to be congratulated.

There are two important by-products of these developments which may be overlooked. First, the analysis of vibration by unit vectors, which affords a measure of the sensitivity or response of any installation due to a given unit of unbalance. Many futile attempts have been made to determine by analysis the resonance characteristics of foundation structures. It is almost impossible to predict these with any accuracy on complicated structures. There has been a great deal of controversy concerning the relative merits of concrete and steel structures. From the standpoint of smoothness of operation, these unit vectors afford a very potent means for direct comparisons of existing structures, which may well guide future design.

Second, the balancing processes under discussion are based on assumptions which do not always apply. Experience has shown that a fairly satisfactory linear relation holds between force and amplitude, and a fairly constant phase relation between cause and effect, so long as the condition of the rotor remains unchanged, or no impactive effects or changes occur in the bearings or structure. In such ideal cases where the rotor responds normally, it has been the writer's experience that operators in the field have little difficulty in arriving sooner or later at a satisfactory balance, each by his own individual and sometimes crude methods.

Rotors which exhibit erratic behavior, however, are difficult to balance by even the refined methods developed. There are many causes for such erratic tendencies, and under these changing conditions it is sometimes impossible to evaluate the unit vectors. In such cases, the stroboscopic vibrometer has proved itself to be a most useful tool in diagnosing the difficulty by continued observations over extended periods, covering all phases of operation of the unit.

JAMES J. RYAN.[5] The solution of the problem of field balancing of rotating machinery proposed by the author and the equipment he has developed to obtain the required data, are ideal. There appears, however, to be a need for more adaptable instruments to measure vibration from a scientific standpoint that may be placed in the hands of less expert operators. A universal oscillo-vibrograph has been developed in the machine design research laboratory of the mechanical engineering department at the University of Minnesota by means of which the data required in the solution of the dynamic-balancing problem presented by Mr. Thearle may be conveniently obtained. This instrument either displays visually or records photographically vibration movements indicating the amplitude of the vibration, the wave form, the frequency and the phase position to large scale. Such instruments promote the methods of scientific analysis in vibration problems, and permit engineers in general to solve problems in the field with little difficulty.

AUTHOR'S CLOSURE

The author appreciates the interest shown in this paper. Much of the discussion calls for no further comment.

Under certain conditions where the rotor exhibits erratic behavior, Mr. Rathbone questions the validity of the assumptions upon which the balancing process is based. It has been the experience of the author, also, that ". . . a fairly satisfactory linear relation holds between force and amplitude, and a fairly constant phase relation between cause and effect, so long as the condition of the rotor remains unchanged, and no impactive effects or changes occur in the bearings or structure."

Regarding rotors which exhibit erratic behavior, such as changes in vibration amplitude and phase caused by changes in load or temperatures, it is obvious that no single combination of balance weights can give smooth operation under all conditions. Observations of both amplitude and phase of the vibration are necessary in order to establish the nature of the variations in vibration. It is clearly an advantage to know the extent and nature of the variations in vibration before attempting to correct for them by balancing. This knowledge may save much time and effort.

---

[1] Published as paper APM-56-19, by E. L. Thearle, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Chief Engineer Turbine Division, Fidelity & Casualty Company of New York, New York, N. Y. Mem. A.S.M.E.

[3] "Turbine Vibration and Balancing," by T. C. Rathbone, Trans. A.S.M.E., APM-51-23, vol. 51, 1929.

[4] "Unusual Vibration of a 25,000-Kw Turbine Generator," by T. C. Rathbone, *Electric Journal*, vol. 25, no. 2, Feb., 1928.

[5] Assistant Professor of Machine Design, Mechanical Engineering Department, University of Minnesota, Minneapolis, Minn., Jun. A.S.M.E.

If, with a knowledge of the nature of the variations in vibration, their cause cannot be eliminated, then a compromise balance must be sought. When this is to be done, observations of the amplitudes and phase angles of the vibration are made over the normal range of operating conditions of the machine. From a plot of these vectors (N and F), the best compromise balance obtainable is readily seen. The balancing process is then completed in the usual way, taking care in the calculations to compare each subsequent vector observed (N_2, F_2, N_3, and F_3) with a value of the original vectors (N and F) corresponding to the same conditions of operation.

It is also obvious that the addition of balance weights to a machine cannot correct effects not caused by unbalance, such as disturbances caused by journals which are not round. However the first observations to be made usually reveal these defects, if they exist, and give information leading to their correction, thus saving much time which might otherwise be spent in attempting to correct, by balancing, for vibrations which are not at all due to unbalance.

High-Pressure Steam and Binary Cycles as a Means of Improving Power-Station Efficiency[1]

LUCIAN A. SHELDON.[2] The essence of Mr. Gaffert's paper is given in Fig. 12 which shows plant performance for steam cycles at various pressures up to the critical pressure, for the diphenyloxide-steam cycle, and for the mercury-vapor-steam cycle. These can be divided roughly into four groups: (1) Steam pressures ranging from 400 lb to 600 lb; (2) high-pressure steam ranging from 1200 lb per sq in. to the critical pressure; (3) the diphenyloxide-steam cycle; and (4) the mercury-vapor-steam cycle. Here we find the poorest economies with the normal steam pressures, the economies of the diphenyloxide-steam cycle about the same as those effected with 1200 lb per sq in. steam pressure, the economies from the use of 2500-lb and critical-pressure steam slightly better, and the mercury-vapor-steam cycle the most economical of all. I believe a brief explanation of the reason for this would be of interest.

The efficiency of any cycle depends upon the temperature range through which we work. In general, the greater this temperature range, the better the efficiency. However, it makes quite a difference whether the higher temperature is obtained by pressure, using saturated vapor, or by superheating. With steam we cannot get the high temperature by means of pressure and hence must resort to superheat. But the energy carried in superheat is only a fraction of that which can be carried in mercury at the same temperature where the temperature is obtained by pressure using saturated mercury vapor. A glance at the temperature entropy diagrams shows this quite clearly. With mercury we approach much more nearly the Carnot cycle and hence get considerably better efficiencies. With the diphenyloxide-steam cycle, as Mr. Gaffert points out, we cannot go much above 800 F as this substance breaks down above this temperature, thus limiting future gains in economy. Mercury, however, is an element and does not break down at high temperatures. Therefore, the upper temperature limit of the mercury-vapor-steam cycle is not limited by the heat-carrying medium but only by the ability of the materials which are available to withstand these temperature conditions. When better materials are produced the temperature range of this cycle can be increased still more and still better economies can be obtained.

In regard to the amount of mercury required, the Kearny boiler contains 300,000 lb of mercury, or approximately 7 lb per kw for mercury and steam. I have not included the kilowatts made from the steam in the water walls as this heat does not pass through the mercury boiler. Estimates made on more recent studies of mercury boilers show that we shall require from 5 to 7 lb of mercury per kilowatt and there is some possibility that in future this may be reduced to as low as 4 lb per kw.

According to conclusion No. 3, Mr. Gaffert feels that the annual rate of production of mercury is so small that there is slight possibility that the mercury-steam cycle will become universal. We all realize that prophecies of this nature are always more or less uncertain. There is apparently plenty of mercury in the world, the yearly production being the amount that is required rather than the amount which is available.

The General Electric Company in the past twenty years has produced an average of about 1,000,000 kw of turbo-generators per year. If we assume that 50 per cent of this turbine capacity would at some future time be mercury, it would require, at the most liberal estimate, 7 lb per kw, 3,500,000 lb of mercury per year, or about 1600 metric tons. From the curve of yearly production of mercury which is published in Minerals Yearbook, we find the variation in yearly output from 2150 metric tons to 5600 metric tons, so that this 1600 metric tons necessary, if half of the General Electric turbines were mercury, would come well within the normal variation of the production curve. Therefore, it would seem that it would not be necessary or advisable for us to slacken our efforts in the mercury process for many years to come because of lack of mercury.

Regarding the operation of mercury-vapor-steam equipment, the record of the Hartford, Kearny and Schenectady plants up to July 1, 1934, is listed in Table 1.

TABLE 1 RECORD OF OPERATION OF MERCURY-STEAM PLANTS

	Hartford	Kearny	Schenectady
Dates	October, 1928, to July 1, 1934	March 27, 1933, to July 1, 1934	Jan. 1, 1934, to July 1, 1934
Hours elapsed	50016	11063	4344
Hours in service	24905	6259	2946
Per cent available	49.8	56	68
Kilowatt-hours	202732000	82245000	29923000
Capacity factor, per cent	40.5	37.2	34.4

The figures in this table cover outages for the entire equipment. That is, they include the furnace and the steam part of the unit as well as the mercury portion. If the mercury portion alone were considered, the availability factor would be very much better. The capacity factor for the Schenectady equipment is rather low because the demand for steam was low and, therefore, the equipment was not called on to carry anywhere near the load it was capable of carrying. The availability factor of the Schenectady unit would have been higher but for the fact that the plant was shut down over week-ends.

The experience and knowledge which we have gained to date lead us to believe that we may expect future mercury-steam equipments to operate at higher availability factors than these first installations and, in fact, to be just as reliable and to operate just as large a percentage of the time as steam equipments.

A. G. CHRISTIE.[3] Mr. Gaffert's conclusion, that the thermal advantages of high steam-pressure and temperature cycles provide a considerable margin for increased capital costs, will stimulate renewed study in their possible application to many systems. The mercury-steam plants offer enticing possibilities from a

[1] Published as paper FSP-56-11, by G. A. Gaffert, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Engineer in Charge of Design, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[3] Prof. of Mechanical Engineering, Johns Hopkins University, Baltimore, Md., Mem. A.S.M.E.

purely thermodynamic point of view. Certainly a station performance of 8600 Btu per kwhr, or 0.67 lb of coal per kwhr, will be attractive in regions where coal costs are high. More data on the commercial performance of stations that are now in operation, together with cost figures for mercury make-up, for repairs and for supervision, are desired before a final appraisal of such systems can be made.

In considering Dr. Gaffert's results, it must be borne in mind that his values apply to base load stations, for he has assumed 70 per cent station average load factor and 8000 hours' operation per year. This raises the question: "Should the new equipment be designed for base-load service?" It is obvious from Dr. Gaffert's results that the possibilities of further improvements in the steam cycle, or in fact in any of the other cycles, are limited until metals can be developed for temperatures well above 1000 F. This temperature, though at present regarded as the safe upper limit for metals, may soon be exceeded, as much study and research are devoted to this subject and new alloys suitable for still higher temperatures may appear at any time. The justification for a new station for base-load service over any considerable period is, therefore, dependent to a large degree upon such developments in metals.

The late Peter Junkersfeld repeatedly called attention to the fact that many new turbine units were installed on the assumption of high capacity factors over a long period of use. Junkersfeld found from analyzing actual station records that few of these units developed the expected capacity factors over their useful life. In other words, expenditures have been made in many new stations that were in excess of the costs warranted by actual performance. The difficulty of financing new utility plants at present should cause any additional expenditures for securing high thermal efficiency to be subjected to the most conservative analysis of actual operating performance.

Dr. Gaffert could advantageously supplement this valuable paper by additional contributory data that he must already have at hand. For instance, he might indicate in the various cases, the heat to the condenser. Such data would clearly show the advantage of high pressure and temperature cycles where cooling water is limited or where cooling towers have to be used. He might also indicate the percentage of the main generator output required in the various cases to supply power to auxiliaries. It would be interesting to learn how this varies with the increased thermal efficiency of the plant.

A. B. Clark.[4] The calculations in Dr. Gaffert's paper are all based on a unit of about 50,000-kw capacity. It would therefore appear doubtful if such small units could be economically constructed for operating at steam pressures of 2500 lb or 3000 lb as shown in Figs. 1 and 2, because the blade height, nozzles, etc., of the turbine unit would be very small, and the gland-packing leakage and other leakages would be correspondingly high.

In practice the heater system would be changed somewhat. Those who have had experience with feedwater under high pressure would not want to put so many heaters under the full feedwater pressure, but would reduce the number to an absolute minimum.

Then again, small heater drain pumps for high temperatures and pressures are not practical. Their use leads to a very unbalanced design that results in low efficiencies and high maintenance costs. If the drains are flashed down, the lower-pressure heaters are not able to extract as much steam from the turbine and would modify the efficiency of the cycle. The application of drain coolers would help in this respect.

Increasing the number of heaters does not necessarily increase the economy. It has been found that the omission of an intermediate heater in one particular case did not change the economy of the unit; in fact, there was a slight gain. The machine in question was a reheating unit, and in such cases it is necessary that sufficient heat be taken out of the steam and converted into work before the steam is extracted. This may not be done easily when a large number of heaters are employed.

It is for such reasons as given above that it is sometimes difficult, except in a general way, to benefit from such a system of curves as presented in Fig. 12, which is probably the reason why they are given.

Some years ago, when determining the heat balance of units operating at 2400-lb and 3200-lb pressure, it was found that when the work of the boiler feed pump was taken into account, there was but 0.5 per cent difference in efficiency and only about 1 per cent gain in the heat charged to the turbine cycle. Owing to the dense steam, it would be expected that a turbine operating on a pressure of 3200 lb would not be quite as efficient as one operating at a lower pressure. These figures are somewhat less than those given by the author.

Binary cycles are very interesting and that of the mercury-steam cycle has been commercially worked out. It is, of course, an evaporation-and-condensation cycle. If, however, a system is used by which concentration by evaporation and dilution by absorption of vapor is considered, there are more materials available. However, they effect more of a solution cycle than a binary cycle. For instance, it is well known that caustic soda when mixed with water generates considerable heat. A small turbine has been run experimentally on steam generated with concentrated caustic soda by diluting it with the turbine exhaust steam, and then reconcentrating the caustic soda. As most chemicals boil at higher temperatures than water, steam generated in such a cycle is in a superheated condition. Thus it would appear possible to generate steam by concentrating a suitable compound and use it in a steam turbine, and then dilute the compound by exhaust or bled steam from the turbine to generate steam for a second steam turbine.

Dr. Koenemann's cycle is something of this order in that it uses ammoniacate. The cycle uses ammonia vapor instead of steam and then by a heat-exchanger device generates steam for a turbine from the heat given up from an ammonia condenser.

It has taken years of careful work to give us the steam tables and there still seems to be some doubt as to their accuracy at the higher pressures and temperatures. Before any new fluid can be accepted, its heat properties, as well as any corrosive properties it may have, must be determined. This is apt to be quite an expensive and lengthy undertaking.

G. A. Hendrickson.[5] For those interested in comparing the cycles considered in Dr. Gaffert's paper with actual plant results, it should be noted that an important item, which amounts to some 10 per cent of the total heat consumption of actual plants, has been omitted. This item may be designated as the plant operating efficiency ratio, or the ratio of the thermal efficiency of routine operation to the thermal efficiency that would be obtained if each apparatus in the plant operated continuously under test conditions at its most favorable load. Among the items that contribute to the reduction of the operating efficiency ratio below unity are: operation of turbines at loads other than maximum efficiency load; redistribution of heat quantities in the turbine due to changes in load; pressure drop; temperature loss; leaks in steam lines; banking loss in the boiler room; loss due to burning out fires; and numerous unaccounted-for losses. Some of these have been considered in Dr. Gaffert's paper and some have not. In consequence, the results given are probably 1000 Btu

[4] Engineer, Sargent & Lundy, Inc., Chicago, Ill., Mem. A.S.M.E.

[5] Engineer, The Detroit Edison Company, Detroit, Mich., Assoc. Mem. A.S.M.E.

per kwhr lower than those obtainable from actual plants operating on similar cycles. This difference should be borne in mind when comparing the paper with actual plant results.

In a paper, "A Thermal Study of Available Steam-Power-Plant Heat Cycles," presented by S. T. Vesselowsky and the writer at the A.S.M.E. meeting held in Chicago, June, 1933, an attempt was made to present steam-plant results comparable with actual operating data. It was pointed out in that paper that for steam plants the regenerative feed-heating cycle without reheat is probably best suited to ordinary conditions where a relatively low-capacity use factor is encountered. In base load plants and other situations where a high-capacity use factor is found, reheating or a binary cycle might be justified. From a comparison of Dr. Gaffert's results with those of Mr. Vesselowsky and the writer it appears that at a given limiting temperature the reduction in heat rate available from the use of reheating is about 1500 Btu per kwhr. A binary cycle using mercury and steam makes possible a further reduction of about 1000 Btu per kwhr. This improvement is not immediately available, however, but must follow out a slow and costly process of development, and in the end the net economic gain is much reduced. In the meantime the type of plant which converts chemical energy in coal into heat and then into mechanical power is slowly approaching a minimum limit in heat rate in the neighborhood of 9000 Btu per kwhr. This limit, which is now in sight, is imposed by the properties of commercial materials available for construction.

In this situation it is pertinent to mention that no decreases in heat rate comparable to those in the past need be expected unless radical changes are made in the basic cycle. In the period 1910 to 1930 the minimum heat rate attainable was reduced by one-half. With plants now operating at around 12,000 Btu per kwhr, the margin for further reduction is small. Only the magnitude of the coal bill makes further development practicable. Papers such as the present point out the most promising possibilities in this development. The same magnitude that makes such work possible, however, also makes imperative the highest accuracy attainable in theoretical comparisons, and this with the least possible confusion. Heat rates chargeable to the operation of the turbine alone are probably the easiest means of comparing different cycles. Such comparisons, however, are not final since they do not give overall performance. For those who wish to make quick comparisons with actual plant data, a plant heat rate estimated from the computed turbine results may be added with little difficulty. The writer believes such a presentation to be much more convenient for the general reader.

J. J. Grebe.[6] On the basis of the same boiler efficiencies, the performance figures given in Dr. Gaffert's paper check closely the values which were published by Dr. H. H. Dow in 1926.

Regarding the diphenyloxide or Dowtherm boilers, these units do not have to have forced circulation. In fact, the first boiler built in 1926 was operated at a heat input of 10,000 Btu per sq ft without any circulation difficulties as long as the boiler pressure was maintained above 35 lb. It is merely necessary to proportion the circulation path so that, first, the vapor volume produced can be discharged at a reasonable velocity, and, second, this vapor has the chance to use its air-lift effect as much as possible. For boilers operating at the maximum allowable temperature and high capacity, and using a small inventory of Dowtherm, it is naturally most economical and practical to use forced-circulation boilers such as illustrated in Fig. 2 of Dr. Gaffert's paper.

It might be suggested to Dr. Gaffert that his study be continued to include a cycle in which mercury and a second fluid other than water be used. A mercury condenser boiler used to vaporize a lower-boiling fluid, such as benzol or alcohol, would give a high-pressure vapor requiring little or no superheat for efficient expansion in a turbine, and would permit condensation at a lower temperature, particularly in the winter when very cold condenser water is available, and still not reach enormous vapor volumes. Such a fluid would also have the advantage of a higher vapor density so that the turbines could operate at a lower speed. It would, however, require a larger condenser surface on account of the decreased heat-transfer rates.

Frank O. Ellenwood.[7] Regarding the high-pressure cycle, the statement is made that for a throttle pressure of 1200 lb, the reheating pressure would lie near 200 lb per sq in., depending on the initial temperature of the steam. As a matter of interest to engineers who have not investigated the question of the optimum reheating pressure, it seems appropriate to mention that the 200-lb reheating pressure, as given, would be close to the best one, provided the throttle temperature is about 800 F. On the other hand, the writer believes that if the throttle temperature were 1000 F and the throttle pressure were 1200 lb, the reheat pressure would then be closer to 100 lb per sq in., for the best thermal results. For the 2500 lb (800 F) throttle steam, the writer agrees with the author in selecting a reheat pressure of 500 lb, as specified in Fig. 1. The heat rate of this 2500-lb station is given as 10,520 Btu per kwhr, which has been checked roughly by the writer. This performance, according to the author, is equaled by the 1200-lb (1000 F) plant without reheating. Such a result implies about $2^1/_2$ per cent better performance of the 1000-F turbine than that estimated by Carter and Ellenwood.[8]

It is particularly interesting to note that the curves in the author's Fig. 12 that are below the value of 9300 Btu per kwhr, represent a region that has seldom, if ever, been entered by the performances of the very best of our internal-combustion engines.

Authors' Closure

We are indebted to Mr. Sheldon of the General Electric Company for presenting the operating records of the three mercury-steam plants now in operation. It seems certain that when this binary cycle is used for a base-load station, the availability will be as great as in the case of ordinary steam plants.

Regarding the availability of mercury as a fluid, it is significant to note, according to Minerals Yearbook, that a high rate of production over a sixteen-year period caused by an increase in demand has resulted in an increase in the price per pound. Admittedly, the present method of recovery is crude and all mercury-bearing ore deposits have not been tapped. Mr. Sheldon's figures showing variation in yearly output from 2150 metric tons to 5600 metric tons are for world production. If, as indicated in the author's paper, an average world-production rate were taken at 3000 metric tons per year, allowing no more mercury than 4 lb per kw installed, approximately 1,650,000 kw could be installed per year. However, since a considerable percentage of this production is absorbed in the chemical industry, the amount available for power is at present entirely inadequate to supply the annual increase in generating capacity required throughout the world. It seems evident, therefore, that the mercury-steam cycle must share the increase in load with other methods of generation.

In answer to Prof. A. G. Christie's remarks relative to the per cent of main generator output required for plant auxiliaries, the author has listed in Table 2 of this discussion figures based upon a study of the various cycles. These figures include power

[6] Director, Physical Research, The Dow Chemical Company, Midland, Mich. Mem. A.S.M.E.

[7] Prof. of Heat-Power Engineering, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

[8] "The Thermal Performance of the Detroit Turbine Using Steam at 1000 F," Trans. A.S.M.E., FSP-56-8, July, 1934.

for preparation of fuel, for driving the boiler feed pumps, hotwell pumps, heater drain pumps, mercury or diphenyloxide boiler feed pumps in the case of binary cycles, and forced and induced draft fans.

TABLE 2 PER CENT OF MAIN GENERATOR OUTPUT REQUIRED FOR AUXILIARIES

Type of cycle	Auxiliary power, per cent
Steam cycle:	
400 lb (800 F)—5 pt. extr.	3.97
1200 lb (800 F)—5 pt. extr.	4.74
[a]2500 lb (800 F)—6 pt. extr.	5.39
[a]3200 lb (800 F)—6 pt. extr.	5.88
Mercury-vapor-steam cycle:	
200 lb (1020-F) hg 500 lb (800-F) steam, 4 pt. extr.	2.67
Diphenyloxide-steam cycle:	
210 lb (800 F) diphenyloxide 730 lb (1000 F) steam, 4 pt. extr.	4.33

[a] Using multicylinder reciprocating boiler feed pumps.

The author agrees with A. B. Clark's comments regarding the arrangement of extraction heaters in the feedwater cycle. It is undoubtedly true, as mentioned in the author's paper, that it would be more expedient to drain all of the heaters in the high-pressure steam cycles back to the deaerating heater to eliminate the complicated problem of handling small quantities of drainage at high discharge pressures. The balance of increase in fixed charges versus the increase in efficiency due to additional extraction heaters will determine the optimum number of heaters to install in any given cycle. The author's paper dealt only with the thermodynamics of the question inasmuch as available costs on high-pressure heaters for extremely high-pressure steam cycles are not available.

Referring to the use of a caustic solution such as potassium hydroxide or sodium hydroxide, this cycle has the advantage that it can store energy in a chemical form until required. However, there appears to date to be no authentic data as to the purity of steam leaving such a caustic solution, and in case there were even slight traces of caustic the resulting corrosion on the turbine plant would become a serious factor in a short time. Dr. Koenemann's cycle, to the author's knowledge at least, has never been put into actual practice. The main difficulty is that ammonia at temperatures approaching 800 to 1000 F is entirely unstable and a considerable percentage breaks up into the constituent gases which are very corrosive.

G. A. Hendrickson of the Detroit Edison Company points out several losses which are ordinarily summed up into the figure "operating ratio" for a given plant. The author prefers to consider auxiliary power, that is power required by all pumps and draft equipment as auxiliary power, and then, depending upon the type of plant as well as operating staff, to allow a certain operating ratio to account for these various losses. This method is in constant practice among various capable designing engineers. In short, it enables comparisons between cycles to be made more readily, but, as pointed out by Mr. Hendrickson, necessitates correction for any particular plant to obtain actual day-to-day performance. Mr. Hendrickson also states that heat rates chargeable to the turbine alone are probably more interesting for comparison. In the author's original paper, heat rates chargeable to the turbine for each cycle are stated on each cycle diagram. A comparison of turbine heat rates for a binary cycle is likely to be quite misleading as the per cent of auxiliary power varies considerably for various "top" fluids, being almost twice as much in the case of the diphenyloxide-steam cycle as in the case of the mercury-steam cycle.

If it were possible to obtain a high capacity as well as maximum allowable temperature without a circulating pump, the efficiency of the diphenyloxide-steam cycle could be slightly improved, as brought out by the remarks of J. J. Grebe of the Dow Chemical Company. It is also quite true that were it possible to find a fluid with temperature entropy characteristics not unlike mercury for use as the "bottom" fluid in a binary cycle, the overall cycle efficiency could be improved somewhat with considerable less capital expenditure for extraction equipment.

Prof. F. O. Ellenwood points out that the best reheat pressure is a function of throttle temperature. For lack of space the author did not include curves showing this effect, but the author's results are exactly in accord with the statements made by Professor Ellenwood regarding the location of the best reheat pressure as a function of the throttle pressure.

A New Theory for the Buckling of Thin Cylinders Under Axial Compression and Bending[1]

STEWART WAY.[2] This paper is interesting in that it provides an explanation of the difference between the buckling loads obtained in practice for thin cylinders and the values arrived at by old theories.

If there were no initial eccentricities in the cylinder, failure would take place with the reaching of the point of elastic instability. To determine this point, it is not necessary to consider large deflections. It appears that in the problem at hand it is necessary to consider large deflection phenomena only because of the assumed presence of initial eccentricities.

The principle that the final buckling shape will be the same as that of one of the components of initial deflection is probably quite sound. However, in certain very improbable cases, it is conceivable that the final buckling shape would be totally different from the initial shape. I have in mind the case of a strut with initial deflection in the form of a full sine wave, which, with axial loading, becomes unstable and buckles to the side.

[1] Published as paper AER-56-12, by L. H. Donnell, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Westinghouse Research Laboratories, East Pittsburgh, Pa.

Rolling-in of Boiler Tubes

By F. F. FISHER[1] and E. T. COPE,[2] DETROIT, MICH.

This paper presents an introductory study of boiler-tube rolling and reviews the methods now in common use. It points out their limitations and explains and proposes specifications for a new method which approximates the ideal. Test results are given which show the superiority of this new method. A few pertinent details of tube rolling are also mentioned, including the over-rolling of tubes as a source of high stress concentration in the sheet metal around the tubes. This high stress concentration may be a source of corrosion fatigue.

Introduction

ALTHOUGH much has been written about boiler shells and tubes, and riveted and welded joints, apparently nothing has been published in English about the method of fastening tubes into boiler shells and tube sheets. It is true that test methods have been applied for determining the water-tightness of these joints, but such tests do not disclose the condition of the joint. Manufacturers of rolling tools furnish instruction booklets briefly explaining the use of the tools, yet engineering publications printed in English contain no information regarding this very important item of boiler design and construction. Although all other parts of the boiler are designed with great care by engineers after careful study of strength of metals, effects of temperature, the permissible oxygen and carbon-dioxide content of boiler feedwater, etc., the production of the joints between the tube ends and the tube sheets or drums has been left largely to the ingenuity of the mechanics who assembled the structure. It appears, therefore, that the expanded tube joint, if its use is to be continued, should be subjected to scientific study, its limitations recognized, and an effort made along scientific lines to standardize field practice so that all the joints in a given boiler may be of maximum strength as determined by such investigation.

The knowledge of what constitutes a satisfactory rolled joint is now lacking. Once this knowledge is available and the correct procedure for rolling the joint has been established, the production of joints of maximum strength and tightness could be controlled. If the tubes could be machined to fit into carefully tooled tube holes, or if the tube ends could be shrink-fitted into the holes, the problem would be relatively easy. But liberal clearances must be allowed to permit easy introduction of the tubes into the holes, particularly in tight places, of which there are many in a modern large-capacity boiler. Commercial practice and the demand for low-cost production puts precise machining out of the question. On the other hand, if the differences in diameters between the tubes and tube holes were constant to a micrometer tolerance and the tube-wall thickness did not vary, the problem of securing uniform joints would be relatively simple. Also, if the surfaces of tube and tube hole which are to produce the joint were of uniform and standard finish, the production of uniformly tight and strong joints would not be so difficult. But when joints are to be made between the surfaces of tube ends and tube holes in which the clearance may have any value from ten-thousandths to sixty-thousandths of an inch in a bank of tubes, where the tube holes are reamed to commercial finish yet the tubes are covered with mill scale or are indifferently cleaned, one should not be surprised if a considerable percentage of the joints shows leaks when the assembled structure is subjected to a hydrostatic-pressure test. Neither should one be surprised if the rerolling operation causes leaks in joints accepted on the first hydrostatic test, nor that such joints fail after a comparatively short period of service.

The production of a joint by expanding a tube into a tube hole entails the execution of two distinct phases of the rolling-in operation. The first consists of stretching the tube wall until there is contact between the outside surface of the tube and the inside surface of the tube hole. A joint does not begin to exist until this contact has occurred. As already noted, the difference in diameter of the tube and the tube hole is by no means constant in any group of prospective joints. Therefore, the amount of expanding of the tube necessary to produce contact between tube wall and tube hole may vary widely from tube to tube. Consequently, this fact must be taken into account when producing a joint. The second phase of the rolling-in operation consists of giving the tube wall further permanent stretch, thus pressing the outside surface of the tube against the inside surface of the tube hole. This pressure alone is the source of fluid tightness in the joint. Also this pressure, coupled with the friction between the two metal surfaces, is the source of the strength of the joint which holds the tube in the tube hole when pressure is applied within the boiler. Since the pressure between the two surfaces is the only variable in any given case entirely within the control of the mechanic doing the assembling, it might be at once concluded that the greater the expansion the better the joint. That this is not true has been demonstrated by tests which show that both the tube wall and the metal surrounding the tube hole are subjected to severe cold working when excessive expansion is practiced. When a tube must be replaced in a tube hole which has been heavily over-expanded, the hole is badly distorted and should be reamed to true it up and to remove the metal which was overstressed at the previous rolling.

The formation of an ideal joint by rolling-in requires:

1 A device for expanding the tube so that it is cylindrical at all times during the rolling-in operation.

2 A means by which the completion of the first phase of the rolling-in will be indicated definitely. This anticipates a mechanical, a magnetic, or an electrical device for positively indicat-

---

[1] General Foreman of Mechanical Erection, The Detroit Edison Company, Detroit, Mich. Mr. Fisher was born near Hempstead, N. Y., educated in public and technical schools in England, Germany, and the United States. He served as shop superintendent, erecting engineer, and master mechanic in shops in the Far East, South America, and the United States. For the past fifteen and a half years he has been connected with the Construction Bureau of The Detroit Edison Company.

[2] Research Department of The Detroit Edison Company. Mem. A.S.M.E. Mr. Cope was graduated in 1907 with the degree of B.S. in M.E. from the Towne Scientific School, University of Pennsylvania, and was granted the degree of M.E. in 1910 by the same university. After graduation he spent three years with the Westinghouse Machine Company of Pittsburgh, Pa., and three years as instructor in the Engineering College at the University of Michigan. Since 1913 he has been connected with The Detroit Edison Company, and since 1919 in the Research Department.

Contributed by the Power Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

ing the establishment of a firm contact between the outside surface of the tube and the inside surface of the tube hole.

3 A means for indicating the amount of deformation of the metal surrounding the tube hole. This metal should not be stressed beyond its elastic limit.

A method of tube expanding based on these requirements should produce fluid-tight joints which show the optimum strength to resist destruction when subjected to fluid pressure. Furthermore, it has been well established that corrosion fatigue occurs at points of high stress concentration. The metal of tubes and tube sheet surrounding the holes in boilers in which the tubes have been heavily over-rolled should be an ideal point for the beginning of this form of failure.

This paper presents the results of a study of the expanding of boiler tubes and proposes a method which, over a five-year period, has produced uniform joints of optimum strength. In this method the severe cold working of the tube and drum metal, because of excessive expanding of the tubes, has been either avoided entirely or reduced to a minimum.

Conventional Methods of Tube Expanding

Three methods of tube expanding are generally accepted. These are the tube-bulge method, the uniform-expander-entrance method, and the measured-energy-input method. These will be reviewed briefly and their limitations pointed out. In the tube-bulge method, a few tubes in a bank, used as samples, are expanded until their outside diameters are increased a definite amount ($^1/_{32}$ in. usually) over the nominal tube-hole diameter. This measurement is made immediately next to the tube sheet or drum on the gas side in a water-tube boiler and the water side in a fire-tube boiler. The amount of tube bulge is measured by a gage of fixed opening. The expanding or rolling operation is carried out until this outside gage fits the bulge of the tube. After the operator gets the "feel" of rolling-in these samples, he proceeds to roll-in the remainder without measurement.

In the second method of tube expanding, the tool is entered a fixed distance into all the tubes in a given bank. This distance is determined by trial on a few tubes and the stop is set. In the uniform-energy-input method all joints are rolled-in with a consumption of a given amount of energy expressed in watthours or horsepower-hours.

In these methods of tube expansion there is no way of knowing definitely when the tube surface contacts the tube hole. Also, there is no way of knowing how much the tube is expanded after contact occurs.

TABLE 1 MEASUREMENTS OF FIVE GROUPS OF TUBES AND TUBE HOLES

Nominal tube size, in.	2	2	3 1/4	4	4
Minimum outside tube diam, in.	1.981	1.994	3.236	4.002	3.987
Maximum outside tube diam, in.	1.991	2.004	3.285	4.013	4.008
Minimum wall thickness, in.	0.186	0.138	0.189	0.204	0.211
Maximum wall thickness, in.	0.213	0.143	0.211	0.256	0.262
Nominal tube-hole diam, in.	2.031	2.031	3.281	4.031	4.031
Minimum tube-hole diam, in.	2.025	2.030	3.267	4.020	4.025
Maximum tube-hole diam, in.	2.040	2.038	3.320	4.041	4.039

Measurements made on five groups of tubes and tube holes revealed the values given in Table 1. The customary allowance of $^1/_{32}$ in. for tube bulge would result in a wide variation in the actual amount of expansion in the cases of the minimum and maximum sizes of the tube holes shown. The holes of minimum diameter might be over-expanded. The tube-bulge method of expansion, therefore, does not meet the second and third requirements of the ideal expansion method. If, on the other hand, these joints had been assembled by the uniform-expander-entrance method, in which no exact account is taken of the variation in clearance between tube and hole, and no allowance is made for the variation in tube-wall thickness, it is obvious that

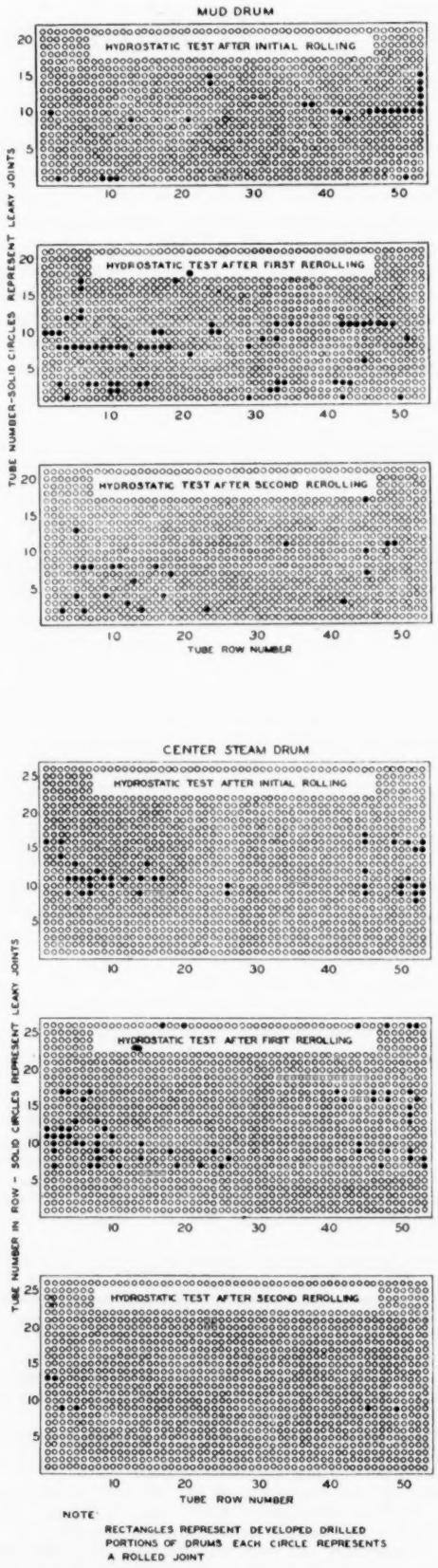


Fig. 1 Record of Rolling-in Joints in Two Drums of a Boiler Using One Conventional Method

the resulting joints would not have been expanded uniformly. If by chance, resulting from random choice of tubes, the smallest tube having the thinnest wall had been expanded into the largest hole, the resulting joint would have been very different from the one resulting from expanding the largest tube having the thickest wall into the smallest hole. In the first case the tube might have been expanded only enough to contact the tube hole, whereas in the second case the joint would have been heavily over-expanded, resulting in severe cold working of the metal of tubes and tube sheet and the setting up of points of very high stress concentration. This second method also falls short of meeting the requirements of the ideal method of tube expanding.

In the uniform-energy-input method no account is taken of the influence of differences in tensile strength of the metal of the tubes and sheets, differences in clearance between tube and tube hole, nor the condition of the rolling tool. When the self-feeding expander is used, the power required to flare the tube is at maximum at the same instant as that required for completing the rolling-in operation of the joint itself. The flare does not constitute an essential portion of the joint, as will be shown later, and the energy required to complete the flare cannot be separated from the total. This method also falls far short of meeting the requirements of the ideal joint.

The consequence of using an uncontrolled method of expansion is shown by the results obtained in the erection of a boiler under the authors' observation. The tubes in this boiler were expanded by one conventional method. The mechanics who did the work were experienced and careful. Fig. 1 shows graphically the results of the rolling-in and testing of the joints. It is to be noted particularly that the second rolling-in both drums produced a greater number of leaks than it was sought to remedy. These leaks were not necessarily in the same joints. The third rolling left the mud-drum joints in scarcely better condition than did the first rolling. The results on both drums indicate that the use of one conventional method of tube rolling with the lack of knowledge of the separate operations involved leads only to imperfectly rolled and probably badly over-rolled tubes and consequently permanently deformed tube and tube-sheet metal in which there are points of high stress concentration with consequent predisposition to corrosion fatigue.

The shape of the entering ends of the expanding-tool rollers is a matter of considerable importance. Since the metal on the inside of the tube wall is cold worked, it is of the highest importance that this cold working should take place with the least possible disturbance of the metal surface. If the entering ends of the rolls are not sufficiently rounded they leave a sharp shoulder at the inner end of the rolled portion. This produces an ideal starting point for cracks or corrosion, as shown by numerous tube failures which have been investigated. With self-feeding expanders, especially when used for rolling-in tubes into thick sheets (such as $1^1/_4$ in. and upward), the metal on the inside of the tube is plowed up by one roller and then rolled down by the succeeding ones. This leaves a spiral mark on the inside of the tube. It is usually not noticeable upon casual inspection, yet constitutes an excellent starting point for corrosion. To correct this condition the boiler erectors in the authors' company have ground the entering ends of the rollers of their expanders to a five-inch radius. A certain amount of cold working takes place, but instead of the metal being plowed into a sharp crest it is moved ahead of the rollers in the form of a gentle undulation. No roller scars are visible on the inside surface of the tubes.

The Elongation Method of Tube Rolling

The elongation method of tube rolling, as developed by the company with which the authors are associated, more nearly fills the specifications for an ideal method than does any one of the other methods noted. In this method advantage is taken of the fact that the tube is loose in the tube hole until the first phase of the rolling-in is completed. It further takes advantage of the fact that during the expanding operation the wall of the tube within the tube sheet is thinned. The metal is squeezed between the expanding tool and the surface of the tube hole and is forced to flow axially. This moves the tube axially a measurable amount, which is a function of the degree of the expansion. The only extra tool required by this method is a dial indicator fitted with the proper-size clamp, so that the indicator may be readily attached to the tubes. The tube is inserted into its tube hole and manually held in place. The dial indicator is attached as shown in Fig. 2. The clamp extends half way around the tube and

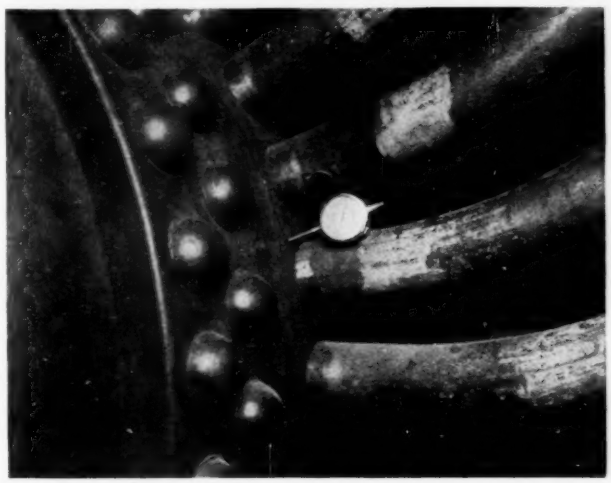


Fig. 2 Dial Indicator Attached to a Tube

bears at three points to prevent rocking. The end of the shaft of the dial indicator rests on the tube sheet or drum. The expanding operation proceeds, using a "parallel" expanding tool. During the first phase the needle of the dial indicator vibrates until the tube has been stretched into circumferential contact with the tube hole. At the instant of this contact the needle comes to rest, indicating that the first phase has been completed, and item two of the ideal method of expanding has been met. Continued expansion further stretches the tube wall into firmer contact with the hole and squeezes it so that axial flow occurs. This axial flow causes movement of the dial-indicator needle. The amount of movement of the needle necessary to produce the best joint has been established by tests which will be described later.

TABLE 2 DIMENSIONS AND CHANGE IN DIMENSIONS OF FOUR TUBES SUBJECTED TO DIFFERENT DEGREES OF ROLLING[a]

Before expanding			After expanding			
Outside diam	Inside diam	Wall thickness	Outside diam	Inside diam	Wall thickness	Elongation
3.996	3.580	0.208	4.010	3.608	0.201	0.010
3.990	3.578	0.206	4.031	3.630	0.196	0.020
4.008	3.586	0.211	4.036	3.642	0.197	0.030
3.995	3.575	0.210	4.031	3.645	0.193	0.040

[a] All dimensions in inches.

The dimensions and change in dimensions of four tubes of a given size, with different degrees of rolling, are given in Table 2. The tube dimensions were taken during a test conducted in connection with the elongation method of tube expanding. These tubes were cleaned, using a mechanical tube-end cleaner having carborundum cutters of 120-grit grade *H*. In place of a regular tube sheet, blocks of boiler plate 10 in. square, $2^1/_4$ in.

thick, and with one reamed tube hole each, were used. Three tubes were expanded to each of several elongations (axial movement caused by the rolling-in operation), all without flare. Each tube assembly was then subjected to hydrostatic pressure until the joint broke and the tube was forced out of the hole. Measurements of the tube diameters and wall thickness were made before expanding and after the assembled tube and block had been separated.

TABLE 3 CHANGES IN DIMENSIONS DURING EXPANDING OPERATION OF THE FOUR TUBES LISTED IN TABLE 2

Elongation, in.	Outside diam increase, in.	Inside diam increase, in.	Wall thickness decrease, in.
0.010	0.014	0.028	0.007
0.020	0.041	0.052	0.010
0.030	0.028	0.056	0.014
0.040	0.036	0.070	0.017

TABLE 4 ASSEMBLIES OF TUBES AND TUBE SHEETS FOR WHICH HOLDING STRENGTH HAS BEEN DETERMINED

Nominal outside diam, in.	Nominal wall thickness, gage	Plate thickness, in.
2	8	1.25
2	11	1.25
3¼	7	2.25
3¼	7	1.00
3¼	11	1.00
4	5	1.25
4	7	2.25
4	7	1.00

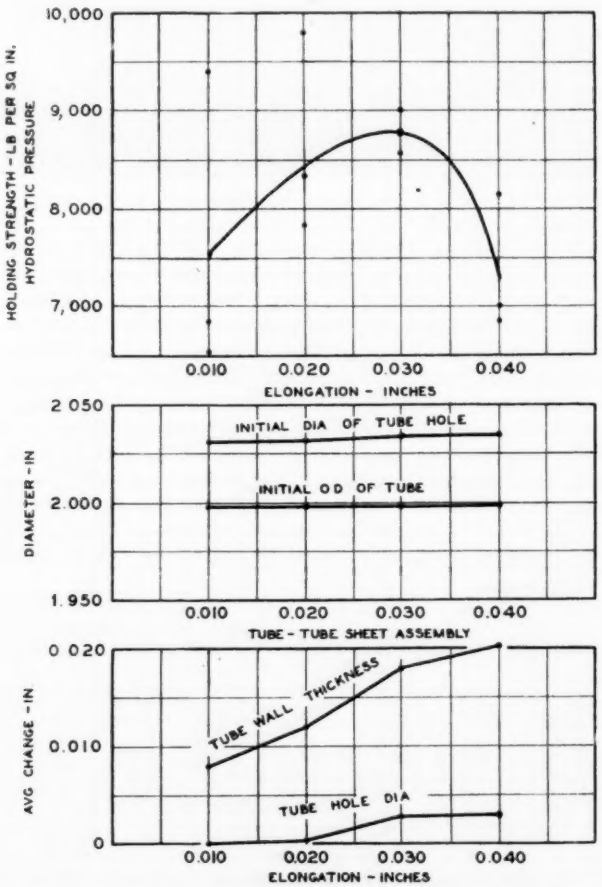


FIG. 3 TEST RESULTS FOR 2-IN., 8-GAGE TUBES ROLLED INTO 1¼-IN. PLATES

During the expanding operation, the changes given in Table 3 occurred. Even though the changes in outside and inside diameter of the tube rolled to 0.020-in. elongation were not consistent with the others in the schedule, the decrease in wall thickness and the axial movement were consistent.

It is evident from Table 3 that the elongation of the tube and the decrease in wall thickness are related. It remained only to establish experimentally the values of the relationship in terms of strength and water-tightness of the joints produced. The experimental determination of the relationship between elongation and the strength of the joint to resist destruction by hydrostatic pressure, called "holding strength," has been found for the assemblies of tubes and tube sheets listed in Table 4.

TEST PROCEDURE AND RESULTS

Sample joints were produced by expanding short lengths (usually 15-in. long) of boiler tube into blocks of boiler plate of the

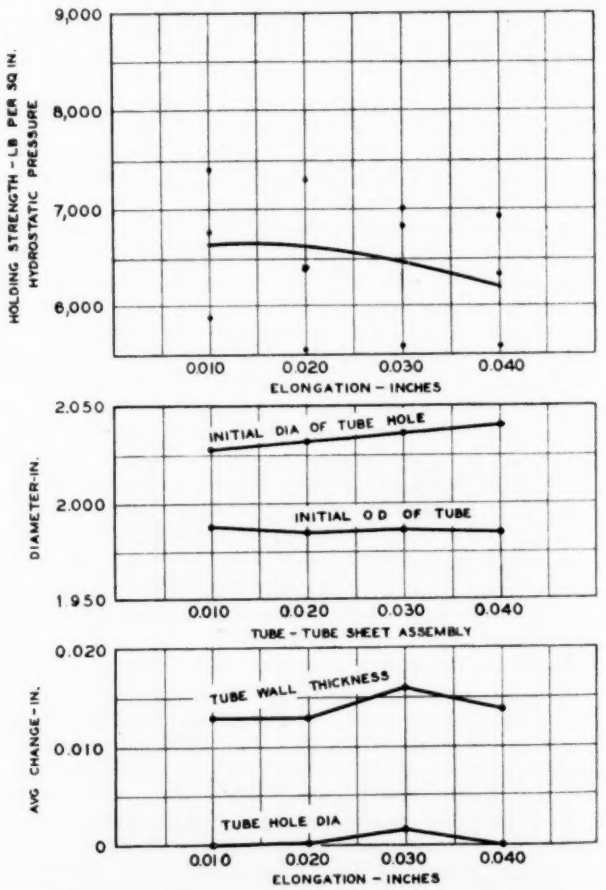


FIG. 4 TEST RESULTS FOR 2-IN., 11-GAGE TUBES ROLLED INTO 1¼-IN. PLATES

thickness indicated in Table 4. Each block of plate had one reamed hole of nominal diameter, finished in the same manner as the tube holes in a boiler drum. The tube samples were usually cut from a single length of tube. One end of each sample was closed by welding in a steel disk. The outside surface of the open end was cleaned of scale and other foreign matter by using a mechanical tube cleaner having carborundum cutters of 120-grit grade *H*. The 3¼-in. and one group of 4-in. tubes were given a supplementary rubbing with No. 1 carborundum cloth. Expanding was done by the use of an air-driven "parallel" expander without flaring rolls. The taper of the rolls was ¼ in. per ft, and that of the mandrel ½ in. per ft, and the feed angle of the rolls was 1½ deg. Dimensions of tubes and tube holes were measured, using appropriate micrometers, both before

expanding and after the tubes had been forced from the tube holes. Three joints were made for each of the combinations for each of several elongations (usually 0.010 in., 0.020 in., 0.030 in., and 0.040 in.). For instance, there were 12 joints made using 2-in., 8-gage tubes in 1 1/4-in. plate, 12 joints using 2-in., 11-gage tubes in 1 1/4-in. plate, etc. Each tube-hole plate was drilled so that a blank flange could be bolted to it. This blank flange carried the necessary connections for the introduction of hydrostatic pressure within the tube. Elongation during expanding was indicated by the dial indicator shown in Fig. 2.

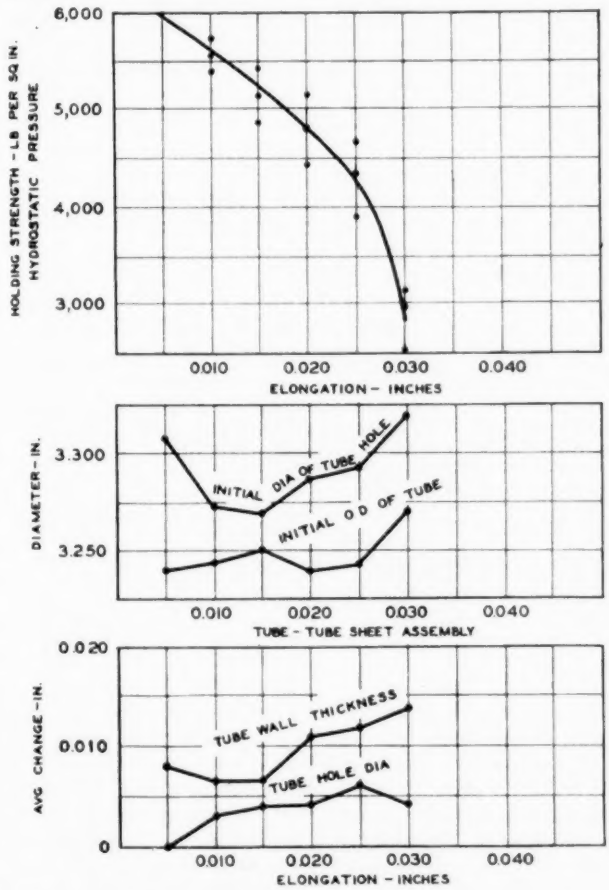


FIG. 5 TEST RESULTS FOR 3 1/4-IN., 7-GAGE TUBES ROLLED INTO 2 1/4-IN. PLATES

The results of these tests are shown on Figs. 3 to 10, inclusive. In these figures the holding strength is expressed in pounds per square inch hydrostatic pressure necessary to cause failure of the joint at room temperature. Failure occurs when the tube slips in the tube hole. The values of holding strength shown on Figs. 3, 4, 8, and 9 are somewhat scattered for each elongation owing doubtless to the fact that the tube surfaces were finished exactly as are those in a boiler. The values are probably such as would be obtained in actual practice. The values shown in Figs. 5, 6, 7, and 10 are not so scattered, doubtless because of the fact that the ends of these tubes were somewhat smoothed with No. 1 carborundum cloth after mechanical cleaning. The clearance (difference in diameters) between the tube and the tube hole is shown by the width of the band between the two curves, "Initial Diameter of Hole" and "Initial OD of Tube." In some cases this band is of almost uniform width, whereas in others, notably Figs. 5 and 6, the width varies considerably. The tube-wall thickness decreases at a relatively uniform rate with increase in elongation, as would be expected, whereas the increase in tube-hole diameter does not follow the increase in elongation. This condition might be expected if the tubes were made from much softer steel than were the tube sheets. In the cases of the tubes and tube sheets represented by Figs. 3, 4, 8, and 9, in which the Rockwell hardness numbers of tubes and tube sheets were obtained, in all instances the tubes were harder, consequently of higher tensile strength than the plates.

The tubes in several boilers have been expanded using the elongation method and the results have been highly satisfactory. Among these are the following:

1 Boiler No. 14, Trenton Channel power plant, operating at 400 lb per sq in. There were no leaks, but only small beads of water appeared on some joints. A light reexpanding of the joints on one drum was carried out and the boiler was passed.

2 Two boilers at the Ford Motor Company, Dearborn, Mich., operating at 1425 lb per sq in. The erector expanded the tubes in the same manner as were those at the Trenton Channel power plant. The tubes on the first boiler were expanded to 0.020-in. elongation. It was found necessary to reexpand a small portion of the tubes in order to pass the hydrostatic test. In the second

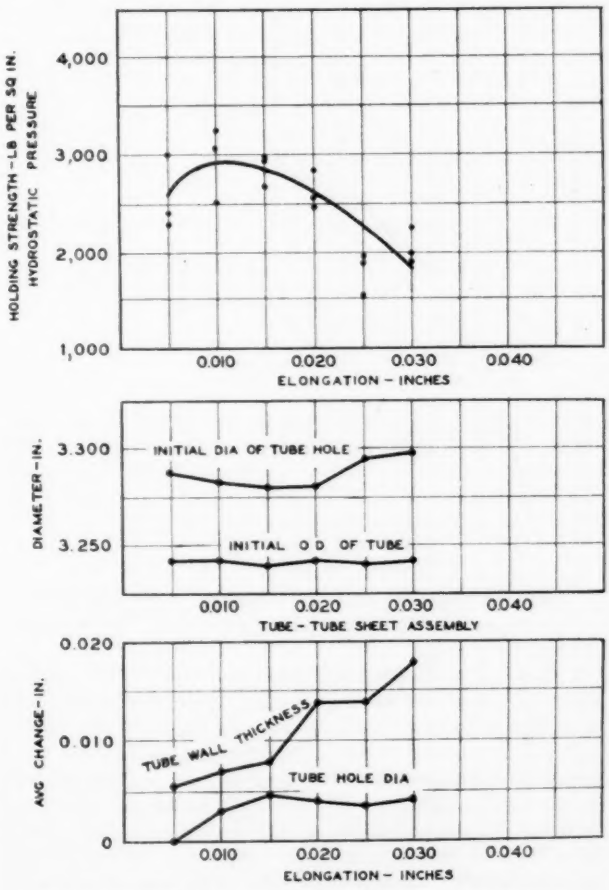


FIG. 6 TEST RESULTS FOR 3 1/4-IN., 7-GAGE TUBES ROLLED INTO 1-IN. PLATES

boiler, the tubes were all expanded to 0.025-in. elongation, and were passed on the first inspection without exception.

3 Two boilers at the Springwells Plant of the Detroit Board of Water Commissioners operating at 400 lb per sq in. Elongation failed to show on the first few tubes expanded. An investigation disclosed the fact that some of the tube holes were tapered. This condition was corrected and the tubes were then expanded to

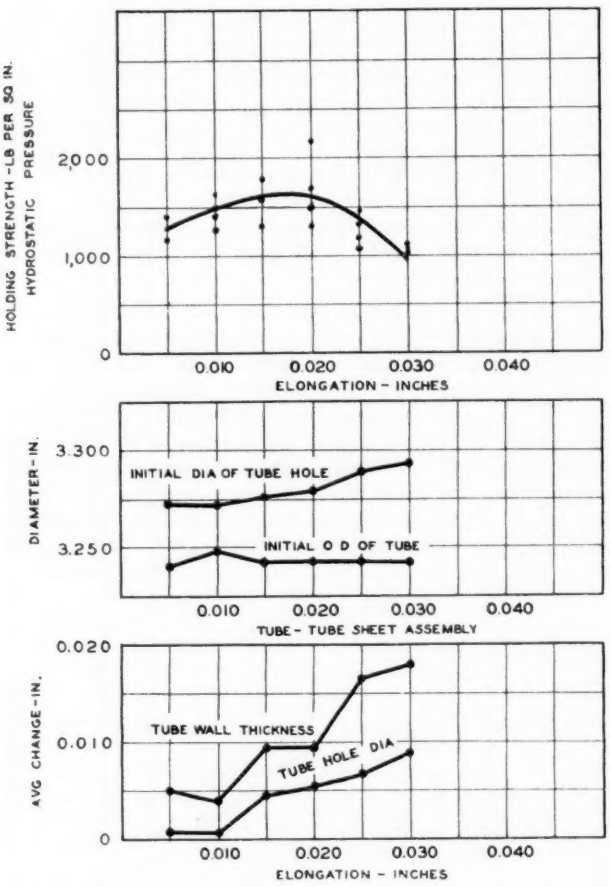


FIG. 7 TEST RESULTS FOR 3¼-IN., 11-GAGE TUBES ROLLED INTO 1-IN. PLATES

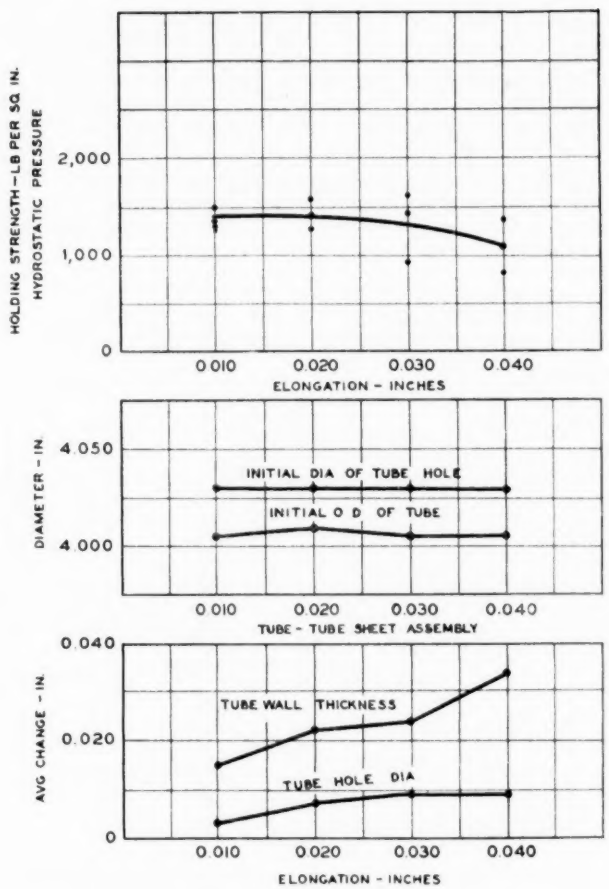


FIG. 8 TEST RESULTS FOR 4-IN., 5-GAGE TUBES ROLLED INTO 1¼-IN. PLATES

0.016-in. elongation. The hydrostatic test showed 25 slight leaks among 532 tubes (1064 joints). These were corrected by light reexpanding. By either of the conventional methods this condition of tapered holes would probably not have been discovered and serious trouble might have developed.

OTHER CONSIDERATIONS

There are several other related matters in the expansion of boiler tubes which should be given consideration. These are discussed briefly as follows:

1 *Effect of the Condition of Finish of Surfaces of Tube and Tube Hole on Tightness and Holding Strength.* When joints of any group are rolled-in to uniform elongation, the tightness and holding strength will depend largely on the finish of the surfaces which are forced into contact. It is obvious that a threaded joint between tube and sheet would produce a holding strength equal to the tensile strength of the net area at the root of the thread. This may be regarded as the strongest type of mechanically assembled joint. The minimum value of holding strength might be developed if the surfaces were polished. Logically, the holding strength of a joint in which the tube surfaces had been cleaned, using a mechanical tube cleaner equipped with carborundum cutters of 120-grit grade *H*, would fall between the maximum and minimum values suggested above. The strength of the threaded joint can be calculated if the form of the thread is known. The holding strengths of the joints in which the surface of the tubes had been polished and those in which the tube surface had been cleaned using a mechanical tube cleaner, were determined by hydrostatic test. In both cases the tube holes were smooth bored. The outside diameter of the tube was 3.996 in., the inside diameter was 3.580 in., it had a hardness of 70 Rockwell *B* (124 Brinell), and an approximate tensile strength of 60,000 lb per sq in. Tests on joints showed an average holding strength to be 2033 lb per sq in. for the one having polished tube ends and a holding strength of 3700 lb per sq in. for those joints in which the tube ends were cleaned by a mechanical tube cleaner. The calculated strength of a threaded joint (25 threads per in.) between this size tube and a tube sheet 2¼ in. thick is 118,800 lb. The total strength of the polished-tube joint was 27,600 lb, and that of the tube of standard finish used in the authors' company was 46,500 lb. It is probable that a machined tube rolled into a roughened hole would show the same holding strength as the roughened tube rolled into a machine-finished tube hole. It is evident from these figures that the condition of the two surfaces is a factor of first importance in the production of joints of high holding strength. It must be concluded that attention to the cleaning of tube ends is an important factor in the production of joints of uniform strength.

2 *The Effect of Hardness of Tube and Sheet Metal on Strength and Tightness of Joints.* Enough data have not been collected to warrant any conclusions in this matter. It merits some study and research.

3 *The Effect of Grooves in the Tube Hole.* Grooved tube holes have been used in an effort to improve the tightness and holding strength of rolled-tube joints. There are, however, few test data available to show the amount of improvement resulting

from this expedient. A. Thum and W. Ruttmann in their "Mitteilungen Nr. 45 der Vereinigung der Grosskesselbesitzer" (Dec., 1933) note in their summary under item 6: "The yield limit (due to a repeated bending movement) can be improved 50 per cent by a groove." Experimental results to establish this statement were not shown. Further investigation of this matter should be made, taking into account the depth, width, and shape of the groove or grooves and their location in the tube hole.

4 *The Function of the Bulge and Flare.* The opinion is held by boiler men that the tightness and strength of the joint depends in part on the bulge and the flare made during the expanding operation. It is true that the flare will prevent the tube from slipping out of the tube hole. When the tube is rolled-in, the backing out of the expander pushes the flare a few thousandths of an inch away from the tube sheet so that the flare does not contribute to the tightness or holding strength of the joint. Also, in many cases the flare is not correctly rolled and its effect on holding strength is therefore doubtful. In the case of flat tube sheets, the initial expanding of both bulge and flare will produce contact completely around the tube. On the other hand, in joints expanded into curved tube sheets excessive work on the flare is necessary to produce contact all around the tube. This produces a rounded edge in the tube hole which makes very difficult the refitting of a new tube in case of replacement. The flare does perform an important function in providing a smooth entrance to the tube which reduces the resistance to flow.

5 *Effect of Lengthening the Tubes Caused by Rolling-in.* The tests which established the elongation method of rolling-in tubes, showed that the tube is lengthened a measurable amount by the rolling operation. The more heavily the tube joint is rolled-in, the greater the lengthening of the tube. If adjacent tubes in a drum or tube sheet are rolled-in to different elongations, the difference in lengthening sets up considerable thrust on the tube which has been rolled the least, or compression on the tube which has been rolled the most. This compression may lead to the bowing of the more heavily rolled-in tube with the possibility of failure because of interference. If it were possible to roll-in all the tubes of a given bank or bundle simultaneously and equally, there would be no difference in lengthening of adjacent tubes. But at present tubes are rolled-in one at a time. The elongation given each tube except the first, imposes a thrust on all tubes that have been previously rolled-in. The presence of accumulation of thrust is especially evident in straight "tack tubes." That this is true is shown by the failure of tack-tube joints during assembling of the boiler, and perhaps more frequent failure after the boiler has been in service for some time. The actual value of this axial loading is a difficult matter to calculate, because of the shapes and arrangements of the tubes in any given boiler. An approximation has been made, however, by actually weighing the axial thrust during the rolling-in operation. This was carried out by having the free end of a short tube, which was being rolled into a 10-in. square plate, exert its axial thrust against the ram on a hydraulic jack. The base of the jack and the plate were rigidly joined together. The hydrostatic pressure shown by the gage on the jack was multiplied by the area of the jack plunger to give the value of the thrust. While this test is not presumed to reproduce the conditions occurring during the erection of a

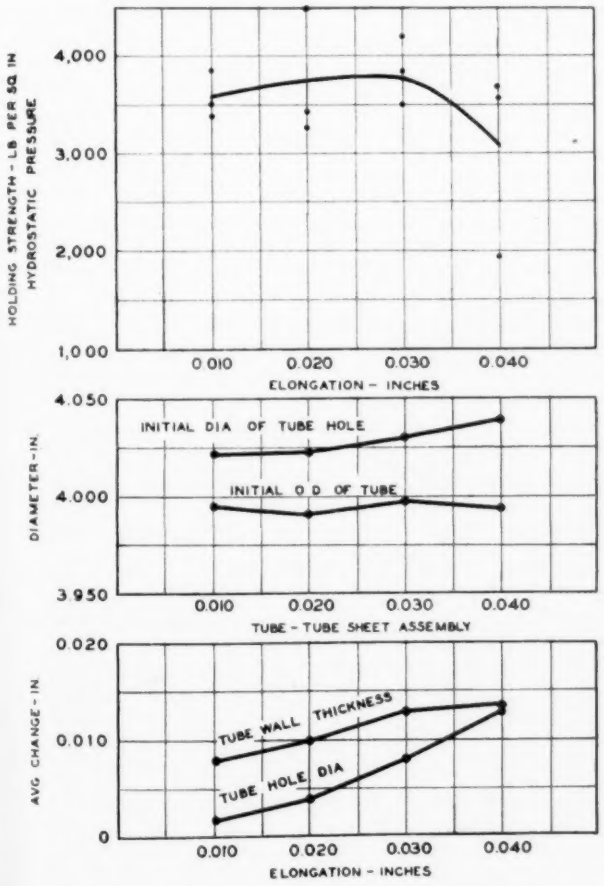


Fig. 9 Test Results for 4-In., 7-Gage Tubes Rolled Into 2¼-In. Plates

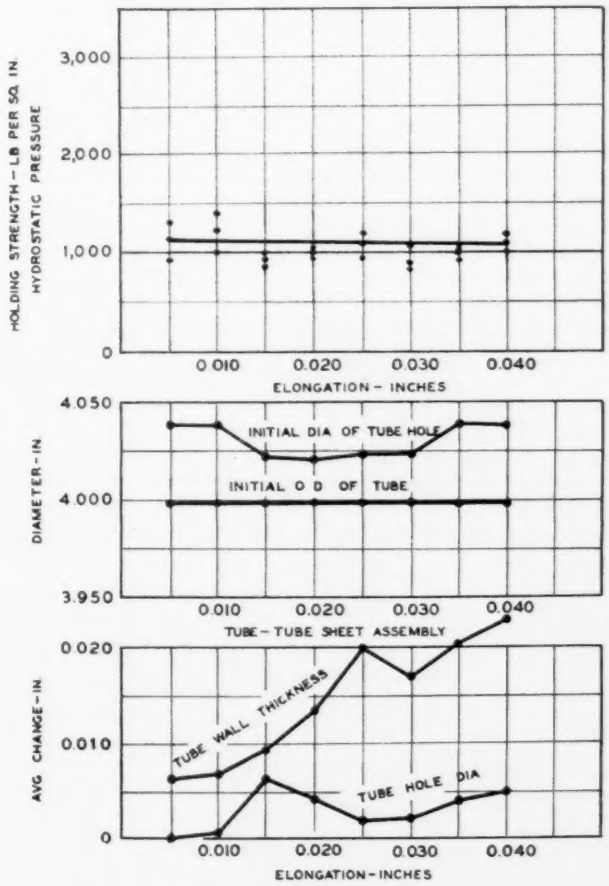


Fig. 10 Test Results for 4-In., 7-Gage Tubes Rolled Into 1-In. Plates

boiler, yet it does indicate the possible magnitude of the thrust. Fig. 11 shows the results obtained in one of the tests described.

The accumulation of thrust, if not controlled, often results in the bulging of the tube sheets, the raising of steam drums off their supports during erection or even the rupture of tubes. Control of cumulative thrust applied to boilers having drums may be effected by either of two procedures. In both cases tack tubes are installed first. They are fully rolled-in at one end and partly rolled-in at the other. Next, the remaining tubes are fully rolled-in at one (usually the upper) end and partly rolled in (usually to 0.005-in. elongation) at the other end. In the first procedure, the partly rolled-in ends are then fully rolled-in proceeding along

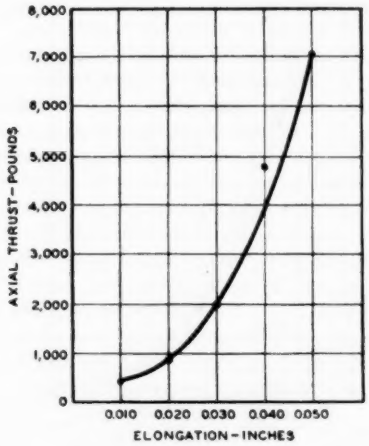


FIG. 11 AXIAL THRUST PRODUCED BY ROLLING-IN SEVERAL $3^1/_4$-IN., 7-GAGE TUBES 1 FT LONG, INTO $1\ ^7/_8$-IN. PLATES. THE DEGREE OF EXPANSION IS EXPRESSED AS ELONGATION

the length of the drum. In the second procedure, tack tubes are rolled-in as in the first procedure. All remaining tubes are fully rolled-in at one end (usually the upper end), the remaining joints are then rolled-in, starting at the middle of the length of the bank and rolling tubes to full elongation toward the ends of the bank. This divides the amount of accumulated elongation and the consequent thrust in half. In long banks, the rolling-in of the remaining tubes is carried out by dividing the length of the bank into four parts and rolling the tubes in each fourth of the length separately. Any of these methods reduces the amount of accumulated thrust and makes less likely the failure of the joints from this cause.

A few tests were made to determine the characteristics of the expanding tool. These tests were conducted by rolling-in $3^1/_4$-in., 7-gage tubes into $2^3/_{16}$-in. plate. The feed angle was changed for the different tests and a record made of the revolutions of the head of the tool and the time in seconds to complete the joint. The results are given in Table 5.

TABLE 5 CHARACTERISTICS OF THE EXPANDING TOOL

Feed angle of expander, deg.	No. of revolutions of expander head	Time to roll—joint, sec.
1	$18^3/_4$	150
$1^1/_4$	$17^3/_4$	110
$1^1/_2$	$17^1/_2$	90
2	12	60

CONCLUSIONS

This paper presents the results of an elementary study of the fundamentals of rolling-in boiler tubes. From these results the following conclusions have been drawn:

1 The rolling-in of boiler tubes may be broken down into two separate phases; in the first the tube is enlarged until it fits the tube hole; in the second the tube is further expanded into tight contact with the wall of the tube hole. Definite control of these two phases of the operation is necessary if the rolling-in is to produce joints of maximum strength and tightness.

2 In the usual methods of rolling-in tubes the two phases are not controlled with the result that uniformity of strength and tightness of joints cannot be assured. Records of such rolled-in joints show that some are under-rolled and others are over-rolled. Over-rolling of joints results in heavy cold-working of the metal of the tube and of that surrounding the tube hole, thus setting up points of high-stress concentration with predisposition to corrosion fatigue.

3 In the "Elongation" method of tube rolling, the two phases of the rolling-in operation are definitely controlled by the use of a dial indicator clamped to the tube being rolled-in. The relationship between elongation and holding strength has been investigated experimentally, using several tube sizes and sheet thicknesses. It appears that the most satisfactory joint is produced when the elongation is about 0.020 in.

4 There are several other matters in the tube-rolling problem which merit investigation. Some of these are: (*a*) The effect of the condition of finish of the surfaces to be forced together in rolling-in a joint. (*b*) The effect of the relative hardness of tube and tube sheet on the tightness and holding strength of the joint. (*c*) The effect of grooves in the tube hole on holding strength and rigidity of the joint.

In this paper little has been said about the tool used for rolling-in the joint. This tool, like most others, has been the result of growth over a period of years. There does seem to be a need for some further development in the tools ordinarily used. In the work done in Germany for the Association of Owners of Large Boilers, A. Thum and W. Ruttmann, the investigators, lay great stress on the effect of rate at which the joint is formed. This rate is a function of rate of feed of the tool and the number of revolutions of the tool to form the joint. They have merely touched on this matter and state that it should be studied further.

Steam-Turbine Leaving Losses and Vacuum Corrections

By LINN HELANDER,[1] MANHATTAN, KAN.

Based upon the discussion of steam flow through non-divergent nozzles with oblique exit faces in "Die Dampfturbinen," by Gustav Flügel,[2] this paper presents a procedure wherewith steam-turbine vacuum corrections may be calculated with a fair degree of accuracy when the design characteristics of the last row of blades and one set of water rates for specified steam conditions and loads are known. The procedure outlined is designed primarily for consulting engineers and power engineers, not for the turbine engineer. On that account certain simplifying assumptions have been introduced. These simplifying assumptions, although perhaps not acceptable where precise results are required, will be found satisfactory for the purposes for which the formulas presented herein have been developed. A comprehensive sample calculation is used to exemplify the procedure outlined.

In addition to a procedure for evaluating vacuum corrections, the paper presents simple formulas for evaluating small increments of isentropic heat drop when the expanding steam is saturated. These formulas give evaluations of small increments of heat drop more accurately than they can be read from ordinary steam charts and may be solved with little if any more labor than is used in reading charts.

Certain formulas are also presented which evaluate the mass rate of steam flow through a blade or nozzle when the critical velocity is reached therein, the mass rate of steam flow through a nozzle or blade when the terminal nozzle or blade pressure is the minimum attainable (that is, when the exhaust-chamber pressure is at or below that corresponding to the limiting vacuum), the rate of change of internal shaft work with a change in the exhaust-chamber vacuum temperature for expansions beyond the critical pressure, and the approximate rate of change of internal shaft work with vacuum temperature when the terminal blade pressure is above the critical pressure. Approximate formulas are given which express the specific volume and latent heat of dry saturated steam in terms of the temperature of the steam. These formulas have been developed for use where the exhaust steam is saturated and where, in addition, the temperature of the exhaust steam does not exceed approximately 140 F.

THE procedure herein proposed for calculating leaving losses and vacuum corrections is based on Flügel's discussion of the flow of steam in nozzles with exit faces not at right angles to the nozzle axis.[2] According to Flügel's analysis, where steam flows through a non-divergent nozzle with an oblique exit face, the velocity of the steam relative to the nozzle at the exit will be directed along the axis of the nozzle so long as the critical velocity is not exceeded; but when the critical velocity is exceeded, the steam leaving the nozzle will be directed, relative to the nozzle, along a line that makes an angle δ with the nozzle axis. Where the pressure in the exhaust chamber does not fall below a certain value P_{vd}, defined as the minimum pressure attainable at the exit boundary of the nozzle, the magnitude of δ can be determined from the continuity equation wherein $M = F_b \sin(\alpha + \delta) V_{ro}/v \sin \alpha$. In this equation, M is the mass velocity of the steam in the nozzle, lb per sec; F_b is the cross-sectional area of the nozzle at the exit face measured at right angles to the axis of the nozzle, sq ft; α is the angle that the nozzle axis makes with the plane of the exit face; V_{ro} is the velocity of the steam measured relative to the nozzle at the exit face, ft per sec; and v is the specific volume of the steam at the exit face; see Fig. 1.

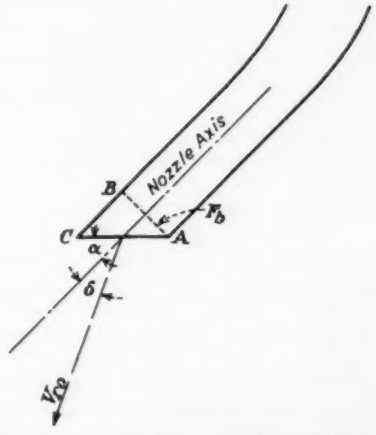


FIG. 1 EXPANSION OF STEAM BEYOND CRITICAL PRESSURE IN A NON-DIVERGENT NOZZLE. WHEN EXPANSION PROCEEDS BEYOND THE CRITICAL PRESSURE THE RELATIVE LEAVING VELOCITY V_{ro} IS DEFLECTED FROM THE NOZZLE AXIS AN AMOUNT δ

The last row of blades of a turbine may be treated as non-divergent nozzles. Then $F_b/\sin \alpha$ becomes the free area of the annulus of the last row of blades, V_{ro} becomes the exit steam velocity relative to the blades, v becomes the specific volume of the steam leaving the blades, M becomes the total mass rate of steam flow through the last row of blades, α becomes the blade angle of the last row of blades, $\sin \alpha$ becomes the gaging of the last row of blades, and $(\alpha + \delta)$ becomes the efflux angle of the last row of blades, i.e., the angle that the relative leaving velocity makes with

---

[1] Head of the Department of Mechanical Engineering, Kansas State College of Agriculture and Applied Science, Manhattan, Kan. Professor Helander was graduated from the University of Illinois in 1915 with the degree of B.S. in mechanical engineering. Since that time he has worked as steam engineer for the Pittsburgh Crucible Steel Company, Midland, Pa.; as general engineer on steam apparatus for the Westinghouse Company, East Pittsburgh and South Philadelphia, Pa.; as a senior engineer for the U.G.I. Contracting Company and the United Engineers and Constructors, Philadelphia, Pa.; and as a consultant. Since 1931 he has been engaged in research and teaching.

[2] "Die Dampfturbinen," by Gustav Flügel, J. A. Barth, Leipzig, 1931.

Contributed by the Power Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

the plane of the exit face. Herein, the efflux angle will be denoted by β. When the exhaust-chamber pressure falls below the value P_{vd}, defined above, the circumferential component of the relative exit velocity, in accordance with Flügel's discussion, remains constant as the exhaust-chamber pressure is reduced, and, since the magnitude of this component determines the magnitude of the turning torque on the blades, the turning torque on the blades likewise remains constant under this condition. Consequently, reducing the exhaust-chamber pressure, when it is already at or below the value P_{vd}, has no effect upon the economy of the turbine. The pressure P_{vd} therefore constitutes the true limiting pressure and the corresponding vacuum constitutes the true limiting vacuum.

In accordance with the interpretations of Flügel's discussion given in the preceding paragraph, the following procedure is suggested for evaluating leaving losses:

(*a*) Where the critical velocity is not exceeded in the last row of blades, assume the efflux angle and the blade angle to be identical.

(*b*) Where the critical velocity is exceeded in the last row of blades, calculate the magnitude of the efflux angle by means of the continuity equation.

In order to employ the continuity equation for evaluating the efflux angle when the critical velocity is exceeded, the relative leaving velocity must be known. To obtain this, the following procedure is suggested:

(*a*) From known exhaust conditions, employing curves and formulas presented herein, determine the critical pressure, the quality of the steam at the critical pressure and the relative velocity at the critical pressure, the latter being directed along the blade axis.

(*b*) Having determined the critical pressure and the quality at the critical pressure, determine the isentropic heat drop from the critical pressure to the blade-terminal pressure, i.e., to the exhaust-chamber pressure if the latter is not below P_{vd}, otherwise to P_{vd}, and therefrom, knowing the efficiency of incremental energy conversion in the last row of blades, the actual heat drop from the critical pressure to the terminal-blade pressure.

(*c*) Having determined the relative velocity at the critical pressure and the actual heat drop from the critical pressure to the blade terminal pressure, calculate the relative exit velocity using the formula

$$V_{ro} = \sqrt{(2gJH' + V^2_{rc})}$$

where H' is the actual heat drop from the critical pressure to the blade terminal pressure, V_{rc} is the relative velocity at the critical pressure, and V_{ro} is the relative exit velocity.

The following procedure is suggested for evaluating vacuum corrections: Assuming that the steam rate of the turbine is known for a specified vacuum, calculate the leaving losses implicit in this steam rate. Add these losses, expressed in Btu, to the internal work done per pound of steam as evaluated from the known steam rate, the mechanical efficiency of the turbine, and the generator efficiency. The result will be the gross internal conversion of heat energy into mechanical energy, including in the latter the kinetic energy of the steam. For convenience, designate this quantity by H'_1. Deduct H'_1 from the heat content of the steam as it enters the turbine in order to find the heat content of the steam as it leaves the last row of blades. From the heat content so evaluated, determine the quality of the steam leaving the last row of blades. Therefrom, knowing the nozzle efficiency of incremental energy conversion in the last row of blades and making use of formulas given herein, determine the actual increase in heat drop that will occur when the vacuum is increased to its new value. Add this increase of actual heat drop o H'_1 and deduct from the result the leaving losses evaluated for the new vacuum. From the result so obtained, knowing the efficiency of the generator and the mechanical efficiency of the turbine, the new steam rate can be determined and therefrom the vacuum correction evaluated. When the exhaust pressure is below the critical pressure, determining the critical pressure and the state of the steam at the critical pressure is one part of the above procedure. That the entire procedure is not difficult, will be seen from the following example which illustrates the manner in which vacuum corrections may be evaluated.

Calculation of Vacuum Corrections

The itemized steps of the following example illustrate the method of calculating for vacuum corrections when, as in Case 1, the critical velocity is not exceeded in the last row of blades, and when, as in Case 2, the critical velocity is exceeded in the last row of blades. In both cases, the problem is to determine the steam rate at a vacuum of 29 in. hg when the steam rate is given for a vacuum of 28½ in. hg. The symbols used in the problem are those tabulated in the nomenclature while the formulas referred to are found in Appendix A.

		Case 1	Case 2
1	Steam flow through last row of blades, lb per hr.	53,000	123,000
2	Dry steam nozzle efficiency, E'_n, of incremental energy conversion in last row of blades, per cent. Assumed for this problem but should be obtained from manufacturer	90	90
3	Gaging = sin α of last row of blades. Should be obtained from manufacturer	0.45	0.45
4	F_b = spouting area of last row of blades = free area of annulus of last row of blades multiplied by sin α, sq ft. To be obtained from manufacturer	9.35	9.35
5	Steam rate, lb per kwhr, with vacuum 28½ in. hg. To be obtained from manufacturer	10.59	9.83
6	Generator and mechanical losses, per cent of gross load. To be obtained from manufacturer or may be estimated from published data	8.7	4.1
7	Internal steam rate, lb per kwhr. Item 5 × (1 — Item 6/100)	9.67	9.43
8	Internal work per lb steam, Btu per lb = 3415/Item 7	353.5	362.5
9	Heat content of steam entering turbine, Btu per lb, from steam tables	1332.4	1332.4
10	Approximate heat content of steam leaving last row of blades, vacuum 28½ in. (Item 9 — Item 8), Btu per lb	979	970
11	Heat content of liquid at a vacuum of 28½ in., Btu per lb, from steam tables	59.7	59.7
12	Latent heat of dry saturated steam at a vacuum of 28½ in., Btu per lb, from steam tables	1040.8	1040.8
13	Approximate quality of steam leaving last row of blades, vacuum 28½ in. (Item 10 — Item 11)/Item 12	0.88	0.87
14	Value of M/F_b, Item 1/Item 4	5670	13,150
15	Critical temperature, deg F, i.e., temperature corresponding to critical pressure, from Fig. 3 using curve for which $E_n/X_c = 0.90$. This assumes E_n = (Item 2) X_c	74.5	101
16	Value of (M/F_b) sin α. Item 14 × Item 3	2550	5920
17	Limiting temperature, deg F, i.e., temperature corresponding to limiting vacuum, from Fig. 3	50	76
18	Temperature corresponding to exhaust-chamber vacuum of 28½ in. hg, deg F, from steam tables	91.8	91.8
19	Estimated quality at critical pressure	....	0.87
20	Specific volume of dry saturated steam at critical pressure from steam tables, cu ft per lb	....	341
21	Approximate relative velocity at critical pressure, ft per sec., Item 14 × Item 19 × Item 20/3600	....	1083
22	Latent heat of dry saturated steam at critical pressure, from steam tables, Btu per lb	....	1036
23	Approximate isentropic heat drop from critical pressure to 28½ in. vac. from formula 4*a*. $(\text{Item 15} - \text{Item 18})\left[1 + \left(\frac{\text{Item 19} \times \text{Item 22}}{\text{Item 15} + 460}\right)\right] - (\text{Item 18} + 460)\log_e\left[1 + \left(\frac{\text{Item 15} - \text{Item 18}}{\text{Item 18} + 460}\right)\right]$	....	14.9
24	Approximate actual heat drop from critical pressure to 28½ in. vac. Btu per lb. Item 23 × Item 2 × (Item 19 + Item 13)/2	....	11.65
25	Approximate velocity of steam relative to blade at exit boundary face. Vac. 28½ in. Ft per sec = $\sqrt{[2g \times 778 \times \text{Item 24} + (\text{Item 21})^2]}$	....	1325
26	Specific volume of dry saturated steam at 28½ in. vac. from steam tables, cu ft per lb	445.3	445.3
27	Estimated sin β = sin efflux angle, at 28½ in. vac. In Case 1, gaging, i.e., Item 3. In Case 2, $\frac{\text{Item 14} \times \text{Item 3} \times \text{Item 26} \times \text{Item 13}}{\text{Item 25} \times 3600}$	0.45	0.48
28	V_b = mean blade speed of last row of blades, ft per sec. From manufacturer	878	878
29	Approximate leaving loss at 28½ in., Btu per lb, from formula 1*b*. See calculation *A* in Appendix B[4]	3.7	9.8

30	Approximate total heat converted into mechanical energy, including therein the kinetic energy of the leaving steam, at 28½ in., vac., Btu per lb (Item 8 + Item 29)	357.2	372.3
31	Revised estimate of heat content of steam leaving last row of blades at 28½ in. vac., Btu per lb (Item 9 — Item 30)	975.2	960.1
32	Revised estimate of quality of steam leaving last row of blades, 28½ in. vac. (Item 31 — Item 11) /Item 12	0.88	0.865
33	Heat content at critical pressure, Btu per lb, Item 31 + Item 24	....	971.75
34	Heat content of liquid at critical pressure from steam tables, Btu per lb	....	68.9
35	Revised estimate of quality at critical pressure. (Item 33 — Item 34)/Item 22	....	0.872
36	Revised estimate of isentropic heat drop from critical pressure to 28½ in. vac. (Item 15 — Item 18) $\left[1+\left(\frac{\text{Item 35}\times\text{Item 22}}{\text{Item 15}+460}\right)\right]$ — (Item 18 + 460) $\log_e\left[1+\left(\frac{\text{Item 15}-\text{Item 18}}{\text{Item 18}+460}\right)\right]$	....	14.9
37	Revised estimate of actual heat drop from critical pressure to 28½ in. vac., Btu per lb. Item 36 × Item 2 × (Item 35 + Item 32)/2	....	11.65
38	Revised estimate of velocity of steam relative to blade at exit boundary face at 28½ in. vac., ft per sec = $\sqrt{[2g\times 778\times\text{Item 37}+(\text{Item 21}\times\text{Item 35/Item 19})^2]}$	....	1325
39	Revised estimate of sin β = sin efflux angle, 28½ in. vac. For Case 1, Item 3 and for Case 2 $\frac{\text{Item 14}\times\text{Item 3}\times\text{Item 26}\times\text{Item 32}}{\text{Item 38}\times 3600}$	0.45	0.479
40	Revised estimate of leaving losses at 28½ in. vac., Btu per lb, from formula 1*b*. See calculation *B* in Appendix B[4]	3.7	9.8
41	Temperature corresponding to exhaust-chamber vacuum of 29 in. hg, deg F	79.1	79.1
42	Isentropic heat drop from 28½ in. vac. in Case 1, from critical pressure in Case 2, to vacuum of 29 in., Btu per lb, for Case 1, (Item 18 — Item 41) $\left[1+\left(\frac{\text{Item 32}\times\text{Item 12}}{\text{Item 18}+460}\right)\right]$ — (460 + Item 41) $\log_e\left[1+\left(\frac{\text{Item 18}-\text{Item 41}}{\text{Item 41}+460}\right)\right]$ For Case 2, substitute Item 15 for Item 18, Item 35 for Item 32, and Item 22 for Item 12	21.2	35.6
43	Estimated actual heat drop from 28½ in. vac. to 29 in. vac. in Case 1, and from critical pressure to 29 in. vac. in Case 2. Item 42 × average quality × Item 2 = Item 42 × Item 32 × Item 2, approximately	16.75	27.70
44	Approximate heat content of steam leaving last row of blades at 29 in. vac., Btu per lb. Case 1, Item 31 — Item 43. Case 2, Item 33 — Item 43	958.45	944.05
45	Heat content of liquid at 29 in. vac., from steam tables, Btu per lb	47	47
46	Latent heat of dry steam at 29 in. vac., Btu per lb	1048	1048
47	Quality of steam leaving last row of blades at 29 in. vac. (Item 44 — Item 45)/Item 46	0.87	0.856
48	Revised estimate of actual heat drop from 28½ in. vac. to 29 in. vac. in Case 1, and from critical pressure to 29 in. vac. in Case 2. Case 1, Item 42 × Item 2 × (Item 47 + Item 32)/2. Case 2, Item 42 × Item 2 × (Item 35 + Item 47)/2	16.7	27.70
49	Velocity of steam relative to blade at exit from last row in Case 2 at 29 in. vac., ft per sec = $\sqrt{\left[2g\times 778\times\text{Item 48}+\left(\frac{\text{Item 21}\times\text{Item 35}}{\text{Item 19}}\right)^2\right]}$	....	1600
50	Specific volume of dry saturated steam at 29 in. vac., from steam tables, cu ft per lb	652.7	652.7
51	Sin β = sin efflux angle at 29 in. vac. Case 1, Same as Item 3. Case 2, $\frac{\text{Item 14}\times\text{Item 3}\times\text{Item 50}\times\text{Item 47}}{\text{Item 49}\times 3600}$	0.45	0.574
52	Cos β = cos $\sin^{-1}$ Item 51	0.893	0.819
53	Leaving losses at 29 in. vac., Btu per lb, from formula 1*b*. See calculation *C* in Appendix B[4].	3.4	20.9
54	Total gross conversion of heat into mechanical energy at 29 in. vac. Case 1, Item 8 + Item 40 + Item 48. Case 2, Item 8 + Item 40 — Item 37 + Item 48	373.9	388.3
55	Net conversion of heat into internal shaft work at 29 in. vac., Btu per lb. Item 54 — Item 53	370.5	367.4
56	Internal steam rate at 29 in. vac., lb per kwhr. 3415/Item 55	9.22	9.30
57	External steam rate at 29 in. vac., lb per kwhr. $\frac{\text{Item 56}}{(1-\text{Item 6}/100)}$	10.10	9.70
58	Per cent reduction in internal steam rate due to change of vacuum. (Item 55 — Item 8)/ Item 55	4.6	1.33

The foregoing procedure assumes that the actual increment of heat drop due to a change of vacuum is equal to the incremental isentropic heat drop multiplied by the nozzle efficiency of incremental energy conversion in the last row of blades. This is correct when the critical pressure has been attained in the last row of blades; otherwise it is an approximation. The procedure assumes also that the influence of leakage losses in the last row of blades can be ignored; likewise variations in E'_n, the dry-steam nozzle efficiency of incremental energy conversion. Although these assumptions are satisfactory for general purposes, the turbine designer very likely will want to take cognizance of the fact that leakage losses influence the result and may increase when the exhaust-chamber pressure is reduced below the critical pressure, also of the fact that E'_n may vary with a change in the exhaust pressure and the rate of steam flow.[3] In evaluating E_n, the nozzle efficiency applicable for the

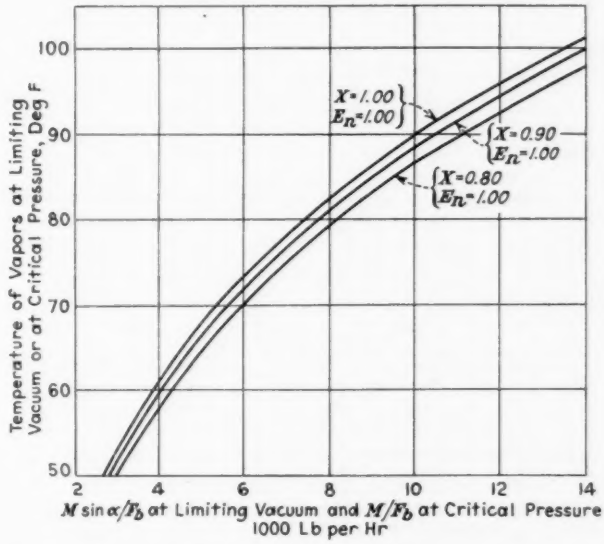


FIG. 2 VALUES OF M SIN α/F_b AT LIMITING VACUUM AND OF M/F_b AT CRITICAL PRESSURE ASSUMING NOZZLE EFFICIENCY OF INCREMENTAL ENERGY CONVERSION IN LAST ROW OF BLADES TO BE UNITY

$(M_d \sin\alpha/F_b = 3600\sqrt{[(k_d g P'_{nd})/(v_{nd} X_d)]}$
$M_c/F_b = 3600\sqrt{[(k_c g P'_{nc})/(v_{nc} X_c)]}$
$k = 1.035 + 0.1\,X)$

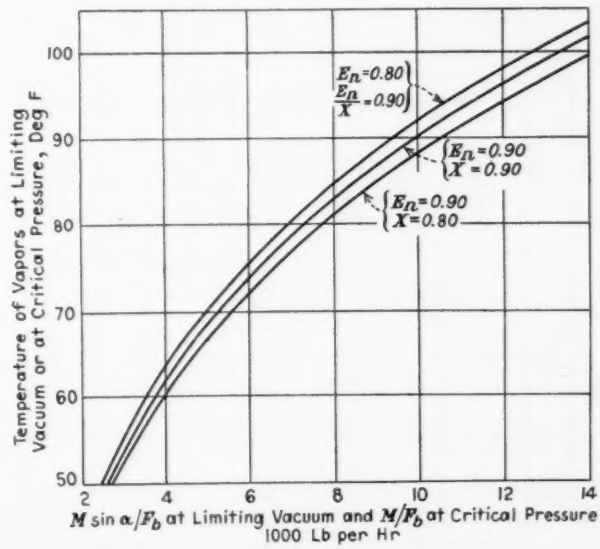


FIG. 3 VALUES OF M SIN α/F_b AT LIMITING VACUUM AND OF M/F_b AT CRITICAL PRESSURE ASSUMING NOZZLE EFFICIENCY OF INCREMENTAL ENERGY CONVERSION IN LAST ROW OF BLADES TO BE AS INDICATED. CURVES OBTAINED BY USE OF FORMULAS [2*b*] AND [8*b*]

[3] "Die Düsencharakteristik," by Gustav Flügel. *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, No. 217, Berlin, 1919.

incremental heat drop, E'_n was multiplied by the average quality of the steam during its incremental expansion. This is consistent with the practice of reducing the efficiency of a stage 1 per

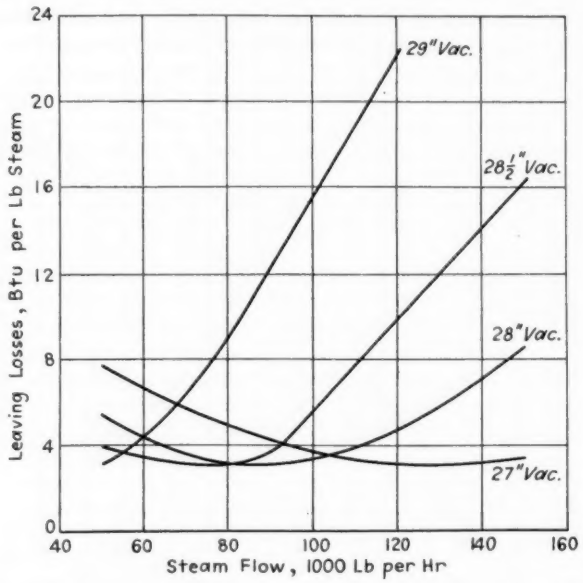


FIG. 4 LEAVING LOSSES ASSUMING $X = 0.88$ AND $E_n = 0.90$ CALCULATED BY USE OF FORMULA [1b]
(Blade speed $V_b = 878$ ft per sec, sin α = gaging = 0.45, and $F_b = 9.35$ sq ft.)

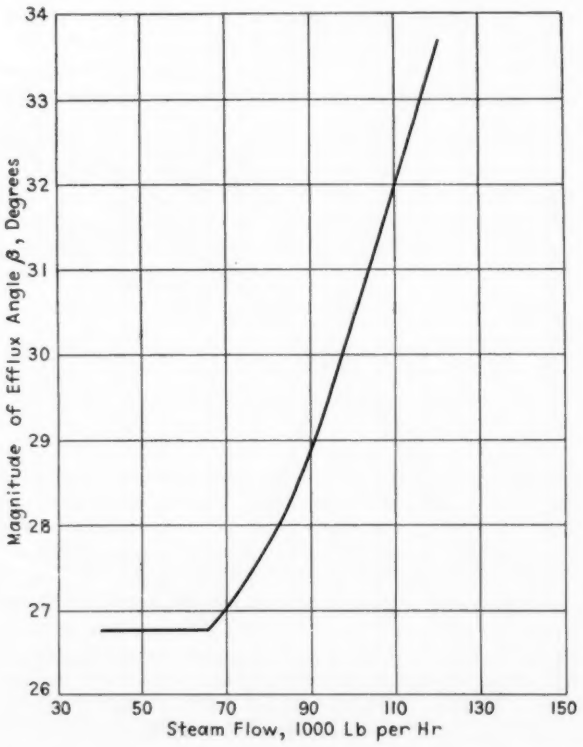


FIG. 5 MAGNITUDE OF EFFLUX ANGLE β CORRESPONDING TO LEAVING-LOSS CURVE FOR A VACUUM OF 29 IN. HG GIVEN IN FIG. 4

cent for each average per cent of moisture in the steam in the stage. Actually, however, the effect of a unit percentage of moisture on the efficiency of incremental energy conversion within a blade may be less than its effect on the over-all stage efficiency, and further it may differ appreciably in the nozzle proper and in the triangular section following section F_b of Fig. 1. Time did not permit of a more comprehensive investigation of the influence of moisture, but a discussion of this will be welcomed.

Fig. 2 shows values of the saturation temperature corresponding to the limiting pressure P_{vd} when the abscissas represent $M \sin \alpha / F_b$ and of the saturation temperature corresponding to the critical pressure when the abscissas represent M/F_b, the nozzle efficiency of incremental energy conversion in both cases being unity. Fig. 3 is the same as Fig. 2 except that the nozzle efficiency of incremental energy conversion is no longer assumed to be unity; also the data plotted in Fig. 3 were obtained from a formula developed by the writer while those plotted in Fig. 2 were obtained from formula [45] in Flügel's "Die Dampfturbinen."[2] The curves of these two figures should agree approximately where the ratios of E_n/X are identical: Fig. 4 shows curves of charac-

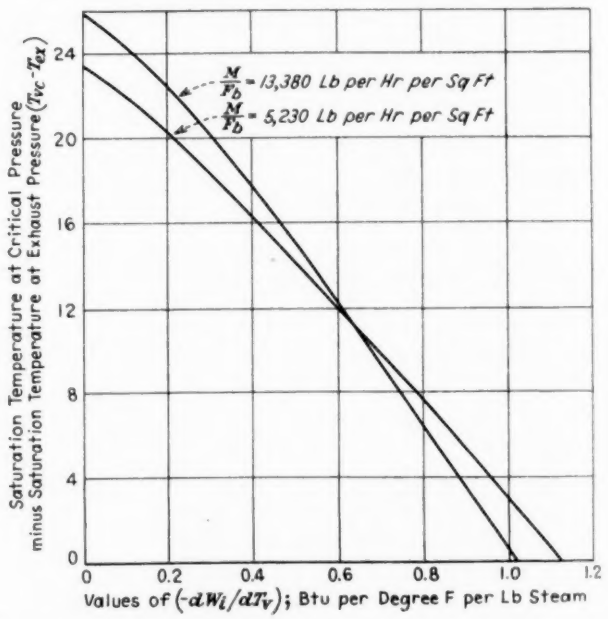


FIG. 6 RATE OF CHANGE OF INTERNAL SHAFT WORK WITH A CHANGE OF VACUUM TEMPERATURE, I.E., INCREMENT OF INTERNAL SHAFT WORK PER POUND OF STEAM PER DEGREE DECREASE IN VACUUM TEMPERATURE WHEN EXHAUST PRESSURE IS BELOW CRITICAL PRESSURE
($E_n = 0.90$, $X = 0.90$, and sin α = gaging = 0.45.)

teristic leaving losses calculated by assuming the quality of the exhaust steam to be 88 per cent and the nozzle efficiency of incremental energy conversion in the last row of blades to be 90 per cent. The spouting area of the last row of blades, F_b, was assumed to be 9.35 sq ft and the gaging was assumed to be 45 per cent in developing Fig. 4 as was assumed in the illustrative example previously presented. Fig. 5 shows values of β, the efflux angle of the last row of blades, corresponding to the data presented in Fig. 4.

From Figs. 2 and 3 it will be seen that the depression of the temperature corresponding to the limiting vacuum below that corresponding to the critical pressure is approximately 25 F when the gaging, i.e., sin α, is 0.45, and 33 F when the gaging is 0.35. The value of 25 F for 45 per cent gaging is verified also by the curves of Fig. 6 which show values of $(-dW_i/dT_v)$, the rate of change of internal work with a change of vacuum temperature, plotted as abscissas and the difference between the satura-

tion temperature at the critical pressure and that at the exhaust-chamber pressure as ordinates. Here the difference between the saturation temperature at the critical pressure and that at the limiting vacuum is the ordinate where $(-dW_i/dT_v) = 0$.

The author is grateful for the many helpful suggestions offered to him during the preparation of this manuscript; to W. E. Caldwell and his associates on the Power Division Executive Committee of the A.S.M.E. for their interest in the paper; and to various members of the engineering staff of Kansas State College who assisted with translations and with the correction of the manuscript.

Nomenclature

a = free fraction of area of annulus of last row of blades
β = efflux angle of last row of blades
α = blade angle of last row of blades
C = specific heat, Btu per lb per deg F
C_w = specific heat of water = 1, approximately
D'_m = mean diam of last row of blades, ft
E_n = nozzle efficiency of incremental energy conversion in last row of blades. E_n is the efficiency with which the next increment of isentropic heat drop selected is converted into relative kinetic energy, i.e., into kinetic energy of the expanding steam measured relative to the blade. Thus

$$E_n(dH) = \frac{d(V_r^2)}{2 \times 778g} \quad \text{or} \quad E_n(\Delta H) = \frac{V_{r2}^2 - V_{r1}^2}{2 \times 778g}$$

where V_r = velocity of steam relative to blade, ft per sec
E'_n = dry-state value of E_n
F_b = spouting area, last row of blades = $a\pi D'_m L_b \sin \alpha$, sq ft
g = acceleration of gravity, ft per sec per sec
H = isentropic heat drop, Btu per lb
H_{12} = isentropic heat drop, state 1 to state 2, Btu per lb
h_{sx} = heat content of steam leaving the last row of blades, the quality of this steam being X; Btu per lb
h_f = heat content of saturated liquid at pressure of steam leaving last row of blades, Btu per lb
j = leaving losses = kinetic energy of steam leaving last row of blades, Btu per lb
k = $1.035 + 0.1X$; $k_c = k$ when $X = X_c$, and $k_d = k$ when $X = X_d$
L_b = blade height, ft
M = steam leaving last row of blades, lb per hr
M_c = steam leaving last row of blades when critical pressure exists at exit section F_b (see Fig. 1), lb per hr
M_d = steam leaving last row of blades when pressure at blade exit section CA of Fig. 1 is that corresponding to the limiting vacuum, lb per hr
P_v = pressure of steam leaving last row of blades, lb per sq in.
P_{vc} = critical pressure of steam in last row of blades, lb per sq in.
P'_{vc} = critical pressure of steam in last row of blades, lb per sq ft
P_{sx} = pressure existing in the exhaust chamber, lb per sq in.
P_{vd} = limiting pressure, lb per sq in.
P'_{vd} = limiting pressure, lb per sq ft
r_v = latent heat of vapors leaving last row of blades, Btu per lb
r_{vd} = latent heat of vapors at limiting pressure, P_{vd}, Btu per lb
r_{vc} = latent heat of vapors at critical pressure, Btu per lb
r_s = Latent heat of steam leaving last row of blades = Xr_v, where X is quality of steam, Btu per lb
s = entropy, Btu per deg F
T_v = absolute temperature of steam leaving last row of blades, deg F
T_{vc} = absolute temperature of steam at critical pressure, P_{vc}, deg F
T_{vd} = absolute temperature of steam at limiting pressure, P_{vd}, deg F
T_{sx} = absolute temperature of steam in exhaust chamber, deg F
V = velocity, ft per sec
V_{ro} = velocity of steam leaving last row of blades measured relative to blade, ft per sec
V_b = mean velocity of blade, ft per sec
V_o = absolute velocity of steam leaving last row of blades, ft per sec
V_{rc} = critical relative velocity of steam in last row of blades, ft per sec
v_v = specific volume of dry saturated vapors leaving last row of blades, cu ft per lb
v_{vd} = specific volume of dry saturated vapors at limiting pressure, P_{vd}, cu ft per lb
v_{vc} = specific volume of dry saturated vapors at critical pressure, P_{vc}, cu ft per lb
W_i = internal shaft work, Btu per lb steam
X = dry fraction of steam leaving last row of blades
X_d = dry fraction of steam leaving last row of blades when terminal pressure in blades is the limiting pressure, P_{vd}
X_c = dry fraction of steam at section F_b of last row of blades when critical pressure P_{vc} exists at that section
z = $5.99 \log_e (T_v/550) + 12{,}200\ (550 - T_v)/550\ T_v$
z_c = value of z at critical pressure, P_{vc}, i.e., when $T_v = T_{vc}$
z_d = value of z at limiting pressure, P_{vd}, i.e., when $T_v = T_{vd}$

Appendix A

The following formulas will be helpful in evaluating leaving losses and vacuum corrections. Their derivation will be given in Appendix B.[4] Definitions of symbols are given in the nomenclature.

I Formulas for evaluating leaving losses.

$$j = \frac{V_{ro}^2 + V_b^2 - 2V_bV_{ro}\cos\beta}{2g \times 778} \quad \ldots\ldots\ldots [1a]$$

$$= \frac{[X(M/F_b)(\sin\alpha/\sin\beta)v_v]^2}{64.9 \times 10^{10}} + \frac{V_b^2}{5.01 \times 10^4} - \frac{XV_b(M/F_b)(\sin\alpha/\sin\beta)v_v\cos\beta}{9.02 \times 10^7} \quad \ldots\ldots\ldots [1b]$$

$$= \frac{[X(M/F_b)(\sin\alpha/\sin\beta)e^z]^2}{29.56 \times 10^5} + \frac{V_b^2}{5.01 \times 10^4} - \frac{XV_b(M/F_b)(\sin\alpha/\sin\beta)e^z\cos\beta}{1.925 \times 10^5} \quad \ldots\ldots\ldots [1c]$$

Note: Formula [1c] expresses j as a function of the terminal temperature of the steam and, therefore, is in such form that the derivative dj/dT_v can be evaluated.

II Mass velocity of steam leaving last row of blades when critical pressure exists at blade exit section F_b.

$$M_c/F_b = \frac{10^3}{X_c e^{z_c}} \sqrt{\left(\frac{X_c r_{vc} E_n}{0.678\left[\frac{9.07 r_{vc}}{T_{vc}} - 1 - E_n + \frac{T_{vc}(1 - 0.55X_c)}{X_c r_{vc}}\right]}\right)} \quad \ldots [2a]$$

[4] Appendix B has been filed in the Archives of the Society and may be obtained upon request.

$$M_c/F_b = \frac{10^3 \times 569}{X_c v_{vc}} \sqrt{\left(\frac{X_c r_{vc} E_n}{\left[\frac{9.07\, r_{vc}}{T_{vc}} - 1 - E_n + \frac{T_{vc}(1 - 0.55 X_c)}{X_c r_{vc}} \right]} \right)} \quad [2b]$$

$$M_c/F_b = 3600 \sqrt{[(E_n/X_c)\ (k_c g P'_{vc}/v_{vc})]}, \text{ approximately} \ldots . [2c]$$

In formula [2c] $k_c = (1.035 + 0.1\, X_c)$, see Flügel's Die Dampfturbinen, page 69.[2] Formula [2c] is derived by recognizing from formula [2a] that $M_c/F_b = (\sqrt{E_n}) \times [(M_c/F_b)$ evaluated for $E_n = 1]$, approximately; also by employing the equation for the velocity of sound.

III Critical velocity of steam in last row of blades.

$$V_{rc} = 130.2 \sqrt{\left[\frac{X_c r_{vc} E_n}{0.678 \left(\frac{9.07 r_{vc}}{T_{vc}} - 1 - E_n + \frac{T_{vc}(1 - 0.55 X_c)}{X_c r_{vc}} \right)} \right]} \quad \ldots . [3a]$$

$$V_{rc} = \frac{(M_c/F_b)\ X_c v_{vc}}{3600} \quad \ldots . [3b]$$

$$V_{rc} = \sqrt{(E_n k_c g P'_{vc} X_c v_{vc})} = 52.5 \sqrt{(E_n k_c X_c T_{vc})}, \text{ approximately} \ldots . [3c]$$

IV Small increments of isentropic adiabatic heat drop when low-pressure saturated steam is the expanding medium.

$$H_{12} = (T_{v1} - T_{v2})(C_{w12} + X_1 r_{v1}/T_{v1}) - C_{w12} T_{v2} \log_e (T_{v1}/T_{v2}) \quad \ldots . [4a]$$

$$H_{12} \text{ (approximate)} = X_1 r_{v1} \log_e (T_{v1}/T_{v2}) \quad \ldots . [4b]$$

$$dH = -dT_v X r_v / T_v \quad \ldots . [4c]$$

where T_{v1}, X_1, r_{v1} = initial absolute temperature, quality, and latent heat, respectively, of the expanding steam; T_{v2} = terminal absolute temperature of the expanding steam, deg F; and C_{w12} = average specific heat of water between states 1 and 2, = unity approximately for temperature range herein considered, Btu per lb per deg.

V Specific volume of dry saturated water vapors between 40 F and 140 F.

$$v_v = 468.5 e^s \quad \ldots . [5]$$

VI Latent heat of water vapors between 40 F and 140 F.

$$r_v = 1344.3 - 0.55\, T_v \quad \ldots . [6]$$

Error = +0.7 Btu when $T_v = (140 + 460)$ F, and
+0.2 Btu when $T_v = (40 + 460)$ F.

VII Rate of change of internal shaft work with change of vacuum temperature.

$$-\frac{dW_i}{dT_v} \text{ (where critical pressure is attained)} = \frac{V_b \sin \beta}{\cos \beta} \left\{ \frac{3600\, E_n r_v F_b}{T_v M (\sin \alpha) v_v} + \frac{M(\sin \alpha) v_v X}{90{,}200{,}000\, F_b} \left[\frac{(1 + E_n)}{T_v} - \frac{9.07 r_v}{T_v^2} - \frac{(1 - 0.55X)}{X r_v} \right] \right\} . [7a]$$

$$-\frac{dW_i}{dT_v} \text{ (Where critical pressure is not attained)} = \frac{E_n X r_v}{T_v} + \left[\frac{(1 + E_n)}{T_v} - \frac{9.07 r_v}{T_v^2} - \frac{(1 - 0.55X)}{X r_v} \right] \left[\frac{\left(\frac{M}{F_b} \frac{v_v X}{468.5} \right)^2}{14.78 \times 10^6} - X \left(\frac{M}{F_b} \right) \left(\frac{v_v V_b \cos \alpha}{468.5 \times 1.925 \times 10^6} \right) \right] \quad \ldots . [7b]$$

(Formula [7b] is approximate.)

VIII Mass velocity of steam leaving last row of blades when limiting pressure P_{vd} exists at exit face of blade; i.e., when exhaust-chamber vacuum is greater than or equal to the limiting vacuum.

$$M_d/F_b \text{ (when } E_n = \text{unity)} = 3600 \frac{\sqrt{(k_d g P'_{vd}/v_{vd} X_d)}}{\sin \alpha}$$

(see formula 45 in Die Dampfturbinen[2] by Flügel) . . . . . [8a]

$$M_d/F_b = \left(\frac{10^3 \times 569}{(\sin \alpha) X_d v_{vd}} \right) \sqrt{\left(\frac{X_d r_{vd} E_n}{\frac{9.07 r_{vd}}{T_{vd}} - 1 - E_n + \frac{T_{vd}(1 - 0.55 X_d)}{X_d r_{vd}}} \right)} \quad \ldots . [8b]$$

$$M_d/F_b = 3600 \frac{\sqrt{[(E_n/X_d)(k_d g\, P'_{vd}/v_{vd})]}}{\sin \alpha} \text{ approximately.} \quad [8c]$$

Principles Underlying the Rational Solution of Automatic-Control Problems

By SERGEI D. MITEREFF,[1] PETERSBURG, VA.

The author presents twelve fundamental equations covering the characteristics of automatic regulators. It is the author's belief that six of these equations completely cover all present forms of commercial regulators. The additional six equations cover modifications and improvements which will permit greatly increased accuracy and flexibility of automatic control and will permit a satisfactory solution of those control problems which could not be handled with complete success by previous types of regulators.

IN SPITE of the widespread use of automatic regulators and governors in all branches of industry, little has been published relative to the fundamental principles of the subject. In the absence of data necessary for rational solutions the majority of automatic-control problems are solved at present by purely empirical methods.

Because of the variety of automatic-control apparatus on the market, it is a difficult task to select the regulator suited exactly to the requirements of a given installation. The adjustment of a regulator after its selection and installation is a problem in itself, inasmuch as the usual cut-and-try method of adjustment is very tedious and unreliable. Therefore, the development of a solid basis for a rational approach to the problem of automatic control is a matter of considerable practical importance.

Principal Factors of an Automatic-Control Installation

Any automatic-control installation can be considered as consisting of two major parts which are, (*a*) the system to be controlled, and (*b*) the automatic-control apparatus added to this system in order to produce the desired result.

The system external to the regulator can be divided into four factors common to any control installation, practically without exception, these are: (*a*) storage of a fluid or energy, (*b*) inflow of a fluid or energy to the storage, (*c*) outflow of a fluid or energy from the storage, and (*d*) a function (temperature, pressure, speed, level, etc.) which is indicative of the amount of fluid or energy in storage and which it is desired to maintain constant.

It should be pointed out that either the inflow or the outflow of fluid or energy is capable of being varied at will, while the other flow varies under the influence of factors which cannot be controlled.

It is evident, therefore, that an automatic-control problem consists essentially of equalizing automatically the rate of flow to the storage with the rate of flow from the storage, this being the necessary condition for maintaining constant both the amount of fluid or energy in the storage and the function indicative of the amount of fluid or energy in the storage.

Taking as an example the speed control of a turbine-generator unit, the energy-storing capacity of the rotating parts will be identified as the storage. The flow of steam to the turbine is the inflow of energy which can be controlled, while the electrical output of the generator is the outflow of energy varying under the influence of the demand which is not under our control. The speed of the turbine is, of course, the function which we desire to maintain constant and which is indicative of the amount of energy stored in the rotating mass.

Analyzing the example of the temperature control of a gas-fired oven, the storage is represented by the heat-storing capacity of the oven. The flow of energy under our control is the inflow of gas to the oven. The uncontrollable flow of energy is the heat loss from the oven by radiation and heat absorption of the product. The temperature in the oven is the function indicative of the amount of heat stored in the oven and which it is desired to maintain constant. These examples are typical and could be extended considerably.

Principal Parts of an Automatic-Control Apparatus

An automatic-control apparatus consists of (*a*) the impulse-receiving element, (*b*) the final operating element, and (*c*) the operative connection between the impulse-receiving element and the final operating element.

The function of the impulse-receiving element is to produce a force or motion proportional to the function to be maintained constant. In a pressure regulator, for instance, the impulse-receiving element is represented by a diaphragm acted upon by the pressure transmitted from the tank through a pilot line. In a thermocouple type of temperature regulator, the impulse-receiving element consists of the thermocouple and the galvanometer. In a speed governor, the flyball pendulum constitutes the impulse-receiving element.

The function of the final operating element is to modify the rate of the controllable flow of fluid or energy. In most cases the final operating element is simply a valve or an electrical contactor.

The function of the operative connection is, first, to provide the desired characteristic of the automatic-control apparatus and, second, to generate a sufficient force for actuating the final operating element.

Characteristic of an Automatic-Control Apparatus

In spite of the fact that the concept of the characteristic of a regulator or governor is one of the most important factors entering into the general problem of automatic control, it has received little attention in literature on the subject. This oversight is very surprising, especially in view of many elaborate attempts to cover such comparatively insignificant factors as "lack of sensi-

---

[1] Jun. A.S.M.E. Mr. Mitereff was born at Pskov, Russia, in 1902. He was a member of Pskov Cadet Corps and was educated at the Russian Naval Academy, Engineering Division, Vladivostok. In 1927 he was graduated with honor from Johns Hopkins University, receiving a B.S. in mechanical engineering. From the time of his graduation from Johns Hopkins until 1933, he was assistant engineer in the power and development sections of the Atmospheric Nitrogen Corporation, Hopewell, Va. He now resides in Petersburg, Va., and is engaged in the preparation of a book on the subject of automatic regulation.

Contributed by the Power Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, American Society of Mechanical Engineers, 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

tiveness" and "sticking," factors which hardly deserve more than a passing remark to the effect that they could and should be kept down to a required minimum in any properly designed automatic-control apparatus.

By the characteristic of an automatic-control apparatus is meant the relationship between the primary impulse actuating the apparatus and its final effect. The best, since the most definite, way of expressing this relationship is by means of a mathematical equation. By the primary impulse is meant the extent of change of a form of energy derived from the variation of the function to be controlled from its value corresponding to the normal value of the controlled function. By the final effect is meant the extent of actuation of the final operating element of the regulator. It should be pointed out that the mathematical equation expressing the characteristic of a control apparatus gives the relationship between the primary impulse and the final effect both measured at the same moment under consideration.

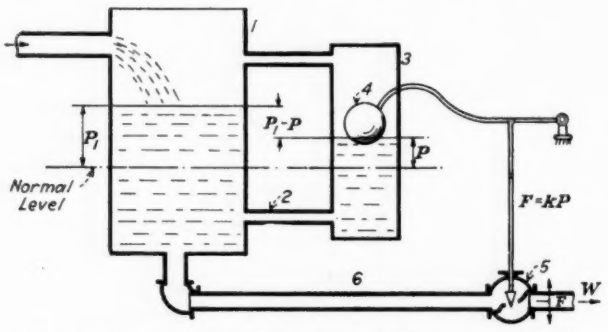


FIG. 1 AN AUTOMATIC LEVEL-CONTROL INSTALLATION WITH ONE PILOT LINE

(The following equations express the relationship between the level in the float chamber and the level in the tank.)

(*a*) $P_1 - P = k_1 \frac{dP}{dT}$; $P_1 = P + k_1 \frac{dP}{dT}$ —Pilot line 2 has fluid friction but no inertia

(*b*) $P_1 - P = k_2 \frac{d^2P}{dT^2}$; $P_1 = P + k_2 \frac{d^2P}{dT^2}$ —Pilot line 2 has inertia but no fluid friction

(*c*) $P_1 - P = k_1 \frac{dP}{dT} + k_2 \frac{d^2P}{dT^2}$; $P_1 = P + k_1 \frac{dP}{dT} + k_2 \frac{d^2P}{dT^2}$ —Pilot line has both fluid friction and inertia

In the above example of the temperature control in a gas-fired oven, the primary impulse takes the form of the value of the thermoelectric current (or potential) generated by the thermocouple by which it deviates from its normal value corresponding to the normal temperature in the oven. The final effect is represented in this case by the distance between the position of the valve admitting gas to the oven at the moment under consideration and the position of the valve at the beginning of operation of the regulator when the temperature was held constant at its normal value.

TIME LAG IN A CONTROL INSTALLATION

One of the greatest practical difficulties encountered in an automatic-control installation is the phase displacement between different factors commonly known as the time lag. The time lag is caused by fluid friction, resistance to heat flow, and inertia forces inherent in any physical layout, and the most frequent factor contributing to the time lag is the fluid friction and inertia of the impulse-transmitting means. For instance, in the case of a pressure regulator, the friction and inertia of the liquid in the pilot line connecting the tank with the regulator distort considerably a perfect correspondence between the pressure in the tank and the pressure as reaching the regulator.

Another source of time lag is the inertia and friction of the fluid in the pipe in which the regulating valve is located. For instance, in the case of the speed control of a hydraulic turbine there exists a phase displacement between the opening of the gate and the rate of water flow through it because of inertia of water in the penstock.

It should be pointed out that the aforementioned types of time lag are very often unavoidable features of the control installation in contradistinction to the time lag frequently encountered in the regulator itself which, as a rule, can be eliminated by correct design. It will be clear, therefore, that since some of the principal causes of the time lag cannot be eliminated, means should be found in such cases to counteract the time lag by selecting a regulator which has a characteristic suitable for just this particular purpose.

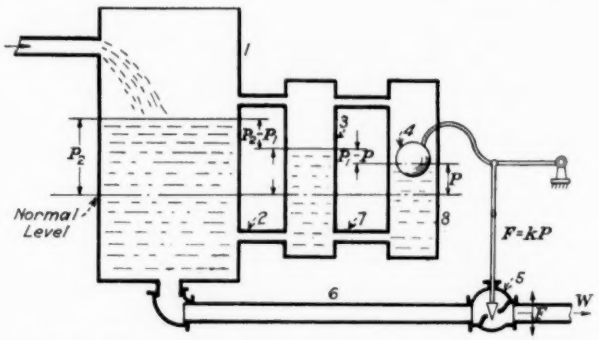


FIG. 2 AN AUTOMATIC LEVEL-CONTROL INSTALLATION WITH TWO PILOT LINES

(The following equations express the relationship between the level in the float chamber and the level in the tank.)

(*d*) $P_2 = P + k_1 \frac{dP}{dT} + k_2 \frac{d^2P}{dT^2}$ —Plot lines 2 and 7 have only fluid friction

(*e*) $P_2 = P + k_1 \frac{dP}{dT} + k_2 \frac{d^2P}{dT^2} + k_3 \frac{d^3P}{dT^3}$ —Of the pilot lines 2 and 7 one has only friction while the other has both friction and inertia

(*f*) $P_2 = P + k_1 \frac{dP}{dT} + k_2 \frac{d^2P}{dT^2} + k_3 \frac{d^3P}{dT^3} + k_4 \frac{d^4P}{dT^4}$ —Both pilot lines have friction and inertia

k_1, k_2, k_3, and k_4 Constants of proportionality depending upon the physical dimensions of the system

In order to pave the way for a rational solution of the problem of the time lag, a typical control installation will be analyzed. In Figs. 1 and 2 is shown an automatic level-control installation where the float (4) is placed in a float chamber connected by a pilot line to the tank (1). Equations (*a*), (*b*), and (*c*) in Fig. 1 express the relationship between the level in the float chamber (3) and the level in the tank (1) if only one pilot line is used. Equations (*d*), (*e*), and (*f*) in Fig. 2 give the relationship between these levels when two pilot lines in series are used. In both of these cases the conditions when the pilot line has only fluid friction, only inertia, and both friction and inertia are covered by these equations.

The time lag arising from the use of a pilot line is typical of any other source of time lag in any other part of controlled system. Therefore, the equations in Figs. 1 and 2 have the virtue of universality and they are applicable to any type of automatic-control installation, whatsoever, including automatic piloting of ships and aircraft.

It will be clear also that in order to obtain the level in the tank from the indication of the level in the float chamber, it is only necessary to design an automatic-control apparatus which would solve automatically the given equations. It is rather surprising that there are at present no commercial automatic controls which have the characteristics capable of performing this task.

The exact mathematical solution of the time lag, caused by inertia of fluid in the pipe (6) as affected by the extent of opening of the regulating valve (5) and the corresponding rate of flow

through it, is rather complex. It could be shown, however, that if we increase the opening of the valve (5) by the amount equal to $k_2 \frac{dP}{dT}$, the rate of flow through the valve (5) will closely correspond to the change of level in the float box.

In other words, by designing a regulator which has the characteristic $F = k_1P + k_2 \frac{dP}{dT}$ we can obtain the relationship $W = kP$, where W = the rate of flow through pipe (6), P = the extent of change of the level in the float box, F = the distance traversed by the valve (5), k_1, k_2, and k = constants of adjustment, and T = time.

In this way the retarding effect of inertia of the fluid in the pipe (6) could be almost completely counteracted. This method of counteracting the inertia of fluid in the pipe in which the regulating valve is located is applicable equally well in the case of the speed governing of a hydraulic turbine, and many other automatic-control installations.

Classification of the Automatic-Control Apparatus

All automatic-control apparatus now in commercial use, practically without exception, can be grouped according to their characteristics (expressed by the equations) under only six classes:

I $F = k_1 \int P dT + (C)$ ..... [1]

II $F = k_1 P$ ..... [2]

III $F = k_1 \int P dT + k_2 P + (C)$ ..... [3]

IV $F + k_2 \frac{dF}{dT} = k_1 P$ ..... [4]

V $k_3 \int F dT + F = k_1 \int P dT + k_2 P$ ..... [5]

VI $F + k_3 \frac{dF}{dT} = k_1 \int P dT + k_2 P + (C)$ ..... [6]

The following classes of automatic-control apparatus are more or less new to the art:

VII $F = k_1 P + k_2 \frac{dP}{dT}$ ..... [7]

VIII $F = k_1 \int P dT + k_2 P + k_3 \frac{dP}{dT} + (C)$ ..... [8]

IX $F = k_1 P + k_2 \frac{dP}{dT} + k_3 \frac{d^2P}{dT^2}$ ..... [9]

X $F = k_1 \int P dT + k_2 P + k_3 \frac{dP}{dT} + k_4 \frac{d^2P}{dT^2} + (C)$ ..... [10]

XI $k_5 \int F dT + F + k_4 \frac{dF}{dT} = k_1 \int P dT + k_2 P + k_3 \frac{dP}{dT}$ [11]

XII $k_7 \int F dT + F + k_6 \frac{dF}{dT} + k_5 \frac{d^2F}{dT^2} =$

$k_1 \int P dT + k_2 P + k_3 \frac{dP}{dT} + k_4 \frac{d^2P}{dT^2}$ ..... [12]

where P = the deviation from its normal value of the primary impulse actuating the regulator as measured at the moment under consideration; F = the final regulating effect of the automatic-control apparatus as measured at the same moment under consideration; T = time; and k_1, k_2, k_3, k_4, k_5, k_6, and k_7 = arbitrary constants of adjustment.

Only six of the above characteristics may be found in existing commercial regulators. The reader can prove this statement by analyzing, in the light of this article, a number of controls with which he may be familiar. The statement applies to all regulators and governors irrespective of either the field of application (control of temperature, pressure, speed, combustion, level, etc.) or the motive power actuating them (electrical, hydraulic, or pneumatic operation). The only limitation in this connection is that the controls selected for analysis should be of a continuous type, because it is obvious that the characteristic of regulators of mere on-and-off type cannot be expressed by a mathematical equation.

The on-and-off type does not include, however, the majority of the electric regulators of the contact-making type, since, even though their operation may be intermittent, there still exists in practically all of these regulators a definite mathematical relationship between the variation of the controlled function and the actuation of the regulating valve.

Elementary examples of the automatic-control apparatus of the first and twelfth classes only will be given in this paper. Space limitations forbid demonstration of all twelve characteristics. The author, however, has placed on file in the archives of

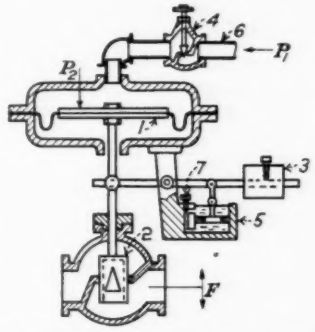


Fig. 3 A Common-Type Pressure-Regulating Valve

the Society, a complete demonstration of all twelve cases where they are available to the more serious students of control problems.

Example of Class I

Consider a pressure regulator of the most common type, such as illustrated in Fig. 3. This regulator consists of the diaphragm (1) moving the regulating valve (2). The pressure to be kept constant is transmitted to the regulator through the pilot line (6), in which is inserted the needle valve (4). When the pressure is held steady at its desired normal value the weight (3) is in balance with the force produced by the pressure acting upon the diaphragm (1) and, therefore, the valve (2) is at a standstill.

Let P_2 = the normal value of the pressure. It is evident that the pressure on the top of the diaphragm (1) is always equal to P_2, provided the valve (2) is in its normal operating position and is neither wide open nor closed tight. Let P_1 = the pressure existing in the line (6) in front of the needle valve (4). By definition, the primary impulse actuating the regulator will be equal to the difference between the pressure before the needle valve (4) and the normal desired pressure always existing in the space above the diaphragm (1).

We may write, therefore,

$$P = P_1 - P_2 \quad \text{..... [13]}$$

It follows from the law of viscous flow through an orifice such as the needle valve (4), that the speed of fluid flow through the valve (4) is proportional to the pressure difference existing across it. It is evident, on the other hand, that the speed of movement

of the valve (2) is proportional to the speed of fluid flow through the valve (4).

Therefore, the following relationship holds:

$$\frac{dF}{dT} = k_1 (P_1 - P_2) = k_1 P \ldots\ldots\ldots\ldots [14]$$

where $\frac{dF}{dT}$ is the mathematical expression of the speed of movement of the valve (2) existing at the moment under consideration, while k_1 is the arbitrary constant of adjustment depending upon the opening of the valve (4).

By multiplying both sides of Equation [14] by dT and integrating it we obtain Equation [1] given previously as expressing the characteristic of the regulator of Class I

$$F = k_1 \int P dT + (C) \ldots\ldots\ldots\ldots [1]$$

In this equation the constant of integration depends upon the position of the regulating valve at the beginning of operation of the regulator.

The operation of the dashpot (5) is equivalent to the operation of the needle valve (4) and either one of them could be omitted without changing the characteristic of the regulator. If the dashpot is used, the constant of adjustment k_1 will depend on the opening of the bypass valve (7).

It will be seen from the above explanation that the Class I regulator is characterized by the fact that the regulator moves the regulating valve with a speed proportional at any given moment to the extent of deviation of the controlled function from its normal desired value as existing at the same moment. This means also that the regulating valve continues to move as long as this deviation exists.

In this as well as in all other examples it is assumed that the parts of the regulator are so designed and so proportioned that the effect of friction and inertia of the regulator on its operation is so small that it can be neglected.

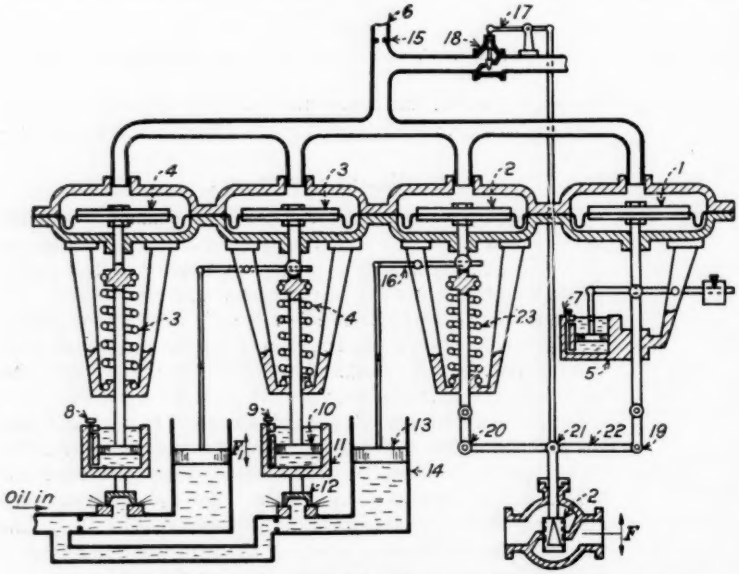


FIG. 11 REGULATOR PROVIDED WITH AUTOMATIC ADJUSTMENT OF THE PRIMARY IMPULSE

EXAMPLE OF CLASS XII[2]

This class of regulator, represented by Fig. 11, is provided with automatic adjustment of the primary impulse by means of the valve (18).

Without this adjustment the characteristic of this regulator is

$$F = k_8 \int P dT + k_9 P + k_{10} \frac{dP}{dT} + k_{11} \frac{d^2P}{dT^2} + (C). \quad [48]$$

This equation is modified by the operation of the valve (18) into

$$F = k_8 \int (P - k_{12}F) dT + k_9 (P - k_{12}F) + k_{10} \frac{d(P - k_{12}F)}{dT} + k_{11} \frac{d^2(P - k_{12}F)}{dT^2} \ldots\ldots [49]$$

By combining the like terms and the groups of constants, we get Equation [12], expressing the characteristic of Class XII.

REMARKS CONCERNING CLASSES VII, VIII, IX, X, XI, AND XII

To the best of the author's knowledge and belief the regulator characteristics of Classes VII to XII, inclusive, are new to the art. The principal difference between the automatic-control apparatus of these new classes and those of the classes widely used at present is the existence of the terms $\frac{dP}{dT}$ and $\frac{d^2P}{dT^2}$ in the characteristic equations. These terms are obtained by the use of a single dashpot or the series combination of two dashpots.

The alternate means available for obtaining these terms are:

1 *The Electric Condenser.* The use of a condenser for this purpose is based on the fact that if we impress a variable potential across a condenser connected in series with a very small resistance, the potential drop across the resistance will be proportional to the rate of change of the variable potential.

2 *Electrical Coil Moving Across a Uniform Magnetic Field.* The use of a coil for the purpose of obtaining the rate of change of a function is based on the law that the potential induced in the coil is proportional to the speed of movement of the coil in respect to the magnetic field.

3 *Orifice.* The use of an orifice is a general case of the use of the dashpot since the purpose of the dashpot is to force the fluid through a resistance (bypass valve) in a volume proportional to the extent of change of the function, thus obtaining pressure drop across the resistance which is proportional to the rate of change of the function. Strictly speaking the use of a condenser is a particular case of the use of an orifice.

4 *Induction Coil.* The fact that the drop of potential across an induction coil is proportional to the rate of change of current passing through it is the basis of the use of such a coil for the purpose of obtaining the rate of change of a function.

5 *Inertia.* The use of inertia is based on the principle that the reaction of a mass is proportional to its acceleration. The precession of a gyroscope should also be classed under this heading.

6 *Step-by-Step Differentiation.*

7 *Differentiating Calculating Machine.* Several of these are available, based on the principle of finding the tangent of a curve.

Theoretically there is no reason why the derivatives of higher orders than the second of the controlled function could not be added to any of the characteristics covered. In actual practice, however, the usefulness of such complicated characteristics is limited to very special cases. Therefore, the twelve characteristics shown are the practical limit of the development in this respect.

[2] A complete demonstration of all twelve classes of regulators has been filed by the author in the archives of the Society. This demonstration includes Figs. 4 to 10, inclusive, Equations [16] to [47], inclusive, and Equations [50] to [55], inclusive.

Solution of the Problem of Automatic Control as a Whole

It is beyond the scope of this article to present a detailed solution of the problem of automatic control in all its various ramifications but there are two basic principles which should be particularly emphasized in this connection.

First, it could be definitely shown that the characteristic of the Class II regulator, Equation [2], would be entirely adequate in all cases of automatic control, provided there were no time lag in any part of the controlled system and provided the regulator performs in perfect accordance with this characteristic. More generally this means that an automatic-control problem is completely solved when the controllable flow of fluid or energy to or from the storage is made to vary strictly in phase with and in proportion to the change of the amount of fluid or energy in storage, or mathematically when $W = kP_2$ (Fig. 2). Under these ideal conditions the variation of the controlled function could be made as small as desired by the simple expedient of increasing the value of the constant of adjustment k_1.

For instance, in the example of the level control shown in Fig. 1, the level in the tower (1) could be held as steady as desired by simply increasing the ratio of the leverage between the float (4) and the valve (5), provided the float is located in the tower to eliminate the time lag in transmission of the impulses and provided the float and the valve are entirely devoid of inertia and friction and also that there is no inertia of the fluid flowing through the pipe (6). In actual practice, however, the above conditions can be only approximated. Therefore, there exists just one optimum value of k_1 in Equation [2] with the result that the attempt to minimize the variation of the controlled function by increasing k_1 beyond its optimum value for this particular installation is invariably followed by a rapid periodic oscillation of the system known as "hunting."

Second, the current methods for eliminating hunting are based on the misconception that a regulator could be stabilized by means of a dash pot or a similar retarding device.

In the discussion of Class IV in the complete paper, filed in the archives of the Society, it is shown that the effect of stabilizers is equivalent to an artificially created time lag. Since hunting is the result of the time lag inherent in the system, the addition of an artificial time lag cannot possibly correct the situation. The only result attained by the stabilizers is to increase the magnitude of fluctuation of the controlled function for a given change in the rate of flow of fluid or power to or from the system. It is admitted, however, that by retarding the response of the regulator, the system may be made apparently more stable from the standpoint of hunting.

The only rational method of combating hunting is by counteracting the time lag. It could be stated without going into a detailed mathematical proof that the addition of the terms $\frac{dP}{dT}$ and $\frac{d^2P}{dT^2}$ to the characteristic of a regulator will counteract the time lag. Whether these terms should be used singly or together depends entirely upon the type of time lag. It is interesting to note that the elimination of hunting by the rational method involves advancing the response of the regulator.

Of course, if hunting is the result of the imperfect operation of the regulator, due either to its excessive inertia or friction, the regulator should be redesigned to make it perform strictly in accordance with its characteristic equation.

There are certain types of regulators, however, in which the inertia effect of some essential part cannot be entirely eliminated even by a most careful design. For instance, in the spring-loaded flyball speed governor the ever-present inertia of the flyballs introduces a phase displacement between the change of the turbine speed and the corresponding change of the angle of inclination of the flyballs.

From the elementary law of the reaction of a mass to its acceleration, and for normal small changes in turbine speed

$$k_3 P_1 + k_4 \frac{d^2P_1}{dT^2} = P \qquad [56]$$

where P_1 = the angle of inclination of the flyballs, P = the extent of change of the turbine speed, k_4 = constant of proportionality depending on the inertia of flyballs, k_3 = constant of proportionality depending on the design of the governor in respect to the rotative speed of the flyballs. By using an operative connection between the flyballs and the throttle valve having the characteristic

$$F = k_5 P_1 + k_6 \frac{d^2P_1}{dT^2} \qquad [57]$$

Since k_4 and k_5 are two perfectly arbitrary constants of adjustment we can make $k_5 = k_1 k_3$ and $k_6 = k_1 k_4$.

Substituting these values of k_5 and k_6 into Equation [57]

$$F = k_1 \left(k_3 P_1 + k_4 \frac{d^2P_1}{dT^2} \right) = k_1 P \qquad [2]$$

It will be seen, therefore, that by using characteristic Equation [57], the unavoidable retarding effect of the inertia of the flyballs can be completely counteracted. Of course, this case is just a particular example of the use of new characteristics for counteracting the time lag in the impulse-transmitting (or rather impulse-generating) means.

It could be shown in a similar manner that if a regulator has an unavoidable series combination of inertia and a fluid friction, as is the case when the flyballs are dampened with a dashpot, the retarding effect of such combination can be counteracted by using characteristic Equation [9]. If in addition to this retarding effect existing in the regulator itself there is also a time lag in the system external to the regulator, an introduction of the derivatives of the higher order than the second in the characteristic of the regulator will be necessary if a precise and stable regulation is to be achieved under all conditions of operation.

Bibliography

"Regelung der Kraftmaschinen," by Dr. Ing. Max Tolle, Julius Springer, Berlin, 1921.

"Regler für Druck und Menge," by Guido Wünsch, R. Oldenbourg, Berlin, 1930.

"The Manual of Instrumentation," by M. F. Behar, Instrument Publishing Company, Pittsburgh, 1932.

Application of the Elastic-Point Theory to Piping Stress Calculations

By S. W. SPIELVOGEL[1] and S. KAMEROS,[2] NEW YORK, N. Y.

The application of the neutral-point theory to piping expansion problems leads to a simplification in their solution. A simple transfer formula introduced by the authors eliminates what would have been a laborious step in the computations and resolves the procedure into the determination of two moments of inertia and the product of inertia of the pipe line with respect to two rectilinear axes through its center of gravity.

THEORETICAL and experimental research of recent years has led to the development of a well-defined technique in the design and mathematical analysis of pipe lines subjected to expansion. Various authors have simplified the tedious labor of calculating the unknown quantities by grapho-analytical procedures or through graphs and tables which furnish either solutions for the most frequently occurring integrals entering into the calculation or direct results for a great variety of pipe-line shapes. A further simplification of the procedure for calculating the three redundant end reactions in fully restrained pipe lines can be obtained by making use of the well-known principle of the elastic-point, or so-called "rigid-bracket," theory. The inherent advantage of this method is particularly apparent when applied to irregular, unsymmetrical pipe lines, the solutions for which lead to unwieldy equations not suitable for graphical representation and consequent utilization for pipe lines of similar form. Where the shape of the pipe line is of such character as to call for individual solution, the elastic-point method has the advantage of simplicity, time saving, and elimination of sources of error.

Mention of the possibilities of the application of the elastic-point theory to pipe-line stresses has been made before in the discussion of technical papers presented to the A.S.M.E., but to the author's knowledge the advantages of the theory have never been fully demonstrated. The application of the method in its original form offers but little advantage over prevalent procedures. The authors have discovered, however, that in conjunction with a transfer formula derived herein, it permits interpretations which remove some of the obstacles which make this subject a specialty for experts. This paper is presented, therefore, in the belief that it will contribute toward opening this field to those not fully equipped with the complex knowledge of statically indeterminate systems.

The authors have purposely not repeated in this paper a discussion of such subjects as flexibility constant, the effect of flattening of curved pipe, and stress-multiplication factor. These subjects have already been covered in papers of the Society. Those who are not conversant with them may have recourse to the bibliography at the end of this paper.

The following discussion leading to the neutral-point theory is along the line of reasoning applied to statically indeterminate problems and is presented for completeness only.

A pipe line situated wholly in one plane with its ends fully restrained and subjected to temperature changes constitutes a statically indeterminate system with three unknown quantities at each of the two terminal points. These unknown quantities are the horizontal and vertical components of the end reactions and two restraining moments, one for each end. To solve this problem, there are available

1 The three fundamental equations of equilibrium; namely,

The sum of all horizontal forces equals zero.....$\Sigma X = 0$
The sum of all vertical forces equals zero.....$\Sigma Y = 0$
The sum of all moments equals zero.....$\Sigma M = 0$

2 Three more equations, which are obtained by considering the distortion in the system caused by the restrictions which prevent the free expansion of the pipe.

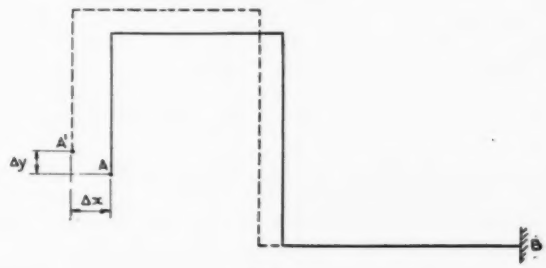


FIG. 1

The pipe line shown in Fig. 1 is subjected to temperature changes. If the end A were made free and end B were securely fixed, the pipe line would become a cantilever which, when heated, would lengthen horizontally the amount Δx and vertically the amount Δy. There would be no angular distortion at the free end and the ultimate shape of the line would be similar to the original shape, the point A merely being translated to A'.

If two unit forces are now applied at the free end and in directions opposite to the expansions Δx and Δy, part of the translatory displacements will be restored and the free end will undergo a rotary motion resulting in an angular displacement. Inasmuch as no angular displacement exists in the original, i.e., the fully restrained system, a counteracting moment must be applied at the free end to hold the pipe element at point A in its position, which in the case illustrated by Fig. 1 is a vertical position.

Let all translatory motions produced by the unit forces be

[1] Assistant Engineer, The Brooklyn Edison Company, Inc. Mr. Spielvogel was graduated in 1919 from the Technische Hochschule in Vienna, Austria, with the degree of C.E. afterward engaging successively in the design of bridges and steel and concrete structures in connection with steam-generating stations and high-pressure steam piping at the "Gutehoffnungshutte," Germany, Electric Bond and Share Company, and Erie Railroad, New York. Since 1925, Mr. Spielvogel has been connected with the mechanical engineering department of the Brooklyn Edison Company, Inc.

[2] Mechanical Engineering Department of the New York Edison Company, Inc. Mr. Kameros was graduated from Columbia College in 1924 with a B.A. degree, and from the Columbia School of Mines, Engineering, and Chemistry in 1926 with an M.E. degree.

Contributed by the Power Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

denoted by δ and all rotary motions by ζ. Give these symbols subscripts such that the first indicates the direction in which the displacement takes place and the second the direction of the unit action (force or moment) causing that displacement. Thus δ_{xy} shall denote the horizontal displacement caused by a vertical unit force, δ_{ym} the vertical displacement caused by a unit moment at the free end, ζ_{my} the angular displacement at the free end caused by a unit vertical force, etc.

If we apply at the free end forces X and Y and a moment M which have such intensities as to move point A', Fig. 1, back to A and restore the original position of the tangent at A, it is evident that these forces and the moment are identical with the actual end reactions. This statement is expressed mathematically by Equations [1], in which the sum of all horizontal move-

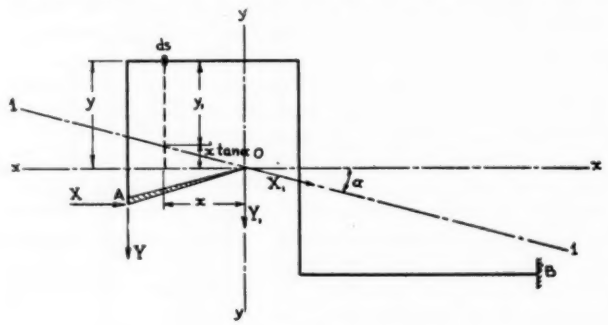


FIG. 2

ments produced by X, Y, and M is made equal to the expansion Δx, the sum of all vertical movements made equal to Δy, and the sum of all rotations made equal to zero.

$$\left.\begin{aligned} X\delta_{xx} + Y\delta_{xy} + M\delta_{xm} &= \Delta x \\ X\delta_{yx} + Y\delta_{yy} + M\delta_{ym} &= \Delta y \\ X\zeta_{mx} + Y\zeta_{my} + M\zeta_{mm} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [1]$$

In Equations [1], Δx and Δy are known. The nine coefficients for X, Y, and M represent deflections or rotations of the free end of the cantilever due to unit loads or a unit moment and can be calculated. Actually only six of these coefficients need be computed because, owing to Maxwell's law of reciprocity, $\delta_{xy} = \delta_{yx}$, $\delta_{xm} = \zeta_{mx}$, and $\delta_{ym} = \zeta_{my}$.

In solving Equations [1], slide-rule accuracy is insufficient. It is no infrequent occurrence that one set of solutions fails to satisfy the equations for the only reason that insufficient significant figures had been developed in solving for X, Y, and M. Furthermore, the moments at points between the two supports are materially affected by slight changes in the values of the end reaction. In applying the principle of the elastic center we gain the advantage that only two of the six coefficients in Equations [1] need be calculated and that X and Y can be found directly from equations which contain one unknown only. In the following discussion, statements will be made, the proof of which will be found in the latter part of this paper.

Release one of the two supports, say support A, Fig. 2, and connect end A rigidly to a bracket leading to the center of gravity O of the line. Let point O be temporarily the support of the pipe line. Loads upon the system or expansions within the system will cause point O, if it were free, to move in the same direction and with the same magnitude as point A if it were free to move. The translatory forces required to nullify the displacements at O have the same intensity as the reacting forces at A. The final result, therefore, is obtained by transferring to point A the reactions found for point O and adding at A the moment caused by the offset of O against A.

A rectangular-coordinate system is now passed through O, for example y-axis vertical, x-axis horizontal. The reactions Y_1 and X_1 at O are then assumed to act along y and axis 1–1, respectively, where axis 1–1 is inclined to the x-axis at an angle α. The angle is so selected as to make axis 1–1 conjugated to axis y–y. This means that the product of inertia or moment of deviation of the pipe line about axes 1–1 and y–y is zero; or, in other words, that the sum of each small element of pipe multiplied by its coordinates with reference to these axes is zero. These axes are not necessarily principal axes.

Angle α is obtained from

$$\tan \alpha = I_{xy}/I_{yy} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where I_{xy} and I_{yy} denote, respectively, the product of inertia and the moment of inertia of the pipe line for the rectangular system of coordinates, that is, with reference to axes x–x and y–y.

As a result of these assumptions, the displacements δ and ζ which carry dissimilar subscripts become zero. Thus,

$$\delta_{xy} = \delta_{yx} = 0; \quad \delta_{xm} = \zeta_{mx} = 0; \quad \delta_{ym} = \zeta_{my} = 0$$

and Equations [1] change to

$$X_1 = \frac{\Delta x_1}{\delta_{11}}; \quad Y_1 = \frac{\Delta y}{\delta_{yy}}; \quad M = 0 \quad \ldots\ldots\ldots\ldots [3]$$

Note that a uniform temperature change causes no moment at point O.

The terms Δx_1 and Δy now denote, respectively, the expansion movements of point O in direction of the inclined axis 1–1 and in the direction of the vertical axis y–y if O were the end of a free cantilever and the terms δ_{11} and δ_{yy} the movements due to unit loads acting at O in the direction of these respective axes.

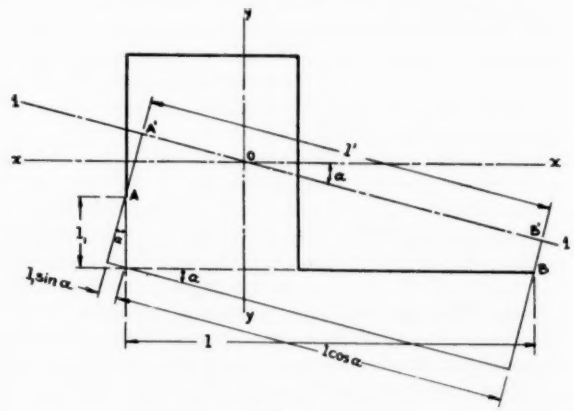


FIG. 3

Referring to Fig. 3 and denoting the coefficient of expansion by c and the temperature change by Δt, the movements are then calculated as follows:

$$A'B' = l' = l \cos \alpha + l_1 \sin \alpha$$

$$\Delta x_1 = l' \cdot c \cdot \Delta t \ldots\ldots [4] \qquad \Delta y = l_1 \cdot c \cdot \Delta t \ldots\ldots [5]$$

When both ends of the pipe line are at the same level $l_1 = 0$ and $\Delta x_1 = l \cos \alpha \cdot c \cdot \Delta t$; and $\Delta y = 0$

The divisor δ_{11} is obtained by summing the products of each length of pipe element ds and the square of the distance y_1 from the inclined axis 1–1 measured as shown in Fig. 2 and dividing the result by EI, the product of the modulus of elasticity of the material and the moment of inertia of the cross-sectional area of the pipe. The divisor δ_{yy} is obtained by summing the products of each length of pipe element ds and the square of the horizontal distance x from y–y and dividing the result by EI.

It is evident, therefore, that δ_{yy} is proportional to the moment of inertia of the pipe line about a vertical axis through the center of gravity of the line and that a similar interpretation may be given to δ_{11} with the modification, however, that the multipliers are the squares of the distances y_1 from the inclined axis 1–1. In both cases the factor of proportionality is $1/EI$. Accordingly

$$\delta_{yy} = \frac{I_{yy}}{EI} = \frac{1}{EI}\int x^2 ds$$

$$\delta_{11} = \frac{I_{11}\cos^2\alpha}{EI} = \frac{\cos^2\alpha}{EI}\int y_1^2\, ds$$

I_{11} denoting what may be called the modified moment of inertia about axis 1–1.

An attempt to calculate I_{11} in accordance with the foregoing definition would soon reveal that the apparent simplification demonstrated by a comparison of Equations [1] and [3] is offset by the necessity of establishing analytical equations for each branch of pipe line and the tedious labor of integrating in this special case, of non-rectangular coordinate axes. The following derivation, however, will show that I_{11} is obtainable directly from a simple transfer formula which contains I_{xx}, I_{yy}, and I_{xy}, all referring to the original, that is, rectangular system of coordinates.

Referring to Fig. 2

$$y_1 = y - x\tan\alpha$$

$$I_{11} = \int y_1^2\, ds = \int (y - x\tan\alpha)^2\, ds$$

$$= \int y^2\, ds - 2\tan\alpha\int x\, y ds + \tan^2\alpha\int x^2\, ds$$

The first integral represents the moment of inertia of the line about axis x–x, the second the product of inertia about axes x–x and y–y, and the third the moment of inertia about axis y–y, so that

$$I_{11} = I_{xx} - 2\tan\alpha I_{xy} + \tan^2\alpha I_{yy} \quad \ldots\ldots\ldots\ldots [6]$$

I_{xx}, I_{xy}, and I_{yy} are all introduced in Equation [6] with positive sign.

Equations [3] can now be written as

$$X_1 = \frac{EI\,\Delta x_1}{I_{11}\cos^2\alpha};\quad Y_1 = \frac{EI\,\Delta y}{I_{yy}} \quad \ldots\ldots\ldots\ldots [7]$$

Summary of Procedure

As a result of the foregoing analysis the procedure for calculating the end reactions may be summarized as follows:

1 Determine the center of gravity of the pipe line

2 Calculate moments of inertia I_{xx}, I_{yy}, and product of inertia I_{xy} of the pipe line about horizontal and vertical axes through the center of gravity of the line

3 Calculate angle of the conjugated axis from Equation [2]

4 Calculate modified moment of inertia I_{11} from Equation [6]

5 Calculate the expansions

(*a*) For pipe supports at same level

$$\Delta x_1 = l\cos\alpha\cdot c\cdot\Delta t;\quad \Delta y = 0$$

(*b*) For pipe supports at different levels

$$\Delta x_1 = (l\cos\alpha + l_1\sin\alpha)\cdot c\cdot\Delta t$$
$$\Delta y = l_1\cdot c\cdot\Delta t$$

6 Calculate reactions at center of gravity from Equations [7]

7 Calculate horizontal and vertical components of X_1

Horizontal component of $X_1 = X_1\cos\alpha = X$
Vertical component of $X_1 = X_1\sin\alpha$

8 Add vertical component of X_1 to Y_1 obtaining

$$Y = Y_1 + X_1\sin\alpha$$

9 Transfer X and Y to left support.

10 Calculate moment at left support due to the offsets of center of gravity to left support A

11 Calculate moments along pipe line and determine the maximum.

Illustration of Procedure

The method of procedure is illustrated by the solution of an example involving a 6-in., 600-lb standard seamless pipe with

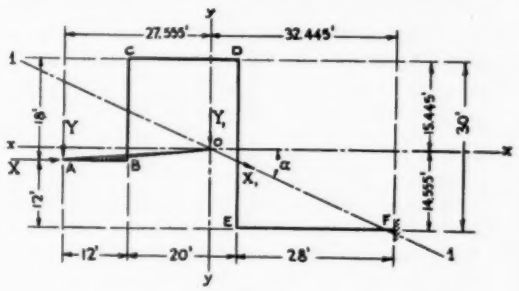


Fig. 4

a coefficient of expansion, $c = 6.14$ in. per 100 ft; a moment of inertia of cross-section, $I = 37$ in.^{4}; and a modulus of elasticity, $E = 25.5 \times 10^6$ lb per sq in. The steam temperature is given as 750 F, and the room temperature as 60 F. The pipe supports are on different levels as shown in Fig. 4.

1 Center of gravity of pipe line

27.555 ft from left end; 14.555 ft from bottom

2 Moment of inertia about axis $x-x$

$$I_{xx} = (12\times 2.555^2) + \left[\frac{18^3}{12} + (18\times 6.445^2)\right] + (20\times 15.445^2)$$
$$+ \left[\frac{30^3}{12} + (30\times 0.445^2)\right] + (28\times 14.555^2) = 14{,}270.667$$

Moment of inertia about axis $y-y$

$$I_{yy} = \left[\frac{12^3}{12} + (12\times 21.555^2)\right] + (18\times 15.555^2)$$
$$+ \left[\frac{20^3}{12} + (20\times 5.555^2)\right] + (30\times 4.445^2)$$
$$+ \left[\frac{28^3}{12} + (28\times 18.445^2)\right] = 23{,}306.667$$

Product of inertia

$$I_{xy} = (12\times -2.555\times -21.555) + (18\times 6.445\times -15.555)$$
$$+ (20\times 15.445\times -5.555) + (30\times 0.445\times 4.445)$$
$$+ (28\times -14.555\times 18.445) = -10{,}317.333$$

3 Angle of conjugated axis

$$\tan\alpha = \frac{I_{xy}}{I_{yy}} = \frac{-10{,}317.333}{23{,}306.667} = -0.4426$$

$$\alpha = -23\text{ deg, }52\text{ min, }30\text{ sec.}$$

4 Moment of inertia about conjugated axis

$$I_{11} = I_{xx} - 2\tan\alpha I_{xy} + \tan^2\alpha I_{yy}$$
$$= 14{,}270.667 - 9123.903 + 4565.776 = 9703.54$$

5 Projected length of pipe line and expansions

$$l' = l\cos\alpha + l_1\sin\alpha$$
$$= 54.87 + 4.856 = 59.726 \text{ ft}$$

$$EI\,\Delta x_1 = \frac{25.5\times10^6\times37}{144}\times\frac{6.14}{12}\times\frac{59.726}{100} = 2{,}002{,}390$$

$$EI\,\Delta y = \frac{25.5\times10^6\times37}{144}\times\frac{6.14}{12}\times\frac{12}{100} = 402{,}315$$

6 $$X_1 = \frac{EI\,\Delta x_1}{\cos^2\alpha\, I_{11}} = \frac{2{,}002{,}390}{0.8363\times9703.54} = 246.75 \text{ lb}$$

$$Y_1 = \frac{EI\,\Delta y}{I_{yy}} = \frac{402{,}315}{23{,}306.667} = 17.26 \text{ lb}$$

7 $$X = X_1\cos\alpha = 246.75\times0.9145 = 225.65 \text{ lb}$$

8 $$Y = Y_1 + X_1\sin\alpha = 17.26 + (246.75\times0.4047) = 117.12 \text{ lb}$$

9, 10, and 11 Bending moments

at A	$(225.65\times 2.555) + (117.12\times 27.555)$	=	+3804 ft-lb
B	$(225.65\times 2.555) + (117.12\times 15.555)$	=	+2397 ft lb
C	$(-225.65\times 15.445) + (117.12\times 15.555)$	=	−1663 ft-lb
D	$(-225.65\times 15.445) + (-117.12\times 4.445)$	=	−4006 ft-lb
E	$(225.65\times 14.555) + (-117.12\times 4.445)$	=	+2764 ft-lb
F	$(225.65\times 14.555) + (-117.12\times 32.445)$	=	− 516 ft-lb

Derivation of Equations

It has been stated that Equations [1] change into Equations [3] if the support is transferred to the center of gravity O of the pipe line and the reactions assumed to act along a vertical axis y–y and its conjugated axis 1–1. This means that all coefficients carrying dissimilar subscripts in Equations [1] become zero. The proof of this follows:

If m_1 and m_y denote the moments produced by loads of 1 pound acting, respectively, along axes 1–1 and y–y, and δ_{xm} is the horizontal movement and δ_{ym} the vertical movement of point O produced by a unit moment at O, then from the theory of elasticity

$$\delta_{xm} = \frac{1}{EI}\int m_1\, m\, ds \quad \ldots\ldots [8]$$

and

$$\delta_{ym} = \frac{1}{EI}\int m_y\, m\, ds \quad \ldots\ldots [9]$$

From Fig. 5 it is evident that $m_1 = 1$ lb $y_1\cos\alpha$, and $m_y = 1$ lb x also $m = 1$ ft-lb (if x and y are in feet).

Hence, substituting these values in Equations [8] and [9] the following are obtained:

$$\delta_{xm} = \frac{\cos\alpha}{EI}\int y_1\, ds; \qquad \delta_{ym} = \frac{1}{EI}\int x ds$$

The products under the integral represent the statical moments of the pipe line about the two axes. Inasmuch as these axes have been selected to pass through the center of gravity, the statical moments are zero. Therefore, the expressions δ_{xm} and δ_{ym} are both zero for these conditions. Since, by the law of reciprocity of deflections and angular changes $\delta_{xm} = \zeta_{mx}$ and $\delta_{ym} = \zeta_{my}$, the terms ζ_{mx} and ζ_{my} are both also zero for the conditions selected.

Taking now the terms δ_{xy} and δ_{yx} and referring to Fig. 5, from the theory of elasticity,

$$\delta_{xy} = \delta_{yx} = \frac{1}{EI}\int m_1\, m_y\, ds = \frac{\cos\alpha}{EI}\cdot\int xy_1\, ds \ldots. [10]$$

The term under the integral represents the product of inertia of the pipe line with respect to the two selected axes. By choosing

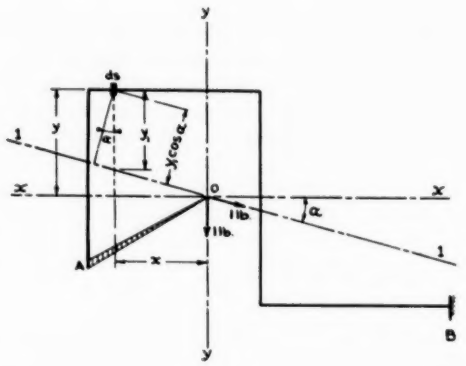


Fig. 5

axis 1–1 to be a conjugated axis of y–y the product of inertia may be made zero and therefore both δ_{xy} and δ_{yx} made zero. Referring to Fig. 5 it is seen that $y_1 = y \mp x\tan\alpha$. Therefore, from Equation [10],

$$\delta_{xy} = \frac{\cos\alpha}{EI}\int (xy \mp x^2\tan\alpha)ds = 0$$

From which $\tan\alpha = \pm\dfrac{\int xy\, ds}{\int x^2\, ds} = \pm\dfrac{I_{xy}}{I_{yy}}$

The last derived equation is Equation [2]. From the above it will be seen therefore that by selecting the two axes as indicated Equations [1] are transformed into Equations [3].

Returning again to Fig. 5 it is seen that

$$\delta_{yy} = \frac{1}{EI}\int m_y\, m_y\, ds = \frac{1}{EI}\int x^2\, ds$$

The term under the integral is the moment of inertia of the pipe line with respect to the y–y axis. Therefore,

$$Y_1 = \frac{\Delta y}{\delta_{yy}} = \frac{EI\,\Delta y}{\int x^2 ds} = \frac{EI\,\Delta y}{I_{yy}}$$ as given in Equations [7]

also

$$\delta_{11} = \frac{1}{EI}\int m_1 m_1 ds = \frac{1}{EI}\int y_1\cos\alpha\cdot y_1\cos\alpha\, ds$$
$$= \frac{\cos^2\alpha}{EI}\int y^2{}_1\, ds$$

The term under the integral is the moment of inertia of the pipe line with respect to axis 1–1. Therefore,

$$X_1 = \frac{\Delta x_1}{\delta_{11}} = \frac{EI\,\Delta x_1}{\cos^2\alpha\int y^2{}_1\, ds} = \frac{EI\,\Delta x_1}{\cos^2\alpha\, I_{11}}$$ as given in Equations [7]

The authors have prepared a number of additional problems of typical pipe lines containing quarter bends or sloping branches. Their solutions as well as the appendix are available upon request either to the authors or to the society headquarters.

Bibliography

1 "Discussion of Neutral-Point Theory as Applied to Arches," by C. B. McCullough in "Reinforced Concrete and Masonry Structures," compiled by C. B. Hool and W. S. Kinne, McGraw-Hill Book Company, 1924, page 468.

2 "Stresses and Reactions in Expansion Pipe Bends," by A. M. Wahl, Trans. A.S.M.E., vol. 50, 1928, paper FSP-50-49, p. 241.

3 "The Elastic Deformation of Pipe Bends," by W. Hovgaard, Publication of the Massachusetts Institute of Technology, vol. 62, no. 64, November, 1926.

4 "Deformation of Plane Pipes and Further Research on Pipe Bends," by W. Hovgaard, Publication of the Massachusetts Institute of Technology, vol. 64, no. 59, December, 1928.

5 "Design of Steam Piping to Care for Expansion," by W. H. Shipman, Trans. A.S.M.E., vol. 51, 1929, paper FSP-51-52, p. 415.

6 "Piping Handbook," by Walker and Crocker, Second Edition, McGraw-Hill Book Company, 1931, Chapter 7.

The Loading and Friction of Thrust and Journal Bearings With Perfect Lubrication

By H. A. S. HOWARTH,[1] PHILADELPHIA, PA.

The author brings together here for the first time important formulas and charts that have been offered by bearing analysts in this field during the past fifteen or twenty years. Some new material is added for completeness, and some references made to earlier fundamental work. Descriptive matter that accompanied the original publication of the charts and formulas has been eliminated as far as practicable. Improved charts derivable from earlier ones, and the rearrangement of others will, it is hoped, prove more useful to the designing engineer than the original publications. The net result is a concise summary of valuable information.

In order that the reader at the beginning may have a fair idea of what is to be found and where in this paper, an explanation of the contents follows:

The first section dealing with journal bearings begins with a concise illustrated description of their several kinds. The side-leakage chart is then explained because from its curves are obtained the correction factors that are necessary for the proper application of the subsequent material to the solving of journal-bearing problems.

Optimum conditions for journal bearings of three classes are shown by charts, tables, and formulas. The titles, captions, and notes on these pages are intended to be sufficient explanation.

General conditions for journal bearings of several classes are next shown by a series of charts which are accompanied by the necessary textual explanation.

Thrust-bearing loading and friction data follow the charts for general conditions for journal bearings. They refer only to Kingsbury thrust bearings, which are made with freely pivoted shoes supporting a thrust collar having an unbroken flat surface. Data are given only for a single set of such surfaces having six pivoted shoes.

The appendixes, of which there are five, contain descriptive matter, the reading of which will help the reader in understanding the first section of the paper. Appendix A contains the symbols and their explanation. Appendix B gives a number of examples together with their solutions, which explain applications of the charts. These are by no means exhaustive. Appendix C discusses viscous flow in general, the viscosity coefficient, the relation between viscosity and friction, and gives useful formulas for several kinds of viscous flow useful to those dealing with bearing problems. Conversion factors are given for several units of absolute viscosity. The symbols for these formulas differ in some respects from those in Appendix A. Appendix D contains an explanation of perfect lubrication as it occurs in bearings, and covers such headings as tapered oil film, side leakage, film thickness, viscosity change within film, the flat-wedge film, the curved-wedge film, optimum conditions, deviations from optimum. This is a valuable introduction to the charts in the body of this paper. Appendix E is a bibliography of the technical literature referred to in or forming parts of the paper.

Journal Bearings

FOR THE purpose of analytical study, journal bearings may be separated into several groups that approximate the forms that are used or should be used. The broadest divisions are (*a*) clearance bearings, and (*b*) fitted bearings.

Clearance bearings are those in which the bearing has a larger radius of curvature than the journal. The angular extent β to which the useful bearing surface surrounds the journal determines whether the bearing is full or partial. The full bearing completely surrounds the journal and $\beta = 360$ deg. Partial bearings are usually less than 180 deg and probably average about 120 deg. Characteristics of partial bearings are given in the charts for a wide range of useful angles β.

Fitted bearings are those in which the journal and bearing have the same radius of curvature. Such a bearing must have an angular extent β less than 180 deg and preferably much less. The characteristics of such bearings are given only up to $\beta = 150$ deg.

The direction in which the load of the journal is applied to the bearing, with relation to the angular limits of the bearing surface, is of considerable importance to the analytical study. This leads to a division of partial bearings into (*a*), centrally loaded and, (*b*) eccentrically loaded. That there may be different degrees of load eccentricity of the latter is seen by comparing Classes *A* with *D*, and *B* with *E*, given in the journal-bearing classification which follows.

Centrally loaded bearings have their angular extent β divided into two equal parts by the line of action of the applied load. In such bearings the leading angle α equals $\beta/2$.

[1] Vice-President and General Manager, Kingsbury Machine Works, Philadelphia, Pa. Mem. A.S.M.E. H. A. Stevens Howarth was graduated from Yale University in 1899 with honors in mechanical engineering. Several years of practical experience with steel mills and manufacturing were followed by teaching of mechanical engineering subjects at Lehigh University and Carnegie Institute of Technology. In 1914 Mr. Howarth became associated with Albert Kingsbury, in his thrust-bearing business, and in 1917 became general manager and chief engineer. He continued in the same position and business with the Kingsbury Machine Works, formed by Dr. Kingsbury in 1921 and incorporated in 1925. In 1928 he became vice-president and general manager. The present paper is the second of a group planned to simplify for designers the various analytical studies of bearings published by Albert Kingsbury, and other members of his organization, including the author. The first paper of this group was published in the A.S.M.E. Trans., vol. 56, 1934, paper MSP-56-2, p. 891.

Contributed by the Machine Shop Practice Division for presentation at the Semi-Annual Meeting, Cincinnati, Ohio, June 19 to 21, 1935, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y. and will be accepted until July 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

OPTIMUM CONDITIONS CLASS-A ECCENTRICALLY LOADED JOURNAL BEARINGS WITH RUNNING CLEARANCE—SIDE LEAKAGE NEGLECTED EXCEPT IN CHART A

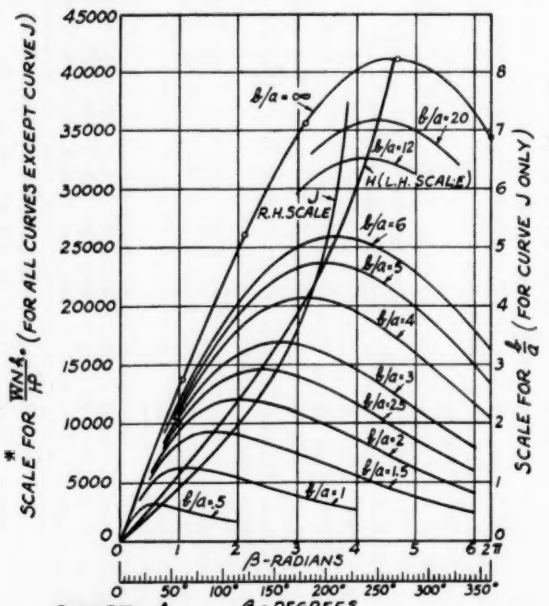


CHART - A

THIS SHOWS FOR CLASS "A" BEARINGS THE VALUES OF THE NON-DIMENSIONAL GROUP WNk_s/Hp AS AFFECTED BY SIDE LEAKAGE, WITH FILM FORMS FOR THE MINIMUM COEFFICIENT OF FRICTION. CURVE J GIVES THE VALUE OF $\frac{k}{d}$ AT WHICH GROUP WNk_s/Hp IS GREATEST FOR EACH VALUE OF β.

* WHEN THE CURVE FOR A GIVEN RATIO OF $\frac{l}{a}$ IS CHOSEN, EACH ORDINATE OF IT WILL BE CORRECT FOR THE OPTIMUM RELATION OF THE SYMBOLS IN THIS GROUP. HENCE THE CORRECTION FACTORS FOR SIDE LEAKAGE ARE ALREADY ALLOWED FOR IN THE HEIGHTS OF THOSE ORDINATES.

	β	60°	120°	180°	270°	360°
1	A	122.3°	66.33°	12.17°	-62.48°	-118.6°
2	α	32.94°	66.49°	101.3°	156.6°	208.6°
3	α/β	.5490	.5541	.5627	.5799	.5796
4	C	.7481	.4821	.3655	.3310	.3379
5	k_m/k_o	2.382	2.305	2.139	1.990	2.021
6	k_i/k_o	1.200	1.209	1.226	1.257	1.266
7	η/k_o	3.970	1.931	1.576	1.495	1.510
8	θ	158.8°	140.8°	127.4°	118.8°	118.6°
9	φ	155.3°	132.8°	113.5°	94.09°	90.00°

TABLE A-1, GEOMETRICAL OPTIMA FOR MAXIMUM LOAD W

	β	60°	120°	180°	270°	360°
10	$W\frac{k_s^2}{\mu N a^3}$	.0201	.0759	.1543	.2635	.2939
11	$F\frac{k_s}{\mu N a^2}$	.0936	.1863	.2768	.4080	.5379
12	$\lambda\frac{a}{k_s}$	4.660	2.453	1.794	1.549	1.830
13	$\lambda\frac{l}{k_s}$	4.880	5.138	5.637	7.297	11.50
14	$Hp\frac{k_s^2(10)^6}{\mu N a^3}$	1.485	2.956	4.392	6.473	8.534
15	$Hp\frac{(10)^6}{WNk_s}$	7.394	3.893	2.847	2.457	2.904
16	$\frac{WNk_s}{Hp(10)^6}$	.1352	.2569	.3513	.4070	.3444
17	$\lambda\sqrt{\frac{\mu N}{P_o}}$	.4671	.4780	.4983	.5620	.7016

TABLE A-2, DYNAMICAL OPTIMA FOR MAXIMUM LOAD W



FIG. "A"
ECCENTRICALLY LOADED CLEARANCE BEARING.

	β	60°	120°	180°	270°	360°
1	A	117.9°	58.04°	1.119°	-72.15°	-121.0°
2	α	33.89°	68.54°	104.7°	161.4°	211.0°
3	α/β	.5649	.5711	.5815	.5977	.5860
4	C	.7496	.4911	.3844	.3520	.3660
5	k_m/k_o	2.586	2.476	2.249	2.086	2.155
6	k_i/k_o	1.248	1.263	1.283	1.292	1.280
7	η/k_o	3.986	1.965	1.624	1.543	1.577
8	θ	156.4°	136.7°	123.1°	117.5°	121.0°
9	φ	151.8°	126.6°	105.8°	89.24°	90.0°

TABLE A-3, GEOMETRICAL OPTIMA FOR MINIMUM FRICTION COEFFICIENT SEE CHART A-3 FOR OTHER VALUES OF β.

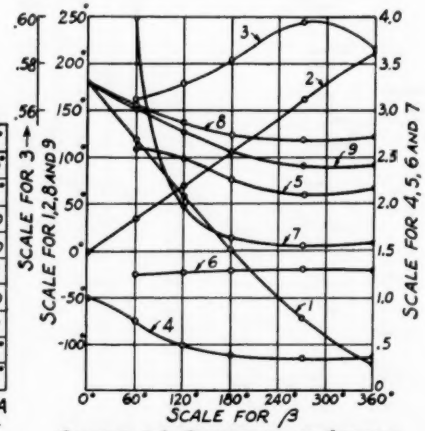


CHART A-3, GEOMETRICAL OPTIMA FOR MINIMUM FRICTION COEFFICIENT

	β	60°	120°	180°	270°	360°
10	$W\frac{k_s^2}{\mu N a^3}$	.0197	.0742	.1507	.2602	.2925
11	$F\frac{k_s}{\mu N a^2}$	.0902	.1790	.2658	.3985	.5327
12	$\lambda\frac{a}{k_s}$	4.584	2.413	1.763	1.531	1.821
13	$\lambda\frac{l}{k_s}$	4.801	5.054	5.540	7.217	11.44
14	$Hp\frac{k_s^2(10)^6}{\mu N a^3}$	1.432	2.840	4.217	6.323	8.452
15	$Hp\frac{(10)^6}{WNk_s}$	7.274	3.829	2.798	2.430	2.890
16	$\frac{WNk_s}{Hp(10)^6}$	.1375	.2612	.3574	.4115	.3460
17	$\lambda\sqrt{\frac{\mu N}{P_o}}$	.4548	.4647	.4841	.5524	.6965

TABLE A-4, DYNAMICAL OPTIMA FOR MINIMUM FRICTION COEFFICIENT. SEE CHART A-4 FOR OTHER VALUES OF β.

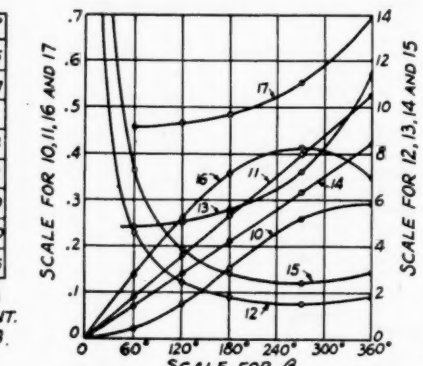


CHART A-4, DYNAMICAL OPTIMA FOR MINIMUM FRICTION COEFFICIENT.

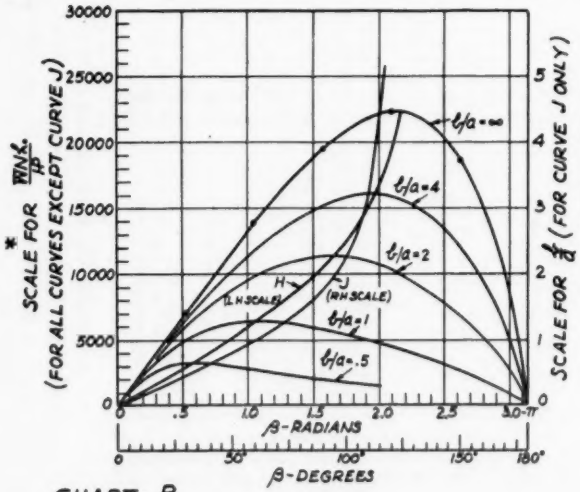


CHART- B

THIS SHOWS FOR CLASS "B" BEARINGS THE VALUES OF THE NON-DIMENTIONAL GROUP $WN\ell/hP$ AS AFFECTED BY SIDE LEAKAGE, WITH FILM FORMS FOR THE MINIMUM COEFFICIENT OF FRICTION. CURVE J GIVES THE VALUE OF ℓ/a AT WHICH GROUP $WN\ell/hP$ IS GREATEST FOR EACH VALUE OF β.

* WHEN THE CURVE FOR A GIVEN RATIO OF ℓ/a IS CHOSEN, EACH ORDINATE OF IT WILL BE CORRECT FOR THE OPTIMUM RELATION OF THE SYMBOLS IN THIS GROUP. HENCE THE CORRECTION FACTORS FOR SIDE LEAKAGE ARE ALREADY ALLOWED FOR IN THE HEIGHTS OF THOSE ORDINATES.

	β	0°	30°	60°	90°	120°	150°
1	A	180°	129.1°	92.21°	64.30°	40.67°	19.39°
2	α	0°	17.47°	35.79°	55.72°	77.76°	101.1°
3	α/β	.5774	.5822	.5964	.6191	.6480	.6739
4	h_A/h_o	2.189	2.176	2.143	2.078	1.969	1.804
5	h_m/h_o	2.189	2.176	2.143	2.306	3.021	5.433
6	h_i/h_o	1.373	1.398	1.480	1.624	1.843	2.131
7	θ_i	180°	150.1°	136.4°	135.2°	142.4°	156.9°
8	φ	180°	146.6°	128.0°	120.0°	118.4°	120.5°

TABLE B-1, GEOMETRICAL OPTIMA FOR MAXIMUM LOAD W

	β	0°	30°	60°	90°	120°	150°
9	$W\frac{10\ell^2}{\mu N a^2}$	0	.0451	.1685	.3309	.4534	.3924
10	$F\frac{h_o}{\mu N a^2}$	0	.0407	.0774	.1073	.1284	.1339
11	$\lambda\frac{a}{h_o}$	∞	9.027	4.597	3.242	2.831	3.413
12	$\lambda\frac{\ell}{h_o}$	4.706	4.727	4.814	5.093	5.930	8.935
13	$HP\frac{h_o(10)^6}{\mu N a^3}$	0	.6455	1.229	1.702	2.037	2.125
14	$HP\frac{(10)^3}{WN\ell_o}$	∞	14.32	7.293	5.144	4.492	5.415
15	$\frac{WN\ell_o}{HP(10)^3}$	0	.0698	.1371	.1944	.2226	.1847
16	$\lambda\sqrt{\frac{\mu N}{P_o}}$	.4311	.4285	.4219	.4171	.4263	.4781

TABLE B-2, DYNAMICAL OPTIMA FOR MAXIMUM LOAD W



FIG "B"
FITTED PARTIAL BEARING

	β	0°	30°	60°	90°	120°	150°
1	A	180°	133.0°	95.59°	66.29°	41.73°	20.08°
2	α	0°	17.84°	36.26°	55.95°	77.45°	99.41°
3	α/β	.5914	.5946	.6043	.6217	.6455	.6627
4	h_A/h_o	2.533	2.501	2.408	2.276	2.122	1.993
5	h_m/h_o	2.533	2.501	2.408	2.486	3.189	5.804
6	h_i/h_o	1.434	1.457	1.528	1.662	1.873	2.165
7	θ_i	180°	154.8°	140.8°	138.1°	144.0°	158.1°
8	φ	180°	150.8°	131.9°	122.2°	119.2°	119.5°

TABLE B-3, GEOMETRICAL OPTIMA FOR MINIMUM FRICTION COEFFICENT SEE CHART B-3 FOR OTHER VALUES OF β.

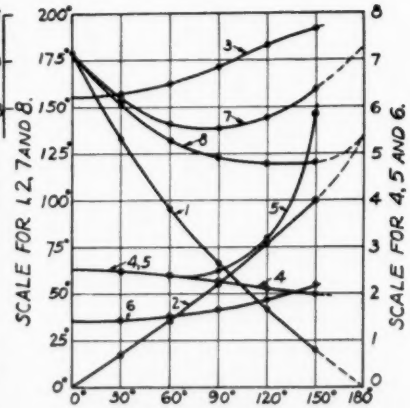


CHART B-3, GEOMETRICAL OPTIMA FOR MINIMUM FRICTION COEFFICENT

	β	0°	30°	60°	90°	120°	150°
9	$W\frac{10\ell^2}{\mu N a^2}$	0	.0443	.1663	.3281	.4504	.3876
10	$F\frac{h_o}{\mu N a^2}$	0	.0393	.0755	.1054	.1267	.1307
11	$\lambda\frac{a}{h_o}$	∞	8.877	4.538	3.214	2.813	3.372
12	$\lambda\frac{\ell}{h_o}$	4.622	4.648	4.752	5.049	5.892	8.829
13	$HP\frac{h_o(10)^6}{\mu N a^3}$	0	.6238	1.197	1.673	2.010	2.074
14	$HP\frac{(10)^3}{WN\ell_o}$	∞	14.08	7.201	5.100	4.463	5.351
15	$\frac{WN\ell_o}{HP(10)^3}$	0	.0710	.1389	.1961	.2240	.1869
16	$\lambda\sqrt{\frac{\mu N}{P_o}}$	.4194	.4177	.4138	.4117	.4221	.4695

TABLE B-4, DYNAMICAL OPTIMA FOR MINIMUM FRICTION COEFFICENT SEE CHART B-4 FOR OTHER VALUES OF β.

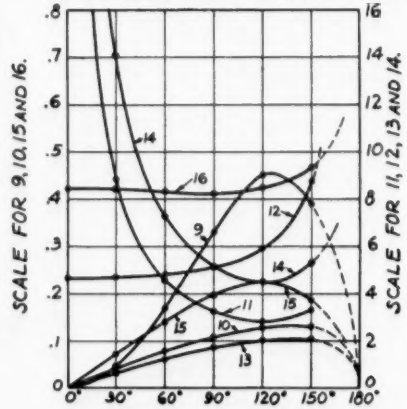


CHART B-4, DYNAMICAL OPTIMA FOR MINIMUM FRICTION COEFFICENT

OPTIMUM CONDITIONS CLASS-C CENTRALLY LOADED JOURNAL BEARINGS WITH RUNNING CLEARANCE—SIDE LEAKAGE NEGLECTED EXCEPT IN CHART C

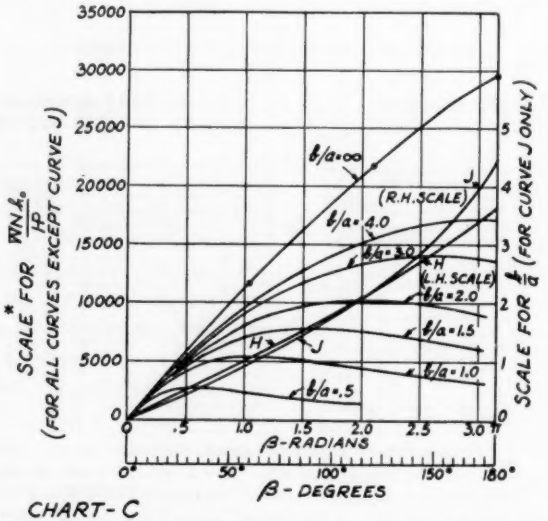


CHART - C

THIS SHOWS FOR CLASS "C" BEARINGS THE VALUES OF THE NON-DIMENSIONAL GROUP $WN\ell_o/Hp$ AS AFFECTED BY SIDE LEAKAGE, WITH FILM FORMS FOR THE MINIMUM COEFFICIENT OF FRICTION. CURVE J GIVES THE VALUE OF ℓ/a AT WHICH GROUP $WN\ell_o/Hp$ IS GREATEST FOR EACH VALUE OF β.

* WHEN THE CURVE FOR A GIVEN RATIO OF ℓ/a IS CHOSEN, EACH ORDINATE OF IT WILL BE CORRECT FOR THE OPTIMUM RELATION OF THE SYMBOLS IN THIS GROUP. HENCE THE CORRECTION FACTORS FOR SIDE LEAKAGE ARE ALREADY ALLOWED FOR IN THE HEIGHTS OF THOSE ORDINATES.

	β	60°	120°	180°
1	A	131.2°	83.66°	37.83°
2	α	30°	60°	90°
3	α/β	.50	.50	.50
4	C	.7466	.4688	.3437
5	ℓ_m/ℓ_o	2.004	1.980	1.937
6	ℓ_1/ℓ_o	1.137	1.144	1.158
7	η/ℓ_o	3.946	1.883	1.524
8	θ_1	162.5°	146.8°	134.3°
9	φ	161.2°	143.7°	127.8°

TABLE C-1, GEOMETRICAL OPTIMA FOR MAXIMUM LOAD W

	β	60°	120°	180°
10	$W\frac{\ell_1}{\mu N a^3}$	.0180	.0673	.1352
11	$F\frac{\ell_1}{\mu N a^2}$	.0982	.1954	.2902
12	$\lambda\frac{a}{\ell_o}$	5.464	2.906	2.147
13	$\lambda\frac{\ell}{\ell_o}$	5.722	6.087	6.744
14	$Hp\frac{\ell_o(10)^6}{\mu N a^3}$	1.559	3.101	4.605
15	$Hp\frac{(10)^3}{WN\ell_o}$	8.670	4.611	3.406
16	$\frac{WN\ell_o}{Hp(10)^3}$	.1153	.2169	.2936
17	$\lambda/\sqrt{\frac{\mu N}{p_o}}$	.5181	.5329	.5581

TABLE C-2, DYNAMICAL OPTIMA FOR MAXIMUM LOAD W

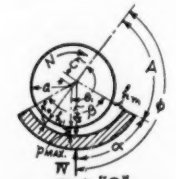


FIG "C"
CENTRALLY LOADED CLEARANCE BEARING

	β	60°	120°	180°
1	A	131.6°	84.39°	38.71°
2	α	30°	60°	90°
3	α/β	.50	.50	.50
4	C	.7615	.4904	.3645
5	ℓ_m/ℓ_o	2.072	2.056	2.021
6	ℓ_1/ℓ_o	1.140	1.148	1.163
7	η/ℓ_o	4.193	1.962	1.574
8	θ_1	163.0°	147.8°	135.7°
9	φ	161.6°	144.4°	128.7°

TABLE C-3, GEOMETRICAL OPTIMA FOR MINIMUM FRICTION COEF. SEE CHART C-3 FOR OTHER VALUES OF β.

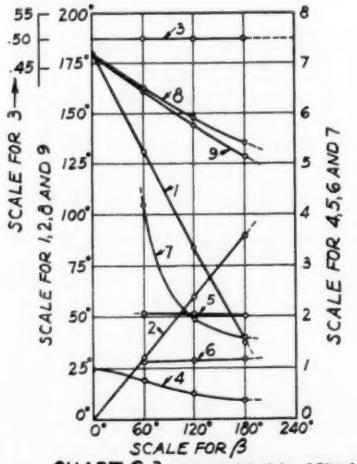


CHART C-3, GEOMETRICAL OPTIMA FOR MINIMUM FRICTION COEF.

	β	60°	120°	180°
10	$W\frac{\ell_1}{\mu N a^3}$	.0180	.0671	.1349
11	$F\frac{\ell_1}{\mu N a^2}$	.0980	.1948	.2890
12	$\lambda\frac{a}{\ell_o}$	5.458	2.902	2.142
13	$\lambda\frac{\ell}{\ell_o}$	5.716	6.078	6.731
14	$Hp\frac{\ell_o(10)^6}{\mu N a^3}$	1.555	3.091	4.586
15	$Hp\frac{(10)^3}{WN\ell_o}$	8.660	4.604	3.399
16	$\frac{WN\ell_o}{Hp(10)^3}$	.1155	.2172	.2942
17	$\lambda/\sqrt{\frac{\mu N}{p_o}}$	.5171	.5317	.5564

TABLE C-4, DYNAMICAL OPTIMA FOR MINIMUM FRICTION COEF. SEE CHART C-4 FOR OTHER VALUES OF β.

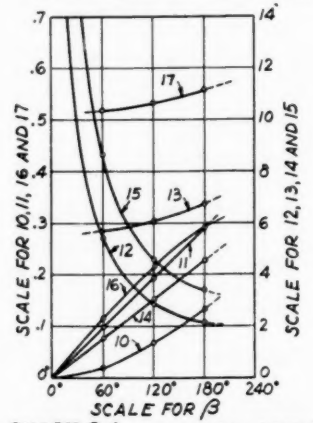


CHART C-4, DYNAMICAL OPTIMA FOR MINIMUM FRICTION COEF.

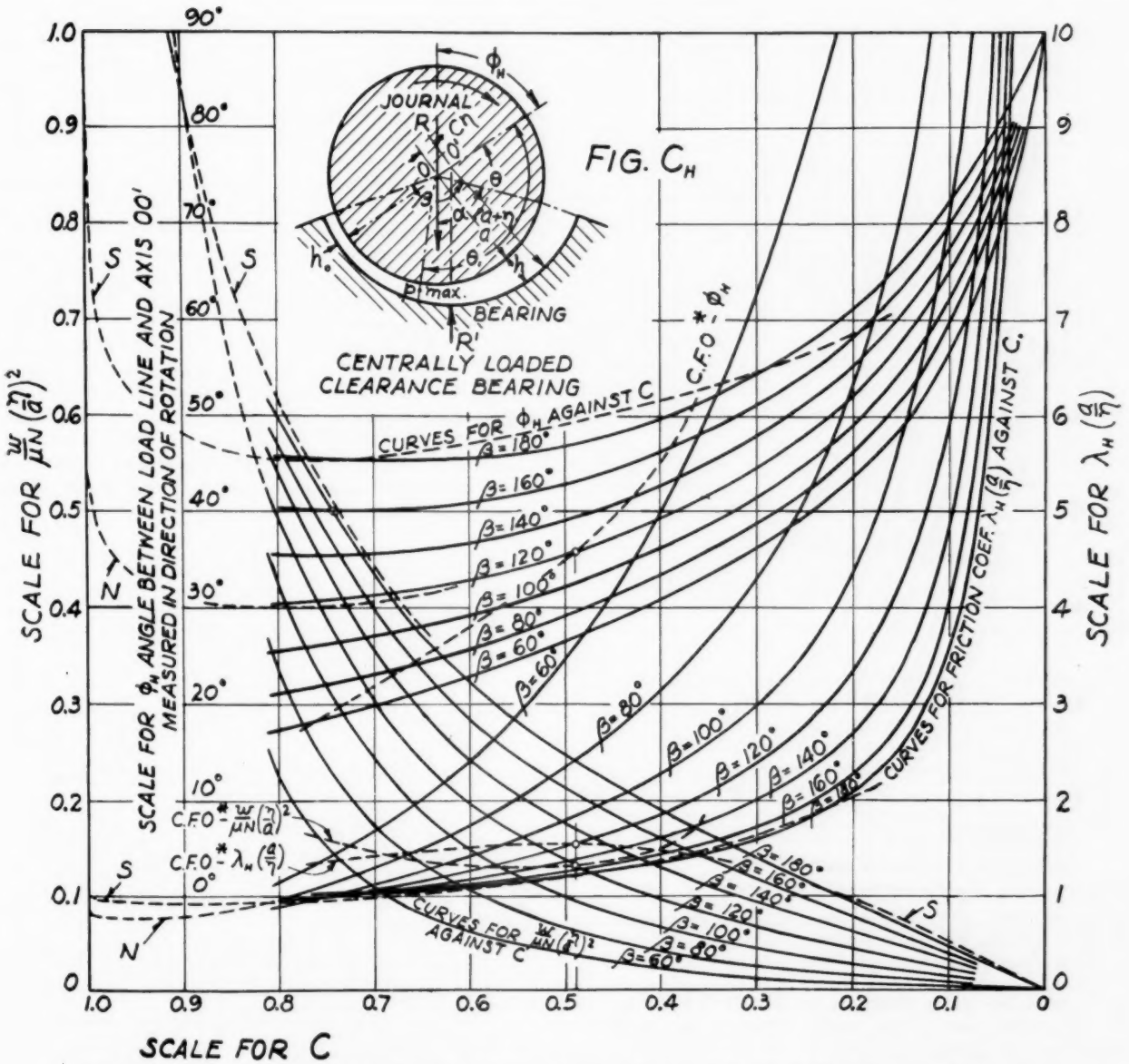


CHART C-5 GENERAL CONDITIONS OF CENTRALLY LOADED JOURNAL BEARINGS (CLASS C) WITH RUNNING CLEARANCE. SIDE LEAKAGE NEGLECTED

Eccentrically loaded bearings have their angular extent β divided into two unequal parts by the line of action of the applied load. The leading angle α is that portion of the angular extent β passed over by a point on the journal as it rotates from the edge of the bearing to a position directly under the load. The ratio of α/β may vary considerably when bearings are in service, because designers do not always need to trouble themselves to analyze individual bearings. There are, however, some ratios that are better than others.

JOURNAL-BEARING CLASSIFICATION

The bearings whose optimum and general characteristics are given in the accompanying charts and tables may be divided into Classes *A* to *F*. The sources of data available for each class are given.

Class A. In Class *A* are found the special series of eccentrically loaded partial bearings with running clearance that Dr. Kingsbury found to be best. In this class the values of α/β vary not only for various values of β but they are different for the two optimum conditions studied by him. In all cases, however, the α/β ratio is appreciably greater than $\beta/2$.

Optimum conditions for this class are given for bearing angles β varying from 0 to 360 deg, in the figures, charts, and Tables *A*, *A*-1, *A*-2, *A*-3, and *A*-4. The letter *A* is used in every case to identify the class. This must not be confused with the symbol *A*. The data are reproduced from the work of Dr. Kingsbury (5).[1]

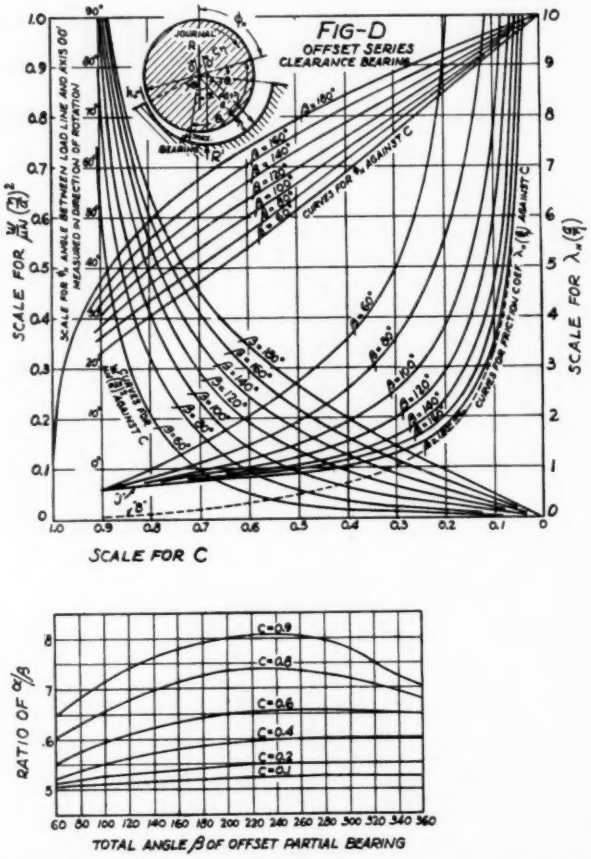
Class B. In Class *B* are found the special series of eccentrically loaded fitted partial bearings that Dr. Kingsbury found to be best. The values of the ratio α/β vary not only for various values of β but they are also different for the two sets of optimum conditions studied by him. In all cases the α/β ratio is appreciably greater than $\beta/2$.

Optimum conditions for this class are given for bearing angles

[1] Numbers in parentheses refer to Bibliography at the end of the paper.

β varying from 0 to 150 deg in figures, charts, and Tables *B*, *B*-1, *B*-2, *B*-3, and *B*-4. The data are reproduced from the work of Dr. Kingsbury (5).

Class C. In Class *C* are found partial bearings of the centrally loaded clearance series whose characteristics were studied by Dr. Kingsbury and by the author. The nominal values of α/β always equal $\beta/2$.



CHARTS *D*-1 (BELOW) AND *D*-2 (ABOVE) GENERAL CONDITIONS FOR OFFSET (ECCENTRICALLY LOADED) JOURNAL BEARINGS (CLASS *D*) WITH RUNNING CLEARANCE. SIDE LEAKAGE NEGLECTED

Optimum conditions for this class are given for bearing angles β varying from 0 to 180 deg, in figures, charts, and Tables *C*, *C*-1, *C*-2, *C*-3, and *C*-4. The data are reproduced from the work of Dr. Kingsbury (5).

General conditions for this class are given in Chart *C*-5, which has been prepared by the author from his earlier work (7) and made more usable.

Class D. In Class *D* are found the special series of offset (eccentrically loaded) partial bearings with running clearance whose characteristics were presented by the author in his graphical analysis (7). This series differs a little from Class *A*.

General conditions for this class are given in Charts *D*-1 and *D*-2 for bearing angles β varying from 60 to 180 deg. Chart *D*-2 is more usable than the original previously published (7).

Class E. In Class *E* are found the special series of eccentrically loaded fitted partial bearings that the author described in his earlier work (7). The values of α/β vary for different bearing angles β. This series differs a little from Class *B*.

General conditions for this class are given for bearing angles β varying from 60 to 120 deg, in Chart *E*-1. This chart is reproduced in improved form from earlier work of the author (7). On this chart are also shown by dotted lines some of the optimum conditions determined by Dr. Kingsbury. It should be noted here that for a fitted bearing the general conditions and the optimum conditions are the same for a given bearing or series when considered from the same point of view. This is true because there is no running clearance to introduce the question of eccentricity.

Classes ACD. The designation, Class *ACD*, has been used for a study made by the author of all classes for a single bearing of angular extent $\beta = 120$ deg and having running clearance (8).

General conditions as to capacity only (no friction data are given) are shown in Chart *ACD*-120. An immense amount of data on the 120-deg-clearance bearing are given on this chart, which is presented in improved form.

Class F. The designation, Class *F*, has been applied to full bearings, which must of course have running clearance. Such a bearing is a limit for the angle β to approach as will be seen by examining the optimum charts and tables given by Dr. Kingsbury for Class *A*. Hence, those charts and tables for values of $\beta = 360$ deg should be looked upon as optimum for the full bearing, especially if there be one common groove for oil supply and discharge located at the limits of the film.

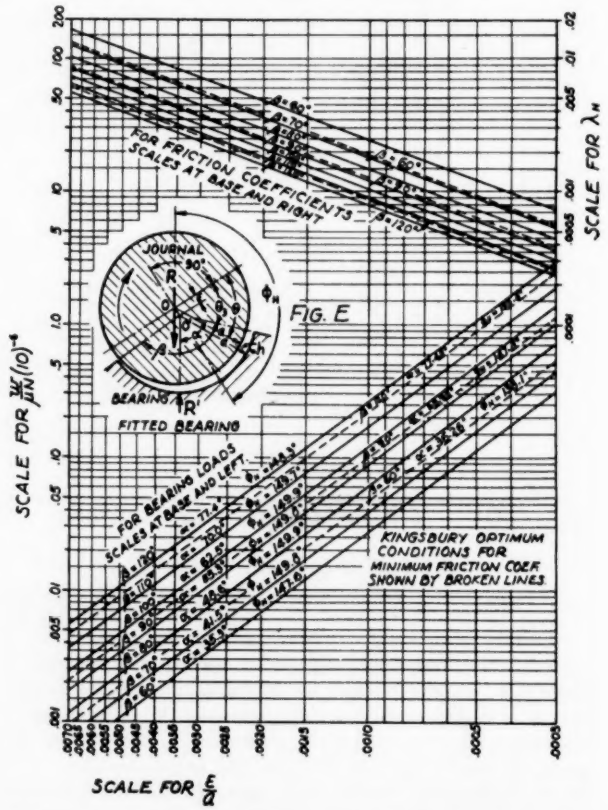


CHART *E*-1 GENERAL CONDITIONS FOR ECCENTRICALLY LOADED FITTED JOURNAL BEARINGS (CLASS *E*) WITHOUT RUNNING CLEARANCE. SIDE LEAKAGE NEGLECTED

General conditions for the full bearing are given in Chart *F*-1 in an approximate manner only because no one has yet been able to produce a continuous film in such a bearing, with all pressures positive. In Chart *F*-1, therefore, the load-carrying portion of the film is assumed to be 180-deg long and to have the load-carrying characteristics of a 180-deg offset partial bearing of Class *D*. For friction, however, the film is assumed to be that of a perfect full bearing. This is an approximation and is

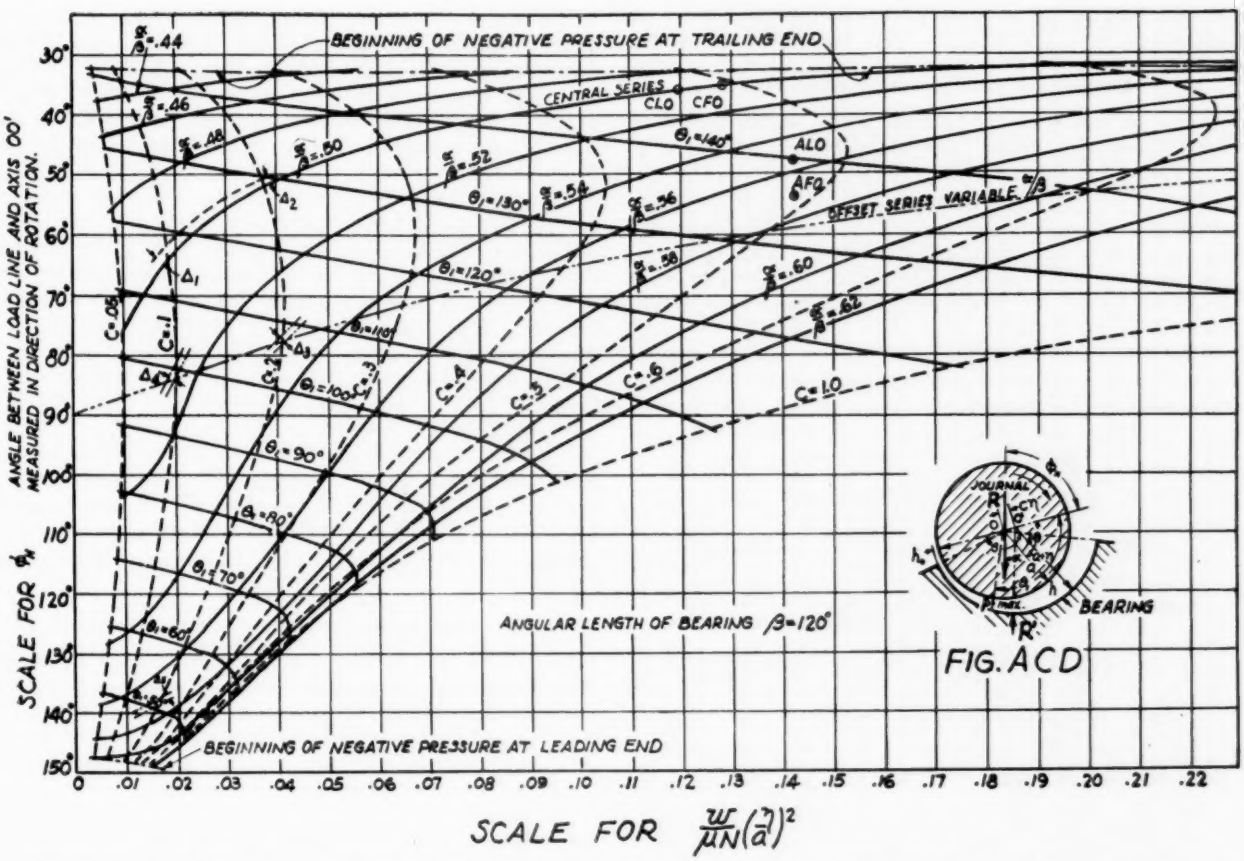


Chart *ACD*-120 General Conditions for 120-Deg Journal Bearings, With Running Clearance, for Capacities Only. Side Leakage Neglected

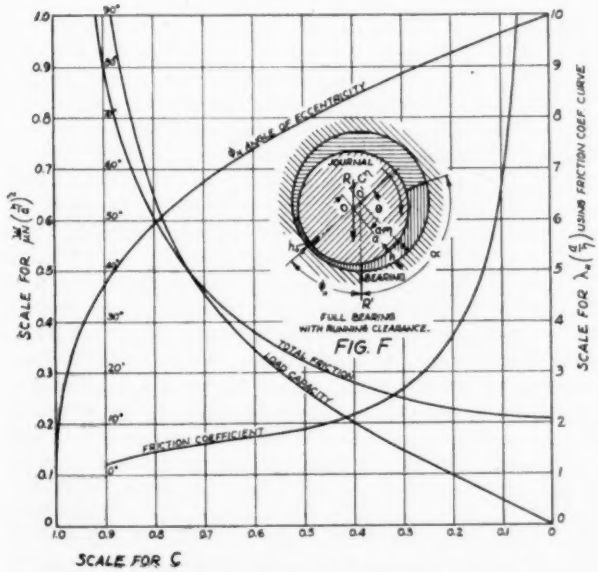
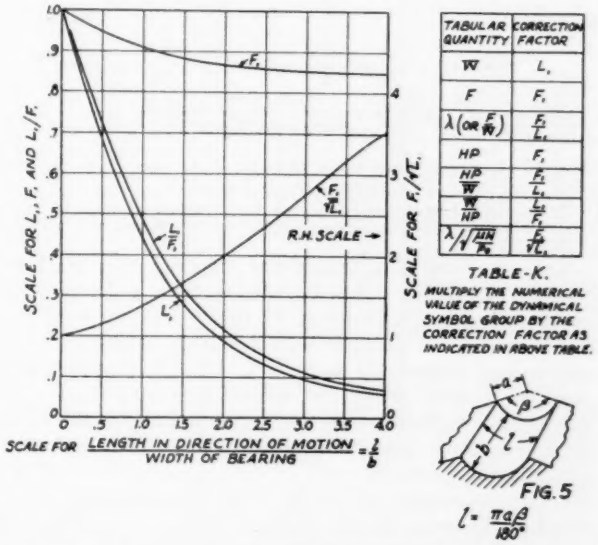


Chart *F*-1 Approximate General Conditions for Full Journal Bearings (Class *F*) With Running Clearance. Side Leakage Neglected

(For horsepower equivalents of total friction for the journal, multiply $\frac{\pi b}{2}(a/\eta)a^2N^2(10)^{-4}$ by the scale reading obtained from the use of the total-friction curve and the right-hand vertical scale. To allow for side leakage multiply by F_1 taken from Chart *KX* for $\beta = 180$ deg.)



TABULAR QUANTITY	CORRECTION FACTOR
W	L_1
F	F_1
λ (OR $\frac{F}{W}$)	$\frac{F_1}{L_1}$
HP	F_1
$\frac{HP}{W}$	$\frac{F_1}{L_1}$
$\frac{W}{HP}$	$\frac{L_1}{F_1}$
$\lambda/\sqrt{\frac{\mu N}{P_a}}$	$\frac{F_1}{YL_1}$

TABLE-K.

MULTIPLY THE NUMERICAL VALUE OF THE DYNAMICAL SYMBOL GROUP BY THE CORRECTION FACTOR AS INDICATED IN ABOVE TABLE.

Chart *KX* Correction Factors for the Influence of Side Leakage Upon the Load-Carrying Capacity and Upon Friction, for Optimum Journal Bearings

(Apply these factors as shown in Table *K*.)

probably near enough to the correct value, for estimating purposes.

Correction Factors for Side Leakage

Correction factors for the influence of side leakage upon load capacity and upon friction are obtainable from Chart *KX*. These are reproduced from Plate *X* in Dr. Kingsbury's paper (5).

These factors apply to optimum film proportions as determined by Dr. Kingsbury but they will be reasonably dependable when applied to film forms not far different from optimum. Even when applied to abnormal conditions they tend to bring the results toward the correct values, and are usually on the safe side.

If the influence of side leakage upon capacity and upon friction were neglected in the solving of bearing problems, the information obtainable from Dr. Kingsbury's optimum-loading and friction charts and from the author's general charts would not agree well with actual conditions determined by reliable tests.

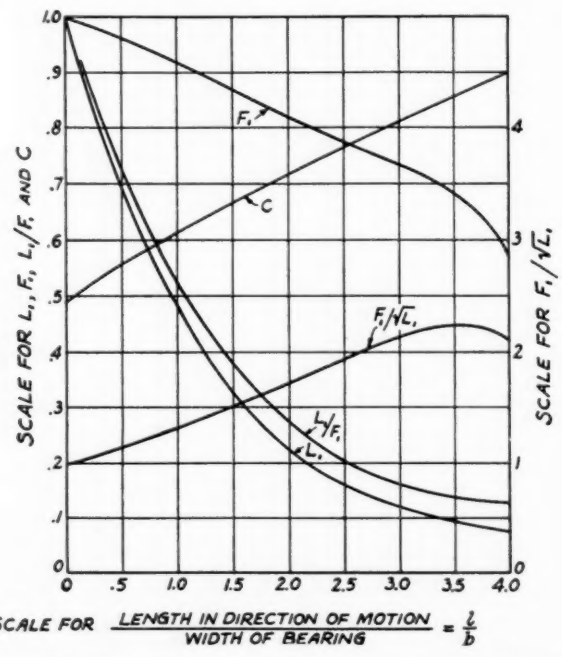


Fig. 1 Optimum 120-Deg Central Bearings for Minimum Coefficient of Friction

(Side-leakage factors obtained by comparing the actual capacities for various values of l/b with the capacity at infinite width when $c = 0.4904$.)

The use of the correction factors is explained by Dr. Kingsbury (5) as follows. "In the dynamical tables and curves" (see charts and tables for Classes *A*, *B*, and *C*) "the numerical coefficients are for the infinitely wide conditions (under which the side-leakage influence is negligible). Thus, for correction for bearings of finite width and given length/width ratio, these (numerical) coefficients are to be multiplied by the proper correction factor (from Chart *KX*) as below." If the symbol group in the dynamical table or chart contains the symbol or set of symbols tabulated in Table *K*, the corresponding numerical value of the group, in the dynamical table or chart, must be multiplied by the corresponding correction factor in Table *K*. The numerical value of the correction factor must be found from Chart *KX* by applying thereto the actual developed proportions of the useful film of the bearing being studied. See examples in Appendix B.

The side-leakage effect upon a 120-deg bearing of Class *C* was given special study by S. J. Needs (15) following a suggestion by Dr. Kingsbury (5) that the optimum film form of a narrow bearing would be different from that of a wide one. Needs found this to be true for the 120-deg Class-*C* bearing. He also found that negative pressures occur near the trailing end of a narrow 120-deg bearing earlier than would be expected if side leakage were negligible, thereby making them equivalent to eccentrically loaded bearings of angular extent less than 120 deg.

A study of these results since made by Needs has yielded a special chart, Fig. 1 herein, that may be used for obtaining correction factors and also optimum eccentricities for this 120-deg Class-*C* bearing, that may be used with the charts and tables of optimum and general conditions herein for the solution of bearing problems, particularly those with length/width ratios greater than 1.5.

Thus we now have two charts from which the correction factors of Table *K* may be obtained. Those in Chart *KX* are on the safe side and applicable to all optimum bearings. Those in Fig. 1 all apply directly to the 120-deg centrally loaded clearance bearing, and they also indicate that the capacities of narrow clearance bearings in general do not fall off as rapidly by becoming narrower, as Chart *KX* would indicate, if the eccentricities are allowed to increase in the manner indicated by Fig. 1.

The Loading and Friction of Journal Bearings

The charts here presented give the optimum and general characteristics of several useful forms of journal bearings that are described under "Journal-Bearing Classification." The headings and captions and notes on each set of charts are as complete as believed necessary to facilitate their use after the reading of the introductory pages of this paper.

The viscosity coefficient μ is assumed to have a constant mean value throughout the film. Although this is not in accordance with the facts, it is found to be satisfactory for the solution of bearing problems and has been so stated by several investigators, including Boswall and Kingsbury.

The charts and tables of bearing characteristics were all prepared under the assumption that the bearings were finite parts of infinitely wide ones and that, therefore, the flow of oil in the film was only in the planes of the direction of rotation. Therefore, the charted and tabulated data represent upper limits of load that a bearing film may carry, and the corresponding friction. Therefore, the correction factors must be applied to the ideal values to bring them in line with the actual capacities of bearings of finite width for which side leakage becomes more and more important as the length-width ratio increases.

Although Dr. Kingsbury's optimum conditions for Classes *A*, *B*, and *C* show ideal relations of viscosities, clearances, eccentricities, loads, and load directions, the designer may want to know how far wrong he is in some particular design that deviates from those optima. In one important case, Class *C*, the journal bearings examined by the author for general conditions are identical with those whose optima were found by Dr. Kingsbury. It was therefore possible to add a series of points to the general conditions, Chart *C*-5, to show the relation thereto of Dr. Kingsbury's optima. Three dotted curves are drawn marked by symbol groups preceded by the letters *C*, *F*, *O*. These letters mean Class-*C* friction optima. For a complete discussion of this comparison see Appendix D on perfect lubrication in the sections on optima and deviations from optima.

Chart *ACD*-120 yields so much loading information about all classes of 120-deg clearance bearings that it may be used for studying the influences upon running position, of various changes in design proportions such as the α/β ratio and the clearance ratio η/a. Side-leakage-correction factors may be applied to this chart as in Example 4, Appendix B, but only for L_1 as in

$p_0 = wL_1$. The farther one departs from good film proportions the less reliable will be the L_1 factor taken from Chart *KX*. In such cases Fig. 1 should be kept in mind.

Chart *ACD*-120 also gives the angular location θ_1 of the maximum oil pressure in the film.

The optima of Dr. Kingsbury for this bearing appear only as four points. Those for the Class-*A* bearing are marked *ALO* and *AFO*, for maximum-load and minimum-friction coefficient, respectively. For Class-*C* bearings they are marked *CLO* and, *CFO*. These afford the user a way to tell how nearly optimum are his actual conditions.

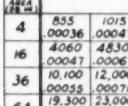
When operating conditions are such that the Chart *ACD*-120 shows negative pressure is likely to appear at the trailing end of the bearing it is best to study such bearings as if they were of the eccentrically loaded Class *A*, or of the offset Class *D*, in either case with a leading angle of $\alpha = 60$ deg, letting the trailing angle $(\beta - \alpha)$ be determined by the class used. See Example 5 in Appendix B.

In Chart F-1 for full bearings they are treated as half bearings so far as carrying capacity is concerned, but as full bearings so

TABLE 1 THRUST-BEARING CAPACITY FOR FREELY PIVOTED SHOES

(Viscosity coefficient = 3.4×10^{-6} Reyn, lb-sec per sq in.)

FACE AREA (SQ. IN.)	SURFACE SPEED, FEET PER SECOND, $= U/12$							
	5	10	20	40	60	80	100	120	140
	CAPACITY, POUNDS PER SQUARE FACE AND MINIMUM FILM THICKNESS, INCH								
4	855 .00036	1015 .00047	1210 .00061	1440 .00079	1590 .00092	1710 .00102	1810 .00112	1890 .00119	1960 .00126
16	4060 .00047	4830 .00061	5750 .00079	6850 .00103	7550 .00119	8150 .00132	8600 .00145	9000 .00155	9350 .00163
36	10,100 .00055	12,000 .00071	14,300 .00092	17,000 .00119	18,750 .00138	20,200 .00154	21,400 .00168	22,400 .00180	23,200 .00192
64	19,300 .00061	23,000 .00079	27,400 .00102	32,500 .00133	36,000 .00155	38,600 .00172	40,900 .00188	42,800 .0020	44,500 .00215
100	31,900 .00066	38,000 .00086	45,200 .00112	53,500 .00145	59,500 .00168	64,000 .00186	67,500 .00205	70,700 .0022	73,500 .00235
144	48,000 .00071	57,200 .00092	68,000 .00119	81,000 .00155	89,500 .00180	96,500 .0020	102,000 .0022	106,500 .00235	111,000 .0025
196	68,000 .00075	81,000 .00098	96,000 .00127	114,500 .00163	126,500 .00190	136,000 .0021	144,000 .0023	150,000 .0025	156,000 .00265
256	92,000 .00079	109,000 .00103	130,000 .00133	155,000 .00172	171,000 .0020	184,000 .0022	194,000 .00245	203,000 .0026	211,000 .0028
324	119,000 .00083	142,000 .00108	168,000 .00138	201,000 .00180	222,000 .0021	239,000 .0023	252,000 .00255	264,000 .0027	274,000 .0029
400	152,000 .00086	181,000 .00112	215,000 .00145	256,000 .00188	283,000 .0022	304,000 .0024	321,000 .00265	336,000 .0028	350,000 .0030



FOR AVERAGE PROPORTIONS OF THRUST BEARINGS, $d = 3b$. FOR ANY RATIO OF b TO d, IF $l = b$ THE BEARING AREA WILL TOTAL $6 \cdot l^2$ FOR SIX SHOES, AND THE TOTAL CAPACITY WILL BE THAT IN THE TABLE TIMES THE NUMBER OF SHOES

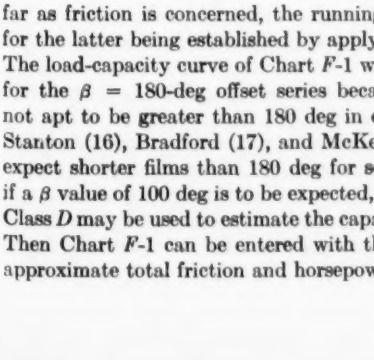
TO FIND THE FEET PER SECOND FROM THE R.P.M.

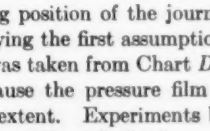
$$\frac{U}{12} = \frac{\pi dN}{12 \times 60}$$

$l \div b$	LOAD CORRECTION FACTOR
0.50	1.567
0.67	1.257
1.00	1.000
1.33	0.732
1.50	0.630
2.00	0.423

TABLE 13 BASED ON ASSUMPTION THAT $l = b$. FOR OTHER PROPORTIONS MULTIPLY THE TABULAR LOAD BY THE CORRECTION FACTOR IN TABLE AT LEFT

* TABLE 13 BASED ON DATA GIVEN IN BULLETINS ON KINGSBURY BEARINGS PUBLISHED BY KINGSBURY MACHINE WORKS INC., PHILA. PA. THE UPPER VALUE IS THE CAPACITY AND THE LOWER ONE IS THE MINIMUM FILM THICKNESS - h_1.





LOADED PLATE RIDING ON A FILM OF OIL. THE LENGTH OF THE PLATE IS "l" AND THE WIDTH IS "b"

far as friction is concerned, the running position of the journal for the latter being established by applying the first assumption. The load-capacity curve of Chart *F*-1 was taken from Chart *D*-2 for the $\beta = 180$-deg offset series because the pressure film is not apt to be greater than 180 deg in extent. Experiments by Stanton (16), Bradford (17), and McKee (18) might lead one to expect shorter films than 180 deg for some conditions. Hence, if a β value of 100 deg is to be expected, the charts and tables for Class *D* may be used to estimate the capacity and the eccentricity. Then Chart *F*-1 can be entered with that eccentricity, and the approximate total friction and horsepower loss estimated. Care must be exercised not to use the friction-coefficient curve on Chart *F*-1 with any other line than the load-capacity line on Chart *F*-1. See Example 7 in Appendix B for application of side-leakage-correction factors to full-bearing problems.

The data available from the charts and tables of this paper, and the problems solved in examples of Appendix B depend entirely upon absolute-viscosity coefficients. No ready means are offered herein for converting absolute into commercial viscosities or vice versa. For such conversions the reader is referred to a recent paper by the author (20) entitled "Current Practice

TABLE 2 CAPACITY, POWER LOSS, AND COEFFICIENT OF FRICTION—KINGSBURY THRUST BEARINGS

(6 shoes, 51-deg subtended angle, viscosity = 3.4×10^{-6} Reyn)

OUTER DIA. OF BEARING INCHES	350 LB./SQ. IN. PRESSURE, SHOE BORE = 1/2 OF OUTSIDE DIA.	
	CAPACITY POUNDS	POWER LOSS HORSEPOWER AT 100 R.P.M.
5	4,380	.0128
6	6,300	.0220
7	8,600	.0350
8	11,200	.0522
9	14,200	.0744
10½	19,300	.118
12	25,200	.176
15	39,500	.344
17	50,800	.501
21	77,300	.944
25	109,500	1.59
29	147,500	2.49
33	191,000	3.66
37	240,000	5.17
45	355,000	9.30
53	493,000	15.2
61	652,000	23.2
73	935,000	39.7

SPEED (R.P.M.)	SPEED FACTOR FOR POWER LOSS	COEF. OF FRICTION AT 350 LB./SQ. IN. PRESSURE
50	.354	.00069
60	.465	.00076
80	.716	.00088
100	1.00	.00098
120	1.32	.0011
160	2.02	.0012
200	2.83	.0014
250	3.95	.0016
300	5.20	.0017
400	8.00	.0020
500	11.2	.0022
600	14.7	.0024
800	22.6	.0028
1000	31.6	.0031
1250	44.2	.0035
1500	58.1	.0038
2000	89.4	.0044
3000	164	.0054

PRESSURE LB. PER SQ. IN.	CORRECTION FACTOR FOR COEF. OF FRICTION
100	.53
175	.71
250	.85
300	.93
350	1.00
400	1.07
500	1.19

in Pressures, Speeds, Clearances, and Lubrication of Oil-Film Bearings," Trans. A.S.M.E., Vol. 56, 1934, paper MSP-56-2, p. 891.

The Loading and Friction of Kingsbury Thrust Bearings

Although loading tables for thrust bearings have been published in bulletins issued by Dr. Kingsbury describing their design and construction, this paper will deal more with their theoretical aspects, and will include the data on their friction losses. See Appendixes C and D for a discussion of the fundamental ideas involved.

Loading of Thrust Bearings. It is found from the study of film forms that a certain one may be looked upon as productive of the least friction, and that the minimum film thickness is a proper criterion for bearing safety. Because of the difficulty involved in measuring film thicknesses, very little of this has been done. That little, however, has indicated that the mathematical theory is practically correct. It is reasonable, therefore, to build up a basic loading table and to mark upon it the minimum film thicknesses that are to be expected under the conditions set forth. Table 1 is based on the experience and practice of Dr. Albert Kingsbury, inventor of the pivoted-shoe thrust bearing, and is supplemented by data derived by means of the mathematical theory of lubrication.

Table 1 is for average thrust bearings used for steam turbines,

centrifugal pumps, ordinary industrial machinery, marine propulsion, and hydroelectric units, where the oil is not heavy. At operating temperature the viscosity coefficient of the lubricant is assumed to be about 0.23 poise. This is equivalent to 23 centipoises or 3.4×10^{-6} Reyn. The commercial viscosity equivalents (when the basic specific gravity = 0.875) are 125 sec Saybolt universal, 110 sec Redwood No. 1, and number 3.7 Engler.

The decimal values, below the capacity figures, are the corresponding approximate estimated minimum film thicknesses, on the median line of the shoe at the trailing edge, when the previously described oil is used. The viscosity given is assumed to be the mean value within the film. The bath viscosity or supply viscosity must be higher by an amount which may be estimated approximately.

The load-correction factors for various shoe-face proportions, are based on a study of side leakage which is discussed in Appendix D.

Friction in Thrust Bearings. In a letter to the editor of *Mechanical Engineering*, April, 1927, the author presented a formula for the friction coefficient for a perfectly lubricated surface moving upon tiltable rectangular shoes, with straight-line motion. This is approximately the condition under which the Kingsbury thrust bearing operates, and is deducible from the equations of Osborne Reynolds. The symbols for this formula which follows are defined in Appendix A, the notations differing somewhat from the original.

$$\lambda = \frac{Fb}{R} = 1.81\, k \sqrt{\left(\frac{\mu U}{l p_0}\right)}$$

The factor k is so applied as to yield a coefficient of friction that takes side leakage of oil into account. The relation of this factor to the length/width ratio of the bearing shoe faces is given in the curve in Table 1. This curve is based upon data published by Michell (9). When there is no side leakage $k = 1$. See also the discussion "Side Leakage Reduces Capacity" in Appendix D, and the Kingsbury Chart KX in the text of this paper and compare curve for F_1/L_1.

Power Losses in Thrust Bearings. Table 2 gives the relations between capacity, speed, and friction, for Kingsbury thrust bearings of various sizes, when the mean viscosity of the oil in the bearing films is 3.4×10^{-6} Reyn. The l/b ratio of the shoes for these tables is 4/3 and the outer diameter of the bearing is 4/3 times the mean diameter. For such a bearing the net area of the segments in square inches, there being six, is one-half the square of the outer diameter in inches. The friction coefficients and power losses are calculated upon the assumption that the correction factor for side leakage is $k = 1.6$. By a study of the formula for λ one may readily make corrections for other conditions than for those assumed.

Appendix A

An effort has been made to standardize the symbols used in the text of this paper so that a given character will have the same meaning throughout for journal bearings and a corresponding meaning for other bearings. The symbols used are in most cases those appearing in Dr. Kingsbury's paper (5). When comparing the present work with original source material care must be used to avoid confusion of symbols.

Nomenclature

a = radius of the journal[3]

A = least angle from line of centers to leading edge of partial bearing, measured in direction of rotation. See exception in study of fitted bearings, Fig. B

b = width of bearing surface; measured along the shaft axis in a journal bearing, or in general at right angles to direction of motion

c = eccentricity factor. When multiplied by the radial clearance η the product $c\eta$ equals the distance from the center of the bearing to the center of the journal

F = the total tangential friction force per unit width of the moving surface, at the surface of the journal, or at mean diameter of a thrust bearing[1]

F_1 = correction factor for friction, with side leakage

h = film thickness at any angular position θ in journal bearings, or any distance x for flat tapered films. For fitted journal bearings $h = \epsilon \sin \theta$. For clearance journal bearings $h = \eta(1 + c \cos \theta)$

h_0 = minimum thickness of the oil film in a bearing. Exception: When the line of centers for a clearance bearing does not intersect the partial bearing surface, h_0 is measured where the bearing surface if extended would cross that line

h_1 = thickness of the oil film at region of maximum pressure

h_A = thickness of oil film at entering edge of the bearing film if less than h_m

h_B = film thickness at trailing edge of partial clearance bearing, when greater or less than h_0

h_m = maximum thickness of the oil film

HP = horsepower friction developed by a journal bearing or a thrust bearing per unit of bearing width

hp = horsepower

l = the length of a bearing face in the direction of motion. In a partial journal bearing, whose angular extent β is expressed in degrees, $l = \pi\beta a/180$. This is useful for determining the length over width ratio l/b used for selecting side-leakage-correction factors

L_1 = correction factor for load-carrying capacity, with side leakage

N = rpm

n = number of pivoted segments in a thrust bearing

O = center of curvature of journal when running on oil film

O' = center of curvature of bearing surface supporting the journal

p = pressure within the oil film where the thickness is h

p_0 = nominal mean pressure acting upon the bearing. In journal bearings this is obtained by dividing the total applied load by the product of the journal diameter and bearing width (axial). This is also expressed by dividing the load $\overline{W}$ (per unit of width) by the journal diameter $2a$. In thrust bearings $p_0 = R/nlb$. See Table 1

P_m or P_{max} = maximum pressure within the oil film of a bearing

R = total load applied by the journal to the bearing, or by a collar to a thrust bearing

R' = total reaction of the bearing against the journal

s = a film-form dimension in Fig. 9

t = time, sec

T = distance apart of plane surfaces. See Appendix C and Figs. 2 to 4

u = velocity of any element of viscous flow, within the film

U = relative velocity of the bearing surfaces. In thrust bearings it is measured on the mean diameter

w = nominal mean unit load a bearing would carry if side leakage could be neglected. See p_0. Their relation is $p_0 = L_1 w$

[3] Exceptions occur in Appendix C on "Viscous Flow" for which the symbols are described by the figures and at end of that section.

$\overline{W}$ = actual load applied by a journal to its bearing, per unit of width (axial) = R/b

Z = character expressing the viscosity coefficient in centipoises. See Appendix C

α = leading angle for partial bearing, measured in direction of motion of journal from leading edge of bearing to line of action of the load

β = angular extent of the bearing surface over which perfect film lubrication is assumed to be maintained. Usually expressed in deg

ϵ = eccentricity of journal axis when running in a fitted bearing

η = radial clearance provided in a clearance bearing. When a = the journal radius, $(a + \eta)$ = radius of curvature of a bearing surface, and $c\eta$ = eccentricity with which the journal axis runs in such a clearance bearing

θ = angle from diameter through line of centers, to point within oil film of a journal bearing where thickness is h. See exception in study of fitted bearing, Fig. B.

θ_1 = angle θ corresponding with location of maximum unit pressure within the oil film

λ = coefficient of friction for the journal surface. In thrust bearings this is the coefficient for the mean diameter of the thrust-collar face

λ' = coefficient of friction for the bearing surface

λ_H = uncorrected coefficient of journal friction in charts of general conditions for journal bearings. $\lambda = \frac{F_1}{L_1}\lambda_H$

μ = coefficient of viscosity. See Appendix C

ϕ = angle relating load line to line of centers of journal and bearing in charts of optimum conditions. See bearing cross-sections, Figs. A, B, and C

ϕ_H = angle relating load line to line of centers of journal and bearing in charts of general conditions. See bearing cross-sections, Figs. C_H, D, E, ACD, and F

Appendix B—Examples

Example 1. Required, a bearing of Class A, of minimum friction, to support a load of 25,000 lb at 1500 rpm, with h_0 = 0.003 in., and with 200 lb per sq in. on the projected area of the journal.

This requires a projected area of 125 sq in. Assume for this example that $b/a = 3$, i.e., that the width of the bearing (b) will be 1.5 times the journal diameter ($2a$). The projected area being $2a^2(b/a) = 125$, it follows that $a = \sqrt{(125/6)} = 4.564$ in. This radius may or may not be sufficient for strength in bending or in torsion, or both; but it will be assumed that it is sufficient.

Referring to Chart A, the least horsepower loss (per unit of width b) when $b/a = 3$, is incurred when $\beta = 153.6$ deg (2.68 radians), for which the value of group $\overline{W}Nh_0$/HP is 16,900. As the ratio of load to power loss is the same for the whole width as for unit width

$$\text{total power loss (HP} \times b) = \frac{25{,}000 \times 1500 \times 0.003}{16{,}900} = 6.66 \text{ hp}$$

This does not include losses in any clearance spaces outside of the film arc β.

The clearance η (see Chart A-3, curve 7) for $\beta = 153.6$ deg is found from the height to the curve, which is 1.72. This is the value of group 7 in Table A-3. Hence $\eta = 1.72\, h_0 = 0.00516$ in. The bore diameter of the bearing is therefore larger than the 9.128-in. diameter of the journal by $2 \times 0.00516 = 0.01032$ in., which makes it 9.138 The clearance ratio η/a is therefore 0.00113.

In order to determine the required oil-viscosity coefficient it is necessary to determine the correction factors. This may be done as follows:

The arc length of the bearing surface, $l = 2.68 \times 4.564 = 12.23$ in.

$$\text{The width } b = 3.00 \times 4.564 = 13.69 \text{ in.}$$

The ratio l/b is therefore 0.893. By reference to Chart KX, several correction factors which may now be determined are: $L_1 = 0.487$; $F_1 = 0.916$; and $L_1/F_1 = 0.532$. The required mean viscosity in the oil film may now be determined from group 10, Table A-4 and curve 10, Chart A-4. For the infinitely wide bearing and $\beta = 153.6$ deg, the height of curve 10 measures 0.117. To this is to be applied the correction factor $L_1 = 0.487$. Hence group 10 = 0.117 L_1. The actual mean load per unit of bearing width is

$$25{,}000/13.69 = 1826 \text{ lb} = \overline{W}$$

Substituting this and other known values in group 10 and solving for μ, we obtain

$$\mu = \frac{1826 \times (0.003)^2}{0.117 \times 0.487 \times 1500 \times (4.564)^3}$$

$$= 2.022 \times 10^{-6} \text{ in.-lb-sec (Reyn)}$$

With the conversion factor 69,000, this viscosity coefficient is 0.1395 poise, which is that of a light machinery oil at about 130 to 140 F. For determination of corresponding commercial viscosities see the viscosity-temperature-conversion charts in the author's recent paper (20).

The coefficient of friction may be found from group 12, Table A-4 and Chart A-4, curve 12.

$$\lambda = \frac{F_1}{L_1} \times 2.00 \times \frac{h_0}{a} = \frac{2.00 \times 0.003}{0.532 \times 4.564} = 0.00247$$

Inspection of Chart A-1 indicates that by increasing the ratio b/a, other things being equal, the relative power loss will be reduced; this is shown more specifically in Example 2. The extent to which b/a may be increased must be limited eventually by considerations of space, cost, and the strength or elasticity of the journal.

In the present example, the angle β might be increased, following curve for $b/a = 3$, on Chart A-1, securing, thereby, a somewhat greater bearing area; but this would involve a relatively greater friction loss, as is obvious on inspection.

Other data for this bearing can be found from the charts and tables, e.g., the leading angle α from group 3 and curve 3.

Example 2. In Example 1, suppose that b/a is taken as 4 instead of 3, the other data remaining unchanged. We then have, referring to the same tables and charts

$$a = \sqrt{(125/8)} = 3.953 \text{ in.}$$

$\beta = 180.8$ deg = 3.155 radians, for the least power loss.

$$\text{Total power loss} = \frac{25{,}000 \times 1500 \times 0.003}{20{,}700} = 5.435 \text{ hp.}$$

Clearance $\eta = 1.622\, h_0 = 0.004866$ in.
$= 0.00123$ in. per in. of diam.

Length $l = 3.155 \times 3.953 = 12.47$ in.
Width $b = 4 \times 3.953 = 15.81$ in.
Ratio $l/b = 0.789$. This determines the correction factors:

$$L_1 = 0.533,\ F_1 = 0.924, \text{ and } L_1/F_1 = 0.577$$

$$25{,}000/15.8 = 1581 \text{ lb} = \overline{W}$$

$$\mu = \frac{1581 \times 0.003^2}{0.1517 \times 0.533 \times 1500 \times (3.953)^3} = 1.90 \times 10^{-6} \text{ in.-lb-sec (Reyn)}$$

This corresponds to 0.131 poise.
The coefficient of friction is found to be

$$\lambda = \frac{F_1}{L_1} \times 1.762 \times \frac{h_0}{a} = \frac{1.762 \times 0.003}{0.577 \times 3.953} = 0.002318$$

These optimum charts and tables may also be used for determining the proper radial clearance or the oil viscosity for a known set of other conditions.

Example 3. What radial clearance η is best for following conditions? Total load R = 25,000 lb, N = 1500 rpm, β = 120 deg, p_0 = 125 lb per sq in., b/a = 4, oil viscosity = μ = 3.4×10^{-6} in.-lb-sec (Reyn). Bearing is centrally loaded which means Class C. Proceed as follows:

From above data it is found that $2a$ = 10 in., b = 20 in., $\overline{W}$ = 1250.

$$l = 10.47 \text{ in.} \qquad l/b = 0.524$$

Correction factors from Chart KX will then be L_1 = 0.672, F_1 = 0.947, L_1/F_1 = 0.710.

Then from group 10 in Table C-4 and by introducing correction-factor L_1, we obtain

$$\frac{\overline{W}h_0^2}{\mu N a^2} = 0.0671\, L_1 = 0.0451$$

This may be solved for h_0, the only unknown. (Hence h_0 = 0.0048.) Thereafter the desired value of η may be found from group 7, Table C-3. It will be $1.962\, h_0 = 0.0094$. The clearance ratio will therefore be η/a = 0.00188. The friction coefficient λ is to be found from group 12, Table C-4, which group equals $2.902\, F_1/L_1$. Hence

$$\lambda = 2.902/0.710 \times 0.0048/5 = 0.00392$$

The procedure for examples under Class-B journals would be similar to that already described, but having reference to the approximate Class-B tables and charts, corresponding to those referred to in Examples 1, 2, and 3.

As explained under "Deviations from Optimum," in Appendix D, it is practicable to explore other than optimum conditions by the use of the charts of general conditions which follow those on optimum conditions. To these, with fair approximation, may be applied the side-leakage-correction factors given on Chart KX. Kingsbury's optimum points have been marked on the charts of general conditions when the bearing class is the same or the chart suitable for showing both conditions. This is practicable for the Chart C-5 of centrally loaded bearings and of Chart ACD-120 which gives loading characteristics of a wide variety of 120-deg bearings.

Example 4. Find the running positions of the journal in the following centrally loaded 120-deg bearing using two different oils whose viscosity coefficients are 3.4×10^{-6} Reyn and 10.2×10^{-6} Reyn, respectively. The total load R = 10,000 lb, the actual nominal pressure p_0 = 125 lb per sq in., the journal diameter = 8 in., the bearing width = 10 in.; the clearance ratio η/a = 0.001, and the speed N = 400 rpm.

The first step in solving this problem is to set down the known values of the symbols as far as practicable. The scale at the left of Chart C-5 represents a symbol group for which all values are known except w. This w is the ideal nominal pressure the bearing would sustain if side leakage could be neglected. It is reduced to the actual nominal pressure p_0 by side leakage. Hence $p_0 = L_1 w$ and the correction factor L_1 must be found from Chart KX. From the problem data the l/b ratio is found to be $8\pi \times 120/10 \times 360 = 0.839$.

This yields the following correction factors: L_1 = 0.51, L_1/F_1 = 0.555, F_1 = 0.92. Hence,

$$\frac{w}{\mu N}\left(\frac{\eta}{a}\right)^2 = \frac{245 \times 1 \times 10^{-6}}{\mu \times 400} = \frac{0.613 \times 10^{-6}}{\mu}$$

First, substituting $\mu = 3.4 \times 10^{-6}$, the group is found to equal 0.180. With this we may enter the Chart C-5 at left. This reaches the loading curve for β = 120 deg, where c = 0.581. Reading down to friction, coefficient curve for β = 120 deg, we find that $\lambda_H \frac{a}{\eta}$ = 1.29. Then reading upward on c = 0.581 to the ϕ_H-curve for β = 120 deg, we find that ϕ_H = 33.2 deg. The actual friction coefficient will be $\lambda = \frac{F_1}{L_1}\lambda_H$. Hence, $\lambda = 1.29 \times 10^{-3}/0.555 = 0.00233$. The journal running position will be on the line ϕ_H = 33.2 deg. Its distance from the center of the bearing will be $c\eta = 0.581 \times 0.001 \times 8/2 = 0.002324$. The minimum film thickness will be $h_0 = (1 - c)\eta = 0.001676$.

Dr. Kingsbury's optimum eccentricity factor is seen from the C, F, O, dotted loading curve $\frac{w}{\mu N}\left(\frac{\eta}{a}\right)^2$ on Chart C-5 to be 0.49 for a 120-deg bearing. For this, h_0 = 0.00204. Hence, for $\mu = 3.4 \times 10^{-6}$, our assumed bearing is not quite as safe as the optimum would be.

Solving this same problem with $\mu = 10.2 \times 10^{-6}$ leads to the following results, the group value for entering the chart being 0.060. Proceeding as before we find c = 0.28, ϕ_H = 46 deg, $\lambda_H(a/\eta)$ = 2.54, λ = 0.00458, and h_0 = 0.00288. Hence, with this heavier oil the friction is greater, as would be expected. The minimum film thickness is also greater than for the lighter oil. An intermediate oil would cause the journal to run with optimum eccentricity, but as the clearance is fixed it is necessary to decide which is the more important, a least film thickness of h_0 = 0.00168, or h_0 = 0.00204 or h_0 = 0.00288. The degree of perfection of the shaft alignment may be the deciding factor. Heavier oil is better when conditions of alignment are not ideal. See film thicknesses noted in loading Table 1 for Kingsbury thrust bearings.

Example 5. In a 120-deg bearing, would negative pressures in the oil film be expected with a 6-in.-diam journal 10 in. wide, carrying 600 lb at 1000 rpm with a clearance ratio of η/a = 0.002, and an oil viscosity of 4.0×10^{-6} Reyn, (*a*) when α/β = 0.44? (*b*) when α/β = 0.52? (*c*) when α/β = 0.62?

First find correction factor L_1 from Chart KX. It will be the one corresponding to $l/b = (6\pi \times 120)/(10 \times 360) = 0.628$. Therefore, L_1 = 0.613 and $w = 600/(6 \times 10 \times 0.613) = 16.30$. Then since $\mu = 4.0 \times 10^{-6}$ the group $\frac{w}{\mu N}\left(\frac{\eta}{a}\right)^2 = \frac{16.3 \times 4 \times 10^{-6}}{4 \times 10^{-6} \times 1000} = 0.0163$. Referring to Chart ACD-120 the answer to (*a*) is that the bearing is close to the region there indicated for negative pressure at the trailing edge. The answer to (*b*) is that the bearing is not likely to have negative pressure. The answer to (*c*) is that negative pressure is likely at the leading edge.[4]

Abnormal conditions were assumed for this bearing in order

[4] An investigation by S. J. Needs shows that negative pressure is to be expected in a finite bearing for conditions where the infinite-width bearing would show all positive pressures. Hence, when the data of a problem show that the running position approaches the upper-limit line of Chart ACD-120, some negative pressure is probable. Such a condition would place the bearing in some other class for which α = 60 deg and $\beta - \alpha < 60$ deg.

to bring out the negative-pressure feature. Note the locations of the optimum points for the Class-*A* bearings near the center of the Chart *ACD*-120. These points are marked *AFO* and *ALO*. For a Class-*C* bearing the optimum points are marked *CFO* and *CLO*. By comparing these points with those located by the problem data some idea of the divergences may be gained.

Example 6. Find the minimum film thickness in a 100-deg fitted bearing running under the following conditions: $N = 300$, $\mu = 3 \times 10^{-6}$ Reyn, total load = 3000 lb, journal diameter = 5 in., and its width = 6 in. The procedure is as follows:

$$l/b = (\pi 5 \times 100)/(6 \times 360) = 0.727$$

Therefore, $L_1 = 0.565$ and $w = 3000/(30 \times 0.565) = 177$. Then $\frac{w}{\mu N} \times (10^{-6}) = \frac{177 \times 10^{-6}}{3 \times 10^{-6} \times 300} = 0.197$, the value with which Chart *E*-1 is entered at the left. Reading across to lower diagonal for $\beta = 100$ deg and then down to ϵ/a the latter is found to be 0.0008. As $h = \epsilon \sin \theta$ (see symbols in Appendix *A*) it is necessary that θ be known for the h_0 location, which is at the trailing end of the bearing. This value of θ is seen from Fig. *E* of Chart *E*-1 to be equal to 270 deg $+ \beta -$ $(\phi_H + \alpha)$. For this problem it is found to be 157.6 deg. Hence $h_0 = 0.0008 \times 2.5 \times 0.381 = 0.00076$.

Other data can of course be obtained. $\lambda_H = 0.00082$. Applying the correction factor $L_1/F_1 = 0.61$ we have $\lambda = \frac{\lambda_H}{L_1/F_1} = \frac{0.00082}{0.61} = 0.00134$ which is the actual coefficient of friction.

(NOTE: This problem could have been solved by means of optimum charts of class-*B* journals, because for fitted bearings all conditions are optimum for a given series. Kingsbury's Class *B* differs a little from the author's Class *E*.)

Example 7. What total friction-power loss and what friction coefficient are to be expected in a full bearing operating under conditions where $2a = 4$ in., $b = 8$ in., $N = 3600$, $\mu = 2 \times 10^{-6}$, total journal load = 3600 lb, and $\eta/a = 0.003$. Proceed as follows: Assume $\beta = 180$ deg for purpose of finding L_1. Then $l/b = (4\pi \times 180)/(8 \times 360) = 0.79$ and $L_1 = 0.53$ and $F_1 = 0.925$ and $L_1/F_1 = 0.575$. $w = 3600/(32 \times 0.53) = 212$ lb per sq in. because $w = p_0/L_1$. Now,

$$\frac{w}{\mu N}\left(\frac{\eta}{a}\right)^2 = \frac{212 \times 9 \times 10^{-6}}{2 \times 3600 \times 10^{-6}} = 0.265$$

Entering Chart *F*-1 at the left with this value and reading across to the load-capacity curve we find the value of $c = 0.5$. Holding this value of c for the other curves, the right-hand scale readings therefore lead to $\lambda_H \frac{a}{\eta} = 1.88$. The total power loss = $3.2 \times \frac{\mu b}{2}\left(\frac{a}{\eta}\right) a^2 N^2 10^{-6} F_1$ hp. The actual coefficient of friction λ may be found from $\lambda = \frac{\lambda_H}{L_1/F_1} = \frac{\lambda_H}{0.575}$. Solving, these give the desired answers to the problem. The total power loss will be 4.1 hp and the friction coefficient $\lambda = 0.0098$. Checking back from λ_H to the total power loss, it is found that the latter is the same if the correction factor is properly introduced as in $\lambda = \frac{F_1}{L_1}\lambda_H$.

Appendix C—Viscous Flow

FUNDAMENTALS

Laminar or viscous flow in a film is that kind in which particles of liquid or other matter move as if they were parts of a laminar structure. Hersey (19) has illustrated it by means of a thick book, with a stiff binding, one of the covers of which is shifted relative to the other as from position *A* to position *B* in Fig. 2. All the leaves will be shifted a little, if held to the binding. The sum of their relative shifting will, if integrated, equal the relative shift of the covers. This represents the viscous flow of oil between parallel plates when one plate moves relative to and parallel with the other in the absence of pressure.

If the book binding be elastic as in Fig. 3, and a pressure P be applied to the leaf edges as in *D* while the covers are held, the leaves will shift to the right from position *C* to position *D*, the greatest shift being at center in *D*. This represents the viscous flow of oil between two fixed parallel plates when subjected to a greater pressure at one end than at the other. In the case of the books, the extent of movement is limited. In the case of a liquid, continuous laminar motion is possible.

The force that opposes the viscous flow of a liquid is called the resistance to distortion. The velocity with which the upper surface of an oil film or the upper cover in *B* of Fig. 2, moves relative to the lower, when compared with the film thickness or the book-thickness T, establishes what is called the rate of distortion. The resistance which such a medium offers to such a rate of change of shape or such a rate of distortion is an indication of its viscosity. The ratio of the unit resistance to the rate of distortion it produces is a measure of the viscosity of the liquid and is called the coefficient of viscosity. The viscosity coefficient enables us to connect the internal forces in the liquid with the external forces that cause it to become distorted. The force acting on a unit area of the upper face of the film, as upon the book cover in Fig. 2, is designated as f. This will cause the upper face to move with a velocity U with respect to the stationary one. The rate of distortion produced will be U/T. The coefficient of viscosity μ equals the ratio of the force f to the rate of distortion U/T which it produces. Thus, we have our first relation of the force, distance, and velocity concerned with viscous flow

$$\mu = \frac{f}{U/T} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

from which

$$f = \mu\, U/T \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

We have here the fundamental formula for the study of the viscous flow of liquids. This equation is so important that it can be used very directly to find the approximate frictional resistance to the rotation of a journal in a bearing, if the surface speed, the film thickness, and the viscosity are known.

FRICTION AND VISCOSITY

This relation was expressed by Petroff (1) in 1883, when he assumed that a shaft ran concentric within its sleeve bearing. Fig. 4 may be thought of as illustrative of the viscous flow of oil in absence of pressure, between fixed and moving parallel plates, their distance apart being T. This is still a useful assumption for high-speed bearings, moderately loaded, whose journals run nearly concentric within their bearings, and the latter use practically complete sleeves.

The tangential friction force at the journal surface, resisting rotation, is

$$2\pi abf = F \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

The surface speed of the journal is

$$U = 2\pi aN/60 \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

In this formula N is revolutions per minute. Combining Equations [3] and [4] with Equation [2] a useful formula [5] is found.

This is independent of unit system employed, so long as but one is used and $N/60$ represents revolutions per second. It is similar to the one proposed by Petroff. The viscosity coefficient μ must be expressed in the system of units employed for rest of equation.

$$F = \frac{\mu\pi^2 a^2 bN}{15T} \quad [5]$$

This friction force F can be again combined with speed and expressed as power loss.

Poises, Centipoises, and Reyns

The relation between the viscosity coefficient in English units

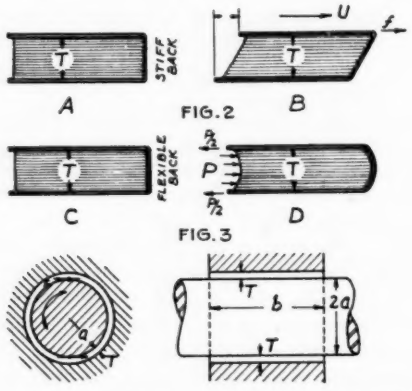


FIG. 2

FIG. 3

FIG. 4, A JOURNAL CENTRAL WITHIN ITS BEARING

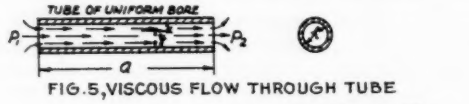


FIG. 5, VISCOUS FLOW THROUGH TUBE

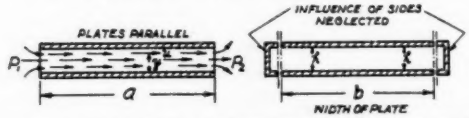


FIG. 6, VISCOUS FLOW BETWEEN PARALLEL PLATES

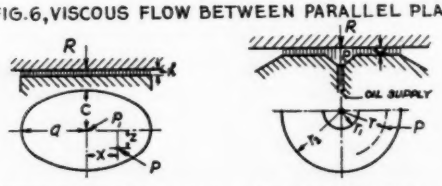


FIG. 7, APPROACHING FACES WITH OIL FILM BETWEEN THEM

FIG. 8, SEPARATION OF FACES BY PRESSURE OIL

(pound-seconds per square inch) Reyn, and metric units (dyne-seconds per square centimeter), poise, may be expressed as follows when μ_E is the former and μ_M the latter.

$$\mu_M/\mu_E = 69{,}000 \quad [6]$$

There appears in literature another unit for viscosity coefficient called the centipoise, designated as Z and having a magnitude of one hundredth part of a poise. The relation between it and the poise is expressed in Equation [7] and that between it and the Reyn in Equation [8].

$$Z/\mu_M = 100 \quad [7]$$

$$Z/\mu_E = 6{,}900{,}000 \quad [8]$$

Viscous Flow Through Tubes

By applying the laws of viscous flow to the flow through tubes, and assuming that there is no molecular slippage of liquid at the tube surface, the law of volume of flow per second is

$$\frac{V_r}{t} = \frac{\pi(P_1 - P_2)h^4}{128\mu a} \quad [9]$$

Referring to Fig. 5, the linear velocity of flow u at any distance y from the side of the tube is

$$u = \frac{(P_1 - P_2)(hy - y^2)}{4\mu a} \quad [10]$$

The variation in this velocity is seen to be parabolic and the velocity of flow, therefore, greatest at the tube axis.

Viscous Flow Between Parallel Plates

Viscous flow between plates that are parallel, Fig. 6, is simply expressed by Equations [11] and [12]. In this case, also, the velocity distribution, transversely, is parabolic. There is assumed to be no slippage at the plate surface.

Volume rate of flow is

$$\frac{V_p}{t} = \frac{bh^3(P_1 - P_2)}{12\mu a} \quad [11]$$

$$u = \frac{(P_1 - P_2)(hy - y^2)}{2\mu a} \quad [12]$$

In the Equations [9], [10], [11], and [12], one system of units only is to be applied and the viscosity coefficient must be in that system also. P_1 and P_2 are the pressures acting upon the liquid at two ends of the tube or plates. The pressure drop from P_1 to P_2 is assumed to be uniform with distance throughout the length a.

Viscous Flow From Approaching Faces

Osborne Reynolds in 1886 (2) investigated the time rate at which such surfaces, if parallel, will approach when the space between them is filled with a viscous liquid, and a constant normal force R is maintained. See Fig. 7. With elliptical plates, no central cavity, no oil supply, and the plate moving downward, the formulas which are applicable are

$$t = \frac{3\mu\pi a^2 c^2}{2R(a^2 + c^2)}\left(\frac{1}{h_1^2} - \frac{1}{h_2^2}\right) \quad [13]$$

$$P_1 = \frac{2R}{\pi ac} \quad [14]$$

$$P = -\frac{2R}{\pi ac}\left(\frac{x^2}{a^2} + \frac{z^2}{c^2} - 1\right) \quad [15]$$

where P_1 = oil pressure at the center, and P = oil pressure at any point with coordinates x and z.

Separation of Parallel Circular Plates by High-Pressure Oil

This condition of viscous flow, illustrated by Fig. 8, may be accomplished by forcing oil into the small recess at the center with sufficient pressure to cause separation and outward radial flow.

Case 1. Circular Plates, Load Carried by Continuous Flow of Pressure Oil. Pressure at Outer Edge of Plate Assumed to be Zero. The formulas applicable to this case are

$$G = \frac{0.0867\, h^3 R}{\mu(r_2^2 - r_1^2)} \quad [19]$$

$$P_1 = \frac{2R \log_e(r_2/r_1)}{\pi(r_2^2 - r_1^2)} \quad [20]$$

$$P = \frac{2R \log_e(r_2/r)}{\pi(r_2^2 - r_1^2)} \quad [21]$$

$$P_1 = \frac{7.35\mu G \log_e(r_2/r_1)}{h^3} \quad [22]$$

$$\frac{P_1}{P_0} = \frac{2r_2^2 \log_e(r_2/r_1)}{r_2^2 - r_1^2} \quad [23]$$

where P_1 = oil pressure supplied to central cavity, P = oil pressure at any radius r, P_0 = specific pressure on the plate produced by the load = $R/\pi r_2^2$. In Equations [19] and [22], G represents U. S. gallons per minute, provided all other units are inches, pounds, and seconds, in which case μ must be introduced as Reyns.

Case 2. Circular Plates, Oil Supply Stopped, Plates Moving Together. The formulas applicable to this case are

$$t = \frac{3\mu\pi(r_2^4 - r_1^4)}{4R}\left(\frac{1}{h_1^2} - \frac{1}{h_2^2}\right) \quad [24]$$

$$P_1 = \frac{2R}{\pi(r_2^2 - r_1^2)} \quad [25]$$

$$P = \frac{2R(r_2^2 - r^2)}{\pi(r_2^4 - r_1^4)} \quad [26]$$

where P_1 = oil pressure in the central cavity, and P = oil pressure at any radius r.

The symbols used in Equations [1] to [26] are as follows, and must not be confused with the symbols that appear in Appendix A

- R = total load on plate
- G = flow of pressure oil, U. S. gpm where other units in the equations are inches, pounds, and seconds, and μ is introduced as Reyns
- μ = absolute viscosity of oil
- h = thickness of oil film
- t = time for moving plate from height h_2 to height h_1, sec

Appendix D—Perfect Lubrication

Tapered Oil Film

The tapered oil film is the secret of the success of the sliding-surface bearing, by virtue of which it attains its great capacity, its very low friction coefficient, and its remarkable durability. The existence of an oil film completely separating the bearing surface from its journal was discovered in 1883 by Beauchamp Tower (3). When this fact came to the attention of Osborne Reynolds (2) he was so intrigued with the idea that he developed the hydrodynamical theory which explains it. He showed that the film of oil must have the shape of a wedge in order to keep the surfaces apart, whether the bearing be flat or cylindrical. In 1896 Albert Kingsbury (4), by actual measurement, discovered that the lubricating film was wedge shaped below the air-borne journal of his test bearing. Further evidence of the necessity for a wedge-shaped film has been since deduced from other analyses and tests, as well as from the observation of wear in bearings not adequately lubricated. A good review of Reynolds' work, including a few needed corrections in the formulas, appears in Kingsbury's paper (4).

Side Leakage. Reynolds, in his bearing analysis, neglected the influence of side leakage upon the pressures and friction that are developed within the oil film. Therefore, he determined the maximum pressures that a bearing could develop with a given film under ideal conditions. His work is most valuable as an upper limit or standard with which actual conditions may be compared. It is generally recognized however that, if all other conditions are held the same, the narrower the bearing surface (perpendicular to the direction of motion) the less will be the pressures developed within a film of given form and thickness. This is true because a larger percentage of the oil drawn in at the front edge, will leak away at the sides of the narrower bearing, before reaching the far end of the film. See fuller discussion following under heading "Side Leakage Reduces Capacity."

Film Thickness

The chief importance in the consideration of film thickness and shape lies in the fact that upon them largely depends the factor of safety of a bearing. If the relative shapes of the two co-operating bearing faces are such as to permit safe starting and stopping under load, then the safe running of the bearing may well depend upon the minimum thickness of the oil film and its shape. This point was discussed by Dr. Kingsbury (5) and led him to propose the minimum film thickness as an important criterion of bearing safety. The reason for this seems obvious when one remembers that a bearing, first, has to maintain its film under changes of load and temperature, second, has to support deflected shafts or misaligned ones and, third, has to pass particles of foreign matter through its film to a reasonable extent, without serious injury to the bearing surfaces.

Film thicknesses are difficult to measure and few reliable tests of heavily loaded bearings are available. Therefore, reliance is usually placed upon calculations, when the bearings so investigated have proved themselves in service. Calculated minimum film thicknesses together with capacities for flat surfaces, based on Kingsbury's thrust-bearing practice, are given in Table 8 in the body of this paper.

Viscosity Change Within Oil Film. The friction generated in a bearing, the surfaces of which are completely separated by an oil film, is produced within the oil itself. This friction is dependent upon the viscosity of the oil, upon the film thickness, upon the speed, and upon the extent and shape of the bearing surfaces covered by the film. See Appendix C. As the oil passes through a bearing film, the heat generated must be disposed of in one way or another. Some passes into the bearing faces while the rest remains in the oil itself and passes out with the oil discharged from the sides and the far end of the bearing film. Oil thus discharged is therefore hotter than when it entered. This rise in temperature reduces the oil viscosity, i.e., makes the oil thinner. A fair idea of the maximum possible rise of temperature within a perfect film may be had by simple calculation involving the mean film thickness, the speed, and the specific heat of oil. It is customary to assume the mean viscosity within the film to be that of the whole film, when making bearing calculations.

The Flat-Wedge Film

This film is shown in three views in Fig. 9, giving the plan view, longitudinal section A-A, and a front view looking into the film in the direction of the entering oil. Dotted lines in the plan view show the oil flow and how side leakage takes place.

A study of this film by means of Reynolds' equations in Kingsbury (5), assumes the oil flow to be all parallel to A-A, in the plan view. This is the direction of motion. This analysis discloses a number of important characteristics of an oil film. The lower or stationary bearing surface of 9-*a* may be assumed to be placeable at will in any desired relation to the upper or moving face of the bearing. The proportions of s to l may therefore be varied at will, as may also be the thicknesses h_0 and h_m. In order to study these variations quantitatively, the load W, per unit of width b, may be assumed fixed. The length l of

the bearing face may also be assumed fixed, and likewise the speed U and the mean viscosity μ. It is assumed that the position of support of the inclined face, if pivoted, may be altered at will to satisfy these imposed conditions. Three different sets of conditions, or cases, will now present themselves, and the results are given below. These are limited here to the conditions in which the influence of side leakage upon the pressure is negligible, as with infinitely wide bearings.

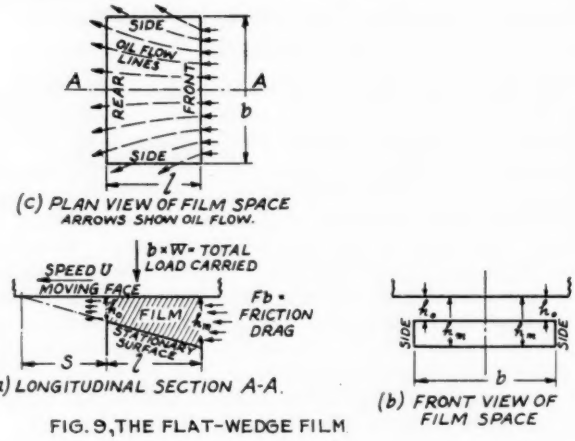


FIG. 9, THE FLAT-WEDGE FILM

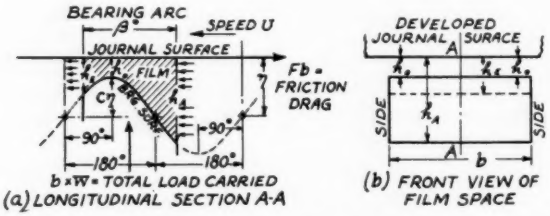


FIG. 10, CURVED-WEDGE FILM OF CLEARANCE BEARING

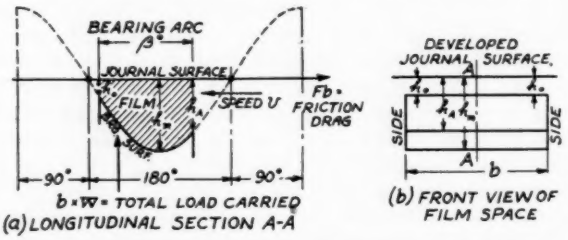


FIG. 11, CURVED-WEDGE FILM OF FITTED BEARING

Case I. ***Minimum Friction Condition.*** The frictional resistance to motion will be a minimum when $s/l = 0.484$. The film proportions in this case will be $h_m/h_0 = (s + l)/s = 3.065$.

Case II. ***Optimum Load Condition.*** The film thickness at h_0 will be greatest when $s/l = 0.841$. This condition was determined by Reynolds. The film proportions in this case will be $h_m/h_0 = (s + l)/s = 2.190$.

Case III. ***Optimum Friction Condition.*** The friction, per unit of least film thickness h_0, will be at minimum when $s/l = 0.652$. The film proportions for this case will be $h_m/h_0 = (s + l)/s = 2.533$. Kingsbury (5) has selected this as the optimum friction condition for flat-surfaced bearings.

The curves for which the preceding-case data are critical points, and upon which the following discussion is based, are given in Fig. 12. They are plotted from Reynolds' equations as given by Kingsbury (5). The case conditions are marked on curves having the same numbers. The symbol groups and the film picture in this chart carries the slope-angle C which does not appear in Fig. 9.

Examination of the curves of Fig. 12 show their critical points to be grouped in such a way that the range of variations of least film thickness and of friction does not exceed 4 per cent. Although any one of the case criteria would be a reasonable choice, Case III is the best from the combined viewpoint of bearing efficiency and factor of safety.

Deviations From Optimum. Side-Leakage Effect Negligible. (Fig. 12.) If a bearing be constructed with the ratio $M = s/l$ appreciably less than 0.484(see curve No. 1) the friction will be considerably increased. The minimum film thickness h_0 would be reduced at the same time. Therefore, a change of proportion in this direction leads more quickly toward higher friction and toward metallic contact. In this direction also the angularity of the bearing faces increases. From a study of curve No. 1, it appears safer to deviate from the minimum friction condition ($M = 0.484$) toward the right, in the direction of nearer parallelism of the bearing surfaces, which follows an increase of s/l.

Thus we are led toward Case III and Case II. In this direction beyond $M = 0.841$ the least film thickness h_0 decreases

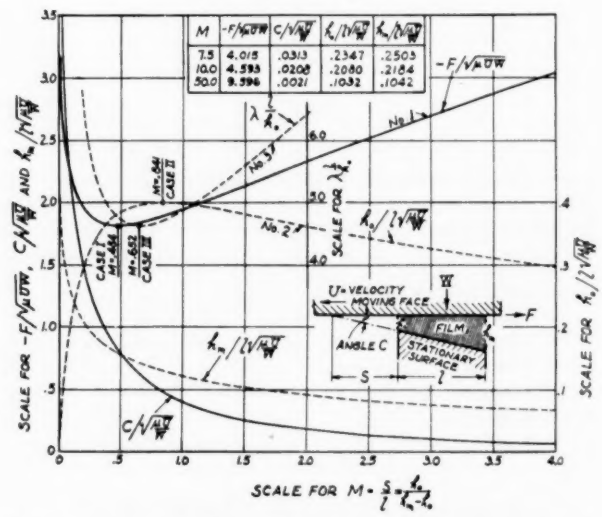


FIG. 12 EFFECT OF DEVIATIONS FROM OPTIMUM CONDITIONS FOR A BEARING WITH FLAT-WEDGE FILMS

slowly, as is seen from curve No. 2, and the friction increases but slowly, as is seen from curve No. 1.

A further study of the curves of Fig. 12, if they were extended beyond the chart limits, would give an idea of how high pure film-friction coefficients could be expected to rise in bearings that have nearly parallel faces, far removed from the best. The values tabulated at the top of the chart show that when the least film thickness h_0 reaches one-quarter of the best value assumed in Case II, the friction will be more than five times the least obtainable. In spite of this high friction, the bearing surfaces might run with perfect lubrication and without wear, if great care is used in producing and maintaining the surfaces.

It seems reasonable to expect that correction factors for side leakage set up for optimum conditions may be used with a fair degree of accuracy for proportions differing materially therefrom, if the deviation is in the direction of nearer parallelism of bearing surfaces.

THE CURVED-WEDGE FILM

Oil-film shapes in journal bearings, when geometrically accurate and simple, are of two forms. Both may be expressed

by a sine-curve equation and are, therefore, easily shown in developed section when the journal surface is represented by a straight line. For convenience of comparison, the angular extent of the bearing $\beta°$ in Figs. 10 and 11 is drawn equal to the length l of the flat wedge film in Fig. 9.

Clearance-Bearing Films. For bearings with running clearance the sine-curve axis is spaced from the sine-curve median line by an amount equal to the radial clearance η as in Fig. 10-*a*. Half of the total height of the sine curve, the half equaling $c\eta$, when deducted from the radial clearance η, equals the minimum film thickness, $h_0 = \eta(1 - c)$. As the minimum thickness h_0 is apt to lie within the film the thickness at the far end has been lettered h_B. The thickness h_A may obviously be less than the maximum film thickness for some film proportions.

Fitted-Bearing Films. When the running clearance is zero the bearing is said to be fitted to the journal. The film-formation limitations in this case are illustrated by Fig. 11. The fitted bearing cannot have an arc β much greater than 90 deg, and an angle of 180 deg is geometrically impossible. In a fitted bearing the maximum film thickness h_m is apt to be within the film limits and not at the entering end. The minimum film thickness h_0 will be at the far end of the film. For this reason the thickness at the beginning of the film has been lettered h_A.

Optimum Conditions for Journal Bearings. For these conditions reference must be made to tables and curves mentioned in "The Loading and Friction of Journal Bearings." See Classes *A*, *B*, and *C*, taken from Kingsbury's paper (5). As curved-wedge films cannot be so well and simply examined mathematically as was done for the flat-wedge film, recourse was had by Kingsbury to electrical methods described in Section V of (6), for checking his analytical study of them. In general the meaning of the results obtained by Kingsbury will be understood by keeping in mind the explanations of optimum conditions previously offered for the flat-wedge film.

General and Special Conditions. For a study of general bearing conditions one is referred to the charts for Classes *C*, *D*, *E*, and *F*, mentioned under the heading "The Loading and Friction of Journal Bearings," adapted from Howarth (7) and (8). Also upon some of these charts there are marked such optimum conditions as determined by Kingsbury (5), for the particular class of bearings studied by Howarth. This discussion is continued and amplified with examples in Appendix B that illustrate the use of the loading and friction charts.

Deviations From Optimum. In view of the many possible series of journal-bearing films, the study of deviations from optimum is here limited to a centrally loaded bearing with 120 deg clearance. In order to study the curved-wedge film of this bearing as fully as done for the flat-wedge film in Fig. 12, the film in the 120 deg bearing was assumed to be a curved wedge extending, as in Fig. 13, from h_{max} at the entering edge to h_0 where the film thickness is least, for which the wedge length is l_0. This length l_0 will usually be a little less than the full arc 120 deg, depending upon operating conditions.

Kingsbury's study of the 120-deg centrally loaded bearing, (see the charts and tables of Class *C*), gives values of dimensionless groups for optimum conditions. On Fig. 13 certain of these groups are used as ordinates, and are also marked along the corresponding curves. The base scale shows the curved-film wedge proportion M for a considerable range, including the optima for minimum friction and for maximum load, as well as the minimum frictional drag for carrying a given load under specified conditions. Curves No. 1, No. 2, and No. 3 on Fig. 13 correspond with those of the same numbers in Fig. 12. There appear in Fig. 13, however, two additional curves. Curve No. 4 shows the clearance-ratio variation and curve No. 5 shows the eccentricity change. The optimum points were obtained from Kingsbury (5) and the curves were plotted from data taken from Howarth (7).

Let it now be assumed that the 120-deg bearing is running under the optimum friction condition, at which $M = 0.943$. The optimum clearance ratio corresponding with this is assumed to be unity when plotting curve No. 4. If, the clearance ratio for this optimum friction condition is increased 20 per cent, the value of M will change, and be no longer that for this optimum condition. Referring to curve No. 4, the point X will be reached for which $M = 0.7$. Following vertically through this point we find from curve No. 1 that the frictional-drag group has changed but little, from curve No. 2 that the capacity group has reduced but little, and from curve No. 3 that the friction-coefficient group has increased a small amount. If next we assume the clearance ratio for minimum friction to have been decreased 20 per cent, we find from curve No. 4 a point Y for which the value of M is about 1.4. For this condition the fric-

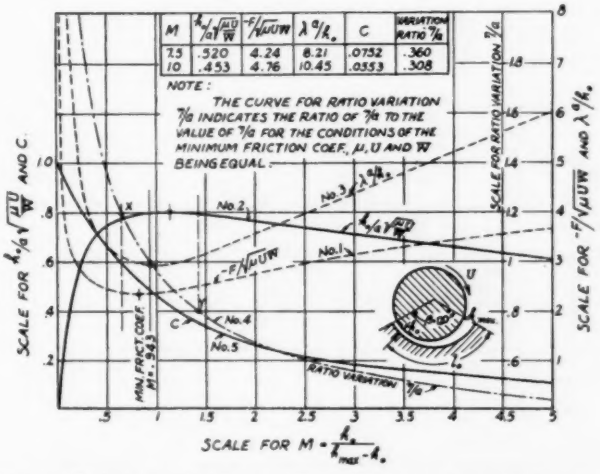


FIG. 13 EFFECT OF DEVIATION FROM OPTIMUM FRICTION-COEFFICIENT CONDITIONS IN A 120-DEG-CLEARANCE BEARING. THIS CHART IS COMPARABLE WITH FIG. 12 FOR FLAT BEARING FACES

tional-drag group as shown by curve No. 1 has increased a little in value, the capacity group as shown by curve No. 2 has decreased but little, and the friction coefficient group shown by curve No. 3 has increased but little. These facts will be found useful when arranging tolerances for the bearing and journal diameters.

In the same way as in Fig. 12, a table was prepared to show increases in friction that would accompany wide deviations from optimum conditions for flat-wedge films. For the same purpose similar figures have been tabulated on Fig. 13 for the curved-wedge film, for values of M beyond the limits of the chart curves. When $M = 10$ the frictional-drag group of curve No. 1 will reach a value of 4.76, which may be compared with its minimum of 2.4. It is not practicable to extend this comparison farther for this group with the data available. For $M = 10$, however, we find that the clearance-ratio reduction from optimum has reached nearly 70 per cent. The friction-coefficient group has increased from the optimum of 2.95 to the larger value of 10.45. The capacity group has decreased from 0.8 to 0.453. It is quite probable that with the value of M equal to 10 or even 20, the bearing would still function properly with pure film lubrication if accurately and carefully made.

From the curves of Fig. 13, another set shown in Fig. 14 has been drawn for more convenient use. The characteristic groups for capacity, frictional drag, and the friction coefficient as well as for the eccentricity are plotted directly over the variation

of the clearance ratio. As before, the clearance-variation figure has been made unity for the optimum condition of minimum-friction coefficient. This chart shows in a more direct way the influence of deviation of the clearance ratio from this optimum. It is clearly seen that small deviations have little influence, as demonstrated already by means of Fig. 13. If, however, the clearance-ratio variation is large, the efficiency of the bearing is sensibly affected, especially if the clearance ratio is reduced. If, for example, the clearance ratio is increased 40 per cent, the group factor for the frictional drag as shown by curve No. 1 increases from 2.4 to 2.5, the group factor for the capacity (as shown by curve 2) decreases from 0.8 to 0.74 and the friction-coefficient factor, as shown by curve No. 3, increases from 2.9 to 3.3. A reduction of the clearance ratio by 40 per cent on the other hand yields more undesirable results. The frictional-drag factor rises from 2.4 to 2.9; the capacity-group factor falls from 0.8 to 0.74; and the friction-coefficient factor increases from 2.9 to 4.

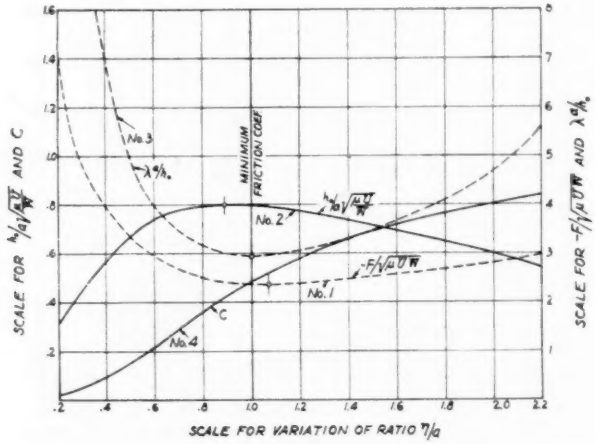


FIG. 14 EFFECT OF DEVIATIONS FROM OPTIMUM FRICTION-COEFFICIENT CONDITIONS IN A 120-DEG-CLEARANCE BEARING

The increase of the friction-coefficient factor is far more noticeable with a reduction of the clearance ratio than with an increase. If, therefore, it is not feasible to design the bearing with the clearance ratio corresponding to the optimum conditions, a larger rather than a smaller clearance ratio should be provided. This conclusion has also been corroborated by practical experience. The relation of curvatures to safety of starting and stopping under heavy loads should not be overlooked, as it leads in the other direction, the choice of a smaller clearance.

SIDE LEAKAGE REDUCES CAPACITY

The influence of side leakage upon the capacity of a perfectly lubricated thrust bearing, and other bearings with wedge-shaped films, is considerable. It has been determined by several investigators, notably by Michell (9), Martin (10), Boswall (11), Kingsbury (6), and Duffing (12). In Fig. 15, the coordinates of which are capacity factor and length/width ratio, the results of several investigators are shown as plotted by Kingsbury who drew this curve to compare them all. The capacity factor shows what percentage of the ideal load can be carried on a film with finite dimensions of length and width. The ideal, with which the bearing of finite width and length is compared, is a bearing having the same finite length (in direction of motion), but which is understood to be a part of one whose width is so great (infinite) that there is no side leakage to reduce the capacity.

The points determined by Michell, by Martin, and by Duffing, apply to a flat-wedge film whose proportions are two to one, i.e., $h_m/h_0 = 2.0$. Kingsbury also found points for such a film. These are all included under the heading (plane surfaces) marked on the curve of Fig. 15. Kingsbury extended the investigation to cover curved-wedge films such as occur in journal bearings of the fitted and the clearance types. By taking a variety of such points for his optimum bearings and comparing their capacities for various values of l/b, with the ideal capacities he found his points to lie fairly well along a line through the points determined only for plane surfaces. By this means Kingsbury found a set of correction factors that could be used with ideal capacity and friction charts and tables for the solving of practical

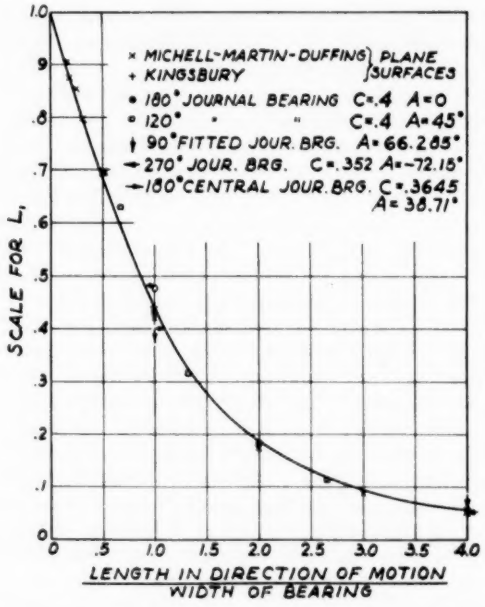


FIG. 15 INFLUENCE OF SIDE LEAKAGE UPON CARRYING CAPACITY OF OIL FILM AS DETERMINED BY ANALYSES OF MICHELL (9), MARTIN (10), DUFFING (12), AND KINGSBURY (6)

bearing problems. See Chart *KX*, and also the examples in Appendix B.

BIBLIOGRAPHY

*1 "New Theory of Friction," by N. P. Petroff, *Engineering Journal* (Russian), St. Petersburg, 1883, pp. 71–140, 228–279, 377–436, 535–564. Abs., *Fortschritte der Physik*, pp. 250–252; German trans., L. Voss, Hamburg, 1887; French trans., *Revue de Mecanique*, vol. 7, 1900, pp. 571–602.

*2 "On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil," by O. Reynolds, Philosophical Transactions of the Royal Society of London, vol. 177, 1886, part 1, pp. 157–234; Papers on mechanical and physical subjects, by O. Reynolds, University Press, Cambridge, London, vol. 2, 1901, pp. 228–310.

3 "First Report on Friction Experiments," by B. Tower, Proceedings, Institution of Mechanical Engineers, 1883, pp. 632–659.

4 "Experiments With an Air Lubricated Journal," by A. Kingsbury, *Journal American Society of Naval Engineers*, 1897, vol. 9, pp. 267–292.

5 "Optimum Conditions in Journal Bearings," by A. Kingsbury, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-7, p. 123.

6 "On Problems in the Theory of Fluid-Film Lubrication, With an Experimental Method of Solution," by A. Kingsbury, Trans. A.S.M.E., vol. 53, 1931, paper APM-53-5, p. 59.

7 "A Graphical Analysis of Journal Bearing Lubrication," by H. A. S. Howarth, Trans. A.S.M.E., vol. 45, 1923, p. 421; vol. 46, 1924, p. 881; vol. 47, 1925, p. 1120.

8 "Journal Running Positions," by H. A. S. Howarth, Trans. A.S.M.E., vol. 51, part 1, 1929, paper APM-51-3, p. 21.

*9 "The Lubrication of Plane Surfaces," by A. G. M. Michell, *Zeitschrift für Mathematik und Physik*, vol. 52, 1905.

10 "The Theory of the Michell Thrust Bearing," by H. M. Martin, *Engineering*, London, Feb. 20, 1920.

11 "The Theory of Film Lubrication" by R. O. Boswall, Longmans, London and New York, 1928, 280 pp.

12 "Die Schmiermittelreibung bei Gleitflächen von endlicher Breite," by G. Duffing, vol. 5, 1930, pp. 839–850; Handbuch der physikalishen u. techn. Mechanik, F. Auerbach and W. Hort, J. A. Barth, Leipzig, vol. 1, 1930, p. 195.

13 "A Graphical Study of Journal Lubrication," by H. A. S. Howarth, Mechanical Engineering, vol. 49, no. 4, 1927, p. 375.

*14 "Zur hydrodynamischen Theorie der Schmiermittelreibung," by A. Sommerfeld, *Zeitschrift für Mathematik und Physik*, 1904, vol. 50, pp. 97–155.

15 "Effects of Side Leakage in 120-Degree Centrally Supported Journal Bearings," by S. J. Needs, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-16, p. 721.

16 "Friction," by T. E. Stanton, Longmans, London, 1923, 183 pp.

17 "Oil Film Pressures in a Complete Bearing," by L. J. Bradford and L. J. Grunder, Pennsylvania State College Bulletin No. 39, 1930.

18 "Pressure Distribution in Oil Films of Journal Bearings," by S. A. McKee and T. R. McKee, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-8, p. 149.

19 "Fundamental Action of Lubricants," by M. D. Hersey, *American Machinist*, June 13 and 20, 1929.

20 "Current Practice in Pressures, Speeds, Clearances, and Lubrication of Oil-Film Bearings," by H. A. S. Howarth, Trans. A.S.M.E., vol. 56, no. 12, 1934, paper MSP-56-2, p. 891.

* The four papers of Petroff (1), Reynolds (2), Sommerfeld (14), and Michell (9), are collected in German translations in "Hydrodynamische Theorie der Schmiermittelreibung." Ostwald's Klassiker No. 218, Leipzig, 1927.

Discussion

Pulsating Air Flow[1]

SANFORD A. MOSS.[2] Professor Bailey is to be congratulated on his distinctly novel and ingenious contribution to the measurement of pulsating flow. Inquiry might be made as to the technique of operating the apparatus. Is it difficult to regulate the valves 6 and 9, shown in Fig. 1, so as to secure constancy of head? Does the rpm of the blower have to be regulated with more than ordinary care in order to preserve constancy once it is established? In order to avoid surface-tension changes, the orifice 8, Fig. 1, was finally made an orifice in a thin plate. Such an orifice has a coefficient of discharge subject to some variation, so that the actual flow may not be at all times proportionate to the square root of the head. It would seem the author's original orifice with rounded approach was more nearly theoretically correct. Could not the orifice be made of stainless steel or bronze, or even of brass artificially seasoned so that it would remain constant with respect to surface tension?

At first thought it would seem that the use of the apparatus is limited to the case where pressure difference, in the air flow being measured, is such as to cause negligible density change. However, it was shown in a paper read at the December, 1927, meeting of The American Society of Mechanical Engineers, that air flow by weight with a differential pressure up to 10 per cent of the initial absolute pressure, varies exactly with the square root of the differential pressure, which would make the instrument usable up to this differential-pressure ratio. It is to be assumed that the instrument could be used with measurements made with an impact tube in the pipe and a static hole in the pipe wall, as well as with the pitot tube with static and impact openings discussed by the author. No doubt the instrument could also be used when air flow is measured by means of a nozzle at the end of a pipe line, with initial pressure measured with a static hole preceding the nozzle, or an impact tube preceding the nozzle, or an impact tube in the nozzle jet.

The author speaks of possible sources of error due to a particular type of connecting pipe used. It would seem that error from this source is always possible, because air in the tubing between the pressure-measuring instrument and the author's apparatus has a period, inertia, and compressibility of its own, which may at times give oscillations other than those of the original jet, or give friction which may dampen some of the original pulsations, as the author mentions. Possibly this effect might be eliminated by filling all of the tubing with water, which is practically incompressible. With this arrangement, the tubes 11 and 12, Fig. 1, would be connected as closely as possible to the pitot tube or other flow-measuring apparatus with as short column of air between as possible. Then there could be tubing of any convenient length between the lower ends of the glass tubes and connections 13, Fig. 1, to the water box, all filled with water. By this means the air column between the original source of pulsation and the water box would be made a minimum, and this presumably would increase the accuracy of the instrument.

One further item, the effect of the inertia of the water flowing through the orifice, may be mentioned. When the pulsation is at a given point in the cycle and the velocity through the orifice is a given amount, and the next phase of the pulsation changes the head on the water orifice, the water itself does not instantly assume the velocity corresponding to the new head but, on account of its mass, must be accelerated, which requires a certain time. For this reason, the pulsation of the water cannot be in phase with the pulsation of the air. Perhaps for low velocities through the water nozzle, and comparatively low frequencies of pulsation, this error is negligible, and perhaps the author has some definite mathematical treatment of it.

P. H. HARDIE.[3] The author, apparently because of the brevity of his paper, has not fully explained many important points regarding the operation of the fluid-velocity meter and pulsating air flow. The writer would like to ask the author to clarify the following points:

(1) The fluid-velocity meter actually consists of three orifices in series, since valves 6 and 9, Fig. 1, are restrictions. They are also subject to pulsating differential pressures. Will the author show mathematically, from the equation of the flow through three resistances in series, how the flow is affected by intermediate pressure pulsations?

(2) Does the water flow change when the area through the gage lines is restricted to such an extent that it effectively eliminates pulsations, the two valves 6 and 9 remaining fixed? If it does, how much change occurs, and do the two water legs become unbalanced?

(3) Was the fluid-velocity meter checked for accuracy by any method other than the cut-out disk? For example, it would be interesting to make a check in a duct having a rotating butterfly valve some distance back from the pitot station, the gas having been previously metered accurately.

(4) How does the author account for such excessive pulsation from the blower on which he made his experiments? Referring to Fig. 5*b*, with the blower alone, V_0/V_{av} is almost 1 at the lowest speed reported, 800 rpm. The values of V_0/V_{av} plot a straight line on semi-log paper. Extrapolation values for lower speeds indicate that at 600 rpm the air reverses and reaches a speed in the reverse direction of 25 per cent of the average forward velocity. This is astounding and should certainly be checked with a vibrograph, phonodyke, or some similar type of recording device which has been successfully used for such records in the past. Has the author made similar measurements on commercial-type fans of larger sizes, and if so, what was the magnitude of pulsation found?

(5) Did the tests on this blower at different speeds indicate that the fan laws apply when the flow pulsated? If they do not, the present practice of checking performance at reduced speeds will have to be discontinued.

In conclusion, it might be well to point out that it is not an established fact that pulsations in the discharge of air blowers assume serious proportions. H. F. Hagen's paper,[4] to which the author makes frequent reference, attributed large errors in pitot-tube velocity measurements to pulsating flow, but it was pointed

[1] Published as paper PTC-56-1, by N. P. Bailey, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Research Engineer, General Electric Company, West Lynn, Mass. Mem. A.S.M.E.

[3] Test Engineer, Research Bureau, Brooklyn Edison Company, Inc., Brooklyn, N. Y. Assoc-Mem. A.S.M.E.

[4] "Pulsation of Air Flow From Fans and Its Effects of Test Procedure," by H. F. Hagen, Trans. A.S.M.E., vol. 55, 1933, paper FSP-55-7.

out in the discussion of Mr. Hagen's paper (fourth paragraph, page 111) that the vibrograph records showed pulsations, the magnitude of which would account for only a very small fraction of the total error. The experiments of Lionel S. Marks, recently reported in A.S.M.E. Transactions for November, 1934, paper PTC-56-2, indicated that the errors attributed to pulsating flow were due to other causes.

LIONEL S. MARKS.[5] The theory of the instrument developed by Mr. Bailey demands that the instantaneous static and impact pressures be exerted at the instrument or that the difference between the two be maintained at the instrument. This does not seem possible of achievement with rapid pulsation, primarily on account of the small size of the static-pressure orifices. The author speaks of increasing the static-orifice area but, even if carried to the limit at which the static pressure will be registered accurately, this would not give an opening adequate for the purpose.

It would seem desirable to verify the indications of this instrument by determining independently the true average air velocity by a nozzle or other means. A possible method of verification would be to measure the same air quantity at two stations, for example, in a duct and, later, in a nozzle of not more than half the duct diameter. With pulsating flow, the volumes determined from the effective velocities at these two stations are different; the volumes determined from the average velocities should be the same.

H. F. HAGEN.[6] The curves of Fig. 5 in the author's paper indicate a possible explanation of the not infrequent reports of tests of fans, particularly propeller fans, that do not follow the accepted fan laws with change of speed. The author's results show an increasing pulsation effect with reducing speed and his explanation of increased damping at higher frequencies is entirely reasonable. The relations between the fan performances wide open, and at the reduced orifices, showing decreased pulsation with increased resistance, agree with the writer's findings in many tests comparing pitot-tube traverse and nozzle volumes.

The nature of air flow from fans is apparently an extremely complicated phenomenon. The author has found indications of severe pulsation in a fan discharge. In Professor Marks' paper[7] are described peculiar spins indicated by a direction tube. A fan test seemingly must guard against both these actions. The new instruments developed by the author, therefore, seem a necessity.

The writer built an arrangement described in the third paragraph of the author's conclusions for the purpose of decreasing noise. There was no difference either in noise or performance between the rotors with blades staggered or in line.

AUTHOR'S CLOSURE

The questions on the theory of the instrument, asked by Messrs. Moss and Hardie, can best be answered by considering what would take place if a cyclic air velocity of amplitude V_a and frequency ω were applied at the open end of the air tube leading to the instrument. The amplitude of the cyclic-pressure wave of the same frequency which would reach the water surface can be shown mathematically to be $V_a \sqrt{P_a E_a}$ where P_a is the mass density of air and E_a is the bulk modulus of elasticity of air. Any damping due to fluid friction would reduce this amplitude; but for simplicity, damping is being ignored.

If the openings from which water can escape or enter the water chamber are small compared with the tube diameter, as they are in the instrument, it can be demonstrated mathematically that the amplitude of the pressure wave of the same frequency which is transmitted to the water, approaches very closely to $V_w \sqrt{P_w E_w}$ where V_w is the amplitude of the cyclic water-velocity wave which results, P_w is the mass density of water, and E_w is its bulk modulus of elasticity. Since the instantaneous pressures of the surface of contact of the air and water must balance, it follows that,

$$V_w = V_a \sqrt{\frac{P_a E_a}{P_w E_w}} = \frac{V_a}{3450}$$

This indicates that for ordinary frequencies of pulsation, the cyclic air motion does not appreciably penetrate the more dense and less elastic water. At very low frequencies, the amount of water that would be discharged or taken in through even the very small orifice used during any half cycle, becomes large enough to cause a rapid and appreciable motion of the water surface. This lower frequency limit is approximately 5 cycles a second for the instrument described.

On the other hand, if a steady air velocity V_a impinges on the open end of the air tube, the resulting water velocity through the orifice would be given by

$$V_w = V_a \sqrt{\frac{P_a}{P_w}} = \frac{V_a}{28.8}$$

From this it is apparent that any cyclic air velocity is scaled down by the action of the water 120 times as much as is a steady air velocity. It is the conviction of the author that the real explanation of the instrument is that the cyclic air velocity does not penetrate the water and that the flow through the orifice is practically steady and is affected very little by the cyclic component.

At low frequencies, this cyclic component of air velocity does penetrate to the orifice, with the result that the water surface in the gage glasses oscillates and the instrument becomes useless. The instrument can probably be designed for slightly lower frequencies by decreasing the orifice size and increasing the size of the gage glasses.

In reply to the question by Dr. Moss on the technique of operating the instrument, it may be said that considerable practice is required to adjust the two valves, watch the two water columns, keep an eye on a stop watch, and handle the graduate for collecting the water; but once the skill is acquired, more consistent results are obtainable than are possible with a manometer because the instrument effectively averages the small variations of speed that usually occur in most testing.

The points involved in questions 4 and 5, asked by Mr. Hardie, can best be cleared up by a brief résumé of experimental work that has been carried on since the paper was published. A series of careful tests at low air velocities showed a consistent discrepancy which caused the method of calibration to be restudied. As explained in paragraph 2, column 2, page 782 of the paper, the head for calibrating the instrument was obtained by holding the two column heights at different levels. When the head for calibration was created by an air pressure which was also connected to a manometer for measurement, and the water levels in the two columns were made equal as they are in actual operation, a different calibration curve was found. It was raised 15 per cent above the one shown in Fig. 2, page 782, at a discharge of 20 cc per minute, and 4 per cent above at a discharge of 100 cc per minute.

A very careful study of the technique previously used in calibrating the instrument revealed that the method followed indi-

---

[5] Professor of Mechanical Engineering, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

[6] Vice-President and Director of Research, B. F. Sturtevant Company, Hyde Park, Boston, Mass.

[7] "Air Flow in Fan Discharge Ducts," by Lionel S. Marks, Trans. A.S.M.E., November, 1934, paper PTC-56-2.

cated a head that was smaller than the actual head by an amount equal to twice the capillary head of the glass tubing used in the instrument. Later, it was found that by properly manipulating the water columns during calibration to eliminate this effect, the same calibration was obtained by the two methods.

When the data used in the calculation of Fig. 5 in the paper were referred to this corrected calibration curve, it was found that for the blower there was no detectable pulsation under any of the conditions of operation. All values of V_{av} were equal to the corresponding values of V_{eff} within a variation of 2 per cent, which may be attributed to lack of refinement of the work.

When the data for the square wave form given in column 1, page 783, were corrected from the new calibration curve, it was found that for the undisturbed jet, $V_{av} = 122.2$, as compared with the value $V_{eff} = 121.5$. With the disk rotating, $V_{av} = 61.0$ and $V_{eff} = 82.5$. This gave a value of $V_{av}/V_{eff} = 0.73$ for the square wave, as compared with the theoretical value of 0.707.

In a like manner, the ratio of V_{av}/V_{eff} for the single-cylinder engine in Fig. 6 of the paper, had an average value of 0.98 when the receiver tank was used, and 0.77 for the direct-intake measurement.

The propeller tests shown in Figs. 7 and 8 of the paper showed no appreciable pulsation when this connection was made.

This experimental blunder in the calibration of the instrument caused the author to draw misleading conclusions about air blowers that he is very happy to correct. It is believed that the field of application of the instrument is primarily the measurement of air to internal-combustion engines and the intake and discharge of reciprocating machines. A comprehensive experimental program is under way, that is designed to improve and simplify the instrument and study in detail the action of pitot tubes, nozzles, and orifices, both on intake and exhaust, when the air is pulsating. It is felt that the work will result in a very useful contribution to the art of air measurement.

The Relative Grindability of Coal[1]

R. M. Hardgrove.[2] The authors' contribution on the grindability of coal has developed some points which may assist toward the standardization of a uniform method for making this determination.

The authors' experience with an Abbé Mill confirms our own experience of seven years ago and also the experience of Yancey, of the Bureau of Mines, in that the results are spotty unless lifting ribs are used.

The development of the 3000-factor for the fraction less than 300 mesh is very interesting. The 1000-factor used in our method was chosen arbitrarily to give results that generally agreed with pulverizing practice. As we learned more about the subject we found that this factor was still too large. In the writer's paper, presented before The American Society of Mechanical Engineers in Chicago, 1933, this question was discussed thoroughly and it was shown that pulverizer capacities at 100 grindability are only about 60 per cent more than those at 50 grindability for air-swept pulverizers, i.e., the range is greater than practical results warrant. The most thoroughly scavenged pulverizers, as used in closed-circuit systems, however, do approach capacities proportional to the grindability scale. The use of the 3000-factor instead of 1000 naturally increases the range of the grindability scale and, therefore, makes the agreement between it and actual pulverizer capacity still more of an arbitrary relation than it is

[1] Published as paper FSP-56-13, by H. J. Sloman and A. C. Barnhart, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Engineer in Charge of Design, Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

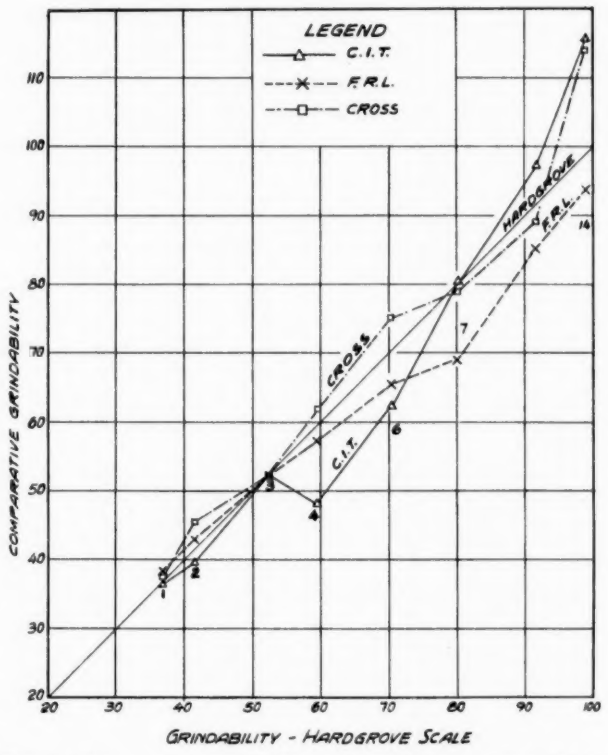


Fig. 1 Comparison of the Results Obtained by the C.I.T., F.R.L., Cross, and Hardgrove Methods of Determining Grindability

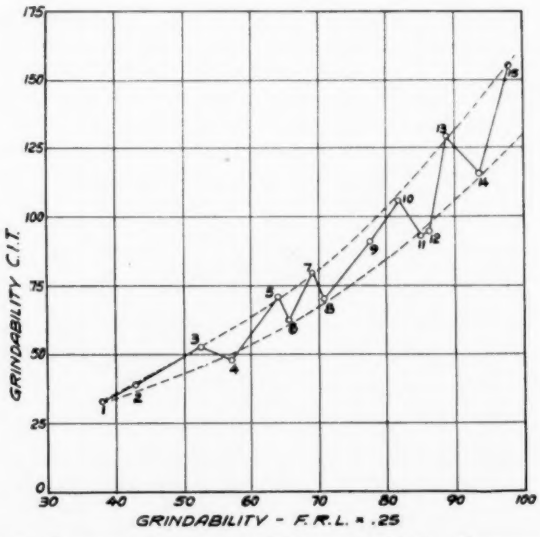


Fig. 2 Comparison of the C.I.T. and F.R.L. Methods of Determining Grindability

now. There does not seem to be any good reason for changing the 1000-factor now in use.

The particle-size distribution differs widely in Figs. 4 and 5. Fig. 4 is that obtained with the roll and would represent a factor of 1300–1500. Fig. 5 is based on an Abbé Mill and represents a factor of 2500–3000. This difference would be expected with these types of machines but the writer cannot understand why the authors use the 3000-factor with the roll machine.

The comparative results in Table 11 are plotted in Fig. 1, using a multiplier of 0.25 for the F.R.L. method and 0.17 for the

Cross method which brings all results together at 52 grindability. From this graph it is evident that samples four and six are too low on the C.I.T. curve.

The general trend of the C.I.T. line shows the wider range, as compared to the writer's, due largely to the 3000-factor used in the C.I.T. method. The high values at 100 grindability are especially noticeable.

In Fig. 2, the C.I.T. values of all 15 samples in Table 11 are plotted against the F.R.L. values as a base.

Assuming the F.R.L. values to be consistent, it would look as though the C.I.T. values fall on two distinct lines. We have obtained similar results in working with an Abbé Mill and it was attributed to either slippage or coated balls, and we rather suspect that the low points in this series are low because the roll, or the plate became coated and thereby cushioned the grinding.

The results are not equal in consistency to the F.R.L., the Cross, the Bureau of Mines, or the Hardgrove methods, and the scale is not nearly as close to the commercial results as the Bureau of Mines or the Hardgrove methods. The Bureau of Mines method has many desirable features and checks very well with the Hardgrove method, but has the one disadvantage of being tedious and costly to run. The time required to make a test by the C.I.T. method as proposed by the authors of the paper being discussed, is even longer than is required for our method. The C.I.T. method would seem to be open to more variation in results depending upon details in manipulating the roller.

G. B. Gould.[3] Although the work of Messrs. Sloman and Barnhart is interesting and valuable, there are two serious objections to the method they propose. It appears that their index is based upon a method of grinding which reduces only a small part of the sample to minus 200-mesh.

While it may be possible to demonstrate by a series of experiments that such a method yields reliable results, it is, in my opinion a dangerous one to rely on in dealing with a material like coal, which is not uniform in structure or hardness. On the contrary, coal, as it is mined and shipped, is a mixture of portions of a seam which often differ materially in these respects.

To take an extreme and impossible condition for purpose of illustration, if a sample composed of 75 per cent diamond chips and 25 per cent fusain were subjected to test by this method, a high grinding index would result, reflecting not at all the great amount of work required to reduce the major portion to the desired size of minus 200-mesh. Coal is a material similar to this hypothetical mixture, only differing from it in the range of hardness of its various portions.

For this reason, we have believed from the beginning of our experimental work on pulverizing-test methods several years ago that one important requirement of a satisfactory and reliable method is that at least 80 per cent of the entire sample of the most easily pulverized coal should be reduced to minus 200-mesh.

The disadvantage of assuming that the reduction in size resulting from a very slight grinding effect can be reliably projected for the whole sample is also present in the method proposed by Hardgrove, but the error, if any, is magnified in the method here proposed by the high value given to the superfines.

Assuming that the values assigned to the sizes below 200-mesh are theoretically correct, the accuracy of the final index depends to a high degree upon the separation and measurement of sizes as small as 300-mesh. From the standpoint of practical commercial laboratory practice, the accurate screening of coal through a 300-mesh screen is out of the question. By striving to simplify and minimize the grinding work, the difficulties of accurately measuring the result are greatly accentuated.

The adoption of a fixed screening period would be found unsatisfactory if the experiments had been extended over a wider range of coals. We tried that but found that some coals, when pulverized, are of a very gummy nature and are screened through even a 200-mesh screen with great difficulty.

The unsatisfactory results reported with the pebble mill by Messrs. Sloman and Barnhart, as well as by Hardgrove, do not indicate any inherent unfitness of this type of mill for the purpose.

The method proposed by Yancey employs the pebble mill with very successful results, and has the advantages of reducing every sample to 80 per cent through a 200-mesh screen, and requires screening only through that one screen. From a practical commercial standpoint, the primary requirement is to reduce the major part of the whole quantity to minus-200-mesh, and Yancey's method measures the work necessary to do that in a simple and direct manner.

Ollison Craig.[4] Manufacturers of pulverized-coal equipment realize keenly the necessity of knowledge concerning the grindability of the various coals to be used in plants in which coal is fired in powdered form.

R. M. Hardgrove in his paper, "Grindability of Coal,"[5] gave the industry definite information which was of very great value. The main value of Mr. Hardgrove's paper lay in the fact that he gave figures of relative grindability for a large number of coals. This was of more practical value than the statement of the method by which the figures were arrived at.

So far as the manufacturers are concerned it is of more importance that the order of coals as to grindability be known than the absolute values be known. Any system which will place coals in their proper order of grindability and give reasonably accurate values and which at the same time makes the values easy to determine, is the most desirable from the manufacturers' viewpoint. Mr. Sloman and Mr. Barnhart have proposed a method which can be used by any manufacturer or user of coal-pulverizing equipment.

It is our opinion that some method should be standardized and adopted by the Society as the recognized method for determining relative grindability of coal and it is certainly desirable that such a method be reduced to the simplest possible form.

Authors' Closure

Mr. Hardgrove's discussion of the development of our 3000-factor for minus 300-mesh coal should be amplified by the following explanation. In our Fig. 4 where minus 300-mesh coal was obtained with the roll and plate, it is true that the factor was low. Because of limited crushing action in the roll test we found that there was not a sufficient quantity of minus 300-mesh coal produced to perform a satisfactory determination of particle-size distribution in that size of coal. In order to obtain a sufficient range of size in minus-300-mesh coal, it was necessary to use the product from a pebble mill. This determination was by the sedimentation-velocity method and was described in the latter half of our paper.

Mr. Hardgrove's discussion of our 3000-factor does not suggest the retention of his 1000-factor, but rather the improvement of performance of commercial pulverizing systems to a point where they may make full use of the properties of the coals which may be fed to the mills. We do not agree that any grindability scale should be brought into line of agreement with existing machine

[3] President, Fuel Engineering Company of New York, New York, N. Y. Mem. A.S.M.E.

[4] Engineer, Riley Stoker Corp., Worcester, Mass. Mem. A.S.M.E.

[5] "Grindability of Coal," by R. M. Hardgrove, Trans. A.S.M.E., vol. 54, 1932, paper FSP-54-5.

performance because the expected future improvement in pulverizers would render such a scale worthless. Mr. Hardgrove has stated that with the use of closed circuits and thoroughly scavenged machines capacities are realized proportional to his grindability scale. Our factor of 3000 was determined by experimentation as the proper one to use for minus-300-mesh coal. While this factor does increase the range of the grindability scales that have been proposed previously, this increase of range is desirable inasmuch as a greater differentiation may be obtained with coals of nearly the same grindability.

In Mr. Hardgrove's Fig. 1, all results are brought together at 52-grindability by the arbitrary choice of factors. In this case it would be expected to find a wider range of grindability with the C.I.T. method because of the use of the 3000-factor. Mr. Hardgrove has plotted his values as a straight line, but should any other set of values from the remaining methods be so plotted, the variation would be quite as apparent as in his graph. This plot has no inherent value and the fact that samples 4 and 6 are too low on the C.I.T. curve, as plotted on his Fig. 1, has no significance.

Mr. Hardgrove's Fig. 2 is plotted with the assumption that the F.R.L. values are consistent and the C.I.T. values are shown as falling on two distinct lines. We have plotted Mr. Hardgrove's eight values from our Table 11 and, in comparison with the F.R.L. method, his values also fall on two distinct lines with greater divergence than the C.I.T. method. Nothing is proved but the arbitrary assumption of the accuracy of the F.R.L. method.

We agree that there is some cushioning between the plate and roll of the C.I.T. method, but we also assert that there is less cushioning and attrition in this method than in any of the other proposed methods. The best that can be hoped for is to reduce cushioning to a minimum inasmuch as it cannot be eliminated entirely.

Contrary to the opinion expressed by Mr. Hardgrove, the variation in the results of the C.I.T. method depends very little upon details in manipulating the roller. This is evident from the results quoted in the paper where three different operators checked very closely when using the same coal.

The time taken to actually perform a test is less than any other proposed method. We stated in the paper that 90 minutes were required to make complete duplicate tests; this included screen analysis, weighing, and calculations. The actual grinding with the roll can be accomplished in less than one minute.

The data obtained by H. F. Yancey et al do not substantiate the opinion expressed by Mr. Gould that it is necessary for 80 per cent of a material to pass through 200-mesh. Doctor Yancey of the U. S. Bureau of Mines, in his paper presented before the A.I.M.E. in February, 1934, showed that plotting cumulative per cent passing 200 mesh against the number of revolutions of the mill gave a straight line up to 50 per cent or more through 200-mesh, depending upon the grinding characteristics of the particular coal, and stated that the slope falls off at higher percentages, probably due to increased weight of unfinished material and accumulation of material harder to grind. The data from Doctor Yancey's paper indicate that the hypothetical case of Mr. Gould's mixture of 75 per cent diamond chips and 25 per cent fusain has no application in coal-grindability testing. Coals which are shipped today do not have large percentages of hard extraneous material as in Mr. Gould's assumption.

Mr. Gould thinks it is impractical to screen coal through 300 mesh and adopt a definite screening period. This might be true if 500-gram samples were used as in the Bureau of Mines method. However, when 20-gram samples were used as discussed in the paper, no difficulty was experienced. The data given in Table 6 show that by simplifying and minimizing the grinding work, the difficulties of accurately measuring the results have not been greatly accentuated.

In regard to the Bureau of Mines method, we agree with Mr. Hardgrove when he mentions the disadvantages of its being tedious and costly to operate. The Bureau of Mines method requires the handling of the sample too many times, resulting in the loss of fine coal and difficulty of cleaning mill and balls. We feel that this method would require more rigid technique than the average consumer would care to provide.

In closing, we again wish to stress the simplicity and ease of duplication of the C.I.T. method, and one which does not call for expensive special apparatus nor a high degree of skill to operate.

Further Experiments on the Variation of the Maximum-Lift Coefficient With Turbulence and Reynolds' Number[1]

E. A. Stalker.[2] The investigations conducted by Dr. Millikan on the value of the maximum-lift coefficient have been very important. As to the behavior of the three airfoils, U.S.N. P6, Clark Y-18, and N.A.C.A. 2412, it is pertinent to consider the stagnation point on the lower surface of the section. A high arching of the mean-camber line will shift the stagnation point rearward so that in the case of the highly cambered section, the flow going over the upper surface will travel a greater distance than in the case of the low mean-camber section. There will consequently be time or distance in which the boundary layer may become turbulent. On a flat plate the turbulence would just be forming at the trailing edge for a Reynolds number of 500,000, the value at which Dr. Millikan's curves begin; but for a flow about a wing nose the turbulence would occur much earlier. It would appear that the highly cambered wing, already having the mechanics for generating its turbulence, would benefit little from additional turbulence introduced into the airstream. It would also appear that a wing section of low mean-camber height would be benefited by a flap to a greater extent that a highly cambered wing. The above observations appear to be borne out by Dr. Millikan's experiments since wing U.S.N. P6 has a value of the mean camber line of 9.2 per cent, while Clark Y-18 and N.A.C.A. 2412 have only 4 and 2 per cent, respectively. The presence of a depressed flap may also be considered as converting a given wing section into a more highly cambered one.

It thus appears that certain assumptions should be made about the behavior of both the upper- and lower-surface stagnation points. It may be that the assumption that the upper-surface stagnation remains at the trailing edge will prove sufficient, but it is, of course, possible to have an orderly travel of this point forward and yet preserve a straight-line lift curve. In fact, this is what experiment actually shows. It seems to be more desirable, however, to use the simpler assumption, at least until the effect of the lower stagnation point is investigated. It might also be that a connection with the location of the most forward center of pressure position could be established.

---

[1] Published as paper AER-56-14, by Clark B. Millikan, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Professor of Aeronautical Engineering, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E.

Some Physical Properties of Water and Other Fluids

By ROBERT L. DAUGHERTY,[1] PASADENA, CALIF.

Absolute and kinematic viscosities of various liquids and gases as functions of temperature are shown by curves plotted on the same diagrams to the same scales so that these properties for different fluids can be readily compared.

Pressure, volume, and temperature relations for water are shown graphically, as are values of the volume modulus of elasticity for a wide range of pressures and temperatures. From recent data presented by Smith and Keyes it was possible to compute values of the specific heat of water for all temperatures up to the critical and for all pressures from saturation up to 350 atmospheres. These values are here shown.

ONE of the important physical properties of all fluids is viscosity but, although much has been published on the subject, the material as presented is tabulated in different units or shown on charts with different scales. Furthermore, different classes of fluids, such as gases and liquids, have been shown separately, so that it is not at all obvious as to how they compare, one with the other.

The writer, therefore, thought it would be of interest to engineers to gather together data on fluids which differed widely and to plot all of it on one chart to the same scale so that their relationships could be seen at a glance. In Fig. 1 are shown the viscosities of such dissimilar fluids as mercury, hydrogen, air, residuum or fuel oil, ammonia, and brine. These are all fluids that are of interest to the engineer. Furthermore the viscosity curves for saturated water and saturated steam have been joined at the critical temperature to illustrate the way in which the viscosities of all liquids and their vapors must coincide at the critical point.

It may be noted that the viscosity of either a liquid or a gas is practically independent of pressure for a moderate pressure range such as is ordinarily encountered in engineering work, but for very high pressures of the order of several thousand pounds per square inch there is an appreciable variation, especially in the case of liquids. But even then the change in viscosity is only a small fraction of the viscosity values shown in Fig. 1.

In most engineering problems one is concerned with the ratio of viscosity to density, and this ratio is called kinematic viscosity. Since the density of all gases varies in a marked manner with the pressure, it is obvious that the kinematic viscosity of a gas is affected greatly by the pressure. Values of kinematic viscosities are shown in Fig. 2, and it is very interesting to compare these values with those in Fig. 1. Thus hydrogen which gives the lowest curve in Fig. 1 appears near the top in Fig. 2, while the lowest curve in Fig. 2 is for mercury. Of course for a gas one

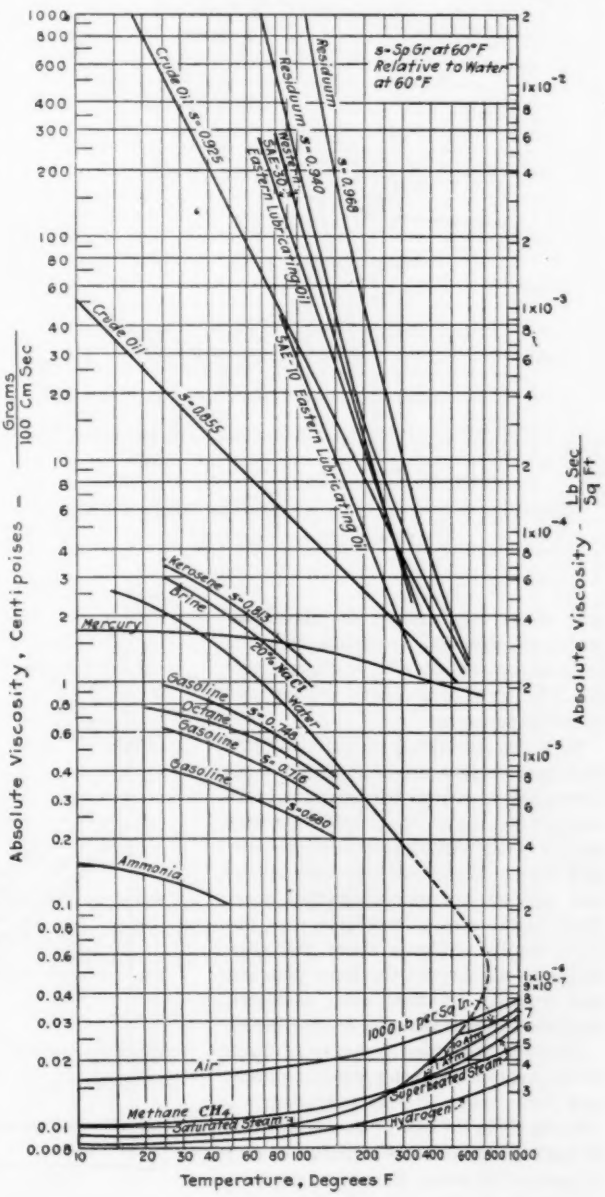


FIG. 1

[1] Professor of Mechanical and Hydraulic Engineering, California Institute of Technology. Vice-President, A.S.M.E., 1928–30. Professor Daugherty was graduated in mechanical engineering from Stanford University in 1909 and received his master's degree in 1914. He was assistant in mechanics at the University during 1907–1908; assistant in hydraulics, 1908–1909; and instructor in mechanical engineering, 1909–1910. From 1910 to 1916 he was assistant professor of hydraulics at Cornell University, resigning to become professor of hydraulics at Rensselaer Polytechnic Institute. Three years later he joined the faculty of the California Institute of Technology as head of the mechanical engineering department. In addition to teaching Professor Daugherty has been actively engaged in consulting work.

Contributed by the Hydraulics Division and presented at the National Aeronautic-Hydraulic Meeting, Berkeley, Calif., June 19, 20, and 21, 1934, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until September 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

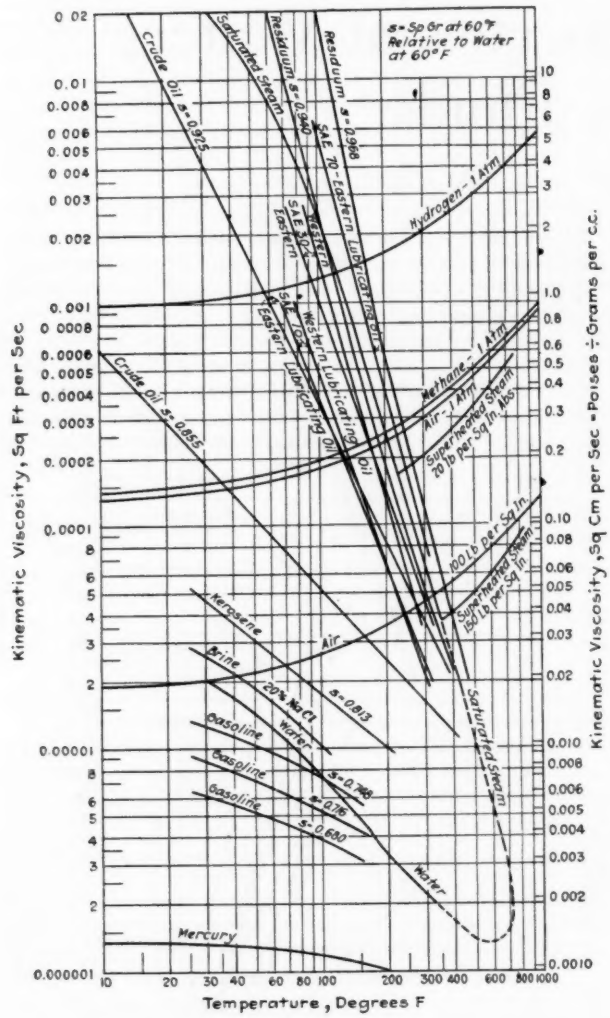


FIG. 2

may draw any number of curves for kinematic viscosity, depending upon the pressure chosen, such as is illustrated in Fig. 2 by air at atmospheric pressure and at 100 lb per sq in.

It is also interesting to note in Fig. 2 that at a temperature of about 200 F the kinematic viscosities of residuum, lubricating oils, heavy crude oils, saturated steam, and air at atmospheric pressure, as well as methane and therefore natural gas, are all approximately the same. Thus in the case of flow in a pipe line they would all have about the same Reynolds number for the same pipe size and velocity of flow and, therefore, would have about the same friction factor.

Another important property of fluids is the relation between pressure, volume, and temperature. This relationship is very simple in the case of the ideal or perfect gas and in so far as actual gases may be assumed to follow the perfect gas laws, the problem offers no difficulties. But vapors, which depart widely from perfect gases, cannot be treated so simply and neither can liquids. So far as our present knowledge goes these relationships are complex and are also purely empirical. In the upper right-hand corner of Fig. 3 is shown the field covered by the known data for water. P. W. Bridgman has thoroughly covered the range from 0 to 95 C from 500 atmospheres up to 12,000 atmospheres, while L. B. Smith and F. G. Keyes have measured these properties from 30 to 360 C and for pressures up to 350 atmospheres. Apparently Bridgman extrapolates back to saturation pressure, while Smith and Keyes extrapolate down to 0 deg temperature. Thus values as given by these two sources do not agree precisely in the field of low temperatures combined with low pressures, and the accuracy of values given by either may be questioned in this range. A limited amount of data has been determined in this low-temperature low-pressure field by several investigators such as Amagat and Tyrer. On the other hand no data at all have been determined for very high pressures combined with very high temperatures. While such a combination might be of great theoretical interest, it is fortunately of small practical value.

The lower portion of Fig. 3 shows Bridgman's values in more detail and to the same scale is shown a small portion of the Smith and Keyes data.

The compressibility of gases is important because of the magnitude of the volume changes. The compressibility of solids is very small, but in many cases of engineering structures even small changes in dimensions may result in stresses so great that they cannot be neglected, small though they be.

In the case of liquids their compressibility is often of small practical importance and hence this property is frequently ignored. However, it is of interest to note, for instance, that water is about 100 times as compressible as mild steel. But air at atmospheric pressure is 20,000 times as compressible as water, and at 300 lb per sq in. it is still 1000 times as compressible.

A practical case where compressibility of water is important is in the study of water-hammer phenomena. The volume modulus of elasticity, which enters into formulas dealing with water hammer, is the reciprocal of the coefficient of compressibility. For a perfect gas it may be shown that its isothermal volume modulus of elasticity is equal to the absolute pressure of the gas, but for other fluids calculation of the values is more involved.

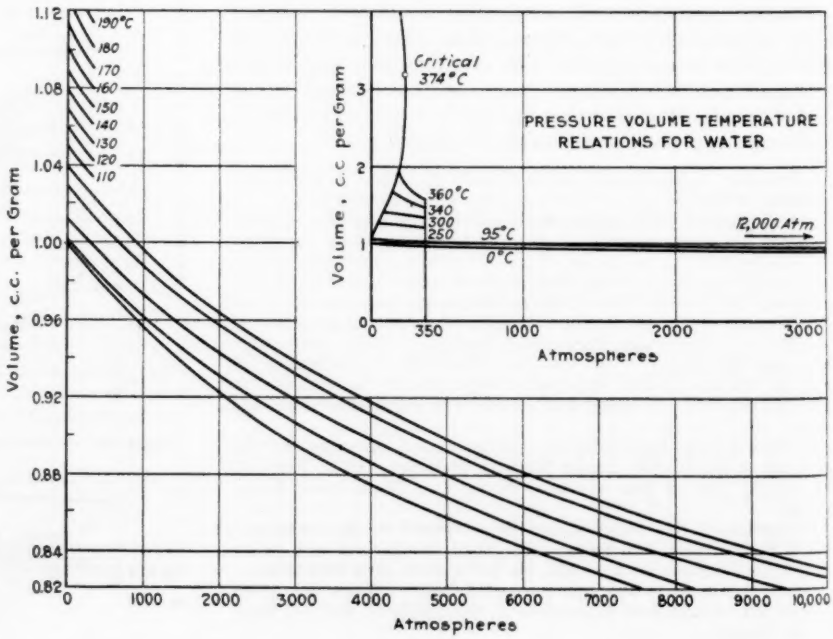


FIG. 3

In Fig. 4 are shown values of the isothermal modulus of elasticity for water at various temperatures and pressures. In the case of water hammer the action is practically adiabatic. The value of the adiabatic modulus may be obtained by multiplying the isothermal value by c_p/c_v, the ratio of the specific heat at constant pressure to that at constant volume. Values of this ratio are given in Fig. 7, which shows that for the temperatures usually encountered in such problems the difference between the two moduli is less than 2 per cent. This is really less than the uncertainty as to the correct value of the isothermal modulus itself, as may be seen in Fig. 5. Also any uncertainty as to the proper value of the modulus for water is less than the uncertainty as to the value to be used for the pipe line in connection therewith. The construction of a pipe line with joints, saddles which restrict movement, and other similar features makes the pipe as a whole behave differently from what it would if it were a homogeneous structure.

According to the Bridgman data, the value of the isothermal modulus at any one temperature increases with the pressure almost as a straight line, even up to 12,000 atmospheres. Hence the lines in Fig. 4 could readily be extended, if desired. From the Bridgman and Tyrer data the value of K at any given pressure is found to increase from 0 deg up to a maximum value at about 50 C and then to diminish again, while the Smith-Keyes data show that it continually decreases at an increasing rate as higher temperatures are reached.

Values of K as a function of temperature according to various observers are shown in Fig. 5. An approximate arithmetical determination of the modulus from the pressure-volume data direct involves very small differences between large quantities. Hence any slight error in the volumes is much magnified when differences are taken. Therefore the derivative of the pressure-volume curve is very much less accurate than the curve itself. This accounts for the discrepancies observed in Fig. 5.

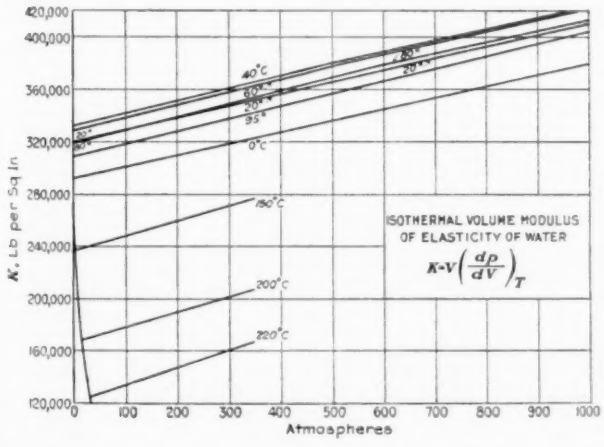


Fig. 4

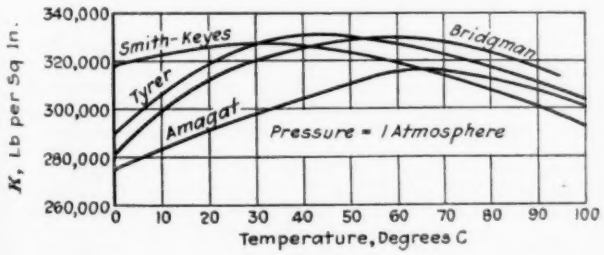


Fig. 5

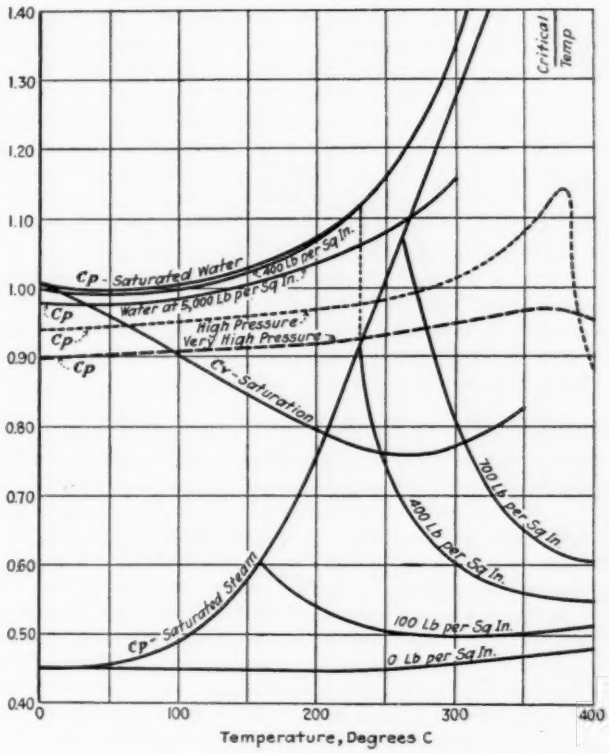


Fig. 6

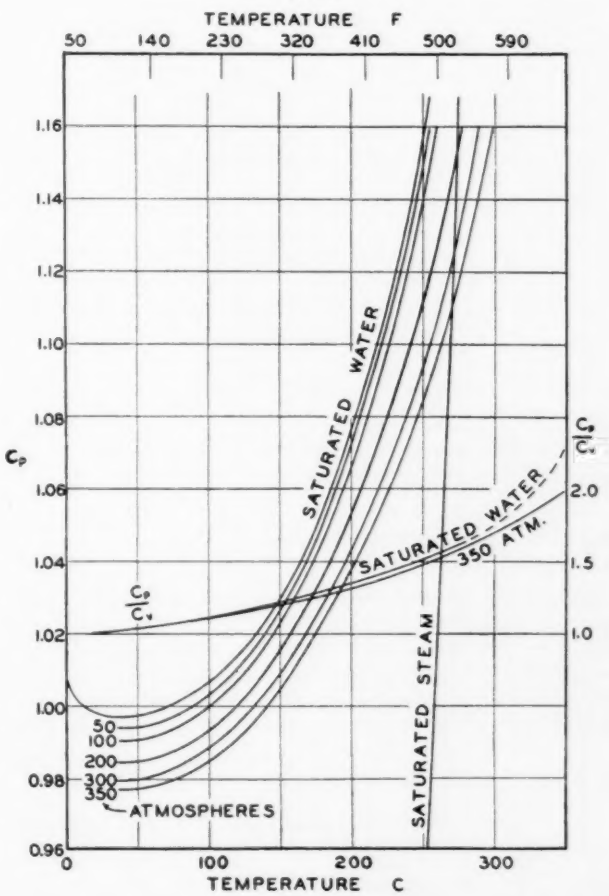


Fig. 7

The values of K so far shown are so-called "instantaneous" values, that is, they apply to an infinitesimal change of volume only. From the definition of $K = V\ (dp/dV)$ may be obtained $dV/V = dp/K$. The integral of dV/V is $\log_e V_2/V_1$, but to evaluate the integral of dp/K it is necessary to resort to graphical methods. Values of $1/K$ may be plotted against simultaneous values of p as obtained from Fig. 4. The area under this curve between the limits of p_1 and p_2 will then be the value of $\log_e V_2/V_1$, from which V_2 may be obtained for any pressure change from p_1 to p_2, assuming V_1 to be known.

If curves of $(1/K_m)$ were constructed as functions of pressure for various temperatures, where K_m is a mean value from some fixed initial pressure p_1, it is obvious that the value of $\log_e V_2/V_1$ could then be obtained by multiplying $(p_2 - p_1)$ by the value of $(1/K_m)$ corresponding to p_2.

There is often given the expression $K_m = V_1 \dfrac{p_1 - p_2}{V_1 - V_2}$ where K_m is a mean value for the pressure range concerned. This equation is not correct theoretically, but it may be shown that if $(V_1 - V_2)$ is 1/10 of V_1 the value of $(V_1 - V_2)$ computed by it will be 4.8 per cent too high. As the change in volume becomes smaller the error in this formula rapidly diminishes, so that it may often be used as an approximation.

Some thermal properties of water and steam are shown in Figs. 6 and 7. The values are for the various specific heats and are given in this paper because they are derived in large part from the pressure-volume-temperature relations such as those in Fig. 3. Since the specific heats involve second derivatives there may be some question as to the absolute accuracy of the values shown, but at least they show the way in which these quantities vary.

The values given in both Figs. 6 and 7 are instantaneous values. Mean values for any given range could be obtained by graphical integration. The saturation curves are of course merely the loci of the terminal points of a series of constant-pressure curves. The saturation curves for both water and steam for the specific heats at constant pressure approach infinity at the critical temperature. On the other hand the specific heat at constant volume may have a large value at the critical temperature, but it is still finite.

For pressures above the critical the specific heat of water at constant pressure is finite at the critical temperature, and the higher the pressure the less the maximum value attained by a given curve. However there is a complete absence of actual data in this region and the dotted curves are merely hypothetical. The critical pressure is about 220 atmospheres or about 3224 lb per sq in. The highest pressure for which data now exist is 350 atmospheres and does not go beyond 300 C, but the dotted curves correspond to very much higher pressures and temperatures than these.

For moderate temperatures the ratio of c_p/c_v is apparently very nearly independent of the pressure. But at the critical temperature the saturation value would be infinite, while that for a pressure above the critical would be finite. Hence the values of this ratio for different pressures must diverge very greatly as the critical temperature is approached.

It is surprising that for a substance as common as water, and of so much importance to us, there is still so much uncertainty about exact values of many of its properties.

Propeller Pumps

By M. P. O'BRIEN[1] and R. G. FOLSOM,[2] BERKELEY, CALIF.

This paper presents a simplified method of design with performance prediction throughout a reasonable portion of the operating range near the design point of propeller or axial-flow pumps. The results obtained by this method are compared with test characteristics for a known pump and very good agreement obtained. Cavitation phenomena are not considered.

The system of computations is based essentially on the propeller-blade element theory as used by Pfleiderer. The assumptions involved are clearly stated and the validity of each discussed. A theoretical proof for the desirability of constant total head being developed over the propeller disk is given. A sample computation is included which clearly illustrates the application to an actual pump.

Equally good agreement was found in computing the characteristics of a propeller fan.

THE theory of propeller pumps may be developed from two essentially different points of view. One treatment starts from a theory applicable to an infinite number of blades and applies a correction for real conditions. The other starts from the lift and drag exerted by streamlined sections and corrects for the interference of adjacent blades. The latter conception forms the basis for the computations included in this paper.

The method of computations is essentially that used by Pfleiderer[3] with, however, modifications which result in a better agreement between the computed and measured head-capacity characteristics. Many of the basic principles involved have been discussed in a recent paper by Spannhake[4] and will not be repeated here. The original aim was to compare measured results with those computed from a very simplified theory and give an indication of the direction and magnitude of the corrections that must be applied. The discrepancies between theory and measurement were found to be so small as to indicate that the simplified theory provides a practical method of design.

Data on a 20-in. pump of the Pfleiderer type were furnished by the Byron-Jackson Company for making the comparison. The method outlined is applicable to any similar type of axial-flow pump, provided the aerodynamic characteristics of the blade sections are known.

Cavitation is an important factor in the operation of propeller pumps. In the computations which follow it is assumed that the pressures are everywhere sufficient to prevent the formation of cavities.

Basic Equations

The fundamental equation for the operation of centrifugal pumps (radial, mixed, and axial flow) is obtained from the principle of torque and angular momentum. As applied to fluid problems, the equation is

$$\Delta T = \frac{\gamma}{g}(V_{u3}r_3 - V_{u0}r_0)\Delta Q \quad [1]$$

If the discharge is imagined to be divided into small elements ΔQ, the quantities $V_{u3}r_3$ and $V_{u0}r_0$ for the elements may vary both radially and with position between the blades at a constant radius. Equation [1] is often stated to be valid only for an infinite number of blades but this view is not correct; it has the same validity as Newton's equations of motion and the reason for assuming an infinite number of blades is that the velocity component V_u can then be specified. This distinction is unimportant perhaps but it places the application of the equation on a somewhat different basis, in that the problem is to find methods for predicting the component V_u rather than to devise corrections which will make Equation [1] valid.

Integrating Equation [1] so as to take into account the variations mentioned, the actual torque is

$$T = \frac{\gamma Q}{g}(V_{u3}r_3 - V_{u0}r_0) \quad [2]$$

Multiplying by the angular velocity and equating the result to the power output, gives as the equation for the head developed

$$H_d = \frac{1}{g}(V_{u3}u_3 - V_{u0}u_0) \quad [3]$$

The simplified theory presented here is based upon the characteristics of airfoils and it is convenient to rewrite the preceding equations in terms of the circulation about the blades so as to obtain the forms commonly used in aerodynamics. The circulation is the line integral of the velocity around a closed path and its value at a constant radius for axially symmetrical flow is

$$\Gamma = 2\pi r V_u \quad [4]$$

The equation for the torque becomes

$$T = \frac{\gamma Q}{2\pi g}(\Gamma_3 - \Gamma_0) \quad [5]$$

---

[1] Associate Professor of Mechanical Engineering, University of California. Assoc-Mem. A.S.M.E. Professor O'Brien received his B.S. in Civil Engineering from the Massachusetts Institute of Technology in 1925, and served as assistant in civil engineering at Purdue University from 1925 to 1926, and research assistant, Engineering Experiment Station, from 1926 to 1927. During 1927 and 1928 he held the Freeman Traveling Scholarship of the American Society of Civil Engineers, and was assistant research engineer at the Royal College of Engineering, Stockholm, Sweden, during the same period. Since 1928 Professor O'Brien has been on the faculty of the University of California.

[2] Instructor, Department of Mechanical Engineering, University of California. Jun. A.S.M.E. Mr. Folsom received the degrees of B.S., M.S., and Ph.D. from the California Institute of Technology in 1928, 1929, and 1932, respectively. Since his graduation in 1928 he has worked for one year as part-time research assistant at Riverside Cement Company, Riverside, California; four years as teaching fellow at the California Institute of Technology; and one year as engineer for the Water Department of the City of Pasadena, California. He has been at the University of California since 1933.

[3] "Die Kreiselpumpen," by C. Pfleiderer, second edition, J. Springer, Berlin, 1932.

[4] "Problems of Modern Pump and Turbine Design," by W. Spannhake, Trans. A.S.M.E., 1934, pp. 225–248, paper HYD-56-1.

Contributed by the Hydraulics Division and presented at the National Aeronautic-Hydraulic Meeting, Berkeley, Calif., June 19, 20, and 21, 1934, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until September 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The circulation about each of the blades is

$$\Gamma_s = (\Gamma_3 - \Gamma_0)/z \quad \ldots\ldots [6]$$

and

$$T = \frac{\gamma Q z \Gamma_s}{2\pi g} \quad \ldots\ldots [7]$$

Including the drag of the blades, Equation [7] becomes

$$T_a = \frac{\gamma Q z \Gamma_s}{2\pi g} + zrL \tan \lambda \cos \beta_\infty \quad \ldots\ldots [8]$$

The lift on a unit length of one blade in an infinite row is

$$L = \rho v_\infty \Gamma_s \quad \ldots\ldots [9]$$

where the numerical value of $V_{u\infty}$ is $V_{u3}/2$. Substituting for Γ_s

$$T_a = \frac{QzL}{2\pi v_\infty} + zrL \tan \lambda \cos \beta_\infty \quad \ldots\ldots [10]$$

Multiplying Equation [10] by the angular velocity and equating the result to the power input gives

$$H_d = \frac{zL\omega}{2\pi\gamma v_\infty} + \frac{\omega z r L \tan \lambda \cos \beta_\infty}{\gamma Q} \quad \ldots\ldots [11]$$

Since L is referred to unit length of blade, the discharge at any radius can be expressed as

$$Q = 2\pi r V_f \quad \ldots\ldots [12]$$

The number of blades is $2\pi r/t$ and the equation for the lift is

$$L = C_L \rho (v_\infty^2/2) c \quad \ldots\ldots [13]$$

Substituting these quantities in Equation [11] and simplifying

$$H_d = C_L \frac{c}{t} \frac{u}{V_f} \frac{v_\infty^2}{2g} \frac{1}{\cos \lambda} \left(\frac{V_f}{v_\infty} \cos \lambda + \sin \lambda \cos \beta_\infty \right) \quad \ldots [14]$$

But $V_f/v_\infty = \sin \beta_\infty$ and

$$H_d = C_L \frac{c}{t} \frac{u}{V_f} \frac{v_\infty^2}{2g} \frac{\sin (\beta_\infty + \lambda)}{\cos \lambda} \quad \ldots\ldots [15]$$

Equation [15] gives the head developed at a certain radius on the assumption that the flow is two-dimensional or, in other words, that radial flow does not occur. Equation [15] provides a method of estimating the velocity terms entering Equation [3], which may be rewritten as

$$H_d = \frac{u}{g} (V_{u3} - V_{u0}) \quad \ldots\ldots [16]$$

for an axial-flow pump. Here, H_d is the head developed at the radius corresponding to u and the velocity terms are average values.

Equations [15] and [16] apply to the conditions at a given radius and the next problem is to determine how H_d should vary radially to give optimum results. The most desirable condition is that H_d be a constant. The stability of the pressure distribution at discharge resulting from a constant developed head will be investigated. Solving Equation [16] for the tangential component at discharge

$$V_{u3} = \frac{gH_d}{u} + V_{u0} \quad \ldots\ldots [17]$$

Assuming that the flow is originally axial, $V_{u0} = 0$. The pressure change between inlet and discharge at any radius is then

$$\frac{\Delta p}{\gamma} = H_d - h + \frac{V_{f0}^2}{2g} - \frac{V_{f3}^2}{2g} - \frac{V_{u3}^2}{2g} \quad \ldots\ldots [18]$$

For pure axial flow, $V_{f0} = V_{f3}$ and the pressure change as a function of radius is

$$\frac{\Delta p}{\gamma} = H_d - h - \frac{gH_d^2}{2\omega^2 r^2} \quad \ldots\ldots [19]$$

Since all of the flow had the same pressure originally, Equation [19] represents the variation in pressure in the plane of discharge.

It has been assumed here that each element of flow remains at a constant radius and it is now necessary to determine whether such a condition is possible with the pressure distribution given by Equation [19]. For motion in circular or spiral paths, the radial-pressure gradient is

$$\frac{dp}{dr} = \rho \frac{V_u^2}{r} \quad \ldots\ldots [20]$$

Substituting $V_u = \frac{gH_d}{\omega r}$ and integrating gives

$$\frac{p}{\gamma} = \text{constant} - \frac{gH_d^2}{2\omega^2 r^2} \quad \ldots\ldots [21]$$

The pressure variation of Equation [19] is seen to be consistent with that necessary for flow at constant radius and it is evident

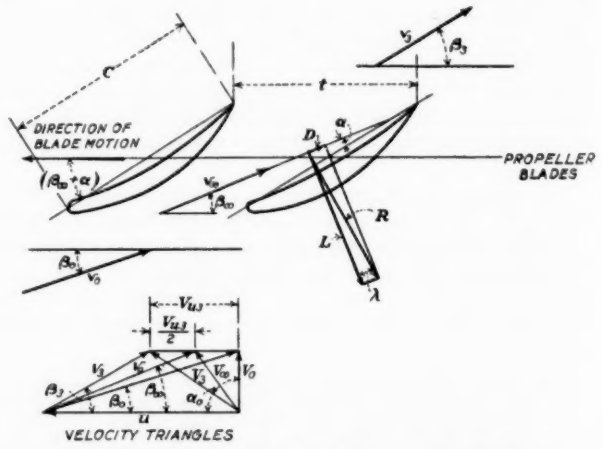


FIG. 1 VELOCITY DIAGRAMS FOR PROPELLER PUMPS

that an axial-flow pump should be designed so as to make H_d a constant. This requirement has not been generally adopted for the design of propeller pumps and fans and, in fact, some methods[5] aim at a constant pressure over the section at discharge, a condition which would result in radial flow.

An interesting feature of Equation [19] is that it shows the necessity for a hub on both fans and pumps. For a constant developed head, the pressure drops rapidly from the tip of the blade inward and will reach very low values if the inner radius is small.

PREDICTION OF HEAD-CAPACITY CHARACTERISTIC

The theoretical treatment of propeller pumps and fans has been carried to such a point as to make application difficult and very few experimental checks on the validity of this theoretical work have been published. Having this in mind, the authors have used a simplified theory to predict the head-capacity curve of a pump which had been tested and for which complete design drawings were available. Certain corrections were omitted deliberately because the primary aim was to find the simplest

[5] "The Propeller-Type Fan," by O. G. Tietjens, Trans. A.S.M.E., vol. 54, 1932, paper APM-54-13, pp. 143–152.

method of design which is also reliable. The good agreement between theory and measurement may have been fortuitous and comparisons for other pumps may show that certain modifications of this simplified treatment are necessary. Recent applications of this method to propeller fans showed good agreement between computed and measured head-capacity curves.

The assumptions upon which the simplified theory is based are as follows:

1 No radial component of velocity in the pump.

2 No rotational component and uniform axial velocity at entrance.

3 Lift and drag coefficients of blade sections are the same as for a single airfoil with an infinite aspect ratio. The angle of attack is corrected for mutual interference.

4 The shock loss on entering the guide vanes depends upon change in the rotational component of velocity.

TABLE 1 GENERAL CHARACTERISTICS OF PROPELLER PUMP OF FIG. 2

Section radius, ft	0.354	0.474	0.594	0.714	0.833
Blade velocity, ft per sec	32.8	44.0	55.1	66.2	77.3
Blade sections, Gött.	No. 387	thick No. 490	No. 490	thin No. 490	thin No. 490
Section multiplication factor	...	1.287	...	0.800	0.655
Approx. N.A.C.A. equivalent	6415	5311	4309	3307	2306
Normal $(\beta_\infty + \alpha)$ N.A.C.A.	38° 48′	25° 55′	19° 24′	15° 21′	12° 47′
c/t	0.736	0.625	0.602	0.581	0.562
Guide-vane entrance angle	54°	60° 40′	66° 25′	69° 13′	71° 48′
Number of impeller blades	3	...	...	...	...
Number of guide vanes	8	...	...	...	...
Pump speed, rpm	885	...	...	...	...
Blade radial clearance	0.03 in.	...	...	...	...

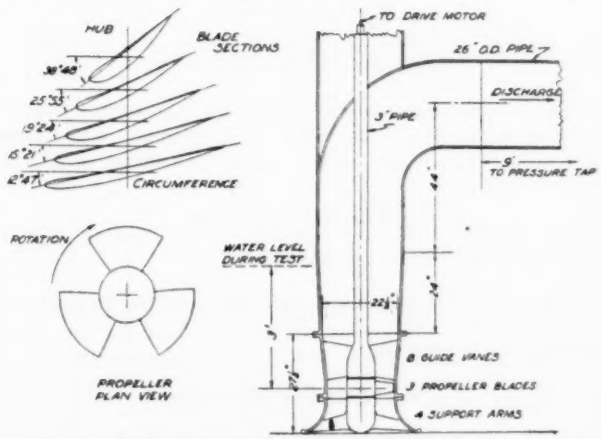


FIG. 2 20-IN. PROPELLER PUMP OF THE PFLEIDERER TYPE

5 The friction losses may be broken into two distinct portions, namely, that caused by the drag of the propeller blades and that resulting from frictional resistance in the stationary water passages.

6 The head developed by the pump as a whole is the weighted average of the heads developed at each radius.

Before discussing the errors introduced by making these assumptions, the method of computation will be illustrated by a numerical example based upon the pump shown in Fig. 2. The general characteristics of this pump appear in Table 1. Application of the method requires the assumption of a discharge and computation of the corresponding head by successive approximation. A direct computation of the head does not appear possible.

EXAMPLE

For a discharge of 12,000 gpm, the axial velocity just previous to entrance to the pump is $V_{f0} = 15.0$ ft per sec. At a radius

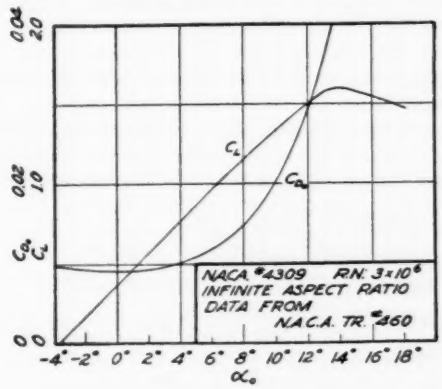


FIG. 3 INFINITE-ASPECT-RATIO DATA FOR N.A.C.A. AIRFOIL SECTION NO. 4309 FROM N.A.C.A. TECHNICAL REPORT NO. 460

of 0.594 ft, the blade profile is a Göttingen No. 490 airfoil section, the characteristics of which may be closely approximated from the N.A.C.A. test results given in Technical Report No. 460. Fig. 3 presents the characteristic curves for the N.A.C.A. airfoil section No. 4309 which is very nearly identical with the No. 490 Göttingen section. Five radii were used in computing the head characteristics, and curves similar to those in Fig. 3 must be known for each section if the blade shape changes. This example will be carried through for one radius only since the method is exactly the same for the other radii.

The operating speed of the pump is 885 rpm, and the peripheral velocity of the blade section is 55.1 ft per sec. The next step is to assume a value of H_d and compute the corresponding head given by Equation [15]:

Assume $H_d = 18$ ft

From Equation [16] with $V_{u0} = 0$

$$V_{u3} = 18 \times 32.2/55.1 = 10.5 \text{ ft per sec}$$

$$v_\infty^2 = V_f^2 + (u - V_{u3}/2)^2 = 15.0^2 + (55.1 - 10.5/2)^2 = 2704 \text{ (ft per sec)}^2$$

TABLE 2 COMPLETE COMPUTATIONS FOR THE 20-IN. PFLEIDERER-TYPE PROPELLER PUMP SHOWN IN FIG. 2

Blade angle at tip	12° 47′			12° 47′			12° 47′			12° 47′			9° 47			15° 17′		
Capacity, gpm	6,000			10,000			12,000			14,000			9,700			14,300		
Axial velocity, ft per sec	7.49			12.47			14.98			17.43			12.11			17.85		
Heads, ft	Hd	h_s	h_p	Hd	h_s	h_p	Hd	h_s	h_p	Hd	h_s	h_p	Hd	h_s	h_p	Hd	h_s	h_p
Section $r = 0.354$ ft	18.5	2.5	2.8	16.6	0.8	0.9	14.7	0.2	0.4	13.0		0.4	15.5	0.7	0.6	14.0	...	0.4
Section $r = 0.474$ ft	27.6	3.9	2.7	20.4	1.0	0.8	16.3	0.2	0.5	13.0	Less	0.5	17.4	0.6	0.7	15.5	...	0.5
Section $r = 0.594$ ft	34.3	4.4	2.3	23.5	1.1	1.0	18.0	0.3	0.8	12.9	than	0.8	18.2	0.4	1.1	17.3	0.1	0.7
Section $r = 0.714$ ft	41.6	4.7	4.7	25.7	0.9	1.6	18.8	0.2	1.3	12.2	0.05	1.2	17.4	0.2	1.6	19.0	0.1	1.2
Section $r = 0.833$ ft	47.2	4.6	5.2	28.5	1.0	2.3	20.0	0.2	1.8	12.1		1.7	16.7	0.1	2.3	21.7	0.1	1.6
$(Hd)_{avg}$, ft	35.8	..	..	23.7	..	..	18.1	..	..	12.6		..	17.3	..	..	17.7	..	..
$(Hd - h_s - h_p)_{avg}$, ft	28.0			21.5			16.6			11.6			15.6			16.8		
h_f, ft	0.4			1.0			1.4			2.0			0.9			2.0		
H, ft	27.6			20.5			15.2			9.6			14.7			14.8		
e_h, per cent	77.1			86.4			84.0			76.2			85.0			83.7		
e (Test overall effi.), per cent	53			77			81			77			76			82		
Test, bhp	81			67			58			48			...			...		

$\tan \beta_\infty = V_f/(u - V_{u3}/2) = 15.0/(55.1 - 10.5/2) = 0.3005$

$\beta_\infty = 16$ deg 44 min

Blade angle $(\beta_\infty + \alpha) = 19$ deg 24 min

Angle of attack, $\alpha = (19$ deg 24 min$) - (16$ deg 44 min$)$
$= 2$ deg 40 min

From Fig. 3, $C_L = 0.64$ and $C_{D0} = 0.0098$

Tan $\lambda = C_{D0}/C_L = 0.0098/0.64 = 0.0153$

$\lambda = 0$ deg 53 min

Cos $\lambda = 0.9999$

$(\beta_\infty + \lambda) = 16$ deg 44 min $+ 0$ deg 53 min $= 17$ deg 37 min

Sin $(\beta_\infty + \lambda) = 0.3026$

$c/t = 0.602$

From Equation [15] $H_d = 0.64 \times 0.602 \times (55.1/15.0)$
$\times (2704/64.4) \times (0.3026/0.9999)$

$H_d = 18.04$ ft

This head agrees with the assumed value within the accuracy of the computation. If the assumed and computed values do not agree, another assumption is made and the computation repeated.

The shock loss on entering the guide vanes is assumed to be

$$h_s = [V_{u3} - (V_{f4}/\tan \alpha_4)]^2/2g$$

$\alpha_4 = 66$ deg 25 min and $\tan \alpha_4 = 2.291$

$h_s = [10.5 - (15.0/2.291)]^2/64.4 = 0.25$ ft

The friction loss through the propeller resulting from the section drag is

$$h_p = C_L \frac{c}{t} \frac{v_\infty^2}{2g} \frac{\tan \lambda}{\sin \beta_\infty}$$

$\sin \beta_\infty = 0.2879$

$h_p = 0.64 \times 0.602 \times (2704/64.4) \times (0.0153/0.2879) = 0.86$ ft

The net head at the radius $r = 0.594$ ft uncorrected for friction in fixed passages is

$$H_d - h_s - h_p = 18.04 - 0.25 - 0.86 = 16.93 \text{ ft}$$

A similar computation is carried through for all five radii. The resulting heads are averaged on the basis of area by plotting head versus r^2 as shown in Fig. 4. The average ordinate is obtained by planimetering the area. For this problem the average head uncorrected for friction in the fixed passages is 16.6 ft.

All other losses in the pump test section can be represented by an equation of the type

$$h_f = K(V_f^2/2g)$$

where K is a characteristic of the water passages and should not vary with either capacity or speed, within the usual range of conditions. The magnitude of K can be approximated from tables of hydraulic friction losses. For the pump under consideration, K has been estimated at 0.4. Therefore, at 12,000 gpm

$$h_f = 0.0062 \times 15.0^2 = 1.4 \text{ ft}$$

The estimated net head is then

$$H = (H_d - h_s - h_p)_{avg} - h_f = 16.6 - 1.4 = 15.2 \text{ ft}$$

The hydraulic efficiency is

$$e_h = 15.2/18.1 = 84 \text{ per cent}$$

The results of all computations appear in Table 2 and a comparison of the computed and measured results is shown in Fig. 6. In addition to the normal angles shown in Fig. 2, and listed in Table 1, the comparison includes two points with the blades rotated about their own radial axes.

Equation [21] gives the pressure distribution which will tend to prevent radial flow and this was also found to be the pressure distribution resulting from the condition, H_d = constant. In this example, the computations indicate that H_d varied materially along the radius producing a radial flow and a corresponding additional energy loss. In Fig. 5, the pressure distribution has been computed at each radius from

$$\frac{p}{\gamma} = H_d - (V_f^2 + V_{u3}^2)/2g$$

and compared with the ideal pressure distribution, assuming $(H_d)_{avg}$ to be the head developed at each radius. The difference between the two curves is a measure of the head available to cause cross-flow.

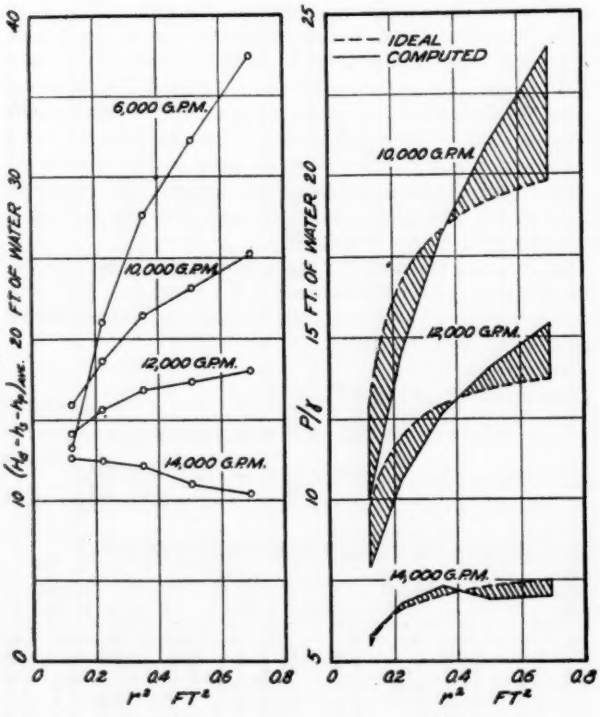


Fig. 4

Fig. 5

FIG. 4 CURVES USED FOR AVERAGING NET HEADS AT THE VARIOUS RADII GIVEN IN THE EXAMPLE

FIG. 5 PRESSURE DISTRIBUTION AT EACH RADIUS OF THE 20-IN. PFLEIDERER-TYPE PUMP

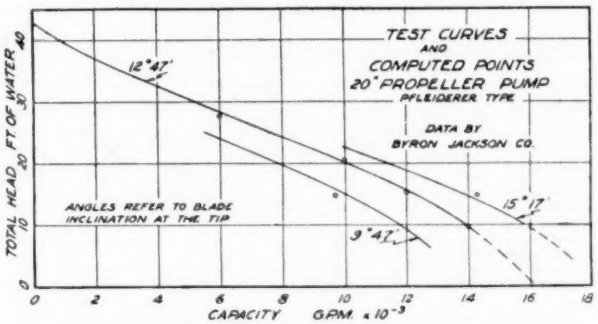


FIG. 6 TEST CURVES AND COMPUTED POINTS FOR THE 20-IN. PFLEIDERER-TYPE PROPELLER PUMP

Limitations of the Simplified Theory

The surprisingly good agreement between the computed and measured head-capacity characteristics should not obscure the fact that the theory, on which the method of computation is based, neglects a number of important factors. The computed results themselves show an inconsistency in the case of the point for 14,000 gpm on the curve for 12 deg 47 min, in that the hydraulic efficiency computed is 76.2 per cent while the measured overall efficiency is 77 per cent.

To emphasize the degree of approximation involved in this simplified theory, the neglected factors are enumerated and discussed.

(*a*) *Leakage Around the Blade Tips.* The leakage expressed as a percentage of the net discharge will decrease with increasing pump size. Assuming that the average pressure difference between the two sides of the blades is equal to the average developed head, the backflow around the blade tips from the upper to the lower side is approximately

$$Q_L = 0.6\, a \sqrt{(2g\, H_d)}$$

For the pump considered in the example the radial clearance was 0.03 in. and the length of each blade was 0.83 ft. Using

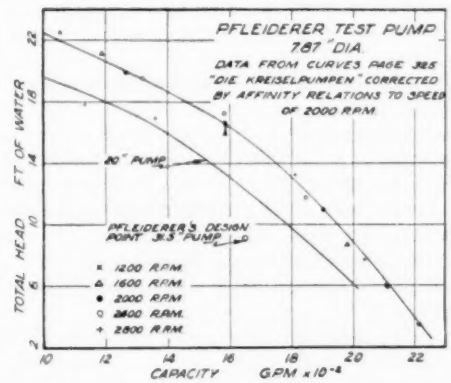


Fig. 7 Head-Capacity Characteristics for 7.87-In. Pfleiderer Test Pump

these values, the total backflow for the three blades is 0.135 cu ft per sec or 0.5 per cent of the net discharge.

In computing the head produced by a 32-in. pump of the same type as considered here, Pfleiderer assumes arbitrarily that the leakage and obstruction of the blades together account for an increase in the axial component amounting to 10 per cent.

A correction for the obstruction of the blades is unnecessary if the head is based upon airfoil characteristics with corrected angles of attack and Pfleiderer's correction really represents leakage only. In Fig. 7, his point for a 32-in. pump is compared with measured values for a geometrically similar 20-in. pump. The difference is believed to result from his excessive correction of the axial velocity.

In the theory of airplane propellers, where the tip of the blade is free, the circulation decreases from the center outward and an induced drag develops. In addition to the added resistance in the direction of motion, there is a change in the effective angle of attack. Because of the small leakage around the blade tips, conditions in a propeller pump with small clearance are believed to correspond closely to an infinite length of airfoil and no correction has been made for induced drag and the accompanying change in the angle of attack.

(*b*) *Interference of Adjacent Blades.* Numachi and Weinig find that in an ideal fluid adjacent blades at certain angles cause an increase in the lift coefficient, while experiments on airplane-propeller sections show a decrease for the same arrangement.[6,7] The magnitude of the effect depends upon the value of c/t and the propeller experiments do not apply to the pump studied so that a correction could not be made on the basis of experimental data. In the example, the method of computing the angle of attack includes a correction for blade interference.

(*c*) *Variation in Blade Section.* The lift and drag coefficients have been selected from data which are applicable to an infinite length of constant section and have been applied to a blade having a section which changes with radius. The method of handling this problem in the design of airplane propellers does not appear to be applicable to pumps with small clearance at the blade tips and no correction has been made.

(*d*) *Uniform Axial Velocity and No Tangential Velocity at Entrance.* This assumption is believed to correspond very closely with the actual conditions because the bell-mouthed inlet section is short and contracts rapidly. Such an inlet is known to give an almost uniform velocity distribution for some distance beyond the end of the converging section. The arms supporting the lower bearings are streamlined and tend to prevent prerotation although they probably do not eliminate it entirely.

(*e*) *Frictional Drag at Walls.* The relative velocity decreases to zero at both the hub and the housing. The combined effect is a rotation of the fluid in planes perpendicular to the shaft and in a direction opposite to that of the impeller. The water moves in a spiral path between the blades and even in a pump developing the same head at every radius, radial flow occurs.

(*f*) *Average Head.* The method of computing the head as the weighted average of the heads developed at each radius seemed at first to be based upon a very unsound assumption but closer study gave it a better appearance without fully justifying it. The axial component of velocity is probably uniform in the discharge plane and loss of head will not result from mixing with elements at different radii. Variations in pressure from the ideal curve result in cross-flow without, however, bringing about a substantial loss of mechanical energy. The remaining factor is the tangential component of velocity. Taking an average of the heads developed at each radius is equivalent to assuming that the dragging effect of concentric layers brings about a radial distribution of rotational velocities which corresponds approximately to the distribution which would result from a constant developed head at all radii. This situation is not the same as for the mixing streams of different axial velocities with the consequent Borda-Carnot shock loss and a rational method has not been found for estimating the loss resulting from the variation in developed head.

The tangential component of velocity also varies between the blades. This effect has been treated extensively by Spannhake[4] and will not be discussed here except to state that the method of computing the tangential velocity gives a properly averaged value.

(*g*) *Limits of Applicability.* As the rate of discharge is increased, negative lifts and heads appear first at the blade tips and develop inward. The method for computing the average head probably will not apply when the heads are partly negative but this limitation is of little practical importance.

At low rates of discharge, the angle of attack of the blades is above the "burble-point" and computation of the head is limited

---

[6] "Airfoil Theory of Propeller Turbines and Propeller Pumps With Special References to the Effects of Blade Interference Upon the Lift and Cavitation," by F. Numachi, The Technical Reports of the Tohoku Imperial University, vol. 8, no. 3, 1929, pp. 136–191.

[7] "Über die Winkelübertreibung von Turbinenschaufeln," by F. Weinig, *Wasserkraft und Wasserwirtschaft*, part 3, Feb. 2, 1934, pp. 25–31.

by the available experimental data on the lift and drag coefficients of the blade sections used. In the extreme case of zero net discharge from the pump as a whole, the outer sections of each blade are developing positive head and delivering a positive discharge which equals in total amount the downward flow through the sections near the hub. In these extreme cases, radial flow is important and the methods of computation suggested here are believed to be inapplicable to them.

(*h*) *Friction and Shock Losses.* Omitting frictional losses on the blade surfaces, which are represented by the drag coefficient, and the shock loss on entering the guide vanes, all other shock and surface-friction losses are represented by a single coefficient which is a characteristic of the water passages between the suction sump level and the point of discharge pressure measurement. Use of such a coefficient is basically correct but determination of its proper value for a given installation is difficult. Direct measurement of the friction loss with the impeller removed would give a value slightly too great because of the curvature of the guide vanes.

The shock loss on entering the guide vanes has been assumed to be proportional to the square of the difference in the tangential components at discharge from the runner and after entrance to the guide vanes. A coefficient of unity for the entering loss alone is probably excessive but there is an additional loss in the recovery of tangential velocity as pressure within the guide vanes.

Affinity Laws

The affinity laws are important aids in the design of centrifugal pumps and it is interesting to investigate their validity for the same pump at different speeds and for geometrically similar pumps of different sizes. Pfleiderer[3] gives data on the head-capacity characteristics of a 7.87-in. pump at five different speeds. The results have been corrected to a speed of 2000 rpm and plotted in Fig. 7. The agreement is very good except for the highest speed at low capacities. The same figure shows the corresponding test data for the 20-in. pump. Apparently these pumps are not strictly geometrically similar. The affinity laws used in making the corrections are exactly the same as for radial- and mixed-flow pumps.

One interesting feature of Fig. 7 is the comparison of the design point computed by Pfleiderer and the test data on a 20-in. geometrically similar pump. The discrepancy is believed to result from his excessive correction of the axial-flow component.

Conclusion

In conclusion it should be pointed out that no theory can be considered satisfactory until it has been found to apply to a wide variety of conditions. Subsequent comparisons of theory and experiment may indicate that the theory should be modified to take into account certain secondary phenomena which have combined in such a way as to neutralize each other in the pump used for comparison here. On the basis of these results, it appears that the modified method of Pfleiderer may be used to predict a portion of the head-capacity curve in the region of the design point.

Appendix

Nomenclature

a = area, sq ft
c = chord length of airfoil, ft
C_L = lift coefficient of airfoil
D = drag per unit of blade length, lb per ft
g = acceleration of gravity, ft per sec per sec
h = elevation head, ft
h_p = head lost in friction in the propeller, ft
h_s = head lost in shock at guide vanes, ft
h_f = all other friction and shock losses, ft
H = measured head developed by the pump, ft
H_d = developed head of the impeller, ft
K = coefficient of friction and shock losses
L = lift per unit of blade length, lb per ft
p = pressure, lb per sq in.
Q = capacity, cu ft per sec
Q_L = leakage, cu ft per sec
r = radius, ft
s = distance, ft
t = blade separation measured along the arc, ft
T = torque, ft-lb
T_a = net torque, ft-lb
u = velocity of a point on the blade, ft per sec
v = velocity of the fluid relative to the impeller, ft per sec
v_∞ = geometric mean value of the relative velocities in front and in rear of the impeller, ft per sec
V = absolute velocity of the fluid, ft per sec
V_u = tangential component of absolute velocity of fluid, ft per sec
V_f = axial component of absolute velocity of fluid, ft per sec
z = number of blades
α_4 = angle between the guide vanes at entrance and the pump axis, deg
β_∞ = angle between v_∞ and the direction of blade motion, deg
γ = weight per unit volume of fluid, lb per cu ft
$\Gamma = \int_0 V \cos(V, s)\, ds$ = circulation, ft sq per sec
Γ_s = circulation about a single airfoil section, ft sq per sec
$\tan \lambda = D/L$
$\rho = \gamma/g$ = density, lb sec per ft^4
ω = angular velocity, radians per sec

Subscripts:

0 — conditions before entrance into impeller
3 — conditions after leaving impeller
4 — conditions just after entrance into guide vanes.

Concerning the Degree of Accuracy of the Gibson Method of Measuring the Flow of Water[1]

By D. THOMA,[2] MÜNCHEN, GERMANY

The paper first discusses the limitations of various methods of measuring water which were in use when the Gibson method was first announced. These limitations of the current-meter method of flow measurement include: The effect of turbulent accessory motions which cause periodic velocity changes in the direction and magnitude of flow; the difficulty in obtaining readings close to the wall of the canal or closed conduit; and the necessity of determining the velocity at several points simultaneously due to temporary changes in distribution; and the expense involved in taking a number of such simultaneous readings. Reference is also made to the use of the pitot tube for taking readings close to the wall of the passage.

The greater part of the paper, devoted to a discussion of the Gibson method of water measurement, includes the derivation of formulas involved in this method of water measurement, its limits of accuracy, and the errors which may arise in its use. These errors, due to (*a*) accessory motions, (*b*) to false valuation of friction, (*c*) to friction of the mercury column, and (*d*) to inertia of the mercury column, are presented with derived formulas and approximate values together with suggestions for minimizing them.

IN ORDER to make the peculiarity of the Gibson method stand out clearly, a short criticism of other methods for measuring large quantities of water will first be given.

When the velocities of water are measured at a sufficient number of points in a cross-section to determine the flow per second through a canal or a closed conduit, the measurement will always be affected by the turbulent accessory motions which cause periodic changes in the velocities both in magnitude and direction. The problem in such measurements is to determine for a given interval of time the average value of the components of velocities vertical to the measuring cross-section.

When measuring with Woltman's current meter, the changes in magnitude of the water velocity are not serious because the revolutions of the meter are sufficiently proportional to the water velocity and, therefore, the average speed of the meter, which is obtained from the test, exactly corresponds to the average value of the water velocity. The changes in the direction caused by the accessory motions, however, are disturbing since the speed of the meter is affected by an additional side component of the flow so that the meter in general no longer indicates exactly the component of the velocity in the shaft direction, upon which it is exclusively dependent when the flow quantity is being determined. Lately, some meters have been invented which are practically independent of these errors within a sufficiently wide angle. However, these meters (blades separately fixed on spokes) are subject to errors through the collection of water plants and other thread-like debris. We hope soon to have current meters which are independent of accessory motions of flow and which will repel the debris like a pointed screw.

One more deficiency in the current-meter measurement is the impossibility of determining the water velocity close to the walls. To obviate this deficiency, special small meters, called wall meters, can be used near the edge of the measuring cross-section. These have been made by L. A. Ott[3] according to the author's directions, and make it possible to determine the velocity of the water one inch from the walls. However, wall meters are still being developed and are not yet commonly available. By using wall meters, and by interpolation of the water velocity in accordance with the new experiments on the distribution of the velocities close to a wall, the limit of the errors is reduced to a fraction of previous values.

Considerable difficulty arises with current measurements because of the need for determining the water velocities at a number of measuring points at the same time, this being necessary because the flow may temporarily undergo slow changes which cause errors, if the measurements are made at many points, one after the other. To repeat the measurements several times in order to decrease the errors takes a very long time and in most cases, therefore, is not done. To fill the requirements for simultaneous measurements is very expensive. For instance, 27 meters were used to determine the discharge of the water at the powerhouse at Aufkirchen on the Mitlere Isar.

If, on the other hand, one uses the pitot tube to determine the water velocity, it is possible by a suitable shaping of the tube (Prandtl's tube) to minimize the errors caused by fluctuations in the direction of flow. It is also possible to make dependable measurements at very short distances from the wall. Pitot tubes, however, give errors caused by fluctuations in the magni-

[1] Translated from "Mitteilungen des Hydraulischen Instituts der Technischen Hochschule München," Bull. 1, 1926, pp. 59–74, by E. B. Strowger, assistant hydraulic engineer, The Niagara Falls Power Company, Mem. A.S.M.E., and P. Olsen, engineering assistant, The Niagara Falls Power Company.

[2] Professor of hydraulic engineering, Technische Hochschule, München, Germany. Mem. A.S.M.E. Dr. Thoma was graduated from the Technical University at Munich, as a mechanical engineer, in 1906. From 1906 to 1908 he was employed as designer by the firm of Briegleb, Hansen & Co., of Gotha, manufacturers of water turbines, governors, etc. For the following two years he was associated with Prof. A. Föppl as assistant in technical mechanics. During this time he remained in touch with Briegleb, Hansen & Co., Gotha, and in 1910 became head of its governor department, remaining with the company for ten years. He became chief engineer of its turbine testing stations, as well as head of the governor department, in 1913. In 1915 he became superintendent of all work on designing water-power machinery and water-power plants. In 1920 Dr. Thoma was appointed professor of technical mechanics of the Technical University of Munich. He resigned in 1921 to become professor of hydraulics, and director of the Hydraulic Institute of the Technical University at München.

Contributed by the Hydraulics Division and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 4 to 8, 1933.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] L. A. Ott, Mathematik Institut, Kempten, Germany.

tude of the water velocity. Because the velocity head is proportional to the square of the water velocity, meters do not indicate, with quickly changing velocities, the average water velocity but the square root of the average square of the velocities, which is always larger than the average velocity. Furthermore, instruments are not yet developed which register the necessary readings when measuring at many points. The pitot tube is, therefore, in general not so applicable as the Woltman current meter for measurements where great accuracy is required.

The accuracy of the total flow, obtained from measurements with current meters, may be assumed to be about 1 per cent when the test is carefully prepared and performed under favorable conditions, and values accurately computed. The most favorable condition exists when the turbulence of the flow does not exceed the turbulence unavoidable even in a straight conduit at velocities above the critical velocity. If the water flow, however, contains strong eddies which are produced, for instance, by bends or by contractions in an improperly made intake, larger errors are possible because even with current meters of suitable shape the disturbing influence of the side components will be too large.

In many cases it is quite impossible to find or to arrange a cross-section in which the flow is steady. Also, the required number of meters and the necessary care to obtain the desired accuracy are often obstacles. However, as the weir, the traveling screen, and other methods for measuring large quantities of water are generally very expensive, one has either to be contented with the lesser accuracy of the current-meter measurement or give up the measurement altogether.

For that reason, a sensation was caused about five years ago (the "salt-velocity method" by Allen was not known at that time) when the American engineer, Norman R. Gibson, presented a new water-measuring method which promised to fill all desires and required only a comparatively simple apparatus.

The Gibson Method of Flow Measurement

The Gibson method is applicable for measurements in closed conduits and is founded on the pressure rise which takes place in such conduits when the water flow is brought to rest. For example, in conducting a test, a steady continuous flow is maintained after which the turbine gates are gradually closed, the increased pressure in the conduit being continuously registered by a pressure-time recorder, designed by Gibson. The difference between the initial water flow and the leakage flow, which goes through the gates when they are closed, is obtained from the pressure diagram. The dimensions of the conduit must, of course, be known. Since the small leakage water loss can be determined with great accuracy by a special test, it is, therefore, possible to determine the water flow per second at the open steady position of the gates.

The rise in the pressure when closing the gates is a result of the retardation which is forced on the water column in the pipe. This depends fundamentally upon the magnitude of the momentum (the impulse) which this water column had at the start and at the end of the closing of the turbine gates. For a pipe with known dimensions, the sum of all momentum quantities of the water particles contained in the pipe, or, expressed in a shorter way, the magnitude of momentum contained in the pipe, is dependent only upon the flow of water per second through the pipe and absolutely independent of the accessory motions. A short calculation leads, therefore, to the presumption that the result from the pressure diagram must be absolutely independent of the accessory velocities, which for other methods might be very disturbing. The sensibility of the pressure-recording apparatus can be increased almost as one desires when requisite care is used in the design and construction of this single instrument. We might hope that we could obtain the water quantity from this very exact pressure diagram without any errors in accordance with the exact physical nature of the method. Then it would be possible to make water measurements for water-power plants with the same general precision as for tank measurements.

The purpose of this paper is to discuss these considerations in detail, to determine the limits of eventual errors, and to find out how to arrange for the measurement in order to minimize the errors.

With the expression "errors" I will, of course, not censure or in any way try to diminish the splendid merits of the man who invented and developed this important method. Neither will I express any doubts about the high efficiencies on the turbines which Gibson has found by using his method in several large American water-power plants. The results which have been obtained in Germany, in both water-power plants and in laboratories, show that Gibson's results are possible of attainment considering the immense size of the American turbines and the hydraulic demands for which careful consideration has been given in the design of all constructional parts in these power houses.

When deducing the general formulas we will neglect the action of gravity and the differences in the pressure that are added to the static-pressure difference corresponding to differences in heights. The introduction of these pressures in general formulas would only lead to unnecessary complexities. We neglect, also, the elasticity of water and of the walls of the pipe.

The water pipe which the water enters from a large reservoir may be supposed to be straight and of uniform cross-section. We use the following symbols: c_0 is the average velocity of the water for steady continuous flow in a cross-section of the pipe at the beginning of the test $= \dfrac{\text{the quantity of water per second}}{\text{area of the pipe}}$; c is the average velocity (in a cross-section) of the flow during the test; and c_1 is the average velocity of the water after the test, i.e., the average velocity of the water corresponding to the leakage. During the test, the pressure p_C in the measuring cross-section C, and the pressure p_A at point A sufficiently distant in the reservoir are registered. (In practical tests the pressure measurement at A is registered by recording the changes in the water level in the reservoir with a water-level gage.)

To understand the fundamental laws for the method, we at first neglect the friction. The velocity of the water is then the same everywhere in the pipe. Accessory velocities do not arise.

If B, Fig. 1, is a cross-section of the pipe close to the inlet where the water velocity is already uniform all over the section, the difference in pressure $p_A - p_B$ is almost completely determined by the instantaneous value of the water velocity in the pipe. As the distance is very short in which the water velocity rises from zero (in the reservoir) to the value c, the acceleration for each water particle, caused by the change in position, is great compared with the change in the water velocity for a fixed point due to the decrease in the water quantity per second in the pipe. The pressure difference $p_A - p_B$ may, therefore, be calculated without any noticeable errors as there should be a steady flow. Therefore, the following holds true

$$p_A - p_B = \gamma \frac{c^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

The mass between the cross-sections B and C is $\dfrac{\gamma f l}{g}$, where f is the cross-sectional area of the pipe; l is the distance $(B - C)$; and γ is the weight of a unit volume of water. For the acceleration the force, $f(p_B - p_C)$, is available. Using the fundamental dynamic law we get

$$f(p_B - p_C) = \frac{\gamma f l}{g} \frac{dc}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

If we cancel f in Equation [2] and add Equation [1] we get the expression

$$p_A - p_C = \frac{\gamma}{g} l \frac{dc}{dt} + \gamma \frac{c^2}{2g}$$

We have now to take into consideration that the change in the water quantity on the way to the cross-section B also has some effect, and we do that by increasing the actual length l. If this corrected length is called L, we get

$$p_A - p_C = \frac{\gamma}{g} L \frac{dc}{dt} + \gamma \frac{c^2}{2g} \quad \ldots\ldots\ldots\ldots [3]$$

This is a differential equation of the first order with respect to c, which it is possible to integrate graphically or analytically when we have to take the changes in $p_A - p_C$ from the diagram obtained by the pressure recorder. One can, for instance, starting from the known water velocity c_1 corresponding to the leakage, reconstruct backward the changes in the water velocity and from that find the water velocity c_0 which was present at the start of the test.

In another method given by Gibson, the change in c is assumed and afterward corrected. This proceeding converges very fast and is very suitable for graphic integration which, of course, in this case is still more to be recommended because the change in $p_A - p_C$ is given graphically.

The chain of reasoning used to derive Equation [3] and on which also Gibson's deduction (1)[4] is founded, is not quite uninterrupted; when deducing Equation [1] it was assumed that the difference in pressure between the points A and B was as big as it should be at steady flow, but in reality there exists a non-stationary flow. There may be some doubt whether it is admissible to use the fundamental dynamic law without any correction on "the mass (existing) between the cross-sections B and C." The "mass" in that way defined does not contain the same individual parts at the end of the test as at the beginning. During the test, outgoing water, by passing the cross-section C, has left the "mass," and new water, by passing the cross-section B, has entered into the "mass." The dynamic fundamental equation refers, however, to a fixed system of mass always consisting of the same particles.

To remove all doubts we shall start from the general equation for the pressure conditions at unsteady flow without friction and eddies. Referring to Fig. 2, the following symbols will be used: x is the distance from one general point on the pipe axis to point A, v is the water velocity on the axis and on the expanded axis from the pipe, p is the pressure, and ϕ the velocity potential, which is defined so that at every instant and in every place $\frac{\delta\phi}{\delta x} = v$. The point A is supposed to lie on the expanded axis from the pipe and so far from the inlet that at this point the square of the velocity is negligible ($v_A^2 = 0$). The cross-section B is as close to the inlet as possible, but at such distance, that at every point of this section, the water velocity is equal to the pipe velocity c. The cross-section C is the measuring section. The values of v, p, and ϕ in A, B, and C will be designated by v_A, p_A, ϕ_A, v_B, p_B, etc., respectively.

The relation between v, p, and ϕ is given by the well-known general equation

$$\frac{\gamma}{g} \frac{\delta\phi}{\delta t} + \gamma \frac{v^2}{2g} = -p + \psi(t)$$

in which $\psi(t)$ is a function of the time alone (not of x) and depends on the way in which ϕ is chosen. To wit, ϕ being defined by $\frac{\delta\phi}{\delta x} = v$, only $\frac{\delta\phi}{\delta x}$ is prescribed for a given flow and any function of the time may be added. Therefore, we are at liberty to stipulate that during the whole test $\phi_A = 0$. In applying the general equation to the point A, we get

$$\frac{\gamma}{g} \frac{\delta\phi_A}{\delta t} + \gamma \frac{v_A^2}{2g} = -p_A + \psi(t)$$

Considering that $\phi_A = 0 =$ constant and therefore $\frac{\delta\phi_A}{\delta t} = 0$, and that $v_A^2 = 0$, we get from this $\psi(t) = p_A$. By introducing this, the general equation is transformed to

$$\frac{\gamma}{g} \frac{\delta\phi}{\delta t} + \gamma \frac{v^2}{2g} = p_A - p \quad \ldots\ldots\ldots\ldots [4]$$

First of all we now have to determine ϕ for the cross-section B. Considering that $\phi_A = 0$ and $\frac{\delta\phi}{\delta x} = v$ we get

$$\phi_B = \int_A^B v\, dx$$

In order to make the integration we express the velocity at any point of the axis between A and B as a fraction of the present pipe velocity c by the equation $v = \alpha c$. Then

$$\phi_B = c \int_A^B \alpha\, dx$$

The value of α for the point in the section B is 1. In order to compute $\int_A^B \alpha\, dx$, the values of α for a sufficient number of other points between A and B must be found. This is easily effected by drawing the streamlines, either by estimating or by one of the several well-known graphical methods. The values of α will be found by considering that α varies inversely as the areas of the cross-sections of the fluid tube confined by the streamlines adjacent to the axis. The integral $\int_A^B \alpha\, dx$ can then be computed. It has a length the dimension of which we will designate by l' and which may be considered as the reduced length of the inlet; which may be longer or shorter than the distance between the cross-section B and the reservoir wall, depending on the shape of the inlet. Therefore

$$\phi_B = l'c \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

The velocity potential in the measuring cross-section C results in

$$\phi_C = \phi_B + \int_B^C v\, dx = \phi + lc$$

and when considering Equation [5] we get

$$\phi_C = l'c + lc = Lc \quad \ldots\ldots\ldots\ldots\ldots [6]$$

By differentiating with respect to time from Equations [5] and [6]

$$\frac{\delta\phi_B}{\delta t} = l' \frac{dc}{dt} \quad \text{and} \quad \frac{\delta\phi_C}{\delta t} = L \frac{dc}{dt}$$

[4] Numbers in parentheses refer to similarly numbered references at the end of the report.

If we now at last combine these values with Equation [4] we get[5]

$$p_A - p_B = \frac{\gamma}{g} l' \frac{dc}{dt} + \gamma \frac{c^2}{2g} \quad [7]$$

and

$$p_A - p_C = \frac{\gamma}{g} L \frac{dc}{dt} + \gamma \frac{c^2}{2g} \quad [8]$$

The last equation is identical with Equation [3]. The considerations made at first, which theoretically were not quite indisputable, have given a correct result. This also shows that the suggestion given in the literature (2) to substitute the last term in Equation [3] by the expression $\frac{c_1^2}{2g} - \frac{(c_1 - c)^2}{2g}$ is entirely out of order.

We shall now study the method, considering friction. For the water-way from the reservoir to the section B, it is possible to use the considerations above for flow without friction, because the friction on the short distance to B, without doubt, may be neglected. In order to follow the conditions between B and C it is convenient to use the law of impulse and momentum. As we know, it is permitted to deal with the momentum which is contained in any space, and also with the momentum which enters or goes out from this space, in the same way as if the momentum were a material substance, which is carried along with the water and is produced or consumed by outside forces.

The momentum which is contained in the space between B and C is indicated by J. If i_B is the momentum which the inflowing water has supplied through the section B during any length of time, $\frac{di_B}{dt}$ is the impulse current through the section B. In the same way the impulse current through section C is signified by $\frac{di_C}{dt}$. From the law of impulse and momentum, it follows that the increase per second in momentum in the space between B and C is equal to the momentum supplied per second through B with the outgoing momentum through C deducted and with all external forces acting in the direction of flow added, i.e.,

$$\frac{dJ}{dt} = \frac{di_B}{dt} - \frac{di_C}{dt} + \Sigma P \quad [9]$$

The next step is to determine J. Suppose dm is the mass of one small water particle, which has the length dx, the cross-section df, and the velocity v. The momentum for this small particle is $dm\,v$ and, because $dm = \frac{\gamma}{g} dx\,df$, it is also equal to $\frac{\gamma}{g} dx\,df\,v$. The momentum for all the water between the cross-sections on the distance dx from each other, therefore, is

$$dJ = \int \frac{\gamma}{g} dx\,df\,v = \frac{\gamma}{g} dx \int v\,df$$

Since $\int v\,df = cf$ it is possible also to write $dJ = \frac{\gamma}{g} cf\,dx$, from which the momentum of the water between B and C is

$$J = \frac{\gamma}{g} cf\,l \quad [10]$$

which is dependent only on the average velocity and, therefore, also on the quantity of water per second. It is absolutely independent of the distribution and all accessory velocities.

Differentiating with respect to time we get from Equation [10]

$$\frac{dJ}{dt} = \frac{\gamma}{g} fl \frac{dc}{dt} \quad [11]$$

It is also easy to express the impulse current through the cross-section B, because the friction has not yet been acting and the velocity is the same at all points. The inflowing mass per second is $\frac{\gamma}{g} cf$ and therefore the impulse current is

$$\frac{di_B}{dt} = \frac{\gamma}{g} fc^2 \quad [12]$$

The impulse current through the cross-section C should, of course, be just as big if the distribution were uniform both in place and time. However, the flow is not uniform at that point on account of the friction. We can divide the instantaneous velocity v at any point in the cross-section into one component v_m which is the average value for the place in question and one component of oscillating velocity v'. This division is shown in Fig. 3, where the full line refers to the instantaneous value of the velocities and the broken line to the average value with respect to time. Accordingly, if we write

$$v = v_m + v' \quad [13]$$

the value of momentum, which during the time dt is discharged through the element df in section C (mass per second times velocity) is equal to

$$\frac{\gamma}{g} (v_m + v')^2\,df$$

Further, if Δt is a short time interval which is large compared with the period for the fluctuations in v', but still short compared with the time during which the turbine gates are closing, we will obtain the value of momentum which during the time Δt is discharged through the cross-section element df as

$$\frac{\gamma}{g} \int_{(\Delta t)} (v_m + v')^2\,df$$

or as

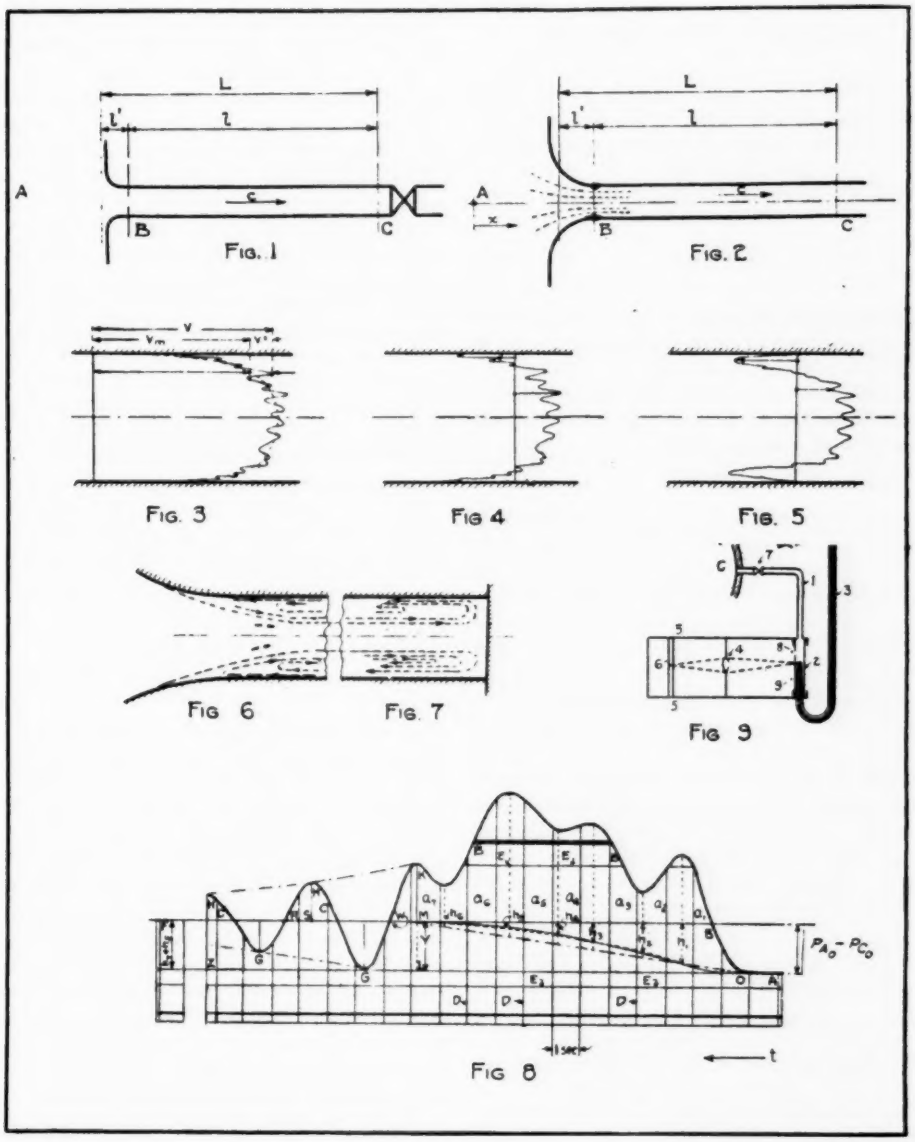
$$\frac{\gamma}{g} df \int_{(\Delta t)} v_m^2\,dt + \frac{2\gamma}{g} df \int_{(\Delta t)} v_m v'\,dt + \frac{\gamma}{g} df \int v'^2\,dt$$

Because v is constant in the element of the cross-section considered, the first integral is equal to $v_m^2 \Delta t$. The second integral is zero, because v_m is constant and v' is the variation in the actual instantaneous velocity from the average value with respect to time so that $\int v'dt = 0$. The outgoing impulse during the time Δt through an element in the section is, therefore

$$\frac{\gamma}{g} \Delta t\, v_m^2\,df + \frac{\gamma}{g} df \int_{(\Delta t)} v'^2\,dt$$

Taking the sum for the whole cross-section, the outgoing impulse during the time t is

[5] The method chosen here for the demonstrations has the advantage that it is also applicable to bent pipes. At the same time it shows that the length in a bend is not to be measured along the pipe axis but along the line in which the water velocity has the average speed. This line lies about $\frac{r^2}{4R}$ closer to the center of the bend than the pipe center line does (r = the radius of the pipe, R = the radius of the bend to the center line of the pipe) and is for that reason shorter. If one does not consider these things the length of the pipe will be used with too high a value and the water flow will be too little. The difference is, however, very small and it will cause (e.g., for the power plant at Queenston, where the pipe has two bends) an error in the water flow of only about —0.1 per cent.



$$\Delta ic = \frac{\gamma}{g}\Delta t \int_{(f)} v_m^2\, df + \frac{\gamma}{g}\int_{(f)} df \int_{(\Delta t)} v'^2\, dt \quad \ldots\ldots [14]$$

The second term on the right side we may consider as the product of $f\Delta t$ and one average value of the square of the variable velocity v' for the whole section, the average taken with respect both to time and place. If we use the symbol v_I^2 for this average value, Equation [14] takes the form

$$\Delta ic = \frac{\gamma}{g}\Delta t \int_{(f)} v_m^2\, df + \frac{\gamma}{g}\Delta t\, f\, v_I^2 \quad \ldots\ldots [15]$$

As the distinction between Δt and dt was necessary only for the definition of the average value v_I^2, it is now possible again to write dt instead of Δt and dic instead of Δic. Equation [15] then takes the form

$$\frac{dic}{dt} = \frac{\gamma}{g}\int_{(f)} v_m^2\, df + \frac{\gamma}{g} f v_I^2 \quad \ldots\ldots [16]$$

For further simplification we presume

$$v_m = c + v'' \quad \ldots\ldots [17]$$

where v'' is then the deviation of the average velocity with respect to time in the considered element of the cross-section from the average velocity in the whole cross-section. By substituting $v_m^2 = c^2 + 2cv'' + v''^2$ in Equation [16] we get

$$\frac{dic}{dt} = \frac{\gamma}{g}\int_{(f)} c^2 df + \frac{2\gamma}{g}\int_{(f)} cv''\, df + \frac{\gamma}{g}\int_{(f)} v''^2\, df + \frac{\gamma}{g} f v_I^2 \quad \ldots\ldots [18]$$

As c is constant, the first integral is simplified to $c^2 f$. The second integral is equal to zero because v'' is the variation from the average value c and, therefore, $\int_{(f)} v''\, df$ is zero. Finally, if we now use v_{II}^2 instead of the average value of the square of v'', i.e., $\int_{(f)} v''^2\, df = v_{II}^2 f$, Equation [18] takes the form

$$\frac{dic}{dt} = \frac{\gamma}{g} f c^2 + \frac{\gamma}{g} f v_{II}^2 + \frac{\gamma}{g} f v_I^2 \quad \ldots\ldots [19]$$

Substituting Equations [11], [12], and [19] in Equation [9] the first term of $\frac{dic}{dt}$ is eliminated by $\frac{di_B}{dt}$ and

$$\frac{\gamma}{g} fl \frac{dc}{dt} = -\frac{\gamma}{g} f v_{II}^2 - \frac{\gamma}{g} f v_I^2 + \Sigma P$$

The forces acting on the water volume consist of (a), the force corresponding to the pressure difference between B and C, and (b), the force R caused by the friction on the pipe wall acting on the water opposite to the direction of flow. By substituting $\Sigma P = f(p_B - p_C) - R$, the equation above takes the form

$$\frac{\gamma}{g} fl \frac{dc}{dt} = -\frac{\gamma}{g} f v_{II}^2 - \frac{\gamma}{g} f v_I^2 + f(p_B - p_C) - R \ldots . \quad [20]$$

In order to make it possible to express $p_B - p_C$ by the observed pressure difference $p_A - p_C$ we have to eliminate $p_A - p_B$ in Equation [7]. Though when deducing Equation [7] we had assumed no friction acting, it is possible to use the equation for actual flow, as it is shown before, because the friction on the way from A to B hardly ever comes to action.

If we substitute

$$p_B - p_C = p_A - p_C - (p_A - p_B) = p_A - p_C - \frac{\gamma}{g} l' \frac{dc}{dt} - \gamma \frac{c^2}{2g}$$

in Equation [20] and again set $l + l' = L$, then it follows that

$$\frac{\gamma}{g} fL \frac{dc}{dt} = f(p_A - p_C) - \gamma f \frac{c^2}{2g} - \frac{\gamma}{g} f(v_{II}^2 + v_I^2) - R \ldots [21]$$

In this expression R, besides v_{II}^2 and v_I^2, is not yet known. The simplest assumption which one can make for the action of the friction during the closing of the gates is that it is of the same magnitude as it should be at a continuous flow with the same quantity of water per second. This premise is also a fundamental point in Gibson's deduction. Because it is possible in this case to assume, with very sufficient accuracy, that the magnitude of the friction at constant flow is proportional to the square of the average water velocity, we therefore get

$$R = \frac{c^2}{c_0^2} R_0 \ldots \ldots \ldots \ldots \ldots \ldots [22]$$

where R_0 is the value at constant flow before the test. R_0 we can determine, considering Equation [21] $\left(\frac{dc}{dt} = 0\right)$ for the steady continuous conditions before the test, to be

$$R_0 = f(p_{A_0} - p_{C_0}) - \gamma f \frac{c_0^2}{2g} - \frac{\gamma}{g} f(v_{II_0}^2 + v_{I_0}^2) \ldots . [23]$$

If we substitute the value of R, which is obtained from Equations [22] and [23] in Equation [21] and arrange in order, we get after a few calculations

$$-\frac{dc}{dt} = \frac{g}{\gamma L}\left[(p_C - p_A) + \frac{\gamma}{g}(v_{II}^2 + v_I^2) + \frac{c^2}{c_0^2}\left\{(p_{A_0} - p_{C_0}) - \frac{\gamma}{g}(v_{II_0}^2 + v_{I_0}^2)\right\}\right] \ldots . [24]$$

The second term within the brackets indicates, for every instant of time, the influence of the accessory motions (existing at that instant). The last term, $\frac{\gamma}{g}(v_{II_0}^2 + v_{I_0}^2)$, takes care of the fact that the friction force, which at continuous steady flow is transmitted from the walls of the pipe to the water, would be judged too high, if the accessory motions were neglected.

If we neglect the accessory velocities, Equation [24] takes the form

$$-\frac{dc}{dt} = \frac{g}{\gamma L}\left[(p_C - p_A) + \frac{c^2}{c_0^2}(p_{A_0} - p_{C_0})\right]$$

The valuation used by Gibson corresponds to this equation.

We could also get this equation if we supposed that v_{II}^2 and v_I^2 during the test decrease proportionally with c^2. The two last terms in Equation [24] should then balance each other. This assumption, however, would not be correct. We notice this very readily if we consider the case when the quantity of water per second is decreased rapidly from the value at the start to zero. All water particles are then for the same short time under the action of the same strong pressure gradient so that the velocity of each particle is subjected to a change by the same amount but not in the same proportion (Fig. 4). The values of v_{II} and v_I have then not changed during the sudden decrease in the water quantity to zero. If the closing time is longer, the action of the wall friction, which tries to keep hold of the back-flowing water, becomes active up to a marked distance from the wall so that the distribution of the water indicated in Fig. 5 exists after the closure. In any case the valuation $v_{II}^2 + v_I^2 = v_{II_0}^2 + v_{I_0}^2$ will come closer to the truth than the presumption given above. If we apply Equation [24] on a complete closure considering the time, we get with $\frac{dc}{dt} = 0$ and $c = 0$

$$p_C - p_A = -\frac{\gamma}{g}(v_{II}^2 + v_I^2)$$

In the measuring cross-section, therefore, a pressure below the normal will exist until the accessory motions have died out. (This does not appear in the diagrams published by Gibson on account of the strong natural period of oscillation in the mercury column.) One understands this action if one considers that the water in the pipe in spite of the decrease of the water-flow to zero has not come to rest. The core of the water column flows forward while an edge current runs backward. Although it is true that the impulse *contained* has become *zero* and remains at zero, that is not the case with the impulse *current* through the section; the measuring cross-section C sends water upward and receives returning water. Because the impulse contained in the pipe above the cross-section does not change any more, it is necessary that this impulse current be counter-balanced by a pressure below the normal in the measuring section.

Of course, these circumstances assist during the closure in keeping the pressure in the measuring section from increasing as much as it would without accessory motions. The blockade of the pressure takes place near the inlet, where the back-flowing water again turns; the back-flowing water at the walls cannot press itself into the reservoir with higher pressures. Therefore, it turns and forces a reduction in the cross-section of the fresh water entering from the reservoir with a corresponding increase in the velocity which affects the additional pressure drop. Fig. 6 indicates this action but it gives, of course, only an imperfect picture because it refers in reality to non-stationary conditions, which cannot be represented completely by drawing current lines, and also because our present means of description of hydraulic actions are not well suited to show the non-stationary conditions.

Also, there is at the lower end of the pipe, an alternation in the direction of flow. If we could measure the average pressure immediately in front of a slide, which shuts off the pipe, Fig. 7, the pressure would not be below normal.

Therefore, the hope that the Gibson method of water measurement (contrary to the methods mentioned above) is independent of accessory motions, is unfortunately unfulfilled. It, therefore, remains for us to determine the order of magnitude of the possible errors.

Because no information is published about the magnitude of v_1^2, one has to resort to an indirect estimation. It is known that in turbulent flow, measurements of the velocity with the pitot tube give, according to the foregoing reasons, a water velocity about 1 to 2 per cent too large. A simple calculation shows that the relative error in the velocity measurement is $\frac{v_1^2}{2v_m^2}$, where v_m is the average velocity at the point of measuring and v_1^2 is the average value of the square of the variable additional velocity varying with respect to time. Within the possible limits for the degree of accuracy required, it is permitted to use the average values in the measuring section (v_1 and c) instead of v_1 and v_m. To avoid too large errors, we obtain for the beginning of a test run

$$v_{I_0}^2 = 0.02\, c_0^2$$

if we cautiously state that the pitot-tube measurements give an error of 1 per cent.

Two methods can be used for the valuation of $v_{II_0}^2$. On one hand, the known formulas for the water distribution in long, straight, smooth pipes, can be used, from which $v_{II_0}^2 = 0.02\, c_0^2$. The conditions are then, however, judged too favorably because the turbine conduits hardly ever are as straight and smooth as the pipes with which the formulas for the water distribution were obtained in the laboratory and also because in most cases the flow is affected by dissymmetries, which are caused by an unsymmetrical inlet, and result in an increase in $v_{II_0}^2$. For these reasons it is perhaps better to base our calculations on the known value α, which gives the proportion between the actual velocity energy flowing through a cross-section and the energy corresponding to the average water velocity. From present information, α ranges between 1.085 and 1.15. The impulse transported, on account of the irregularity in the distribution in the water velocity, is affected in the proportion of about 3 to 1 less than the energy transported. One can, therefore, presume $v_{II_0}^2 = 0.028$ to $0.05\, c_0^2$ or, as an average, about $v_{II_0}^2 = 0.04\, c_0^2$. Therefore

$$v_{II_0}^2 + v_{I_0}^2 = 0.06\, c_0^2$$

for the beginning of the experiment.

If we choose the afore-mentioned valuation

$$v_{II}^2 + v_I^2 = v_{II_0}^2 + v_{I_0}^2$$

and further suppose that c, during the observed time, decreases uniformly from c_0 to the end value zero, then in Equation [24], the expression

$$\frac{\gamma}{g}\,(v_{II}^2 + v_I^2) - \frac{c^2}{c_0^2}\,\frac{\gamma}{g}\,(v_{II_0}^2 + v_{I_0}^2)$$

(which may be considered as the term for the average error for the whole observed time) is equal to $\frac{2}{3}\,\frac{\gamma}{g}\,0.06\, c_0^2$.

The circumstance that $v_{II}^2 + v_I^2$ does not stay quite invariable but instead lies between $v_{II_0}^2 + v_{I_0}^2$ and $\frac{c^2}{c_0^2}\,(v_{II_0}^2 + v_{I_0}^2)$ we take into account by decreasing the factor $\frac{2}{3}$ to $\frac{1}{3}$. By that we get the average value of the error as

$$0.02\,\frac{\gamma}{g}\,c_0^2 = 0.04\gamma\,\frac{c_0^2}{2g}$$

From the pressure diagram published by Gibson, it is not possible to get $\frac{c_0^2}{2g}$, because from the diagrams the resulting value of $\frac{p_{A_0} - p_{C_0}}{\gamma}$ is composed of the velocity head and the friction-loss head. In one numerical example given by Gibson, which refers to a pressure diagram not reproduced here, there is a note from which it appears that two thirds of $\frac{p_{A_0} - p_{C_0}}{\gamma}$ belongs to the velocity head and one-third to the friction-loss head. If we suppose that this proportion exists also for Gibson's pressure diagram (3) reproduced in Fig. 8, it follows that the error in the result for this pressure diagram, caused by the term for the error, is —1.5 per cent, while the error for the diagram in the numerical example is —0.9 per cent. The true water quantities should for that reason have been about 1.5 per cent and 0.9 per cent greater, respectively, than given by Gibson.

However, Gibson reports errors averaging +0.1 per cent from tests made on his method in the hydraulic laboratory at Cornell University. During the tests, the actual quantity of water was determined by the only indisputable method, measuring by means of a calibrated tank.

Therefore, there must be other sources of errors, which act in the opposite direction. In reality, the assumption made when deducing Equation [24], that the friction during the closing is just as great as at continuous steady flow with the same amount of water, Equation [22], is not quite true. If, for instance, the condition after complete stoppage of the water flow is considered, the friction according to this supposition should be zero. In reality, however, a downward directed friction force is carried over on the water from the pipe wall which touches the backflowing edge current. This condition is also active during the flow, and decreases the value of friction below the amount which should be present for equal quantities of water at steady flow.

If the errors which are made by this valuation of the friction are estimated to be between $\frac{1}{20}$ and $\frac{1}{10}$ of the friction at continuous flow at the beginning of the measurement, the result obtained from the pressure diagram, Fig. 8, would then be affected with an error from +0.9 to +1.8 per cent for this reason, and the pressure diagram in the numerical example with an error from +0.6 to +1.1 per cent.

It must be taken into consideration that the statements for the errors are founded on very rough estimates. Thus it is possible as a final outcome to say only that, when calculating the results from the diagrams given by Gibson, two opposite acting errors arise, of which the first has an order of magnitude of —1 per cent, and the second +1 per cent.

One more objection, which, however, does not refer to Gibson's method, itself, but to the calculation of the results from the measurements, is connected with the apparatus used by Gibson for recording the pressure diagram. This objection, therefore, has to be taken into closer consideration. The main parts of the pressure-recording apparatus are shown in Fig. 9. The glass tube 2 is connected through the water-filled tube 1 to the conduit at the measuring cross-section C; the lower end of the glass tube is connected by a U-tube to the smaller riser pipe 3. The riser pipe and the glass tube are filled with mercury as illustrated in the figure. The pressure fluctuations in the measuring section cause oscillations in the mercury column which are recorded photographically by illuminating the glass tube from the right. The lens 4 casts through the narrow vertical slot 5–5, an image of the mercury column on the photographic film 6 which is moved in a horizontal direction by a clock-work mechanism. A cord pendulum, not shown in the figure, covers the slot for a short time during each second and in that way marks the seconds on the film.

This arrangement has considerable experimental advantages

because, on one hand, only friction from fluid exists and because, on the other hand, the instrument can be calibrated easily before each test. Calibration is accomplished by adjusting different pressures in the tube, 1, by means of the valve 7, the pressures being measured by determining the difference in height between the levels of the mercury columns, which are photographically recorded. The pictures of two fine wires, 8 and 9, which are stretched across the glass tube, serve as marks for the measurements and consequently one is independent of accidental displacements of the film in a vertical direction.

Although this arrangement has great experimental advantages it also has the disadvantage of having a long natural period of oscillation. Consequently, the instrument does not show the instantaneous value of the pressure in the measuring section. It also oscillates a long time after the turbine gates are completely closed. To separate the influence of the oscillations which occur after closure, Gibson first determines the point at which the recorded pressure line runs into the line for the damped oscillations which remain after entirely closing the gates. The point k in Fig. 8, indicates the time when the gates reached the closed position. Gibson now considers, as the end of the authoritative pressure diagram for the calculations, the time at which the line for the damped oscillations first reaches a maximum or minimum; the position of this maximum or minimum is thereby exactly determined by a small additional calculation. One might now ask if this determination of the end position of the diagram causes errors. Of the terms standing on the right side in the equation for $\frac{dc}{dt}$, Equation [24], the first one predominates. For that reason, it is of great importance to take correctly from the diagram the time integral of the pressure difference or the time integral of the pressure in the measuring section, because the pressure in the inlet changes very little and furthermore is recorded by an independent apparatus. If the motion of the mercury column should take place without friction, the stipulations selected by Gibson for the limits of the diagram should correspond to the real facts; the impulse in the mercury column is zero at the end as well as at the start of the observation, because at both times it is at rest and, therefore, the content of the diagram surface corresponds to the time integral of the pressure in the measuring section. When, however, in reality the fluid friction of the motion of the mercury column has to be overcome, there is also impulse absorbed by the friction force during the observation. When, as in this case, it is permitted to consider the friction proportional to the velocity of the mercury column, it is easily shown that the total impulse absorbed by the friction is dependent only on the difference in the pressure at the beginning and at the end of the observation.

In order to prove this specifically, we simply indicate with p the pressure in the measuring section and with p', the pressure recorded by the apparatus. The error in the result, which is obtained if one takes into consideration only the predominating first term in Equation [24], is

$$\Delta c = \frac{g}{\gamma L} \left(\int p \, dt - \int p' \, dt \right)$$

It is possible to write the equation for the motion of the mercury column in the following form

$$a \frac{d^2p'}{dt^2} + k \frac{dp'}{dt} + p' = p \quad \ldots\ldots\ldots\ldots [25]$$

where a is a coefficient depending on the mass of the mercury column, its cross-section, etc., and k is a friction coefficient which we might consider constant at deflections which are not too large. By integrating Equation [25] once for the time between the beginning (index 0) and the end (index 1) of the observation:

$$a \left[\left(\frac{dp'}{dt} \right)_1 - \left(\frac{dp'}{dt} \right)_0 \right] + k \, (p_1' - p_0') + \int_{t_0}^{t_1} p' \, dt = \int_{t_0}^{t_1} p dt$$

In this expression the first term is zero because the mercury column is at rest at the beginning and at the end of the observation. The third term on the left side is the time integral for the pressure diagram which, therefore, is too small by the amount $k(p_1' - p_0')$. It is possible to calculate the heretofore unknown constant k from two quantities which we can take from the diagrams, namely the time T for the natural period of the apparatus and the proportion ψ, in which each deflection of the damped oscillation stands to the following deflection on the same side. If one indicates the (almost) constant pressure, which appears after complete closing of the gates by p_2, the general solution of Equation [25] results in

$$p' = p_2 + e^{-\frac{k}{2a} t} \left[A \sin \sqrt{\frac{1}{a} - \frac{k^2}{4a^2}} \, t + B \cos \sqrt{\frac{1}{a} - \frac{k^2}{4a^2}} \, t \right]$$

and, after some calculations the expression

$$k = \frac{T \log_e \psi}{2\pi^2 + \frac{(\log_e \psi)^2}{2}}$$

The second term in the denominator one can neglect for the values of ψ here occurring. The time integral of the pressure difference according to this is too small by an amount of

$$\frac{T \log_e \psi}{2\pi^2} (p_1' - p_0')$$

$p_1' - p_0'$ we can read from the diagrams, but no unit for measuring is given. One can, nevertheless, give the proportional error in the quantity of water.

From the first diagram (Fig. 8) $\psi = 1.54$, $T = 3.8$ sec, and the error in the water quantity = —0.83 per cent.

For the second diagram (numerical example, Gibson's Fig. 10) $\psi = 1.57$, $T = 4.0$ sec, and the error in the water quantity = —0.62 per cent.

The true quantity of water has in both cases been greater than it was calculated to be according to these values. In both diagrams the first extreme value of the recorded pressure which appears after the complete closing of the gates is a maximum. If the first recorded extreme value should have been a minimum the error should have been positive and, concerning the amount, much smaller. From Gibson's publications in which the calculations of the test on the method at Cornell University appear, it is not possible to see in which direction the error has been acting.

In the deductions above it was presumed that only the time integral of the recorded pressure was of any importance, corresponding to the excluding of the three last terms in Equation [24]. Of these, the two last ones are very small correction terms, but the term $\frac{c^2}{c_0^2} (p_{A_0} - p_{c_0})$ is worthy of consideration.

It is now easy to understand that an error is added by the inertia of the mercury column which tends to increase apparently the quantity of water; at the beginning of the observation the recorded pressure stays below the real pressure and this causes the water velocity at the beginning and, therefore, also the above-mentioned term to be judged too great. It is not possible to determine the value of this latter error as exactly as the errors covered before. To get some idea of the valuation, the condition for the recording apparatus was calculated, presuming the simple case that $\frac{dc}{dt}$ is constant during the observation. By this it was

shown that the relative error in the quantity of water calculated according to Gibson's direction, is

$$+\frac{1}{2\pi^2}\left(\frac{T}{T_s}\right)^2 \frac{p_{A_0} - p_{c_0}}{p_m}$$

where T is the time for the natural period of the apparatus, T_s is the time for closing the gates and p_m is the average value of

$$p_c - p_A + \frac{c^2}{c_0^2}(p_{A_0} - p_{c_0})$$

during the observation. Of course, the retardation of the water velocity during the observation is in reality not uniform, but nevertheless it is possible to use the above formula to determine the magnitude of the errors.

By the formula, it follows, for the first diagram Fig. 8, that the error in the water quantity = +0.3 per cent. For the second diagram (numerical example, Gibson's Fig. 10) the error in water quantity = +0.09 per cent. These are, therefore, unimportant and disappear compared with the other errors.

To sum up, it can be said the four sources of errors have been ascertained to be:

		Order of magnitude, per cent
1	Error due to accessory motions	—1
2	Error through false valuation of friction	+1
3	Error through friction of the mercury column	—0.5 to —1
4	Error through inertia of the mercury column	+0.1 to +0.3

Besides these sources of error (as in every method) there are still others such as those caused by lack of precision of instruments, errors in measurement of the pipe line, inaccuracies in evaluation, etc. These errors, however, may with sufficient care be reduced to insignificant magnitude and need not be separately treated here since they affect every water measurement in a similar manner.

Of the above four errors it is possible to eliminate the third one in the calculations of the result. The fourth is so insignificant that a rough valuation is sufficient. The errors Nos. 1 and 2 are determinative in fixing the accuracy of the measurement. But these errors, also, are small so that one might well say in drawing a conclusion regarding the Gibson measurement in its existing form, that the accuracy under favorable circumstances is approximately equal to the aforementioned current-meter measurements taken with great care under favorable circumstances.

For the review of the specified magnitude of errors it should be noted that they are drawn only from the diagrams published by Gibson and furthermore that in the evaluation of error No. 1 it has been assumed that the stream is as smooth as can be practically attained in a straight pipe. In case, however, the measurement section lies behind a bend as occasionally appears to have been the case (e.g., at the Queenston Plant, denoted by Gibson as a typical plant) greater amounts of error No. 1 have to be reckoned with. When, for example, one-tenth of the measured cross-section is taken up by the dead water area adjoining a bend, v_{11}^2 increases $\frac{1}{9}$, i.e., from about 0.04 to 0.15. A measurement section with smooth current is desired for the Gibson water measurement method even as for the current-meter measurement. For the test at Cornell University, the measuring section lay in a long straight stretch; the conditions in this respect were also particularly favorable.

On the other hand it must not be forgotten that the Gibson method looks back on only a brief period of development and that improvements are possible which must be founded on experiments. Through tests in a single experimental laboratory certainly no judgment can be attained of the expected accuracy under other conditions, because it remains indeterminate in what way the opposing errors have neutralized one another. One should seek to separate the individual errors. This can be done in various ways. If, for example, under otherwise permanent conditions, the length L be increased, error No. 1 is increased to a lesser degree than error No. 2 (the magnitude of the accessory motions will not increase after L has exceeded a definite value depending upon the pipe diameter). On the other hand, one may isolate error No. 2 if the pressure difference between two points on the pipe is used instead of the pressure difference between the measured section and the reservoir. (This method of water measurement has already been applied by Mr. Gibson.) If we have for the first measuring section a sufficiently long straight "runway" and, back of the second measuring section, a sufficiently long outlet stretch (because of the back current) then the accessory motions are of the same magnitude in both sections and their influence on the pressure disappears. In present plants these conditions can only seldom be attained. We can, however, attain them in a special experimental installation. Also the influence of a bend in front of the measuring cross-section, the disturbance due to too close proximity to the outlet, and other conditions can only be determined in an experimental installation. In contrast to the present conditions where only a rough valuation of the magnitude of the errors is possible, one will then be able to give the corrections which must be applied to cancel the error in the final results. Since the errors are not large in themselves, no very great relative accuracy in such corrections is necessary.

One must not be startled by the considerable work which will be necessary for the attainment of this perfection. Even for such simple measuring apparatus as, for example, the scale, or the compass being affected by the ship's movements, it was necessary to do an exceedingly great amount of work in the theoretical and also in the practical field in order to attain present-day perfection. One must not expect that it will be different with the new water-measuring method. Furthermore, sufficient incentive for work is not lacking here. The Gibson method has in many cases great practical advantages over other methods; it also makes possible a measurement under conditions where the other methods could not be applied. Therefore, we should strive for a further step in accuracy since the most exact measurements are of the highest importance to the technical advancement of water-turbine construction as a supplement to, and check of, experiments in experimental laboratories.

References

(1) "The Gibson Method and Apparatus for Measuring the Flow of Water in Closed Conduits," by N. R. Gibson, Trans. A.S.M.E., 1923, vol. 45, p. 343.

(2) *Zeit. V.D.I.*, 1924, p. 366.

(3) "The Gibson Method and Apparatus for Measuring the Flow of Water in Closed Conduits," by N. R. Gibson, Trans. A.S.M.E., 1923, vol. 45, Fig. 8, p. 362.

Experimental and Practical Experience With the Gibson Method of Water Measurement

A Discussion of Prof. D. Thoma's Paper on "The Degree of Accuracy of the Gibson Method of Water Measurement (1)

By N. R. GIBSON[1] and E. B. STROWGER[2], NIAGARA FALLS, N. Y.

The Gibson method is based on the equation of impulse and momentum applied to an enclosed column of water in motion. It is applicable in testing hydraulic power plants where the turbine is supplied with water through a closed conduit and means, such as turbine gates, are available for interrupting the flow of the water. To apply the method, it is necessary to obtain pressure-time diagrams, which show the changes with respect to time that occur in the conduit during and after the closure of the turbine gates. There are two kinds of diagrams: (*a*) simple diagrams, in which the changes of pressure at one point in the conduit are recorded, and (*b*) differential diagrams, in which the difference between the changes of pressure at two points in the conduit are recorded.

Professor Thoma (1)[3] has discussed quite thoroughly, in an accompanying paper (Trans. A.S.M.E., 1935, HYD-57-4), the conditions of water flow which may affect the degree of accuracy of the Gibson method of water measurement, when using the simple application method. In general, he has considered four errors due to (No. 1) accessory motions, or extra currents due to turbulence, (No. 2) false valuation of the friction in the conduit during closure, (No. 3) friction of the mercury column, and (No. 4) the inertia of the mercury column.

In discussing the Thoma article, the authors have first given a somewhat more detailed proof of the fundamental equation applying to the method and have supplemented the derivation with a graphical representation of the terms of the equation on the diagram itself. Test results on a long pipe are shown where both kinds of diagrams were taken and where the agreement between the two sets of diagrams is within 0.2 per cent. The test results are discussed and a detailed study is made of the effects of the four errors.

The authors conclude that the residual error is probably within the limits of precision possible when large quantities of water are being measured.

THE Gibson method is based on the equation of impulse and momentum applied to an enclosed column of water in motion. It is applicable in testing hydraulic power plants where the turbine is supplied with water through a closed conduit and means, such as turbine gates, are available for interrupting the flow of the water. To apply the method, it is necessary to obtain pressure-time diagrams, which show the changes of pressure with respect to time that occur in the conduit during and after the closure of the turbine gates. There are two kinds of diagrams:

Simple diagrams—in which the changes of pressure at one point in the conduit are recorded

Differential diagrams—in which the difference between the changes of pressure at two points in the conduit are recorded.

Prof. Thoma's Conclusions

Prof. Thoma has discussed quite thoroughly the conditions of water flow which may affect the degree of accuracy of the Gibson method of water measurement when using the "simple" application of the method. In general, he has considered and enumerated four errors due to (No. 1) accessory motions or extra currents due to turbulence, (No. 2) false valuation of friction in the conduit during the closure, (No. 3) friction of the mercury column, and (No. 4) the inertia of the mercury column. The first two are regarded as about 1 per cent in magnitude and are negative and positive, respectively. The last two are of opposite sign also, but are of the order of 0.5 of 1 per cent. Under favorable conditions, Prof. Thoma regards the accuracy of current-

[1] Vice-President and Chief Engineer, The Niagara Falls Power Company, Niagara Falls, N. Y. Mem. A.S.M.E. Mr. Gibson was graduated from the University of Toronto with the degree of B.A.Sc. in 1904. His professional career includes engagements with the Ontario Power Company of Niagara Falls, the firm of Smith, Kerry & Chace, Consulting Engineers, and The Niagara Falls Power Company. He became vice-president and chief engineer of the latter company in 1926. After the merger of this company with others, he was appointed vice-president and chief engineer of the Buffalo, Niagara, and Eastern Power Corporation in 1929 and vice-president of Niagara Hudson Power Corporation in 1931. In 1930, Mr. Gibson was awarded the Elliott Cresson Gold Medal by the Franklin Institute, Philadelphia, for his invention of the Gibson Method and Apparatus for measuring the flow of fluids in closed conduits. In 1931, he received an honorary degree, D.Eng., from the University of Toronto.

[2] Hydraulic Engineer, The Niagara Falls Power Company, Niagara Falls, N. Y. Mem. A.S.M.E. Mr. Strowger was graduated from the University of Rochester in 1918 with the degree of B.S. in Mechanical engineering. He was employed from 1918 to 1919 by the Interlake Engineering Company of Cleveland, Ohio, and in 1919 became connected with the Niagara Falls Power as draftsman and designer. In 1928, he became assistant hydraulic engineer of The Niagara Falls Power Company and in 1929 became associated with the engineering staff of the Buffalo, Niagara, and Eastern Power Corporation.

[3] Numbers in parentheses refer to similarly numbered references at the end of this paper.

Contributed by the Hydraulics Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, December 4 to 8, 1933, New York, N. Y.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

meter measurements as about 1 per cent, so that he concludes as follows:

Of the above four errors, it is possible to eliminate the third one in the calculations of the result. The fourth is so insignificant that a rough valuation is sufficient. Errors Nos. 1 and 2 are determinative in fixing the accuracy of the measurement. But these errors also are small so that one might truly say in forming a conclusion regarding the Gibson measurement in its existing form that the accuracy under favorable circumstances is approximately equal to the aforementioned current-meter measurements taken with great care under favorable circumstances.

Friction Neglected

In the following discussion the equations given, when identical with those of Prof. Thoma, will have the same number assigned to them as those in the Thoma article. First neglecting friction, the velocity of the water in the pipe is uniform and no accessory motions arise.

Considering Fig. 1, if A is a point sufficiently distant in the reservoir, then it can be readily shown that the supernormal pressure existing at any point C in the conduit caused by a retardation forced on the water column is given by

$$p_A - p_C = \frac{\gamma}{g} L \frac{dc}{dt} + \gamma \frac{c^2}{2g} \quad [3]$$

where c is the velocity in the conduit, γ the weight of a unit volume of water, and L is the "equivalent length" of the water conduit. If B is a point near the inlet where the pipe section is typical and if we draw the stream line going through A and designate by u the velocity at any point of the stream line, then the equivalent length to point B is l' and may be found from

$$\frac{1}{c} \int_A^B u \, dx = l' \quad [3a]$$

where dx is a small length.

Prof. Thoma points out that Equation [3] is the fundamental equation for pressure at section C neglecting friction and accessory motions. He then shows that this is the pressure recorded by the Gibson apparatus if no friction or accessory motions exist and concludes therefrom that with no accessory motions and with no friction the Gibson method is rigidly correct. The inertia of the mercury and the friction of the mercury in the U-tube in the Gibson apparatus will be considered later on and it will be shown that they have only a small influence upon the measurement. The following is a demonstration of Prof. Thoma's rigorous proof of the Gibson method. Let us designate A as a point on the extended axis of the pipe as shown in Fig. 2, ϕ as a measure of velocity potential expressed as $\frac{\delta\phi}{\delta x} = v$ or $\phi = \int v \, \delta x$, p, as before, the pressure at any point, and dt any small interval of time during the closure, then

$$\frac{\gamma}{g} \frac{\delta\phi}{\delta t} + \gamma \frac{v^2}{2g} = p_A - p \quad [4]$$

The velocity potential at B may be written

$$\phi_B = \int_A^B V \, dx = l'c \quad [5]$$

and that at C is

$$\phi_C = \phi_B + \int_B^C V \, dx = \phi_B + lc = l'c + lc = Lc \quad [6]$$

Then

$$\frac{\delta\phi_B}{\delta t} = l' \frac{dc}{dt} \quad \text{and} \quad \frac{\delta\phi_C}{\delta t} = L \frac{dc}{dt}$$

Substituting in [4], we get

$$p_A - p_B = \frac{\gamma}{g} l' \frac{dc}{dt} + \gamma \frac{c^2}{2g} \quad [7]$$

and

$$p_A - p_C = \frac{\gamma}{g} L \frac{dc}{dt} + \gamma \frac{c^2}{2g} \quad [8]$$

This equation is identical with Equation [3], whence the foregoing conclusion.

Graphical Explanation of Equations [3] and [4]

Fig. 3 shows graphically the equality expressed in Equations [3] and [4] neglecting accessory motions and friction and using a diagram made by a gage having zero inertia and zero friction.

Accessory Components of Velocity and Friction Considered

Now considering friction and the accompanying accessory components of velocity or "accessory motions," Prof. Thoma shows that the Gibson method, as applied by the use of the simple diagram, is not entirely independent of these motions. We may designate the value of momentum contained in the space between B and C as J, and we may define i_B as the value of momentum supplied from $t = 0$ to $t = t$ through section B by the inflowing water, and i_C similarly as the value of momentum which has gone out through section C from $t = 0$ to $t = t$. If we coin a new term for the first derivative of i_B with respect to time and call this derivative the "impulse current" through section B, then the impulse current is the momentum gained or lost per second and has, therefore, the dimension of a force. The increase per second in the value of momentum contained in the space between B and C is equal to the value of momentum supplied per second through B with the value of momentum through C deducted and with all external forces acting in the direction of flow added. This is expressed as follows

$$\frac{dJ}{dt} = \frac{di_B}{dt} - \frac{di_C}{dt} + \Sigma p \quad [9]$$

To determine J, we must integrate the expression for the momentum of a small particle of mass. This may be expressed as

$$dm \, V = \frac{\gamma}{g} dx \, df \, V$$

Then

$$dJ = \int \frac{\gamma}{g} dx \, df \, V = \frac{\gamma}{g} dx \int V \, df$$

But

$$\int V \, df = cf$$

so that

$$dJ = \frac{\gamma}{g} cf \, dx$$

and

$$J = \frac{\gamma}{g} cfl \quad [10]$$

where f is the cross-sectional area of the conduit. The momentum contained between B and C depends only upon the quantity of water flowing per second and is independent of the distribution and side components of flow. By taking the first derivative of

Equation [10], we get an expression for the left-hand side of Equation [9]

$$\frac{dJ}{dt} = \frac{\gamma}{g} fl \frac{dc}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

The impulse current through section B is $\frac{di_B}{dt}$ which equals the product of mass per second by velocity because in this section all particles have the same velocity c

$$\frac{di_B}{dt} = \frac{\gamma}{g} cf\, c = \frac{\gamma}{g} fc^2 \quad \ldots\ldots\ldots\ldots\ldots [12]$$

To get an expression for $\frac{di_C}{dt}$ in Equation [9], consider Fig. 4. The flow is not uniform with respect to both time and cross-section. The instantaneous velocity at any point in the section is v. Prof. Thoma divided this instantaneous velocity into two components, v_m the average value of velocity for the point in question, and v' the component of oscillating velocity. The full line in Fig. 4 refers to the instantaneous values of velocity and the broken line to the average value with respect to time.

$$v = v_m + v' \quad \ldots\ldots\ldots\ldots\ldots\ldots [13]$$

The value of momentum discharged through the element df in section C in the time dt is

$$\frac{\gamma}{g} (v_m + v')^2\, df\, dt$$

Taking a short interval of time Δt, then this expression for the element becomes

$$\frac{\gamma}{g} \int_{\Delta t} (v_m + v')^2\, df\, dt$$

or

$$df \frac{\gamma}{g} \int_{\Delta t} v_m^2\, dt + \frac{2\gamma}{g} df \int_{\Delta t} v_m v'\, dt + \frac{\gamma}{g} df \int_{\Delta t} v'^2\, dt$$

The first integral is equal to $v_m^2\ \Delta t$, the second is zero since v_m is a constant and $\int v'\, dt = 0$. The expression for the outgoing impulse then becomes

$$\frac{\gamma}{g} \Delta t\, v_m^2\, df + \frac{\gamma}{g} df \int_{\Delta t} v'^2\, dt$$

For the whole cross-section Δi_C becomes

$$\Delta i_C = \frac{\gamma}{g} \Delta t \int_f v_m^2\, df + \frac{\gamma}{g} \int_f df \int_{\Delta t} v'^2 dt \ldots\ldots [14]$$

If we use the symbol V_I^2 as the average value, for the whole cross-section, of the square of the variable velocity v' with respect to both time and cross-section, then we may write

$$\Delta i_C = \frac{\gamma}{g} \Delta t \int_f v_m^2\, df + \frac{\gamma}{g} \Delta t\, f\, V_I^2 \ldots\ldots\ldots [15]$$

Changing Δt to dt since the distinction was made in order to define v_I, Equation [15] becomes

$$\frac{di_C}{dt} = \frac{\gamma}{g} \int_f v_m^2\, df + \frac{\gamma}{g} f V_I^2 \ldots\ldots\ldots\ldots [16]$$

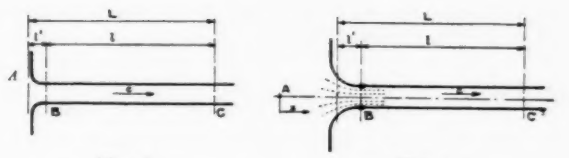


FIG. 1
(Thoma's Figure 1.)

FIG. 2
(Thoma's Figure 2.)

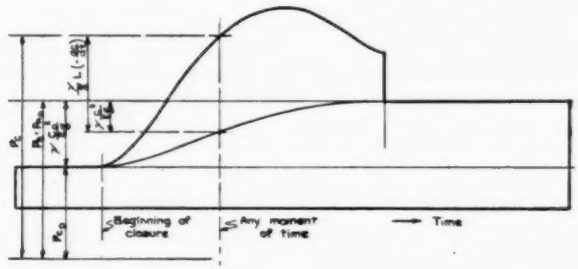


FIG. 3

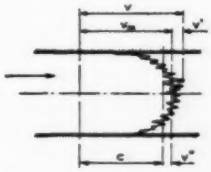


FIG. 4
(Thoma's Figure 3.)

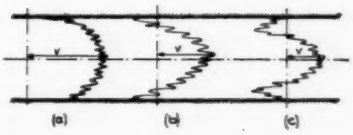


FIG. 5
(Thoma's Figures 4 and 5.)

For further simplification we may take

$$v_m = c + v'' \quad \ldots\ldots\ldots\ldots\ldots [17]$$

where v'' is the deviation of the *average* velocity with respect to time in the *element* considered, from the *average* velocity in the whole cross-section. This relation is shown graphically in Fig. 4. Equation [16] then becomes

$$\frac{di_C}{dt} = \frac{\gamma}{g} \int_f c^2 df + \frac{2\gamma}{g} \int_f cv''\, df + \frac{\gamma}{g} \int_f v''\, df + \frac{\gamma}{g} f\, V_I^2 . \quad [18]$$

The first integral becomes $c^2 f$, the second is equal to zero since $\int_f v''\, df$ is zero. Equation [18] then becomes

$$\frac{di_C}{dt} = \frac{\gamma}{g} fc^2 + \frac{\gamma}{g} f\, V_{II}^2 + \frac{\gamma}{g} f\, V_I^2 \ldots\ldots\ldots [19]$$

Where V_{II}^2 is the average value of the square of v'' over the cross-section.

We now have expressions for $\frac{dJ}{dt}$, $\frac{di_B}{dt}$, and $\frac{di_C}{dt}$ which may be substituted in Equation [9]. If this is done, we get

$$\frac{\gamma}{g} fl \frac{dc}{dt} = -\frac{\gamma}{g} f\, V_{II}^2 - \frac{\gamma}{g} f\, V_I^2 + \Sigma P$$

Since the forces ΣP acting on the water column consist of the pressure difference between B and C and the force R caused by friction, then $\Sigma P = f(p_B - p_C) - R$ and we may write

$$\frac{\gamma}{g} fl \frac{dc}{dt} = -\frac{\gamma}{g} f\, V_{II}^2 - \frac{\gamma}{g} f\, V_I^2 + f(p_B - p_C) - R \ldots [20]$$

Since the friction on the way to section B may be neglected we

may use Equation [7] to eliminate $(p_B - p_C)$ from Equation [20] as follows, remembering that $l + l' = L$

$$p_B - p_C = (p_A - p_C) - (p_A - p_B) = p_A - p_C - \frac{\gamma}{g} l' \frac{dc}{dt} - \frac{\gamma c^2}{2g}$$

and then

$$\frac{\gamma}{g} f L \frac{dc}{dt} = f (p_A - p_C) - \frac{\gamma f c^2}{2g} - \frac{\gamma}{g} f (V_{II}^2 + V_I^2) - R \ldots [21]$$

To evaluate R, it is assumed that during the closing of the turbine gates the friction is the same as it should be at a continuous flow with the same quantity of water per second. The friction is proportional to the square of the average velocity so that

$$R = \frac{c^2}{c_0^2} R_0 \ldots [22]$$

where R_0 is the initial value of R immediately before the test and similarly c_0 is the initial velocity. When $R = R_0$, then $\frac{dc}{dt} = 0$, so that

$$R_0 = f (p_{A_0} - p_{C_0}) - \gamma f \frac{c_0^2}{2g} - \frac{\gamma}{g} f (V_{II_0}^2 + V_{I_0}^2) \ldots [23]$$

The first item of the right member represents the initial difference in pressure between A and C, the second, the initial velocity head pressure, and the third represents the amount by which the actual friction is smaller than would be found if the accessory velocities were neglected. This is evident when Bernoulli's equation is written for the two points in question, using the average velocity c only. Substituting the value of R obtained from Equations [22] and [23] in Equation [21] we get

$$-\frac{dc}{dt} = \frac{g}{\gamma L} \left[(p_C - p_A) + \frac{c^2}{c_0^2} (p_{A_0} - p_{C_0}) + \frac{\gamma}{g} (V_{II}^2 + V_I^2) - \frac{c^2}{c_0^2} \frac{\gamma}{g} (V_{II_0}^2 + V_{I_0}^2) \right] \ldots [24]$$

In this equation, the third item on the right, $\frac{\gamma}{g} (V_{II}^2 + V_I^2)$, indicates for every interval of time the influence of the accessory velocities on the force available for the retardation $-\frac{dc}{dt}$ and the fourth term, $\frac{c^2}{c_0^2} \frac{\gamma}{g} (V_{II_0}^2 + V_{I_0}^2)$, corresponds to the decrease from the value of friction at continuous *steady* flow.

If accessory velocities or accessory motions be neglected, then Equation [24] becomes

$$-\frac{dc}{dt} = \frac{g}{\gamma L} \left[(p_C - p_A) + \frac{c^2}{c_0^2} (p_{A_0} - p_{C_0}) \right]$$

which expresses the fundamental relations as used in the delineation of a pressure-time diagram of the Gibson method. This equation states that the water-hammer pressure resulting in a time dt, $\frac{\gamma L}{g} \frac{dc}{dt}$, is equal to the pressure difference between points A and C plus the pressure still necessary to maintain the velocity and friction heads at this time. This is shown graphically in Fig. 3.

Prof. Thoma mentions that this equation could also be obtained if we suppose that V_{II}^2 and V_I^2 decrease during the closure proportionally with c^2. If this were true the last two terms in Equation [24] would balance each other. As in Fig. 5, he shows a section (a) representing the initial conditions of flow, a section (b) representing the flow at an instant during a sudden closure, and a section (c) typical of conditions near the end of a relatively long closure.

During a sudden closure "all water particles are then for the same short time under the action of the same strong pressure rise so that the velocity of each particle changes in value corresponding to the velocity at the beginning; the values V_{II} and V_I have then not changed during the sudden decrease in the water quantity to zero." With a long closure, however, the distribution of velocity may change considerably according to Prof. Thoma. In any case, however, consider $V_{II}^2 + V_I^2 = V_{II_0}^2 + V_{I_0}^2$ which Prof. Thoma says will come closer to the truth than the assumption that V_{II}^2 and V_I^2 decrease proportionally with c^2. This is probably an approximation of what happens. If we consider a complete closure, then in Equation [24], c is equal to zero and $\frac{dc}{dt}$ is equal to zero at the end of the closure so that

$$p_C - p_A = -\frac{\gamma}{g} (V_{II}^2 + V_I^2)$$

represents the pressure below normal at C at the end of the closure due to the accessory velocities.

The relative errors in the velocity measurement using a pitot tube at any one point in the cross-section may be expressed as $\frac{v_1^2}{2v_m^2}$ where v_1^2 is the average value of the square of the variable additional velocity varying with respect to time. This may be shown by referring to Equation [13] as follows:

$$v^2 = v_m^2 + 2v_m v' + v'^2$$

$$\int_{\Delta t} v^2 dt = \int_{\Delta t} v_m^2 \, dt + 2 \int_{\Delta t} v_m \, v' \, dt + \int_{\Delta t} v'^2 \, dt$$

The first integral of the right-hand member equals $v^2{}_m t$, the second equals zero because v_m is a constant with respect to time and by definition $\int v' dt = 0$, and the third equals $v_1^2 \, \Delta t$ by definition. Then

$$\frac{\int_{\Delta t} v^2 dt}{\Delta t} = v_a^2 = v_m^2 + v_1^2$$

But v_a^2 is a measure of the pitot tube reading and v_m^2 is a measure of the true velocity. The relative error in the velocity measurement is

$$\frac{v_a - v_m}{v_m} = \frac{v_a - \sqrt{v_a^2 - v_1^2}}{v_m}$$

or

$$\frac{v_a - v_m}{v_m} = \frac{v_a - v_a \left[1 - \left(\frac{v_1}{v_a} \right)^2 \right]^{1/2}}{v_m} = \frac{v_a - v_a \left[1 - \frac{1}{2} \left(\frac{v_1}{v_a} \right)^2 + \frac{1}{8} \left(\frac{v_1}{v_a} \right)^4 - \frac{1}{16} \left(\frac{v_1}{v_a} \right)^6 + \ldots \right]}{v_m}$$

Neglecting the terms in the expansion where the exponent is greater than two, the expression becomes

$$\frac{v_a - v_m}{v_m} = \frac{1}{2} \frac{v_1^2}{v_a v_m}$$

But we may equate $v_a = v_m$ with small error in this fraction, so that we may write

$$\text{Relative error} = \frac{1}{2}\frac{v_1^2}{v_m^2}$$

If one states that the pitot-tube measurements are in error by 1 per cent and remembering that v_m changes to c_0 and v_1 changes to V_I when we are considering the whole cross-section, then, at the beginning

$$V_{I_0}^2 = 2\ (0.01)\ c_0^2 = 0.02\ c_0^2$$

In evaluating $V_{II_0}^2$ it is known for water distribution in long, straight, smooth pipes that $V_{II_0}^2 = 0.02\ c_0^2$. But Prof. Thoma thinks this condition too favorable because turbine conduits are hardly ever as straight and smooth as the pipes with which the formulas for the water distribution were obtained in the laboratory and also because in most cases the flow is affected by dissymmetries caused by an unsymmetrical inlet. For this reason he estimates that $V_{II_0}^2 = 0.04\ c_0^2$. Then with these assumptions

$$V_{II_0}^2 + V_{I_0}^2 = 0.06\ c_0^2$$

and assuming

$$V_{II}^2 + V_I^2 = V_{II_0}^2 + V_{I_0}^2$$

and furthermore assuming a uniform change in c from c_0 to zero during the closure, then, in Equation [24], the expression

$$\frac{\gamma}{g}\ (V_{II}^2 + V_I^2) - \frac{c^2}{c_0^2}\frac{\gamma}{g}\ (V_{II_0}^2 + V_{I_0}^2)$$

which may be considered the average error for the whole observed time is equal to

$$\frac{\frac{\gamma}{g}\left[c - \frac{1}{3}\frac{c^3}{c_0^2}\right]_{c_0}^{0}}{c_0}\ (V_{II_0}^2 + V_{I_0}^2)$$

which becomes

$$\frac{2}{3}\frac{\gamma}{g}\ (0.06)\ c_0^2$$

To take into account the variation in $(V_{II}^2 + V_I^2)$ during the closure between $(V_{II_0}^2 + V_{I_0}^2)$ and $\frac{c^2}{c_0^2}\ (V_{II_0}^2 + V_{I_0}^2)$. Prof. Thoma reduces the factor $\frac{2}{3}$ to $\frac{1}{3}$, obtaining as the average value of the error

$$0.02\ \frac{\gamma}{g}\ c_0^2 = 0.04\ \gamma\ \frac{c_0^2}{2g}$$

The error in cfs may be expressed as

$$E_1 \text{ in cfs} = -\frac{K}{SF}\ \frac{0.04\ c_0^2}{2g}\ T_s\ S\ r = -\frac{0.02\ T_s\ c_0^2}{F}$$

where K is the calibration constant of the apparatus, (2) equals $\frac{g}{r}$, S is the horizontal length in in. corresponding to 1 sec of time, F equals $\Sigma\frac{l}{a}$, T_s is the length of the diagram in sec, and r is the vertical height in in. corresponding to one ft of pressure change in the conduit. The relative error in per cent may be expressed as

$$E_1 \text{ in per cent} = -(100)\ \frac{0.02\ T_s\ c_0^2}{F}\ \frac{SF}{KA} = -\frac{2\ T_s\ S\ c_0^2}{KA} \quad \ldots [26]$$

where A represents the net area of the diagram in square inches.

Taking the diagram used in the numerical example given in Mr. Gibson's paper (3) which is run No. 15 of the test on unit No. 16 of station 3-B of The Niagara Falls Power Company, made August 17, 1920, the measured quantity of water is 1708 cfs and the average error in height of diagram in ft of water pressure is $(0.04)\ \frac{9.06^2}{64.4} = 0.051$ ft. The error in water quantity with the above assumptions is $-\frac{(0.02)\ (17.1)\ (8.97)^2}{1.939} = -14.2$ cfs and the error in per cent is —0.84.

Taking the diagram shown in Fig. 8 of Prof. Thoma's article (1), which is run No. 1 of the test on unit No. 2 of station 3-A of The Niagara Falls Power Company made December 11, 1921, the measured quantity of water is 608.5 cfs and the average error in cfs is $-\frac{(0.02)\ (11.51)\ (9.77)^2}{3.67} = -6.0$ or a percentage error of —0.98 instead of —1.5 as obtained roughly by Prof. Thoma.

We come now to a consideration of the foregoing in respect to the differential application of the Gibson method in which the difference in pressures at two measuring sections is recorded. Under these conditions V_{II}^2 and V_I^2 at each measuring section must be considered. These quantities vary alike for the two sections where the upper one is located a reasonable distance from the inlet and where the area of the penstock is nearly the same at each section so that the distribution of velocities at each section is approximately the same.

For the differential diagram, Equation [24] may be written as follows for the measuring sections C_1 and C_2

$$-\frac{dc_2}{dt} = \frac{g}{\gamma L_2}\left[(p_{C_2} - p_A) + \frac{c_2^2}{c_{20}^2}\ (p_{A_0} - p_{C_{20}}) + \frac{\gamma}{g}\ (V_{II}^2 + V_I^2) - \frac{c_2^2}{c_{2_0}^2}\frac{\gamma}{g}\ (V_{II_0}^2 + V_{I_0}^2)\right]$$

$$-\frac{dc_1}{dt} = \frac{g}{\gamma L_1}\left[(p_{C_1} - p_A) + \frac{c_1^2}{c_{10}^2}\ (p_{A_0} - p_{C_{10}}) + \frac{\gamma}{g}\ (V_{II}^2 + V_I^2)\ \frac{c_1^2}{c_{10}^2}\frac{\gamma}{g}\ (V_{II_0}^2 + V_{I_0}^2)\right]$$

Combining these equations we get

$$-\left(\frac{\gamma L_2}{g}\frac{dc_2}{dt} - \frac{\gamma L_1}{g}\frac{dc_1}{dt}\right) = (p_{C_2} - p_{C_1}) + \frac{c_2^2}{c_{20}^2}\ (p_{A_0} - p_{C_{20}}) - \frac{c_1^2}{c_{10}^2}\ (p_{A_0} - p_{C_{10}}) - \frac{c_2^2}{c_{20}^2}\frac{\gamma}{g}\ (V_{II_0}^2 + V_{I_0}^2) + \frac{c_1^2\gamma}{c_{10}^2\ g}\ (V_{II_0}^2 + V_{I_0}^2)$$

But

$$\frac{c_2}{c_{20}} = \frac{c_1}{c_{10}} \quad \text{and} \quad \frac{dc_2}{dt} = \frac{dc_1}{dt}$$

so that

$$\frac{c_2^2}{c_{20}^2}\frac{\gamma}{g}\ (V_{II_0}^2 + V_{I_0}^2) = \frac{c_1^2}{c_{10}^2}\frac{\gamma}{g}\ (V_{II_0}^2 + V_{I_0}^2)$$

and

$$-\left(\frac{\gamma L_2}{g}\frac{dc_2}{dt} - \frac{\gamma L_1}{g}\frac{dc_1}{dt}\right) = -(L_2 - L_1)\ \frac{\gamma}{g}\frac{dc_2}{dt}$$

$$= \left[(p_{C_2} - p_{C_1}) + \frac{c_2^2}{c_{20}^2}\ (p_{A_0} - p_{C_{20}}) - \frac{c_1^2}{c_{10}^2}\ (p_{A_0} - p_{C_{10}})\right] \ldots [27]$$

TABLE 1 TEST OF UNIT NO. 20, STATION 3-C, THE NIAGARA FALLS POWER COMPANY, JUNE 22, 1924

Run 10 (Differential diagram)

Planimeter readings	Differences	Gross area	Slot correction	Net area	Net area corrected for shrinkage
17.103					
	4.087				
13.016		8.171	0.135	8.036	8.086
	4.084				
8.932					

$K = 40.23$ $F = 0.2522$

$S = 0.4645$

$Q = \dfrac{(40.23)\ (8.086)}{(0.4645)\ (0.2522)} = 2777$

Leakage = 16

Total discharge 2793 cfs

Headwater elevation = 560.19
Tailwater elevation = 341.60
Gross head = 218.59

$h_f + h_v$ = 2.06
Net head = 216.53
Generated kw = 46,933

Quantities for net head = 213.5 ft
Q = 2773 cfs
kw = 45,920

This is the general equation for the differential diagram with the areas at the measuring sections nearly the same. If these areas are equal then $c_{20} = c_{10}$ and $c_2 = c_1$ and the last two terms simplify to

$$\frac{c_2^2}{c_{20}^2}\,(p_{A_0} - p_{C_{20}}) - \frac{c_2^2}{c_{20}^2}\,(p_{A_0} - p_{C_{10}}) = \frac{c_2^2}{c_{20}^2}\,(p_{C_{10}} - p_{C_{20}})$$

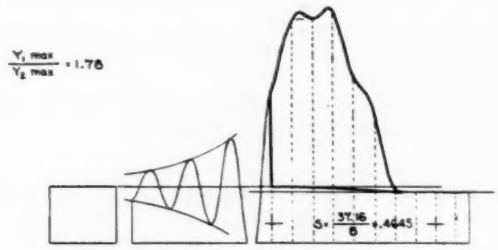


FIG. 6

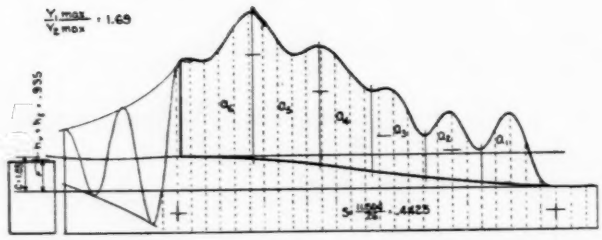


FIG. 7

TABLE 2 TEST OF UNIT NO. 20, STATION 3-C, THE NIAGARA FALLS POWER COMPANY, JUNE 15, 1924

Run 9 (Simple diagrams)

	Planimeter readings	Diff.	Mean area corrected for slot	A	r	$(1-r)$	$(1-r)^2$	$\frac{(1-r)^2}{C}$
a_1	28.106							
	26.807	1.299	2.601					
	25.505	1.302	0.070	2.531	0.088	0.912	0.832	0.857
			2.531					
a_3	30.804							
	29.457	1.347	2.698					
	28.106	1.351	0.036	5.193	0.181	0.819	0.671	0.691
			2.662					
a_2	10.065							
	8.300	1.765	3.527					
	6.538	1.762	0.045	8.675	0.302	0.698	0.488	0.504
			3.482					
a_4	14.953							
	12.505	2.448	4.888					
	10.065	2.440	0.043	13.520	0.471	0.529	0.280	0.288
			4.845					
a_5	18.989							
	15.246	3.743	7.492					
	11.497	3.749	0.040	20.972	0.730	0.270	0.073	0.075
			7.452					
a_6	26.590	3.800						
	22.790	3.801	7.601	28.570				
	18.989		0.003	0.166				
			7.598	28.736				
		40.450						
Total net area		26.070	14.380	28.784				
		11.666	14.404	0.237 slot correction				
				28.547				

Leakage area = $\dfrac{(16)\ (1.699)\ (0.4425)}{(72.56)} = 0.166$

Area corrected for shrinkage = $(28.547)\ \dfrac{54.83}{54.53} = 28.704$

$S = 0.4425$
$K = 72.56$
$F = 1.699$
$Q = \dfrac{(72.56)\ (28.704)}{(0.4425)\ (1.699)} = 2770$
Leakage = 16
Total discharge 2786 cfs

$h_v + h_f$ (from net head piezometer) = 7.31
$h_v = \dfrac{(2786)^2}{(153.94)}\ \dfrac{1}{64.4} = 5.08$
h_f = 2.23
Headwater elevation = 559.43
Tailwater elevation = 341.38
Gross head = 218.05
$h_f + h_v$ = 2.29
Net head = 215.76
kw generated = 46,640
h_f to Gibson tap = 0.49 in. on diagram

Quantities for net head = 213.5 ft
Q = 2770 cfs
kw = 45,900

and Equation [27] becomes

$$-(L_2 - L_1)\,\frac{\gamma}{g}\,\frac{dc_2}{dt} = \left[(p_{C_2} - p_{C_1}) + \frac{c_2^2}{c_{20}^2}\,(p_{C_{10}} - p_{C_{20}})\right] \ldots [28]$$

Equations [27] and [28] are independent of accessory velocities and so, as stated above, where the measuring sections are equal or nearly so, and where the upper measuring section is located at a reasonable distance from the inlet, then the differential diagram is independent of accessory velocities.

Fig. 6 is a differential diagram taken on June 22, 1924, during the test of unit No. 20 of station 3-C of The Niagara Falls Power Company. Fig. 7 is a simple diagram taken on June 15, 1924, during another test on the same unit. The two diagrams were taken as closely as possible at the same gate opening and the same power output. In the case of the differential diagram ($L_2 - L_1$) is 87.25 ft and in the case of the simple diagram L is 599.26 ft, as shown in Fig. 8.

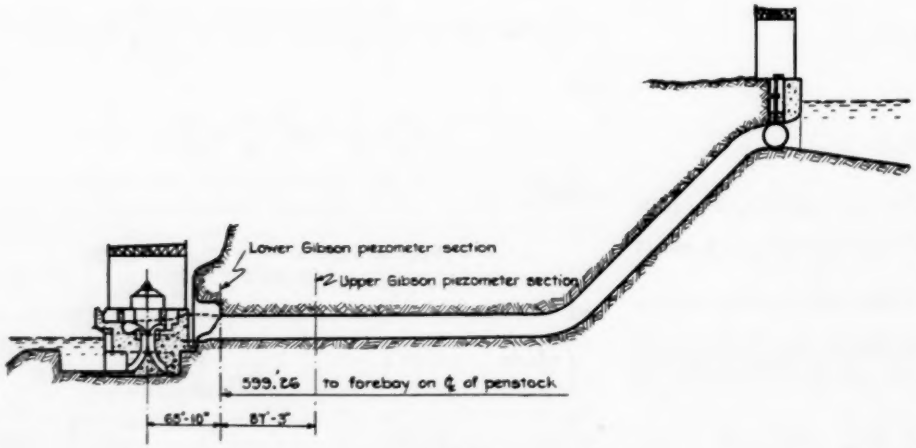


FIG. 8

Table 1 shows the computations for the differential diagram and Table 2 the computations for the simple diagram. They show that when equalized for the same net head, the quantities of water as determined by these two different applications of the Gibson method agree almost exactly.

The common power-discharge curve for the two tests of this unit is shown in Fig. 9, in which the quantities obtained by measurements using the simple diagrams are plotted by points enclosed in circles and those obtained from the differential diagrams are enclosed in triangles. The average divergence of the differential points from the line established by the single points is within +0.20 per cent. The agreement between the two tests then is within 0.20 per cent.

This would point experimentally to one of two conclusions. Either the positive and negative errors of both the simple diagrams and differential diagrams are of equal magnitude and cancel one another or leave the same residual; or Prof. Thoma's estimates of the magnitudes of these errors are too large. (1) Later it will be shown that when the differential method is used, errors No. 2 and No. 4 are small and are opposite in sign to error No. 3, resulting in a very small residual error.

Error Due to False Valuation of Friction

The assumption that the friction during the closing is just as great as at continuous steady flow with the same amount of water possibly is not strictly true. Prof. Thoma points out that a downward friction force is exerted upon the back-flowing edge current during the last stage of the closure. This force tends to make the diagram too large and, therefore, causes a positive error in the result opposed to the negative error due to the accessory velocities. Estimating the average error in the friction from one-twentieth to one-tenth of the friction obtaining when $c = c_0$, then the error in the quantity of water in square inches on the diagram varies from $+\frac{1}{20} h_{f_0} T_s S$ to $+\frac{1}{10} h_{f_0} T_s S$. Expressed as a relative error in per cent, this becomes

$$E_2 = [(+5) \text{ to } (+10)] \frac{h_{f_0} T_s S}{A} \quad \ldots\ldots\ldots\ldots [29]$$

where h_{f_0} is the friction expressed in inches of ordinate on the diagram. In the simple diagram, h_{f_0} is measured from forebay to measuring section and, in the case of the differential diagram, is measured between taps.

For the diagram used in the numerical example this error becomes

$$(+5) \frac{\left(\frac{0.69}{3}\right)(17)\ (0.4723)}{16.21} = +0.57 \text{ per cent}$$

to

$$(+10) \frac{\left(\frac{0.69}{3}\right)(17)\ (0.4723)}{16.21} = +1.14 \text{ per cent}$$

This error for the diagram shown in Fig. 8 of Prof. Thoma's article (1) with the same assumptions as to friction would be from

$$(+5) \frac{(0.485)(1.20)(11.5)(0.728)}{18.20} = +1.3 \text{ per cent}$$

to

$$(+10) \frac{(0.485)(1.20)(11.5)(0.728)}{18.20} = +2.7 \text{ per cent}$$

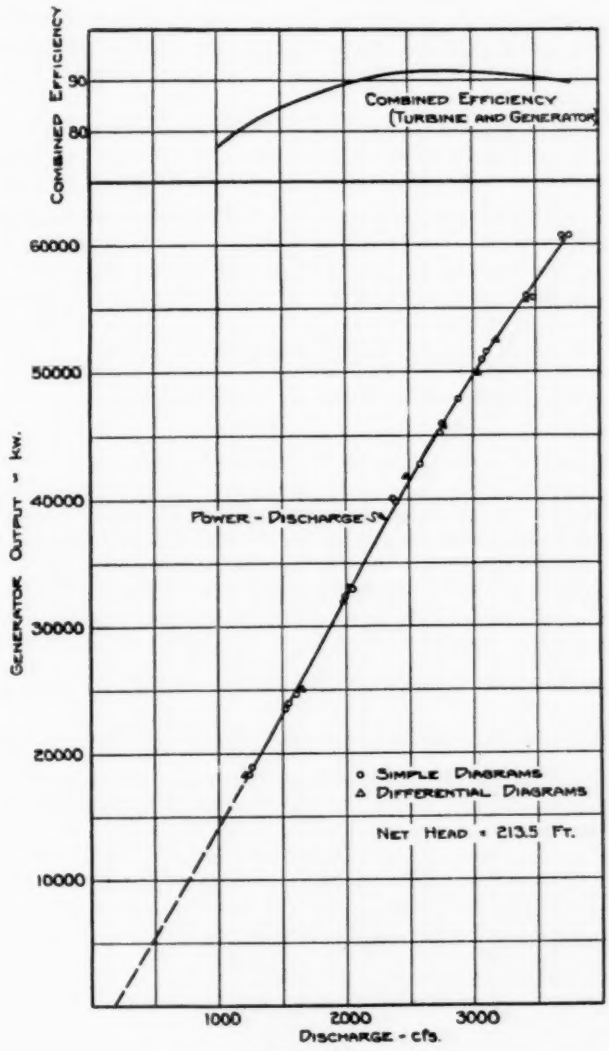


Fig. 9

Prof. Thoma gets 0.9 to 1.8 per cent for these values because he assumes that 33 per cent of $\frac{p_{A_0} - p_{C_0}}{\gamma}$ is the value of the friction head at $c = c_0$ instead of 48.5 per cent as is actually the case. These are, of course, very approximate estimates of the errors.

In the study of error No. 2, it is difficult to accept Dr. Thoma's coefficient $\frac{1}{20}$ to $\frac{1}{10}$. This error could only exist to any appreciable degree where the "velocity front" in the conduit is attenuated. Most penstocks of modern plants are so large that the velocity front is almost square.

As will be pointed out later, adopting 0.075 as a mean value given by Prof. Thoma for the coefficient to be used in evaluating error No. 2 and evaluating errors No. 3 and No. 4 in accordance with the theory advanced, we find the mean residual error at point of maximum efficiency in 35 cases examined would be 0.43 per cent without regard to sign but only +0.06 per cent if positive and negative values are taken into account. Thirteen cases gave positive results and 22 cases negative results.

In many cases the factor $\frac{p_{A_0} - p_{C_0}}{\gamma}$ on the differential diagram is not greater than $^1/_8$ to $^1/_4$ in. on diagrams from 4 to 8 in. long

and containing 20 to 50 sq in. of impulse area. Even in the small differential diagram shown in Fig. 6, the error due to false valuation of friction would, according to this estimate, amount to only

$$\left[\left(+\frac{1}{20}\right)\text{ to }\left(+\frac{1}{10}\right)\right]\frac{(100)(0.11)(6.02)(0.4645)}{8.086} = 0.2\text{ to } 0.4\text{ per cent.}$$

In a great many instances where differential diagrams have been used, a close check has been obtained both where the closures have been of long and of short duration. The error due to a false valuation of friction must be well under the figures suggested by Prof. Thoma.

Error Due to Friction of the Mercury Column

The equation for the motion of the mercury column is

$$a\frac{d^2p'}{dt^2} + k\frac{dp'}{dt} + p' = p \quad \ldots\ldots [25]$$

where p' is the pressure recorded by the apparatus, p the pressure in the measuring section, a a constant depending upon the mass of the mercury column, its cross-section, etc., and k a friction coefficient which may be considered constant for small deflections. The friction is a function of the velocity of the mercury. The symbol p' represents ordinates on the diagram and, therefore, represents the position of the mercury surface, so that $\frac{dp'}{dt}$ represents the velocity of the mercury and $k\frac{dp'}{dt}$ the frictional force. If we let p_2 represent the (nearly) constant pressure existing after the closure, then for the damped harmonic oscillations we get

$$a\frac{d^2p'}{dt^2} + k\frac{dp'}{dt} + p' - p_2 = 0$$

Let $(p' - p_2) = p''$; then

$$a\frac{d^2p''}{dt^2} + k\frac{dp''}{dt} + p'' = 0$$

The general solution of this equation is (4)

$$p'' = e^{\frac{-k}{2a}t}\left[A\sin\left(\sqrt{\frac{1}{a} - \frac{k^2}{4a^2}}\right)t + B\cos\left(\sqrt{\frac{1}{a} - \frac{k^2}{4a^2}}\right)t\right]$$

or

$$p' = p_2 + e^{\frac{-k}{2a}t}\left[A\sin\left(\sqrt{\frac{1}{a} - \frac{k^2}{4a^2}}\right)t + B\cos\left(\sqrt{\frac{1}{a} - \frac{k^2}{4a^2}}\right)t\right] \quad \ldots\ldots [30]$$

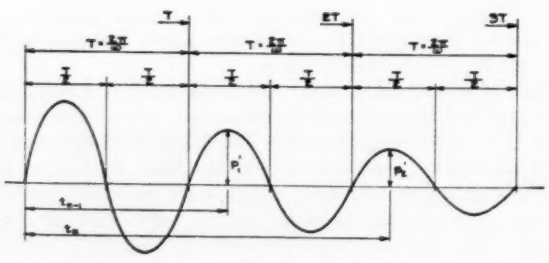


Fig. 10

Fig. 10 represents a damped harmonic vibration as given by this equation with the constant p_2 equal to zero so that the axis of abscissas divides the figure symmetrically. The figure is also drawn for $p' = 0$ when $t = 0$ so that the constant B becomes zero.

The value of k in the above equation may be obtained by making a few calculations. In Equation [30], when $t = 0$ then $p' = 0$ and we may make $p_2 = 0$. Then

$$0 = e^{\frac{-k}{2a}0}\left[A\sin\left(\sqrt{\frac{1}{a} - \frac{k^2}{4a^2}}\right)0 + B\cos\left(\sqrt{\frac{1}{a} - \frac{k^2}{4a^2}}\right)0\right]$$

Therefore, $B = 0$ and Equation [30] becomes

$$p' = e^{\frac{-k}{2a}t}A\sin\left(\sqrt{\frac{1}{a} - \frac{k^2}{4a^2}}\right)t \quad \ldots\ldots [31]$$

Let
$$w = \sqrt{\frac{1}{a} - \frac{k^2}{4a^2}} \quad \ldots\ldots [32]$$

And

$$\beta = \frac{k}{2a} \quad \ldots\ldots [33]$$

Then

$$p' = e^{-\beta t}A\sin w\,t \quad \ldots\ldots [34]$$

When $t = n\,T$, $p' = 0$, so that $w\,nT = n\,2\pi$ or

$$T = \frac{2\pi}{w} \quad \ldots\ldots [35]$$

For a maximum or minimum value of p'

$$\frac{dp'}{dt} = -\beta e^{-\beta t}A\sin w\,t + e^{-\beta t}A\,w\cos w\,t = 0$$

or $w\cos w\,t - \beta\sin w\,t = 0$ or $\tan w\,t = \frac{w}{\beta}$

Whence the condition for a maximum or minimum is

$$w\,t_n = \tan^{-1}\frac{w}{\beta} + (n-1)\,2\pi$$

$$t_n = \frac{1}{w}\tan^{-1}\frac{w}{\beta} + (n-1)\frac{2\pi}{w} = \frac{1}{w}\tan^{-1}\frac{w}{\beta} + (n-1)\,T \text{ and}$$

$$t_{n-1} = \frac{1}{w}\tan^{-1}\frac{w}{\beta} + (n-2)\frac{2\pi}{w} = \frac{1}{w}\tan^{-1}\frac{w}{\beta} + (n-2)\,T$$

$$p_1' = e^{-\beta t_{n-1}}A\sin w\,t_{n-1}$$

$$p_2' = e^{-\beta t_n}A\sin w\,t_n$$

$$\psi = \frac{p_1'}{p_2'} = \left[\frac{e^{-\beta t_{n-1}}A\sin w\,t_{n-1}}{e^{-\beta t_n}A\sin w\,t_n}\right] = \left[\frac{e^{-\frac{\beta}{w}\tan^{-1}\frac{w}{\beta} - \beta(n-2)\,T}}{e^{-\frac{\beta}{w}\tan^{-1}\frac{w}{\beta} - \beta(n-1)T}}\right]$$

$$\psi = e^{-\frac{\beta}{w}\tan^{-1}\frac{w}{\beta} - \beta(n-2)T + \frac{\beta}{w}\tan^{-1}\frac{w}{\beta} + \beta(n-1)T}$$

$$\psi = e^{(-\beta n + 2\beta + \beta n - \beta)T} = e^{\beta T} \quad \ldots\ldots [36]$$

Then we have

$$\psi = e^{\beta T} \quad \ldots\ldots [36]$$

$$T = \frac{2\pi}{w} \quad \ldots\ldots [35]$$

$$w = \sqrt{\frac{1}{a} - \frac{k^2}{4a^2}} \quad \ldots\ldots [32]$$

$$\beta = \frac{k}{2a} \quad \ldots\ldots [33]$$

$$\frac{1}{a} - \frac{k^2}{4a^2} = \frac{4\pi^2}{T^2}$$

$$4a - k^2 = 16\,\frac{\pi^2 a^2}{T^2}$$

$$\frac{16\pi^2}{T^2}\,(a^2) - 4a + k^2 = 0$$

$$a = \frac{+4 \pm \sqrt{16 - \frac{64\,\pi^2 k^2}{T^2}}}{\frac{32\pi^2}{T^2}} = \frac{+1 \pm \sqrt{1 - \frac{4\,\pi^2 k^2}{T^2}}}{\frac{8\pi^2}{T^2}}$$

$$\psi = e^{\frac{k}{2a}T} \quad \text{or} \quad \log \psi = \frac{k}{2a}\,T$$

$$\log \psi = \frac{kT}{2}\left[\frac{8\pi^2}{T^2}\,\frac{1}{\left[1 \pm \sqrt{1 - \frac{4\,\pi^2 k^2}{T^2}}\right]}\right]$$

$$\log \psi = \frac{4k\pi^2}{T\left[1 \pm \sqrt{1 - \frac{4\pi^2 k^2}{T^2}}\right]}$$

$$T\,(\log \psi) \pm T\,(\log \psi)\sqrt{1 - \frac{4\pi^2 k^2}{T^2}} = 4\,k\pi^2$$

$$\pm\, T\,(\log \psi)\sqrt{1 - \frac{4\pi^2 k^2}{T^2}} = 4\,k\pi^2 - T\,(\log \psi)$$

$$T^2\,(\log \psi)^2\left(1 - \frac{4\pi^2 k^2}{T^2}\right) = 16\,k^2\pi^4 - 8\,k\pi^2\,T\,(\log \psi) + T^2\,(\log \psi)^2$$

$$-\frac{4\pi^2 k^2}{T^2}\,T^2\,(\log \psi)^2 = 16\,k^2\pi^4 - 8\,k\pi^2 T\,(\log \psi)$$

$$-4\pi^2\,(\log \psi)^2\,k - 16\pi^4 k + 8\pi^2\,T\,(\log \psi) = 0$$

$$\left[-\pi^2\,(\log \psi)^2 - 4\pi^4\right]k = -2\pi^2\,T\,(\log \psi)$$

$$k = \frac{2\pi^2\,T \log\,(\psi)}{4\pi^4 + \pi^2\,(\log \psi)^2}$$

$$k = \frac{T\,(\log \psi)}{2\pi^2 + \frac{(\log \psi)^2}{2}} \qquad \text{[37]}$$

By integrating Equation [25] between the start and end of the closure, i.e., between $t = 0$ and $t = t_1$ we get

$$a\left[\left(\frac{dp'}{dt}\right)_1 - \left(\frac{dp'}{dt}\right)_0\right] + k\,(p_1' - p_0') + \int_{t_0}^{t_1} p'dt = \int_{t_0}^{t_1} pdt$$

The first term of this expression is zero because the mercury column is at rest at the beginning and at the end of the observation. The third term is the time integral for the pressure diagram which is evidently too small by the amount $k\,(p_1' - p_0')$ due to the friction of the mercury.

Neglecting the second term in the denominator of Equation [37], we get for the expression $k\,(p_1' - p_0')$

$$\frac{T \log_e \psi}{2\pi^2}\,(p_1' - p_0')$$

The relative error in per cent is

$$E_2 = -\frac{100\,TS\,(\log_e \psi)\,(p_1' - p_0')}{2\pi^2\,A} \qquad \text{[38]}$$

Evaluating T, ψ, and $(p_1' - p_0')$ from the diagram of Prof. Thoma's Fig. 8, $T = 3.74$ sec, $\psi = 1.61$, $(p_1' - p_0') = 2.80$ in. on diagram, so that

$$E_2 = -\frac{(100)\,(3.74)\,(0.728)\,(0.4783)}{2\pi^2\,(18.20)} = -1.02 \text{ per cent}$$

Evaluating T, ψ, and $(p_1' - p_0')$ from the numerical example given in Mr. Gibson's paper, $T = 4.0$ sec, $\psi = 1.55$, and $(p_1' - p_0') = 2.33$ in. on diagram, so that

$$E_2 = \frac{(100)\,(4.0)\,(0.4723)\,(0.438)}{2\pi^2\,(16.21)} = -0.60 \text{ per cent}$$

It should be remembered that these numerical values apply to the simple diagram. In the differential diagram the corresponding error is reduced because only a small amount of mercury is used and consequently the natural period of oscillation T is smaller. Moreover the quantity of $(p_1' - p_0')$ is smaller as the forebay surge is eliminated from the diagram and with approximately equal measuring sections there is little or no velocity head in the diagram ordinate. Furthermore by damping the gate stroke toward the end of the closure, this quantity may be made extremely small.

Error Due to Inertia of the Mercury Column

Considering the term $\frac{c^2}{c_0^2}\,(p_{A_0} - p_{C_0})$ in Equation [24], Prof. Thoma says: "At the beginning of the observation the recorded pressure stays below the real pressure and this causes the water velocity at the beginning and, therefore, also the above-mentioned term to be judged too great." The relative error in the determination of the water quantity assuming the theoretical case where the destruction of velocity is uniform from the beginning to the end of the closure, i.e., $\frac{dc}{dt}$ is constant, is given as

$$\left[\frac{1}{2\pi^2}\left(\frac{T}{T_s}\right)^2 \frac{p_{A_0} - p_{C_0}}{p_m}\right]^*$$

where T is the time for the natural period of the apparatus, T_s the duration of closure in seconds and p_m the average value of

$$p_C - p_A + \frac{c^2}{c_0^2}\,(p_{A_0} - p_{C_0})$$

or

$$p_m = \frac{-\gamma L}{g}\left(\frac{dc}{dt}\right) = \frac{-\gamma L}{g}\,\frac{c_0}{T_s}$$

This expression gives the maximum value of the error because in a practical case $\frac{dc}{dt}$ cannot be constant but is smaller at the start of the closure. This results in a closer agreement between the curve recorded by the surface of the mercury and the curve of true pressure.

We may derive this expression for the limiting relative error due to the inertia of the mercury column as follows. With the assumption of uniform destruction of velocity, the actual pres-

* This expression is approximate, the correct one being

$$\frac{5}{8\pi^2}\left(\frac{T}{T_s}\right)^2 \frac{p_{A0} - p_{C_0}}{p_m}$$

sure-time diagram occurring at section C is shown in Fig. 11 by the area $AGFEDCBA$ having a constant height $\frac{-\gamma L c_0}{g T_s} = p_m$ and a length T_s. The actual diagram as recorded in this case would be $AMLKJDCBA$. The mercury lags behind the pressure line $AGFE$ at the start and during the closure oscillates about GFE, in this case in a damped harmonic curve. The area of the diagram in either case considered above the horizontal base AX is the same. If we substitute the sine curve $AQPON$ for the damped harmonic curve (i.e., neglect the friction in the mercury column), then by comparing the net area of the diagram as calculated by the Gibson method with the theoretical net diagram,

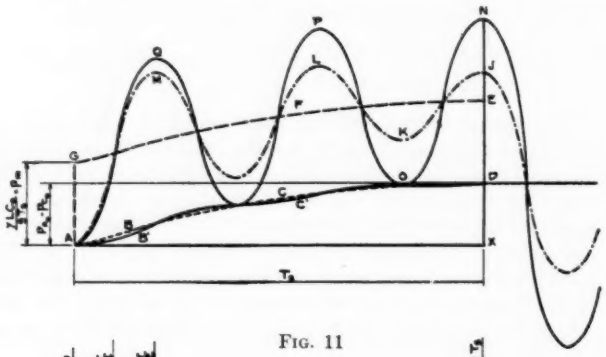


FIG. 11

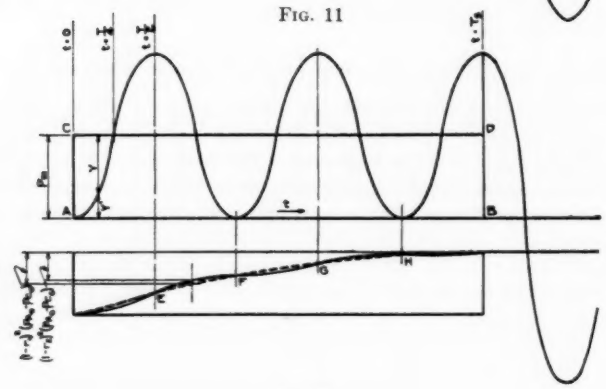


FIG. 12

an expression is obtained for the error due to inertia. The difference, of course, is due to the difference in the location of the recovery line on the diagram. In the case of the theoretical diagram this location is $ABCD$, and in the case of the actual diagram the location is $AB'C'D$. The actual recovery line oscillates about the true one making plus and minus errors which tend to compensate although the largest divergence occurs at the start of the diagram.

Inasmuch as the line GFE is parallel with $ABCD$ and the sine curve $AQPON$ oscillates about GFE, we may assume that the ordinate $(p_{A_0} - p_{C_0}) = \overline{XD}$ is zero for the diagram and reconstruct the figure as shown in Fig. 12. This will simplify the calculations. By integrating both theoretical and actual diagrams in Fig. 12 from $t = 0$ to some point $t = t$, we may then find the value of r or $\frac{c}{c_0}$ at this point. The difference in the value of $(1 - r)^2(p_{A_0} - p_{C_0})$ for the two diagrams gives the error in the location of the recovery line and this expression integrated over the length of the diagram gives the error due to the inertia of the mercury.

The general equation for the sine curve with CD as the axis is $y = A \sin kt + B \cos kt$. We may evaluate constants A, B, and k as follows: When $t = 0$ then $y = -p_m$, so that $-p_m = 0 + B$, or $B = -p_m$. Similarly when $t = \frac{T}{4}$ then $y = 0$, so that $0 = A \sin k\frac{T}{4} - p_m \cos k\frac{T}{4}$, or $\tan \frac{kT}{4} = \frac{p_m}{A}$. Also when $t = \frac{5}{4}T$ then $y = 0$, so that $0 = A \sin \frac{5}{4}kT - p_m \cos \frac{5}{4}kT$, or $\tan \frac{5}{4}kT = \frac{p_m}{A}$. This results in the relation $\frac{5}{4}kT - \frac{kT}{4} = 2\pi$, or $k = \frac{2\pi}{T}$. We may, therefore, write

$$y = A \sin \frac{2\pi}{T} t - p_m \cos \frac{2\pi}{T} t$$

Substituting $k = \frac{2\pi}{T}$ in the relation $A \sin k\frac{T}{4} = p_m \cos k\frac{T}{4}$, we find that $A = 0$, so that we may now write

$$y = -p_m \cos \frac{2\pi}{T} t$$

Transferring the axis from CD to AB and using y' for the ordinate to the curve, we may write the relation

$$y' = p_m + y = p_m - p_m \cos \frac{2\pi}{T} t$$

The area under the curve to any point t is

$$\int_0^t y'dt = \int_0^t p_m dt - \int_0^t p_m \cos \frac{2\pi}{T} t\, dt =$$

$$p_m t - p_m \frac{T}{2\pi} \sin \frac{2\pi}{T} t = p_m \left(t - \frac{T}{2\pi} \sin \frac{2\pi}{T} t\right)$$

The total diagram area is $p_m T_s$. The true ratio r_1 as used in delineation is $r_1 = \frac{p_m t}{p_m T_s} = \frac{t}{T_s}$ and $(1 - r_1)^2 = 1 - \frac{2t}{T_s} + \frac{t^2}{T_s^2}$. The approximate ratio r_2 as used in the delineation of the diagram as recorded by the mercury is

$$r_2 = \frac{p_m \left(t - \frac{T}{2\pi} \sin \frac{2\pi}{T} t\right)}{p_m T_s} = \frac{t}{T_s} - \frac{T}{2\pi T_s} \sin \frac{2\pi}{T} t$$

and

$$(1 - r_2)^2 = 1 + \frac{t^2}{T_s^2} + \frac{T^2}{4\pi^2 T_s^2} (\sin \frac{2\pi}{T} t)^2 - \frac{tT}{\pi T_s^2} \sin \frac{2\pi}{T} t$$

$$- \frac{2t}{T_s} + \frac{T}{\pi T_s} \sin \frac{2\pi}{T} t$$

The error in the location of the recovery line is then

$$\left[(1 - r_2)^2 - (1 - r_1)^2\right] (p_{A_0} - p_{C_0})$$

$$= (p_{A_0} - p_{C_0}) \left[1 + \frac{t^2}{T_s^2} + \frac{T^2}{4\pi^2 T_s^2} \left(\sin \frac{2\pi}{T} t\right)^2 - \frac{tT}{\pi T_s^2} \sin \frac{2\pi}{T} t\right.$$

$$\left. - \frac{2t}{T_s} + \frac{T}{\pi T_s} \sin \frac{2\pi}{T} t - 1 + \frac{2t}{T_s} - \frac{t^2}{T_s^2}\right]$$

This expression reduces to

$$(p_{A_0} - p_{C_0}) \left[\frac{-tT}{\pi T_s^2} \sin \frac{2\pi}{T} t + \frac{T^2}{4\pi^2 T_s^2} \left(\sin \frac{2\pi}{T} t\right)^2 + \frac{T}{\pi T_s} \sin \frac{2\pi}{T} t\right]$$

The error in diagram area to any time t is

$$(p_{A_0} - p_{C_0}) \int_{t=0}^{t=t} \left[\frac{-tT}{\pi T_s^2} \sin \frac{2\pi}{T} t + \frac{T}{4\pi^2 T_s^2} \left(\sin \frac{2\pi}{T} t \right) + \frac{T}{\pi T_s} \sin \frac{2\pi}{T} t \right] dt$$

which reduces to

$$(p_{A_0} - p_{C_0}) \left[-\frac{T^3}{4\pi^3 T_s^2} \sin \frac{2\pi}{T} t + \frac{T^2 t}{2\pi^2 T_s^2} \cos \frac{2\pi}{T} t - \frac{T^3}{16\pi^3 T_s^2} \cos \frac{2\pi}{T} t \sin \frac{2\pi}{T} t + \frac{T^2 t}{8\pi^2 T_s^2} - \frac{T^2}{2\pi^2 T_s} \cos \frac{2\pi}{T} t + \frac{T^2}{2\pi^2 T_s} \right]$$

If we integrate from $t = 0$ to $t = T_s$, we will obtain the total amount of the error for the measurement. T_s can be taken as a multiple of T in evaluating $\cos \frac{2\pi}{T} T_s$ and $\sin \frac{2\pi}{T} T_s$ since at the end of the diagram the mercury is at rest.[4]

$$(p_{A_0} - p_{C_0}) \left[-\frac{T^3}{4\pi^3 T_s^2} \sin \frac{2\pi}{T} T_s + \frac{T^2 T_s}{2\pi^2 T_s^2} \cos \frac{2\pi}{T} T_s - \frac{T^3}{16\pi^3 T_s^2} \cos \frac{2\pi}{T} T_s \sin \frac{2\pi}{T} T_s + \frac{T^2 T_s}{8\pi^2 T_s^2} - \frac{T^2}{2\pi^2 T_s} \cos \frac{2\pi}{T} T_s + \frac{T^2}{2\pi^2 T_s} \right]$$

This expression reduces to

$$(p_{A_0} - p_{C_0}) \frac{T^2}{2\pi^2 T_s} + \frac{T^2}{8\pi^2 T_s}$$

or

$$(p_{A_0} - p_{C_0}) \frac{5}{8} \frac{T^2}{\pi^2 T_s}$$

The relative error then is

$$\frac{(p_{A_0} - p_{C_0}) \frac{5}{8} \frac{T^2}{\pi^2 T_s}}{p_m T_s} = \frac{5}{8\pi^2} \left(\frac{T}{T_s} \right)^2 \frac{(p_{A_0} - p_{C_0})}{p_m}$$

Expressed as a percentage this becomes

$$E_4 = (+100) \left(\frac{5}{8} \right) \frac{1}{\pi^2} \left(\frac{T}{T_s} \right)^2 \frac{(p_{A_0} - p_{C_0})}{p_m} \quad \ldots\ldots [39]$$

or, for the simple diagram

$$E_4 = (+100) \left(\frac{5}{8} \right) \frac{1}{\pi^2} \frac{T^2 S}{T_s A} (p_{A_0} - p_{C_0}) \quad \ldots\ldots [40]$$

and, for the differential diagram

$$E_4 = (+100) \left(\frac{5}{8} \right) \frac{1}{\pi^2} \frac{T^2 S}{T_s A} (p_{C_{10}} - p_{C_{20}}) \quad \ldots\ldots [41]$$

It should be noted that this is the maximum error due to the inertia of the mercury column since $\frac{dc}{dt}$ has been assumed equal to a constant. In a practical case the velocity is not destroyed at a constant rate. At the beginning of closure, the turbine gates move slowly and the rate of velocity change is small. The mercury in the manometer follows the pressure change in this case more closely than in the case assumed.

In the simple diagram of Prof. Thoma's Fig. 8 (1), the error in the water quantity is then not more than

$$E_4 = (+100) \left(\frac{5}{8} \right) \frac{1}{\pi^2} \frac{(3.74)^2 (0.728)}{(11.51)(18.20)} (1.29) = +0.39 \text{ per cent.}$$

In the simple diagram of the numerical example, the error in the water quantity is not more than

$$E_4 = (+100) \left(\frac{5}{8} \right) \frac{1}{\pi^2} \frac{(4.0)^2 (0.4723)}{(17.1)(16.21)} (0.798) = +0.14 \text{ per cent.}$$

In the process of delineation the areas a_1, a_2, a_3, etc., are so chosen that the points E, F, G, H (Fig. 12) are determined. These points have the same location on either recovery line of Fig. 12. The error in the recovery line of a given diagram due to inertia of the mercury thus varies as the line progresses along the diagram, the net result being a positive error, the maximum possible value of which is extremely small. Furthermore, by using the differential measurement, this error tends to be inappreciable because (*a*) of the small amount of mercury used, which reduces the value of T in Equation [41] and (*b*) the relatively smaller value of $(p_{C_{10}} - p_{C_{20}})$ in Equation [41] as compared with $(p_{A_0} - p_{C_0})$ in Equation [40] due to the elimination of the influence of the forebay surge and to the tendency to reduce the recovery of velocity head.

TABLE 3 THEORETICAL CAUSES OF ERROR INVOLVED IN THIRTY-FIVE TESTS MADE WITH DIFFERENTIAL APPLICATION OF THE GIBSON METHOD

Key No.	Unit	Cause No. 2, per cent	Cause No. 3, per cent	Cause No. 4, per cent	Residual of causes, per cent
1	1	+0.36	−0.32	+0.03	+0.03
1	2	+0.30	−0.28	+0.02	+0.04
2	3	+0.66	−0.30	+0.17	+0.53
3	3	+0.46	−1.21	+0.39	−0.36
4	3	+0.12	−0.50	+0.09	−0.29
4	3	+0.30	−0.49	+0.11	−0.08
4	6	+0.21	−0.61	+0.08	−0.31
4	6	+0.29	−0.45	+0.09	−0.07
4	6	+0.38	−0.56	+0.12	−0.06
4	7	+0.22	−0.90	+0.12	−0.56
4	7	+0.31	−0.59	+0.14	−0.14
4	7	+0.41	−0.49	+0.15	+0.07
4	7	+0.38	−0.57	+0.17	−0.02
4	8	+0.19	−0.58	+0.12	−0.27
5	2	+0.22	−0.93	+0.03	−0.68
5	3	+0.24	−1.01	+0.03	−0.75
5	4	+0.22	−1.00	+0.03	−0.75
6	1	+3.09	−0.28	+0.07	+2.88
6	3	+2.25	−0.27	+0.06	+2.04
7	1	+1.07	−0.30	+0.02	+0.79
8	1	+0.44	−0.28	+0.02	+0.18
8	2	+0.00	−0.10	+0.00	−0.10
9	8	+0.06	−0.18	+0.05	−0.07
10	1	+0.28	−0.38	+0.02	−0.08
10	2	+0.25	−0.38	+0.01	−0.12
11	1	+0.00	−0.09	−0.02[a]	−0.11
11	2	+0.00	−0.13	−0.03[a]	−0.16
12	1	+0.32	−0.96	+0.04	−0.60
12	2	+0.13	−0.97	+0.03	−0.81
13	1	+0.55	−0.70	+0.09	−0.06
14	1	+1.24	−1.02	+0.40	+0.62
15	1	+0.79	−0.43	+0.06	+0.42
15	2	+0.72	−0.33	+0.07	+0.46
16	1	+0.67	−0.54	+0.06	+0.19
16	3	+0.62	−0.46	+0.06	+0.22
Average of 35 units without respect to sign				0.043 per cent	
Average of 35 units with respect to sign				+0.06 per cent	

[a] Static line below running line.

Application of Theory of Errors to Actual Tests

The above theory of errors has been applied to a large amount of test data obtained during the last few years. All the diagrams considered have been of the differential type and, consequently, error No. 1 (1) may be assumed equal to zero. The calculated values of errors Nos. 2, 3, and 4 (1) and of the resulting residual error at the point of maximum efficiency of the unit

[4] In Figs. 11 and 12, T_s is not taken as a (whole) multiple of T, but T_s is made equal to 2.5 T. The computation made herein refers to the case that $T_s = nT$, n being a whole number. The case that $T_s = \left(n + \frac{1}{2} \right) T$ gives the same relative error.

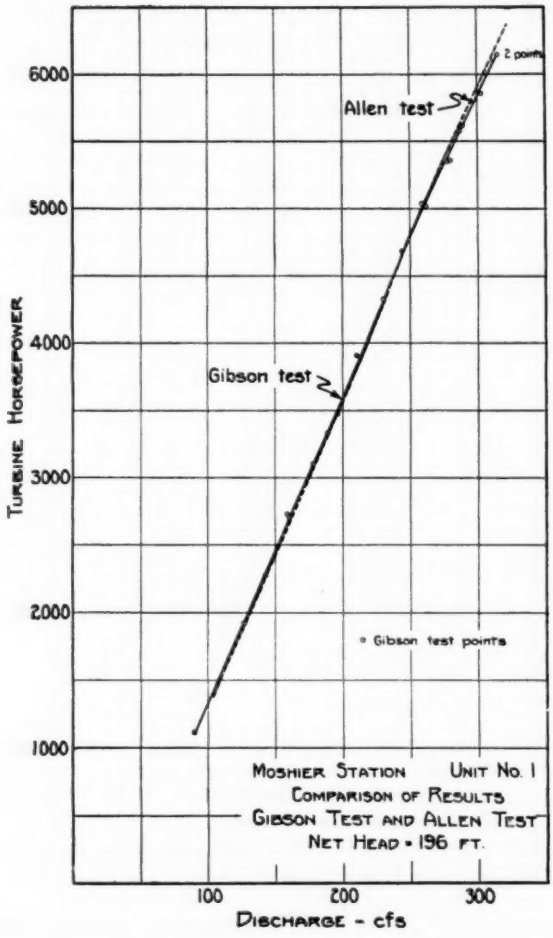


Fig. 13

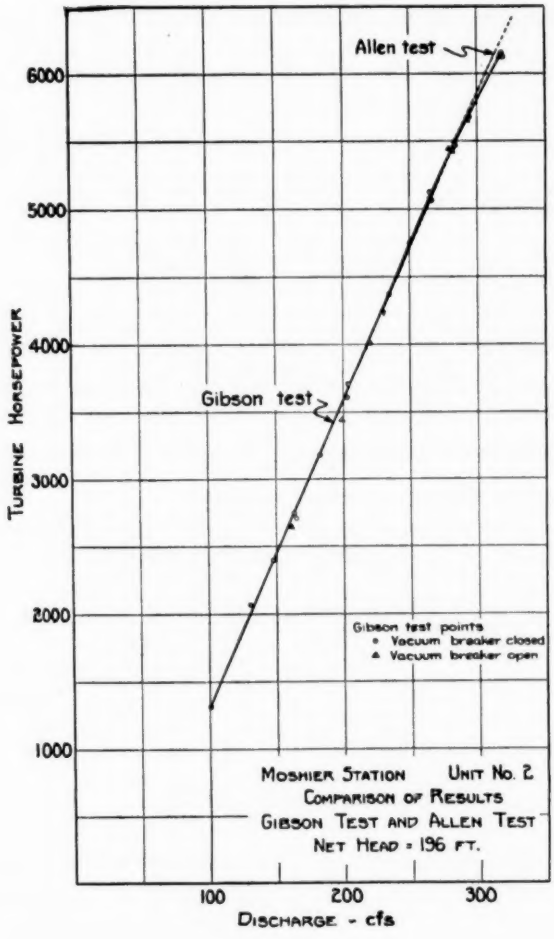


Fig. 14

tested, were found to be as given in Table 3. In the case of error No. 2, 0.075 was adopted as a mean value of the coefficient given by Prof. Thoma. From this investigation the mean residual error at the point of maximum efficiency in 35 cases examined would be 0.43 per cent without regard to sign and +0.06 per cent considering sign, as shown in Table 3.

Considerable experimental work has been carried on from time to time in the hope of being able to find the magnitude of the residual error experimentally, but it seems probable that it is too small to be within the range of experimental observation. It is well known that, even in laboratory work, it is difficult to get results to agree exactly when the same model is tested in two different laboratories.

In confirmation of this conclusion attention may be called to Figs. 13, 14, and 15, which give comparative test results of three units by both the Allen and the Gibson methods. The remarkable agreement between the two measurements is apparent. In the case of Moshier unit No. 1 correction for the theoretical errors puts the two curves in slightly better agreement, and in the case of Moshier unit No. 2 the reverse is true, although the agreement either with or without the theoretical corrections is very good. In the case of the 1900-hp unit shown in Fig. 15, the correction for theoretical errors tends to make the Gibson curve depart from the Allen curve by a greater amount. It is possible, of course, that both tests may be in error because, as pointed out by de Haller (5), there are theoretical reasons for believing that the Allen method gives too small quantities when the velocity front is attenuated. The magnitude of this

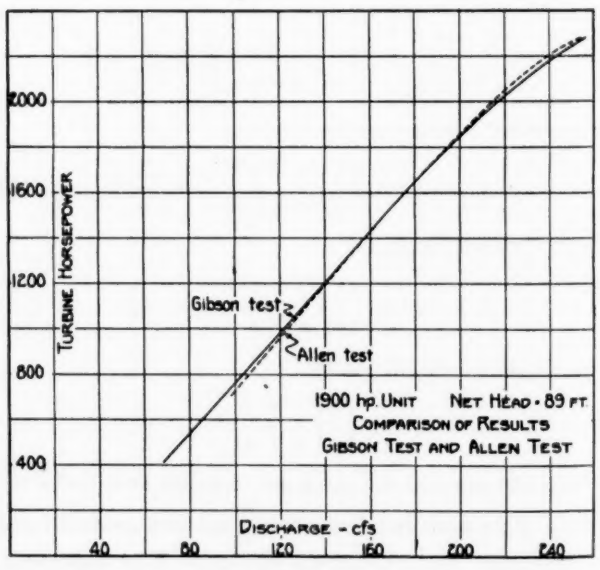


Fig. 15

error also may be so small that it cannot be determined experimentally. However, it would be a remarkable coincidence if two methods differing so much in principle should have errors in the same direction and in like degree. In addition to these two

cases, we have comparisons with volumetric measurements in the original Cornell tests, and other comparisons involving weirs and pitot tube measurements.

The Gibson tests at Cornell University which were compared with volumetric determinations have been analyzed with respect to the various theoretical errors discussed herein. The diagrams taken were of the simple type so that error No. 1 is included. The results of this analysis are given in Table 4.

It is evident that in this series of tests involving 19 individual determinations, the residual error is calculated to be —0.36 per cent assuming a coefficient of 0.075 for determining error (6). The attempt to correct for the theoretical errors results in increasing the divergence between the volumetric and the Gibson results from the value of +0.2 per cent to +0.5 per cent. This indicates that the theoretical corrections are not applicable to the extent estimated.

It will be interesting to tabulate the theoretical errors involved in the diagrams shown in Figs. 6 and 7. These two diagrams were taken on unit 20 of Schoellkopf station 3-*C* at Niagara Falls, N. Y. One was a differential and the other a simple diagram. Without correction for theoretical errors the results as given in Table 5 were obtained.

TABLE 4 COMPARISON OF VOLUMETRIC, GIBSON AND ADJUSTED GIBSON MEASUREMENTS RECORD OF CORNELL TESTS MADE UNDER APPROXIMATELY THE NIAGARA CONDITIONS

Run No.	Cause no. 1	Cause no. 2	Cause no. 3	Cause no. 4	Residual of four causes, per cent	Discharge—cfs: Volumetric	Gibson	Adjusted Gibson	Per cent variation: Volumetric to Gibson	Volumetric to adjusted Gibson
45	−0.06	+0.05	−0.62	+0.12	−0.51	20.27	20.31	20.41	+0.2	+0.69
46	−0.07	+0.09	−1.00	+0.14	−0.84	20.42	20.52	20.69	+0.5	+1.32
47	−0.07	+0.16	−0.78	+0.16	−0.53	20.45	20.65	20.76	+0.9	+1.52
Mean of series					−0.63	20.38	20.48	20.62	+0.5	+1.18
5	−0.18	+0.29	−0.43	+0.12	−0.20	30.38	30.51	30.57	+0.4	+0.62
6	....	....	....	....	....	30.63	....	....	....	....
7	−0.18	+0.26	−0.25	+0.12	−0.05	30.69	31.15	31.17	+1.5	+1.56
Mean of series					−0.13	30.57	30.82	30.87	+0.8	+0.98
8	−0.20	+0.09	−0.66	+0.16	−0.61	42.24	42.17	42.43	−0.2	+0.45
9	−0.21	+0.10	−0.70	+0.15	−0.66	41.94	42.28	42.56	+0.8	+1.48
10	−0.22	+0.09	−0.32	+0.13	−0.32	42.22	42.09	42.23	−0.3	0.00
12	−0.20	+0.13	−0.67	+0.16	−0.58	...	41.96	42.20	...	....
Mean of series					−0.54	42.13	42.12	42.35	0	+0.52
30	−0.24	+0.10	−0.44	+0.13	−0.45	40.36	40.41	40.59	+0.1	+0.57
31	−0.22	+0.12	−0.51	+0.13	−0.48	40.36	40.90	41.10	+1.3	+1.83
32	−0.25	+0.21	−0.36	+0.13	−0.27	40.98	40.88	40.99	−0.2	0.00
Mean of series					−0.40	40.57	40.73	40.89	+0.4	+0.80
15	....	....	....	....	....	46.82	...	...	...	....
17	−0.26	+0.09	−0.23	+0.13	−0.27	46.98	46.60	46.73	−0.8	−0.53
16	−0.30	+0.13	−0.28	+0.12	−0.33	46.95	46.60	46.75	−0.7	−0.43
Mean of series					−0.30	46.92	46.60	46.74	−0.7	−0.38
33	−0.28	+0.24	−0.51	+0.18	−0.37	50.30	49.95	50.13	−0.7	−0.34
34	−0.28	+0.32	−0.49	+0.20	−0.25	50.66	51.47	51.60	+1.6	+1.86
35	−0.29	+0.37	−0.27	+0.16	−0.03	50.93	51.15	51.17	+0.4	+0.47
Mean of series					−0.22	50.63	50.85	50.97	+0.4	+0.67
Total of means					.....	231.20	231.60	232.44		
Mean of means					−0.36				+0.2	+0.53

The causes referred to in this table are the possible causes of error discussed by Prof. Thoma. The discharges referred to as "adjusted Gibson" are the discharges obtained from the Gibson diagram corrected for Prof. Thoma's causes of error.

The value given for cause No. 2 is the average of the limits proposed by Prof. Thoma.

TABLE 5

	Cfs	Net head	Generator kw
Fig. 6—differential	2793	216.53	46933
Fig. 7—simple	2786	215.76	46640

When equalized to the same net head, the discharge quantities agree within 0.1 per cent. If the theoretical errors are so calculated, the divergence then becomes about 0.7 per cent.

Referring to Prof. Thoma's Fig. 8 (1), and to the numerical example (6) mentioned by Prof. Thoma, the individual errors and the residual error have been calculated as given in Table 6.

TABLE 6

	Thoma Fig. 8 (1) station Schoellkopf 3-*A* unit no. 2	Gibson's numerical example station Schoellkopf 3-*B* unit no. 16
E_1	−1.0	−0.8
E_2	+1.9	+0.9
E_3	−1.0	−0.6
E_4	+0.4	+0.1
Theoretical residual error	+0.3	−0.4

Conclusions

(*a*) The differential method is independent of error No. 1 (1) due to accessory motions of the water particles, where the measuring sections are equal or nearly so and where the upper measuring section is located at a reasonable distance from the inlet. This error where existing, is negative.

(*b*) In the study of error No. 2 (1), which may be due to assuming that the recovery of friction head during closure is proportional to the square of the average velocity, it is difficult from our analysis to accept Prof. Thoma's coefficient of $\frac{1}{20}$ to $\frac{1}{10}$. This error could only exist in appreciable degree where the "velocity front" in the conduit is attenuated. Most penstocks of modern plants are so large that the velocity front is almost square. This error is positive.

(*c*) Error No. 3 (1), which is concerned with the friction of the mercury column in the pressure measuring apparatus and of the balancing water column in the connecting piping and penstock, is reduced in the case of the differential measurement. This is true because only a small amount of mercury is used and, consequently, the natural period of oscillation T is smaller. Moreover, the quantity $(p_1' - p_0')$ is smaller as the forebay surge is eliminated from the diagram and with approximately equal measuring sections there is little or no velocity head in the diagram ordinate. Furthermore, by damping the gate stroke toward the end of the closure, this quantity may be made extremely small.

(*d*) The maximum error due to the inertia of the mercury has been considered, herein, by assuming that the rate of change of velocity with respect to time is a constant. Actually the velocity is not destroyed at a constant rate since at the beginning of closure, the turbine gates move slowly and the rate of velocity change is small. The mercury in the manometer gage consequently follows the pressure change more closely than under the assumption made.

(*e*) Finally, it has been demonstrated that with the differential

diagram errors Nos. 2 and 4 (1) are small and are opposite in sign to error No. 3 (1) resulting in a very small residual error. Although errors Nos. 3 and 4 (1) may be theoretically computed from the diagram itself, this should not be done without reference to error No. 2 (1).

Until experimental verification of the foregoing theory of errors has been established, we conclude that the residual error is probably within the limits of precision possible in the measurement of large quantities of water. This conclusion is confirmed by the tests made in comparison with other methods of measurement and, indirectly, by a great many tests involving both the simple and differential application of the Gibson method.

REFERENCES

1 "Über den Genauigkeitsgrad des Gibsonschen Wassermessverfahrens," by Professor D. Thoma. Published in " Mitteilungen des Hydraulischen Instituts der Technichen Hochschule München," Bull. No. 1, 1926, pp. 59–74.

2 Trans. A.S.M.E., 1923, page 29.

3 Ibid., 1923, page 343.

4 "Calculus," by Davis and Brenke, p. 366. The Macmillan Co., New York, N. Y.

5 "Considerations theoriques sur la mesure des debits d'eau par la methode d'Allen," par P. de Haller, Helv. Phys. Acta, 1930.

6 "The Gibson Method and Apparatus for Measuring the Flow of Water in Closed Conduits," by N. R. Gibson, Trans. A.S.M.E., 1923, pp. 343–376.

Observations on the Use of Current Meters for Precise Flow Measurement[1]

By L. A. OTT,[2] KEMPTEN, GERMANY

This paper discusses the circumstances surrounding the development of current meters in Central Europe, and the application of these meters in the testing of large water-power plants. Among the important steps presented in the application of these instruments are, the use in parallel of a plurality of meters, the adoption of simultaneous electric recording by a chronograph, the consequent adoption of water-free contact mechanisms and elimination of the multiplicity of meter types, and standardization of practice.

Reference is made to the practice of attempting to avoid the necessity for properly converting unfavorable measuring sections by using two types of meter having different degrees of error in resolving oblique flow. It is pointed out that precise testing presupposes equally precise rating of meters with duplication of the exact supporting means used in field tests. Standard values for the degree of accuracy of the instruments used in Germany are given, and the accuracy actually obtainable in practice is discussed in the light of comparative tests which have been made.

THE testing of large water-power plants, together with developments in the construction and use of current meters for precise flow measurement, have become farther advanced in Central Europe than elsewhere because of certain favorable circumstances. First, engineers and manufacturers recognized and accepted the fact at an early date that in the construction of hydrometric vanes, the axis of rotation is best placed parallel with rather than normal to the flow. This eliminated unprofitable differences of opinion as to the advantages and disadvantages of cup and screw meters with the result that rapid progress was made. Second, just at the time when the necessity arose for exact water measurements in large water-power plants, we had in Dr. Epper, chief of the Swiss Hydrographic Office, a pioneer in water-measurement technique, who was versed in the solution of problems of measurement in civil and mechanical engineering fields, and who, from his own experience in the testing of meters by towing tests, was able to give instructions for standard procedure. Third, Dr. Epper and other engineers active in the development of water-measurement technique, could always depend on the willing cooperation of a firm with considerable experience in this field, the management of which considered the invention and production of good water-measuring instruments as its life task.

At the present time, it is the predominant opinion of engineers that the simultaneous reading of a number of meters, the indications of which are electrically recorded on a chronograph, affords the most accurate method of measuring water flow in large power plants. This method of water measurement, first applied in 1908 by Prof. Reichel at the testing station for water motors at the Technische Hochschule of Berlin (1),[3] but only introduced in general practice about 1922, is comparatively simple to apply, even in the largest low-head power plants.

The necessity for the use in parallel of a plurality of meters fortunately did not require the development of a new type of instrument in Europe. On the contrary, it reduced the number of types previously used. Among other meters which disappeared from use were the "Weichsel," Epper-Ott Meter IV (2), and the "Iller," Epper-Ott Meter IX, with guard ring (3).

These meters, although very good instruments, disappeared because they possess electric-contact mechanisms accessible to the water. Meters sufficiently free from mutual influence to permit simultaneous operation must have water-free contact, such as found in the "Rhein," Mensing-Ott Meter VI (4), and the "Texas," Ott Meter V (5). Of these two meters, the latter finally has proved itself superior as a result of special advantages derived from the axle bearing of its propeller. Since water measurements in large European plants during the past five years have been conducted with this Ott Meter V, it may be considered as the present European standard.

This meter, like all Ott meters, can be furnished with either a spoke propeller or a spoon propeller. The spoke propeller is a cylindrical screw with three blades, confined by straight lines, and attached to the boss by means of spokes. The spoon propeller is a conical screw with only two blades which form one solid piece with the boss. Although the conical screw has an advantage in that it will not entrap grass and is self-cleaning, this is frequently less important in flow measurements than the advantage of the cylindrical screw with spokes which, with the same diameter and pitch, possesses a smaller error of measurement in a flow which is somewhat oblique to the axis of the instrument.

Since no meter has yet been constructed which correctly records the velocity component normal to the measuring section of obliquities of flow exceeding about 15 deg, great emphasis is placed in Europe on the importance of using a proper measuring section having smooth parallel flow. If such a section is not available, guiding walls and decks are installed and, prior to undertaking tests, preliminary runs are made to prove the acceptability of the measuring section. If the water filaments run reasonably close to parallel in a measuring section, but as a whole run obliquely at a known angle, then the meter axis is not placed normal to the profile but parallel with the flow and the results of the measurement are afterward multiplied by the cosine of the direction angle.

To avoid the necessity of converting unfavorable measuring profiles by guide walls, tests are sometimes made with two instruments with propellers of different cosine characteristics. Corrections are then derived from the difference in results from these two meters. Such tests, however, have not been employed in Europe up to the present because the method doubles the work

[1] Translated by Dr. J. R. Zwickl, chief engineer, refrigerating machinery division, Baldwin-Southwark Corp., Philadelphia, Pa., Assoc-Mem. A.S.M.E., and L. F. Moody, professor of hydraulic engineering, Princeton University, Princeton, N. J. Mem. A.S.M.E.

[2] Mathematik Institut, Kempten, Germany.

Contributed by the Hydraulics Division and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 3 to 7, 1933.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Numbers in parentheses refer to similarly numbered references at the end of the report.

of testing and calculating, and greatly increases the duration of the observations for the separate metering points over the time required in the normal method of measuring regular flow. In the normal method, it is a recognized fact that the individual results do not give the actual mean velocities at the individual metering points because of the variation of the stream picture. However, determinations of the exact water velocities at the individual metering points are not important inasmuch as the individual variations equalize out in the final result. In the two-meter method it is probably necessary, as Prof. D. Thoma has pointed out, to determine the actual mean velocities at the separate metering points, with an increase in observation time required for this purpose.

Meter Rating

The high accuracy of measurement necessary for turbine testing, presupposes a correspondingly high accuracy of meter rating, which must be accomplished with the meter supported during the towing tests in exactly the same manner as when it is being used for taking measurement readings. It must also be noted that during the rating tests it is essential that the water be brought to rest between each two runs if accurate results are to be obtained. If a large number of meters are to be calibrated in a limited period, several meters should be towed simultaneously in the rating flume rather than towing each meter individually and shortening the pauses between separate runs. At Kempten, four meters are usually rated at one time, with a fifteen-minute period allowed for bringing the water in the rating flume to complete rest after the test has been made. For larger flumes a longer period of time is required.

A graphical method is employed for recording the results of the rating tests. Although plotting revolutions per second or feet per second as abscissas and feet of travel per revolution as ordinates is convenient in recording the results of meter-rating tests, the method is not well suited for determining the accuracy of the rating or for comparing different ratings because the physical inter-relation between the water velocity, the rotation of the meter vane, and the mechanical and hydraulic resistances occurring in the instrument (for instance, the influence of oil viscosity in the ball bearings) will disappear. It is better, with the same abscissas, to plot as ordinates the corresponding velocity slip (6), that is, the difference, $\Delta = v - kn$, between the velocity v of the towing car and the axial advance of the screw calculated from k, the hydraulic pitch of the propeller, and n, its number of revolutions per second. The properties of the instruments, the special influences of the supporting devices, and the towing procedure are then more clearly defined than in the first method of plotting mentioned. In a good instrument, the scattering of the individual observation points compared to the faired curve should not exceed 0.007 fps over the entire velocity range from 0.4 fps to 10 fps.

If we take this scattering of the rating test points as a measure of the accuracy of measurement of a meter under favorable flow conditions, we see that a relative accuracy of measurement of 1 per cent presupposes a water velocity of about 0.7 fps. For this reason it is specified in the German standards for water measurement that the water velocity in the measuring section should not be less than 0.25 meter per sec or 0.8 fps. In spite of the numerous comparative tests already conducted with various methods of measurement, and in particular the direct volumetric measurement, the degree of accuracy actually attainable in practice still appears to be uncertain. This is not surprising since checking accuracies of measurement of more than 99 per cent is a very delicate matter; particularly the direct volumetric measurement of the large volumes of water which have to be handled in such comparisons, even when a sufficiently large container is available, appears in general not to be a simple matter but, on the contrary, a very difficult problem.

References

(1) *Zeit. V.D.I.*, 1908, vol. 52, p. 1835.

(2) "Effect of Turbulence on the Registration of Current Meters," by David L. Yarnell and Floyd A. Nagler, Trans. Am.Soc.C.E., 1931, vol. 95, p. 772, Fig. 4. Also, "The Rating and Use of Current Meters," by Carl Rohwer, Tech. Bull. No. 3, May, 1933, Colorado Experimental Station, Fort Collins, Plates 7 and 23.

(3) *Schweizerische Bauzeitung*, 1927, vol. 89, p. 102.

(4) "Effect of Turbulence on Registration of Current Meters," by David L. Yarnell and Floyd A. Nagler, Trans. Am.Soc.C.E., 1931, vol. 95, p. 772, fig. 5.

(5) "Research Institute for Hydraulic Engineering," by H. Rouse, Trans. A.S.M.E., 1932, paper HYD-54-3, Fig. 9. Also, "Making Water Measurements With Current Meters," by Sigmund J. Bitterli, *Power*, Jan. 19, 1932, p. 103, fig. 2.

(6) "Hydraulics for Engineers," by R. W. Angus, Pitman, London, 1931, p. 115, fig. 59.

The Psychrograph

By A. M. NORRIS,[1] BALTIMORE, MD.

Problems in air-conditioning can be solved quickly and accurately by the use of a psychrograph developed by the author. In this paper, representative problems of the type met with in air-conditioning work are solved on the psychrograph, accompanied by explanations, and illustrate the ease with which the chart can be used in the various cases presented.

THE AUTHOR of this paper has long been impressed by the importance of the relationship expressed by the ratio of sensible-heat input to total-heat input in all air-conditioning calculations. Since a simple proportion will plot as a straight line on ordinary coordinate paper, it was apparent that certain problems could be solved graphically if a psychrometric chart were constructed with equal units of sensible heat and equal units of total heat as coordinates.

The psychrograph is such a chart and was designed for the graphic solution of problems. It has been printed on letter-size sheets, convenient for filing with other calculations; thin paper being used to permit blue printing, should more than one copy of a solution be desired.

An increase in legibility was achieved by inclining the axis of the abscissas and using a vertical axis for the ordinates. This had the effect of lengthening those lines on the psychrograph which indicate dry-bulb temperatures, and, consequently, of increasing the spacing between all of the other inclined and curved lines which cross them.

While the framework of the psychrograph consists of a vertical system of lines representing *total* heat in Btu per pound of dry air and vapor present, crossed by an inclined system of lines representing *sensible* heat in Btu per pound of dry air, the chart has been so designed as to show those functions which are commonly mentioned in the statement of an air-conditioning problem. Therefore, dry-bulb temperatures are shown along the corresponding lines of sensible heat, and wet-bulb temperatures along those of total heat.

It is recognized that, theoretically, dry-bulb lines drawn on such a frame work will have a very slight curvature, due to the sensible heat in the vapor under conditions of partial saturation, but this sensible heat in the vapor is seldom, if ever, considered in an air-conditioning problem; and the specific heat of steam, superheated at very low pressures, seems to be a matter of debate among the authorities. It was therefore disregarded, as its introduction would carry with it complications in calculation of air quantities required, which are not warranted by the possible accuracy of a commercial air-conditioning estimate nor by the accuracy of the instruments which are available for the measurement of results. In the preparation of the psychrograph, Goodenough's "Table of Mixtures of Air and Saturated Water Vapor" was used.

In Fig. 1, the gradient of the dry-air line, a section of which appears on the upper left-hand margin, was determined by referring points on it to the total-heat scale, the total heat in a pound of dry air at any temperature being identical with the sensible heat per pound given in Goodenough's table for that temperature. Within the limits of the chart, this line is straight, as the specific heat of dry air is assumed constant at 0.2416 Btu. Other lines on the psychrograph were plotted from tables calculated by the author, using accepted formulas.

There remains only the scale, *ratio of sensible-heat to total-heat lines.* This scale was constructed by using the heat-ratio-reference center, at the lower left-hand side of the chart, as the origin, and plotting the simple proportion $\frac{Sensible\ heat}{Total\ heat} = N$, for various values of N, on the two heat scales which form the basis or skeleton, of the chart.

The lines so obtained will pass through the origin, or heat-ratio-reference center and points on the scale, *ratio of sensible to total-heat lines.* These lines are only directional in their nature. If any point on the chart is taken and heat added to or deducted from the air represented by this point in a definite ratio of sensible heat to total heat, the direction of change of state must be the direction of the heat-ratio line corresponding to the specific ratio.

The following theorems apply:

Theorem 1. A line drawn parallel to any given heat-ratio line, through any given point on the chart, will be the locus of all changes of psychrometric condition in the air represented by the given point, when heat is added to or deducted from this air in the given ratio of sensible heat to total heat. From this follows:

Theorem 2. When two samples of air are mixed, the locus of all possible psychrometric conditions, for mixtures in varying proportions, must be the straight line joining the two points which represent the psychrometric qualities of the two samples, since one sample must lose heat and the other gain heat in identical quantities and ratios of sensible to total heat; and the two points and the point representing the mixture will all lie on the same heat-ratio line parallel.

An exception to Theorem 2 occurs when the straight line joining the two points crosses and recrosses the saturation curve. In this case, the locus of possible mixtures will follow the saturation curve instead of crossing it, and the quality of mixture will be somewhat uncertain, if less than saturated, as a portion of the vapor present may condense out of the mixture and fail to re-evaporate.

Acknowledgment is made by the author to W. A. Minkler for modifications in the original chart which have increased its legibility and, consequently, materially simplified its use.

Example 1. The use of the psychrograph as an ordinary psychrometric chart is shown in Fig. 1, in which a single point has been taken. Reference to the figure will show that the dry-bulb temperature is 80 F; wet-bulb temperature, 67 F; relative humidity, 51 per cent; dewpoint, 60.4 F; grains of vapor per lb of dry air, 78; cu ft per lb of dry air plus vapor present, 13.84; sensible heat, 19.33 Btu; total heat, 31.10 Btu.

It will readily be seen that if any two of the foregoing char-

[1] Chatard & Norris, Inc. Mem. A.S.M.E. Mr. Norris received the degree of M.E. from Stevens Institute of Technology in 1907 and was then for three years connected with the Consolidated Gas, Electric Light & Power Co., as test engineer. In 1910 he became a member of the firm of Chatard & Norris, Baltimore, Md., contracting engineers and manufacturers' agents. In 1935 he became associated with the engineering office of D. D. Kimball of New York City.

Contributed by the Process Industries Division and presented at a meeting of the Metropolitan Section of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., October 22, 1934.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until September 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expression of their authors, and not those of the Society.

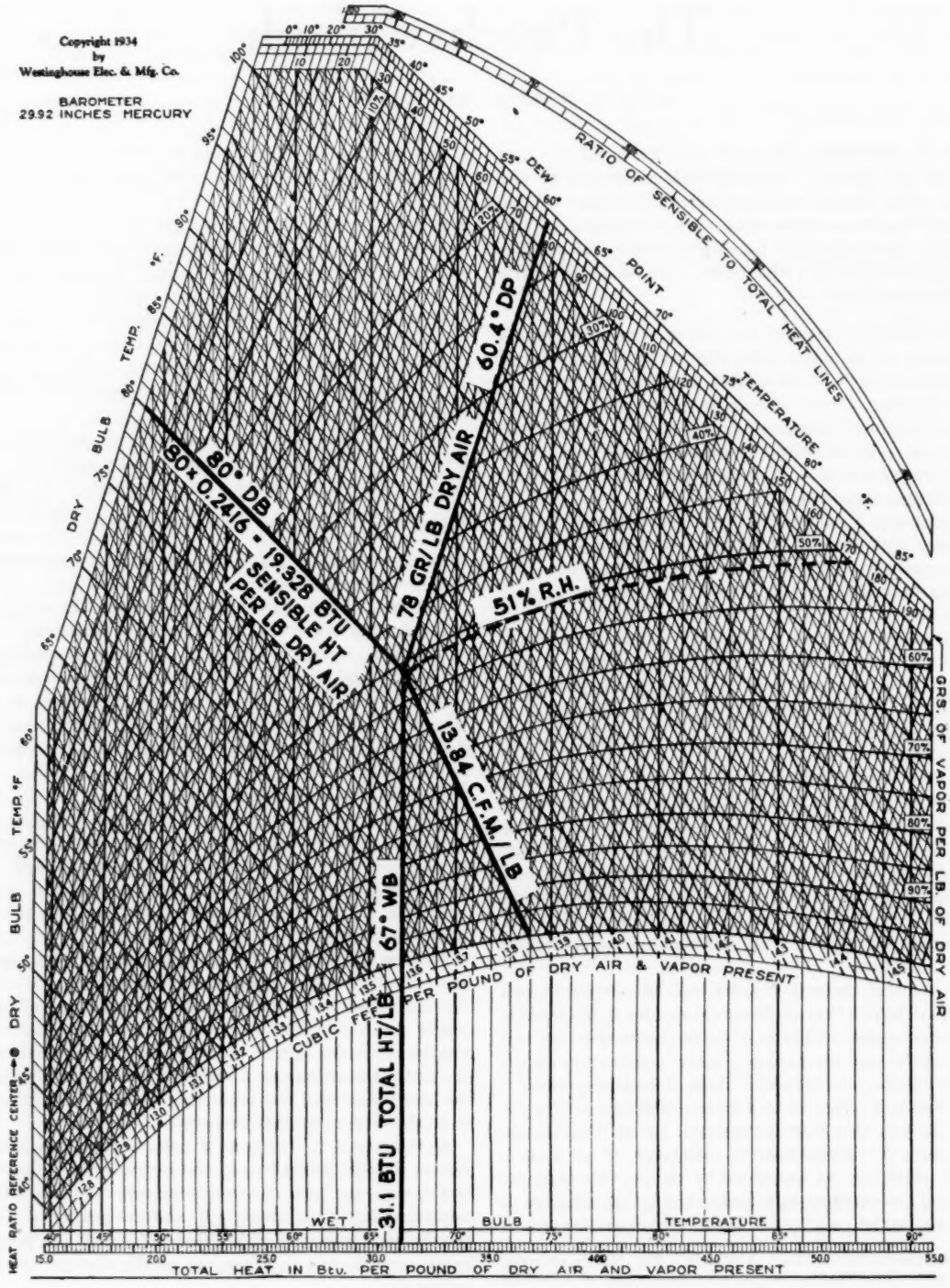


FIG. 1

acteristics are known, and the lines representing them drawn, the intersections of these lines will fix all the other characteristics for the sample.

Example 2. In this problem, in which the dewpoint temperature of 60 F and dry-bulb temperature 75 F, are given. It is desired to find the relative humidity should the dry bulb be increased to 90 F with no moisture added (dewpoint constant).

Referring to Fig. 2, locate the point of intersection of the slant line representing 60 F dewpoint and the slant line representing 75 F dry-bulb temperature.[2] At this point, move obliquely upward and to the right, along the 60 F dewpoint line, to its intersection with the 90 F dry-bulb line. The relative humidity here is read as about 37 per cent. Incidentally, the wet-bulb temperature has increased from about 65 F to approximately 70 F.

Example 3. Given air at 80 F dry-bulb temperature and 58 F wet-bulb temperature, to find the increase in total heat when

[2] When solving problems it is assumed that reference will also be made to Fig. 1 which is a reproduction of the psychrograph.

56 grains of moisture per lb of air are added and the dry-bulb temperature remains at 80 F.

Referring to Fig. 3, locate the point of intersection of the slant line representing the 80 F dry-bulb temperature and the vertical line representing the 58 F wet-bulb temperature. From this point, project upward to the right along the grains per lb line where 36.5 grains per lb is read. The total heat may be read as 24.8 Btu per lb directly from the wet-bulb temperature.

If 56 grains per lb be added, the mixture will contain 36.5 + 56 = 92.5 grains per lb. Locate this quantity on the right-hand scale and then project downward to the left to the intersection with the 80 F dry-bulb temperature line. From this intersection project vertically downward where total heat for the new condition is read as 33.3 Btu per lb. Thus, with the dry bulb remaining constant, the total heat increased from 24.8 to 33.3 Btu per lb, or 8.5 Btu with an increase in moisture content of 56 grains per lb.

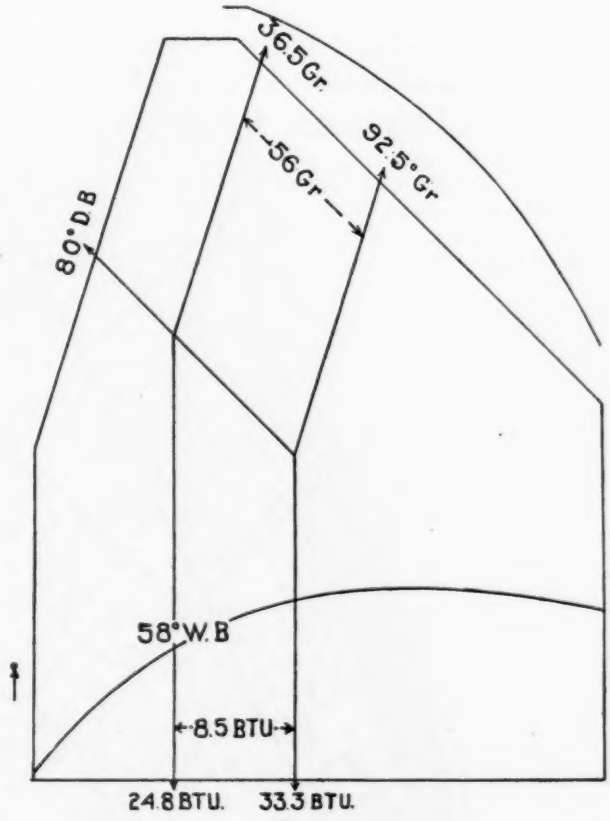


FIG. 3

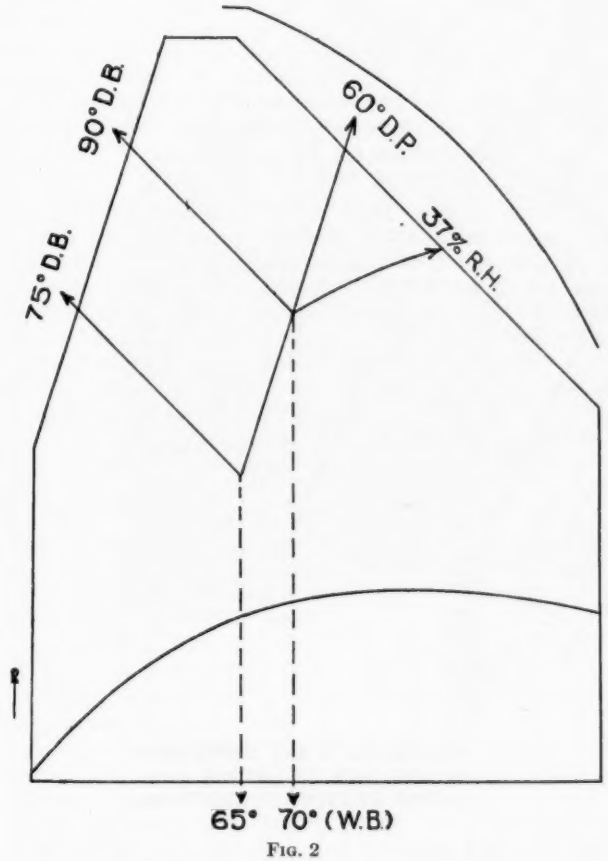


FIG. 2

Example 4. Fig. 4 shows the solution of the problem in which is given air cooled from 80 F dry-bulb temperature and 65 F wet-bulb temperature (about 45 per cent relative humidity) to 60 F dry-bulb temperature and 55 F wet-bulb temperature (72.5 per cent relative humidity), to find the latent heat and sensible heat extracted in cooling.

This example demonstrates a graphical solution that cannot be made directly on any other psychrometric chart. Locate the two points representing the conditions of the air at the two sets of dry-bulb and wet-bulb temperatures. Project vertically downward and read the total heat for the two points as 29.6 and 23.0 Btu per lb of air. Connect the two points by a straight line. Draw a line parallel to this line through the heat-ratio-reference center to the curved scale marked, *ratio of sensible to*

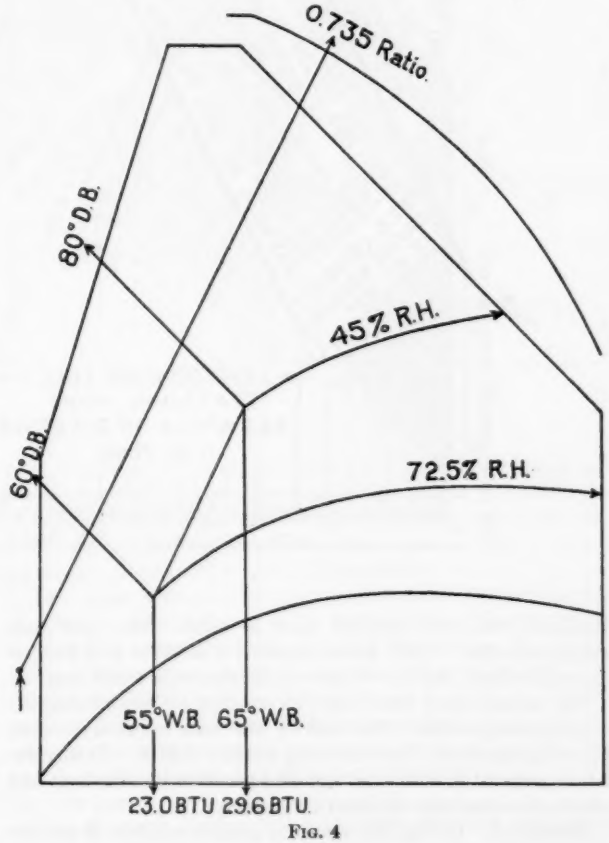


FIG. 4

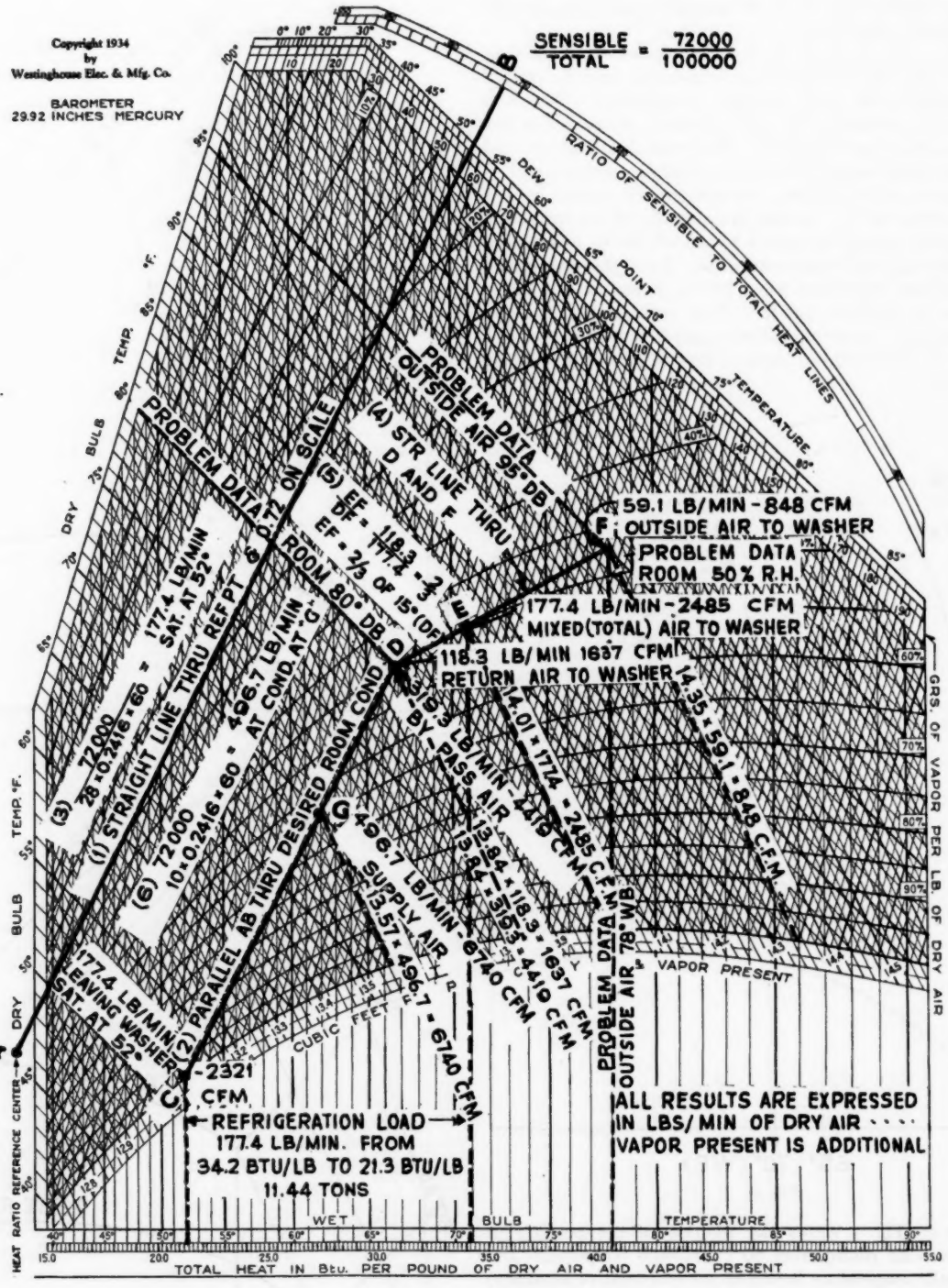


FIG. 5

total-heat lines, and read the ratio as 0.735. The total heat extracted is 29.6 — 23.0 = 6.6; then 6.6 × 0.735 = 4.85 Btu per lb, sensible heat, and 6.6 — 4.85 = 1.75 Btu per lb, latent heat.

The sensible-heat value may be checked by multiplying the dry-bulb-temperature difference by the heat required to raise the temperature of 1 lb of air 1 deg which is 0.2416. This calculation gives 20 × 0.2416 = 4.83 Btu per lb of sensible heat and checks the value from the chart very closely.

Example 5. In Fig. 5 is shown a graphic solution of an air-conditioning problem in which it has been assumed that a by-pass system, using a washer, will be installed.

The outside air is assumed at 95 F dry-bulb and 78 F wet-bulb temperatures, and an allowance of 848 cfm or 59.1 lb per min of outside air for ventilation purposes has been assumed. The desired room condition is 80 F dry-bulb temperature and 50 per cent relative humidity. It is also assumed that a heat estimate has been made and it has been found that the heat load, exclusive of the ventilation air which passes first through the washer,

but inclusive of any infiltration, is 72,000 Btu per hr of sensible heat and 100,000 Btu per hr of total heat, giving a ratio of 0.72.

The heat estimate must be broken down as already outlined because we are interested in the quantity of sensible and total heat liberated in the conditioned space. This heat must be disposed of, or carried out, by the conditioned-air supply, which supply must be raised from its condition at the outlets to the desired room condition by picking up this heat.

The surplus heat above delivery condition in the air for ventilation, which passes first through the washer, is removed in the washer, and thereafter it becomes simply a vehicle for the removal of room heat along with the reconditioned recirculated air. It is also assumed that the desired difference between room and supply air is 10 deg. The problem is solved as follows:

Find point B, on the scale, *ratio of sensible to total-heat lines*, corresponding to the ratio of 0.72. Through B and the heat-ratio-reference center A, draw a straight line AB. Find the desired room condition at D, representing 80 F dry-bulb temperature and 50 per cent relative humidity. Through point D draw a straight line CD, parallel with AB and intersecting the saturation curve. In accordance with Theorem 1, this line CD will be the locus of all possible delivery airs which will give the desired room condition, since the delivery air must pick up heat in the ratio $\frac{sensible}{total} = 0.72$ to finally reach this desired room condition, and therefore the point G at 70 F dry-bulb temperature will be the desired quality of the delivery air. This delivery air may be obtained by mixing room air at condition D and saturated air at condition C in the proper proportion, which will be 10 parts of saturated air at condition C to 18 parts of room air at condition D, since the heat lost by the air at condition D equals 10 deg drop times the quantity of air at condition D in pounds of dry air per unit of time, multiplied by the specific heat of 1 lb of dry air, and this in turn will equal the heat gained by the air at condition C, which equals 18 deg rise times the quantity of air at condition C in pounds of dry air per unit of time, multiplied by the specific heat of 1 lb of dry air. Eliminating common factors and transposing, it may be expressed as: The quantity of air at condition C is to the quantity of air at condition D as 10 is to 18 or as DG is to CG.

Since all of the refrigeration is applied to the saturated air, which must have sufficient capacity to remove 72,000 Btu per hr of sensible heat, this value divided by the temperature rise of 28 deg of the saturated air to 80 F, by the specific heat of a pound of air (0.2416) and by 60 min, will give 177.4 lb per min of air saturated at 52 F as required.

It is also possible to figure the delivery air at 70 F in the same manner by dividing 72,000 Btu per hr by 10 deg temperature difference by specific heat of a pound of air (0.2416) and by 60 min to obtain 496.7 lb of delivery air per min. Then 496.7 — 177.4 = 319.3 lb per min of room air through the bypass to give the desired mixture, and the ratio $\frac{177.4}{319.3} = \frac{10}{18}$.

By Theorem 2, the condition of the air to the washer may be obtained by drawing a straight line through the point D, representing the room condition, and point F, representing the outside-air condition of 95 F dry-bulb and 78 F wet-bulb temperatures. Under the terms of the problem, there must be 59.1 lb per min of outside air, and the air through the washer is 177.4 lb as already stated. Then 177.4 lb — 59.1 lb = 118.3 lb of return or room air which must mix with the outside air to pass through the washer. Since the temperature difference between D and F is 15 deg and the ratio of outside air to return air is $\frac{59.1}{118.3} = \frac{1}{2}$, EF must be twice the distance DE, and the point E falls on the 85 deg dry-bulb line. This is another way of stating the relationship from that given on the graphic-solution diagram, but it will be clear that $DF \times 118.3 = EF \times 177.4$; or $\frac{EF}{DF} = \frac{118.3}{177.4} = \frac{2}{3}$, which is to say, EF equals $\frac{2}{3}$ of 15 deg or 10 deg.

All of the results given have been expressed in pounds per minute of dry air, the vapor present being additional. Reference to the diagram will indicate the method of converting all of these quantities into cubic feet per minute. For instance, at the point F, a pound of dry air plus the vapor present will occupy 14.35 cu ft and 14.35 × 59.1 lb = 848 cfm.

In the same manner, the cubic feet per pound for point E was determined to be 14.01; for point D, 13.84; for point G, 13.57 and for point C, 13.08.

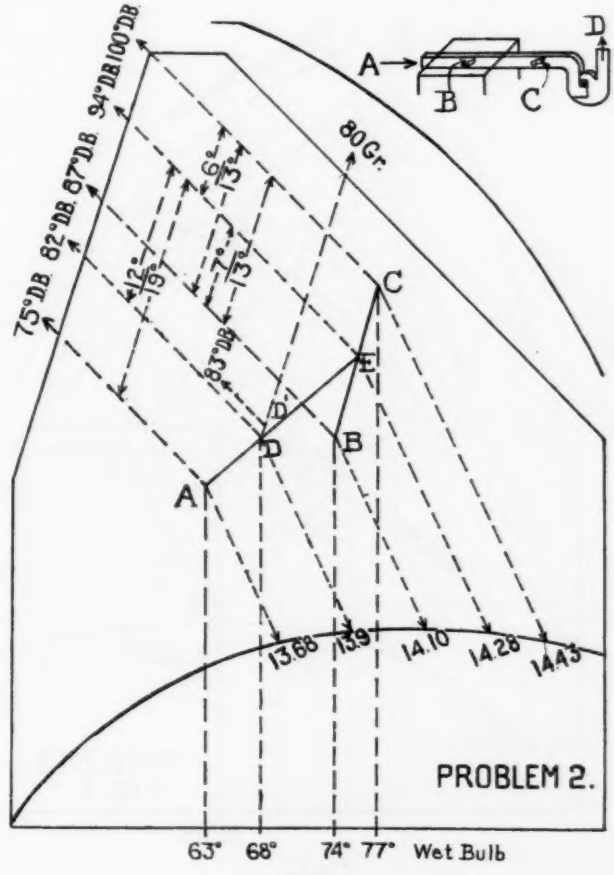


FIG. 6

It will be seen that the mixtures check out on the cubic feet per minute basis, as 848 cfm of outside air mixed with 1637 cfm of room or return air will total 2485 cfm of air to the washer, and 4419 cfm of room air mixed with 2321 cfm of washer air will result in a mixture of 6740 cu ft of delivery air.

The refrigeration required may be figured by projecting point E, which represents the air to the washer, to the Btu scale at the bottom of the chart, and obtaining 34.2 Btu per lb; also project point C, which is the saturated air, to the Btu scale, and read 21.3 Btu per lb; which is to say that each pound will have 12.9 Btu removed from it in passing through the washer. Since 177.4 lb per min of air at condition C passes through the washer and from each pound we remove 12.9 Btu, the total refrigerating load will be 11.44 tons.

It is, of course, obvious that this problem has been worked out

in far greater detail than is usually required. This was done to illustrate the complete information which can be obtained from a graphic solution.

Example 6. In Fig. 6 is shown the graphic solution of a problem in detecting air leakage.

The fact that the point on the psychrograph representing a mixture of two different samples of air will fall on a straight line joining the points representing the air samples and the fact that the dry-bulb temperature difference, between the mixture and the original samples, determines the proportion of the mixture, have been made use of in the solution of this problem.

The small drawing in the upper right corner indicates the original data. A duct drawing air from chamber A in which the condition is dry-bulb temperature 75 F, wet-bulb temperature 63 F, passes through chamber B, in which the condition is 87 F dry-bulb temperature, 74 F wet-bulb temperature, and through chamber C, in which the condition is 100 F dry-bulb temperature and 77 F wet-bulb temperature. Assuming perfect insulation, the delivery air at point D should be 75 F dry-bulb temperature and 63 F wet-bulb temperature, or the same as at A. Actually, the condition at point D is 82 F dry-bulb temperature and 68 F wet-bulb temperature and the anemometer reading at this point gives 13,900 cfm, which is equal to 1000 lb per min of dry air plus vapor present.

The problem is to find the leakage into the duct in compartments B and C. It will be obvious that this leakage must be a mixture of air at condition C and air at condition B and must therefore fall on the straight line BC; and that this mixture must combine with air at condition A to give the condition at D. Since the condition at D is known and has been plotted on the chart, the straight line passing through A and D, prolonged to cut line BC at E, will fix the leakage mixture. Since there are 1000 lb of air per min being delivered at D, the amount of air drawn in at A must be equal to $\frac{DE}{AE} \times 1000 = \frac{12}{19} \times 1000 = 632$ lb per min. Hence the quantity of leakage at condition E must be 368 lb per min or 1000 lb less 632 lb, and the quantity of air at condition B plus the quantity of air at condition C, must equal 368 lb per min, divided in the ratio $C = 7/13 \times 368 = 198$ lb per min and $B = 6/13 \times 368 = 170$ lb per min. The quantities expressed in pounds per minute may be converted into cubic feet per minute by the use of the scale of cubic feet per pound of dry air and vapor present, which will give the following readings: Point D, 13.9 cu ft per lb; point A, 13.68 cu ft per lb; point E, 14.28 cu ft per lb; point C, 14.43 cu ft per lb; point B, 14.10 cu ft per lb.

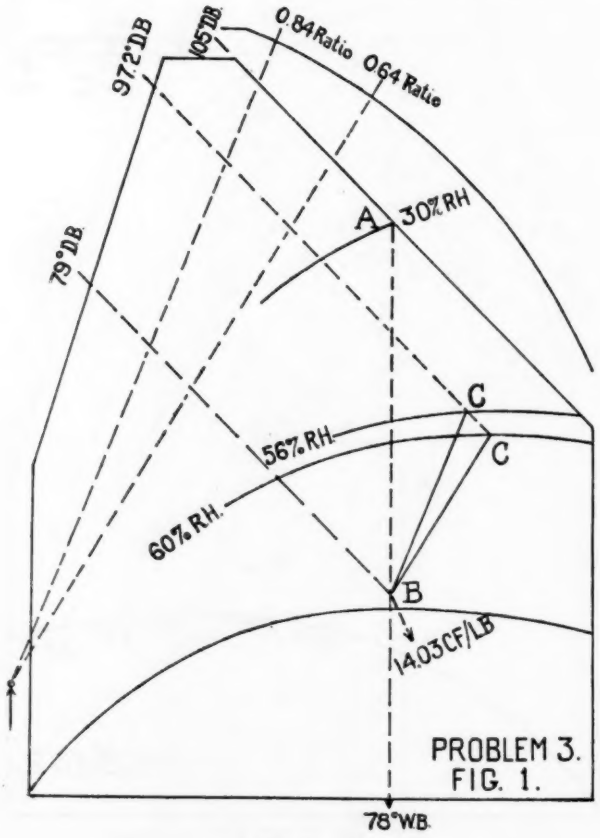


FIG. 7

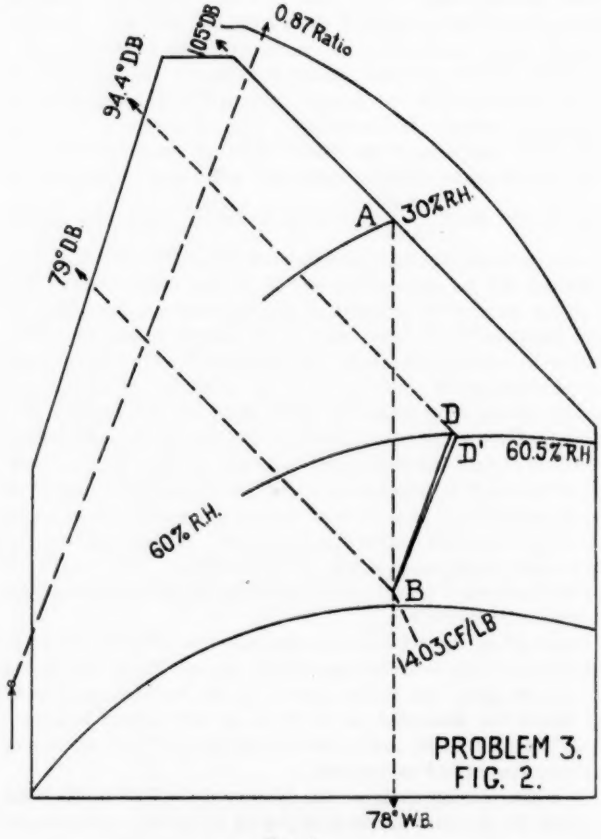


FIG. 8

Multiplying out the respective quantities expressed in pounds, gives 13,900 cfm at point D; 8645 cfm at point A; 5255 cfm at point E; and $A + E = 13{,}900$ cfm. Also 2857 cfm at point C; 2397 cfm at point B; and $C + B = 5254$ cfm.

It will also be noted that the same answers might have been obtained graphically had we used any other combination of the mixtures. In other words the line AB might have been drawn representing the mixtures of airs at conditions A and B and the line DC prolonged to cut AB at E', or a line AC might have been drawn and the line DB prolonged to cut AC at E''. In any case the calculated results would be identical.

For the purpose of this problem, we have assumed that there will be no transmission of heat through the duct walls other than by air leakage. However, any transmission of sensible heat through the duct walls can be calculated and reduced to a temperature rise for the air flowing through the duct expressed in

degrees of dry-bulb temperature. Let us assume that the air leaving the fan has a condition D' and has picked up 1 deg due to temperature head across duct walls. We may follow the line of constant moisture content *from* D' for 1 deg F dry bulb to point D and this point may be used for the solution of the problem just as it has been used in the problem as given.

It is the belief of the author that a great deal of information concerning air flows can be obtained by the intelligent use of wet- and dry-bulb thermometers.

Example 7. In this problem, which is graphically solved in Figs. 7 to 9, it is desired to select a standard washer and to maintain a constant relative humidity of 60 per cent in the room.

The heat estimate has revealed the following data: Sensible heat due to temperature difference T between the outside and room temperatures is 150 Btu per min per deg difference; sensible heat from lights, power, and people (a constant) is 400 Btu per min; latent heat is approximately 300 Btu per min (exact determination may depend on final obtainable room condition, which is not known at this stage of the problem; maximum observed temperature outside is 105 F; minimum outside relative humidity is 30 per cent; no return air is in circulation.

There are standard washers available in sizes of 2000 cfm, 5000 cfm, 7500 cfm, and 10,000 cfm. The washers are equipped with bypass of air around spray chamber and are guaranteed to have capacity to bring the leaving air to within 1 deg of the spray temperature.

It is desired (1) to select a standard washer which will maintain a constant relative humidity of 60 per cent in the room; (2) having selected the washer, to determine the room temperature when the outside air is 80 F dry bulb and 67 F wet bulb, the relative humidity in the room to be maintained at 60 per cent; (3) to determine the proportion of outside air at 80 F dry bulb and 67 F wet bulb which must be mixed with washed air to maintain 60 per cent relative humidity in the room. To solve:

(1) *To determine the spray temperature* for maximum conditions, plot 105 F dry bulb and 30 per cent relative humidity at A and read approximately 78 F wet bulb which will be the temperature which the spray will assume. The washer guarantee then fixes the leaving-air condition at B, with a dry-bulb temperature of 79 F and a wet-bulb temperature of 78 F. Since the air has neither gained nor lost total heat in passing through the spray the wet bulb must remain constant at 78 F.

Let the temperature rise of the circulated air, from condition B to the room temperature, equal T'.

Then T (the temperature difference between outside and room) + T' (the temperature rise from condition B to room temperature) must equal 105 deg (the outside temperature) minus 79 deg (the temperature at condition B), or 26 deg.

$$T + T' = 26 \therefore T' = 26 - T$$

The sensible-heat input to the room will be $150\,T + 400$.

The trial of the 5000-cfm washer is made in Fig. 7. The 5000-cfm washer will deliver $\frac{5000}{14.03} = 356$ lb of air per min.

Then $150\,T + 400 = T' \times 356 \times$ specific heat of air, as sensible-heat input, must equal the heat picked up by the circulated air. Therefore, if the 5000-cfm washer is selected, $150\,T + 400 = (26 - T)\,356 \times 0.2416$, or $T = 7.8$ deg.

Plot C, $105 - T = 97.2$ F on the 60 per cent relative-humidity line and draw BC which is parallel with the 0.64 heat-ratio line.

It will be seen that the 5000-cfm unit will not have sufficient humidifying capacity, since from the problem data for the room at 97.2 F dry-bulb temperature, sensible heat = $150\,T + 400 = 1570$ Btu per min, latent heat = 300 Btu per min, and total heat = 1870 Btu per min. Then $N = \frac{1570}{1870} = 0.84$

Draw $N = 0.84$ through B to C' at 97.2 F dry bulb and read 56 per cent relative humidity. Therefore try the next larger size washer.

The trial of the 7500-cfm washer is made in Fig. 8. Locate A and B as in Fig. 7.

As the 7500-cfm washer will deliver $\frac{7500}{14.03} = 534$ lb of air per min, then

$$150\,T + 400 = (26 - T)\,534 \times 0.2416, \text{ or}$$
$$T = 10.6 \text{ deg}$$

Plot D, $105 - T = 94.4$ F dry bulb, on the 60 per cent relative-humidity line and draw BD.

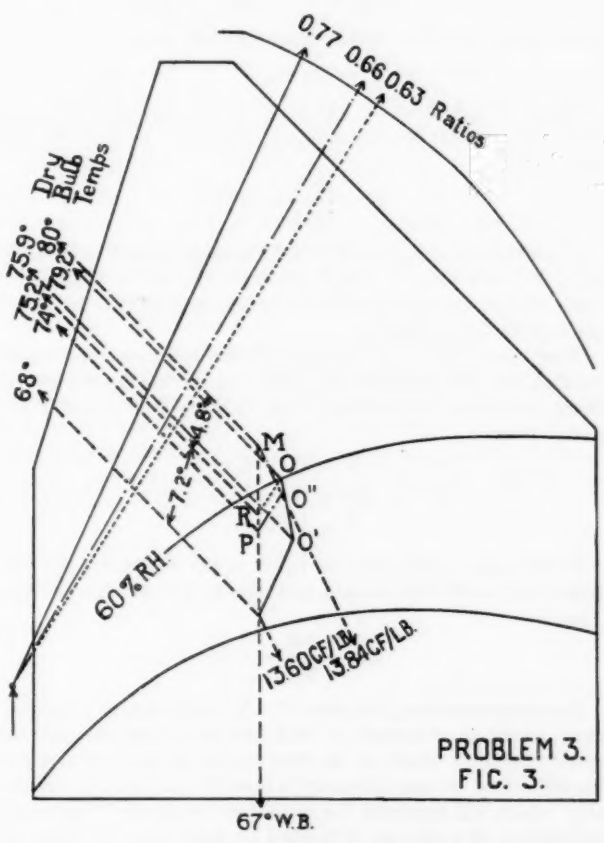


FIG. 9

In this case, sensible heat = $150\,T + 400 = 1990$ Btu per min, latent heat = 300 Btu per min, and total heat = 2290 Btu per min. Then $N = \frac{1990}{2290} = 0.87$. Therefore, draw the heat-ratio line $N = 0.87$ through B to D' to show that the 7500-cfm unit will have capacity to humidify to 60.5 per cent relative humidity.

Use of the bypass to mix the dry outside air with the practically saturated washer air will maintain the desired condition of 60 per cent at a room temperature between 94 F and 95 F and the 7500-cfm unit will be the correct selection, if the room condition obtained does not result in a material change in the latent-heat load from the assumed value of 300 Btu per min.

An increase in the latent-heat-load calculation for the final room condition will have the effect of reducing the value of N, swinging D' in a clockwise direction, which will only result in the

use of a greater amount of dry bypass air to maintain the desired room humidity.

It is, therefore, necessary only to recheck the work when the latent-heat load is materially decreased as a result of a preliminary determination of the room condition.

(2) *To determine the room temperature* as shown in Fig. 9 when the outside air is at 80 F dry-bulb and 67 F wet-bulb temperatures with the 7500-cfm unit operated to maintain 60 per cent relative humidity in the room, locate point M representing the outside-air condition at 80 F dry bulb and 67 F wet bulb, the point S representing the condition of the air leaving the spray chamber at 68 F dry bulb and 67 F wet bulb and solve the equations

$$T + T' = 12 \therefore T' = 12 - T$$

$$150\ T + 400 = T' \times 536 \times 0.2416$$

Substituting, $150\ T + 400 = (12 - T)\ 129$, or

$$T = 4.1 \text{ deg}$$

Then, sensible heat = $150\ T + 400$ = 1015 Btu per min, latent heat = 300 Btu per min, and total heat = 1315 Btu per min. Then

$$N = \frac{1015}{1315} = 0.77$$

Draw the heat-ratio line $N = 0.77$ through point S to cross the $80 - T = 80 - 4.1 = 759$ F dry-bulb line at O'. This represents the approximate condition obtainable if all the air passes through the spray chamber.

Now, assuming that one-half of the circulated air passes through the spray chamber, the mixed air leaving the washer discharge will have the condition P or 74 F dry bulb and 67 F wet bulb. Then

$$T + T' = 6 \therefore T' = 6 - T$$

$$150\ T + 400 = (6 - T) \times 129, \text{ or}$$

$$T = 1.3 \text{ deg}$$

In this case, sensible heat = $150\ T + 400$ = 595 Btu per min, latent heat = 300 Btu per min, and total heat = 895 Btu per min. Then

$$N = \frac{595}{895} = 0.66$$

Proceed as above to plot point O'', which represents the approximate condition obtainable if half the air passes through the spray chamber. Since O'' is close to the 60 per cent relative-humidity line, we may disregard a possible curvature in the line $O'O''$ which will represent the conditions obtained by successive proportions of a mixture of washed air and room air, from all washed air to half washed air and half room air, and we may extend the straight line $O'O''$ to cut the 60 per cent relative-humidity line at O at a dry-bulb temperature of 79.2 F.

Hence at 60 per cent relative humidity $T = 0.8$ deg and we may solve for the difference t between the outside temperature and the temperature of the mixed air leaving the washer to produce a room condition of 79.2 F dry bulb, 60 per cent relative humidity as follows:

$$150\ T + 400 = (t - T) \times 129$$

Substituting $T = 0.8$

$$150 \times 0.8 + 400 = (t - 0.8)\ 129, \text{ or}$$

$$t = 4.8 \text{ deg}$$

Under this room condition, sensible heat = $0.8 \times 150 + 400$ = 520 Btu per min, latent heat = 300 Btu per min, and total heat = 820 Btu per min. Then, $N = \frac{520}{820} = 0.63$.

It will be seen that these results check as the mixed air leaving the washer will have a condition at R of $80 - t = 80 - 4.8 = 75.2$ F dry bulb and 67 F wet bulb, and the heat-ratio line, $N = 0.63$, drawn through the point R, will pass through point O.

(3) It will also be seen from the diagram that:

x lb of outside air at condition M will cool through 4.8 deg in heating y lb of spray-chamber air at condition S through 7.2 deg. Hence the ratio, by weight, $\frac{\text{spray-chamber air}}{\text{outside air}} = \frac{4.8}{7.2} = \frac{2}{3}$; or $\frac{2}{5}$ of the total air delivered is spray-chamber air = 213.6 lb per min and $\frac{3}{5}$ of the total air delivered is outside air = 320.4 lb per min making the total air delivered 534 lb per min.

These values expressed in pounds of dry air per minute may

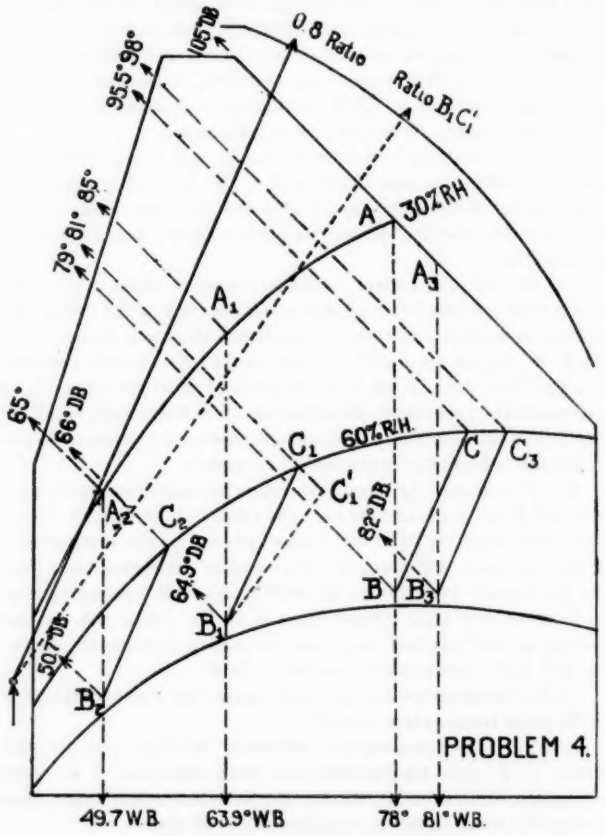


FIG. 10

be converted to cubic feet per minute by multiplying each by its corresponding cubic feet per pound of dry air as taken from the chart.

$$213.6 \times 13.60 = 4434 \text{ cfm}$$
$$320.4 \times 13.84 = 2905 \text{ cfm}$$
$$534 \times 13.74 = 7337 \text{ cfm}$$

In this problem we have assumed the fan delivery as constant in pounds per minute in order to use similar appearing equations. Of course, corrections can be made from actual fan data for varying conditions of intake, but the method of solution remains unchanged.

Example 8. In this problem, which is plotted in Fig. 10, it is desired to determine design conditions in an air-washer installation to maintain constant relative humidity. It is assumed: (1) that no refrigeration is available; (2) that the washer water is recirculated hence the washer water must acquire the tempera-

ture of the wet-bulb of the air passing through the washer; (3) that no room air is recirculated, all the air from the washer being taken from outside; (4) that it is desired to maintain a constant relative humidity and the lowest room temperature which is consistent with the maintenance of this relative humidity; and, (5) that the control mechanism will start and stop the sprays.

The plant should be designed to take care of some maximum outside condition of dry-bulb temperature and minimum outside relative humidity.

It should be kept in mind that the spray can neither add nor subtract any heat to or from the air which passes through it, as long as this air remains at a constant wet-bulb temperature. The spray will, however, change some of the sensible heat in the air to latent heat.

In the diagram, it is assumed that the plant has been designed to meet a condition of 105 F dry bulb and 30 per cent relative humidity outside, that the desired room relative humidity is 60 per cent and that the unit selected is just sufficient in capacity to humidify to 60 per cent when all of the air is washed. Reference to the preceding example, illustrating the selection of a washer, will show that this is a special case, as usually the washer will have some excess capacity for humidification, to be corrected by bypassing or closing the spray-water valve. The special case is chosen because it represents the minimum washer which will meet the outside conditions specified.

The method of determining the room temperature and the slope of the heat-ratio or N line has been fully explained in the preceding example and it is assumed that in this case the resulting room temperature is 95.5 F and that $N = 0.8$. This assumed relationship is indicated by $A\ B\ C$, where A represents the outside condition, 105 F dry bulb, 30 per cent relative humidity, AB represents the outside wet-bulb temperature and consequently the spray temperature, B represents the condition of the air leaving the washer, BC represents the 0.8 heat-ratio line, and C represents the resulting room condition at 95.5 F dry bulb and 60 per cent relative humidity.

It may be demonstrated that this installation will be adequate for any other probable condition inside the limits given as follows:

Now assume an outside condition of 85 F dry bulb and 30 per cent relative humidity. Construct a new figure A_1, B_1 and C_1, in which B_1C_1 is the heat-ratio line 0.8. It will be seen that this will result in a temperature difference between outside and inside of only 4 deg or less than in the basic case. This reduced temperature difference will result in a decrease in the sensible-heat load and consequently in a decrease in the heat ratio with a consequent rotation of the heat-ratio line in a clockwise direction around B, and in a decrease in the temperature rise of the air to absorb the sensible heat. Let the new heat-ratio line be B_1C_1'. It will be seen at once that under these outside conditions the apparatus will now have surplus capacity for humidification and that the sprays may be shut off for part of the time.

There is also shown on the chart a diagram $A_2B_2C_2$ for an outside condition of 65 F and 30 per cent relative humidity and one $A_3B_3C_3$ for 105 F and 36 per cent relative humidity. Again, in each case the sensible-heat load is reduced below that in the basic case and, therefore, in each of these cases the N lines or heat-ratio lines B_2C_2 and B_3C_3 will be swung in a clockwise direction and shortened by a reduction in the sensible-heat pickup This reduction must be handled by the fixed quantity of circulated air, and, therefore, in each case the sprays will be on and off and the washer will have excess capacity.

These examples show (*a*), that a reduction in dry-bulb temperature, with constant relative humidity, results in a decrease in required capacity; and (*b*), that an increase in relative humidity with constant dry-bulb temperature, results in a decrease in required capacity. They prove the correctness of the original statement that the plant should be designed to take care of some maximum condition of outside dry-bulb temperature and minimum outside relative humidity.

Discussion

A Mathematical Solution of the Rotor-Balancing Problem[1]

E. L. THEARLE.[2] In general, a balancing weight placed on any one end of a rotor affects the vibration of both pedestals. When balancing some very long rotors, however, it may be convenient to assume that the vibration of any one pedestal is influenced only by balance weights placed in the nearest correction plane. The author of the paper under discussion is to be congratulated on his solution of the problem as it exists for such cases where the end of a machine may be treated independently, and when vibration phase angles cannot be conveniently measured.

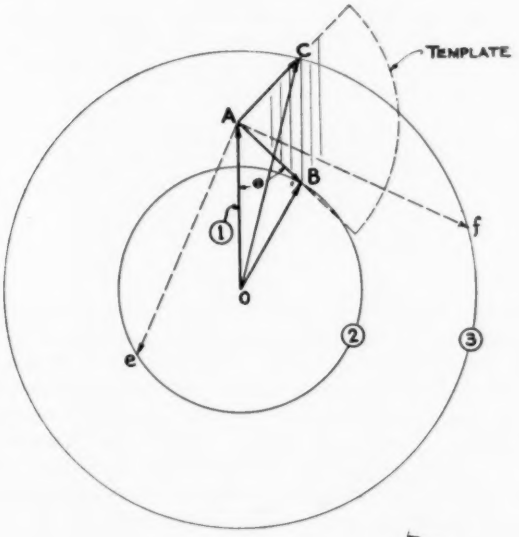
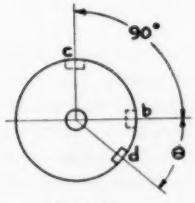


FIG. 2



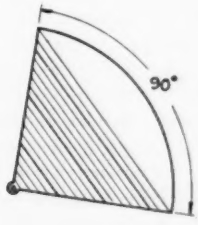


FIG. 3

A simple solution of the problem by a somewhat different method is herewith presented. The rotor is run, without a trial weight, and the vibration amplitude is measured. The amplitude is represented by the vector OA in Fig. 1, which is drawn in any direction with a length proportional to the measured amplitude. A trial weight on any known amount is placed on the rotor in any position b, Fig. 2. The rotor is again run and the vibration amplitude is measured. The circle 2, Fig. 1, is drawn with its center at O, and with a radius proportional to this second amplitude reading. The same trial weight is then shifted, say 90 deg, in a counter-clockwise direction, to the new position c. After the third run amplitude is measured, the circle 3 is drawn with a center at O, and a radius proportional to this third amplitude.

The vectors AB and AC, representing the effects of the trial weight in positions b and c, respectively, must be 90 deg apart in the same respect as the trial-weight positions, equal in length, and must terminate on the circles 2 and 3, respectively. Their correct positions are readily determined by swinging a ruled template, such as shown in Fig. 3, about the point A in Fig. 1 until its edges intersect the two circles at points equidistant from A.

Then the effect of the trial weight, in position b, for instance, is the vector AB. It is obvious from Fig. 2 that if this trial weight is shifted in a clockwise direction, from position b, through the angle θ, and the amount of the weight is multiplied by the ratio OA/AB, its effect will be equal and opposite to the original vibration OA, which will, therefore, be annulled.

Of course, the angle through which the trial weights are shifted need not be 90 deg but may be any convenient angle, with the template made accordingly.

This method has the important practical advantage that all construction may be kept within the bounds of a single sheet of paper while, when several measured amplitudes are almost equal, the method presented in the paper herein being discussed may involve striking arcs with very long radii.

The two possible solutions mentioned by the author are made apparent by this method, as shown by vectors e and f, Fig. 1. Possibly the quickest way of determining which solution is correct is to try the easier one.

TANDY A. BRYSON.[3] In discussing Mr. Bromberg's paper, it is suggested that there may be many occasions when it is inconvenient to use a drawing board for the graphical solution, in which case definite mathematical formulas are of value.

In Fig. 4 let eU be the vector which represents the amplitude of vibration caused by the unknown unbalance U located at the unknown angle φ from an arbitrary reference line O-X which may be marked on the rotor. In this term eU, U represents the mass of unbalance in ounces, pounds, etc., which is located at some assumed radius r from the axis of rotation; while e is a composite factor to convert the term into an expression of unbalance in units of amplitude of vibration. The factor e, therefore, embraces consideration of the radius of the unbalanced mass, the necessary conversion factors, and the stiffness of the rotor supports. For any particular test set-up this factor is a constant because it may be assumed that all

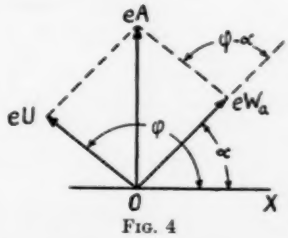


FIG. 4

[1] Published as paper APM-56-14, by Jacob Bromberg, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] General Electric Company, Schenectady, N. Y. Assoc-Mem. A.S.M.E.

[3] Consulting Engineer, American Tool and Machine Company, Boston, Mass. Mem. A.S.M.E.

test weights are placed at the same radius and that the rotor supports are not stressed beyond the range of Hooke's law of proportionality.

In like manner, eW_a represents the amplitude which would be caused by the test weight W_a placed on an otherwise balanced rotor at the radius r and at the known angle α from the line of reference. Obviously, eW_a may not be measured by test since it is blanketed by the effect of the unbalance. However, it is possible to measure the length of eA, the resultant of eU and eW_a, thus obtaining one element of the data from which U and φ may be computed.

Writing the equation for eA

$$\overline{eA}^2 = \overline{eU}^2 + \overline{eW}^2_a + 2e^2UW_a\cos(\varphi - \alpha)$$
$$= \overline{eU}^2 + \overline{eW}^2_a + 2e^2UW_a(\cos\alpha\cos\varphi + \sin\alpha\sin\varphi)$$

Since this equation contains three unknowns, U, e, and φ, it is necessary to apply three separate test weights, W_a, W_b, and W_c, at the constant radius r (or to make suitable correction if the radial location change) and at known angles, α, β, and λ, from the reference line and to measure the amplitudes eA, eB and eC caused by each in cooperation with the unbalance. Using equal test weights at the three different positions and solving the equations, there results

$$\tan\varphi = \frac{(\overline{eA}^2 - \overline{eC}^2)(\cos\alpha - \cos\beta) - (\overline{eA}^2 - \overline{eB}^2)(\cos\alpha - \cos\lambda)}{(\overline{eA}^2 - \overline{eB}^2)(\sin\alpha - \sin\lambda) - (\overline{eA}^2 - \overline{eC}^2)(\sin\alpha - \sin\beta)} \quad \ldots\ldots\ldots [1]$$

Letting

$$x = (\cos\alpha - \cos\lambda)\cos\varphi - (\sin\alpha - \sin\lambda)\sin\varphi$$
$$y = \cos\alpha\cos\varphi + \sin\alpha\sin\varphi$$

Solve for U

$$U^2 + 2WU\frac{(\overline{eA}^2 - \overline{eC}^2)y - \overline{eA}^2x}{\overline{eA}^2 - \overline{eC}^2} = -W^2$$

$$e^2 = \frac{\overline{eA}^2 - \overline{eC}^2}{2WUx}$$

While these formulas appear somewhat forbidding, their evaluation is actually quite simple if the test data and the values of the trigonometric functions are placed in orderly tabulation such as is shown in Table 1. The use of the listed data will be made clear by application to the author's first problem, wherein equal test weights of 23 oz were used.

TABLE 1 TABULATION OF DATA

$W_a = W_b = W_c = W = 23$ oz; $W^2 = 529$

$\overline{eA}$	= 30[a]	$\overline{eB}$	= 21	$\overline{eC}$	= 19
$\overline{eA}^2$	= 900	$\overline{eB}^2$	= 441	$\overline{eC}^2$	= 361
α	= 25.71	β	= 64.29	λ	= 102.84
$\cos\alpha$	= 0.901	$\cos\beta$	= 0.434	$\cos\lambda$	= −0.222
$\sin\alpha$	= 0.434	$\sin\beta$	= 0.901	$\sin\lambda$	= 0.974
$\overline{eA}^2 - \overline{eB}^2$	= 459			$\overline{eA}^2 - \overline{eC}^2$	= 539
$\cos\alpha - \cos\beta$	= 0.467			$\cos\alpha - \cos\lambda$	= 1.123
$\sin\alpha - \sin\lambda$	= −0.540			$\sin\alpha - \sin\beta$	= −0.467

[a] Unit of measurement = 0.0001 in.

Substituting in Equation [1], the values listed in Table 1

$$\tan\varphi = \frac{539(0.467) - 459(1.123)}{459(-0.540) - 539(-0.467)} = -75.6$$

The negative sign indicates that the unbalance is located in either the second or fourth quadrants. Since the test weight $W_c(Sc)$ in the second quadrant caused a reduction in amplitude, it acted opposite to the unbalance which, in consequence, is in the fourth quadrant. Accordingly, $\varphi = 270.36$ deg, $\cos\varphi = 0.0132$, and $\sin\varphi = -0.9999$. Therefore

$$x = 1.123(0.0132) - 0.540(0.9999) = 0.555$$
$$y = 0.901(0.0132) - 0.434(0.9999) = -0.422$$

$$U^2 + 46U\frac{539(-0.422) - 900(0.555)}{539} = -529$$

$$U^2 - 61.8U = -529$$

$$U = 51.57 \text{ oz (or } 10.23 \text{ oz)}$$

$$e^2 = \frac{539}{46(51.57)0.555} = 0.410$$

$$e = 0.640$$
$$eU = 0.640(51.57) = 33.0$$

The test of the rotor without trial weights showed $eU = 29$, which, when compared with the computed amplitude of 33, serves as a measure of the test and computation accuracy. (It should be said here that the above are slide-rule computations.) The comparison also indicates that $U = 51.57$, rather than 10.23; although proof of this fact may also be obtained by inserting the computed values in the original equation for $\overline{eA}^2$.

The above method may be used for any three locations of the trial weights. However, it is desirable and usually possible to locate the trial weights in such manner as to simplify the computations to a large degree. By making $\alpha = 0$, $\beta = 90$, and $\lambda = 180$, the formulas reduce to

$$\tan\varphi = 1 - 2\frac{(\overline{eA}^2 - \overline{eB}^2)}{(\overline{eA}^2 - \overline{eC}^2)};\quad U^2 - 2WU\frac{(\overline{eA}^2 + \overline{eC}^2)}{(\overline{eA}^2 - \overline{eC}^2)}\cos\varphi = -W^2$$

$$e^2 = \frac{\overline{eA}^2 - \overline{eC}^2}{4WU\cos\varphi}$$

THOMAS C. RATHBONE.[4] The author describes his method for determining the amount and location of the balance correction at one end of the rotor as applicable to the test results obtained on the conventional dynamic-balancing machine and proposes it as an improvement over the mechanical solution devised by Prof. G. B. Karelitz. Professor Karelitz actually devised his system for use on turbine generators under operating conditions in the field, as well as on test machines, and the writer, being associated with Professor Karelitz during these early attempts at field balancing, can vouch for the successful application of his method on a number of installations. In the writer's opinion, the semi-automatic graphical method may be more readily used for averaging the data by the man in the field.

The balancing method described by the author is based on vibration amplitudes alone. Actually, vibration is characterized by its phase relation to the disturbing force, as well as by its amplitude, and is of equal importance, otherwise, no vector solution would be tenable. During the earlier investigations of field balancing, the writer devised a method for solving the problem by means of the phase relation alone, which required but two trial runs with known test weights. The method will be found satisfactory by using the so-called scribed high-spot with the rotor in a dynamic-balancing machine, performing the scribing at 20 to 30 per cent above the critical speed. Shaft scribings to determine phase relations on a rotor under actual operating conditions in the field were found to be so erratic and unreliable that special apparatus to determine the phase of vibration by stroboscopic means were resorted to.

Fig. 5 of this discussion represents the end of the rotor under consideration and the vibration vectors involved. Before any

[4] Chief Engineer, turbine division, Fidelity & Casualty Company of New York, N. Y. Mem. A.S.M.E.

trials are made, the phase relation between the upper extremity of the vertical component of vibration for example, and some data on the rotor are determined. This direction is represented by OA. (In the case where the test is made in the dynamic-balancing machine, OA represents the scribed high-spot location.) OA represents the directional characteristic of the initial unknown unbalance. The length of this vector, representing quantity, is

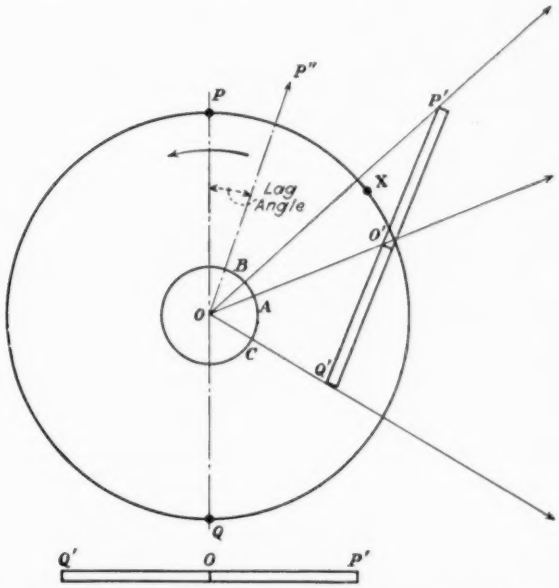


FIG. 5 METHOD OF DETERMINING UNBALANCE BY PHASE RELATION OF VIBRATION TO ROTOR

to be determined, as well as the lag angle between this vector and the causative force.

A known test weight is then inserted at any position, such as P, and a trial run made to determine the new phase position resulting from the combination of the known force and the unknown unbalance. Let OB represent the new high spot or phase position. Next, the known weight is removed from P and inserted at Q, 180 deg from P, and the second trial run made, resulting in a phase shift defined by OC.

The vectors representing the two trial weights are equal and opposite. The solution is quite simply obtained by means of a scale or straight edge divided into two equal parts with points, O', P' and Q' as illustrated in Fig. 5, representing the known weights. By holding the point O' on the line OA, the scale is maneuvered until a position is found where P' falls on OB and Q' falls on OC. There is only one position possible.

The amount of initial unbalance is determined directly by the relation of the length OO', representing this unbalance, to the scale length $O'P'$ or $O'Q'$ representing the known test weight.

The location of the initial unbalance is determined by the lag angle, which is found by comparing the actual direction of one of the test weights, as OP, with its resultant vector direction, $O'P'$. This angle POP'', laid off ahead of OO', locates the unbalance at OX, and the problem is solved.

It should be mentioned that neither the method based on amplitudes alone, or on phase relation alone, reduces the problem to its ultimate simplicity. By utilizing both the amplitude and phase data, the amount and location of the unknown unbalance at one end can be determined by a single trial run. The writer has used this method with success on the dynamic-balancing machine, and has utilized this principle in determining the amount and location of the correction at both ends of the rotor simultaneously, where each bearing is influenced by forces emanating from the other bearings.

JAMES J. RYAN.[5] The solution of the rotor-balancing problem as given by the author appears to be both unique and fundamental. Considering one end of a rotor, mounted in a Lawaczek-Heyman balancing machine, with the other end fixed at the bearing, and applying the mathematical solution as given, the correct weights may be determined. However, it is not necessarily true that, with the rotor out of the balancing apparatus and in its own more or less flexible bearings and pedestals, the theoretical balancing procedure as given is fundamentally correct. Any system of field balancing (balancing a rotor in its own pedestals) is subject to considerable error unless the static forces and the dynamic moments are each taken into account by balancing on both ends of the rotor simultaneously. Such a procedure would consist in placing identical weights at the same angular positions on each end of the rotor, and, if an accurate solution is required regardless of the analytical effort necessary, the same weights should be placed at 180 deg removed from one another for additional trial runs. The data obtained are compiled and analyzed[6] in some such a manner as described by the author, and several machines have been successfully balanced by the method referred to.

A method that should be used more often in engineering work has received very little attention in the writings of vibration engineers. Its origination has been lost in antiquity, although it is the fundamental of fundamentals, and its simplicity makes it available for use to every one. It may be termed the "sine curve of unbalance." If a weight is added to the balance plane of a rotor at various positions around the periphery of the balance ring, the resulting amplitudes of vibration for that end of the rotor when plotted against the weight position will approximate the locus of a sine curve. The trough of the sine curve will indicate the position for balance. The inverse ratio of the amplitude of the sine curve to the mean ordinate of the sine curve multiplied by the weight of the unbalance weight used in the trials will give the approximate weight to be added at the indicated position for balance. This is the simplest method of balancing on record, and is more accurate than most because of its simplicity. Since, any three points plotted as coordinates determine a sine curve (as for a circle), only three trial runs are necessary to obtain the required data. It is preferable to place the trial weights at 90-deg positions for ease in off-hand sketching of the sine curve.

Using this method of procedure on the data as set forth by the author in the first example, Fig. 6 of this discussion is obtained, wherein the approximate location of the balancing position is between balance plug holes 27 and 28, and the amount of unbalance is calculated to be 49.5 oz. Professor Karelitz, in his paper[7] from which the example was taken, obtained the same balance location, and gives the exact amount of unbalance to be 56 oz.

As stated above, balancing on one end of a rotor is unreliable in many cases, especially where a dynamic unbalance couple exists. Thus the procedure is to place identical weights at the same position on each end of the rotor, shifting the weights 90 deg for the three trial runs, and measuring the amplitudes of motion of the two pedestals for each trial. The two sine curves obtained, sketched on the same graph, will establish the balancing condition for the rotor in most cases.

---

[5] Assistant Professor, Machine Design, Mechanical Engineering Department, University of Minnesota, Minneapolis.

[6] "Field Balancing of Rotors," by James J. Ryan, *The Electric Journal*, December, 1928, vol. XXV, no. 12, pp. 596–604.

[7] "Field Balancing Rotors at Operating Speed," by G. B. Karelitz, *Power*, Feb. 7 and 14, 1928.

Fig. 7 of this discussion represents the sine curves of unbalance for a large rotor considerably out of balance with respect to the dynamic couple and indicates the effectiveness of this method of balancing. The data in Table 2 were taken from trial runs on the machine.

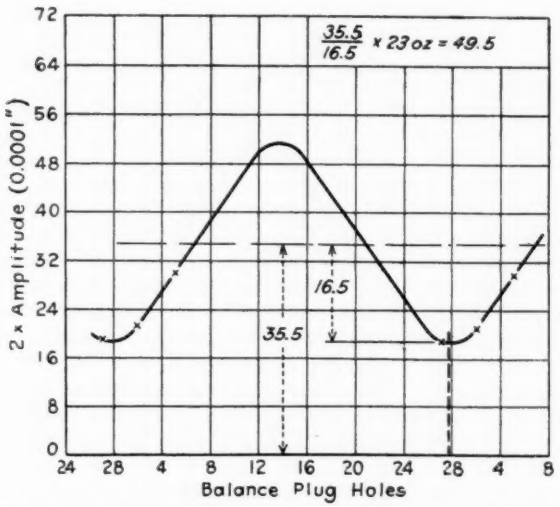


FIG. 6 THE APPROXIMATE LOCATION OF THE BALANCING POSITION AS ASCERTAINED FROM THE DATA IN THE AUTHOR'S FIRST EXAMPLE, USING THE WRITER'S METHOD OF PROCEDURE

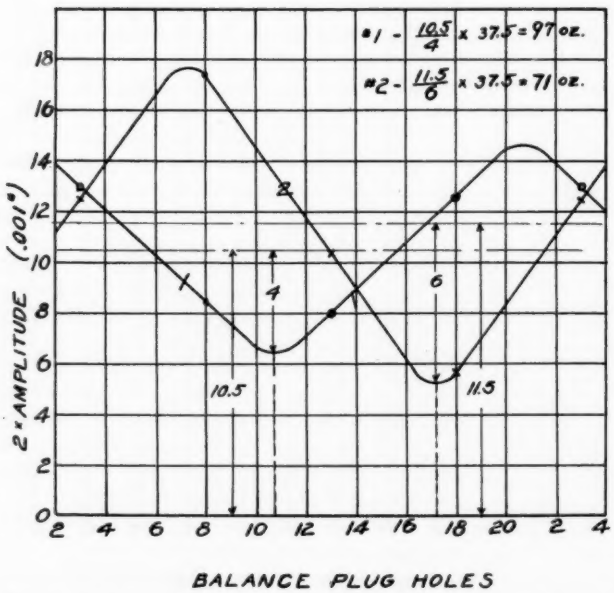


FIG. 7 THE SINE CURVES OF UNBALANCE FOR A LARGE ROTOR CONSIDERABLY OUT OF BALANCE WITH RESPECT TO THE DYNAMIC COUPLE

The curves obtained indicate balance weights of 97 ounces distributed at balance plug holes 10 and 11 on the number 1 end, and 71 ounces distributed between holes 17 and 18 on the number 2 end. The error may be estimated, for the rotor was operated satisfactorily with 87 ounces in hole 9 on the number 1 end, and 95 ounces distributed between holes 18 and 19 on the number 2 end, after repeating the above procedure with smaller trial weights, leaving the original balancing weights in position. Using this method of analysis, the original run with no trial weights has little meaning.

TABLE 2

Trial run	Balancing plane	Balancing wt., oz.	Balance ring location	Maximum amplitudes (1/1000 in.)
Initial	1	None	...	3.0
Initial	2	None	...	5.0
A	1	37.5	13	8.0
A	2	37.5	13	10.4
B	1	37.5	3	13.0
B	2	37.5	3	12.5
C	1	37.5	18	12.6
C	2	37.5	18	5.7

This method of balancing enables the operator to balance a machine on the job, using a vibrograph, a slide rule and cross-section paper. He is guided definitely as to further procedure. The method is elementary and efficient.

AUTHOR'S CLOSURE

E. L. Thearle's solution is of the same type as Professor Karelitz', i.e., it involves the use of a special mechanical device. It nevertheless has the important advantage of using a set of parallel lines permanently drawn on a template, while the set of concentric circles or polygons, proposed by Professor Karelitz involves considerable preparatory drafting in each separate case. The measurement without a trial weight is included by Mr. Thearle in his solution which involves three basic runs for measurement of vibration amplitude; this makes a fourth run necessary in order to try the "easier" solution, to use Mr. Thearle's term.

Mr. Bryson's analytical solution takes the polar coordinates of the counterbalance weight for the unknown of the problem and also uses such coordinates for the given trial weights. This may be a luckier choice than Cartesian coordinates used by the writer.

The author solved numerically the same case chosen by Mr. Bryson, following his own method as shown in Appendix 2 of the paper. The results for x and y were

$$x_1 = -0.676;\ y_1 = 51.94;\ x_2 = -0.132;\ y_2 = 10.18$$

In order to compare this result with Mr. Bryson's we determine $\tan \varphi = y_1/x_1 (= y_2/x_2) = 51.94/-0.676 = -76.86$, or $\varphi =$ 90 deg, 44 min, and 44 sec. In Mr. Bryson's notation who locates the initial unbalance and not the required balancing weight, as the author does, φ should be 270 deg, 44 min, 44 sec, or 270.75 deg. The magnitude of the unbalance appears as 51.95 oz and 10.17 oz, respectively, while for distances O''_1S_a and O''_2S_a we obtained 47.10 and 20.85. The amplitude resulting from initial unbalance alone could now be predicted as $0.0030 \times 51.95/47.10 = 0.0032$ in. in the first case and as $0.0030 \times 10.17/20.85 = 0.00148$ in. in the second (in complete accordance with the numbers evaluated by direct measurements in Fig. 3 of the paper) and the correct solution can now be pointed out. The writer's number (0.0032 in.) is somewhat closer to the observed amplitude (0.0029 in.) than Mr. Bryson's (0.0033 in.), but the error is still about 10 per cent of the magnitude in question. The discrepancy is due not so much to inaccuracies of slide-rule computations, as to the inaccuracies of the amplitude observations and of the general theory of this kind of vibrations. Also, Mr. Bryson's remark, that the choice of the correct solution can be made "by inserting the *computed* values in the original equation for $\overline{eA}$" is incorrect, unless $\overline{eU}$ in this equation is to be computed from the *observed* amplitude at the (fourth) no-trial-weight test. There exists no means for separating the physically correct solution from the basic three amplitude measurements alone.

The comparison showed that the writer's solution, being somewhat more symmetrical than Mr. Bryson's and also more convenient for finding the quadrant where the unbalance is located, is nevertheless much more complicated. Still more painstaking

would be the more general case of unequal trial weights which, incidentally, has been left out by Mr. Bryson. However, even his version can hardly ever have a chance of being used by a practitioner for actual unbalancing. Here, graphical methods are the best.

It should be noted, that the case of almost equal amplitudes hardly warrants the apprehension expressed for it by Mr. Thearle and Mr. Bryson. Where two of the three amplitudes happen to be close to equality, they will usually be considerably different from the third amplitude, thus giving convenient radii for the two arcs required for the solution. When all three amplitudes are almost equal, both solutions are almost identical as will be shown later, and are to be looked for in a small region around the center of the circle passing through points S_a, S_b, S_c (or center of the log of balancing—for equal trial weights). Also replacing an arc of a too long radius by a straight line perpendicular to the corresponding side of triangle $S_aS_bS_c$ and passing through the respective point U would not seriously affect the accuracy of the solution. All these statements may be verified in the example of Figs. 3 and 4 of the paper.

Mr. Rathbone's remark, that the phase relation of the vibration to the disturbing force is of no less importance than the length of the vector giving the amplitude, is theoretically quite true. This, however, does not affect the possibility and desirability of solutions which make use of amplitudes alone. Direct observations of the phase-lag angle by means of high-spot scribing are sometimes unreliable, as Mr. Rathbone himself has pointed out.[8]

The question of whether or not a semi-automatic graphical method of solving the balancing problem is preferable to a possibly somewhat involved geometrical construction hardly allows a general answer and may be left open, inasmuch as the writer's method does not intend to replace any former attempt to attack the same problem. The writer would only take the liberty to observe that the engineers dealing with this problem seem to show a certain propensity to improvised mechanical devices, which are used sometimes in cases where a geometrical construction would unquestionably be simpler. Such a case is that treated by Mr. Rathbone in Fig. 5 of this discussion, where line $P'Q'$, which is to be halved by the vector OA at the point O', could be found by drawing through any point P' on OB a line $P'R'$ parallel to OC (intersecting OA at R'), and then drawing a line $R'Q'$ parallel to OB (intersecting OC at Q') and connecting P' with Q'.

In the case of Mr. Thearle's Fig. 1, a construction replacing his template with the set of drawn lines would be very complicated. His method, like Professor Karelitz' and the writer's, has the advantage of using amplitudes alone.

Of course, Mr. Rathbone's results obtained from two and even from one run on the machine by simultaneous observations of amplitude and phase are remarkable.

The method of drawing a sinusoidal curve representing the amplitude in terms of the location of a constant trial weight, used in this discussion by Professor Ryan, has been pointed out by Professor Stodola[9] (but without Professor Ryan's interesting details). The question of whether it is possible to sketch offhand sinusoids on the basis of three points (especially in a combination similar to that in Fig. 6 of this discussion) must be left to the experience and ability of the individual draftsman. In the present case, Professor Ryan's result, as compared with those of Professor Karelitz and the writer, can claim for tolerable accuracy only as to the location of the unbalance. Still Professor Ryan's method may be of use for quick preliminary orientation.

In conclusion, the writer points out some further properties of the construction described in his paper.

1 It is clear, that the centers O_a, O_b, and O_c lie upon one and the same straight line, which is perpendicular to the segment $O''_1O''_2$ and crosses it at its middle.

2 In the case of all three amplitudes being exactly equal ($m = n = p$) the three circles would become straight lines perpendicular to the sides S_aS_b, S_bS_c, and S_cS_a, respectively, and passing through their middles. We would obtain *one* solution, corresponding to the center of the circle circumscribing triangle $S_aS_bS_c$. Formulas [3] of the paper, in which the second determinants in the numerators now vanish, would become $x = d_y$; $y = -d_x$. Returning now to the general case, d_x and d_y in formulas [3] allow a simple geometrical interpretation, namely, d_y and $-d_x$ are the coordinates of the center of the circle passing through the vertexes of the triangle $S_aS_bS_c$.

3 Let us now translate parallelly the coordinate axes from O into this center (say O'), as a new origin. In the new system of coordinates $X'Y'$, the numbers a, b, and c (distances between point O and points S_a, S_b, and S_c) are replaced by $a' = b' = c' = R$ (radius of the circumscribing circle). The first determinants in the numerators of formulas [3] of the paper now vanish and we have in the new notation

$$x' = -Ke'_y;\ y' = Ke'_x \qquad [3']$$

(K, not K', because, according to formula [1] of the paper, K is an "invariant" of the transformation, depending on differences of coordinates and on constant numbers m, n, and p.) It follows immediately from formula [3'], that points O''_1 and O''_2 lie upon a straight line through the center O'.

4 Equation [4] of the paper now becomes

$$A'K^2 - B'K + C' = 0 \qquad [4']$$

where $A' = 3(e'^2_x + e'^2_y)$ and $C' = a'^2 + b'^2 + c'^2 = 3R^2$.

From well-known properties of quadratic equations it follows that

$$K_1K_2 = C'/A' = R^2/(e'^2_x + e'^2_y)$$

The product of K_1K_2 can only be positive, i.e., K_1 and K_2 can only be both positive or both negative. It now follows from formula [3'], that x'_1 also has the same sign as x'_2 and y'_1 the same sign as y'_2. Points O''_1 and O''_2 are located on the same side of center O'.

5 From squaring our last equation we obtain

$$(K_1K_2)^2(e'^2_x + e'^2_y)^2 = R^4 \qquad [5']$$

From formula [3'] we obtain by squaring

$$x'^2_1 = K_1^2e'^2_y;\ x'^2_2 = K_2^2e'^2_y;\ y'^2_1 = K_1^2e'^2_x;\ y'^2_2 = K_2^2e'^2_x$$

whence by addition

$$x'^2_1 + y'^2_1 = K_1^2(e'^2_x + e'^2_y);\ x'^2_2 + y'^2_2 = K_2^2(e'^2_x + e'^2_y)$$

and, by multiplying the last two equations,

$$K_1^2K_2^2(e'^2_x + e'^2_y)^2 = (x'^2_1 + y'^2_1)(x'^2_2 + y'^2_2)$$

By substituting the right-hand side of the latter equation in Equation [5'], we obtain

$$(x'^2_1 + y'^2_1)(x'^2_2 + y'^2_2) = R^4, \text{ or}$$

$$|\sqrt{(x'^2_1 + y'^2_1)}| \cdot |\sqrt{(x'^2_2 + y'^2_2)}| = R^2;$$

$$(O'O''_1)(O'O''_2) = R^2$$

[8] "Turbine Vibration and Balancing," by T. C. Rathbone, Trans. A.S.M.E., vol. 51, 1929, paper APM-51-23, p. 267.

[9] "Dampf- und Gasturbinen," by A. Stodola, fifth or sixth edition, p. 356, Fig. 398.

From this equation and from the two properties of points O''_1 and O''_2 which have been emphasized, it follows that *points O''_1 and O''_2 are poles with respect to the circle circumscribing the triangle $S_aS_bS_c$.*

For the particular case of equal trial weights, this property of the two solutions of the balancing problem has been pointed out by Prof. J. P. Den Hartog, with reference to the construction here described, in his recent treatise on mechanical vibrations.[10]

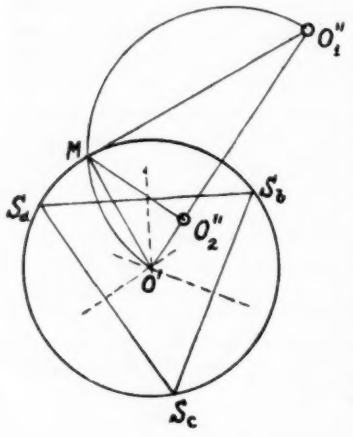


FIG. 8

This polarity may be used for finding the physically correct solution in cases where a solution obtained, for instance, by one of the methods pointed out in the present discussion, happens to be the false one. If O''_2 is known, (Fig. 8 of this discussion) draw O''_2M perpendicular to $O'O''_2$, intersecting circle O' at M, then draw MO'_1 perpendicular to $O'M$, intersecting $O'O''_2$ at O''_1. If O''_1 is known, draw a half-circle upon the diameter $O'O''_1$, intersecting circle O' at M, then draw MO''_2 perpendicular to $O'O''_1$, intersecting $O'O''_1$ at O''_2.

The present paper was written more than three years ago, but circumstances beyond the author's control hindered its publication. In the meantime Mr. L. P. Kroon of the Westinghouse Electric & Manufacturing Company found independently an essentially identical solution of the balancing problem. When it had been called to his attention that there was an unpublished paper in existence working out the same subject, he very generously conceded priority to the author and also helped to bring about the publication of the present contribution.

I am also indebted, for encouragement or help, to J. G. Baker and L. M. Tichvinsky, both of the Westinghouse Electric and Manufacturing Co., and to Prof. J. P. Den Hartog, Prof. F. M. Lewis and Prof. E. O. Waters.

Influence of Bends or Obstructions at the Fan-Discharge Outlet on the Performance of Centrifugal Fans[1]

R. D. MADISON.[2] Where the air leaves a fan with substantial uniformity in direction and velocity it is reasonable to suppose that an elbow placed directly on the fan discharge would offer no specially different type of flow or resistance from what it would if placed elsewhere in the duct system. The tests reported in the paper under discussion show this to be the case.

Unfortunately most fans are not so ideally suited to such a test. Although it was specifically stated that only one fan was tested it was not representative of the majority of fans on the market today. The writer refers to the large diverging chamber from the point of cut-off to the fan discharge, approximating $1^1/_2$ diameters of pipe. Such fans are often used in the power-plant field and are special fans. The expanding portion is more often built as a part of the duct work. It is still a matter of opinion what the tests would have shown had the elbow been placed immediately at the discharge of a conventional fan.

Where the air leaves a fan with non-uniform flow it is reasonable to suppose that the behavior of an elbow will be affected if placed adjacent to the fan discharge. It is known that there is more tendency to spiral flow in single-inlet fans, particularly those with large outlets. The writer has been making tests on elbows following such fans and expects to present these data in the near future when the tests are completed. At the present time sufficient data have been obtained to say that fan performance can be materially changed if the conditions are especially bad.

The writer has observed that the pressure loss may be different whether the elbow turns to the right or left of the line of discharge. In fact such observations have been noticed where the elbow is 11 diameters from the fan discharge. The type of flow is especially severe in the case of a reverse elbow at the outlet of a forward curved blade fan which has a "drop cut-off," that is, an outlet materially larger than the area at the point of cut-off. The static-pressure curve of this type of fan and elbow shows a large drop in pressure beyond the peak of efficiency and is concave upward from there to free delivery. The pressure is unstable near free delivery, a condition caused only by the location of the elbow. It is interesting to note that when one diameter of duct was interposed between this fan and elbow the pressure curve again followed the characteristic trend, although the elbow loss was somewhat higher than when subjected to uniform flow. Whether one says that the fan characteristic is affected by such arrangement or that simply the elbow loss is variable, the fact remains that if the usual elbow loss were used in figuring an installation the expected flow would not be obtained.

The effect of one diameter of duct between a fan and elbow is cited here to show that it can have a remarkable stabilizing effect on the flow of air approaching the elbow. In view of this, it is a matter of opinion what the authors would have found in their tests had the specially long outlet not been used in their work. While it is known that single-inlet fans are likely to cause more rotation in the air leaving the discharge than corresponding double-inlet fans, the rotation in the latter is not nullified by the opposing inlets as the author suggests in another paper[3] but is greatly reduced due to the absence of friction along the central plane of the fan. Two vortices are set up in opposite direction and in lesser amounts.

While it is not the best practice to place an elbow directly at the fan discharge, still it is frequently done in ventilating work where space is at a premium. The behavior of the elbow will thus depend upon the condition of flow at its entrance, being better in some fans than in others. The reverse elbow is to be discouraged in any case and the turn used that follows the direction of fan rotation. The practice of using turning vanes if it becomes necessary to place the elbow at the fan outlet, is good. Here again, however, full flow across the elbow inlet must exist or the expected performance may not be obtained.

It would be interesting to know whether the authors tested the elbow at some distance from the fan and what the pressure loss was. Curve Fig. 6 would indicate this to be about 0.4 velocity heads but from the writer's tests he would expect this to be about 0.25 velocity heads for this elbow. Wirt's tests[4] at the General Electric Company show losses of about 0.425 velocity

[10] "Mechanical Vibrations," by J. P. Den Hartog, McGraw-Hill, 1934, pp. 246–248 and 380.

[1] Published as paper FSP-56-12, by L. S. Marks, J. H. Raub, and H. R. Pratt, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Research Engineer, Buffalo Forge Company, Buffalo, N. Y. Assoc-Mem. A.S.M.E.

[3] "Air Flow in Fan-Discharge Ducts," by L. S. Marks, Trans. A.S.M.E., 1934, paper PTC-56-2, p. 876.

[4] "New Data for the Design of Elbows in Duct Systems," by Loring Wirt, *General Electric Review*, June, 1927.

heads for an elbow of this type, and the writer feels that it was owing to the method of test where a nozzle preceded the elbow and a uniform velocity was effective over the whole cross-sectional area. This condition does not exist in practice. Besides the tests were made at very high velocities in a 3-in. elbow. The writer's tests check much more closely with those of Busey[5] which were made at the Buffalo Forge Company, although they are slightly higher.

C. E. Peck.[6] The results found by the authors of the paper under discussion are valuable in that they verify the general opinion that the influence of bends or obstructions at the discharge of the fan has little effect on the fan performance. However, it is important to note, as pointed out by the authors, that their conclusions are based on a fan of the centrifugal type with a volute housing so designed that the air being discharged from the housing has a reasonably definite direction and fairly uniform distribution at the outlet.

The axial-flow type of fan or propeller type of fan discharging directly into a duct system may give to the air quite different types of flow characteristics such that bends, sudden enlargements, or obstructions will affect the fan performance appreciably. For instance, the air leaving a high-capacity propeller-type blower has very large rotational-velocity components which maintain themselves at large distances beyond the fan in the duct work. The rotational velocities vary from the center to the wall of the duct. With these complicated directions of flow existing, any obstruction or bend, or change in duct cross-section at the fan discharge or even at some distance from the fan discharge may appreciably affect the fan delivery and efficiency.

The performance of a propeller fan with properly designed guide vanes at the fan discharge is such that the air leaving the vanes is essentially parallel with the axis of the duct. With this condition and uniform velocity distribution the effect of bends or obstructions beyond the guide vanes would probably be small.

In general, when a fan of any type is applied to a duct system and the fan is provided with a volute case or guide vanes or some device which produces uniform flow in a given direction the effect of duct shape and obstructions beyond the fan is negligible. However, when fans are applied in special cases, such as the cooling of electrical machinery where volutes and guide vanes can be rarely used, the effect of obstructions at the fan outlet is more noticeable. The effect of the close proximity of end windings and the rotor of the electrical machine must be considered and usually such obstructions greatly influence the fan performance.

Authors' Closure

The investigations of Mr. Madison show that the pressure drop in an elbow depends on the uniformity of flow of the air approaching it but do not appear to demonstrate any influence of the elbow on the performance of the fan. If the character, location, or orientation of the elbow is such as to change the discharge pressure at the fan outlet, the condition under which the fan is operating will change to some other point on the fan-performance curve. If the elbow actually affects fan operation, the performance curve of the fan will change. Mr. Madison has apparently investigated a combination of fan and elbow for a fan which gives a spiral discharge flow and shows that the performance of this combination varies with the location and orientation of the elbow. The authors believe that this variation results from actions in the elbow alone and that the fan is unaffected as long as it has a complete fan casing.

In the authors' tests the elbow was not tested at some distance from the fan because the air flow at the fan-casing outlet was found to be practically uniform over the whole cross-section.

The authors agree with Mr. Peck that if the fan casing is absent or incomplete, the effect of obstructions in close proximity to the fan may be considerable.

[5] "Loss of Pressure Due to Elbows in the Transmission of Air Through Pipes or Ducts," by Frank L. Busey, A.S.H.&V.E. Trans., 1913.

[6] Power engineering department, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. Jun. A.S.M.E.

Calibration of Rounded-Approach Orifices[1]

R. E. Sprenkle.[2] Mr. Smith's data are timely, particularly since some can be connected directly to other pertinent data which greatly extends the scope of usefulness. For instance, the writer's Fig. 1 shows the data from Mr. Smith's large oil-flow nozzle plotted together with data from a Bailey 3.06-in. × 1.836-in. water-flow nozzle. While at the junction point of the two sets of data, or at a Reynolds number of approximately 35,000, a possible separation by approximately $^1/_2$ per cent exists, there is no question but that the data from the Bailey nozzle are a real continuation of the data of Mr. Smith's nozzle, and that a single smooth curve represents the complete data of the two when plotted against Reynolds' number.

This despite the fact that the Bailey nozzle used pipe-line connections instead of the impact and throat type of connections used by Mr. Smith. Pipe-line taps place the inlet static connection into the pipe wall at a distance of one pipe diameter preceding the nozzle inlet, and the outlet static connection into the pipe line back of the nozzle throat, as shown in Fig. 13 of Mr. Buckland's paper,[3] "Fluid Meter Nozzles."

Further, there is a distinct difference in size between the two nozzles, as well as the use of entirely different flowing fluids in obtaining these calibration data. When to all of these is added the difference in the physical set-up and the fact that the different experimenters involved were working entirely independently of each other, this agreement becomes all the more noteworthy.

Since the water curve obtained from the Bailey nozzle flattens out at about 600,000 Reynolds' number, and continues flat up to the maximum test point of about 900,000 Reynolds' number, there is little reason to doubt the projection of the curve as a perfectly flat line to Reynolds' number of much greater value, possibly to infinity. Such tests, using steam flow, are now scheduled to be made shortly on the Bailey nozzle, better to show the validity of this assumption.

In view of this the curve has been extrapolated to a Reynolds number of over 3,000,000 so as to cover the useful range of steam, air, or other low-viscosity fluids, and it is felt that this same curve could be extrapolated further if desired without being in error more than plus or minus $^1/_2$ per cent. Likewise, the same curve should apply to any flow nozzle of this general structure, provided the diameter ratio of the throat to the inlet-pipe diameter does not exceed 60 per cent.

Mr. Smith's medium- and small-sized nozzles do not line up either with his large one or with the Bailey nozzle, possibly as a result of differences in relative roughness of the throat finish. Then too, recent researches on orifices show the improbability of making orifice throats less than 0.5 in. so as to conform with larger diameter throats. This same observation applies to nozzle throats as well. Thus, such small sizes must be considered in a

[1] Published as paper RP-56-10, by J. F. Downie Smith, in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Bailey Meter Company, Cleveland, Ohio. Assoc-Mem. A.S.M.E.

[3] "Fluid Meter Nozzles," by B. O. Buckland, Trans. A.S.M.E., 1934, paper FSP-56-14.

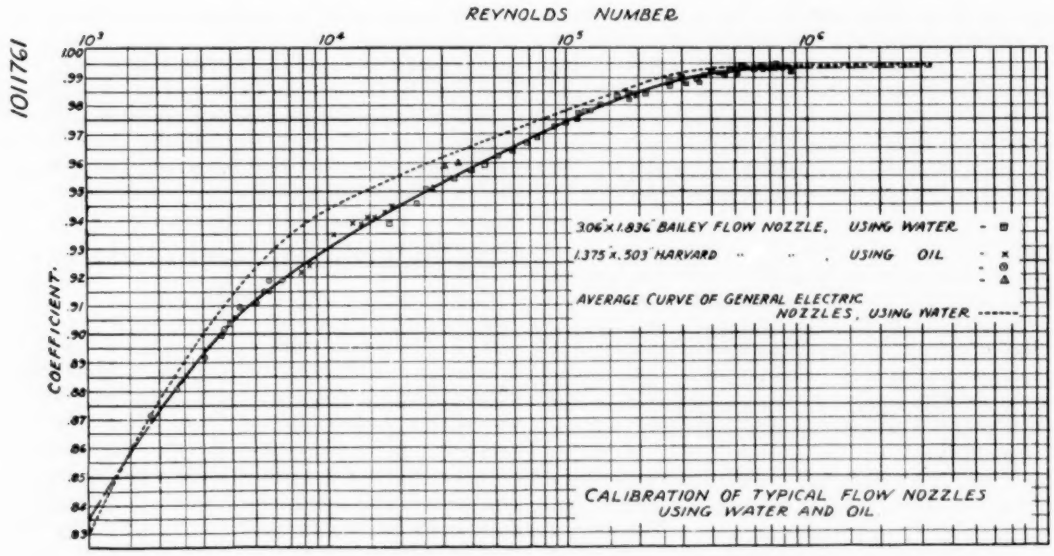


FIG. 1 CALIBRATION OF TYPICAL FLOW NOZZLES USING WATER AND OIL

class by themselves and cannot be grouped with larger ones without the application of some correction factor to compensate for these variations in finish.

An interesting comparison with the average curve of the General Electric nozzles, described by Mr. Buckland,[3] is made possible by placing this average curve on the writer's Fig. 1. The maximum difference between the two curves is about 1 1/2 per cent, and it occurs at a Reynolds number of approximately 10,000. At higher and lower Reynolds' numbers, particularly at the higher Reynolds number, the agreement between the two curves is better than 1/2 per cent. Because of the rather spotty data of the General Electric nozzles at Reynolds' numbers less than 50,000, it is believed by the writer that the heavy curve shown in his Fig. 1, as developed by Mr. Smith and the Bailey nozzle, is slightly more favorable. However, if a mean line were drawn between the two, the maximum deviation would not exceed 3/4 per cent, with the average between 1/4 per cent and 1/2 per cent at the more useful ranges which is an accuracy that will conform to most test specifications.

RONALD B. SMITH.[4] While the results of this paper serve as further confirmation of the Reynolds criterion it would seem to me that this is the ideal application of the sharp-edged orifice rather than the nozzle. It is extremely difficult to reproduce accurately the author's approach radii on nozzles 1/4 in. and 3/16 in. in diameter so that the coefficients cannot be applied to other nozzles with certainty. For instance, the author's coefficients of the geometrically similar large and medium-size nozzles do not agree. In addition, the use of a 1/16-in. throat hole in only a 1/8-in. nozzle must result in some abnormality in the flow.

Apart from the author's research it may be of interest to point out that in the regions of laminar flow the use of a pitot tube at 0.15 diam from the pipe wall offers no advantages over the usual static hole as far as accuracy is concerned. While the author does not mention the straight length upstream of the nozzles, let us assume that it is sufficiently long so that a parabolic profile is nearly developed. Although this would require considerable length it is approached in about 50 diameters.[5] Now the impact tube at 0.15 diam from the wall measures practically the average velocity for the parabolic (or the one-seventh turbulent) profile, yet it is known that the kinetic energy of the parabolic profile is twice the square of the mean velocity. Inasmuch as the development of the flow equation is essentially a balance of energies it would then be more rational to locate the tube at the rms velocity point when there is a laminar flow. This would be at about 0.3 diam.

ED S. SMITH, JR.[6] The author's data cover a range of Reynolds' numbers of present interest for the nozzle having an impact tube in the inlet. The curve in general parallels that for the Herschel Standard venturi tube, falling consistently several per cent below it.

The writer considers an impact tube, spaced only 0.15 diam from the pipe wall as tested by the author, to be a poor pitot on account of the steep velocity gradient so near the wall. It would seem that this tube location would be unduly liable to error at low Reynolds' numbers where the velocity-distribution curve has a parabolic form, i.e., an extended nose in the center. The use of straightening vanes is indicated in this flow régime.

In spite of the foregoing objection, the author's tests show an excellent correlation of coefficient with Reynolds' numbers, thus establishing the relation usefully for the particular nozzle-impact tube forms used.

SANFORD A. MOSS.[7] This paper shows a great deal of precise flow-measurement work, and is a distinct contribution to our knowledge of the properties of rounded-approach nozzles. One of the contributions is evidence in the matter as to whether or not Reynolds' number is a proper criterion for the plotting of nozzle coefficients. The author's curves in Figs. 3 and 4 do not at all coincide as they would if Reynolds' number were a complete criterion. The curves are also a little lower than the Reynolds number curve given by Mr. Buckland.[3] It has been suggested that the coefficient of various nozzles might be brought together if "Head" were used as the abscissas, rather than the Reynolds number, and this is worth investigating. Mr. Smith uses as the

[4] Turbine Engineering Department, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. Jun. A.S.M.E.

[5] "Aero- and Hydromechanics," by Tietjens-Prandtl, vol. 2, pp. 25–28.

[6] Hydraulic Engineer, Builders Iron Foundry, Providence, R. I. Mem. A.S.M.E.

[7] Research Engineer, General Electric Company, West Lynn, Mass. Mem. A.S.M.E.

ordinate of his curves, on Figs. 3 and 4, the velocity coefficient whereas Mr. Buckland uses flow coefficient as the ordinate for his curves, which is the coefficient occurring in the theoretical formula for weight flow. Might it not have been a little more useful for computations involving use of the nozzle, as well as easier in the computations for finding the coefficient, if Mr. Smith had also done this?

RICHARD G. FOLSOM[8] and J. A. PUTNAM.[9] Rounded-approach orifices with cylindrical downstream sections were developed with a view to obtaining a flowmeter having a constant-discharge coefficient near to unity. The added feature of the impact tube was introduced to simplify the flow equation in that it automatically takes into account the velocity of approach, when placed in the correct position. Such an arrangement has considerable value when metering gaseous fluids. However, in handling liquids the law of continuity is so simple that the additional constructional and experimental difficulties of the impact tube far overshadow its advantages.

Mr. Smith's paper and other recent publications[10,11] clearly illustrate the characteristics of this type of metering device at both low and high values of Reynolds' number. The discharge coefficient drops rapidly at low values of Reynolds' number similar to the corresponding characteristic of the simple diaphragm-orifice.

Fig. 2 of this discussion shows the calibration curve of a small square-edged orifice used for metering in the mechanical laboratories of the University of California and which is comparable with the small orifice used by Mr. Smith. The meter conforms in general with the I.S.A. 1930 orifice, except that the diameter is less and the edge is thicker than the tolerance limits set by the I.S.A. The pressure connections are placed so that accidental burrs can have no effect. A disadvantage of the meter used by Mr. Smith is the position of the static-pressure connection in the high-velocity section where errors due to burrs will be a maximum.

Although the nozzle-impact-tube meter coefficients are higher, they vary as much as the coefficients of the simple orifice in the region of low Reynolds' number. At high Reynolds' number both types have a constant coefficient.

Since there is no choice on the basis of discharge-coefficients, the simpler sharp-edged orifice proves to be the most satisfactory meter under operating conditions. For accurate work, all small meters must be calibrated in place.

R. J. S. PIGOTT.[12] Mr. Smith is to be congratulated on the excellent consistency of his test work. In comparison with other tests on small nozzles, the scatter of points is quite noticeably less than usual.

[8] Instructor, Department of Mechanical Engineering, University of California, Berkeley, Calif. Jun. A.S.M.E.

[9] Teaching Assistant, Department of Mechanical Engineering, University of California, Berkeley, Calif.

[10] "Regeln für Die Durchflussmessung mit genormten Düsen und Blenden," V.D.I., 1932.

[11] Détermination du Coefficient de Débit de Tuyères et Orifices Noyés," by MM. P. Leroux et Deullin, *Annales des Mines*, ser. 13, vol. 4, no. 11, 1933.

[12] Staff Engineer, in charge of engineering, Gulf Research & Development Corporation, Pittsburgh, Pa. Mem. A.S.M.E.

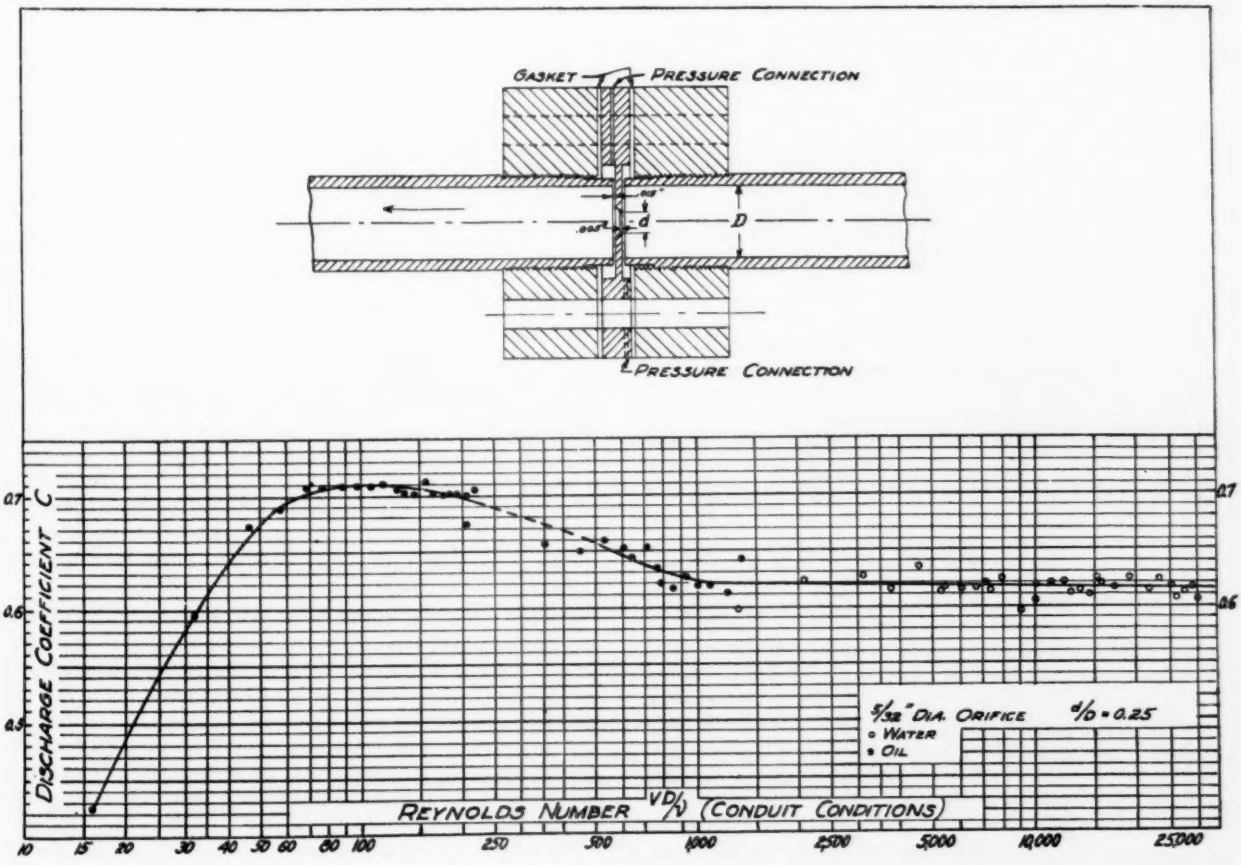


FIG. 2 THE CALIBRATION CURVE OF A SMALL SQUARE-EDGED ORIFICE USED FOR METERING IN THE MECHANICAL LABORATORIES OF THE UNIVERSITY OF CALIFORNIA

It is assumed that the pipe used with these nozzles was drawn brass, although it is not so stated in the paper. The point is of importance, as the roughness of the preceding pipe has a definite effect upon coefficients.

Regarding the use of the impact tube located at 0.15 diam, the writer used this location in 48-in. pipe in 1910, for steam sampling. It gives a fairly good average velocity reading for fairly large Reynolds numbers, but it is certainly not rigorous. Such an impact tube does have an influence on the coefficient, since it disturbs the flow into the nozzle.

With regard to the dip in the coefficient curve, for the medium orifice it is possible that the impact tube for this particular set-up was at a critical position. There is also another cause for disturbances in this region; the plots are usually based on Reynolds' number in the throat. Turbulence usually begins at $dv\rho/\mu = 1200$ to perhaps 2500. But at this time, the upstream section is in viscous flow, and full turbulence for the whole nozzle does not occur until $dv\rho/\mu$ in the throat is greater than 3300 for the large nozzle, 6600 for the medium, and 13,200 for the small nozzle. Indeed, if the flow is very smooth in the upstream pipe, turbulence may not be fully established until about twice the foregoing values. Consequently, there is some instability in the orifice in this range; it may be detected by a tendency of the head gages to oscillate, since there are at least two possible extreme values for the differential. Very likely the dip is due to a combination of the impact tube and the unstable condition.

The writer would assign even the small variations in coefficient to three factors: (*a*) non-similarity; (*b*) difference in relative roughness; (*c*) the impact tube.

For the past ten years, it has become customary to plot venturi-tube, disk-orifice, and nozzle coefficient against number, assuming geometrical similarity, because the venturi was the same general shape, the orifice was flat and had a sharp edge, or the nozzle was the same contour. This practice is unfortunate, as it is not fair to the Reynolds number comparison. It is a fact that almost no completely similar nozzles have been tested. If the same 12-in. steel pipe is used for a five-, four-, and three-inch nozzle, although the nozzles themselves are similar in contour, the assembly is *not* similar. Further, if a 12 by 3 and a 4 by 1 nozzle are compared, using steel pipe in both cases, they are dissimilar; the four-inch pipe is *relatively* much rougher; so is the one-inch nozzle, with the same polish.

Mr. Smith's nozzles are approximately similar only. In the fluid-meters-nozzle research at present proposed, we intend to establish full geometrical similarity, varying roughness and diameter ratio. In this way, those deviations hitherto all charged to test variation will certainly be reduced to test errors purely, without the additional scatter resulting from dissimilarity.

These small nozzle tests are very much needed to fill out the lower Reynolds number region, and are particularly timely for the writer.

Author's Closure

Mr. Moss has pointed out that the three curves given in the paper do not coincide and on this basis concludes that Reynolds' number R is not a complete criterion to use as abscissas. It would appear to the author, however, that such a conclusion from the present data is unwarranted. In the first place, the orifices are not quite geometrically similar, with the result that a dimensional-analysis treatment of the problem would not give us C as a function of R only. Other dimensionless groups involving lengths would unquestionably enter.

The fact that these curves are slightly lower than those of B. O. Buckland can be explained perhaps by the lack of similarity in the two pieces of apparatus, and by his use of static pressure upstream, whereas the author used impact pressure upstream. Mr. Moss's suggestion to plot C against head rather than R has been tried. It yields points with the present data that are scattered considerably more than where R is used.

Mr. Moss points out that the author uses the simple equation $v_{theor} = \sqrt{2gh}$ rather than $v_{theor} = \sqrt{\frac{2gh}{1 - r^4}}$ used by B. O. Buckland where r is the ratio of throat diameter to upstream diameter, and states that the latter is the coefficient occurring in the theoretical formula for weight flow. He has overlooked the fact that there is a difference in set-up in the two cases. Using static pressure upstream, the latter of the two formulas is correct, but if an impact tube is to be used, the former equation is the correct one, provided that the impact tube is placed at the proper place to get the desired velocity head.

Ronald B. Smith, Ed S. Smith, Jr., and R. J. S. Pigott state that the use of an impact tube at 0.15 diam from the pipe wall offers no theoretical advantages over the usual static hole. The author agrees with this. R. B. Smith's logic in discussing the kinetic-energy relations cannot be questioned. Strangely enough, the author used reasoning quite similar in discussing a paper presented by Prof. L. S. Marks on air flow in fan ducts a few days before these present discussions were presented.

When these orifices were installed the question of possible errors arising in coefficient due to erroneous readings of velocity upstream was investigated. Perhaps a résumé of the conclusions reached at the time would be illuminating. If an error of 50 per cent in velocity upstream were made, the effect on the velocity at the throat of the big orifice was 1.6 per cent, for the medium orifice it was 0.1 per cent, and for the small orifice it was entirely negligible. These conclusions are based on the assumption that an error of 50 per cent in the velocity upstream would be shown up in the flow through the orifice, but even this assumption is unjustified, since an error in the velocity upstream obtained for each orifice during calibration would be mainly counterbalanced in the use of the orifice during tests on flow measurement. Thus the small errors mentioned are considerably larger than any mistakes which would occur in the use of such an instrument, and the conclusion that the introduction of an impact tube would lead to negligible errors was borne out by the results obtained.

Now there is no single point upstream which would give the proper impact pressure over a wide range of Reynolds' number, particularly if the flow may change from viscous to turbulent, but the movement of the position of the impact tube at every reading would have introduced many complications and, in view of the small errors introduced, as previously mentioned, the distance 0.15 diam was adopted, as recommended in the Power Test Codes Tentative Draft, series 1933, Instruments and Apparatus, part 2, p. 13, and previously recommended by Sanford A. Moss in a verbal communication.

R. E. Sprenkle has presented a most remarkable verification of the author's data in his curve, especially as the apparatus used and liquid flowing in each case were different. Such close agreement is very gratifying.

Several discussers, including R. J. S. Pigott and R. B. Smith, have asked about the relative roughness of the orifices and approach pipes. That question the author cannot answer quantitatively. All orifices were made of composition metal (a brass) and were machined as smoothly as our shops could make them, templates being used in turning and polishing. The finish was bright and apparently glassy-like in smoothness. The pipes on each side of the nozzles were standard iron pipes, of ordinary roughness.

Mr. Pigott's comments on the dip noticed with the medium orifice are interesting, and obviously true; but they do not explain fully why this dip was found with only the one orifice.

Messrs. Folsom and Putnam state that the static-pressure connection in the author's apparatus is at the high-velocity section where burrs would have a maximum effect. The removal of burrs formed was not a serious matter, and the data obtained would seem to demonstrate that any irregularity left had little effect on the coefficients of the orifices.

The determination of the static pressure at this point has the definite advantage that the pressure is obtained under relatively stable conditions. A static-pressure connection immediately after the orifice, as used by Messrs. Folsom and Putnam, is not desirable, as this is the position of unstable turbulence. The eddies formed by the fluid immediately after passage through the orifice are very troublesome, although it is possible that in the extreme corner they would have little effect. The graph shown by the discussers has points departing by as much as 3 per cent from the mean line drawn, in the region of ordinary operation. In many tests this deviation is not allowable. It is true that over a range of Reynolds' number (conduit conditions) of from 1000 to 30,000, as shown in the discussers' graph, the coefficient is relatively steady, and this has definite advantages if a rough automatic measuring device is to be used. Otherwise, however, it is not a difficult matter to calculate the Reynolds number and pick the coefficient from the proper graph.

The V-Notch Weir for Hot Water[1]

H. N. Eaton.[2] This paper illustrates, in an interesting way, the fact that frequently, by varying one of the physical quantities involved in a physical phenomenon, we can determine what would be the effect of varying a different physical quantity which is also involved in the phenomenon. In the present instance, the effect on the coefficient of the V-notch of varying the head acting on the notch is used to show what would be the effect, over a limited range of the coefficient curve, of varying the kinematic viscosity of the water flowing through the notch. The advantage of this procedure lies in the fact that it is much more difficult to vary the kinematic viscosity of the water than to vary the head on the notch, at the same time controlling the conditions carefully enough to obtain accurate measurements. This expedient has been utilized to advantage in other branches of engineering and physics, particularly in aerodynamics, and the writer is interested to see an application of it made to hydraulics.

The process of reasoning by which the author arrives at his plot of C against $h/\nu^{3/2}$ appears to be correct, but the following treatment is suggested as a more direct and logical one.

We start with the customary formula for the V-notch

$$Q = Ch^{5/2} \dots\dots\dots\dots\dots\dots [1]$$

where Q is the volume rate of flow, h is the head on the notch, measured above the vertex, and C is the coefficient of discharge of the notch.

We wish to determine how C is affected by the different physical and geometrical quantities which are involved in the phenomenon.

The following quantities may affect the flow Q, and hence the coefficient C: The head h, the acceleration of gravity g, the density of the water ρ, the viscosity of the water μ, the surface tension of the water s, the width of the approach channel b, the height of the vertex of the notch above the floor of the approach channel H, the width of the crest of the notch plate w, the angle of the notch α, the factor of a relative roughness k of the upstream surface of the notch plate, and to a lesser extent the roughness of the walls of the approach channel.

We can express this dependence as follows:

$$f_1 (Q, h, g, \rho, \mu, s, k, \alpha, H, b, w) = 0 \dots\dots\dots [2]$$

where f denotes "function of."

From these eleven significant quantities we can form $n-i$ dimensionless products, where n is the number of significant quantities and i is the number of physical dimensions required to express these quantities (in this case three—mass, length, and time). Hence, eight dimensionless products result, by means of which we can express our relationship as follows:

$$f_2 \left(\frac{Q}{g^{1/2}h^{5/2}}, \frac{Q\rho}{h\mu}, \frac{s}{\rho g h^2}, k, \alpha, \frac{h}{H}, \frac{h}{b}, \frac{h}{w} \right) = 0 \dots\dots [3]$$

The particular forms of the products we choose depend upon the particular relationships we wish to study, and for different purposes we can express the same relations in many different forms. A concrete illustration of this will be given later in this discussion in passing from Equation [4] to Equation [5].

The first three of the dimensionless products chosen above were designed to separate clearly three different effects: first, the balance between the inertial and gravitational forces expressed by the product $\frac{Q}{g^{1/2}h^{5/2}}$; second, the balance between inertial and viscous forces, expressed by $\frac{Q\rho}{h\mu}$; and third, the balance between surface tension and gravitational forces expressed by $\frac{s}{\rho g h^2}$. The fourth variable, k, is a dimensionless roughness factor, and the last four are purely geometrical factors which express the form, but not the size, of the notch and the approach channel.

The first three dimensionless products, because of the particular force ratios which they represent, correspond, respectively, to the Froude, Reynolds, and Weber numbers. However, the names "Froude number" and "Reynolds number" should not be applied to the first two, because these names are used in a more restricted sense to apply, respectively, to the square of a velocity divided by a length and the acceleration of gravity, and to the product of a length and a velocity divided by a kinematic viscosity. It has been suggested to the writer by Dr. L. B. Tuckerman of the National Bureau of Standards that the names "generalized Froude number" and "generalized Reynolds number" be applied to these two dimensionless products. The name "Weber number" is usually applied to the dimensionless product $v^2l\rho/s$, which represents the balance between inertial and surface-tension forces, instead of the form given above. This name has not yet become as fixed in its usage as have "Froude" and "Reynolds" numbers, and, since these two forms of the Weber number both take account of the effect of surface tension, no distinction will be made here.

The surface tension of the water affects the coefficient curve only at very low heads and is probably of no significance over the range discussed by the author of the paper. Consequently, the dimensionless product $s/\rho g h^2$, will be omitted from consideration in what follows. It is interesting to note that, if surface tension can be ignored, the density then appears only in combination with the viscosity in the form of the ratio μ/ρ, which we call the kinematic viscosity, ν, and hence, under this condition, the density and viscosity need not be included separately in [2] but can be replaced by the kinematic viscosity.

The width of the approach channel and the depth of the floor below the vertex of the notch will affect the coefficient at high

[1] Published as paper RP-56-9, by Ed S. Smith, Jr., in the October, 1934, issue of the A.S.M.E. Transactions.

[2] Acting Chief, Hydraulic Laboratory Section, National Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

heads through their effect on the stream lines and on the velocity of approach, unless the dimensions of the approach channel are sufficiently large. It has been shown experimentally that the width of the approach channel does not have any effect on C as long as $b > 8h$.[3] The writer has seen no satisfactory data illustrating the effect of proximity of the floor. Barr gives two curves for a 90-deg notch which indicate that, for a head of 3 in., the depth H must be greater than $3h$ and for a head of 4 in. H must be greater than $4h$. The discussion which follows is applicable only to a V-notch for which the width b and the depth H of the approach channel are so great that they exert no measurable effect on the coefficient even at the highest heads used. This restriction is justifiable here without further consideration, since it is not the purpose of this discussion to set limits to the regions within which the effects discussed are appreciable, but to indicate a more direct method of reaching the conclusion arrived at by the author of the paper.

In addition to the restrictions which have already been set upon the problem, we shall assume that we are dealing with a notch having a given angle α and a given crest width w and that the variation in the relative roughness of the notch plate and the walls of the approach channel with changes in head can be neglected.

With the restrictions thus established we can simplify Equation [3] to

$$f_2\left(\frac{Q}{g^{1/2}h^{5/2}}, \frac{Q}{h\nu}\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

However, for our present purposes, another of the infinite number of possible combinations of these two dimensionless products will be more convenient, and we replace $Q/h\nu$ by $\frac{g^{1/3}h}{\nu^{2/3}}$ which we do by dividing $Q/h\nu$ by $Q/g^{1/2}h^{5/2}$ and taking the two-thirds power of the result. We also introduce the coefficient C in the first dimensionless product in [4] by means of [1], and [4] now becomes

$$f_4\left(\frac{C}{g^{1/2}}, \frac{g^{1/3}h}{\nu^{2/3}}\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

With our relation in this last form, we can see, as we could not easily see before, that it is legitimate to plot $\frac{C}{g^{1/2}}$ against $\frac{g^{1/3}h}{\nu^{2/3}}$, and if none of the physical quantities which we have neglected affects C measurably over the range of our plot, we shall get a single curve. Furthermore, as long as the value of g remains constant, as it will in the practical use of the notch, we can drop it from consideration and can plot C against $h/\nu^{2/3}$, as the author has done.

Whether or not all of the physical quantities which have been left out of consideration in deriving Equation [4] actually have no measurable effect on C over the range of the curve given by the author is a question which the writer will not attempt to settle, since this was not the purpose of the discussion.

H. S. Bean.[4] In this paper, Mr. Smith proposes the use of certain arbitrary ratios as parameters against which to plot his coefficients for V-notch weirs. Of course, one is at liberty to use any such parameter as may suit his pleasure or convenience, but in passing them on to others, attention should be called to any dependence that such parameters may have upon the system of units used. Let us examine the proposed parameters for their dependence upon units, referring to the author's equations as they are numbered in this paper.

The basic flow relation as expressed by Equation [7] is

$$Q = Ch^{5/2}$$

While expressing an experimental result, this equation is really a definition of C. C need not be a constant, and, in the general case, will not be constant, but will have a definite and different value for every pair of values of Q and h. It is interesting to note that in order for Equation [7] to balance dimensionally, C must have the dimensions $L^{1/2}\ T$, which are the same as those for $g^{-1/2}$. Thus we find that, as here defined, C is not independent of the units used. Since this equation is used as a basis for Equations [9] and [10], it follows that they also are dependent upon the units used.

This same conclusion may be obtained directly by noting the dimensions of the ratios given by [10]. For example, the dimensions of the first ratio in [10] are $\left(\frac{L^3}{T}\right)^{2/3}\frac{T}{L^2} = \frac{T^{2/3}}{L^{1/3}}$

Thus, while there is true correspondence between Equations [8] and [10], this correspondence is not general; that is, it depends upon the particular units being used. In this connection it is well to note that the author expresses density, viscosity, and surface tension in cgs units, while his weir dimensions, velocity, and flow are in English units. Therefore, in making use of any of the relations given by the author, we must be careful either to use the same combination of units which are given in the paper, or to convert the relations to other units which we might prefer to use.

Author's Closure

The discussers of the paper have stressed the fact that the operators and coefficients used therein are not dimensionless and that both English and cgs units have been used. These informalities were deliberately introduced for greater convenience in use and, because of the need for brevity, this was done without explanation.

The most important point of Mr. Eaton's independent, mathematical analysis is the confirmation of the true correspondence between the coefficient C and the operator $h/\nu^{2/3}$. Mr. Bean's check of the true correspondence between Equations [8] and [10] is appreciated.

It is regretted that the discussion of the paper included no data on triangular weirs using liquids of different viscosities, such as are awaited from the University of California project 273 being conducted by Mr. Carson under the direction of Prof. M. P. O'Brien.

It should be noted that all the figures in the paper are on double-logarithmic paper, even though the ordinates are to a large scale in Figs. 1 and 3. This explanation is needed to clarify the use of the dot-and-dash line in Fig. 1 as representing the exponential Equation [14].

An additional reference which should also be given is: "Flow of Water Over a V-Notch," by Joseph Tarrant, Trans. A.S.M.E., Vol. 50, 1928, paper HYD-50-8, p. 25.

The author concludes by calling attention to the new and useful operator $h/\nu^{2/3}$ as a basis for correlating weir coefficients, not only for hot water but also for oils in the "look boxes" of refineries. The value of the contribution is entirely from the engineering, rather than the scientific, viewpoint.

---

[3] "Experiments on the Flow of Water Over Triangular Notches," by J. Barr, *Engineering*, London, vol. 89, 1910. See also "Hydraulics and Its Applications," by A. H. Gibson, D. van Nostrand Co., New York, N. Y. Third edition, pp. 162 and 163. See also Trans. A.S.C.E., vol. 93, 1929, p. 1134, Fig. 60*d*, where Prof. W. S. Pardoe gives a curve which indicates that the width may be as low as $3.4h$ without affecting C.

[4] Physicist, Chief Gas Measuring Instrument Section, National Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

Fluid-Meter Nozzles[1]

RONALD B. SMITH.[2] The problem of establishing reliable nozzle coefficients and a standard flow-measuring technique for acceptance-test work is of particular concern to the Power Test Codes Committee at the present time. In the writer's opinion the shape that is chosen matters but little providing only that the nozzle can be easily reproduced and accurately installed. The important point is to choose a standard which has been so thoroughly verified that its characteristics under all probable test installations are accurately established.

One of the nozzles under consideration as a standard is the V.D.I. profile, some of the characteristics of which Mr. Buckland compares with the G.E. nozzle. The V.D.I. nozzle is the outgrowth of the nozzle used for the past 30 years for flow-measurement work by the I. G. Farbenindustrie. Within recent times it has been adopted as standard by the International Standards Association and now, as a result, is generally known as the I.S.A. nozzle. Since 1928, largely at the request of the V.D.I., several thousand laboratory calibrations of the nozzle have been made, with the result that its flow coefficients, for pressure drops down to the acoustic and for area ratios ranging from zero to 0.6, have been established under a wide variety of conditions with an accuracy generally within plus or minus 0.5 per cent. The flow coefficient of the I.S.A. nozzle is constant over a greater range than is usually the case with a full flowing nozzle. For instance, in the author's Fig. 14 it is evident that the I.S.A. nozzle can be used to a 50 per cent lower range than the G.E. nozzle before one must resort to cut-and-try methods in the calculations.

By attempting to compensate for the different locations of the pressure taps the author concludes that the coefficient of the G. E. nozzle is $1^1/_4$ per cent higher than the I.S.A. This result is based on the assumption that a pressure measurement in the throat of a nozzle and a pressure measurement in the pipe two nozzle diameters downstream are identical, and are equivalent to the atmospheric pressure with a freely discharging jet. This opinion appears untenable from analysis of the very tests that the author quotes to support it, namely, the work of Stach. Except for the smallest nozzle, Stach's coefficients show less than 0.3 per cent difference between measurements of the I.S.A. nozzle when discharging freely and when operating in a pipe with the usual corner taps.[3] This slight difference can be explained by the fact that Stach used pressure-chamber openings smaller than standard. Thus, the result of Stach's work is to indicate that the pressure measurement in the downstream corner is equivalent to the pressure for discharge into an infinite chamber.

If we compare the pipe and throat-tap measurements on a Moss-Johnson nozzle as reported by Sprenkle,[4] we must conclude that a throat tap reads the pressure about $1^1/_2$ per cent high.

Downstream from the nozzle and along the pipe wall there is a pressure fall. It is interesting to note that this distribution, as in Fig. 1 of this discussion, on a Moss-Johnson nozzle which is practically the same as the G.E. nozzle, is similar to the results reported by Witte on the I.S.A. nozzle. For instance, the minimum pressure is $1^1/_2$ per cent of the differential pressure and it occurs (for $m = 0.25$) about $^3/_4$ diam downstream from the mouth. Because the same quantitative phenomenon is observed with an orifice it seems probable that the distribution is produced by viscous effects at the boundary of the jet.

In combination with Sprenkle's results, the pressure-distribution curve leads one to suspect that throat pressures are 1 per cent higher than corner pressures, and that, as a result, if the nozzles are compared on this basis, there will be practically no difference in their coefficients. That there is no actual difference between the two nozzle coefficients when the pressures are measured in the same manner has been proved by Witte.[3] For instance, using corner taps, which includes some impact pressure and therefore accounts for a low coefficient, he measures with an area ratio 0.25:

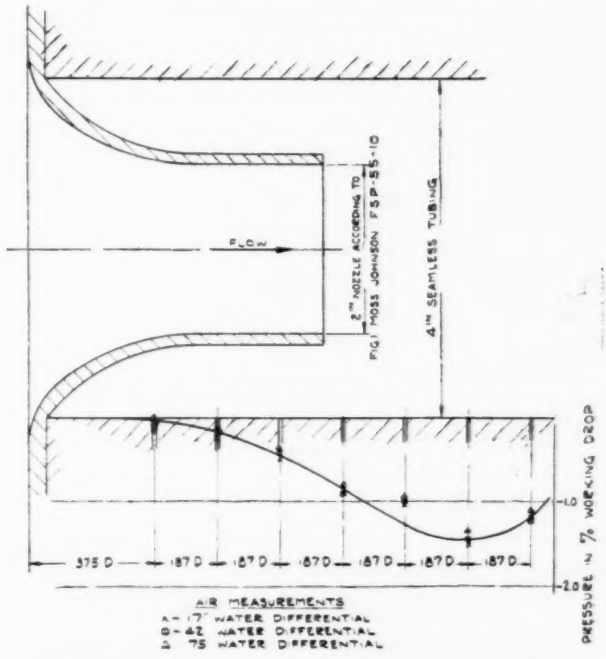


FIG. 1 PRESSURE DISTRIBUTION IN A MOSS-JOHNSON FLOW NOZZLE

For the I.S.A. nozzle	= 0.9765
For the Moss-Johnson nozzle	= 0.974
For the same Moss-Johnson nozzle with throat taps	= 0.991

Thus the relative magnitudes in the author's Fig. 14 are misleading.

Mr. Buckland suggests that the rapid fall of the coefficients of the I.S.A. nozzle below the operating region is the result of contact loss in the nozzle. That there is a temporary contact loss in an I.S.A. nozzle where the approach radius is tangent to the throat is known. Not so well known is the fact that nozzles of the G.E. shape also show contact loss at this point. I have observed the phenomenon many times on a Moss-Johnson nozzle by coating the inner surface with lampblack and kerosene and studying the streak lines that are produced by the air. However, in neither nozzle is it of serious importance nor would it be termed a vena contracta, since contact is reestablished within $^1/_4$ in. downstream. The peculiar slope of Mr. Buckland's coefficients in Fig. 8 of his paper between Reynolds' numbers of 10^4 and 4×10^5 may be the result of this phenomenon.

R. E. SPRENKLE.[5] Mr. Buckland's paper will be of material assistance in familiarizing engineers with the fact that the Ameri-

[1] Published as paper FSP-56-14 by B. O. Buckland, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Turbine Engineering Department, Westinghouse Electric and Manufacturing Company, South Philadelphia, Pa. Jun. A.S.M.E.

[3] "Neuere Mengenstrommessung zur Normung von Dusen und Blenden," by R. Witte, *Forschung auf dem Gebiete des Ingenieurwesens*, September-October, 1934.

[4] "A System for the Measurement of Steam With Flow Nozzles for Turbine Performance Tests," by S. A. Moss and W. W. Johnson, Trans. A.S.M.E., vol. 55, 1933, paper FSP-55-10, p. 145.

[5] Bailey Meter Company, Cleveland, Ohio. Assoc-Mem. A.S.M.E.

can style of flow nozzle is a real precision instrument and that its accuracy and reliability well merit its use as a standard of measurement.

Our experience in building and using several thousand flow nozzles of all sizes and for all kinds of flow-metering service, has shown that the use of pipe-line connections at both the nozzle inlet and outlet, as shown in Fig. 13 of the paper, is the simplest and most dependable method of measuring the pressure differential across the nozzle. This experience covers a span of nearly twenty years during which many weighed-water or other tests have proved the adequacy of the commercial nozzle of this design as a means for measuring water, steam, air, gas, and other flow rates.

Pipe taps into the wall back of the nozzle throat instead of into the throat itself, possess some real advantages. First, this location is in a protected zone out of the path or contact with the stream lines of the flowing fluid, and thus not susceptible to errors in static-pressure measurement due to small localized eddies, whirls, or other disturbances such as may, and often do, exist along the throat surface. Moreover, being in a zone where velocities are comparatively low, there is even less chance of this pressure measurement being in error.

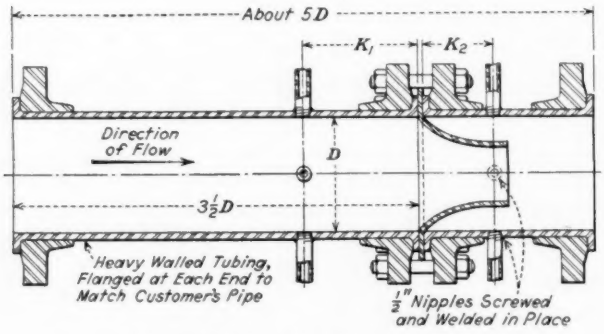


FIG. 2 A TYPICAL NOZZLE PIPE SECTION WITH PRESSURE CONNECTIONS AND NOZZLE PROPERLY LOCATED

From the standpoint of physical application, the nozzle with the outlet connection made back of, instead of into, the throat, allows the use of a much thinner flange, with a consequent reduction in pipe spread to provide for its insertion between existing flanges. The nozzle proper is easier to build because of the omission of the piezometer chamber and internal connection passages which are required to provide for throat taps.

One of the outstanding advantages of the nozzle, shown in the author's Fig. 13, is that extreme care does not need to be taken in drilling the outlet connection into the pipe wall. True, this must be done in the field, as Mr. Buckland states, but due to its protected location, it is much less difficult to make than the inlet pipe connection which is used by both nozzle types.

The throat tap connection is admittedly difficult to make unless all possible precautions are taken. This point is brought out not only by Mr. Buckland himself, but also in the discussion of the Moss-Johnson paper in 1932 by the present writer, in which comparative tests made with both throat and pipe taps in a special nozzle in the Bailey Meter Company laboratory were described in detail. In that discussion, we demonstrated the difficulty, in fact, almost impossibility, of getting the separate throat-tap pressure readings to check each other, as compared with the ease of obtaining a very satisfactory agreement between the different outlet pipe tap readings. The elimination of this job of making satisfactory throat connections more than compensates for the labor of providing for this connection in the field.

While the pipe tap back of the throat cannot be calibrated as an integral part of the nozzle itself, neither can the inlet-pipe connection which is used with both types of nozzles. And of the two, the inlet connection is the more susceptible to changes in the flow state, being immediately adjacent to the path of the stream lines. As such, it is the most important connection to be included in any integral nozzle-assembly calibration. The truly correct and proper method is to calibrate the nozzle with the section of pipe in which it is to be used, and thus both pressure connections are included in the assembly and all possible installation vagaries eliminated. A typical nozzle pipe section with pressure connections and nozzle properly located, is shown in Fig. 2 of this discussion.

The first reason given for the use of the throat instead of the pipe-line connections, was that the lack of geometrical similarity of the external shape of the nozzle used might produce erroneous results were pipe taps used. We would point out that even along the internal surfaces over which the fluid passed, complete geometrical similarity did not exist. True, the test nozzles were in themselves, geometrically similar in form, but when installed in the pipe lines, the assemblies with the pipe were not geometrically similar by widely varying amounts. To attain complete similarity, the curvature must begin at the same relative point with reference to the inside pipe wall on each nozzle. In all but one of the G.E. nozzles, the distance of the junction of the curvature with the straight flange section as measured from the inside of the pipe wall, varied from 4 per cent to 27 per cent of pipe diameter, and in this one case, this point was actually up in the holding flange by an amount equal approximately to 5 per cent of the pipe diameter. Since the flow must pass over these surfaces, this lack of similarity is likely to produce a larger spread between coefficients of different nozzles of various sizes than would have resulted from pipe-tap measurements made in a region where this lack of similarity was relatively unimportant.

That there can be no *complete* geometrical similarity between nozzles of different diameter ratios, is quite apparent but nevertheless not always fully understood. In fact, only when nozzles of the same diameter ratio are used in different sizes of pipes can such similarity be obtained, and even then the relative pipe roughness may not be quite the same. Since various diameter-ratio sizes must be used for practical metering, it is useless to expect complete agreement of calibration data on a similarity basis.

An improvement can be made, in the attaining of better similarity between different diameter-ratio sizes, by always placing the beginning of the curvature at the surface of the internal pipe wall and then so shape the curvature to the one-quarter ellipse by making the minor axis equal to $(D - d)/2$ instead of $5/8\ d$. With increasing diameter ratios, this ellipse becomes flatter but there are no humps or irregular surfaces over which the fluid must flow and thus no marked or sudden deviations from the natural flow path.

A comparison of calibration data from nozzles of the shape just described, using pipe taps, with the General Electric nozzles would be of interest. In Fig. 11, Mr. Buckland shows such a comparison between the data from a 12-in. × 7.554-in. Bailey Meter Company nozzle, and the average G.E. curve. The agreement of one with the other, is about as perfect as can be expected. However, the range of Reynolds' number used with the 12-in. nozzle was rather small; so to extend the curve to lower limits, a 3-in. pipe size, 60 per cent diameter-ratio nozzle was recently calibrated in our laboratory in Cleveland.

The calibration data obtained from a Bailey Meter Company 3.06-in. × 1.836-in. nozzle is shown in Fig. 3 of this discussion, as well as that from the 12-in. × 7.554-in. nozzle. In passing,

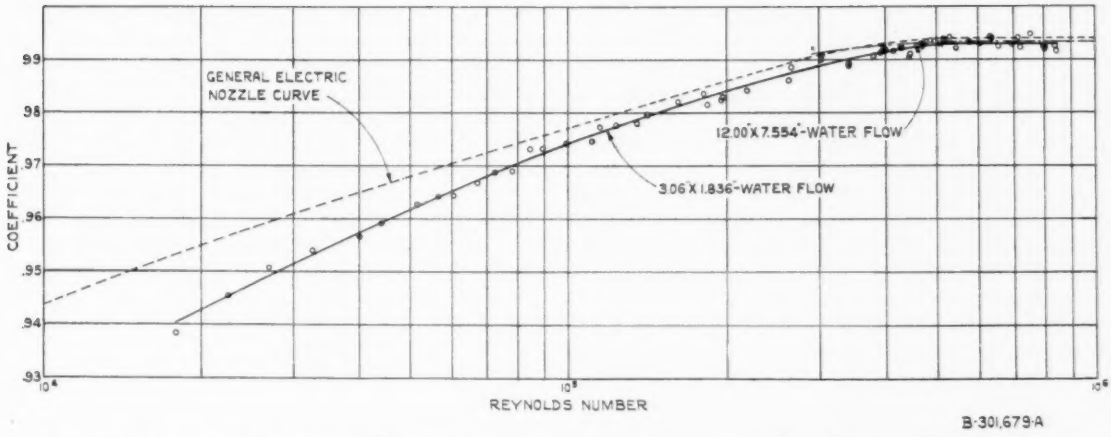


FIG. 3 CALIBRATION OF TWO BAILEY METER COMPANY FLOW NOZZLES COMPARED WITH AVERAGE CURVE OF GENERAL ELECTRIC NOZZLES

it should be noted that the 3-in. nozzle was made of highly polished brass and calibrated in smooth brass tubing, using water flow while the 12-in. nozzle was made of steel and calibrated in a commercial steel tubing, also using water as the flowing fluid. As will be noted, these two nozzles, checked each other almost perfectly through a range of Reynolds' number from 500,000 to the highest point calibrated and deviated from each other at the most about $^1/_4$ per cent at a Reynolds number of 300,000 or the lowest point tested on the 12-in. nozzle.

In Fig. 3 of this discussion, we have also shown by the broken line, the average calibration curve from Mr. Buckland's Fig. 8, as a comparison with calibrations of two Bailey Meter Company nozzles. Despite the higher diameter ratio, and the fact that both the Bailey nozzles shown used pipe-line pressure connections back of the nozzle throat instead of into the throat, and further, that the internal contour of the nozzle shape was not precisely the same, the General Electric type and the Bailey type checked each other from 0.1 per cent to 0.3 per cent over a working range of Reynolds' number of from 100,000 to 1,500,000, or the highest tested point. Whether or not the increased deviation at lower values of Reynolds' number is due to the difference between the location of the outlet pressure connections, to the small difference in the shape, or to experimental errors, is a question we cannot adequately answer at this time.

It is sufficient to add that through the useful range of Reynolds' number, or from 100,000 up, and with nozzles of diameter ratios not materially exceeding 60 per cent, either of the two types of American nozzles can be used for the purposes outlined in this paper with an accuracy that is certainly well within plus or minus 1 per cent, and with actual calibration within plus or minus $^1/_2$ per cent, provided proper precautions are taken both as to obtaining undisturbed flow through the nozzle, and in the design, construction, and installation of the nozzle assembly itself.

R. J. S. PIGOTT.[6] In examining Mr. Buckland's paper, one notes a tendency to call various nozzles, or orifices, geometrically similar, when as a matter of fact, they are not. It is not enough to use nozzles that are similar in contour because, for rigorous comparisons, it is necessary also to have similarity in pressure taps, polish of nozzle, upstream pipe, and orifice ratio. This complete condition has practically never been observed in tests as yet, and until it is, we shall not be able to get the full value of Reynolds' criterion comparisons. Scatter of points is much wider than can be assigned correctly to experimental errors, or to any departures from the "single line" theory; and the whole situation for studying the proper relations is somewhat confused.

The long series of experiments on orifices conducted by the joint A.G.A.-A.S.M.E. meter committee shows that the upstream roughness, orifice ratio, and tap location have very noticeable effects upon coefficient, quite in line with theory. While some advance the thought that pipe roughness has no effect on a nozzle, theory clearly indicates there ought to be some effect. If this thought were correct, neither the sharp-edged orifice nor the venturi should show roughness effect; but we know that they do show such differences.

Mr. Buckland has recognized this point in his Fig. 9 wherein the curves better approach full similarity, by eliminating orifice-ratio effects; pipe and nozzle relative roughness remaining the same.

The writer has been working for some time on a method of predicting coefficients, and finds that there is a definite relation between the ratio of "surface area" washed by the fluid between taps, to the area of throat, and the coefficient at any Reynolds number. The relative loss, or $(1 - C)$, is directly related to the pipe flow friction. The writer has for some time used the coordinates $(1 - C)$ vs. $dv\rho/\mu$ on double-log paper. Mr. Ed Smith has also used the same type of coordinates. It gives some very very valuable analytical indications which the semi-log graph of C vs. $dv\rho/\mu$ is incapable of showing.

One other point in nozzle testing has not been given sufficient attention. On a curve showing throat Reynolds' number, we would expect complete viscous flow below $R = 1200$. But above that point, the nozzle is in mixed flow until the upstream section also is fully turbulent. With an orifice ratio of 0.50, the minimum value of throat R for complete turbulence is 2500, and higher for smaller ratios. In addition, there is apparently a stronger tendency for a convergent nozzle to stay in the viscous region at higher values than in parallel sided pipe. As a consequence, many nozzles tested by Mr. Buckland and others cannot be safely considered in fully turbulent flow until values of possibly $R = 30{,}000$ to 40,000 have been passed. There is, therefore, a considerable range in which the flow is somewhat unstable, and the scatter of test points will usually be a little wider.

With regard to a supposed critical Reynolds' number at which the coefficient becomes constant, the writer is inclined to doubt any such value of R as 10^5. In pipe flow, such flattening does not take place until $R = 2 \times 10^6$ to 4×10^6. What appears to be a flat coefficient is merely due to rate of change much smaller than the test accuracy can show. A logarithmic graph

[6] Staff Engineer, Charge of Engineering, Gulf Research & Development Corp., Pittsburgh, Pa. Mem. A.S.M.E.

of $(1 - C)$ shows this condition very plainly. A sloping line at 11 deg or 12 deg, corresponding to the smooth-pipe conditions, will fit this cloud of points quite as well as a horizontal line. In this region, a precision of plus or minus 0.5 per cent in the tests means a variation of 40 to 80 per cent of $(1 - C)$. It is, of course, futile at present to attempt to prove this point, until still better test accuracy can be attained.

The A.S.M.E. Special Research Committee is undertaking an extensive program of investigation on this subject, with the original intention of comparing the proposed I.S.A. or Witte nozzle, with the type discussed in Mr. Buckland's paper. In order to determine more closely those factors not too clearly defined at present, such as roughness and diameter-ratio effects, the program will cover full-range tests on a preferred-number series of both sizes and ratios, with geometric similarity as fully developed as possible. Tests will be made in full with water, but duplicated so far as necessary with steam and air. Funds for this work are to be collected, as is usual in A.S.M.E. research undertakings, from interested industries.

W. S. Cooper.[7] The writer believes that Mr. Buckland should have given more data on the performance of these nozzles in actual field tests. High order of accuracy in measuring flow-rates is demanded on acceptance tests by builders and purchasers of turbine and boiler-room equipment, and direct measurement (by weighing) of condensate and feed-water flow rates has heretofore been considered the only reliable means. After all, the field of application of the nozzle will lie in the replacement of the more expensive direct-weighing method, and it is under such circumstances that a knowledge of the nozzle's performance characteristics is desired.

There is doubt in the writer's mind as to whether such accuracy as claimed by the author with laboratory tests could be obtained with the piping situation usually encountered in the average power plant. Furthermore, liquid flow in most power-plant piping is of a pulsating nature since the fluids are handled either by centrifugal or by reciprocating pumps. Pulsation was probably entirely absent or eliminated in the laboratory tests where the fluid was probably supplied by standpipes.

The writer had occasion recently to conduct field tests on one of the General Electric Company's nozzles described by the author. This nozzle was the one with proportions shown in the seventh line of Table 1 in Mr. Buckland's paper, namely, the 12.01-in. × 5.016-in. nozzle. The laboratory test results reported by the author for this nozzle are shown in his Fig. 8, the plotted points appearing as plus signs. The writer's tests were conducted in conjunction with two condenser acceptance tests where the main condensate was weighed with an accuracy within 0.1 per cent on carefully calibrated scales.

The nozzle was inserted in series with the 12-in. main condensate test header which delivered the condensate from the condenser under test to the weighing tanks. With respect to the piping, the nozzle was located in as favorable a situation as will be found in the field. The nozzle was inserted at a point corresponding to about 110 ft of approach piping (which would tend to minimize pulsation) and the nozzle itself was preceded by 14 ft of straight piping of uniform size. The downstream side of the nozzle consisted of $7\frac{1}{4}$ ft before the first obstruction was reached.

The pressure differential across the nozzle was read from two mercurial single-column cistern-type manometers. Both manometers were connected to the same upstream static-pressure tap located in a horizontal plane 12 in. before the entrance edge of the nozzle. The low-pressure side of one manometer was connected through an internal port to the piezometer ring in the throat of the nozzle, while the low-pressure side of the other manometer was connected to a downstream static tap in the pipe at a transverse plane passing through the discharge end of the nozzle. This double arrangement was furnished to provide check readings of the nozzle differential. Each manometer was read by a separate observer. It was found that there was practically no difference between the two sets of readings.

The results of the writer's test are shown in Fig. 4 of this discussion. The horizontal line at $C = 0.994$ is that portion of the author's blanket curve from his Fig. 8 that applies to the range of Reynolds' number used by the writer. The plotted points indicate the spread of the nozzle coefficient, C, as determined under field conditions. The estimated maximum error on this field test is about 2.4 per cent and the estimated probable error is 0.5 per cent. It should be noted, however, that these field results

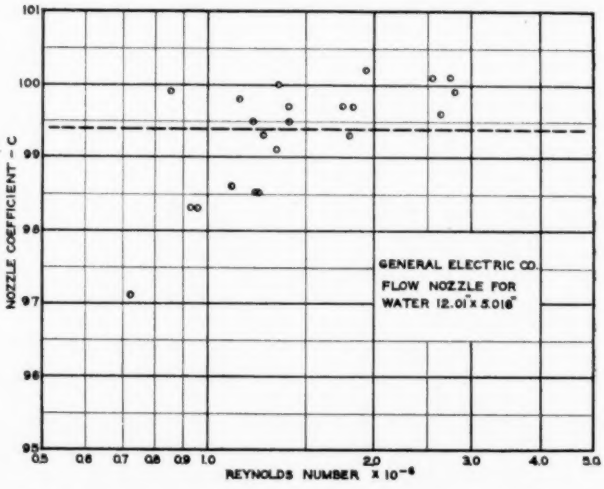


Fig. 4 Performance of General Electric Company Flow Nozzle During a Field Test

(Each point represents a $1\frac{1}{2}$-hr run during which the main condensate from a steam condenser was also very accurately weighed after passing through the nozzle. The dashed line at $C = 0.994$ is that portion of the curve in Mr. Buckland's Fig. 8, corresponding to the above field data. The flow nozzle tested in this field test was the same as the 12.01-in. × 5.016-in. nozzle tested by the author.)

were obtained under unusually favorable circumstances. It is probably true that in the average run of cases where the nozzle could be used, results would not be so reliable as in this case.

Sanford A. Moss[8] and W. W. Johnson.[9] There are of course a good many ways in which flow may be measured with laboratory precision, and the paper by Mr. Buckland is a good example of one of them. It is to be noted that the work was carried out with great care and with a test set-up especially made for the flow measurement, and with all details arranged so that certainty of accuracy was the primary consideration. This puts the work in a wholly different territory from flow measurement made by the usual commercial flow meter which must be suitable for permanent, simple installation and maintenance in a commercial pipe line with a small pressure drop, and with an instrument which gives direct reading of flow. None of these considerations can be allowed to influence the precise flow measurement with certainty of accuracy, which is the author's purpose.

Of course, future research may show that some of the details used by the author might be altered to give as nearly as possible,

[7] Research Bureau, Brooklyn Edison Company, Brooklyn, N. Y. Assoc-Mem. A.S.M.E.

[8] Research Engineer, General Electric Company, West Lynn, Mass. Mem. A.S.M.E.

[9] Mechanical Engineer, General Electric Company, West Lynn, Mass. Mem. A.S.M.E.

measurements corresponding to theoretical flow. For instance, it might be that the throat taps should have a longer parallel portion, such as is shown in the author's Fig. 4 or Fig. 12. However, the close agreement of the author's points shows that his throat-tap measurements must be very good. His Fig. 9 shows a very close agreement between flow coefficients for different values of m, the ratio of nozzle to pipe area. This seems to indicate that the theoretical allowance for velocity of approach takes full account of the effect of pipe diameter, with the possible exception of the $^1/_8$ per cent mentioned which is much less than the errors of observation. The spread of the curves in Fig. 11 of Mr. Buckland's paper is evidence in the discussion as to whether or not Reynolds' number is a proper criterion for abscissas for flow coefficients for different conditions, as was discussed in the Moss-Johnson paper referred to by the author. We thought that our tests with different temperatures and pressures of steam were brought together better by using differential pressure divided by absolute initial pressure as abscissas, and it has also been proposed to use head as abscissas. The droop of the Moss-Johnson curve, No. 1 in Fig. 11, may be due to some such considerations. It must, of course, be admitted that it may also be due to observational irregularities because of the very small flow and small differential pressures at the beginning of the curve. Some of the data given in the Moss-Johnson paper seem to indicate that there was a definite difference in the coefficient with throat taps and with pipe taps, such as for the author's Fig. 13 and that with pipe taps the discharge coefficients are lower. This being the case, the flow coefficients for Fig. 13 seem very high.

E. D. Dickinson.[10] Mr. Buckland's paper is a confirmation of the principles behind the growing opinion that precise measurements of fluids can be obtained by the use of properly proportioned nozzles. The proportions of the nozzle itself constitute but one factor contributing to the accuracy of the results. It is essential that certain precautions be taken. When these precautions are taken, tests can be reproduced with absolute fidelity and the results can be depended upon to be as accurate as laboratory tests.

I do not hold a brief for any particular method of measuring flow by nozzles. However, I have relied upon the flow nozzle for obtaining accurate measurements of steam flow for a period of years and the results have confirmed my contention that precise measurements of flow can be obtained with greater reliability and at less cost by the use of a nozzle similar to that described by Mr. Buckland than by any other recognized method. We have run a great many tests, both of research nature and on commercial machines, where precise results were obtained and valuable information secured that could not have been possible had we not had at our disposal a calibrated flow nozzle similar to Mr. B. O. Buckland's and as described by Dr. S. A. Moss and Mr. W. W. Johnson in their paper[4] presented at the A.S.M.E. Annual Meeting in December, 1932.

Author's Closure

Ronald B. Smith states that Stach's tests (on the coefficients of the V.D.I. Normdüse discharging into the atmosphere) show less than 0.3 per cent difference between measurements of the I.S.A. nozzle when discharging freely and when operating in a pipe with the usual corner taps. He summarizes the situation by saying that the result of Stach's work is to indicate that the pressure measurement in the downstream corner is equivalent to the pressure for discharge into an infinite chamber.

I cannot agree with Mr. Smith's interpretation of Stach's data. The data show clearly that the coefficient of the Normdüse is the same when discharging into the atmosphere as it is when installed in a pipe when the downstream pressure is measured at the point of *minimum pressure on the pipe wall*. In order to clear up this point I shall reproduce Stach's calibration results together with Witte's measurements of pressure difference between the point of minimum pressure on the pipe wall and the downstream corner tap. Table 1 of this discussion shows flow coefficients and pressure differences taken from the papers by Witte and Stach.

TABLE 1 FLOW COEFFICIENTS AND PRESSURE DIFFERENCES ON THE V.D.I. NOZZLE AS GIVEN BY WITTE AND STACH

m	Press. diff. Δ (Witte)	α	α_a (Stach)	$\alpha_m = \alpha - \Delta/2$
0.10	0.010	0.989	0.984	0.984
0.20	0.016	0.999	0.993	0.991
0.30	0.020	1.016	1.010	1.006
0.40	0.023	1.045	1.036	1.034
0.50	0.026	1.096	1.078	1.083

m is the ratio of nozzle area to pipe area.
Δ is the difference between the downstream corner-tap pressure and the minimum pressure on the pipe wall expressed as a fraction of the difference between the up- and downstream corner-tap pressures.
α is the flow coefficient of the Normdüse in a pipe, using corner taps.
α_a is the flow coefficient when discharging into atmosphere, using upstream corner tap and the atmosphere.
α_m is the flow coefficient using the upstream corner tap and the minimum pressure on the pipe wall. It is obtained by subtracting $\Delta/2$ from α, since the fraction Δ is nearly twice as large as the difference produced in the coefficient by using the minimum pressure on the pipe wall instead of the corner-tap pressure.

A comparison of α_a and α_m shows them to be about equal, much more closely so than are α and α_a. I, therefore, conclude that the coefficient of the Normdüse is the same when discharging into the atmosphere as it is when installed in a pipe with the downstream pressure measured at the point of minimum pressure on the pipe wall.

Mr. Smith states that the relative magnitudes in Fig. 14 of the paper are misleading. As defined in the paper this figure is a comparison of the coefficient curve of the G. E. nozzle with the coefficient curve of the V.D.I. Normdüse. The V.D.I. nozzle coefficients have been corrected to what they would be if the upstream pressure had been measured one pipe diameter upstream from the nozzle face and the downstream pressure at the point of minimum pressure on the pipe wall. As Mr. Smith points out, Witte[11] compared the two nozzles by calibrating them both with corner pressure taps. Witte finds that under these conditions the coefficient of the G. E. nozzle is $^1/_8$ per cent lower than that of the V.D.I. nozzles. It is true, that in the light of these recent tests by Witte, Fig. 14 of the paper shows too large a difference between the two coefficients in the range where the coefficients are independent of Reynolds' number. It is also true that in accordance with the Bureau of Standards tests the magnitude of this difference shown in Fig. 14 is correct. However, whatever the correct relation between the coefficients may be in the range where they are independent of Reynolds' number, the same variation of the coefficients with Reynolds' number is given by Witte as is shown in Fig. 14. The coefficient of the V.D.I. nozzle rises much more abruptly with increasing Reynolds' number than that of the G.E. nozzles between Reynolds' numbers of 10^4 and 10^5.

It should be of interest to note in this connection that the usual conditions met in testing a 10,000-kw turbine will require the use of a flow nozzle about $1^1/_4$ in. in diameter in a 4 in. pipe, and the operation of the nozzle in a range of Reynolds' numbers from 10^4 to 10^5. This is very close to $m = 0.09$ and right in the range of the rapid rise of coefficient of the Normdüse. With Reynolds' numbers higher than this range, the Normdüse is as useful for flow measurements as any other carefully calibrated device but

[10] Designing Engineer, Turbine Engineering Department, General Electric Company, West Lynn, Mass. Mem. A.S.M.E.

[11] "Neuere Mengenstrommessungen zur Normung von Düsen und Blenden," by R. Witte, *Forschung auf dem Gebiete des Ingenieurwesens*, September-October, 1934.

at this point of rapid rise in coefficient I would rather use a different nozzle.

I want to thank Mr. Smith for his data on the pressure distribution along the pipe wall on the Moss-Johnson nozzle.

I would like to thank Mr. Sprenkle for his data on the 3 × 1.8-in. nozzle. It would be interesting to have the dimensions showing the location of Mr. Sprenkle's pressure taps so that his data could be more readily compared with other available data.

I want to thank Mr. Pigott for his suggestion regarding a method of plotting flow coefficients.

In answer to Mr. Cooper's question regarding the use of the nozzles in the field, I have made seven turbine-performance tests in which the flow was measured only by means of flow nozzles. In these tests none of the Btu-rate or water-rate points scattered from an average curve more than ± $^1/_2$ per cent. Each of these tests consists of approximately 15 or more points. These results obtained by the use of flow nozzles are much more satisfactory than the example shown by Mr. Cooper.

In the case he cites (The performance tests of the turbine units Nos. 7 and 8 of the Brooklyn Edison Company at Hudson Avenue), the main object was the measurement of the turbine and condenser performance by means of weigh tanks. The flow nozzle was a secondary consideration and was, therefore, neglected. It was not until the tests of the second unit that readings on the manometer were taken often enough. The points of the test on the second unit (No. 8) are in my opinion the only acceptable ones. In fact I would rather use only the last 6 of these. During these last 6 points the manometers were read every $^1/_2$ min. If Mr. Cooper will consider only the points taken on the second unit (No. 8) the results will check our calibration curve much closer. These points are given in Table 2 of this discussion.

TABLE 2 FLOW COEFFICIENTS OF 12 IN. × 5 IN. NOZZLE AS DETERMINED BY WEIGH TANKS DURING A TURBINE TEST

Log_{10} of Reynolds' number	Coefficient of discharge, C
6.125	0.991
6.145	0.995
5.931	0.999
6.056	0.998
6.041	0.986
6.261	0.997
6.286	1.002
6.441	1.001
6.405	1.001
6.124	1.000
6.145	0.997

TABLE 3 CALIBRATION RESULTS OF A 2.8812-IN. × 5.762-IN. FLOW NOZZLE (WATER TEMPERATURE, 69 F)

(Data by Prof. W. S. Pardoe)

Coefficient	Reynolds' number
0.9540	31310
0.9665	42490
0.9745	67090
0.9785	89450
0.9785	109580
0.9868	153190
0.9869	183380
0.9900	216920
0.9905	249230
0.9920	248230
0.9930	284010
0.9920	323150
0.9910	355570
0.9920	391350
0.9945	485280
0.9950	588150
0.9950	686540
0.9950	726800
0.9945	907940
0.9955	590300

In the use of flow nozzles for precise testing it is absolutely essential that the fluctuations in flow be slow enough for the manometer to follow the pressure changes and also for the observers to follow the manometer. I have not yet found a plant where these conditions could not be satisfied by some extra manipulation, as for example, either operating the pumps at different suction levels or using hand control of the flow.

It is true that, with respect to the location, the installation of the nozzle during the tests referred to by Mr. Cooper was entirely satisfactory but the conditions of flow during these tests were not. The flow fluctuated rather widely and rapidly.

Dr. Moss will be interested in the calibration results given in Table 3 on a new nozzle in which the pressure taps are brought straight out from the throat.

Mr. Dickinson's statement that the fluid nozzle is a practical device for testing turbines confirms my own experience.

Since writing this paper, I have obtained a calibration on a 2.8812-in. × 5.762-in. nozzle. This nozzle was welded into a 9-ft length of seamless steel tubing. It was made with four separate throat taps and a flat exit face very much like the nozzle shown in Fig. 12 of the paper. The nozzle and the tube were calibrated together. Table 3 gives the results of the calibration.

Flow Distribution in Forced-Circulation Once-Through Steam Generators[1]

H. J. KERR.[2] The authors' paper confirms and extends the information presented by the writer in his paper,[3] "Once-Through Series Boiler for 1500 to 5000 Lb Pressure," in which the effect of inlet feedwater and outlet steam temperature on the instability of circuits was shown in diagrams. The value of resistances in stabilizing the flow and to compensate for unequal heat absorption in the different circuits was pointed out.

With reference to the authors' paper, the freedom from deposits in the test apparatus above 2500 lb pressure, irrespective of steam temperature, is worthy of note. Does this mean that above this pressure, steam to turbines will not need to show a purity represented by a resistance of 1,000,000 ohms to permit of continuous operation?

In determining the friction factor for a given Reynolds' number, the authors have used a straight-line projection on logarithmic coordinates of the known viscosity values of water up to 320 F. Probably this is a fair approximation though it does not agree, above 500 F, with Hevesy's values as given in Landolt and Bornstein tables. There may be some question as to the special point shown in Fig. 4 of the paper being discussed, checking in the case of steam as it apparently does with water.

Dealing with the question of stability in the boiler proper, the authors, in Fig. 8, show the effect of inlet-water temperature. These curves can be considered as a magnification of a small section of the curves in Fig. 7 of the writer's paper[3] previously referred to. I believe it would be clearer if the curves were extended over a greater temperature range, thus showing the reversal of direction which takes place, as it is, of course, impossible for the 200-F water curve to continue indefinitely in the direction shown, although it will continue in this direction until the tube is burned.

Fig. 9 of the paper under discussion shows the value of resistances in stabilizing flow. I have found, however, after talking to several engineers, that the significance of the dropping pressure with increasing enthalpy is not well understood. Per-

---

[1] Published as paper FSP-56-16 by H. L. Solberg, G. A. Hawkins, and A. A. Potter, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

[3] "Once-Through Series Boiler for 1500 to 5000 Lb Pressure," by H. J. Kerr, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-1a.

haps the authors would be willing to explain this further in their closure.

Referring again to the authors' paper, Figs. 11 and 12 show quite clearly the effect of equalizers on stability in various locations. In this connection, Fig. 12, circuit *C*, shows the effect of unequal heat application to any superheater. This figure indicates that a change in heat application, to different circuits, of the order of plus or minus 5 per cent from the average, produces a change in enthalpy in the leaving steam of some 40 Btu or a change in temperature of some 70 deg. This condition prevails with any superheater and necessitates careful design of the unit to prevent this variation from reaching serious proportions. Fig. 14 shows clearly that properly designed series-type boilers do not need to be operated at the critical pressure.

In general, the authors' conclusions cannot be disputed, provided that in (1) a limit is placed on inlet feedwater and outlet steam temperature; in (2) the added work on the feed pump due to resistance will not be serious in a properly designed boiler; in (3) and (5) the tubes are horizontal or proceed upward. Perhaps (4) should read more definitely, "Economizer and superheater circuits are definitely stable."

I agree with the authors that a unit designed with a separator drum at the end of the evaporating zone simplifies the problem of once-through series boilers. This type can be considered as the end point of the steaming-economizer units, many of which are now in service. We have built and operated such units to the end point.

There are, however, certain advantages in the straight-through unit without drums for high pressures. The problems involved are stabilization of flow, feedwater, and control.

R. C. H. Heck.[4] The authors have shown excellent judgment in places where a choice of procedure had to be made, as in proportioning between liquid and vapor for the mixed current, Fig. 6. It is interesting, as well as technically valuable, to have a theoretical discussion give so clear and consistent a reason for instability already observed in the vaporizing section of the fluid path.

The following questions concerning the flow of liquid water came up while reading the paper: (1) What is the length of the tube section used for the experimental determination of Table 1? (2) Does the survey of data on the viscosity of water up to 320 F agree in results with the tabulation in International Critical Tables, converted to fahrenheit base by McAdams in his Heat Transmission? (3) May we have a formula or plot of the extrapolation beyond 320 F?

Authors' Closure

Referring to the questions raised by Professor Heck, the dimensions of the test section which was used for measuring friction factors are shown in Figs. 1 and 2, of the paper. The distance between piezometer connections is four feet.

The data on the viscosity of water at temperatures up to 320 F, as given in the International Critical Tables, have been converted to fahrenheit base by McAdams in his book, "Heat Transmission," and these were used by the authors in calculating the results which are presented in the paper. The data are plotted in the accompanying Fig. 1 as a solid line with the extrapolated curve indicated as a dotted line.

Mr. Kerr refers to the data on the viscosity of water by G. von Hevesy ("Die Beweglichkeit der Ionen, die dem Lösungsmittel Eigen Sind," published in *Zeitschrift für Electrochemie*, Vol. 27, January, 1921). Hevesy states definitely that the values for the viscosity of water were determined experimentally at a maximum temperature of 156 C and that values for the higher temperatures were calculated on the assumption that there is a relationship between viscosity and electrical conductivity. Experiments have failed to establish the validity of this assumption and his data are not given in the International Critical Tables.

Mr. Kerr suggests that the author's Fig. 8 should be extended to higher temperatures. The maximum ordinate of these curves is 1000 F which is the upper limit of the Keenan steam tables. Extension of these curves to higher temperatures would involve either direct extrapolation of the curves or extrapolation of the steam-table data. The authors prefer to confine their calculations and curves to the range of the steam tables.

Mr. Kerr suggests that the significance of the decreasing pressure drop with increasing enthalpy, as shown in Fig. 9, should be explained in more detail. These curves were calculated by assuming 400 F feedwater, 2500 lb per sq in. outlet pressure and constant heat input. Pressure drops were calculated for flow rates sufficient to give final steam temperatures of 670 to 1000 F. The left curve which is nearly vertical, indicates that high flow rates, resulting in low final steam temperatures, will produce practically the same pressure drop as low flow rates and high final steam temperatures. In other words, several parallel circuits which have equal pressure drops and equal heat absorption may deliver steam at widely different flow rates and final steam temperatures. Such circuits are unstable and overheated tubes may be expected. The addition of an inlet resistance which is supplied with water at a constant temperature and therefore has a pressure drop which is approximately proportional to the square of the flow rate, has a stabilizing effect on circuits in which the pressure drop is nearly independent of flow. The pressure drop through the combined circuit varies as some power of the flow rate which approaches two as the value of the inlet resistance is increased, as shown in Fig. 9. The farther these curves depart from the vertical, the greater will be the change in pressure drop

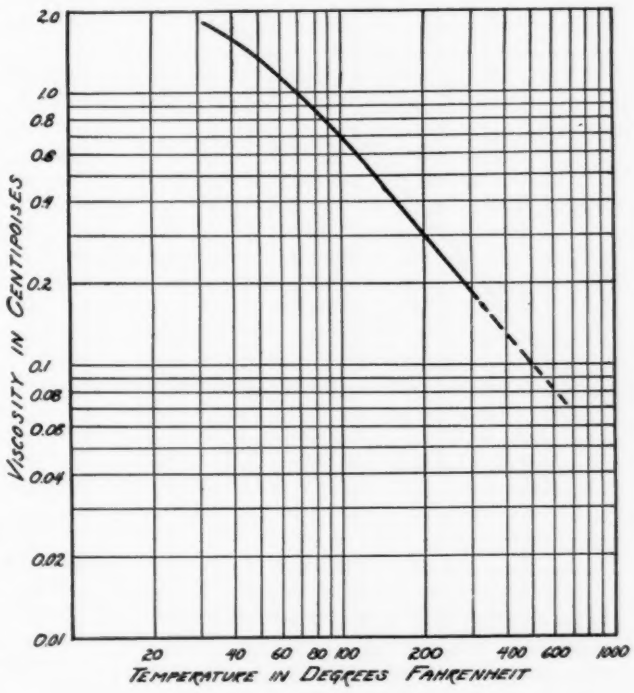


Fig. 1 Viscosity of Water
(Solid curve is based on International Critical Tables. Dotted curve represents extrapolated values.)

[4] Professor of Mechanical Engineering, Rutgers University, New Brunswick, N. J. Mem. A.S.M.E.

required to alter the flow and the final steam temperature, and the greater will be the stability of the circuit.

It is a source of gratification to the authors that their conclusions are substantiated by the extensive experience of Mr. Kerr.

Current Practice in Surface Broaching[1]

MILLARD ROMAINE.[2] In view of the advantages of surface broaching as shown by the examples brought out in Mr. Geschelin's paper a question naturally arises as to why this method has not been more universally used. Before attempting to answer that question it might be well to analyze the various advantages of surface broaching.

The most important advantage is the extreme rapidity of the operation, as compared with other methods, resulting in lower labor cost, etc. The very low time per piece usually obtainable by surface broaching is traceable to two main reasons.

The first of these is that a work piece can usually be chucked more quickly for broaching than it can be chucked for milling. This is due to the fact that the forces set up by broaching are in two very definite directions, one in line with and one normal to the path of the broach. The holding and clamping of the piece is, therefore, greatly simplified. In many cases no clamping at all is necessary. For example, the Houde shock-absorber part shown in Fig. 15 of Mr. Geschelin's paper is not clamped at all. Hence on many broaching operations the handling time of the work is greatly reduced over that necessary for other methods. The second reason for the low unit time for surface broaching is due to the rate of speed at which the cutter passes over the work. This rate is from 12 to 25 or more times as fast as the rate used in milling. In both cases the distance traveled is equal to the length of the cut plus the distance across the cutter. This latter is considerably greater for a broach than for a milling cutter but the speed more than makes up for the difference, with the result that the cutting time for the broaching method is considerably faster than for milling.

A second important advantage of the broaching method lies in the low tool cost usually obtained. In comparison with milling, the first cost of the broaching tools is rather high. Where face mills are used for milling operations, the difference in original cost of cutting tools is rather obvious; where complicated cutter gangs are used on milling it is not quite so obvious, but even so, the first cost of the milling-cutter set-up is rather less than that of the broach. The milling cutters themselves on the arbor may be compared to the broach inserts and, generally speaking, the cost of the inserts is more than the cost of the milling cutters. However, the cutter arbor and the spacing collars for the milling-cutter gang, which perform the same functions of supporting and properly locating the cutting tools proper as the subplates and inserts on the broaching set-up, cost considerably less. As a result the first cost of tooling equipment on a broaching machine is generally more expensive than the first cost on the milling machine. This is, of course, to some extent offset by the fact that the holding fixtures are usually cheaper on the broach. However, when it comes to comparing the total actual cost of the tools when used on high production, the broaching tools show a much lower total cost. This is due, of course, to the extremely long life between grinds obtainable on the broach. The broaching cutters usually run for days while the milling gang runs for hours. Very interesting figures are given by E. S. Chapman, President of the Amplex Mfg. Company, in his paper presented at the Production Meeting of the Society of Automotive Engineers on October 11, 1934. In two cases where direct comparisons could be made, the broaching-tool cost was one-third of the milling-cutter cost in one case and one-ninth of the milling-cutter cost in the other case. All of the broaching-tool costs mentioned by Mr. Chapman were rather low in cost per piece, varying from $0.0036 in the highest case down to as low as $0.00057 in the other case.

The long cutter life resulting in these low tool costs is due to a number of factors, among which may be mentioned the rigidity of the average set-up, and the fact that the tooth on the broach, taking a chip of constant thickness, does not have to build up that chip from zero thickness to a maximum, as is the case with the ordinary milling cut. Since the broaching cutter moves in a straight line, it is not necessary to provide a clearance angle that will prevent dragging on the heel of the cutting land. This allows the use of lower clearance and rake angles. The result is a stronger cutting edge and one which is also capable of dissipating heat a little more rapidly. Another reason for long cutter life lies in the fact that the broaching cutter operates at about half or less the cutting speed of the milling cutter. When cutting materials require the use of a coolant, it is very easy to apply this coolant to a broaching cutter. In the case of a milling cutter, centrifugal action makes it difficult to get the coolant where it will do the most good. Another and very important reason for the long life of broaching tools is the fact that the roughing teeth remove the metal and the finishing teeth do the finishing. It is quite frequently necessary to sharpen the milling cutter which is still capable of removing metal, simply because the finish produced is not satisfactory, the teeth having been damaged by the roughing. On the other hand, on the broaching cutter the finishing teeth remove a minimum of stock, do not encounter any scale, generally speaking, and should naturally last much longer.

Another advantage of the broaching method lies in the facts that the machine employed is of simple construction, that its fixtures have a low first cost, and that the total investment required for a given production is generally lower with broaching equipment than it is with milling equipment.

A fourth advantage is effected as a result of the simplicity of the broaching equipment, inasmuch as this makes for low maintenance costs.

When we consider the foregoing advantages we are surprised, therefore, to find that this method has been restricted almost entirely to the automotive and other high-production industries.

An analysis of some of the operations quoted will show, perhaps, the reason for this fact. Published data reveal the fact that stock removed by the broaching operation apparently does not exceed a maximum of $1/4$ in. at any time. Another interesting observation which may be made is that in almost every case where broaching equipment is used, the stock is removed from a forging or from a surface which has been machined previously and that in very few cases is stock removed from castings where much variance in stock removal is to be expected. In other words it is apparent that the broaching method has been applied to parts where, first, comparatively little stock is removed ($1/4$ in. or less but usually around $1/8$ in.) and, second, where this stock removal is held within fairly close limits. Now it is only in high-production industries that rough parts come through which require an average stock removal of $1/8$ in. or less; industries with comparatively low-production volumes not being able to afford either forging dies or patterns and foundry equipment required to hold a part to set limits. As a result their parts come through with as much as $1/2$ in. stock. The removal of this large amount

---

[1] Published as paper MSP-56-1, by Joseph Geschelin, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Sales Engineer, The Cincinnati Milling Machine Company, Cincinnati, Ohio. Assoc-Mem. A.S.M.E.

of stock, plus the fact that it may vary, has two main effects upon the economic use of the broaching method.

The first of these is evidenced in the fact that removal of the additional stock means increasing the stroke of the broaching machine, thus necessitating a larger machine for this purpose. The length of travel of the cutter past the work either in the case of a milling machine or a broaching machine is equal to the length of the work piece plus the distance across the tool. In the case of a milling machine this distance is constant regardless of how much stock there is to be taken off; if excess stock is encountered, the operator of the milling machine can get through the cut with very little difficulty by slowing down his feed rate. On a broaching operation, however, if a piece of work which normally has $^1/_8$-in. stock on it and requires a broach 30 in. long, came through with $^1/_4$ in. of stock to be removed, a broach 60 in. long should be provided which, of course, almost doubles the length of the travel required to produce the piece. This indicates that the broaching force remains the same and the speed of the broach travel remains the same, but the distance of the travel has to be much more. This, therefore, means a much larger machine because of the greatly increased stroke.

The second main effect that the removal of large amounts of stock has on the use of the broaching method is one concerning the time for passing each piece over the machine, the time being controlled by the broaching tool which must be designed to take care of the maximum amount of stock. In the case of a milling operation the removal of additional stock has no particular effect on the cutter, the cut is simply taken at a slower feed rate, with no bad effects, as a rule, on the cutter itself. However, in the case of a broach operation, if the broaching tool is designed for removing a maximum of $^1/_8$-in. stock and a piece is encountered having $^1/_4$ in. stock, that extra $^1/_8$ in. must be taken off by the first tooth of the broach which, being practically impossible, involves considerable hazard. Extra unlooked for amounts of stock, therefore, may, and do, quite frequently cause breakage of the tools and the work. Therefore, the only safe way to design broaching equipment is to design the broaching tools for the maximum amount of stock which is to be expected and to provide a machine with sufficient stroke to take care of this maximum amount of stock. This means, of course, that the time for each piece passed over the machine is the same for the piece that has the maximum amount of stock. On the other hand, on the milling machine most of the pieces are produced at the high rate of production and only the occasional piece with the excess stock is produced at the lower feed rate.

These economic considerations are exemplified by the sectional broach shown in Fig. 3 of Mr. Geschelin's paper, wherein the broach is designed for a 48-in. stroke, the broaching cut being about 42 in. long. This length is required because of the necessity of sometimes removing as much as $^7/_{32}$ in. stock which in turn is necessary because this first broaching operation is used as a corrective operation for the weight and balance of connecting rod being produced. If it were possible to hold all of the connecting rods to a stock removal of only $^1/_8$ in., this job could be done at a considerably faster rate or a much smaller machine could be used. As the operation is at present set up, about 90 per cent of the connecting rods are machined only by the last few sections of the broaching tool.

It would appear, therefore, that the increase in use of surface-broaching machines is going to depend more or less upon the development in the low-production industries of forging and foundry practice to a point where the amount of stock to be removed and the variation in the amount of stock from piece to piece is held to fairly close limits.

Another thing that militates against the use of a broaching method in low-production industries is the rather high cost of the tooling equipment. The broach holders, subplates (particularly if they are provided with wedge adjustment), and inserts cost considerably more than arbor gangs and the amortization of these extra costs over the expected quantity of pieces to be produced usually makes a broaching operation for a limited number of pieces, when analyzed to include this factor, considerably greater than the milling method; although the actual labor cost to perform the operation is much lower by broaching. Another point to be considered is that even in high production there are many pieces the shape of which preclude the use of the broach. In order to broach a surface on any part, the surface must lie in such a position on the work that a broach of considerable length can be passed across it. Sometimes it is not possible to meet this condition which eliminates the broach possibility.

In conclusion we would say that the determination of whether a given operation should be performed by broaching or milling can only be made after a careful analysis of both methods taking all factors into consideration, both from a mechanical standpoint and from the standpoint of the economic factors involved, which analysis, of course, requires a rather complete knowledge of both methods.

ROBERT T. KENT.[3] The question has been raised as to the application of surface broaching to other than mass-production fields. The tenor of the discussion seems to incline to the idea that the cost of equipment will bar surface broaching unless production is in sufficient quantity to keep the equipment in operation for a major portion of the time. The writer is of the opinion that quantity of production has little or nothing to do with the question, except as one factor that must be considered along with others in arriving at the answer. The question is answered by determining what method of production gives the lowest unit cost. In ascertaining the unit cost, everything that enters into cost, such as operating cost, taxes, interest, repairs, maintenance, set-up time per piece, and others, must be considered. When the problem is analyzed in this manner, there can be no question as to which method is best for any particular set of conditions. This method applies to all problems of production, as well as to surface broaching versus milling.

The writer is fortified in this opinion by a case within his own experience. A machine that was distinctly mass-production equipment was offered for a certain class of work. The output of this machine was such that its production in three to five hours was sufficient to supply the manufacturer's needs for the week. At first it seemed foolish to make an investment in equipment that would stand idle the greater portion of the time, but an analysis on the basis of unit costs showed that the investment was profitable and resulted in a net reduction of the overall cost of the work.

Collapse by Instability of Thin Cylindrical Shells Under External Pressure[1]

G. C. PRIESTER.[2] Equations developed from purely theoretical and ideal conditions are often found to be too cumbersome for practical application. To develop an equation which will meet practical conditions and involve simple calculations to a limited degree of accuracy requires familiarity with the ideal re-

[3] Consulting Engineer, Verona, N. J. Mem. A.S.M.E.

[1] Published as paper APM-56-20, by Dwight F. Windenburg and Charles Trilling, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Professor of Materials of Engineering, University of Minnesota, Minneapolis, Minn.

lationships and a working knowledge of its experimental limitations.

The authors of this paper have given an excellent interpretation of the theoretical formulas involved and have set up an empirical formula which conforms to the former. This is a decided advance in our present theory of the collapse of thin tubes under external pressure.

In all problems of stress analysis it is desirable to have a complete theoretical analysis of the problem and sufficient experimental data to verify the general law. Whenever the experimental data fail to conform to the theoretical formula, either the theory is inadequate, or the methods of testing procedure have not been properly controlled.

The authors have shown why their experimental results do not agree exactly with the theoretical values. They have set up an equation which takes into account the necessary theoretical considerations and which agrees, within practical limits, with experimental data. They have stated the limitations of the equation. The paper includes a detailed explanation of a method of procedure for the solution of a specific problem.

W. Hovgaard.[3] In 1921 I drew the attention of the Bureau of Construction and Repair to the work done in this field by von Mises, von Sanden, and Günther in connection with the design of German submarines during the War. Since then I have been in touch with the Experimental Model Basin regarding this matter, and have corresponded with Mr. Windenburg about it.

The paper gives a review of all the formulas proposed for determining the strength of cylindrical tubes against collapse by instability and shows very clearly the relations between them. Von Mises's formula, [6], is shown by the experiments to be the most accurate.

The authors have, however, gone beyond the results attained by previous workers in this field by constructing a much simpler and yet quite accurate formula, given as [9] in the paper. Following von Mises, the envelope of the n-curves was obtained by differentiating formula [8] and eliminating n. But while von Mises applied this method to a simplified form of formula [8] so that it was valid only near the origin, Mr. Windenburg derived the formula by direct differentiation of formula [8] itself, and then by a skilful approximation simplifying the equation of the envelope. Mr. Windenburg also pointed out an error in von Mises formula, [2]. For the practical engineer formula [9] is of great value because it gives the critical pressure without a knowledge of n, which can be found independently from Equation 1. The experimental verification carried out by the Model Basin gives the engineer an assurance of the soundness of the formula.

It appears, from Table 3, that for short vessels the formula gives a rather high value of the critical pressure and hence of the permissible working pressure.

E. F. Miller.[4] The paper explains the method in which the last draft of Proposed Rules for Construction of Unfired Vessels Subjected to External Pressure was reduced to such a simple usable form. The value of any technical contribution is measured by its usefulness. When complex formulas can be simplified to such an extent by a series of approximations without appreciably affecting their accuracy, it represents a real contribution for the use of the design engineer.

The proposed rules for construction of external-pressure vessels is limited in its scope to the simpler forms of vessels. The usefulness of the code would therefore be considerably extended by the inclusion of the major part of the present paper as an appendix to the code for use in designs that fall outside the scope of the proposed code.

W. P. Roop.[5] This paper effectively summarizes a large amount of valuable work. In connection with its use in design, the following questions occur to me:

1 To what extent does existence of hard surface layers in the thin sheets used in the tests affect the experimental results?

2 Is there any theoretical or experimental indication of deflections exceeding those which would occur in simple compression, due to elastic buckling below the yield point? The rigidity of other thin-metal structures with which I have recently been concerned is materially affected by such action.

3 Is it possible to obtain a useful approximation describing the behavior of a flat plate under hydrostatic and coplanar load by considering D to approach an infinite value?

4 Formula [9] is understood to represent a well substantiated result of experimental and theoretical analysis. Disregarding all questions of loss by corrosion, excess weight due to the requirements of convenience, and inadvertent departure from specifications, what margin is considered appropriate to cover the spread between ultimate and working pressures? The sources of uncertainty I have in mind are error in the formula, unhomogeneity of material, and risk of injury causing excessive out-of-roundness.

C. O. Rhys.[6] This paper treats the three classes of tubes considered in a convenient, thorough and clear manner. The way in which the different formulas agree with each other and with the results of experiments is very conclusive from the viewpoint of correctness and applicability.

The one exception to the above is the large difference between formulas [A] and [B]. It is interesting to note that while [A] is derived from theoretical considerations for a radial load only, [B] is the result of experiments which appear to have been made on tubes subjected to both radial and axial loads. (See Trans. A.S.M.E., Vol. 53, paper APM-53-17b, p. 232.) It might be expected that this difference in loading would affect the resulting formulas, but an inspection of Table 1 of the present paper shows that differences in collapsing pressure for the two kinds of loadings are only important for very short tubes and almost disappear when $L/D=2$. From the reference given above, it seems probable that [B] is based on data from a number of tubes for which $L/D = 8$ or more. It can be seen from Fig. 1 that the critical length, which would correspond with $n = 2$, is in the region of $L/D = 8$ to 10. All this appears to show that the difference between [A] and [B] is in no way dependent on the difference in loading, and emphasizes the importance of departures from a geometrically perfect form.

Formula [9] is derived in a very interesting way from formula [8]. It automatically does away with the necessity for finding the proper value of n in formula [8] or in any of the other formulas in which n appears. That it reduces to zero instead of to [A] when L becomes infinite, seems quite in order. As already pointed out formula [A] is for radial loads only while formula [9] is for both radial and axial loads. For the latter case, the load required to bring about failure would become almost nothing when the tube became very long; in fact, it would approach a very long column under a slight buckling load.

As shown by Equation [28], for the proper set of coordinates, formula [9] becomes identical with Euler's equation for long columns. This effect would not occur in the case of formula [A], which is independent of length.

To put the matter in another way, formula [A] applies to tubes

[3] Brooklyn, N. Y.

[4] Mechanical Engineer, Westinghouse Electric & Manufacturing Company, South Philadelphia Pa., Jun. A.S.M.E.

[5] Lieutenant Commander (CC), U.S.N.

[6] Standard Oil Development Co., Elizabeth, N. J.

beyond the critical length, and loaded with radial pressure only. Once the critical length (or length beyond which the collapsing section receives no support from the ends) is exceeded, the collapsing pressure is independent of length.

Formula [9] and Equation [28] apply to tubes loaded with both radial and axial pressure. When the tube gets very long, it appears reasonable to suppose that the axial load would cause failure by buckling, as in a column, before the radial load would cause failure by the kind of collapse considered in formula [A].

The mathematical work in the derivation of formula [9] and in the determination of the number of lobes has been gone over and found to be correct.

F. V. Hartman[7] and R. G. Sturm.[8] The several formulas given by the authors for the collapsing pressure of circular thin-walled tubes whose lengths are shorter than the critical length are all based on the ideal type of end constraint, namely, simply supported ends which "tend merely to maintain the circularity of the tube without restricting the slope of the tube walls."

There is some recognition given, to the fact that the ends of the walls may be fixed or restricted against the change in slope, but it is simply stated that such restraint makes for added safety. For certain proportions of cylinders the effect of fixity of the ends may amount to several times as much as the collapsing pressure of simply supported cylinders. Since in many cases economy demands that excessive unknown factors of safety be eliminated wherever possible, it seems that more consideration should be given to the effect of fixation on the strength of cylinders.

The authors give 36 tests in confirmation of their final choice of a formula to be used in design of pressure vessels. Unfortunately, however, there has been no mention made of what the conditions of the ends of the test specimens were. Such a comparison may lead to the same difficulties as would be encountered in tests of columns whose end restraint is unknown. It should be appreciated that the way in which end fixity affects the strength of tubes is not the same as the way that end fixity affects the strength of columns, but for short tubes end fixity may be as important as it is for columns. Certainly one would not test a number of columns to compare with Euler's theoretical column curve for columns with round ends by simply cutting columns to different lengths and testing them in a machine where the end conditions are unknown.

From a study of reference 3 at the end of the paper, it is noted that some of the cylinders were relatively long with a number of intermediate wire stiffening rings, while others seemed to be unstiffened tubes. In this reference, a photograph of a tube 16 in. in diameter and 40 in. long with inside wire-frame stiffeners is shown. It is evident from the photograph that the shell tipped at the stiffener which seems to indicate that this form of stiffener offers very little restraint to the shell. It would be valuable to know if all of the specimens were supported in this manner.

In the photograph just referred to, the lengths of lobes 1 and 2 appear to be longer than some of the other lobes. Since the lobe formation did not extend entirely around the shell it would be of interest if the authors would explain the method used in determining the number of lobes found by experiment.

The out-of-roundness, while not emphasized in the present paper, is indicated in Table 3. It is interesting to note that for model no. 63 which is the case of maximum out-of-roundness relative to thickness, the actual collapsing pressure by experiment was 43 per cent greater than the critical pressure for an ideal tube computed by the recommended formula. For model no. 66, which has the least out-of-roundness relative to the thickness, the actual experimental result was only 84 per cent of that computed by formula [9]. It would seem likely that conditions other than out-of-roundness are responsible for the variations noted. Out-of-roundness expressed in relation to thickness seems to be misleading. On the basis of thickness, model no. 63 is about 16 times more out-of-round than model no. 66, but on basis of diameter, the former is only about 6 times more out-of-round than the latter.

From Table 3, it may be noted that there is a general tendency for the collapsing pressure determined by experiment to be greater than that computed by formula [9] in the ranges where the collapsing pressure is small. In this range one usually expects to find the best agreement between theory and experiment. On the other hand, the experimental values are definitely less than the computed values in the higher ranges of collapsing pressure. Since some of these discrepancies are in the neighborhood of 40 per cent, it seems that an explanation should be sought. Perhaps the conditions of end restraint may be the answer.

D. B. Wesstrom.[9] The authors have presented in a concise, logical, and convenient form a discussion of the more important formulas relating to external-pressure vessels subject to collapse by instability. The experimental results shown in Table 3 and Fig. 2 are of immense value in demonstrating the soundness of applying these rational formulas to practical problems.

The impression is given in the paper that formula [6] for tubes under both radial and axial pressure is on the side of safety in all cases, being slightly oversafe for tubes with radial pressure only, as represented by formula [1]. This is not strictly true for pressure-vessel design. Vacuum towers generally have internal equipment of considerable weight and sometimes have a large concentrated load of auxiliary equipment placed on the top head. In addition there are wind stresses which, while they vary as he distance from the neutral axis of the tower, may be considered fairly uniform over at least one or two lobe lengths of the circumference. The same may be said of deadweight stresses where vessels are supported horizontally.

The longitudinal stresses due to deadweight and wind load may exceed the simple stress due to axial pressure, and in extreme cases may be several times the simple pressure stress. Thus for practical purposes formula [6] really represents only the average case, as does formula [9] likewise, and this justifies the use of a fairly high factor of safety in any general code for the design of such vessels.

In discussing the ideal end conditions for tubes shorter than the critical length, the authors state that any fixation makes for added safety. It is probable that they have reference only to fixation against rotation. Fixation against radial contraction may have the opposite effect. In a vessel of ideal design, the stiffening rings will contract along with the shell although not to the same extent. If the stiffening rings were made heavier, the difference in radial contraction would be increased, and higher bending stresses would be set up in the shell plate at and near the points of support. It is a question whether they would be sufficiently high to lower the collapsing pressure, but certainly they do not make for added safety.

The writer was interested in the fact that formulas [3] and [8] give values of p one-third higher than formula [A] when L becomes infinite, and endeavored to find the reasons for this. In the case of formula [3] this is brought about by dropping the unity term within the first pair of brackets in formula [2]. When L becomes infinite the only members left within the brackets are

[7] Experimental Job Shop, Aluminum Company of America, New Kensington, Pa. Mem. A.S.M.E.

[8] Research Engineer Physicist, Aluminum Company of America, New Kensington, Pa.

[9] Bloomfield, N. J. Jun. A.S.M.E.

$n^2 - 1$, and as n equals only 2 under these conditions, the unity member compares favorably with n^2 and should be retained. Formula [3] applied to a tube of infinite length would then check formula [A].

A similar situation arises in formula [8] because of too much simplification. Referring to formula [6], this is due partly to replacing $n^2 - 1$ by n^2 in the denominator at the end of the equation, and partly to dropping the second and third terms within the braces. When written out, neglecting powers of ρ higher than the first, these terms become

$$-2\mu_1 n^2 + \mu_2 = -2n^2 - \rho(3 + \mu)n^2 + 1 + \rho(1 + \mu)$$
$$= -2n^2 + 1 - \frac{(3 + \mu)n^2 - 1 - \mu}{n^2\left(\frac{2L}{\pi D}\right)^2 + 1} \quad \text{[a]}$$

The only simplification that might properly be made would be to neglect $(-1 - \mu)$ in the numerator of the third term in formula [a]. Otherwise the three terms should be retained within the braces in formula [6] in developing formula [8]; also the unity term in the aforementioned denominator should be kept. Under these conditions, formula [8] would check formula [A] for tubes of infinite length.

It is noted that formulas [5] and [9] reduce to zero for tubes of infinite length. It is believed impossible to modify them to agree with formula [A] for this extreme case without robbing them of the simplicity which was sought in deriving them. Despite this, the writer believes that formula [9] of the U. S. Experimental Model Basin is an outstanding contribution to the subject, as evidenced by its agreement both with the theoretical exact formula [6] and with experimental results, as shown in Tables 2 and 3, respectively. Formula [9] has the added advantage that a determination of n is unnecessary. The value of n is of little practical interest to the pressure-vessel designer.

The authors make it clear that formula [9], along with the others, is applicable only to tubes shorter than the critical length where collapse will occur by instability. The writer feels this should be emphasized by some suggestions for determining the range of application of formula [9] for any given t/D ratio in a practical problem. Obviously one method is to calculate the collapsing pressure by simple crushing

$$p = 2S(t/D) \quad \text{[b]}$$

where S is the hoop stress at which failure by simple crushing is assumed to take place; also to calculate the collapsing pressure for a tube longer than the critical length, by formula [A] or [B]; and reject all values of p obtained from Equation [9] that do not lie within this range.

Another method, in some respects more useful, is to determine the limiting values of L/D for which formula [9] can be applied. Setting formula [9] equal to formula [b] and solving, the lowest L/D value to be used in formula [9] is found to be

$$\left(\frac{L}{D}\right)_{\min} = 0.45\left(\frac{t}{D}\right)^{\frac{1}{2}}\left[\frac{2.69\,E\,(t/D)}{(1 - \mu^2)^{\frac{3}{4}}\,S} + 1\right] \quad \text{[c]}$$

which for $\mu = 0.3$ becomes

$$\left(\frac{L}{D}\right)_{\min} = 0.45\left(\frac{t}{D}\right)^{\frac{1}{2}}\left[\frac{2.89\,E}{S}\left(\frac{t}{D}\right) + 1\right] \quad \text{[d]}$$

The highest L/D value to be used in formula [9] is found in a similar manner by placing this equation equal to formula [A]

$$\left(\frac{L}{D}\right)_{\max} = \frac{1.21(1 - \mu^2)^{\frac{1}{4}}}{(t/D)^{\frac{1}{2}}} + 0.45(t/D)^{\frac{1}{2}} \quad \text{[e]}$$

Formula [e] varies but little over a wide range of μ, and for $\mu = 0.3$ reduces to

$$\left(\frac{L}{D}\right)_{\max} \frac{1.18}{(t/D)^{1/2}} + 0.45(t/D)^{1/2} \quad \text{[f]}$$

If formula [B] is used as the basis of comparison the constant 1.18 in formula [f] is replaced by 1.56. In either case, the second term in formula [f] can be neglected.

In cases where the most economical design consists in using the minimum plate thickness, the required t/D ratio can be computed from formula [b] and the resulting value inserted in formula [d]. This will give the maximum value of L/D that can be used without entering the region of instability and thus making necessary the use of a heavier shell plate.

Formula [d] and formula [f] with a constant of 1.56 have been tested against the chart in the A.S.M.E. Code rules in reference 4 and found to check the intersections of the straight lines when prolonged. The straight lines on the chart are connected by transition curves, representing the intermediate region mentioned in the beginning of the paper. Thus these formulas do not check the chart exactly, but are useful in constructing such a chart, or similar charts.

T. McL. Jasper.[10] The authors of the paper have presented experimental data which are associated with very slender tubes under collapsing conditions. The available data in this field are meager and those supplied by the authors are a welcome addition to the results available on very thin tubes. In such tubes, in general, unless they are relatively very short, the yield-strength properties of the steel do not enter into the problem.

The writer's work on collapse has been with commercial tubes

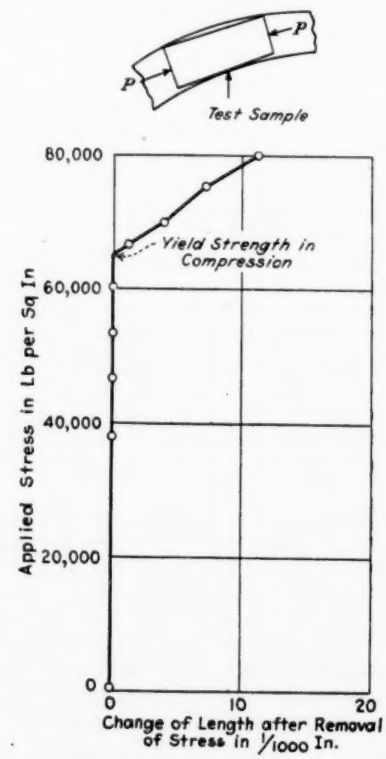


Fig. 1 Showing Yield Strength in Compression

[10] Director of Research, A. O. Smith Corp., Milwaukee, Wis. Mem. A.S.M.E.

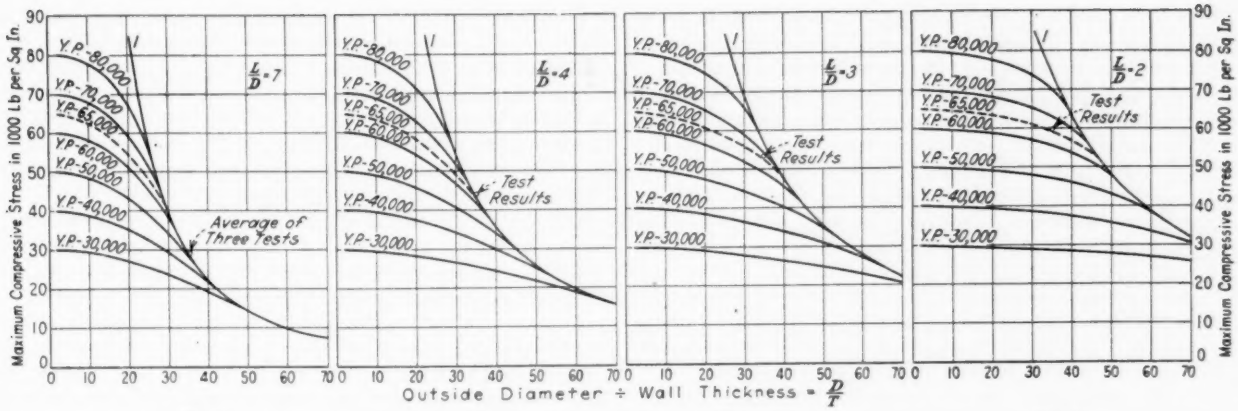


Fig. 2 Fig. 3 Fig. 4 Fig. 5

FIG. 2 RELATION BETWEEN MAXIMUM COMPRESSIVE STRESS AT COLLAPSING PRESSURE AND D/T, FOR HOMOGENEOUS TUBES OF STEEL HAVING VARIOUS YIELD-POINT VALUES

NOTE: Tubes are 7 diameters in length and of uniform roundness and thickness.
1—Curve calculated from Sturm's equation.

FIG. 3 RELATION BETWEEN MAXIMUM COMPRESIVE STRESS AT COLLAPSING PRESSURE AND D/T FOR HOMOGENEOUS TUBES OF STEEL HAVING VARIOUS YIELD-POINT VALUES

NOTE: Tubes are 4 diameters in length and of uniform roundness and thickness.
1—Curve calculated from Sturm's equation.

FIG. 4 RELATION BETWEEN MAXIMUM COMPRESSIVE STRESS AT COLLAPSING PRESSURE AND D/T FOR HOMOGENEOUS TUBES OF STEEL HAVING VARIOUS YIELD-POINT VALUES

NOTE: Tubes are 3 diameters in length and of uniform roundness and thickness.
1—Curve calculated from Sturm's equation.

FIG. 5 RELATION BETWEEN MAXIMUM COMPRESSIVE STRESS AT COLLAPSING PRESSURE AND D/T FOR HOMOGENEOUS TUBES OF STEEL HAVING VARIOUS YIELD-POINT VALUES

NOTE: Tubes are 2 diameters in length and of uniform roundness and thickness.
1—Curve calculated from Sturm's equation.

which are relatively thick and in which the physical as well as the elastic properties of the steel enter.[11] The equation set up by R. G. Sturm in combination with the compression yield points of the steel under collapse is used in the writer's work and a very close check-up has been obtained which confirms the use made of curves such as result from the aforementioned work. I am presenting five typical curves which show results which have not heretofore been published.

Fig. 1 of this discussion shows a curve indicating the yield point in compression of the material. The specimen from which these test results are taken is relatively small but very accurately made and consists of a rectangular prism taken in the direction of stress which causes collapse. The yield strength in compression is obtained by applying successive loads to the prism and plotting the stress applied against change of length after each successive load is removed. The writer's Figs. 2 to 5 show collapse tests made from tubes from which the steel in Fig. 1 was taken and are the figures shown in the paper referred to. It will be noticed that the (D/T) value is constant, but that the actual length has been varied to produce (L/D) values of 7, 4, 3, and 2, respectively. These diagrams were designed using Sturm's equation and take into consideration the end conditions represented by a vessel. The test vessels were the actual length and the ends were actual plates welded to the cylinders.

By varying the length of the test vessel it has been possible to check both the elastic phase as well as the phase of the problem controlled by the physical properties of the material.

MASON S. NOYES.[12] It was hoped that the work of the A.S.M.E. Special Research Committee on the Strength of Vessels Under External Pressure, would yield results of general application. The data published in the first paper, APM-53-17a, and the present one, leave at least three items as yet not investigated.

The first item is that of shape. No consideration has been given to any other form than the cylinder, as stated in the first paragraph of the "Proposed Rules," yet these proposed rules do not read "for the Construction of Unfired *Cylindrical* Pressure Vessels Subjected to External Pressure." Again, assuming that the authors are interested chiefly in shell strength of submarines, one wonders why oval tubes of relatively small difference in major and minor diameters (similar to the usual modern submarine-hull form) were not tested also, in conjunction with some formula including a diameter-ratio factor. Investigation should lead eventually to the elongated oval, the triangular, and even to the nearly rectangular cross-sections used for condenser shells, to none of which the formulas listed in the present paper are applicable.

The second item is that of wall openings. If the tube or cylinder is to be used it must have branches (or inlets and outlets) at some point not always located on the heads. What is the effect of side openings on the number and location of lobes?

The third item is that of material. Reference to the authors' Tables 1 and 3 reveals that the material has been limited to steel or other similar material having a well-defined modulus of elasticity. The proposed rules already mentioned include only steel specifications, in referring to material. The values given in the present paper are around 30 million, showing that an excellent grade of steel has been utilized. On the other hand, under certain conditions of external pressure, tubing of good size is used, which is made of copper. This metal has no definite value for E, the modulus of elasticity varying with the "temper" or amount of cold work done on it and with the temperature conditions, nor has it a well-defined value for μ, its Poisson's ratio.

It is hoped that the investigators will not stop at the present point in their research, which seems to have covered steel cylinders so carefully, but will continue in an effort to get at least a partial answer to the effects of the above items.

AUTHORS' CLOSURE

The authors wish to express their appreciation for the many helpful and instructive discussions of their paper. These dis-

[11] "The Collapsing Strength of Steel Tubes," by T. M. Jasper and J. W. W. Sullivan, Trans. A.S.M.E., vol. 53, 1931, paper APM-53-17b, p. 219.

[12] Bureau of Engineering, Navy Building, Washington, D. C. Assoc-Mem. A.S.M.E.

cussions help to clear up doubtful points and make the paper of more value to those wishing to use it as the basis of design.

It is apparent that there is still some slight misunderstanding as to the application of the column analogy to the collapse of pressure vessels. Messrs. Hartman and Sturm make frequent use of the term "end fixation" in a pressure vessel, comparing it with the end fixation in a column. These two types of fixation are not really comparable. An analogy may be drawn between a pressure vessel and a column by considering the shell of the pressure vessel to be made up of a series of columns, one-half lobe length long, extending end-to-end circumferentially around the vessel. These "columns" are probably pin-ended since at the buckling pressure the ends are unstable and cannot offer a resisting moment. The column ends in the shell certainly are not related to either the heads or stiffening rings of the vessel.

It would probably be more instructive to shift the analogy from columns to plates. It is well known that the strength of a long plate subjected to a compressive load on its ends is influenced by the degree of fixation at the edges (not ends) of the plate. The theoretical buckling load for a plate with built-in edges is shown by Timoshenko[13] to be about 75 per cent greater than for a plate with simply supported edges. The fixation at the stiffening rings or heads of a pressure vessel is analogous to this edge fixation of a plate but it has not been evaluated theoretically. It is reasonable to suppose that the variation in the strength of a pressure vessel due to fixation at the heads will be no greater than the difference between the strength of a plate with fixed edges and one with simply supported edges. It certainly does not seem likely that "the effects of fixity of the ends (of a pressure vessel) may amount to several times as much as the collapsing pressure of simply supported cylinders." The authors have tested model pressure vessels with stiffening rings of many shapes and sizes ranging from extremely light stiffeners to bulkheads, and, whatever the effect of fixity at the stiffener may be, variations in this effect are not evident from the data. Consequently, either the fixation must have negligible effect or at least it must be of about the same degree in all cases.

It might be well to point out that for thick pressure vessels which fail at high stresses, the degree of fixation at the heads of a pressure vessel may be important for it is comparable to the fixation at the ends of a beam. However, this does not apply to vessels which collapse by instability, since the buckling is the result of tangential rather than longitudinal stresses.

Mr. Wesstrom points out that the authors' statement, "any fixation makes for added safety" applies only to fixation against rotation. He points out also that fixation against radial contraction (the authors did not consider this "fixation") may have the opposite effect. Since resistance of the stiffening ring to radial contraction materially affects only the longitudinal bending stress, it probably has small effect upon the strength of pressure vessels which collapse by instability.

Messrs. Hartman and Sturm have called attention to discrepancies between the experimental data and the theoretical formula [9] given in the paper. The explanation for this has already been given in the paper. The collapsing pressures computed by the theoretical formula [9] in Table 3 of the paper are too high in the region of high stresses because the value $E = 30 \times 10^6$ lb per sq in. was used in the computation instead of the correct value of the effective modulus. The collapsing pressures computed by the theoretical formula [9] are too low in the region of low stresses because the formula predicts critical buckling pressures which may be considerably below the ultimate collapsing pressures recorded in Table 3 for the experimental models. It seems unnecessary to call in an unrelated and poorly defined condition of end constraint to help account for these variations.

Regarding model no. 63, which Messrs. Hartman and Sturm indicate has the most out-of-roundness relative to the thickness although the collapsing pressure was 43 per cent greater than the theoretical pressure, it should be noted that an ultimate collapsing pressure considerably above the critical buckling pressure might well be expected for such a long thin model. Moreover, the experimental determination of the collapsing pressure of this particular model was subject to considerable error, since this collapsing pressure of 10 lb per sq in. (the lowest of the tested models) was observed by a 300 lb per sq in. Bourdon-tube gage.

The authors expressed out-of-roundness as a fraction of the thickness rather than of the diameter because in column theory the eccentricity of a column is expressed in relation to the radius of gyration which is analogous to the thickness of a pressure vessel (with a constant multiplier; see Equation [29] of the paper and preceding paragraph). The column eccentricity is not related to the length of the column which is analogous to the diameter (multiplied by a constant) of the pressure vessel.

The authors were asked to explain the methods used in the experimental determination of the number of lobes. The "polar diagram" as illustrated in Fig. 4 of reference 3[14] of the paper shows the exact condition of the shell just prior to collapse. Since such a diagram shows incipient bulges throughout the entire circumference, an examination of the diagram leaves little question as to the actual number of lobes. When satisfactory diagrams are not obtained, however, it is necessary to estimate the number of lobes from measurements of the average lobe length of the failed model. This method is unsatisfactory since neither the lobe length nor the number of lobes found in a failed vessel correspond necessarily to the lobe length and number of lobes in the vessel at the time of unstable collapse.

The collapsing pressures obtained by any instability formula must be checked by a stress calculation to make certain that the proportional limit of the material has not been exceeded. Mr. Wesstrom has outlined a method for determining the region of applicability of formula [9] of the paper. He points out that his formulas check Fig. U-21 of the A.S.M.E. Pressure Vessel Code. This is to be expected since his method is essentially that used in plotting the curves of that figure. However, it should be noted that it is best to limit formula [9] to stresses below the proportional limit rather than below the yield point. This can be readily done by substituting s_{prop} for s_{yield} in Mr. Wesstrom's Equation [d]. Formula [9] is not directly applicable for smaller values of L/D than those given by the modified Equation [d] and should be used only with extreme caution.

The authors believe Mr. Rhys has attached too much importance to the effect of axial loading on the collapsing strength of a pressure vessel. He is correct in the statement that a tube under both radial and axial pressure if increased in length indefinitely will eventually fail as a column or strut under the end load. This is purely academic, however, for the end load resulting from uniform external pressure is so small that in no practical case could it cause buckling of the tube as a strut. This end load, nevertheless, has an appreciable influence on the buckling of the shell by the formation of waves in a circumferential belt. Formula [6] of the paper is obtained by modifying formula [1] to include the effect of end load on this type of collapse. It does not consider the possibility of the tube buckling as a strut. Consequently it does not (and should not be expected to) reduce to zero when $L = \infty$ but to formula [A], given in the paper. The fact that formula [9] *does* reduce to zero for $L = \infty$ is due solely

[13] "Stability and Strength of Thin-Walled Constructions," by S. Timoshenko, Proceedings of the Third International Congress for Applied Mechanics, Stockholm, Aug., 1930, Eq. [5], p. 4, Table 4, p. 5.

[14] "Strength of Thin Cylindrical Shells Under External Pressure," by H. E. Saunders and D. F. Windenburg, Trans. A.S.M.E., vol. 53, 1931, paper APM-53-17a, p. 207.

to the nature of the approximations involved and not to the fact that it includes end load.

It might be noted that the analogous column length of the pressure vessel is a quarter-circumference for $n = 2$ (tubes longer than the critical length), and that this column length does not change if the tube be increased in length. Hence the column analogy is in accord with the fact that the strength of a tube longer than the critical length is independent of the length.

In regard to the four questions raised by Lieutenant Commander Roop the following can be said:

1 It is believed that hard surface layers in the material have very little effect on experimental results. As shown by the theoretical formulas given in the paper, the elastic modulus and Poisson's ratio are the only properties of the material upon which the strength of vessels which collapse by instability depends, and these properties in steel are but little affected by cold working of the material.

2 The actual measurements of deflections during the conduct of tests (reference 3 of the paper, Fig. 4 for example)[14] show deflections prior to collapse far in excess of the deflection due to uniform contraction of the shell.

3 It does not seem possible to obtain an approximate equation for the strength of a flat plate under hydrostatic and coplanar load by considering D to approach an infinite value. The t/D and L/D ratios both become zero in this limiting case, and the collapsing pressure in the various formulas is expressed in terms of these ratios in such a way that it too vanishes.

4 In general the selection of a suitable factor of safety is largely a matter of judgment. Fig. 2 of the paper shows the spread of points representing collapsing pressures of test models. The spread for commercial pressure vessels would probably be somewhat greater. A factor of safety of 2 is ample to take care of variations in thickness, variations in physical properties of the material, and ordinary variations from circular form. However, an addition to this factor is required to give added life to the vessel, to insure against accidental overloads, and to insure against other unforeseen accidents which might precipitate failure. Consequently, the choice of a suitable factor of safety depends upon the individual case. R. T. Stewart[15] gives some excellent general rules for the selection of a factor of safety.

Mr. Jasper offers some very interesting theoretical and experimental data. However, as he points out, his data are for thick tubes most of which collapse at stresses near the yield point while the authors have confined themselves in the present paper to collapse by instability only.

Mr. Noyes regrets that the paper is not more general in scope. The authors felt that a general review, discussion, and evaluation of the merits of the many instability formulas dealing with thin circular cylindrical-pressure vessels was desirable, and the purpose of the present paper was merely to present such a survey.

The A.S.M.E. Special Research Committee on the Strength of Vessels under External Pressure has considered questions of the type raised by Mr. Noyes. Sufficient experimental data are not now available to check the theoretical formulas commonly used to determine the strength of pressure vessels of elliptical cross-section.

Rules for the design of nozzle openings now form part of the A.S.M.E. Code for Unfired Pressure Vessels (Paragraph U-59). Nozzle openings which meet these requirements should not materially affect the strength of the shell.

The theoretical formulas in the paper were checked by steel models, but they apply to vessels constructed of any material provided the correct value of the effective modulus is used. The Special Research Committee is now engaged in formulating rules for circular cylindrical vessels made of copper, aluminum, and various alloys. It is hoped that these rules will be ready for publication in the near future.

[15] "Collapsing Pressure of Bessemer Steel Lap-Welded Tubes, Three to Ten Inches in Diameter," by R. T. Stewart, Trans. A.S.M.E., vol. 27, 1906, p. 815.

The Test Performance of Hudson Avenue's Most Recent Steam-Generating Units[1]

P. W. Keppler.[2] This paper and a similar recent one[3] on Hell Gate direct-fired units make possible an engineering comparison between the two methods of firing. Both methods deal with recently designed, large units which are using practically the same fuel (Eastern semi-bituminous).

In Fig. 1 of this discussion heat input is plotted against the many differences in losses that are connected with the coal-burning equipment, the following abscissa scales selected are at the same time convenient and accurate for comparison: 20 to 80 lb of (14,000 Btu) coal burned per sq ft of projected grate area per hr, and 10,000 to 40,000 Btu fed into the furnace in the form of pulverized coal per cu ft of furnace volume per hr. The maximum reported fuel-burning rate for The Hudson Avenue Station is 76.2 lb, and 39,500 Btu input for Hell Gate, nearly coinciding on our scale.

The losses that can be plotted as reported are: fly-ash loss of pulverized coal, and cinder, ashpit and unburned-gas loss of the stoker. The unaccounted-for losses will be dealt with later. Curve No. 1 of Fig. 1 in this discussion shows an advantage for pulverized coal throughout, rising from 0.9 per cent at 20 lb (10,000 Btu) to 3.7 per cent at 70 lb (35,000 Btu), and then falling to 1.6 per cent at 80 lb (40,000 Btu). This drop is due to the rapid rise in fly-ash loss from 2.2 per cent at 31,700 Btu to 8.7 per cent at 39,500 Btu. This in turn is due to lack of induced draft. It should here be kept in mind that the Hell Gate boiler was guaranteed for a capacity of only about 27,000 Btu input, whereas the Hudson Avenue boiler was built for an even higher maximum capacity (of shorter duration), than the 24-hr capacity reported, and was therefore not limited by induced draft. The actual excess air however was not so much greater for the stoker boiler, 13.3 per cent against 10.3 per cent. This indicates that this stoker requires less excess air than the pulverized-coal equipment under question. Nevertheless it is practically certain that this pulverized-fuel equipment would be capable, if given enough excess air, of continuing its low cinder-loss curve even beyond 39,500 Btu, at which point it would only be 2.8 per cent instead of 8.7 per cent. This could be shown, possibly by dotted lines, but the writer would rather confine himself as much as possible to facts and let the reader draw his own conclusions.

Excess-air losses are shown on curve No. 2 of Fig. 1 of this discussion. Tests on representative equipment indicate an increase in dry-gas losses of 0.05 per cent, per cent of excess air, and on this basis, although excess air differs as much as 5 per cent, the difference in losses is rather small.

The sensible heat in the slag was also estimated (curve No. 3). A specific heat of 0.2, and a slag temperature of 2500 F is here assumed. The maximum slagging loss at 13,000 Btu,

[1] Published as paper FSP-56-15 by P. H. Hardie and W. S. Cooper, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Testing Engineer, Hell Gate Station, The United Electric Light and Power Company, New York, N. Y. Jun. A.S.M.E.

[3] "Characteristics of Large Hell Gate Direct-Fired Boiler Units by W. E. Caldwell, Trans. A.S.M.E., vol. 56, 1934, paper FSP-56-2.

where 45 per cent of the ash is reported to be slagged, is only 0.11 per cent.

The unaccounted-for and radiation losses seem decidedly in favor of the stoker. Accuracy may be gained by "weighting" these for input, for they appear to be uniform, regardless of rate of driving. The resulting "weighted" average loss is 1.37 per cent for the stoker compared with 4.04 per cent for the pulverized-coal equipment. Some of the radiation loss is returned to the boiler by the incoming combustion air being heated above the outside-air temperature. This correction may be applied with good accuracy to the Hell Gate boiler, for during the test it ran alone in its own boiler-house extension, which is practically a building by itself, and its combustion air is taken from the top of this building. On the Hudson Avenue boiler this is not exactly the case but fortunately the correction is much smaller due to the air being taken from the bottom of the building; a considerable error in this estimate would be comparatively small. The heat thus returned is 1.03 per cent for the Hell

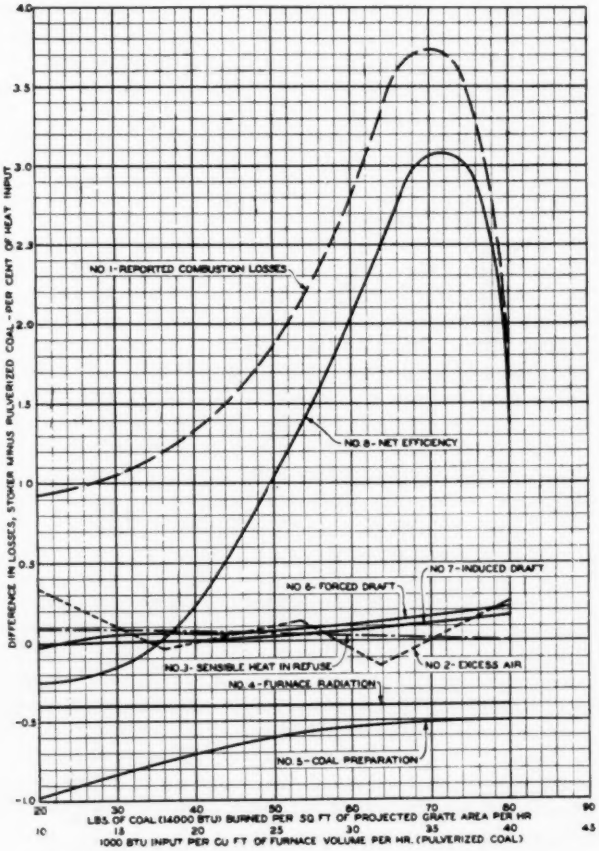


FIG. 1 EFFECT OF FUEL-BURNING EQUIPMENT ON TEST EFFICIENCY, STOKERS (HUDSON AVENUE) VERSUS PULVERIZED COAL (HELL GATE)

Gate boiler, compared to 0.22 per cent for the Hudson Avenue boiler, reducing the actual radiation plus unaccounted-for loss to 1.15 per cent and 3.01 per cent, respectively. This loss can only be radiation for the stoker tests, excluding test error. For the pulverized-coal tests it may be carbon lost in the slag or unburned-gas loss together with radiation, again excluding test error. But other tests at Hell Gate prove that this slag contains no combustible matter. And the sum of CO_2 plus O_2 is within 0.2 of the amount corresponding to complete combustion of the coal used; it is also known that the solution used for oxygen determination fails to take out the last tenth or two of oxygen. No CO was found during the entire test. Furthermore, the presence of unburned hydrocarbons is extremely unlikely for they meet with considerable turbulence, high temperature, and possibly 200 per cent excess air as they are blown off near the burner. The furnace is also very large and comparatively free from stratification. Other elaborate tests with combustion Orsats have shown unburned gases to be very rare with turbulent burners. The writer therefore believes that this loss is practically all radiation for both groups of tests. The radiation surface per unit heat input is 1.6 times greater for the Hell Gate boiler; this includes low temperature air ducts, etc., but the insulation on some of these is correspondingly thin, or left off altogether, so that their radiation may be just as intense as from hot surfaces that are usually well insulated or protected by water walls. This extra surface, largely due to the use of more heat recovery apparatus, would raise the Hudson Avenue radiation to 1.84 per cent. It should also be considered that the Hell Gate setting is far less compact and well ventilated, surrounded by windows, and subject to bad drafts from the coal-conveying system, some of which have since been eliminated. Furthermore the adjacent boiler was shut down, so that this one boiler was subject to radiation from all four sides and had to heat up a boiler room, half of which was shut down. The outside temperature averaged only 37 F, compared to 49 F for the stoker tests. This accounts at least for a good portion of this difference between 1.84 and 3.01 per cent. But the stoker should get credit for the smaller furnace required and for the protection against radiation it affords by its grate area as well as its coal hopper. By assuming a uniform heat emission and by determining the various surfaces, it was estimated how much the radiation loss would increase in the stoker boiler if its furnace were replaced by a pulverized-coal furnace, larger according to the difference in heat release, and radiating from all five sides. The resulting increase is found to be 0.4 per cent and is therefore credited to the stoker (curve No. 4).

Auxiliary power requirements must also be considered. The conservative modern heat rate of 14,000 Btu input per kw is assumed throughout. To compare fan-power requirements, a static fan efficiency of 60 per cent, and electrical drive with 80 per cent motor efficiency are used for both cases. For calculating forced-draft power consumption, the burner pressure of the pulverized-coal equipment is compared with the windbox pressure of the stoker. For induced draft the average setting resistance of the two is used for both, so that the combustion equipment is merely held responsible for the difference in head and volume caused by differences in excess air.

There is a marked difference between mill power and stoker-motor power as shown in Fig. 1 of this discussion by curve No. 5, which varies from 1.2 per cent at 20 lb (10,000 Btu) to 0.5 per cent at 80 lb (40,000 Btu). Forced and induced draft (curves Nos. 6 and 7) show only small differences.

Curve No. 8 shows the resulting net difference. It is seen that if the bulk of the stoker output can be raised below 40 lb per sq ft, and if the higher rates of driving are used only during rare emergencies, the resulting operating efficiency of stokers should be about equal to that of comparable pulverized-fuel equipment.

If the appreciable unburned-gas losses could be reduced by the use of overfire air, the resulting increase in fan power, particularly at low outputs, should be comparatively small. If this were feasible the stoker would be definitely more efficient at moderate rates of driving and its disadvantage beyond these would be reduced.

The writer has purposely confined himself to thermal efficiency, but it must of course be kept in mind that many other important factors would enter into a complete comparison.

JOSEPH GERSHBERG.[4] To conduct tests of a stoker-fired boiler in a satisfactory manner one has to cope with, among other things, two requirements of the fuel bed. First, it must have a thickness and a contour proper for a given make of stoker and kind of coal used if the best sustained boiler efficiency at a chosen rate of evaporation is to be realized. Second, its condition at the beginning and at the end of a run must be identical if demands of test accuracy are to be met. Deviation from the first requirement is commonly designated as the fire being too heavy or too thin. Non-fulfilment of the second requirement results in an error both in the boiler efficiency as well as in the "radiation and unaccounted-for loss."

Ordinarily it takes several hours to obtain a proper fuel bed for a chosen rating and to reestablish it toward the end of the run. Since under present art of testing, the fuel-bed condition is estimated by eye, it becomes extremely difficult to avoid large errors in boiler efficiency and "radiation and unaccounted-for loss," particularly at light loads. With a large stoker surface it takes only a few inches of fuel-bed thickness to cause an error of several per cent in these items of the heat balance. Zero or negative values of "radiation and unaccounted-for loss" are quite common at very light loads.

A method of testing which would eliminate or reduce these errors is certainly welcome. The authors followed a procedure which they contend "has the advantages that the testing period is reduced, and the influence on the results of possible differences in fuel-bed thickness between the beginning and end of runs is minimized." Analysis of results in the energy balance of their Table 1 does not seem to bear out the last contention. Appended curves of these results (Fig. 3 of this discussion) show that with the exception of "radiation and unaccounted-for loss" all other losses behave in a normal way, indicating that, as far as combustion is concerned, it was of somewhat similar quality during the ascending as well as the descending order of loading. However, the curve of "radiation and unaccounted-for loss" shows a very erratic behavior. Since this item in this test represents only the radiation loss and test errors, the curve should be at least a horizontal line if not sloping downward with the increase of energy output, provided the errors are small and fairly constant. One would expect the errors due to estimates of the fuel-bed thickness to be the smallest at the very high ratings, namely in runs 9 and 10. The average of "the radiation and unaccounted-for loss" for these runs is about 2.2 per cent. Taking this value as constant for the entire series of runs one would readily notice (see Fig. 2*B* of this discussion) that all points of the first eight runs except run 3 fall below it. This result indicates that in every one of these runs except run 3, the fuel bed was thinner at the end than at the beginning of a run by average amounts shown in Table 1 of this discussion. According to these values the thickness of the fuel bed with which the test was started in run 1 was reduced at the end of run 8 by 18.3 in., or 15.7 in. if the "unburned

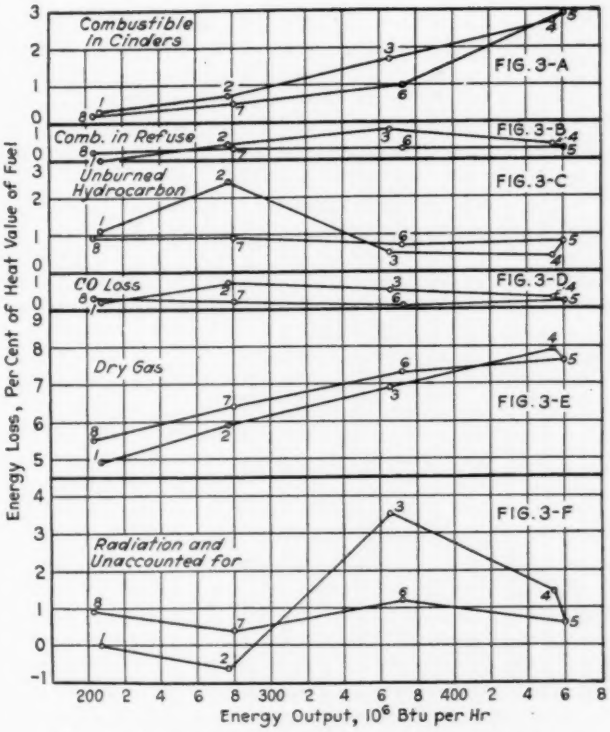


FIG. 3 APPENDED ENERGY-BALANCE CURVES DRAWN FROM TABLE 1 OF PAPER FSP-56-16

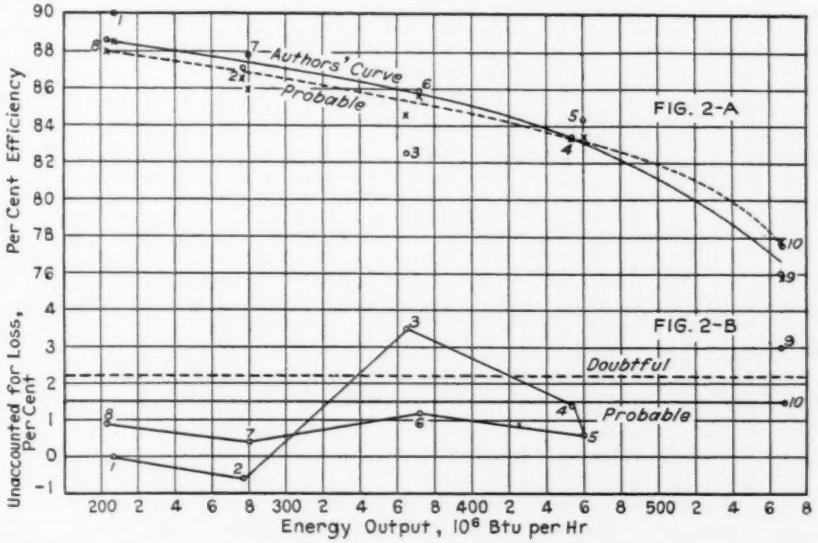


FIG. 2 APPENDED TEST-PERFORMANCE CURVES DRAWN FROM RESULTS GIVEN IN TABLE 1 OF P. H. HARDIE AND W. S. COOPER'S PAPER FSP-56-15

[4] Chief Testing Engineer, United Electric Light & Power Company, New York, N. Y. Mem. A.S.M.E.

TABLE 1 APPENDED RADIATION AND UNACCOUNTED-FOR LOSS

Run No.		1	2	3	4	5	6	7	8	9	10
		Difference in average fuel-bed thickness, in.									
Radiation and unaccounted-for loss	2.2%	—2.4	—4.5 —1.9[a]	+ 2.6	—2.0	—3.9	—1.9	—2.6	—1.3	2.6	—2.3
	1.5%	—1.6	—3.0 —0.9[a]	+ 4.0	—0.2	—2.2	—0.6	—1.6	—0.6	5.0	0

Difference in average fuel-bed thickness at the beginning and end of a run was computed for the inclined stoker area =

$$\frac{694 \text{ sq ft projection area}}{\cos 25 \text{ deg}} = 765 \text{ sq ft}$$

Coal density = 60 lb per cu ft

[a] Corrected to 0.9 per cent energy loss due to unburned hydrocarbons instead of reported 2.4 per cent (see Table 1, item 72, of the paper), which seems to be in error (see Fig. 3-*C* of this discussion).

hydrocarbons loss" (see Fig. 3*C* of this discussion) in run 2 is taken as 0.9 per cent instead of 2.4 per cent, which seems to be in error. So large a reduction in thickness, however, is unlikely. A critical examination of results of runs 9 and 10 tends to regard 1.5 per cent of "radiation and unaccounted-for loss" established in run 10 as the most probable value. This loss in run 9 is 3.0 per cent. The discrepancy of 1.5 per cent might be due to two possibilities: First, an error of only 6 per cent in the sampling of cinders would suffice to cause indicated 0.5 per cent difference in the loss due to combustible in cinders. Second, run 9 was started with a too thin fire which was built up toward the end of this run and thereafter was maintained approximately constant throughout the 10th run. This may account for the remaining 1 per cent discrepancy.

With 1.5 per cent "radiation and unaccounted-for loss," Table 1 of this discussion shows much smaller reductions in thickness of the fuel bed for individual runs than with 2.2 per cent. Accordingly, the thickness of the fuel bed existing at the beginning of the first run needed to be decreased by only 5.8 in., or more likely, by 3.7 in. (if corrected for error in run 2 as explained previously) at the end of run 8, to make possible errors in boiler efficiency for individual runs. Fig. 2*A* of this discussion shows the probable efficiency curve as compared with that given in the paper.

This analysis leads to a conclusion that the authors' procedure of uninterrupted testing for a series of *different* loads does not minimize the errors of individual runs due to differences in fuel-bed condition at the beginning and end of each run. The procedure becomes effective only when it is applied to a series of runs for the *same* load, the entire series being treated as a single run. In other words the duration of a run is to be prolonged, as it is practiced whenever accuracy of testing is sought, particularly, at light loads.

Now with reference to the shortening of the testing period, the authors' claim of this advantage in their method of testing can be conceded if loads are changed in one order only, either ascending or descending. In other words, there must be no check runs. There is a considerable saving in time if only a few hours, say three or four, are spent for the establishment of a proper fuel bed at the beginning of each run instead of 24 hours required for a check run.

E. C. M. Stahl.[5] The combustion-control meters used at Hudson Avenue are of the type which maintain a predetermined steam-flow–air-flow ratio, or in effect a predetermined per cent "total air" (ratio of actual air supplied to theoretical air required for complete combustion). For this reason the operating department at Hudson Avenue has been interested in determining the total air at which highest combustion efficiency can be obtained.

The elementary method of establishing the proper quantity of air to be supplied is to determine by means of a field Orsat the point at which carbon monoxide disappears from the flue gases. This method has been found to be inadequate due to the fact that complete analysis of the flue gas indicates the presence of unburned gases considerably in excess of that shown by the field Orsat. In Fig. 4 of this discussion the relation between the losses in boiler efficiency as found by the two methods of flue-gas analysis is indicated. This relation was brought out by Bert Houghton in his paper before the Third International Conference on Bituminous Coal. It should be noted that the loss incurred under operating conditions varies markedly with the per cent total air supplied.

The curve in Fig. 5 of this discussion has been plotted from test data to demonstrate that the cinder loss also varies with total air supplied. An increase in the percentage of total air at any rating increases the velocity of the air through the fuel bed with a resultant increase in the quantity and size of cinder carried away. It appears that the impact and hence the cinder loss varies as some power of the air velocity.

Fig. 4 of this discussion shows the combined effect of total air on principal combustion losses. As the per cent total air is increased for the purpose of reducing the losses due to incomplete combustion, the dry-gas and cinder losses increase rapidly. Obviously, the most efficient combustion conditions exist at the point where the summation of all the losses is a minimum. This best point will occur in the vicinity of *A* in Fig. 4. If, on the other hand, the per cent total air is set at some point *B* as indicated by analysis with the field Orsat or at some arbitrary point *C* in an attempt to avoid possibility of incomplete combustion, unnecessary combustion losses will result. The location of the point *A* varies with the burning rate and should be determined

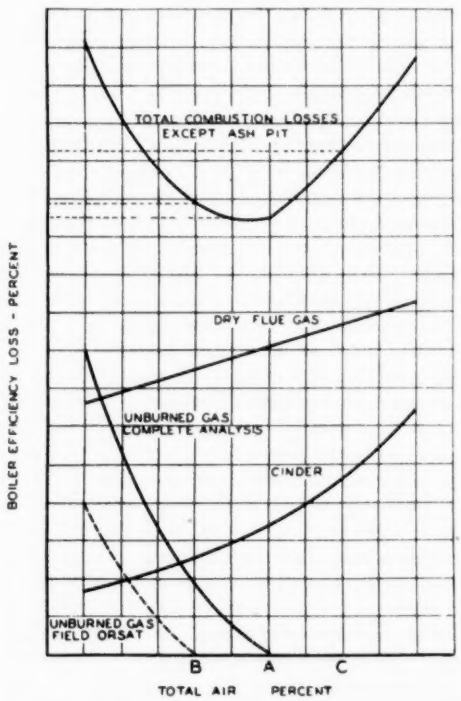


Fig. 4 The Relation Between the Losses in Boiler Efficiency as Found by the Two Methods of Gas Analysis

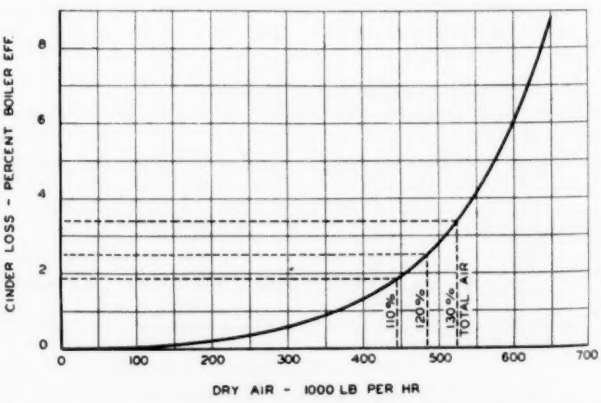


Fig. 5 Variation of Cinder Loss With the Ratio of Actual Air Supplied to the Theoretical Air Required for Combustion

[5] Operating Superintendent, Brooklyn Edison Company, Inc., Brooklyn, N. Y. Mem. A.S.M.E.

at several boiler loads so that combustion meters may be set to indicate a per cent total air that varies with steam flow in a manner to embrace the loci of a series of points determined as point *A*. Since the magnitude of these losses varies with each stoker and furnace installation, highest combustion efficiency can be obtained only if the variable combustion losses are determined by use of methods similar to those described by the authors.

H. F. Lawrence.[6] The writer believes it has been conceded rather generally that the presence of CO in the gases of combustion indicates the presence of unburned hydrogen and hydrocarbons, particularly if the CO is present in appreciable quantities. The writer also believes it has been conceded generally that the unburned hydrogen and hydrocarbons increase greatly with increased coal-burning rates per square foot of grate surface. Referring to the author's paper, note that run 10 which was at the highest capacity, showed the greatest loss due to CO and also the least loss due to unburned hydrogen and hydrocarbons. Note, also, that run 9, which was at the same rate as run 10, showed less than half the CO loss and four times the hydrocarbons loss as compared with run 10.

Another outstanding feature of these tests was the procedure of continuous operation with no recess between runs. Fig. 6 of this discussion shows graphically the changes in load with the percentage of increase at each change upward. The increase in

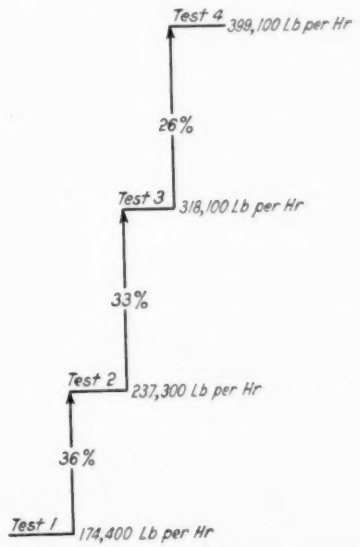


Fig. 6 Graphical Representation of Load Changes of No. 74 Boiler Tests, Hudson Avenue Station

load from test 1 to test 2 was 36 per cent; from 2 to 3, 33 per cent, and from 3 to 4 it was 26 per cent. These changes in load, rather than being small, were large and it is quite remarkable that changes of this magnitude were made and conditions stabilized within five minutes.

In addition to the tests reported, many peak-load tests of five hours duration were made, during which the water was not weighed, but measured on the regular station instruments which had previously been calibrated. As these were merely load tests, no attempt was made to calculate efficiencies. On November 16, 1933, the maximum five-hour rate was run at a load of 607,000 lb of steam per hr and 632,000 lb of steam per hr for a maximum three-hour period. Flow-meter charts transposed

[6] American Engineering Company, Philadelphia, Pa. Mem. A.S.M.E.

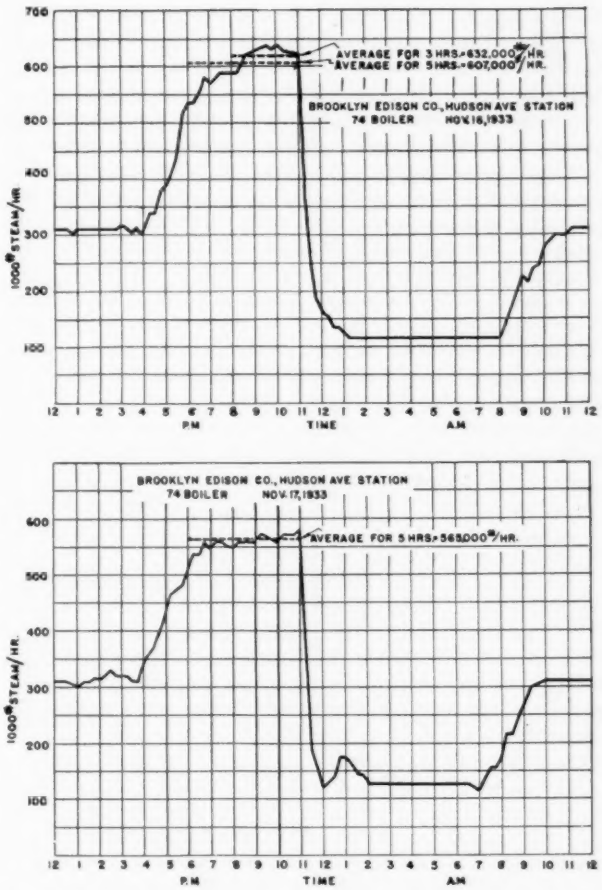


Fig. 7 Flow-Meter Charts of Hudson Avenue No. 74 Boiler Tests Transposed to Straight-Line Graphs

to straight-line graphs are shown for the runs of November 16 and 17, 1933, in Fig. 7 of this discussion.

David Brownlie.[7] The authors have carried boiler-plant testing to such an advanced stage that it is interesting to consider what remains to be done in this field. Certainly one point is the determination, by some type of recording apparatus if necessary, of the amount of sulphur dioxide and sulphur trioxide in the combustion gases. Until a few years ago no method had been developed for determining the amount of dust and acid-sulphur compounds being discharged from the stack. Now, however, the determination of the dust content is of primary importance, and we are rapidly getting to the point where it will be just as essential to know the amount of sulphur compounds in the combustion gases.

In this connection there is already one power station in Great Britain, namely Battersea, London, that is operating on the principle of the complete removal of the sulphur compounds, involving, it is believed, the pumping of 2000 tons of scrubbing water every 24 hours. The Fulham (London) station and the Tir John (Swansea) station, both now being erected, are also to be operated on similar lines, while at the Barton station (Manchester) experimental work on one boiler has been in progress for some time and the same applies to the Billingham Works of Imperial Chemical Industries Ltd.

It seems to me also that in these tests at the Hudson Avenue station, a further minor refinement would have been the installa-

[7] Consulting Technical Chemist, London, England. Mem. A.S.M.E.

tion of continuous duplex gas-analyzing apparatus for the determination of both the CO_2 and the CO (unburnt products) and also the CO_2 and the oxygen.

Another main item that calls for comment is the method of expressing the efficiency. Thus, for example, the "efficiency of the steam-generating unit" is given in the authors' paper as varying from 76.2 per cent to 90 per cent, while separate items are included for the power consumption of some of the auxiliaries. Although the matter is not very clear, presumably this 76.2 to 90 per cent steam-generating efficiency is the gross efficiency only, based on the total evaporation to the steam in the boilers, without regard to the consumption of steam by auxiliaries in the boiler house itself. I am informed by Messrs. Hardie and Cooper, however, that the total auxiliary power used, but not including the feed pump, corresponds to 1.3 per cent to 2.3 per cent of the total evaporation of the boilers.

In 1922 I published suggestions for an International Boiler Test Code, one of the points of which was that the efficiency of boiler plants should be expressed as the true net thermal efficiency, that is, the efficiency based upon the amount of steam available for useful work outside the boiler house itself, and not upon the actual production of steam by the boiler plant.

That is, to use a very simple example by way of illustration, if a boiler plant is evaporating 100,000 lb of water per hr and 5000 lb of water per hr is being used to drive auxiliaries such as boiler-feed pumps, forced- and induced-draft fans, mechanical stokers, and ash and clinker conveyers, then the net thermal efficiency must be based upon 95,000 lb evaporation and not on 100,000 lb.

Messrs. Hardie and Cooper might have included in the results of their tests, the actual net thermal efficiency and not only the gross efficiency. They inform me that they do not consider the feed pump as an "auxiliary" in the ordinary sense of the term, since it performs its own distinct function. Frankly, I cannot agree with them, although from the theoretical point of view the matter is difficult.

Finally, it is impossible to work any boiler plant today without expending energy in forcing the feed water into the boilers and, in my opinion, all the energy used must be deducted in calculating the net thermal efficiency.

Obviously, as I have pointed out for many years, we have long required a proper International Boiler Test Code drawn up by engineers and fuel technologists who have had practical experience in this field of engineering.

A. G. Christie.[8] The value of this series of tests lies in the complete analysis of the energy distribution in the steam-generating unit at the various loads. Certain interesting deductions of both practical and academic interest may be drawn from these data.

A comparison of the authors' item 74, "Energy loss due to combustible in cinders" with item 73, "Energy loss due to combustible in ashpit refuse," indicates that the former averages several times the latter. This would indicate that more consideration must be given on future tests to the sampling of the flue gases for cinders than to sampling the clinker-grinder refuse. The authors have developed a method of sampling the flue gases for cinders that appears to give correct data.

The energy loss due to radiation and that unaccounted-for, is remarkably small. This indicates that proper wall construction and insulation can reduce heat-conduction losses into the boiler room to a small value.

The draft at the induced draft fans at full steaming capacity of 500,000 lb per hr is high amounting to 14.3 in. The plenum pressure at the same load is 6 in. The total pressure is 20.3 in. which accounts for the comparatively large power consumption of the fans. The authors have not charged any of the boiler output to these fans. It would be interesting to know the net efficiency after allowance for auxiliary power.

No data are presented on the performance of the cinder catchers in the removal of cinders from the flue gases. Since observations were undoubtedly made on this equipment, the authors might indicate the relative effectiveness of the cinder catcher.

The heat balance shows that the loss from unburned hydrogen and hydrocarbons averages several times the loss due to CO. Much care has been devoted in the past to the analysis of the flue gases for CO while no attempts have generally been made to detect the hydrogen and hydrocarbon losses. Evidently laboratory practice and test procedure must be revised to include the accurate determination of the larger losses due to hydrogen and hydrocarbons.

The gross efficiencies of the steam-generating unit are excellent at all loads. The performance of the stoker, as indicated by the low ashpit and cinder losses, was an important factor in securing these results. A rate of combustion of 75 lb per sq ft of grate area per hr for two consecutive periods of 24 hr deserves especial comment, particularly in view of the overall efficiencies obtained under these conditions.

The authors state that both boiler and stoker were put in first-class operating condition for these tests. It would be interesting to know at certain outputs how the average daily efficiency compares with test efficiency.

Items 61 and 62 in the authors' Table 1 appear to be based on boiler surface alone and do not include the very effective water walls. The rates of heat generation and transfer as given are consequently much higher than actually occur in the boiler due to the high rates of evaporation in the water wall. Why not include the area of the water walls with the boiler surface in setting up such average figures?

Frank O. Ellenwood.[9] In this paper we have an energy balance of a steam-generating unit that gives, for the first time to the writer's knowledge, a very close approximation of the radiation from the unit, because the authors have determined several losses that have usually been treated as unaccounted-for. It is especially significant to observe that the loss due to the unburned hydrocarbon was low; but, in the writer's opinion, it should be noted that such a result would not have been obtained had the furnace not been provided with ample distance between the fuel bed and the first row of boiler tubes, a distance in this case of from 30 to 40 ft. When a boiler is set too close to the fuel bed, as has been done in some stoker installations, this loss becomes of much greater magnitude, as does the cinder loss also. It would be of special interest to many engineers if the authors could give information regarding these points for some of the other units that have boiler tubes closer to the fuel bed.

From the energy balance, and also from the curve in Fig. 5, it is apparent that the cinder loss is the serious one when extremely high rates of firing are employed, such as 75 lb of coal per hr per sq ft. That this unit was able to operate at this phenomenal rate for 48 hr and still have no furnace trouble of any kind is a remarkable testimonial to the builders and operators. If stokers are to be operated at such high rates for prolonged periods of time, studies should be directed to possible means of reduction of the cinder loss.

The tabulation of the data appeals to the writer as being exceptionally clear, concise, and technically correct; his only suggestion for a slight improvement in the tabulation would be

[8] Professor of Mechanical Engineering, Johns Hopkins University Baltimore, Md. Mem. A.S.M.E.

[9] Professor of Heat-Power Engineering, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

that it should also include the net efficiency for each run. This point becomes of increasing importance in making tests of large modern steam-generating units, whose performances it may be desirable to compare even though their fuel-burning equipment and auxiliaries may be entirely different.

A long step forward is represented by the technical progress that is exemplified by the design and testing of this steam-generating unit when compared with that of the so-called "boiler tests" of a few years ago. When one observes that the unit had no air preheater, the results of the tests become still more impressive, and one again realizes that high net efficiencies of steam-generating units are not confined to powdered-fuel installations.

Authors' Closure

Mr. Keppler has made a careful and detailed comparison of the performance of the pulverized-fuel unit at Hell Gate with the stoker-fired unit at Hudson Avenue with allowances for those differences which might influence the comparative results. Any such method of correction as Mr. Keppler used, however, requires a great many assumptions, some of which are subject to differences of opinion. The authors believe that there is other pertinent information not mentioned by Mr. Keppler which should be considered even though it is not amenable to numerical evaluation. For instance, the Hell Gate unit is equipped with an air preheater and economizer while the Hudson Avenue unit has only the latter. The Hell Gate tests were of relatively short duration in that they varied from ten hours at the lowest load to two hours at maximum load, and no check runs were made.

The impression might be gained from Mr. Keppler's statements regarding the guaranteed capacity of the two units that the maximum test load on the Hell Gate unit was higher in proportion to the unit's size than that on the Hudson Avenue unit. As a matter of fact, the maximum test loads on the two units per square foot of boiler surface were the same.

The authors fail to see the justification of Mr. Gershberg's method of correcting the test results to an assumed constant value of "radiation and unaccounted-for," and his attributing the variation to differences in fuel-bed thickness between the beginning and end of runs. It is interesting to note, however, from Fig. 2 of this discussion that his "probable" efficiency curve agrees with the authors' curve within the limits of test accuracy in spite of the liberties he has taken with the original data.

It is very easy to talk about having the condition of the fuel bed, its contour, thickness, etc., the same at the beginning and end of a run, as Mr. Gershberg does, but it was the inherent impossibility to estimate by eye with any degree of accuracy these conditions on a stoker of the size of the one under discussion which led to the adoption of the continuous method of testing. The authors do not believe that on this test the variation in the "radiation and unaccounted-for" was abnormal and apparently none of the other discussers thought so. Mr. Lawrence, who would be most interested in obtaining reliable results, calls the continuous method of operation an "outstanding feature of these tests."

Mr. Gershberg suggests that a preliminary period prior to each run would eliminate the necessity of check runs. The authors consider that check runs are always desirable and especially so when testing steam-generating units because, aside from the question of testing accuracy, the results are subject to changes in the order of one or two per cent as a result of operational factors.

Mr. Stahl has very clearly pointed out that accurate gas analysis is essential to the proper setting of the boiler meters. In this connection, the authors have felt for several years that the true rôle of the Orsat apparatus in formal tests of steam-generating units was that of guiding the stoker operators during test rather than that of providing a basis for the flue-gas computations.

Referring to the variation of unburned H_2 and hydrocarbons, Mr. Lawrence points out that these tests did not sustain the prevailing opinion that the presence, or absence, of CO in the exit gases reveals the presence, or absence, of these unburned gases. Other tests[10] have also fully demonstrated that this conclusion does not apply to the combustion situation encountered at Hudson Avenue.

Mr. Brownlie and Professors Christie and Ellenwood have expressed the opinion that the net efficiencies should have been included in the tabulated results even though the approximate percentages of the energy output consumed by the boiler auxiliaries at the lowest and highest test loads were stated in the last paragraph under "Test Data and Results," page 839 of the paper. The net efficiencies for each run are given in Table 2 of this discussion.

TABLE 2 NET EFFICIENCIES OF STEAM-GENERATING UNIT

Run number	1	2	3	4	5
Net efficiency, per cent	88.8	86.1	81.1	82.0	82.8
Run number	6	7	8	9	10
Net efficiency, per cent	84.2	86.6	87.3	74.4	75.7

It should be noted that there is no standard set of assumptions as to what primary data shall be employed in computing net efficiencies. For example, a difference of opinion exists as to whether the feed pump shall be regarded as an auxiliary. While Mr. Brownlie prefers to consider the feed pump as a boiler auxiliary, Professor Ellenwood[11] takes the opposite point of view. The authors regret that they are unable to report for Mr. Brownlie's information the energy consumption of the feed pump because it was not measured during test.

Another reasonable difference of opinion among engineers is whether the electrical energy consumed by the auxiliaries shall be charged on the basis of the average station heat rate which includes standby losses, etc., or whether the combined heat rate of the test boiler and the turbine which it supplies during each test is the proper one to use. The latter would conform more nearly to the practice in the case of steam-driven auxiliaries but the combined heat rate of a single boiler and turbine for each test period is not generally available from the station records.

The net efficiencies given in Table 2 of this discussion were computed excluding the boiler feed pump as a boiler auxiliary and using the average net fuel rate of the station. The details of the procedure used are described in Barnard, Ellenwood, and Hirshfeld's, "Heat Power Engineering," part 2, page 443.

In answer to Professor Christie's inquiry concerning the efficiency of the cinder catcher, the "wet" type of cinder catcher used on this unit has shown test efficiencies of 80 to 90 per cent at ordinary loads.

Professor Christie mentions the authors' statement about putting the boiler and stoker in prime operating condition and asks how much effect this had upon improving the efficiency. The cleaning of the boiler and maintenance work on the stoker were the same as is done at regular intervals and, therefore, resulted in small improvement over average performance. The extra supervision of the stoker operation, however, aided by the Orsat readings may have resulted in greater improvement than the cleaning of the boiler. For average daily operating efficiencies reference should be made to Table 5 of "Ten Years of Stoker

[10] "Reducing the Unaccounted-For Losses in Boiler Tests," by W. F. Davidson, Report presented to the Fuels Division, A.S.M.E., at the Chicago Meeting, 1931.

[11] "Elements of Heat Power Engineering," by Barnard, Ellenwood and Hirshfeld," John Wiley and Sons, New York, N. Y.

Development at Hudson Avenue,"[12] by J. M. Driscoll and W. H. Sperr.

The authors agree with Professor Christie that Items 61 and 62 in Table 1 of the paper would have been more nearly correct had the water-wall surface been included in reporting the steam generated and heat transferred per square foot of boiler surface.

The comparative information requested by Professor Ellenwood regarding cinder loss in boiler units having low and high settings may also be found in Fig. 11 of the paper by Driscoll and Sperr previously mentioned. It will be noted from this figure that the height of the setting has little or no effect upon the cinder loss. However, a marked reduction in the hydrogen and hydrocarbon losses was obtained on this unit as compared to older units with lower setting. Possibly some of this improvement was due to the zoned-air control.

The Elastic Properties of Steel at High Temperatures[1]

LEO H. HALL.[2] It seems that caution is needed in using Mr. Versé's method for measuring temperature when applied to static tensile tests.

In the first place, a wire supported and heated as the author describes, cannot be at uniform temperature throughout, since heat is being conducted away at the supports and at the loading point without a corresponding loss at intermediate points. In the writer's experience in measuring electrical resistivities at high temperatures, this lack of uniformity of temperature has sometimes been observed to increase rather than decrease by insulating the wire and, in a metal such as steel which has a fairly high temperature-resistance coefficient, it is sufficient to vitiate results obtained by the use of this method, at least in measuring resistance.

This means that the temperature read by resistance in a set-up such as the author describes is not a true temperature, but a composite of the temperatures of different parts of the wire. The elastic properties observed are also composites of those existing over a range of temperatures. Since the elastic properties do not vary linearly with resistance, it is evident that errors may be introduced by the use of this method. The magnitude of the error depends on the temperature gradient along the wire, which is in turn determined by its specific heat, conductivity, diameter, length, and surface roughness. That it is not insignificant may be judged from the fact that in similarly suspended insulated wires, differences of temperature of more than 100 C have been measured between points 2 ft apart, neither point being at the support.

Another source of possible error lies in the fact that the electrical resistivity of some metals is known to vary between the unstressed and the highly stressed conditions, at least at room temperature. It is the writer's recollection that some very considerable variations have been observed. So far as he knows, no tests have been made to determine whether such variations persist at higher temperature, or whether they increase under that condition.

While the method used by Mr. Versé is highly ingenious and has the merit of great simplicity, it should be borne in mind that it is limited as to accuracy and, that under some conditions, it entails considerable error. It would be of interest to learn how the author calibrated his device.

AUTHOR'S CLOSURE

Mr. Hall's remarks were expected and, in an unabridged thesis presented to the University of Michigan, they were answered as follows:

1 Regarding the non-uniformity of temperature along the specimen, it is true that such an error is present in our measurements but, however, it is not of important magnitude. In accurate measurements made in the physics department of the University of Michigan, where a similar method is used for the measurement of the coefficient of expansion of metals, it was determined that the effect of the end supports is not felt over a distance larger than 2 in. from the support. The length of the wire being 125 in., and an averaging effect taking place in both the measurements of E and of the electrical resistance, the resultant error is very small. Indeed, if E and the electrical resistance should vary linearly with temperature, there would be no error at all.

2 Regarding the possible variation of the electrical resistance with the stress condition, the stresses were kept very low in our tests so as to reduce this error to a negligible value.

3 The preliminary calibration was made on a sample of the wire at the following standard temperatures:

Freezing point of water	0 C
Boiling point of water	100 C
Boiling point of naphthalene	219 C
Boiling point of sulphur	445 C

For these calibrations, the standard equipment for the calibration of resistance pyrometers was used. The electrical resistance of the sample as well as that of the wire was measured by means of a potentiometer and a standard resistance. The standard resistance was made of an alloy the resistance of which varied but little with the temperature. Besides, this resistance was very small so as not to undergo any appreciable heating, and more, over it was kept at a constant temperature by a flow of compressed air. In every case the length of the wire was measured at 25 C, so that the calibration automatically takes care of the thermal expansion.

Finally, it is pointed out that Mr. Hall's remarks do not pertain to our static-torsion tests, which also showed the existence of Professor Timoshenko's high-temperature strain-hardening effect and the advantages of Dr. Everett's unloading method.

---

[12] "Ten Years of Stoker Development," by J. M. Driscoll and W. H. Sperr, Trans. A.S.M.E., vol. 57, no. 2 (February, 1935), paper FSP-57-3.

[1] Paper by Guy Versé, published in the January, 1935, issue of the A.S.M.E. Transactions.

[2] Mem. A.S.M.E.

Photoflow Method of Water Measurement

By W. M. WHITE[1] and W. J. RHEINGANS,[2] MILWAUKEE, WIS.

This paper is written to reestablish the pitot tube as a useful and accurate instrument for measuring the flow of water. Difficulties in the measurement of water by the pitot tube have been caused by errors introduced in the measurement of the static pressure and such errors as are caused by changing distribution of flow across the measuring section while a traverse is being made.

Recent tests indicate the proper form of piezometer to use in order to determine the true static pressure of flowing water. The errors caused by changing distribution of flow can be eliminated by the photoflow method described in the paper. This method involves the photographic recording of the differences in pressure of pitot-tube points, positioned throughout the cross-section of flow, and piezometer connections positioned to indicate the pressure across the flow section. By making a simultaneous record of all the readings across the section, errors caused by changing distribution of flow are eliminated and a large number of measurements can be taken in a short interval of time.

The photoflow coefficient as determined by volumetric measurement for highly disturbed flow in a closed conduit was found by test to be 0.971. A coefficient of 0.976 for the photoflow method as applied to the flows normally encountered in closed conduits should give an accuracy of 0.5 per cent.

THE photoflow method[3] of water measurement is the photographic recording of the differences in pressures of pitot-tube points positioned throughout the cross-section of flow, and piezometer connections positioned to indicate the pressure across the section of flow. The method as here described has particular reference to closed circular conduits although it is not limited in its application to this specific case.

Reference is made to a paper by one of the authors[4] in which it was determined by a series of experiments that the pitot-tube point, if constructed as an orifice bounded by a surface of revolution, transformed velocity impinging exactly axially into pressure exactly according to the law $V = \sqrt{(2gh)}$, regardless of the form or extent of its exterior walls. At the time this paper was presented, there was considerable controversy regarding the pitot-tube formula, some advocating $V = \sqrt{(2gh)}$ while others, notably the late William Kent, author of "Kent's Mechanical Engineers' Handbook," and Prof. I. P. Church, author of "Mechanics of Engineering," both maintained that the formula should be $V = \sqrt{(gh)}$. The conclusiveness of the experiments in the paper referred to was indicated by the fact that Kent and Church changed the pitot-tube formula from $V = \sqrt{(gh)}$ to $V = \sqrt{(2gh)}$ in the subsequent editions of their books. From then on, $V = \sqrt{(2gh)}$ was generally accepted as the true pitot-tube formula.

The same paper also pointed out that one of the reasons for the necessary application of a coefficient in connection with the pitot tube was because of the difficulty in securing the real pressure properly applicable to the case in question. A recent paper presented to the Society[5] sets forth information as to the form and shape and arrangement of piezometer openings by which the direct water pressure may be measured in conduits. A study of this paper and of the various piezometer connections discussed will indicate one of the real causes of former difficulties in the measurement of water by the pitot tube.

The information contained therein, together with the tests recorded in "The Pitot Tube; Its Formula"[4] show why a pitot tube with the piezometer located in certain positions on or near the pitot-tube point, has to be rated and why a coefficient, which can properly be called the piezometer coefficient, must be applied to the readings. It was also found that tubes with piezometer openings placed in the path of the water are extremely sensitive to angular flow and subject to considerable error whenever other than straight flow is encountered.

Sufficient published data are now available to enable one experienced in the art to construct proper piezometer connections on the periphery of a straight cylindrical conduit such that the pressures of the water throughout a given cross-section may be properly indicated, thus eliminating all necessity for rating the pitot tube to obtain the piezometer coefficient and eliminating possibility of error in the static-head readings caused by angular flow.

The present art provides means for determining the correct velocity head in water flowing in straight conduits when the water flows in straight stream lines. However, in actual practice, straight streamline flow is never encountered. The particles of water move spirally, intermingle, and roll from center of pipe to pipe wall and back to center continuously, but without great loss of position in the travel of the flow. It has been shown that the introduction of a salt solution at a section in a conduit will diffuse throughout the mass of water within a reasonably short distance in the travel of the water, but the salt remains substantially together as a plug in its forward travel through the conduit. This is indicated by the fact that the salt solution at the outer periphery of the conduit will lag only a short distance behind the solution in the center of the pipe at a section which

---

[1] Manager and Chief Engineer, Hydraulic Department, Allis-Chalmers Manufacturing Company. Mem. A.S.M.E. Dr. White received his M.E. degree from Tulane University and served as engineer with the New Orleans Drainage Board from 1899 to 1902, and as hydraulic engineer and chief engineer in the I. P. Morris Department of Wm. Cramp & Sons Ship and Engine Building Company from 1902 to 1911, where he was in charge of hydraulic work for the company, including design and building of large hydroelectric installations among which was the plant at Niagara Falls. Since 1911, Dr. White has served in his present capacity with the Allis-Chalmers Mfg. Company. In 1930, he received the degree of Doctor of Science from Tulane University.

[2] Test Engineer, Hydraulic Department, Allis-Chalmers Manufacturing Company. Mr. Rheingans was graduated from the University of Wisconsin in 1920 with a B.S. degree in civil engineering and entered the employ of the Allis-Chalmers Manufacturing Company the same year where he has been engaged in testing and research work in the Hydraulic Turbine Department.

[3] Method patented by D. W. Proebstel, October 11, 1927, U. S. Patent 1,645,449.

[4] "The Pitot Tube; Its Formula," by W. M. White, The Journal of the Association of Engineering Societies, vol. 27, August, 1901, p. 35.

Contributed by the Hydraulics Division and presented at the Annual Meeting, New York, N. Y., December 4 to 8, 1933, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] "Piezometer Investigation," by C. M. Allen and L. J. Hooper, Trans. A.S.M.E., vol. 54, 1932, paper HYD-54-1.

may be a thousand feet or more from the point of introduction. Since the velocity of the water at the center of the conduit is considerably greater than at the sides, this can only mean that there is a sidewise motion of the water in its flow and that accordingly the particles of water must move at greater absolute velocity than the average axial flow along the conduit. The pitot-tube point, therefore, encounters not axial flow but angular flow.

If the pitot-tube points recorded the true pressure corresponding to the resolved axial velocity of such angular flow no coefficient of any kind would be required since now correct pressure in the section of flow can be secured. The pitot tube, however, does not indicate the direct resolved velocity and it has been found necessary to introduce a coefficient for this peculiarity of the pitot-tube point.

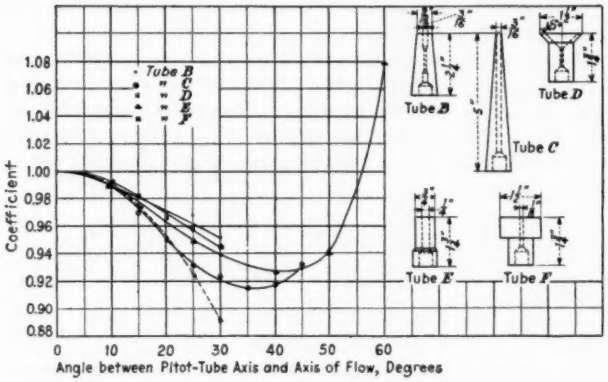


FIG. 1 PITOT-TUBE COEFFICIENTS FOR VARIOUS ANGLES OF FLOW SHOWING WHY COEFFICIENTS MUST BE APPLIED TO MEASUREMENTS IN CONDUITS

In the paper "The Pitot Tube; Its Formula,"[4] the author records the fact of his experimentation "with nozzles of different shapes moving through water and placed at different angles with reference to the line of motion." In a carefully prepared paper presented to the Engineers' Society of Western Pennsylvania[6] is recorded a very complete series of experiments on the coefficients for different shapes of pitot tubes for various angles of flow. Fig. 1 shows the results of these experiments and the outlines of the tubes used. From Fig. 1 it will be seen that for certain types of pitot tubes (*B*, *E*, and *F*) the coefficient varies from 1.00 to 0.96 for a maximum angle of flow of 25 deg, which angle is greater than can be ordinarily expected in even the very worst conditions of flow. It will be noted in the cases where the experiments were carried out to large angles that the value of the coefficient does not decrease in proportion to the change in angularity but beyond 40 deg it actually reverses and starts to increase.

Recent tests made on the exact design of pitot-tube point used for the Little Falls Pumping Station tests hereinafter described, showed similar angular-flow coefficients and characteristics. To determine these coefficients, this pitot-tube point was placed at the center of a 12-in.-diam pipe on the discharge end of a centrifugal pump and was mounted so that it could be rotated to any desired angle from the axis of the pipe. The flow was made as straight as possible with baffles.

The discharge of the pump was adjusted to maintain a constant velocity of 5 fps at the center of the pipe. Velocity-head readings were then taken at 0 deg and for every 5-deg angle of the axis of the pitot-tube point from the axis of the pipe on both sides of the centerline up to about 50 deg. These tests were repeated four times.

The indicated velocities were computed, averaged for the four tests, and plotted against the angles as shown in Fig. 2. The resolved velocity at a given angle was obtained by multiplying the axial velocity by the cosine of the angle. The measured velocity was then computed as a percentage of the resolved velocity and was plotted as shown.

Fig. 3 shows an outline of the Little Falls tube and a comparison of its angular-flow coefficients with those of the similar pitot tube *B* shown in Fig. 1. The close agreement of the results obtained by these two differently conducted experiments is rather remarkable.

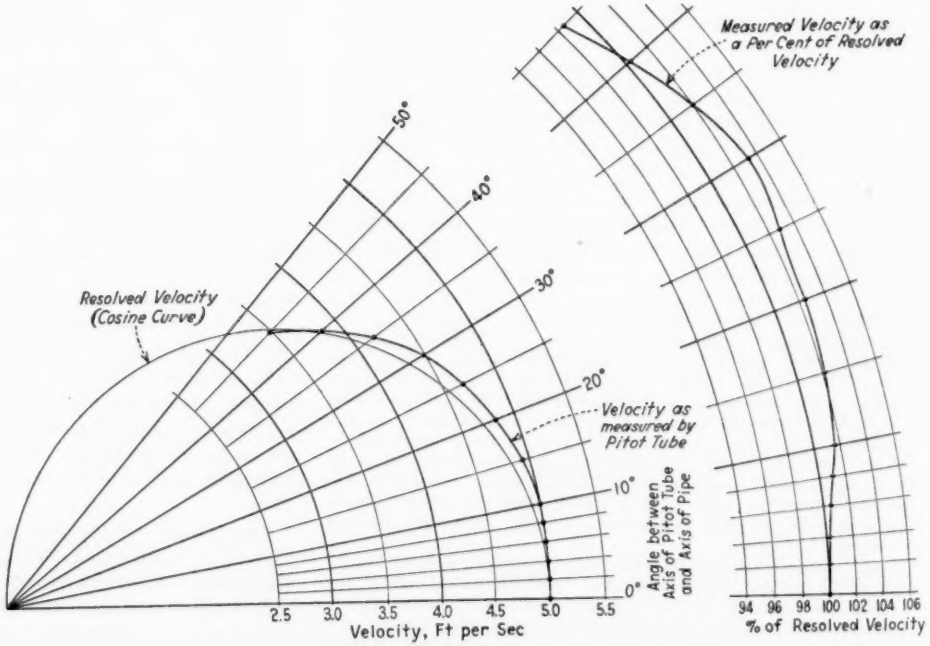


FIG. 2 PITOT-TUBE MEASUREMENTS AT VARIOUS ANGLES IN THE CENTER OF A 12-IN. DIAM PIPE WITH 5 FPS CONSTANT VELOCITY

In addition to the variation in direction of flow, turbulence also affects the readings of the pitot tube because of the variation of velocity from instant to instant, accompanied by an oscillation of the water column in the tube and its connections.

The pitot-tube reading will not give the exact head corresponding to the average velocity over an interval of time but will tend to indicate the mean head which corresponds to the mean square of the velocity. This must theoretically always be in excess of the

[6] "Measurement of the Velocity of Flowing Water," by Lewis F. Moody, Proceedings of the Engineers' Society of Western Pennsylvania, vol. 30, no. 4, May, 1914.

head corresponding to the mean velocity, and has the effect of decreasing the pitot-tube coefficient of measurement.

However, Moody in the paper[6] previously referred to states that tests made with oscillating velocity showed that the effect on the pitot tube was very slight.

Although we now have exact means for indicating, recording, and determining axial flow, the experiments cited all show that when the angular flow encountered in all conduits is being measured, a coefficient must be applied which lies not so much to the pitot tube but rather to correct for flow conditions.

The worst flow conditions at the point of the pitot tube and an angular flow of an even 30 deg shows that the coefficient of a suitable pitot-tube point such as dimensioned in Fig. 3 will not be less than 0.97. Since the angular flow in conduits at velocities usually encountered is probably more than 10 deg, in which event the coefficient is about 0.98, the maximum range of the coefficient for the photoflow method as herein described consequently lies within narrow limits.

It is therefore claimed by the authors that the adoption of 0.976 as the coefficient for use with the photoflow method in circular conduits as here set forth should have an accuracy of 0.5 per cent because of the narrow range limit of the coefficient for various conditions of flow.

The many calibrations of the pitot tube with a known velocity of flow and with properly constructed piezometers have shown coefficients of measurement of about 0.976 as most generally applicable for normal flow conditions.

The calibration of the photoflow method set forth in this paper, based on volumetric measurement of water, determines the coefficient of 0.970 to 0.973 for the case of extremely disturbed and cross-current flow, and represents therefore about the lower limit of the coefficient for this type of tube in circular conduits.

The photoflow method was originally developed by D. W. Proebstel who used it for the first time to measure the discharge at the Bull Run Plant of the Portland Electric Power Company in 1924.[7] The feature which distinguishes this method from the ordinary pitot-tube traverse is the photographic recording during a fraction of a second of all the readings of multiple pitot-tube points rigidly fixed throughout the measuring section.

This has numerous advantages. Foremost of these is the recording of all the pitot-tube readings simultaneously, thereby eliminating the inaccuracies inherent with the traverse methods because of the changing distribution of flow found at all sections within a conduit. Even though the rate of flow is kept absolutely constant during the period of time required for making a determination of the quantity flowing, there is a continual shifting around and a rearrangement of the velocity at any given section. Tests show that the variations of velocity within the section may become quite large, particularly in disturbed or turbulent flow, where instances of a 30 to 50 per cent change of velocity at one point have been recorded, without an appreciable change in the rate of flow.

The velocity changes may take place during the comparatively short time of a few seconds, but whether the change at one point is fast or slow, there must always be an immediate and opposite change at some other point or points in order to maintain the continuity of flow in the conduit.

The frequency of the changes is usually very irregular. In some of the flows observed the velocity distribution would go through a complete cycle in 2 min and again it might take 10 min. Even then it was found that with observations taken at 2-min intervals, no two sets of flow-distribution records out of ten observations were ever exactly alike within the possible limits of accuracy.

[7] "Measuring Water Flow in Conduits," by D. W. Proebstel, *Electrical World*, April 4, 1925, p. 711.

This changing distribution of flow occurring at all sections within the conduit has heretofore been the source of considerable error in the pitot-tube traverse method or in any other method in which velocities across a section are determined in rotation. The time required to make the traverse is usually such that the flow distribution may pass through several complete cycles before all readings are completed. Velocities in one portion of the section are likely to be determined during one phase of the cycle, and those in another portion during a different phase. Thus, accurate determination of the average velocity is nearly impossible with the former methods.

By use of the photoflow method, however, the velocities across the entire measuring section are recorded simultaneously and the possibility of obtaining values of velocities at different points at varying phases of the distribution cycle is eliminated. It is

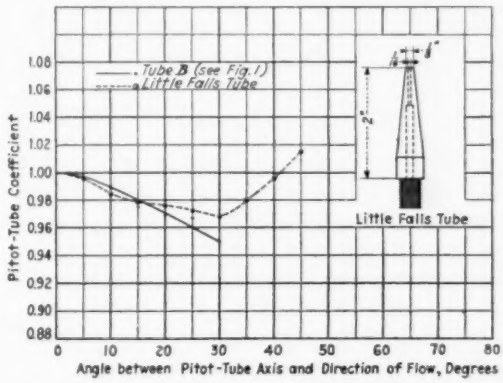


FIG. 3 COMPARISON OF PITOT-TUBE COEFFICIENTS FOR VARIOUS ANGLES OF FLOW, SHOWING NARROW RANGE OF COEFFICIENT FOR LARGE VARIATION OF ANGLE

therefore possible to make accurate determinations of the rate of flow with this method no matter how great the changes in velocity at any point within the measuring section.

In addition to the large variations in velocity at points within the measuring section, there are also the small fluctuations of high frequency found on all water columns connected to conduits containing flowing water.

Whenever these fluctuations are caused by variations in the static pressure, the pitot-tube and piezometer water columns will all move up and down together. With the photoflow method the speed of the camera can be adjusted by the use of artificial light so as to make the moving water columns appear to be standing still, and a true and simultaneous reading of all the connections will be obtained. Since the piezometer readings are subtracted from the pitot-tube readings to obtain the velocity head, fluctuations common to all the connections such as produced by variations in the static pressure will therefore have no effect on results obtained by the photoflow method.

The high-frequency fluctuations in the water columns, which may be caused by small variations at individual points in the section, will have a tendency to cancel each other. With a large number of pitot tubes the possible error from this source becomes relatively small.

Another advantage of the photoflow method is the short period of time required to record the readings. Photographs of the different velocities can be taken at the rate of about two per minute and for special cases a motion-picture camera can be used. This means that twenty determinations of the mean velocity across the measuring section can be made during a normal 10-min test run. Since, for the very worst conditions of flow, ten such determinations would give a high degree of accuracy, this method is capable of far greater accuracy than a pitot-tube

or a current-meter traverse where it sometimes takes several hours to make one such determination of the average velocity, and where the cost of making ten determinations at each gate setting tested would be excessive.

The number of measurements of the mean velocity of the entire section which can be made at regular intervals during a short period of time will also permit accurate measurements to be made of such fluctuating flow as will sometimes be encountered.

The cost of making a photoflow test is exceptionally low. No expensive equipment is required. The cost of the films or plates is negligible. Developing the plates and printing the pictures at the site saves time and reduces this item of expense. The pitot-tube points can be turned out in quantity lots at a

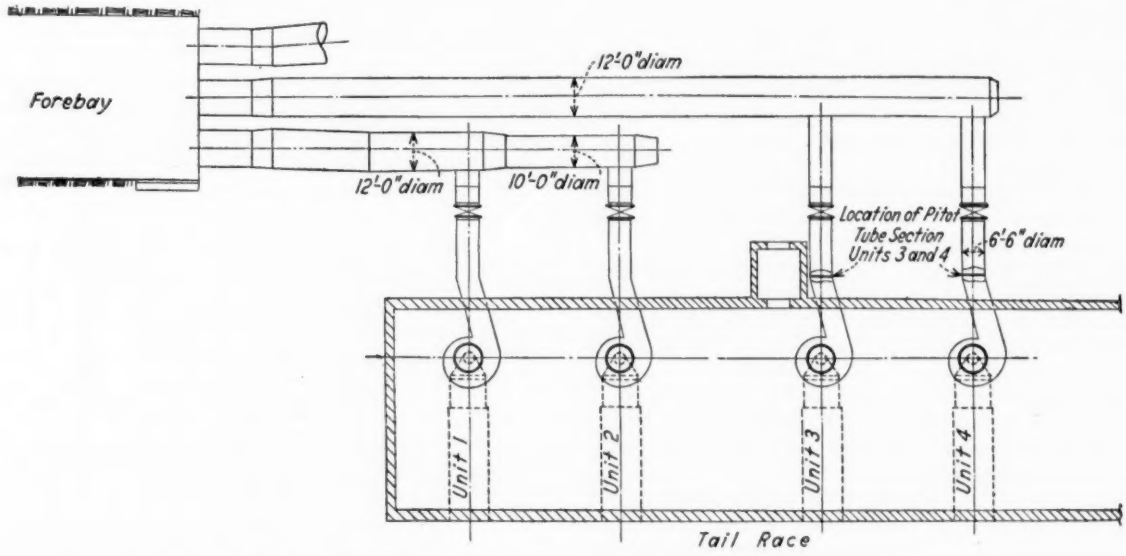


FIG. 4 GENERAL PLAN VIEW OF LITTLE FALLS PUMPING STATION

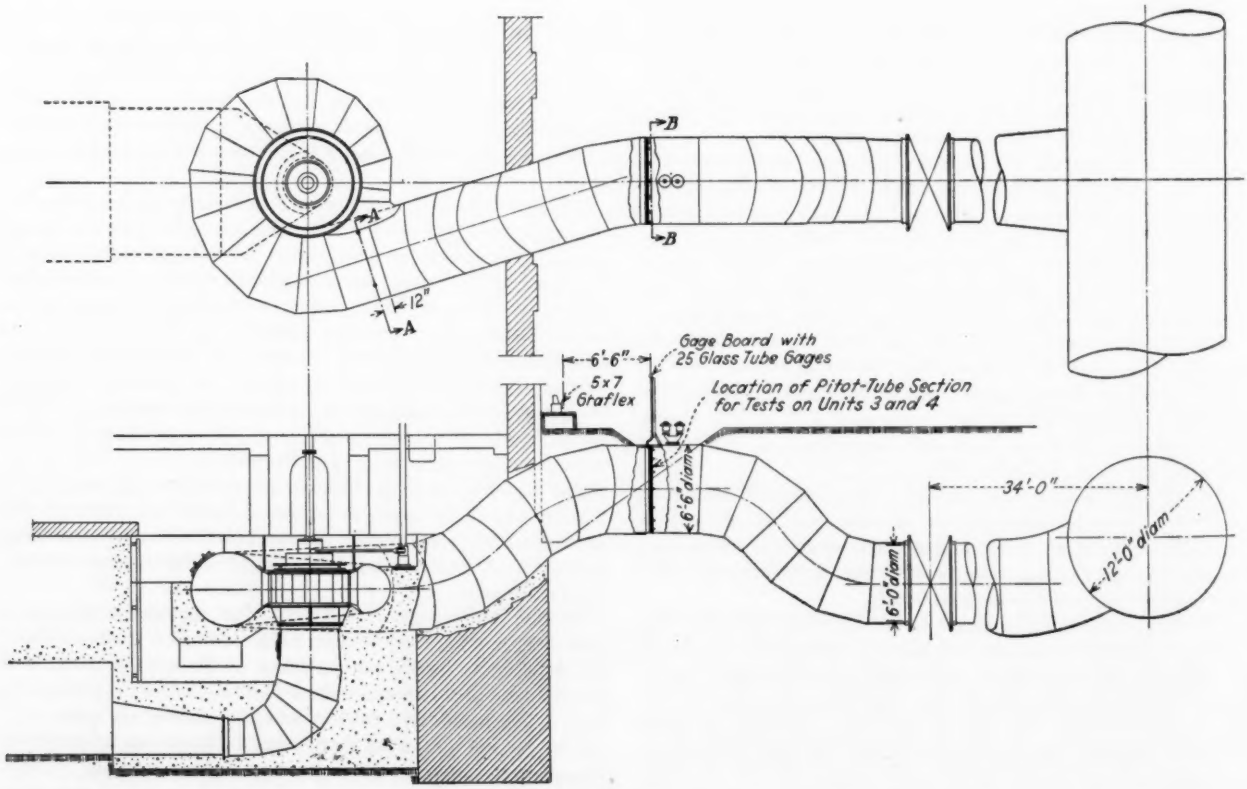


FIG. 5 CROSS-SECTION OF THE LITTLE FALLS PUMPING STATION SHOWING THE LOCATION OF PITOT-TUBE SECTION. NOTWITHSTANDING THE DISTURBED FLOW THE PHOTOFLOW METHOD GAVE CONSISTENT RESULTS AT THESE SECTIONS

nominal cost. The installation of the pitot tubes, the gridwork upon which they are fastened, and the gage connections have been found to be less costly than the usual current-meter rigging. The operating personnel can be reduced to one man who can take the photographs, develop the plates, print the pictures, and compute the discharge. Therefore, from a viewpoint of cost, the photoflow method compares favorably with any other method of measuring quantity of water.

An example of the application of the photoflow method under extremely unfavorable conditions is the acceptance test made at the Little Falls Pumping Station at Little Falls, New Jersey, on two 900-hp, 32-ft head, propeller-type turbines. Fig. 4 is a general plan view of the plant from the forebay to the tailrace. Fig. 5 is a section of the penstock and turbine showing the location of the pitot tubes.

The contract with the Passaic Valley Water Commission specified that the water be measured by current meter, salt titration, or pitot tube. Local conditions limited the current-meter measuring section to a narrow tunnel in the tailrace underneath the power station. Because of the disturbed flow in these tunnels, it was questionable whether current-meter measurements would prove to be satisfactory. Salt titration was considered to be too complicated for this installation. Therefore, only the pitot-tube method remained. The choice of a measuring section was limited. The large 12-ft-diam penstock was buried underground and under buildings and was practically inaccessible. The velocities in this penstock were low, being about $2^1/_2$ fps for one turbine at full gate. The $6^1/_2$-ft connecting penstock was full of bends and elbows but finally the short, straight section on the upper bend was selected, as is indicated in Figs. 4 and 5.

Those familiar with pitot-tube measurements would say that this was a very poor place for the use of pitot tubes. However, as will be shown later, the results obtained fully justified the confidence placed in the photoflow method as a means for measuring turbulent flow.

In the photoflow method, the number of pitot tubes to be used depends upon the size of the section, the nature of the flow, and the accuracy desired. In this case the measuring section was divided into three equal areas by three concentric circles. A pitot-tube point was located at the center of area of each of these circles on three equally spaced diameters, giving a total of 18 points located in the center of eighteen equal areas, as shown in Fig. 6. An additional point was located in the center of the penstock. It was assumed that the 18 points in this size of penstock were sufficient to give consistent results in highly turbulent flow. The test results obtained proved that this assumption was correct.

The pitot-tube points were mounted on a streamlined gridwork with their impact ends 2 in. upstream from the plane of the grid. This was for the purpose of getting them away from the influence of the obstruction to flow incidental to the grid. Where the supports have to be made fairly heavy, this distance should be increased.

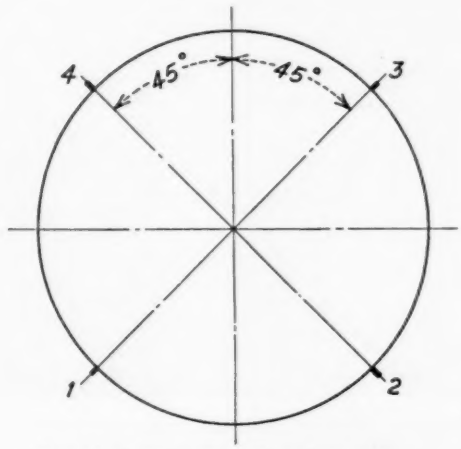
Six piezometers for determining the static head at the pitot-tube section were located in the wall of the penstock and were equally spaced around the circumference, as shown in Fig. 6.

All of the pitot-tube points and piezometers were connected to separate glass-tube gages on a single gageboard. The upper ends of the glass tubes were connected to a common manifold. Water-column elevations in the glass tubes were adjusted by means of air pressure in this manifold. For low pressures, a vacuum in the manifold can be used to raise the water columns. For very low velocities oil can be used above the water instead of air to increase the differential readings.

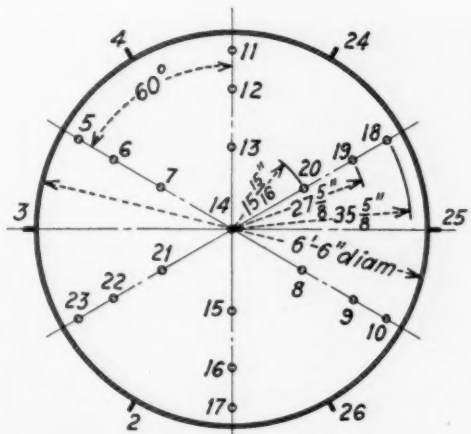
Shut-off valves, blow-off valves, and drain valves were provided for manipulation of the gages although none of the connections were throttled. A 5 in. × 7 in. Graflex camera was used for recording the readings. This was found to be about the smallest size print easily readable without enlargement.

Figs. 7 and 8 are photographs taken during a test run at 0.65 gate on Unit No. 4. The gages are numbered from left to right starting with No. 2 on the left, and correspond to the numbering shown in Fig. 6.

Tables 1 and 2 show the calculations made from these photographs in arriving at the average velocity for the section.



Section A-A showing Location of Piezometers for Measuring Pressure Head on Turbines



Section B-B showing Location and Numbering of Pitot Tubes and Piezometers

FIG. 6 PIEZOMETER AND PITOT-TUBE SECTIONS FROM FIG. 5 SHOWING DISTRIBUTION OF PITOT TUBES IN EQUAL AREAS AT THE MEASURING SECTION

The six piezometer readings were first averaged and then subtracted from the pitot-tube readings to obtain the velocity head. The velocity for each point was computed and the average indicated velocity obtained by taking the arithmetical mean. Pitot tube No. 14 was not included in this average because it was at the center of the penstock and not at the center of area of one of the 18 equal areas. Multiplying the average indicated velocity by the pitot-tube coefficient and by the area of the measuring

section gave the discharge. The pitot-tube coefficient was determined by volumetric measurement as will be shown later.

Figs. 7 and 8 and Tables 1 and 2 show the large variation in velocity which occurred at certain points, such as Nos. 8, 12, and 20, without affecting the total discharge. For instance, the variation in velocity for Figs. 7 and 8 at point No. 12 was 18 per cent, and several other points had as much as 10 per cent variation, but the difference in total computed discharge was found to be only 0.2 per cent.

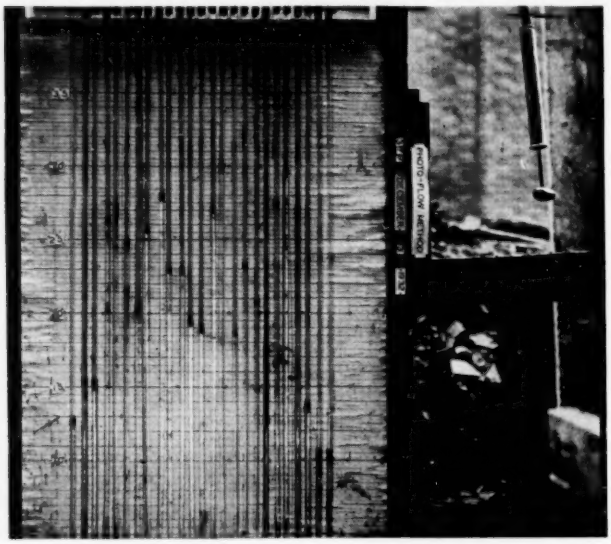


FIG. 7 PITOT-TUBE AND PIEZOMETER READINGS DURING A 0.65-GATE TEST ON UNIT NO. 4 AT LITTLE FALLS PUMPING STATION. THE WATER COLUMNS ARE NUMBERED FROM LEFT TO RIGHT STARTING WITH NO. 2 ON THE LEFT AND ENDING WITH NO. 26 ON THE RIGHT. COLUMNS 2, 3, 4, 24, 25, AND 26 ARE PIEZOMETER READINGS. THE REMAINDER ARE PITOT-TUBE READINGS

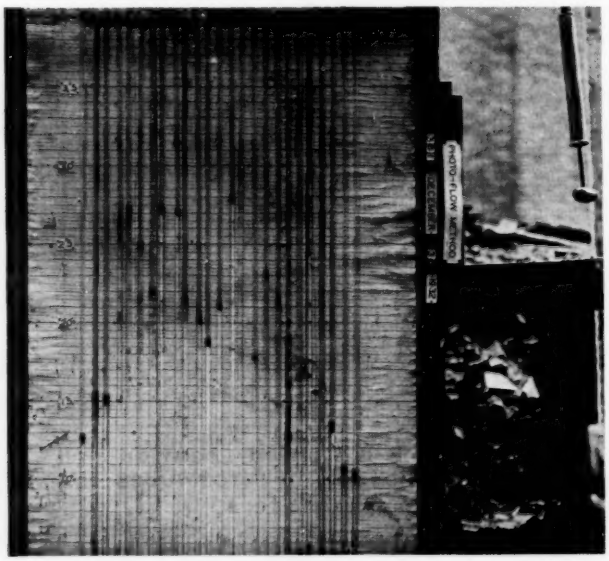


FIG. 8 THIS PHOTOGRAPH WAS TAKEN SEVERAL MINUTES AFTER THE ONE SHOWN IN FIG. 7 DURING THE SAME TEST RUN. COLUMNS 8, 12, AND 20 SHOW THE VARIATION IN VELOCITY WITHOUT AN APPRECIABLE CHANGE IN DISCHARGE

With the pitot tubes located on three straight lines across the measuring section, it was possible to draw velocity curves for

TABLE 1 COMPUTATIONS FOR READINGS SHOWN IN FIG. 7

Gage number	Pressure gage rdgs	Pitot-tube gage rdgs	Velocity head, ft	Velocity, fps	
2	1.299	...	...	...	
3	1.571	...	...	...	
4	1.555	...	...	...	
5	...	2.652	1.322	9.24	
6	...	2.741	1.411	9.54	
7	...	2.508	1.178	8.72	
8	...	2.062	0.731	6.87	
9	...	2.580	1.250	8.98	
10	...	2.819	1.489	9.80	
11	...	2.316	0.986	7.98	
12	...	2.312	0.982	7.96	
13	...	1.960	0.630	6.38	
14	...	1.903	0.573	6.08	Center
15	...	2.730	1.400	9.51	
16	...	3.003	1.673	10.39	
17	...	1.875	0.545	5.93	
18	...	2.349	1.019	8.11	
19	...	2.100	0.770	7.05	
20	...	1.790	0.460	5.45	
21	...	2.760	1.430	9.60	
22	...	2.975	1.645	10.30	
23	...	2.740	1.410	9.54	
24	1.395	...	...	...	
25	1.081	...	...	...	
26	1.078	...	...	...	
	7.979			151.35	
Average =	1.330				

151.35/18 = 8.41 average indicated velocity; angular-flow coefficient = 0.97; 8.41 × 0.97 = 8.16 average velocity; area = 33.13 sq ft; 33.13 × 8.16 = 270.4 cfs.

TABLE 2 COMPUTATIONS FOR READINGS SHOWN IN FIG. 8

[Gage number	Pressure gage rdgs	Pitot-tube gage rdgs	Velocity head, ft	Velocity, fps	
2	1.292	...	...	...	
3	1.561	...	...	...	
4	1.548	...	...	...	
5	...	2.669	1.345	9.31	
6	...	2.742	1.418	9.56	
7	...	2.520	1.196	8.78	
8	...	2.228	0.904	7.64	
9	...	2.739	1.415	9.55	
10	...	2.728	1.404	9.52	
11	...	2.170	0.846	7.39	
12	...	2.045	0.721	6.82	
13	...	1.900	0.576	6.10	
14	...	2.125	0.801	7.19	Center
15	...	2.820	1.496	9.82	
16	...	2.990	1.666	10.35	
17	...	1.790	0.466	5.48	
18	...	2.338	1.014	8.09	
19	...	2.152	0.828	7.31	
20	...	1.910	0.586	6.15	
21	...	2.841	1.517	9.87	
22	...	2.948	1.624	10.25	
23	...	2.690	1.366	9.39	
24	1.377	...	...	...	
25	1.096	...	...	...	
26	1.069	...	...	...	
	7.943			151.28	
Average =	1.324				

151.28/18 = 8.405 average indicated velocity; angular-flow coefficient = 0.97; 8.405 × 0.97 = 8.15 average velocity; area = 33.13 sq ft; 33.13 × 8.15 = 270 cfs.

TABLE 3 DISCHARGES AS DETERMINED BY PHOTOFLOW METHOD ON UNIT NO. 4 AT LITTLE FALLS PUMPING STATION

Gate	Photo no.	Discharge, cfs	Average discharge	Max. variation in discharge, per cent	Probable error by method of least squares, per cent
0.400	413	121.5	...	...	...
	414	122.6	...	...	...
	415	121.5	...	...	...
	416	123.4	...	...	...
	417	122.3	...	...	...
	418	123.2	122.4	1.6	0.20
0.575	364	216.2	...	...	...
	365	219.0	...	...	...
	366	218.1	...	...	...
	367	218.4	...	...	...
	368	219.2	...	...	...
	369	220.0	218.5	1.7	0.16
0.700	394	301.1	...	...	...
	395	298.8	...	...	...
	396	299.1	...	...	...
	397	299.4	...	...	...
	398	299.8	...	...	...
	399	301.3	299.9	0.8	0.09

three diameters and determine the average velocity by integrating the area underneath the curves. This was done for a number of tests but the results checked so closely with those obtained by taking the average of the 18 readings at the centers of equal areas that the latter method was used for all tests with a considerable saving of time and work.

It was found that six photographs for each test run were sufficient to give consistent and accurate results. With a test-run duration of 10 min this meant a photograph every 2 min. Table 3 shows the discharges as determined from these photographs at each of three gate settings. The variation of 1.7 per cent between the maximum and minimum discharge at 0.575 gate may seem rather large at first glance but by taking an average of six measurements the probable error was reduced to 0.16 per cent as computed by the method of least squares. By increasing the number of photographs taken during any one run, the probable error could be reduced still more but this was considered unnecessary for these tests.

As an indication of the consistency of results obtained with the photoflow method, the discharges were plotted against the gate opening as shown in Fig. 9. The small deviation from a smooth curve is all the more remarkable in view of the poor measuring section and the widely fluctuating distribution of flow as was

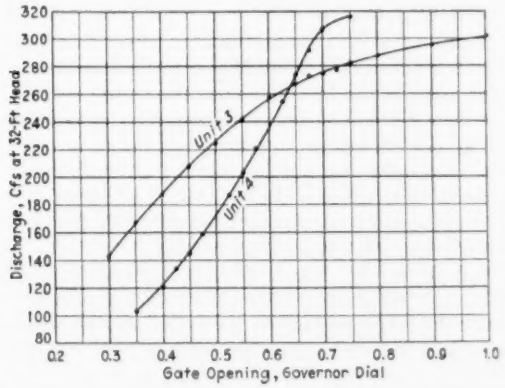


FIG. 9 DISCHARGE CURVES BY THE PHOTOFLOW METHOD ON TWO UNITS AT LITTLE FALLS PUMPING STATION, SHOWING CONSISTENCY OF RESULTS OBTAINED BY THIS METHOD

plainly evident in these tests. Unit No. 4 has an adjustable blade runner which accounts for the difference in the two discharge curves.

One of the interesting features of these tests was the static-head reading of the six piezometers. An analysis of the records in Figs. 7 and 8 and in Tables 1 and 2 shows that although there was considerable variation in the static pressure, in each instance the average of two readings diametrically opposite was the same as the average of any two other opposite readings. Furthermore, the difference between, say, piezometer No. 6 (Fig. 6) and No. 25 was the same as the difference between No. 2 and No. 26, etc. This is shown graphically in Figs. 10 and 11 where the pressures are plotted against both the horizontal and vertical distances across the penstock.

The full lines connecting the readings are practically parallel and indicate the relation between the readings in a given direction. The dash lines connect readings diametrically opposite and show the close agreement of the averages at the center of the penstock. This led to the conclusion that the pressure across the penstock was largely dependent upon the centrifugal force produced by a change of direction of the path of the water. Theoretically this pressure varies as the square of the radius of the curved path, but since the distance across the penstock is small compared to this radius, very little error is introduced by assuming that the pressure varies directly as the radius.

Using this latter relationship to determine the actual pressure at each pitot tube, new velocities were computed for several of the tests but the final average velocity thus obtained checked very closely with that obtained by using the average of all the piezome-

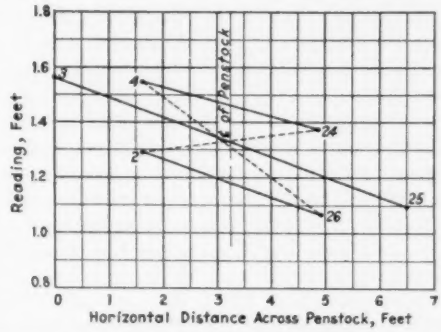


FIG. 10 INTER-RELATION OF PRESSURE READINGS TAKEN FROM FIG. 8, SHOWING WHY PIEZOMETERS FOR CLOSED CONDUITS SHOULD ALWAYS BE LOCATED IN PAIRS, DIAMETRICALLY OPPOSITE EACH OTHER

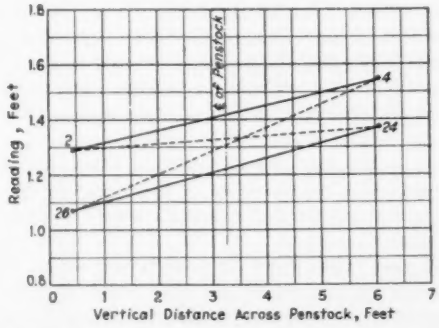


FIG. 11 INTER-RELATION OF PRESSURE READINGS TAKEN FROM FIG. 8

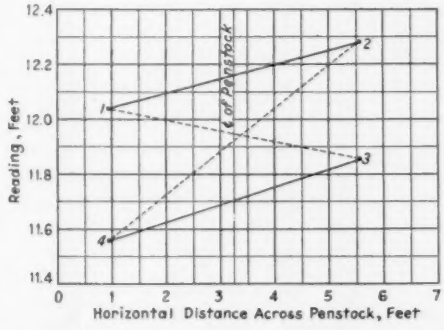


FIG. 12 INTER-RELATION OF PRESSURE READINGS AT SECTION *A-A*, FIG. 6, WITH A DISCHARGE OF 286 CFS. THIS IS ANOTHER INDICATION WHY PIEZOMETERS IN CLOSED CONDUITS SHOULD BE LOCATED DIAMETRICALLY OPPOSITE EACH OTHER

ter readings as was done in the computations shown in Tables 2 and 3.

Fig. 12 shows the pressure readings of the four piezometer connections at section *A-A*, Fig. 6, at the entrance to the scroll case. This is another example of the relation between pressure reading and radius of curvature of the path of the water. In this case the pressures are affected by the bend in the penstock directly upstream from the section and by the beginning of the scroll-case curvature.

In measuring the static pressure in closed conduits it is, therefore, most important that the piezometers be located diametrically opposite each other and that an even number of piezometers such as two, four, or six be used instead of one, three, or five, etc.

The contract with the Passaic Valley Water Commission for the Little Falls Pumping Station units called for a bonus and penalty on overall efficiency. It was, therefore, essential that the measurement of discharge as well as the power and head measurements be as accurate as possible in their absolute values. Since the photoflow method was being used in a section with considerably greater disturbed and turbulent flow than ever attempted before with pitot tubes, it was decided to determine the coefficient of measurement of the pitot-tube points in place by volumetric measurement of the water. It was found that the forebay with an area of 70,000 sq ft could be isolated from the river by means of head gates and used as a huge volumetric measuring tank. The maximum possible drawdown of the forebay was 4 ft. Thus, with a maximum discharge of 300 cfs through one of the turbines it was possible to calibrate the photoflow method for a period of 15 min for each drawdown, which proved to be sufficient to give accurate results.

The volume of the forebay was determined by first making a drawdown and then filling it through a calibrated venturi meter.

Leakages into and out of the forebay were reduced to less than 1 per cent of the maximum discharge through the turbine by making all the gates and valves connected to the forebay as tight as possible. In addition, a leakage determination was made before and after each test run, thereby reducing the possible error from this source to a negligible amount.

Four hook gages were placed in stilling boxes at various stations for measuring the elevation of the water in the forebay. By the aid of verniers the water elevations were read to the nearest one thousandth of a foot.

Readings were taken every 30 seconds and were all synchronized with the photoflow pictures by a system of light flashes. The fluctuations of elevation caused by waves set up by the drawdown and fill-up were not very large, and were of such low frequency that readings every 30 seconds accurately determined their outline.

The close agreement between the elevations as measured at the four stations indicated that the probable error from this source was relatively small.

In filling the forebay through the venturi meter to determine the forebay area at the different elevations, the penstock-gate valves leading to all the turbines were closed, but the unit under test was left motoring on the line. After the water in the forebay had reached the desired elevation the proper penstock-gate valve was opened, the turbine gate set to give the required rate of flow at the pitot-tube section, and the drawdown test was made to determine the photoflow coefficient.

The area of the forebay at various elevations was determined six times. The probable error of the average of the six determinations was estimated at less than 0.25 per cent.

Nine determinations of the photoflow coefficient were made with the pitot tubes located in the penstock of Unit No. 3, and seven determinations of the coefficient with the pitot tubes located in Unit No. 4. Tests were made with different rates of flow but the results indicated that the coefficient did not change with a change in velocity. The average coefficient for Unit No. 3 for all rates of flow measured was 0.973 and for Unit No. 4 was 0.970. Because of the large number of measurements the probable error was less than 0.25 per cent.

The results agree with the previous statement that highly disturbed and turbulent flow would have a coefficient close to 0.970.

In conclusion, it can be stated that the results obtained with the photoflow method so far have been very satisfactory and indicate its possibilities as an accurate, quick, and economical means of measuring the flow of water. The excellent results obtained with this method in the tests described herein show that it can be used under conditions heretofore thought to be entirely unsuited for pitot-tube measurements.

Although the method has been used only in closed conduits, it should give satisfactory results in open conduits. There apparently is a large field for its use in the measurement of the large quantities of water used by low-head turbines.

Pitot-Tube Practice

By EDWARD S. COLE,[1] NEW YORK, N. Y.

The author reviews the history of the pitot tube and, on the basis of experimental results, discusses the accuracy of the pitot-traverse method of water measurement. In connection with the practical application of the pitot tube to flow measurement, the author discusses the calibration of the tube, corrections for the effect of the projected area of the rod, and the errors caused by angularity of flow in a pipe. The degree of angularity of flow in the normal pipe line was investigated by means of a current-vane indicator and a motion-picture camera. Reference is made to investigations of the effect of eddying flow on pitot-tube readings as conducted by means of a series of ratings made with three pitot tubes of similar design used under three different conditions of flow, viz: Still water, the smooth or parallel flow of a large venturi throat, and the irregular or eddying flow of a 12-in. pipe line. In making the comparative ratings, coefficient corrections are made for the projected area of the pitot tube in the 12-in. pipe. The author shows that with normal flow in a pipe line, the manometer readings will vary through a series of cycles. These cycles, with pitot tubes in a 40-in. and a 12-in. pipe, were observed with a motion-picture camera.

IT IS the purpose of this paper to present some of the results of long experience with the so-called pitot-traverse method of water measurement in the hope that engineers may more widely recognize its accuracy.

In order to avoid repetition later on in this paper, it may be advisable to define the terms "simple" and "combined" pitot tubes. The simple type consists of one forward-facing tube which reads the dynamic head when used in an open stream. When used in a pipe under pressure it reads the dynamic plus the static head and in order to read velocity, the static head must be deducted by means of wall piezometers. The combined type is usually made up of two tubes; one reading the dynamic head, the other the static head. The combined type may have different forms, although variations in design are chiefly concerned with the static orifice.

History of the Pitot Tube

The history of the pitot tube began in 1730 when Henri Pitot[2] using a bent glass tube in the River Seine discovered that the height to which water rose in his tube was proportional to the square of the velocity of the stream. Aside from its extreme simplicity, this device was novel in that it measured velocity without introducing the time element. Contrary to general belief, Pitot used the combined form with double tubes and described it in his paper,[2] Fig. 1. The second or static tube had its opening in the end of a straight pipe pointing directly across the flow and therefore it must have been subject to errors. It may not be generally known that Pitot in his paper of 1732 described quite clearly an application of his tube for measuring the speed of a small sail boat which he navigated on the Seine. Since then there have been several attempts to make a successful ship log on this principle. A pitot log was used on the Great Lakes by Nicholson, probably fifty years ago, but owing to one defect or another it did not come into extensive use.

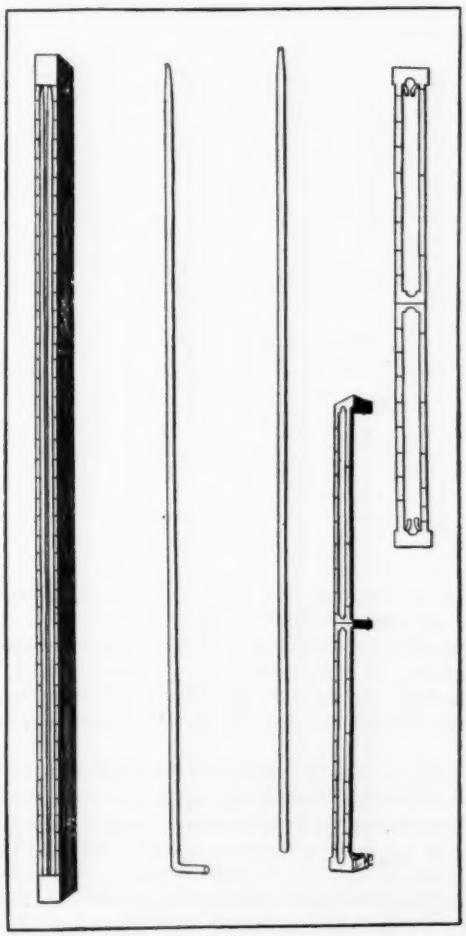


Fig. 1 Drawing of Pitot's Original Tube, 1732

Pitot's combined tube was improved one hundred and twenty-five years later by Darcy, another French engineer who in one form turned the static tube downstream and in another gave it only lateral openings which more nearly read the true static pressure, Fig. 2. Darcy also reduced the vibrations in the water

[1] President, The Pitometer Company, Engineers. Mem. A.S.M.E. Mr. Cole received the degree of M.E. from Cornell University in 1894 and from that date until 1902 served on the staff of John A. Cole, consulting engineer, Chicago, working on the design, construction and management of city water-works plants in the Middle West. In 1896 he developed the pitometer. As a member of the Department of Water Supply, Gas, and Electricity of New York, N. Y., he was in charge of water-supply studies with the pitometer during 1902 and 1903. Since that time he has conducted pitometer investigations throughout the country.

[2] "Description d'une machine pour mesurer la vitesse des eaux courantes, et le sillage des vaisseaux," by H. Pitot, Memoires de l'Academie Royale des Sciences, November 12, 1732.

Contributed by the Hydraulic Division and presented at the joint meeting of the Hydraulic Division, A.S.M.E. and the Power Division, A.S.C.E., New York, N. Y., January 17, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

columns by using orifices of much smaller area than the tubes.[3]

Bazin, another French engineer, made extensive use of the Pitot-Darcy tube.[4]

Hiram F. Mills of Boston was probably the first American engineer to use the pitot tube, making studies of the flow of

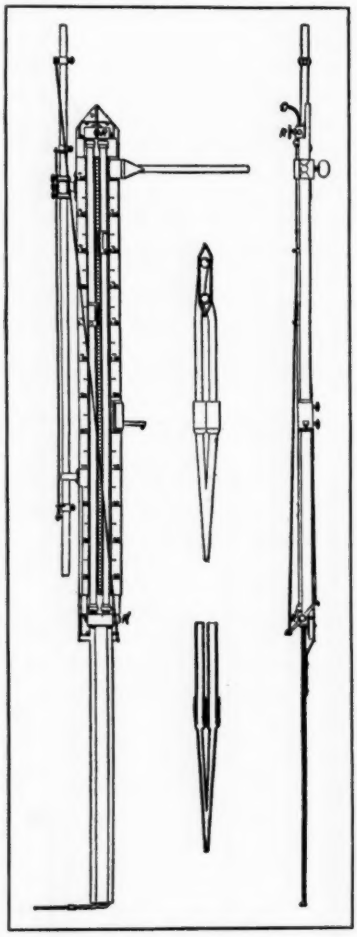


FIG. 2 DARCY'S TUBE, 1858

water in a 12-in. cast-iron pipe at Lawrence, Mass. in 1875. These experiments unfortunately have never been published. The original notes came into the hands of the late John R. Freeman and are now being searched for the exact form of pitot tube used by Mills and the method of its calibration. There can be no doubt that he was the first American experimenter in this field.

It is believed that Mills preferred the simple form of pitot tube with static pressure taken from wall piezometers as distinguished from the combined type used by Pitot, Darcy, and Bazin.

Mills wrote[5] that he "made measurements of the quantity of water drawn by many of the water wheels of the Lawrence Mills in the feeding penstocks by instruments devised by me in 1877 and used by my assistants under the direction of John R. Freeman during 1878 and 1885, and later under the direction of Richard A. Hale. These instruments indicated the velocity at 16 or 18 points in the section at the same moment."

[3] *Annales des Ponts et Chausses*, 1858.

[4] Ibid., 1890.

[5] Discussion by H. F. Mills of "Experiments at Detroit, Mich., of the Effect of Curvature Upon the Flow of Water in Pipes," by Williams, Hubbell, and Fenkell, Trans. A.S.C.E., vol. 47, 1902, p. 203.

John R. Freeman[6] followed Hiram Mills in the use of the pitot tube and in 1888 adopted the simple type for use in measuring fire-stream jets finding an accuracy of within 0.25 per cent. "This investigation established the fact that the pitot tube is an instrument of great precision for the measurement of high velocities."[7]

Henry Flad, a distinguished engineer of St. Louis, about 1888 carried on pitot-tube studies, and probably was the first to attempt the use of photography for recording the manometer deflections but in this he was said to have been unsuccessful.

The author's first work with the pitot tube began in the fall of 1896 at Terre Haute, Ind., where a practical form of instru-

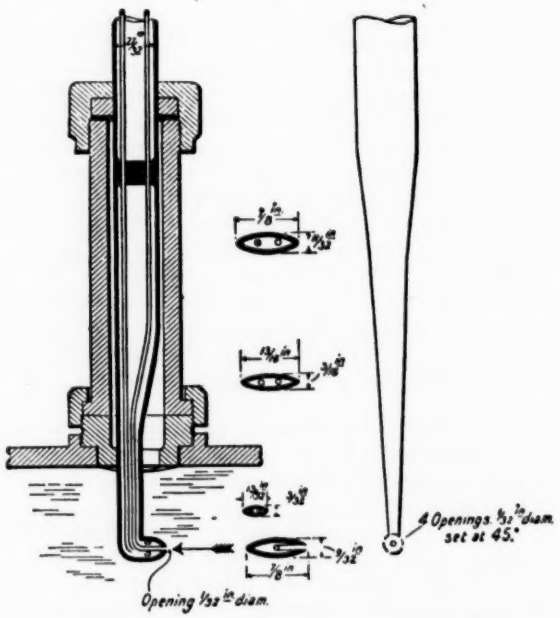


FIG. 3 TUBE USED BY WILLIAMS, HUBBELL, AND FENKELL IN 1897

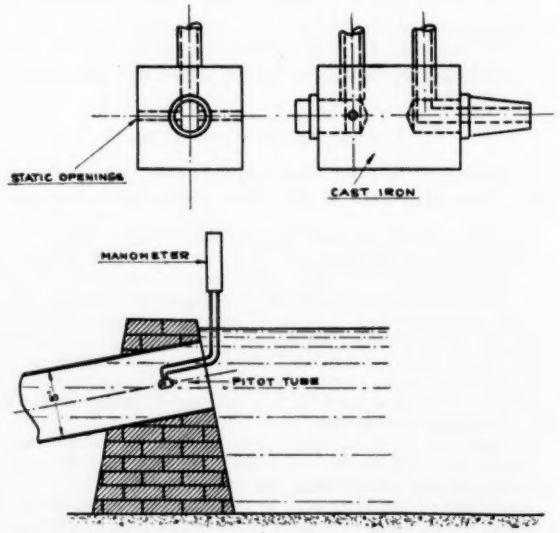


FIG. 4 TUBE USED BY W. B. GREGORY AT CORNELL UNIVERSITY IN 1894

[6] "Experiments Relating to Hydraulics of Fire Streams," by John R. Freeman, Trans. A.S.C.E., vol. 21, 1889, p. 411.

[7] "A Treatise on Hydraulics," by M. Merriman. Eighth edition. John Wiley & Sons, Inc., New York, p. 103.

ment was developed for use in pipes under pressure. A description of this development was published in the A.S.C.E. Transactions, vol. 47, 1902, page 275.

The work of Williams, Hubbell and Fenkell, which began at Detroit in 1897, resulted in a combined form of pitot tube for use in pipes under pressure,[8] Fig. 3.

Gregory and Mason in tests conducted in connection with their thesis, at Cornell University in 1894, used a pitot tube, Fig. 4, introduced at the open end of a 5-ft steel pipe.

Later Gregory and Maltby developed a form of combined pitot tube at Ithaca, N. Y., in 1903, Fig. 5. Although not the most convenient form for inserting into a pipe under pressure it is a highly successful instrument which has been extensively used by Professor Gregory in his professional practice and by the New Orleans Sewage and Water Board for the past thirty years.

White's studies of the pitot tube in 1901 developed the theory of the simple dynamic orifice and demonstrated the formula to be $h = V^2/2g$ rather than $h = V^2/g$ which had been urged by some writers at that time.

Pardoe, at the University of Pennsylvania in 1908, succeeded Professor Easby in pitot-tube experiments. Prof. C. G. Hyde, of the University of California, made careful calibration tests of the

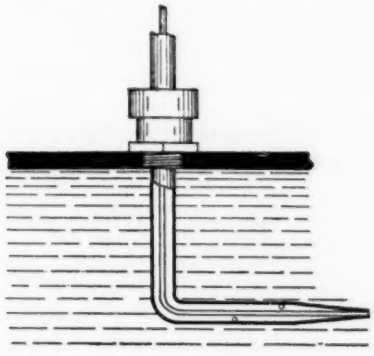


FIG. 5 PITOT USED BY GREGORY IN 1903

combined pitot tube, Fig. 6, in 1915. E. C. Murphy[9] made careful ratings of the pitot tube in moving water as well as in still water. Stanton studied velocity variation close to the pipe wall. Boyd and Judd[10] in 1904 repeated Freeman's work with jets. They reported that "discrepancies in pitot tubes are due to failure to get the true static pressure rather than to any error in the pitot tubes themselves."

The theory of the dynamic tube or orifice has been well developed by Moody and Rogers[11] and others. Its law is $V = c\sqrt{(2gH)}$.

The theory of the static orifice is more complicated than that of the dynamic orifice and is best represented by laboratory observations such as were made by Allen and Hooper at Worcester Polytechnic Institute.[12] In its ideal form the wall piezometer is simple enough, but in practice the author found it difficult to secure ideal conditions and prefers to follow Darcy and Bazin in obtaining the static head as in the combined type, i.e., along with the dynamic orifice.

DEVELOPMENT OF THE PITOMETER

At the time of the author's experimental work with pipes under pressure at Terre Haute, Ind., water-works plant, there was little available information on the pitot tube. Darcy's published results related entirely to open-stream use. A crude sketch in Carpenter's "Experimental Engineering,"[13] Fig. 7, suggested the use of an impossible combination of tubes and of manometers.

The few published references to pitot-tube accuracy were discouraging. One author[14] wrote, "Pitot's tube has been but little used and is in general regarded as an imperfect instrument for velocity determinations."

After many attempts it was seen that a practical pitot-tube method for measuring the flow of water in pipes under pressure must embody (1) a suitable means for inserting the tube into the pipe under pressure, (2) a practical and accurate method for calibrating the instrument, (3) a differential manometer free from air which contains a liquid

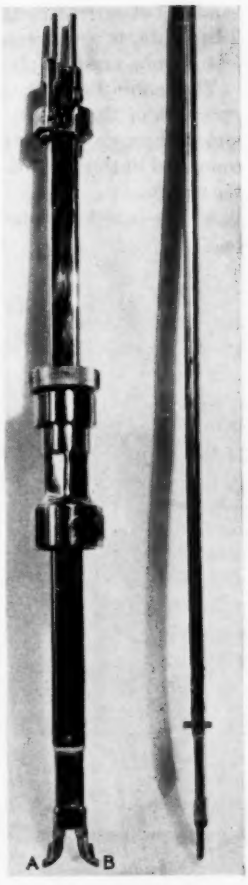


FIG. 6 AUTHOR'S PITOT TUBE FOR PIPES, 1896

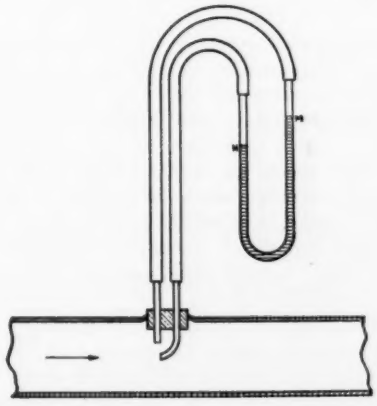


FIG. 7 CARPENTER'S SKETCH OF PITOT TUBE FOR HIGH PRESSURES, 1892

permitting the magnification of the deflections at low velocities, (4) a method of integrating the flow from a velocity traverse, and (5) a means for the frictionless recording of manometer deflections where a continuous record of flow is necessary.

With the extensive literature now available regarding flow measurement in pipes under pressure, it is difficult to realize the

[8] "Experiments at Detroit, Mich., on the Effect of Curvature Upon the Flow of Water in Pipes," by Williams, Hubbell, and Fenkell, Trans. A.S.C.E., vol. 47, 1902, p. 1.

[9] Discussion by E. C. Murphy of "Experiments at Detroit, Mich., on the Effects of Curvature Upon the Flow of Water in Pipes," by Williams, Hubbell, and Fenkell, Trans. A.S.C.E., vol. 47, 1902, p. 197.

[10] "Pitot Tubes; With Experimental Determinations of the Form and Velocity of Jets," by J. E. Boyd and H. Judd, *Engineering News*, vol. 51, 1904, p. 318.

[11] "Measurement of the Velocity of Flowing Water," by L. J. Moody, Proceedings, Engineers' Society of Western Pennsylvania, vol. 30, no. 4, May, 1914. Also discussion of this paper in vol. 30, no. 5, June, 1914.

[12] "Piezometer Investigation," by C. M. Allen and L. J. Hooper, Trans. A.S.M.E., vol. 54, 1932, paper HYD-54-1.

[13] "Experimental Engineering," by R. C. Carpenter. Third edition, John Wiley & Sons, Inc., New York, 1892, p. 266.

[14] "A Treatise on Hydraulics," by M. Merriman. Fourth edition, John Wiley & Sons, Inc., New York, 1890.

handicap of early experimenters in this field since, lacking published data, it was necessary to work out each of these essential features of a practical pitot tube for pipes.

The calibration of the instrument was obtained by float measurement of the velocity in a trough, the instrument being inserted through the bottom. Considerable difficulty was encountered in this method as the floats did not run uniformly down the trough.

A pitot tube following Professor Carpenter's sketch, Fig. 7, of

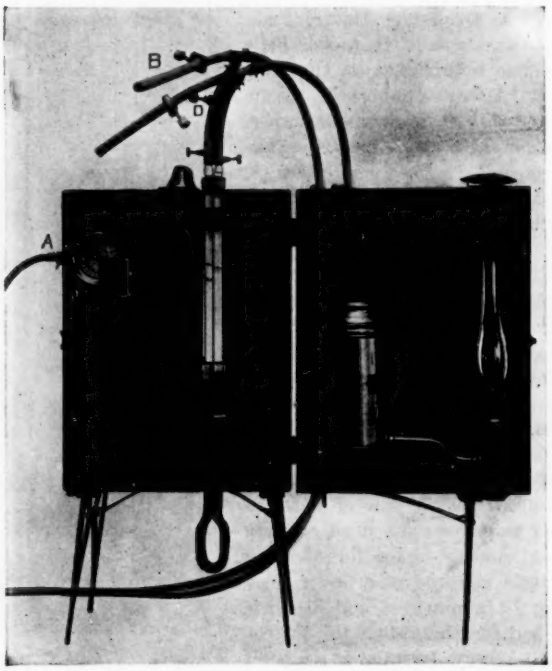


FIG. 8 PHOTO-RECORDER WITH PRISM ATTACHMENT DEVELOPED IN 1897 AND 1908 FOR RECORDING MANOMETER READINGS

course, gave highly erratic readings but the manometer caused the first concern. This crude form, with air above the measuring liquid, and without means for removing it, was the only suggestion available at that time. Such a form with its rapid U-tube fluctuations was quite impossible to read with any precision. It was only when all air was excluded from the manometer and connections, allowing the water to fill the U-tube above a heavier and immiscible liquid, that reliable readings were obtained. In later years, cases were discovered in which failures of pitot tubes were caused largely by this troublesome presence of air in the manometer.

It was evident that the readings were extremely small at the low velocities of 1 fps or lower commonly found in water-works mains but with carbon tetrachloride properly mixed with benzine to give a specific gravity of 1.25, a differential of four to one was available, which proved to be most useful.

After overcoming U-tube troubles it was next in order to discover why the hook-and-point type of pitot tube gave such erratic and inconsistent results on calibration. After many attempts, the Darcy plan of turning the static tube downstream was tried and immediately the calibration points began to fall within more narrow limits and a practical form, Fig. 6, was at last available.

Many years of experience with this combined type of pitot tube or pitometer has shown that it has the unique advantage of reversibility. A ready and rapid means of checking its own reading is afforded by rotating through 180 deg. This is of great practical value in the field where it is usually difficult to obtain a zero check.

Later, in 1897, during a systematic field survey carried out with the newly developed pitometer at Terre Haute, the need was felt for continuously recording the U-tube deflections, and photography was tried with poor results until the proper method of focusing the U-tube on the sensitized paper was found. At first one leg of the manometer was placed before a slot on the other side of which was a revolving drum carrying the sensitized paper. Light from an oil lamp passing through the glass tube focused upon the slot, thus leaving a record of the liquid height.

With slightly colored liquid partially intercepting the light, a gray-and-black record was formed. With mercury as the measuring liquid for high velocities the light was entirely intercepted giving a white-and-black record. Errors resulting from the vertical angularity of the light rays were compensated for by properly spaced notches in the slot plate which automatically ruled lines upon the photo record corresponding to the true half deflections in the U-tube itself.

After some ten years of use this device was improved by the addition of a double prism between the lamp and the U-tube so

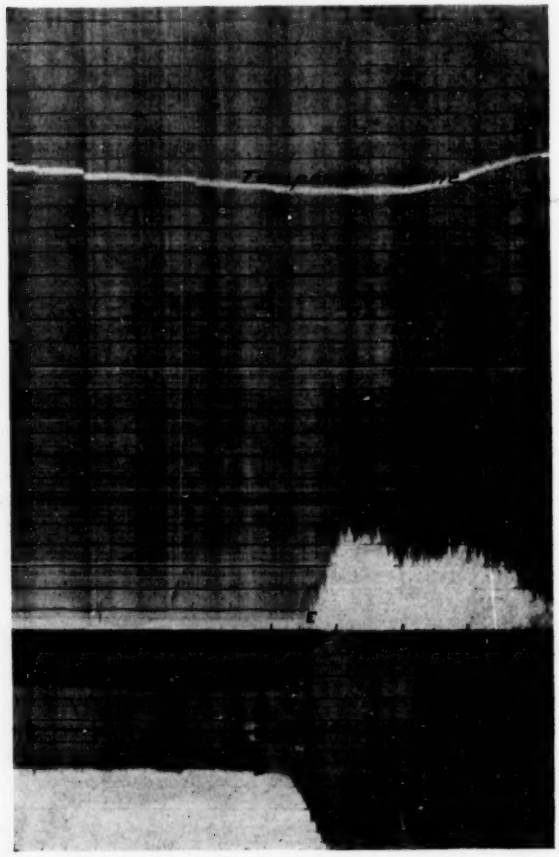


FIG. 9 A REDUCED SECTION OF A PRISM PHOTO-RECORD, RECORDING BOTH LEGS OF U-TUBE MANOMETER

that the liquid height in both legs of the manometer was recorded to 3 in., Figs. 8 and 9. Pressure and temperature changes were also recorded along with the velocity variations. The original design of the tube with its upstream and downstream orifices has been improved upon from time to time, but the essential principles of the pitometer developed in 1896, have been retained.

Practical Application to Flow Measurement

From nearly 40 years of experience it has become apparent that certain fundamentals must be considered in order to insure accuracy of flow measurements. Some of these are intimately related to calibration while others are details in technique of operation.

The following items must be considered in connection with calibration:

1 For important tests the tube must be calibrated.

2 Proper correction must be made for the effect of the projected area of the rod.

3 The effect of angular and eddying flow in the normal pipe line may be corrected by a cosine-reading type of tube such as shown in Fig. 29.

The following details of operation which comprise the pitot-traverse method are important:

(*a*) A proper selection of the gaging point must be made.

(*b*) The pipe factor, i.e., ratio of mean to center velocity, is of fundamental value and is determined by traversing the pipe. The mean velocity of the traverse is obtained by ring integration, and the value for the center velocity is taken from the traverse curve.

(*c*) All readings of the deflection should be made throughout a full cycle of flow and careful attention given to the U-tube and its connections, and the specific gravity of the measuring liquid.

(*d*) For large pipes attention should be given to the possible effect of vibration of the pitot tube.

1 Calibration

The determination of c in the formula $V = c\sqrt{(2gH)}$ must be determined by calibration. Calibrations may be made in moving water or in still water, and present indications lead the author to believe that with the proper correction for the projected area of the rod, when used in pipes, calibrations by either of these methods will give the same results.

In moving water the pitot-tube readings are compared with velocity as given by floats in an open stream or, if the rating is in a pipe, the pitot tube itself is used to compute the mean velocity from a velocity traverse, which is then compared with the mean velocity given by weir or tank measurement.

A still-water rating may be made by moving the pitot tube ahead of a boat drawn at known speeds or by driving it through still water by a revolving boom or a car running over a tank.

Before describing the calibration tests of the author's tube, it may be interesting to present a summary of some available calibration data on both types of pitot tube. This summary is given in Table 1.

Calibration Research, 1930. In order to determine accurately what errors are introduced in the manufacture of the tubes and in their calibration, seven different tubes were tested at the Alden Hydraulic Laboratory, Worcester Polytechnic Institute, in 1930. Five of the tubes conformed to a standard while two others differed slightly in curvature of tips. These tubes were calibrated in the 16-in. throat of the 36 × 16-in. venturi meter, and in the 12-in. line. The results are as shown in Fig. 10.

TABLE 1 SUMMARY OF PITOT-TUBE CALIBRATION DATA

Experimenter	Method of calibration	Type of tube	Coefficient	Remarks
Darcy, Bazin	Still water, floats in moving water, and measurement of known flow	Combined (Fig. 2)	1.00	*Annales des Ponts et Chausses*, 1858 and 1890.
Gregory and Maltby	.....	Combined (Fig. 5)	1.00	Used and calibrated by the New Orleans Sewage & Water Board.
White	Studies of pitot tube	Simple	1.00	Tube not calibrated but $V = \sqrt{(2gH)}$ proved in which case $c = 1$.
Pardoe	Weighing-tank measurement of flow in pipes	Simple	1.00	University of Pennsylvania, 1908.
Pardoe	8-in. and 4-in. pipes, flow by weighing tank	Simple	0.982	University of Pennsylvania, 1934.
Boyd and Judd	.....	Simple	1.00	*Engineering News*.[10] Freeman's work repeated, 1904.
Alden Hyd. Lab., Worcester P. I.	16-in. throat of venturi meter, 12-in. steel pipe	Simple	0.994	Impact orifice, Fig. 6, 1930.
Darcy, Bazin, or Murphy as summarized by Parker[1]	Floats in moving water	Darcy	1.006	92 ratings.
	Moving water—weir	Darcy	0.993	87 ratings.
	Floats in still water	Darcy	1.034	32 ratings.
	Moving water	Combined	0.875	Both orifices directed against current but pressure orifice plugged and a small hole 0.04-in. diam, pierced laterally.
	Still water	Combined	0.864	
	Floats in still water	Combined	0.998	Impact orifice directed against the current, pressure orifice facing downstream.
	Still water	Combined	0.991	

[1] "Control of Water," by P. M. Parker, D. Van Nostrand Company, New York, 1913, p. 69.

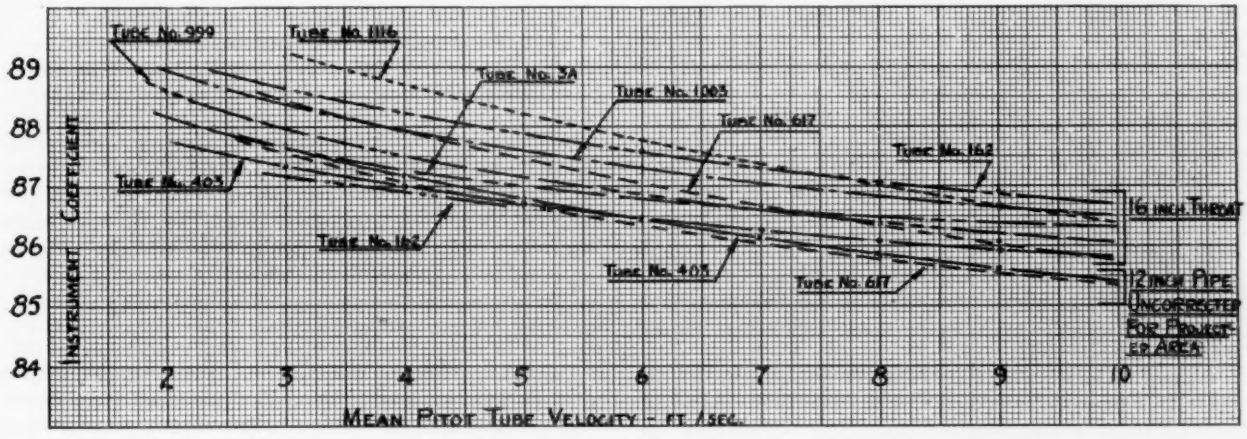


FIG. 10 RESULTS OF CALIBRATION TESTS OF PITOT TUBES AT ALDEN HYDRAULIC LABORATORY, WORCESTER P. I., 1930

(NOTE: Tubes nos. 999 and 1003 do not fit template.)

An explanation of the 1930 tests will aid in making comparisons of the various calibrations presented.

The 16-in. throat of a 36-in. venturi meter has long been used for calibrating purposes at this laboratory because of the smooth flow and flat velocity curves with their high pipe factor of about

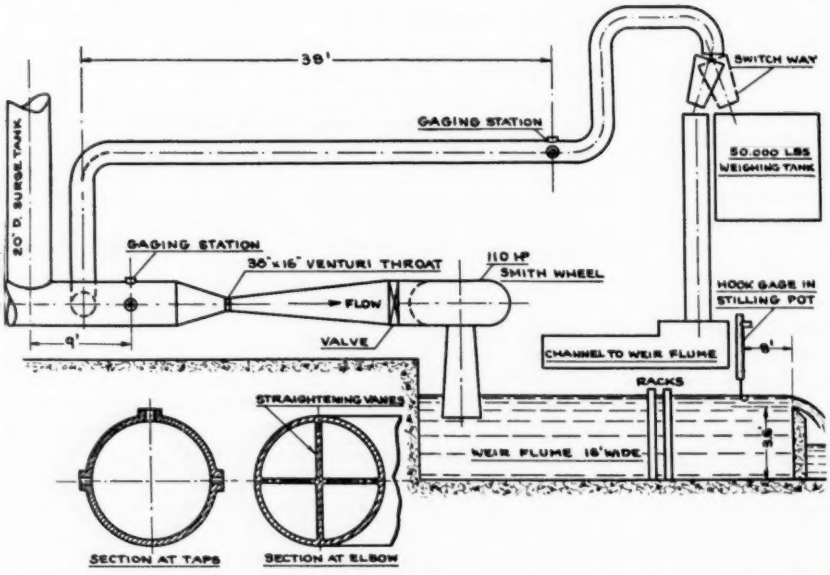


FIG. 11 APPARATUS USED FOR RATING PITOT TUBES AT ALDEN HYDRAULIC LABORATORY

server, and the figures set up on a calculating machine. Ten readings of head were taken for each position of the rod shown in Fig. 6, i.e., with *A* tip upstream and with *B* tip upstream. The head on the weir was read and then the discharge was adjusted to a new value for the next test.

Variations in velocity under the uniform head available were so small that the mean of the readings was assumed to be the mean head for each run instead of taking the average of the square roots of the U-tube deflections.

The test procedure in the 12-in. line calibrations was practically the same as in the 16-in. throat with the exception that the mean velocity was obtained by weighing-tank measurements. The piping layout for these two gaging stations is shown in Fig. 11.

The 12-in. steel pipe line is believed to afford fairly normal flow conditions under the constant laboratory head of about 30 ft. The effect of two standard pipe elbows in different planes at the entrance was minimized by the use of straightening vanes as shown by the excellent symmetry of the velocity traverses in Fig. 13.

As may be noted in Fig. 10 there was a difference in coefficient between the 16-in. throat and the 12-in. line and further

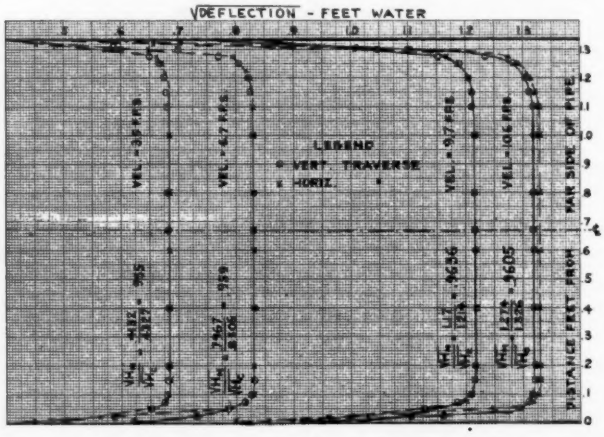


FIG. 12 TRAVERSE CURVES OF FLOW IN 16-IN. VENTURI THROAT, ALDEN HYDRAULIC LABORATORY, 1934

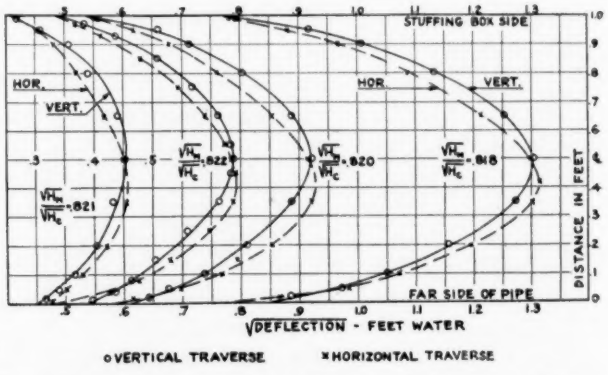


FIG. 13 TRAVERSE CURVES OF FLOW IN 12-IN. PIPE. GAGING STATION 38 FT DOWNSTREAM FROM STRAIGHTENING VANES. ALDEN HYDRAULIC LABORATORY, 1934

0.96. For traversing, the pitot tube was inserted through 1-in. cocks 90 deg apart in the bronze throat piece, and for calibration the orifices were located 4 in. in on the horizontal diameter. The flow was measured over a 10-ft contracted weir which had been calibrated by the weighing tank, using the increment method as in Fig. 11.

Pipe factors, described in detail later, for the 16-in. and 12-in. sections, were carefully determined by means of double traverses before and after the series of tests. Sample traverse curves are shown in Figs. 12 and 13.

The test procedure in the 16-in. throat was as follows: After the discharge had been set to a desired value, about 10 to 15 minutes were allowed to elapse to secure steady-flow conditions in the penstock and over the weir. The U-tube deflections with the pitot tube at the gaging point were then read in feet of water at the rate of about ten times per minute by an experienced ob-

investigations of the reasons for the discrepancy were thought necessary.

Calibration Research, 1934. In 1934 it was decided to have three of the pitot tubes (rods 162, 403, 617, for which calibration curves are shown on Fig. 10) which were calibrated in 1930 recalibrated at the same gaging stations in the Alden Hydraulic Laboratory. To these calibrations was added a calibration made in still water. An 84-ft lattice steel boom was suspended from a ball bearing on top of a vertical shaft. This shaft was imbedded in the top of a boulder about 60 ft from one shore of a three-acre pond. The water surrounding the station was normally 6 to 7 ft deep. The pitot tube mounted on the boom swept through a circle of 36.11 ft radius. The speed of the boom was obtained from contacts made at each revolution and recorded on a chronograph.

The pitot heads were measured by water columns under partial vacuum, and the difference in heads was read about 15 times a

minute by an observer who rode the boom. These were at once set up on a calculating machine which gave the mean deflection for the run. The speed of the boom was regulated by a special governor and as its variation was small it was considered that the mean U-tube head was close to the true head for each run. The centrifugal effect was balanced by using equal lengths of hose and by having the U-tubes at equal radii. Care was taken to check the zero readings at intervals by stopping the boom.

The possibility of circulation effects in the water caused by the rotation of the pitot tube was carefully studied. No appreciable amount of circulation was evidenced under the most severe tests. One series of tests was made in which the rotation of the boom was reversed. The elapsed time from the moment the speed in one direction was changed until full speed in the reverse direction was attained amounted to 3 or 4 min. The pitot heads read when the boom was going in one direction substantially agreed with the heads read with the boom going in the reverse direction. Other tests were made in which the coefficient of the pitot tube obtained by the boom rating was compared under various conditions of rating. Obstructions were placed on the opposite end of the boom so that considerable disturbance of the water was created. Apparently this disturbance died down very quickly for the coefficients of the pitot tube remained the same regardless of the conditions thus introduced.

On account of the size, shape, and depth of the pond the coefficients obtained by the boom calibration were not subject to error caused by circulation of the water.

The scattering of the points in the boom ratings, Figs. 14 and 15, was due in part to varying conditions of wind and weather on the pond. That more uniformity of readings is possible with the boom method is shown by the test points in Fig. 30 which were obtained under more favorable conditions.

The results, Figs. 14, 15, and 16, showed agreement between the boom and the venturi throat, while the calibration in the 12-in. line was about 2 per cent lower. It was decided at this time that the amount of correction of the area of the projected rod should be carefully determined.

2 *Correction for Projected Area of Rod*

For many years a correction had been applied to the pipe area to compensate for the projected area of the rod. This correction was arrived at by more or less arbitrary methods and was applied by deducting from the pipe area an amount equal to the full area of the rod in the direction of flow, when the orifices were at the center of the pipe.

Correction at Alden Hydraulic Laboratory, 1930. For the calibrations made at the Alden Hydraulic Laboratory in 1930 a somewhat different analysis was made, and a revised correction amounting to one-third of the area of the projected rod when the orifices were at the center of the pipe was arrived at in the following manner.

When the instrument is on the far side of the center of the pipe, the area of the pipe is reduced and the mean velocity by that section is increased above the value when the instrument is not present. Assuming that the ratio of the point velocity to mean velocity remains constant, the point velocity obtained will be too great by the ratio of the area of the pipe to the area of the pipe minus the effective area of the pitometer.

If the point velocity obtained were to be corrected at the centers of rings of equal area, for any given ring there would be a plus correction on one side of the center and a minus correction on the other side, assuming the readings are to be corrected with respect to the pitometer at the center. The average amount by which the two velocities in a given ring exceeds the true values would be equal to the amount by which the measured center velocity exceeds its true value without the instrument in place.

Since the pipe factor is the ratio of the average velocity in the centers of rings of equal area to the center velocity, the value of the pipe factor is not changed by the presence of the instrument. However, the shape of the traverse curve may be distorted, the measured velocities being higher on the far side of the center of the pipe.

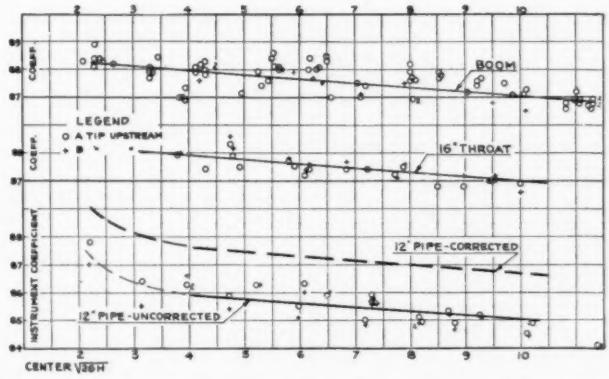


FIG. 14 CALIBRATION OF PITOT TUBE NO. 403, ALDEN HYDRAULIC LABORATORY, 1934

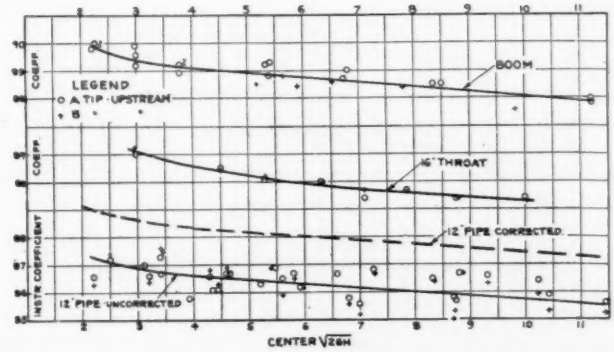


FIG. 15 CALIBRATION OF PITOT TUBE NO. 617, ALDEN HYDRAULIC LABORATORY, 1934

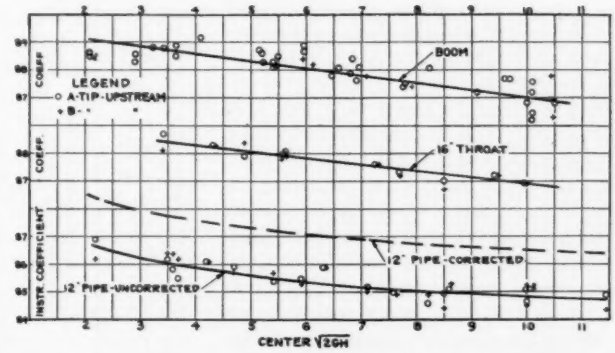


FIG. 16 CALIBRATION OF PITOT TUBE NO. 162, ALDEN HYDRAULIC LABORATORY, 1934

Although the value of the pipe factor is not affected by the presence of the instrument in the pipe, the value of the coefficient is affected and should be corrected when the rating is done in small pipes. Assuming the instrument is set at the center of the pipe, the mean velocity at a given discharge is increased above its actual value without the instrument in the pipe, by an amount depending on the projected area of the instrument set at the center.

The theory which resulted in a correction of one-third the

projected area of the rod was based on the assumption that only the trailing orifice was affected by the presence of the rod. It was assumed that the upstream orifice was not affected and since the trailing orifice is about one-half as efficient as the upstream orifice, it contributes one-third of the total deflection of the instrument.

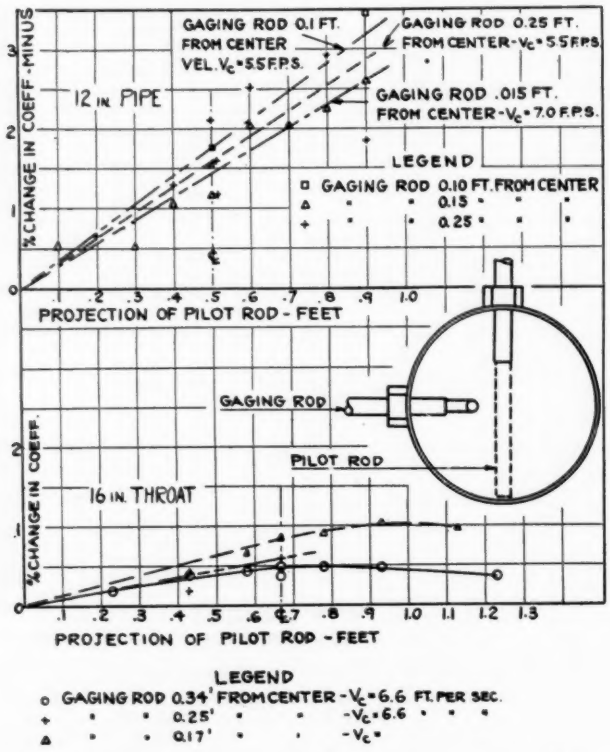


FIG. 17 METHOD FOR DETERMINING PITOT-COEFFICIENT CORRECTIONS FOR PROJECTED AREA OF ROD IN A 12-IN. PIPE AND IN A 16-IN. VENTURI THROAT

Correction Studies at Alden Hydraulic Laboratory, 1934. In 1934 a preliminary test was made at the Alden Hydraulic Laboratory to determine whether the correction might be larger than had been assumed. The method was to locate a pitot tube as an index rod at some point in the pipe and without changing its position to move a second or dummy tube across the pipe on another diameter in the same section. It was indicated by the first pitot tube, that the further the dummy rod was projected the greater would be the velocity indicated by the index tube. It was apparent that the dummy rod would disturb the flow past the index tube if it were too close and various positions of the indicating rod were tried to note the effect of the disturbance. The results are shown in Fig. 17 and, although being far from conclusive chiefly because of disturbance effects, they indicated at the time that the correction for the rod in the 12-in. line was greater than one-third the projected area.

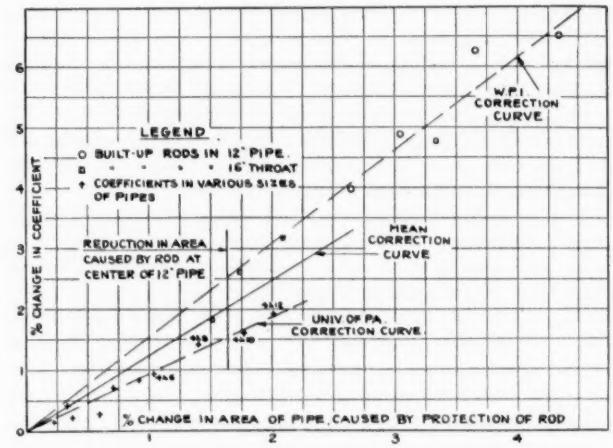


FIG. 18 CORRECTION OF PITOT-TUBE COEFFICIENT FOR CHANGES IN AREA OF PIPES CAUSED BY THE PROJECTED AREA OF THE ROD (NOTE: 12-in. pipe corrected by mean correction curve, Fig. 24.)

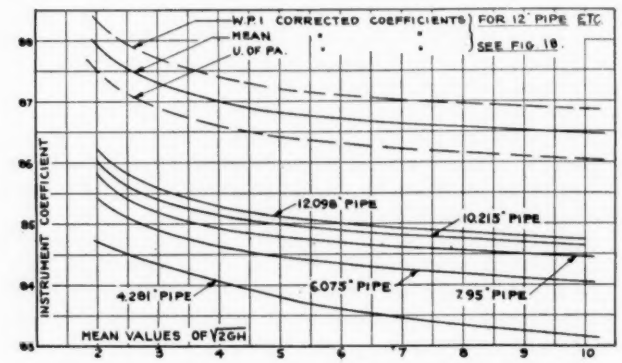


FIG. 19 CALIBRATION CURVES OF ROD NO. 617, UNIVERSITY OF PENNSYLVANIA, 1934

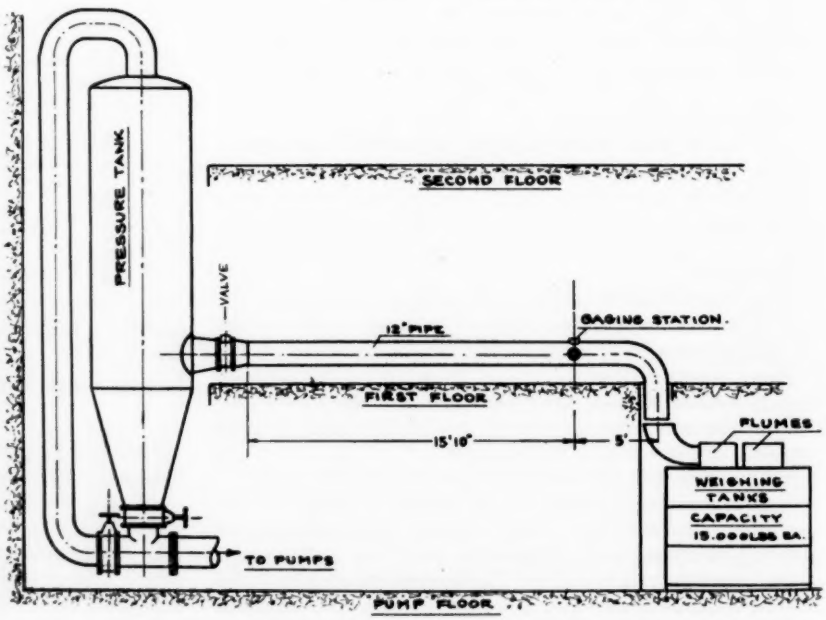


FIG. 20 SKETCH OF APPARATUS FOR RATING PITOT TUBES AT UNIVERSITY OF PENNSYLVANIA, 1934

A set of tests was then devised which would give some quantitative data on the amount of the correction. The method was to build up the cross-section of the rod in such a way as not to disturb the flow past the orifices, but to make an appreciable change in the net area of the pipe. The rod was built up by pieces of 1-in. dowel which were split and made to fit either side of the rod. The lower end of the dowel was tapered to a point 1 in. from the orifices. The section thus built up was wrapped with friction tape and shellacked. The cross-sectional area of the built-up rod in the direction of flow was determined by means of calipers.

The procedure was to build up the cross-sectional area of a calibrated rod and then to recalibrate it in both the 12-in. line and the 16-in. throat at velocities of 8, 9 and 10 fps. The resulting change in coefficient was assumed to be due entirely to the change in area of the pipe caused by the increased cross-sectional area of the rod. Four built-up sections were tried, the largest being 1 in. in diameter. The results are shown in Fig. 18.

University of Pennsylvania Tests. A third method for arriving at the proper correction seemed desirable and this was carried out at the University of Pennsylvania. Two of the three rods (Nos. 162 and 617) calibrated at the Alden Hydraulic Laboratory, were rerated by Professor Pardoe at the University of Pennsylvania in pipes of 4, 6, 8, 10, and 12 in. diameter. Fig. 19 shows Professor Pardoe's calibrations for rod No. 617. The method of calibration was different in some respects from the calibration at the Alden Hydraulic Laboratory. The piping layout is shown in Fig. 20 and it may be noted that the gaging station for the 12-in. pipe is only 15 diameters below a contraction. The distribution of velocity at the gaging station is illustrated by the traverse curves shown in Fig. 21. Instead of obtaining the indicated mean velocity by readings of the center velocity and pipe factors (ratio of mean to center velocity) as was done at the Alden laboratory, the procedure was to measure the mean velocity by traversing and to compare it with the true mean velocity as determined by weighing-tank measurements. The flow was well regulated by means of centrifugal pumps discharging into a large tank with an overflow to maintain a constant head. During the traversing, continuous weighing-tank measurements were made to determine the true mean velocity. The average of the square roots of the deflections obtained at the center of area of 16 rings of equal areas for the 12-in. pipe times $\sqrt{(2g)}$ was taken as the mean velocity by the pitot tube. The ratio of the mean $\sqrt{(2gH)}$ thus obtained to the mean velocity determined by the weighing tank is the coefficient. As was expected the coefficient in each pipe was different, although it should be noted that the coefficient obtained in the 12-in. line agreed with the coefficient obtained in the 12-in. line at Worcester, thus giving a remarkable check between the two series of investigations.

For the University of Pennsylvania tests, the ratio of the area of the cross-section of the

DIAMETER OF PIPE - INCHES	4.281	6.075	7.450	10.213	12.098
AREA OF PIPE - SQ. FEET	0.100	0.202	0.345	0.571	0.798
AREA OF ROD, WHEN HALF-WAY ACROSS PIPE - SQ. FEET	.00365	.00521	.00773	.01067	.0131
REDUCTION IN AREA OF PIPE, CAUSED BY AREA OF ROD - %	3.65	2.58	2.24	1.865	1.64

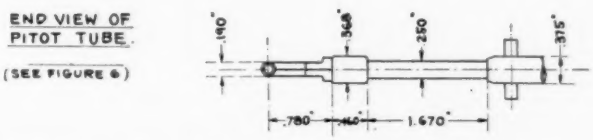


FIG. 22 AREA OF PITOT TUBES WHEN HALF WAY ACROSS VARIOUS-DIAMETER PIPES AND THE REDUCTION IN AREA OF THE PIPE CAUSED BY THE AREA OF THE PITOT ROD

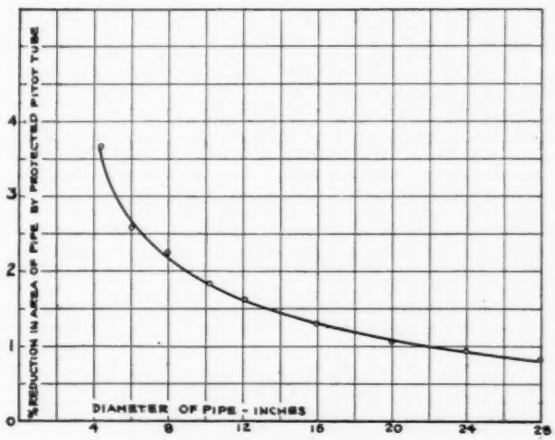


FIG. 23 GRAPH OF VALUES IN FIG. 22

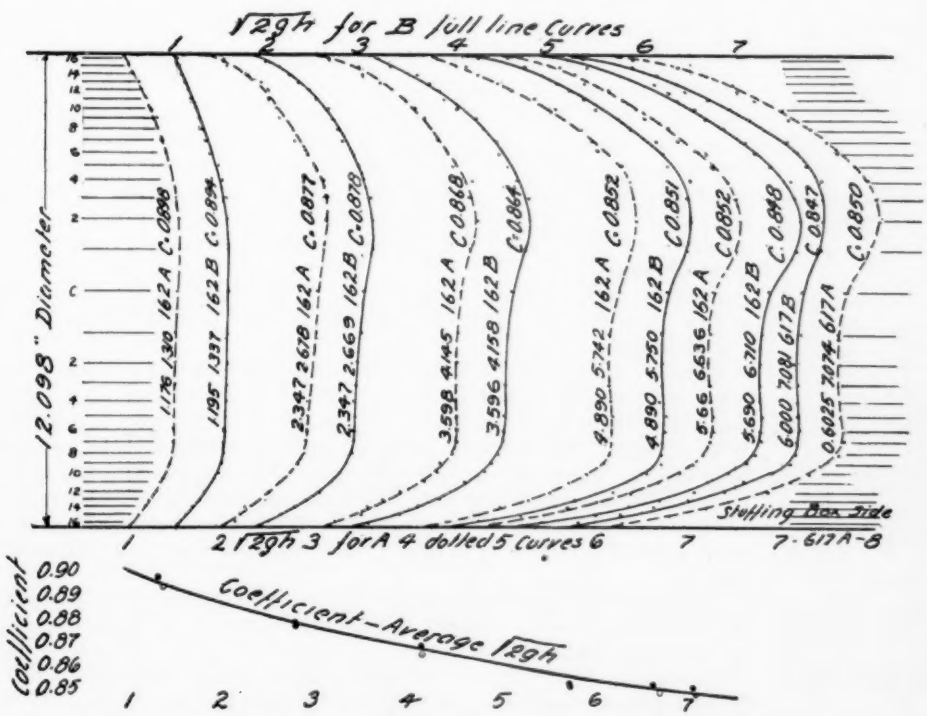


FIG. 21 TRAVERSE CURVES SHOWING DISTRIBUTION OF VELOCITY AT THE GAGING STATION SHOWN IN FIG. 20. GAGING STATION IS 15 DIAMETERS BELOW CONTRACTION

normal rod in the direction of flow, with the orifices at the center of the pipe, to the area of the pipe, for each size pipe was computed and is shown in tabular form in Fig. 22. The values are plotted in Fig. 23. A sample computation for the 4.281-in. diam pipe follows:

Length of projected rod, orifices at the center = 2.14 in.
Cross-sectional area of rod in direction of flow:

Orifices	0.780 in.	× 0.190 in.	= 0.1482 in.
Fillet	0.160 in.	× 0.290 in.	= 0.0464 in.
Fillet	0.231 in.	× 0.368 in.	= 0.0850 in.
Fillet	0.070 in.	× 0.290 in.	= 0.0203 in.
Connecting tubes	0.900 in.	× 0.250 in.	= 0.2250 in.
Projected length =	2.141 in.		

Area of projected rod in direction of flow = 0.5249 sq in.
Area of projected rod = 0.5249/144 = 0.00365 sq ft
Area of 4.281-in. diam pipe = 0.100 sq ft
Area of projected rod/area of pipe = 0.00365/0.100 = 0.0365 = 3.65 per cent

Similar computations give a ratio of 1.64 per cent for the 12.098-in. pipe, a difference of 2.01 (3.65 — 1.64 = 2.01)per cent from the ratio for the 4.281-in. pipe. The coefficient obtained in these two pipes differed by 1.9 per cent at velocities of 8.9 and 10 fps. Fig. 18, the University of Pennsylvania correction curve, was determined by points obtained in this manner, and plotted with the percentage of change in coefficient as the ordinate, and the change in ratio of areas as the abscissa.

For the W.P.I. correction curve, Fig. 18, the cross-sectional area of the built-up rod, as well as the ratio of this area to the area of the pipe, was computed by the same procedure as shown in the sample computation for the 4.281-in. pipe. The per cent change in coefficient resulting from the change in area of the pipe caused by the built-up rod, was plotted, as above, against the change in the ratio of projected rod areas to pipe area.

It should be noted that the two correction curves shown in Fig. 18 do not give the same results, and the difference is probably due to errors which are present in each method. For the University of Pennsylvania curve, an error is introduced in the determination of the area of the orifices. That this error may be large is shown by the fact that in the sample computation, the area of the orifices is 28 per cent of the total projected area of the rod. However, an error of this size is not introduced by inaccurate measurement, but rather by an assumption that the entire area of the orifices is effective in reducing the area of the pipe. Fig. 6 shows the open construction of the tips and although some investigation has been made it is not definitely known what part of the orifices is effective. If the area of the orifices were neglected in the computation of the area of the projected rod, the University of Pennsylvania curve would be raised above the W.P.I. curve.

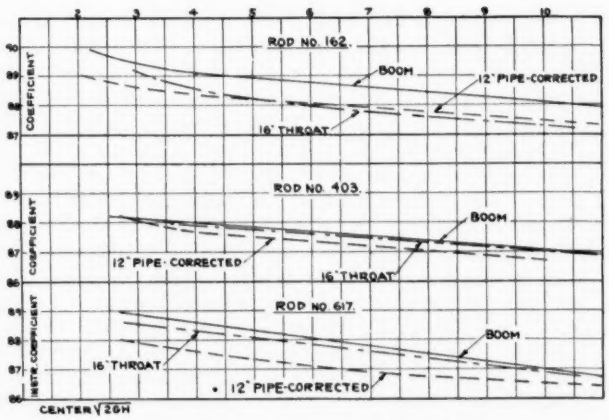


FIG. 25 COMBINED RATINGS OF THREE PITOT TUBES BY THREE METHODS, ALDEN HYDRAULIC LABORATORY

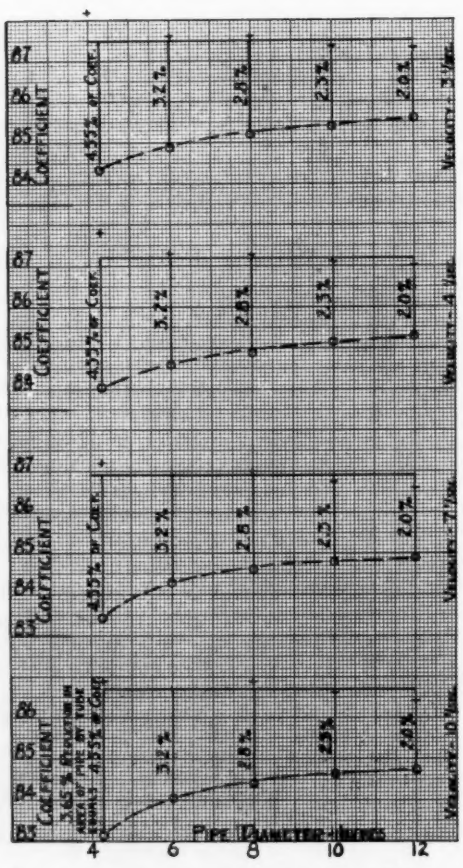


FIG. 24 METHOD OF CORRECTION FOR PROJECTED AREA OF PITOT TUBE

The error in W.P.I. curve is of a different nature because the area of the orifices is relatively small compared to the area of the projected rod, and is probably introduced by local disturbance on the orifices. Although the built-up sections were designed to produce a minimum amount of disturbance, it is possible that some disturbance was still introduced.

For lack of better information the mean of the University of Pennsylvania and the W.P.I. curves has been used in this paper.

Use of Correction Curve. The use of the correction curve is made clearer by a study of Fig. 19, the calibration of rod No. 617 in various-size pipes made at the University of Pennsylvania. In Fig. 24 the coefficient obtained for a given velocity is plotted against the pipe diameter. It will be noticed (Fig. 19) that for any velocity the coefficient decreases as the pipe diameter decreases. In other words, as the ratio of projected rod area to pipe area becomes greater the coefficient becomes smaller.

A series of ordinates has been added, Fig. 24, to show the corrected coefficient at any given velocity. These ordinates are obtained from Figs. 23 and 18 in the following manner:

From Fig. 23 we see that in a 12-in. pipe the ratio of the area of the projected rod to the area of the pipe is 1.64 per cent. Entering the mean curve on Fig. 18 with 1.64 per cent on the abscissa we find the corresponding change in coefficient is 2.0 per cent. The coefficient obtained in the 12-in. pipe should then be raised 2.0 per cent. The coefficient in the other sizes of pipe can be corrected by following the same method. The result will be that at a given velocity the corrected coefficient will be the same in any size of pipe. The corrected coefficients at each velocity can then be plotted to give a coefficient curve which will hold for any velocity.

In practice it might be more convenient to use the coefficient which applies to the pipe in which one is working. For reasons which will be apparent later it has, however, been desirable to determine the corrected coefficient in the 12-in. line.

Correction in 16-In. Throat. For these tests the orifices were located 4 in. in on the horizontal diameter and no correction for the area of the projected rod was necessary.

Correction for Other Types of Tube. So far as the author is aware this correction

for the projected area of the rod has never before received proper attention, and this may account for apparent inconsistencies in pitot-tube calibrations.

Where the pitot tube is supported on a streamlined rod passing entirely across the pipe so that a traverse involves no change in the projected area, the author believes some correction for the presence of the pitot tube must still be made if it is to be used in another size of pipe or is to be compared to still-water or large venturi-throat rating.

Application of Correction to 1934 Calibrations. The mean correction curve which has been adopted was used to correct the calibrations made in the 12-in. pipe at the Alden laboratory, and in the various sizes of pipe at the University of Pennsylvania. Figs. 14, 15 and 16 show the test points obtained in the Alden laboratory calibrations as well as the corrected 12-in. line coefficient. Fig. 25 is a summary of all the calibrations and shows the agreement obtained by calibration in still water, the 16-in. throat, and in the 12-in. line.

The calibrations made at the University of Pennsylvania with corrections by the three different methods are shown in Fig. 19.

3 *Angularity and Eddying Flow*

Angularity of flow in pipes has been much discussed. Theoretically, the simple pitot tube over-reads in angular flow, but this error is small as shown by the fact that the difference between the value of $V \cos a$, and $V\sqrt{(\cos a)}$ is but 0.2 per cent with $a = 5$ deg, and this paper will attempt to show that the mean angularity of turbulent flow in the normal pipe line may not be as great as 5 deg. It should be observed in passing that tests to determine the angularity characteristics of a pitot tube, i.e., rotating the tube about its axis, must be made under proper conditions. Tests made by rotating the long tip of a simple pitot tube at the center of a 12-in. pipe might throw the point of the tip a considerable distance from the center and into a lower velocity, thus requiring a greater angle to produce a given coefficient. When in turn the degree of angular flow in a pipe is inferred from a comparison of coefficients, it is evident that the inferred angularity may be much too large.

In order to study angular flow in pipes, the device shown in Fig. 26 was constructed with a cross-shaped vane free to move with the water in any direction up to about 30 deg from the pipe axis. The friction and inertia of this indicator was made as small as possible, its shaft being mounted in stainless-steel ball bearings. Its behavior was observed under normal flow conditions, and also with distorted flows as produced by bends or a nearby partly opened gate.

With normal flow, readings failed to show an average angularity as great as 5 deg. The indicator was in constant and rapid vibration through small angles with an occasional jump to 10 or 15 deg for an instant only as was shown by motion pictures. It has been pointed out by Moody[11] that angularities of short duration had little or no effect on the pitot tube.

Professor Allen[15] gives data for computing the angular flow as shown by the spread of salt solution. His tests, which were made in a 40-in. penstock, indicate a maximum angularity of 6 deg, which value is practically constant at 5 and 10 ft downstream from the injection tube.

The most conclusive evidence showing the small effect of angularity or eddying flow on the pitot tube is to be found in the close agreement of the three methods of calibration herewith presented.

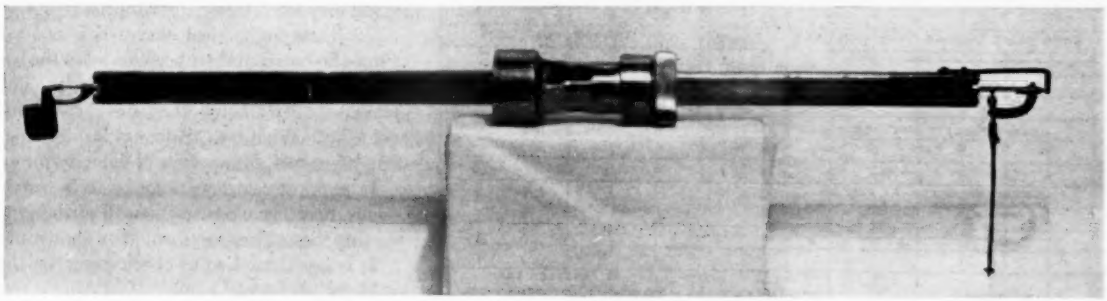


FIG. 26 INDICATOR FOR STUDYING ANGULAR FLOW IN PIPES

The angularity characteristics of the author's pitot tube are shown in Fig. 27. It will be noted that when rotated through an angle of 5 deg, the tube over-reads the cosine value by 1 per cent. The fact that the same coefficient was obtained in still water, in the smooth flow of the venturi throat, in the turbulent flow of the Alden laboratory 12-in. line and in the turbulent flow of the University of Pennsylvania 12-in. line, indicates that turbulent or eddying flow such as encountered in normal pipe lines does not subject the author's pitot tube to appreciable errors.

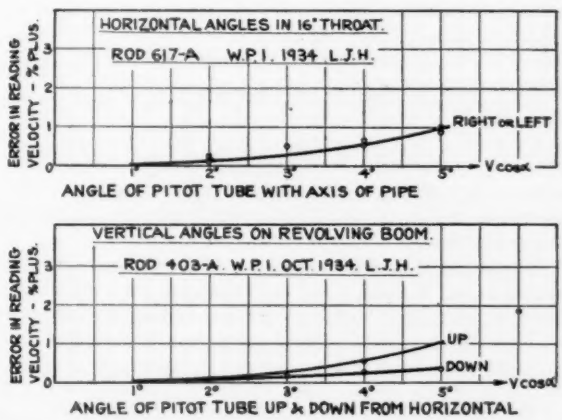


FIG. 27 ANGULARITY ERRORS OF REVERSIBLE PITOT TUBE

In view of this evidence it seems that the existence of angles of 20 or 30 deg as claimed by some writers needs substantial proof.

THE PITOT-TRAVERSE METHOD

Broadly speaking, this method may include gagings made with the simple pitot tube and wall piezometers, but this paper is chiefly concerned with the combined type. With proper attention to details of operation, flow measurements by readings of the center velocity are made rapidly and accurately.

---

[11] "How Water Flows in a Pipe Line," by C. M. Allen, *Mechanical Engineering*, vol. 56, February, 1934, pp. 81–84.

(a) *Selection of the Gaging Point.* Although many important gagings have been made it is difficult to lay down a simple rule for guidance in selecting the gaging point. Actual conditions must govern and sometimes it is necessary to accept the best compromise location and make trial velocity traverses of the pipe to indicate the suitability of the gaging point. Here, as elsewhere, there is no substitute for experience.

In general, the longest possible straight length of pipe upstream should be selected, and not too close to a downstream bend or fitting. Sometimes ten diameters upstream, and two diameters downstream will give good results, depending on conditions of the pipe line and flow.

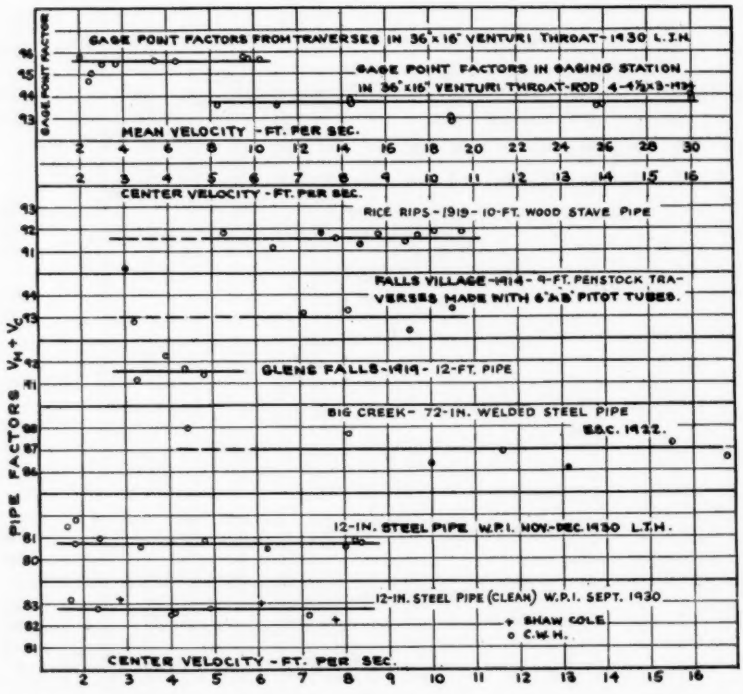


FIG. 28 COMPARISON OF PIPE FACTORS FOR LARGE PENSTOCKS AND SMALLER-DIAMETER PIPES

The pipe factor or ratio of mean to center velocity has great practical value, because, when once obtained for a given gaging point in a pipe line, it holds good for a considerable time or until the roughness or tuberculation of the pipe wall has increased depending on the nature of the pipe and its coating, and upon the chemical character of the water. Sometimes the pipe factor will hold good for years, as for example in the cast-iron mains of a water works carrying the relatively hard water of the Great Lakes cities.

Although the pipe factor may be shown to have a theoretical value of 0.83 upon the assumption that the velocity traverse is a semi-ellipse and cylinder, it seems useless to attempt to predict pipe factors in pipes of a given age or condition, for they vary from 0.70 to 0.90 according to local conditions and with some forms of vortex or spiral flow with depressed center velocity the pipe factor may even exceed unity.

If the pipe factor changes at all with ordinary velocities it does so very slowly. A comparison of determinations in water-power penstocks and in smaller pipes is shown in Fig. 28.

The pipe factor gives a ready means of making short tests, for example, in a penstock when interpolating between traverses made at several gate openings, for repeat measurements at gaging points of a city distribution system using the established pipe factor in connection with readings of center velocity, and also for the continuous recording of flow in a pipe line. With the pipe factor it is only necessary to make a continuous record of center velocities.

(b) *Traversing and Ring Integration.* Traversing and the ring method of computing mean velocity from the velocity traverse are now well described in many textbooks and need not be repeated in detail here. The errors of computation are small if as many as ten equal-area rings are used and if two diameters at right angles are traversed. It is well to have one diameter in the plane of an upstream bend in order to judge the lack of symmetry in velocity distribution.

With unsymmetrical curves it is not safe to assume the horizontal traverse to have the same ratio of mean velocity to center velocity as the vertical traverse, even though they have the same center velocity. With two traverses 90 deg apart there are, of course, four values of velocity for each ring.

In making velocity traverses it is important to secure readings close to the wall of the pipe and to do this the orifices are formed as shown in Fig. 6.

It is good practice to check back to the center velocity at frequent intervals during a traverse in order to avoid readings which are not comparable because of a change in the cycle of flow variation. When traversing with an unsteady flow, as in water mains or power penstocks under variable load, it is convenient to use a second pitot tube set near the point of maximum velocity. The function of this second tube, called a "pilot meter," is to indicate flow while the traverse is being made so that readings may be taken only at some predetermined rate.

Fluctuations in velocity may be very large in water-works service mains, but in water-power penstocks with wheels at a fixed gate opening the velocity variation through a full cycle is usually relatively small. It is sometimes important to dampen large variations in the manometer by throttling the connections. To do this without affecting the mean reading requires care; and the form of pinch cock used on the rubber hose should compress it symmetrically as to inlet and outlet to obtain the best results in producing a mechanical average.

The method of ring integration used at the Alden laboratory for the tests described in this paper was as follows:

Traverses were made on two diameters, 90 deg apart. Readings were taken at the respective centers of area of ten rings of equal area. The position of the center of area for each ring was determined as follows: Ring No. 1 = $R\sqrt{(0.05)}$, ring No. 2 = $R\sqrt{(0.15)}$, ring No. 3 = $R\sqrt{(0.25)}$, ring No. 4 = $R\sqrt{(0.35)}$, etc., where R = radius of pipe. The square roots of the pitot-tube deflections for each diameter were plotted to a large scale and smooth curves were drawn through the test points. From each curve twenty readings of the square roots were taken at the respective centers of area, making forty readings, and these were averaged to obtain the mean square root for the gaging section. This method is more accurate than that by the use of the midpoint of equal areas or by the average of the boundary readings of the rings of equal areas.

With mean velocity thus obtained the pipe factor is computed, which in a case of straight pipe lines is a criterion of pipe-wall roughness; and in any pipe line reveals the suitability of flow conditions at the gaging point.

Repeat traverses at the same location have frequently been found to check the pipe factor within 0.2 per cent and this is ex-

cellent assurance of the accuracy of the pitot-traverse method. Further assurance is to be found in the agreement between the Alden laboratory calibration made by pipe factors, and the University of Pennsylvania calibration made by constant traversing.

(*c*) *Reading of Center Deflections.* In obtaining the center velocity head or deflection which by the use of the pipe factor may be converted to mean velocity, it is important to have the manometer and all connections from the orifices to the manometer leak-tight and entirely free from air. The glass U-tube or manometer should be clean so that the measuring liquid has a well-shaped meniscus and the specific gravity of the liquid should be determined in the U-tube itself, at the temperature of use. Readings of the manometer deflection should be made at frequent intervals throughout a full cycle of flow. These precautions, of course, apply to traversing as well.

In reading manometer deflections with eddying flow in normal pipe lines there are pulsations in the U-tube which take place in cycles. If we record the readings through a full cycle we have the mean of the squares of the varying velocities and it has been claimed that this results in over-reading, but as the error is but 0.15 per cent with 5 per cent change in velocity[16] we may disregard it where fairly uniform flow is found.

A motion-picture record of pitot-tube deflections throughout typical velocity cycles was made both for 12-in. and 40-in. pipe lines with normal flow conditions under the uniform head available at the Alden Hydraulic Laboratory. This record showed two things, first, that the velocity changes are very small and, second, that these changes follow in well-marked cycles.

In these tests three pitot tubes were set at three points at the same section in a 40-in. pipe and at two points in a 12-in. pipe, each having exactly the same length of hose connections. No throttling of the lines was employed, yet the U-tube readings changed very slowly.

Variations caused by load changes may or may not follow a cycle but in important tests the load is always made as steady as possible.

In small pipes such as are found in city water works the pipe is tapped under pressure on the vertical diameter in the usual way; the pitot tube is attached to the 1-in. corporation cock and its manometer is connected. The air is blown off from the U-tube and connections. The pipe is traversed usually with but five equal-area rings and the pipe factor is computed rapidly with the aid of a prepared form for the field notes. Where it is only necessary to know relative flows or changes, a single traverse of five rings is quite sufficient. The pipe diameter is determined by means of a special caliper.

In large pipes it may be necessary to traverse an alternative location in order to select the best point. Two diameters should be traversed, and the use of 15 rings is recommended. The pilot meter is usually of great assistance in securing a good traverse.

(*d*) *Vibration of the Pitot Tube.* With large-diameter pipes and moderate velocities, or with moderate-diameter pipes and high velocities, attention should be given to possible vibrations of the pitot tube. Slight vibrations do not seem to affect the readings of velocity head but large vibrations may increase them. It is well to check readings near the far side of the pipe by inserting a short pitot from that side so that its tips may stand close to the end of the vibrating rod. Oval sections with 2-in. major axis and 1-in. minor axis with tips as shown in Fig. 6 have been used successfully in pipes 12 ft in diameter without extra support.

The Heavy-Duty Rod

The pitot tube shown in Fig. 29 is a heavy-duty rod which has been used on ships with success. Its strength and rigidity have been demonstrated in practice where with 4 ft of unsupported projection, velocities of 50 fps have failed to disturb it. Because of its great strength, this rod, without the aid of extra supports within the pipe, is specially adapted for high velocities and large pipe diameters.

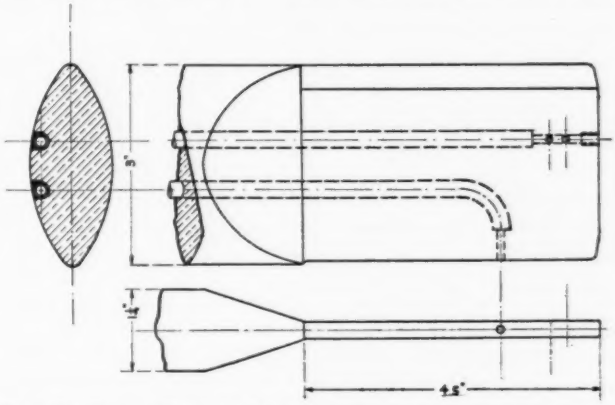


Fig. 29 Cosine Type of Heavy-Duty Pitot Tube for Large-Diameter Pipes and High Velocities

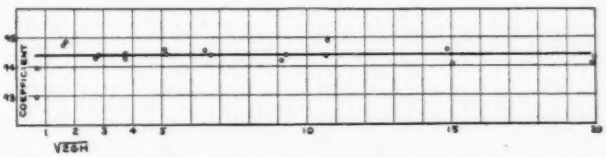


Fig. 30 Still-Water Rating of the Pitot Tube Shown in Fig. 29

Because the pitot tube shown in Fig. 6 over-reads the cosine value by 1 per cent when rotated through an angle of 5 deg, considerable experimental work has been done in the last 2 years at the Alden Hydraulic Laboratory to develop a tube which will read the cosine to 5 or 6 deg and at the same time have a fixed coefficient over a great range of velocities. Figs. 30 and 31 indicate the degree of success which has been attained by the use of the heavy-duty rod. Many calibrations of this type with its practically uniform coefficient were made at the Worcester revolving-boom rating station to a maximum speed of 20 fps and this speed was later extended to 50 fps when the tube was installed on a ship and rated over a measured mile.

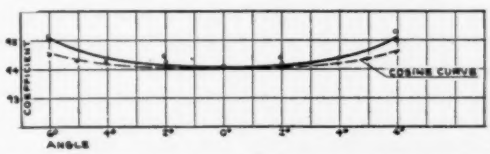


Fig. 31 Cosine Curve for Heavy-Duty Pitot Tube

Summary

(*a*) The pitot tube, both simple and combined, is well adapted for the measurement of flow in pipes under pressure.

(*b*) The coefficient of the simple pitot tube is practically unity for normal flow.

(*c*) The coefficient of the combined pitot tube (D'Arcy type) is also practically unity for normal flow.

(*d*) The coefficient of the reversible type (Fig. 6) is as shown by the calibration curves, Figs. 19 and 25, and ranges from 0.870 at 10 fps to 0.885 at 3 fps and is practically unaffected by angularity to 3 deg.

(*e*) The average angularity of flow in normal pipe lines is com-

[16] "Applied Hydro- and Aeromechanics," based on lectures by L. Prandtl and O. G. Tietjens. Translated by J. P. Den Hartog, McGraw-Hill Book Company, Inc., New York, 1934, p. 47.

monly less than 5 deg, although sudden angular fluctuations are occasionally as high as 15 deg.

(*f*) The pitot tube, Fig. 6, reads close to the cosine of angular flows to 3 deg. The heavy-duty type of pitot tube, Fig. 29, reads close to the cosine of angular flows to 6 deg, Fig. 31, and has a practically uniform coefficient from 3 to 50 fps.

(*g*) Pitometer ratings made in still and moving water agree within 0.5 per cent of their mean values, when the proper correction is made for the area of the protruding tube under usual conditions of angularity and eddying flow when rated in a pipe.

(*h*) This method in common with all other methods of water measurement should be used by trained and experienced men.

Research Investigation of Current-Meter Behavior in Flowing Water

By S. LOGAN KERR,[1] PHILADELPHIA, PA.

This paper investigates the inconsistencies of flow measurements in closed flumes by means of current meters. It describes (*a*) the construction of a flume in which two meters were installed side by side and tested under varying conditions of flow, (*b*) the velocity distribution in the flume, and (*c*) the establishment of the true velocity plane across the face of the meters by means of a pitot tube. The pitot-tube coefficient was established for each flow condition, thus providing an accurate means of establishing the actual velocity existing in front of the current meter.

IN ORDER to investigate certain inconsistencies in the measurement of flow in large hydroelectric plants by means of current meters, a research program was undertaken by the I. P. Morris Division of Baldwin-Southwark Corporation in their hydraulic laboratory at Eddystone, Pa. The original investigation included the study of available literature, principally a paper by Messrs. Nagler and Yarnall at the University of Iowa, where current meters were placed in flowing water and oscillated back and forth, and also set at various angles with the flow, in an attempt to establish their behavior under turbulent conditions. It is felt, however, that this particular study did not actually simulate conditions under which current meters are frequently used in connection with measurements of the efficiencies of hydroelectric units.

The resolution of turbulence into the two elements, angularity and variations in forward flow, seems to omit the very important factor of the variation of velocity across the face of the meter and the rapid fluctuation of velocity distribution in turbulent water.

To investigate this phenomenon properly, it was decided to construct a flume in which two current meters could be installed side by side and tested under varying conditions of flowing water. Means were arranged for visual observation of the degree of turbulence present at the metering section. The original program was expanded considerably to include the investigation of velocity distribution in the flume, and particularly the establishment of the true velocity plane across the face of the meter by means of a pitot tube. All of the discharge measurements were checked by a calibrated weir, which was normally employed for testing model turbine units and other apparatus. The weir was calibrated by measuring the rate of rise in level in the sump tank while maintaining a constant flow over the weir.

Fig. 1 shows the logarithmic calibration curve of the weir and the comparison of the calibration with the Rehbock formula. For the lower flows the weir shows slightly higher discharge than the formula, but for flows from 7 cfs to 9 1/2 cfs (the range during the tests of the current meters) the weir checks the Rehbock formula very closely.

The pitot-tube coefficient was established for each flow condition, and hence provides accurate means of establishing the actual velocity existing in front of the current meter.

Description of Apparatus

The test flume as shown in Fig. 2 was 11 ft 8 1/2 in. overall, and had a bell mouth at the intake with two sets of racks to straighten out the flow. The first set consisted of 25 boards, 7/8 in. thick on 1 3/8 in. centers, located in a vertical plane. The second set consisted of 24 steel plates 1/16 in. thick on 9/16 in. centers, and were placed 6 in. downstream from the vertical racks and extended a total of 10 in. downstream. Two removable screens with fine mesh were installed about 24 in. further downstream to remove from the flow any remaining disturbances. These screens were cleaned periodically to make certain that no foreign matter was clogging them and changing the flow distribution in the channel. At a point 2 ft 7 1/4 in. downstream from the lower end of the horizontal racks was an aperture, in which removable baffle plates could be installed. The various baffle plates used in the tests are shown in Fig. 2 as arrangements *A*, *B*, *C*, and *D*. These baffles consisted of steel plates arranged with dowels to center them, thus placing them in exactly the same position for each test.

The current meters were installed 4 ft 10 in. downstream from the baffles and a safety-glass window was located in the top of the flume immediately above the propellers of the meters.

For the first series of tests a system of yarn streamers was installed about 12 in. upstream from the meters and located in three horizontal planes, at 1/4, 1/2, and 3/4 of the height of the channel. This placed the middle group of yarns on the horizontal centerline of the meters and the upper and lower groups beyond the periphery of the meters. On the first traverses of the pitot tube, it was found that these yarn streamers would follow closely the true stream lines and conform to the contour of the tube. When the accurate pitot-tube determinations were in progress, the yarn streamers and wire supports were removed to avoid any possible interference with the flow.

Illumination was provided on both sides of the flume to permit a visible study of the flow conditions in the metering section.

Each pair of meters employed consisted of one right-hand and one left-hand meter of the type manufactured by Dr. Ott in Germany and loaned for this investigation by the Safe Harbor Water Power Corporation. Two of the meters were the so-called type 1 (spoke type) and the other two were designated as type 2 (screw type). The meter bodies, as used in service conditions, were also employed, supported by a bar made of hardwood of the same shape as the supporting bars used in the field. The testing flume discharged into the weir channel approximately 2 ft downstream from the centerline of the meters. The flow then passed

[1] Research Engineer, I. P. Morris Division, Baldwin-Southwark Corp. Mem. A.S.M.E. Mr. Kerr was graduated from the University of Pennsylvania in 1921 with the degree of Bachelor of Science and in 1924 with degree of Mechanical Engineer. Upon graduation he entered the I. P. Morris Division of the William Cramp & Sons Ship & Engine Building Co. as research assistant. In 1924 he was appointed assistant hydraulic engineer and in 1927 assistant chief engineer. Since 1929 he has been research engineer in charge of experimental and field tests and development work on hydraulic turbines, governors, valves, and other equipment.

Contributed by the Hydraulics Division and presented at the Annual Meeting, New York, N. Y., December 4 to 8, 1933, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

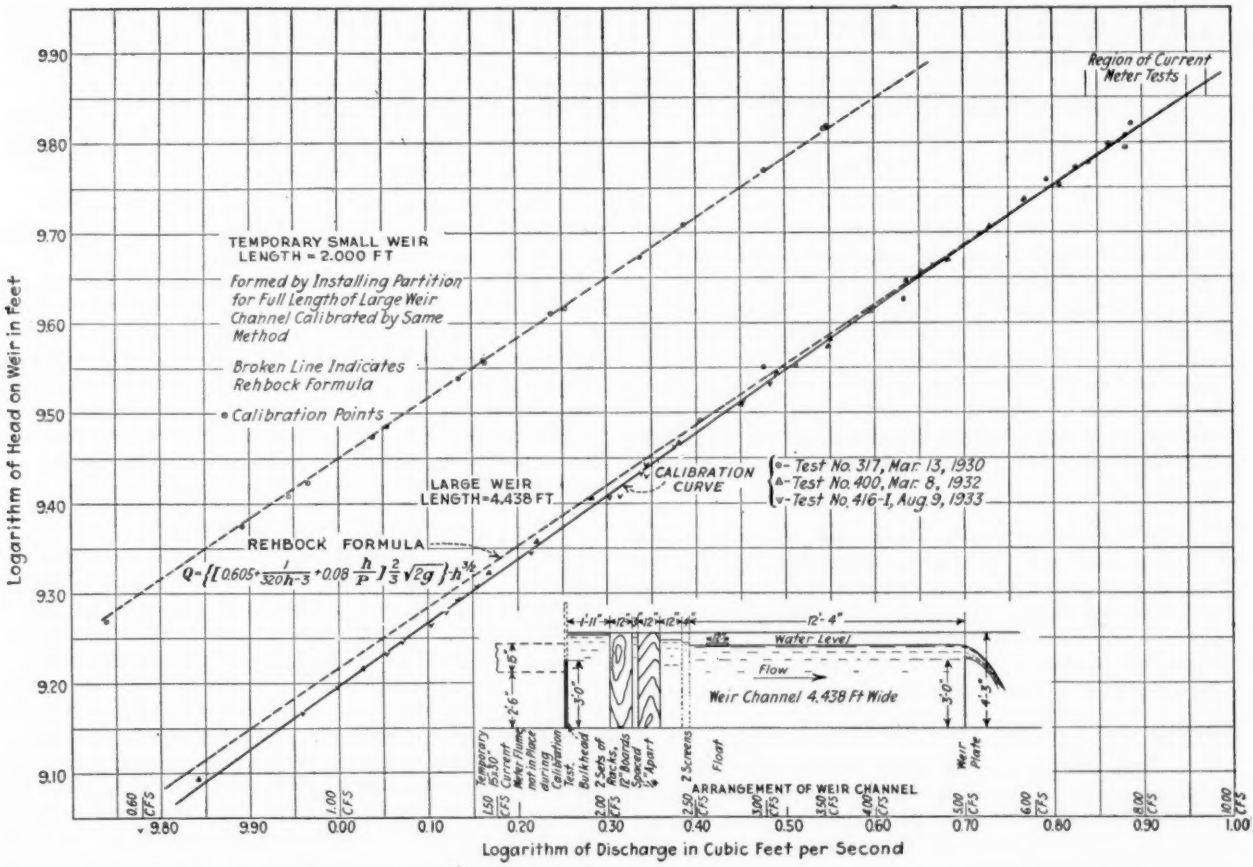


FIG. 1 LOGARITHMIC CALIBRATION CURVE OF THE CALIBRATED WEIR AND A COMPARISON OF THE CALIBRATION WITH REHBOCK FORMULA

through a series of racks and screens and over the calibrated weir.

The water was discharged into the sump and pumped back into the head tank, passing through a controlled orifice into the pool in which the current-meter flume was installed. The recirculation of flow was accomplished by a high-speed propeller-type pump and arrangements were made to maintain the speed of this pump constant by using a separate rotary-converter unit in the plant sub-station, thus giving freedom from voltage fluctuations.

Fig. 3 is a view looking downstream through the flume before the installation of the racks and screens, showing the location of the two meters.

METER CALIBRATIONS

Each of the meters employed had been carefully calibrated in the tangent still-water rating flume at the Bureau of Standards in Washington. These rating curves were used to determine the velocity of flow as registered by the meter. Additional calibrations were made in Washington to give the degree of registration of the forward velocity of the meter when the meter itself was placed at different angles with the direction of movement of the rating car. These were the so-called "angular still-water ratings" of the meters, in contrast to the normal straight-line ratings. The angular ratings were determined for several forward velocities and a general average calibration curve was prepared which was used to compute the so-called "angular correction" for field testing, using the two-type meter method. The meters gave differing results, depending upon the quadrant in which the angle was measured, that is, whether the flow approached from the top or bottom or whether it approached from the right or left.

As these variations usually were less than 1 per cent, the average correction curve should reduce the error to within the accuracy of the flow measurements. These details have been dealt with at length by J. M. Mousson in his discussion[2] of Hunter Rouse's 1932 paper.[3] The difference in registration, or angular calibration, is sometimes considered as a means of establishing the correction for angular flow in the metering section.

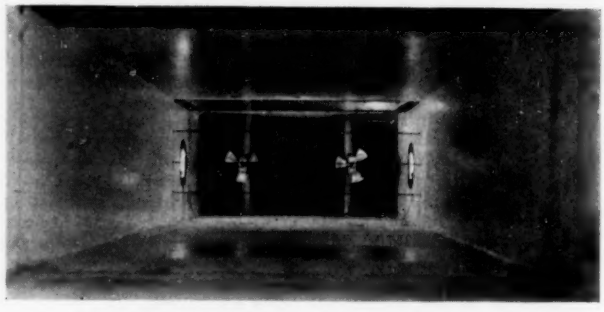


FIG. 3 DOWNSTREAM VIEW THROUGH THE FLUME BEFORE INSTALLATION OF RACKS AND SCREENS SHOWING LOCATION OF THE TWO METERS

[2] Discussion of "Research Institute for Hydraulic Engineering and Water Power," by Hunter Rouse, Trans. A.S.M.E., 1933, vol. 55, paper HYD-55-3, p. 33.

[3] "Research Institute for Hydraulic Engineering and Water Power," by Hunter Rouse, Trans. A.S.M.E., 1932, vol. 54, paper HYD-54-3, p. 27.

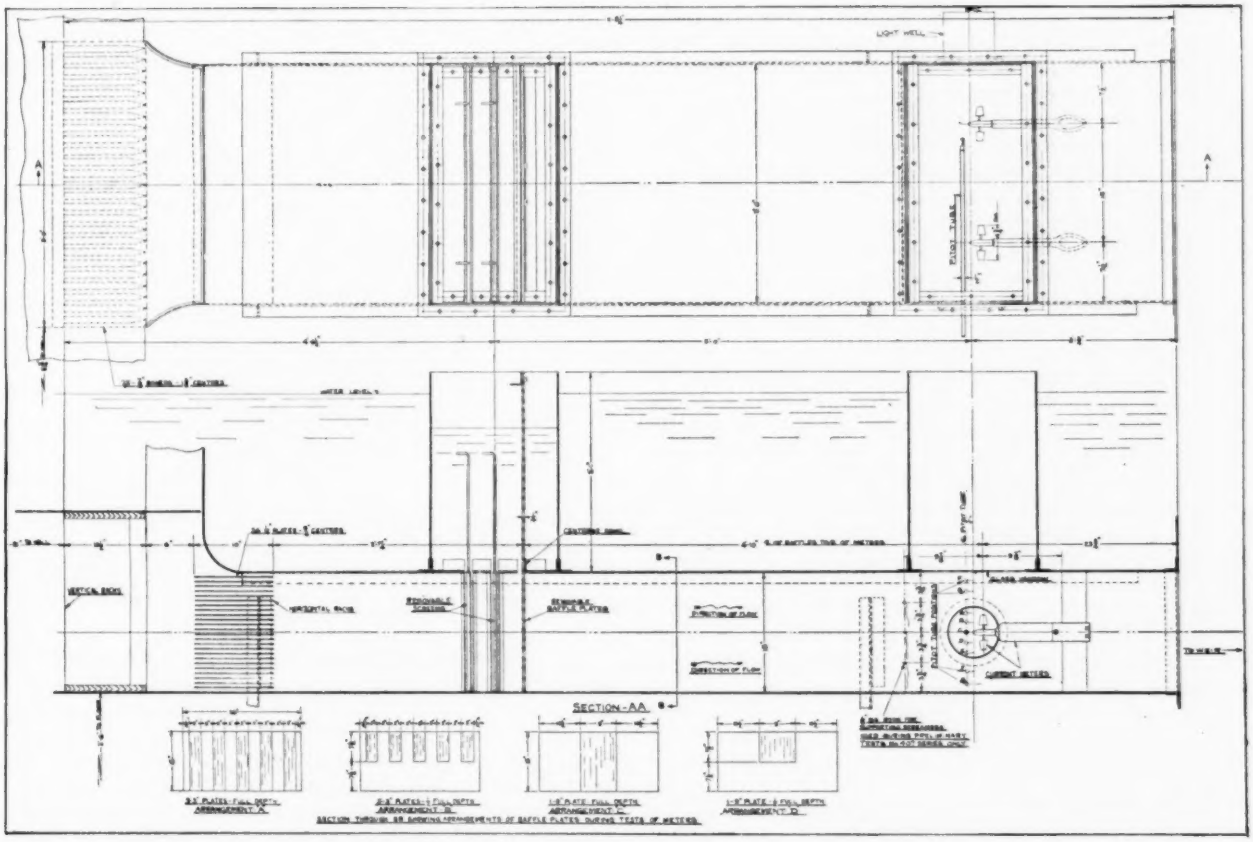


FIG. 2 ARRANGEMENT OF THE CURRENT-METER TESTING FLUME

DESCRIPTION OF TESTS

The first series of tests consisted in observing the behavior of the current meter with various types of flow in the test flume. The flow distribution was not measured but each arrangement of baffle, namely, *A*, *B*, *C*, and *D*, was employed for several different velocities. Tests were also made with smooth flow with no baffles in place. The meters were interchanged, that is, the right-hand meter and left-hand meter were reversed in their positions and both types of meters were tested in this manner. It was found that the differences in registration between the two types of meters, referred to their angular still-water calibrations, indicated very pronounced angularity of flow which did not appear to be present from visual observation. These differences in registration are given in Table 1.

As a result of this preliminary series of tests it was apparent that further research was necessary to establish the flow distribution in the flume and particularly to determine the average velocity immediately in front of the meter so that this value could be checked against the current-meter registration and the degree of over or under registration thus determined.

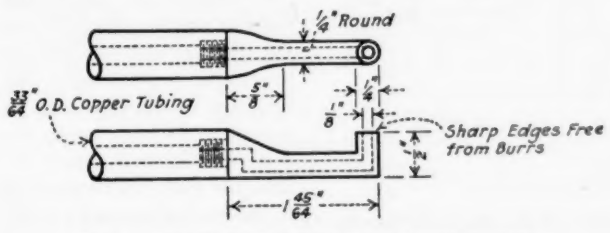


FIG. 4 DETAIL OF THE PITOT-TUBE CONSTRUCTION

TABLE 1 REGISTRATION DIFFERENCES BETWEEN THE TWO TYPES OF METERS

	Type of flow	Diff. type 1 meter minus type 2 meter over type 1, %	Percentage correction type 1 meter (added), %	Apparent angularity, degrees
1	Smooth flow preliminary tests	3.14	0.99	11.7
2	Smooth flow final tests.....	3.66	1.13	12.5
3	Five 3-in. baffles half depth..	3.42	1.08	11.8
4	One 9-in. baffle half depth...	5.18	1.59	14.8
5	Five 3-in. baffles full depth..	8.56	2.60	18.8
6	One 9-in. baffle full depth...	17.02	5.07	25.3

PITOT-TUBE TRAVERSE

A pitot tube conforming to the type commonly used in penstock work was installed in the section immediately upstream from the meters. This pitot tube had a single dynamic-pressure hole. The static pressure was taken at four points, each of which was connected to an individual gage glass. Three of these static-pressure holes were in the bottom of the flume, one located on the centerline of the flume and the other two on the centerlines of the current meters. The fourth static-pressure hole was located on the left-hand wall of the flume looking downstream to check any possible difference due to side velocities or eddy currents.

The tests under this series were really in two parts. The first series was made to establish the average velocity of flow impinging on the current meters while the second group was made to establish the average pitot-tube coefficient under actual flow conditions existing.

DESCRIPTION OF PITOT-TUBE APPARATUS AND TEST METHODS

Fig. 4 shows the pitot-tube construction. Fig. 5 is an elevation of the flume with the location of each individual pitot-tube traverse point designated and with the positions of the me-

ters shown. Fig. 6 illustrates the method of determining the average velocity in front of the meter by plotting five horizontal traverse curves, each having five individual pitot-tube determinations of velocity. With this grid of points established, the area in front of the current meter was divided into five annular rings of equal area in a manner similar to the method employed in measuring the velocity in closed conduits. The velocity curves were then plotted on the eight radial planes and the velocities determined at the center of each annular ring. This established eight velocity points for each of the five annular rings, or a total of 40 velocity points, the average of which was used as the mean velocity in front of the meter.

Additional tests were made for certain of the runs to establish the velocities adjacent to the walls in order to avoid any uncertainty in regard to the velocity distribution existing between the last pitot-tube point and the wall of the flume. These studies were made very carefully at points taken $^1/_8$ in. from the wall and four other points at intervals of $^1/_4$ in. apart until the zone about 1 in. from the wall was thoroughly explored.

During the first runs several very interesting facts were brought to light. It was found that pitot-tube traverses could be checked very well even in turbulent flows, providing there had been no interruption in the flow. The usual shutdown at noon-time was eliminated, tests being carried straight through without interruption until the entire set of traverse points had been measured, even though it was necessary at times to continue testing until evening.

It appeared that the general condition of flow could be duplicated, but the exact distribution varied enough to make it necessary to avoid interruptions when determining the pitot-tube coefficients.

Pitot-Tube Traverse

In order to give a clear picture of the velocity distribution under the three types of flow, isometric drawings have been prepared showing the velocity points. Fig. 7 shows the conditions of test 410–2 for smooth flow at the mean velocity of 2.6 fps. Fig. 8 shows a similar arrangement for the five 3-in. baffles at one-half depth, corresponding to baffle arrangement *B*, Fig. 2.

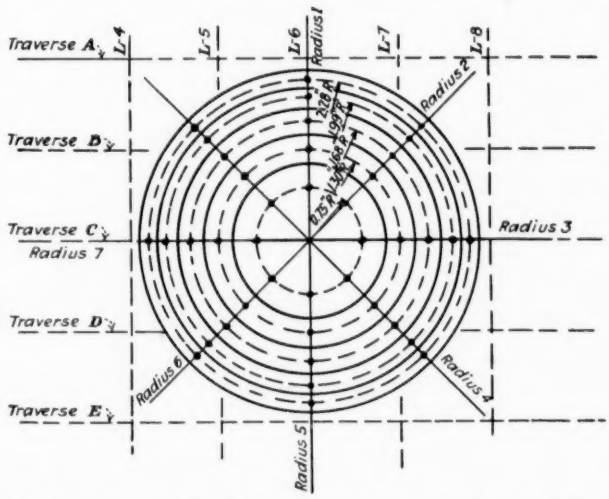


Fig. 6 Traverses Used and Location of Pitot Tube in Equal-Area Rings at Which Velocities Were Determined

(Diagram of 4³/₄ in. diam. circular area in front of meter. Heavy dots show points in five rings of equal area at which velocities by pitot tube were determined. Average of these forty velocities was used as mean velocity in front of meter.)

Fig. 9 shows the arrangement for test 410–3, which had five 3-in. baffles placed full depth in the flume, as indicated in baffle arrangement *A*, Fig. 2. It will be noted from these drawings that the velocity plane in front of the meter was a very irregular warped surface, which results in an unequal distribution of velocity across the plane of the current meter. This is particularly true when turbulence is present or when distortion of flow is encountered with the baffles at one-half depth, as shown in Fig. 8.

The wall velocities were investigated for the last 1 in. from the wall by pitot-tube readings taken very close together, the closest reading being taken $^1/_8$ in. from the wall. Fig. 10 shows these curves plotted to a large scale for the average flume velocity of 2.2 fps with the five 3-in. baffles at one-half depth (baffle arrangement *B*, Fig. 2). It will be noted that the velocities adjacent to the right-hand and left-hand walls of the flume were very similar in shape. These curves also agree very closely with the formula developed for wall velocities by von Kármán and Prandtl:

$$V = (\text{a constant}) \times X^{1/7} \ldots [1]$$

where X = the distance from the wall.

This agreement is indicated by the curves in dashed lines shown in Fig. 10, derived from formula [1].

In some of the tests at the higher flows, the velocities near the left-hand wall were difficult to determine because of the tendency of the pitot tube to oscillate. In such cases, the wall velocities for the last $^3/_4$ in. from the wall were determined by the von Kármán and Prandtl formula.

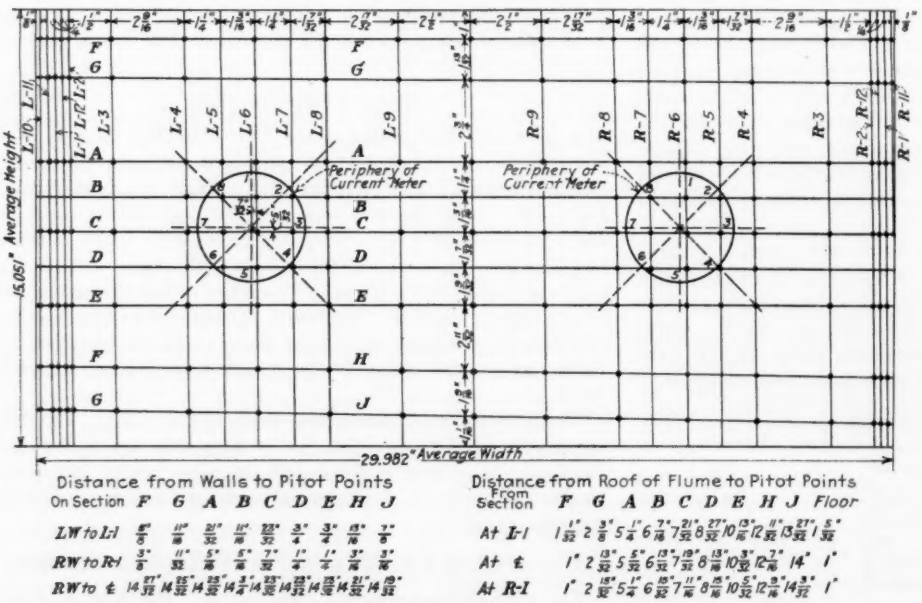


Fig. 5 Section of the Flume Looking Downstream Showing Cross-Sectional Dimensions and Location of Pitot-Tube Observation Points

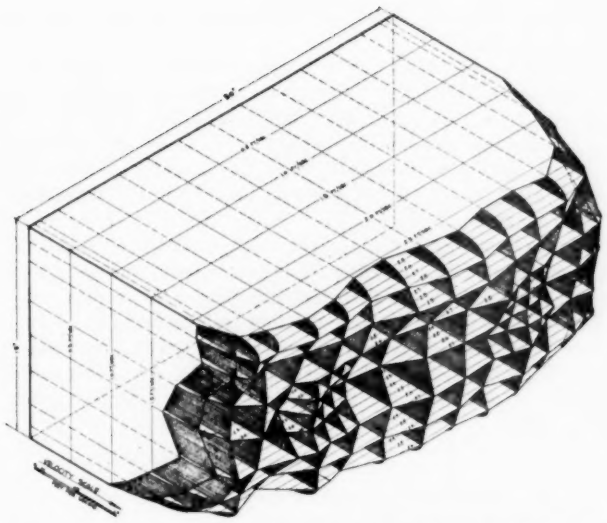


FIG. 7 VELOCITY DISTRIBUTION IN THE FLUME WITH NO BAFFLES FOR SMOOTH FLOW AT 2.6 FPS—CONDITIONS FOR TEST 410–2

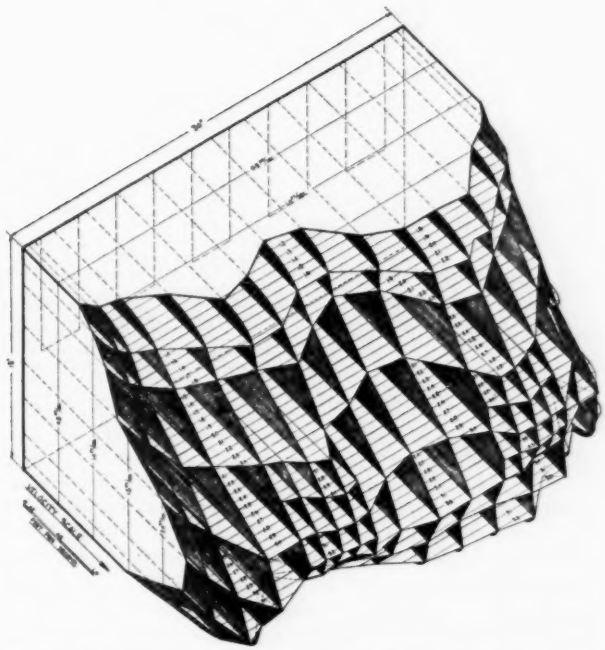


FIG. 8 VELOCITY DISTRIBUTION WITH BAFFLES AT ONE-HALF DEPTH AND A MEAN VELOCITY OF 2.6 FPS

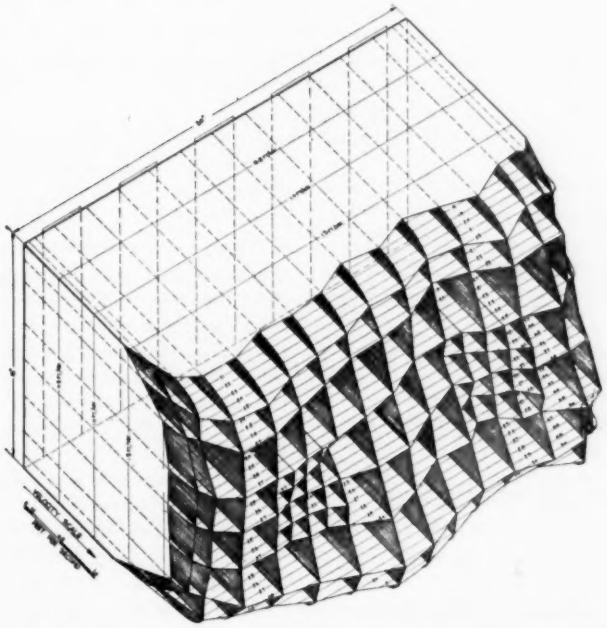


FIG. 9 VELOCITY DISTRIBUTION WITH FIVE 3-IN. BAFFLES AT FULL DEPTH AND MEAN VELOCITY OF 2.6 FPS—CONDITIONS FOR TEST 410–3

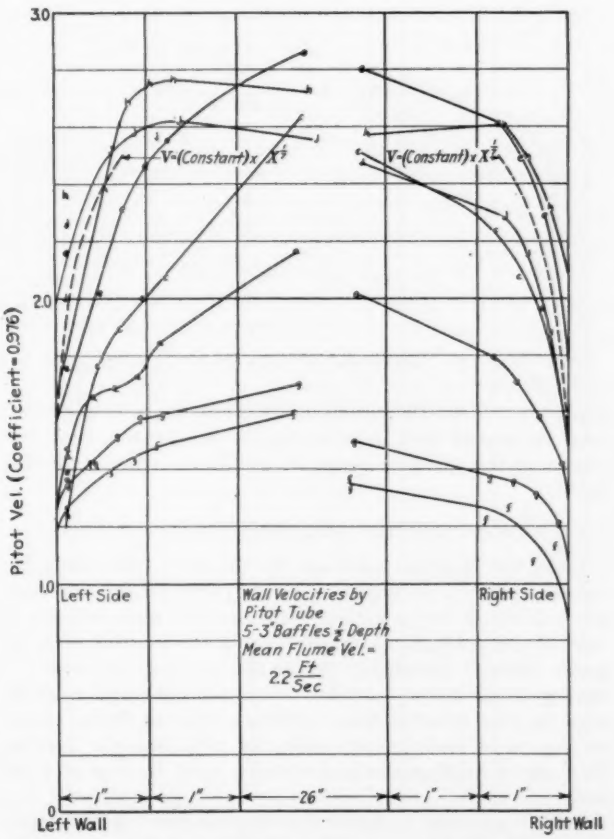


FIG. 10 WALL VELOCITIES WHEN THE AVERAGE FLUME VELOCITY IS 2.2 FPS WITH FIVE 3-IN. BAFFLES SET AT ONE-HALF DEPTH

RESULTS

Table 2 gives a comparison of two independent calculations of the pitot-tube coefficient in tests 410–2 to 410–7, inclusive, for the various flow conditions outlined. The formula used was $V = C\sqrt{(2gh)}$, the values of the coefficient C being given in Table 2.

In Table 2 it will be noted that the pitot-tube coefficient varies with different flow conditions and with velocities. In order to relate this to previous experience, data as shown in Fig. 11 is included in this paper. Early tests in a closed pipe, made by F. H. Rogers at the University of Pennsylvania in 1909, show a marked variation of pitot-tube coefficient increasing from 0.969 at a velocity of 2 fps up to 1.00 at a velocity of 15 fps. In the range in which we are most interested, namely, 2 to 6 fps, an average coefficient of 0.976 appears to be reasonable for smooth flow free

TABLE 2 COMPARISON OF PITOT-TUBE COEFFICIENTS FOR VARIOUS FLOW CONDITIONS

Test no.	Laboratory calculations	Independent calculations	Flow conditions
410–2	0.969	0.965	Smooth flow, mean flume velocity 2.6 fps. For sides, formula [1] was used since it corresponded closely to exp. results on 410–7. For bottom and top, avg of 410–7, right side, was used
410–3	0.953	0.948	Five 3-in. baffles at full depth. Mean flume velocity 2.6 fps
410–4	0.939	0.939	Five 3-in. baffles at full depth. Mean flume velocity 2.2 fps
410–5	0.957	0.962	Five 3-in. baffles at half depth. Mean flume velocity 2.6 fps
410–6	0.934	0.938	Five 3-in. baffles at half depth. Mean flume velocity 2.2 fps. Curves at side observed. Top and bottom taken as avg of right side
410–7	0.976	0.975	Smooth flow, mean flume vel. 3 fps. Curves at sides observed. Top and bottom taken as avg of right side

from turbulence. This coefficient also corresponds to the value designated in the Machinery Builders Test Code for pitot tubes in closed conduits. Additional values were secured from tests made for the S. Morgan Smith Company at Worcester Polytechnic Institute by Prof. C. M. Allen, in which the coefficient varied from 0.96 to 0.98, but averaged very close to 0.976.

When plotting the results of the tests for the 410 series, it was found that the smooth-flow coefficients averaged very closely to the mean curve of the University of Pennsylvania tests, shown as the I. P. Morris curve in Fig. 11. When turbulence was introduced, however, it was found that the difference between the determinations was much greater and that no single curve could be drawn through the points. By reference to the previous tests, a band of coefficients was established by drawing lines through the maximum and minimum values, similar in shape to the mean curve for smooth flow, and denoting the zone between these two curves as the probable range of coefficients under turbulent flow.

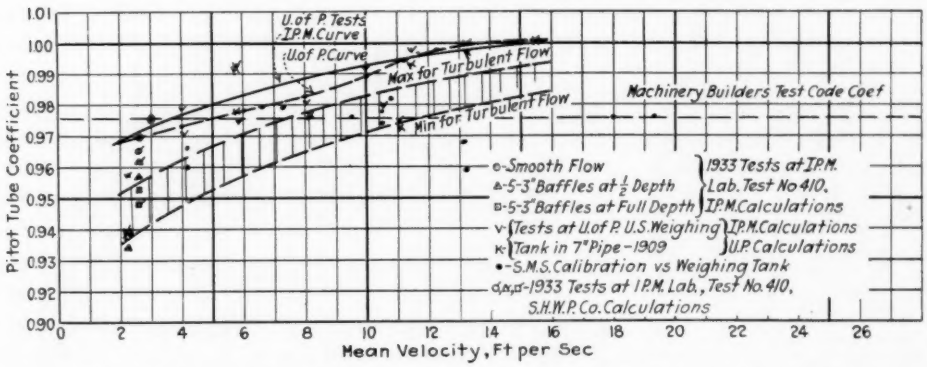


FIG. 11 VARIATIONS OF PITOT-TUBE COEFFICIENT WITH MEAN FLUME VELOCITIES

COMPARISONS WITH CURRENT-METER REGISTRATIONS

From the velocities measured by the pitot tube, using the correct coefficient as determined for the particular flow condition and velocity, it was possible to determine the mean velocity in front of the meter and compare this with the registration of the meter obtained periodically during the pitot-tube traverse. A reading of the meter was taken after each individual traverse, after the pitot tube had been withdrawn from the flume. Eight or nine meter readings were taken for each complete traverse, these meter readings checking within a small fraction of 1 per cent.

Table 3 gives the results of this determination for both right-hand and left-hand meters. The type 1 meter alone was used for tests 409 and 410, but three runs were made with type 2 meter, which is included for reference. The data in this table have been plotted in Fig. 12.

The ratio of velocity by pitot tube to velocity by meter, taken from the last column of Table 3, are plotted as ordinates, while the velocities of the current meter established from the still-water calibrations made at the Bureau of Standards in Washington are plotted as abscissas. This curve, therefore, shows the meter coefficient for different velocities and for both smooth and turbulent flows.

It will be noted that a band about 1 per cent wide, slightly on the side toward over-registration, results when smooth flow exists in the channel. When turbulent flow exists, the meters have a definite tendency to over-register and also the band of points is nearly twice as wide as that for smooth flow. It is interesting to note that in no case with turbulent flow did the meter under-register, as compared with the absolute-velocity determination by pitot tube.

CONCLUSIONS

From the study of these tests certain conclusions can be drawn which should be carefully considered in making determinations of discharge under field conditions. The so-called ideal conditions for current-meter measurement could be outlined somewhat as follows:

1 The measuring section should be rectangular in form and a sufficient distance downstream from any change in direction or change in area to insure smooth flow lines, with a velocity in excess of 1 fps and not over 8 fps.

2 The section should be free from turbulence caused by racks, supports, piers, or changes in section which would cause oblique flow or turbulence.

Where such conditions exist, it is reasonable to expect that the discharge determination would be accurate to within 1 per cent of the true discharge, using the still-water rating of the current meter as a basis.

With the type 1 meter in turbulent-flowing water, similar to that which existed in the test flume, there seems to be a distinct tendency toward over-registration, as compared with either the still-water rating or with the actual registration of the meter in smooth-flowing water. The results of this investigation, shown in Fig. 12, indicate that turbulence may cause the meters to register from 2 to 5 per cent excess discharge.

An objection might be raised that, because of the relatively small size of the flume, the reduction in area caused by the current meters would result in a higher velocity at the section containing the meters than at the section of the pitot-tube traverses. A correction was made, computed from the area of the current-meter vanes and hubs, by adding 0.66 per cent to the velocities determined by the pitot tubes. Assuming, however, that this correction is in error, and that for smooth flow the coefficient of the meter should be unity, it is evident that for turbulent flow the meter would still over-register by approximately $2\frac{1}{2}$ per cent. This method of comparison is equivalent to using a rating of the current meters derived from their actual behavior in place in the flume under smooth-flow conditions, instead of their still-water ratings. That is, if the smooth-flow ratings, based on actual measurements of the discharge by calibrated weir, are applied to

TABLE 3 COMPARISONS BETWEEN CURRENT-METER AND PITOT-TUBE REGISTRATIONS, TESTS 409 AND 410

Run No.	Mean flume vel.	Flow condition	Pitot coeff. from test 410	Left—Meter No. 5764-1-*L*: Avg pitot vel. observed	Avg pitot vel. + 0.66%	Avg meter vel.	Ratio pitot vel. to meter vel.	Right—Meter No. 5759-1-*R*: Avg pitot vel. observed	Avg pitot vel. + 0.66%	Avg meter vel.	Ratio pitot vel. to meter vel.
409–1	3.080	Smooth	0.976	3.151	3.172	3.182	0.9969	2.962	2.982	3.042	0.9803
–2	2.590	Smooth	0.969	2.694	2.712	2.718	0.9978	2.498	2.514	2.557	0.9832
–3	2.578	5 3-in. baffles at full depth	0.953	2.782	2.800	2.817	0.9940	2.367	2.886	2.882	1.0014
–4	2.200	5 3-in. baffles at full depth	0.939	2.027	2.040	2.140	0.9534	2.438	2.454	2.530	0.9699
–5	2.600	5 3-in. baffles at 1/2 depth	0.957	3.084	3.104	3.173	0.9783	3.090	3.110	3.178	0.9786
–6	2.200	5 3-in. baffles at 1/2 depth	0.934	2.493	2.509	2.571	0.9759	2.467	2.483	2.540	0.9776
410–1	3.000	Smooth	0.976	3.130	3.151	3.145	1.0019	2.931	2.950	3.012	0.9794
–2	2.600	Smooth	0.969	2.767	2.785	2.807	0.9922	2.531	2.548	2.589	0.9842
–3	2.600	5 3-in. baffles at full depth	0.953	2.569	2.586	2.693	0.9603	2.665	2.683	2.767	0.9697
–4	2.200	5 3-in. baffles at full depth	0.939	2.100	2.114	2.196	0.9626	2.291	2.306	2.415	0.9549
–5	2.600	5 3-in. baffles at 1/2 depth	0.957	3.152	3.173	3.282	0.9668	3.054	3.074	3.175	0.9682
–6	2.200	5 3-in. baffles at 1/2 depth	0.934	2.633	2.650	2.753	0.9626	2.530	2.547	2.655	0.9594
–7	3.000	Smooth	0.976	3.176	3.197	3.184	1.0041	2.941	2.960	2.980	0.9933
				Left—Meter No. 5764-2-*L*				Right—Meter No. 5759-2-*R*			
409–7	3.000	Smooth	0.976	2.821	2.840	2.813	1.0096	2.726	2.744	2.751	0.9975
–8	2.600	5 3-in. baffles at full depth	0.953	2.809	2.827	2.596	1.0890	2.960	2.960	2.714	1.0980
–9	2.600	5 3-in. baffles at 1/2 depth	0.957	3.126	3.147	3.162	0.9953	3.140	3.161	3.193	0.9900

Average ratio for smooth flow = 0.991.
Average ratio for all baffles = 0.970.

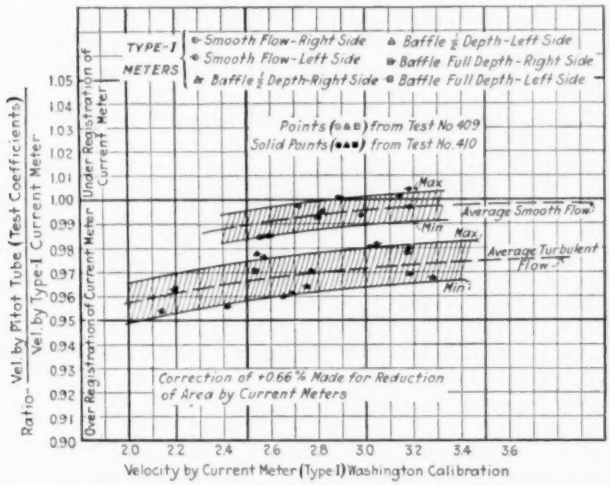


FIG. 12 COEFFICIENTS OF TYPE 1 METER. POINTS PLOTTED FROM DATA IN TABLE 3

the turbulent-flow measurements, the results will exceed the observed flow by approximately 2 1/2 per cent.

Where angular flow occurs, in combination with turbulence, the accuracy of water measurements by current meters is still in doubt as no tests are available to indicate that correct results can be secured under such conditions.

In conclusion, it should be stated that the tests made in the I. P. Morris Laboratory at Eddystone, Pa., were undertaken in an attempt to clear up some of the elements of doubt which have existed in regard to current-meter measurements, and to explain the discrepancies which have resulted when current-meter measurements are made under conditions which are not ideal. It is felt that further research and studies are necessary, and these should include the determination of the true-velocity plane in front of the current meter under varying conditions of flow. Such determinations would clear up some of the unexplained differences between the results secured at Eddystone and those secured in other flumes and in open channels. Another feature which should be established is the behavior of current meters with both turbulent and oblique flow present simultaneously in the metering section. From such determinations it might be possible to fix more definitely the limiting conditions under which current meters should be employed for precise flow measurements.

PERSONNEL

These tests were conducted at the Baldwin-Southwark Corporation, I. P. Morris Division Laboratory at Eddystone, Pa., by K. W. Beattie of the research department, under the direction of the author. The planning of the tests and investigations was done by a committee consisting of F. H. Rogers, chief engineer, R. E. B. Sharp, hydraulic engineer, L. F. Moody, consulting engineer, and the author, as representatives of the I. P. Morris Division. In addition the S. Morgan Smith Company was represented by J. D. Scoville, hydraulic engineer, and the Safe Harbor Water Power Corporation was represented by L. M. Davis and E. T. Schuleen of the test department.

Water Gaging for Low-Head Units of High Capacity

By J. M. MOUSSON,[1] BALTIMORE, MD.

In the field of hydroelectric construction there has been in recent years considerable advance in the development of low-head units of high specific speed. This tendency in design has been conducive to the development of low-head sites which were at one time considered uneconomical. It became, therefore, increasingly important to find methods of measuring the discharge of low-head plants where the water quantities are large and the water passages particularly short and unsuited to other methods of testing. American engineers had developed methods of measurement of flow in long penstocks and then attempted to extend these methods to plants with shorter water passages. European engineers have shown a preference for current meters, at one time commonly used on both sides of the Atlantic for measurements in large intakes.

The paper discusses the status of the current-meter method in Europe at the time this method was selected for the tests at Safe Harbor and the application of the two-type meter method. It shows how this method was applied at the outset and how certain modifications were made for increasing the reliability and decreasing the time and effort required for making tests. These made possible new methods of compilation which have been developed to facilitate greatly this phase of the testing program. The successful application of these methods should make current-meter measurements more feasible than ever before.

In preparation for these tests, investigations were conducted at the Bureau of Standards and elsewhere to determine the characteristics and limitations of the meters under various conditions of uniform, oblique, pulsating, and turbulent flow.

It has been shown that water passages with parallel sides for metering and approach sections do not insure parallel flow, and that even under as nearly ideal conditions as it is practical to secure, the use of two types of meters will indicate an effective angularity. However, it is believed that accurate measurements of oblique flow can be made by the use of two types of meters, each affected to a different extent by angularity. The use of this method will greatly enlarge the field of current-meter testing.

There are pointed out the advantages of making index tests in conjunction with current-meter measurements; not only to extend them to a wide range of operating conditions, but also as a measure of the consistency of the discharge measurements.

Introduction

ON COMPLETION of the first four main units of the initial installation at Safe Harbor, it was essential that careful measurements be made of the water used for the purpose of accurate accounting, for determining the characteristics of the units preparatory to laying out loading schedules, and for many economic studies required in connection with operation. Inasmuch as the water passing through the wheels constitutes a relatively large proportion of the total river flow, accurate measurement is particularly desirable for the purpose of providing a reliable flow record and for dispatching water to the plants below.

The four main turbines are of the Kaplan type whereas the two service units have Francis runners. The Kaplan turbines have a capacity of 42,500 hp at a rated head of 55 ft, a speed of 109.1 rpm, and a maximum discharge of approximately 9000 cfs. A typical cross-section through the power house on the center line of a main unit is shown in Fig. 1. The water enters through three intake bays. Twelve such units are provided for in the plans for the ultimate development. The two service units are rated at 3100 hp and 180 rpm under a head of 55 ft.

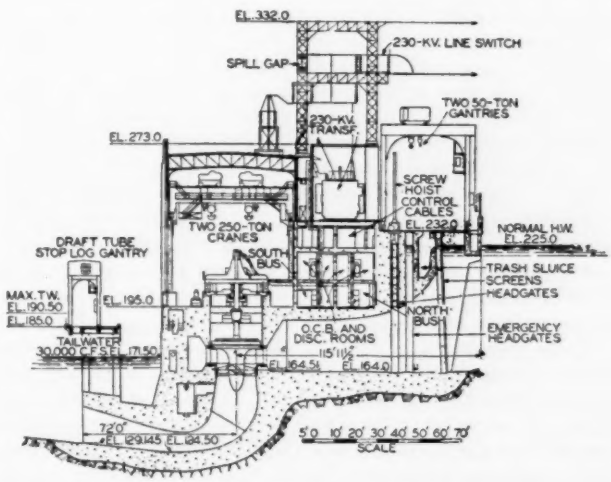


FIG. 1 TYPICAL CROSS-SECTION THROUGH THE SAFE HARBOR POWER HOUSE

The testing program provided for discharge measurements on two of the main units and one service unit. These tests were carried out during the summer and fall of 1932 and 1933.

Selection of Method

Before selecting the method to be used, a careful study was made of the history and development of many methods of water measurement. The weir, traveling screen, and salt-titration

[1] Safe Harbor Water Power Corporation, Baltimore, Md. Mr. Mousson was graduated in 1926 from the Swiss Federal Institute of Technology, Zurich. He was designing engineer with the Pennsylvania Water & Power Co., and the Electric Bond and Share Co. from 1928 to 1929. Since that time, he has served on the engineering staff of the Safe Harbor Water Power Corporation as designing engineer, sponsor, and hydraulic engineer.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., December 4 to 8, 1933, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

methods could be eliminated as impractical on account of the tremendous quantities of water to be gaged and the impossibility of erecting the necessary structures at reasonable cost.

In general, the field of turbine testing in America, for the last decade at least, had been dominated by the Gibson and the Allen methods. The increasing success that these experimenters had, even outside of the field of long pipe lines, where their methods were eminently suited, had brought them more nearly into the domain of the methods used where the water passages are particularly short, as in low-head plants.

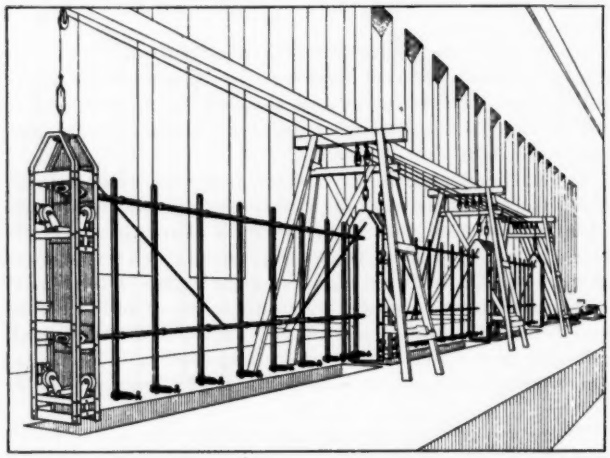


FIG. 2 CURRENT-METER FRAMES AND HOIST USED AT RYBURG-SCHWOERSTADT

Although current meters have been used in America, the results were on the whole rather unsatisfactory, principally because too extensive measurements were attempted with too few meters, inadequate supporting frames, and often improper location of the measuring section, particularly if in the tailbay.

During the years prior to 1931, marked improvements had been made in the art of current-meter measurements which were not generally appreciated in America because of the limited application that had been made in this country. These improvements were not confined to the meters alone, but applied also to the apparatus, set-ups, and procedures. The simultaneous use of a large number of meters had led to the adoption of movable supporting frames. Graphic recorders had found successful application for making a permanent record, avoiding thereby the need of a large corps of observers.

Large propeller units were more highly developed at an earlier date in Europe than in America, and stimulated a keener demand for a method that would successfully measure unusually large quantities of water. The layout of these plants precluded the use of a weir and the shortness of the water passages in proportion to the width put them in a range to which the American methods had not been extended at that time.

Inasmuch as Safe Harbor resembles similar European plants in this respect, it is only natural that careful consideration should be given to the methods that were used abroad. The engineers connected with the Safe Harbor tests fully appreciated the possibility that the Gibson and Allen methods might eventually be applied to similar layouts, but in view of uncertainty in this respect, it was decided to use that method for which the greatest amount of accumulated experience was then available.

CURRENT-METER PRACTICE IN EUROPE

It is well to consider the stage of development that the current-meter method had reached at the time the Safe Harbor tests were contemplated. This may best be done by describing briefly the test equipment and procedures used in 1931 at Ryburg-Schwoerstadt, on the Rhine between Germany and Switzerland.

These tests were confined to one of the four Kaplan turbines. Each of these turbines discharges a maximum of about 11,650 cfs under a head of 35.2 ft. Three current-meter frames, one for each intake bay, were provided. Each of these frames supported ten current meters. They were raised and lowered by three hand winches, one for each frame, supported by a temporary wooden structure as in Fig. 2. The position of the frames was indicated by an arrow mounted on the intake deck and by markings on the hoisting ropes.

The current meters were right hand, Ott V Texas type equipped with spoke-vane propellers. False steel roofs extending from the racks to the upstream edge of the stop-log slots were installed in the intakes to straighten the flow approaching the measuring section, Fig. 3. The roofs were sloped, however, approximately 10 deg toward the downstream side, and the plane of measurement was assumed to be flush with the downstream edge of the temporary structure. The area of the water passage at the plane of measurement in each intake bay was rectangular in shape, 23.0 ft wide and 29.1 ft high.

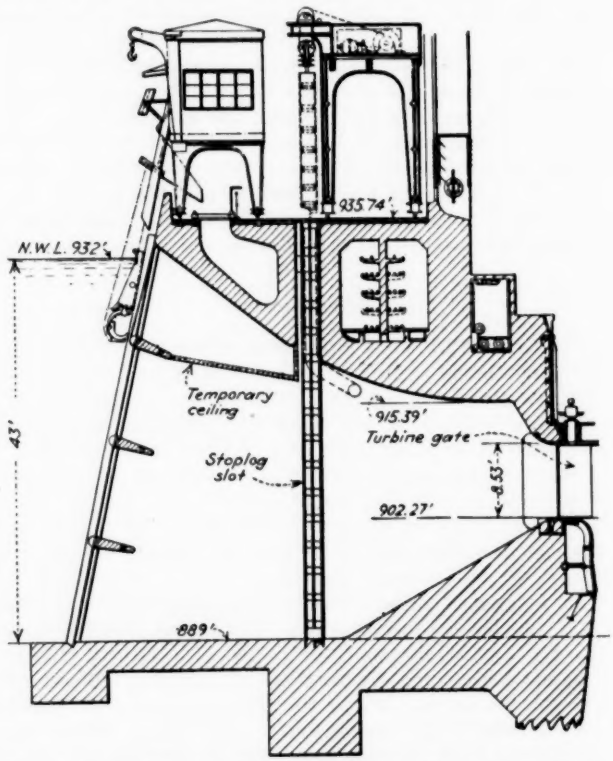


FIG. 3 CROSS-SECTION THROUGH INTAKE WITH FALSE ROOF, RYBURG-SCHWOERSTADT

During each test the frames were brought to 12 different elevations, and held for about 1 min. The spacing of the meters as well as the various frame positions were chosen so that the areas represented by each point of measurement would be small near the side walls, intake bottom, and intake roof, but progressively greater toward the center. During each test requiring between 22 and 24 min, 360 measurements were taken.

One graphic recorder was provided for each intake opening. The circuits for the commutators of the current meter were connected individually to ten pens, while one pen on each recorder registered the time in seconds.

A total of 24 tests were made under almost identical head conditions. The measurements were divided into groups of four or five runs, each group covering the range of peak efficiency for a given fixed runner-blade setting and variable gate opening.

Selection of Apparatus and Design of Equipment

(*a*) *Current Meters.* An extensive study was made of all data then available regarding the various current-meter designs and their respective behavior in still as well as in flowing water. Considering the fact that current meters had not been extensively used for discharge measurements on turbines in America the wealth of data was surprising. These data concerned practically all known current meters of American and European design of both the cup and propeller type such as the Price, Gurley, Hoff, Haskell, Pegram, Fteley, Ott, Stoppani, and Amsler.

At the outset it was realized that the principal difficulty to be feared was the effect of the bell-mouth. With intakes as shown in Fig. 1, it would be necessary either to correct for the convergence of the stream lines approaching the gate slots where measurement would be made, or else to erect some means of securing parallel flow by a false roof.

The A.S.M.E. Test Code, now in course of revision,[2] specifies on page 18, paragraph 101: "Meters of two types shall be used having opposite characteristics in running water; that is, one type of meter retarded by oblique flow and the other type accelerated by oblique flow."

The first consideration, therefore, was to choose from the available types two suitable meters, i.e., a type which would over-register slightly in oblique flow, and a second which would under-register approximately by the same amount, so that the true discharge would be derived by averaging the readings of the two.

As a result of our study, no suitable meter could be found which would consistently over-register for a given angle of obliquity whatever the plane of approach. The cup meters which have the tendency to over-register, could not satisfy the requirements as these meters very obviously are affected differently depending upon the plane of approach.[3] Some tests were actually made using cup-type meters simultaneously with propeller-type meters, but it is obvious that because the over-registration of cup meters is dependent upon the plane, this cannot be a reliable means of doing more than indicating that oblique flow exists in the section.

In view of this fact it was proposed to substitute in place of the meters called for in the Power Test Code, two types of meters, both of which under-register but to different extents.

The object was then to provide first a meter termed type 1, which should register in oblique flow, as nearly as possible the actual flow times the cosine of the angle of inclination, i.e., the component in the direction of the axis of the meter; and another meter called type 2 which should differ as much as possible from the characteristics of type 1, but still have a stable and well-defined relation between the amount of registration and the angle of inclination to the flow. It is the sole purpose of this type-2 meter to measure the amount by which the type-1 meter is affected by angularity.

The meter which at that time best satisfied the requirements for the type 1 was the Ott V Texas type with three square vanes mounted on radial spokes. These meters had a nominal pitch of 25 cm, Fig. 4. An example of the painstaking care and forethought on the part of the manufacturer is his provision of plaster molds for the propellers so that whenever desired a check could be made of the propeller shape to see that it had not become damaged or altered, even in the slightest degree.

The decision reached for the first type of meter naturally influenced the selection of the second and the advantage of having clamps, bodies, axles, ball bearings, and all spare parts interchangeable made the decision obvious.

In spite of all the information which the manufacturer had, it was deemed necessary that some experimental work should be done before being satisfied that the meters he had to offer were the most suitable. The fact that there is a rating station at the factory greatly simplified the situation, as valuable time could thus be saved. Based on the results of his test, the preliminary decision to use the type-1 meter was adhered to, while the Ott V Texas meter, equipped with a propeller of conical screw shape and 50-cm pitch, was selected for the type 2, Fig. 5.

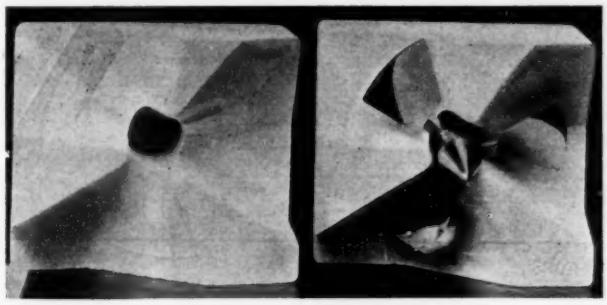


Fig. 4 Type-1 Propeller With Plaster Mold

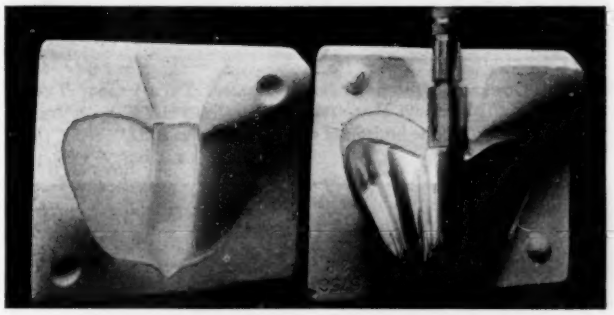


Fig. 5 Type-2 Propeller With Plaster Mold

In European practice it is customary to observe an established relation, which is that the number of stations should be between 4.3 and 7.6 times the square root of the area of the section in square feet. After carefully considering various possible ways of dividing the intakes, it was decided to use nine meters of each type in each intake, thus requiring, with one spare of each kind, 56 meters. Each propeller was furnished with its plaster mold.

It had been decided that, in order to avoid error arising from the effect of eddies with axis parallel to that of the meter, it would be necessary to use right-hand and left-hand meters alternately placed, a principle rigidly adhered to throughout the tests.

(*b*) *Supporting Frames for Current Meters.* The three frames supporting the meters were built to travel in both head-gate and emergency-gate slots. The frames consist of two structural-steel ends, to which two horizontal cross-members are bolted. These horizontal cross-members consist of chrome molybdenum, streamline, seamless steel tubing 2 × 4.75 in., heat-treated to give high tensile strength, Fig. 6. The special material made it unnecessary to have diagonal bracing, which would have set up possible undesirable disturbances. The end girders

[2] Power Test Code, Subcommittee No. 18, Ely C. Hutchinson, chairman.

[3] L. A. Ott's discussion of "Research Institute for Hydraulic Engineering and Water Power," by Hunter Rouse, Trans. A.S.M.E., 1933, vol. 55, no. 10, p. 41, paper HYD-55-3.

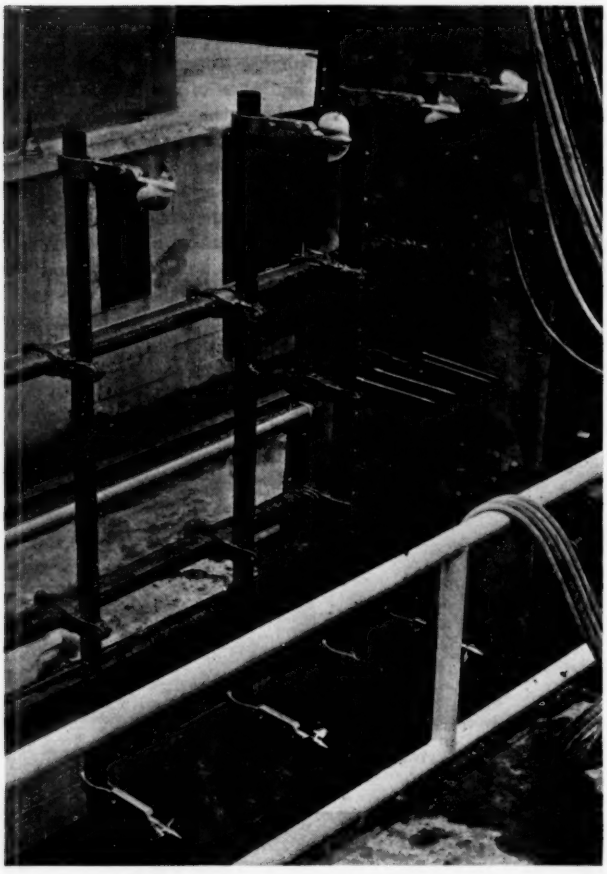


FIG. 6 CURRENT-METER SUPPORT FRAME, TYPE-1 AND TYPE-2 METERS MOUNTED SIMULTANEOUSLY
(Note horizontal row of pitot tubes.)



FIG. 7 CURRENT-METER FRAME HOIST. TYPE-2 METERS MOUNTED NEAR BOTTOM OF VERTICAL FRAME RODS

are equipped with rollers. Those on the downstream side are mounted rigidly, while those on the upstream side and adjacent to the piers are mounted on arms so that they can be forced outward by springs to eliminate vibration and to hold the frames firmly in position in spite of any variation in the width of the slots. The structural-steel ends are closed off on the water-passage side by means of a steel plate to eliminate disturbances along the piers. Special gages were constructed for inspection at the factory and during field assembly to check the alignment of the meter supporting rods.

(*c*) *Current-Meter Hoist and Hoist Supports.* In order to simplify lowering and raising the frames, as well as to reduce the personnel during the tests, a fully automatic hoist was specially designed. The somewhat high initial cost of this was well justified, as it was foreseen that it would allow a great reduction in testing time required as well as in office work. Furthermore, as the ultimate development is intended to comprise 12 units, the testing equipment could be regarded as permanent investment to take care of all future requirements.

The hoist consists of a double-drum, worm-geared, electric motor-driven unit designed to lower and raise the three current-meter frames simultaneously.

The hoist is capable of raising or lowering the frames at speeds of 2, 3, 4, and 6 fpm, Fig. 7. A mechanically driven elevation indicator geared to the pinion driving the drums is provided to show the position of the meter frames. This elevation indicator enables the operator of the hoist to bring the frames to any desired position to the nearest one-hundredth of a foot. The indicator is also equipped with a contact electrically connected to pens of the graphic recorders to indicate every tenth of a foot of travel. This arrangement permits the determination of the speed at which the meter frames are raised or lowered by comparison with the time impulses recorded simultaneously, and provides a permanent record of the true position of the current meters at any time during the tests.

(*d*) *Graphic Recorders and Circuits.* The graphic recorders must have a sufficient number of pens to register simultaneous impulses from 54 current meters as well as from the elevation indicator and a timing device. Motor-driven Esterline-Angus production recorders with multipliers and quick-trip attachment solved this problem satisfactorily. High chart speed is essential to prevent confusion of the current-meter record at high velocities.

A typical record is shown in Fig. 8. The sequence of the long and short contacts for each cycle of ten revolutions of the propeller enables the observer to determine the direction of flow. The short break in the long contact is useful in interpreting the record for very low velocities. As no commercial instrument could be found with 56 pens, it was necessary to get two recorders, Fig. 9, one accommodating two charts and the other a single chart. Both recorders have 20 pens for each chart, which were assigned one to each intake bay. Of the 20 pens provided per chart, 18 could be used to register the meter contacts, while the two end pens were connected to the contact-making devices for timing and indicating the elevation of the meter frames, Fig. 10.

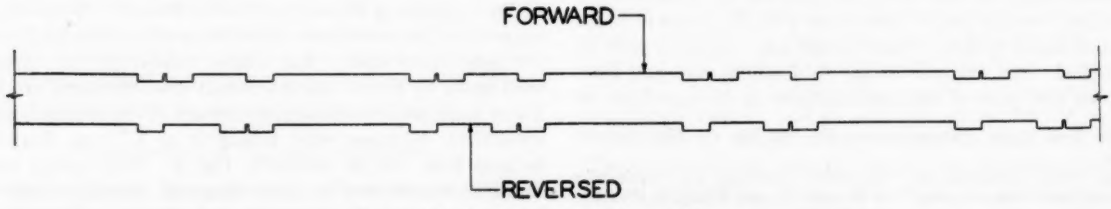


FIG. 8 TYPICAL CURRENT-METER RECORD

The graphic recorders were installed in a housing placed on a standard-gage railroad flat car. This provision was made so that any section of the intake deck could be cleared at short notice for operating purposes.

A condenser and resistor bank containing 60 units is used in the current-meter circuits to prevent sparking of the contacts. Each unit consists of a 0.24-μf condenser and a 10-ohm resistor. A four-cell, Philco, type P.M.T., 13-plate battery, supplies current for the meter circuits at 4 volts.

To insure proper coordination of the test personnel, a signal system of electric bells was installed.

(*e*) *Timing Device.* As it was necessary to provide a source of time impulses, serious consideration was given to various spring-wound portable timers. It developed, however, that these types of apparatus would introduce an uncertainty in time of about 0.2 per cent. Considering that this error would impair the accuracy of the tests, advantage was taken of a practically absolute source of time.

In the control room of the power plant there is installed permanently a Warren pendulum clock for frequency regulation. A photoelectric cell and an automobile spotlight focused on this cell were placed in such positions that the pendulum cut the beam of light at every swing, Fig. 11. This arrangement gave a precision far in excess of requirements, as the pendulum can be regulated to accumulate an error of less than one second a day. The wiring diagram for the timing device is shown in Fig. 10.

(*f*) *Pitot Tubes.* It is well known that the velocities may vary considerably in the vicinity of the boundaries of the measuring section. Special provisions were made to determine them by pitot tubes, thus avoiding the small error which would result from extending the velocity curves beyond the last meter position.

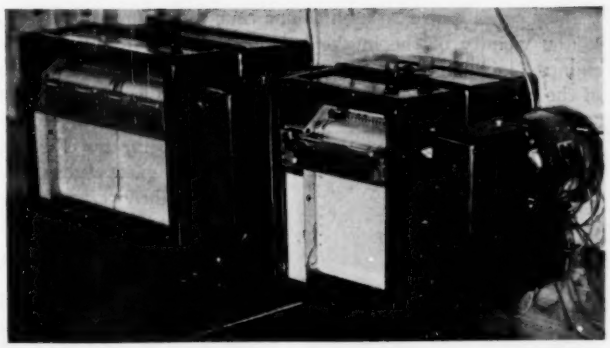


FIG. 9 GRAPHIC RECORDERS

A row of four pitot tubes was installed near each end of the current-meter frame. The tubes were mounted on short lengths of streamline tubing half way between the cross-members of the frames as shown in Fig. 6. The pitot tubes of one row are spaced so that two are between the intake wall and the first current meter, while two others are between the first and second current meters. The pitot tubes as arranged permit measurements of relative values of velocity which may be coordinated with the current-meter measurements. Reinforced rubber hose connected the tubes with a gage board located in the trash sluice, see Fig. 12. This gage board consists of three groups of eight glass tubes each, one group for each intake bay. A glass tube on either side of the three groups is used for static-head readings.

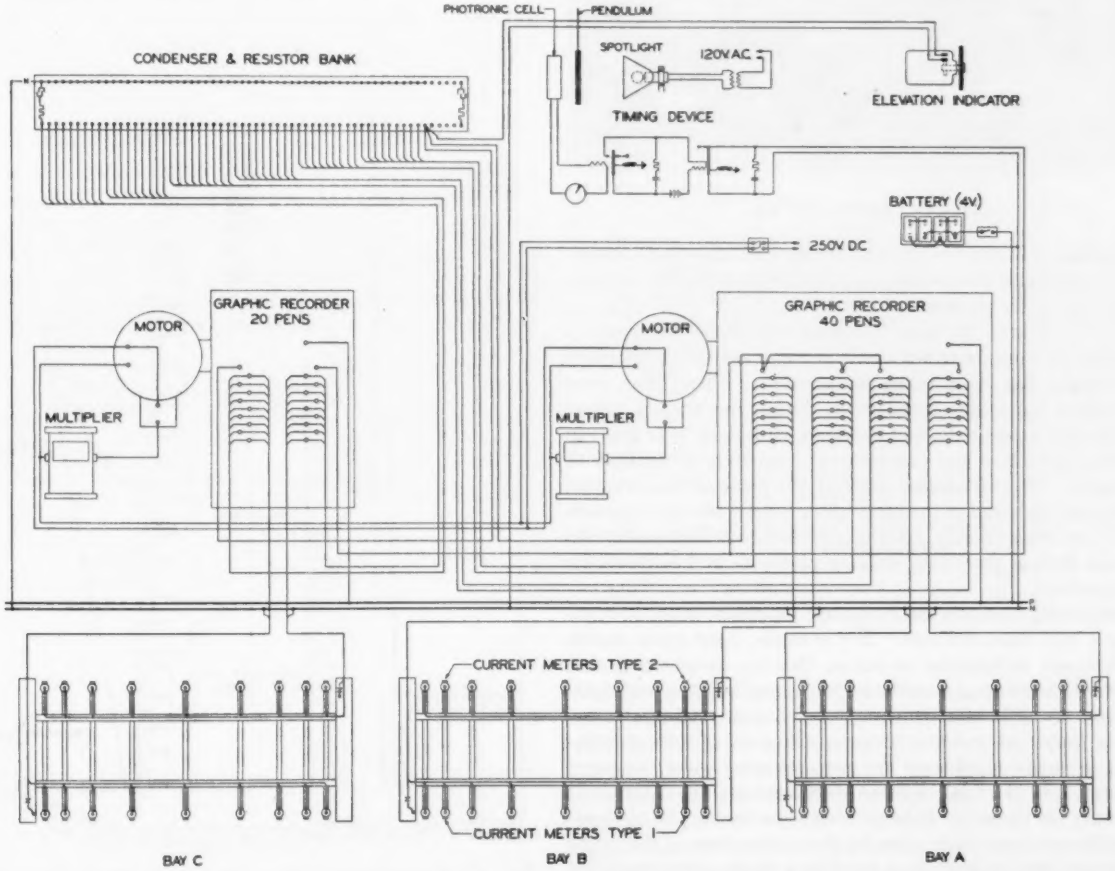


FIG. 10 WIRING DIAGRAM

Pitot tubes were also used at one time to explore the velocity in the vicinity of the intake bottoms. A streamline rod was clamped in a vertical position to the horizontal cross-members of the current-meter frames. This vertical rod was made of the same tubing as the horizontal frame members, see Fig. 13. Four of the pitot tubes previously described were mounted on the lower end of the rod.

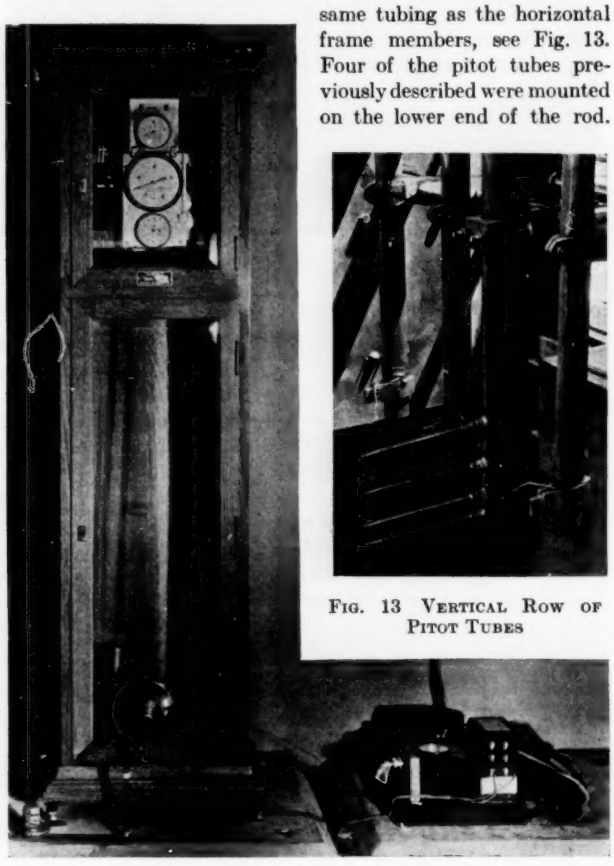


FIG. 11 TIMING DEVICE

FIG. 13 VERTICAL ROW OF PITOT TUBES

The spacing provided two pitot tubes between the first current-meter position and the bottom, and two between the first and second current-meter positions.

(*g*) *False Work.* In most European tests the effectiveness of false work to straighten out the flow approaching the plane of measurement has been taken for granted. There have been cases where temporary installations within the intakes proper were thought inadequate, and elaborate structures were made in the forebay, such as pier extensions or temporary elongations of the intakes. On still another occasion the plane of measurement was located upstream in the head-water canal where an approach of this type was available, making it necessary to have costly and elaborate bridges providing working platforms and supports for the apparatus.

Installations of a temporary nature naturally have their advantages and disadvantages. For example, head-water canals are often not rectangular in shape, thereby complicating test equipment, measuring procedures, and compilation of test data. Not only are elaborate temporary installations undesirable because of their cost, but also because, with most of these installations, the turbine discharges are not measured under operating conditions. In the final analysis, discharges are not determined exclusively for the sake of the problems concerning the turbines, but on the contrary, even more for the performance of the entire installation, that is, individual units as a whole under operating conditions.

Keeping in mind the characteristics of the current meters selected for Safe Harbor and the physical outlines of the intakes, it was thought possible to omit temporary installations of any kind, for it was believed that an evaluation of the obliquity of flow could be made.

It was thought desirable, however, to install for the first test a false roof in the intake of one of the units shown in Fig. 12. The advantage to be derived from such a plan was the possibility of tying in the two-type current-meter method with the conventional one-type method.

The false roof installed under the bell-mouth of each intake bay of one unit was made of wood. This roof was placed in a level position as it was believed that by so doing the flow would be straightened out more satisfactorily. The temporary structure was drawn tight against the concrete by means of steel cables extending down the emergency-gate slots and the trash-rack openings. It was also fastened to wooden fillers placed within the emergency-gate slots to prevent undesirable disturbances from being carried downstream to the plane of measurement. To improve further the flow conditions, a temporary wooden bell mouth covered with sheet metal was attached to the trash racks. This in turn had an extension reaching upstream to be flush with the intake piers, since hydraulic model tests in connection with the spillway design for Safe Harbor had shown that the most favorable conditions of discharge could be obtained by keeping the piers and the spillway nose flush with each other.

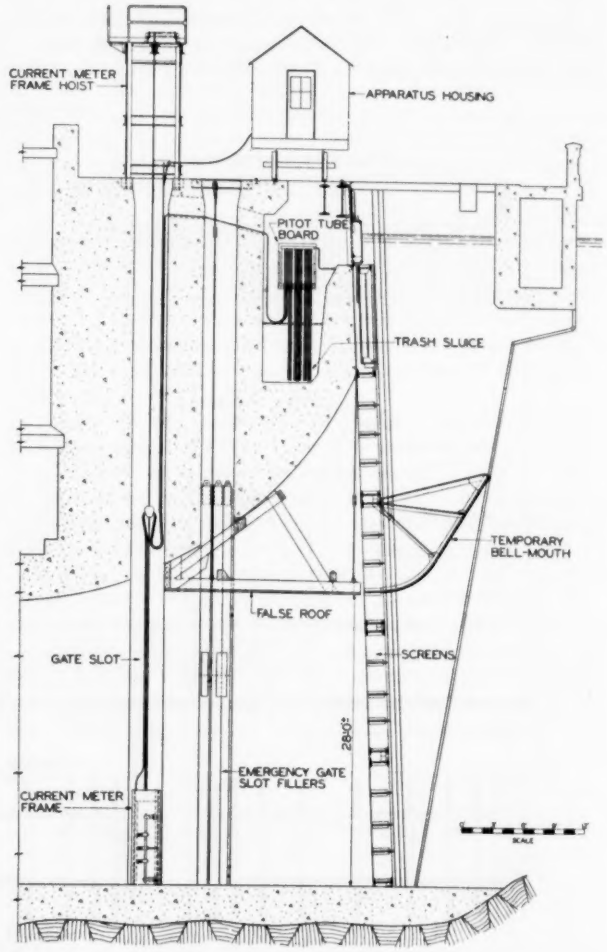


FIG. 12 CROSS-SECTION THROUGH INTAKE WITH FALSE ROOF IN PLACE AND APPARATUS SET-UP

Calibration of Current Meters and Special Investigations

(a) *Calibration at the Bureau of Standards.* As a first step it was necessary to determine the usual still-water calibration at various constant speeds. A minimum of 18 runs for each rating was made to cover adequately the velocity range from 0.5 to 7.5 fps.

Instead of rating tables, or the usual calibration curves, giving revolutions per second vs. velocity in feet per second as in Fig. 14, curves were plotted to give pitch in feet per revolution vs. revolutions per second as in Fig. 15. The advantage of such a curve is that it may be read with greater precision. By this means the effect of personal equation in reading the curve is practically eliminated.

A number of the meters used at Safe Harbor were calibrated by the manufacturer before shipment and recalibrated on arrival at the Bureau of Standards. These ratings agreed within 0.1 per cent throughout the useful velocity range.

Following the tests in the field, a substantial number of the meters of each type were recalibrated as required, whereas the propellers of all the meters were carefully checked against the plaster molds.

These meters use oil-lubricated ball bearings, but the prime function of the oil is to keep water out of the bearings, rather than to lubricate. Nevertheless, there is some effect of viscosity. A record was kept of the water temperature in the rating tank, and ratings were made in the winter as well as the summer to secure temperature variations from about 5 C to 26 C. In addition to repeated ratings with one meter at various water temperatures, calibrations were made with various blends of oil other than the light ice-machine oil supplied by the manufacturer. These blends consisted of sperm oil and kerosene.

The result of this investigation indicated that the viscosity of the oil does affect the meter and that this effect in turn depends upon the rate of rotation. It was interesting to see that within the limit of the experiments, and any one rate of rotation, the effect varies in a straight-line relationship with the oil viscosity, Fig. 16.

The oil mixture consisting of 10 per cent sperm oil and 90 per cent kerosene proved very satisfactory and no appreciable effect of temperature will be felt in practice.

To determine the effect of oblique flow, four type-1 and six type-2 meters were inclined in a vertical plane by steps of 5 deg up to and including 30 deg. As the average characteristic of all meters of each type was the ultimate object, these meters were

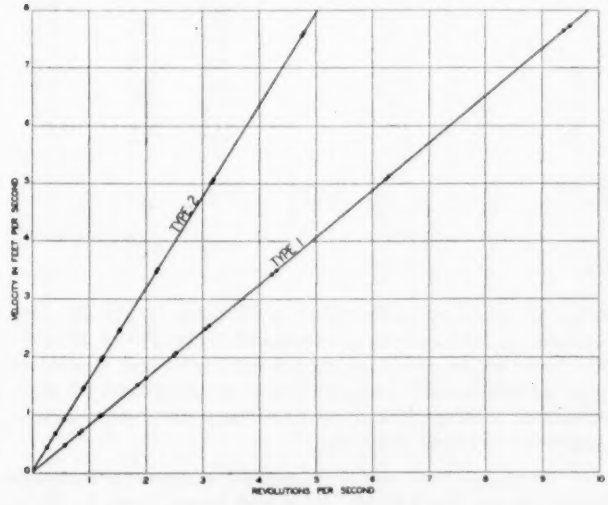


Fig. 14 Conventional Calibration Curves

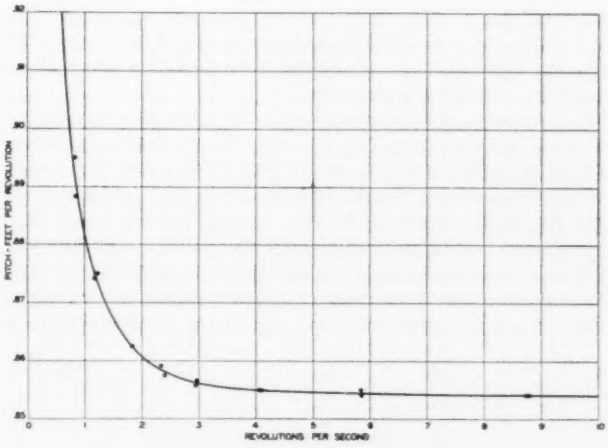


Fig. 15 New Type of Calibration Curve

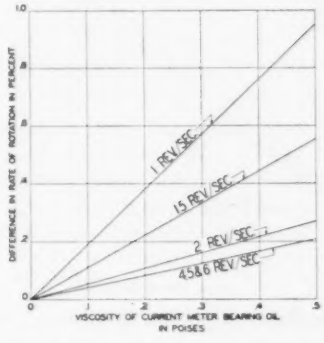


Fig. 16 Correction Curves for Rate of Rotation and Varying Viscosity of Bearing Oil

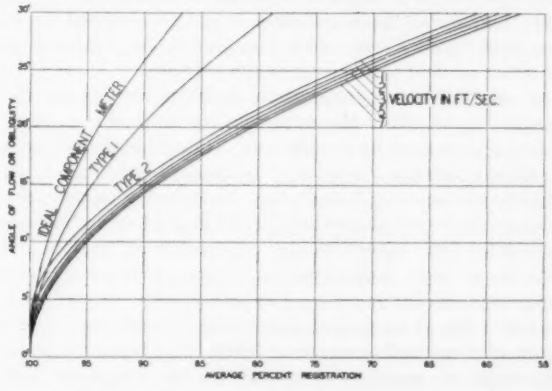


Fig. 17 Average Oblique-Flow Characteristics of Type-1 and Type-2 Meters

selected from the previous tests in order to have long, short, and medium pitches; although there was no great difference between the extremes.

The effect of velocity upon the oblique-flow characteristics was investigated subsequently. One current meter of type 1 was rated for angles of 10, 20, and 30 deg, while 10, 15, 20, 25, and 30 deg were used for type 2. For these tests, medium-pitched meters were chosen. It was found that for a given obliquity, the amount of under-registration of type 1 was independent of the velocity; whereas for type 2, the under-registration increases for the higher velocities, Fig. 17.

Having determined the average characteristic in the plane in

which oblique flow under test conditions could be expected to predominate, one meter of each type was turned in a horizontal plane to show that the angularity of flow had the same effect regardless of the plane of approach.

(*b*) *Principle of Discharge Correction Based on Two-Type Meter Method.* By replotting Fig. 17, using as ordinates the cosines instead of angles, it was discovered that for curves of average per cent registration for the two types of meter, straight lines were obtained as shown in Fig. 18. It may be emphasized that this characteristic is of fundamental importance, as it indicates that the meters, in average, are integrating correctly all cosine components regardless of variation in obliquity and that both meter types furnish a registration corresponding to an identical mean

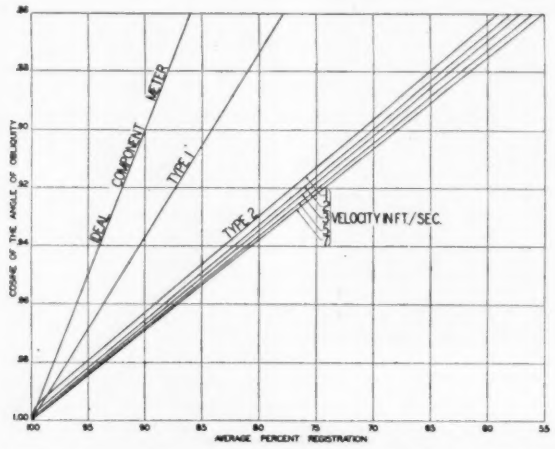


FIG. 18 AVERAGE REGISTRATION OF TYPE 1 AND TYPE 2 VS. COSINE

cosine of obliquity. If the characteristics as shown in Fig. 18 were curves instead of straight lines, and even if the under-registration of type 2 were a constant times the under-registration of type 1, different mean ordinates would be obtained for each meter type which would make it impossible to bring the two-type meter method to a correct analytical basis.

The straight-line average characteristics for oblique flow, shown in Fig. 18, led to the correction curves shown in Fig. 19 which obviously must be straight lines. Considering their derivation, these correction curves may be used to combine the total discharges measured by means of the two types of meters instead of limiting their application to the individual velocity components obtained by both meters at the same metering station. This conclusion is most important, as a correction of individual velocity components determined at each station with both types of meters is almost a practical impossibility, because an enormous amount of computation would be involved in pairing individual observations to apply a correction to the individual type-1 measurements.

In addition it must be considered that variations in flow exist which make it impossible to obtain identical components with the same meter and at the same station for two consecutive measurements under identical physical discharge conditions. Experience has shown that individual velocity components obtained with one type of meter may vary more than 20 per cent for identical locations and equal discharges, but that the difference in total discharges obtained from many individual measurements is less than 0.1 per cent. From this it appears that although individual corrections could be attempted, they would be erroneous if applied to obtain the obliquity at one particular location.

It must be emphasized that the wholesale correction based on mean velocities or discharges determined with each meter type, made the two-type meter method possible, both from a practical and theoretical point of view.

(*c*) *Current-Meter Measurements at Holtwood.* It was considered advisable to make some actual water measurements in a laboratory by means of the two-type meter method previous to measurements in the field. It was specified that the laboratory selected for these preliminary tests should have water-gaging facilities calibrated by the Gibson or Allen method. The Holtwood hydraulic laboratory of the Pennsylvania Water & Power Company furnished a satisfactory set-up, as the venturi meter here installed to measure the discharge had been calibrated by means of the Allen salt-velocity method under excellent physical conditions. The measuring section for the current meters was located in the open steel flume on the downstream side of the laboratory.

Two series of tests were made, one with the approaching water straightened, as much as possible, upstream from the plane of measurement by means of stilling racks, and a second group, without the racks, under comparatively rough-flow conditions showing visible signs of turbulence and obliquity. From

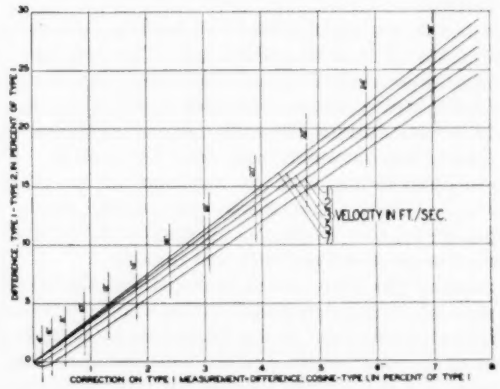


FIG. 19 CORRECTION CURVES, USING TYPE-1 AND TYPE-2 METERS

Table 1 it may be seen that the two-meter method, as developed, gave satisfactory results.

(*d*) *Analysis of Tests Made at the Obernach Research Station.* An analysis in detail was made of the tests conducted in 1930

TABLE 1

Test group	Run No.	Current-meter type	Departure from venturi, per cent		Diff. type 1 and type 2 in per cent of type 1	Correction to type 1, per cent (see Fig. 19)	Departure of corrected discharge, per cent
1 (smooth flow)	1	1	−0.03	avg = −0.24	0.51	0.20	+0.17
	2	1	−0.45				−0.25
	3	2	−0.75				....
2 (rough flow)	4	1	−0.59		1.68	0.49	−0.10
	5	2	−2.28				....

at the Obernach research station in Germany by the Research Institute for Hydraulic Engineering and Water Power, Munich. This investigation, based on original data, furnished, in addition to an independent check on conclusions resulting from the Holtwood tests, a comparison of the two-type meter method against volumetric measured discharge.[4]

[4] J. M. Mousson's discussion of "Research Institute for Hydraulic Engineering and Water Power," by Hunter Rouse, Trans. A.S.M.E., 1933, vol. 55, no. 10, paper HYD-55-3.

Incorporation of Index Method in Testing Program

It was recognized at the outset that it would be desirable to reduce the number of test runs to a minimum without impairing the accuracy of the measurements and without curtailing important and necessary observation regarding the characteristics of the turbines. In the case of Kaplan units this requires more information than for any other runner type, since the proper cam which establishes the correct relation between wicket gate and runner-vane positions cannot be based on model tests. It is necessary, therefore, to run through a series of tests to obtain the characteristics of the runner for numerous fixed-blade angles and variable gate.

In addition, it was realized that the index method would increase the effectiveness of the current meters. Index methods consist simply of the determination of the relative amount of water discharged by a turbine by observing the pressure differences between two or more points in the turbine setting. Seven piezometer openings were provided in the wheel case of each Safe Harbor main unit. There are two piezometers of the Peck type built in a stay vane of the speed ring, three for a Winter-Kennedy system in the walls of the scroll case, and two located in one of the piers of the intake. A Winter-Kennedy system of three piezometers is installed in each of the service units.

In certain tests made abroad, previously referred to, the entire characteristics of the unit were determined by means of 24 current-meter measurements. Each blade position was tested separately as though the unit were a fixed-bladed propeller runner. Inasmuch as there were but five points determined on each individual blade-angle curve, there was opportunity for considerable latitude and judgment in drawing the final curve which would represent the performance if the blade and gates were held always in the theoretically correct relation to each other. A further disadvantage is that this shows simply the performance when conditions are right, and not the performance of the unit when actually operating with the blades and gates held in relation to each other by the cam.

To determine the shape of the cam for a wide range of heads, as will occur at Safe Harbor, requires an extensive testing program which would have been exceedingly expensive and laborious with the current meters alone; hence the index method proved a helpful means of determining the correct relation prior to the current-meter tests. The incorporation of the index method confined the use of current meters to testing on the cam, except for a few runs to check the shape of two of the multitude of curves obtained by the index method.

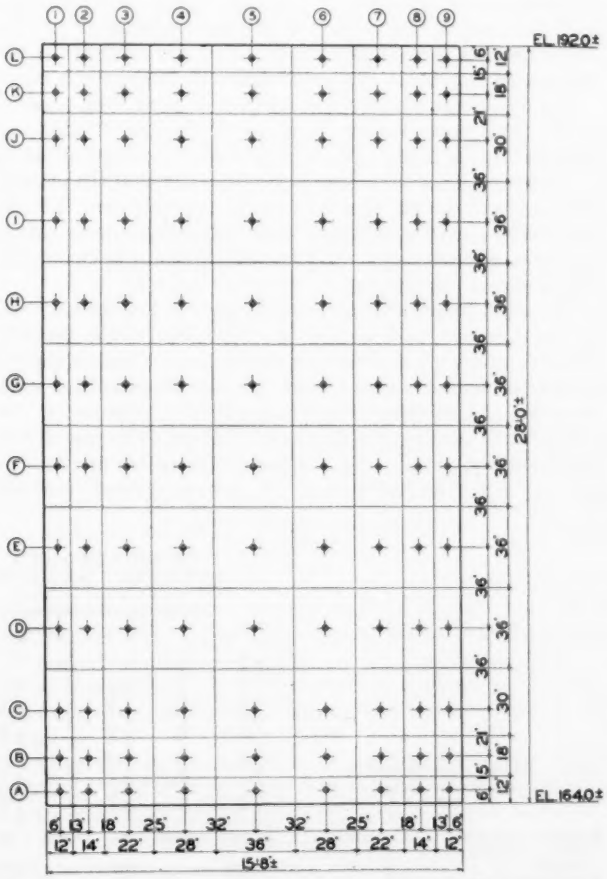


Fig. 20 Location of Metering Stations in Plane of Measurements for Point Method

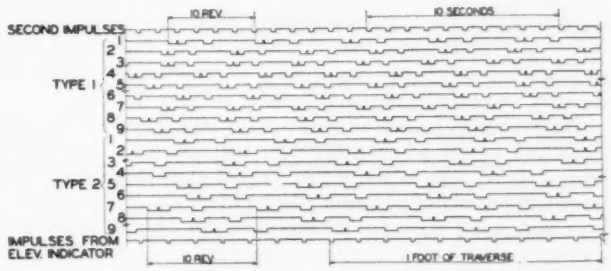


Fig. 21 Typical Chart Section, One-Fifth Actual Size

Testing Procedures and Computation

(*a*) *Point Method.* The current-meter apparatus was first installed in the gate slots of the one unit which was provided with the false roof. To obtain the most suitable flow conditions for test, the adjacent units were operated so as to carry about the same load. During all current-meter tests, piezometer readings were taken.

Both types of current meters were mounted 6 ft apart on the meter-support frames, the type-1 meters at the bottom and the type 2 at the top of the vertical rods. The metering stations are shown in Fig. 20. The type-1 meters covered the entire intake area, whereas the type-2 meters could only be brought to the positions *D* to *L*, inclusive. It was believed that the accuracy of the type-2 measurements would not be impaired materially by the fact that the latter could not be placed at the three lowest positions, *A*, *B*, and *C*, as it was assumed that the obliquity in the vicinity of the intake bottoms would be small. The data obtained by the type-1 current meters for the positions *A*, *B*, and *C* were combined with the type-2 measurements above these three horizontals to obtain the type-2 discharges. The meters were kept at each location 80 sec. The average duration of one complete test run was approximately 30 min.

The individual velocities were obtained from the graphic-recorder charts after first determining the chart speed by means of the second impulses, Fig. 21. The meter revolutions were counted and measured to the even ten, as the meter-contact wheels make one revolution for every ten revolutions of the current-meter propeller. The revolutions per second were then multiplied by the corresponding pitch, in feet per revolution, obtained from the individual meter-calibration curves, Fig. 15.

(*b*) *Vertical-Integration Method.* It is obvious that the conventional point method is exceedingly laborious, since for one complete run 324 type-1 and 243 type-2 velocities must be determined and 60 diagrams must be plotted and planimetered.

The enormous amount of time required for testing and computation warranted a detailed study to simplify these procedures.

It was suggested that tests be made by moving the frame at a constant speed over the total height of the intake and thereby causing the current meters to integrate all velocities encountered over the total length of the traverse. It was realized that, if this method should prove successful, that is to say, if results were obtained identical with the point method, enormous savings could be effected both in time required for actual testing and in computing the results.

It was anticipated that, in letting the meters integrate all velocity measurements in one vertical traverse, mean velocities would be obtained corresponding to the same mean velocities as determined for the point method by means of the vertical velocity diagrams, and thereby eliminate the determination of individual velocity measurements and render unnecessary the plotting and planimetering of vertical velocity diagrams.

In moving the frames an angularity is introduced which varies according to the lifting speed and the velocity of the water. For relatively high lifting speeds and one type of current meter, discharges would be obtained which are too small, although the influence is less than may be expected. In case this additional obliquity should have an appreciable effect in spite of relatively low lifting speed, it was expected that the two-type current-meter method would successfully solve the problem, as obliquities whether natural or artificial are evaluated.

On the other hand, the current meters themselves showed characteristics favorable to an integration method. Plotting velocity in feet per second vs. revolutions per second for both types of meters in question, curves are obtained which are practically straight lines, as in Fig. 14. In spite of the increasing friction and slip at low velocities, the average of a number of ordinates taken at random from these curves falls so close to the lines themselves that an error is introduced which is, from a practical point of view, entirely negligible.

To prove the workability of the vertical-integration method it was thought advisable to make a subsequent series of test runs on the same unit with the false roof in place, and then analyze these data independently of those obtained by the point method. The lifting speeds used were 4 and 6 fpm. This procedure cut the original time of 30 min, necessary for one run by the point method, down to about 8 or even 5 min, respectively. In the course of one test run the meter frames were either lowered or raised. When raised, the frames disappeared completely into the gate slots above the flow passage and likewise started out from within the slots when the frames were lowered.

Various ways of compiling the data were studied. The revolutions of the type-1 current meters were counted between the bottom and top of the intake bays, the limits being established by the record of the elevation indicator shown in Fig. 21. Because of the length of the record, it was considered satisfactory to estimate, at the ends of the charts, the number of revolutions to the even unit by extrapolation beyond the last contact indicating an even ten revolutions. Similarly the time was estimated to one-tenth of a second.

Since the meters could be lowered no further than 6 in. above the bottom of the intake, it was necessary to determine the velocities in this area by some other means. The meters were held at this lowest elevation for about 6 sec, that is to say, at least the number of seconds which would have been required to traverse these 6 in. The additional number of revolutions was then multiplied by a factor determined by the pitot-tube measurements. This number of revolutions was then added to the number counted as previously described.

The type-2 current-meter registrations were counted between elevation 170.5 and the top of the intake bay as lower and upper limits, respectively. In order to tie in the type-1 and type-2 measurements near the bottom, it was necessary to determine the number of revolutions of the type-1 meter between the bottom and elevation 170.5. The type-2 discharge was obtained by adding the discharges indicated in the areas above and below elevation 170.5 by the type 2 and type 1, respectively, Fig. 20.

A comparison between the results obtained by the two methods, that is, by point and vertical integration, showed them to be absolutely identical.

(c) *Separate Measurements With Type-1 and Type-2 Meters.* The enormous savings over the point method justified the extension of the testing program to other units. First, one of the main units without a false roof in the intake was investigated. The plane of measurements was moved upstream into the emergency-gate slots to save all expenditures which would have been necessary to remove the intake gates and screw-stem gate hoist, and to provide fillers for the emergency-gate slots in the piers and ceiling over the emergency-gate slot openings to obtain suitable flow conditions further downstream. It was thought that in taking a chance and placing the plane of measurements in the emergency-gate slot, the full benefit of the two-type current-meter method would be investigated under conditions where the expected obliquity was considerable.

Still further savings in computation were possible by making these tests with each type of current meter alone, mounted near the bottom of the vertical support rods shown in Fig. 7. The separate application of the two types of meters made it necessary to plot the discharges obtained against a common standard such as piezometer deflection, power output, or gate opening before a discharge correction could be applied. It may be emphasized that this method is advisable in any case, whether or not the two meters are used simultaneously, as the spreading of the test points may thereby be eliminated before the correction is made.

The advantage over the first method of using both types of meters simultaneously was that the evaluation of discharge indicated by the type-1 meters between the bottom and elevation 170.5 was no longer necessary as the total height of the plane of measurements could be explored with the type-2 meter. The determination and planimetering of three discharge diagrams, that is, one for each intake bay per test run, could thereby be eliminated. The respective savings may be seen from Table 2.

The rapid progress made in computing the test data for this second main unit made feasible an investigation of the discharge characteristics of one of the two service units. Besides the important economies of time achieved by the adoption of the vertical-integration method, engineering and labor expense for the actual water gagings were kept at a minimum by the use of full-automatic equipment. Table 3 shows the number of men employed for the measurements. The benefit derived from the

TABLE 2

Data to be obtained for one complete test	Point method. Type 1 and 2 simultaneously	Vertical-integration method, type 1 and 2 meter: Simultaneously: Data to be determined	Simultaneously: Saving over point method, per cent	Separately: Data to be determined	Separately: Saving over point method, per cent
Contact indications to be counted	40,000	15,500	61	12,400	69
Contact indications to be scaled on charts	606	....	100	....	100
Velocities to be determined	567	81	86	54	91
Velocity vectors to be plotted	648	81	88	54	92
Diagrams to be plotted and planimetered	60	9	85	6	90

TABLE 3

Equipment and Personnel	Safe Harbor Types 1 and 2 current meters		Ryburg-Schwoerstadt
	Separate	Simultaneous	
Equipment in use at one time:			
Current meters	27	54	30
Graphic recorders	1	2	3
Piezometers	4	4	..
Personnel:			
Graphic recorders	1	2	3
Index method	1	1	..
Hoisting	1	1	6
Cable guards	1	2	3
Personnel total	4	6	12

type of equipment and testing procedures may readily be seen in comparing the number of men used at Safe Harbor with the number of men at Ryburg-Schwoerstadt. In spite of the more elaborate equipment, considerable economies were accomplished at Safe Harbor.

Results and Conclusions of Initial Tests

(*a*) *Unit With False Roof.* The results of the tests made by means of the point and vertical-integration method with both meter types simultaneously and carried out at the unit provided with a false roof are shown in Fig. 22. The discharges indicated by each type of meter were plotted against the piezometer deflections simultaneously obtained, using a double logarithmic scale. It is to be observed that the two curves are very distinct

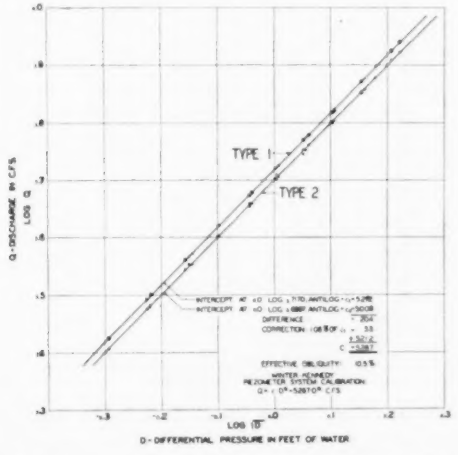


Fig. 22 Results of Tests Made on Unit With False Roof

and are parallel to each other, indicating that there is a constant percentage by which the type-1 and type-2 discharges differ from each other. By means of a correction curve similar to that shown in Fig. 19, the correction to be applied to the type 1 was found to be 1.06 per cent, which corresponds to an effective obliquity of 10.5 deg.

Values of such magnitude seem surprising in view of the care that had been taken to straighten out the flow approaching the metering section. It should be recalled, however, that even under the conditions which were made as nearly ideal as possible in the experiments conducted by the Research Institute at Obernach, the behavior of the meters indicated an effective angularity of 9 deg.

It must be emphasized that for this particular unit the obliquity observed is to a considerable extent an effective angularity of individual water particles and flow filaments, the latter accentuated by the screens rather than a general obliquity of flow. The latter may be exemplified by the position of a thin streamer and the former by the fluttering of the tail end. A similar phenomenon has been observed by Prof. C. M. Allen who demonstrated that salt injections in straight pipe lines spread very rapidly, even when flow is straightened out as much as possible.[5]

Since the angularity observed in these tests did not greatly exceed that which was found for the practically ideal conditions at Obernach, it appears that the flow lines had been straightened out very well by the use of the roof. In spite of this precaution, there remains an influence which may cause a single-type meter to under-register by more than 1 per cent. It is essential, therefore, in order to get correct results to make use of the two types of meter and not to depend entirely upon any means of straightening out the flow.

Since a wide range of velocities must be integrated by a meter traversing the intake, or even in a fixed position, an investigation was made at the Bureau of Standards to see if the accuracy was impaired by this phenomenon. Tests were made by varying the speed of the car carrying the meter through the tank. Most of the runs were made with two and four complete cycles of speed variation. For these tests the meters showed a slight tendency to under-register; the reason for this will be apparent when we consider that the speed of rotation is not strictly proportional to the velocity of the meter through the water. For velocities normally encountered in testing, and even for a considerable range between the high and the low portions of the cycles, the average effect can be shown to be very slight, but nevertheless always in the direction to under-register the flow.

Another series of runs was made for a half cycle of speed variation by gradually decelerating or accelerating the car during the whole length of the run. The inertia of the moving parts caused the meter to read slow when speeding up and fast when slowing down. Similar tests were conducted with the meter in an inclined position and the same results were obtained.

This work led to many other investigations and considerable interest has centered about the question of the ability of the meters to perform correctly under the conditions of test. For instance, it was thought that there might be some disturbing effect from the horizontal girders of the racks, Fig. 12.

It is contended by some that the conditions under which a meter operates in practice are by no means comparable with the rating in still water. Others hold the opinion that the meter receives myriads of impulses, each one of which is similar to movement through still water at some angle. Consequently, because it has not been found possible to make a meter rate fast in still water, regardless of the angle at which it is turned to the direction of travel, with the possible exception of that small range of angularity where type 2 shows a slight tendency to over-register at low velocity, it is argued that these meters would always indicate less than the true flow.

Of course it must be recognized that an eddy having its axis roughly parallel with that of the meter might cause gross over-registration, but the effect of this same eddy on a meter having the opposite direction of rotation would have an equally opposite effect. It was for this reason that left-hand and right-hand meters were used in alternate positions.

These tests at the Bureau of Standards have taken other interesting forms, consisting of deliberately disturbing the water ahead of the meter by rakes and planks. Care must be exercised not to set up a motion of the water either with or against the meter, or to set up an actual swirl either in the direction the meter is turning or in the opposite direction. It cannot be said that these particular tests were conducted under ideal conditions, but they have served to demonstrate the ability of a meter to plow through highly turbulent water without any appreciable effect upon the accuracy of registration.

[5] N.E.L.A. Engineering National Section, Minutes of Hydraulic Power Committee Meeting, Edgewater Beach Hotel, Chicago, October, 1932.

It is believed that, considering the great care taken in the observations, and the consistency of the test points when plotted as shown in Fig. 21, as well as the subsequent investigations regarding the behavior of the meters under various conditions in the rating tank, the interpretation of the results in determining the correction to be applied to type-1 meter appears to be entirely justified.

(*b*) *Unit Without False Roof.* The determination of the effective obliquity and the discharge correction from the data obtained by means of the vertical-integration method and separate application of type 1 and type 2 for the unit without the false roof was more difficult. If the difference in discharges registered by the two types of meters were used, a correction of 3.8 per cent should have been applied to the type-1 discharge measurements. It did not appear reasonable that the correction for the unit without the false roof would be so much larger than the correction for the unit with the roof, even though the plane of measurement had been moved upstream into the emergency-gate slots.

A careful restudy of the chart records obtained by the vertical-integration method disclosed the fact that some type-2 meters had stopped rotating in all three intake bays in the vicinity of the ceiling. It was thought possible that the convergence of the streamlines might have exceeded the angle at which the meter stalls.

To determine this angle accurately, further tests were made at the Bureau of Standards. By extending the range of obliquity above 30 deg it was found that the type-2 meter stalled for an angularity greater than about 34 deg regardless of the velocity. This is, of course, a fault which it is desirable to eliminate.

Since the departure from the cosine and consequently the tendency to stall increases with the pitch, it is possible to remedy this condition by constructing a meter with a shorter pitch, Fig. 18. At the time of purchase it was thought that the angularity would not exceed the range of satisfactory performance of this type-2 propeller. For the sake of accentuating the difference and securing greater accuracy in making the correction, a meter was obtained which would differ as much as possible from the type 1.

(*c*) *Service Unit.* Great difficulties were also encountered at this unit which prevented the obtaining of a true discharge by combining type-1 and type-2 measurements. Aside from exceedingly low velocity, excessive angularity in a horizontal direction was encountered. This additional obliquity was caused by the fact that no main units were installed at that time on the river side of this service unit. Since the water for all the units has to approach the intakes from the river side, it is obvious that the water for the house units must flow almost parallel to the longitudinal axis of the power house.

In spite of all these unfavorable circumstances, the type-1 measurements appeared to be satisfactory as the performance range was not exceeded either by angularity or by low velocity. The type-2 meters, however, stopped almost over the whole vertical half of the intake toward the river side. Where the stoppage occurred for the type 2, the type 1 indicated very low velocity. To come to any conclusions regarding the magnitude of correction was impossible, and it was realized that the performance range of the type-2 meter had also been exceeded at this unit.

Subsequent Tests

The results of the tests which were carried out during 1932, as previously described, showed clearly the limitations of the equipment.

To remedy this condition a variety of current-meter propellers of old and new design were investigated at the Ott laboratory. The object was to find a suitable propeller to be used in conjunction with the type 1 and which would operate satisfactorily even under extreme oblique-flow conditions. Most desirable was a spoked-vane propeller design, since it had been indicated by the oblique-flow calibration of the type 1 that variations in velocity would not change the oblique-flow characteristic. This investigation, however, was not confined to spoke-vane propellers having various pitches and vane shapes, but it included conical-screw propellers of the two-vane and three-vane type. The upper and lower limits in pitch were 70 and 15 cm, respectively. Based on the results, it appeared that the most favorable solution would be to replace the type-2 meters of conical shape and 50-cm pitch with a spoked-vane propeller of 15-cm pitch, which will be called type 3 hereafter. Its oblique-flow characteristic lay between the type 1 and the cosine line, Fig. 17, and in view of this it was expected that the type-3 meter would indicate more water, that is to say, would under-register less than the type 1. The stalling angle of the type 3 was found to be above 70 deg. It was apparent that the relation between the cosine of the angle of obliquity and the amount of registration was a straight line and consistent from 0 deg to well above 60 deg. The high degree of relative precision obtained during the previous discharge measurements, to be discussed later, indicated that even in case the oblique-flow characteristic would not differ a substantial amount, reliable results could be expected by the two-type meter method employing type-1 and type-3 meters.

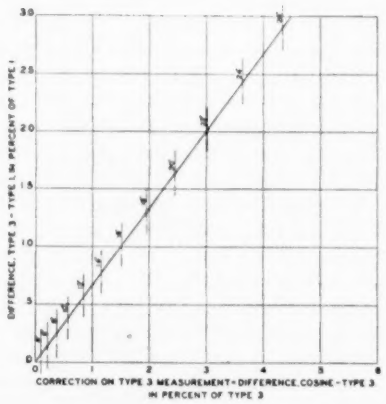


Fig. 23 Correction Curves Using Type-3 and Type-1 Meters

Discharge measurements were made first at the unit previously tested without false work, employing the type-1 and type-2 meters separately, and using throughout the vertical-integration method. The computation of the discharges was simplified considerably by substituting an analytical method for the tedious process of planimetering the discharge diagrams. The weight given each current meter was half the distance between the two adjacent ones on either side. For the first and second meters from the wall, the proper weights were obtained from the pitot-tube investigation in proximity of the walls.

The correction to be applied on the type-3 measurements was found to be 1.23 per cent, indicating an effective obliquity of 14 deg (see Fig. 23) which corresponds to a correction of 2.09 per cent on the type-1 meter.

Additional tests of the same unit were made but without the trash racks in place. The results indicated that the effective obliquity was slightly less when compared with the one previously obtained. The discharges, however, as indicated by the two-type meter method were identical for both series of tests.

The service unit was also retested employing a like procedure. The results indicated a discharge correction of 1.60 per cent on the type 3, corresponding to an effective obliquity of 16.5 deg.

Consistency and Relative Precision of Water Gaging

The simultaneous application of two methods of water measurement, such as current-meter and index methods, permits an analytical determination of the consistency of the test results. A measure of the consistency is the average departure of the ratio of the discharge to the square root of piezometer deflection from the mean value of this ratio, given in Table 4. Where index-method facilities are not available, the consistency of the water gagings comprising the whole discharge range cannot be determined, but it will be confined to one particular discharge arrived at by repeated tests under identical conditions.

It is customary abroad to evaluate whenever possible the probable error of the measurements by means of the method of least squares. It must be emphasized, however, that the term "probable error" is misleading since the measurements usually are not compared with an absolute standard such as volumetric measurements. It is obvious that any method except the latter has inherent errors and, as long as the probable error of the test results is determined independent of an absolute standard, departures are obtained which are relative and not absolute.

TABLE 4 DETERMINATION OF CONSISTENCY AND RELATIVE PRECISION OF TYPE-1 CURRENT METERS AND INDEX TESTS, UNIT WITH FALSE ROOF

Test run	Q for type-1 meter cfs. uncor. for head	Square root of standard deflection, $\sqrt{D}$	$Q/\sqrt{D}$	Departures δ	Departures Per cent	δ^2
1	4752	1.677	2834	—16	0.56	256
2	3687	1.295	2847	— 3	0.11	9
3	6571	2.311	2843	— 7	0.25	21
4	8729	3.042	2869	+19	0.67	362
5	8401	2.954	2844	— 6	0.21	36
6	7875	2.767	2846	— 4	0.14	16
7	7426	2.607	2848	— 2	0.07	4
8	6651	2.328	2857	+ 7	0.25	49
9	5987	2.105	2844	— 6	0.21	36
10	5994	2.112	2838	—12	0.42	144
11	5887	2.062	2854	+ 4	0.14	16
12	5289	1.851	2857	+ 7	0.25	49
13	4726	1.655	2855	+ 5	0.18	25
14	4174	1.465	2849	— 1	0.04	1
15	3632	1.272	2855	+ 5	0.18	25
16	3115	1.089	2860	+10	0.35	100
17	2657	0.933	2848	— 2	0.07	4
			Avg = 2850		Avg = 0.24	$\Sigma\delta^2$ = 1153

Consistency or average departure of one measurement = $\pm 0.24\%$
Mean departure of one measurement = $\pm[\sqrt{[\Sigma\delta^2/(n-1)]} = \pm \sqrt{(1153/16)} = \pm 8.5 = \pm 0.30\%$
Mean departure of all measurements = $\pm \sqrt{[\Sigma\delta^2/n(n-1)]} = \pm \sqrt{[1153/(17 \times 16)]} = \pm 2.2 = \pm 0.08\%$
Relative precision of all measurements = $0.674 \times 0.08\% = \pm 0.054\%$

Table 4 shows the determination of the relative precision of the type-1 measurements made by the vertical-integration method on the unit with the false roof. The high degree of relative precision may be attributed to various reasons, such as the elimination of the personal equation in obtaining the data in the field and the new type of rating curve. In addition, it is believed that the second impulses obtained from the Warren master clock helped to eliminate one source of error which often consistently impairs the accuracy of testing procedures.

It is noteworthy that identical relative precisions were obtained for the point and the vertical-integration method on the unit with the false roof. Since the same number of measurements and identical piezometers were used for the derivation of the relative precision and in addition the same average ratios of discharge to square root of deflection were found, the two methods must be of like accuracy.

Critical Review

Experience gained at Safe Harbor shows that back flow is likely to occur as soon as the minimum section or the section of parallel boundaries has been reached. Even in slightly converging passages, such as used at Ryburg-Schwoerstadt, reversals have been observed. It would be far better, from this point of view, to make current-meter measurements at low-head power plants without the false roof where the flow is converging to a high degree as in the Safe Harbor units.

In the light of the test results obtained at the unit with the false roof in the intake, it is believed that even where a parallel approach is provided to the gaging section, two types of meter must be used to allow for the effective obliquity.

It should not be concluded that the necessity of two meter types would make this method uneconomical. It has been found at Safe Harbor that by means of the integration procedures, far-reaching savings may be made over the point method. For instance, the discharge for two runs by means of the integration method with both meters used separately could be computed by one man in a day, whereas at least four days were consumed for only one type of current meter for the point method.

It is not necessary to confine the integration method to procedures with vertical traverses. Where the depth of water passages is small in comparison to the width, horizontal traverses may be of advantage. In case current meters should be used in penstocks, in spite of the advantages of the methods sponsored by Gibson and Allen, the current meters may be mounted on a supporting rod pivoted in the penstock center. By means of a suitable gear mechanism, the rod may revolve so that the current meters describe a circular path. A position indicator, geared to the driving mechanism of the rod, permits the determination of the rod position from the chart records of the graphic recorders.

It has been shown that the measurements with both types of current meter may be made separately to good advantage. Should the two-type meter method be contemplated elsewhere, it would be necessary to provide only one set of bodies including axles for the two sets of propellers. Following this suggestion, a considerable saving may be made in the cost of graphic recorders and other equipment.

With reference to the current meters themselves it would be of advantage to select two meter types having a ratio of under-registration somewhat more suitable than the ratio of type 1 and type 3. Here the correction to be applied on the type-3 measurements is one and one-half times the difference between the indications of type 1 and type 3. It is believed that two spoke-vane propeller types having 15-cm and 30-cm pitch, respectively, would be more desirable since the correction on the shorter-pitch meter would be about equal to the difference between the discharges indicated by the two types of current meter.

The index method of testing has been extremely valuable, especially in the determination of the proper cam shape, as well as in conjunction with the current-meter measurements to show the consistency of the test results. It may be emphasized, however, that at Safe Harbor, the power company determined the proper shape of the cam, in spite of the fact that the cam is an integral part of the turbine. As the purchaser is naturally interested in obtaining the best possible cam shape, he may relieve the turbine manufacturer of this phase of the turbine design.

This should not become the customary procedure and by no means should it be necessary for the sake of economy to sacrifice testing on the cam for the tests to determine the cam shape. It is believed that final acceptance tests of Kaplan units should always be made on the cam which will be used under normal operating conditions.

Based on the experience gained in connection with the Safe Harbor tests, it appears that the effective angularity is much

greater than was previously expected and that measurements may be made to good advantage at locations of converging flow. It would be bad policy, therefore, to set a definite limitation regarding the obliquity and say that the measurements are unreliable when the angularity is more than a certain amount. The limitation of angularity should not be specified, but attention should be called to the fact that the angularity should not exceed the performance range of either of the two meters used.

At the present stage of development in the art of water measurement, it is believed that economies which may be achieved through improved testing procedures and through the omission of temporary installations, as well as the possibility of gaging the discharge under operating conditions, make the current-meter method more attractive and economically more feasible. It is hoped that the record of the Safe Harbor tests, as outlined in this paper, will contribute to a better understanding of the problems involved in water gagings by means of current meters. Since in recent hydroelectric construction in the low-head field there is a marked tendency toward further shortening of the length of water passages, which may render other methods of measurements more difficult, a careful study of current-meter methods is, at this time, very desirable.

Acknowledgments

The publication of this paper was made possible by the support received from J. A. Walls, president, Safe Harbor Water Power Corp. The author wishes to express his appreciation for the cooperation rendered by Walter F. Stutz of the U. S. Bureau of Standards, Washington, D. C., under whose supervision the current-meter investigation was carried out at the Bureau. The interest shown by the engineers of the S. Morgan Smith Company and the I. P. Morris Division of Baldwin-Southwark Corporation, turbine manufacturers, is also appreciated. The joint effort made by all engineers of the Safe Harbor Water Power Corp. engaged in these tests has contributed an important part in the advance made in the art of water measurements.

BIBLIOGRAPHY

1 "Messen und Teilen von Wasser in Niederdruckanlagen" (Gaging and Dividing of Water in Low Head Power Plants), by S. Bitterli. Verhandlungen der Schweiz. Naturforschenden Gesellschaft, Bern, 1922, vol. 2, pp. 331–339.

2 "Wassermessungen bei Grosskraftanlangen" (Water Measurements at Large Hydroelectric Power Stations), by H. F. Canaan, Wasserkraft-Jahrbuch, 1930.

3 "Fluegel-Wassermessungen in Druckrohrleitungen" (Water Measurements by Means of Current Meters in Penstocks), by H. Dufour, *Schweizerische Bauzeitung*, vol. 83, 1934, p. 39.

4 "Vergleichstarierungen der Hydrometrischen Fluegel" (Comparative Calibrations of Water Current Meters), by S. Kolupaila, Report of the fourth hydrological conference of the Baltic States, Section IX, Leningrad, U.S.S.R., Sept., 1933.

5 "Water Gaging for Kaplan Units of High Capacity at Safe Harbor," by J. M. Mousson, *The Baltimore Engineer*, vol. 8, 1934, nos. 9 and 10; *Canadian Engineer*, vol. 66, 1933, no. 23 and vol. 67, 1934, no. 1.

6 "Wassermessungen in einen Grosskraftwerk" (Water Measurements at a Large Power Development), by J. M. Mousson, *Zeit V.D.I.*, vol. 78, 1934, no. 46, pp. 1343–1346.

7 "Wassermessungen bei Wasserkraftanlagen" (Water Measurements at Water Power Plants), by L. A. Ott, Wasserkraft-Jahrbuch, 1924, pp. 263–265.

8 "Résultats des essais effectués sur les groupes électrogènes de l'usine Chancy-Pougny" (Results of Tests Made on the Hydroelectric Units at Chancy-Pougny), by P. Perrochet, *Schweizerische Bauzeitung*, vol. 87, 1926, p. 241.

9 "Wassermengenbestimmung mit Bandchronograph im Kraftwerk Faal a.d. Drau" (Water Measurements With Graphic Recorder at the Faal Power Plant on the Drau River), by P. Perrochet and C. Montadon, *Schweizerische Bauzeitung*, vol. 91, 1928.

10 "Misure di portate negli impianti idroelettrici eseguite col metode Gibson" (Measurements by Gibson Method for Determination of Discharge at Hydroelectric Plants), by E. Scimemi, *Energia Elettrical*, vol. 8, December, 1931, no. 12, pp. 1029–1036.

11 "Der Genauigkeitsgrad von Fluegelmessungen bei Wasserkraftanlagen" (The Precision of Current Meter Measurements at Water Power Plants), by A. Staus, Julius Springer, Berlin, 1926.

12 "Effect of Turbulence on the Registration of Current Meters," by D. L. Yarnell and F. A. Nagler, Trans. A.S.C.E., December, 1929, vol. 55, pp. 2611–2640, bibliography pp. 2636–2640.

Note: This bibliography is necessarily incomplete, since only publications were compiled which had direct bearing on the subject. For a more complete list of references see the Engineering Index.

Draft-Gear Action in Long Trains

By O. R. WIKANDER,[1] PITTSBURGH, PA.

This paper is a study of the coupler pressures, car speeds and accelerations in a long train, produced by external forces acting on same.

It is based on the similarity in behavior of such a train and of an elastic bar of the same length as the train, the same mass per unit length and the same yield, when subjected to longitudinal compressive or tensile forces.

Following the method explained in the paper, the pressure and speed diagrams are derived for the various periods through which the waves are passing, the beginning and ending of each period being marked by certain events. Formulas are derived for the maximum bar pressures and tensions, for the locations at which they occur, and for the maximum accelerations or retardations of the particles.

It is shown how these formulas may be applied to freight trains of known characteristics, and the general requirements are derived which should be met by a freight-car draft gear under given service conditions.

Well-known elementary calculations, based on the official ARA Draft Gear Tests and supplemented by the formulas given in the paper, permit the design of draft gears suitable for the given service conditions.

Introduction

WHEN a long train is subjected to external forces, accelerating or retarding, the work performed or consumed by these forces is partly used to accelerate or retard the moving masses, partly to overcome train resistance, partly to compress the draft gears which connect adjacent cars, and to some extent to stretch or compress the car bodies themselves.

The action of the external forces produces different coupler pressures or tensions, car speeds, accelerations, and retardations in different parts of the train, depending on the nature of the forces, as well as on the elastic and mass properties of the train.

Investigations of the action of external forces are of fundamental importance for the solution of draft-gear and air-brake problems and for a better understanding of the inter-relation of the action of these important parts of the equipment of long trains.

In the following investigation the action of the external forces is analyzed according to a method particularly suited for the purpose, but which apparently has never been applied before to any great extent to problems of this nature.

This method is based on the theory of wave transmission in elastic solid mediums. Its use in solving train dynamic problems was suggested in a paper by L. H. Donnell.[2]

According to this method, a long train is assumed to be mechanically equivalent to an elastic bar of the same length, the same mass per unit length, and the same yield under compression or tension. This assumption is only fairly correct because (1) no allowance is made for the free train slack, (2) the elastic bar is assumed to follow Hooke's law which states that strain is proportional to the stress, while friction draft gears only approximately follow this law, (3) the bar is supposed to be perfectly elastic, while the dampening effect of friction draft gears reduces the recoil forces in a long train, and (4) the elastic bar is assumed to have a uniformly distributed mass, while the mass distribution in a long train is not always even approximately uniform.

In order not to overburden this paper with special cases the following investigation of an elastic bar has been limited to the case of a bar with uniform elastic and mass properties.

The case of partial application of uniformly distributed forces has been included because it corresponds to the action of the type-K air brake, which is in general use at the present time.

It would seem that a long train could always be represented by an elastic bar with both ends free and that the investigation of elastic bars with one or both ends fixed would have no bearing on train dynamic problems and might have been omitted.

A fixed end of a bar corresponds, however, to a very sudden and great increase in its wave resistance and is comparable to the condition when a pressure wave traveling in an empty train strikes a group of fully loaded cars, and represents the extreme consequences of such a condition.

They show that the investigation of an elastic bar with uniform mass and elastic properties and both ends free only applies to trains with fairly uniformly distributed loads. Unusual load distribution, such as long trains composed of alternating strings of empty and loaded cars, require special investigation, which can be carried out by the use of the formulas given for reflected waves.

1 LONGITUDINAL WAVES IN AN ELASTIC BAR

Graphic Illustration of Dynamic Action in Long Bars

In a long bar the pressures, the particle speeds, accelerations, and retardations change not only from point to point in the bar but also from moment to moment at each particular point. It is therefore difficult to represent the action of the external forces in diagrams with only two coordinates.

The method herein used to derive and represent the dynamic conditions in a long bar at a certain moment involves the use of two diagrams, one directly below the other, and, taken together, they portray the dynamic conditions of the bar at that moment.

---

[1] Mechanical Engineer, Ring Spring Department, Edgewater Steel Company, Pittsburgh, Pa. Mem. A.S.M.E. Mr. Wikander received the degree of mechanical engineer in 1895 from the Technical Academy, Chemitz, Saxony, and the degree of electrical engineer in 1899 from the Technical University, Karlsruhe, Germany. He was affiliated with the electrical laboratory of the Société Decauville, Corbeil, in 1899; the Compagnie d'Électricité Thomson-Houston de la Méditerranée, Paris, in 1900, as electrical engineer; the National Tube Company, McKeesport, Pa., in 1903, as assistant chief electrician; and with the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., from 1904 to 1906 and later from 1909 to 1911. In 1905 Mr. Wikander was appointed chief of the experimental department, General Electric Company of Sweden but the following year became commercial attaché at the Swedish legation of Buenos Aires, Argentina. From 1911 to 1918 he was employed as designing and later sales engineer, with the Hall Steam Pump Company, Pittsburgh, Pa. In 1918 he became consulting engineer for the SKF Industries, Inc., New York, N. Y. He practiced as consulting engineer from 1922 to 1925 in which year he became affiliated with the Edgewater Steel Company.

Contributed by the Railroad Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "Longitudinal Wave Transmission and Impact," by L. H. Donnell, Trans. A.S.M.E., 1930, vol. 52, paper APM-52-14-153.

The upper diagram usually represents the compression or tension in the bar, and is called the pressure diagram, while the lower one usually represents the speed of the particles at each point of the bar. If not otherwise qualified the word "speed" denotes the speed of the particles of the bar. In these diagrams the horizontal base line represents the length of the bar.

Several sets of diagrams may be shown in the same drawing, thus illustrating the gradual change in the dynamic conditions in the bar during a certain period of time.

Where the propagation speed of pressure or particle speed is constant the horizontal distance between two points in the pressure or speed diagram represents the time required for a pressure or speed wave to travel the distance between the points.

Since the change in the length of a bar, produced by longitudinal forces, is negligible as a rule, the base lines of the pressure and speed diagrams are of constant length.

A pressure at any point of a bar is represented by a positive ordinate or an ordinate above the base line. A tension is represented by a negative ordinate or an ordinate below the base line in the pressure diagram.

A concentrated external force is represented as an ordinate at its point of application, positive or above the base line if the force acts in the direction from left to right, and negative or below the base line if it acts in the opposite direction.

A distributed external force is represented by a curve in the pressure diagram and the ordinate at any point of this curve is equal to the algebraic sum of the external forces applying to parts at the left of this point, forces acting in the direction from left to right being considered positive and those acting in the opposite direction being considered as negative.

The speed of particles at any point of the speed diagram is represented by a positive ordinate, or an ordinate above the base line if the particles move from left to right, and by a negative ordinate, or an ordinate below the base line if they move in the opposite direction.

Further details as to the design and use of the pressure and speed diagrams will be given in the course of the discussion of the various problems.

Speed of Pressure Propagation in an Elastic Bar

It is known from the theory of sound that a pressure acting on an elastic body of uniform mass and elastic properties will propagate itself through the body at a constant speed, depending only on these properties.

When applied to an elastic bar of uniform cross-section and constant modulus of elasticity, this speed is expressed by the equation

$$V = \sqrt{(k/m)} \quad \ldots\ldots [1]$$

where V = speed of pressure propagation, fps; k = stiffness constant or modulus of elasticity of the material in lb per sq in. multiplied by the cross-sectional area of the bar in sq in.; and m = mass per ft of length.

Example. If Equation [1] is applied to a bar of 1 sq in. sectional area weighing 3.4 lb per ft, $k = 30 \times 10^6$ lb per sq in., $m = 3.4/32.17 = 0.1058$ = mass per ft, and $V = \sqrt{(30 \times 10^6/0.1058)} = 16{,}800$ fps.

Speed of Particles in a Perfectly Elastic Bar

A pressure wave passing through a bar produces a speed of the particles in the direction of the wave. This speed depends on the pressure to which the particle is subjected and on the physical characteristics of the bar, i.e., on m, its mass per foot of length, and on k, its stiffness constant. The relation between these quantities is expressed by the equation

$$c = \frac{f}{\sqrt{(mk)}} \quad \text{or} \quad f = c\sqrt{(mk)} \quad \ldots\ldots [2]$$

where c = speed of the particles in the direction of the wave, ft per sec, and f = pressure to which the particles are subject, lb.

The denominator $\sqrt{(mk)}$ is a constant which depends only on the physical properties of the bar. Thus, the bar pressure and the speed of the particles are proportional. A pressure wave therefore produces a speed wave of the same speed as itself.

If the end of a bar is subjected to a tension force, the tension wave will move in the direction opposite to that of the force, while the speed of the particles always has the same direction as the force which creates it.

The relationship expressed by Equation [2] applies only to waves produced in a stationary bar and only to waves which have not been reflected.

The expression $\sqrt{(mk)}$ has a certain analogy to an electric resistance and will for the sake of convenience be called the wave resistance of the bar. (This quantity corresponds to the acoustic resistance of a medium, as defined by H. Brillié, in *Le Génie Civil*, August 23 and 30, 1919.)

If the wave resistance of an elastic bar is designated by R_w, Equation [2] can be written as

$$c = f/R_w \text{ or } f = cR_w \quad \ldots\ldots [2a]$$

Reflected Pressure and Speed Waves

If the mass or the stiffness constant of a bar suddenly changes the wave resistance will also change and the wave motion will be disturbed.

Referring to Fig. 1, AB is an elastic bar, the part AC of which has the mass m per unit length, the stiffness constant k, and therefore the wave resistance $R_w = \sqrt{(mk)}$. The part CB of this bar has the mass m_1 per unit length, the stiffness constant k_1, and therefore the wave resistance $R_{w1} = \sqrt{(m_1k_1)}$.

Assuming that a constant force f, acting from A to B, is suddenly applied at the end A, a bar pressure f and a speed of the particles c are created, both of which travel from A to C at a wave speed V (see Equation [1]). After passing through the intermediate shapes represented by the rectangles over AC' and AC'', these waves both take the shape of the rectangle over AC as shown in Fig. 1 at the time they reach the point C.

Because of the change in wave resistance, which occurs at C, further development of the pressure wave is disturbed and, if the wave resistance is increased at C, the pressure wave proceeds as shown in Fig. 2, in which the rectangle over AC is taken from Fig. 1. The bar pressure at C, Fig. 2, increases an amount f', producing a reflected pressure wave of the height f' superimposed on the original pressure wave of the height f and traveling back from C to A. The total pressure f_1 at C, Fig. 2, produces a transmitted wave which travels from C to B at the speed $V' = \sqrt{(k_1/m_1)}$.

The speed diagram, Fig. 2A, corresponds to the pressure diagram in Fig. 2. Increase in the wave resistance at C causes the particle speed to drop the amount c' or from c to c_1. This drop c' in particle speed is reflected and travels from C toward A at the wave speed V, while the remaining speed c_1 is transmitted from C to B at the wave speed V'.

The particle speed c_1 at C can be expressed in two ways

$$\left.\begin{aligned} c_1 &= (f - f')/R_w \\ \text{or } c_1 &= f_1/R_{w1} \end{aligned}\right\} \quad \ldots\ldots [3]$$

The pressure at C is

$$f_1 = f + f' \quad \ldots\ldots [4]$$

If $\sqrt{(m_1k_1)}/\sqrt{(mk)}$, the ratio of the wave resistances of the two parts of the bar, be designated as r, the forces f' and f_1, and the corresponding particle speeds c' and c_1 which they create, may be expressed as follows:

$$f' = \frac{r-1}{r+1} f \quad [5]$$

$$f_1 = \frac{2r}{r+1} f \quad [6]$$

$$c' = \frac{r-1}{r+1} c \quad [7]$$

$$c_1 = \frac{2}{r+1} c \quad [8]$$

If r is greater than unity in Equation [5], f' will be positive, causing an increase in the bar pressure above its original value f. If, however, r is less than unity in this equation, f' will be negative and will cause a decrease in the original bar pressure. The pressure f_1 is always positive.

In case the bar has a free end at the point, the ratio r will be equal to zero and

$$f' = -f \quad [9]$$

which means that the original pressure wave will be reflected, changing its sign. Also, if $r = 0$ is substituted in Equation [6], $f_1 = 0$, or the pressure in the transmitted wave will be zero.

The particle speed produced by this additional force f' will be c' which, when deducted from the original speed c, gives for $r = 0$

$$c - c' = c - (-c) = c + c = 2c \quad [10]$$

Therefore, it is seen that the speed of particles reaching a free end of a bar is also reflected but does not change its sign.

Likewise, the foregoing equations prove that for a fixed end of the bar, corresponding to a value of $r = \infty$, the additional pressure

$$f' = f \quad [11]$$

and the speed of particles changes to

$$c - c' = c - c = 0 \quad [12]$$

The bar pressure is reflected from a fixed end of a bar without change of sign while the speed of particles is reflected with change of sign.

If $r = 1$, there is no change in the wave resistance and f' as well as c' are equal to zero, while $f_1 = f$ and $c_1 = c$, as might be expected.

The rules which govern the reflection of pressure and speed waves at the end of a perfectly elastic bar may be expressed as follows:

1 A pressure wave reaching the free end of a bar is reflected, changing its sign.

2 A pressure wave reaching the fixed end of a bar is reflected without change of its sign.

3 A speed wave reaching the free end of a bar is reflected without change of its sign.

4 A speed wave reaching the fixed end of a bar is reflected, changing its sign.

Formation of Pressure and Speed Waves in a Perfectly Elastic Bar

(A) *Single External Force, Instantaneously Applied*

Referring to Fig. 3, assume that a constant force f is instantaneously applied at A of a perfectly elastic bar AB so as to act from A to B.

Such a force would be created by an infinite mass striking the end A at a speed c, which, multiplied by the wave resistance of the bar $\sqrt{(mk)}$ would produce the force f, expressed by Equation [2].

The material at the end of the bar to which the force is applied would be compressed, setting up a pressure in the bar which may be represented by the vertical line AC. Under the continued application of the force the compression will travel as a wave along the bar at a speed determined by the physical properties of the bar. See Equation [1].

The pressure distribution in the bar at successive intervals of time can be represented by the rectangles $ACC'A'$, $ACC''A''$, and $ACDB$. The last rectangle represents the pressure in the bar at the instant that the front of the pressure wave reaches the end B.

It is interesting to note that the ratio of the speed of particles c as given by Equation [2] to that of pressure and speed transmission V as given by Equation [1] can be expressed as

$$c/V = f/k \text{ or } c = (f/k)V \quad [13]$$

and is therefore equal to the ratio of the bar pressure to the stiffness constant of the bar.

The diagram in Fig. 3 can therefore be considered as representing either the internal pressures or the speed of particles produced as the pressure f travels from one end of the bar to the other. From the moment that the pressure reaches the end of the bar the pressures and the particle speeds can no longer be represented by the same diagram. The subsequent changes in pressures and speeds will depend on whether the end B is free or fixed.

If this end is free, the pressure will drop to zero and a tension wave traveling from B to A at the speed V will gradually reduce the pressure diagram to the rectangles $AA''C''C$ and $AA'C'C$ until it disappears into the vertical AC.

If at this moment the force f continues to act, it will produce a new pressure wave which will again travel to the end B of the bar and again recede to zero, this process continually repeating itself as long as the force f continues to act.

The speed diagram which is identical with the pressure diagram, Fig. 3, except for the scale, until the front of the wave reaches the end B of the bar, is illustrated in Fig. 3*A*, in which the rectangle $ABDC$ has been taken from Fig. 3.

The sudden drop of the pressure at the end B will increase the speed of the particles at this point to the value $BE = 2BD$. The wave front DE of the speed wave will travel from B to A in unison with the receding pressure wave and take the consecutive positions $E''C''$ and $E'C'$ until it reaches the position CF and the speed diagram takes the shape of the rectangle $ABEF$. If the force f continues to act, the speed of the particles at the point A will increase the amount $FG = CF = AC$, and the speed-wave front GF will travel from A to B, taking the intermediate positions $G'E'$ and $G''E''$ until it reaches the position EH. The speed diagram takes the shape of the rectangle $ABHG$.

If the end B of the bar AB is fixed the pressure diagram is represented by Fig. 3*A*. When the front of the pressure wave reaches the fixed end B it is reflected without change of sign from B to A, the wave front DE taking the intermediate positions $C''E''$ and $C'E'$ until it reaches the position CF. The pressure diagram takes the shape of the rectangle $ABEF$.

If an infinite mass moving at the speed c produced the pressure f, it would suddenly increase the pressure to $3f$, or from AC to AG, Fig. 3*A*, as the reflected pressure wave reaches A. Upon its next return this force would increase to $5f$. The increases in force continue in this manner until the bar pressure becomes infinite.

Therefore, if the end B of the bar is free, Fig. 3 represents the pressure on the bar and Fig. 3*A* represents the speed of the particles, while if the end B is fixed, the pressure is represented by Fig. 3*A* and the speed by Fig. 3.

(B) Single External Force Gradually Applied and Removed

Referring to Fig. 4, AB is a long bar of uniform cross-section and constant modulus of elasticity. If a compressive force acting longitudinally at the end of the bar is gradually and uniformly increasing from zero to the value f during the time t, it will create a pressure wave, represented by the triangle ACD, traveling from A to B at the pressure-propagation speed V. The inclined cross-sectional lines represent the consecutive positions of the wave front. The vertical AD represents the force f while the horizontal AC represents the distance L the pressure has traveled in the time t.

Every particle of the part AC of the bar has, under the influence of the force f, been given a certain speed c which according to Equation [13] is smaller than the speed of pressure transmission in the same ratio as the force f is smaller than the stiffness constant of the bar.

The time t during which the pressure f was attained does not affect the resulting speed of the particles. The diagram ACD represents the speed c of the particles at the time t as well as the internal pressure in the bar, although the scales of course are different.

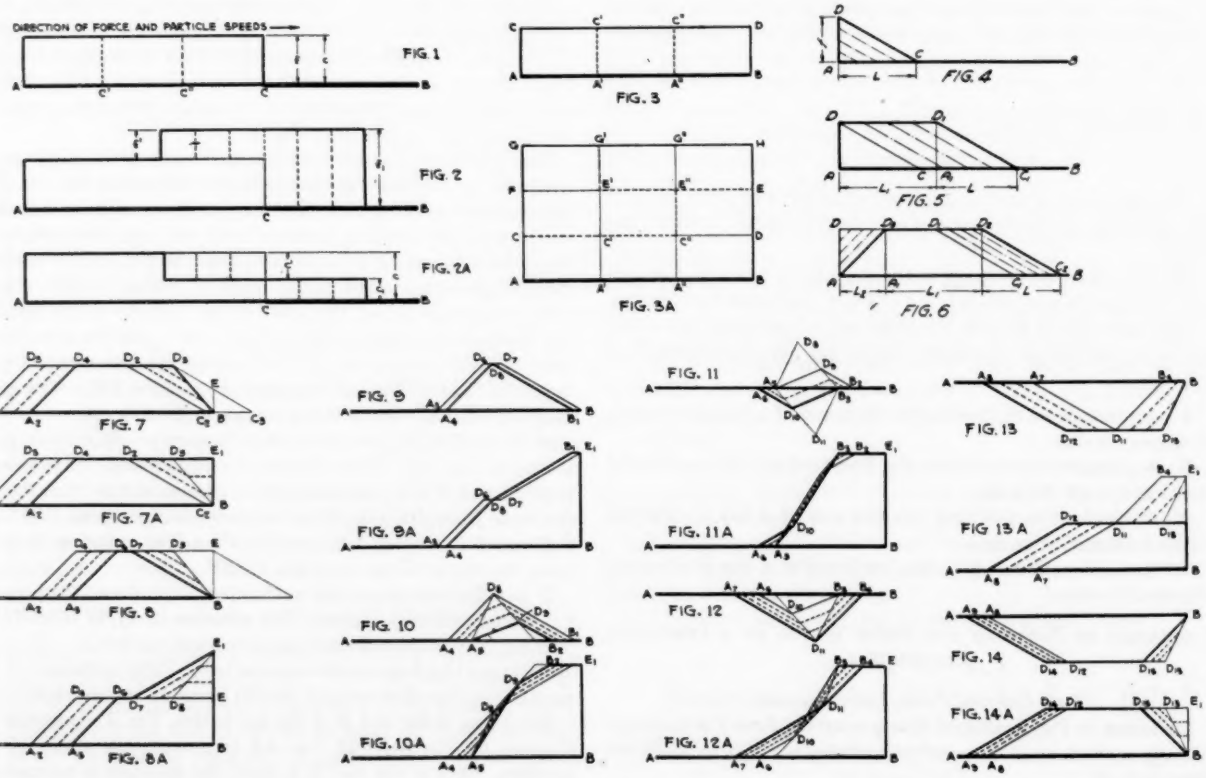
If the force f remains constant for a certain length of time t_1, referring to Fig. 5, the wave front CD, taken from Fig. 4 and shown in thin lines, will then have traveled the distance DD_1 and have reached the position C_1D_1. The pressure distribution in the bar is then represented by the area AC_1D_1D. The inclined cross-sectional lines represent the consecutive fronts of the pressure area during the time t_1. It also represents the speed of the particles, as previously explained.

With reference to Fig. 6, assume that the force f decreases gradually to zero during the time t_2. If the material of the bar is perfectly elastic or free from mechanical hysteresis, the pressure area AC_1D_1D, which has been taken from Fig. 5 and is shown in thin lines, will, during the time t_2, have traveled the distance L_2 without changing its shape and will be represented by the area $A_1C_2D_2D_3$. While the force f decreases during the time t_2, the pressure and speed line AD_3 will be formed and the particles in AA_1 of the bar will be retarded gradually until those at A return to a standstill, after the time t_2. At this time the pressure and speed diagram has taken the shape $AC_2D_2D_3$ and is moving from A to B at the constant speed V without changing its shape.

At the instant illustrated by Fig. 6 the pressure (and speed) history during the interval t_2 is represented by the successive positions of the dot-and-dash lines. It starts from the point D when the pressure f begins to decrease and progresses to the position AD when the force f at A has become zero. In the meantime the point D, at which the decrease in pressure began, has traveled to D_3.

When the quadrangle $AC_2D_2D_3$ passes beyond the point B, it changes its shape in different ways for the pressure and the speed diagram, the change in shape depending on whether the end B of the bar is free or fixed.

When the End B of the Bar AB Is Free. Referring to Fig. 7, it is assumed that the pressure and speed wave, represented by the quadrangle $AC_2D_2D_3$ taken from Fig. 6 and shown in light full lines, has traveled until it would have reached the position $A_2C_3D_3D_4$ if the bar AB had extended far enough. The part BC_3E of this quadrangle, which would extend beyond the free end B, is reflected and changes its sign. Hence, an equivalent area, represented by the triangle BD_3E is deducted from the remaining pressure area, forming a resulting pressure diagram represented by the quadrangle $A_3BD_3D_4$. The inclined dashed lines within the area $AA_2D_4D_3$ represent intermediate positions of the rear of the wave as it progresses from AD_3 to A_2D_4. The dashed lines within the area $BC_2D_2D_3$ represent intermediate positions of the wave front as it advances from C_2D_2 to BD_3.



The corresponding speed diagram is shown in Fig. 7*A*. In this case, the part of the wave which would project beyond the free end *B* is reflected without change of sign and, adding itself to the, as yet, unreflected part, passes through the intermediate shapes, shown in dashed lines within the quadrangle BC_2D_2E, until the speed wave takes the shape of the quadrangle $A_2BE_1D_4$ at the time the pressure diagram assumes the shape shown in Fig. 7.

Figs. 8, 9, 10, 11, 12, 13, and 14 show consecutive shapes of the pressure diagrams which are produced when the wave is reflected at the free end of a bar. Likewise Figs. 8*A*, 9*A*, 10*A*, 11*A*, 12*A*, 13*A*, and 14*A* show consecutive shapes of the corresponding speed diagrams.

When the End B of the Bar AB Is Fixed, the diagram representing the speed of the particles is as shown in Fig. 7, while Fig. 7*A* is the pressure diagram. The latter part of the pressure diagram, Fig. 7*A*, which would extend beyond the end *B* is reflected without change of sign and, after the front of the wave has passed through the intermediate shapes represented by the dashed lines in Fig. 7*A*, the pressure diagram assumes the shape of the quadrangle $A_2BE_1D_4$. Referring to the speed diagram, Fig. 7, the part BC_3E which extends beyond the fixed end *B* of the bar is reflected, changes its sign and is deducted from the, as yet, unreflected part of the diagram. The front of the speed wave passes through the intermediate shapes, represented by the dashed lines in Fig. 7, until the speed diagram takes the shape of the quadrangle $A_2BD_5D_4$, shown in the same figure.

Figs. 7, 8, 9, 10, 11, 12, 13, and 14 are the speed diagrams and Figs. 7*A*, 8*A*, 9*A*, 10*A*, 11*A*, 12*A*, 13*A*, and 14*A* are the corresponding pressure diagrams, derived for a bar with the end *B* fixed. Corresponding points in the speed diagrams and pressure diagrams are marked with the same symbols.

(*C*) *Uniformly Distributed External Forces Gradually Applied*

Referring to Fig. 15, *AB* is a perfectly elastic bar subjected to external longitudinal forces acting from *A* to *B*, the application beginning at *A* and gradually progressing to *B* at a constant rate of force application, corresponding to the gradual application of the brakes in a long train. It is proposed to determine the pressures produced by these forces as well as the speeds imparted to the particles of the bar.

The formation of the waves will be somewhat different, depending on whether the ratio of the speed of pressure transmission to the rate of force application is greater than, equal to, or smaller than unity. The reflection of the waves from the ends of the bar also will be different, depending on whether the ends are free or fixed.

In every case the formation of the waves can be subdivided into a number of distinct periods, the transition from one period to the other being marked by certain events. Diagrams, showing the dynamic conditions of the bar at each transition point and at two intermediate points during each period, as well as analytical expressions of various quantities, will be used to illustrate the formation and to compute the magnitude of the pressure and speed waves in the bar. Pressure and speed diagrams have been investigated for the cases listed in Table 1.

Case 1, Period 1. This period begins with the application of the force and continues until the pressure wave reaches the rear end of the bar. Let the vertical *BO* in Fig. 15 represent the total longitudinal force which is to be gradually and uniformly applied over the bar *AB*.

TABLE 1 CASES AND CONDITIONS OF PRESSURE AND SPEED WAVES INVESTIGATED

Case 1	$p > 1$	End *A* free	End *B* free
Case 2	$p = 1$	End *A* free	End *B* free
Case 3	$p < 1$	End *A* free	End *B* free
Case 1*A*	$p > 1$	End *A* free	End *B* fixed
Case 2*A*	$p = 1$	End *A* free	End *B* fixed
Case 3*A*	$p < 1$	End *A* free	End *B* fixed
Case 1*B*	$p > 1$	End *A* fixed	End *B* free
Case 2*B*	$p = 1$	End *A* fixed	End *B* free
Case 3*B*	$p < 1$	End *A* fixed	End *B* free
Case 1*C*	$p > 1$	End *A* fixed	End *B* fixed
Case 2*C*	$p = 1$	End *A* fixed	End *B* fixed
Case 3*C*	$p < 1$	End *A* fixed	End *B* fixed

After a certain time t the force application may have progressed to a point C'' on the bar *AB*. The longitudinal force distributed over the part AC'' of the bar is represented by the vertical $C''D''$. The force acting at each point of the part AC'' divides into a tension force and a compression force of equal magnitude. This results in a uniformly distributed force $C''E''$ equal to $\frac{1}{2}\ C''D''$, producing a compression wave, plus a uniformly distributed force $C''F''$, of the same magnitude as $C''E''$, producing a tension wave.

Since the speed of pressure transmission V is assumed to be greater than the rate of force application V_1, the compression produced when the external force began to act at *A* will have traveled to a point G'' beyond the point C'' by the time the force application reaches this latter point. The pressure distribution produced by the force $C''E$ is then represented by the triangle $AE''G''$. If the bar had extended beyond *A*, the tension wave would have traveled a distance $AH'' = AG''$ and the tension distribution produced by the tension force $C''F''$ would have been represented by the triangle $AF''H''$.

The part $AH''J''$ of this tension triangle, which extends beyond the free end *A* of the bar, is reflected, changes its sign, and forms the pressure area $AJ''K''$. If the tension area $AF''J''$ is subtracted from the pressure area $AJ''K''$, a pressure area $F''J''K''$ remains. Adding this area to the pressure area $AE''G''$, the resulting pressure diagram is represented by triangle $AG''L''$.

The corresponding speed diagram, shown in Fig. 15*A*, is similarly constructed with the exception that the areas of the triangles $AE''G''$ and $AF''J''$ and that of the reflected triangle $AJ''H''$ are all added together, forming the speed diagram $AG''L''M''$.

After the pressure propagation has reached the rear end *B* of the bar, the pressure diagram has taken the form of the triangle *ABL*. The point *L* is the intersection of the extended line AL'' with the line *BL* drawn parallel to $G''L''$. The corresponding speed diagram is represented by the area *ABLM* in Fig. 15*A*. The point *L* has the same location in Fig. 15*A* as in Fig. 15.

This condition represents the end of the period. From the geometric relationships in these diagrams the following analytical expressions may be derived.

The vertical *CD* in Fig. 15 represents the total applied external force $= f_1 l_1$. The maximum occurring internal pressure at the end of this period is represented by the vertical *CL* and can be expressed as

$$P_1 = f_1 L \frac{1}{p+1} \qquad [14]$$

The location of the point *C* at which P_1 occurs is equal to *AC* or

$$l_1 = (1/p)L \qquad [15]$$

The acceleration of the part *AC* of the bar, to which the external forces have been applied, is equal to the accelerating force, represented by the vertical *CD*, less the retarding force, represented by the internal pressure *CL*, divided by the mass, represented by the horizontal *AC*, or

$$a_1 = \frac{CD - LC}{AC} = \frac{f_1}{m} \frac{1}{p+1} \qquad [16]$$

The acceleration of the part *BC* of the bar which has not as yet been reached by the force application is represented by the internal pressure $CL = P_1$ divided by the mass of the part *BC* of the bar on which it acts. This acceleration is

$$a_2 = \frac{CL}{BC} = \frac{f_1}{m}\,\frac{p}{p^2-1} \quad [17]$$

The time at which the pressure P_1 is reached may be expressed as

$$t_1 = L/V \quad [18]$$

and the speed, which the part AC has acquired after the time t_1, represented by the vertical CL in Fig. 15A, may be expressed as

$$c_1 = \frac{f_1 L}{mV}\,\frac{1}{p+1} \quad [19]$$

Comparing Equation [14] for P_1 and Equation [19] for c_1, it will be noted that the vertical BO in Fig. 15 represents the sum f_1L of all the external forces ultimately acting on the bar, while the same vertical distance in the speed diagram would represent the expression f_1L/mV or the speed which the unit force f_1 acting on the unit mass m would impart to the mass during the time t_1.

The momentum of the bar at the time t_1 may be expressed as the average external force applied to the bar during the time t_1, multiplied by the time or

$$Q_1 = \frac{f_1 l_1}{2}\,t_1 = \frac{f_1 L^2}{V}\,\frac{1}{2p} \quad [20]$$

Up to this point the formation of the wave has been independent of whether the end B of the bar is free or fixed and, therefore, the above formulas apply to both Case 1 and Case 1A.

Case 1, Period 2. This period begins at the time the pressure wave reaches the free rear end of the bar and continues until the reflected wave meets the crest of the original one.

Referring to Fig. 16, the pressure diagram ABL has been taken from Fig. 15 and the speed diagram $ABLM$ in Fig. 16A has been taken from Fig. 15A. Following the same procedure as outlined under Period 1, it will be found that the pressure diagram changes from the triangle ABL at the beginning of this period, takes the intermediate forms represented by the areas $ABB'L'$ and $ABB''L''$, and assumes the form of the triangle ABL_1 at the end of this period. The corresponding speed diagrams change from the quadrangle $ABLM$ at the beginning of this period, takes the intermediate forms, represented by the areas $ABN'B'L'M'$ and $ABN''B''L''M''$, and assumes the form of the rectangle $ABNM_1$ at the end of this period.

It is interesting to note that every particle in the bar has a uniform speed $c_2 = C_1L_1$ in Fig. 16A at the time of the maximum occurring bar pressure $P_2 = C_1L_1$ in Fig. 16.

This pressure can be expressed as

$$P_2 = f_1 L\,\frac{2p}{(p+1)^2} \quad [21]$$

and occurs at the distance AC_1 from point A, which is

$$l_2 = L\,\frac{2}{p+1} \quad [22]$$

The acceleration of the part AC_1 of the bar is still a_1, as given in Equation [16], while that of the part BC_1 of the bar is

$$a_3 = 2a_2 = \frac{f_1}{m}\,\frac{2p}{p^2-1} \quad [23]$$

The time t_2 at which the pressure P_2 is reached may be expressed as

$$t_2 = \frac{l_2}{V_1} = \frac{L}{V}\,\frac{2p}{p+1} \quad [24]$$

The speed c_2 is equal to the distance C_1L_1 or

$$c_2 = a_1 t_2$$

or

$$c_2 = \frac{f_1 L}{mV}\,\frac{2p}{(p+1)^2} \quad [25]$$

The momentum of the bar at the end of this period, computed as the product of its mass m and its uniform speed c_2 can be expressed as

$$Q_2 = \frac{f_1 L^2}{V}\,\frac{2p}{(p+1)^2} \quad [26]$$

It has been found, as the equations developed for the above Period 1 and Period 2 of Case 1 have indicated, that every specific value of the various quantities P, l, a, t, c, and Q can be expressed by a function of the constants of the bar which is the same for all cases and periods multiplied by a function of the propagation ratio p. The function, however, will vary according to cases and periods.

It has been shown how such specific values are constructed or computed from the constants for Periods 1 and 2 of Case 1.

For the purpose of the present investigation, the maximum value of P, its location l, and the maximum value of the acceleration or retardation a are the most important quantities. Therefore, the following analysis will be confined generally to the study of these quantities.

Case 1, Period 3. This period begins at the time the pressure wave, which has been reflected from the free rear end B of the bar, meets the crest of the original wave, and continues until the force application has reached the end B.

Following the procedure previously outlined, the pressure diagram Fig. 17, which at the beginning of this period is represented by the triangle ABL_1, takes the intermediate shapes represented by the quadrangles $ABL_1'L_2'$ and $ABL_1''L_2''$ and assumes the shape of the triangle ABL_2 at the end of the period. The corresponding speed diagram Fig. 17A changes from the rectangle $ABNM_1$ at the beginning of this period, takes the intermediate shapes represented by the areas $ABN'L_1'L_2'M_2'$ and $ABN''L_1''L_2''M_2''$, and assumes the shape of the area $ABN_1L_2M_2$ at the end of this period.

The maximum internal pressure occurring at the end of this period is represented by the vertical C_2L_2 in Fig. 17 and can be expressed as

$$P_3 = f_1 L\,\frac{p(2-p)}{p+1} \quad [27]$$

It occurs at the distance AC_2 from point A which is

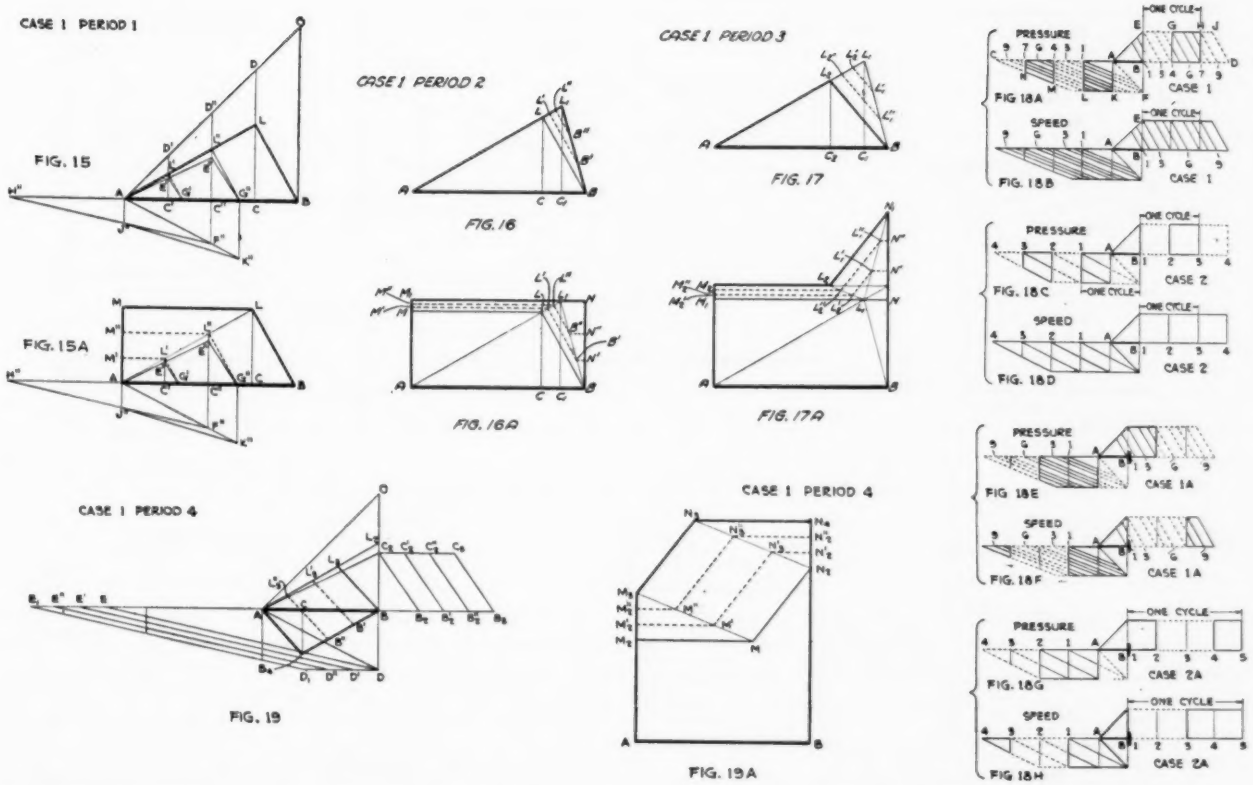
$$l_3 = L(2-p) \quad [28]$$

The acceleration of the part C_2B of the bar at the end of this period is

$$a_4 = \frac{f_1}{m}\,\frac{2p-1}{p^2-1} \quad [29]$$

Summary of Analysis of the Remaining Periods and Cases. It is believed that the foregoing analysis of the first three periods of Case 1 illustrates sufficiently the application of the method to other periods and cases and that a reproduction of all the diagrams of individual periods of the remaining cases will not be necessary.

For the development of any desired period in any of the cases listed in Table 1, the diagrams shown in Figs. 18A to 18H are helpful. Referring to Fig. 18A, the horizontal AB is the bar to



which the uniformly distributed external forces are gradually applied, acting from A to B at a uniform speed of application.

The horizontal AD in Fig. 18A represents the accumulated travel of the front of the pressure wave back and forth between A and B, from the time it originated at the front end A of the bar. The horizontal AC represents the accumulated travel of the front of the tension wave, also originating at the end A. Each of these lines is divided into sections within which the original wave retains its sign, alternating with sections in which a change of sign occurs. The inclined lines, which intersect the horizontal AD in the points 1, 2, 3, etc., represent the advancing front of the compression wave at the end of the 1st, 2nd, 3rd, etc. periods, respectively. Likewise, the inclined lines, which intersect the horizontal AC in the points 1, 2, 3, etc., represent the front of the tension wave at the end of the 1st, 2nd, 3rd, etc. periods, respectively.

In the pressure diagram Fig. 18A, both the compression and tension waves change their sign every time they pass the free ends A and B of the bar. The horizontals AD and AC may therefore be divided into sections within which the original wave retains its sign, alternating with sections within which a change of sign takes place. In view of the fact that ends A and B are both free, each such section is equal to the length AB of the bar.

The pressure diagram at the end of any period of Case 1 will be found if the areas between the inclined line AE in Fig. 18A and the wave fronts above the base line CD are algebraically added to the areas between the inclined line AF and the wave fronts below the base line, keeping in mind that the sign is changed so as to be negative in the areas $BEG4$ and $7HJD$ above the line CD and so as to be positive in the areas $AKL1$, and $4MN7$ below the base line CD. The addition of these areas should be carried out in such a way that the points 4 and C come at the end A and points 1 and 7 at the end B of the bar. All the areas enclosed by full lines are positive while those formed by dashed lines are negative.

The speed diagram at the end of any period is constructed from the diagram in Fig. 18B in the same manner, except that all the areas are added together arithmetically since the speed waves are all positive and all are reflected from the ends of the bar without change of sign.

In Figs. 18A to 18H, inclusive, the vertical height of the pressure diagrams represents one-half of the total applied external force or $\frac{1}{2}f_1L$. The height of the speed diagrams or the speed represented by the verticals $BE = BF$ may, according to Equation [2], be expressed as

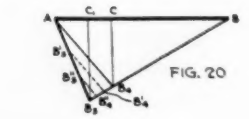
$$c = \frac{f_1 L}{2\sqrt{(mk)}} \qquad [30]$$

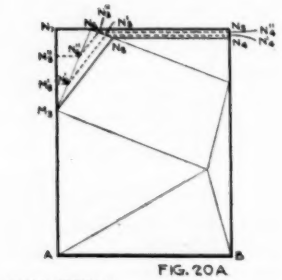
and is equal to the speed of the particles. This speed will be produced by a pressure equal to one-half of the external forces in a bar of the wave resistance $\sqrt{(mk)}$.

During the first period the unreflected compression and tension areas in all the pressure diagrams have opposite signs, the former being positive and shown in full lines above the line AB, while the latter are negative and shown in dashed lines below the base line AB. The corresponding unreflected waves in all the speed diagrams are both positive during the first period and shown in full lines.

The signs of the compression and tension waves during the following periods depend on whether the ends of the bar are free or fixed. In Figs. 18A to 18H, inclusive, the sections within which the diagram areas are positive are shown in full lines. Those within which they are negative are shown in dashed lines. With the help of such diagrams, the pressure and speed diagrams for all the cases and periods in Table 1 have been investigated, but it is not considered necessary to reproduce them all.

The diagrams for the Cases 1, 2, and 2A may serve as examples. Figs. 19 and 19A and Figs. 20 and 20A represent Case 1, Periods 4 and 5, respectively. Figs. 21 and 21A and Figs. 22 and 22A

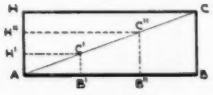

FIG. 20


FIG. 20A

CASE 1 PERIOD 5


FIG. 21

CASE 2 PERIOD 1


FIG. 21A

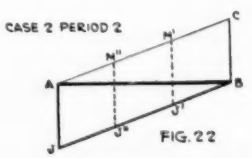
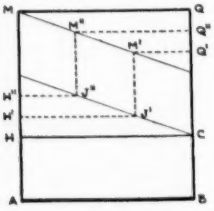

CASE 2 PERIOD 2

FIG. 22


FIG. 22A

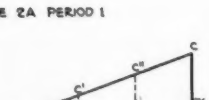
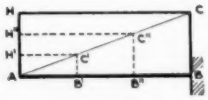

CASE 2A PERIOD 1

FIG. 23


FIG. 23A

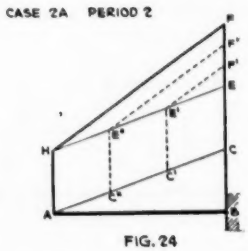
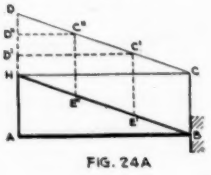

CASE 2A PERIOD 2

FIG. 24


FIG. 24A

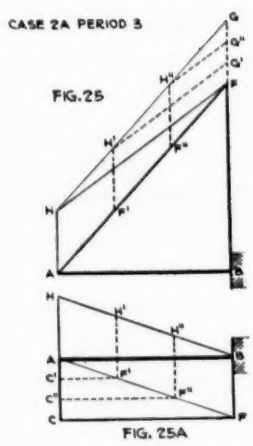

CASE 2A PERIOD 3

FIG. 25

FIG. 25A

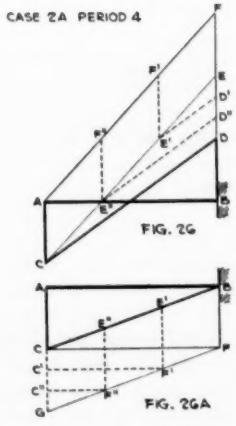

CASE 2A PERIOD 4

FIG. 26

FIG. 26A

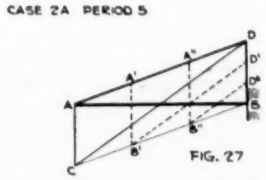
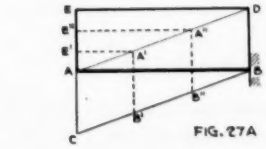
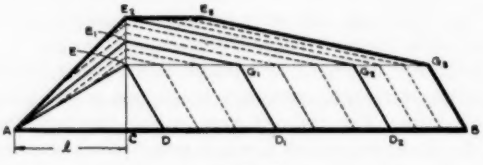

CASE 2A PERIOD 5

FIG. 27


FIG. 27A


FIG. 28

represent Case 2, Periods 1 and 2, respectively. Figs. 23 and 23*A* to Figs. 27 and 27*A*, inclusive, represent Case 2*A*, Periods 1 to 5, respectively.

In each of these figures the diagram at the beginning of the period is shown in light full lines. Two intermediate shapes of the diagram are shown in dashed lines. The diagram at the end of the period is shown in heavy full lines. The diagram at the end of one period is of course the same as that at the beginning of the following period and both are designated by identical symbols.

In all the cases it will be found that from a certain point on the pressure diagrams are regularly repeated in cycles of a certain number of periods each. This applies also to the speed diagrams, with the qualification that in Cases 1, 2, and 3, where both ends of the bar are free, the speed of the bar as a whole increases continuously under the influence of the applied external forces. The number of periods in each cycle and the number of the period at the end of which the cycles begin are given in Table 2 for all the cases listed in Table 1.

TABLE 2 CYCLES OF THE PRESSURE AND SPEED WAVES

Case number	No. of periods per cycle	Period no. at end of which cycles begin	No. of periods up to and including the first cycle
1	6	3	9
2	2	1	3
3	6	2	8
1*A*	12	3	15
2*A*	4	1	5
3*A*	12	2	14
1*B*	12	3	15
2*B*	4	1	5
3*B*	12	2	14
1*C*	6	3	9
2*C*	2	1	3
3*C*	6	2	8

The analysis of the pressure and speed waves for the type of force application under consideration thus involves an examination of a total of 108 sets of diagrams. For the purpose of this investigation it is considered sufficient to give the summary of such an examination. This summary is contained in Tables 3 and 4.

In the preceding analysis it was always assumed that the external forces, beginning at A, acted in the direction from A to B, in which case the conditions listed in Table 3 would apply.

If the external forces should begin to be applied at A but should act in the opposite direction, or from B to A, all the preceding diagrams would change in such a manner that every point of them would move vertically across the line AB to an equidistant position. The various values for Table 3 would still apply, but their meaning would change according to Table 4.

The changes in the force application do not affect Table 1.

(*D*) *Partially Applied Uniformly Distributed External Force Acting on a Perfectly Elastic Bar*

1 Both Ends of Bar Free. Referring to Fig. 28, AB is a perfectly elastic bar subjected to a uniformly distributed external force, acting from A to B, its application beginning at A and progressing at a uniform rate up to point C.

Partial Force Application for $p > 1$. At the time the force application reaches the point C, the pressure diagram will be represented by the triangle ADE. By the use of the previously

TABLE 3 CONDITIONS IN ELASTIC BAR WHEN PRESSURE IS APPLIED FROM *A* TO *B*

TABLE 4 CONDITIONS IN ELASTIC BAR WHEN PRESSURE IS APPLIED FROM *B* TO *A*

Line No.	TABLE 3	VALUES FOR TABLE 3 AND TABLE 4												TABLE 4
1	Case number	1	2	3	1*A*	2*A*	3*A*	1*B*	2*B*	3*B*	1*C*	2*C*	3*C*	Case number
2	Max bar pressure, multiply values in this line by f_1L	$\frac{2p}{(p+1)^2}$	$1/2$	$\frac{p}{2}$	2	2	2	$\frac{2}{(p+1)^2}$	$1/2$	$1/2$	$2-p$	1	1	Max bar tension, multiply values in this line by f_1L
3	Distance from front end of the bar at which max pressure occurs, multiply values in this line by L	$\frac{2}{p+1}$	1	$\frac{1+p}{2}$	1	1	1	$\frac{2}{p+1}$	1	$\frac{1+p}{2}$	1	1	1	Distance from front end of the bar at which max tension occurs, multiply values in this line by L
4	No. of earliest period during which max bar pressure occurs	2	1	2	7	2	7	2	1	2	3	1	3	No. of earliest period during which max bar tension occurs
5	Max bar tension, multiply values in this line by f_1L	$\frac{(3-p)p}{2(1+p)}$	$1/2$	$1/2$	$\frac{2-p}{2}$	$1/2$	$\frac{(3-p)p}{2(1+p)}$	2	2	2	$2-p$	1	1	Max bar pressure, multiply values in this line by f_1L
6	Distance from front end of the bar at which max tension occurs, multiply values in this line by L	$\frac{p-1}{2}$	0	$\frac{1-p}{2}$	$\frac{p-1}{2}$	0	$\frac{1-p}{2}$	1	0	0	0	0	0	Distance from front end of the bar at which max pressure occurs, multiply values in this line by L
7	No. of earliest period during which max bar tension occurs	5	2	5	11	4	11	6	2	6	4	1	4	No. of earliest period during which max bar pressure occurs
8	Max acceleration (or retardation) of particles of bar, multiply values in this line by f_1/m	$\frac{3p-1}{p^2-1}$	∞	$\frac{1}{1-p}$	$\frac{1}{p-1}$	∞	$\frac{1}{1-p}$	$\frac{1}{p-1}$	∞	$\frac{1}{1-p}$	$\frac{1}{p^2-1}$	$\frac{1}{\infty}$	$\frac{1}{1-p^2}$	Max acceleration (or retardation) of particles of bar multiply values in this line by f_1/m
9	No. of earliest period during which max acceleration (or retardation) occurs	5	1	2	5	1	5	8	1	8	3	1	3	No. of earliest period during which max acceleration (or retardation) occurs

described method, it is found that under the influence of the applied force, the pressure diagram will take the intermediate shapes $AD_1G_1E_1$, $AD_2G_2E_2$, and the final shape $ABG_2E_2E_1$, the latter being attained at the time the front of the pressure wave reaches the end B of the bar. The vertical CE_2 in the pressure diagram, Fig. 28, represents the maximum occurring bar pressure and the greatest bar pressure that the applied force can produce, regardless of the length of the bar AB.

If the external force applies to the part AC of the bar AB as shown in Fig. 29, the reflected pressure wave will just reach the crest of the original one at the time the latter reaches its maximum value CD.

Referring again to Fig. 28, it will be seen that the maximum pressure produced by a force applied over the part l of a bar never can exceed the value represented by the vertical CE_2 which may be expressed as

$$P = f_1 l \quad \ldots\ldots [31]$$

The greatest length l_4, represented by the distance AC in Fig. 29, which still permits the formation of such a pressure may be expressed as

$$l_4 = L \frac{2}{3+p} \quad \ldots\ldots [32]$$

and the corresponding maximum bar pressure, represented by the vertical CD in Fig. 29, is

$$P_4 = f_1 l_4 = f_1 L \frac{2}{3+p} \quad \ldots\ldots [33]$$

The maximum bar pressure for a force application extending over the whole length of the bar is represented by the vertical C_2D_1 in Fig. 30 and is given as P_2 in Equation [21]. It occurs when the force application has proceeded the distance AC_2 in Fig. 30, given as l_2 in Equation [22]. Also, P_2 is larger than P_4 for all values of p greater than unity.

For all values of l between the limits $0 < l < l_4$, the maximum bar pressure P is expressed by Equation [31]. For values of l within the range $l_4 < l < l_2$, the maximum value of P is expressed by the equation

$$P = f_1[l(p-1) + 2L] \frac{1}{2(1+p)} \quad \ldots\ldots [34]$$

Partial Force Application for $p = 1$. By using the same method, the pressure diagrams shown in Fig. 31 are obtained for $p = 1$. The broken line $ABCD$ represents the form of the pressure diagram at the moment the maximum obtainable bar pressure ED first occurs. This maximum pressure can be expressed as

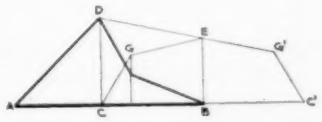


FIG. 29

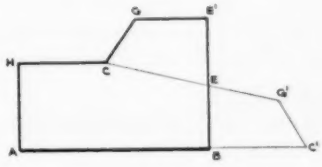
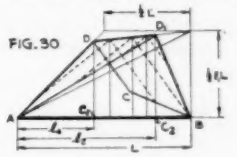
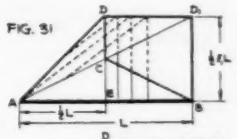
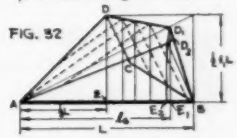


FIG. 29A







$$P_4 = 1/2 f_1 L \quad \ldots\ldots [35]$$

For all values of l between zero and $1/2L$, the corresponding value of P is expressed by Equation [31].

For all values of l between the limits $1/2L < l < L$, the value of P is equal to P_4 as given in Equation [35].

The pressure diagrams for intermediate values of l are shown in dashed lines in Fig. 31. The corresponding maximum bar pressures DE and BD_1 are both equal to P_4 as given in Equation [35].

For $l = L$ the pressure diagram takes the shape of the triangle ABD_1 in Fig. 31 at the time P reaches its maximum value P_4.

Partial Force Application for $p < 1$. By using the same procedure as previously outlined, the pressure diagrams shown in Fig. 32 are obtained.

For values of l between the limits $0 < l < 1/2L$, the value of P is that given in Equation [31]. In this case, $l = 1/2L$ represents the length of the greatest part of the bar to which the external force may apply and still produce the maximum pressure ED which the force can possibly create, regardless of the length of the bar.

This pressure is equal to P_5 as given in Equation [35]. The corresponding pressure diagram is represented by the quadrangle $ABCD$, Fig. 32.

In case of a uniformly applied force over the whole length of the bar, the maximum bar pressure D_2E_2, equal to $\frac{1}{2}pf_1L$, is reached when the pressure propagation has reached the distance AE_2, equal to $\frac{1}{2}(1 + p)L$, which values are found in Table 3 (Case 3, Period 2). The corresponding pressure diagram is represented by the triangle ABD_2 in Fig. 32.

It will be seen from this equation that the value of P gradually increases from the value of P_4 as expressed by Equation [33] to the maximum value of P_2 as expressed by Equation [21] along the straight line DD_1 in Fig. 30 when the part l, to which the external force applies, increases from the value of l_4 as given by Equation [32] to l_2 as given by Equation [22].

Referring to Fig. 30, it will therefore be seen that the heavy full line ABD_1D represents the maximum bar pressures that can be produced by an applied force of f_1 lb per ft of bar, regardless of the part of the bar to which it is applied.

The shape of the pressure diagrams $ABCD$ and ABD_1 are shown in full lines at the time the bar pressure reaches its maximum value for $l = l_4$ and $l = l_2$. Diagrams for intermediate values of l are shown in dashed lines. They represent the shapes of the pressure diagrams when the external force is applied to the bar AB over intermediate distances at the time the maximum bar pressure is reached for each application.

It is interesting to note that in case of partial force application over the length l_6 of the bar, equal to AE_1 in Fig. 32, and expressed by the equation

$$l_6 = L\frac{2}{3-p} \quad \ldots\ldots\ldots\ldots\ldots\ldots [36]$$

the bar pressure exceeds the value from Table 3 and amounts to P_6, where

$$P_6 = f_1L\frac{1}{3-p} \quad \ldots\ldots\ldots\ldots\ldots\ldots [37]$$

represented by the vertical D_1E_1 in Fig. 32.

For values of l between the limits $\frac{1}{2}L < l < l_6$, the maximum obtainable bar pressure

$$P = f_1\frac{L - l(1-p)}{1+p} \quad \ldots\ldots\ldots\ldots\ldots\ldots [38]$$

and, for such values of l, the corresponding values of P are located on the inclined line DD_1 in Fig. 32. Pressure diagrams for such values of l are shown in dashed lines. For values of l between the limits $l_6 < l < L$ the corresponding values of P are located on the straight line BD_1 in the same figure.

Partial Force Application in Different Directions. In the preceding analysis of partial force application it was shown that if the external force is applied from A to B, beginning at A, and if the application of this force proceeds at a uniform rate in that direction, the maximum bar pressures possibly created, regardless of the length of the part of the bar to which the external force applies and of the propagation ratio, will always fall within the quadrangle ABC_1M_1 shown in Fig. 33 in heavy full lines, provided both ends of the bar are free.

The maximum occurring coupler tensions will be confined within the quadrangle ABM_2G_2 also shown in heavy full lines in Fig. 33. If the applied forces act from B to A the bar pressures will remain within the quadrangle ABM_1G_1 and the bar tension within the quadrangle ABC_2M_2, both shown in dashed lines. It will be seen that regardless of the direction of the applied forces and of the rate at which the application proceeds, the bar pressure or tension will, in case of a bar with both ends free, never exceed the value of $\frac{1}{2}f_1L$.

2 One End of Bar Is Free, One End Fixed. Referring to Fig. 34, AB represents a perfectly elastic bar subjected to gradually and uniformly applied external longitudinal forces acting from A to B. They begin to apply at the end A and proceed toward the end B at a constant rate of application. These forces may apply over the whole length of the bar or over only a part of it.

If the end A is free, while the end B is fixed, the analysis shows that the bar pressures may attain any value within the quadrangle ABD_1G_1 and the bar tensions may attain any value within the triangle ABG_2, in Fig. 34. If the external forces apply in the opposite direction, bar tensions will predominate and may assume any value within the quadrangle ABD_2G_2, while the bar pressures will be confined within the triangle ABG_1. It is interesting to note that if one end of the bar is fixed, the maximum possibly occurring bar tensions and pressures are equal to $2f_1L$ or four times greater than if both ends were free.

3 Both Ends of the Bar Are Fixed. Referring to Fig. 35, it is assumed that both ends of the perfectly elastic bar AB are fixed.

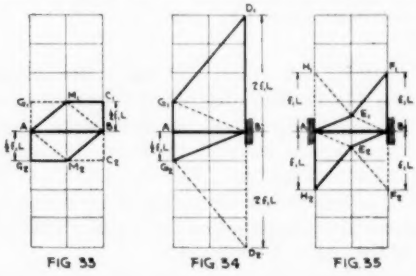


FIG 33 FIG 34 FIG 35

It is subjected to gradual and uniform external longitudinal forces which begin to apply at end A and proceed toward end B at a constant rate of application. They may apply over the whole length of the bar or only a part thereof.

If the forces act from A to B, the maximum bar pressures will fall within the quadrangle ABF_1E_1 and the maximum bar tensions within quadrangle ABE_2H_2. If the forces act in the opposite direction, the maximum bar pressures will fall within the quadrangle ABE_1H_1 and the maximum bar tensions within the quadrangle ABF_2E_2. In either case and regardless of the direction of the forces, the bar pressures will always fall within the polygon $ABF_1E_1H_1$ and the bar tensions within the polygon $ABF_2E_2H_2$.

In the center of the bar the pressure or tension will never exceed $\frac{1}{4}f_1L$ or one-half that which may be attained in a bar with both ends free. The maximum occurring bar pressure or bar tension, which only occurs at the ends of the bar may attain the value f_1L or twice that obtainable in a bar with both ends free.

2 BRAKING AND STARTING OF A LONG TRAIN

(A) Braking of a Bunched Train

The brakes apply first on the first car and then the brake application proceeds from car to car through the length of the train. After the brakes have begun to retard the first car, the second car runs relatively faster for a certain time. The temporary speed difference between the two cars is equalized by compression of the draft gears as the second car overtakes the first. This condition is repeated between the second and third cars, and so on throughout the train.

Period of Increasing Coupler Pressure

If in a long train which has no free slack, all the cars have the same weight, and all the draft gears offer a resistance proportional

TABLE 5 EQUATIONS FOR VALUES OF P_m, a_{max}, C_{min}, AND q

Propagation ratio, $p = V/V_1$	Maximum coupler pressure P_m	Eq No.	Maximum car retardation a_{max}	Eq No.	Minimum safe draft-gear capacity C_{min}	Eq No.	Gear travel q corresponding to C_{min}	Eq No.
$p > 1$	$nF\,\frac{2p}{(p+1)^2}$	[49]	$\frac{F}{M}\,\frac{3p-1}{p^2-1}$	[52]	$\frac{1}{M}\left(\frac{nFL_1}{V_1}\right)^2\frac{1}{(p+1)^4}$	[55]	$\frac{nFL_1^2}{V_1^2M}\,\frac{1}{p(p+1)^2}$	[58]
$p = 1$	$nF/2$	[50]	∞	[53]	$\frac{1}{M}\left(\frac{nFL_1}{4V_1}\right)^2$	[56]	$\frac{nFL_1^2}{4V_1^2M}$	[59]
$p < 1$	$(nF/2)p$	[51]	$\frac{F}{M}\,\frac{1}{1-p}$	[54]	$\frac{1}{M}\left(\frac{nFL_1}{4V_1}\right)^2$	[57]	$\frac{nFL_1^2}{4V_1^2M}\,\frac{1}{p}$	[60]

to the amount of compression to which they are subjected, the train can be considered as a bar of uniform mass and elastic properties. The coupler forces and car retardations, which occur in such a train under the influence of external forces such as a progressive braking, can be computed from the formulas derived in Part 1 for a perfectly elastic bar, with both ends free.

The formulas in Part 1 were derived under the assumption that the external forces acted on a bar at a standstill, accelerating its particles. In the case of braking of a long train, however, the external forces retard the moving cars.

Instead of the symbols f_1, L, and m, which are used in the various equations for elastic bars, it is convenient to use the corresponding symbols which express characteristics of a long train. These symbols are given in Equations [39] to [44], inclusive.

$$f_1 = F/L_1 \quad [39]$$

$$f_1L = Fn \quad [40]$$

$$m = M/L_1 \quad [41]$$

$$L = nL_1 \quad [42]$$

$$f_1/m = F/M \quad [43]$$

$$R_w = \sqrt{(mk)} = \frac{\sqrt{(CM)}}{q} \quad [44]$$

k = stiffness constant

The stiffness constant is the force required to stretch a bar twice its original length.

The force required to stretch a car and its two draft gears the length $2q$ is equal to the ultimate reaction of the gears. Assuming a straight compression line, beginning at zero, the force $2C/q$ will stretch the car the amount $2q$ and the stiffness constant

$$k = \frac{2C}{q}\,\frac{L_1}{2q} = \frac{L_1}{q^2}\,C \quad [45]$$

The pressure-propagation speed V can be expressed by substituting in Equation [1] the value of k and m as given in Equations [45] and [41], respectively.

$$V = \sqrt{\frac{k}{m}} = \sqrt{\left(\frac{L_1^2C}{q^2M}\right)} = \frac{L_1}{q}\sqrt{\frac{C}{M}} \quad [46]$$

The value of the draft-gear travel q should include the strain of one coupler shank plus the strain in one-half the length of a center sill when subjected to the force P. The corresponding potential energy should be included in the value of the draft-gear capacity C, so as to take into account the combined elastic action of the car bodies and the draft gears.

If solid steel blocks were used as connections between the couplers and the center sills of the car bodies, the whole train would act as a solid steel bar, and a retarding force acting on the first car would be transmitted through the whole length of the train at a speed of 16,800 fps. This speed is over 15 times as fast as the speed of sound in air, 1080 fps. The rate of brake application on the other hand depends on the travel of air pressure from one car to another and therefore cannot under any circumstances exceed the speed of sound. On the average freight train in the type-K brakes the rate of emergency brake application amounts to about 620 fps, but in trains with the new type-AB brakes, the rate of propagation of emergency application amounts to 960 fps. The use of conventional draft gears decreases the pressure-transmission speed from 16,800 fps to values in the neighborhood of the rate of brake application of the average freight train.

Applying the formulas given in Part 1 to brake action in a long train, it will be noted that the propagation ratio p is an important factor in determining coupler forces and car accelerations occurring in a long train. Substituting the value of V from Equation [46] in the propagation ratio $p = V/V_1$

$$p = \frac{V}{V_1} = \frac{L_1}{qV_1}\sqrt{\frac{C}{M}} \quad [47]$$

$$q = \frac{L}{pV_1}\sqrt{\frac{C}{M}} \quad [48]$$

The substitution of Fn, the equivalent of f_1L, Equation [40], in line 2 of Table 3, gives Equations [49], [50], and [51] for cases 1, 2, and 3. The substitution of F/M, the equivalent of f_1/m, Equation [43], in line 8 of Table 3, gives Equations [52], [53], and [54] listed in Table 5.

The minimum safe draft-gear capacity, always assuming a straight compression line, beginning at zero, can be expressed

$$C_{min} = P_m q/2$$

or

$$P_m = 2C_{min}/q \quad [61]$$

Substituting in this equation the value of P_m from Equations [49], [50], and [51] and the value of q from Equation [48], Equations [55], [56], and [57] are obtained. These equations are listed in Table 5. The gear travel q corresponding to C_{min} is obtained by the substitution of the value of C_{min} from Equations [55], [56], and [57] for C in Equation [48].

The maximum coupler pressure P_m, the maximum car retardation a_{max}, the minimum draft-gear capacity C_{min}, and the corresponding gear travel q are expressed by Equations [49] to [60], inclusive. The equation to use depends on whether the propagation ratio is greater than, equal to, or smaller than unity. These equations, listed in Table 5, are based on the assumption that the draft gears are powerful enough to cushion elastically all occurring coupler forces. If the gears go solid during the passing of the pressure wave the latter will be disturbed and the coupler forces will increase about twice as fast as they would otherwise.

Example. A freight train of 150 seventy-ton cars is subjected to emergency braking from the front end of the train. Other conditions of the problem are as follows:

Weight of car, empty	50,000 lb
Weight of car, loaded	210,000 lb
Length between coupler faces	50 ft

Brake pressure	60% of light weight 30,000 lb
Coefficient of friction	25 per cent
Braking force	0.25 × 30,000 lb 7500 lb
Rate of emergency brake application	930 fps

From the conditions of the problem it is observed that $n = 150$, $F = 7500$ lb, $V_1 = 930$ fps, $L_1 = 50$ ft, $M = 1550$ for empty cars, and $M = 6500$ for loaded cars.

The values of P_m, a_{max}, C_{min}, and q can be computed for various propagation ratios for empty and loaded cars from Equations [49] to [60], inclusive. Sets of values of these quantities,

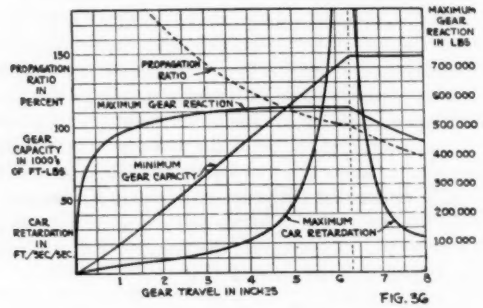


FIG. 36

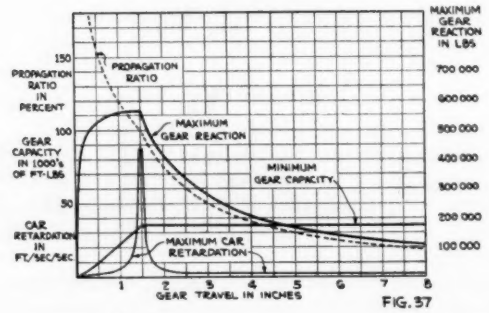


FIG. 37

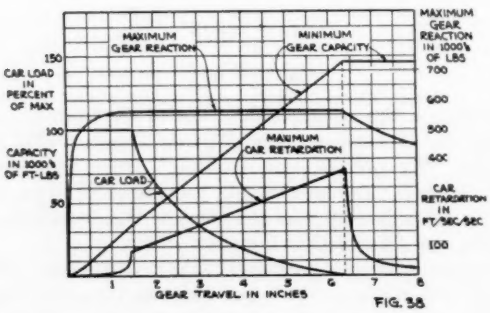


FIG. 38

corresponding to various values of p, can be plotted against any one of them as the independent variable.

Using the length of travel as the independent variable, the curves for the empty train have been plotted in Fig. 36 and those for the fully loaded train in Fig. 37. Fig. 38 is a composite diagram made up from Figs. 36 and 37 and also shows the maximum values P_m, a_{max}, C_{min}, and q for intermediate or partial train loadings.

Fig. 36 shows that if the gears shall not go solid during emergency braking of the 150 empty cars, the gear capacity and the gear reaction must not fall below certain minimum limits represented by the curves. The gear-capacity curve shows that the minimum required gear capacity increases with increasing gear travel, practically along a straight line beginning at zero up to a capacity of about 147,000 ft-lb. A draft gear must have this capacity if its travel is $6^5/_{16}$ in. and it shall not go solid during emergency braking of the empty train. A travel increase above $6^5/_{16}$ in. does not cause any further increase of the minimum gear capacity.

If, however, the gear capacity is limited to, say, 30,000 ft-lb, corresponding to a gear travel of $1^7/_{16}$ in., a longer gear travel will cause the gears to go solid during an emergency brake application of an empty train of 150 cars. A gear of the standard AAR travel of $2^5/_8$ in. would, according to Fig. 35, need a capacity of 60,000 ft-lb to prevent it from going solid in such service.

The gear-reaction curve in Fig. 35 shows that the gear reactions increase very rapidly with increasing gear travel, reaching a value of about 540,000 lb at the standard AAR travel of $2^5/_8$ in. and its maximum value of 562,500 lb at the gear travel of $6^5/_{16}$ in. Beyond this travel the maximum occurring gear reaction decreases rapidly. The maximum value of the gear reaction is obtained when the value of p in either one of Equations [49] or [51] is equal to unity. In that case they both take the form of Equation [50], $P_m = nF/2$. The value of P_m in this equation depends only on the number of cars n and the retarding force acting on each car. This gear reaction is the same for empty, partially loaded, and fully loaded cars and is equal to one-half the retarding force of the whole train or 562,500 lb. In order to cushion this force a draft gear should have an ultimate yielding resistance which would exceed this figure.

However, it is permissible to reduce this figure by an amount about equal to the initial resistance of the gear. Up to the amount of this latter resistance the gear has a very high spring constant and the corresponding force will be transmitted through the train theoretically at the same speed as through a solid steel bar, or at about 17,000 fps. The effect on the coupler pressures is assumed to be somewhat similar to that when only a part of the full air pressure is admitted to the brake cylinders for a short period, as is done in the new type-AB brake. A coupler force about equal to the amount of the initial gear compression will very rapidly equalize part of the speed difference of the front and rear cars and the final coupler pressure will decrease a corresponding amount. A gear of 100,000-lb initial and 500,000-lb ultimate reaction will thus be approximately equivalent to a gear with no initial and 600,000-lb final reaction. It will be superior in the respect that it keeps the maximum coupler pressure lower.

The car-retardation curve in Fig. 36 shows that the maximum car retardation during an emergency brake application of an empty train will gradually increase with increasing gear travel, at first slowly, reaching the comparatively harmless value of about 11 ft per sec per sec at the standard AAR gear travel of $2^5/_8$ in., then faster and faster until it reaches a theoretically infinite value at a gear travel of $6^5/_{16}$ in. In practice this value will be limited to the retardation produced by the maximum gear reaction of 562,500 lb acting on the mass of one empty car, weighing 50,000 lb (corresponding to a mass of 1550). This retardation amounts to 562,500/1550 = 363 ft per sec per sec, or more than ten times gravity, which of course is very high.

Fig. 37 shows the corresponding curves for fully loaded cars. The minimum required gear capacity increases practically proportionally to the gear travel until the latter reaches the critical value of $1^1/_2$ in. after which the gear capacity remains constant at the value of about 35,000 ft-lb. At a gear travel of 1.2 in. the required gear capacity amounts to about 28,000 ft-lb. At the standard AAR gear travel of $2^5/_8$ in. the required gear capacity amounts of course to the previously given value of about 35,000 ft-lb. The curve showing the gear reaction is similar to that for empty cars, but reaches the same maximum of 562,500 lb at a gear travel of only $1^1/_2$ in. For longer gear travels, the gear reaction decreases rapidly as shown by the curve.

Fig. 37 shows that the maximum occurring car retardation for loaded cars increases at first slowly with increasing gear travel but rises rapidly as the gear travel approaches $1^1/_2$ in. at which point the retardation is equal to the maximum gear reaction of 562,500 lb acting on the mass of the fully-loaded car or 562,500/6500 = 86.5 ft per sec per sec, which retardation is nearly three times the acceleration of gravity and undesirably high.

Fig. 38 shows the superimposed curves of Figs. 36 and 37. The inclined line, representing the minimum gear capacity for intermediate loads, shows that for every gear travel between $1^1/_2$ in. and $6^5/_{16}$ in. there is a partial loading at which the minimum required gear capacity will reach the values indicated by the inclined line. For the standard AAR gear travel of $2^5/_8$ in., the required minimum gear capacity will, for about 43 per cent partial loading, amount to about 60,000 ft-lb.

The maximum gear reaction for intermediate loads, represented in Fig. 38 by the horizontal line at the load of 562,500 lb, shows that for every gear travel between $1^1/_2$ in. and $6^5/_{16}$ in. there is some partial-loading condition at which this reaction will be reached.

The maximum car retardation for intermediate loads is represented in Fig. 38 by an inclined line marked accordingly and extending from the value of 86.5 for fully loaded cars at $1^1/_2$ in. gear travel to the value of 363 ft per sec per sec for empty cars at a gear travel of $6^5/_{16}$ in. It will be noted that the maximum car retardation increases in direct proportion to the gear travel between the previously given values of the latter. Below the gear travel of $1^1/_2$ in. the maximum car retardation drops rapidly.

For the standard AAR gear travel of $2^5/_8$ in. and assuming that the proper gear capacity of 60,000 ft-lb were available, the maximum car acceleration would, for about 43 per cent partial loading, reach the value of 150 ft per sec per sec which approaches five times the acceleration of gravity and is undoubtedly high, even though it would act only for a very short time.

Special Cases of Braking of a Bunched Train

The formulas in Table 5 are based on the action of an elastic bar with both ends free. They apply to trains with fairly uniformly distributed loads and with brakes gradually applying to all cars of the train. The greatest possibly occurring coupler pressure in such trains is expressed by Equation [50].

(*a*) *Partial Brake Application.* With the type-K air brake, the one in general use on American railroads, the brakes apply only to the first 70 or 80 cars, or to about the first half of a long train of 150 cars. The question whether under such partial brake application greater coupler pressures will be produced because of the running in of the unretarded rear end of the train is answered by the previously presented investigation of partial force applications to elastic bars. It is shown that while the maximum occurring coupler pressure may be displaced to any point within the rear half of the train, its magnitude will not increase above the value given by Equation [50] as long as the wave resistance of the train is uniform.

(*b*) *Uneven Load Distribution.* If a group of fully loaded cars is placed at the rear end of a long train of empties and emergency braking is applied from the front end of the train, the coupler pressures will materially increase at the rear end, causing the draft gears to go solid. The wave resistance of this train may reach such high values that the pressure distribution will approach that represented by Case 2*A*, line 2, in Table 3. In that case the coupler pressure increases to four times that given by Equation [50]. The subsequently produced coupler tensions will not be increased above those occurring in case of uniform load distribution, as shown in Table 3, line 5.

If the group of loaded cars were placed at the front end of the train, the condition represented by Case 2*B* would be approached and no increase in the coupler pressures would be produced. The subsequently produced coupler tensions would, however, according to Table 3, line 5, be four times as large as in the case of uniform load distribution, if the draft gears had 100 per cent recoil.

If the group of fully loaded cars were evenly divided between the front and rear ends of the long train of empty cars, the worst conditions would be represented by Case 2*C* and the maximum occurring coupler pressures and tensions might, according to lines 2 and 5 of Table 3, approach twice those in case of uniform load distribution.

These considerations illustrate the importance of a fairly uniform distribution of the load of a train over its whole length and indicate that load concentrations toward the ends of the train are particularly undesirable.

(*B*) Braking of a Stretched Train (With Free Slack)

The free slack between coupled cars is measured by the amount of relative car movement which is obtainable with no change in the position of the various elements which make up the cushioning device.

All new cars, irrespective of draft-gear design, are subject to about the same amount of free slack when standard coupler specifications are followed. On such cars the free slack is about $^7/_8$ in. per car, but it increases as a result of wear in service and permanent deformation of couplers, attachment, and draft gears.

When the *free slack* of a train is taken up under the influence of a brake application, the occurring impacts will close the draft gears to a certain extent, thus producing an additional approach between impacting cars, which is called the *controlled slack*. The *total slack* of a train consists thus of the free and the controlled slack.

If all the slack is controlled, the train is bunched and the preceding analysis applies to the brake action on such a train.

In case the cars are separated by a certain amount of free slack, the principle of the elastic bar should be applied with the understanding that the take-up of the slack will produce car impacts, not expressed in the preceding equations for coupler pressures.

Because of the time interval between the application of the brakes on adjacent cars, the front cars will first begin to retard and the free slack between the cars will gradually be taken up, beginning at the front end of the train. If a train has free slack it will, when subjected to braking from the front end, begin to bunch at that end. The rear end of the bunched portion will always be free and any coupler pressure which reaches the cars at this point will be reflected, changing its sign. The coupler forces actually generated in the bunched portion of the train will be smaller than in a train without free slack, where the coupler forces continually increase even after the wave reaches the free rear end of the train. If the free slack is too great the impact between the bunched portion of the train and the individual cars, separated by free slack, may however become so violent as to cause momentary excessive coupler pressures.

It will be assumed in the following that each car which impacts the bunched part assumes the speed of the latter, without rebound.

Referring to Fig. 39, the circles 1 to 12, inclusive, denote railway cars, counted from the front end of a moving train, which cars are separated by supposedly equal amounts of free slack s_1 per car. The braking forces of F lb per car gradually applied to these cars, beginning at the first car and progressing in the direction of the rising numbers, retard the originally uniform movement of the cars. The horizontal or x-axis represents the time, counted from the application of the brakes to the first car. The vertical or y-axis represents the train slack. The tangent

of the angle which the straight line OA forms with the x-axis is a measure of the brake-propagation speed V_1. The length of the horizontal dashed line between the car number on the vertical axis and the straight line OA represents the delay of brake application to the individual car after the brakes have been applied to the first car.

When the braking force F is applied to the first car of a long train moving at a uniform speed it will subject the car to a retardation a. The movement of the retarded car in relation to the

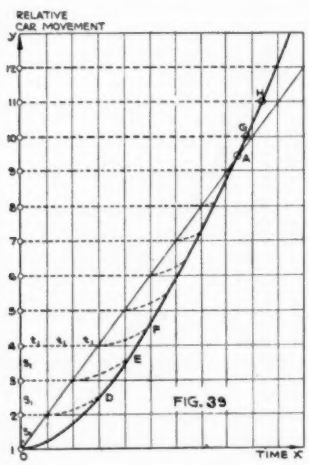


FIG. 39

as yet unretarded part of the train is represented by the curve OD in Fig. 39 and may be expressed by the equation

$$y = (a/2)x^2 \quad \ldots\ldots [62]$$

representing a parabola with its apex in the point O.

The speed lost by the car is expressed by the derivation

$$\frac{dy}{dx} = ax \quad \ldots\ldots [63]$$

After the time t_1, or the interval between the applications of the brake to adjacent cars, the first car will have suffered the loss of speed at_1. From then on both cars will be retarded at the same rate a and will impact at the relative speed

$$c_1 - c_2 = at_1 \quad \ldots\ldots [64]$$

This impact speed is independent of the amount of slack between the cars, provided the slack is sufficient to allow the braking force to apply to the second car before the latter is struck by the first car.

As the draft gears are assumed to have a liberal amount of absorption, any impact of one car with a string of several cars will cause them all to take the same speed and they will move on together in a bunched group. The speed of this group will at any time be represented by the slope of the curved line $ODEFA$ in Fig. 39. This group of retarded cars strikes the also retarded individual cars in points D, E, and F until the curve reaches its point of intersection with the inclined line OA at A, at which point the take-up of the slack catches up with, and passes beyond, the brake application. Beyond that point the group of retarded cars will strike only unretarded ones. The coordinates of the point A are x_1 and y_1. The nature of the curve changes at this point and the part $ODEFA$ for which $x < x_1$ and the part beyond the point A for which $x > x_1$ will therefore be analyzed separately.

(*a*) $x < x_1$. Consider first the part of the braking action represented by the curve $ODEFA$, or before the slack take-up has reached the point of brake application. At any time x during this period, the group of retarded cars will have a total mass of $M(x/t_1)$ and since they have been retarded during the time x by an average force of $^1/_2Fx/t_1$ their average loss in speed c_x can be computed from the momentum equation

$$M\frac{x}{t_1}c_x = {}^1/_2F\frac{x}{t_1}x$$

Therefore, for all values of $x < x_1$

$$c_x = \frac{Fx}{2M} \quad \ldots\ldots [65]$$

The brakes apply to car number $(x/t_1 + 1)$ at the time x, and it begins to retard. Before this car collides with the others, the latter have formed a group moving in unison, and the retardation of all braked cars is uniform. Therefore, the collision takes place at the impact speed c_x given in Equation [65].

At all times the braked cars have suffered an average loss of speed of c_x. The corresponding loss in travel of all braked cars during any time element would therefore be equal to the number of cars braked during time element times their average loss in speed c_x times the duration of the time interval.

Let Y = loss in travel of all braked cars at the time x. Then

$$\frac{dY}{dx}dx = \frac{x}{t_1}c_x dx = \frac{Fx^2}{2Mt_1}dx$$

or after integration

$$Y = \frac{F}{6Mt_1}x^3 + \text{constant}$$

The integration constant is 0 since for $x = 0$, Y must also be equal to zero. Therefore

$$Y = \frac{F}{6Mt_1}x^3 \quad \ldots\ldots [66]$$

The coordinates x_1, y_1 of the point A should satisfy Equation [66] and also be located on line OA, Fig. 39. For this point therefore

$$Y = \frac{F}{6Mt_1}x_1^3 \text{ and } \frac{y_1}{s_1} = \frac{x_1}{t_1} \text{ or } y_1 = x_1\frac{s_1}{t_1}$$

Y can also be expressed as

$$Y = \frac{y_1}{2}\frac{y_1}{s_1} = \frac{y_1^2}{2s_1}$$

From the preceding three equations x_1 and y_1 may be determined. Accordingly

$$x_1 = \frac{3Ms_1}{Ft_1} \text{ and } y_1 = \frac{3Ms_1^2}{Ft_1^2} \quad \ldots\ldots [67]$$

The number n_1 of the car at which point A is located may be computed as

$$n_1 = \frac{y_1}{s_1} + 1 = \frac{3Ms_1}{Ft_1^2} + 1 \quad \ldots\ldots [68]$$

(*b*) $x > x_1$. Beyond the point A in Fig. 39, the coordinates of which are x_1 and y_1, the group of retarded cars will strike only unretarded cars. Assume that the point A has been passed and that the bunched group, consisting of cars 1 to 10 inclusive, is moving along the line GH striking the, as yet, unretarded car 11

at the point H on the curve x, y. The equation for the curve x, y beyond the point A can be determined as follows:

The loss in momentum of the bunched cars at the time x can be expressed either as the product of the mass of the group of cars by their loss in speed, or as the product of the average retarding force by the time during which it has been acting. Hence

$$M \frac{y}{s_1} c_x = F \frac{x}{t_1} \frac{x}{2}$$

which equation, since $c_x = dy/dx$, may be written as

$$y \frac{dy}{dx} = \frac{F}{M} \frac{s_1}{2t_1} x^2 \quad \text{.....[69]}$$

which becomes, after integration

$$y^2 = \frac{F}{M} \frac{s_1}{t_1} \frac{x^3}{3} + \text{constant}$$

By substituting in this equation, the values of x_1 and y_1 from Equation [67], the integration constant is found to be zero. Therefore

$$\left.\begin{aligned} y &= \sqrt{\left(\frac{Fs_1}{Mt_1} \frac{x^3}{3}\right)} \\ \text{or} \qquad x &= \sqrt[3]{\left(\frac{3Mt_1y^2}{Fs_1}\right)} \end{aligned}\right\} \quad \text{.....[70]}$$

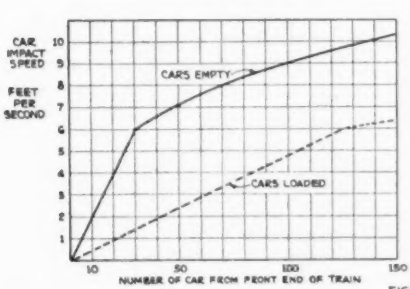


FIG. 40

Substituting the value of x from Equation [70] in Equation [69] and solving for dy/dx

$$c_x = \frac{dy}{dx} = \sqrt[3]{\left(\frac{9}{8} \frac{Fs_1y}{Mt_1}\right)} \quad \text{.....[71]}$$

For the car number n the value of $y = (n - 1)s_1$ and therefore

$$c_x = c_{max} = \sqrt[3]{\left(\frac{9}{8} \frac{Fs_1^2(n-1)}{Mt_1}\right)} \quad \text{.....[72]}$$

or if L_1/V_1 is substituted for t_1 in Equation [72] and it is solved for s_1

$$s_1 = \sqrt{\left(\frac{8ML_1}{9FV_1(n-1)} c^3_{max}\right)} \quad \text{.....[73]}$$

If the slack s_1 per car is known, the corresponding maximum car impact speed for a train of a certain number of cars of known mass can be computed by using Equation [72]. If the permissible car impact speed is known, the maximum permissible slack can be computed from Equation [73].

Example. Assuming a draft-gear recoil of not over 50 per cent, it is required to determine the impact speeds between adjacent cars in a 150-car train, each car weighing 50,000 lb empty and 210,000 lb loaded, and having an average of 4 in. of slack between adjacent cars. It is given that the emergency braking rate is 7500 lb and that it is applied at the front end of the train at a propagation speed of 600 fps. The length of cars between pulling faces of the couplers is 50 ft.

From the preceding discussion it is found that $M = 1550$ for empty cars and 6550 for loaded cars, $L = 50$ ft, $F = 7500$ lb per car, $V_1 = 600$ fps, $t_1 = 0.0833$ sec, $s_1 = 0.333$ ft, and $F/M = 4.84$ for empty cars and 1.154 for loaded cars.

Referring to Fig. 39, the coordinates x_1 and y_1 for point A are given by Equation [67]. The value of $x_1 = 2.48$ sec for empty cars and 10.4 sec for loaded cars. According to Equation [68], $n_1 = 31$ for empty cars and 126 for loaded cars.

The corresponding value of the impact speed is computed from Equation [65] for all values of $x < x_1$. For $x = x_1$, the impact speed $c_x = (F/2M)x_1 = 6$ fps for loaded as well as for empty cars. The impact speed gradually increases from zero to this value, which is reached at car n_1.

For values of $x > x_1$, the impact speed is given by Equation [71] which shows that c_x is a parabolic curve, the maximum value of which is expressed by Equation [72]. Therefore, for $n = 150$

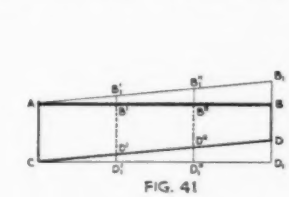
$c_{max} = 1.938 \sqrt[3]{(n-1)} = 10.3$ fps for empty cars, and

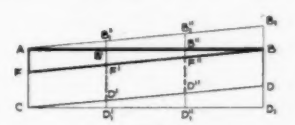
$c_{max} = 1.200 \sqrt[3]{(n-1)} = 6.38$ fps for loaded cars.

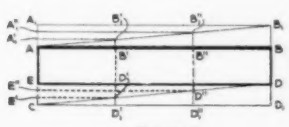
The values of c_{max} for all values of n up to 150 are plotted for empty and loaded cars in Fig. 40.

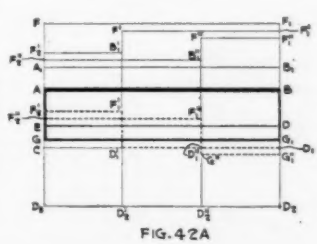
(C) Starting of a Stretched Train

The action of a locomotive in starting a stretched train corresponds to that of a constant pulling force P_L acting at the end of an elastic bar. The frictional resistance, opposing the motion of each car, corresponds to a gradually applied and uniformly distributed force, acting in the direction opposite to the motion and applying to each car at the moment the car begins to move. The rate of force application is therefore equal to that of pressure transmission, and the propagation ratio of the frictional force is equal to unity. The action of these forces is represented by Case 2, as given in Table 3.


FIG. 41


FIG. 42


FIG. 41A


FIG. 42A

Referring to Fig. 41, the pressure diagram created by the locomotive drawbar pull P_L would, acting alone and after passing through the intermediate shapes $AB'D_1'C$ and $AB''D_1''C$, take the shape of the rectangle ABD_1C. The pressure diagram created by the friction forces, acting alone, would, after passing through the intermediate shapes $AB'B_1'$ and $AB''B_1''$, take the shape of the triangle ABB_1. Superimposing these two diagrams, the resultant diagram, after passing through the intermediate shapes

$AB'D'C$ and $AB''D''C$, takes the shape of the quadrangle $ABDC$ at the end of the first period.

Referring to Fig. 41*A*, the speed diagram created by the drawbar pull P_L, acting alone, would, after passing through the intermediate shapes $AB'D_1'C$ and $AB''D_1''C$, take the shape of the rectangle ABD_1C. The speed diagram created by the friction forces, would, after passing through the intermediate shapes $AB'B_1'A_1'$ and $AB''B_1''A_1''$, take the shape of the rectangle ABB_1A_1. Superimposing these two diagrams, the resulting speed diagram would, after passing through the intermediate shapes $AB'D'E'$ and $AB''D''E''$, take the shape of the rectangle $ABDE$ at the end of the first period.

If the locomotive drawbar pull P_L were acting alone, the pressure diagram, Fig. 42 (a reproduction of the pressure diagram in Fig. 41), would, at the beginning of the second period, have the shape of the rectangle ABD_1C and would, after passing through the intermediate shapes $AB''D_1''C$ and $AB'D_1'C$, vanish in the vertical AC at the end of the period.

If the frictional forces were acting alone, the pressure diagram ABB_1 would, after passing through the intermediate shapes $BF''B_1''A$ and $BF'B_1'A$, take the shape of the triangle ABF at the end of the second period. Superimposing these diagrams, the resultant pressure diagram will, after passing through the intermediate shapes $ABF''D''C$ and $ABF'D'C$, take the shape of the triangle ABF, in which BF is parallel with AB_1.

Referring to Fig. 42*A*, the speed diagram which would have been created by the locomotive drawbar pull P_L acting alone, and which at the end of the first period was represented by the

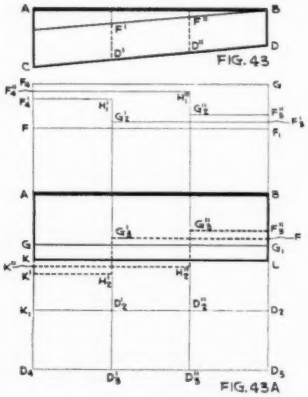


FIG. 43

FIG. 43A

rectangle ABD_1C taken from Fig. 41*A*, would, after passing through intermediate shapes $ABD_2D_2''D_1''C$ and $ABD_2D_2'D_1'C$, take the shape of the rectangle ABD_2D_2 at the end of the second period, CD_2 being equal to AC. The frictional forces, acting alone, would, at the beginning of the second period, have created the speed diagram AA_1B_1B taken from Fig. 41*A*. This diagram, after passing through the intermediate shapes $ABF_1''F''B_1''F_2''$ and $ABF_1'F'B_1'F_2'$, would, at the end of the second period, have taken the shape of the rectangle ABF_1F. Superimposing the speed diagrams created by the drawbar pull and the frictional forces, it will be seen that the resultant speed diagram, which at the beginning of the second period had the form $ABDE$, takes the shape of the rectangle ABG_1G at the end of the second period after passing through the intermediate shapes $ABG_1''G''F_1''F_2''$ and $ABD_1D_1'F_1'F_2'$.

During the third period the pressure diagram shown in Fig. 43 will, after passing back through the intermediate shapes $ABF'D'C$ and $ABF''D''C$, again take the shape of the quadrangle $ABDC$. Period 4 will be the same as Period 2, Period 5 the same as Period 3, and so on.

Referring to Fig. 43*A*, the speed diagram which would have been created by the locomotive drawbar pull acting alone had, at the beginning of the third period, the shape of the rectangle ABD_2K_1. After passing through the intermediate shapes $ABD_2D_2'D_3'D_4$ and $ABD_2D_2''D_3''D_4$, this diagram would have taken the shape of the rectangle ABD_3D_4 at the end of the third period, D_2D_3 being equal to AC in Fig. 42*A*.

The rectangle ABF_1F in Fig. 43*A* which has been taken from Fig. 42*A*, represents the speed diagram which, at the end of the second period, would have been created by the frictional forces, acting alone. These forces would, during the third period and still acting alone, produce the speed diagrams $ABF_3'G_2'H_1'F_4'$ and $ABF_3''G_2''H_1''F_4''$, followed by the rectangular shaped diagram $ABGF_4$ at the end of the third period. Superimposing the speed diagrams in Fig. 43*A*, which would have been created by the drawbar pull and the frictional forces, the resultant speed diagram, which at the beginning of the third period had the shape ABG_1G, taken from Fig. 42*A*, will take the shape of the rectangle $ABLK$ at the end of the third period after passing through the intermediate shapes $ABF_5'G_3'H_2'K'$ and $ABF_5''G_3''H_2''K''$.

Referring to Fig. 42, AC represents the drawbar pull P_L of the locomotive. The vertical BB_1 is, according to Table 3, Case 2, for an elastic bar, equal to $^1/_2f_1L$. By substituting the corresponding constants for a train of n cars, each car being subjected to a frictional resistance of f_2 lb, it is found that this force is equal to $^1/_2f_2n$. The other forces in the diagram can be computed from the geometrical relationship to these two quantities. The force BD, acting at the end B of the bar, at the end of the first period, can therefore be expressed as

$$P = P_L - {}^1/_2f_2n \quad \ldots\ldots\ldots\ldots\ldots\ldots [74]$$

Referring to Fig. 41*A*, the wave resistance of the train should be known in order to compute the car speed corresponding to the pressures in the preceding paragraph. Then the car speed AC in Fig. 41*A* can, according to Equation [2], be expressed as

$$c = f/\sqrt{(mk)} = P_L/R_w = P_Lq/\sqrt{(CM)} \quad \ldots\ldots [75]$$

The starting speed of the last car, represented by the vertical BD in Fig. 41*A*, can be expressed as

$$c_n = \frac{1}{R_w}(P_L - {}^1/_2f_2n) = \frac{q}{\sqrt{(CM)}}(P_L - {}^1/_2f_2n) \ldots [76]$$

The other forces and speeds in all these diagrams can be computed from their geometrical relationship to those given in the preceding equations.

Example. A locomotive, exerting a drawbar pull P_L of 80,000 lb is hauling a train of 150 cars which weigh 50,000 lb empty and 210,000 lb loaded. The frictional resistance equals 18 lb per ton. Therefore, $f_2 = 450$ lb for an empty car and 1890 lb for a loaded car. The draft-gear travel is 0.22 ft, and the draft-gear capacity is 18,000 ft-lb.

It is desired to know the speed which the train has acquired at the time the tension and speed waves reach the end of the train and also the force which starts the last car.

The mass M of an empty car = 1550 lb, while for a loaded car M = 6500. The wave resistance of the train $R_w = \frac{\sqrt{(18{,}000 \times 1550)}}{0.22} = 24{,}000$ lb per ft per sec for an empty train and $= \frac{\sqrt{(18{,}000 \times 6500)}}{0.22} = 49{,}150$ lb per ft per sec for a loaded train.

The pull starting the last car, represented by the vertical BD in Fig. 42, is, according to Equation [74], $(P_L - {}^1/_2f_2n) = 80{,}000 - {}^1/_2 \times 450 \times 150 = 46{,}250$ lb for a train of empty cars. For a train of loaded cars $(P_L - {}^1/_2f_2n) = 80{,}000 - {}^1/_2 \times 1890 \times$

150 = —62,000 lb, the negative value showing that the drawbar pull of 80,000 lb is insufficient to start the stretched loaded train.

The train speed acquired at the time the tension wave reaches the last car of the empty train is $c_n = (1/R_w)(P_L - {}^1/_2 f_2 n) = 46{,}250/24{,}000 = 1.93$ fps.

(*D*) Starting of a Bunched Train (With Free Slack)

The method used in analyzing the braking of a train with free slack can also be applied to this problem.

Any train with draft gears of low or moderate recoil will form a group of all started cars, and the latter will be assumed to move as a unit as they pick up, one by one, the cars which are standing still. The work performed by the locomotive as measured at the drawbar is, during the starting period of the train, mainly used (1) to accelerate the cars from a standstill, and (2) to overcome the frictional resistance of the cars during this period.

Assume that the first $(n - 1)$ cars have attained a uniform speed c_n at the time the last car is suddenly brought up to this speed from a standstill. The draft gears between the last two cars will in that case be subjected to an impact not exceeding that of a car moving at the speed c_n and suddenly stopped against a bumping post, the impact being taken up by two series-connected gears.

Therefore, the speed c_n can with fair accuracy be determined from the equation

$$P_L s_1 n = \frac{(n-1)M}{2} c_n^2 + \frac{(n-1)n}{2} s_1 f_2 \quad \ldots\ldots [77]$$

Solving this equation for c_n

$$c_n = \sqrt{\left[\frac{s_1}{M}\left(2P_L \frac{n}{n-1} - nf_2\right)\right]} \quad \ldots\ldots\ldots [78]$$

The minimum capacity of the draft gear C_{min} which will take up the impact of a car with a speed of c_n, two gears being connected in series, is expressed by the equation

$$2C_{min} = \frac{M}{2} c_n^2 + f_2 s_1$$

$$\text{or} \qquad C_{min} = \frac{s_1}{2}\left(\frac{n}{n-1} P_L - f_2 \frac{n-2}{2}\right) \quad \ldots\ldots [79]$$

Example. Assume that the train in the preceding example had a total slack of 100 ft distributed over the 150 cars, or $s_1 = 0.667$ ft. The speed at which the stretched train would pick up the last car, expressed by Equation [78], is

$$c_n = \sqrt{\left[\frac{0.667}{1550}\left(2 \times 80{,}000 \times \frac{150}{149} - 150 \times 450\right)\right]}$$

= 6.35 fps for the train of empty cars.

For loaded cars, this formula gives a negative value, indicating that the locomotive is not powerful enough to start such a train, regardless of the amount of free slack. In practice an increase in slack will decrease the frictional resistance of the cars and thus facilitate starting.

The minimum draft-gear capacity required to prevent the gears from going solid when starting the empty train with 100 ft of slack is, according to Equation [79]

$$C_{min} = \frac{0.667}{2}\left(\frac{149}{150} \times 80{,}000 - 450 \times \frac{148}{2}\right) = 15{,}700 \text{ ft-lb}$$

Acknowledgments

The author wishes to express his gratitude to the Edgewater Steel Company for sponsoring this investigation, to L. H. Donnell and L. H. Fry for helpful advice, and to J. H. Vogelsang for valuable assistance in construction of diagrams and checking of calculations.

Nomenclature

a, a_1, a_2, a_3, a_4 = acceleration or retardation of particles of an elastic bar, ft per sec per sec; = acceleration or retardation of cars in a long train in ft per sec per sec

C = capacity of one draft gear (plus that of one-half car body), ft-lb

$c, c_1, c_2, c', c_n, c_x$ = speed, or loss of speed, of particles in an elastic bar, fps; = speed or loss of speed of cars in a long train, fps; = relative impact speed between cars, fps

F = retarding force per car during emergency braking, lb

f = external force applied to an elastic bar, lb

f', f = longitudinal compression or tension occurring in an elastic bar

f_1 = external force applied per foot of an elastic bar, lb; = pressure transmitted in an elastic bar after a change in wave resistance, lb

f_2 = average frictional resistance per car during the starting of a long train, lb

k, k_1 = stiffness constant or modulus of elasticity of the material of an elastic bar in lb per sq in. multiplied by its cross-sectional area, lb; = $(L_1/q^2)C$ = stiffness constant of a long train, corresponding to that of an elastic bar and thus equal to the force required to stretch it to twice its length, lb

L = length of an elastic bar, ft

L_1 = length of one car between pulling faces of couplers, ft

l = distance from the front end of an elastic bar at which the pressure P occurs, ft; = the length to which a uniformly distributed external force applies, ft

l_1 = distance from the front end of an elastic bar at which the pressure P_1 occurs, ft (Case 1, Period 1)

l_2 = distance from the front end of an elastic bar at which the pressure P_2 occurs, ft (Case 1, Period 2)

l_3 = distance from the front end of an elastic bar at which the pressure P_3 occurs, ft (Case 1, Period 3)

l_4 = length of the greatest part of an elastic bar to which a uniformly distributed external force may apply and still produce the maximum pressure which the force, acting over the length l_4 can possibly create, regardless of the length of the bar, ft

l_6 = distance from front end of an elastic bar, at which the pressure P_6 occurs, ft (Case 3, Period 2)

M = mass per car

m, m_1 = mass per ft length of an elastic bar, or mass per ft length of a long train $(= M/L_1)$

n = number of cars in train

n_1 = number of cars from front of train at which the slack take-up reaches and passes beyond the brake application

P = longitudinal compression or tension in an elastic bar, lb
P = coupler pressure or tension in a long train, lb
P_1 = maximum occurring internal bar pressure at end of Case 1, Period 1, lb
P_2 = maximum occurring internal bar pressure at end of Case 1, Period 2, lb
P_3 = maximum occurring internal bar pressure at end of Case 1, Period 3, lb
P_4 = maximum internal bar pressure produced by external forces, applied over part l_4 of the bar (for $p > 1$), lb
P_5 = maximum obtainable internal bar pressure with uniformly applied external forces in bar with both ends free, lb
P_6 = maximum internal bar pressure at end of Case 3, Period 2, lb
P_m = maximum internal pressure in an elastic bar, lb
P_L = drawbar-pull of a locomotive, lb
p = propagation ratio = V/V_1
Q_1, Q_2 = momentum of moving particles of a bar
q = travel of a draft gear plus compression of one-half of the car body, ft
R_w, R_{wl} = wave resistance of a bar, lb per ft per sec
R_w, R_{wl} = wave resistance of a long train, lb per ft per sec
r = $\sqrt{(m_1k_1)}/\sqrt{(mk)}$ ratio of the wave resistances of two parts of an elastic bar
s_1 = average slack per car, ft
t, t_1, t_2 = time intervals, sec
V, V' = speed of pressure transmission in elastic bars, ft per sec, or
V, V' = speed of pressure transmission in long trains, ft per sec
V_1 = rate of external force application in an elastic bar, ft per sec
V_1 = rate of brake application in a long train, ft per sec
x = time elapsed since brakes applied on car 1, sec
x_1 = value of x when brakes apply on car n_1, sec
Y = loss in travel of all braked cars at time x, ft
y = slack take-up at the time x, ft
y_1 = value of y when brakes apply on car n_1

Locomotive Tractive Effort in Relation to Speed and Steam Supply

By E. G. YOUNG,[1] URBANA, ILL., AND C. P. PEI,[2] CHAMPAIGN, ILL.

The problem of estimating with reasonable accuracy the amount of tractive force exerted by a locomotive at running speeds enters every tonnage rating, economy study, and locomotive-operating investigation. Its importance is recognized, and many investigators have approached a rational solution by determining the hourly steam supply and the steam consumption per horsepower-hour, and from these the horsepower and tractive force. The steam rates assumed have been generally correlated with the speed only. Indirectly this requires the defining of the resulting speed-tractive-effort relation as "maximum tractive effort," "all-day tractive effort," or similar terms. This paper points out that the variation of the steam rate with the cut-off is of equal importance, but that a chart showing the variation of steam rate with the two factors, speed and cut-off cannot be used directly to obtain horsepower without an additional process. A more direct attack is made by (*a*) estimating the actual weight of steam used per revolution in cylinders of various sizes, with varying speeds, cut-offs, and working pressure; and (*b*) by correlating the speed, cut-off, and mean effective pressure, from which the tractive force corresponding to the cylinder horsepower is directly calculated. Each relation calculated gives the four values of speed, tractive effort, cut-off, and steam consumption for a given operating condition, and the calculation of a number of selected points results in a chart presenting a set of curves showing the tractive effort which may be expected at various speeds for specified conditions of boiler output and cut-off. This type of chart permits the user to see the economy to be expected at any condition within the probable working range of the locomotive and to determine the validity of any single speed-pull curve, involving a specified program of cut-off and boiler output for each speed.

THE current interest in the performance and economy of the steam locomotive has focused attention on the old problem of the relation between speed and tractive effort, and has resulted in some very interesting and important contributions to its clarification. In previous discussions of the problem, two general forms of solution have been offered, of which one may be considered as purely empirical, and the other semi-rational.

The empirical solution is found in the various forms of speed-factor tables or curves. These, in general, express the tractive effort available at any speed in terms of a fraction of the rated tractive effort; such tables or curves are obtained by taking the averages of the test data for a number of locomotives, either singly or arranged in groups. These averages are valid, as a means of generalization, only when (1) the conditions of the tests are similar and rigorously defined, and (2) the average results are derived from tests of similar locomotives.

It follows that this generalization is only valid as a means of predicting the performance of another locomotive when the conditions under which it is to be operated correspond with the test conditions from which the average relation between speed and tractive effort was obtained. So far as the authors know, no such restrictions have ever been placed upon the use of any speed-factor method. In general, the relation is developed as one between the tractive effort and speed only, ignoring other factors that affect the tractive-effort development.

The type of solution that has been termed semi-rational usually proceeds on the basis of determining a horsepower output by dividing an estimated boiler evaporation by an anticipated steam rate or water rate, and then calculating the tractive effort from its relation to the horsepower and speed. An assumed firing rate, or boiler output, or tractive effort at a given speed may be the starting point from which the other quantities are determined. This method is desirable from the standpoint of simplicity, but the use of an anticipated water rate is an extremely indirect process and a very complicated one if the variation of the water rate with both the speed and cut-off, a fundamental fact of locomotive performance, is taken into account. Most of the semi-rational processes have made the water rate vary with the speed only.

In most of the methods of estimating tractive effort, either empirical or semi-rational, the final relation between the tractive effort and speed is represented by a speed-pull curve. This representation of tractive effort by a single speed-pull curve implies that, at a given speed, the locomotive can be operated to exert the amount of tractive effort corresponding to that speed as defined by the speed-pull relation, and conversely, for a given amount of tractive effort developed, the locomotive can attain a speed corresponding to that tractive effort. A further implication is that there is only one tractive effort obtainable at a given speed, and only one speed attainable at a given tractive effort. The absurdity of the latter assumption is clearly indicated in some of the recent investigations attempting to define the tractive effort as *maximum* tractive effort, *performance* tractive effort, or *average* tractive effort. These arbitrary definitions are, at the best, merely expedients of expression but do not possess significant meanings. The reasons are quite apparent: For a given locomotive, there are at least three sets

[1] Research Professor, University of Illinois. Mem. A.S.M.E. Mr. Young was graduated from the University of Illinois in 1913, and received his M.S. degree in 1916 and M.E. degree in 1923. Previous to his present connection he held positions with the American Locomotive Company and the Chinese Government Railways.

[2] Engineer, New York Air Brake Company, Watertown, N. Y. Mem. A.S.M.E. Mr. Pei was graduated with the B.S. degree from Tangshan Engineering College, China, in 1921. In 1923 he received the M.S. degree and in 1932 the M.E. (Professional) degree from the University of Illinois. He was formerly associated with the mechanical departments of the Nickel Plate Railroad, the Chicago, Milwaukee, St. Paul & Pacific Railroad, and the Northern Pacific Railroad.

Contributed by the Railroad Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of *maximum* tractive efforts, (1) the one limited by the adhesive weight of the locomotive, (2) the one based on the maximum cylinder output, and (3) still another one based on the maximum boiler output. In so far as the *performance* tractive effort is concerned, it is based upon the premise that the locomotive is operated in daily service with little variation so that a representative relation between the speed and tractive effort may be established for that particular mode of operation. The *average* tractive effort is really a mathematical quantity and it is useful only when the deviation of the actual from the average tractive effort is within such limits as to give the average tractive effort a physical significance.

A single relation between the tractive effort and speed for a given locomotive is thus an inadequate representation of facts. These facts can only be represented by taking the major variables into account and establishing a series of speed-pull curves on bases that can be rigorously defined. The variables for a given case may be separated into two distinct classes, which, for the present purpose, may be called the *physical factors* and the *operating factors*.

When a train is started over a division, the work which the locomotive must do at every point is already fixed by the tonnage and the inherent resistances of the train, and the speed requirements as known from schedules and train orders, by the profile of the road, and also the weather conditions. The locomotive assigned to the run will presumably be able to meet the requirements imposed upon it, as far as the capacity is concerned, at least, and if the tonnage ratings are well made, with reasonable economy as well. To the physical factors imposed by the train, track, and weather, may be added the limitations relating to the locomotive itself, its dimensions, proportions, and weight. When the crew gets aboard, they have under their control only a certain latitude in speed variations, the throttle opening, the cut-off, and the firing rate. Eliminating the practice of partial throttle operation, because of its inherent inefficiency, there remain only four operating variables, namely, speed, cut-off, firing rate (or steam supply), and the tractive effort. Of these four factors, fixing any two determines the other two, and consequently the entire operation of the locomotive, and therein lies the skill of the engine crew. Any two of the four operating factors, such as firing rate and cut-off, may be shown (as in the following paragraphs) to be dependent, if the others are assumed as independent. Hence, the tractive effort-speed relation must be defined by a family of curves, rather than a single relation.

The firing rate and cut-off are not independent factors because the two are related to each other through the total boiler output determined by the firing rate on the one hand, and the total steam requirement for the cylinders determined by the cut-off and speed on the other hand. Therefore, it follows that, at a given speed, the cut-off at which the locomotive may be operated is determined by the available steam supply, which is, of course, determined by the firing rate. Conversely, the firing rate is determined by the steam demand of the cylinders as determined by the cut-off and speed. This leads to the conclusion that the relation between tractive effort and speed is specifically defined only when qualified by the operating factors of firing rate or cut-off, or the combination of both.

Since the firing rate and cut-off are the only factors under direct control, either of these may be selected as the starting point in a performance analysis. It is the purpose of this paper to consider the consumption of steam in locomotive cylinders at various speeds and cut-offs, thus determining the steam requirement. No attempt will be made to relate the steam supply to the firing rate since this relationship rightfully belongs to the study of boiler performance. With the determination of steam requirement and assuming that it can be met by the steam supply from the boiler, it remains to obtain the correlation between the steam supply and the mean effective pressure, from which the cylinder tractive effort is calculated. The latter is a physically non-existent quantity, but it is extremely useful as a basis of comparison. The cylinder tractive effort may be defined in relation to the proportion of the locomotive by the familiar expression

$$T = p\frac{d^2s}{D} \qquad [1]$$

where T = cylinder tractive effort, lb; p = mean effective pressure, lb per sq in.; d = diameter of cylinder, in.; s = length of stroke, in.; D = diameter of driving wheels, in. The quantity d^2s/D is constant for any given locomotive and will be designated as Z.

The cylinder horsepower may also be calculated from the mean effective pressure as follows:

$$h = p\left(\frac{4as}{12 \times 33{,}000}\right)N \qquad [2]$$

where h = cylinder horsepower; a = area of piston, sq in.; and N = speed, rpm. The second factor on the right-hand side of Equation [2] is constant for any given locomotive and will be designated as Y. Equations [1] and [2] apply to conventional two-cylinder simple locomotives.

Another useful relation expresses horsepower in terms of T and V, where V is the speed in miles per hour:

$$\text{Since} \quad V = \frac{60ND\pi}{12 \times 5280} \quad \text{and} \quad a = \frac{d^2\pi}{4}$$

$$h = p\left(\frac{12 \times 88V}{D\pi}\right)\left(\frac{d^2\pi s}{12 \times 33{,}000}\right) = \left(\frac{pd^2s}{D}\right)\left(\frac{V}{375}\right)$$

$$h = \frac{TV}{375} = \frac{pZV}{375} \qquad [3]$$

Many methods of making tractive-effort estimates have been proposed which depend on the relation

$$h = E/W \qquad [4]$$

where E = hourly evaporation, lb, and W = water rate in lb per ihp-hr. This is, of course, a true relation, but in view of the difficulty of estimating W, the process is extremely indirect. The fact is well known that at starting, with low speed and maximum cut-off, the water rate is high, of the order of 30 lb; also that at high speed and short cut-off, minimum values are obtained, of the order of 15 lb or lower. It is evident that in speeding up a train, the water rate passes through this entire range, and several investigators have correlated water rate with speed. This process, however, is inadequate, since at the same speed, operation at a considerable range of cut-offs is possible, depending on the steam supply; but the cut-off is also a factor in determining water rate, of equal importance with the speed. Sufficient data are now available so that a representative diagram can be shown for water rates for locomotives working at about 200 lb per sq in. boiler pressure. When it is drawn as a surface, relating the water rate in pounds per indicated horsepower-hour, speed in revolutions per minute, and cut-off in per cent of stroke, the shape is as shown in Fig. 1. Water-rate curves are usually shown plotted against speed, as represented by the lighter lines in Fig. 2. The group of points through which various lines of equal cut-offs are drawn in Fig. 2 have been in some investigations merely enclosed by an area of steam performance, and in other cases, a still less valid representation has been made in which all values for a given speed are averaged and the average

points connected. It is obvious that the average relation will depend more on the number of varying combinations of speed and cut-off than upon the specific results.

If a relation between speed and water rate, independent of cut-off, is assumed, such a relation will cut across the curves in Fig. 2, as shown by the heavy line *XY*. Many such relations have been assumed, each correct in itself, provided the locomotive is operated through the program of combinations of speed and cut-off upon which the water-rate relation is based. But the forecasting of such a program as a generalization can only be a matter of opinion, and the actual results in any given case will be closely related to the method of operation governed by the variations in operating conditions. The suggestion of a definite and precise correlation between speed and cut-off is open to the same criticism as applied to the single-line representation between speed and tractive-effort relationship.

If any fixed curve *XY* in Fig. 2 is to be discounted as an unsatisfactory generalization, the scheme of computing the tractive effort from the water rate must also be rejected, as the general relation shown by the light lines, Fig. 2, cannot be used directly. For example, it is desired to calculate the tractive effort of a locomotive of known proportions at a given speed; let the conditions be defined by assuming a cut-off. From Fig. 2, a water

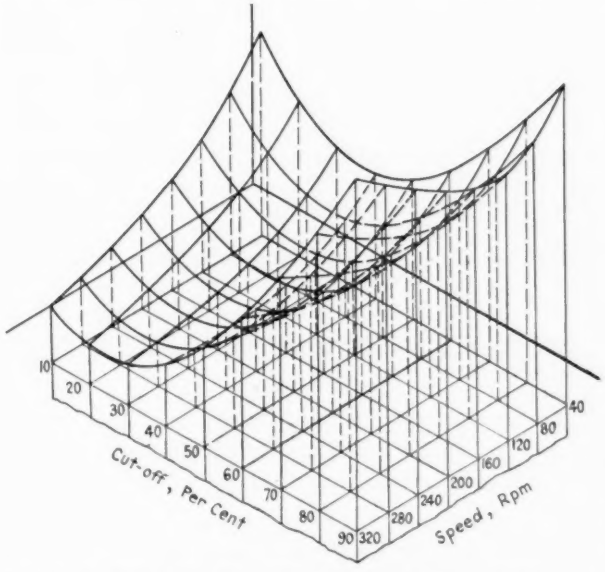


Fig. 1 Water-Rate Speed-Cut-Off Diagram Drawn as a Surface

rate may be assumed accordingly, but an additional process must be provided before the horsepower or tractive effort can be determined. Similarly, if a steam supply is assumed and the steam available per revolution is determined, but without the additional process, horsepower still remains unknown.

The proposed method is a more direct one, by means of which may be calculated:

(*a*) The total amount of steam used per hour when the cylinder dimensions, working pressure, speed, and cut-off are known or assumed.

(*b*) The cut-off at which the locomotive may be operated when the pressure, cylinder dimensions, speed, and steam supply are known.

(*c*) The relation between speed, cut-off, and mean effective pressure as a fraction of the admission pressure, so stated that with any two quantities given, the third one may be determined.

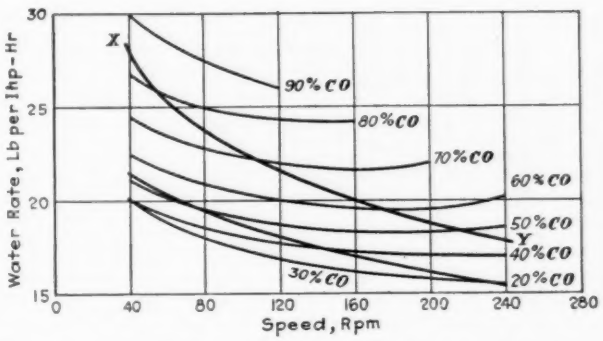


Fig. 2 Locomotive Water-Rate Curves. Line *XY* Represents Relation Between Speed and Water Rate, Independent of Cut-Off

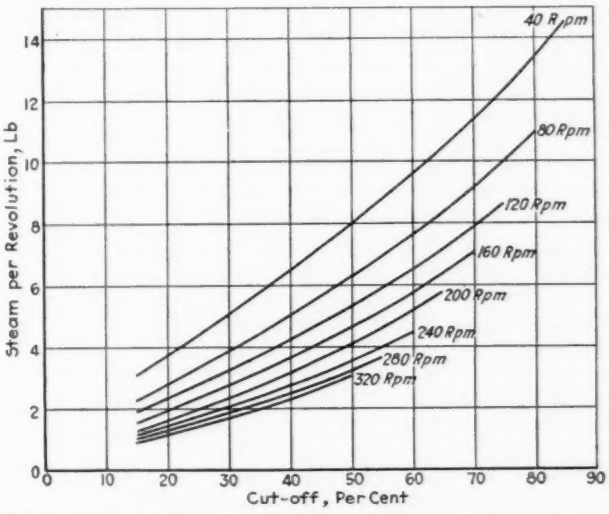


Fig. 3 Relation Between Steam Consumption per Revolution, Cut-Off, and Rpm for Two Cylinders Each With a Volume of 10.45 Cu Ft. Working Pressure 205 Lb per Sq In.

The process of estimating the tractive effort as applied to a locomotive of known dimensions would be (1) assume the speed and cut-off; (2) determine the steam used per revolution when the cylinder dimensions, working pressure, speed, and cut-off are known or assumed as in (*a*); (3) determine the steam required per hour; (4) determine the admission pressure by finding the probable loss between the boiler and the cylinders; and (5) find the mean effective pressure from (4) and (*c*) as given previously.

If, in place of assuming the cut-off, the steam supply is assumed, the process is (1) assume speed and steam supply per hour; (2) find the steam per revolution from the hourly supply and speed; (3) find the cut-off which may be used with this amount of steam per revolution from (*b*) as given previously; (4) determine the admission pressure as in (4) of the preceding paragraph; and (5) find the mean effective pressure as in (5) of the preceding paragraph.

The development of the process depends entirely on specific information obtained from the laboratory-test results, eliminating the uncertainties resulting from average steam output, average speed, average tractive effort, and average water rate usually obtained in road tests. The laboratory data are by no means as complete as might be desired, and in many cases inconsistencies make the determination of certain quantities most difficult. However, enough data are available in published

TABLE 1 STEAM PER REVOLUTION FOR VARIOUS LOCOMOTIVES

Rpm	Locomotive (cyl. vol., press.)	Cut-off 20% Lb	20% Ratio[a]	30% Lb	30% Ratio	40% Lb	40% Ratio	50% Lb	50% Ratio	60% Lb	60% Ratio	70% Lb	70% Ratio	80% Lb	80% Ratio	90% Lb	90% Ratio
40	I-1s (14.6, 250)	6.6	1.69	8.6	1.65	10.7	1.62	13.0	1.60	15.6	1.61	...	...	...	...	...	...
	L-1s and K-4s (10.45, 205)	3.9	1.00	5.2	1.00	6.6	1.00	8.1	1.00	9.7	1.00	...	...	...	...	15.8	1.00
	H-8sb (8.5, 205)	3.4	0.87	4.7	0.90	5.2	0.79	6.4	0.79	7.6	0.78	...	...	...	...	9.3	0.59
	Mo. Pac. 1690 (8.5, 200)	...	...	...	...	...	...	...	...	...	...	...	...	...	...	12.0	0.76
80	Baldwin 60,000 (11.7, 350)	...	...	...	...	...	...	8.5	1.37	10.8	1.44	13.4	...	...	...	20.2	1.68
	Baldwin 60,000 (11.7, 250)	...	...	...	...	...	...	...	...	...	...	9.2	...	...	...	14.0	1.17
	I-1s	4.3	1.59	6.6	1.58	7.6	1.52	9.5	1.53	...	...	...	...	...	...	...	...
	L-1s and K-4s	2.7	1.00	3.8	1.00	5.0	1.00	6.2	1.00	7.5	1.00	...	...	...	...	12.0	1.00
	H-8sb	2.4	0.89	3.4	0.89	4.5	0.90	5.5	0.89	6.6	0.88	...	...	...	...	...	...
	Mo. Pac. 1690	...	...	3.1	0.82	3.9	0.78	...	...	...	...	...	...	...	...	...	...
120	Baldwin 60,000 (11.7, 350)	...	...	...	...	...	...	6.9	1.30	8.8	1.35	11.0	...	13.6	...	17.2	...
	Baldwin 60,000 (11.7, 250)	...	...	...	...	...	...	...	...	...	...	...	...	8.8	...	...	...
	I-1s	3.5	1.52	4.8	1.50	6.2	1.48	7.9	1.49	9.7	1.49	...	...	...	...	...	...
	L-1s and K-4s	2.3	1.00	3.2	1.00	4.2	1.00	5.3	1.00	6.5	1.00	...	...	...	...	...	...
	H-8sb	2.0	0.87	2.7	0.84	3.7	0.88	4.5	0.85	...	...	...	...	...	...	...	...
	Mo. Pac. 1690	...	...	...	...	...	...	...	...	4.8	0.74	...	...	...	...	...	...
	E-6s-89 and E-3sd (6.5, 205)	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Purdue No. 4 (3.5, 170)	...	...	1.5	0.47	...	...	2.0	0.38	...	...	...	...	...	...	...	...
160	Baldwin 60,000 (11.7, 350)	...	...	...	...	...	...	5.8	1.26	7.4	1.30	9.4	...	...	...	...	...
	Baldwin 60,000 (11.7, 250)	...	...	...	...	...	...	...	...	...	...	5.8	...	...	...	...	...
	I-1s	...	...	3.75	1.34	5.0	1.39	6.6	1.44	...	...	...	...	...	...	...	...
	L-1s and K-4s	...	...	2.8	1.00	3.6	1.00	4.6	1.00	5.7	1.00	...	...	...	...	...	...
	H-8sb	...	...	2.4	0.86	3.2	0.89	4.0	0.87	...	...	...	...	...	...	...	...
	Mo. Pac. 1690	...	...	2.2	0.79	2.8	0.78	3.4	0.74	4.2	0.74	...	...	...	...	...	...
	E-6s-89 and E-3sd	...	...	2.2	0.79	2.9	0.81	3.7	0.81	...	...	...	...	...	...	...	...
	Purdue No. 4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
200	K-4s	1.7	1.00	2.5	1.00	3.4	1.00	4.2	1.00	5.4	1.00	...	...	...	...	...	...
	E-6s-89	1.4	0.82	2.0	0.80	2.7	0.79	3.5	0.83	4.4	0.82	...	...	...	...	...	...

[a] Ratio of the steam consumption to that used by *L-K* combination at the same speed and cut-off.

form to make useful generalization possible, and it is hoped that the remaining gaps may be filled by additional tests.

Fig. 3 shows the relation between the steam per revolution, the cut-off, and the revolutions per minute, for the two Pennsylvania Railroad locomotives, class L-1s[3] and class K-4s.[4] The data from these two locomotives overlap only in the tests at 120 and 160 rpm, but the results for these speeds are so consistent as to make it legitimate to consider the combined results as coming from a single locomotive, tested over a wider range of speed and cut-off conditions than was ever used in a series of locomotive laboratory tests. These locomotives have cylinder volumes, including clearance, of 10.7 and 10.2 cu ft (one cylinder), respectively. No different relations may be drawn at the overlapping speeds for the two locomotives, and hence the volume of 10.45 cu ft may be taken as representative of the two. It remains to determine what relation is borne to the steam used in the 10.45-cu ft cylinders and, at 205 lb per sq in. boiler pressure, by that used in cylinders of other volumes and with steam at other pressures. This relation is referred to as the *L-K* combination.

Test reports are also available for a series of locomotives[5] using superheated steam and with cylinder volumes from 6.5 cu ft to 10.7 cu ft, all working at the same boiler pressure of 205 lb per sq in.; of a locomotive with three 8.0-cu ft cylinders and 200 lb per sq in. boiler pressure;[6] of two locomotives with cylinder volumes of 14.6 cu ft and 250 lb per sq in. boiler pressure;[7] and of the Baldwin 60,000[8] with one high-pressure cylinder

TABLE 2 CONDENSED SUMMARY OF DATA IN TABLE 1

Locomotive	Cylinder volume Cu ft	Ratio to *L-K*	Working pressure Lb per sq in.	Ratio to *L-K*	Conditions of comparison	Steam ratio to *L-K*
P.R.R. E-3sd and E-6s-89	6.5	0.63	205	1.00	All speeds, all cut-offs	0.80
Missouri Pacific, 1690	8.0	0.75	190[a]	0.93	All speeds, all cut-offs	0.77
P.R.R. H-8sb	8.8	0.82	205	1.00	All speeds, all cut-offs	0.87
Baldwin 60,000	11.7	1.10	335[a]	1.65	All speeds, all cut-offs	1.50
			200[a]	0.95		1.06
P.R.R. I-1s	14.6	1.36	250	1.21	All cut-offs, speeds:	
					40 rpm	1.60
					80 rpm	1.50
					120 rpm	1.50
					160 rpm	1.40
Purdue No. 4	3.5	0.33	170	0.83	Speeds:	
					120 rpm	
					160 rpm	0.45
					Cut-offs:	
					30%	
					40%	

[a] In calculating the ratios, attention was paid to the actual range of boiler pressures in the tests. On this basis, the working pressure for the Missouri Pacific locomotive 1690 is 190 lb per sq in., and for the Baldwin 60,000 it is 335 and 200 lb per sq in.

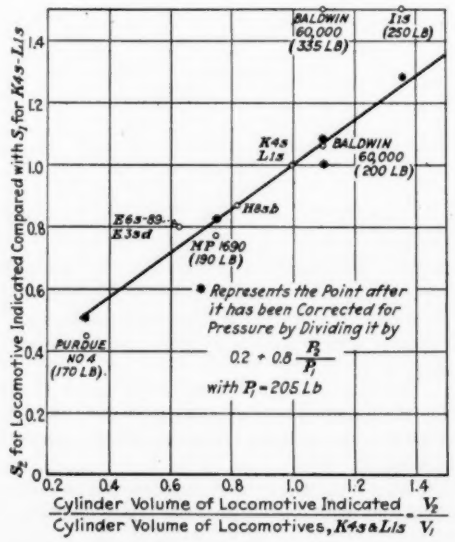


FIG. 4 AVERAGE RATIO OF STEAM USED BY VARIOUS LOCOMOTIVES TO THAT USED BY THE L-K COMBINATION

[3] Pennsylvania Railroad Bulletin No. 28.

[4] Pennsylvania Railroad Bulletin No. 29.

[5] Pennsylvania Railroad locomotives, E-3sd, E-6s-89, E-6s-51, K-2sa, H-8sb. See Pennsylvania Railroad Bulletins Nos. 11, 21, 27, 18, and 10, respectively.

[6] Missouri Pacific locomotive No. 1690, *Railway Age*, June 30, 1925.

[7] Pennsylvania Railroad locomotives I-1s-790 and I-1s-4358, see Pennsylvania Railroad Bulletins Nos. 31 and 32, respectively. These two locomotives differed in that the latter had a type-E superheater and also a feedwater heater, but the resulting difference in performance does not enter into consideration of this paper.

[8] Baldwin Locomotive Works publication, "Locomotive No. 60,000."

of 11.7 cu ft volume, with boiler pressures of 250 and 350 lb per sq in. Some data are also available for the performance of Purdue No. 4 locomotive which has a cylinder volume of 3.5 cu ft. All volumes are those of one cylinder, including clearance. Curves similar to those of Fig. 3 were drawn throughout the range of performance for all of these locomotives; these curves are not presented in this paper but in Table 1 are shown the values for steam consumption per revolution from smooth curves drawn through the actual points, and also the ratio of the steam consumption to that used by the *L-K* combination at the same speed and cut-off. Table 2 is a condensed summary of the data in Table 1.

The values in Table 2 are plotted in Fig. 4. The circles represent the various locomotives, showing for each its average ratio of steam used to that for the steam consumption of the *L-K* combination, plotted against the volume ratio. The points representing locomotives with 205 lb per sq in. working pressure show the use of less steam than that for the *L-K* combination in a ratio related to the smaller cylinder volume. The points representing locomotives with other than 205 lb per sq in. pressure show a use of more or less steam dependent both on the cylinder volume and working pressure. The first step in relating these is to reduce the varying pressure points to the values they would have if 205 lb per sq in. pressure were used. By trial, it was found that when the values of these points were divided by the quantity

$$0.2 + 0.8(P_2/P_1)$$

where P_2 is the actual working pressure of the locomotive under consideration, and P_1 is the working pressure of the *L-K* combination, or 205 lb per sq in., the corrected points fall close to the straight line drawn through the points for the 205 lb per sq in. pressure. The equation for the latter line shows that, if the steam for the *L-K* combination with cylinder volume V_1 is taken as unity, the steam for a locomotive with some other cylinder volume V_2 is

$$0.3 + 0.7(V_2/V_1)$$

Hence, the steam per revolution S_2 for a locomotive with pressure P_2 and volume V_2 as compared with the steam S_1 used by the *L-K* combination with pressure P_1 and volume V_1, may be expressed as

$$\frac{S_2}{S_1} = \left(0.3 + 0.7\,\frac{V_2}{V_1}\right)\left(0.2 + 0.8\,\frac{P_2}{P_1}\right) \quad [5]$$

This is a purely empirical relation based on test results. The variations in the ratios for any given locomotive within its own range of performance are not sufficiently consistent for any other process to define this relation, and in general, the variation secured by using the formula proposed for finding S, the steam used per revolution, is less than the variation in the test data.

The value of S thus secured represents the steam used per revolution for both cylinders of the conventional two-cylinder simple locomotives; for three-cylinder simple locomotives such as the Missouri Pacific No. 1690, the steam used per revolution is 50 per cent greater than S, or 1.5 S, and for locomotives with only one high-pressure cylinder, as in the case of Baldwin 60,000, the steam used per revolution is equal to 0.5 S.

After determining the amount of steam per revolution it becomes necessary to find the admission pressure, in order that the mean effective pressure may be later determined. From the preceding process, S (= S_2) is known, and the total steam used by the cylinders in pounds per hour is

$$E_c = 60\,NS \quad [6]$$

where E_c = steam used by the cylinders, lb per hr, N = rpm, and S = steam used by the cylinders per revolution, lb.

The pressure of the steam as delivered to the cylinders varies with the steam flow, and with the area of all the passages from the boiler to the cylinders. These areas are at least four in number, and there are no data nor theory to evaluate their effect. However, the loss in pressure, for any given steam supply, is closely related to the capacity of the boiler, and this in turn to its dimensions, so that a satisfactory correlation is found between the loss in pressure, the steam flow, and the heating surfaces. This relation is well represented by the expression

$$P_d = 0.6(E/H)^{1.6} \quad [7]$$

where P_d = loss in pressure between the boiler and branch pipe, lb per sq in., E = total steam consumption, lb per hr, and H = total heating surfaces (outside), including superheater, sq ft. It is obvious that the total steam consumption E is made up of the steam used in the cylinders E_c and the steam used by the auxiliary devices E_a, or

$$E = E_c + E_a$$

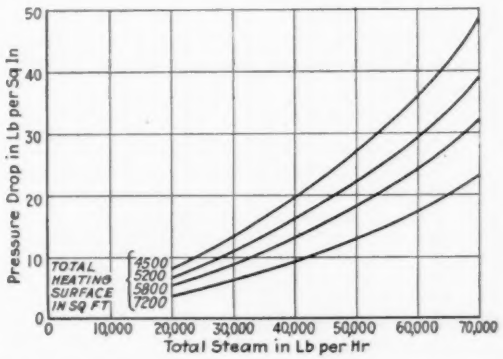


FIG. 5 STEAM PRESSURE DROP BETWEEN BOILER AND BRANCH PIPES

The curves from which Equation [7] is derived are shown in Fig. 5. They are valid only up to the point at which the boiler pressure itself begins to fall off.

For any simple locomotive operating at a given speed, the work in the cylinders depends on the mean effective pressure, which in turn depends on the admission pressure, the cut-off, and the steam distribution generally. No attempt can be made to estimate the effects of the variations in steam-distribution design; and the arrangements of the conventional engines are such that this variable may be eliminated. The effect of the diagram factor is so consistent that the mean effective pressure is, for any given speed and cut-off, a very definite proportion of the admission pressure. In Fig. 6 (6*a* to 6*e*, inclusive) the ratio of the mean effective pressure to the admission pressure at the various cut-offs is shown for the Pennsylvania locomotives L-1s, K-4s, H-8sb, and I-1s, the Illinois Central locomotive No. 1742,[9] and the Missouri Pacific locomotive No. 1690 for the usual range of test speeds. The curves accurately represent the range and value of the ratios. In the case of the four lower speeds, there are several curves available for comparison; for the higher speeds, the information from Pennsylvania class K-4s tests is used. These curves are assembled in a composite diagram in Fig. 7, which may be taken as representing the relation between the ratio of mean effective pressure to admission pressure, and the cut-off in per cent of stroke, at the various

[9] University of Illinois, Engineering Experiment Station Bulletin No. 220.

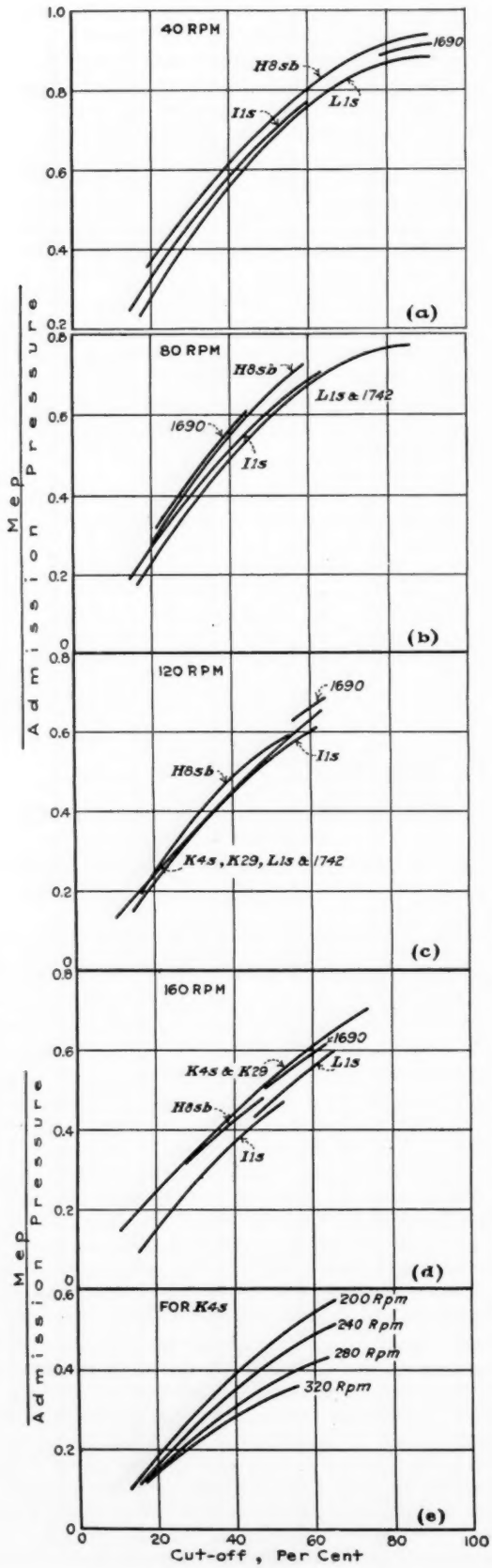


FIG. 6

speeds in revolutions per minute, for locomotives with conventional types of cylinders and valve gears and using superheated steam.

ESTIMATING TRACTIVE EFFORT

The process of estimating the tractive effort and horsepower of a locomotive may now be summarized:

1 Given the dimensions of the locomotive, the speed and cut-off, the procedure is:

(*a*) From Fig. 3 find the steam per revolution for the locomotive used as standard of comparison, having a cylinder volume of 10.45 cu ft and a working pressure of 205 lb per sq in., at the required speed and cut-off.

(*b*) Find the steam per revolution for the pressure and cylinder volume of the locomotive considered by modifying the steam per revolution found under (*a*) by the use of Equation [5].

(*c*) Find the steam used by the cylinders per hour by using Equation [6].

(*d*) Find the total steam used per hour by adding estimated amount of steam used by the auxiliary devices.

(*e*) From the total steam demand and the heating surfaces, estimate the pressure drop between boiler and cylinders by using Equation [7] or Fig. 5.

(*f*) Find the admission pressure by deducting the pressure drop (*e*) from the boiler pressure.

(*g*) Estimate the ratio of mean effective pressure to admission pressure for the given speed and cut-off from Fig. 7, and obtain mean effective pressure by multiplying the admission pressure (*f*) by this ratio.

(*h*) Calculate the tractive effort by Equation [1].

(*i*) Calculate cylinder horsepower by Equations [2] and [3].

2 Given the steam supply and the speed, the procedure is:

(*a*) If the steam supply is given as the net amount to the cylinders, the steam per revolution is obtained directly by dividing the steam supply E_c by 60 N, otherwise the total steam supply must be reduced by an estimated amount of steam used by the auxiliary devices in order to arrive at the net amount of steam to the cylinders.

(*b*) Reduce the actual steam per revolution (*a*) by the use of Equation [5] to find the amount of steam per revolution required by the locomotive used as the basis of comparison.

(*c*) Knowing the speed and the steam per revolution (*b*), find the cut-off at which the locomotive may be operated for that speed from Fig. 3.

Further procedure is the same as items (*d*) to (*i*), both inclusive, previously outlined under case 1, since the speed, cut-off, and total steam supply are now known.

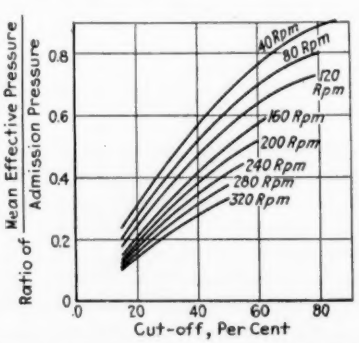


FIG. 7 COMPOSITE DIAGRAM OF THE RELATION BETWEEN RATIO OF MEAN EFFECTIVE PRESSURE TO ADMISSION PRESSURE AND THE CUT-OFF IN PER CENT OF STROKE AT VARIOUS SPEEDS
(For locomotives with conventional-type cylinders and valve gears and using superheated steam.)

As a means of showing the application of the method which has just been outlined, the cylinder tractive effort and cylinder horsepower of a locomotive of assumed dimensions will be de-

FIG. 6 (LEFT) RATIO OF MEAN EFFECTIVE PRESSURE TO ADMISSION PRESSURE AT VARIOUS CUT-OFFS FOR DIFFERENT LOCOMOTIVES

termined for a series of assumed values of cut-off and steam supply:

1 Locomotive dimensions:

Cylinders (two)	26 × 28 in.
Driving wheel diameter, in.	80
Boiler pressure, lb per sq in.	240
Total heating surfaces, including superheater, sq ft	6000
Engine constant for tractive effort	237
$Z = (26^2 \times 28)/80 = 237$	
Engine constant for horsepower	0.150

$$Y = \frac{4(\pi/4)26^2 \times 28}{12 \times 33{,}000} = 0.150$$

2 From Equation [5], the steam used per revolution S for this locomotive having 26 × 28-in. cylinders, or a cylinder volume of 9.5 cu ft, including clearance estimated at 10 per cent, and 240 lb per sq in. pressure, may be related to that used by the comparison locomotive (L-K combination) with a cylinder volume of 10.45 cu ft and 205 lb per sq in. working pressure by the ratio

$$S = [0.2 + 0.8(240/205)][0.3 + 0.7(9.5/10.45)]S_1$$
$$S = 1.064\,S_1$$

3 An example of calculations for a given cut-off is given in Table 3. The calculations of cylinder horsepower and water rate, in addition to the tractive effort, are carried out as a check on the validity of the result.

4 A similar set of calculations based on steam supply may be made as shown in Table 4.

Similar calculations have been carried out with the cut-off assumed at 75, 60, 45, and 30 per cent, and with assumed values

TABLE 3 EXAMPLE OF CALCULATIONS FOR A GIVEN CUT-OFF

(a)	Cut-off assumed, per cent	75
(b)	Speed assumed, rpm	40
(c)	Steam per revolution for the L-K comparison locomotive with a cylinder volume of 10.45 cu ft and a working pressure of 205 lb per sq in. from Fig. 3, lb	12.4
(d)	Steam per revolution for the locomotive with a cylinder volume of 9.5 cu ft and a working pressure of 240 lb per sq in., (c) × 1.064, lb	13.2
(e)	Steam to cylinders per hour, (d) × (b) × 60, lb	31700
(f)	Estimated total steam used, including auxiliaries, lb	35000
(g)	Pressure drop between boiler and branch pipes, calculated by Equation [7] with E = 35,000 and H = 6000, lb per sq in.	10
(h)	Admission pressure, 240 — (g), lb per sq in.	230
(i)	Ratio of mean effective pressure to admission pressure for 40 rpm and 75 per cent cut-off, from Fig. 7	0.86
(j)	Mean effective pressure, (h) × (i), lb per sq in.	198
(k)	Cylinder tractive effort, (j) × 237, lb	47000
(l)	Cylinder horsepower, (j) × 0.150 × (b)	1188
(m)	Water rate, (e)/(l), lb per ihp-hr	26.7

TABLE 4 EXAMPLE OF CALCULATIONS BASED ON STEAM SUPPLY

(a)	Steam to cylinders, assumed, lb per hr	40000
(b)	Total steam required, including estimated steam for auxiliaries, lb per hr	44000
(c)	Speed, assumed, rpm	240
(d)	Steam per revolution for this locomotive, (a)/60(c), lb	2.78
(e)	Steam per revolution based on the comparison locomotive, (d)/1.064, lb	2.61
(f)	Cut-off corresponding to 240 rpm and 2.61 lb of steam per revolution, from Fig. 3, per cent	38
(g)	Pressure drop, from Equation [7], lb per sq in.	14
(h)	Admission pressure, 240 — (g), lb per sq in.	226
(i)	Mean-effective-pressure ratio corresponding to 240 rpm and 38 per cent cut-off, from Fig. 7	0.328
(j)	Mean effective pressure, (h) × (i), lb per sq in.	74
(k)	Cylinder tractive effort, (j) × 237, lb	17500
(l)	Cylinder horsepower, (j) × 0.150 × (c)	2560
(m)	Water rate, (a)/(l), lb per ihp-hr	15.6

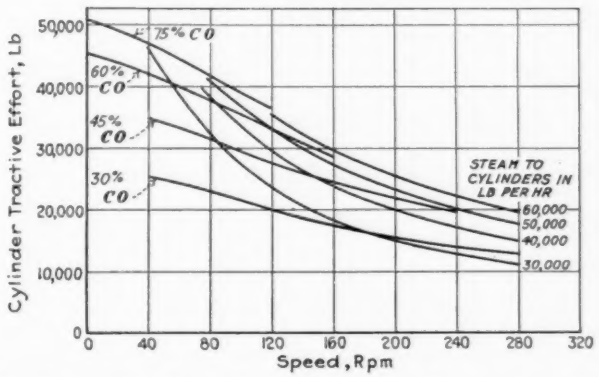


FIG. 8 TRACTIVE-EFFORT-SPEED RELATION AT VARIOUS CONSTANT CUT-OFFS AND FOR VARIOUS RATES OF CONSTANT STEAM SUPPLY TO THE CYLINDERS

of 30,000, 40,000, 50,000, and 60,000 lb of steam per hr to the cylinder. For every assumed cut-off or steam supply, the calculations are made for speeds of 40, 80, 120, 160, 200, 240, and 280 rpm, omitting some of the obviously impractical combinations of speed and cut-off. The calculated results are plotted in Fig. 8, showing the relation between the cylinder tractive effort and speed in two sets of curves; one for the tractive-effort-speed relation at various constant cut-offs, and the other for various rates of constant steam supply to the cylinder.

The proposed methods of calculation extend only to a speed of 40 rpm. The determination of tractive effort at low speeds rests on the fact that for the first few revolutions of the driving wheels, a very small steam supply will be adequate to fill the cylinder, so that for zero speed when, so to speak, the locomotive is merely leaning against the train, the actual cut-off and the dimensional constants of the locomotive determine the tractive effort. Reasonable assumptions with regard to release and compression pressures give the following mean effective pressures for the maximum cut-off given, at zero speed:

Maximum cut-off, per cent	Ratio of mean effective pressure to boiler pressure
90	0.97
80	0.93
70	0.87
60	0.80

The advantages of the method proposed and of the resulting figure, Fig. 8, may be mentioned briefly as:

1 The process takes the physical facts of the transformation from potential into kinetic energy fully into account.

2 The chart places before the user a full exhibit of the capabilities of the locomotive.

3 If such a chart is to be used as the background for a single-line speed-pull curve, it is immediately apparent what program of evaporative capacity and cut-off is expected of the locomotive.

4 A basis is furnished for estimating the economy of the locomotive under any conditions, including those far inside of what may be considered its capacity.

Discussion

The Design and Performance of an Axial-Flow Fan[1]

C. KELLER.[2] Noise measurements as plotted in the authors' Figs. 19 and 20 agree with the writers' recently published observations[3] which were made on the advice of Dr. J. Ackeret during experimental investigations of single-stage axial blowers. Among other things, these tests deal with the cause of noise and it might be interesting to compare both the results obtained and another method of investigation with the paper under discussion.

The pronounced increase of the intensity of the noise measured by the authors near the inflection point on the static-pressure line, and also the following sudden decrease, has been confirmed by all our tests on four different impellers. In our estimation, the noise in this region is caused by vigorous detachment

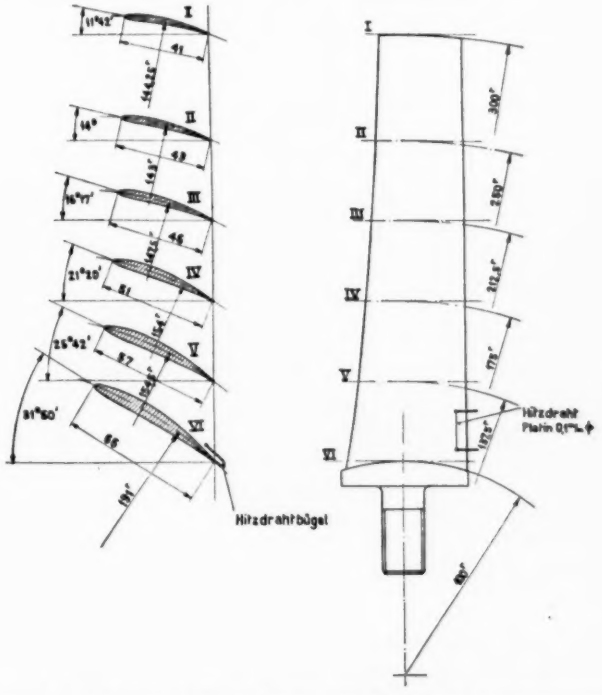


FIG. 1 AN IMPELLER BLADE WITH THE HOT-WIRE ARRANGEMENT (*Hitzdrahtbügel* = Flexible Copper Wires; *Hitzdraht Platin* 0.1 *m/m* ϕ = Hot Wire Platinum 0.1 *mm* ϕ.)

of the air stream on the various wing parts. With a decreasing flow volume, the profiled impeller wings are loaded progressively (the lift coefficient c_a of the profile increases). With a decreasing volume a point is reached where the impeller can no longer produce the required pressure ($c_a > c_{amax}$). As a result surging starts which is a well-known phenomenon in blower and compressor design, and the pulsation of flow volume and pressure gives a pronounced noise.

The hot-wire instrument makes it possible to investigate exactly the important flow phenomena on a moving wing by measuring the velocities on the spot where detachment of the stream occurs. A brief description of this investigation method is as follows:

A thin platinum wire 0.1 mm diam and 20 mm long is attached 2 mm above the surface of the rear side of a wing by means of 1-mm flexible copper wires as shown in Fig. 1 of this discussion. This arrangement does not cause any disturbances in the flow passing along the blade. The ends of the wires are fastened to thin insulated copper wires which are inserted in the wing and lead through the hub to the shaft end where they are connected to a battery by means of collecting rings, copper brushes, and rheostats.

During normal operation, the air passes over the wing surface without detachments and the platinum wire, through which passes a current i, is cooled by the air which has a velocity w. With a larger angle of attack (small volume) the eddies and backflow on the trailing side of the wing increase, starting at the rear edge, until finally the flow is wholly detached and the hot wire lies in a region of eddies. The wire, no longer subjected to the

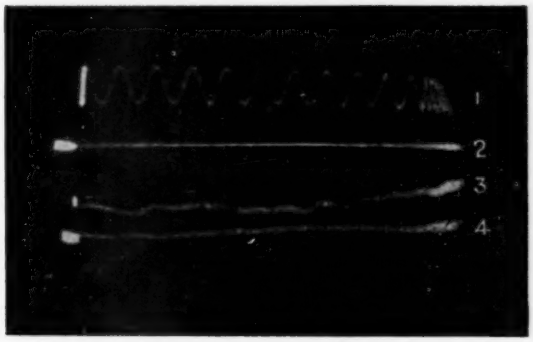


FIG. 2 OSCILLOGRAPH OF CHANGING-VOLTAGE DIFFERENCE BETWEEN HOT-WIRE ENDS DUE TO DETACHMENT OF FLOW FROM WING

(Line 1—50-cycles alternating current as oscillograph scale; line 2—normal flow w over the wing profile; line 3—beginning of detachment at the rear edge of the wing; line 4—flow detached from the wing profile.)

regular air flow w is cooled less than in normal operation and therefore its temperature rises. The electric resistance of the thin wire changes accordingly and a different voltage drop between the ends is the result. This voltage difference, after being duly increased by amplifier tubes, can be made visible by a cathode-ray oscillograph. This allows the observation of eventual periodic oscillations of detachments, if the hot wire has not too much inertia.

Tests have been made with a total difference of between two and three volts at the hot wire. The current in the hot wire amounted to approximately 3 amperes. A detachment of the stream corresponded to a difference of between 0.2 and 0.3 volt. For the leading tension of the electron ray of a Cossor tube which was used as a cathode-ray oscillograph, two amplifier tubes transformed the voltage difference from the hot-wire end 100 times to between 200 and 300 volts. By using the time abscissa of the

[1] Published as paper AER-56-13 by Lionel S. Marks and John R. Weske, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Chief Engineer, Caloric Research Laboratories, Escher Wyss Engineering Works, Ltd., Zurich, Switzerland.

[3] "Axialgebläse vom Standpunkt der Tragflügeltheorie," by Dr. C. Keller. Communication from the Aeronautical Institute of the Federal High School, Zurich, Switzerland, 1934.

Cossor tube, the oscillations of the voltage difference can be observed during a certain time, as illustrated in Fig. 2 of this discussion.

The upper sine curve represents 50 cycles alternating current, photographed on the same plate with the oscillograph curves, and serves as a calibrating scale. The distance between lines 2 and 4 corresponds to the voltage difference between the cold and warm hot wires (adherent and detached flow). Line 2 is very accurate and straight over the whole period of exposure. It shows the constant-voltage difference at the hot wire during normal operation of the fan with a closely adherent stream over the profile and indicates correspondingly uniform cooling. The air velocity w was then approximately 60 m per sec (3000 rpm). As soon as detachment is starting at the rear edge, the velocity w, and therefore the wire cooling, is changed. Now the voltage difference follows the irregular line 3. The unstable condition, prevailing near the hot wire is represented by oscillations of the wire-voltage difference. No regular frequency could be observed in this case. The angle of attack was about 11 deg when the detachment started.

Any further increase of the angle of attack (smaller volumes) causes a complete detachment of the flow and the hot wire lies fully in a dead space of flow. The wire, in comparison to the previous position which gave line 3, is heated still more and the change of potential difference is also greater. This case is illustrated by line 4. Contrary to line 3, where detachment of the flow is only beginning with alternating detachment and adherence, the temperature of the wire is almost constant which results in the constant-voltage curve, similar to line 2.

The three lines, 2, 3, and 4 correspond to the varying degree of noisiness as observed by Marks and Weske; line 2 representing minimum noise; line 3 representing an increase of noise with a decrease of efficiency; and line 4 representing lower noisiness after inflection point in the static-pressure curve.

R. G. Folsom[4] and M. P. O'Brien.[5] Many papers dealing with industrial equipment include little or no information on the details of construction and the authors are to be commended for presenting sufficient material to permit a technical analysis of the results. It is regrettable that restricted publication space made necessary the omission of the additional data referred to, especially those concerning the rotational velocity at discharge.

The writers have developed a quantitative method of predicting the head-capacity curves[6] of axial-flow pumps which agrees with test data on a commercial propeller pump. Since the flow through a propeller fan is practically incompressible, the same method of prediction should be valid and the information presented by the authors in their Figs. 3, 4, 5, 6, and 9 makes possible an application of this method.

The procedure used by the writers starts from the airfoil theory of Pfleiderer[7] modified in such a way as to give a portion of the head-capacity curve near the design point provided that the aerodynamic characteristics of the blade sections are known. The modification consists simply in taking a weighted average of the net total head developed at each radius and subtracting the friction and shock losses in the inlet and discharge ducts. Empirical fan-test constants are unnecessary in these computations, all coefficients being obtained from hydraulic tables.

[4] Instructor, Department of Mechanical Engineering, University of California, Berkeley, Calif. Jun. A.S.M.E.

[5] Associate Professor, Department of Mechanical Engineering, University of California, Berkeley, Calif. Assoc-Mem. A.S.M.E.

[6] "Propeller Pumps," by M. P. O'Brien and R. G. Folsom, presented at the Aeronautics-Hydraulics Meeting of The American Society of Mechanical Engineers, Berkeley, Calif., June, 1934.

[7] "Die Kreiselpumpen," by C. Pfleiderer, Second edition, J. Springer, Berlin, 1932.

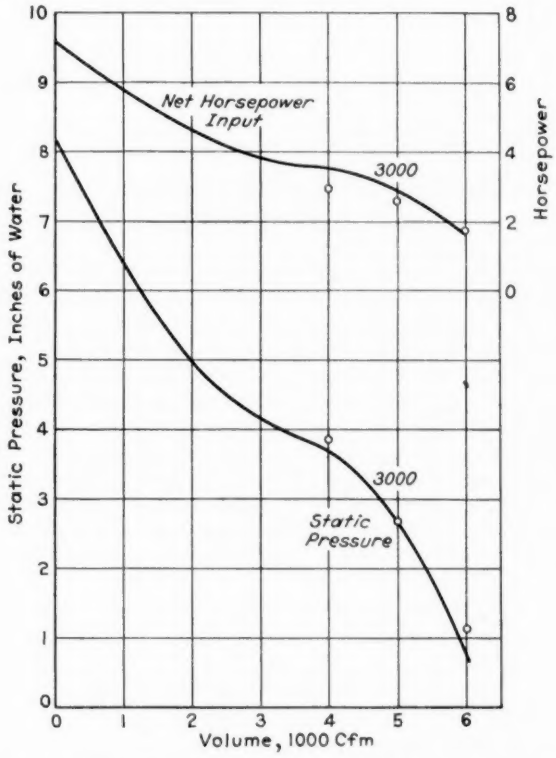


Fig. 3 Comparison of Computed Results With Marks and Weske's Test Curves of Capacity Versus Static Pressure and Power

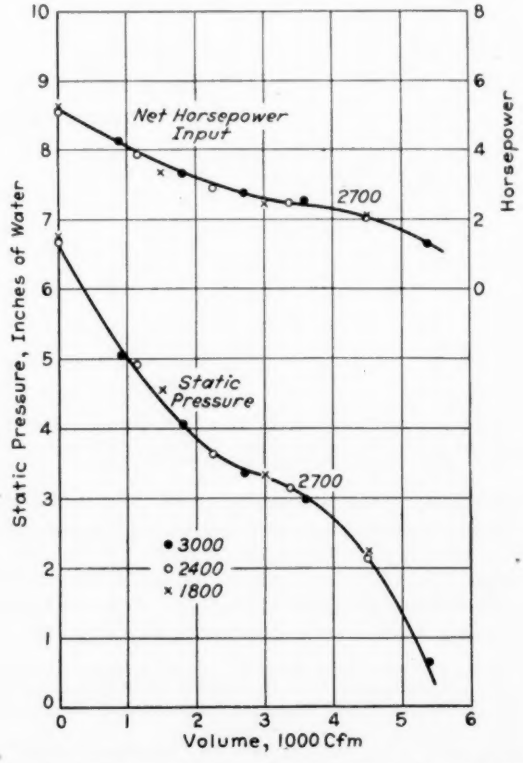


Fig. 4 Application of the Usual Affinity Laws to the Curves Given in Fig. 9 of Marks' and Weske's Paper

Fig. 3 in this discussion of the paper compares the computed results with the authors' test curves of capacity vs. static pressure and power. The discrepancy between the computed and measured heads is less than the probable error of the computations. The computed net power input is lower than that estimated by the authors on the basis of friction losses measured with a plain cylindrical hub. The good agreement indicates that the method used by the writers is satisfactory for design although it may not be theoretically sound in all details.

With regard to the bend in the head-capacity curves at which the authors observed that the fan became noisy, it is interesting to note that the airfoil theory shows this point to correspond approximately to the "burble-point" of the blade section. At greater rates of discharge, pumping is primarily the result of the lift of the blade sections while at lower rates the drag predominates. The bend appears to be the transition stage between these two conditions. "Burbling" starts at the hub and progresses outward as the capacity is decreased and its partial development along the blades probably causes the secondary flows observed by the authors.

Fig. 4 of this discussion shows the application of the usual affinity laws to the curves given in Fig. 9 of the paper under discussion. Theoretically these relationships should hold for axial-flow pumps and fans and it appears that they are dependable over the range of speeds investigated.

The authors state, "The fan can be designed so that the same amount of work is done on each particle of air, at the flow conditions corresponding to the point of maximum efficiency." Subsequently it is stated that, "The angle of attack was selected so as to give constant pressure on each blade element." The work per pound of the fluid is the total head (pressure head plus velocity head). It can be shown that the assumption of a constant total head and constant pressure difference at all radii are incompatible. It can also be shown that the radial-pressure variation resulting from a constant total head at all radii gives a stable type of flow and for this reason the assumption of a constant total head appears to be preferable.

ALEXEY J. STEPANOFF.[8] The design finally adopted by the authors resembles very closely that of the axial-flow pumps already well established and for which considerable data are available. The design of the fan for which data have been presented was based on the use of certain constants obtained from a preliminary design and study of its performance. This preliminary fan, in turn, was built "following closely a design which has given good performance." Aerodynamic principles involved were given due consideration.

This is a normal procedure of commercial design of pumping machinery, resulting in the marked progress in performance observed during the last few years. The authors deserve credit for improving efficiency of the final design as compared with that used for reference.

Attempts have been made lately by several investigators to apply the test data obtained with airfoils in the wind tunnel to the design of axial-flow pumps, which substantially is the basis of the so-called "airfoil theory" of axial-flow pumps. There are, however, several handicaps to making this theory practical, such as: correction for mutual blade interference, finite blade length, variable pitch, and for the effect of a revolving series of blades instead of the parallel movement dealt with in aerodynamics. In addition to that, the performance of the diffusion case with inevitable losses should be accounted for, because it is in the performance of the complete unit that the designer is interested, and not of the impeller alone. The determination of corrective factors to account for all these influences experimentally is much more complicated than the building and testing of an actual model of the pump or fan, which is the procedure followed by the authors.

The number of variables to observe (head, capacity, and efficiency) in such a model test are the same as those with the airfoils (lift, drag, and angle of attack), and the testing procedure and equipment are much simpler. To make the results independent of the actual dimensions and speed of the pump, various factors were introduced (ratio of velocities or velocity heads) which correspond to the lift and drag coefficients in the airfoil theory. The factors for axial pumps, if plotted against specific speed as abscissas, fall in line with those for semi-axial flow (coni-flow), mixed-flow, and radial-flow centrifugal pumps, for which data have been compiled for years by the designer. It is very unlikely that designers will recalculate these data to conform to the airfoil theory.

No doubt, the charm of the airfoil and circulation theories lies in the fact that for certain simplified conditions those theories give solutions which agree very closely with the observed results, which is a very rare happening in the field of formal hydrodynamics.

An advanced pump or fan designer can enjoy the beauty of the airfoil and circulation theories and derive benefit from the achievements of modern hydrodynamics without trying directly to apply them to the actual design.

While the writer was studying this paper, several points were called to his attention. The fan efficiency, based on total head as calculated by the authors, corresponds to pump-hydraulic efficiency, neglecting leakage and disk friction. This was found by the authors to decrease with the speed. With pumps, efficiency is a maximum at a certain speed, below and above which efficiency drops slowly. At lower speeds the decrease in efficiency is caused by the decrease in mechanical efficiency at smaller outputs. This could not be observed by the authors, as mechanical losses were excluded in their calculations of the fan efficiency.

At higher speeds the difference between the velocities and pressures on the front and back side of the impeller blades becomes too great to perform the equalization of these at discharge and entrance edges with a minimum loss. The blade becomes overloaded. With pumps this is followed by cavitation if the suction pressure or submergence is not sufficient to suppress the local formation of cavities on the suction side of the blades.

The increase of the blade area, either by extending the blades or by increasing the number of blades, will increase the optimum speed of the pump or fan. This will also reduce the noise of the unit as a result of decreased pressure and velocity differences throughout the unit.

The authors have selected the profiles for the impeller blade with a straight front, or constant vane angle. The good performance of this fan proves, contrary to statements sometimes made, that the increase of the vane angle from the leading to the trailing edge is not absolutely necessary for an efficient generation of head. To make the generation of head possible with this shape of blade, the fluid must enter the blade with a sudden deflection or shock, but this shock is in the direction of flow, resulting in little or no loss of energy. On the other hand, by making the blade angle at the discharge equal to or smaller than that at the entrance the blade end is unloaded (becomes nonactive), resulting in uniting of the two streams on both sides of the blade with a minimum of loss.

Although the authors have chosen profiles with very thick entrance edges for the diffusion vanes, hoping to improve the flow for a wider range of capacities, the efficiency curves are not any flatter than found normally for this type of impeller.

The 0.5 ratio of the hub diameter to the impeller diameter used by the authors is greater than normally encountered in

[8] Byron Jackson Company, Berkeley, Calif. Mem. A.S.M.E.

pump designs. Large impeller hubs require an unusually long diffuser tapering hub for an efficient recovery of the velocity head.

The best distance between the impeller and guide vanes for axial pumps was found by Pfleiderer to be about 3 per cent of the impeller diameter. The authors' optimum clearance between the impeller and guide vanes is 5 per cent. The difference is probably caused by the fact that the authors' diffusion vanes have much heavier entrance edges than those of the Pfleiderer pump.

The authors deviate from the established methods of testing and terminology. This, while having no particular advantages, may lead to confusion. The close comparison of the performance of the fan presented in the paper with available data is impossible, as, while the authors based the fan efficiency on the power input less mechanical losses, the latter are not given in the paper. The statement in the paper that the pressures, volumes, and efficiencies, as determined by the methods of the Standard Test Code at another laboratory not named in the paper, may only be explained by the inaccuracy of the test data.

Authors' Closure

The preliminary investigations referred to in the paper had the same object as that discussed by Dr. Keller, namely, the exploration of the flow through the fan blades. For this purpose the test arrangement consisted of pitot tubes rotating with the fan and of a pressure seal by which the pressures were transmitted from the rotating elements to a stationary gage board.

In the region beyond the stalling point the conditions of flow around a rotating blade are more complicated than those of two-dimensional flow. The air in the stagnant zone at the back of the blade is dragged along by the blade at approximately the rotative speed of the blade. It thereby is subjected to centrifugal acceleration and tends to move radially outward. Stalling begins at the root of the blade; when it starts it has a disturbing effect upon the neighboring zones further away from the center line.

Stalling is only one of the causes of the rapid increase of noise near the inversion point of the fan-characteristic curve. Our measurements of axial velocity immediately in front of the fan wheel (by stationary pitot tubes) show that there is a displacement of the flow as the discharge volume decreases which results in diminishing flow near the tip of the blade. This may go so far as to cause reversal of flow at low discharge volumes. It is at the point where this reversal of flow first appears that the highest peak of the noise curve occurs.

It is gratifying to find the agreement between the computations of Mr. Folsom and Professor O'Brien and the actual performance of the fan. The difference between the two methods of design (constant total head and constant pressure difference) disappears when the fan and guide vanes are considered as one system and when the axial velocity leaving the system is constant over the cross-section.

Mr. Stepanoff suggests that the fan designed by the authors was a modification of the fan used in the preliminary investigations. This is not correct. The fan was designed on aerodynamic principles and departed from the previous design in a number of fundamental points, as stated in the paper. The combined corrective factor for the various influences to which Mr. Stepanoff refers (mutual blade interference, etc.) was actually determined experimentally, as he advocates, and was used in the design. The authors agree that two-dimensional air-foil theory is inadequate for a rational analysis of the flow through an axial-flow impeller. Air-foil theory, however, supplies useful data for the selection of the proper shape of the blade section. For the design of the blade as a whole, circulation theory as applied to propellers (particularly the work by Goldstein and others) supplies necessary and useful data for the practical design.

Air Flow in Fan-Discharge Ducts[1]

H. F. Hagen.[2] The author has made a valuable contribution toward the explanation of the discrepancies that appear between the volume measurements made by use of the pitot tube and those made with a flow nozzle. However, the quantitative use of the direction tube implies that the air has been assumed to have a smoothly rotating flow without ring vortices or radial motion. To validate this would require extensive investigation. The supporting evidence of nozzle agreement and egg-crate action is inferential and, while convincing, too meager to be conclusive, as the author himself states.

Fan literature describes two fundamental spins; a spin of uniform moment of momentum having a constant total head throughout the range of flow, and the vortex with a constant angular velocity. The first type of spin can occur in any fluid flow. The second type can be produced only by an expenditure of energy.

The flows indicated by the direction tube are of neither of these fundamental forms. Inasmuch as each blade produces vortices, it is pertinent to consider whether or not the apparent spins are due to a pattern of individual blade vortices, particularly as such vortices have a theoretical permanence. If such a vortex pattern exists, the square or pulsation effect cannot be ignored.

The author's experiments with more precise metal diaphragms are open to the objection that the small amplitude of their very precision makes them as susceptible to the minute displacements of the ordinary sound wave as they are to the relatively large displacements of air pulsation. The rubber diaphragm used by the writer indicated pulsations too great to be ignored. Professor Bailey's paper[3] also shows a severe pulsation in a fan delivery.

The writer has experimentally verified the author's work. The curves of Fig. 13 in the paper were checked in the writer's laboratory concurrently with the author's investigation. Later, an "egg crate" was used with his predicted results. In another case, the amount of spin shown by the direction tube was predicted from a difference between inlet and outlet volume measurements. However, complications arose. Two fans that showed a variation of approximately 10 per cent between pitot tube and nozzle produced agreement in subsequent testing throughout the whole range of the fan characteristic. It is to be noted that the nozzle volumes were the consistent ones.

The direction tube developed by the author is a new instrument of great promise. The writer has had opportunity to use it in an examination of the flows from two double-inlet experimental fans. One of these indicated a spin similar to those shown by the author for single-inlet fans, but of only 12-deg inclination. The other showed a double vortex of 22-deg maximum spin. Nozzle checks have not yet been made and further investigation is required along these lines.

To the best of the writer's knowledge, in every instance where the direction tube has been used in a fan duct it has indicated rotational flow. The author's deductions from the writer's reported results on double-inlet fans may require revision. Direction-tube tests made in cases that show agreement between pitot tube and nozzle may also prove illuminating.

---

[1] Published as paper PTC-56-2, by L. S. Marks, in the November, 1934, issue of the A.S.M.E. Transactions.

[2] Vice-President and Director of Research, B. F. Sturtevant Company, Hyde Park, Boston, Mass.

[3] "Pulsating Air Flow," by Neil P. Bailey, Trans. A.S.M.E., vol. 56, 1934, paper PTC-56-1, p. 781.

These test experiences have been mentioned to emphasize the complicated nature of the whole subject and to indicate that the author's unquestionably important discoveries do not provide an entirely complete explanation.

The author's work with the various types of pitot tubes is interesting. The writer has tested five different tubes of the type shown in the author's Fig. 4 and found curves similar in type to that of the author but with individual differences of velocity pressure of 3.5 per cent. Errors of this order frequently occur in pitot-tube work even on the inlet side of fans where there is no reason to suspect a spin. There appears to be an error of this order in the author's calibrations. Presumably the impact ends of the tubes of Figs. 2, 3, and 4 are identical. Allowing for signs plus or minus from the unity line a similar gradient can be secured from the published curves for impact-pressure variation which should be the same for the three. Such curves plotted from Figs. 2 and 4 agree closely, but the curve from Fig. 3 does not check within 4 to 6 per cent of the velocity head in the important 15-deg to 30-deg range. Has the author an explanation of this apparent change in the impact gradient?

Incidentally, it should be noted that, in rectangular ducts where the pitot tube is not conventionally moved toward the center line of the duct, the tubes of Figs. 2 and 3 would interchange characteristics and that possibly the symmetrically distributed holes of Fig. 4 are to be preferred.

It is worthy of remark that the author put his reliance, and very properly, on the nozzle measurement. In all the work done by both the author and the writer, the nozzle has been reliably consistent and has been used as a measure of the correctness of other methods. The only real objection to the nozzle is a commercial one of cost. This objection has been overcome by the development of accurate but inexpensive sheet-metal forms.

H. S. Bean.[4] The discussion, in the paper, of errors due to non-axial flow is of particular interest and value. It may be well to note that errors of the nature and approximate magnitude described have been observed in some tests made at the National Bureau of Standards on small household fans such as are used in the summer. Such results lead one to suspect many of the pulsation errors reported in Mr. Hagen's 1933 paper[5] and in Professor Bailey's current paper[3] were really errors due to non-axial flow.

In 1926 and 1927, the Gas Measurement Committee, Natural Gas Department, American Gas Association, conducted a series of tests at Buffalo, N. Y., to determine the effects on orifice-meter indications of various types of pipe fittings placed on the inlet side of the orifice. One of the "fittings" used was a combination of three elbows so connected as to determine two perpendicular planes. Presumably, such a combination of angles produced a whirl in the gas stream. The effect on the orifice meter varied from practically zero to more than 13 per cent low (i.e., the indication of the meter was less than it should have been), depending upon the size of orifice and its distance from the angles. Moreover, with an orifice of 50 per cent diameter ratio, the effect of these angles was very apparent even with over 50 pipe diameters of straight pipe between the orifice plate and the nearest angle.

While so far as is known, there have been no tests of just this kind made with venturi tubes and flow nozzles, it seems reasonable to expect that the effect on them would be similar. If this expectation is right, one may well question the accuracy of nozzle measurements such as mentioned by Professor Marks, when the approach stream has a whirling motion.

In these Buffalo tests, the use of straightening vanes, "egg crates" as Professor Marks called them, was tried as a means of overcoming the adverse effects produced by the fittings. In all but a few instances the straightening vanes were completely effective. While several different designs of vanes were tried, the most effective had about the same relative dimensions as those used by Professor Marks.

Professor Marks states that a pitot-tube traverse made 2.5 pipe diameters downstream from the straightening vanes was as satisfactory as at 9. This is less than the recommendation made in Part 3 of the Fluid Meters Committee Report, which specified that the distance between the vanes and the orifice should be between 4 to 10 pipe diameters, depending on the diameter ratio of the orifice used.

Near the end of the paper Professor Marks states: "It is concluded that, with 2.5 diameters of duct past the pitot-tube station, there is no disturbance of the pitot-tube readings resulting from the presence of obstructions at the end of the discharge duct." In other words, any disturbance produced by such obstructions does not travel back upstream as much as 2.5 pipe diameters. This lends support to the recommendation of the Fluid Meters Committee that there should be at least from 2 to 4 pipe diameters of unobstructed pipe between the downstream pressure tap and any fitting.

J. F. Downie Smith.[6] In his paper, Professor Marks has explained the desirability of changing the position of the outermost pitot-tube station in a circular duct in order to make the A.S.H.&V.E. test codes come into closer agreement with theory and practice. His point was that a better average velocity would be obtained by the method he suggests. Since he has recommended a change in the test codes, another recommendation may be considered simultaneously with it.

The procedure in fan testing at present is to divide the duct into several equal-area strips, and, at the center of each, to measure (among other things) impact and static pressures. From the data, the velocity of the air at each point is calculated and the velocities are then averaged to get the average velocity over the section. This answer is correct within the required accuracy. But when this average velocity is used in the calculation for the velocity head of the air at the section, the answer obtained is wrong. The kinetic energy obtained from $\frac{V^2_{avg}}{2g}$ is not the average kinetic energy of the air.

Consider a small plane section of the duct with area dA perpendicular to the duct axis, a mean air velocity V at this point and density ρ. The kinetic energy of the material passing this section would be

$$\frac{WV^2}{2g} = \frac{dA\,V\,\rho\,V^2}{2g} = \frac{dA\,\rho\,V^3}{2g}$$

Over the entire section, the kinetic energy would be $\int_0^A \frac{\rho V^3}{2g}$ dA, and the kinetic energy for a unit weight of the material would be:

$$\frac{1}{AV_{avg}\,\rho_{avg}} \int_0^A \frac{\rho V^3}{2g}\,dA = \frac{1}{2g\,AV_{avg}} \int_0^A V^3\,dA$$

Now for a parabolic distribution of velocity, as occurs in isothermal streamline flow, the average kinetic energy obtained in

[4] Physicist Chief, Gas Measuring Instruments Section, National Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

[5] "Pulsation of Air Flow From Fans and Its Effect on Test Procedure," by H. F. Hagen, Trans. A.S.M.E., vol. 55, 1933, paper FPS-55-7, p. 105.

[6] Instructor in Mechanical Engineering, Harvard Engineering School, Harvard University, Cambridge, Mass. Assoc-Mem. A.S.M.E.

the above manner can readily be shown to be $\frac{V^2_{avg}}{g}$ instead of $\frac{V^2_{avg}}{2g}$ as has been used in the test codes. The latter is only half the size of the former. Fortunately, in fan work, the flow is seldom streamline, and for turbulent flow the error is much less than with viscous flow, amounting to only a few per cent. Unfortunately, it is not a constant amount and varies with the turbulence.

But in fan testing, the velocity head at each of the several points specified in the ducts is usually obtained or can be easily calculated. Then it appears better to average the velocity heads obtained at these various points and thus compute the velocity head of the air at this section. From this velocity head we can unfortunately get no data about discharge unless flow is viscous or of known turbulence. But the discharge can be obtained by calculating the average velocity as formerly.

Therefore, the specific recommendations are: (1) Calculate the volumetric discharge by averaging the velocities as specified in the test codes; (2) calculate the velocity head of the discharge by averaging the velocity heads obtained at the several specified test points.

Ed S. Smith, Jr.[7] The writer agrees generally with the author's conclusion that a "new definition of velocity head" is needed in the art of testing fans discharging into ducts but suggests that this new definition be made suitable for extension to the testing of pumps and turbines for liquids as well as gases.

This new definition that the writer suggests was proposed by George E. Lyon of Rensselaer Polytechnic Institute in 1921.[8] In view of the lack of published discussion, it may be well to state that, about 1930, Dr. Dryden, of the U. S. Bureau of Standards, independently proposed this to the writer in a conversation on metering of fluids by large-ratio nozzles.

The velocity head

$$h_b = \frac{\text{Total kinetic energy per second}}{\text{Mass per second}} = \frac{E}{M} \quad [1]$$

The total kinetic energy per second $= \int_0^R 2\pi r\, dr\, V\rho\, \frac{V^2}{2g}$ for a circular duct where r is the radius, V is the velocity, and ρ is density. This simplifies to:

$$E = \frac{\rho\pi}{2g} A_1 \quad [2]$$

where A_1 is the area of a graph in which V^3 is plotted as ordinate against r^2 as abscissa for a known velocity distribution.

$$M = \int_0^R 2\,\pi\, r\, dr\, V\, \rho$$

or

$$M = \rho\pi A_2 \quad [3]$$

where A_2 is the area of a graph in which V is plotted as ordinate against r^2 as abscissa for a known velocity distribution, as is well known. Thus the average velocity head is simply

$$h_b = \frac{\rho\pi}{2g} A_1 \div \rho\pi\, A_2 = \frac{1}{2g}\frac{A_1}{A_2} \quad [4]$$

Rectangular ducts may be treated in a somewhat similar, graphical manner. In the above analysis, only the velocity components along the axis of the duct have been considered.

It seems proper to credit the fan with any useful energy that it has added to the flowing stream as determined from the difference in total energy between a point in the inlet and one in the discharge duct. The total energy is to be taken as the sum of the energy of the static pressure, the kinetic energy of translation and rotation, and the total heat of the fluid itself. The useful portion of these energies must be determined from the portion of the energy increase that can reach the point of application and there be of service. The same problem is met in the sale of steam, e.g., where the flow-measurements are differently weighted when the fluid is to be used for heat or power.

As a practical matter, it seems fair to insist that the rotational energy be converted into that of translation or omitted from consideration. Consistent with this, it would seem desirable to measure the energy of translation at a point far enough distant from the fan, in view of any straightening egg-crates present, so that the velocity distribution is so nearly normal that further losses due to velocity readjustments will not have appreciable effects.

At such a point of normal velocity distribution, the mechanical viscosity is practically uniform across the central portion of the duct, thus producing a parabolic velocity distribution there. With smooth walls, the velocity builds up as the seventh root of the distance from the walls. The parabolic curve, extending outward from the center, and the seventh-root curve, extending inward from the walls, overlap for a generous range providing that suitable constants are used to adjust the above curves to follow the actual curve most closely. The seventh-root curve, taken alone, gives a sharp peak to the velocity-distribution curve, at the center of the duct, which entirely fails to fit the physical situation. It is best not to attempt to use a single law (parabolic, seventh root, or other) as basic in drawing general conclusions, but rather to make only such use of them as is mathematically convenient and correct.

The centrally located tube is the only fixed pitot tube that is considered worthy of use by the writer, and then only after it has been calibrated in situ at the various rates by velocity-distribution traverses, or otherwise. Any averaging or other pitot tubes located near the walls of the duct are in a region where the velocity-distribution gradient is too steep for accuracy or reliability, particularly for test purposes where guarantee performance is being checked.

Where the characteristic of the fan is such that the head first increases and then decreases with an increase in quantity, expansible fluids may have violent pulsations on the rising portion of the curve. The author's conclusion that pulsation errors are relatively small is evidently based on experience limited to fans having a steadily falling characteristic. With a rising characteristic, it is generally necessary to throttle the suction to avoid pulsations so severe as to render meaningless any velocity readings obtained by differential-type meters. It would seem desirable to supplement the pitot tube with a hot-wire velocity meter. While blowers having a rising characteristic may be considered as unworthy of mention, still they should fall under any system of test methods that lays any claim to comprehensiveness.

It may prove advantageous to rate such fans with a liquid instead of a gas. The writer has found it convenient to check the characteristic of a liquid pump by measuring the flow of air it produced. Of course the power required works against testing air fans with water as much as it favors testing large water pumps with air. However, the liquid-test procedure should find some use in model tests run to develop a design that avoids the objectionable rising characteristics of some former

[7] Hydraulic Engineer, Builders Iron Foundry, Providence, R. I. Mem. A.S.M.E.

[8] "Flow in Conical Draft Tubes of Varying Angles," by G. E. Lyon, Trans. A.S.M.E., vol. 43, 1921, paper no. 1830, p. 1245.

model. "An Investigation of the Performance of Large Centrifugal Pumps Using Air as a Medium," an advanced thesis by Miguel A. Quinones, Rensselaer Polytechnic Institute Bulletin No. 48, September, 1934, admirably confirms the potential value of such procedure, besides establishing the present accuracy of such a method.

A. PETERSON.[9] It may not be amiss to emphasize one important conclusion that may be reached from reading Professor Marks' paper and that is the possibility of serious errors in test results by use of the pitot tube. It is significant that Professor Marks uses the flow nozzle as a standard for comparison. This immediately brings up the question why should not the flow nozzle be specified also in the test code for fans.

The writer is a member of Committee number 10, his work having been confined mainly to the compressor section of the code which incidently has passed through its final steps for publication.

Before adoption of the flow nozzle for measuring volume, we had discussions and controversies with builders of commercial flowmeters such as thin-plate orifices, pitot tubes, etc., but the Committee took the stand that the most accurate method to determine volumes, with the knowledge of the art at that time, was the flow nozzle.

In the code as written, the door is, however, open to other methods of volume measurements but the test is not then strictly in accordance with the A.S.M.E. Code although it may happen to be just as accurate as a test according to the code.

In trying to have the flow nozzle also adopted in the fan Code in preference to the pitot tube, considerable opposition was raised by many fan manufacturers mainly on account of the expense of testing large fans by the flow-nozzle method.

After reading Professor Mark's paper and Mr. Hagen's paper,[5] I am more than ever convinced that the flow nozzle also should be specified in the fan code.

The pitot tube naturally has its place and can be covered in the fan code in the same manner as was done in the compressor code in permitting by mutual agreement other methods for volume measurements, but the flow nozzle should be the standard method.

In this connection, it may be well to emphasize the report of the Special Committee appointed to study in detail the principal objects and purposes of the A.S.M.E. Power Test Codes.

This report contains the following statement: "The Codes shall be so definite that they may be made part of commercial agreements and shall serve as a measure of fulfilment of contract obligations in so far as the items covered in these codes are concerned as well as a means of preventing disputes between parties to a test."

Professor Berry, who was a member of this Committee stated in the November, 1932, issue of *Power:* ". . . . the A.S.M.E. Test Codes are planned for use in acceptance testing and must therefore be so drawn that their results can be defended in litigation that may arise from a disagreement. The methods presented in such a code must be only those that are recognized as yielding the most dependable results attainable in the present state of the art."

I believe the foregoing quotations together with Mr. Hagen's paper[5] and Professor Marks' paper, herein being discussed, give ample justification for the Fan Code Committee to specify the flow nozzle for volume measurements.

If the fan is of such size that the expense of the test set-up becomes prohibitive, test of a model may be resorted to, which practice is followed by the hydraulic-turbine people and to some extent also by the centrifugal-pump manufacturers.

[9] Chief Engineer, Pump and Compressor Department, DeLaval Steam Turbine Company, Trenton, N. J. Mem. A.S.M.E.

R. D. MADISON.[10] The author has presented here a real contribution to the study of air flow in ducts, and has offered constructive suggestions for the changes in fan-test procedure. The use of the honeycomb straightener with a short test duct is to be commended both from the standpoint of economy of set-up and accuracy of test.

It is comforting to know that the author's opinion is that pulsation is responsible for only a small part of test errors. While pulsation still has to be dealt with it is not likely to abolish the use of the pitot tube and traverse. The pitot has been somewhat discredited of late and without real justification. The author has pointed out that with a straightener there seems to be little choice between the Prandtl tube and that adopted by the A.S.H.&V.E. Tests may prove that a modification of them both is the answer. The writer believes that larger static holes and a blunter, more elliptical-shaped nose in the A.S.H.& V.E. tube is desirable.

The writer has used straighteners and found them desirable from all viewpoints. In recent elbow tests he used a honeycomb approximately 5 in. square and 10 in. long and satisfactorily reduced spiral flow to a negligible amount. However, if shock loss at entrance is to be neglected, a length of 15 in. (3 diameters) would have been preferable. Contrary to what some may think, a thin-edged straightener does not "equalize" flow, although the static pressures, as read, are much more uniform. In fact, the writer has found that the velocities are apparently more uneven due to the more accurate reading of the static pressure. A straightener will introduce a pressure loss both by reason of the friction of the walls and by the shock loss at entrance. The former should not be charged to the fan. If there is a large amount of spiral flow there will be a large shock loss and this would and should automatically penalize the fan since spiral flow serves no useful purpose in any installation. It is the writer's opinion that a length of duct approximately 6 diameters could be used with a straightener—3 diameters between the fan and center line of the straightener, $1^1/_2$ diameters between the straightener and the pitot station and $1^1/_2$ diameters between the pitot station and end of the duct where a symmetrical throttling device is situated.

The subject of the use of rectangular ducts is too quickly dismissed by the author without considering some of its advantages. Most fans have square or rectangular outlets and are connected to ducts of similar shape. It would not only simplify the duct to keep it the same size as the fan outlet but it would change the flow lines less and facilitate the fabrication of the straightener and air-throttling device. It is true that there is a larger percentage of boundary to area in a rectangular duct than in a round one but there is also a greater percentage of readings taken near the boundary. The round-duct traverse presupposes a fairly uniform velocity distribution across the duct. This is not always attained, especially in single-inlet fans. In fact, the "pattern" of the velocity distribution may change radically with different points of rating with the same fan operated at constant speed. In the rectangular traverse the points are more uniformly distributed over the whole area and therefore more likely to be representative under all conditions. The author feels that the outermost point in the round traverse is not correct and should be moved slightly nearer the boundary. If tests prove that this is necessary and the same condition exists in the rectangular traverse, a corresponding change could be made. As a result of the observations, the writer believes that with the use of a honeycomb straightener there is every justification for the use of rectangular test ducts.

[10] Research Engineer, Buffalo Forge Company, Buffalo, N. Y. Assoc-Mem. A.S.M.E.

W. H. CARRIER.[11] Although the pitot tube has always been recognized as a very convenient and simple instrument for determining fluid flow, the accuracy of the results obtained by it has heretofore been questioned. However, as a result of Professor Marks' research, it is possible to use this instrument with confidence.

There may be some question as to the proper distance from the fan discharge to take static-pressure readings. It is well known that a considerable length of piping on the fan discharge has an equalizing effect which actually increases the observed static pressure in many types of fans. It is also true, of course, that there is a certain frictional loss in this duct work. It is debatable whether an addition to a fan which improves its apparent efficiency should not be penalized by whatever frictional loss occurs in producing the additional efficiency. In a straight length of discharge duct there is normally this conversion of "static" energy, which will naturally occur in the duct, and the frictional loss of the duct is calculated from the fan outlet, so the performance of the fan would normally be taken on the present basis providing, of course, the correct coefficient of friction were allowed.

I believe that users of fan equipment would prefer to have the static-pressure readings taken closer to the fan outlets, as suggested by Professor Marks, as frequently this static pressure is the only usable pressure in an installation, and it is always desirable to get a better fan performance than expected rather than a poorer one.

On the other hand, fan manufacturers would probably like to see their fans given credit for performance under the most ideal conditions.

It is my opinion that only improvement in fan practice and design would result from adopting the suggestion of Professor Marks that the static-pressure readings be taken one diameter from the fan discharge. In the end this should be a benefit to all concerned, and should be looked at from a broad rather than a narrow point of view.

AUTHOR'S CLOSURE

The air flows shown in Figs. 9 to 14 of the paper have all the characteristics of a continuously distributed vorticity. The second type of vortex described by Mr. Hagen is a particular case of distributed vorticity. Any arbitrary continuous distribution of vorticity may appear in a fluid. A free vortex (where the vorticity is concentrated on a line) will, through the action of viscosity and turbulence, diffuse its vorticity through the fluid in a continuous way just as heat diffuses by conduction through a body. The distribution of angular velocity indicated by Fig. 14 suggests such a type of diffusion.

Mr. Hagen's experience with fans, which sometimes show good agreement between pitot-tube and nozzle-volume measurements and at other times show a 10 per cent discrepancy, would appear to result from the commonly observed instability of performance of certain fans. A traverse with a direction tube would presumably have given the explanation for the phenomenon.

The suggestion by Mr. Hagen that rotational flow has always been found when a direction-tube traverse has been made is probably accurate. In the double-inlet fan mentioned in the paper, two opposing vortices are set up which, at $2^1/_2$ diameters from the fan, have a maximum whirl angle of 22 deg. At 9 diameters distance the maximum angle has decreased to 5 deg, a magnitude which has no perceptible influence on the pitot-tube readings.

The author does not believe that substantially identical pitot tubes will register individual differences of velocity head of the magnitude apparently found by Mr. Hagen. The five tubes used by Mr. Hagen vary sufficiently in dimensions to justify the expectation of some appreciable differences in their indications. The dimensions of Mr. Hagen's tubes, as well as those of the A.S.H.&V.E. standard tube, are such as to give an appreciable error in velocity-head indications. Since the writing of this paper, investigations have been made which will permit the design of a pitot tube giving negligible error for a correctly oriented tube and avoiding appreciable differences between the indications of different tubes.

Variation in impact head in the author's measurements, which Mr. Hagen deduces from Figs. 2, 3, and 4 of the paper, represents probably some variation in operating conditions. The author was interested at first only in the variation of velocity head with yaw; the static pressures were obtained at a later date when the desirability of knowing them developed. The conditions for the two sets of observation were not identical.

In the early part of his paper, the author recommends the use of round ducts for accurate work; in the rest of the paper the use of a round duct is assumed. With a rectangular duct and whirling flow, the mean static pressure can be determined as in a round duct by traverses along two center lines with spacing of observation locations as in the usual rectangular-duct traverses.

H. S. Bean suggests that whirling flow is to be expected in a nozzle if the air approaches it with whirling flow. This is certainly the case and has been observed by the author. As the axial velocity is increased through the nozzle, approximately inversely as the ratio of areas, and the tangential component is unaffected, the angle of whirl should be reduced in the nozzle. This is precisely what was observed. The fan whose performance is shown in Fig. 11 of the paper was provided with a nozzle at the end of a 10-diameter discharge duct and a traverse was made with a direction finder of the air leaving the nozzle. The whirl angle was found to vary in a fairly uniform manner from zero at the center to about 7 deg at the periphery. For these angles the correction factor, Fig. 8, is practically unity.

The discussion of J. F. D. Smith is concerned with a point which is not touched upon in the paper. He is unquestionably right in his proposal if the mean velocity head is the quantity desired. However, it should be pointed out that the mean velocity is the quantity usually of interest to the fan engineer and that the useful product of the fan is the velocity head corresponding to the mean velocity rather than the mean velocity head. In other words, a flow of uniform velocity across the whole cross-section is preferable to the same total flow with a velocity which varies across the cross-section. The latter, however, would show the higher mean velocity head.

When the author stated that there was need of a new definition of velocity head, he had in mind the question of non-axial flow and not the variation of velocity across the cross-section as suggested by Ed S. Smith, Jr. Mr. Smith's discussion of this point is in effect the same as that of J. F. D. Smith and is answered above. The author agrees with his conclusion that the useful portion of the energy given to the air is that which can reach the point of application and be of service there.

The pulsations referred to by Mr. Smith are those resulting from instability in the fan performance which, as he says, is common in fans having the characteristics which he describes. The author has not made any reference to this kind of pulsation. His discussion applied only to the type of pulsation described in Mr. Hagen's paper,[5] which is thought to be present in all fans and at all times and which results presumably from the discontinuous discharge from the fan blades or other similar source. With the type of instability to which Mr. Smith refers there is no definite performance of the fan; the conditions in the fan are changing violently and erratically.

The author agrees with Mr. Peterson as to the greater reli-

[11] Chairman of the Board, Carrier Engineering Corp., Newark, N. J. Mem. A.S.M.E.

ability of the nozzle for volume measurements; in addition, the measurements consume much less time. As Mr. Hagen suggests, the development of rectangular sheet-metal nozzles may reduce the cost of nozzles so much as to remove the objection on the score of expense. The author is now investigating the rectangular sheet-metal nozzle to ascertain its coefficients and general reliability. Meanwhile, the use of egg-crate straighteners with an adequate length of duct and the proper specification of pitot tube should make pitot-tube volume measurements reliable.

The recent work on pitot tubes, to which the author has already referred in his reply to Mr. Hagen's discussion, does not entirely support the suggestions of Mr. Madison for increasing the accuracy of the pitot tube. The results of this new work will be published before long. Mr. Madison is quite right in stating that an egg-crate straightener will not equalize flow; it will only take out the rotational component. If the axial component of flow approaching the straightener is greater at the bottom of the duct than at the top, it will remain greater after passing through the straightener. It is also true that the equalization of flow takes place more rapidly when the air is whirling. Consequently, if equalized flow is desired, it is preferable to have a long run of duct preceding the straightener rather than following it. The experience of the author indicates that the increase in accuracy resulting from equalized flow before the pitot-tube station is very slight, but obviously there may be conditions of velocity distribution in which a pitot-tube traverse will not average the velocity with sufficient accuracy.

The advantages of rectangular ducts are well stated by Mr. Madison and on the score of cost and convenience, except for the long time required for readings, they are desirable. For accuracy, however, the author thinks that round ducts are preferable. If rectangular sheet-metal nozzles prove satisfactory, the use of rectangular ducts would certainly be indicated.

The Economics of Preheated Air for Stokers[1]

JAMES W. ARMOUR.[2] The authors, in their paper, by correcting for stoker area, ash content of the fuel, and the burning rate, have brought out very forcibly that preheated air is the major factor in causing high maintenance costs of stokers.

They have pointed out some ways in which maintenance with preheated air can be reduced and of these there are two in particular in which the stoker designer is interested. First, the design of the part itself and, second, the material which is used in the part.

Many operating companies attempt to reduce maintenance by buying cast-iron parts from a local foundry. This practice has hindered progress in solving some of the problems relating to stoker maintenance. The design of a part with reference to its maintenance is very important and is well illustrated by experiments which are being conducted in connection with the grate surface of the Harrington traveling-grate stokers.

A new grate surface was designed and installed on a stoker operating on anthracite and coke breeze. After a year's operation it appeared that these grate castings were an improvement over the old design from the standpoint of maintenance and appeared to have other desirable features with which we are not concerned as far as this particular discussion applies. As a result of this year's trial, we then undertook to install this grate surface in a number of other plants. One plant, burning bituminous coal on a traveling-grate stoker, had an installation requiring a rather high burning rate in quite a small furnace. Our grate castings lasted approximately one month. A change in design improved the life of these parts so that some of them have now been in service under exactly the same conditions in the same plant for nearly two years. This case is an exceedingly good illustration of what can be done on design alone, although such a vast improvement can hardly be expected in most cases.

The question of material has received rather lukewarm attention in the past. A number of so-called "heat-resisting" irons have been put on the market with rather indifferent results. We know it is not possible to add a small amount of alloying metals to cast iron and materially increase the melting point of the iron, so we rather look askance at any claims for inexpensive "heat-resisting" iron. It would be very desirable if castings, having a melting point higher than any furnace temperature which could be produced in a stoker-fired furnace, were available at a reasonable cost. Such material does not appear to be available at the present time at a price which would warrant its use.

There are, however, materials available which appear to give longer life and at a sufficiently reasonable price to justify further investigation. We are now conducting an investigation into the economics of using alloyed cast iron to determine just how much improvement can be made without increasing the cost beyond an economical point. This study has not progressed far enough to draw final conclusions, but it does show enough promise so you might be interested in a brief description of the trials.

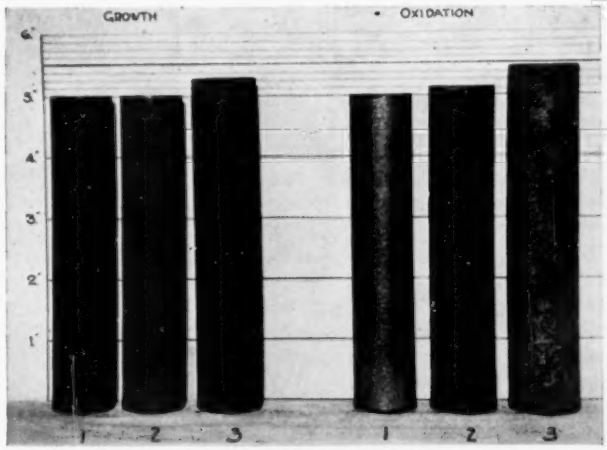


FIG. 1 THE EFFECT OF GROWTH, AND GROWTH PLUS OXIDATION ON TEST SAMPLES OF CAST IRON FOR STOKER PARTS

(Sample 1 is an alloyed cast iron costing about four times the cost of ordinary cast iron. Sample 2 is an iron which can be produced for approximately 25 per cent more than ordinary cast iron. Sample 3 is the ordinary run of cast iron.)

Realizing that fuel-bed conditions with underfeed stokers are very hard to maintain uniform and that burnouts of tuyère blocks may occur from causes having no bearing on the material of the tuyère blocks, we are confining our field experiments to a traveling-grate stoker where the fuel-bed conditions are fairly uniform. Aside from actual melting of the grate surface, we believe it is generally accepted that furnace castings break down primarily due to growth of the castings and rapid oxidation at elevated temperatures. Our experiments have, therefore, developed along the lines of obtaining growth-resistant iron and, in some cases, an iron which will resist oxidation. These factors may go together in the same iron.

Eleven different mixtures were finally decided upon for a trial, these selected mixtures being based on recommendations of

[1] Published as paper FSP-56-17, by R. E. Dillon an d M. D. Engle in the December, 1934, issue of the A.S.M.E. Transactions.

[2] Vice-President and Engineering Manager, Riley Stoker Corporation, Worcester, Mass. Mem. A.S.M.E.

various concerns and metallurgists interested in this particular problem. At the time the castings were poured test bars were also made, there being two test bars of each mixture. Enough grate castings of each mixture were made so as to cover a fairly large area, to eliminate the possibility of a condition which might occur at any isolated spot affecting the trial. All the various eleven mixtures were installed on one traveling-grate stoker. Such a stoker is quite suitable for this test for, as the grate castings travel through the furnace and then back again into the front, they are subjected to repeated heating and cooling which is conducive to obtaining the greatest amount of growth and oxidation. These grate castings are now in service.

The test samples have been subjected to laboratory tests which consisted of packing one sample in a box of iron filings so as to exclude the air and repeatedly heating it to 1500 F, and allowing it to cool. The other sample was subjected to the same heating and cooling process but in an oxidizing atmosphere.

The laboratory tests have indicated that it is possible to obtain a casting which has very little growth over a period of 50 cycles of repeated heating and cooling. As a rough estimate it is anticipated that castings can be produced for about 25 per cent increased cost which will give at least 50 per cent increased life. Such an estimate is subject to proof by the field tests which have not progressed far enough to draw any conclusions. The writer's Fig. 1 gives some idea of the effect of growth and growth plus oxidation of the laboratory samples.

Regarding the tuyères, it is possible to make them of ordinary cast iron with that part exposed to the fire of a more expensive heat-resisting iron. With such an arrangement it might be possible to use the higher alloyed irons without increasing the cost of the entire casting to a point where it does not become economical.

The authors have mentioned the use of water sprays under the stoker. Steam has also been used with some success, but the use of either water or steam means a reduction in operating efficiency and, while this may relieve an existing condition, it would hardly be desirable for application on a new installation. The use of water-cooled tuyères has been tried and shows some possibilities, providing the heat absorbed by the water can be economically recovered. All such devices add to the complications of stoker design which stresses the point that where high preheated-air temperatures are desired, high maintenance cost of firing equipment can be avoided by the use of pulverized coal.

JOHN VAN BRUNT.[3] While the general opinion has been that highly preheated air would cause higher maintenance on stokers, individual opinions differ as to what temperatures constitute highly preheated air. Some stoker manufacturers do not take exception to 400 F or even 500 F, while others are more conservative and set the limit at 300 to 325 F. It has been my personal judgment that 325 F should be the limit unless the operating company is willing to accept the higher maintenance that accompanies higher air temperatures.

This paper is particularly valuable in that it enables one to assign a specific figure to the maintenance cost due to preheated air instead of estimating or guessing. However, there are several thoughts and questions that came up as a result of a study of the author's curves, answers to which might be of general interest:

(1) In Fig. 1, why is the inference made that the maintenance is higher with 75-F air than with 200-F air?

(2) In Fig. 2, showing the relation of stoker area to maintenance, it is possible that the shape of curve *A* is due in part to the fact that the larger stokers are usually operated at higher combustion rates and with low-ash coal. If curve *B* were further corrected for ash content of coal the corrected curve should be flatter, as there appears to be no other logical reason why size alone should affect maintenance to the extent shown.

(3) The indications of Fig. 3 are logical, in that higher combustion rates are accompanied by higher fuel-bed temperatures.

(4) Fig. 4 shows expected results, since ash serves as a protection to fuel-bearing surfaces. The effect of ash content in coals used on chain-grate stokers is marked. The downward slope of curve *B* at the left may be questioned, inasmuch as it indicates that a zero-ash coal would be less damaging to grate than a coal with 8 per cent ash. An explanation of why this curve *B* touches zero at 14 per cent ash would be interesting.

(5) Curves *A* and *B* in Fig. 5 do not compare with the accepted understanding of the effect of air in cooling the grate. One would expect curve *B* to start at the left at about $7^1/_2$ cents.

(6) The values indicated by the points of Fig. 7 are probably due to the higher-volatile coals which are usually higher in ash than low-volatile coals. Perhaps the higher percentage of fixed carbon or coke in the low-volatile coals may be a factor.

(7) In Fig. 10, summarizing the conclusions, curve *B* shows the same characteristic rise at low air temperatures as curve *A* but to a greater degree; this I think requires further explanation, as I do not think it is safe to infer that 200-F air will result in lower maintenance cost than 100-F air, to the amount of about 2 cents per ton. Logically, curve *B* should start at zero cents at 50 F and rise in a straight line tangent to curve *B* as plotted at about 235 F.

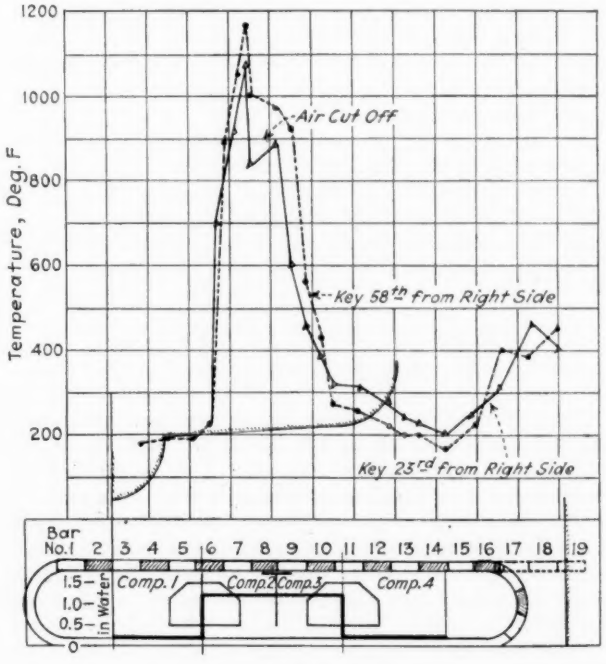


FIG. 2

The explanation of these peculiarities may lie in factors that were not available to the authors, such as banking hours, sudden dropping of load, composition of ash, fusion temperature of ash, and differences in operating.

Some time ago we had occasion to investigate the temperatures of the grate surface of traveling-grate stokers. Fig. 2, in this discussion, shows the temperatures reached by the stoker keys at various points along the travel of the grate. The maximum temperature, it will be noted, occurs at the point where ignition has penetrated through the fuel bed to the grate, which

[3] Vice-President in Charge of Engineering, Combustion Engineering Company, Inc., New York, N. Y. Mem. A.S.M.E.

is about 40 in. from the front end of the stoker. As soon as ash starts to form, the grate becomes protected from the incandescent fuel and the temperature rapidly drops to 300 F, and falls still further as the fuel bed is burned out.

From this curve it is easy to understand why preheated air rapidly increases the maintenance. With 300-F preheated air, the temperature of the stoker grate at the point where it enters the furnace would probably be in the neighborhood of 200 F higher, and the peak temperature would probably also be 200 F higher. The base of the peak would be broader, consequently the key would be subject to a higher temperature and for a longer period of time. It would drop to about 500 F at a position corresponding to 300 deg on the slide. Our experience indicates that a short peak temperature of 1000 F or thereabouts does not result in unusually high grate maintenance, but if for any reason the high temperature is sustained for any considerable time the maintenance increases very rapidly.

M. J. Selaya.[4] The maintenance costs and comparisons set down in the paper deal with equipment dating from 1925 up to about 1931. Boiler-room installations of eight and ten years ago were vastly different in construction and design from the modern equipment of today. As a result of research and development, the modern underfeed stokers operate with much lower maintenance costs than the author's general figures indicate.

Advancement in the use of preheated air for stoker-fired boilers has been somewhat retarded during the last three or four years. The authors might well have mentioned that retrenchments during this time by operating companies and others who in the past have shared in the development of boiler-room equipment, have limited comparisons, especially for the higher preheated-air temperatures.

Inasmuch as records of stoker maintenance by different companies are not kept by one set method, a general breakdown of the figures given in the paper would indicate other conclusions than those presented. Station labor can either be direct on the stoker involved or the average for the station which would include labor and overhead, even at times when no replacements are necessary. One case is known where an operating company uses a fixed labor charge of 10½ cents per ton for maintenance. It is suggested that due to the many methods of bookkeeping in use, the material cost only would be a truer index for comparisons.

In 1925 an underfeed stoker was built and installed for 500 F air temperature. In an analysis covering an eight-year period, the maintenance costs were divided into 64 per cent for material and 36 per cent for labor. Another installation with the same 500-F air was made to demonstrate its ability to burn some of the cheaper coals carrying ash with low-temperature softening characteristics. The maintenance on this stoker over an 18-months period, using the same fixed station-maintenance charge as on the older stoker, was divided into 49 per cent for material and 51 per cent for labor. The maintenance on extension grates, which was excessive on the first stoker, was eliminated to negligible figures through the use of a new material, and the total maintenance on material was reduced by 50 per cent over the older stoker. It is worthy of note that the fixed labor charge per ton is more than the material charges.

Stoker designers have compensated for the reduced temperature differential between the air and the iron, and they have studied the composition of the metal to eliminate cracking and growth. One case is known where a 75 per cent reduction in maintenance cost was effected by the exclusive use of manufacturer's parts. It is probable that this item would affect the figures shown by the author.

[4] American Engineering Company, Philadelphia, Pa.

The authors offer a conclusion or prediction that up to 300 F the preheating of air supplied to underfeed stokers does not seem to increase maintenance above reasonable figures, but that there is a sharp increase above this point. The data available beyond 350 F is limited, and for this reason should not be taken too seriously. When the opportunities for studying the effects of high-temperature preheated air are exhausted, the maintenance figures will be lowered.

The opinion is offered that correct operation is an important factor. This is well demonstrated by a plant which burns low-grade fuel up to 50 lb per sq ft of projected grate area with air temperatures up to 400 F, yet has a maintenance cost on material of 2 cents per ton. This raises the question as to whether some of the figures quoted by the author do not include charges for experiment and test periods during which the equipment was subjected to unusual conditions, or times when the operators were inexperienced.

The main topic of this paper deals with the effect of preheated air on the underfeed stoker, although figures are presented on the chain-grate stoker. This is an unfortunate comparison because in most cases, chain-grate stokers burn coals having high ash content peculiar to certain localities. Underfeed stokers burn a great variety of coals. Studies indicate that no definite usable data can be derived from comparisons given for the different types of stokers. A direct comparison can only be made by burning the same classes of coal on the different types of stokers.

George C. Eaton.[5] At the Edgar Station, North Weymouth, Mass., experience has shown that air temperatures in excess of 400 F are very damaging to a stoker because the distortion which results from such temperatures causes the stoker parts to crack. This appears to be consistent with the data plotted in Fig. 8.

During a test at the Edgar Station, a coal with approximately 11 per cent ash was burned and it was noticed that the stoker was damaged less than when New River coal containing 4 per cent to 6 per cent ash was used. On the other hand, we apparently had less stoker maintenance during a test run when mixtures of petroleum coke and New River coal were burned, the ash content of the mixtures being between 3½ and 4½ per cent. These apparently opposite results are for but very short periods of time and are by no means reliable, although Fig. 4 seems to give them foundation.

Considerable success has been attained at the Edgar Station in reducing stoker-maintenance costs by the use of ribbed side-plates to prevent warping, and by the development of an improved extension-grate design embodying a fixed and a moving section, no part of the grate extending at any time over the shingle grate.

J. S. Bennett.[6] Although this paper will probably result in engineers making a more careful analysis of the high maintenance cost of stokers, it unintentionally might have the effect of discouraging the use of higher preheated-air temperatures by continued reference to past performances without stressing at the same time the value of study and effort to solve the problem.

Stoker manufacturers have been considering this problem for a number of years and many innovations have been introduced to reduce maintenance. One of these is a water-cooled stoker in which the tuyères and overfeed sections are converted into air-admitting water-wall areas connected to the boiler circulation. Four of these stokers are in regular service and have

[5] Head of Mechanical Technical Engineering Division Generating Department, The Edison Electric Illuminating Company, Boston, Mass. Jun. A.S.M.E.

[6] Mechanical Engineer, American Engineering Company, Philadelphia, Pa. Mem. A.S.M.E.

burned 11,000 tons of Illinois coal with an ash-fusing temperature below 1900 F. None of the water-cooled areas of the stoker have shown any signs of overheating. As a result of such developments, greater advancement along this line can be expected, all of which will result in reduced maintenance cost of firing equipment that is exposed to high preheated-air temperatures demanded by modern equipment.

T. E. PURCELL.[7] The authors have developed a curve showing the relation between air temperatures and stoker-maintenance costs. However, even after several corrections have been applied to the data, the costs at 350 F of preheated air, for example, still vary by more than 15 cents per ton. This shows the inconsistency which still prevails in the stoker-maintenance-cost situation.

Stoker-maintenance costs as used in the paper apparently include all costs of repairing the entire stoker and drive whether or not the repairs can possibly be affected by preheated air. We should be mostly concerned with the effect of the hot air on the maintenance of the grate section where the damage actually takes place rather than on the entire installation.

In addition to the factors mentioned in the paper, there are others which have significant effects on the final cost. Among them are the design and age of the stokers, the fusion temperature of the ash, sulphur content of the coal, the local labor rate, the cost differential between materials purchased locally or from stoker manufacturers, banking boilers, the skill and alertness of the operators, etc. We cannot hope to correct for all these factors but nevertheless their existence complicates the problem.

The paper presents corrections for some factors, among them being grate area and burning rate. The trends in the correction curves are logical, but I wonder if the curves have any real significance. Incidentally I may add that, owing to better design, the largest stokers our company operates have the lowest maintenance cost even though the average burning rate is higher than it is on the smaller stokers. Perhaps if the cold-air stokers as well as the preheated-air stokers had been used in developing these correction curves, more points would have been obtained to confirm or perhaps to contradict their trend.

There is some satisfaction, however, in the fact that the uncorrected curve for maintenance cost vs. air temperature does not differ greatly in slope from the one corrected for many of the variables. On the other hand, it is disturbing that design decisions must still be based on incoherent data, even though the corrected curve is used.

R. L. BEERS.[8] It seems to me that the authors do not go into sufficient details as to the fuels being used. It is generally conceded that chain-grate stokers are widely used for free-burning, non-coking, high-ash coals, such as those mined in Illinois and Indiana; and they are not suitable for the coking, low-ash fuels such as those mined in West Virginia, Pennsylvania, and Eastern Kentucky. For the latter fuels, the underfeed stoker has proved without question to be best suited. The paper does not state whether any of the chain-grate stokers were burning low-ash Eastern fuels, but if data on such plants were available it is believed that the maintenance costs would be very much higher than for the underfeed and the efficiency would be much lower. With a low-ash, low-fusing-and-coking coal a chain-grate stoker will not stand up even with air at room temperature unless a high percentage of excess air is allowed to pass through the fuel bed at the rear end. Preheated air and a chain-grate stoker with this fuel would make an unsatisfactory combination.

It is difficult to make comparisons on the results of stoker installation as there are so many variable factors that enter. Two plants, exactly alike, will produce different results. The skill of the operators, load conditions and variation in the fuels may give entirely different results. The ash percentage is not always the important point. The burning characteristics of the fuel, the fusing temperature and quality of the ash are also very important. To make the comparison just, the two types of stokers should be operating under the same types of boilers with equal load conditions and with the same fuels.

It seems to me that the maintenance figures given for the underfeed stokers are very high. Possibly there are some factors entering that are not given in the paper. Quite frequently most of the damage to an underfeed stoker occurs during short periods and not under normal operation. This might occur when starting up quickly from a banked fire without having the retorts filled with green coal or in carrying very high peak loads.

The maintenance is no doubt important but there are other items, such as efficiency and flexibility that may more than offset this item. Furthermore, in making maintenance comparisons, the maintenance of ignition and mixing arches, not used with underfeed stokers but required by chain-grate stokers, should be included. The maintenance of these arches may be an appreciable item. Also, they cut down the radiant-heat transfer from the fuel bed to the boiler tubes, tending to lower the boiler efficiency and capacity.

H. E. KLEFFEL.[9] In order to simplify the problem of ascertaining the main factors that contribute to the maintenance costs of stokers, I would suggest we use the amount of fixed carbon burned per square foot of grate surface per hour as a basis of comparison, correcting for area of grate surface and temperature. This, I believe, would be a better basis of comparison than using the pounds of coal burned per square foot per hour, the per cent volatile, per cent ash, and the excess air.

The authors have indicated that stations burning coals high in ash and volatile content appear to have lower maintenance costs than stations burning coal with low-ash and volatile content. This may be explained by the fact that with coals high in ash and volatile there is less fixed carbon in the coal to be burned on the grate surface, the volatile being distilled at relatively low temperatures and burned in the furnace chamber, the coke being carried along with the ash burning on the surface of the grate until the carbon content is consumed, the ash affording protection to the grate surface, if it does not become molten. Ash having a relatively low fusing temperature if allowed to become molten will contribute to higher maintenance costs and stoker outage, this being aggravated by high air temperatures.

If the use of the fixed carbon burned per square foot of grate surface per hour, as a factor shows a logical relationship between maintenance and air temperature, it might be of further interest to make a comparison between the maintenance of chain-grate and underfeed stokers on this same basis, using the effective grate area. By that I mean the total grate area through which air passes for combustion, for underfeed stokers being approximately 80 per cent of the projected area and for chain grates over twice the projected area of the fuel bed.

This may afford a clue to the effectiveness of alternate cooling for the large-area chain-grate surface and also indicates the high duty imposed on the underfeed-stoker's stationary-grate surface at high combustion rates and provides data for the stoker designer for future consideration.

[7] General Superintendent of Power Stations, Duquesne Light Company, Pittsburgh, Pa. Mem. A.S.M.E.

[8] Vice-President in Charge of Engineering, Detroit Stoker Company, Detroit, Mich. Mem. A.S.M.E.

[9] New York, N. Y. Assoc-Mem. A.S.M.E.

I. E. MOULTROP.[10] It has been rather disappointing to find that there is a definite upper limit to the preheating of air if the maintenance costs are to be kept within reasonable bounds. When the design of preheaters reached a practical stage, it followed that the way was clear for the station designer to go to almost any limit in forcing the capacity of the stoker and boiler in order to keep down capital expense without sacrificing too much in heat rejected in the stack. Now we find that this hypothesis is not true and the station designer must very carefully consider this point in planning a low-cost, economical station.

In reference to maintenance expense, I believe that some of this is due to lack of induced-draft fan capacity. Many station designers in the past have erred in not installing sufficiently large induced-draft fans. Now that we have substituted a fan for the chimney, the fan is the limiting factor and if the forced-draft fan is run beyond the capacity of the induced-draft fan the result is bound to be a certain bottling-up of heat in the furnace which must be detrimental to both the metal parts of the stoker and the refractories of the furnace.

Here is a most urgent need for intensive research work to produce a material to be used in those stoker parts subjected to the hot air and heat from the fuel bed, which will stand up for a reasonable length of time and which will not clog up with slag adhering to the upper surface of the tuyères, and which will be reasonably cheap in first cost.

If we believe some of our optimistic business friends, the time is coming very soon when the utility companies will have to think about increased capacity and it would be a pity to have to use stokers handicapped by an absurdly low limit of air temperature.

B. C. MALLORY.[11] It is interesting to note that the maintenance of stokers with cold air is as much as with average air temperatures of 300 F. It would be interesting to know the average length of time the various stokers ran continuously for it seems that this may have a bearing on the maintenance cost. In our case, each stoker had three outages during the year 1933. That meant that whenever the stokers were out of service all parts that did not appear able to stand four months more service had to be renewed.

We have found that in changing from one kind of coal to another, it may be necessary to carry a differently shaped fuel bed, and that maintenance costs go up until the new coal has been burned for several months. Such an item should be considered as a charge against the possible saving in the difference in coal prices.

Current Practice in Pressures, Speeds, Clearances, and Lubrication of Oil-Film Bearings[1]

ALBERT H. DALL.[2] The author's pressure-velocity diagram is an ingenious summation of current practice as related to Mr. Kingsbury's optimum data on 120-deg bearings.[3] The viscosity alignment charts, together with Mr. Kingsbury's data, constitute design criteria sufficient for any oil-film bearings operating at optimum conditions of load or friction. However, the designer must choose the minimum film thickness and from this choice determine the bearing clearance and the viscosity of the oil to be used. It is regarding this choice of minimum film thickness that the writer wishes to cite a few experimental facts which it is hoped will be pertinent to this discussion.

It has been stated repeatedly in bearing literature that the minimum oil-film thickness must be governed by practical considerations. These considerations may be summed up briefly as being the initial degree of perfection of the two surfaces involved, and the distortion of these surfaces by whatever stresses may be set up in use. Another deciding factor is that of maintenance or decay of the quality of these surfaces with usage.

Recent advances in the art of precision grinding have made it possible commercially to produce finishes on journals, which are comparable to lapped surfaces. Such surfaces are now being produced in mass-production industries such as the automobile industry. Very accurate and smooth bearing surfaces can now be produced by diamond boring or scraping. The initial quality of the surfaces can therefore be attained without excessive cost. The maintenance of these surfaces is essential if a small minimum film thickness is chosen.

Experiments at the Cincinnati Milling Machine Company have shown that the use of the so-called high-lead bronzes assists greatly in the maintenance of the original surfaces even when used with comparatively soft-steel journals.

The process by which this combination of materials tends to maintain, and in some cases improve the quality of the surfaces, can be explained on the basis of chemical facts. Mineral oils in general are neutral. However, it has been found that the acidity of oils increases with use. This acid reacts with the free lead in the bearing to form a metallic soap which coats the surface of the journal and bearing. This coating protects the surfaces when contacting occurs for short periods, such as starting, instantaneous over-loading or in short periods of oil deficiency. The production of sludge is very slight except in cases of excessive moisture. Thus the sole advantage of the so-called extreme-pressure lubricants is partially realized without any of the inherent disadvantages of this type of lubricant.

If the bearing is designed so that the distortion in service is of a negligible order, the use of the foregoing methods of producing and maintaining the surfaces may well modify the past practice of bearing clearance. As the author points out, the coefficient of friction and, therefore, the power loss varies directly with the minimum film thickness for any bearing running under optimum conditions. It is obvious, therefore, that any means which tend to decrease this minimum will be desirable.

AUSTIN KUHNS.[4] The oil-film type of bearing is astonishingly reliable and efficient. It is capable of excessive overloads and considerable misalignment and abuse without signs of distress, provided it is supplied with sufficient lubrication.

As a result there is a wide diversity of opinions by different students about the best proportions for these bearings, every one of which is based upon the irrefutable argument of successful operation. The power losses through the bearings are so small and so difficult to measure that the relative merits between them can hardly be distinguished.

It is the belief of the writer that Mr. Howarth's work will result not only in simplification and standardization but that it will lead us to better designs, especially to shorter bearing lengths which can compete more successfully with the antifriction bearing. Every experienced engineer has observed oil-

[10] Chief Engineer of Company and Superintendent of Construction Bureau, The Edison Illuminating Company of Boston, Boston, Mass. Mem. A.S.M.E.

[11] Technical assistant to the Chief Engineer, Lynn Gas & Electric Company, Lynn, Mass. Jun. A.S.M.E.

[1] Published as paper MSP-56-2, by H. A. S. Howarth, in the December, 1934, issue of the A.S.M.E. Transactions.

[2] Engineer, Research Department, Cincinnati Milling Machine Company, Cincinnati, Ohio. Jun. A.S.M.E.

[3] "Optimum Conditions in Journal Bearings," by A. Kingsbury, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-7.

[4] Farrel-Birmingham Company, Buffalo, N. Y.

film bearings carrying extraordinary loads and has been led to the conclusion that the sizes used today, even though considerably reduced from the standards of a decade ago, are far larger than necessary.

The writer recalls one gear drive which he inspected a short time ago which had been "starved" of oil to the point where the bearing metal had melted and run away except for a few small fins which, due to subsequent copious lubrication, had supported the journal loads for a considerable period of time. Each high speed bearing was $3^1/_2$ in. in diameter rotating at 500 rpm resulting in a rubbing speed of 458 fpm. The bearing pressure figured approximately 3500 lb and the combined length of the unmelted babbitt strips did not exceed $^3/_4$ in. The writer has records of a number of installations which because of changes in driving units have been operating continuously under overload conditions of 100 to 300 per cent. In no instance have the bearings failed.

It is the hope of the writer that Mr. Howarth will not only place great weight on his emphasis of the absolute importance of correct and sufficient lubrication but that he will be able to simplify the specifications for the lubricating oils and establish an expression which will include the desired characteristics in terms of fundamentals which will avoid the use of trade names. Such a specification cannot be expressed in words of one syllable but the importance of reliable and efficient bearings and compactness in design justifies its study by intelligent engineers and their familiarity with its use in practice.

From a study of the material given in Mr. Howarth's paper and a comparison with our own practice, it is clear that the journal clearances which we use are less than the average. As manufacturers of gear and power-transmission products, the proportions and clearances of our bearings are adjusted for that particular class of service. Also, for convenience and as an aid to standardization we use as far as possible bearings for which we already have drawings and patterns.

The average clearance which we use for bearings larger than 3.5 in. is 0.001 in. per in. of diameter. For the smaller bearings, the clearance is specially selected for the particular type of service.

Our own specifications of the lubricating oil are today identified by S.A.E. numbers. Selection of the oil is based more upon a consideration of the gear-tooth loads than upon bearing clearances and rubbing speeds. For the quantity of oil required we specify as a minimum 0.007 gal per min per in. of projected area.

Generally speaking S.A.E. 40 is used with bearings over 8 in. in diameter and S.A.E. 30 for bearings under 8 in. in diameter. For the higher speeds S.A.E. 30 is used entirely.

Naturally we have given considerable thought to the supply and distribution of the lubricating oil to the bearing surfaces. We are confronted with different requirements imposed upon us by practical consideration of our designs, and must arrange our bearings for ring oiling, collar oiling, gravity or pressure circulation as the individual installation requires.

It is comparatively simple to deliver the lubricating oil to the bearings, but it requires careful study to distribute this oil from the entering ports to the areas of contact or pressure. This distribution is accomplished by the action of the rotating journal in the bearings. It is assisted by cutting grooves or reservoirs with long sloping chamfers ahead of the pressure areas. The arrangement of these grooves and chamfers has provided an outlet for a great deal of practical ingenuity in design. Every arrangement is reliable and effective if it distributes the oil in sufficient quantity and facilitates its pick-up by the rotating journal.

In permissible bearing pressure, expressed in pounds per inch of projected area, consistency in our designs is confused by the necessity for standardization and utilization of existing drawings and patterns.

For the heavy-duty bearings where the rubbing velocity is below 2000 fpm the pressure per inch of projected area lies between 60 and 80 lb, although occasionally for practical reasons pressures as high as 100 lb are used. The usual method of lubrication is by ring or by collar.

At the higher velocities where positive oil circulation is included and where the rubbing speeds are well in excess of 2000 fpm the bearing pressures per inch of projected area are seldom under 120 lb and are frequently as high as 160 lb.

The relation between bearing length and diameter is influenced by the dimensions of the supporting housing. Usually the length is 1.5 times the diameter for the slower speeds and from 1 to 1.25 times the diameter for the higher speeds.

In discussing the design of oil-film bearings, it frequently is customary to ignore the rigidity of the housing in which the bearing shells are fitted and the accuracy with which the entire unit is machined. Presumably it is assumed that these items will be as nearly perfect as possible but unfortunately experience has shown that serious misalignment of the bearings occurs under operating conditions due to weakness in the housing. As bearings become shorter this tendency to weakness is increased.

In closing, the writer wishes to point out the rapid reduction in the viscosity of the lubricating oil with the rise in bearing temperature. The temperature usually falls between 130 F and 150 F which introduces a wide variation in operating conditions. It is the writer's opinion that far too little consideration has been given by designers to the close control of this factor of temperature.

E. S. Pearce.[5] Our research in the bearing field has been confined to the railway-car journal bearing. This bearing belongs, for want of a better classification, to the no-clearance, single-directional-load class of bearings.

In our five years of work we have found so much that was contrary to the accepted theories and practices concerning the design, operation and maintenance of this type of bearing that we feel quite sure the margin of performance between what is now being obtained and what could be accomplished is great.

Analyzing the paper by Mr. Howarth in the light of the foregoing experience, it seems quite logical to conclude that the situation we have found is not unique to the particular field of bearing application which we have been investigating. Practices and theories of design seem to be predicated more on opinion than on facts. While the bearing is a vital part of the construction, the design has hinged more on what has worked before than on what could or might be found to work.

Fig. 1 in the author's paper, when analyzed from the standpoints of the railway-car bearing, brings to light some very interesting comparisons. For example, in the case of the $5^1/_2 \times 10$-in. journal bearing supporting a total load of 16,375 lb, we find that the total bearing area could be a maximum of 45 sq in. If we applied the author's method of calculating projected area for arriving and the value P, pressure in lb per in., this area would be $49^1/_2$ sq in. But on account of the fact that the bearing is an arc of less than 180 deg, the area can never exceed 45 sq in.

With this area of 45 sq in. and the total load of 16,375 lb, P, the pressure in lb per sq in., is 364 lb. V, the rubbing speed in feet per minute, has a value of $134^1/_2$ fpm at 10 mph, and 941.5 fpm at 70 mph of a 36-in. car wheel with a $5^1/_2 \times 10$-in. journal. Under this comparison, the bearing would fall in the group of

[5] President, Railway Service & Supply Corp., Indianapolis, Ind. Mem. A.S.M.E.

stationary-engine bearings at 10 mph, and in the group in which it is shown, railway cars, at 30 mph and above.

This bearing with 45 sq in. and a unit load of 364 lb per sq in. at 10 mph has a coefficient of friction of 0.00610. Expressed in another way, it offers a rolling resistance of 1.86 lb per ton. This same bearing by special construction limited to a crown area or bearing area of $22^1/_2$ in., or a unit load of 728 lb per sq in. at a speed 70 mph with a velocity of 941.5 fpm, has a coefficient of friction of 0.00449, or a rolling resistance of 1.38 lb per ton. This bearing under its high loads and high speeds would then fall in the class of gas-engine main bearings on the chart of the author's Fig. 1. Under both conditions cited, the performance of the journal bearing is not a limiting factor in the operation of the unit equipment of which it is a part.

The above comparison by no means represents extremes. The same car-journal bearing may have to operate at pressures as low as 100 lb per sq in. and at journal velocities under 134 fpm. Under another condition it may operate in excess of 1000 lb per sq in. and at journal speeds of 1000 fpm or any combination of the two. Whereas, in all other groups of bearings illustrated in Fig. 1, the bearings operate under more or less constant loads at constant journal speeds.

F. P. Dahlstrom.[6] In the determination of bearing capacities and clearances we have profited much by the study of papers and bibliographies submitted to the Society by the members of the A.S.M.E. Special Research Committee on Lubrication. In addition, we should also include those submitted by the individual members to other publications, for example, the *S.A.E. Journal*, and the Bulletins of the Pennsylvania State College.

It is realized that most of the papers so mentioned deal with hydrodynamic principles of bearing operation, that is, with conditions which occur when the shaft is rotating in its bearing.

While much has been done in the study of boundary conditions of lubrication, in which the phenomena are determined solely by the nature of the lubricant and the component bearing materials, the results cannot yet be used by those who design industrial bearings. For example, regardless of the intrinsic lubricating value of many lubricants, we are limited in the case of automobile, Diesel-engine, turbine, and rolling-mill bearings to the use of an oil that is stable and that will separate readily from water. So far as the writer knows, the only available lubricant which meets this requirement is a highly refined mineral oil.

In determining the safe carrying capacities of bearings we agree with many other designers that the minimum oil-film thickness is the important criterion. Methods of its calculation have been used by us based on the data submitted by Professor Karelitz and the members of Mr. Kingsbury's organization.

From the design data which has been submitted, we note that in many cases the clearance ratio varies from 0.001 to 0.002. These values are the result of many years of experience. From calculations which we had made of minimum oil-film thickness resulting from various clearance ratios it seems that the clearance ratios noted are also justified by theory.

Having established a clearance ratio (and assuming that the usual L/D ratio is 1) our bearings are now geometrically similar. This statement leads logically to the pioneer work of M. D. Hersey in which he showed that the performance of geometrically similar bearings could be compared by a characteristic number now recognized as ZN/P. Subsequent careful work by S. A. McKee and T. R. McKee at the Bureau of Standards showed consistently that the breakdown of the oil film occurs when the value of ZN/P is less than a certain amount. The actual value in part depends on the bearing metal used and may be 5, or less. In determining the safe carrying capacity of a bearing it seems reasonable to state that the minimum oil-film thickness will be sufficiently large if the operating characteristic ZN/P is greater than 10, where Z is the viscosity in centipoises, N the speed in revolutions per minute and P the unit bearing pressure in pounds per square inch of projected area.

As one possible suggestion for future work: In so far as the hydrodynamic theory of lubrication of journal bearings is concerned all investigators are dealing with the same phenomenon, that is, one cylinder rotating within another. It would thus seem that several of the results of experimental work done to date might still be further correlated. (After hearing the papers and discussions presented at the 1934 A.S.M.E. annual meeting, the writer realizes that this is considered important by others.) As an example of what is meant, the writer calculated the theoretical eccentricity of a journal bearing for various values of ZN/P. For each value of eccentricity the frictional resistance of the bearing was next estimated. A curve of coefficient of friction vs. ZN/P was then plotted and this curve much resembled those obtained experimentally by S. A. and T. R. McKee.

In presenting the chart shown in Fig. 1, Mr. Howarth has given the engineer a very useful tool for checking his bearing designs.

The values of P and V for a given condition are first located on the chart and from this located point a diagonal line is extended to the scale for Mr. Kingsbury's optimum conditions. One interesting conclusion is now apparent. Having established the optimum condition, the value of V can be altered at will, and if we alter the value of P accordingly, the points so obtained will lie in the same diagonal line. This means that for any given optimum condition the capacity of a bearing is proportional to, that is, it increases with, the speed.

The diagonal lines in the chart which give the product of P and V are at right angles to the construction line noted for determining optimum conditions. This means that if $P \times V$ is assumed to be constant, the capacity of a bearing decreases with the speed.

This second statement is directly opposite to the first. Mr. Howarth has stated in his paper that the PV rule by itself has little to recommend it.

The rule, PV = a constant, was proposed by Thurston seventy years ago. Thurston's constant, 60,000, was based on a bearing coefficient of friction 0.10. Thus, the formula means that, with a constant load, the size of the bearing must be increased as the speed is increased in order to dissipate the heat developed, and, judging from the coefficient of friction, the formula is not one for designing bearings, but really is one for designing brake-shoes.

The chart which Mr. Howarth has shown, which is obviously the result of much study, will be of much value to us in designing lubricated bearings.

George B. Karelitz.[7] Mr. Howarth undertook a formidable task, indeed, in attempting to collect and correlate the bearing practice of manufacturers of various types of machinery. It is interesting to note that experience, as well as trial-and-error method which have prevailed in designing bearings, often brought results in agreement with those which might be recommended on the basis of the hydrodynamic theory. For instance, the rule making the clearance in high-speed bearings equal to $0.002D$ was established by practice. It can be observed from data on friction that with an increase of clearance the friction decreases substantially up to a clearance value of $0.002D$, after which the gains in friction are not great; at the same time, the minimum oil-film

[6] Engineer, Morgan Construction Company, Worcester, Mass.

[7] Professor of Mechanical Engineering, Columbia University, New York, N. Y.

thickness does not vary appreciably with the ratio of the clearance to diameter in the range from 0.001 to 0.002.

Referring to Table 5 in the paper, on the basis of the computations given by Boswall, and side-leakage correction factor by Michell, a convenient formula for minimum oil-film thickness in thrust bearings can be derived:

$$h_o = \sqrt{\frac{1}{15}\frac{\mu U l}{P}}$$

where h_o is the minimum film thickness in in.
μ is the absolute viscosity of oil in lb sec per sq in.
U is the surface speed in in. per sec
l is the length of shoe in in.
P is the load in lb per sq in. of shoe

This formula is in close agreement with film-thickness values given in Table 5.

Referring to the desirable practice of selecting oils by absolute viscosity rather than by commercial viscosity, it will be necessary to produce first a reliable and simple instrument to measure the absolute viscosity. The latest attempt to bring such an instrument on the market was described in *The Engineer* (London) vol. 158, July 27, 1934, p. 95. However, until an instrument comparable in simplicity to a Saybolt viscosimeter is available, a serious and practical obstacle will stand in the way of abolishing the Saybolt viscosities.

EVERETT M. BARBER.[8] Mr. Howarth has assembled a wealth of useful and practical data on bearing design and practice. In Fig. 1, he has so correlated these practical data with the hydrodynamical theory of lubrication that it is possible to determine the bearing proportions and oil viscosity that are required to give optimum bearing performance over a wide range of load and speed conditions.

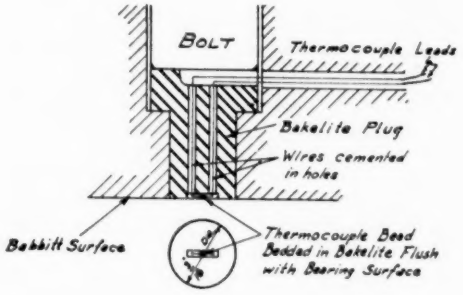
The selection of a lubricant that will have the proper viscosity at the running temperature of the bearing is of fundamental importance in the application of these data. To aid in this selection, the author has provided a very satisfactory method of obtaining the commercial viscosity specifications for the lubricant when the required absolute viscosity and the running temperature of the bearing are known. The required absolute viscosity is readily obtained from the author's Fig. 1. However, the running temperature of the bearing, i.e., the temperature at which the lubricant must have this viscosity, must be obtained from direct measurements on the bearing in question, or from experience with other similar bearings.

In speaking of the viscosity corresponding to the running temperature of the bearing, the author, of course, means the viscosity corresponding to the temperature existing in the load-carrying oil film in the clearance space between the bearing and the journal. This temperature is difficult to measure. However, it is the temperature at which the lubricant is used; and if reliable results are to be obtained, it is important that the oil viscosity be chosen so that it will correspond to this temperature.

Several years ago while engaged in a lubrication-research project at the Engineering Experiment Station of the Pennsylvania State College, the writer attempted to measure the temperature in the oil film of an experimental bearing.[9] Two methods of temperature measurement were employed, as follows:

(1) A copper-constantan thermocouple, insulated in a bakelite plug, was placed in the bearing so that the junction of the couple was flush with the babbitt surface of the bearing. In fact, the plug which was $^3/_{16}$ in. in diameter, and the thermocouple bead which was bedded into the face of it, formed a part of the bearing surface.

(2) A mercury thermometer was inserted in an oil-filled thermometer well which was so placed that the bulb of the thermometer was about $^5/_{16}$ in. from the babbitt surface of the bearing.



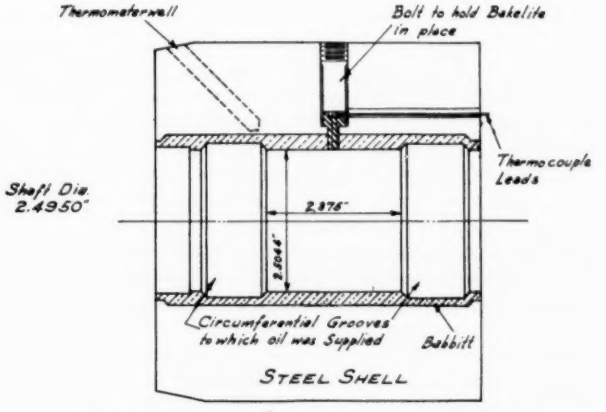


FIG. 1 SCHEMATIC DIAGRAM OF TEST BEARING

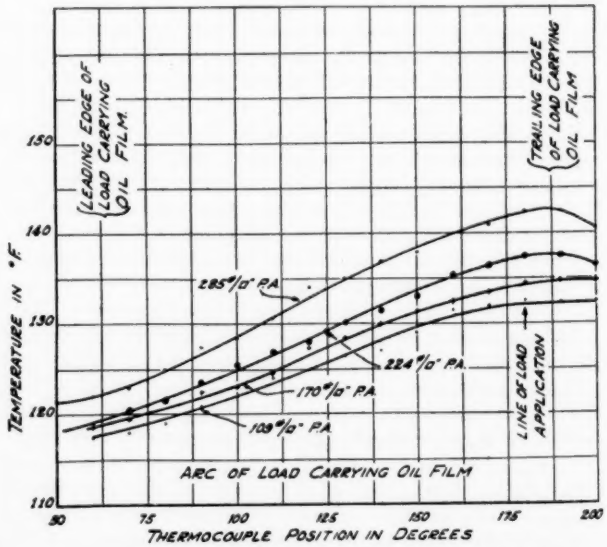


FIG. 2 TEMPERATURE MEASURED BY THE THERMOCOUPLE PLOTTED AGAINST THE CIRCUMFERENTIAL POSITION AT WHICH THE TEMPERATURE MEASUREMENTS WERE MADE

(The temperature was measured by a copper-constantan thermocouple insulated in a bakelite plug in the test bearing. Viscosity of test oil: 320 Saybolt sec at 100 F; 48 Saybolt sec at 210 F; A.P.I. gravity 22.6 at 60 F. Speed, 200 rpm.)

[8] Mechanical Engineer, The Texas Company, Beacon Research Laboratory, Beacon, N. Y.

[9] "Investigation of Journal Bearing Performance," by E. M. Barber and C. C. Davenport, Bulletin No. 42, The Pennsylvania State College.

TABLE 1 COMPARISON OF TEMPERATURES AND CORRESPONDING VISCOSITIES MEASURED BY THE THERMOMETER AND THERMOCOUPLE IN THE TEST BEARING UNDER A WIDE RANGE OF OPERATING CONDITIONS

Test number	1	2	3	4	5	6	7	8	9	10	11	12
Speed (rpm)	800	800	800	800	1600	1600	1600	1600	2400	2400	2400	2400
Load (lb/sq in. of projected area)	109	160	209	281	106	127	188	244	109	170	224	285
Eccentricity factor	0.798	0.855	0.880	0.915	0.770	0.825	0.859	0.880	0.750	0.787	0.817	0.862
Thermocouple temp:												
Leading edge, F	100	101	102	103	119	115	120	124	121	123	123.5	126
Trailing edge, F	113	115	116	117	128	125	131	139	133	135	138.0	142
Viscosity corresponding to thermocouple temp:												
Leading edge (centipoises)	64.5	63.8	62.5	59.0	36.0	40.2	35.0	32.0	34.5	33.0	33.5	29.0
Trailing edge (centipoises)	59.0	54.3	53.5	50.6	28.3	30.5	26.0	21.5	25.0	23.0	22.0	19.9
Average of viscosity at leading and trailing edges	61.8	59.0	58.0	54.8	32.2	35.4	30.5	26.8	29.8	28.0	27.8	24.4
Temperature measured by thermometer, F	96	96.5	96.5	97	114	107	116	115	115	115	113	117
Viscosity corresponding to thermometer reading	74.0	73.5	73.5	72.0	42.0	52.5	39.0	41.5	41.5	41.5	43.8	38.0
Excess viscosity corresponding to thermometer over average viscosity corresponding to thermocouple, per cent	19.7	24.5	26.7	31.4	31.7	48.3	27.9	54.8	39.3	48.3	57.6	55.7

A schematic diagram of the bearing is shown in the writer's Fig. 1. The enlarged insert shows the arrangement of the thermocouple. The position of the thermometer well is also indicated. The bearing itself was a cylindrical steel shell lined with babbitt, with the principal dimensions shown.

Except for the circumferential grooves to which the oil was supplied at either end, the bearing surface was plain. The bearing was so arranged that it could be moved circumferentially without releasing the load, and being a plain cylinder, it always presented substantially the same surface to the load. With this arrangement it was possible to obtain a complete circumferential traverse of the oil-film temperatures with the one thermocouple.

In the writer's Fig. 2 are shown several of the curves of temperature (as measured by the thermocouple) plotted against the circumferential position at which the temperature measurements were made. It will be noted that around the arc of the pressure-carrying oil film there was a very marked and steady increase in temperature. This temperature increase was, of course, due to the cumulative heating effect of the work of friction as the oil was carried through the clearance space. On the "off," or unloaded, side of the bearing the oil was cooled to the inlet temperature again by mixing with fresh oil from the supply grooves.

As the thermocouple was moved from position to position around the bearing in 10-degree increments, the temperature change recorded by the thermocouple was as nearly instantaneous as it was possible to balance the potentiometer, and over a period of as much as thirty minutes the reading for any given position did not change appreciably.

In view of this responsiveness and steadiness of the readings, it is believed that the thermocouple was adequately insulated from the surrounding bearing shell and recorded very nearly the oil-film temperature. Obviously, this method could not measure the temperature gradient from the surface of the bearing to the surface of the journal. However, it apparently measured a temperature that was a reasonable average of the film temperature at each point around the circumference of the bearing.

Table 1 gives a summary of similar measurements made at speeds of 800, 1600, and 2400 rpm, with loads of about 100 to 300 lb per sq in. of projected bearing area. Texaco Regal Oil C, having a viscosity of 320 sec at 100 F; 48 sec at 210 F; and an A.P.I. gravity of 22.6 deg at 60 F, was used in all tests. In addition to the test conditions, the table gives the inlet and outlet (of the load-carrying oil film) temperature and the corresponding viscosity and the numerical average of the inlet and outlet viscosity. It also gives the temperature as measured by the thermometer, and the corresponding viscosity. The final line gives in percentage the amount by which the viscosity corresponding to thermometer reading exceeds the average viscosity as measured by the thermocouple.

R. O. Boswall[10] has shown that the error introduced by assuming a constant viscosity equal to the numerical average of the inlet and the outlet is negligible. Therefore, the percentages which are listed in the last line of Table 1 represent the error that would have been involved in the viscosity measurement had the viscosity been taken as that corresponding to the temperature measured by the thermometer.

An extremely high degree of accuracy in measuring the viscosity of the lubricant in the oil film is not essential because the film thickness and the coefficient of friction do not vary directly as the first power of the viscosity, but, as the author's figures indicate, to a considerably less important extent. However, the differences of 20 to 60 per cent between the viscosity measured as that corresponding to the temperature recorded by a mercury thermometer placed very close to the bearing surface, and the average of the inlet and outlet viscosities corresponding to the temperatures as measured by the thermocouple, represent a significant difference. These data indicate that it probably is not satisfactory merely to stick a thermometer in the oil reservoir or against the bearing shell, as is frequently done, and call the reading so obtained the running temperature of the bearing.

The temperature measurements made by the thermocouple at the leading and trailing edges of the load-carrying oil film in this experimental bearing should correspond to the oil temperatures that occur in the chamfered reliefs usually placed at the leading and trailing ends of commercial bearings. In this case the average viscosity of the oil film could be taken as equal to the numerical average of the viscosities corresponding to the temperatures measured at these points. The writer has no data on this point and it is offered merely as a possible satisfactory procedure.

In Figs. 1 and 3 of Mr. Howarth's paper, he has produced a very useful and practical tool for bearing designers and lubrication engineers and his definition of bearing running temperature and a satisfactory method of measuring it should be of considerable interest to those who desire to apply these data to practical bearing problems.

R. J. S. Pigott.[11] The writer has read with a great deal of satisfaction this further valuable contribution on bearing lubrication. Engineering, as a whole, is as yet too largely an empirical, rather than rational, branch of science, and the design of bearings has hitherto been one of the worst cases of empirical methods.

When we consider that the behavior of bearings is influenced by length, diameter ratio, clearance, speed, location of oil feed, quantity of oil, viscosity, load, misalignment, and last, but by no means least, heat-removal by other means than the lubricant, the complexity of analytical methods is apparent, particularly since none of the factors is completely independent of one or more of the others.

The writer has been convinced for some time that the mere use of pressure and rubbing speed was a relatively loose and incomplete criterion, the use of a friction coefficient even worse; in fact, often quite misleading. It is always useless to predicate the

[10] Theory of Film Lubrication," by R. O. Boswall, Longmans Green & Co., New York.

[11] Staff Engineer, in charge of Engineering, Gulf Research & Development Corporation, Pittsburgh, Pa. Mem. A.S.M.E.

design of any engineering device on two factors, when six or eight are involved. For example, it was customary, for a century, to treat pipe flow as a function of velocity and size only; consequently water, steam, gas and air formulas all differed, and there was no correlation. Reynolds' basic contribution was neglected for thirty years, and then swept the field. By taking into account density, viscosity, and finally roughness, we can now use a single formula for any fluid from molasses to hydrogen.

The writer had occasion to attempt the direct computation of liquid leakage and friction in rotary pumps and motors. Many of the cases involve conditions very similar to those in oil-film bearings. The results were so close to test performance that the latter can be predicted from a paper design within one or two per cent of the actual test. Much of the work will prove useful in bearing analysis.

We have also carried out analyses and tests in the laboratory on flow from a point source to a surrounding field edge, and the results of this work will ultimately be available for analysis of the effect of oil feeds and grooves.

It is clear that the rationalization of bearing design must come along the path taken by Mr. Kingsbury, Mr. Howarth, and their associates. But to render the final development authoritative and beyond question, it seems to the writer that we must have one more device—a bearing- and lubricant-testing machine that will test actual bearings, and yield results that do not require interpretation or extrapolation. The fault that may be found with most testing machines so far used is that they use idealized bearings or sometimes a structure not even faintly resembling a bearing; in many cases, results on one machine do not agree with those on another. Another objection is that none of the machines permits a real pressure, temperature, or leakage search; many do not even give journal displacement. We are developing a machine that we hope will overcome these disadvantages.

Mr. Howarth should not only be congratulated, he should be heartily thanked, for another valuable study of the subject.

A. F. Underwood.[12] The collected material that is presented in Mr. Howarth's paper is interesting in that it shows the lack of uniform bearing design. This may be a necessity when it comes to actual bearings in practice because our experience has shown that bearings which do not conform to the best theoretical designs often run better than when mathematically correct. We do not mean to imply that the theory should be neglected but rather that it is best used as an indicator, especially in the design of new bearings. As a specific example, theory shows that the bearing should be rigid to maintain the correct C/D ratio. Yet it is well known that the flexible cap of an automobile-engine connecting rod will sustain a much higher load than the upper half of the rod which is much more rigid. The reason for this apparent anomaly is that, in the case of the cap, the oil film is distributed over a larger area of babbitt, thereby increasing the fatigue life of the material.

In spite of the nearly universal use of the pressure-velocity factor for bearing design, it is not a criterion of bearing performance. It does become a measure of the heat loss in a bearing if it is in turn multiplied by the coefficient of friction. A number of factors affect the coefficient of friction, and, unless these factors all balance so as to give comparable values of coefficient of friction, the use of the pressure-velocity factor in bearing design may be very misleading.

Author's Closure

In addition to the distortion of bearing surfaces by loading conditions, as pointed out by Mr. Dall, there is the distortion caused by temperature change. With this in mind, Mr. Dall's three considerations which influence one's choice of minimum film thickness become (1) initial perfection, (2) subsequent distortion, and (3) durability.

Durability depends upon careful application of the lubricant and the protection of the bearing films against injury and contamination which obviously means that laboratory conditions must be realized in service. In other words, the bearings must be made fool-proof.

Several interesting comments offered by Mr. Kuhns include definite additions to the author's chartable data. With regard to the specification of lubricating oils, the author indorses Mr. Kuhns' hope that sometime it will be easier to obtain oil by simple description instead of by brand name. The S.A.E. numbers which he mentions are a step in the right direction but they do not go far enough. These numbers are given in Fig. 3 of this discussion, which has been revised from Fig. 7 of the paper, to cover the range needed to show the S.A.E. winter oils 10-W and 20-W. Further refinements are inevitable in the description of oils by numbers that have a real meaning. The addition of 10-W and 20-W to the S.A.E. numbers is but a step in that direction.

Mr. Pearce has given useful data that may well be added to that already charted. The author's use of the projected area of the journal as a basis of comparison for journal bearings was based upon its simplicity. No simple basis of comparison would afford an absolutely true picture of the facts, because of the variety of angular reactions of the bearing films to the journal for different bearing arcs, and because of pressure variations throughout the film. Hence, it is believed by the author that as long as the bearing-arc limitations are kept in mind for each class of bearing, the simpler method of tabulation is the better one to use.

The friction coefficients given by Mr. Pearce seem to include also the rolling friction of the wheel on the rail, if the author interprets correctly the statement, "Expressed in another way, it offers a rolling resistance of 1.86 lb per ton." If such is the case, direct comparisons with optimum bearing conditions are difficult to make.

The wide variation in operating load upon a bearing is not limited to the railroad field. It is found in every machine tool in which the rate of material removal is inconstant. In fact, in a machine tool the bearings are subjected to greater variation of load than in a rail car because while the latter has the minimum though considerable dead weight of the car body, bearings in a machine tool carry loads that are almost wholly due to the working process undertaken, which vary from zero to the maximum. For these reasons the tabulation of data is made on the basis of the normal operating loads the bearings are designed to carry.

The author appreciates the value of plotting test results on the basis of ZN/P as suggested by Mr. Dahlstrom, who recognizes Mr. Hersey as having pointed out the value of such a combined variable in 1915. Mr. Kingsbury[3] has given definite optimum ratios of the friction coefficient λ to a similar combined variable $\sqrt{(\mu N/P_0)}$ for each of his bearings. These range from 0.45 to 0.53 for clearance bearings with arcs varying from 60 to 120 deg. Transforming the μ to Z will give ratios of $\lambda/\sqrt{(ZN/P)}$ for optimum bearings, lying between 2.3 and 2.8×10^{-4} when corrected for side leakage under the assumptions that the bearings are square, i.e., the length is equal to the width. Out of this ratio we cannot obtain any useful low points for reference, but merely a straight line extending to the origin, unless we select a clearance ratio or a minimum film thickness with relation to journal diameter. Proceeding to do this, we find that the value of the friction coefficient for the square

[12] Power Plant Section, Research Division, General Motors Corporation, Detroit, Mich.

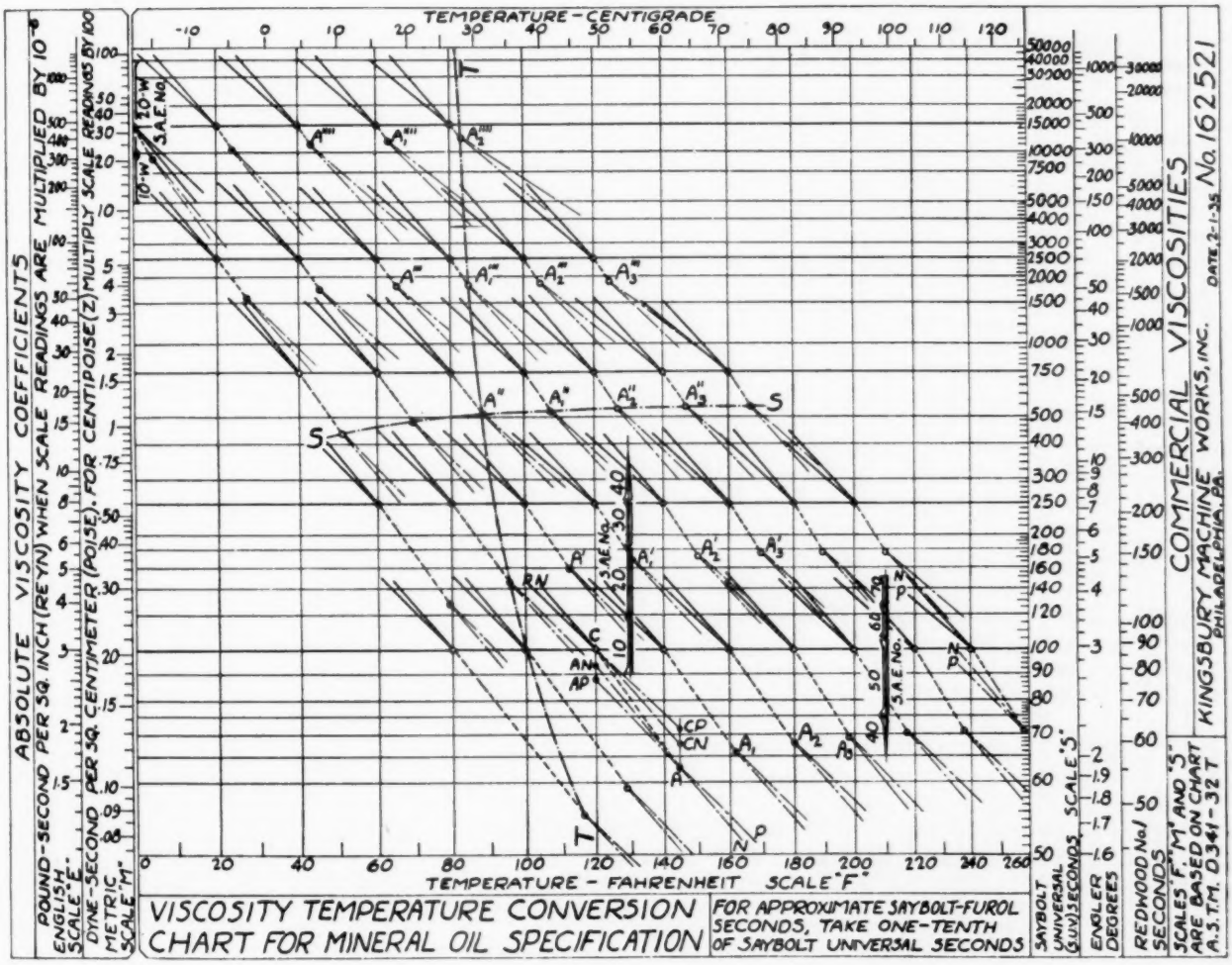


FIG. 3 VISCOSITY-TEMPERATURE CONVERSION CHART FOR MINERAL-OIL SPECIFICATION

(A revision of Fig. 7 of the paper extended to include all the S.A.E. numbers now in use. The intersections for determining the location of the dot-and-dash lines were calculated mathematically as against a graphical method employed in Fig. 7.)

bearing for optimum conditions will be from 2.4 to 3.0 times the clearance ratio. Hence, there is no theoretical bar to the coefficient of friction reaching zero.

Therefore, when a limit is arbitrarily placed upon the value of ZN/P, below which we dare not go, or are unable to go, we admit that there is a similar limit in the perfection of workmanship that is practicable or possible. The author, therefore, believes that it is wise to face the whole problem at once, and not to overrate the correlation of data on charts, the coordinates of which are merely the coefficients of friction λ and the combined variable ZN/P. The clearance ratio must be known to properly interpret such data.

A formula for minimum film thickness, such as derived by Professor Karelitz from works of Michell and of Boswall for flat surfaces, can be deduced readily for clearance or fitted journal bearings from symbol groups No. 10 published in Mr. Kingsbury's "Optimum Conditions in Journal Bearings."[3] A proper constant or series of constants, if more than one be needed, for determining desirable actual minima, must be based on experience. Mr. Kingsbury's experience with his thrust bearings yields the minimum film-thickness values which are given in Table 5 of the paper.

The point stressed by Mr. Barber is an important one and requires considerable space for its complete discussion. It was this fact that led the author to limit the paper to the relation of load and friction to the mean viscosity of the film. This was a definite step forward and it did not seem wise to try to cover adequately at this time the variations of viscosity throughout the grooves and baths of bearings.

The experiments to which Mr. Barber refers were made with a shell designed to simulate the lubrication conditions in a full bearing so far as they can be brought about in a finite bearing operating in an atmosphere of moderate pressure. The shell was designed to explore the distribution of pressures and temperatures and the shore lines of the film natural to such a bearing. Hence, it would be difficult to apply the results reliably to a partial bearing formed between two oil channels. These channels can be used for cooling the bearing, or merely for oil supply, when the cooling is done by water or by conduction to the air. Hence it would be better to seek more comparable temperature-distribution data by experimenting on shells designed to operate like those of the bearings in the machine that is subject to investigation.

The machine that Mr. Pigott mentions as being developed by the Gulf Research and Development Company will be studied with great interest by bearing engineers when it is ready for the market. Resulting from the experience possessed by Mr. Pigott and others of his company, it is to be expected that this will not be "just one more machine" to be set aside as inadequate.

The statement of Mr. Underwood that a connecting-rod cap

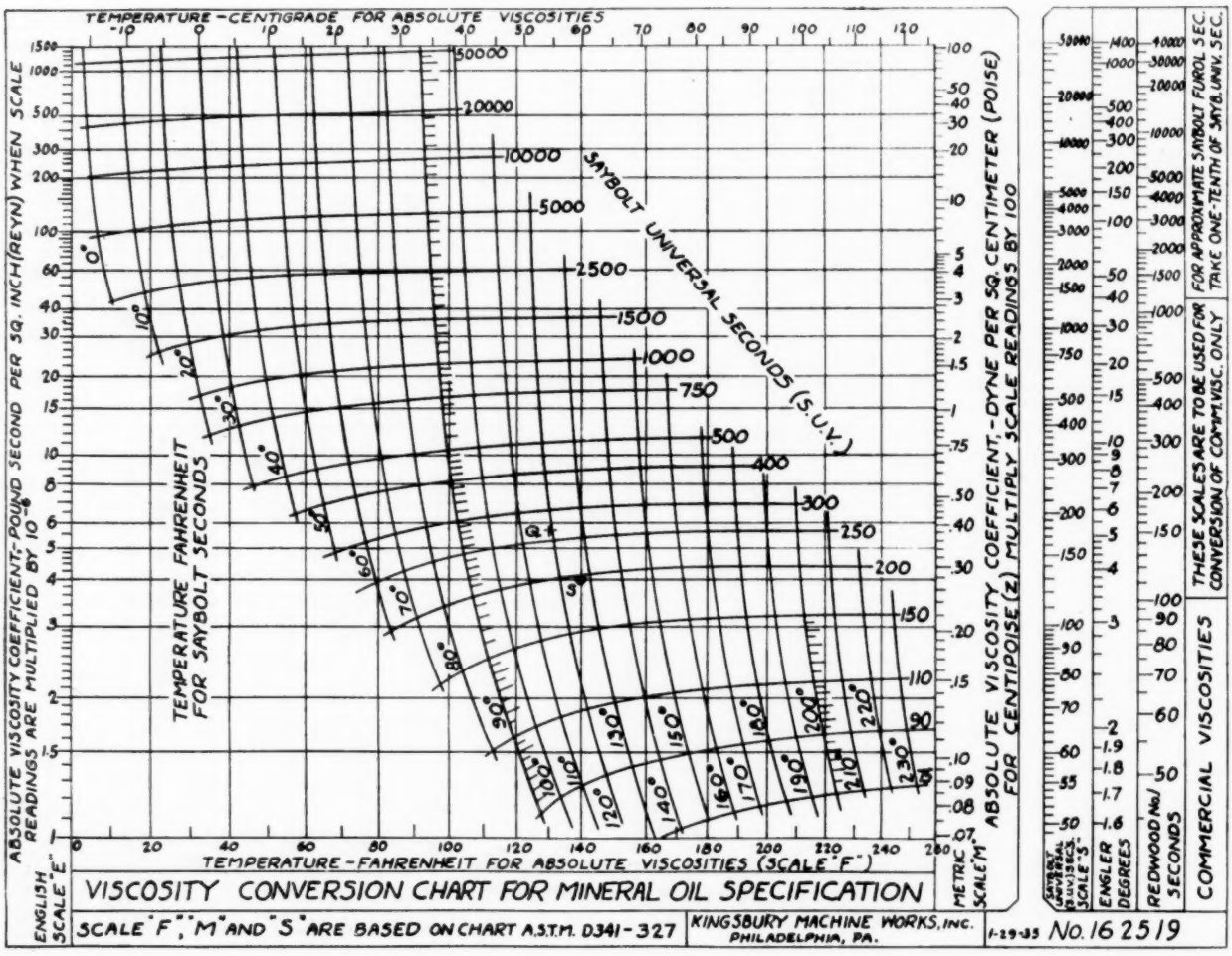


FIG. 4 VISCOSITY-CONVERSION CHART FOR MINERAL-OIL SPECIFICATION

(A revision of Fig. 6 of the paper extended to include all the S.A.E. numbers now in use. A mathematical, rather than a graphical method was also employed for determining the scales of this figure.)

carries load better than the stiffer part backed by the rod is quite interesting. The difference may be due partly to better alignment and partly to the hammock-like surface it presents to the pin. This suggests that a better rod design might be one with a head consisting of two caps held to the rod with a pair of bolts.

In addition to Fig. 3 of this discussion, offered by the author to extend the range of Fig. 7 of the paper to include all the S.A.E. numbers now used, there is also offered a revision of Fig. 6 of the paper. This new chart, Fig. 4 of this discussion, covers the same field as Fig. 3 of the discussion, and has been revised, as was the latter, to be more accurate than the original. The intersections which determine the locations of the dot-and-dash lines of Fig. 7, and thereby the special scales of Fig. 6, were located graphically, and were, therefore, subject to minor errors because of near parallelism. Therefore, a more accurate way of locating them was sought with the result that J. F. Spiegel, one of our engineers, evolved a mathematical method for calculating the intersections that is much more accurate than the graphical method. The redetermination of these intersections occupied a number of weeks. The results, however, warranted the time expended because Fig. 3 and Fig. 4 of this discussion are apt to be widely used. This seems reason enough for repeating them. Obviously, if the paper is ever rewritten only the better set of charts need be included.

Tractive Effort of Steam Locomotives (Locomotive Ratios—II)[1]

A GIESL-GIESLINGEN.[2] It is gratifying that Dr. Lipetz in response to the discussion evoked by his paper of two years ago,[3] has furthered the development of his method for evaluating locomotive tractive effort in order to make it adaptable to cylinders with widely varying proportions. The earlier paper[3] stated that his method was suited to locomotives of normal modern proportions and, assuming only limited fluctuations in relative dimensions of boiler and machinery, a single curve of moduli was developed to answer this stated purpose.

Meanwhile, locomotives have been cited which, with relation to boiler capacity, have cylinders that are as much as, or more than, 50 per cent larger than those upon which the curves in the earlier paper[3] were based. In the paper being discussed, Dr. Lipetz established correction curves in order to accommodate such cases. The writer fully endorses the use of a standard curve of moduli for locomotives with normal cylinder proportions, and of

[1] Published as paper RR-56-6, by A. I. Lipetz, in the December, 1934, issue of the A.S.M.E. Transactions.

[2] Assoc-Mem. A.S.M.E. New York, N. Y.

[3] "Horsepower and Tractive Effort of Steam Locomotives" (Locomotive Ratios), by A. I. Lipetz, Trans. A.S.M.E., vol. 55, 1933, paper RR-55-2, p. 5.

correction curves expressing the deviations in case cylinders of other proportions are provided. However, he suggests some changes in the author's curves.

The author's Fig. 5 shows his correction curves as a function of the locomotive characteristic K, which established a yardstick for the relation between boiler capacity and cylinder size. (See author's Equation [10].) It is obvious that larger cylinders mean a smaller value of K. The author assumes that no corrections should be applied for values of K equal to 14.26 and over, but if the cylinders are enlarged so as to result in values of K smaller than 14.26, then his corrections begin and rise very sharply at first and then at a diminished rate. This is not logical.

To better illustrate the case, reference is made to the writer's accompanying Fig. 1 which was drawn for the New York Central 4-6-4 type locomotive, class J-1. In this graph, the abscissa shows the values of K, each point corresponding to a certain cylinder volume and $K = 15.28$ corresponding to the locomotive's actual cylinder of 25 in. diameter. The heavy line indicates the Lipetz correction curve for 50 rpm. According to Dr. Lipetz we can now enlarge the cylinders until $K = 14.26$ and $d = 25.9$ in., without having to apply any correction to tractive power or performance. But if we increase the diameter only $^1/_2$ in. further to 26.4 in., tractive power according to the author should jump 6.7 per cent for 50 rpm.

We are also allowed to assume a smaller cylinder, perhaps equaling the proportions of locomotive No. 1 in the author's Table 4 for which $K = 16.88$. Therefore, $d = 23.8$ and, according to the author, no correction would be applied. If we would now enlarge the cylinder again, the author's correction curve for 50 rpm would imply that a diameter increase of 2.1 in. (a volume increase of 18.5 per cent) will not influence tractive power, but if we add another 2.1 in., reaching 28-in. diam, tractive power will jump 16 per cent. This is clearly impossible. The correction curve cannot have an edge at $K = 14.26$ but must be a smooth curve of some kind and, if $K = 14.26$ actually characterizes the cylinder proportions for which the 1932 moduli are valid, the proper correction curve will cross the zero line at that point as indicated by the tentative broken line for $n = 50$ rpm.

How large shall the correction be for any point where K differs from 14.26? In his 1932 paper[3] the author develops his method by first obtaining a curve of steam production varying with revolutions per minute in accordance with well established (even though not quite undisputed) experience, and then obtaining horsepower and tractive effort by establishing a curve of steam consumption per indicated horsepower-hour also drawn against revolutions per minute. These two curves were then combined into one representing the Lipetz moduli. The assumption of a definite figure for steam consumption per indicated horsepower-hour at a given speed implies (among other things) that there is, for each given number of revolutions per minute, a definite relation between the volume of the consumed steam and cylinder volume; this will be the case if $K = \frac{E_e}{Vp_b}$ is the same for all engines. If an engine has larger cylinders, then a smaller cut-off must be used to consume the given steam volume and, at least at moderate speeds, better utilization of the steam and higher output will result. The amount of steam generated by the boiler is practically uninfluenced by cylinder size, and if we do not wilfully change the basic curve of steam production established in the 1932 paper,[3] then the changing rate of steam consumption per indicated horsepower-hour is the only factor determining the correction curves for the Lipetz moduli. It is interesting to note that the Kiesel formulas as presented by Mr. Vincent in his discussion of the 1932 paper give a very good answer to this problem: by a simple transformation of the KV-formulas and by making $P_1 v$ const. 506, we can write the KV-formula in the following form:

$$C_i = 13.9 + 58.9 \frac{H}{VP_1 n}$$

where C_i is the steam consumption (excluding auxiliaries), lb per ihp-hr; H is the boiler steam available for the cylinders, lb per hr; V is the combined volume of all cylinders, cu ft; P_1 is the boiler pressure minus 10 lb; and, n is the speed, rpm. If we substitute 96 per cent of the boiler pressure for P_1, and the denominations used by Dr. Lipetz, then we obtain

$$C_i = 13.9 + 61.3 \frac{E}{Vp_b n}$$

where $\frac{E}{Vp_b}$ corresponds to the locomotive characteristic K established by Dr. Lipetz. Within the scope of the Lipetz

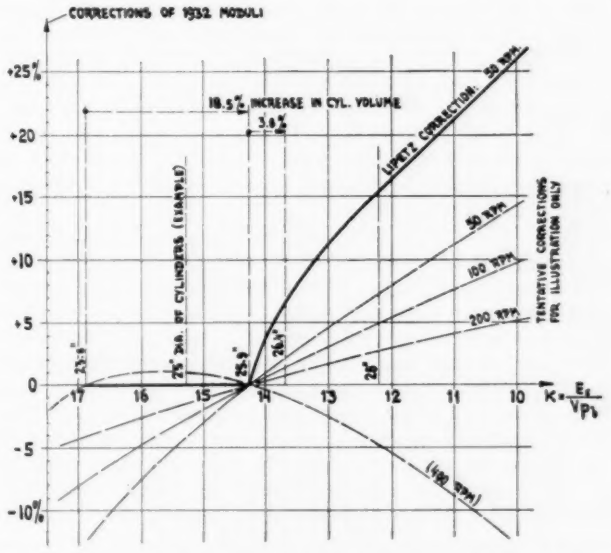


FIG. 1 TENTATIVE CORRECTION CURVES FOR LIPETZ' 1932 MODULI AT 50, 100, AND 200 RPM

moduli, the KV-formula for steam consumption so developed very well represents the trend of steam consumption for varying factors on the right side of the equation, and it is helpful in calculating the correction curves for the Lipetz moduli. Thus, correction curves for 50, 100, and 200 rpm are tentatively shown in Fig. 1 of this discussion. A tentative curve for 400 rpm is illustrated as an example to indicate that the trend will be reversed at very high speeds; this region is, however, entirely outside of the scope of both the present Lipetz moduli and the KV-formula.

Dr. Lipetz, in his present paper, does not limit himself to corrections derived from the changes in the rate of steam consumption. He applies much greater corrections at low speed (50 rpm), arbitrarily chosen to suit the performance curves presented by Mr. Vincent. Thus, for locomotives with large cylinders he abandons the boiler evaporation curve used as a basis in his 1932 paper, and leaves us without a clue as to what is expected from the boiler at low speed, except for saying that the boiler has to be "forced." The explanation for this forcing as given on page 926 of the author's paper is purely psychological: it is claimed that the engineer is induced to force the boiler at low speeds. But certainly there is no necessity of

forcing it because the cut-off could be shortened sufficiently, just as rather small cylinders will permit forcing at higher speeds if the engineer should choose to lengthen the cut-off. To clear up this confusing situation we have to go back to the premises in the author's 1932 paper wherein Dr. Lipetz defined his performance curves as those obtained when a good engineer pilots the engine so as to develop a "reasonable maximum" performance. Later he determined overall thermal efficiency for such a case and found that it lay in the neighborhood of 6 per cent over the whole range of speed. This gave the reasonable result that an engine crew operating a locomotive at a good high rate without overstraining it, deriving its judgment from such observations as: ease in holding the steam pressure, clearness of the stack gases, sound of the exhaust, etc., is operating it at a fairly constant rate of overall efficiency. Under these conditions an average curve of steam production is obtained which at low speeds (50 rpm) remains considerably under the figures which hold at higher speeds (200 rpm), not as an absolute necessity perhaps, but as a good practical result in average cases. By taking such an average evaporation curve as a function of revolutions per minute and relating it to the Cole evaporation, a definite basis of steam production is obtained, to be used uniformly for all locomotives with modern boilers. This method is very practical. Two different locomotives can be compared on such a basis because it is known how much steam each one is expected to generate, and if one of them should fall short of power it is possible to determine whether the fault lies with the boiler or with the steam engine.

With the author's 1934 correction curves, this important advantage has been lost. Each cylinder size, for values of K less than 14.26, implies a different and unknown rate of steam production; even though correction curves for the corresponding steam production could be established, our basis of comparison would become too involved to be practical. Moreover, the supposition is now that steam engine and boiler are working at a "forced" rate at low speeds while they are working at a "reasonable" rate at high speeds. There is none or only a small reserve capacity at low speeds and 15 to 20 per cent reserve capacity at high speeds, while with the 1932 basis of a definite evaporation curve we knew that we had the reserve capacity over the whole range of speed.

That the engine crew is "induced" to work the boiler harder if the locomotive has larger cylinders is true to a limited degree, but this applies to the whole range of speed. Even if this influence were of any importance we had better neglect it in order not to upset our basis of comparison. But there certainly cannot be an inducement for the engineer to operate the larger cylinder with an even greater cut-off than the smaller cylinder as implied by the author's correction curve for $n = 50$; see Fig. 1 of this discussion. It is seen therefrom that a 3.8 per cent increase in cylinder volume (from $d = 25.9$ in. to 26.4 in.) can give the 6.7 per cent increase in performance corresponding to the author's curve only if the cut-off for the larger cylinder is lengthened by about 3 or 4 per cent, which is not logical. The author's corrections in the neighborhood of $n = 50$ have apparently been brought about by the desire to suit the performance curves presented by Mr. Vincent. This, however, was not necessary because we knew that the Vincent curves correspond to a state of greater forcing with small or no reserve capacity over the whole range of speed. Incidentally, this is brought out well by the author's example of the Boston & Albany locomotive. If the new correction curves would be applied to it, the calculated performance at $n = 50$ would be 15 per cent over the road performance and therefore excessive. Dr. Lipetz explains that since this engine has 60 per cent limited cut-off the cylinders could be called about normal, although $K = 12.27$, and he indicates that no correction should be applied. But since he applies the full correction to the Pennsylvania Class I-1s locomotive with only 55 per cent limited cut-off, and likewise to all other examples, it would be inconsistent not to do the same for the Boston & Albany locomotive. However, the 1934 correction method would destory the coincidence between calculated and test figures if this were done.

In conclusion, the writer makes the following recommendations:

1 To use a uniform curve of steam production for all locomotives. It might be investigated whether the 1932 curve is not a few per cent too low at low speeds, and whether it is not falling off slightly too fast at high speeds, but it should be retained in principle.

2 To apply corrections for cylinder size merely in accordance with the rate of steam consumption per indicated horsepower-hour.

3 To adopt the inclined Vincent curve of cylinder tractive effort and to use the straight line only if adhesion is the limiting factor. However, I feel with the author that the "transition curve" is too arbitrary and may better be omitted.

In this way the point of intersection between cylinder and boiler tractive effort will not be at 50 rpm as arbitrarily chosen by Dr. Lipetz for all large-cylinder locomotives, but its location will be a function of cylinder size as it should be.

The great merit of the Lipetz moduli lies in the ease with which they can be adapted to progress in locomotive design. Perhaps we should not try to adjust the moduli to all the locomotives which have been cited by Dr. Lipetz and his discussers. Some of these are no longer modern in their steam engines and all have widely differing draft appliances. With all our progress in steam-locomotive design we have not yet come to sufficiently uniform and accepted design rules. For instance, I cannot agree with Dr. Lipetz on his definition of over-cylindered locomotives; under-cylindering is more frequent. But a large-cylinder locomotive requires a better design of valve gear, steam passages and draft appliance and also better balancing and lighter working parts than we usually have. Under-cylindering covers up deficiencies in design, but of course it manifests itself in deficient performance and economy.

Here is a splendid opportunity for the Association of American Railroads. Individual builders and railroads have not sufficient staff nor the means to fully investigate the subject, nor are their views likely to be generally accepted. If the Association diligently assimilates the information now available here and abroad, the writer ventures to say that we would have standardized, highly efficient designs ready before a new rush in locomotive building starts. We would eliminate a great deal of the uncertainties which today manifest themselves in so many unjustified miscellaneous differences in locomotive design and proportions and cause an entirely unnecessary waste of money in manufacture, upkeep, and operation of our modern locomotives.

Dr. R. Eksergian.[4] The Cole method for the power proportioning of locomotives was at its introduction of fundamental value in rationalizing the relations between boiler and cylinder capacity and coordinating horsepower and tractive-effort speed relations. It gave a measuring stick of the performance of a locomotive in terms of a horsepower unit.

With improvements in the design and proportions of locomotives, and the requirements of more accurate data as to power performance, the inadequacy of the Cole method was felt by all locomotive engineers. For instance, the unit horsepower by

[4] Edward G. Budd Manufacturing Co., Philadelphia, Pa. Mem. A.S.M.E.

the Cole method was based on a cylinder steam consumption (including auxiliaries) of 20.8 lb of steam per hp-hr, and the 100 per cent boiler was such as to evaporate sufficient steam to supply the cylinders in developing a rated cylinder horsepower at 1000 fpm piston speed.

Aside from lower steam consumption in modern power, one difficulty of the method, however, was that it gave no real procedure of estimating the variation of horsepower against speed. For instance, at the lower speeds, with longer cut-offs, the cylinder steam consumption increases and there is a decrease of the evaporative yield of the boiler. At piston speeds around 250 fpm, the possible horsepower might fall considerably below 60 per cent of the rated horsepower at 1000 fpm.

Dr. Lipetz, in his Locomotive Ratios I,[3] recognized the need for a modification of the Cole method and its rationalization to a consistency with the performance of modern power. While it was recognized that cylinder steam consumption is a function of cut-off, speed, and proportions of cylinder design, Dr. Lipetz also brought out the point that evaporative yield was a function of decreasing speed at the lower speeds. This latter deduction was based on the fact that in locomotives investigated, the actual steam consumption of the cylinders fell short of the evaporative capacity of the boiler at the higher speeds. Using the Cole evaporation value as unity, he introduced a coefficient β to take care of this varying evaporative yield. The evaporative yield at 50 rpm was about 65 per cent of the maximum yield at 200 rpm. Considering the increased cylinder steam consumption at the low speeds, it was possible to evaluate a modulus M_p used in the present paper, which, when multiplied by the Cole evaporation value gives the horsepower.

The variation of the modulus M_p really accounts for the variation in horsepower against speed, and was deduced from actual performance data of road locomotives, nearly all of which had long starting cut-offs. In large-size cylinders with limited cut-off, the variation in the total modulus M_p or the corresponding tractive-force modulus M_t may be different. We can have, in effect, the shifting of the maximum part of the horsepower curve to a lower speed range and the criterion of the performance of such engines at 200 rpm no longer holds. The original early cut-off locomotives were designed for slow drag service and their performance became very much limited at the higher rotative speeds, under which conditions their boiler performance was undoubtedly inadequate.

A matter of considerable importance to the writer appears to be whether or not in long cut-offs at the low-speed range, the power percentage of horsepower when compared with the peak horsepower of 200 rpm is due more in part to evaporation deficiency or to limitation of cylinder performance itself. After all, we know only that at 50 rpm the horsepower is roughly a given per cent of a rated horsepower at 200 rpm. With due allowance for increased steam consumption at the lower speeds, the total steam demand of the cylinders still falls short of the boiler evaporation at its normal rating at 200 rpm. But, do we know whether this is chargeable to cylinder limitations or reduced evaporation at the lower speeds? Now, assuming the boiler proportioned to the cylinders, or vice versa, at 200 rpm, then it is usually found that the cylinder tractive effort falls short of the boiler tractive effort, assuming the boiler evaporation constant at its rated value at 200 rpm. If the cut-off is lengthened, wire-drawing becomes excessive and the card factor results in a mean effective pressure practically the same as with a somewhat earlier cut-off. The increased steam consumption for the longer cut-off practically yields no increase of tractive force. In such a case it would appear that the cylinder performance may possibly be the limiting factor in a low-speed range of 50 rpm and not the boiler.

It can be seen, therefore, that the relative proportions of boiler and cylinders in the low-speed range may not be entirely limited to an assumption of limited evaporation of the boiler alone with due allowance for increased steam consumption of the cylinders. It would appear that we might have cylinders with considerable variation in steam consumption with very little change in the maximum mean effective pressure at the low speed of 50 rpm and in such a case, even with an unlimited boiler capacity, the tractive force could not be increased.

One difficulty appears to be that cylinders proportioned for one operating high speed, say 250 rpm, and consistent with the evaporation for that speed, may be quite different from cylinders proportioned against evaporation for operating speeds at 180 rpm. In the latter case, assuming the same evaporation and roughly the same cut-off for minimum steam consumption (neglecting the difference of wire-drawing for the two speeds) the cylinders will be larger than the former, so that the author's K will be smaller.

As an extreme case, let us consider a locomotive with an operating speed at 90 mph, with 84 in. drivers and thus an operating rpm around 360. With orthodox but long-travel valves, economical cut-off may be around 20 per cent or lower. Assuming a suitable card factor for wire-drawing, condensation effect, etc., we arrive at given cylinder proportions. Such cylinders may be inadequate for starting conditions, resulting in a high factor of adhesion. The boiler capacity would be sufficient to sustain long cut-offs for a considerable range at the low speeds, but wire-drawing would cause a drop in tractive force irrespective of boiler capacity. In this case, with low operating speeds say from 40 to 80 rpm, the tractive effort would appear more limited by cylinder size than boiler adequacy.

With locomotives as discussed by Dr. Lipetz operating at 200–250 rpm, the demand on the boiler usually comes at a critical speed around 50 rpm. With limited cut-off locomotives, and, therefore, with large-size cylinders required for starting conditions, the cylinder steam consumption would be too great and therefore inconsistent with the evaporative capacity of the boiler at 200–250 rpm. With the lower steam consumption effected by the shorter starting cut-off, the rated starting tractive effort can be nearly maintained at 50 rpm with less demand on the boiler than for longer starting cut-offs. On this basis a boiler proportioned for this condition would be inadequate for high operating speeds.

With increasing tendencies toward higher operating speeds from 200 to 250 rpm coupled with the desire for maintaining more economical steam consumption at the lower speeds by the use of shorter cut-offs in this latter range, the danger of inadequate boiler capacity is apparent particularly when the boiler adequacy is based on its sufficiency at 50 rpm. Dr. Lipetz has given us an interesting and most ingenious analysis for providing boiler adequacy for this latter condition. If the rated tractive effort T_r gives the tractive effort T_{i50} based on boiler adequacy, this condition will insure adequate boiler capacity for operating speeds around 200–250 rpm.

While the writer is more or less in agreement with Dr. Lipetz' analysis, he would appreciate the author's consideration of the following point. Reviewing the basic analysis

$$P_i = \frac{E_e(1-x)}{s_h} = \frac{E_e}{S_e} = \frac{\beta E_e}{S_e} \text{ for the power}$$

$$T_i = \frac{375P_i}{V} = \frac{375P_1}{\frac{n}{336}D} = \frac{126{,}000\,\beta\,E_e}{nDS_e}$$

$$M_t = \frac{126{,}000\,\beta}{nS_e},\ T_i = M_t\frac{E_e}{D}$$

From test data at a given n, he obtains βE_c, this data being obtained mostly for locomotives with a long starting cut-off. Knowing the power developed or tractive force, he obtains S_a (or S_h). Taking $\beta = 1$, at say 200 rpm, he obtains E_c and compares or corrects for Cole's original evaporation constant E_c. Therefore, at some lower speed he obtains β from the power developed and the steam consumption S_a. At 50 rpm, the tacit assumption is that the power developed is entirely limited by the boiler evaporation capacity so that knowing S_{a50}, the coefficient β_{50} is obtained and likewise the modulus M_t, which also could be obtained directly. If now we assume the locomotive limited by cylinders rather than by boiler capacity at 50 rpm, then β_{50} would not necessarily be the true evaporative coefficient. With large cylinders and more economical cut-off at starting, aside from S_a being decreased, if the true β_{50} were greater than assumed by the author's deduction, M_t would be greater and therefore the boiler adequacy $\frac{T_{i50}}{T_R}$ would appear improved.

Now, of course, as the speed increases where there can be no question about cylinder-performance limitations, the values of the author's β obtained must greatly increase in precision. After all, the question of boiler adequacy is more dependent on conditions at operating speeds. With large-size cylinders, economical cut-offs at these higher operating speeds would be around 20–25 per cent or the same as with long cut-off locomotives. For this reason adequate boiler capacity would demand somewhat larger boilers for speeds around 200–250 rpm and the ratio of horsepower to initial tractive force would increase as compared with long starting cut-offs.

With slow heavy drag service as on grades, etc., the power developed at such low speed ranging from 50–80 rpm, is of particular interest. In this case it is of importance to ascertain whether cylinder performance limitations or boiler capacity is the more critical limitation.

The writer would prefer the analysis of performance simply on the formulas

$$P_i = \frac{\beta E_c}{S_a}, \; T_i = \left[\frac{126{,}000\,\beta}{nS_a}\right]\frac{E_c}{D}$$

with more test considerations as to β, S_a for different speeds, cut-offs, etc., and further analysis of limiting values of M_t at very low speeds. It would be interesting to consider E_c for $\beta = 1$ at several operating speeds, and the variation of β and S_a for different speeds relative to the particular operating speeds considered.

LAWFORD H. FRY.[5] For judging locomotive proportions Dr. Lipetz offers his locomotive characteristic K which is equivalent to E_c/Vp_b, where E_c is the Cole boiler evaporation. Unfortunately, Dr. Lipetz does not give in either the present paper nor in his earlier paper[3] complete information for computing this Cole boiler evaporation.

The writer believes it is unnecessary and undesirable to use an assumed figure such as E_c in comparing boiler and cylinder proportions. Dr. Lipetz concedes something in this direction by saying: ". . . .a constant very similar to K, but with the boiler evaporation replaced by H_e, the evaporative surface in square feet

$$\delta = H_e/Vp_b$$

may also be very useful for comparative purposes."

The writer suggested, in a paper read before the New York Railroad Club in 1903, a boiler factor very similar to the above. This was $B = T_r/H_e$, where T_r is the rated tractive effort in pounds. B and δ are inversely proportional to each other. B has the practical advantage that it is computed from the rated tractive effort, and an additional calculation to find the cylinder volume is unnecessary. δ has a slight advantage in the fact that it is directly proportional to the boiler capacity, so that a large value of δ indicates a large boiler in proportion to the cylinders.

If B is used, it is convenient to note that it measures the pounds of rated tractive effort which must be developed by each square foot of heating surface when the locomotive is working in full gear.

To bring the comparison to a basis of revolutions per minute instead of miles per hour, the writer proposed multiplying B by D, the driving-wheel diameter. This gives

$$BD = \frac{T_r D}{H_e}$$

It can be shown that if the locomotive is working in full gear, the factor BD is proportional to the foot-pounds of work which must be developed at each revolution by each square foot of heating surface. The lower the value of BD, the higher the speed at which the rated tractive effort can be developed. A low value of BD indicates a locomotive with large boiler capacity designed for high-speed service.

For many years the technical papers when publishing accounts of new locomotives gave values of B and BD. This has provided a large amount of information available for the comparison of various locomotive designs.

If it is decided that a newer method of comparison is desirable, the writer would prefer to see Dr. Lipetz' δ used instead of K. It seems to him desirable to make the comparison simple and direct, involving only definite dimensions of the locomotive and avoiding the introduction of any assumed values such as a calculated boiler evaporation.

Discussion of the second phase of the subject, that is the prediction of the tractive power which can be developed at various speeds, cannot be made in detail without full information as to the method of computing the Cole boiler evaporation. This is not given in either of Dr. Lipetz' papers. Dr. Lipetz shows, however, that his method gives computed tractive-power curves which are surprisingly close to the actual curves derived from experiment.

H. RUBENKOENIG.[6] Undoubtedly there is necessity for revision of some of Cole's ratios and other formulas used in calculating tractive effort for locomotives. The generally accepted formula

$$T = KPC^2S/D$$

for determining what is variously designated as cylinder tractive effort, starting tractive power, theoretical tractive force, etc., does not seem to hold for some of the newer locomotives built during the last two or three years.

K in this formula depends upon whether or not the valve has a limited cut-off, and whether or not auxiliary starting ports are used. One of the recently built heavy passenger locomotives of the 4-8-4 class is described as having a rated tractive effort of 69,800 lb. If K is taken as 0.85 (the proper coefficient for 90 per cent cut-off), the tractive effort from the foregoing equation becomes

$$T = 0.85 \times 260 \times 28^2 \times 31/77 = 69{,}300 \text{ lb}$$

[5] Railway Engineer, Edgewater Steel Company, Pittsburgh, Pa. Mem. A.S.M.E.

[6] Professor of Railway Mechanical Engineering, Purdue University, Lafayette, Ind. Assoc-Mem. A.S.M.E.

This locomotive is described as having a valve gear which gives 70 per cent maximum cut-off, but no mention is made of auxiliary starting ports. Assuming auxiliary starting ports, the value of K is 0.78, which when applied to the foregoing formula gives a tractive effort of 64,100 lb. In other words, the rated tractive effort is more than 5000 lb too high if the generally accepted formulas for calculating tractive effort be used.

Since predictable locomotive performances at speeds above starting speeds are based upon data which include suitable corrections being made from the basic figure of capacity tractive effort, it seems that some revision of calculation methods is necessary. It would be interesting to know what assumptions were made in deriving the figure for rated tractive effort of the locomotive previously mentioned.

The 1932 and 1934 papers by Dr. Lipetz are distinct contributions to the subject of locomotive tractive effort and horsepower, and they indicate the direction in which locomotive designers may work in improving the efficiency of the modern steam locomotive.

W. A. Pownall.[7] Dr. Lipetz states that drafting arrangements in particular may cause disagreements or differences between the tractive-effort curves and actual performances. The Wabash railroad has had considerable experience during the past few years with the Kiesel front end which uses an annular-ported exhaust nozzle of large area and a cylindrical draft netting, extending from the stack to the bottom of the boiler, without the usual draft or deflector plates.

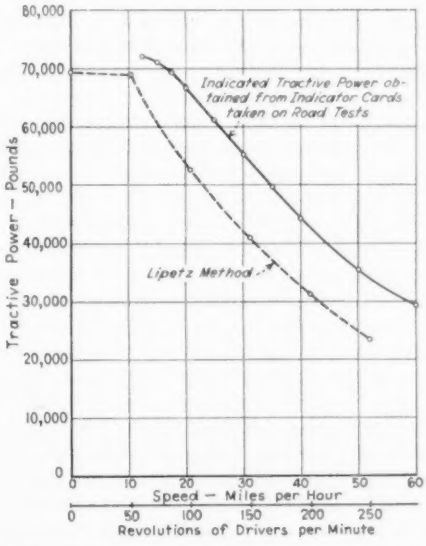


Fig. 2 Cylinder Tractive-Power Curve of a Mountain-Type Locomotive With Kiesel Front End Compared With Lipetz' Calculated Tractive Effort

Fig. 2 of this discussion shows the cylinder-tractive-power curve obtained on a mountain-type locomotive equipped with a Kiesel front end. For comparison, the tractive-effort curve as calculated by Dr. Lipetz' method is also plotted on this figure. The curve as obtained from indicator cards on road tests was not an extreme condition, but was obtained on a large number of runs. It will be noted that at 40 mph, the engine in question actually developed about 35 per cent more tractive power than shown by the Lipetz method. This engine is equipped with a type-E superheater and a feedwater heater.

Fig. 3 of this discussion shows a similar curve for a Mikado-type locomotive equipped with a type-A superheater and without a feedwater heater. The difference between the actual road results and the Lipetz curves is not as pronounced as with the mountain-type locomotive, the excess of the actual tractive power over the Lipetz curve being 17 per cent at 30 mph and 14 per cent at 40 mph. These results are worth attention in showing the increased actual locomotive capacity that can be obtained by drafting arrangement other than the conventional.

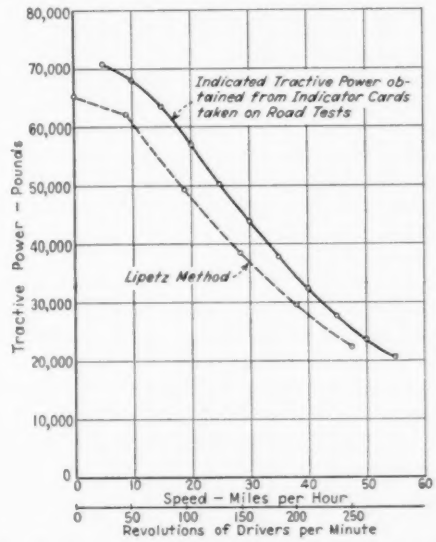


Fig. 3 Cylinder-Tractive-Power Curve of a Mikado-Type Locomotive Compared With Lipetz' Calculated Tractive Effort

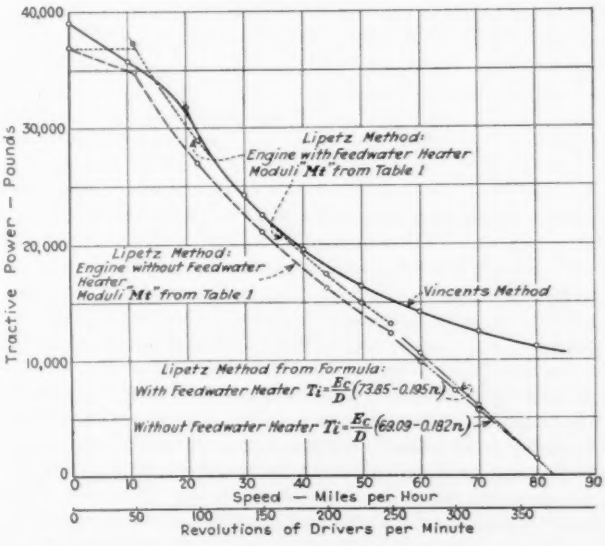


Fig. 4 Cylinder Tractive-Effort of a Pacific-Type Locomotive as Derived From the Vincent and Lipetz Methods

In view of the present interest in high-speed passenger trains there is a definite demand for a knowledge of the draw-bar tractive effort that can be developed by steam passenger locomotives at speeds between 70 to 100 mph.

Fig. 4 of this discussion shows the cylinder tractive effort of a Pacific-type locomotive as derived from the Vincent and Lipetz methods. For speeds above 60 mph, the Lipetz formula would produce a tractive-effort curve considerably lower than the

[7] Mechanical Engineer, Wabash Railway Company, Decatur, Ill.

Vincent curve and also lower than has been obtained in actual service. For example, if the engine resistance were deducted from the Lipetz cylinder tractive effort, there would be nothing remaining to pull the train at about 75 mph, while the engine for which these curves are shown, actually pulled a seven-car train weighing 560 tons, exclusive of the engine and tender, at a speed of from 80 to 83 mph on level track, at which speed it developed an approximate cylinder tractive-effort of 9680 lb. It may be that Professor Lipetz does not intend his formula to be used at driver speeds of over 250 rpm. This actual performance is somewhat less than the value indicated by the Vincent curve and considerably higher than that calculated from the Lipetz formula.

There should be some fairly definite information developed as to available draw-bar tractive effort of locomotives at speeds between 70 and 100 mph.

Author's Closure

While Dr. Giesl-Gieslingen reiterates his agreement with my 1932 method, as given in the previous paper,[3] he takes exception to my 1934 modifications and rejects them summarily. He is in favor of the fundamentals of my 1932 theory and admits the value of the moduli, but thinks that I rather rashly abandoned the strict consistency of that method in order to satisfy the curves presented by Mr. Vincent. I must, therefore, so to speak, defend myself against myself, and this makes my task rather difficult. However, I am afraid that Dr. Giesl-Gieslingen misunderstood the underlying thought of the 1934 modifications and probably forgot the principal object of the 1932 method.

Dr. Giesl-Gieslingen maintains that my theory of 1932 was based, on the one hand, on a certain evaporation curve which he considers more or less correct, although maybe somewhat conservative, and, on the other hand, on a steam-consumption curve with which he also is more or less in agreement. As pointed out in both the 1932 paper and in its closure, the procedure which I followed in the first part of my 1932 paper was used only in order to establish the correct and logical relations between phenomena in a locomotive, which led to a simple formula. I checked it later against all locomotive-test data at my disposal at that time. In other words, while the 1932 method had a theoretical basis, it was purely empirical, and in proving the practicability of the 1932 moduli, I did not attempt to defend the absolute correctness of figures, or curves, of evaporation and steam consumption on which my formula was based. I admitted that the Cole evaporation figure E_c, evaporation-coefficient β and steam-consumption curves may be only approximately correct, but as the formula was later checked against test results and the moduli, specially introduced for that purpose, were corrected so as to conform with these results, the formula, which in its final form was very simple, must have been right and could be used for practical purposes. The user need not go back to the underlying data and examine them. This made the method very simple, as it required the use of only one formula with constant moduli depending upon the rotative speed of the wheels, and at the same time very flexible, because the moduli could be changed when conditions warranted that.

In the 1934 modifications I followed the same principle. In studying the test results of locomotives brought to my attention in Mr. Vincent's discussion of my 1932 paper, I realized that all the locomotives on which I had data at my disposal in 1932 had long cut-offs, and that the locomotives analyzed by Mr. Vincent, especially those for which my 1932 moduli showed discrepancies, were types with limited cut-offs. In my 1932 paper I also considered, among others, the Boston & Albany A-1 locomotive and found that it showed fairly good agreement with my 1932 moduli, although it was of the type with limited cut-off. In analyzing this, I found that the good results of my 1932 moduli, in application to this locomotive, were due to the fact that while the locomotive had limited cut-off, its cylinder sizes were not excessive and very close to the sizes they should have had for a limited cut-off of 60 per cent ($K = 12.27$ instead of $K = 13.09$) while all other locomotives with limited cut-offs which he cited had excessively large cylinders, larger than the limited cut-off would require. The conclusion was, therefore, evident: it was necessary to modify my moduli for locomotives with large cylinders, irrespective of whether or not they were types with limited cut-off or full cut-off. I found a method for doing it and disclosed it in my 1934 paper. Again in order to prove that this method was correct and practicable, I showed that its application to all locomotives on which data were available, both with small and large cylinders, gives very good results. This fact was fully realized and appreciated by Lawford H. Fry in his discussion, and I am very grateful to him for pointing out this fact. It was not a question, as Dr. Giesl-Gieslingen thinks, of satisfying Mr. Vincent's curves. It simply was the question of showing that the locomotives for which Mr. Vincent had test data and which, he thought, were best represented by his formula, could be satisfied by my moduli, if proper modifications are made.

Dr. Giesl-Gieslingen evidently overlooked this point entirely. He thinks that the curve on Fig. 5 of the paper is arbitrary. While it is true that no formula was given for this curve, it was arrived at through a long study of test results and was later checked against all locomotives for which test data could have been obtained. It is the author's opinion that it has more value than a curve like the one shown by Dr. Giesl-Gieslingen in Fig. 1 of this discussion, for which no attempt was made to show its application to any locomotive. Only when a formula has been verified for a reasonable number of locomotives may it be considered seriously, and not before, no matter how scientifically the curve may appear to have been derived.

As an example of another misunderstanding by Dr. Giesl-Gieslingen, the aforementioned Boston & Albany A-1 locomotive can be cited. He says "Lipetz explains that since this engine has 60 per cent limited cut-off the cylinder could be called about normal, although $K = 12.27$, and he indicates that no correction should be applied." What I said was that notwithstanding the fact that this engine has a limited cut-off of 60 per cent, $K = 12.27$ instead of 13.09, if the proper relations of cylinder sizes with respect to the correct α for a cut-off of 60 per cent is taken into consideration. But I did not say that corrections should not be made. They certainly should be made, as they were made in all other locomotives with large cylinders, irrespective of whether they were types with long cut-off or limited cut-off (see, for instance, Texas & Pacific locomotive G-lb, Fig. 8), as 12.27 is less than 14.26. Dr. Giesl-Gieslingen further states "since he applies the full correction to the Pennsylvania class I-ls with only 55 per cent limited cut-off, and likewise to all other examples, it would be inconsistent not to do the same for the Boston & Albany locomotives." Yes, it would be inconsistent, and the correction should be made, but while the I-ls locomotive has a limited cut-off of 55 per cent, very close to 60 per cent of the A-1 locomotive, the cylinders are so large that the K for this locomotive is only 8.53, whereas it should be, according to formula [9a] of the paper, $0.75/0.0596 = 12.55$. Therefore, the cylinders of this locomotive are about 47 per cent larger, and a greater correction is necessary.

As to the Boston & Albany A-1 locomotive, after the correction for the tractive effort at 50 rpm is made, its value will lie above the rated tractive effort, and according to the rules given in the paper, the curve with this point at 50 rpm should be cut off. The remaining part of the curve will not differ much from the test curve.

Dr. Giesl-Gieslingen discusses what I would call "discontinuity

of functions," when he cites in the beginning of his discussion various values of Fig. 5 for smaller and larger cylinders. He refers to the figures "jumping" suddenly by 6.7 and 16.0 per cent, but he forgets that he also jumps $^1/_2$ in. and 2.1 in. in cylinder sizes, or 4.0 and 16.6 per cent in piston areas, respectively. Every continuous function would do that.

Dr. Giesl-Gieslingen is surprised that the Lipetz correction curve for 50 rpm should have an edge at $K = 14.26$. There are many cases like that in engineering practice. A loose tire on a locomotive wheel puts no strain in the wheel rim or in the tire, but if the bore of the tire is several hundredths of an inch smaller than the outside diameter of the wheel, the stress suddenly goes up and it will continue going up with the increase in this difference, most probably with an edge in the curve. The effect of large locomotive cylinders is of about the same nature. If they are too small, there is plenty of steam for them, just as there is plenty of clearance in a loose tire around a wheel, and T_{i50} (indicated tractive effort at 50 rpm) cannot be increased, as a rule, because in small-cylinder locomotives it is equal to T_r (rated tractive effort), corresponding to a maximum cut-off of 85 per cent. However, if the cylinders are larger than what the ordinarily generated amount of steam for the tractive effort at 50 rpm would require, naturally more steam can be utilized by the cylinders, if a certain forcing of the boiler is permitted. Then a modification is, of course, necessary, and the law of modification may go up suddenly, following a curve with an edge. This is what probably happens, and there is nothing illogical in that.

I do not think that it is necessary to go through Dr. Giesl-Gieslingen's other statements except possibly one, in which he says that the reader is left without a clue as to what he might expect from the boiler when it is forced. Dr. Eksergian in his discussion touches on this point in connection with the coefficient β. In order to amplify Dr. Eksergian's thought, and to reply fully to Dr. Giesl-Gieslingen's statement, I am going to present several charts which recently came to light. They were not known to me at the time when the present paper was prepared. A very interesting book,[8] recently published on a 2-10-2 locomotive, series FD, developed and built in U.S.S.R., came into my possession. For the development of this engine the locomotives built in America in 1931 for U.S.S.R. were taken as examples and, therefore, the locomotives are in many respects similar to regular American-built engines. They have large grates, steel fireboxes, firebrick arches, type-E superheaters, feedwater heaters, stokers, boosters, etc. They differ only in having fireboxes of the Belpaire type instead of the round-top shape. A 2-8-4 passenger-type locomotive, series JS, has also been developed and built by the U.S.S.R. In this type, boilers and cylinders are identical with those of the FD engines, the principal difference being in the chassis: frames, wheels, boxes, etc.

TABLE 1 PROPORTIONS AND RATIOS OF U.S.S.R. SERIES FD AND JS LOCOMOTIVES

Locomotive	Russian 2-10-2 FD Class	Russian 2-8-4 JS Class
Cylinder dimensions		
Diameter, d, in.	26.4	26.4
Stroke, s, in.	30.3	30.3
Heating surface, H_e, sq ft	3177	3177
Boiler pressure, p_b, lb per sq in	213.4	213.4
Cole evaporation, E_c, lb per hr	44859	44859
Superheater type	E	E
Feed water heater	Yes	Yes
Max indicated ratio, α_{max}	0.78	0.78
M_t at $n = 50$	65.6	65.6
Driving wheel diameter, D, in.	59	72.8
Boiler adequacy, $a_b = \frac{T_{i\,50}}{T_r}$	0.837	0.837
Boiler percentage (Cole)	0.809	0.809
Cylinder volume, V, cu ft	19.20	19.20
$\delta = \frac{H_e}{VP_b}$	0.775	0.775
$K = \frac{E_c}{VP_b}$	10.95	10.95

The principal dimensions of these two engines are shown in Table 1 of this discussion, which is compiled in the same manner as Table 4 of the paper. In view of the similarity between these two types and American engines, the test results of these U.S.S.R. locomotives are of great interest to us for the present purpose.

The book[8] just referred to describes in detail the U.S.S.R. 2-10-2, Series FD locomotive, and in Appendix 1 gives results of very elaborate tests in the form of charts according to the method worked out years ago by Professor Lomonossoff in Russia.[9] Fig. 602 of the book is reproduced in this discussion as Fig. 5. The abscissas are speeds in kilometers per hour; the ordinates are indicated tractive efforts in kilograms. The full-line curves correspond to constant evaporation, the figures on them showing kilograms of steam per hour consumed by the cylinders (excluding auxiliaries), the steam being referred to one square meter of outside heating surface. The dashed curves correspond to constant cut-off, the figures on them representing percentages of cut-off. In Fig. 6 of this discussion the constant-evaporation curves of Fig. 5 are reproduced in English measurements. The notations

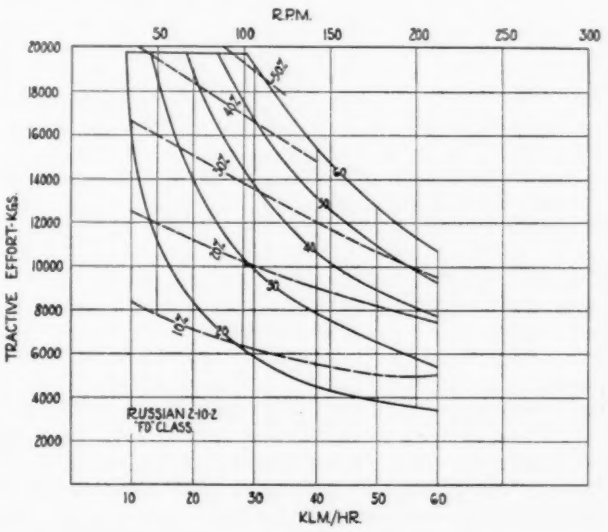


FIG. 5

(Full-line curves are constant-evaporation lines, the figures on them representing kilograms of steam consumed by cylinders per hour, referred to a square meter of outside heating surface. Dashed curves are constant-cut-off lines; figures on them represent percentages of cut-off.)

on the axes are made to conform with the practice of the author's 1932 and the present papers. Full lines are constant-evaporation curves, figures on them representing pounds of total steam generated by the boiler per square foot of outside heating surface per hour. These figures were obtained from the metric-evaporation figures by adding 10 per cent for auxiliaries, as this amount of steam has not been included in the Russian curves. In addition, the author's 1932 and 1934 curves have been added to the chart.

The locomotive characteristic K, as can be seen from Table 1 of this discussion, is equal to 10.95, and corrections have been made in accordance with Fig. 5 of the paper. It is interesting to note that the 1934 curve follows, as to the general trend, all other curves, but it lies outside of all of them, even that which corresponds to the maximum evaporation of 13.5 lb of steam per sq ft of outside heating surface. By extrapolation it can be found that the 1934 curve corresponds to an evaporation of about 14.2.

[8] "Locomotive Felix Dzerjinsky" (published in Russian), Moscow, 1934.

[9] "Object of Locomotive Tests and Their Method," by G. Lomonossoff, St. Petersburg, 1914.

Now, the Cole evaporation figure for this locomotive is 44,859 lb per hour. The outside heating surface is 3177.08 sq ft. The evaporation per sq ft is, therefore

$$E_c/H_e = 44{,}859/3177.08 = 14.15$$

very close to the foregoing figure of 14.2. This agreement is remarkable, or, using Mr. Fry's expression, "surprising." In reality it is natural, if we consider that the Series FD locomotive is of, what may be called, American design, and has, as it will be

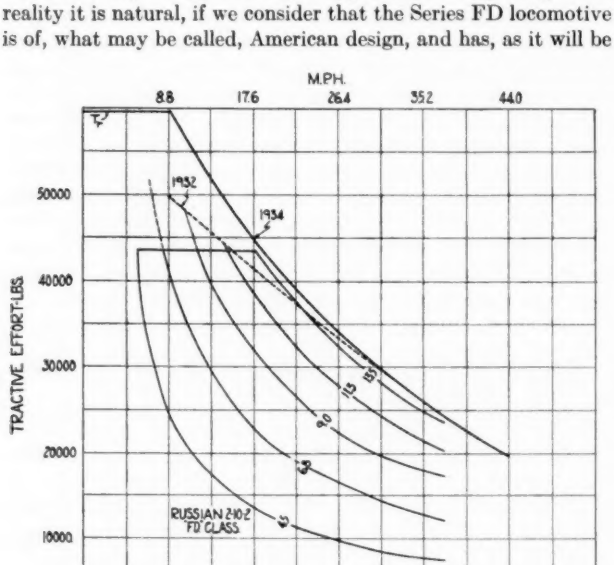


FIG. 6

(Full-line curves are constant-evaporation curves, the figures on them representing pounds of steam generated per square foot of outside heating surface per hour.)

shown later, the regular circular exhaust nozzle, for which the author's curves have been established. The above agreement also shows that the 1934 curve for the Series FD locomotive follows the Cole evaporation figure with a β equal to 1.00 for all speeds; in other words, in order to obtain the 1934 tractive effort at low speeds, the boiler has to be forced. This answers Dr. Giesl-Gieslingen's and Dr. Eksergian's questions as to how high β at low speeds must be. For the Series FD locomotive with $K = 10.95$ it is evidently 1.00. For other cylinder ratios it may be different. It can be figured out approximately from the modification curve by following the trend of Dr. Eksergian's thoughts and the formulas given in his discussion.

In this connection Fig. 6 of this discussion also shows that the 1932 curve would intersect the tractive effort curves at other lower evaporations. For instance, an evaporation of 11.3 would intersect the 1932 curve at about 83 rpm. An evaporation of 9.0 would permit a speed of about 59 rpm, and at 50 rpm the 1932 curve would probably correspond to an evaporation of somewhere between 6.8 and 9.0. It can be found by interpolation to be 8.01, which is about 56.7 per cent of 14.15, which is close to the β assumed by the author for 50 rpm, if we remember that the coefficient β was established as an average of data from many locomotives.

About the same time (1933) the Russian Government developed and built the Series JS passenger locomotive. This locomotive, as has already been stated, has the same boiler as the Series FD locomotive, and has cylinders of the same size. It differs only in the size of driving wheels, which were about 73 in. in diameter instead of 59 in. As our locomotive formulas are based on the boiler evaporation, and the modifications are based on the cylinder dimensions, these two locomotives should give the same power curves, differing only with respect to the tractive efforts in so far as they are affected by the size of driving wheels. Boiler evaporation should be exactly the same. For purposes of comparison, Figs. 7 and 8 of this discussion have been prepared. They correspond to Figs. 5 and 6 for the FD locomotive. Again, Fig. 7 is an exact reproduction of Fig. 12 of the publication on the JS locomotive,[10] while Fig. 8 is the same as Fig. 7 converted to English measurements, with the author's 1932 and 1934 curves added. Furthermore, in order to compare the relation of the 1934 curve on Figs. 6 and 8 of this discussion the highest constant-evaporation curves of Fig. 8 are omitted and the 13.5 curve is retained as the highest, the same as in Fig. 6. It can

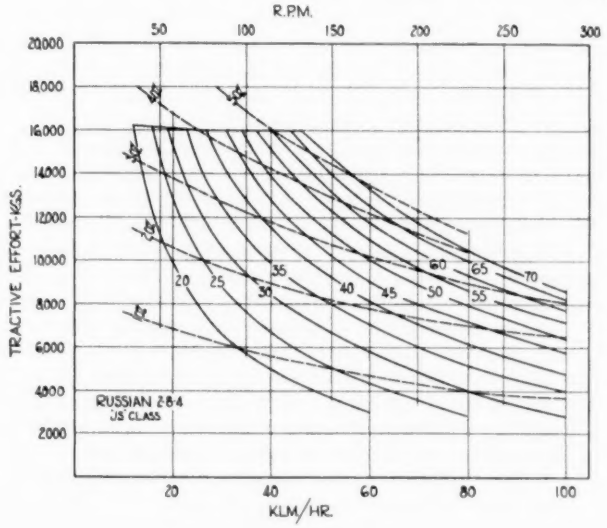


FIG. 7

(Full-line curves are constant-evaporation lines, the figures on them representing kilograms of steam consumed by cylinders per hour, referred to a square meter of outside heating surface. Dashed curves are constant-cut-off lines; figures on them represent percentages of cut-off.)

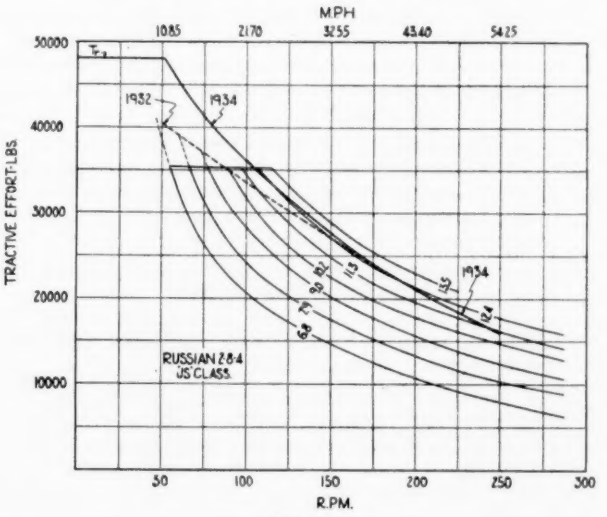


FIG. 8

(Full-line curves are constant-evaporation curves, the figures on them representing pounds of steam generated per square foot of outside heating surface per hour.)

[10] "Principal Test Results of the First Type 2-8-4, Series JS Locomotive" (in Russian), Moscow, 1934.

be seen, however, from comparing Figs. 5 and 7 that the maximum evaporation on the tests of the JS locomotive was much higher than on those of the FD locomotive; 70 against 60 kg per sq meter per hr, or 16.7 per cent higher. This result was somewhat puzzling, but a careful study of the tests of the second locomotive showed that the two locomotives which are supposed to be alike, when actually tested, had different exhaust nozzles: the FD locomotive had the ordinary round nozzle, while the JS locomotive had a special nozzle with separate exhausts from the right and left cylinders, and four round openings for the escape of steam which had a total area 32 per cent larger than that of the single opening of the round nozzle used on the FD locomotive.[11]

This difference probably explains the higher evaporation and may also explain why the 1934 curve, which in Fig. 8 of this discussion has in general the same characteristics as the one described in reference to the FD locomotive (Fig. 6), is switched so as to coincide more with the 12.4 evaporation curve than to be coincident with the 14.2 curve. This is the result of reducing the back pressure of the JS locomotive and, incidentally, reducing the minimum steam consumption of the latter locomotive, per equal power which was 15.5 lb per rail horsepower, as compared with 17.4 of the FD locomotive.[12]

It is also interesting to note that notwithstanding the differences just mentioned, the trend of the 1934 curve and its relation to the constant-evaporation curves is about the same in Fig. 8 for the JS locomotive as in Fig. 6 for the FD locomotive, and that in the JS locomotive it also corresponds to a constant evaporation (about 12.4), which, however, because of better efficiency is lower than the Cole evaporation per square foot of outside heating surface (14.15). In this case β has a constant value of about 0.88. Thus, the tractive effort of a JS locomotive, if tested under the usual American conditions, would give a $1/0.88 = 1.14$ increase ratio for the same amount of steam, and a further increase due to 16.7 per cent higher evaporation, a total increase in the ratio of $1.14 \times 1.167 = 1.33$, or a 33 per cent increase, with the same "strain of forcing," so to speak.

The above explanations, I think should satisfy Drs. Giesl-Gieslingen and Eksergian as to the question of what evaporation the modifications of 1934 actually represent, and should also corroborate Mr. Pownall's statement as to the increase in locomotive capacity due to the use of a drafting arrangement other than the conventional. Mr. Pownall's figure of a 35 per cent increase is about the same as the 33 per cent increase cited previously.

In order to finish with Dr. Giesl-Gieslingen's discussion, a few words should be said about his suggestions. Of course, one should welcome the recommendation that some institution, for instance, the Association of American Railroads, should accumulate and assimilate all possible information now available on results of locomotive tests. The author himself has gone even further and suggested some time ago that a central institution, the best of which at present is the Association of American Railroads, should conduct a similar research of its own.[13] This is now on the program of the Association, and the author understands that tests with modern locomotives with a view to establishing fundamental data for locomotive ratios are now being contemplated. The author hopes that the tests will soon be made and he is confident that their results will help to confirm, or modify, his moduli and his correction curve (Fig. 5 of the paper).

As to Dr. Giesl-Gieslingen's recommendations 1, 2 and 3, they have no bearing on this paper. The recommendations are not new and are too general. Besides, the recommended tests when carried out, cannot obviate the necessity of having a simple method by which a reasonably correct evaluation of the power and tractive effort of a locomotive could be made without going to elaborate tests and calculations. This was the object of the author's papers.

Dr. Eksergian's apprehension as regards the difficulties with β which we may run into, if we follow the author's suggestion regarding boiler adequacy at 50 rpm, probably can be answered better by saying that while the adequacy, as shown in the paper, comes out close to 1.00 instead of Cole's 100 per cent, the author has not stated definitely what percentage he would recommend for either drag, limited cut-off, or high-speed locomotives. This, following his usual policy, has not been made for the reason that no test data are yet available on high-speed locomotives. His feeling, however, is that 1.00 and greater figures will serve just as well with respect to the boiler adequacy, as Cole's 100 per cent, or larger, boiler. For locomotives with large-size cylinders, both coefficients automatically will come out smaller. Following his line of reasoning, the cylinders have to be dimensioned on the basis of starting tractive effort and he does not expect that any difficulty will arise at high speeds on account of the insufficiency of cylinder dimensions, as long as the boiler is given sufficiently large proportions; corresponding to a boiler adequacy of 1.00, but the larger, the better, especially for high-speed locomotives.

As an example, the high-speed streamlined 4-4-2 locomotive recently completed by the American Locomotive Company for the Chicago, Milwaukee, St. Paul & Pacific could be cited. The Cole boiler-percentage figure for this locomotive is 108.8. The author's boiler adequacy is 1.116, which indicates the close approximation of both coefficients to their respective normal values (100 and 1.00 per cent). The author does not advise that the Cole evaporation figure, with which railroad mechanical men have become familiar, should be replaced by the new conception of boiler adequacy, although he thinks that the latter imparts greater definiteness to the meaning of boiler proportions, for reasons explained in the paper. The main reason for the author's suggestion of this new conception was only because in some quarters it was felt that when the Cole speed factors are replaced by the author's moduli, the Cole boiler-percentage figure is lost sight of. The author, therefore, tried to establish something which could take the place of the Cole boiler-percentage figure, if his moduli are used for the evaluation of horsepower and tractive effort instead of the Cole factors. If it is desired to figure the Cole boiler percentage, it can be done, but then additional calculations of some of Cole's figures, which are not needed for the author's method, will have to be made, without arriving at a substantially different figure, except that one (the Cole) is in per cent, while the other (the author's) is in fractions.

Reverting to Mr. W. A. Pownall's discussion, it can be added to what has been already said, that the discrepancies which he noticed between his curves can undoubtedly be explained by the difference in drafting arrangements. The 1932 and 1934 methods correspond to the ordinary round nozzle and A.A.R. smokebox arrangement, as it has been brought out by the agreement between the author's moduli and test results. The Kiesel front may be giving better results, and probably permit forcings of the boiler, without corresponding reduction in the efficiency, similar to the case with the drafting arrangement on the U.S.S.R., Series JS locomotive.

As to the use by Mr. Pownall of the author's analytical formula

---

[11] "Principal Test Results of the First Type 2-8-4, Series JS, Locomotive" (in Russian), Moscow, 1934, p. 41.

[12] Compare Fig. 34 of "Principal Test Results of the First Type 2-8-4, Series JS, Locomotive," with Fig. 609 of "Locomotive Felix Dzerjinsky" previously referred to.

[13] "Coming Motive Power and Locomotive Research," by A. I. Lipetz presented before the Canadian Railway Club, April 9, 1934, concluding paragraph.

for very high speeds and revolutions per minute (up to 360), this has not been intended by the author. The formulas are simplifications of the moduli. They give approximately the same results and should not be used for speeds higher than the moduli themselves, namely, 250 rpm.

In his 1932 and 1934 papers the author hesitated to extend his curves beyond 250 rpm simply for lack of data. When the tests to which Mr. Pownall refers are published, with all details which would permit judging the efficiency, evaporation and other features of the performance, the curves could be extended for higher speeds. The advantage of the author's method is the ease with which the modifications of moduli can be made if dictated by test results. Simply as an example, but not as a matter of recommendation, the author may say this in reference to higher speeds:

A comparison of his 1934 curves with the curves on Fig. 8 of this discussion and others show a great probability that his moduli for 250 rpm may have been slightly underestimated. It is probable that at higher speeds, especially with better drafting arrangements, the supposition of Mr. Cole, that after a certain limit of speed the horsepower remains constant, is true. If this should be so, say, after 200 rpm, the Mp moduli would be as given in Table 2 of this discussion.

TABLE 2 VALUES OF THE M_p MODULI FOR SPEEDS ABOVE 200 RPM

Revolutions per minute, n	200	250	300	350
Locomotives with feedwater heaters, Modulus $M_p \times 1000$	54.0	54.0	54.0	54.0
Locomotives without feedwater heaters, Modulus $M_p \times 1000$	50.5	50.5	50.5	50.5

Then accordingly, on the basis that Mn = constant, the M_t moduli should be as follows:

Revolutions per minute, n	200	250	300	350
Locomotives with feedwater heaters, Modulus M_t	34.1	27.4	22.8	19.6
Locomotives without feedwater heaters, Modulus M_t	31.8	25.2	20.9	17.9

As the boiler can always be forced to a certain extent, especially at high speeds, it may be safe to use the moduli of Table 2 in all cases when curves have to be extended for high speeds, before reliable test data are available. Of course, they will have to be watched very carefully and checked against test data, and as we are soon going to have high-speed locomotives on many railroads, from which high-speed test results undoubtedly will be obtained, this can be easily done.

In view of the fact that some readers may be tempted to extend the formulas, given at the end of the paper, to speeds higher than 250 rpm, as Mr. Pownall has done, the author thinks it advisable to give formulas corresponding to the preceding tabulation of constant moduli M_p and hyperbolic moduli M_t. In such a case, strictly speaking, it must be noticed that the constant-horsepower line should be tangent to the horsepower curve at its maximum point; then the hyperbola will follow smoothly the straight line of the tractive effort. The approximate speed at which this takes place for formulas [13a] and [13b] is 190 rpm (the exact values are 189.4 and 189.9 rpm).

Incidentally, at the rotative speed of 190 rpm, the modification y is equal to zero. Thus, the following formulas can be recommended:

For locomotives equipped with feedwater heaters and speeds up to 190 rpm

$$T_i'' = \frac{E_e}{D}(73.85 - 0.195n)(1 + y) \quad \ldots\ldots [16]$$

where E_e is the Cole evaporation figure, lb per hr, D is the diameter of drivers, in. and n is rpm. Modification coefficient y should be figured in accordance with formula [15], which is here repeated

$$y = \left(\frac{30}{n + 10} - 0.15\right)\frac{15 - K}{7} \quad \ldots\ldots [15]$$

This formula eliminates from Fig. 5 the sharp edge at K = 14.26, so objectionable to Dr. Giesl-Gieslingen, and replaces it by a blunter point at K = 15.0. A number of small-cylinder locomotives, practically all of any importance, will be thus included.

At 190 rpm $y = 0$, and $T_i''_{190} = \frac{E_e}{D}36.8$

The corresponding horsepower will be (see formulas [5] and [9] of the 1932 paper)

$$P_i''_{190} = \frac{T_i'' \times V}{375} = \frac{T_i'' \times nD}{375 \times 336.134} = \frac{E_e}{D} \cdot \frac{36.8 \times 190 \times D}{126{,}050} = E_e \times .05547$$

Thus, modulus $M_p \times 1000$ = 55.47 instead of 54.0 for the maximum horsepower at 200 rpm, a difference of +2.7 per cent. This represents the approximation of the formula and the error in replacing 190 for 200 rpm as the speed at which the horsepower is maximum.

For speeds higher than 190 rpm, the tractive effort, the horsepower remaining constant, will evidently be

$$\frac{P_i'' \times 375 \times 336.134}{nD} = \frac{E_e \times 0.05547 \times 126{,}050}{nD} = \frac{6992\ E_e}{nD}$$

Thus, the tractive effort, for speeds above 190 rpm, can be figured in accordance with formula

$$T_i'' = \frac{E_e}{D}\frac{6992}{n} \quad \ldots\ldots [17]$$

For locomotives without feedwater heaters the corresponding members of the formulas will be reduced in the ratio of 100:107. They will be, for speeds up to 190 rpm

$$T_i' = \frac{E_e}{D}(69.09 - 0.182n)(1 + y) \quad \ldots\ldots [18]$$

and for speeds higher than 190 rpm

$$T_i'' = \frac{E_e}{D}\frac{6557}{n} \quad \ldots\ldots [19]$$

The author wants to emphasize again that these formulas are approximate. They are of such a nature as not to be easily memorized, and have to be looked up for references. Therefore, it might be just as well to use moduli instead of the formulas. In case a simple formula for memorizing should, nevertheless, be desired, the one given in the paper (14′) could be used. It is here repeated

$$T_i = \frac{E_e}{D}(75 - n/5) \quad \ldots\ldots [14']$$

The modification for locomotives with K less than 15.0, in accordance with formula [15], will have to be made.

This formula is also good up to 190 rpm. At this speed the value in brackets is 75 — 38 = 37. The corresponding formula for speeds above 190 rpm would be $T_i = E_e \times 7030/Dn$, because $37 \times 190 = 7030$. We can simplify the formula by making

$$T_i = \frac{E_e}{D}\frac{7000}{n} \quad \ldots\ldots [20]$$

which is approximately the same as formula [18] and sufficiently simple to be remembered.

Curves of formulas (14′) and (20) intersect at n = 200. At

this speed the factor in the brackets in formula [14′], which is M_v is equal to $75 - \frac{200}{5} = 35$, instead of 34.1 according to the previous tables. This is sufficiently close for an approximate calculation.

For locomotives without feedwater heaters the values of formulas [14′] and [20] must be reduced in the ratio of 100 : 107.

Mr. Fry, who is in general agreement with the author's method, brings out a point which may be of interest. He refers to his contributions made about thirty years ago and indicates that instead of Cole evaporation (E_c), the outside (evaporating) heating surface (H_e) could be used. This is approximately correct, because the ratio between these two figures, namely, the average Cole evaporation per hour per square foot of outside heating surface, differs very little. For locomotives given in Table 4 of the paper and in Table 1 of the present closure, it fluctuates between 11.50 (No. 3) and 14.72 (No. 1). For more modern locomotives it varies between 12.91 (No. 10) and 13.96 (No. 17). An average figure of probably 13.5 could be adopted, but it would still be an approximation and would in many cases destroy the good agreement between curves and test results, to which reference was made by Mr. Fry himself.

Mr. Fry's objection is mainly due to the fact that he had difficulty in computing the total Cole evaporation figure. As is known, Cole's figure is based on 55 lb of steam per hr per sq ft of direct heating surface (firebox plus brick arch tubes plus syphons), and average evaporations for tubes and flues, depending upon their size, length, and spacing. Tables of these specific evaporations are given in the locomotive handbook published by the American Locomotive Company in 1917, and reference to it was made in the author's 1932 paper. New figures regarding tubes of $3^1/_2$ in. outside diameter (without superheater units) and $3^1/_2$-in. flues (with superheater units), which came into use after the publication of the A. L. Co. handbook, in pounds of steam per hour per square foot of outside heating surface for various lengths of tubes and flues and various spacings, have been given by the author in Appendix No. 2, page 16 of the 1932 paper.[3] The author is surprised at Mr. Fry's statement that "Lipetz does not give in either the present paper or his earlier paper complete information for computing this Cole boiler evaporation." While this is literally correct, proper references to the A. L. Co. handbook were made. It is true that the locomotive handbook is now becoming very rare, but it has been in use by practically all railroads for years, and copies can be found in libraries. A new edition is contemplated.

Apart from the reason of the momentary difficulty in finding access in some cases to the A. L. Co. handbook, there is no good argument in favor of using the heating surface, as approximation, instead of the Cole evaporation figure, as the latter, when the basis of computation is available, can be determined easily. However, if a change should be desired, there would be no difficulty in substituting the total evaporating heating surface (H_e) for the Cole evaporation (E_c), if the author's moduli are increased in the ratio of, say, 13.5, which, as we saw above, represents the average evaporation figure for modern locomotives in pounds per hour per square foot of total evaporating heating surface. However, correction curve (Fig. 5 of the paper) would have to be accordingly modified, so as to use the author's δ instead of K. This would be another example of the flexibility of the author's method, which can be easily adjusted to simplifications when they are thought necessary, or to new data when they become available.

There remains Prof. H. Rubenkoenig's discussion, which is directed more to the starting tractive effort. He refers to auxiliary starting ports, for which he would like to have a coefficient in the tractive-effort formula. The author's paper did not consider the tractive effort at the moment of starting. This differs from the maximum, or the rated tractive effort, which can be indefinitely developed at a certain speed (about 50 rpm), depending upon the capacity of the boiler. Under these conditions, the author gave in his discussion coefficients for the rated tractive effort in his closure on page 33 of the 1932 paper.[3] The author does not think that the auxiliary ports can increase materially the tractive effort and, therefore, he did not give any corrections for that.

Before closing the author would like to call attention first to the maximum boiler tractive effort, and, second, to the types of locomotives for which his method and moduli should be used. Some railroad engineers claim that they are not interested in the "performance" curves obtained under conditions defined by the author in his 1932 paper and its closure, but in the maximum-tractive-effort curve, which represents the top values at various speeds under hardest working conditions. On some roads they are called "capacity curves." The author did not attempt to give these curves, because they depend upon the specific properties of a locomotive and the individual method of operation, which cannot be easily generalized. At least, there are not yet sufficient data, even on modern American locomotives, to predict with accuracy what the maximum curve can be. The author attempted to give a general method for the evaluation of the performance curve which corresponds to usual every-day methods of operation with a reasonable efficiency. His curves are lower than what occasionally can be materialized, but they have the advantage that they can be safely developed with reasonable efficiency and without excessive maintenance. The maximum curve depends upon the extremes in efficiency and maintenance to which the operator is willing to go, and cannot be adopted generally. This curve has to be established by the operating railroad from actual experience. But even then, it might be found advantageous to compare the test curve with the performance curve, and when enough information is accumulated by the railroad on a number of locomotives, a method of evaluating particular locomotives can be worked out by a comparison with the basic performance curves.

The other point, closely connected with the first, refers to the designs of locomotives which were used as a basis when the method and moduli were established. It was called the modern locomotive, under which term it was understood a locomotive with sufficiently large grate area, a brick arch, type-E or a well-proportioned type-A superheater, 225 to 300 lb per sq in. boiler pressure, simple expansion cylinders, properly designed valve motion according to latest practice, round exhaust nozzle, and A.A.R. smokebox arrangement. It would be erroneous to use the formulas for locomotives which differ from the described standard design. If some improvements are being tried out, say, for instance, an improved drafting arrangement, tests should be made and the difference would indicate the gain resulting from this improvement. The same would apply to poppet valves and similar innovations. It is evidently incorrect to compare actual test data from locomotives of other, especially non-American, designs, with these formulas, as has been done, for instance, by Mr. André Chapelon in his very informative article on the French High Speed 4-8-0 Type Locomotive of the Paris-Orleans Railway. He checks the high power obtained from these locomotives against the Lipetz formula and shows that the power is more than twice the maximum obtained from the formula.[14] In this case the boiler was made up mostly of tubes of the French Serve type, which have $2^9/_{16}$ in. OD and have inside ribs. The engine has four cylinders compound, rotary-cam Lentz-Dabeg poppet valves, and a double Kylchap drafting arrangement. The author does not understand how the Cole evaporation figure

[14] *Revue Generale des Chemins de Fer*, March, 1935, p. 281.

could be computed for a boiler with Serve tubes and how the Lipetz curve could be plotted. The increase in power as compared with the Lipetz formula is probably due to the high forcing of the boiler and to the enumerated improvements.

The author based his moduli on tests with modern American locomotives He has not extended them yet to other types of locomotives, like high pressure, double expansion, etc., mainly for lack of data, and secondly, because the main object of the paper was to show that the method based on boiler evaporation, rather than that based on cylinders (Cole method), gives more consistent results in regard to establishing the power of a locomotive.

The Intake Orifice and a Proposed Method for Testing Exhaust Fans[1]

R. D. Madison.[2] Fig. 11 shows a proposed arrangement which seems to apply particularly to axial-flow fans mounted in ducts. For wall-mounted disk fans the box arrangement shown in Fig. 10 is not only desirable but it must be used if the flow lines through the fan are to duplicate actual conditions. For the centrifugal fan without inlet boxes, the fan should not be tested on the inlet. For centrifugal fans with inlet boxes, the method of providing the box (or boxes) with bell-mouthed entrance duct (as shown on plate *G* of the N.A.F.M. and A.S.H. & V.E. codes) seems to have everything to commend it, since a short discharge duct should be provided in any case. The applicability of the intake orifice to fan testing, as far as code procedure is concerned, is still open to discussion. Although the intake orifice is much simpler than the rounded nozzle to construct, its application is quite limited.

Two points should be mentioned in the use of the inlet orifice which the authors have not fully covered. No provision is made for eliminating chance spiral flow that may be present in the test room and which will be accelerated as the orifice is approached. An open door or the fan's own discharge may provide this spiral impetus. Even a simple form of crisscross straightener a short distance ahead of the orifice should suffice. Practically all wind tunnels are equipped with some such device.

Again, as the orifice approaches 100 per cent pipe diameter, the flange should extend appreciably beyond this point to keep the flow lines at entrance substantially constant. The coefficient for an open-end pipe is different from an orifice in a flat plate and when the pipe is tested with full pipe opening the flange should be sufficiently extended to approximate flat-plate conditions. It would seem more practical, in the case of the 100 per cent orifice, to use a tap at full pressure recovery and the corresponding coefficient of discharge. This would require a less critical placing of the tap.

The diagram, Fig. 4*a*, is likely to be confusing to those not familiar with air flow. The vena contracta is shown more than one pipe diameter from the orifice to correspond with the readings obtained at the pressure taps. Now the actual vena contracta is in the neighborhood of $^1/_2$ orifice diameter from the orifice plate and the flow approaching it corresponds closely to the conventional rounded nozzle. From this point to that of maximum recovery pressure there is considerable turbulence and some of the outer mass of air is flowing in a direction opposite to the central stream. The static pressure is not necessarily uniform over any given cross-section and the static-pressure taps do not record the static pressure except in the outer region adjacent to them. The gradual rise in pressure from the lowest point to that at the orifice plate is probably due to a slowing up of the counter current flow and an attendant increase in static pressure. While the positions of the taps recommended by the authors are logical from the standpoint of stability of readings and are, therefore, the most accurate to use, there is no need to attempt to correlate the flow as indicated in Fig. 4*a*. To do so will only lead to confusion.

[1] Published as a paper PTC-56-3, by N. C. Ebaugh and R. Whitfield, in the December, 1934, issue of the A.S.M.E. Transactions.

[2] Research Engineer, Buffalo Forge Company, Buffalo, N. Y. Assoc-Mem. A.S.M.E.

Authors' Closure

The authors cannot agree with Mr. Madison in the statement that a centrifugal fan without inlet boxes should not be tested on the inlet side. Single-inlet exhaust fans which are usually directly connected to a duct should be tested on the inlet side as such a set up will exactly duplicate the way in which the fan is intended to be used. The inlet orifice is a simple and accurate device for such tests.

The test room used was so large compared with the orifice area that there was no perceptible air motion a few feet away from the plate. The possibility of spiral flow in the room ahead of the plate was remote indeed. The use of air-flow straighteners would hardly be feasible with the intake orifice and it should not be used if there is an unusually bad approach condition.

The statement was made in the paper that the flange on the end of the pipe was acting as the orifice in the case of the 100 per cent orifice. The authors agree with Mr. Madison that the coefficient would be different if this flange were absent. The 100 per cent orifice is not recommended for measuring purposes and was only investigated as a matter of interest.

No distances are marked on Fig. 4a but the discussion of the behavior of the air below the plate must be accepted by the authors since it comes from one who must be considered one of our authorities on air-flow behavior in general.

Pulverized Fuel-Burning Experiences at Buzzard Point Station[1]

E. G. Bailey.[2] The boilers and furnaces at Buzzard Point station represent a new type of steam-generating unit in which have been incorporated the best features of existing installations together with additional features which make it an outstanding development. The furnace is completely water cooled, the lower portion being of the stud-tube construction; vertical turbulent burners are used, which direct the flame down onto the liquid-slag bath; a furnace slag screen divides the furnace into two portions, a lower combustion chamber and an upper cooling chamber which is covered with smooth iron blocks to obtain maximum cooling of the gases before entering the convection surface; straight tube sections of the header type resemble conventional design, but the head is greatly increased permitting high ratings per section; the superheater is supported directly from vertical circulating tubes and is divided into two sections with a desuperheater between to give flat steam temperature; the economizer is entirely within the setting and supported on vertical circulating tubes.

Burning coal in pulverized form presents two major problems, the first of which is to burn practically all of the carbon in the limited space and time available with the high liberations dictated by economy. This is not easy to do with the low-volatile coal used at Buzzards Point but the low unburned-carbon loss reported in the paper confirms the correctness of the burner and

[1] Published as paper FSP-56-18, by H. G. Thielscher, in the December, 1934, issue of the A.S.M.E. Transactions.

[2] Vice-President, Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

furnace design. Recent tests have shown average combustible in the flue dust of 4.0 per cent corresponding to a heat loss of 0.07 per cent at a low rating of 68,000 lb per hr of steam and at a high rating of 313,000 lb per hr of steam the combustible in the flue dust averaged 10.8 per cent corresponding to a heat loss of 0.34 per cent.

The second problem is to dispose of the ash. The vertical burners direct the ash directly against the liquid-ash floor where a large portion is caught. The walls of the lower chamber are wetted with slag and trap additional amounts of ash. The furnace slag screen, which is studded and covered with refractory, catches still more of the ash as the gases impinge on the tubes in passing through. In this way the coarsest, stickiest and most objectionable portion of the ash is caught in the furnace and tapped out as liquid slag. The dust collectors remove practically all of the remaining dust and the appearance of the stacks is very good indeed.

The rate of heat release of 36,800 Btu per hr per cu ft of total furnace volume is no doubt the highest yet obtained from low-volatile coal with a satisfactory low-carbon loss. It is possible to further increase this rate. The furnace slag screen is an important factor in this design. It forms a barrier which shields much of the radiant heat of the flame adjacent the burners and of the slag pool from reaching the boiler tubes, thereby permitting the tapping of ash of higher fusing temperature and reducing the slagging of tubes in the boiler bank.

The proportioning of heating surface between the furnace, boiler and superheater was influenced more than it should have been, evidently, by data obtained from coal of higher volatile and other characteristics different from that being burned. The spacing of the furnace slag-screen tubes might also be further improved in subsequent designs for similar coal. It should be noted, however, that these units are designed for a maximum output of 412,000 lb steam per hr each, corresponding to a heat release of 41,600 Btu per hr per cu ft. This rating cannot be obtained with only one turbine in the station.

The steam scrubbers installed in these boilers have proved satisfactory, proof of which is evidenced by turbine performance.

The modifications required on the burners were necessitated by the combination of high preheat on the secondary air with a readily coking coal. Reducing the number of primary nozzles and increasing their width has been quite successful. The practice of running cold air through an idle burner a few minutes just before putting it into service is a simple procedure which is quite helpful in preventing coking.

This design of boiler unit is well adapted for wide application, especially for capacities considerably above that for which these units are designed.

A. G. Christie.[3] One is impressed with the high ratio of water-wall area to true boiler-surface area in the Buzzard Point boilers. Evidently the greatest advantage is taken of heat transfer by means of radiant heat. In fact, the furnace itself is water cooled practically to the same extent as a Scotch Marine boiler.

The low-volatile coals of Virginia and West Virginia have comparatively high-fusing-point ash. One would not expect these to be best suited for use in slag-bottom furnaces. Apparently the fluid ash can only be tapped with ease under certain favorable high-load and high-furnace-temperature conditions. It is rather surprising to note that the use of various fluxing materials has not resulted in more fluid slag. This must have been due to lack of proper mixing in the slag. Normally such flux material should have combined with the ash and increased fluidity. The author states that these fluxes were introduced in a powdered state. Possibly they were carried over to the electrostatic precipitator instead of entering the slag. Why not try the introduction of limestone in lumps?

Apparently no difficulty has resulted from the pressure and temperature employed, viz., 670 lb and 835 F. This should encourage designers to use still higher steam pressures and temperatures on the next station to be built. It may be noted that some difficulty is being experienced in this station to secure sufficient superheat.

Fig. 3 indicates an interesting distribution of the temperature drop of the gases. About one quarter goes to the water walls, another quarter to the boiler surface proper, a third quarter to the superheater, and the final quarter to economizer and air heater. This emphasizes the growing importance of the secondary heat-recovery apparatus under modern operating conditions of high pressure and temperature.

There are rumors that difficulties have been experienced with the horizontal tubes in the air preheater. Possibly the author may be willing to tell us of these experiences.

The performance of the hydraulic fan couplings as shown in Table 3 indicates too slow a response to load changes. This coupling has many distinct advantages but needs further development to improve its speed of response to load changes.

The rates of heat release in this furnace are higher than has generally been used in the past. No ill effects from bad slagging have been noted. This would indicate that still higher rates of heat release with correspondingly smaller furnaces may be considered for future installations where the furnace is water cooled throughout and a slag-bottom furnace employed. At present the space above the dust screen must serve to a considerable extent as a secondary combustion chamber with flames penetrating completely through the dust screen. The author states that at times there was a tendency to slag the lower tubes of the boiler itself. This suggests that conditions might be improved by wider spacing of the tubes in the dust screen with more rows of tubes in depth in this dust screen and with the whole dust screen placed just below the tubes of the boiler proper. What advantages are claimed for the present position of the dust screen intermediate between the primary and secondary combustion chambers?

The need of dust catchers in the flue gases is evident from the author's statement that 50 per cent of the ash was caught in the electrostatic precipitators.

Little comment can be made on the boiler efficiency or station performance. These are excellent results.

M. K. Drewry.[4] Cited experiences with slagging raise the general question as to whether freedom from undesirable deposits in any slag-tap installation can ever be sustained under varying conditions of load and with various types of coal. It is true that ash accumulations are limited in the high-temperature zone by fluidity of the molten ash, and in the lower-temperature zone by mechanical breakage due to weight. However, at some intermediate point plastic sticky ash is always present near some heat-absorbing surface and, at that point, accumulation of adherent slag is without limit.

Why ash of higher fusion temperature should cause clogging farther into the boiler tubes of the Buzzard Point installation was not readily apparent until some study indicated that clogging moves progressively in the direction of gas flow until equilibrium of gas temperatures and ash melting point occurs.

Variable insulation of surfaces ahead of the superheater causes lack of uniformity in its performance, which suggests a second reason for using superheat-regulating equipment.

[3] Professor of Mechanical Engineering, Johns Hopkins University, Baltimore, Md. Mem. A.S.M.E.

[4] Assistant Chief Engineer of Power Plants, Milwaukee Electric Railway & Light Company, Milwaukee, Wis. Mem. A.S.M.E.

One of the major merits of pulverized-fuel-firing equipment is its adaptability to firing any kind of coal as the market changes. The moderate-temperature installations appear the best answer to the variable-coal, variable-load problem.

Many operating companies have formed the policy of using at least two mills or feeders per furnace, guarding against momentary interruption of all coal supply to the furnace. Buzzard Point employs automatic relighting with gas, the failure of which is perhaps as improbable as the simultaneous failure of two mills.

E. L. Hopping.[5] The author discusses the use of hydraulic couplings with automatic combustion control, for varying speeds on forced- and induced-draft fans and it is noted that because of the slow response of the coupling with the Weir type of control it was necessary to apply additional automatic control to the draft dampers, supplementary to fan speed change.

The Philadelphia Electric Company is now installing a 165,000-kw turbo generator with two 600,000 lb per hour pulverized-fuel-fired boilers in their Richmond Station and are providing hydraulic couplings for operation of forced- and induced-draft fans and variable-speed condensate pumps. The decision to use this speed-changing method was based partly on the experience at the Buzzard Point Station, together with experience gained by installation of a hydraulic coupling with hand-operated Weir control on one of the present test boilers in the older section of Richmond Station. It was realized that Weir control would be too sluggish for the best operation with automatic boiler regulation, and the coupling manufacturers agreed to make comparative tests on a coupling using Weir control in one case and a small auxiliary pump either for removing or replacing oil in the coupling in the other case. These tests indicated that with the auxiliary pump it was possible to reduce the time required for a given speed change to about one-third of that resulting from Weir operation.

It is expected that a satisfactory rate of speed change will be secured on fans for the new Richmond boilers without supplementing the speed control with automatic damper regulation.

Ten Years of Stoker Development at Hudson Avenue[1]

J. S. Bennett.[2] The authors have performed a most useful service in presenting the results of their studies in analyzing losses sometimes disregarded. The figures given on cinder losses are particularly interesting. In the writer's opinion, certain warnings should be uttered against applying these figures too literally. At 75 lb per sq ft, the cinder loss is shown to be nearly 8 per cent. Assuming that the cinder loss is entirely eliminated and that the unit under these conditions is operating at 83 per cent efficiency, the efficiency would be increased by only 6.5 points, not 8 points, because only 83 per cent of the heat in the cinders would be transferred to the water and steam.

Considerable work has been done in developing means for minimizing this loss. The authors point out that a certain proportion of the cinders was trapped in the last pass of the boiler and returned above the ash pit. This improved the performance somewhat, but the method of cinder trapping in the last pass was inefficient as provisions had not been made in the original design for cinder return. The cinders were fed in above the fire, which had a tendency to recirculate part of them. It has been found far more effective to return cinders with the coal feed at the front of the stoker. Cinder trapping can be provided in any new design, with the cinders returned by gravity to the stoker coal hopper. This is particularly simple in the case of double-fired boilers. The writer's Fig. 1 shows an installation of this type put into service recently. In this arrangement, the cinders enter at the bottom of the retort and are covered by fresh coal. They cannot be recirculated, and are fed up slowly from the bottom of the retort where they burn in the same manner as the rest of the fuel bed. Other schemes for cinder return are shown diagrammatically in Figs. 2 and 3 of this discussion. Assuming that 80 per cent of the cinders discharged were trapped and returned as shown, at a coal-burning rate of 75 lb per sq ft, the cinder loss at Hudson Avenue would have represented only 1.6 points in efficiency and these, if burned, would represent an increase in efficiency of 1.3 points.

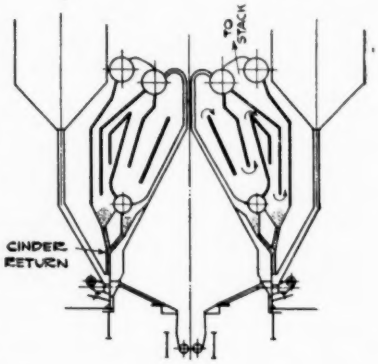


Fig. 1

The authors have emphasized the fact that their experience has been obtained with one grade of coal. It might be interesting to compare the results obtained with other coals using preheated air. The curve shown in Fig. 11 of the authors' paper is reproduced in Fig. 4 of this discussion. The cinder loss of a coal selected for test because of its tendency to produce cinder losses and operating with preheated air where the cinder recovery was somewhat more effective, is shown by the curve marked *B*. The same job, burning a Pittsburgh-district coal, without any cinder return, gave a very much reduced cinder loss, marked *A*. These curves show that the cinder loss is influenced very strongly by the coal used. The low-volatile, high-Btu coals used quite generally in New England and New York City, ordinarily show much higher cinder losses than the coals with higher volatile matter or coals containing ash with lower fusing temperatures.

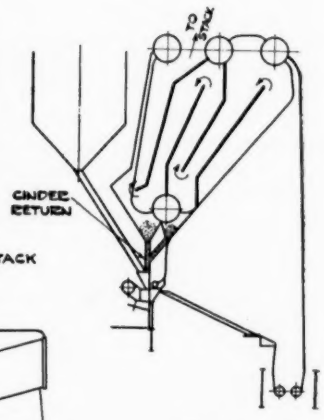


Fig. 2

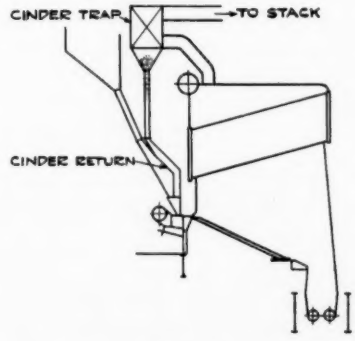


Fig. 3

The authors imply that their experience shows that a short underfeed stoker with a long overfeed section will permit higher sustained fuel-burning rates than the full underfeed-type stoker.

[5] Mechanical Engineer, Philadelphia Electric Company, Philadelphia, Pa. Mem. A.S.M.E.

[1] Published as paper FSP-57-3, by J. M. Driscoll and W. H. Sperr in the February, 1935, issue of the A.S.M.E. Transactions.

[2] Mechanical Engineer, American Engineering Company, Philadelphia, Pa. Mem. A.S.M.E.

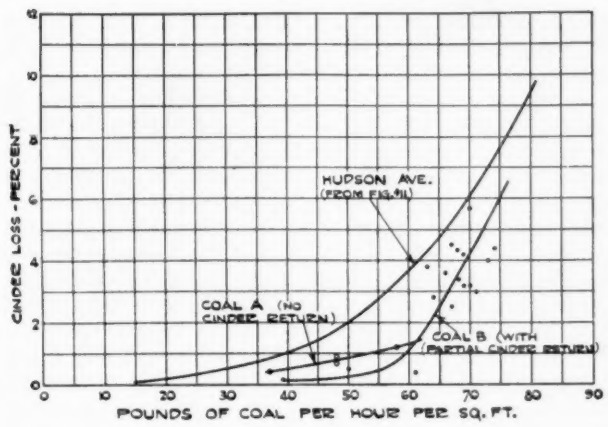


FIG. 4

This conclusion is based on a comparison of relatively small stokers of one make with the longest stokers ever built by a different manufacturer and underfeed stokers of still another make built ten years ago. Where comparisons have been made directly between machines of the same size of the two types, this improvement in coal-burning capacity has not been shown.

Where sustained high fuel-burning rates are required, it is poor policy to rely upon the skill and agility of the operator to maintain the fuel bed. It is better to provide means for distributing the air that will indicate and correct fuel-bed weakness long before the human eye can detect the trouble. Zoned-metered-air control provides this means.

V. G. GAHNKIN.[3] The paper shows that long underfeed stokers are apparently limited to an efficient maximum burning capacity of 55 lb of coal per hr per sq ft of grate area, and that to get higher combustion rates it is necessary to add either a movable link-grate section or to introduce an air-zone method of controlling the windbox pressure.

The improvement effected by air-zone control in the efficiency of boiler No. 74 at high ratings and the increase in capacity from 350,000 to over 500,000 lb per hr continuous rating is very remarkable. The air-zone control by permitting regulation of air pressure to just overcome the fuel-bed resistance in each section makes it possible to reduce excess air and carry higher ratings with the same thickness of fuel bed without danger of lifting portions of the fuel bed from the grate. This should permit the building of larger and more efficient stokers in the future.

The authors have made a comparison of stoker performance in their Fig. 10 based on the heat absorbed by the boiler unit per square foot of projected grate area. It seems that a better comparison of stoker performance could be made on the basis of pounds of coal burned per square foot of projected grate area and thus eliminate the effect of the efficiency of heat absorption by the boiler unit since the effect of the difference in the heating values of the coal is small. In Fig. 5 of this discussion, a comparison of the stoker performance on boilers Nos. 54 and 74 has been made on this basis and shows a much closer agreement on overall efficiency than that shown in the authors' Fig. 10.

The stoker losses, namely, the loss due to combustible in ash and the cinder loss, closely agree for both stokers. At a combustion rate of 75 lb per hr, the combustible-in-refuse loss is about 0.6 per cent higher on No. 54 boiler, while the cinder loss is lower by 0.8 per cent, making the combined loss practically the same for both units.

[3] Chief Testing Engineer, Brooklyn Edison Company, Brooklyn, N. Y. Mem. A.S.M.E.

The difference in overall efficiencies between boilers Nos. 74 and 54 is due therefore to the losses which depend upon the design of furnace and heat-absorbing surfaces. There is 25 per cent more furnace volume per square foot of grate area in No. 74 than in No. 54 boiler; hence, the heat release per cubic foot for a given burning rate is higher for the No. 54 installation, being 65,000 Btu per hr per cu ft for No. 54 and 48,000 for No. 74 at a combustion rate of 75 lb per hr. There is a greater possibility therefore of having an incomplete combustion loss due to CO, H_2 and hydrocarbons in the smaller furnace at the higher ratings. The loss due to incomplete combustion of CO was about the same for both installations. The loss due to unconsumed hydrogen and hydrocarbons was not determined in the No. 54 boiler test.

There is more waterwall, superheater and economizer surface in boiler No. 74 than in No. 54 and the per cent of heat absorbed by each of them is different in both installations, resulting in a higher overall efficiency of No. 74 at high ratings, although the dry-gas loss is greater on boiler No. 74 than on No. 54.

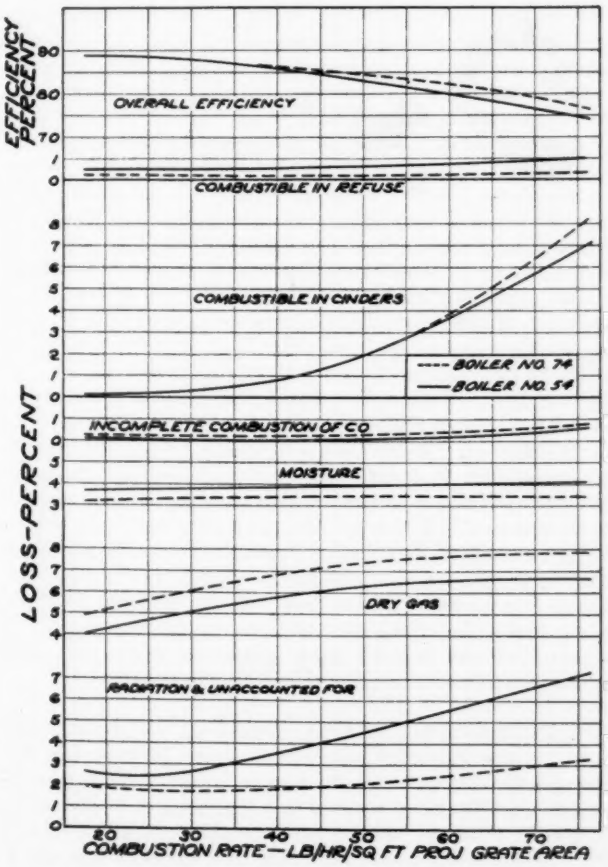


FIG. 5 COMPARISON OF TEST PERFORMANCES OF BOILERS NOS. 54 AND 74, HUDSON AVENUE STATION

The only losses which differ a great deal in the two installations are the radiation and unaccounted-for losses. Even if the hydrocarbon and hydrogen losses determined on No. 74 are added to the radiation and unaccounted-for losses, at a combustion rate of 75 lb per hr, the total is as much as 4 per cent less than on No. 54. If a correction is applied, to take care of radiation and unaccounted-for loss difference, the efficiency curves of both installations become identical, having an efficiency of about 75 per cent at a combustion rate of 75 lb per hr.

In order to show how nearly equal the actual performances of the two stoker and boiler installations are when compared on the same combustion rate, refer to the authors' Table 5. The yearly operating figures for efficiency and combustion rates are very close for the Westinghouse and Taylor stokers. The average combustion rate for 1932 and 1933 for the fifth and sixth rows of boilers was 24.4 lb per hr; for the seventh and eighth rows, the combustion rate was 25.1 lb per hr and their respective efficiencies were 86.0 and 86.3 per cent. These figures include also the banking hours and the low-steaming periods which decrease the actual combustion rates by 15 per cent. For the operating range up to combustion rates of 40 lb per hr, both installations are identical in efficiency. Higher combustion rates are only run for short periods during the peak, and therefore affect the yearly efficiency only slightly.

In conclusion, it might be said that both installations are doing equally good work and are equally efficient when based on the same combustion rates. The fifth and sixth rows are simpler in design and are liked by the stoker operators. The operators are, however, becoming used to the more complex air-zone control of the seventh and eighth rows and have already developed a high operating efficiency.

P. W. Keppler.[4] The authors state that for the very long stokers of units Nos. 7 and 8, 55 lb per sq ft of total projected grate area was found to be the maximum fuel-burning rate obtainable for long periods without zoned air control. They also state that for shorter underfeed stokers of this make, more capacity would undoubtedly be obtainable, but that experience had not been gained regarding this at Hudson Avenue. At Hell Gate, many capacity tests were run at high fuel-burning rates. The peak for long periods was reached on June 25, 1925, on boiler No. 63, when 78 lb of coal per hr per sq ft of total projected grate area were burned for 12 hours. The stoker had a projected length of nearly 16 ft and a total projected grate area (including pit) of 378 sq ft. The heat content of the coal was 14,333 Btu so that 78 lb of this coal are equivalent to 80 lb of 14,000-Btu coal. The limitation to capacity was induced draft, not the stoker; in order to obtain the required gas flow, the furnace had to be put under 0.4-in. pressure, so that the fire could not be observed. With ample induced draft, affording an opportunity to carefully watch and regulate the fire, still higher fuel-burning rates should be obtainable. Furthermore, great improvements in stoker design have been made since this test. The capacity of shorter stokers of this type, if operated without zoned-air control, is likely to be limited more by smoke and combustion losses than by lack of fuel-burning ability.

T. E. Purcell.[5] The large amount of field development following the successive installations serves to exemplify the uncertainty with which results can be predicted for stoker installations. When it is realized that the successive installations were only a step in advance of the previous installation, and that the development was carried on in a single station with a specific type of coal, emphasis must be placed on the fact that there is a dire need for more knowledge concerning the functions of the stoker and the process of burning coal thereon. This paper, it seems to me, points to the great need for cooperation and combined effort on the part of designers, manufacturers, and users of stokers to the end that sufficient knowledge may be made available so that stoker performance can be predicted with some degree of certainty. It appears quite evident to us that each installation must be designed for the specific grade of coal to be used, and that stokers cannot be designed to burn varieties of coal with entire satisfaction and with the best results.

Experience with underfeed stokers at the Colfax and James H. Reed Power Stations may be used as an illustration. The combustion rate in pounds of coal per hour per square foot of grate surface for the stokers in the Colfax Power Station is from 15 per cent to 20 per cent less when burning coal from the Sewickley seam than with coal from either the Pittsburgh or Freeport seams, all three seams being common to the Pittsburgh district. On the other hand, the stokers at the James H. Reed Station, which are of a later design equipped with the link-grate overfeed section and adjusted to burn Sewickley coal, will consume equal quantities of either Sewickley or Pittsburgh coal; but severe burning of the stoker parts and high maintenance cost accompany the use of Pittsburgh coal. Unquestionably, to burn Pittsburgh coal to the best advantage, the stokers would require considerable alteration. The stokers at the James H. Reed station, although prototypes of the link-grate stokers developed at Hudson Avenue and later successfully applied in the High Bridge Station of the Northern States Power Company, required major alterations in the field before they were considered satisfactory operating units.

The authors seem to be fully convinced that stokers provide the better method of burning coal in the Hudson Avenue Station. After ten years of experience with both stokers and pulverized-fuel equipment at Colfax and four years of experience with stokers at Reed, the writer is not prepared to believe that one method is definitely superior to the other. Each new installation merits special investigation of both methods of firing, giving due consideration to the latest developments. Each method of firing has its advantages and disadvantages. In my opinion, a great advantage in pulverized-fuel firing over stoker firing lies in the greater consistency in the results obtained. Uniform performance can be expected, when burning coals having widely varying characteristics, in pulverized-fuel furnaces with either horizontal or vertical firing and with more or less water-cooling surface exposed to the furnace. On the other hand, as stated above, stoker operation is influenced greatly by the characteristics of the coal, and continuous attention and adjustments must be given the equipment. Even then consistent fuel-bed conditions and maximum steaming capacity are not always assured.

I. E. Moultrop[6] and G. C. Eaton.[7] The history of the development of the Hudson Avenue stokers during the past ten years contains much valuable information. One must applaud the courage of the Brooklyn Edison Company engineers who, after carefully considering the merits of the various types of fuel-burning equipment then available, decided in 1922 on the use of underfeed stokers, especially when one considers that the fuel they were to burn was high-grade Eastern bituminous coal. While such coal is a very excellent bituminous coal, it is probably more difficult to handle on an underfeed stoker than a lower-grade coal having a higher ash content. In the light of today's knowledge, no one could criticize the decision, but at that time it certainly required very careful consideration and the courage of one's convictions.

The first stokers installed at Hudson Avenue were very good machines as of that time. The story of the various steps taken to improve the stoker and its performance during the decade from 1924 to 1934 makes very fascinating reading, especially to

[4] Testing Engineer, Hell Gate Station, United Electric Light and Power Company, New York, N. Y. Jun. A.S.M.E.

[5] General Superintendent of Power Stations, Duquesne Light Company, Pittsburgh, Pa. Mem. A.S.M.E.

[6] Chief Engineer, Edison Electric Illuminating Company, Boston, Mass. Mem. A.S.M.E.

[7] Head, Mechanical Technical Engineering Division, Generating Department, Edison Electric Illuminating Company, Boston, Mass. Jun. A.S.M.E.

those who have been through a similar experience and have given much thought to the art of burning fuel for power purposes.

These developments appear to have been a simple procedure when one reads about them, but it should be borne in mind that to accomplish such results, much patience, courage and painstaking study is required to meet and solve the many discouraging situations. Much money also was needed.

The improvements in boiler-unit performance with each installation are remarkable and praiseworthy, the most unusual following the installation of the zoned-and-metered-air equipment on the Taylor stokers. This improvement is noteworthy, not only because of the ability of the equipment to burn very much larger quantities of coal per square foot of grate, but its ability to do this with a relatively flat efficiency curve. A stoker equipped with zoned-air control is really in a class by itself. With further experience in the use of this equipment, an even better performance with the stoker of the future may be confidently expected.

The high combustion rate obtained with stokers having air-control is bound to cause a considerable cinder loss. This should not be considered a serious drawback. With suitable cinder catchers, much of the cinder rejected by the furnace can be reclaimed and burned in the ash pit, materially reducing the cinder loss. Even if this loss must be looked upon as a more or less serious item, it should be remembered that one cannot get something for nothing. If, with a given piece of equipment, much greater capacities are to be obtained, it will usually be at the expense of something which is not desired. If tremendous overloads can be obtained from a given size stoker by running it at very high ratings, the capital investment is kept down and the overhead charges thereby reduced. The savings in these overhead charges which apply to every minute in the life of the plant will offset many times the relatively small cinder loss for the very short time that the overload capacity is required.

It is interesting to note that No. 74 stoker is installed under approximately the same boiler surface as No. 54 stoker, the former giving about double the steam output of the latter. To be sure, No. 74 stoker has an economizer surface 50 per cent greater than that of No. 54, which would compensate for some of this increased capacity.

The maintenance costs of the various stokers are instructive to the reader. There are several questions which come to mind. Because of the many changes and experiments made on all the stokers after they were installed, much of the maintenance expense was no doubt borne by the manufacturer. Were the maintenance costs given corrected for this fact? Were units 1 to 3 and 1 to 4, inclusive, operated during 1932 and 1933, respectively? No costs were given for these unit periods. Maintenance costs of occasionally used units are often of interest. The lack of noteworthy difference between maintenance costs of similar units with and without preheated air is unexpected. Have the authors any explanation for this lack of difference?

The discussion about the omission of air preheaters on the later installations is very interesting and the arguments for their non-use seem to be sound. It is our opinion that if the Brooklyn Edison Company were to build another station now, they would find it desirable to add a certain amount of preheater surface because certainly added economy could be obtained thereby. However, one must not lose sight of the fact that there is now a very definite limit to the amount of preheating desirable with underfeed stokers.

The authors mention that the first installation at Hudson Avenue did not contemplate the use of fans, the thought of the designers being that the chimney draft would be sufficient. It is the opinion of many that the high-capacity plant of today requires the use of both forced- and induced-draft fans. There is a point which the average designer is apt to overlook, namely, that the capacity of both types of fans, particularly the induced-draft, must be ample. Probably more errors in judgment have been made in determining the capacity of the induced-draft fan than in almost any other part of a given installation. The size of this fan fixes the absolute limit of boiler-unit capacity.

Fig. 12 is difficult to read. This group of curves would be very much more valuable to the average reader if its ordinate were doubled. The over-lapping of lines and areas is very confusing. The freedom with which the authors change from boiler numbers to row numbers, from output in terms of millions of Btu per hour per square foot of stoker area to output in pounds of coal burned per hour per square foot of stoker area, is also very confusing to the average reader. If the references and terms could be narrowed down, considerable hunting through the paper would be saved the reader.

Authors' Closure

Mr. Bennett's point that any recovery of cinder will be utilized at some efficiency less than 100 per cent in the event all the cinder is returned to the stoker is important and must be taken into consideration when evaluating this loss. In reference to his statement regarding the Hudson Avenue installation, "The cinders were fed in above the fire, which had a tendency to recirculate them," tests made since obtaining the results presented in the paper indicate:

1 Approximately 46 per cent of the total cinder produced and leaving the stoker was trapped in the boiler hoppers and returned to the furnace.

2 Approximately 25 per cent of that cinder trapped was recirculated, this percentage tending to decrease at the higher burning rates.

3 Between 35 and 40 per cent of the total cinder produced remained in the ash pit to be burned. From the ash-pit combustible-in-refuse loss, reported in a recent paper on the performance of the Hudson Avenue stokers,[8] it is evident that this 35 to 40 per cent returned is completely burned.

Since the presentation of the paper, a permanent system of drain lines has been installed on another row of boilers in the station, running from the dry-type cinder catchers through the rear furnace walls to the ash pit. These lines have operated in a satisfactory manner, conducting the entire catch of the cinder trap back to the furnace.

Mr. Bennett's design providing for the return of cinder to the stoker feed hopper, particularly when it can include the drains from efficient cinder catchers, is an inviting one. It will be interesting to follow the operating experience with the design in Fig. 1 of the discussion.

In considering the significance of the cinder loss, the point brought out by Messrs. Moultrop and Eaton regarding the usual short duration of high-capacity loadings and the investment saving by working equipment at high rating is important in judging the economic value of this loss.

Mr. Gahnkin raised a number of points regarding Fig. 10 of the paper. The efficiencies of the generating units in this figure were shown plotted against output because this is a usual manner of indicating efficiency of steam-generating equipment. The output values were derived directly from Fig. 3 of the paper to illustrate the effect of the reduction of results to a unit grate-area basis. It is true that the extent of heat-absorbing surface of the boiler and economizer affects the overall performance, and for this reason the dry-gas loss was illustrated in Fig. 10 along with the efficiencies in order to show the relatively small differ-

[8] "The Test Performance of Hudson Avenue's Most Recent Steam-Generating Units," by P. H. Hardie and W. S. Cooper, Trans. A.S.M.E., November, 1934, vol. 56, no. 11, paper FSP-56-15.

ence in the magnitude of this loss at comparable outputs per square foot of grate for the three installations provided with economizer or air-heater surface. Using the coal input as abscissa as suggested by Mr. Gahnkin, and basing calculations on the same acceptance-test data used for the reports in the paper, the difference in efficiency of the two equipments, No. 74 and No. 54, at a coal-burning rate of 60 lb per sq ft per hr is about 3.75 points on the efficiency scale. This difference in favor of No. 74 would be further increased slightly if correction were made for difference in dry-gas loss chargeable against the heat-recovery equipment. This would be a part of the difference indicated between the curves in the lower part of Fig. 10. Beyond the rate of 62 lb per sq ft per hr, the efficiency data for 24-hr duration runs are not available for No. 54 boiler.

In comparing the relative flatness of the efficiency curves shown by Mr. Gahnkin in Fig. 5 of the discussion with the curves in the original paper, it should be noted that the use of input as an abscissa tends to flatten the curve because the input increases more rapidly than the output with falling efficiency. Also the vertical scale used by Mr. Gahnkin is smaller, the overall effect of the two changes being the equivalent of spreading the horizontal scale on the original curves about tenfold while maintaining the same vertical scale. The choice of scale appears to be largely a matter of preference depending on whether emphasis is to be placed on small differences or on general similarities.

Mr. Gahnkin's discussion of the variation in furnace volume, heat release and loss due to unburned hydrocarbons is in general agreement with the authors' paragraph 6 under "Discussion of Results" in the paper. However, Mr. Gahnkin apparently favors crediting the stoker with those parts of the unburned-gas and unaccounted-for loss which are chargeable to less favorable furnace size and design. Unfortunately it is not possible to isolate these parts from the parts of the same losses which may properly be chargeable to the stoker itself. It appears a proper apportionment could only be made by a test comparison of otherwise identical units of equipment having different furnaces but having fuel and other operating conditions identical. The conclusion drawn by Mr. Gahnkin that the efficiency curves for both stokers is identical throughout the entire range does not appear to be supported by the data except under conditions of the assumption made by him. The approximate equality in the lower ranges of rating as quoted by Mr. Gahnkin is not at variance with efficiency differences indicated by the author in Fig. 10 of the paper.

Mr. Keppler's statement gives data on the capacity of a short stoker. The Hell Gate stoker cited, of nearly 16 ft projected length, has an underfeed section approximately 9 ft in length as compared with the 13-ft underfeed section of the stokers on the No. 4 unit at Hudson Avenue, which are the shortest stokers tested there without draft limitation. The example of the Hell Gate stoker is a definite instance of the statement in the paper that the capacity limitations discussed do not apply to shorter stokers. It is the belief of the authors, based on the observations of many tests, that ability to operate at high ratings for short periods even up to 12 hours is not an indication that the same rating can be carried continuously for 24 hours. Nevertheless, the Hell Gate tests, if repeated at approximately this load and duration, must be accepted as an indication of the ability of the shorter stoker in comparison with the data presented on the long stokers.

Mr. Purcell's comment that each stoker installation must be designed for the specific coal to be used is an important one from the viewpoint of the prospective user of large stokers. From the relation of his experiences, the statement seems well founded on fact. The authors know of other installations where numerous troubles developed in the initial application, and stoker-design changes were required before satisfactory operation was attained.

The authors do not wish any inference to be drawn from the paper regarding expressions on the superiority of stokers or pulverized-fuel firing, but would agree with Mr. Purcell that each new installation merits special investigation of both methods of firing for the particular conditions and the taking into consideration of the latest developments of each method. This was done at Hudson Avenue. In this connection, the tabulation of yearly boiler efficiencies given in Table 5 in the paper is believed to compare not unfavorably with pulverized-fuel installations.

Regarding the first question raised by Messrs. Moultrop and Eaton on maintenance, it can be said that in general the manufacturer was assigned a single stoker on which to do his experimenting and he carried the maintenance expense of all parts which were not standard in the other stokers. The figures quoted do not contain any of the costs which the manufacturer carried on the experimental unit.

In reference to the operation of Units 1 to 3 and 1 to 4, inclusive, in the years 1932 and 1933, respectively, the number of operating hours of these stokers and boilers in these years was so low as to make unit maintenance costs of no value for comparison.

As to the difference in maintenance cost of the stokers with and without preheated air, the only feature which the authors can point to as being different from many other preheated-air installations is the relatively low temperature of air. The average temperature is 350 F while the maximum is 380 F. It should be noted that it was in 1931 that the No. 4 row was first operated with link-grate stokers. In 1932 the maintenance costs for the No. 4 row rose sharply, largely because the operating hours for 1932 was less than one third the number for 1931. The authors hesitate to predict just how the maintenance for row No. 4 and row No. 5 would have compared had they continued in parallel operation for several years at approximately the same number of service hours and coal-burning rates.

The comments on Fig. 12 and on limiting the use of terms are well taken.

A New Method of Investigating Performance of Bearing Metals[1]

C. S. Cole.[2] There are many sides to this bearing question—the properties of the bearing, of the lubricant, and the mechanical design of the bearing itself. In the first of these phases the Copper & Brass Research Association undertook an intensive study last year at Battelle Memorial Institute, on the properties of six of the more commonly used materials. The results of this work were published in a paper[3] presented before the American Foundrymen's Association in June, 1934. These data are available either in the Transactions of that Society or as preprints which this Association would be glad to send any one interested. It would be most interesting to see the correlation of these studies with tests such as Mr. Connelly is making.

L. M. Tichvinsky.[4] The paper by Mr. Connelly presents an interesting manner of attacking the complicated problem of wear of bearing metals. Metal-to-metal contact occurs fre-

[1] Published as paper IS-57-1 by John R. Connelly in the January, 1935, issue of the A.S.M.E. Transactions.

[2] Engineering Assistant, Copper & Brass Research Association, New York, N. Y. Mem. A.S.M.E.

[3] "A Study of Six Bearing Bronzes," by O. E. Harder and C. S. Cole.

[4] Research Laboratories, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

quently in small and medium-sized bearings[5] at the moments of starting and stopping of the machines. The rate of wear, therefore, in some bearings may be very large and special provisions should be made in order to secure a safe performance of the bearing.

It is known that the present methods of wear measurements are still insufficient for obtaining complete wear criteria. This may be explained by the great number of parameters constituting the function of wear of metals. The New International Association for the Testing of Materials discussed in 1930 in Zürich the problem of wear of metals. The wear there was expressed[6] as follows:

$$W = f(S, H, M, MS, P, p, v, F, f, L, t)$$

where S is the state of the surface
H = hardness
M = molecular forces
MS = microstructure
P = full load
p = specific load
v = peripheral velocity
F = force of friction
f = coefficient of friction
L = work of abrasion deformations
t = temperature at the rubbing surfaces.

This large number of variables will still increase if properties of lubricants used will be included and duration of test considered.

Referring to the author's paper, it seems that it will be more advisable to have the rate of wear plotted against time instead of unit pressure.

The coordination of action of those involved in wear testing of metals will approach the time of optimum wear-test methods which may be accepted by engineering institutions.

Author's Closure

This new method utilizes a new mode of attack and makes possible the investigation of certain characteristics of bearings heretofore undetermined.

The author wishes to emphasize the value of and need for some central clearing agency for work in bearings. Much work has been done by independent investigators. Mathematicians, chemists, and metallurgists have all contributed. Yet the fundamental problems remain unsolved and no particular theory has received general acceptance.

It is hoped that the A.S.M.E. Main Research Committee will see fit to establish a subcommittee on bearings to act as a coordinating body for all the work on bearings which will include metals and other solids, lubricants, and the various physical factors.

Unnecessary duplication will be largely avoided and more economical coordinated research made possible.

[5] Large bearings are usually provided with a high-pressure pump for the purpose of leading the oil under the journal and lifting it at the moments of starting and stopping.

[6] See First Communications of the New International Association for the Testing of Materials, 1930, Group D; "New Scientific Methods of Subdivided Quantitative Wear Testing of Metals," by A. K. Zaitzeff, p. 86.

The Design and Performance of a High-Pressure Axial-Flow Fan

By LIONEL S. MARKS[1] and THOMAS FLINT,[2] CAMBRIDGE, MASS.

This paper deals with the design and performance of an eight-bladed axial-flow fan to give high pressures and an approximately constant horsepower characteristic. The performance in the laboratory indicates the attainment of a static pressure of 22.2 in. of water at a peak efficiency of 77.5 per cent, with a peripheral speed of 40,000 fpm. The no-flow horsepower is only 16 per cent greater than at peak efficiency. The performance is compared with that predictable from wind-tunnel tests of a cascaded series of air foils.

A PAPER[3] published in the A.S.M.E. Transactions in 1934 discussed the design of a three-bladed axial-flow fan and gave results of tests conducted with it. This fan gave a peak efficiency of 80 per cent and could develop a static pressure (according to tests at 18,000 fpm peripheral velocity) of 18.5 in. of water at the permissible peripheral speed of 4000 fpm. There was one feature about the performance of this fan which is shared with many other fans but which was regarded as unsatisfactory, namely, the horsepower output increased steadily as the air flow was restricted until at the no-flow condition it was approximately 125 per cent greater than at peak efficiency.

The present paper deals with an attempt to design a fan (1) for still higher pressures, approximating the practical limits for the axial-flow type and (2) with a horsepower input which does not rise materially above its value at peak efficiency. It was realized that in order to obtain these objectives it would probably be necessary to accept an efficiency lower than that of the three-bladed fan, but this did not seem important as long as the efficiency was high.

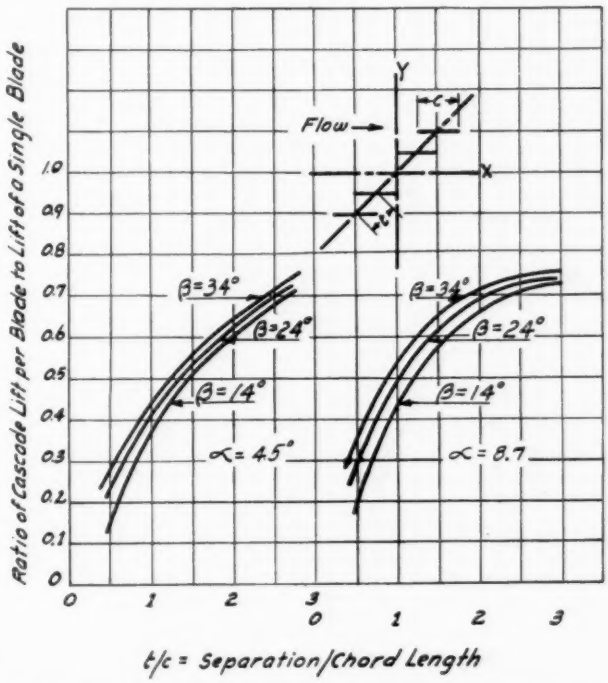


Fig. 1 Lift Coefficients for a Cascade Series of Airfoil Sections

Various considerations indicate that a larger hub would be necessary and a larger number of blades desirable. It was decided arbitrarily to use eight blades.

Cascade Tests

It was proposed to base the design on airfoil theory and wind-tunnel constants. No wind-tunnel data were found in the literature on the effect of mutual blade interference in a cascaded series of airfoils. To supply this information, S. Ober, Associate Professor of Aeronautical Engineering, Massachusetts Institute of Technology, conducted a series of wind-tunnel tests on a cascade of five identical airfoils with boundary walls at both ends (representing the hub and casing of a fan) and determined the lift and drag of the central airfoil which was free from the boundary walls. The Gottingen No. 429 profile with a straight chord was investigated, and tests were made for various stagger angles β, for two angles of attack α, and for a series of values of the ratio of separation along the line of stagger to chord length t/c. Ratio of the lift observed to the lift on a single blade is shown in Fig. 1.

According to Professor Ober, these tests are neither extensive nor accurate enough to warrant complete confidence. The fact that they are strongly at variance with the theoretical values of Numachi[4] would seem to invalidate Numachi's theory rather than cast doubt on the test results.

---

[1] Professor of Mechanical Engineering, Harvard University. Mem. A.S.M.E. Professor Marks was born in Birmingham, England. He received the degree of B.Sc. from the University of London in 1892 and M.M.E. from Cornell University in 1894. He was with the Ames Iron Works, Oswego, N. Y., in 1894 and then went to Harvard University as instructor in mechanical engineering. In 1900 he was made assistant professor and in 1909 was advanced to his present position. Professor Marks is the author of "Steam Tables and Diagrams," "Gas and Oil Engines," "Mechanical Engineers' Handbook," and "The Airplane Engine," and has contributed numerous articles to the technical press.

[2] Designer of conveying equipment, J. W. Greer Company, Cambridge, Mass. Jun. A.S.M.E. Mr. Flint was graduated from Harvard University with a B.S. degree in 1928, and received his M.S. degree the following year from the same University. He has held positions as assistant foreman in the cold-rolling department, American Steel & Wire Company, 1929–1930; engineer, lead extrusion department, Simplex Wire & Cable Company, 1930–1933; and assistant in mechanical engineering, Harvard Graduate School of Engineering, 1933–1935.

[3] "The Design and Performance of an Axial-Flow Fan," by L. S. Marks and J. R. Weske, Trans. A.S.M.E., vol. 56, 1934, paper AER-56-13, pp. 807–813.

Contributed by the Aeronautics Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of the paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] "Aerofoil Theory of Propeller Turbines and Propeller Pumps With Special Reference to the Effects of Blade Interference Upon the Lift and the Cavitation," by F. Numachi, Tech. Reports, Tohoku Imperial University, Sendai, Japan, vol. 7–8, no. 3, 1927–1928, pp. 411–469.

In view of these uncertainties the proposed basis of design was abandoned. The analysis of the actual fan performance on the basis of the coefficients given in Fig. 1 shows, however, that the cascade-test results cannot be much in error and indicates the desirability of further work of this kind as the basis of axial-flow-fan design.

Design Method Used

The design method decided upon was based on the equations recently presented by O'Brien and Folsom,[5] although, subsequently,

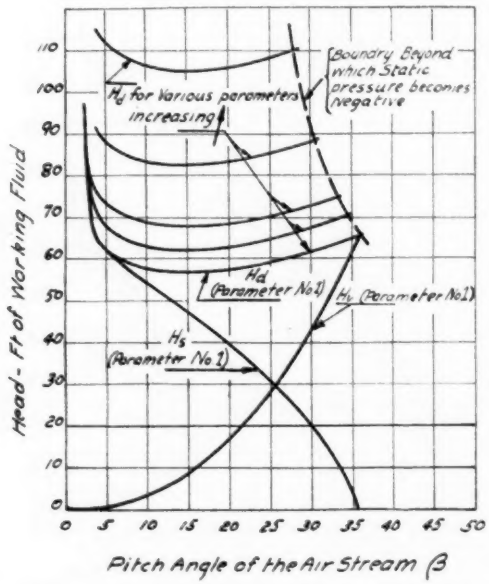


Fig. 2 Variation of Developed Head, Velocity Head, and Static Head With Pitch Angle of the Air Stream for one Particular Value of the Parameters C_L, c/t, and λ as Taken From O'Brien and Folsom's[5] Equations
(U = 100 fps and g = 32.2 ft per sec per sec.)

it was found to be less accurate than the method employing the results of the cascade-tests. In a discussion[6] of the paper[2] on the design and performance of an axial-flow fan, O'Brien and Folsom showed that these equations gave results agreeing satisfactorily with the observed performance. On this somewhat meager basis, it was decided to utilize the O'Brien and Folsom procedure. As stated by them, "The method of computation is essentially that used by Pfleiderer[7] with, however, modifications which result in a better agreement between the computed and the measured head-capacity characteristics."

In these equations, the following symbols are used and apply to any cylindrical section:

H_d = the total or developed head, ft of the working fluid
H_v = velocity head, ft of the working fluid
H_s = static head, ft of the working fluid
U = the tangential velocity of the blade section, fpm
V_{u3} = the tangential velocity of the air at exit, fpm
V_f = the axial velocity of the air at entrance, fpm
C_L = the lift coefficient with infinite aspect ratio
C_D = the drag coefficient with infinite aspect ratio
c = the chord length, ft
t = the circumferential distance between blades, ft
β = the pitch angle of the air stream, or the angle between the geometric mean value of the relative velocities in front of and behind the blade and the direction of blade motion
λ = $\tan^{-1}(C_D/C_L)$.

Combining the equations of O'Brien and Folsom

$$H_d = \frac{2U^2}{g} \frac{\left[C_L \frac{c}{t} \frac{\sin(\beta+\lambda)}{\cos\lambda}\right]}{\left[4\sin\beta\cos\beta + C_L \frac{c}{t} \frac{\sin(\beta+\lambda)}{\cos\lambda}\right]} \quad [1]$$

$$H_d = H_v + H_s \quad [2]$$

$$H_v = (V^2_{u3} + V_f^2)/2g \quad [3]$$

$$V_{u3} = H_d\, g/U \quad [4]$$

$$V_f = (U - \tfrac{1}{2} V_{u3}) \tan\beta \quad [5]$$

A graphical representation of Equation [1] is shown in Fig. 2, in which H_d is plotted against β. A series of H_d curves is given for constant values of C_L, c/t, and λ. For the lowest H_d curve shown, there are added corresponding velocity-head and static-pressure curves. At values of β greater than that at which the H_d and H_v curves intersect, the static pressure becomes negative. The bounding broken curve on the right gives the values of β at which the static pressure becomes zero.

The velocity of the air stream at entrance V_f is assumed constant at every cylindrical section. As the tangential velocity of the blade section varies with the radius, the pitch angle of the air stream must increase from blade tip to hub as seen from Fig. 3. Fig. 2 shows that the static pressure diminishes continuously to zero as β is increased. The greater the combined value of the parameter determining H_d, or the greater C_L, c/t, and λ, the sooner this effect occurs. In order to keep H_d constant at every cylindrical section and thus avoid losses from re-

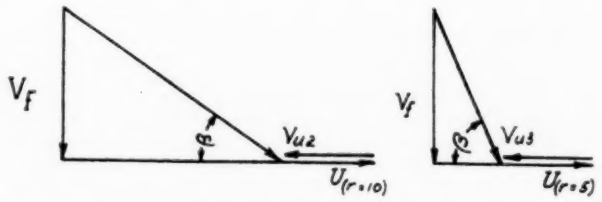


Fig. 3 Variation of Pitch Angle of the Air Stream With Tangential Velocity of Airfoil

circulation through the fan, it is necessary to increase C_L and c/t. It is obvious, therefore, that for efficient high-pressure operation a large hub must be used.

There are three ways by which the head developed H_d may be maintained as the tangential velocity decreases toward the hub:

1 By choosing increasingly thick or increasingly curved profiles. This will increase somewhat the value of C_L with an attendant slight increase in C_D.

2 By increasing the angle of attack α. This angle is the difference between β and the pitch angle of the blade section, at any cylindrical section. This method increases the value of C_L rapidly but soon begins to increase C_D at an even greater rate, as shown in Fig. 4.

3 By increasing the length of the blade-section chord. This method produces a reasonable increase in lift without excessive

[5] "Propeller Pumps," by M. P. O'Brien and R. G. Folsom, Trans. A.S.M.E., vol. 57, July, 1935, paper HYD-57-3, pp. 197–202.

[6] Discussion by M. P. O'Brien and R. G. Folsom of "The Design and Performance of an Axial-Flow Fan," by L. S. Marks and J. R. Weske, Trans. A.S.M.E., vol. 57, August, 1935, p. 344.

[7] "Die Kreiselpumpen," by Carl Pfleiderer, Julius Springer, Berlin, 1932.

increase in drag and was the method selected for the design of this fan. Another important reason for choosing this method in preference to the conventional method (increasing the angle of attack) was that it rendered it possible to design the fan so that no blade section had a pitch angle greater than the angle of attack at which that section would "burble" or break down. It was hoped that the elimination of the break-down point, with its attendant increase in drag, would effectively prevent the rise of horsepower input at low capacity.

After selecting a convenient speed for the fan design, an approach velocity V_f was selected so as to give reasonable capacity. The profile section was chosen from those listed in the N.A.C.A.

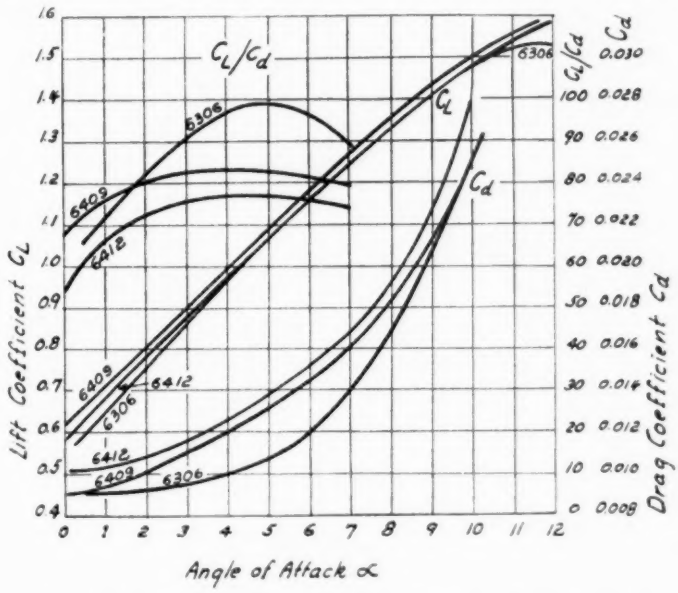


FIG. 4 PROPERTIES OF THE N.A.C.A. PROFILES USED FOR THE AXIAL-FLOW FAN

Report No. 460, having in mind high efficiency (high lift-drag ratio), a high value of the angle of attack at the burbling point, and adequate strength for the inside section. The properties of the three sections selected are shown in Fig. 4.

The outside section was designed first, the outside diameter of the fan being determined by existing laboratory equipment. At each section, the angle of attack used was that for maximum lift-drag ratio at the design point of the fan. The value of t for any cylindrical section was determined by the number of blades. From this point on, a cut-and-try procedure was followed. A value of H_d was chosen somewhat higher than the desired static pressure. The tangential velocity of the air at exit V_{u3} was found from Equation [4], β from Equation [5], H_s from Equation [3], and H_s from Equation [2]. A weighted average, based on the square of the radius of each section designed, was taken of the static heads developed at the three sections designed. These were (1) the outside section with a radius of 9.65 in., (2) the section next to the hub with a radius of 6.76 in., and (3) a section midway between these two with a radius of 8.2 in. If this average gave the desired static head, the original H_d was used; if not, a new calculation was made. When the proper value for H_d was found, Equation [1] was solved for c for each section. The hub diameter was taken as that at which the pitch angle of the blade exceeded the angle of attack at which burble occurs.

The section at the hub and the center section were designed in the same way, keeping H_d and V_f constant for all sections.

Each of the static heads resulting was corrected for section-drag loss H_p, which equals $C_L(c/t)(V^2/2g)$ (tan λ/sin β), where $V^2 = V_f^2 + (U - \frac{1}{2} V_{u3})^2$.

No correction was made for losses from friction in the air passages because of the shortness of the passages.

The details of the fan design are given in the pattern drawing, Fig. 5. Its arrangement in the casing is shown in Fig. 6. The stationary nose is supported by three streamline struts. The stationary streamline continuation of the fan hub is centered in the casing by a spider of five radial blades and contains the ball-bearings which support the shaft at the fan end. The guide vanes are located between the fan and the radial blades. The guide vanes were designed after the fan was built and are based on measurements of air-discharge angles from the fan when operating at 3600 rpm and at capacities of 3000 and 4000 cfm. The angles were determined by the use of a directional tube,[8] and also by threads, and were measured at the three design sections. The measured angles are listed in Table 1. The inside and outside section readings are affected by boundary conditions. Guide vanes were made with angles corresponding to those measured at 4000 cfm and are shown in Fig. 7.

TEST RESULTS

The arrangements for the fan tests were substantially the same as those described in the paper by Marks and Weske[3] and illustrated in Fig. 6 of that paper. The only notable change is that calibrated orifices were substituted for the nozzle for measuring air and the fan shaft was halved in length, which made it possible to operate with speeds up to 3600 rpm.

Complete tests were made at 3600 rpm both with and without guide vanes and at 2400 rpm with guide vanes. Fig. 8 gives a comparison between the performance with and without guide vanes, while Fig. 9 gives a comparison between the performance at the two speeds.

The horsepowers shown in Figs. 8 and 9 and used in the calculations of efficiency are net horsepowers obtained by subtracting the friction horsepower from the dynamometer horsepower. Friction horsepower was determined by replacing the fan with a bladeless hub of the same form and weight

TABLE 1 MEASURED AIR-DISCHARGE ANGLES AT THREE DESIGN SECTIONS

Radius, in.	Discharge angle, deg	
	3000 cfm	4000 cfm
9.65	58	40
8.21	52	35
6.76	57	38

and observing the dynamometer horsepower required to rotate it at various speeds. When operating at 2400 rpm, the horsepower input is low and the friction horsepower is about 20 per cent of the dynamometer horsepower. Any considerable error in determining the friction horsepower would have an appreciable effect on the calculated value of the net horsepower at this speed. At 3600 rpm this source of possible error is negligible, since the friction horsepower is then a small fraction of the net horsepower. The question of possible error of the horsepower determinations at 2400 rpm is raised here because the peak efficiency at this speed is 82.5 per cent as compared with 77.5 per cent at 3600 rpm. This same variation of peak efficiency with speed was found in the three-bladed fan[3] and was commented upon as having been previously observed in the operation of propeller and centrifugal pumps.

[8] "The Determination of the Direction and Velocity of Flow of Fluids," by L. S. Marks, *Journal of the Franklin Institute*, vol. 217, 1934, p. 207.

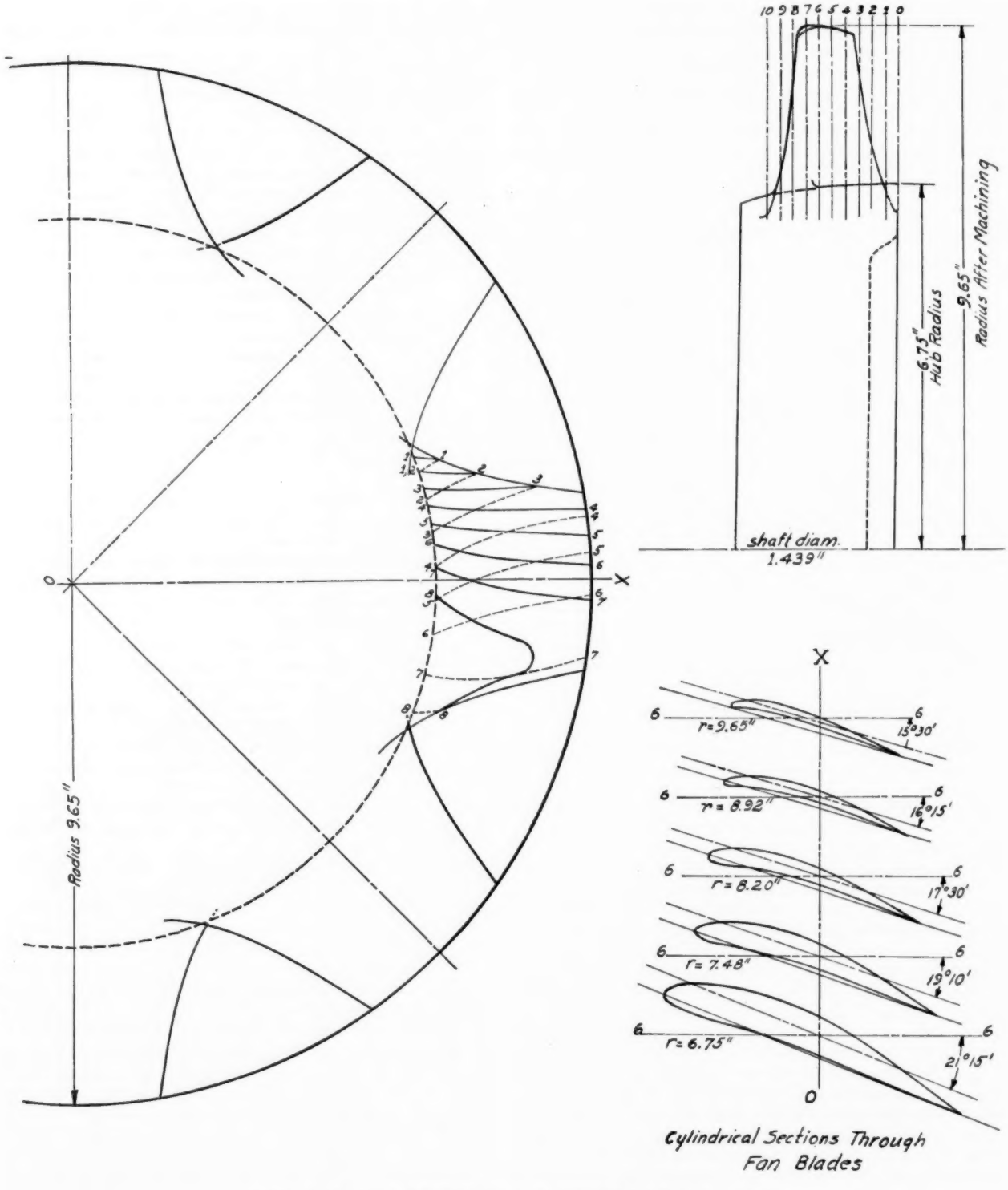


FIG. 5 PATTERN DRAWING OF THE AXIAL-FLOW FAN TESTED

The following observations may be made on the test results:

1 The static pressure at 3600 rpm and at peak efficiency is 4.7 in. of water, which corresponds to a static pressure of 22 in. of water with a tip speed of 40,000 fpm. This is an increase of 22 per cent over the static pressure obtained in the three-bladed fan.[3]

2 The net horsepower at no-flow is 16 per cent greater than the net horsepower at peak efficiency and compares with an increase of 125 per cent for the three-bladed fan.[3] This is a most important result since it permits the use of driving equipment of only half the power of that necessary for the three-bladed fan.[3]

3 The peak efficiency (total efficiency) is 77.5 per cent as compared with 80 per cent for the three-bladed fan.[3] However, a comparison of peak efficiencies alone may be misleading; the efficiency is maintained better with the fan described in this paper than with the three-bladed fan.[3] For example, at half the capacity giving peak efficiency, the present fan has an efficiency of 54.5 per cent as compared with 45 per cent for the three-bladed fan. At capacities above that giving peak efficiency, the efficiency falls off very rapidly.

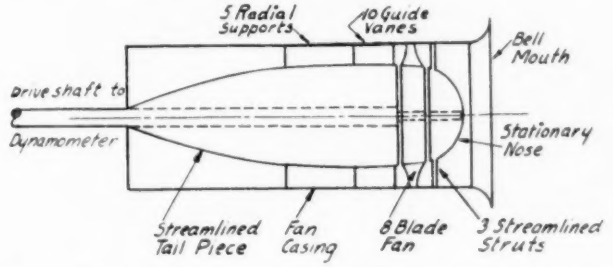


FIG. 6 FAN ASSEMBLY WITH STATIONARY NOSE AND TAIL PIECE IN ITS CASING

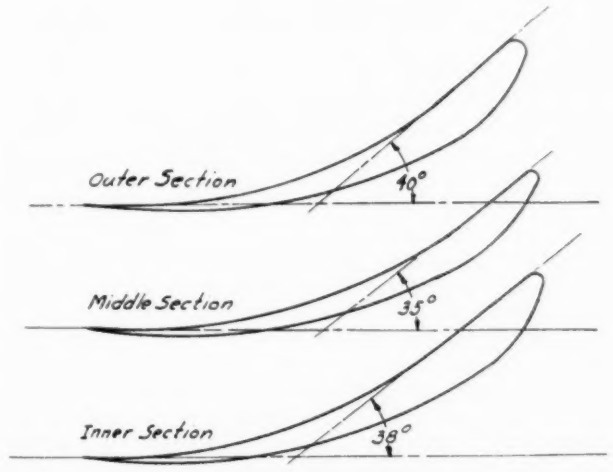


FIG. 7 GUIDE-VANE SECTIONS

DISCUSSION OF TEST RESULTS

It is interesting to make a comparison of the test results with those predictable from the cascade-test data given in Fig. 1. As a sample procedure utilizing these data, take the outside section with a diameter of 19.3 in.; a N.A.C.A. 6306 blade section as shown in Fig. 4; a volume of 4000 cfm; an entrance velocity V_f of 65 fps; a speed of 3600 rpm; and a section tangential velocity U of 303 fps.

Assume H_d = 520 ft of air.

From Equation [4], V_{u3} = 55.4 fps.

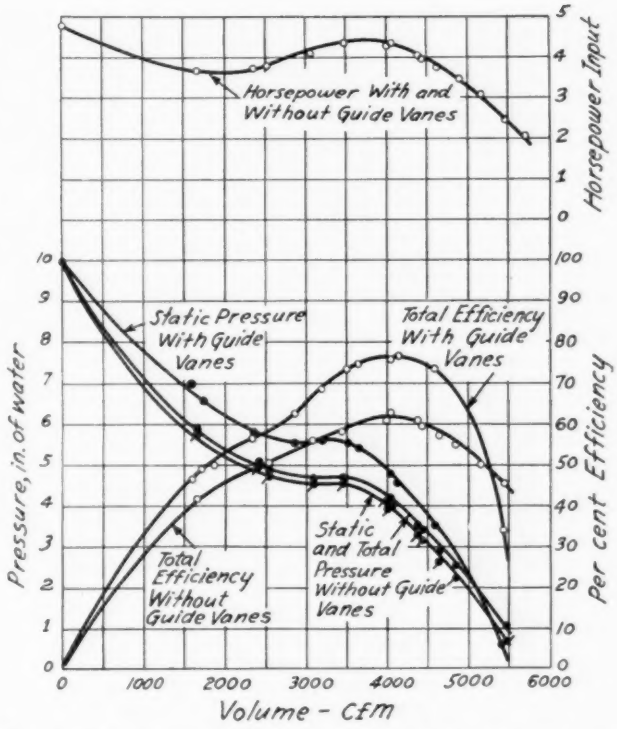


FIG. 8 PERFORMANCE CURVES OF AXIAL-FLOW FAN WITH AND WITHOUT GUIDE VANES

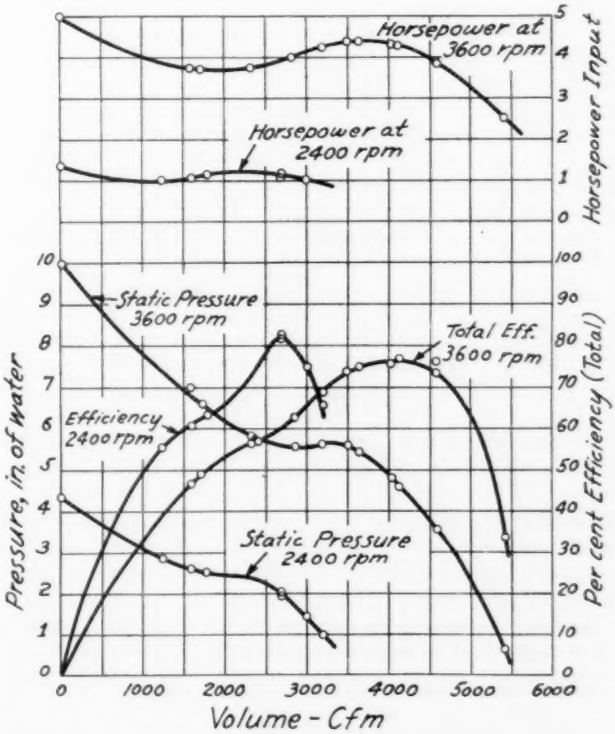


FIG. 9 PERFORMANCE CURVES OF AXIAL-FLOW FAN, WITH GUIDE VANES, AT 2400 AND 3600 RPM

From Equation [5], $\beta = 13$ deg 16 min, and since the blade pitch angle $(\beta + \alpha) = 15$ deg 30 min as seen from Fig. 5, $\alpha = 2$ deg 14 min.

From Fig. 4, $C_L = 0.775$ and $C_D = 0.0093$.

Interpolating in Fig. 1, the correction factor k for mutual blade interference is 0.610. The correction factor for C_D is negligible.

The lift coefficient normal to the long axis of the blade section

$$C_n = (kC_L \cos \alpha) + C_D \sin \alpha = 0.472.$$

The lift coefficient parallel with the axis of the fan

$$C_a = C_n[\cos (\beta + \alpha)] - C_t [\sin (\beta + \alpha)]$$

where $C_t = C_L \sin \alpha - C_D \cos \alpha = 0.455$

The velocity of the air stream relative to the blade section

$$V = \sqrt{[V_f^2 + (U - \tfrac{1}{2} V_{w3})^2]} = \sqrt{79{,}900} \text{ fps}$$

The thrust per blade $T = C_a \times V^2 \times d \times$ area of the blade lying between two cylindrical sections bordering closely the design section, where d is the density of the working fluid.

Multiplying T by the number of blades and distributing it over the area enclosed by the two bordering cylindrical sections

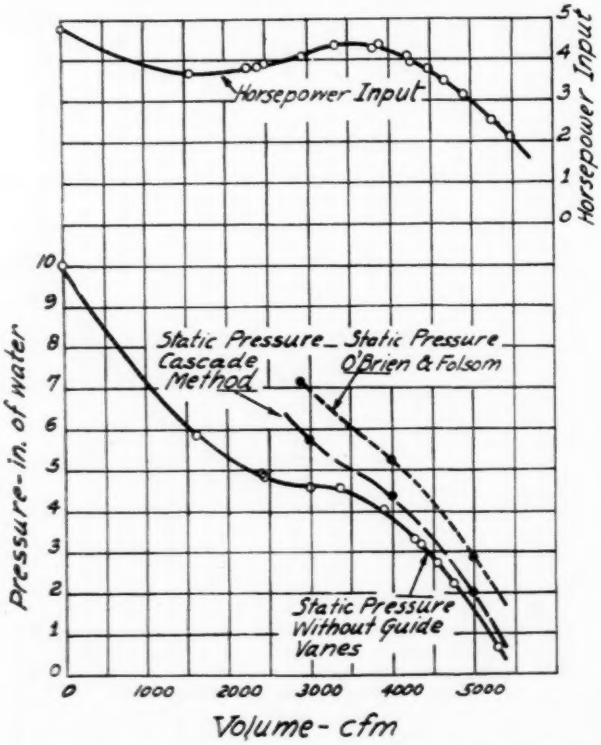


FIG. 10 STATIC PRESSURES OBTAINED IN A FAN WITHOUT GUIDE VANES COMPARED WITH THOSE PREDICTABLE BY TWO DESIGN METHODS

gives the developed head H_d, which for this particular case is found, after two more trial assumptions for H_d, to be 6.4 in. of water. The static head H_s is determined from Equation [2] and a weighted average is then obtained.

The static pressures as obtained by this procedure, together with those which were predicted from the O'Brien and Folsom formulations,[5] are shown in Fig. 10, the experimentally determined static pressures being given by the solid line. Both calculated curves show a strong similarity to the test curve but neither of them follows the shape of the test curve in the range of pressures above the point of inflection of that curve. For static pressures below the inflection point, the cascade method gives results which come very close to those of the test curve and the difference may well be ascribed to factors such as clearance and leakage losses, for which no corrections have been made.

It is interesting to note that the inflection in the test curve comes at a volume for which the angle of attack for all the blade sections is considerably below that at which break-off occurs,

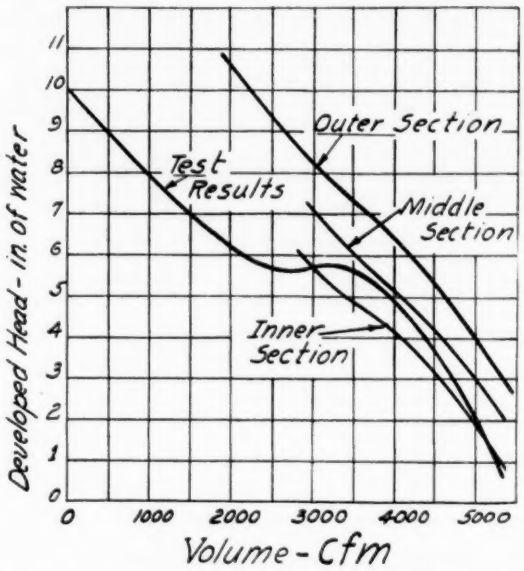


FIG. 11 DEVELOPED HEAD OBTAINED FROM TEST COMPARED WITH THEORETICAL VALUES AT THREE BLADE SECTIONS

and is at a point where the horsepower curve is falling. This would seem to preclude the interpretation of this phenomenon as the breakdown point of the various blade sections.

In Fig. 11 are given the values of the developed head H_d at the three design sections as calculated from the cascade-test data. The test results for total head are also shown. The calculated developed heads do not have the same value at the three design sections under any condition of operation. The sections were designed by the procedure given by O'Brien and Folsom[5] in order to have a common value of developed head at 2900 cfm. At the peak efficiency, the developed head is seen to be 6.4 in. of water and falls to 4.2 in. at the hub. Such a pressure difference would result in recirculation between the blades, and therefore indicates one method of improving the efficiency.

The air-discharge angles at the middle section of the blades, as calculated from the cascade tests, are in very close agreement with the measured angles, being better than 0.25 deg. As calculated by the O'Brien and Folsom method,[5] they are about 8 deg 30 min too large. Near the hub and the tip a comparison is useless, as the boundary effect is unknown.

The noise intensity of this fan was not measured, but it was comparatively low when operated at capacities down to the inflection point of the static-pressure curve (2900 cfm at 3600 rpm). Below this point a penetrating wailing noise developed suddenly and persisted down to the no-flow condition.

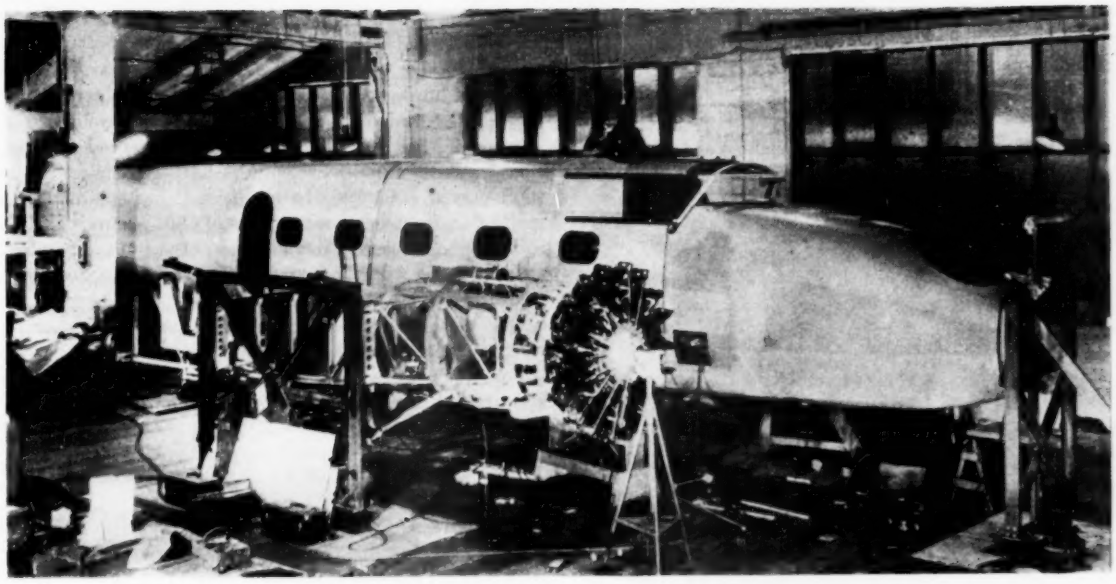


FIG. 1 ARC-WELDED ENGINE NACELLE ON BOEING TRANSPORT PLANE

Reliability of Aircraft Welds

BY N. F. WARD,[1] BERKELEY, CALIF.

This paper outlines the effects of fusion welding upon the dependability of structures in aircraft, due consideration being given to the existing conditions of welding practice found in this field. Obviously, the fusion welding of steel is of primary importance, since this is accepted practice. For the purposes of analysis, specimens prepared by experienced aircraft welders were subjected to physical tests and microscopic examination. With this information certain guiding principles in design for welding are outlined.

INTRODUCTION

ADEQUATELY designed airplanes, when fabricated by processes which produce safe carriers requiring a minimum of maintenance, represent the primary and substantial contribution to air transportation. The design of modern aircraft is guided largely by service requirements and economical fabrication processes which are consistent with structural stability.

The modern airplane is no simpler in structural form than its predecessor the tubular fuselage. In fact, the great variety of joints and fittings built in the all-metal airplane with prestressed covering, imposes additional fabrication costs which were absent in the welded airplanes of former years. Fusion welding, although contributing a reliable structural unit with favorable physical properties and the minimum of excess weight, is a fabrication process which is largely displaced by other methods with the advent of modern all-metal construction. The principal reason for the lessened emphasis on fusion welding is the vitiating effects of welding heats upon the alloys of thin section with further difficulties, in many instances, where low-melting-temperature alloys are required. Undoubtedly this field for structural application is inept for fusion welding, but entirely within the field of resistance welding. The successful use of resistance welding of thin sections in stainless-steel structures has been demonstrated by performance. Further extension of this type of welding to other metals is to be expected. However, fusion welding is utilized effectively from a structural and economical standpoint in the modern all-metal transport plane for such structural units as engine mountings, nacelles, shown in Fig. 1, fittings, and the landing gear.

Due to the complexity of structural units such as shown in Fig. 2, it is apparent that the usual fusion welding demands a precise skill during placement which only a manual operator can exercise. The degree of reliability in the weld is reduced to adequate training and experience of the welding personnel. At the present time, training is given at the aircraft manufacturing plants and only after successful welding of aircraft parts such as exhaust stacks, tanks, and fittings, is the experience recognized as sufficient to permit the operator to do structural welding. Through the efforts of the American Welding Society procedure

[1] Assistant Professor, University of California. Assoc-Mem. A.S.M.E. Professor Ward was graduated from Cornell University in 1921 with an M.E. degree and received his M.M.E. degree from the same University in 1927. From 1921 until 1927 he was employed as instructor in machine design and heat-power engineering at Cornell University, leaving that position to become assistant professor in mechanical engineering at the University of California. He also served as lecturer at the Boeing School of Aeronautics, Oakland, Calif., and consulting engineer for the American Tractor Equipment Company, Oakland, Calif.

Contributed by the Aeronautic Division and presented at the National Aeronautic-Hydraulic Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS at Berkeley, Calif., June 19, 20, and 21, 1934.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until December 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

in aircraft welding has been revised and improved on the basis of experience, so that if closely adhered to, it should produce welds as reliable as human experience can devise.

Significance of Static Test

A study of failure under static loading such as encountered in the standard hardness and tension tests, although not simulated

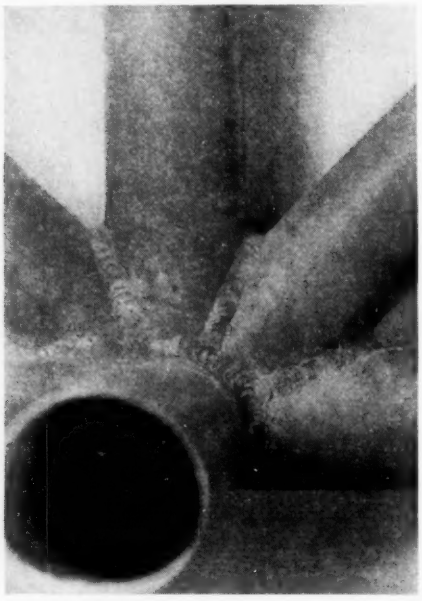


Fig. 2 Typical Gas-Welded Cluster Joint

in actual service, serves with the aid of microscopic examination to detect inherent characteristics of welded joints. When these qualities in the weld and its immediate vicinity are recognized, suitable application of this type of joint is assured.

For specific data in this connection, butt-welded tubes of cold-drawn chrome molybdenum S.A.E. 4130 steel were used. The tubes were 1 in. in diameter and 3 in. long, and had a wall thickness of 0.049 in. They were butt-welded with oxyacetylene torch and low-carbon-steel welding rod to form a specimen with an overall length of 6 ft. The welded tubes were fitted with steel plugs in both ends and inserted far enough to absorb the crushing load of the friction grips in the 30,000-lb Olsen Universal testing machine. The results of the commercial tests of the tubes, summarized in Table 1, are indicative of the reliability in static tension of the usual butt-type weld.

TABLE 1 TENSION TESTS OF BUTT WELDS IN 1-IN. TUBES

	Ultimate stress, lb per sq in.	Ductility in 2 in., %	Remarks
Minimum.....	87,800	9.4	Broke in weld
Maximum.....	93,300	9.4	Broke in annealed section, see Fig. 3
Avg of six.....	91,200	9.4	....

TABLE 2 COMPARISON OF HEAT-TREATED SPECIMENS OF BUTT-WELDED AND UNWELDED TUBES

	Ultimate stress, lb per sq in.	Ductility in 2 in. %	Remarks
Minimum.....	129,000	6.25	Broke in weld
Maximum.....	153,300	9.4	Broke in tube
Avg of six.....	142,400	7.3	....
Original tube as received.....	156,000	25.5	Broke at mid-section

After subjecting another group of butt-welded specimens to a combination of normalizing treatment at 1700 F hardening in water from 1700 F and tempering at 900 F, the results shown in Table 2 were obtained under static tension.

Prior to the tension tests and after the tubes were welded, a survey of hardness with a Rockwell hardness tester was made. The average values of three readings in the weld and at 1/4-in. intervals either side of the weld were observed as indicated in Fig. 3. After heat-treatment the butt-weld hardness increased to Rockwell B-82 and the hardness either side of the weld remained uniform at average values of C-6. These tests indicate a softening or annealing in the region of the outer envelope of the welding flame which produces sufficient heat to recrystallize the cold-drawn microstructure. In the heat-treated tubes the variation in hardness was adjusted with the exception of the weld, where an increase in hardness was evident.

For static tests the indentation of the hardness penetrator appears to have little or no effect on the tube strength. Yet for

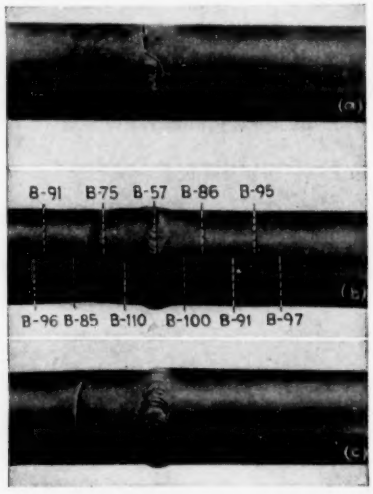


Fig. 3 Fractures in Tensile Specimen. Hardness Numbers at 1/4-In. Intervals Before Tensile Tests Are Shown on the Center Specimen

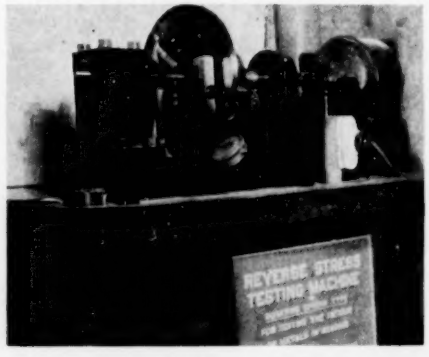


Fig. 4 Fatigue-Testing Machine for Tubes Under Bending Stress

the dynamical loading, encountered during fatigue tests, the indentation inhibits a weakness not already in the tube as welded. For this reason it was thought inadvisable to determine the hardness on fatigue specimens. Since the fatigue tests alter the physical structure of the metal, hardness determination after the test would be of little value. It remains for the microscopic examination to explain the allocation of the fracture as a result of the fatigue tests.

Fatigue Tests of Tubes

With usual service conditions inducing repeated stresses, the endurance of welded tubes under repeated loading is a logical test. Due to limitations of grip size on the fatigue-testing machine which was available, it was necessary to use 1/2-in. chrome-molybdenum S.A.E. 4130 tubes with 0.049-in. wall thickness. Since the smaller tubes are more difficult to weld accurately, the results should represent less favorable performance than the average. In other words, the minimum values for design are indicated by these tests.

The fatigue machine shown in Fig. 4 is of the cantilever type in which one end of the specimen is held rigidly while the other end is guided and loaded with an 80-lb indicator spring. The spring load deflects the specimen and induces the maximum stress at the inner side of a rigid clamp. In the tube tests, the maximum stress is in the region affected by the welding heats. The mounting of the tubular specimen requires the steel plug on the clamped end to prevent collapse of the tube under the grip pressure. A similar plug is inserted in the opposite end which engages the ball-bearing housing through which the spring load is applied.

Each heat-treated specimen, as welded with oxyacetylene flame, was tested for a given spring deflection which was measured on the dial gage shown in Fig. 4. By means of a direct-connected 1/4-hp motor, the single specimens were rotated at 1725 rpm until fractured. This was possible by subjecting each specimen to a different stress by proper spring adjustment. The results are plotted on the curves in Fig. 6. For maximum deflection the specimens fractured as low as 62,000 reversals. For stresses of low intensity some of the specimens did not break after 3,000,000 reversals. In every instance the specimens as welded broke adjacent to the weld bead as shown in Fig. 5. On which side of the weld bead failure would occur wasn ot predictable. Some failed on the side nearest to the rigid grip and an equal number of other specimens failed on the opposite side of the weld bead, as shown by comparing Fig. 5*c* with Fig. 5*b*. The fracture in the unwelded tube may be seen in Fig. 5*a*.

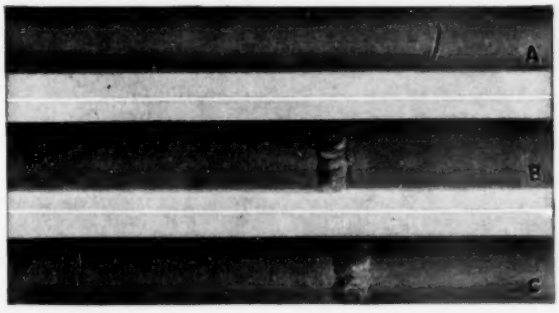


Fig. 5 Fatigue Failure of 1/2-In. Cr-Mo Steel Tubing
(*a*, tube as received; *b* and *c*, Oxyacetylene-welded tubes.)

Several observations for butt-welded tubes during these comparative tests show that loads inducing high stress are accompanied by failure during a wide variation of cyclic frequency.

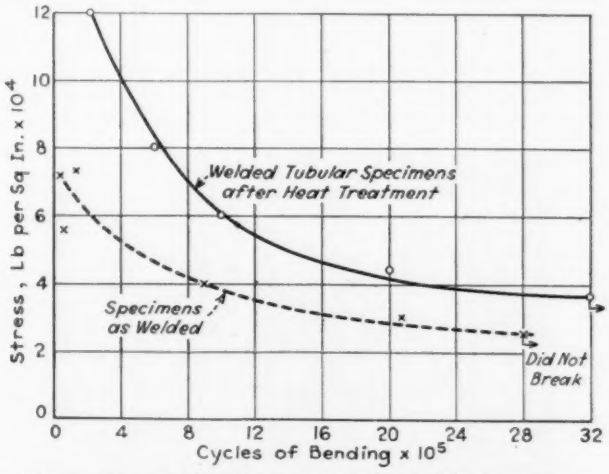


Fig. 6 Fatigue-Test Curves of 1/2-In. Cr-Mo Oxyacetylene-Welded Tubes With 0.049-In. Wall Thickness

The heat-treated specimens develop a more uniform response to fatigue in bending. Improvement of the endurance limit by heat-treatment of welded tubes in these tests amounted to approximately 40 per cent. The ratio of endurance strength to static tensile strength of the tubes as welded was 1/4, while for welded tubes after heat-treatment the ratio was 1/2.

Within probable errors of the test, this comparison between the tubes as welded and the welded tubes after heat-treatment maintains a close parity for the ratio of endurance limit to ultimate tensile strength. However, the fatigue failure in the butt-welded tube without subsequent heat-treatment has a tendency to localize failure in the immediate vicinity of the weld as indicated by the fractures in Fig. 5. The fatigue failure in the heat-treated tubes appeared most frequently at least the diameter of the tube from the weld. Obviously, the desire in service is to preserve these critical sections against concentration of load by any of several types of construction, such as gusseting or reinforcing with straps or telescoping the tubes or using a fishmouth joint.

Where these welds fail has been demonstrated by physical tests. Why the welds are more reliable in certain sections and vitiated in others is revealed in the microstructure of the weld and its vicinity.

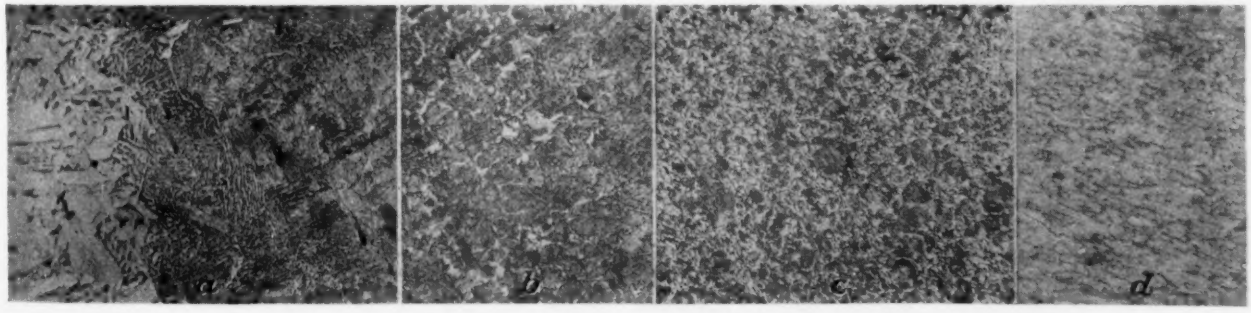


Fig. 7 Microstructure in Oxyacetylene-Weld (× 130)
(*a*, weld matrix at left with transition beginning at the right; *b*, transition; *c*, annealed section 1/2 in. and 3/4 in. from weld; *d*, original cold-drawn tube.

Influence of Welding Heats

Progressing from the weld outward toward the parent metal, the metallurgical changes in the weld may be seen in Fig. 7. The large areas of free ferrite are present with less carbon matrix than seen in the original tube in Fig. 7*d*. The weld metal is a typical structure found in low-carbon-steel castings cooled slowly from fusion temperatures. The precipitation of large ferritic masses accounts for the soft weld. The heat used in placing the weld has been moderate, since no excess of oxide and nitride contamination is evident. The avoidance of high temperature in placing the weld is essential to keep the gas (oxygen and nitro-

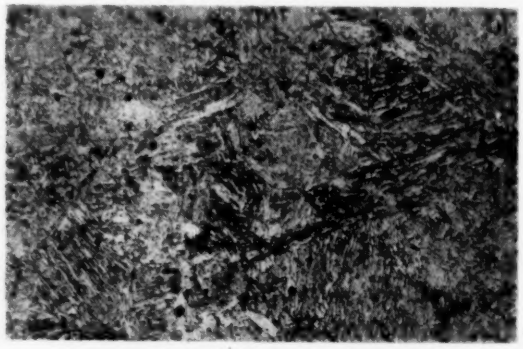


Fig. 8 Microstructure in Transition Zone of Heat-Treated Weld

gen from the air) content at a minimum. These gases develop a porosity and form oxides and nitrides which embrittle the weld if present in combination.

The transition zone from weld to tube appears in Figs. 7*a* and 7*b*. The microstructure is large grained and of tough consistency which is harder than the weld except at grain boundaries, which are soft ferrite. The growth of large-grain structure is a natural accompaniment of welding heats. The precipitation of tough constituents in this transition zone is largely due to rapid conductivity of heat from the weld to the cooler tube. The cross-section of a weld thus affected with the aircraft welding torches persists for $^1/_2$ in. either side of the welds in varying intensity, which depends upon the adjustment of the torch flame and its position. The larger grain of the weld in comparison with the transition zone represents a source of weakness with reversal of loading, since it yields more rapidly than the hard tough metal adjoining it in the transition zone.

Interposed for a distance between $^1/_2$ in. and $^3/_4$ in. between the transition zone Fig. 7*a*, *b* and the original plate 7*d* is found the recrystallized section Fig. 7*c* in which refinement of the grain has been produced by contact of a portion of the welding flame or the heat as conducted from the weld to the cooler plate or from both causes. The existence of this structure explains the fracture in this area for sound weld during static-tension tests, rather than in the weld. Briefly, the unit stress is least in the weld because of its greater section. The softened section (Figs. 3 and 7*c*) strain-hardens with the application of load but suffers a greater reduction in cross-sectional area than adjoining sections, thus developing greater stress intensity which persists under load until the ultimate stress is reached and pronounced reduction in cross-sectional area continues until fracture occurs.

Since the base or parent metals in aircraft structural members are cold-drawn or rolled, they reach their yield point at a greater load than for the annealed section. Where there is a weld fracture under static-tension load, the specimen has been misaligned or the weld has been overheated and consequently embrittled by contamination of oxides and "cold shuts."

Summary of Physical Changes in Oxyacetylene Weld

In tubing of chrome-molybdenum steel the welding heat is beneficial to certain sections and detrimental to others. Since chrome-molybdenum steel hardens as it cools in air from fusion temperatures, the static strength of the areas adjacent to the weld approaches or exceeds that of the cold-drawn sections. Use of mild-steel tubing for welded joints effectively prevents as wide a variation in hardness, but the strength averages half that of chrome-molybdenum steel. The fatigue resistance of the fused areas is reduced because of the large grain. Interposed between this zone and the cold-drawn section is an annealed section in which sound welds finally fracture under static loading.

Welded-tube sections which undergo proper heat-treatment, i.e., normalizing hardening and drawing, develop high ultimate strength, as is evident from the values in Table 2. This heat-treatment develops a welded structure as reliable as the original

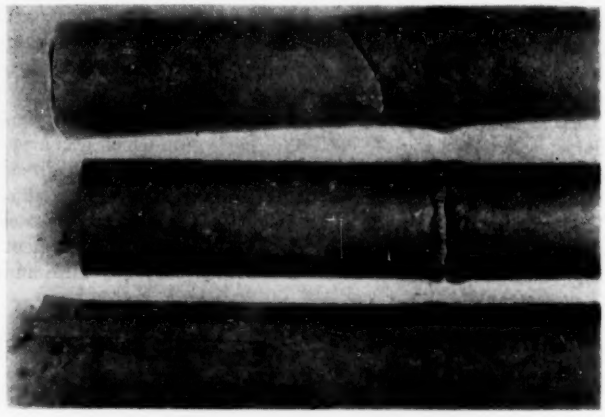


Fig. 9 Typical Failure of Oxyacetylene-Welded Tubes After Heat-Treatment

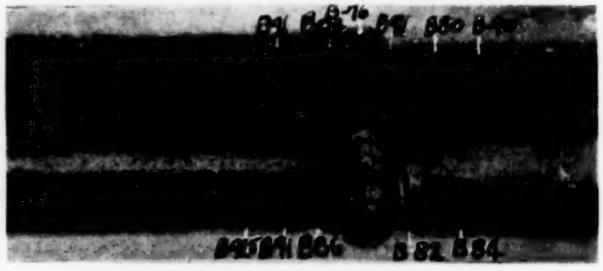


Fig. 10 Static-Tension Failure in Arc-Welded Tube. Hardness Values Taken at $^1/_8$-In. Intervals

by adjusting inequalities of the crystalline matrix and producing uniform strength and ductility. The microstructure changes at the weld zone resulting from this heat-treatment are shown in Fig. 8. The position and type of fracture for the static-tension tests is shown for selected specimens in Fig. 9.

Characteristics of Arc Welds

Metallic arc welding with direct current and low-carbon-steel electrode has had limited application in airplane structures. For adequate fusion of the tube or plate the bead deposit is greater than for oxyacetylene welding. The concentrated heat evolved from the arc localizes the critical zone in close proximity to the weld beads. The annealed section, which is less extensive than that for the oxyacetylene welds, develops within $^1/_4$ in. of the welds. The variation in hardness appears to be less marked

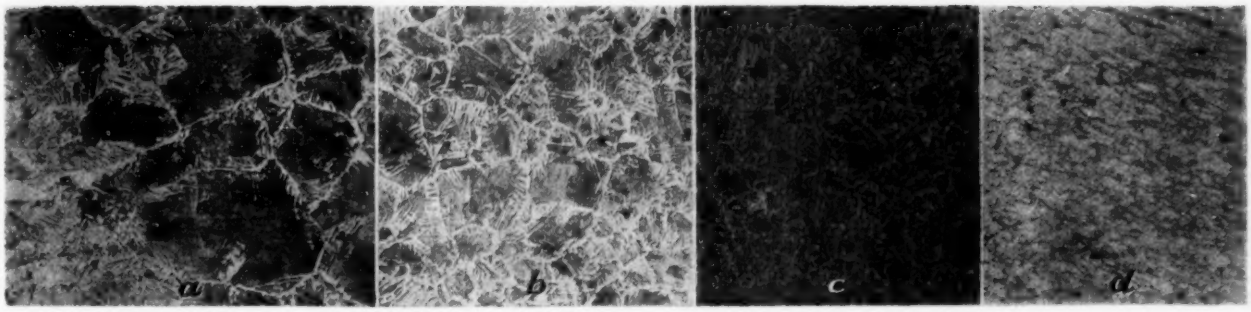


FIG. 11 TYPICAL MICROSTRUCTURE IN 1-IN. TUBE ARC WELDED WITH DIRECT CURRENT AND LOW-CARBON FILLER ROD (× 130)
(*a*, weld matrix showing on the right large-grained sorbite with ferrite boundaries; *b*, transition zone; *c*, partially annealed zone 1/16-in. from weld; *d*, original cold-drawn tube.)

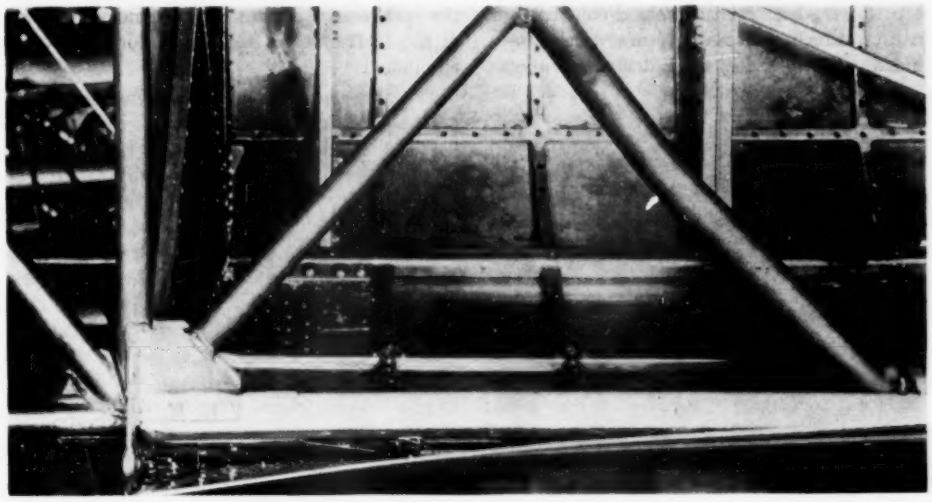


FIG. 12 STATIC FAILURE IN BEADING OF ARC-WELDED TUBE. LOAD FACTOR = 14½

than for the oxyacetylene-welded tube. Comparison of Rockwell hardness values for the electric arc weld is indicated at ⅛-in. intervals in Fig. 10. The failure under static-tension loading is evident from Fig. 10.

The transformations in the weld and tube for a representative arc-welded specimen are shown in Fig. 11. The weld structure at the left Fig. 11*a* is a typical steel matrix as cast, tapering into a sorbitic zone of large grain bounded by soft ferrite. The retention of high sorbitic content is probably due to the large temperature gradient between the weld and its environs which produces faster chilling than is present in oxyacetylene welding. There is evidence in Fig. 11 that grain diminishes in size until the annealing temperature is reached about ¼ in. from the fusion zone. Then the original cold-rolled structure is resumed with lowering of the temperature below that required for recrystallization. Fracture in static tension for the butt-welded tubes occurs in this annealed zone adjacent to the weld for reasons given under the discussion for oxyacetylene welding. The zones of fusion and transformation are very similar for both the oxyacetylene and direct-current arc welds.

The frequency with which failures occur outside of the weld when tested with the weld bead intact for a joint properly designed and welded, is evidence that the strength is not dependent upon the welds so much as the amount of weakening due to the welding process.

OTHER FACTORS AFFECTING RELIABILITY OF WELDED JOINTS

Temperature Stresses. There are many conditions arising from mechanical shrinkage as the welds cool. A brief for the solution of the problems encountered could not be attempted in this short treatise. The principal difficulty with shrinkage in welds during solidification is the adjustment of the residual stresses during cooling or immediately after the weld is placed. Usually the intensity of stress due to shrinkage is localized in the sections which cool unevenly. For example, the hotter areas are weak and the stress is transferred from the cool, stronger areas. Where the correct sequence of deposition of the weld bead and tack welding is used, the shrinkage is absorbed sufficiently so that subsequent normalizing, stress-relief annealing or peening adjusts the residual stresses in larger areas. In this way the stress intensity is distributed but never eliminated. Abrupt change of sections, such as heavy reinforcement in the weld bead, is undesirable not only because of the diminished fatigue resistance at the juncture of the bead and the tube, but also because of localization of shrinkage in this larger mass which retains its temperature during a longer period than the thin tube. Weld beads which are flush with the tube and rough or undercut are prone to weakness in fatigue and represent doubtful security in a structure.

Significance of Welding Knowledge for Design. The general conclusions which may be drawn from the foregoing facts about fusion welding in aircraft leads to the proportioning of the structural members so that stresses of a concentrated nature are not imposed at the critical section. By the critical section is meant that at which failure originates due to predisposition to structural weakness under repeated tension, compression, shear loading, or their combination. Clarifying examples may be cited.

The structural member or members in a fuselage are under tension, compression, and bending loads of varying intensity. The tube size, or plate thickness, is calculated for one or several predominant stresses. In a typical instance of a tube where a large ratio of length to diameter is used, the welds at the ends can safely develop the tube strength, unless the joints receive heat treatment; otherwise the crushing strength in the annealed section of the weld is reduced below that of the tube. This critical section requires suitable reinforcement, such as a strap or gusset. With adequate weld reinforcement the tube should bend in the center. Since the stress is seldom equally distributed in the final welded unit the probable failure occurs near the center section which, if very critical, transfers intensity to the tube ends. The static test on a fuselage shown in Fig. 12 indicates a collapse of the tube under a load factor of $14^1/_2$ with the end welds remaining intact.

Further extension of the analysis for the welded construction explains why application of heat unnecessarily near critical sections should be avoided. For instance, holes drilled for injecting lionoil to prevent internal corrosion of structural tubes are located usually beyond the first critical section and the drill holes are filled with screw plugs to avoid softening by flame or arc heat which produces areas susceptible to collapse near the critical section. Welding of telescoped tubes or fishmouth is preferred to butt-welded tubes because of the added stiffness and stability afforded in the joint. Deposition of relatively smooth concave weld beads is done where vibration is a predominant consideration. These show the place of accurate knowledge of weld performance as briefly outlined in the preceding discussion.

Acknowledgments

The author wishes to take this opportunity to express his appreciation to Mr. Retan of the Boeing School, for his assistance in preparing several of the representative weld specimens; to the Boeing School of Aeronautics, for the supply of tubing used in the specimens; and to the Department of Engineering Materials of the University of California, for the use of physical testing equipment.

The Viscosity of Water and Superheated Steam[1]

By G. A. HAWKINS,[2] H. L. SOLBERG,[3] and A. A. POTTER,[4] LAFAYETTE, IND.

The authors present in this paper data on the viscosity of water and superheated steam at temperatures and pressures used in modern power-plant practice. The authors constructed an apparatus to measure the viscosity of water and superheated steam at pressures up to 3500 lb per sq in. abs and at temperatures up to the critical in the case of water and between 800 F and 1000 F for superheated steam. The apparatus is described and the results obtained with it are reported in this paper.

THE increasing use of dimensional analysis in correlating data and solving problems in the fields of heat transfer and fluid friction has made viscosity a very important physical property of fluids. In making a survey of the literature preparatory to an investigation of flow distribution in forced-circulation once-through steam generators (1),[5] the authors found that data on the viscosity of water and superheated steam are very meager or non-existent at the temperatures and pressures which are used in modern power-plant practice. The viscosity of water is known quite accurately at temperatures below 212 F and some determinations have been made at temperatures up to 320 F. von Hevesy (2) calculated the viscosity of water at elevated temperatures on the assumption that viscosity is a function of electrical conductivity, but this assumption has not been verified experimentally. M. DeHaas (3) used a capillary-tube viscometer for determining the coefficient of viscosity of water up to 153 C. The viscosity of superheated steam at atmospheric pressure is known with reasonable accuracy. Speyerer (4) has determined the effect of pressure on the viscosity of superheated steam at pressures up to a maximum of ten atmospheres.

In view of the absence of experimental data beyond the above limits, the authors decided to construct a suitable apparatus to measure the viscosity of water and superheated steam at pressures up to 3500 lb per sq in. abs and at temperatures up to the critical in the case of water and between 800 F and 1000 F for superheated steam. The results are reported in this paper.

Since this investigation was started Schiller (5) has published viscosity determinations. Schiller obtained the relation for calculating the viscosity of steam in terms of that of water, by observing the velocity at which the discharge coefficient of a nozzle shows an abrupt increase for water and steam and then equating the two Reynolds numbers. Schiller's curves show one test point at 30 atmospheres pressure, four test points at 25 atmospheres and other points at lower pressures with a maximum temperature of about 540 F.

Selection of Type of Viscometer[6]

In deciding upon the type of viscometer to be used in this investigation, where pressures of 3500 lb per sq in. and temperatures of 1000 F were contemplated, the matter of safety was given first consideration because of the high energy storage per unit of volume of hot compressed water. The apparatus had to be suitable for a wide range of viscosities and reasonable in cost. Adaptability to the use of automatic recording devices was desirable in order to reduce the human element in the collection of the more essential data. The high pressures involved, the effect of high temperatures on the elastic properties of suspensions, difficulties in measuring small pressure differences accurately, and of maintaining definite surfaces of separation between liquids and condensable vapors, resulted in discarding most of the types of viscometers which have been used for viscosity determinations.

The Lawaczeck (6) viscometer was selected as being the most suitable type for the range of conditions encountered in these tests. This viscometer was developed in 1919 and consists essentially of a metal weight falling in a tube closed at its lower end and having a diameter which is slightly greater than that of the weight. As the weight falls, the liquid is made to flow with streamline motion through the annular space between the weight and the tube. There are three distinct resistances to the fall of the weight through the liquid. The form of the streamlines causes a so-called "head resistance" which can be decreased to a minimum by using a long weight and a small clearance. As the liquid flows through the annular space, a pressure gradient is set up similar to the case of the flow of a fluid through an annular channel. This resistance is fixed by the area of the cross-section and the rate of fall of the weight. The third resistance is a viscous drag due to the relative movement of the two cylindrical walls. Neglecting the head resistance, the formula for this viscometer is

---

[1] Progress Report A.S.M.E. Special Research Committee on Critical-Pressure Steam Boilers.

[2] Assistant, Engineering Experiment Station, Purdue University. Jun. A.S.M.E. Mr. Hawkins received part of his college training at the Colorado School of Mines and the degrees of B.S. and M.S. in mechanical engineering and also his Ph.D degree from Purdue University.

[3] Associate-Professor of Mechanical Engineering, Purdue University. Mem. A.S.M.E. Mr. Solberg holds the degrees of B.S. and M.S. from Purdue University. He has been for the last 14 years on the teaching staff of Purdue University where in his present position he is in charge of the instruction in power-plant engineering and thermodynamics. He is author of numerous articles on power engineering in the technical press.

[4] Dean of Engineering and Director of Engineering Experiment Station, Purdue University. Past-President A.S.M.E. Dr. Potter is a graduate of Massachusetts Institute of Technology; he has had 32 years' experience teaching and practice; he was connected with the General Electric Company during the earlier development of the steam turbine. For 15 years he was Professor and Dean of Engineering at Kansas State College, and since 1920 has held his present position. He is author of books, articles, and papers on thermodynamics and heat engineering.

[5] Numbers in parentheses refer to similarly numbered references in the bibliography given at the end of this paper.

Contributed by the Power Division and Research Committee for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] The description of the development of the viscometer and the values for the viscosity of water are based on a thesis submitted by G. A. Hawkins in partial fulfillment of the requirements for the Ph.D. degree at Purdue University, June, 1935. The work on the viscosity of steam was completed subsequent to the submission of this thesis.

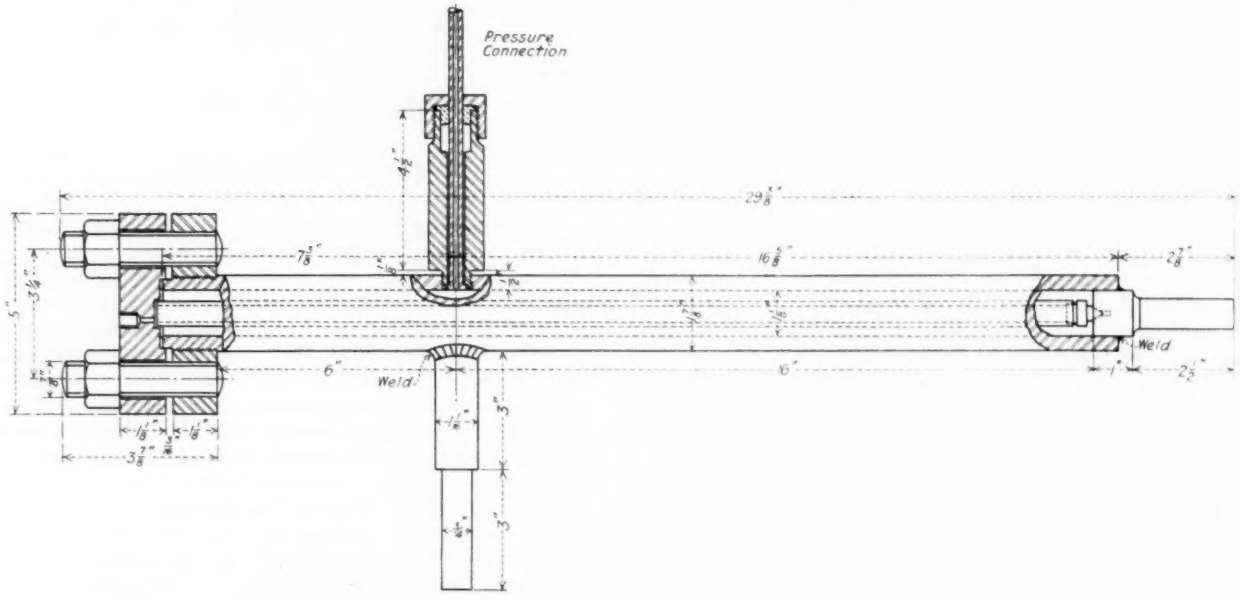


FIG. 1 DETAIL OF VISCOMETER AND PRESSURE CONNECTION

$$\mu = \frac{t}{s}(\sigma - \rho)\frac{\delta^3 d}{3(d + 2\delta)^2 - (2\delta)^2} \quad \ldots\ldots\ldots [1]$$

where μ is the absolute viscosity; σ is the density of the weight or fall-body; ρ is the density of the fluid; δ is the annular space between the fall-body and the tube; d is the diameter of the fall-body; s is the fixed distance through which the weight falls inside the tube; and t is the time required for the weight to fall the fixed distance s.

Because of the difficulty of measuring the dimensions accurately, the instrument is calibrated by using a fluid of known viscosity. Using a calibration constant C determined in this manner, Equation [1] is changed to the form

$$\mu = C(\sigma - \rho)t \quad \ldots\ldots\ldots [2]$$

Bridgman (7) in 1926 used a method similar to this for determining the viscosity of liquids at very high pressures but at low temperatures. In 1933, Stakelbeck (8) used a Lawaczeck viscometer for measuring the viscosity of carbon dioxide, ammonia, and sulphur dioxide, at the suggestion of R. Planck.

DESCRIPTION OF THE VISCOMETER

The viscometer, shown in Fig. 1, consists of an accurately bored, stainless-steel vertical tube through which a weight, or fall-body, of slightly smaller diameter is moved under the influence of gravity. The tube is placed concentrically within a seamless steel pipe capable of holding safely the maximum pressures to be used. This outer pipe or pressure vessel, hereafter called the container, was closed at the lower end by welding and was fitted with flanges at the upper end to allow for insertion of the tube and fall-bodies, and for periodic inspection. The top was closed with a blank flange and was provided with a $^1/_4$-in. vent valve. Two trunnions were secured to the container to serve as a support about which the apparatus could be rotated through an angle of 180 deg. One of the trunnions was provided with a packed pressure connection to which the pressure gage was attached. Distilled water was forced into the container by means of a hydraulic pump. The temperature of the fluid could be regulated by means of an electric heating coil which was wound around the container and thoroughly insulated. Fig. 2 shows the assembly

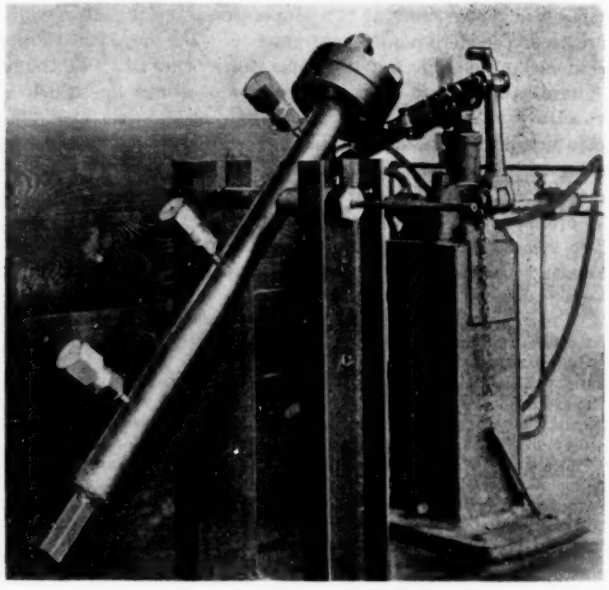


FIG. 2 VISCOMETER TEST ARRANGEMENT

of the container as mounted in its bearings. The three boxes for the electrical connections to the two timing coils are also shown. Three $^1/_8$-in. iron pipe taps were provided in the container, one at the center and one near each end, for the insertion of thermocouples.

The tube, 23 in. long and 0.658 in. outside diameter, was made by the Winchester Repeating Arms Company from a piece of Carpenter Steel Company No. 8 alloy containing 18 per cent chromium, 8 per cent nickel and a small percentage of selenium to facilitate machining. The bar was drilled, reamed four times and polished to 0.410 in. internal diameter with a high-speed lap coated with emery and oil. Periodic inspection of the tube has shown that it successfully resists corrosion and oxidation when in contact with steam and water at high temperatures. One end of the tube was threaded with a standard pipe tap and fitted

with a stainless-steel plug which centered it in a conical-shaped socket in the bottom of the container so as to insure coaxial alignment of the tube with the container when the two were assembled.

Details of the fall-bodies which were used in this investigation are shown in Fig. 3. They were drilled hollow to reduce their weight and the outside diameter is such that the Reynolds number of the fluid flowing into the annular space between the fall-body and the tube never exceeded 1300. The time required for a fall of 14 in. varied from 10 minutes to as low as 2 seconds, about 50 per cent of the tests involved a time of less than 1 minute. Fall-body No. 28 was used for determining the viscosity of water and this was checked by fall-bodies Nos. 30 and 31 in order to establish the fact that the results are independent of the shape of the ends of the fall-body. Fall-body No. 35 was used for determining the viscosity of superheated steam, which was checked by fall-body No. 33.

Pressures below 1500 lb per sq in. were measured by means of calibrated Bourdon gages. A Bailey self-loading, deadweight gage was used for higher pressures.

Temperatures were measured by iron-constantan thermocouples welded into the bottom of a steel well which was threaded to fit a $^1/_8$-in. pipe tap, as shown in Fig. 4. The thread on the thermocouple body was sufficiently long so that the hot junction was very close to the tube which contained the fall-body. Twenty thermocouples were made and calibrated in the physics laboratory of Purdue University against a platinum, platinum-rhodium thermocouple which had been checked by the National Bureau of Standards. Two of the three thermocouples were removed in rotating pairs at regular intervals and replaced by others, the third one being left in place as a check on the new one. Thermocouples were also checked periodically by boiling water in the viscometer with the vent valve open to the atmosphere. The thermocouples were connected to a very sensitive high-resistance Wilson-Maeulen millivoltmeter; the couple resistances were measured with a Wheatstone bridge.

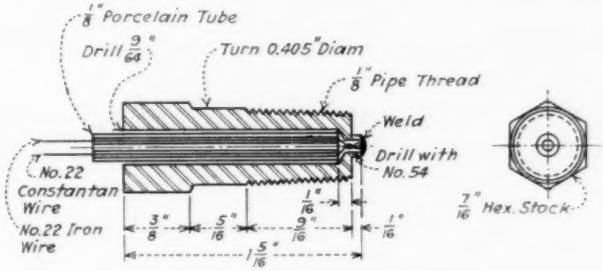


Fig. 4 Design of Thermocouples

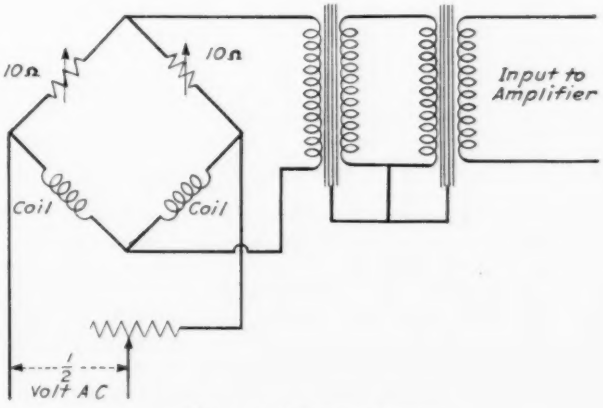


Fig. 5 A-C Bridge Circuit

Timing Equipment

The principal difficulty which was encountered in developing the viscometer was in devising an accurate means of timing the rate of fall of the body which was dropping through the tube. Among the schemes which were investigated and abandoned may be mentioned the use of an X-ray and Geiger-Müller counter, an X-ray and sensitive screen, and an oscillator, receiver, and oscillograph. An a-c bridge amplifier and recording milliammeter were finally selected as the most satisfactory solution to the problem.

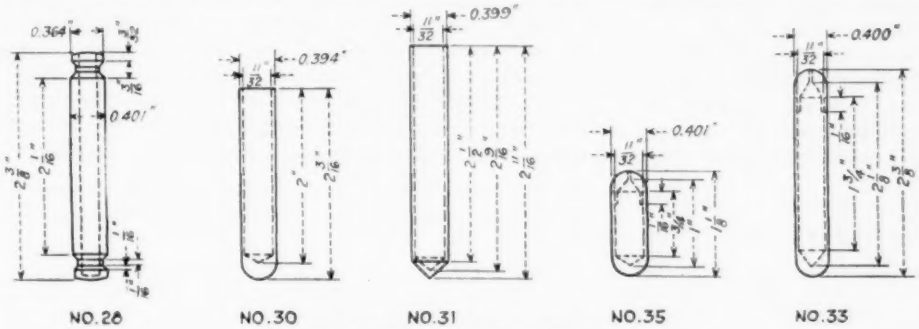


Fig. 3 Details of Fall-Bodies

The a-c bridge circuit is shown in Fig. 5. The two coils, indicated in the circuit diagram, were wound on the tube through which the fall-body dropped and were spaced about 14 in. apart. When the fall-body passed through either coil, the circuit was unbalanced sufficiently to swing a pen across a strip chart on the recording milliammeter. Each coil consisted of a single layer of 60 turns of copper wire wound on a sheet of mica rigidly held in place on the tube by two steel clamps which also served as terminals for the ends of the coils. Each turn was separated from the adjacent turn by a small asbestos thread. After winding, the coils were examined for defects with a magnifying glass. They were as nearly identical as it is possible to make them. The coils were connected in series and the three leads were brought out in asbestos tubing through the pressure connections shown in Fig. 2. Two identical 10-ohm rheostats were used for balancing the bridge. An objectionable 60-cycle disturbance which was being picked up by the bridge, was eliminated by connecting the output of the bridge to two transformers, as shown in Fig. 5. A constant voltage had to be impressed on the bridge in order to obtain satisfactory results.

Fig. 6 illustrates the circuit of the amplifier. It consisted of two 57 amplifiers and one 45 triode power-amplifier tube. In order to eliminate the d-c part of the power output of the amplifier, the leads of the output were connected to the primary of a small transformer. The secondary output, consisting of the a-c part of the amplifier output, was rectified by a small Rectox rectifier. The rectifier output was connected through a double-pole double-throw switch to an indicating milliammeter having a range of 0 to 100 ma and to an Esterline-Angus recording milliammeter having a range of 0 to 5 ma.

The recording-meter chart of the Esterline-Angus milliammeter

was driven by a synchronous motor from the power lines of the Public Service Company of Indiana. System frequency control in the case of this public utility is obtained from Chicago and is excellent as shown by the records from recording frequency meters in the electrical-engineering meter laboratory of Purdue University. The effect of any momentary variations in frequency was averaged out by taking from 5 to 40 readings at each test point. Chart speeds of 3/4, 1 1/2, 3, 6, and 12 in. per hr and per min were

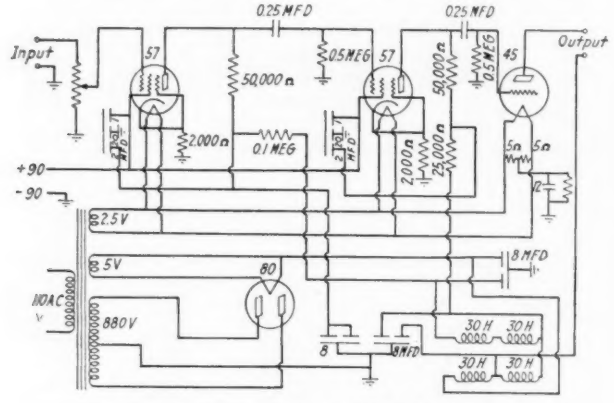


FIG. 6 CIRCUIT DIAGRAM OF THE AMPLIFIER

obtainable from the recorder. Charts were driven at 12 in. per min except where the time of fall exceeded five minutes when lower chart speeds were used. With a chart speed of 12 in. per min, it was possible to scale the time to about one-tenth of a second. The chart speed was checked periodically by recording with a stopwatch the time between two interruptions of the bridge circuit and scaling the distance on the chart.

Fig. 7 shows a typical chart. The two humps are due to the unbalance created in the bridge circuit as the fall-body passed through the coils. The time of fall was scaled from each chart at five different points such as *aa'*, *bb'*, etc. in Fig. 7. If any of the five time measurements differed from the average by more than 1 per cent, the test was discarded.

Some difficulty was experienced at first in securing symmetrical curves from the two coils because of unequal heating of the viscometer. A variation in temperature of 2 F between the two coils altered their resistances sufficiently to unbalance the circuit and destroy the symmetry of the curves. Proper arrangement of heating coils on the container solved this problem. No tests were made unless the readings of all three thermocouples agreed within 1 deg.

A fall-body was suspended from a chemical balance in one of the coils while the input to the bridge circuit was varied in order to check the possibility of these coils disturbing the rate of fall of the body. No deflection of the balance was obtained until the voltage impressed on the bridge circuit was 30 times that used in the test.

The coils were mounted 14 in. apart on the tube and far enough from each end of the tube so that the fall-body had accelerated to a uniform velocity before it entered the tube section which was surrounded by the coil.

CALIBRATION TESTS

A comparison of Equations [1] and [2] shows that the term C in Equation [2] is a function of the dimensions of the fall-body, tube, and distance between timing coils. These dimensions cannot be measured with sufficient accuracy and are also subject to variation with temperature. The effect of compression of the steel due to pressure was neglected as the error so introduced was less than 0.02 per cent at 3500 lb per sq in. The effect of temperature may be treated as a linear function of the temperature without introducing an error of more than 0.5 per cent at 1000 F. The term C for each fall-body was determined by timing the fall when a fluid of known density and viscosity was placed in the viscometer and heated to various temperatures. The results were expressed by an equation of the form

$$C = a + bt$$

where a and b are constants and t is the temperature of the fluid in degrees fahrenheit. These calibration constants for the various fall-bodies are given in Table 1.

TABLE 1 CALIBRATION CONSTANTS OF FALL-BODIES — $C^a = (a + bt)$

Fall-body, No.	a	b
28	0.3628	0.000086
30	0.2525	0.000057
31	0.3560	0.000060
35	13.4200	0.019500
33	19.2700	0.016500

NOTE: t is in degrees F.

[a] See Equation [2].

Since fall-bodies Nos. 28, 30, and 31 were used to measure the viscosity of water, their constants were determined by using water at temperatures up to 212 F within which range the viscosity and density of water are known quite accurately. These values for viscosity were obtained from the International Critical Tables.[7] These calibration constants subsequently gave data on the viscosity of water which were identical for the three fall-bodies and which were in excellent agreement with the published

[7] International Critical Tables. Published for the National Research Council, McGraw Hill Book Company, New York, 1929, vol. 5, p. 10.

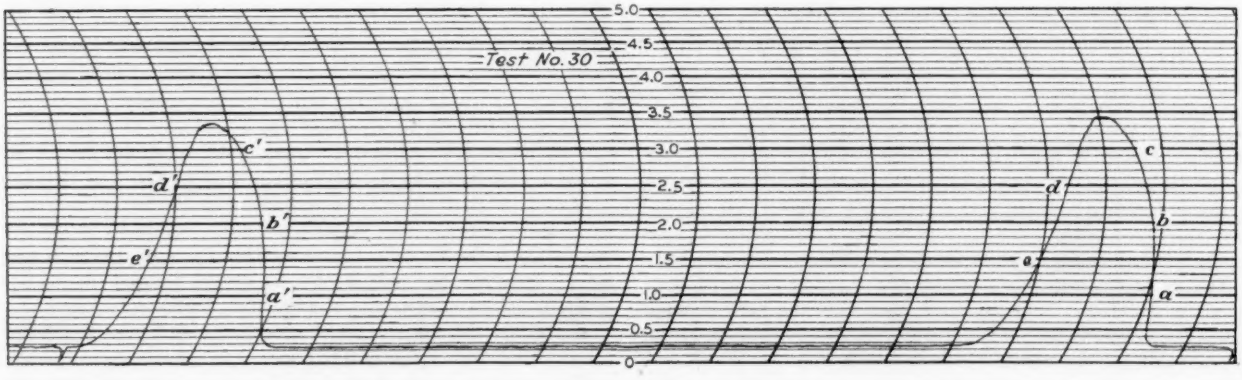


FIG. 7 RECORDING CHART FOR DETERMINING FALLING TIME OF THE WEIGHTS

results of other investigators up to 320 F, which is the upper limit of previous investigations.

The constants for fall-bodies Nos. 35 and 33 which were used in the superheated-steam tests were determined by using dry air at atmospheric pressure and temperatures up to 580 F the viscosity data for the above range having been obtained from the International Critical Tables.[8] Superheated steam up to 500 F at atmospheric pressure was also used for calibrating the constants. The range of pressures and temperatures investigated by the various fall-bodies overlapped sufficiently to obtain a check on the accuracy of the constants.

Test Procedure

In manipulating the apparatus, the fluid in the viscometer was heated to the desired temperature by controlling the electrical input to the heater and was maintained at the desired pressure by manipulating the vent valve or the hand-

[8] International Critical Tables. Published for the National Research Council, McGraw-Hill Book Company, Inc., New York, 1929, vol. 5, p. 2.

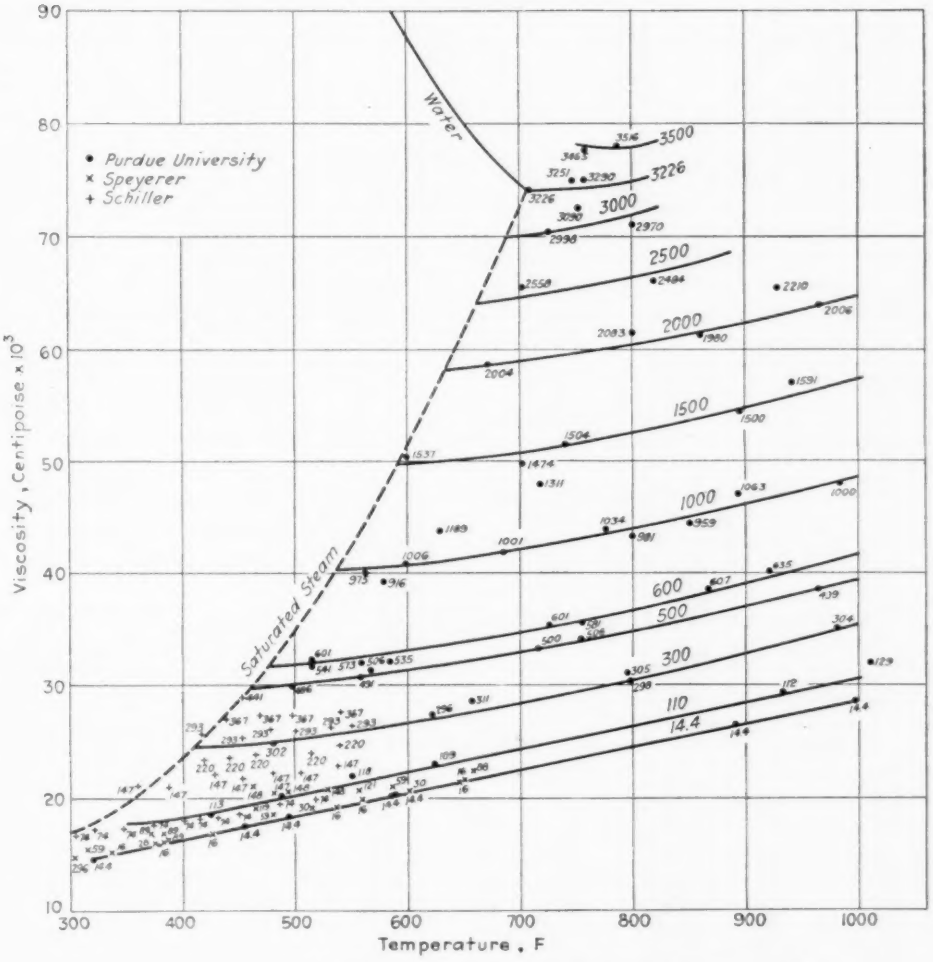


Fig. 9 Viscosity of Water and Superheated Steam
(Small numerals refer to pressure.)

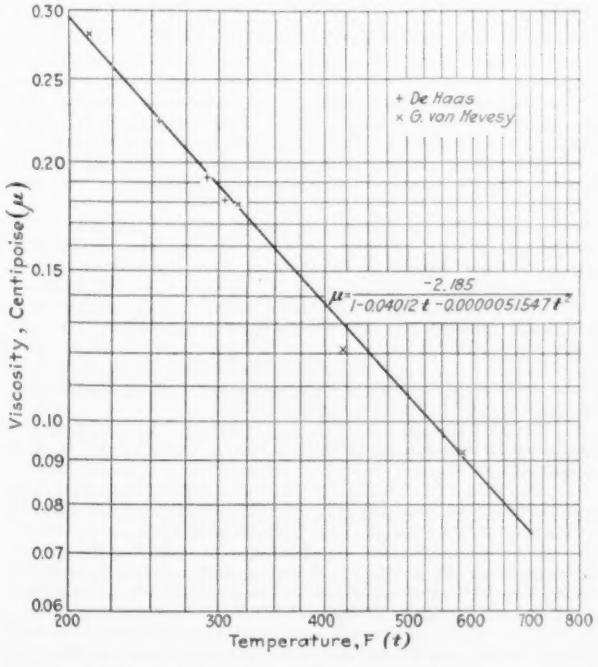


Fig. 8 Viscosity of Saturated Water

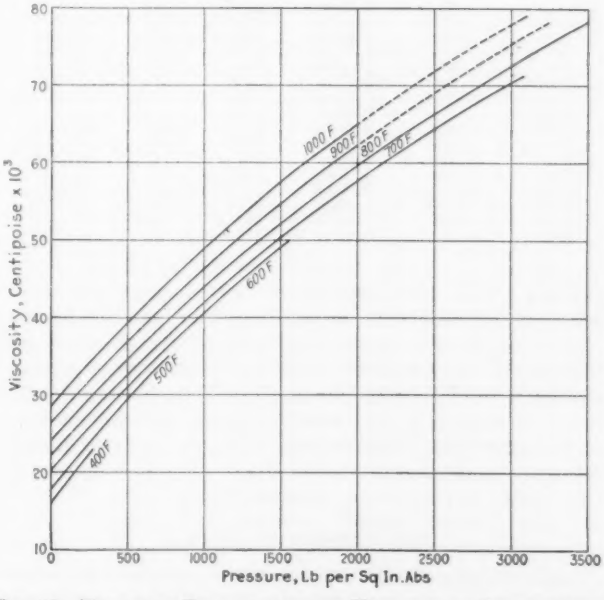


Fig. 10 Effect of Pressure on the Viscosity of Superheated Steam

TABLE 2 VISCOSITY OF SATURATED WATER

Pressure, lb per sq in. abs	Temp, deg F	Viscosity, centipoise
105	331.4	0.170
460	458.5	0.128
850	525.0	0.101
1000	544.6	0.097
1250	572.3	0.092
1505	594.0	0.088
2000	635.6	0.082
2500	668.0	0.078
3000	695.2	0.075
3226	706.1	0.074

TABLE 3 EFFECTS OF PRESSURE ON THE VISCOSITY OF SUB-COOLED WATER

Pressure, lb per sq in. abs	Centipoise at 200 C 392 F	250 C 482 F	300 C 572 F	350 C 662 F	374 C 705 F
500	0.140	...	...	...	...
1000	0.139	0.110	...	...	...
1500	0.138	0.110	0.095	...	...
2000	0.141	0.111	0.096	...	...
2500	0.140	0.110	0.096	0.077	...
3000	0.139	0.109	0.095	0.076	...
3226	0.141	0.109	0.095	0.077	0.074
3500	0.140	0.110	0.095	...	...

TABLE 4 TYPICAL DATA ON VISCOSITY OF WATER

Test no.	Pressure, lb per sq in. abs	Temp, deg F	Viscosity, centipoise
515	2495	574	0.096
516	2500	572	0.096
517	2495	572	0.095
518	2500	572	0.096
519	2500	572	0.096
520	2505	574	0.096
521	2500	572	0.096
522	2500	572	0.096
523	2500	572	0.095
Average	2500	572	0.096

TABLE 5 VISCOSITY OF SUPERHEATED STEAM

Pressure, lb per sq in. abs	Temp, deg F	Viscosity, centi-poise	Pressure, lb per sq in. abs	Temp, deg F	Viscosity, centi-poise
14.4	585	0.0202	975	562	0.0400
14.4	890	0.0265	981	799	0.0432
14.4	997	0.0286	1000	983	0.0480
109	622	0.0230	1001	684	0.0419
111	487	0.0201	1006	598	0.0408
112	932	0.0294	1034	776	0.0438
113	424	0.0186	1063	894	0.0470
118	550	0.0220	1189	628	0.0438
129	1009	0.0320	1311	718	0.0479
296	621	0.0274	1474	702	0.0498
298	799	0.0303	1500	895	0.0545
302	480	0.0249	1504	740	0.0515
304	980	0.0351	1537	598	0.0504
305	794	0.0310	1591	941	0.0571
311	656	0.0286	1980	860	0.0614
486	497	0.0299	2004	672	0.0586
491	558	0.0307	2006	965	0.0641
499	964	0.0386	2083	800	0.0614
506	563	0.0315	2210	928	0.0654
506	567	0.0314	2484	818	0.0661
534	583	0.0320	2558	702	0.0656
541	514	0.0317	....	...	....
573	558	0.0320	2970	801	0.0710
581	765	0.0355	2998	726	0.0704
601	726	0.0352	3091	752	0.0725
601	514	0.0322	3226	706	0.0740
607	866	0.0384	3251	756	0.0749
635	920	0.0400	3290	746	0.0749
916	579	0.0391	3465	759	0.0775
959	851	0.0445	3516	788	0.0780

operated pump. The switch in the heater circuit was opened and after conditions had reached equilibrium and the readings of the three thermocouples checked within 1 deg, the viscometer was rotated on its trunnions into the vertical position where it was held by a stop. The driving motor of the recording milliammeter was started and the time of fall was recorded automatically on the strip chart. The viscometer was then rotated through 180 deg to permit the fall-body to return to the starting position after which the process was repeated.

The Viscosity of Water

Using fall-body No. 28, the viscosity of saturated water was measured from room temperature to the critical temperature of 706 F. The results are tabulated in Table 2 and plotted in Fig. 8. The effect of pressure on the viscosity of compressed (sub-cooled) water was determined by increasing the pressure on the water and repeating the test procedure. The results are shown in Table 3. Each tabulated set of values in Tables 2 and 3 is the average of from 5 to 15 individual determinations. A typical set of test results for one pressure and temperature is shown in Table 4. The original data are too voluminous for presentation in this paper, but may be consulted in the office of the Engineering Experiment Station of Purdue University.

Check tests were made over the entire range of pressures and temperatures with fall-bodies Nos. 30 and 31. No difference in results was obtainable with the three fall-bodies.

Keenan's (9) values for the density of compressed water were used in calculating the results.

The viscosity of saturated water between 200 F and the critical temperature may be represented by the empirical equation

$$\mu = -2.185/(1 - 0.04012t - 0.0000051547t^2)$$

where μ is centipoises, and t is degrees F.

The results of this investigation show that at pressures up to 3500 lb per sq in. abs, the effect of pressure on the viscosity of water is negligible, but that viscosity depends upon the temperature.

The Viscosity of Superheated Steam

The viscosity of superheated steam was determined by filling the viscometer with distilled water, then boiling and discharging the excess liquid and vapor through the vent to the atmosphere until the desired pressure and temperature were attained. The instrument was then rotated into the vertical position and clamped, and the time of fall was recorded automatically.

Fall-body No. 35 was used to determine the viscosity of superheated steam for all of the test points while fall-body No. 33 was used to check a number of the previous tests.

The results of these tests are tabulated in Table 5. Each point is the average of from 5 to 40 separate determinations.

Figs. 9 and 10 show graphically the results of the investigation over the entire range of pressures and temperatures from atmospheric pressure to the upper limit of the Keenan steam tables. The authors believe that these results are sufficiently accurate so that they may be used with confidence in engineering calculations.

Bibliography

1 "Flow Distribution in Forced-Circulation Once-Through Steam Generators," by H. L. Solberg, G. A. Hawkins, and A. A. Potter, Trans. A.S.M.E., vol. 56, 1934, paper FSP-56-16, pp. 841–849.

2 "Die Beweglichkeit Der Ionen, Die Dem Lösungsmittel Eigen Sind," by G. von Hevesy, *Zeitschrift für Elektrochemie*, vol. 27, 1921, p. 21.

3 "Measurements Concerning the Coefficient of Viscosity of Methyl Chloride in Absolute Measure Between the Boiling Point and the Critical State," by M. DeHaas, Communications from the Laboratory of Physics at the University of Leiden, no. 12, 1894.

4 "Die Bestimmung der Zähigkeit des Wasserdampfes, by H. Speyerer, *Forschung auf dem Gebiete des Ingenieurwesens*, part 273, V.D.I. Berlin, 1925.

5 "Bestimmung der Zähigkeit von Wasserdampf," by W. Schiller, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 5, March-April, 1934, p. 71.

6 "Ueber Zähigkeit und Zähigkeitsmessung," by F. Lawaczeck, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 63, no. 29, 1919, pp. 677–684.

7 "The Effect of Pressure on the Viscosity of Forty-Three Pure Liquids," by P. W. Bridgman, Proceedings of the American Academy of Arts and Sciences, vol. 61, no. 3, February, 1926.

8 "Ueber die Zähigkeit verschiedener Käeltemittel im flüssigen und dampfförmigen Zustand in Abhängigkeit von Druck und Temperatur," by H. Stakelbeck, *Zeitschrift für die Gesamte Kaelte-Industrie*, vol. 40, March, 1933, pp. 33–39.

9 "Thermal Properties of Compressed Liquid Water," by J. H. Keenan, *Mechanical Engineering*, vol. 53, no. 2, February, 1931, pp. 127–131.

Stresses in Three-Dimensional Pipe Bends

By WILLIAM HOVGAARD,[1] NEW YORK, N. Y.

The paper presents first an algebraic method of calculating the couples, forces, and stresses in a three-dimensional pipe line fixed at the ends, consisting of straight parts at right angles to each other connected by quarter bends. The solution is complete in that it takes account of bending, torsion, compression, and shearing. It comprises a special discussion of the rotations and deflections of quarter bends when subjected to forces and couples acting normal to their plane, with particular application to the three-dimensional pipe line. The author also gives an analysis of the stresses at the terminal sections and in the bends, together with a discussion of the strength criteria and the permissible working stresses. A numerical example is given to illustrate the application of the formulas.

1—INTRODUCTION

IN a two-dimensional or plane pipe bend subjected to changes of temperature, the reactions at the anchorages are in the plane of the bend. The calculation of the couples and forces acting on any section of the pipe is relatively simple, and has been discussed by the author in four papers[2,3,4,5] published in the Journal of Mathematics and Physics of the Massachusetts Institute of Technology, and in two other papers.[6,7]

When a pipe is so shaped as to lie in three planes, even if at right angles to each other, the problem is more complicated, not only because three coordinates must be used instead of two, but also because torsional couples are produced and the curved parts deflect out of their plane. When the pipe is fixed at the ends, as here assumed, there are at each terminal three component reactions instead of two and three component couples instead of one. Torsional as well as bending stresses are produced and have to be combined with direct and shearing stresses.

When three-dimensional bends are not of simple geometrical form, it is necessary, as in the case of plane bends, to use graphical methods of solution, but when the pipe consists of straight parts and circular quadrants, and provided there are not too many bends and tangents, it is possible and, it is believed, preferable to use the algebraic method, which must always underlie all graphical solutions. It is the object of this paper to present a fairly rigorous solution for the simple case where the pipe has three straight parts and two quarter bends, all situated in three planes at right angles to each other. The method has been successfully applied in practice not only to this case, but also to a pipe having four straight parts and three quarter bends.

It is assumed that the curved parts of the pipe are bent to a radius of curvature not less than four or five times the diameter of the pipe, in which case it is unnecessary to take account of the fact that the neutral axis of a curved bar does not go exactly through the centroid of the section.

On the other hand, we include the effect of the flattening of the circular section which takes place when a curved pipe is bent in its own plane and which may cause a very marked increase in the angular deflection of the bend.[3] This flattening practically does not occur when a curved pipe is bent out of its plane nor is it produced by torsion, so that when calculating the deflections of a pipe bend under such actions, we can regard the pipe as a curved solid bar with the same axis as the pipe and with the same transverse and polar moments of inertia.

The Plane Bend

In developing the following formulas, the general method and notation adopted in the author's paper on deformation of plane pipes[3] are used, but it has been necessary or expedient in dealing with the curved parts of the pipe to extend the method considerably.

Referring first to Fig. 1, let

- M = a couple acting as indicated by a suffix
- R = radius of curvature of the axis of a pipe
- t = $2h$ = wall thickness of the pipe
- r_i = internal radius of the pipe
- r_o = external radius of the pipe
- r = $\frac{1}{2}(r_i + r_o)$ = radius of the middle surface of the pipe wall
- A = sectional area of the transverse section of pipe wall
- I = moment of inertia of the area of the transverse section about the neutral axis
- I_o = polar moment of inertia of the transverse section = $2I$
- E = modulus of elasticity
- G = modulus of rigidity. Taking Poisson's ratio as 0.30, we have: $G = E/2.6$ and $GI_o = EI/1.3$
- S = any point on the axis of the pipe
- ψ = angle between the radius to S and some fixed radius of reference

---

[1] Professor Emeritus, Massachusetts Institute of Technology. Mem. A.S.M.E. Professor Hovgaard was graduated from the Danish Naval College in 1879 with the rank of sub-lieutenant and from the Royal Naval College, Eng., in 1886. From 1888 to 1900 he served at the Royal dockyard, Copenhagen, Denmark, alternating with sea duty as a naval officer except for a period between 1895 and 1897 when he was manager of the Burmeister and Wain shipyard, Copenhagen, in charge of the design and construction of the Imperial Russian yacht *Standart* and other vessels. From 1901 until 1933 Professor Hovgaard was in charge of the course of Naval Construction at Massachusetts Institute of Technology. He was in the technical service of the U. S. Bureau of Construction and Repair in 1917 and 1918, and served as consultant in the same Bureau from 1919 to 1926. Professor Hovgaard also served on Government committees concerning the Navy airship Shenandoah and the Army airship RS-1.

[2] "The Elastic Deformation of Pipe Bends," by William Hovgaard, Journal of Mathematics and Physics, vol. 6, no. 2, 1926, pp. 69–118, Massachusetts Institute of Technology.

[3] "Deformation of Plane Pipes," by William Hovgaard, Journal of Mathematics and Physics, vol. 7, no. 3, 1928, pp. 198–238, Massachusetts Institute of Technology.

[4] "Further Research on Pipe Bends," by William Hovgaard, Journal of Mathematics and Physics, vol. VII, no. 4, 1928, pp. 239–299, Massachusetts Institute of Technology.

[5] "Tests on High-Pressure Pipe Bends," by William Hovgaard, Journal of Mathematics and Physics, vol. 8, no. 4, 1929, pp. 293–344, Massachusetts Institute of Technology.

[6] "Deflections and Stresses in Pipe Bends," by William Hovgaard, Proceedings of the World Engineering Congress, Tokyo, vol. 3–4, 1929, pp. 1–30.

[7] "Bending of Curved Pipe," by William Hovgaard, Proceedings of the Third International Congress for Applied Mechanics, Stockholm, vol. II, 1930, pp. 331–341.

Contributed by the Power Division for presentation at the Annual Meeting, New York, N. Y., December 2 to 6, 1935, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$d\psi$ = angle of a small element of the pipe axis comprised between two consecutive transverse sections

$\Delta d\psi$ = change in $d\psi$ caused by flexural strain

s = length or girth of axis from the point of reference to S

ds = $Rd\psi$

K = a coefficient applied to the angular deflection $\Delta d\psi/d\psi$ to allow for the flattening of the transverse section of a curved pipe

$$K = (48h^2R^2 + 10r^4)/(48h^2R^2 + r^4) \quad \ldots\ldots [1]$$

Writing

$$2hR/r^2 = \lambda \quad \ldots\ldots [2]$$

and substituting λ in Equation [1]

$$K = (12\lambda^2 + 10)/(12\lambda^2 + 1) \quad \ldots\ldots [1a]$$

Since the flexibility of a pipe bend is directly proportional to K, this coefficient shall be called the "flexibility factor."

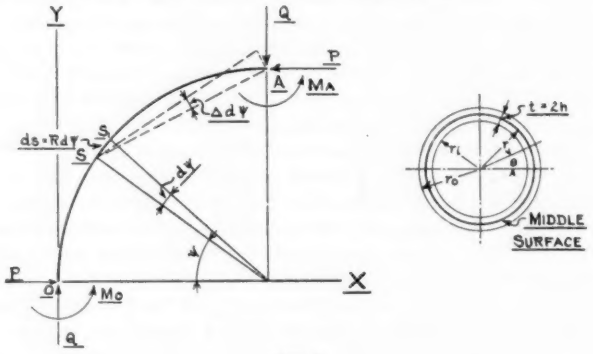


FIG. 1

For brevity, a direction parallel with the X-axis will be called the X-direction, and similarly, the rotation about an axis in the X-direction will be called an X-rotation; a couple with an axis in the X-direction will be called an X-couple; and a linear displacement in the X-direction will be called an X-displacement. Analogous terms are used in connection with the Y and Z axes, an XY-rotation being the rotation in which OX turns toward OY, the same as Z-rotation. The other rotations are similarly referred to. As seen from Fig. 6, a right-handed system of coordinates has been adopted in which the XY-, YZ-, and ZX-rotations are positive.

Axes of couples are indicated in the diagrams by arrows drawn so that when looking along the shaft toward the head, the rotation is clockwise. Thus an arrow drawn in the positive X-direction represents a positive couple and vice versa.

Before dealing with three-dimensional pipe bends, it is desirable first to analyze the case of plane circular quarter bends fixed at one end and subjected to terminal forces and couples acting normal to the plane of the bend at the other end. This problem occurs in three-dimensional bends, and in particular we wish to determine the rotations and the linear displacement normal to the plane of the bend at the free end. The analysis is here carried out in a simple and direct manner, disregarding the displacement of the free end in the plane of the bend, which is of the second order relative to that normal to the plane.

2—QUARTER BENDS UNDER THE ACTION OF FORCES OR COUPLES NORMAL TO THEIR PLANE

CASE I—QUARTER BEND FIXED AT ONE END AND LOADED WITH A FORCE NORMAL TO ITS PLANE AT THE FREE END

Referring to Fig. 2, the bend is represented by its axis, which forms a circular quadrant OA. The bend, which is spoken of as being in a horizontal plane, is fixed at O and loaded by an upward force F at its free end A.

Consider an element ds of the pipe at S. The radius CS forms an angle ψ with the X-axis. The force produces a bending

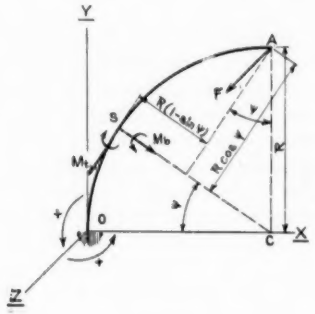


FIG. 2
(Case I.)

moment M_b with axis along the radius at S, and a twisting moment M_t with axis along the tangent at S. These moments can be expressed as

$$\left.\begin{aligned} M_b &= FR\cos\psi \\ M_t &= FR\,(1 - \sin\psi) \end{aligned}\right\} \quad \ldots\ldots [3]$$

The moment M_b produces rotation of the element in a vertica tangential plane so that

$$\Delta d\varphi_b = \frac{M_b}{EI}\,ds = \frac{M_bR}{EI}\,d\psi$$

The moment M_t produces a twist of the element

$$\Delta d\varphi_t = \frac{M_t}{GI_o}\,ds = 1.3\,\frac{M_tR}{EI}\,d\psi$$

By resolving these rotations along the X-direction, we obtain for the X-rotation of the element

$$\Delta d\varphi_{xs} = \left[\frac{M_b}{EI}\cos\psi - \frac{1.3\,M_t}{EI}\sin\psi\right] Rd\psi$$

Hence the total rotation at A in the YZ-plane becomes

$$\Delta\varphi_{xA} = \frac{FR^2}{EI}\left[\int^{\frac{\pi}{2}} \cos^2\psi\,d\psi - 1.3\int_0^{\frac{\pi}{2}} (1-\sin\psi)\sin\psi\,d\psi\right] = 0.506\,\frac{FR^2}{EI} \quad \ldots\ldots [4]$$

The Y-rotation of the element is

$$\Delta d\varphi_{ys} = -\left[\frac{M_b}{EI}\sin\psi + \frac{1.3M_t}{EI}\cos\psi\right] Rd\psi$$

and the total rotation of the pipe at A in the XZ-plane becomes

$$\Delta\varphi_{yA} = -\frac{FR^2}{EI}\left[\int_0^{\frac{\pi}{2}} \sin\psi\cos\psi\,d\psi + 1.3\int_0^{\frac{\pi}{2}} \cos\psi\,(1-\sin\psi)\,d\psi\right] = -1.150\,\frac{FR^2}{EI} \quad \ldots\ldots [5]$$

It will be noticed that we have here used the couples M_b and M_t and not M_x and M_y, although we wish to find the X-rotation

and Y-rotation. The reason for this is that it is simpler first to find the pure-bending rotation and the pure twist because the corresponding moments of inertia of the pipe section are easily determined. After that we obtain the desired rotations by resolving in the X-direction and the Y-direction. In other words, we resolve rotations instead of couples.

The linear vertical deflection Δz at A is determined as the algebraic sum of that due to M_b and that due to M_t. This deflection can be expressed as

$$\Delta z_A = \int_0^{\frac{\pi}{2}} \frac{M_b}{EI} R \cos\psi \, R d\psi + 1.3 \int_0^{\frac{\pi}{2}} \frac{M_t}{EI} R (1 - \sin\psi) \, R d\psi$$

$$= \frac{FR^3}{EI}\left[\int_0^{\frac{\pi}{2}} \cos^2\psi \, d\psi + 1.3 \int_0^{\frac{\pi}{2}} (1 - \sin\psi)^2 \, d\psi\right]$$

$$= +1.248 \frac{FR^3}{EI} \ldots\ldots [6]$$

where the positive sign indicates an upward displacement.

CASE II—QUARTER BEND FIXED AT ONE END AND SUBJECTED TO A TWISTING MOMENT M_z AT THE FREE END—FIG. 3

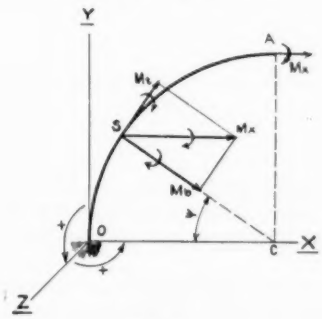


FIG. 3
(CASE II.)

From Fig. 3 it is seen that

$$\left.\begin{aligned} M_b &= M_z \cos\psi \\ M_t &= M_z \sin\psi \end{aligned}\right\} \ldots\ldots [7]$$

The X-rotation of the element is

$$\Delta d\varphi_{xs} = \left[\frac{M_b}{EI} \cos\psi + 1.3 \frac{M_t}{EI} \sin\psi\right] R d\psi$$

and

$$\Delta\varphi_{xA} = \frac{M_z R}{EI}\left[\int_0^{\frac{\pi}{2}} \cos^2\psi \, d\psi + 1.3 \int_0^{\frac{\pi}{2}} \sin^2\psi \, d\psi\right]$$

$$= +1.806 \frac{M_z R}{EI} \ldots\ldots [8]$$

The Y-rotation of the element is

$$\Delta d\varphi_{ys} = -\left[\frac{M_b}{EI} \sin\psi - 1.3 \frac{M_t}{EI} \cos\psi\right] R d\psi$$

and

$$\Delta\varphi_{yA} = -\frac{M_z R}{EI}\left[\int_0^{\frac{\pi}{2}} \sin\psi \cos\psi \, d\psi - 1.3 \int_0^{\frac{\pi}{2}} \sin\psi \cos\psi \, d\psi\right]$$

$$= +0.150 \frac{M_z R}{EI} \ldots\ldots [9]$$

It is seen that the major rotation is in the same direction as the acting couple. The Y-rotation is only about 8 per cent of the X-rotation.

The vertical deflection of the free end is

$$\Delta_{zA} = \int_0^{\frac{\pi}{2}} \frac{M_b}{EI} R \cos\psi \, R d\psi - 1.3 \int_0^{\frac{\pi}{2}} \frac{M_t}{EI} R(1 - \sin\psi) \, R d\psi$$

$$= \frac{M_z R^2}{EI}\left[\int_0^{\frac{\pi}{2}} \cos^2\psi \, d\psi - 1.3 \int_0^{\frac{\pi}{2}} (1 - \sin\psi) \sin\psi \, d\psi\right]$$

$$= +0.506 \frac{M_z R^2}{EI} \ldots\ldots [10]$$

where the positive sign again indicates an upward displacement.

CASE III—QUARTER BEND FIXED AT ONE END AND SUBJECTED TO A COUPLE M_y AT THE FREE END, HAVING ITS AXIS NORMAL TO THE TANGENT AT A—FIG. 4

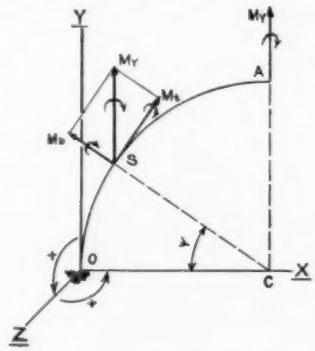


FIG. 4
(CASE III.)

It is seen from Fig. 4 that

$$\left.\begin{aligned} M_b &= M_y \sin\psi \\ M_t &= M_y \cos\psi \end{aligned}\right\} \ldots\ldots [11]$$

The rotation of the element in an X-direction at S is

$$\Delta d\varphi_{xs} = \left[-\frac{M_b}{EI} \cos\psi + 1.3 \frac{M_t}{EI} \sin\psi\right] R d\psi$$

and

$$\Delta\varphi_{xA} = -\frac{M_y R}{EI}\left[\int_0^{\frac{\pi}{2}} \sin\psi \cos\psi \, d\psi - 1.3 \int_0^{\frac{\pi}{2}} \sin\psi \cos\psi \, d\psi\right]$$

$$= +0.150 \frac{M_y R}{EI} \ldots\ldots [12]$$

Rotation of the element in a Y-direction at S is

$$\Delta d\varphi_{ys} = \left[\frac{M_b}{EI} \sin\psi + 1.3 \frac{M_t}{EI} \cos\psi\right] R d\psi$$

and

$$\Delta\varphi_{yA} = \frac{M_y R}{EI}\left[\int_0^{\frac{\pi}{2}} \sin^2\psi\, d\psi + 1.3\int_0^{\frac{\pi}{2}} \cos^2\psi\, d\psi\right] = +1.806\,\frac{M_y R}{EI} \quad [13]$$

These rotations are the same as found in case II, and we have again a major and minor rotation; but the X-axis and the Y-axis are interchanged.

The vertical deflection is

$$\Delta z_A = -\int_0^{\frac{\pi}{2}} \frac{M_b}{EI} R\cos\psi\, R d\psi - 1.3\int_0^{\frac{\pi}{2}} \frac{M_t}{EI} R\,(1-\sin\psi)\, R d\psi$$

$$= -\frac{M_y}{EI}\left[\int_0^{\frac{\pi}{2}} \sin\psi\cos\psi\, d\psi + 1.3\int_0^{\frac{\pi}{2}} (1-\sin\psi)\cos\psi\, d\psi\right] = -1.150\,\frac{M_y R^2}{EI} \quad [14]$$

Case IV—Quarter Bend Fixed at One End and Subjected at Any Point S to a Couple Equal to That Which Would Be Produced by a Force F Acting Normal to the Bend at O—Fig. 5

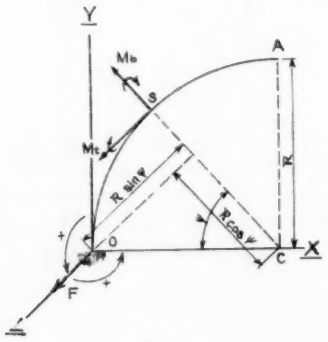


Fig. 5
(Case IV.)

The case is fictitious inasmuch as a force F applied at O would not actually produce a couple at S since O is fixed, but as will be explained later, it is here desired to determine the effect at A of a couple of just that magnitude at any point S. The result is the same as if F were acting downward at A in conjunction with a certain couple.

It is clear that the rotation at the A-end must be of the same amount as in case I, but since the force in case I was applied at the A end, $\Delta\varphi_{zA}$ becomes $\Delta\varphi_{yA}$ in case IV, and vice versa. Thus

$$\Delta\varphi_{zA} = -1.150\,\frac{FR^2}{EI} \quad [15]$$

$$\Delta\varphi_{yA} = +0.506\,\frac{FR^2}{EI} \quad [16]$$

Equation [15] gives the twist or X-rotation at A, while Equation [16] gives the Y-rotation at A.

The linear deflection at A normal to the plane will not be the same as in case I. In case IV, $M_b = FR\sin\psi$, and $M_t = FR(1-\cos\psi)$

$$\Delta z_A = -\int_0^{\frac{\pi}{2}} \frac{M_b}{EI} R\cos\psi\, R d\psi + 1.3\int_0^{\frac{\pi}{2}} \frac{M_t}{EI} R(1-\sin\psi)\, d\psi$$

$$= -\frac{FR^3}{EI}\left[\int_0^{\frac{\pi}{2}} \sin\psi\cos\psi\, d\psi - 1.3\int_0^{\frac{\pi}{2}} (1-\sin\psi)(1-\cos\psi)\, d\psi\right] = -0.408\,\frac{FR^3}{EI} \quad [17]$$

In cases I to IV, we have supposed the acting force and the axis of the acting couple to be in the positive X- or Y-direction and the axes of all the couples are so drawn as to make the rotation clockwise. The direction in which the arrows representing the couples M_b and M_t are to be drawn can thus be determined readily as can also the sign to be given to the rotations and the displacement at A. If the acting force or couple acts in the negative direction, all the rotations and all the displacements change sign, and we have only in the final results to substitute $-F$, $-M_z$, $-M_y$ for F, M_z, M_y, provided the coordinate axes are placed in the same position relative to the fixed and free ends of the quadrant. The orientation of the quadrant in space is indifferent, but if we change the name of the coordinate axes as in the second quarter bend of Fig. 6, we must realize clearly which end of the quadrant is assumed to be fixed and exactly how the acting force or couple is applied. This is indicated in Fig. 8.

3—Analysis of the Three-Dimensional Pipe Bend

General

Pipe bends are often attached to parts of the machinery which are anchored at points a certain distance from the terminals of the pipe, these parts of the machinery themselves being subject to heat expansion.

In such cases, linear displacements of the terminals will take place which must be added algebraically to those caused by heat expansion of the pipe itself.

Fig. 6 shows diagrammatically the general lay-out of such a pipe in the ideal case here considered, where all the bends are circular quadrants associated with straight tangents at right angles to each other. The ends O and A are regarded as fixed in direction and subject only to the displacements caused by the machinery parts to which they are attached. Thus, reaction forces and couples exist at those points, and it is the first object of the following analysis to determine these reactions.

Referring to Fig. 6, O is taken as the origin in a right-handed rectangular system of coordinates. The forces and couples (P, Q, N) and (M_{Ox}, M_{Oy}, M_{Oz}) at O, and ($-P$, $-Q$, $-N$) and (M_{Ax}, M_{Ay}, M_{Az}) at A are the reactions. The forces are considered positive when acting in the positive direction of the coordinate axes and the couples are considered positive when turning in the directions XY, YZ, and ZX. In Fig. 6, all the reaction couples are shown as being positive, but the forces at A, which are known to be equal and opposite to those at O, are shown as being negative.

The radius of the bends is R, and the straight lengths of pipe are L_y, L_z, and L_x, respectively. Also, $L_y + R = H$, $L_x + 2R = L$, and $L_z + R = T$.

The Y-direction shall be referred to as vertical, and the X- and Z-directions as horizontal.

At any section S (x, y, z), the couples acting on the part of the pipe farthest from the origin, but expressed in terms of the reactions at the origin, are

$$\left.\begin{aligned} M_x &= M_{Ox} + Qz - Ny \\ M_y &= M_{Oy} + Nx - Pz \\ M_z &= M_{Oz} + Py - Qx \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [18]$$

The couples acting at S on the part of the pipe nearest the origin, expressed in terms of the reactions at A, are of the form

$$M'_x = M_{Ax} + Q(T - z) - N(H - y) = M_{Ax} + QT - NH - Qz + Ny \ldots\ldots [18a]$$

which should be equal to M_x in magnitude, but of opposite sign.

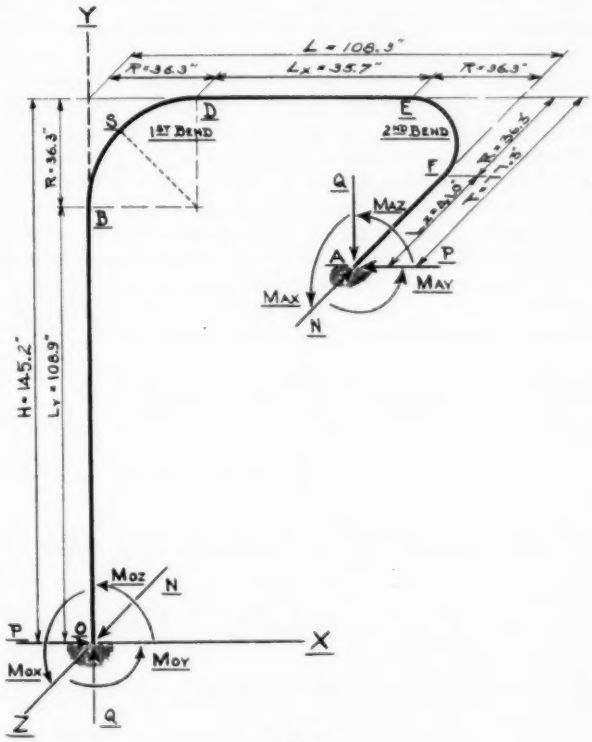


FIG. 6

Now, for equilibrium of the whole pipe taking moments about O, we must have,

$$M_{Ax} + QT - NH + M_{Ox} = 0$$

and substituting in Equation [18a] we get

$$M'_x = -M_{Ox} - Qz + Ny$$

which is equal to $-M_x$ according to the first of Equations [18]. Also M'_y and M'_z, expressed in terms of the reactions at A, are found to be equal to $-M_y$ and M_z, respectively, expressed in terms of the reactions at O.

In calculating the rotations and displacements of the A-end of the pipe, it would seem natural, as in cases I, II, and III, to find the deflections of the element at any point in terms of the reactions at A, that is, in terms of the couples (M'_x, M'_y, M'_z), but since the expressions for those couples, as given by Equation [18a], are more cumbersome than those for (M_x, M_y, M_z) in Equations [18], it is preferred in the following to substitute the latter, simply using $-M_x$, $-M_y$, and $-M_z$ instead of M'_x, M'_y, and M'_z. This consideration led to case IV.

There are six fundamental equations between the reactions and the deflections at each end of the pipe, three for the rotations and three for the linear displacements. Since both ends are assumed to be fixed in direction, the rotations of A relative to O are equal to zero and, therefore, the sign of the bending moments in the equations is indifferent, so that we can use the expressions in Equations [18] as they stand without changing their sign.

In forming the equations for the displacements, it must be borne in mind that the heat expansions at A do not actually take place; they are prevented by the reactions, which may be imagined to produce the displacements Δx_A, Δy_A, Δz_A in negative directions. Hence, while M_x, M_y, M_z must be entered with a negative sign, it is also necessary to enter Δx_A, Δy_A, Δz_A with a negative sign on the other side of the equations, and thus the minus signs cancel one another in this case. (Compare Equations [22] and [23] of this paper with the second and third Equations of [13′] on page 209 of the author's paper on the deformations of plane pipes.[3])

Since the pipe is assumed to be of uniform section and material, the product EI is constant and can be omitted in the equations for the rotations, each of which is equal to zero. In the displacement equations, EI has to be entered, and for the sake of simplicity, is placed as a factor of Δx_A, Δy_A, and Δz_A on the right-hand side of the equations.

It has been shown in case II that a pure or constant couple M_x, applied to a quarter bend, such as for instance the first bend, produces a major rotation (in that case a twist) of the free end in the X-direction, and a minor rotation in the Y-direction. Also, as shown in case III, M_y produces a major rotation in the Y-direction and a minor twist in the X-direction both having the same sign. The minor rotations will be neglected in the equations of rotation, where their inclusion would complicate the problem so as to make the solution impracticable. They can however be included without difficulty in the equations of displacement, and when on this basis the reaction forces and couples have been determined, a more accurate recalculation can be made, if necessary, by substituting these values in the small rotation terms and including them in the rotation equations.

We have seen in cases II and III that a constant couple acting on a quarter bend produces a displacement Δz as well as rotations of the free end. In connection with the three-dimensional pipe we shall refer to that displacement as "local," although the whole pipe beyond the bend partakes in it, but this term is adopted in order to distinguish it from the displacements of A produced by the rotations of the ends of the bends which act with a leverage on the end of the pipe.

Actually the couples, which act on the quarter bends, are not constant throughout the quadrant. Referring to Fig. 7, consider $M_x = M_{Ox} + Qz - Ny$ in the first quarter bend. Here $z = 0$ and $y = L_y + R \sin \psi$, so that M_x is composed of a constant couple $M_{Ox} - NL_y$ and a varying couple $-NR \sin \psi$. Also M_y furnishes a constant couple M_{Oy} and a varying couple $+NR(1 - \cos \psi)$. The effect of the two varying couples is the same as that of a force N acting in the Z-direction at B, since this gives $M_x =$

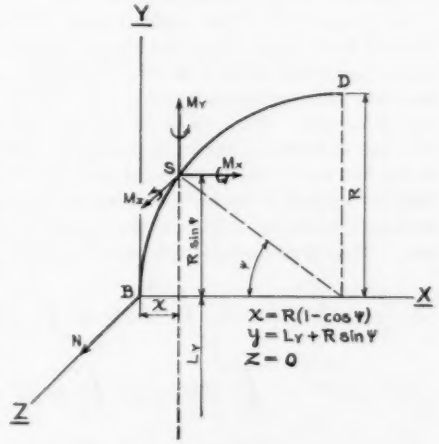


FIG. 7

(First bend, vertical projection.)

$NR \sin \psi$, and $M_y = + NR\ (1 - \cos \psi)$ and this is precisely the problem considered in case IV where F takes the place of N. We have then the rotations produced by the varying part of the couples M_x and M_y at the point D of the first bend, which are $-1.150\ NR^2$ in the X-direction, and $+0.506\ NR^2$ in the Y-direction.

The displacement Δz produced at D by the force N is, according to case IV, $-0.408NR^3$, which is in the negative Z-direction, and arises from the varying parts of both the M_x and M_y couples. Hence the varying parts, like the constant parts of these couples, produce a local displacement of the quarter bend at D and two rotations.

The M_z couple acts in the plane of the first bend and is dealt with as in a plane problem. Only rotations in the XY-plane are produced, and the displacements at A can be calculated directly by integrating the effect of each element ds as in plane bends.

All that has been said here about the first bend applies in principle to the second bend EF, Fig. 8, where M_y takes the place of

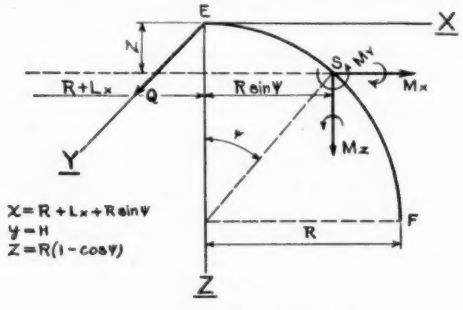


FIG. 8
(Second bend, horizontal projection.)

M_z, and M_z and M_x act in planes normal to the bend. The variable couple is here due to Q acting in the positive direction at E, which is regarded as fixed.

While the straight parts of the pipe can be dealt with by the same general method as used in plane bends, it is expedient to deal with each of the quarter bends separately. The local rotations at the endpoints D and F are determined from cases II, III, and IV and are simply added to the other terms in each of the equations of rotation. In the displacement equations we enter separately the local displacements normal to the plane of each bend and then the displacements caused at A by the rotations at the ends of the quarter bends. This somewhat complicated analysis will be explained in some detail.

In forming the three equations of rotation, the complete integral from O to A for the major component couple is first written, after which are added the integrals for the rotations about the same axis, produced in the quarter bends by the other component couples. While both M_y and M_z produce X-rotation in the first and second bends, respectively, and, therefore, appear in the first equation, M_z does not produce Y-rotation and hence does not appear in the second equation. Similarly M_y does not produce Z-rotation and is therefore absent from the third equation. Thus the equations of rotation become

$$\int_O^A M_x ds + \int_B^D M_y ds + \int_E^F M_z ds = 0 \ldots\ldots [19]$$

$$\int_O^A K M_y ds + \int_B^D M_x ds = 0 \ldots\ldots [20]$$

$$\int_O^A K M_z ds + \int_E^F M_x ds = 0 \ldots\ldots [21]$$

The integrations are with respect to ds, which in the straight parts of the pipe are equal to dx, dy, dz. In the bends $ds = Rd\psi$. The integrations follow the axis of the pipe.

It will be observed that the flexibility factor K is entered only under the integrals where the couple acts in the plane of one of the bends. In all other parts of the system, K is equal to unity. In the first bend, for instance, K is taken into account only in connection with M_z, in the second bend only in connection with M_y.

In forming the displacement equations consider first the X-direction. It is clear that M_x cannot produce any X-displacements in the straight parts and, as explained previously, in the bends except in the first bend where it produces a minor Y-rotation, which with a leverage T causes A to move in the positive X-direction. The couples M_z and M_y must be considered for the whole pipe. The couple M_z acting at any point S, causes the element at that point to rotate and, with a lever $(H - Y)$, to produce a negative X-displacement at A. The couple M_y rotates the element through an angle which, with a lever $(T - z)$, gives a positive X-displacement of A.

The algebraic sum of these displacements will be equal to the heat expansion Δx_A. The equations for the displacements in the Y- and Z-directions are formed in the same manner. Since M_x does not act in the plane of any of the bends, it does not carry the K-factor. The algebraic sums of the displacements are

$$-\int_O^A K(H - y) M_z ds + \int_O^A K(T - z) M_y ds + T \int_B^D M_x ds = EI\, \Delta x_A \ldots\ldots [22]$$

$$\int_O^A K(L - x) M_z ds - \int_O^A (T - z) M_x ds - T \int_B^D M_y ds - L_z \int_E^F M_x ds = EI\, \Delta y_A \ldots\ldots [23]$$

$$\int_O^A (H - y) M_x ds - \int_O^A K(L - x) M_y ds - (L_z + R) \int_B^D M_x ds = EI\, \Delta z_A \ldots\ldots [24]$$

X-ROTATION

The rotation in the YZ-plane is caused chiefly by the first term in Equation [19], containing M_z, which affects all parts of the pipe. The second and third terms of Equation [19], containing M_y and M_z, can produce no rotation in the X-direction in the straight parts of the pipe, and in the bends only where they act normal to the plane of the bends.

Substituting Equations [18] in Equation [19], we obtain

$$\int_O^A (M_{Oz} + Qz - Ny)\, ds + \int_0^{\pi/2} [M_{Oy} + NR\,(1 - \cos \psi)]\, Rd\psi + \int_0^{\pi/2} [M_{Oz} + PH - Q(R + L_z + R \sin \psi)]\, Rd\psi \ldots\ldots [19a]$$

since $z = 0$ and $x = R(1 - \cos \psi)$ in the first bend, and $y = H$, and $x = (R + L_z + R \sin \psi)$ in the second bend.

For the straight part of the pipe OB, where $z = 0$, we obtain

$$\int_0^{L_y} (M_{Oz} - Ny) dy$$

For the first bend, Fig. 7, where $y = L_y + R \sin \psi$, and $x = R\,(1 - \cos \psi)$, we obtain

$$\int_0^{\pi/2} [M_{Oz} - NL_y - NR\sin\psi]\,Rd\psi + \int_0^{\pi/2} [M_{Oy} + NR(1-\cos\psi)]\,Rd\psi$$

From case II we know that the constant couple $(M_{Oz} - NL_y)$ produces an X-rotation at A of $+1.806\ (M_{Oz} - NL_y)R$, and from case III, that the couple M_{Oy} produces an X-rotation of $+0.150\ M_{Oy}R$, but as previously stated, the latter will be neglected, at least preliminarily.

There remain the terms

$$-\int_0^{\pi/2} NR^2\sin\psi\,d\psi + \int_0^{\pi/2} NR^2(1-\cos\psi)\,d\psi$$

which are such as would arise from a force N acting normal to the bend at B. This is easily recognized as case IV, where a force F acting at O in Fig. 5 produces an X-rotation at the point A of $-1.150\ FR^2/EI$. Hence we obtain here for the point D a term $-1.150\ NR^2$,

For the straight length $L_x = DE$, extending from $x = R$ to $x = R + L_x$, we still have $z = 0$, but $y = H$. Thus M_x which in this instance is torsional, becomes $(M_{Ox} - NH)$ and we obtain a term

$$+1.30\int_R^{R+L_x} (M_{Ox} - NH)dx$$

The second bend, Fig. 8, is in a horizontal plane and only the first and third integrals in Equation [19a] need to be considered. Here $y = H$, $x = R + L_x + R\sin\psi$, $z = R(1-\cos\psi)$ and we get the terms

$$\int_0^{\frac{\pi}{2}} [M_{Oz} + QR(1-\cos\psi) - NH]Rd\psi + \int_0^{\frac{\pi}{2}} [M_{Ox} + PH - Q(R+L_x) - QR\sin\psi]\,Rd\psi$$

For the constant part of the M_x-couple $(M_{Oz} - NH)$, we get according to case III a major term $+1.806\ (M_{Oz} - NH)R$, and for the constant part of the M_x-couple according to case II a minor X-rotation, $+0.150\ [M_{Ox} + PH - Q(R + L_x)]R$, which again is neglected in the preliminary calculations.

The variable terms in Q give

$$\int_0^{\frac{\pi}{2}} QR^2(1-\cos\psi)\,d\psi - \int_0^{\frac{\pi}{2}} QR^2\sin\psi\,d\psi$$

which are recognized as expressing the effect of a force Q acting upward at E in the same way as F acts at O in case IV, in which $\Delta\varphi_{yA} = +0.506\ FR^2/EI$. Since OY in case IV corresponds to OX in the present case, we get an X-rotation of $+0.506\ QR^2$.

For the length $L_z = FA$, extending from $z = R$ to $z = R + L_z = T$, where again $y = H$, we obtain from the first term in Equation [19a]

$$\int_R^T (M_{Ox} + QZ - NH)dz$$

We can now write Equation [19a] as

$$\int_0^{L_y} (M_{Oz} - Ny)dy + 1.806\ (M_{Oz} - NL_y)R - 1.150\ NR^2 + 1.30\int_R^{R+L_x} (M_{Oz} - NH)dx + 1.806\ (M_{Oz} - NH)R + 0.506\ QR^2 + \int_R^T (M_{Ox} + Qz - NH)dz = 0 \ldots [19b]$$

Carrying out the integrations and arranging the terms, we obtain the final equation for the X-rotation

$$[3.612R + 1.30L_x + L_y + L_z]\ M_{Oz} + \tfrac{1}{2}\ T^2Q - [\tfrac{1}{2}\ L_y^2 + 1.806\ (L_y + H)\ R + 1.150R^2 + (1.30L_x + L_z)H]N = 0 \ldots [25]$$

If it is desired to allow for the minor rotations caused by M_y and M_z in an eventual recalculation, the following terms

$$0.150\ M_{Oy}\ R + 0.150\ R\ [M_{Ox} + PH - Q(R + L_x)]$$

should be added to the left-hand side of Equation [25]. Substitute the numerical values of M_{Oy}, M_{Ox}, P, and Q as determined by the calculations before these corrections were made; this means simply an addition of a numerical quantity.

Y-Rotation

From Equations [18] and [20] we have

$$\int_O^A K\ (M_{Oy} + Nx - Pz)ds + \int_0^{\frac{\pi}{2}} [M_{Ox} - N\ (L_y + R\sin\psi)]\ Rd\psi = 0 \ldots [20a]$$

and applying this to the various parts of the pipe we find that

$$1.3\int_0^{L_y} M_{Oy}\,dy + 1.806\ M_{Oy}\ R + 0.506\ NR^2 + \int_R^{R+L_x} (M_{Oy} + Nx)dx + KR\int_0^{\frac{\pi}{2}} [M_{Oy} + N(R + L_x + R\sin\psi) - PR(1-\cos\psi)]\,d\psi + \int_R^T (M_{Oy} + NL - Pz)dz = 0 \ldots [20b]$$

The flexibility factor is entered only for the second bend where M_y acts in the plane of the bend.

After integration and transformation Equation [20b] becomes

$$[1.806\ R + 1.571\ KR + L_x + 1.30L_y + Lz]\ M_{Oy} - [0.571\ KR^2 + \tfrac{1}{2}\ T^2 - \tfrac{1}{2}\ R^2]P + [0.506\ R^2 + 2.571\ KR^2 + RL_x + 1.571\ KRL_x + \tfrac{1}{2}\ L_x^2 + LL_x]N = 0 \ldots [26]$$

In a recalculation, the numerical value of a term $+\ 0.150R\ (M_{Ox} - NL_y)$ should be added to allow for the action of M_x on the first bend (case II).

Z-Rotation

From Equations [18] and [21] we derive the equation

$$\int_O^A K\ (M_{Ox} + Py - Ox)\ ds + \int_0^{\frac{\pi}{2}} [M_{Oz} + QR(1-\cos\psi) - NH]\ Rd\psi = 0 \ldots [21a]$$

from which we obtain

$$\int_0^{L_y} (M_{Ox} + Py)dy + KR\int_0^{\frac{\pi}{2}} [M_{Ox} + P\ (L_y + R\sin\psi)]\ d\psi + \int_R^{R+L_x} (M_{Ox} + PH - Qx)dx + 1.806\ [M_{Ox} + PH - Q(R + L_x)]\ R - 1.150\ QR^2 + 1.30\int_R^T (M_{Ox} + PH - QL)dz = 0 \ldots [21b]$$

The factor K appears in the integral for the first bend, where M_z acts in the plane of the bend. The term $-1.150\,QR^2$ is caused by Q in M_z and M_x in accordance with case IV, where the X-rotation corresponds to the Z-rotation in the present case.

Carrying out the integrations and transposing, we obtain for the Y-rotation

$$(1.806\,R + 1.571\,KR + L_x + L_y + 1.30\,L_z)\,M_{Oz} + (KR^2 + 1.571\,KRL_y + 1.806\,RH + \tfrac{1}{2}\,L_y^2 + HL_x + 1.30\,HL_z)P - (2.956\,R^2 + 2.806\,RL_x + \tfrac{1}{2}\,L_x^2 + 1.30\,LL_z)Q = 0 \ldots [27]$$

In a recalculation, a term $+\,0.150R\,(M_{Ox} - NH)$ must be added to the left-hand side in order to allow for the action of M_x on the second bend (case III). This is the only term caused by the second integral in Equation [21a].

X-Displacement

Here again the various terms obtained successively from Equation [22] for all the different parts of the pipe are to be added algebraically in order to obtain the displacement of the A-end. From Equations [18] and [22] we obtain

$$-\int_O^A K(H-y)\,(M_{Oz} + Py - Qx)ds + \int_O^A K(T-z)\,(M_{Oy} + Nx - Pz)\,ds + T\int_0^{\frac{\pi}{2}} [M_{Ox} - N(L_y + R\sin\psi)]\,Rd\psi = EI\,\Delta x_A \ldots [22a]$$

where the last integral represents the displacement due to a minor Y-rotation of the first bend caused by M_x.

The first and second integrals of Equation [22a] run through the whole length of the pipe, but, as explained previously, the bends are treated as independent parts.

In the vertical stem OB, displacements in the X-direction are produced by M_z. At every point, the rotation of an element acts on A with a lever $(H - y)$, giving the term $-\int_0^{L_y} (H-y)\,(M_{Oz} + Py)dy$. At the same time, M_y produces a torsion in OB which at B acts with a lever T relative to the point A, giving a displacement $1.30\,M_{Oy}\,L_y\,T$.

The straight part DE does not produce any X-displacement, but the part FA is bent by M_y and gives a term $+\int_R^T (T-z)\,M_y\,dz$.

The first bend is subjected to M_z and the second bend to M_y acting in their respective planes, and give rise to two terms, obtained by multiplying the corresponding rotation terms in Equations [21b] and [20b], respectively, with $R\,(1 - \sin\psi)$ and $(T - z)$ under the integral signs. Both terms carry the coefficient K. See Figs. 7 and 8.

In the first bend, the Y-rotation $+\,1.806\,M_{Oy}\,R$ given in Equation [20b] acts with a lever T. The minor Y-rotation caused by M_z in the third integral of Equation [22a] acts with the same lever.

From the second and third integrals in Equation [22a] there results for the first bend a term which is $+0.506\,NR^2T$. This expresses the effect of the rotation called $\Delta\varphi_{yA}$ in case IV, given in the third term of Equation [20b]. It acts on A with a leverage T.

Thus Equation [22a] becomes

$$-\int_0^{L_y} (H-y)\,(M_{Oz} + Py)\,dy + 1.30\,M_{Oy}\,L_y\,T + \int_R^T (T-z)\,(M_{Oy} + NL - Pz)dz - KR^2\int_0^{\frac{\pi}{2}} (1-\sin\psi)\,[M_{Oz} + P(L_y + R\sin\psi) - QR\,(1-\cos\psi)]\,d\psi + KR\int_0^{\frac{\pi}{2}} (L_z + R\cos\psi)\,[M_{Oy} + N(R + L_x + R\sin\psi) - PR(1-\cos\psi)]\,d\psi + 1.806\,M_{Oy}\,RT + 0.506\,NR^2\,T + 0.150\,(M_{Ox} - NL_y)\,RT = EI\,\Delta x_A \ldots [22b]$$

Carrying out the integrations and rearranging the terms, we obtain for the X-displacement

$$0.150\,RTM_{Ox} + (1.30\,L_yT + 1.806\,RT + 1.571\,KL_zR + KR^2 + \tfrac{1}{2}\,L_z^2)\,M_{Oy} - (HL_y + 0.571\,KR^2 - \tfrac{1}{2}\,L_y^2)\,M_{Oz} - (\tfrac{1}{2}\,HL_y^2 + 0.571\,KL_yR^2 + 0.430\,KR^3 + \tfrac{1}{6}\,T^3 + \tfrac{1}{3}\,R^3 + 0.571\,KL_zR^2 - \tfrac{1}{3}\,L_y^3 - \tfrac{1}{2}\,TR^2)P + (0.506\,R^2T + 2.571\,KL_zR^2 + 1.50\,KR^3 + KL_xR^2 + 1.571\,KL_xL_zR + \tfrac{1}{2}\,LL_z^2 - 0.150\,RT)\,N = EI\,\Delta x_A \ldots [28]$$

Y-Displacement

From Equations [18] and [23] we can derive

$$\int_O^A K(L-x)\,(M_{Oz} + Py - Qx)ds - \int_O^A K(T-z)\,(M_{Ox} + Qz - Ny)ds - T\int_0^{\frac{\pi}{2}} [M_{Oy} + NR\,(1-\cos\psi)]\,Rd\psi - L_z\int_0^{\frac{\pi}{2}} [M_{Oz} + PH - Q(R + L_x + R\sin\psi)]Rd\psi = EI\,\Delta y_A \ldots [23a]$$

The first and second integrals of Equation [23a] run through the whole length of the pipe. The third and fourth integrals provide for M_y in the first bend and M_z in the second bend, respectively, which couples act normal to the plane of the bends in both cases.

The displacements caused by the deflections of the straight parts are obtained by applying the proper leverages to the respective X- and Z-rotations as given in Equations [19b] and [21b].

The first bend is subject to Z-rotation in its own plane, acting with a leverage $[L - R(1 - \cos\psi)]$, and causing a Y-displacement at A.

The permanent couples $(M_{Oz} - NL_y)$ in the first bend and $(M_{Oz} - NH)$ in the second bend, which come from the second integral in Equation [23a], cause major rotations in the bends at D and F, respectively, and, as explained in cases II and III, carry the factor 1.806. They act on A with a leverage T and L_z, respectively, causing a negative Y-displacement.

The permanent couples also produce local displacements in the second bend at F. The couple $(M_{Oz} - NH)$, which corresponds to M_y in case III, causes a local Y-displacement of $-1.150\,(M_{Oz} - NH)R^2$, and the couple $(M_{Oz} + PH)$, which corresponds to M_x in case II, causes a local displacement at F of $+0.506\,(M_{Oz} + PH)R^2$.

In the third integral of Equation [23a], M_y causes a minor X-rotation of the first bend, giving a negative term $-0.150RTM_{Oy}$ In the fourth integral, M_z causes a minor X-rotation of the second bend, giving a negative term $-0.150\,RLz\,[M_{Oz} + PH - Q(R + L_x)]$.

The second and third integrals of Equation [23a] give together an X-rotation of the first bend due to the force N acting at B. According to case IV, this rotation is $-1.150\,NR^2$, as given in Equation [19b], and acting at A with a lever T, it gives an upward Y-displacement equal to $+1.150\,NR^2T$.

Similarly the second and fourth integrals of Equation [23a] give together an X-rotation of the second bend at F due to Q acting at E. According to case IV, and as given in Equation [19b], this rotation is equal to $+0.506\,QR^2$, which gives a downward displacement at A equal to $-0.506\,QR^2L_z$. These two integrals also, according to case IV, due to the action of Q, cause a local Y-displacement at F equal to $-0.408\,QR^3$.

We thus arrive at the following equation for the Y-displacement

$$L\int_0^{L_y}(M_{Oz}+Py)dy - T\int_0^{L_y}(M_{Oz}-Ny)dy - 1.30\,L_xT(M_{Oz} - NH) - \int_R^T(T-z)\,(M_{Oz}+Qz-NH)\,dz + \int_R^{R+L_x}(L-x)\,(M_{Oz} + PH - Qx)dx + KR\int_0^{\pi/2}[L-R\,(1-\cos\psi)]\,[M_{Oz} + P(L_y+R\sin\psi) - QR(1-\cos\psi)]d\psi - 1.806\,(M_{Oz} - NL_y)RT - 1.806\,(M_{Oz}-NH)RL_z - 1.150\,(M_{Oz} - NH)R^2 + 0.506\,(M_{Oz}+PH)R^2 + 1.150\,NR^2T - 0.506\,QR^2L_z - 0.408\,QR^3 - 0.150\,RL_z\,[M_{Oz}+PH - Q(R+L_x)] - 0.150\,RTM_{Oy} = EI\,\Delta y_A \quad [23b]$$

After integration and transformation we obtain

$$-(TL_y + 3.612\,L_xR + 2.956\,R^2 + 1.30\,TL_x + \tfrac{1}{2}\,L_x^2)M_{Oz} - 0.150\,RTM_{Oy} + (LL_y + \tfrac{1}{2}\,LL_x + 1.571\,KRL + 0.506\,R^2 - 0.571\,KR^2 - 0.150\,RL_x)\,M_{Oz} + [\tfrac{1}{2}\,LL_y^2 + \tfrac{1}{2}\,HL_x^2 + HLR_x + 1.571\,KRLL_y + KLR^2 - 0.150\,HRL_x - \tfrac{1}{2}\,(K-1)HR^2]P - (\tfrac{1}{6}\,L_x^3 + 0.850\,R^2L_x + 0.850\,RL_x^2 + 0.571\,KLR^2 + 0.408\,R^3 + \tfrac{1}{6}\,L_x^3 + 0.506\,R^2L_x + \tfrac{1}{2}\,RL_x^2 - 0.355\,KR^3)Q + [\tfrac{1}{2}\,TL_y^2 + 1.806\,TRL_y + 1.150\,R^2\,(T+H) + 1.806\,RHL_x + 1.30\,THL_x + \tfrac{1}{2}\,HL_x^2]N = EI\,\Delta y_A \quad [29]$$

Z-Displacement

From Equations [18] and [24] we can derive

$$\int_O^1(H-y)\,(M_{Oz}+Qz-Ny)ds - \int_O^A K(L-x)\,(M_{Oy}+Nx - Pz)ds - (L_z+R)\int_B^D[M_{Oz}-N(L_y+R\sin\psi)]\,Rd\psi = EI\,\Delta z_A \quad [24a]$$

The straight parts OB and DE give rise to the first three terms in Equation [24b], and do not require any explanation.

The couple M_y, acting in the plane of the second bend with a leverage equal to $R\,(1-\sin\psi)$, carries the factor K. No other Z-displacements are caused by the second bend as can be seen from Fig. 8.

The first bend is acted upon by a permanent couple $(M_{Oz} - NL_y)$ and a varying couple $-NR\sin\psi$. It is also acted upon by a permanent couple M_{Oy} and a varying couple $NR\,(1-\cos\psi)$. The first permanent couple produces a local displacement at D, which according to case II is equal to $+0.506\,(M_{Oz} - NL_y)R^2$. It also produces a Y-rotation acting with a leverage $(L_z+R) = (L-R)$ as given in the third integral of Equation [24a], giving a term $-0.150\,(M_{Oz} - NL_y)\,(L-R)R$. The second permanent couple M_{Oy} gives according to case III a local displacement at D equal to $-1.150M_{Oy}R^2$, and a Y-rotation with leverage $(L_z+R) = (L-R)$, giving a term $-1.806\,RM_{Oy}\,(L-R)$.

Finally the two varying couples, containing N, give, according to case IV, a local displacement at D equal to $-0.408NR^3$, and since they cause a Y-rotation equal to $+0.506\,NR^2$, they give moreover a negative displacement at A equal to $-0.506\,NR^2\,(L-R)$.

Thus Equation [24a] becomes

$$\int_0^{L_y}(H-y)\,(M_{Oz}-Ny)dy - 1.3\,M_{Oy}\,L_yL - \int_R^{R+L_x}(L-x)\,(M_{Oy}+Nx)dx - KR^2\int_0^{\pi/2}(1-\sin\psi)\,[M_{Oy} + N(R+L_x+R\sin\psi) - PR(1-\cos\psi)]\,d\psi + 0.506\,(M_{Oz} - NL_y)R^2 - 0.150\,(M_{Oz}-NL_y)\,(L-R)R - 1.150\,M_{Oy}R^2 - 1.806\,M_{Oy}R(L-R) - 0.408\,NR^3 - 0.506\,NR^2\,(L-R) = EI\,\Delta z_A \quad [24b]$$

After integration and transformation Equation [24b] becomes

$$(HL_y + 0.356\,R^2 - \tfrac{1}{2}\,L_y^2 - 0.150\,L_xR)M_{Oz} - (1.30\,LL_y + 2.956\,R^2 + 2.806\,RL_x + \tfrac{1}{2}\,L_x^2 + 0.571\,KR^2)M_{Oy} - (\tfrac{1}{2}\,HL_y^2 + 0.914\,R^3 + \tfrac{1}{2}\,LL_x^2 + 1.506\,R^2L_x + 0.785\,KR^3 + 0.571\,KR^2L_x + 0.656\,L_yR^2 - 1/3\,L_x^3 - 1/3\,L_y^3 - 0.150\,LL_yR)N = EI\,\Delta z_A \quad [30]$$

Solution of the Equations

The fundamental equations can be written in the following forms

$$A_1M_{Oz} + A_5Q + A_6N = 0 \quad [31]$$

$$B_2M_{Oy} + B_4P + B_6N = 0 \quad [32]$$

$$C_3M_{Ox} + C_4P + C_5Q = 0 \quad [33]$$

$$D_1M_{Oz} + D_2M_{Oy} + D_3M_{Ox} + D_4P + D_6N = EI\,\Delta x_A \quad [34]$$

$$E_1M_{Oz} + E_2M_{Oy} + E_3M_{Ox} + E_4P + E_5Q + E_6N = EI\,\Delta y_A \quad [35]$$

$$F_1M_{Oz} + F_2M_{Oy} + F_6N = EI\,\Delta z_A \quad [36]$$

where the composition of the coefficients A to F with suffixes can be obtained from Equations [25] to [30], inclusive. For a given lay-out of the pipe line, all the quantities that enter into the coefficients are known and their numerical values can be computed.

The solution for the unknown forces P, Q, and N, and for the unknown couples M_{Oz}, M_{Oy}, and M_{Ox} is best obtained by first eliminating the couples. Determine the value of each of the couples from Equations [31], [32], and [33], and substitute the values of the couples in Equations [34], [35], and [36]. This will result in three equations of the form given in the following Equations [37].

$$\left.\begin{aligned}\alpha_1P + \beta_1Q + \gamma_1N &= EI\,\Delta x_A\\ \alpha_2P + \beta_2Q + \gamma_2N &= EI\,\Delta y_A\\ \alpha_3P + \beta_3Q + \gamma_3N &= EI\,\Delta z_A\end{aligned}\right\} \quad [37]$$

from which the unknown forces, P, Q, and N, can be determined.

If these values of the forces so obtained are substituted in Equations [31], [32], and [33], the couples acting at O can be found.

We are now able to determine the couples acting at any point (x, y, or z) of the pipe by substituting the values of the component couples and forces acting at O in Equations [18]. Thus, the couples at the A-end can be found by putting x, y, and z in Equations [18] equal to L, H, and T, respectively. When reversing their sign we have the reaction couples

$$\left.\begin{aligned} M_{Ax} &= -(M_{Ox} + QT - NH) \\ M_{Ay} &= -(M_{Oy} + NL - PT) \\ M_{Az} &= -(M_{Oz} + PH - QL) \end{aligned}\right\} \quad [38]$$

It is necessary to examine separately the conditions in the bends in order to determine the maximum stresses in these parts. At any point of a bend, one of the three component couples M_x, M_y, M_z acts in the plane of the bend, and stresses caused thereby require the use of the flexibility factor K for their determination. The two other component couples are resolved into one couple M_b with axis in the radius of the bend, causing bending of the pipe. Another couple M_t, with axis tangent to the pipe at the point, constitutes a twisting couple. By means of these couples the bending and torsional stresses can be determined since the corresponding moments of inertia of the section, I and I_o, are readily found.

In the first bend where M_z is acting in the plane of the bend, Fig. 6, the expression for the couple is

$$M_z = M_{Oz} + P\,(L_y + R\sin\psi) - QR(1 - \cos\psi) \quad [39]$$

Differentiating with respect to ψ, this couple is found to be a maximum when

$$\tan\psi_{max} = P/Q \quad [40]$$

Since P and Q are positive, M_z becomes a maximum somewhere within the bend. The maximum value of this couple can be found by substituting from Equation [40] into Equation [39], whence

$$M_{z\,max} = M_{Oz} + PL_y + R\left[\sqrt{(P^2 + Q^2)} - Q\right] \quad [39a]$$

The maximum M_z, however, depending on the sign of M_{Oz}, may be smaller numerically than the couples M_{Bz} and M_{Dz} acting at the points B and D, respectively. The value of M_{Bz} at B where $\psi = 0$ is

$$M_{Bz} = M_{Oz} + PL_y$$

while at D, where $\psi = \pi/2$

$$M_{Dz} = M_{Oz} + PL_y + R(P - Q)$$

It is necessary in all cases to ascertain the value of $M_{z\,max}$. The bending couple normal to the plane of the bend is

$$M_b = M_x\cos\psi - M_y\sin\psi = [M_{Ox} - N(L_y + R\sin\psi)]\cos\psi - [M_{Oy} + NR\,(1 - \cos\psi)]\sin\psi$$

Therefore

$$M_b = (M_{Ox} - NL_y)\cos\psi - (M_{Oy} + NR)\sin\psi \quad [41]$$

The maximum value of M_b occurs when

$$\tan\psi_{b\,max} = -\frac{M_{Oy} + NR}{M_{Ox} - NL_y}$$

The twisting couple is

$$M_t = M_x\sin\psi + M_y\cos\psi = (M_{Ox} - NL_y)\sin\psi_{max} + (M_{Oy} + NR)\cos\psi_{max} - NR \quad [42]$$

the maximum value of which occurs when

$$\tan\psi_{b\,max} = \frac{M_{Ox} - NL_y}{M_{Oy} + NR} = -\frac{1}{\tan\psi_b}$$

showing that M_t has its maximum in a plane at right angles to M_b. Hence, one of these maxima falls outside the quadrant.

The forces P, Q, and N, acting at any section, are best resolved in directions normal and tangential to the transverse section. They produce direct compression and shear, where

$$F_N = P\sin\psi + Q\cos\psi$$

$$F_s = \sqrt{(P\cos\psi - Q\sin\psi)^2 + N^2}$$

Again it is necessary to determine the maximum values, which for F_N occurs at the same section as that of M_z, while F_s has its greatest value at B and D.

Similar equations can be established for the second bend, where M_y acts in the plane of the bend, and M_x and M_z act at right angles to it.

4—THE STRESSES

Stresses at the Terminal Sections O and A

The greatest stresses are found in the terminal sections and in the bends. The longitudinal bending stress p_{1b} at the O-end of the pipe is generally the most important. This stress is caused by the resultant couple of M_{Ox} and M_{Oz}, here denoted by M_O, where

$$M_O = \sqrt{(M_{Ox}^2 + M_{Oz}^2)} \quad [45]$$

Since there is no flattening effect, the flexibility factor K is equal to unity, and the corresponding maximum stress is

$$p_{1b} = M_O r_o/I \quad [46]$$

where r_o is the radius to the outer surface of the pipe.

The steam pressure P produces a longitudinal stress, which is assumed to be uniformly distributed over the section of the pipe wall. This stress is approximately

$$p_{1s} = Pr_i/2t \quad [47]$$

where r_i is the internal radius of the pipe. This equation is given on page 336 of the author's paper "Tests on High-Pressure Pipe Bends."[5]

Finally, there is a uniform longitudinal stress due to the compressive reaction Q which can be expressed as

$$p_{1c} = Q/A \quad [48]$$

but which is generally small and can be neglected in many cases.

Thus, the total longitudinal stress becomes

$$p_1 = p_{1b} + p_{1s} + p_{1c} \quad [49]$$

which has its maximum value at the outer surface in the plane of M_O.

The steam pressure also produces a transverse or "hoop" stress which is assumed to be uniformly distributed over the section of the pipe wall. This stress can be expressed as

$$p_{2s} = Pr_i/t \quad [50]$$

Where the pipe is straight, as it is at O, no other transverse

stresses exist, so that, denoting the total transverse stress by p_2, we have $p_{2s} = p_2$ in this case.

The torsional couple M_{Oy} produces a shearing stress which has its maximum value in the outer surface. It acts in the plane of the section and is directed everywhere normal to the radius. This stress

$$q_t = M_{Oy} r_o / 2I \quad \ldots\ldots [51]$$

A shearing stress q_f is also produced by the resultant of the forces P and N where

$$q_f = \frac{\sqrt{(P^2 + N^2)}}{A} \quad \ldots\ldots [52]$$

This stress acts in the same plane as q_t and is added directly to it at the point where the two stresses act in the same direction. It is generally small and can be neglected in many cases. The sum of the shearing stresses produced by the torsional couple M_{Oy} and the resultant of the forces P and N is

$$q_t + q_f = q$$

The stresses at the A-end of the pipe are determined in the same manner.

In addition to the stresses here enumerated, bending and shearing stresses are produced by the deadweight of the piping between hangers and supports. These stresses are to be included in the p_1-stresses and the q-stresses of Equations [49], [51], and [52]. Their values depend on the arrangement of the points of support and are determined by usual methods. With proper design these stresses are generally small and in order not to lengthen the paper they are omitted in the present analysis.

Stresses in the Bends

The curved parts of the pipe require special treatment. The component couple acting in the plane of a bend produces a flattening of the section, causing a peculiar distribution of the longitudinal stresses and the introduction of relatively high transverse stresses. The component couples normal to the plane of a bend produce twisting as well as bending, as previously described in cases I to IV.

In the first bend, that is, the bend nearest the origin, the couple M_s acts in the plane of the bend and has its maximum value as given by Equation [39a] in the section determined by Equation [40]. The stress in the middle surface of the section so determined is generally a maximum, not at the top and bottom of the section, but at points the radius of which forms an angle θ with the neutral axis of the section as shown in Fig. 1. As explained on page 328 of the author's paper "Tests on High-Pressure Pipe Bends,"[5] the maximum value is found by applying a factor β to the ordinary expression for the longitudinal stress, thus

$$p_{1b\max} = \beta(M_{s\max} r/I) \quad \ldots\ldots [53]$$

where

$$\left.\begin{aligned} \beta &= 2/3\ K \sqrt{\ } [(6\lambda^2 + 5)/18] \quad \text{when } \lambda < 1.472 \\ \beta &= (12\lambda^2 - 2)/(12\lambda^2 + 1) \quad \text{when } \lambda \gtrless 1.472 \end{aligned}\right\} \ldots [54]$$

The value of λ is given in Equation [2], while the value of K is given in Equation [1a].

Usually $\lambda < 1.472$ and the maximum stress occurs at an angle θ from the neutral axis, such that

$$\sin\theta = \sqrt{[(6\lambda^2 + 5)/18]} \quad \ldots\ldots [55]$$

and hence

$$\beta = 2/3\ K \sin\theta \quad \ldots\ldots [54a]$$

When $\lambda \gtrless 1.472$, the maximum value of p_{1b} occurs at the top and bottom of the section.

The value of β varies little from unity except in thin-walled bends of sharp curvature, but θ may be far from a right angle.

The longitudinal bending stresses due to M_b, given by Equation [41], which act in a plane normal to that of the bend, have their maxima at the points which are in the neutral axis relative to M_s, and since $p_{1b\max}$ due to $M_{s\max}$ rarely falls at points less than 45 deg from the neutral axis, these maxima can be considered separately.

The steam pressure produces a longitudinal stress as given by Equation [47], while the reactions P and Q cause a normal stress on the transverse section

$$p_{1c} = (P \sin\psi + Q \cos\psi)/A \quad \ldots\ldots [48a]$$

The transverse stress due to the steam pressure is p_{2s} as given by Equation [50], but in addition there is a transverse bending stress p_{2b} due to the flattening of the section with a maximum at $\theta = 0$ deg, and $\theta = 90$ deg, as explained on page 340 of the author's paper "Tests on High-Pressure Pipe Bends."[5] This latter stress is

$$p_{2b} = \frac{18\ M_s \lambda r}{I(12\lambda^2 + 1)} \quad \ldots\ldots [56]$$

In the outer surface this is a tensile stress, while at the inner surface at the neutral axis for M_s it is a compressive stress, and is reversed in sign at the top and bottom of the section. Compared with the longitudinal stresses in the bend as given by Equation [53], the transverse stresses are very high, and bear a definite ratio to them. If μ be that ratio, then

$$\mu = \frac{p_{2b}}{p_{1b\max}} = \frac{81\ \lambda}{(6\lambda^2 + 5)^{3/2}\sqrt{2}} \quad \ldots\ldots [57]$$

For ordinary values of λ, between 0.5 and 1.2, this ratio varies from 1.80 to 1.36, so that the transverse bending stress is on the average 60 per cent greater than the longitudinal bending stress.

There is also a smaller compressive stress in the neutral axis due to the flattening of the section which reduces the tensile stress at the outer surface. It is expressed by

$$p_{2c} = \frac{2Mr^2}{RI} \frac{(6\lambda^2 + 1)}{(12\lambda^2 + 1)} \quad \ldots\ldots [58]$$

This equation is the same as Equation [25] on page 83 of the author's paper "The Elastic Deformation of Pipe Bends."[2]

Combining these stresses, the total transverse stress, which will generally be a maximum at the outer surface in the neutral axis, is

$$p_2 = p_{2b} + p_{2s} + p_{2c} \quad \ldots\ldots [59]$$

The twisting moment M_s, given by Equation [42], causes a stress, which at the outer surface is given by Equation [51]. The shearing force F_s causes a small shearing stress which generally can be neglected.

The stresses in the second bend are analyzed in the same manner.

Strength Criteria and Working Stresses

Strength criteria and working stresses are among the most important and the most difficult of the pipe-line problems. The difficulty is enhanced by the recent increase in steam pressures and temperatures and by the use of new alloys of steel, with which experience is still very limited. The higher steam pressures produce hoop stresses which can no longer be neglected. At high temperatures the modulus of elasticity and the stress at the yield point are reduced considerably and this, as well as the peculiar nature of each particular alloy, must be taken into account in determining the working stress.

All the stresses so far discussed act in a plane tangential to the cylindrical surface through the point under consideration, and, in combining the stresses, we consider a small element in that plane. If there is no load on the element normal to its plane, we have the first order of a simple case of plane stresses. Referring to Fig. 9, p_1 acts in the X-direction, p_2 in the Y-direction, and the shearing stress q acts on the sides of the element.

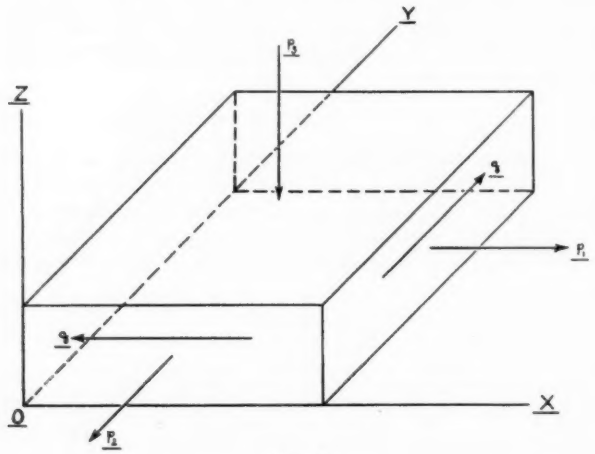


FIG. 9

If we take into account the radial stress p_3, produced by the steam pressure P, we have three-dimensional stress. Actually p_3 is small, being equal to P at the inner surface and decreasing to zero at the outer surface of the pipe wall, but since it has been taken into account by some investigators, it will be included in the following discussion.

For a state of combined stress several strength criteria have been proposed, but we shall only consider the maximum-shear theory and the strain-energy theory,[8] of which the latter appears to be the most rational. Either theory leads to a certain function of the component stresses, so determined that when it is equal in magnitude to the yield point of the material, plastic flow is supposed to occur. This function is referred to as the equivalent stress, which will be noted by p_{eq}. It may or may not represent an actually existing stress, but it is clear that under working conditions the equivalent stress should be smaller than the stress at the yield point of the material.

The Maximum-Shear Theory. According to this theory plastic flow of the material begins to take place when the maximum shearing stress reaches a certain magnitude, regardless of the state of stress. Hence, this stress, must have the same magnitude in the most general case as in the simple case when a straight bar, subject to uniaxial stress, reaches the yield point.

In a state of plane stress, that is, when $p_3 = 0$, or when p_3 is very small, the maximum shearing stress, expressed in terms of the ordinary component stresses, is in the general case

$$q_{max} = \tfrac{1}{2}\sqrt{[4q^2 + (p_1 - p_2)^2]} \quad [60]$$

In a straight bar subject to a simple tension, $q = 0$, $p_2 = 0$, and therefore $q_{max} = p_1/2$.

When p_1 reaches the yield point p_0,

$$q_{max} = p_0/2 \quad [60a]$$

Equating the expressions for q_{max} as given in Equations [60] and [60a], we obtain

$$p_0 = \sqrt{[4q^2 + (p_1 - p_2)^2]} \quad [61]$$

The expression on the right-hand side of Equation [61] is the equivalent stress, and is expressed as

$$p_{eq} = \sqrt{[4q^2 + (p_1 - p_2)^2]} \quad [62]$$

which, if adopted as the working stress, should then be smaller than p_0.

If we take p_3 into account, we have three-dimensional stress and it is known that in such a case the maximum shearing stress is equal to one-half the difference between the greatest and the smallest of the three principal stress, denoted here by σ_1, σ_2, and σ_3. Let $\sigma_1 > \sigma_2 > \sigma_3$, then

$$q_{max} = (\sigma_1 - \sigma_3)/2 \quad [63]$$

Since there are no shearing stresses parallel with the plane of the element, p_3 is a principal stress, and therefore $p_3 = \sigma_3$. The two other principal stresses are found from the formula

$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \tfrac{1}{2}(p_1 + p_2) \pm \tfrac{1}{2}\sqrt{[4q^2 + (p_1 - p_2)^2]} \quad [64]$$

The value of q_{max} from Equation [63] is now equated to its value expressed by Equation [60a] in order to obtain for the equivalent stress

$$p_{eq} = \sigma_1 - \sigma_3 = \tfrac{1}{2}(p_1 + p_2) + \tfrac{1}{2}\sqrt{[4q^2 + (p_1 - p_2)^2]} - p_3 \quad [65]$$

This value of the equivalent stress must be smaller than p_0. At the outer surface of the pipe $\sigma_3 = p_3 = 0$, and $p_{eq} = \sigma_1$ which may be greater than p_{eq} as given by Equation [62] for plane stress.

The Strain-Energy Theory. According to this theory, plastic flow occurs when the energy of pure deformation reaches a certain limit, excluding the energy due to change of volume, regardless of the state of stress. Where plane stress occurs, assuming $p_3 = 0$, the energy of deformation per unit volume can be expressed as

$$w = (1/12G)[(p_1 - p_2)^2 + p_1^2 + p_2^2] + (1/2G)q^2$$

where p_1, p_2, and q are the total longitudinal, transverse, and shearing stresses, respectively, and G is the modulus of rigidity. Since w must be equal to the energy at the plastic limit under all conditions, it must also be equal to the energy in a bar under simple tension when the yield point is reached. Under these conditions $p_2 = 0$, $q = 0$, and $p_1 = p_0$. Therefore

$$w = p_0^2/6G$$

Equating these expressions for w, the yield point is found to be reached when

$$p_0 = \sqrt{(p_1^2 + p_2^2 - p_1p_2 + 3q^2)}$$

Thus the equivalent stress in this case is

$$p_{eq} = \sqrt{(p_1^2 + p_2^2 - p_1p_2 + 3q^2)} \quad [66]$$

which, in order to avoid plastic flow, must be smaller than p_0.

If we can disregard q as being insignificant, p_1 and p_2 become principal stresses and Equation [66] takes the form

$$p_{eq} = \sqrt{(p_1^2 + p_2^2 - p_1p_2)} \quad [66a]$$

If the transverse stress p_2 can also be disregarded, we have the case of a simple longitudinal stress, where $p_{eq} = p_1$, and therefore $p_1 < p_0$.

In three-dimensional stress, plastic flow will occur according to the strain-energy theory when

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2p_0^2$$

[8] "Zur Theorie plastiche Deformation und der hierdurch im Material hervorgerufenen Nach spannungen," by H. Hencky, *Zeitschrift fur angewandte Mathematik und Mechanik*, vol. 4, no. 4, 1924, pp. 323–334.

Hence

$$p_{eq} = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3)} \quad [67]$$

which should be made smaller than p_0. Here $\sigma_3 = p_3$, while σ_1 and σ_2 can be found from Equation [64].

If σ_3 is neglected

$$p_{eq} = \sqrt{(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)}$$

which is identical with Equation [66*a*], as it should be, since in that equation $p_1 = \sigma_1$, and $p_2 = \sigma_2$.

The Working Stress. The yield point of the material will be used in this paper as the standard of reference, and it is suggested that under working conditions the equivalent stress determined by the strain-energy method as given by Equations [66] or [67] should be not greater than about one-half of the stress at the yield point, that is

$$p_{eq} \leqq \tfrac{1}{2}\, p_0 \quad [68]$$

In the author's paper "Tests on High-Pressure Pipe Bends," page 344, it was recommended, on the basis of a number of full-scale tests with pipe bends, to use the longitudinal stress at the middle surface of the pipe wall as strength criterion in the curved parts. It was suggested also that this stress, p_1 in Equation [49] should not be allowed to exceed 16,000 lb per sq in. This means that p_{1b}, the stress due to bending alone, should not be greater than $16{,}000 - (p_{1s} - p_{1c})$, but as previously shown, the transverse bending stresses due to flattening of the bends is much higher than $p_1 b$ and might thus become excessive. It was argued, however, that these high stresses were strictly a local phenomenon; they occurred only within very limited regions of the inner and outer surfaces of the pipe wall, and did not seem to affect the general behavior of the pipe. Although that rule led to results which were in accordance with good engineering practice, it appears advisable under more modern conditions of pressure and temperature in all cases to use the equivalent stress as the strength criterion, as recommended previously. If this is done, the transverse stresses will be taken into account, not only in the straight parts of the pipe, but also in the bends. Possibly a somewhat higher working stress may be allowed in the bends than in the straight parts.

These rules are believed to be conservative, but in view of recent rapid developments they must be regarded as suggestions. Before an accurate and fully reliable basis of design can be established, covering all modern conditions, there is required further practical experience over extended periods, and correlated with a theoretical analysis of the stresses. In any case it seems justifiable, in determining the working stress, to take advantage of the higher yield point in alloy steels, while at the same time due account must be taken of the reduction in the stress at the yield point and in the modulus of elasticity at higher temperatures.

5—NUMERICAL EXAMPLE

The numerical analysis for a three-dimensional three-arm pipe, in which the arms are at right angles to each other and connected by right-angle bends, as shown in Fig. 6, is here given to illustrate the application of the formulas presented in this paper. Both ends of the pipe are fixed in position and direction, and when steam is admitted, expansion takes place, not only of the pipe itself but also of certain parts of the machinery to which the ends of the pipe are attached.

The pipe is made of alloy steel, has a modulus of elasticity of 24×10^6 lb per sq in. at 850 F, and a yield point of approximately 39,000 lb per sq in. at 800 F. The temperature rise in the pipe is assumed to be 850 F, at which the expansion of the pipe per inch of length $\delta = 0.006083$ in. Other data are: Working pressure $P =$ 400 lb per sq in.; outer radius of pipe $r_o = 3.644$ in.; inner radius of pipe $r_i = 3.403$ in.; radius of middle surface $r = 3.523$ in.; thickness of pipe wall $2h = t = 0.241$ in.; sectional area of pipe wall $A = 5.33$ sq in.; moment of inertia of A about a diameter $I = 33.14$ in.^{4}; and the radius of pipe bend $R = 36.3$ in. From these data it is found that

$$\lambda = tR/r^2 = 0.705$$

and that

$$K = (12\lambda^2 + 10)/(12\lambda^2 + 1) = 2.29$$

From the dimensions given in Fig. 6, it is observed that $H = 145.2$ in.; $L_y = 108.9$ in.; $L_x = 35.7$ in.; $L = 108.3$ in.; $L_z = 41.0$ in.; and $T = 77.3$ in.

The dimensions for expansion are: $H_e = 156.1$ in.; $L_e = 96.8$ in.; and $T_e = 82.2$ in. Further, $EI\,\Delta x_A = 468.4$; $EI\,\Delta y_A = 755.3$; and $EI\,\Delta z_A = 397.8$.

The coefficients A to F in Equations [31] to [36] are computed from the expressions in Equations [25] to [30].

A sample calculation of the A-coefficients in Equation [25] gives

$3.612\,R = 131.2$	$\tfrac{1}{2}L_y^2 = 5930$
$1.30\,L_x = 46.4$	$1.806\,(L_y + H)R = 16{,}670$
$L_y = 108.9$	$1.150\,R^2 = 1515$
$L_z = 41.0$	$1.30\,L_x H = 6740$
	$L_e H = 5955$

$A_1 = 327.5 \quad A_3 = \tfrac{1}{2}T^2 = 2988 \quad A_4 = -36{,}810$

The other coefficients are found in the same manner from Equations [26] to [30], inclusive, finally furnishing equations, for this illustration, in which the numerical values of the coefficients replace the algebraic coefficients. The fundamental Equations [31] to [36], inclusive, for this numerical example can now be written with numerical coefficients, which are

$$327.5\,M_{Ox} + 2988\,Q - 36{,}810\,N = 0 \quad [31]$$

$$427.5\,M_{Oy} - 4223\,P + 20{,}700\,N = 0 \quad [32]$$

$$407.1\,M_{Oz} + 47{,}330\,P - 13{,}940\,Q = 0 \quad [33]$$

$$421.0\,M_{Ox} + 26{,}060\,M_{Oy} - 11{,}780\,M_{Oz} - 808{,}000\,P + 956{,}300\,N = 468.4 \times 10^6 \quad [34]$$

$$-22{,}110\,M_{Ox} - 421.6\,M_{Oy} + 27{,}820\,M_{Oz} + 2{,}798{,}000\,P - 336{,}500\,Q + 2{,}382{,}000\,N = 755.2 \times 10^6 \quad [35]$$

$$10{,}150\,M_{Ox} - 25{,}390\,M_{Oy} - 791{,}000\,N = 397.8 \times 10^6 \quad [36]$$

From Equations [31], [32], and [33], the couples can be expressed in terms of the forces, and substituting these values in Equations [34], [35], and [36], we obtain Equations [37] with numerical coefficients, which are

$$\left.\begin{aligned} 817.7\,P - 407.3\,Q - 257.7\,N &= 468.4 \times 10^3 \\ -439.1\,P + 817.8\,Q - 83.1\,N &= 755.3 \times 10^3 \\ -2507\,P - 92.6\,Q + 157.9\,N &= 397.8 \times 10^3 \end{aligned}\right\} \quad [37]$$

from which the reaction forces $P = 1729$ lb, $Q = 1916$ lb, and $N = 640$ lb, are obtained.

Substituting these values of P, Q, and N in Equations [31], [32], and [33] and solving for the couples, we obtain $M_{Ox} = +54{,}410$ in-lb, $M_{Oy} = -13{,}800$ in-lb, and $M_{Oz} = -135{,}400$ in-lb.

The reaction couples at A as determined from Equation [38] are

$$M_{Ax} = -(M_{Ox} + QT - NH) = -109{,}590 \text{ in-lb}$$

$$M_{Ay} = -(M_{Oy} + NL - PT) = +78{,}280 \text{ in-lb}$$

$$M_{Az} = -(M_{Oz} + PH - QL) = +91{,}900 \text{ in-lb}$$

A corrective calculation was carried out by substituting the values of the forces and couples at O just determined in the expressions for the minor terms omitted in the rotation equations and adding the numerical quantities so found as constants on the left-hand side of these equations. Equations [34], [35], and [36] for the displacements were left unaltered, the minor terms having been included already. The solution of the six equations could now be obtained and were found to give values for the reaction forces and couples which differed from those previously obtained by less than 1 per cent, except that M_{Oz} was —134,670 in-lb, instead of —135,400 in-lb, i.e., about 2 per cent smaller.

Another calculation was made by which the minor terms were omitted in all six fundamental Equations [31] to [36], inclusive, and again the results were practically the same. However, it is recommended to follow the method applied above, by which the minor terms are omitted in the rotation equations but included in the displacement equations.

Determination of Stresses at the O-End

From Equation [45]

$$M_O = \sqrt{(M_{Ox}^2 + M_{Oz}^2)} = \sqrt{(54{,}410^2 + 135{,}400^2)} = 145{,}900 \text{ in-lb}$$

The longitudinal stress at the outer surface due to bending is obtained from Equation [46]

$$p_{1b} = (M_O r_o/I) = (145{,}900 \times 3.644)/33.14 = \pm 16{,}040 \text{ lb per sq in.}$$

The longitudinal stress due to the steam pressure, from Equation [47], is

$p_{1s} = Pr_i/2t = (400 \times 3.403)/(2 \times 0.241) = +2825$ lb per sq in.

From Equation [48] is obtained the longitudinal stress due to end reaction, which is

$$p_{1c} = Q/A = -1916/5.33 = -359 \text{ lb per sq in.}$$

Hence, the total maximum longitudinal stress at the outer surface, found by applying Equation [49], is

$$p_1 = p_{1b} + p_{1s} + p_{1c} = 16{,}000 + 2825 - 359 = 18{,}500 \text{ lb per sq in.}$$

The hoop stress due to steam pressure, from Equation [51], is

$$p_{2s} = p_2 = Pr_i/t = 400 \times 3.403/0.241 = 5650 \text{ lb per sq in.}$$

The shearing stress due to twisting moment, from Equation [51] is

$$q_t = \frac{M_{Oy} r_o}{2I} = \frac{13{,}880 \times 3.644}{2 \times 33.14} = 763 \text{ lb per sq in.}$$

According to Equation [52], the shearing stress due to the reaction forces P and N are

$$q_f = \frac{\sqrt{(P^2 + N^2)}}{A} = \frac{\sqrt{(1729^2 + 640^2)}}{5.33} = 346 \text{ lb per sq in.}$$

Hence,

$$q = q_t + q_f = 763 + 346 = 1109 \text{ lb per sq in.}$$

The equivalent stress, according to the strain-energy theory is then found by applying Equation [66], where

$$p_{eq} = \sqrt{(p_1^2 + p_2^2 - p_1 p_2 + 3q^2)} = \sqrt{[18{,}500^2 + 5650^2 - (18{,}500 \times 5650) + (3 \times 1109)^2]} = 16{,}530 \text{ lb per sq in.}$$

According to the maximum-shear theory, Equation [62], the equivalent stress is $p_{eq} = 13{,}040$ lb per sq in.

Stresses at the A-End

The resultant couple of M_{Ax} and M_{Ay} here denoted by M_A, is

$$M_A = \sqrt{M_{Ax}^2 + M_{Ay}^2} = \sqrt{(109{,}590^2 + 78280^2)} = 134{,}700 \text{ in-lb}$$

The corresponding maximum stress is

$$p_{1b} = M_A r_o/I = 134{,}700 \times 3.644/33.14 = 14{,}810 \text{ lb per sq in.}$$

The longitudinal stress due to steam pressure is the same as at O, which is 2825 lb per sq in. The uniform longitudinal stress due to the compressive reaction N is

$$p_{1c} = N/A = -640/5.33 = -120 \text{ lb per sq in.}$$

Therefore

$$p_1 = 14{,}810 + 2825 - 120 = 17{,}500 \text{ lb per sq in.}$$

We have again

$$p_2 = p_{2s} = 5650 \text{ lb per sq in.}$$

The shearing stress due to the twisting moment is

$$q_t = \frac{M_{Ax} r_o}{2I} = \frac{91{,}900 \times 3.644}{2 \times 33.14} = 5052 \text{ lb per sq in.}$$

While the shearing stress due to the reaction forces at A is

$$q_f = \frac{\sqrt{(P^2 + Q^2)}}{A} = \frac{\sqrt{(1729^2 + 1916^2)}}{5.33} = 484 \text{ lb per sq in.}$$

Therefore

$$q_A = q_t + q_f = 5052 + 484 = 5540 \text{ lb per sq in.}$$

and the equivalent stress, according to the strain-energy theory, Equation [66] becomes

$$p_{eq} = \sqrt{[17{,}500^2 + 5650^2 - (17{,}500 \times 5650) + (3 \times 5540^2)]} = 18{,}200 \text{ lb per sq in.}$$

This still falls below one-half the stress at the yield point, $\frac{1}{2}p_0 = 19{,}500$ lb per sq in.

According to the maximum-shear theory, Equation [62] $p_{eq} = 16{,}200$ lb per sq in., which is again appreciably lower than according to the strain-energy theory.

In most cases the direct stress p_{1c} and the shearing stress q_f can be safely neglected as being inside the limits of accuracy of the computations.

The stresses at O and A are greater than anywhere else in the straight parts of the pipe as can be ascertained by means of Equations [18], so that it remains only to investigate the stresses in the bends.

Stress in the First Bend

In the first bend M_s acts in the plane of the bend and is a maximum when

$$\tan \psi_{max} = P/Q = 1729/1916 = 0.902$$

Therefore $\psi = 42$ deg 3 min.

The maximum value of M_s as obtained from Equation [39a] is

$$M_{s\max} = M_{Oz} + PL_y + R[\sqrt{(P^2 + Q^2)} - Q] = 135{,}400 + (1729 \times 108.9) + 36.3\,[\sqrt{(1729^2 + 1916^2)} - 1916] = 77{,}000 \text{ in-lb}$$

The bending moment normal to the plane of the bend at any point S as obtained from Equation [41] is

$$M_b = (M_{Ox} - NL_y)\cos\psi - (M_{Oy} + NR)\sin\psi$$

which is a maximum when

$$\tan\psi_{b\max} = -\frac{M_{Oy} + NR}{M_{Ox} - NL_y} = -\frac{-13{,}880 + (640 \times 36.3)}{54{,}410 - (640 \times 108.9)} = 0.613$$

Substituting this value in Equation [41] we find

$$M_{b\max} = (-15{,}280 \times 0.852) - (9360 \times 0.523) = -17{,}900 \text{ in-lb}$$

The twisting moment at S, according to Equation [42], is

$$M_t = (M_{Ox} - NL_y)\sin\psi + (M_{Oy} + NR)\cos\psi - NR$$

which is a maximum when

$$\tan\psi_{t\max} = -1/\tan\psi_{b\max}$$

This places the maximum outside the quadrant and we have to determine the value of M_t where $\psi = 0$, and $\psi = \pi/2$. It is found that for $\psi = 0$, $M_t = M_{Oy} = -13{,}880$ in-lb, and for $\psi = \pi/2$, $M_t = 38{,}520$ in-lb. Hence, M_t has its greatest value at D, and since M_s has its maximum at $\psi = 42$ deg, and M_b at $31\frac{1}{2}$ deg, and since they are all small or moderate as compared with the couples at A and O, it is unnecessary to consider any combination of the three couples. It is of interest, however, to investigate the stress caused by $M_{s\max}$, which is found from Equation [53].

The maximum stress occurs at a point in the section, the radius of which forms a certain angle θ with the neutral axis. This angle can be found from Equation [55], in which $\lambda = 0.705$. Therefore

$$\sin\theta = \sqrt{[(2.982 + 5)/18]} = 0.666$$

and since $K = 2.29$, the value of β can be found from Equation [54a], where

$$\beta = 2/3\ K\sin\theta = 2/3 \times 2.29 \times 0.666 = 1.017$$

The maximum stress can now be determined from Equation [53], where

$$p_{1b\max} = (1.017 \times 77{,}000 \times 3.523)/33.14 = 8320 \text{ lb per sq in.}$$

It is unnecessary further to peruse the study of the longitudinal stress in this case, since it is known to be much smaller than the transverse stress. From Equation [56] we have at the outer surface in the neutral axis

$$p_{2b} = \frac{18\,M_s\lambda r}{I\,(12\lambda^2 + 1)} = \frac{18 \times 77000 \times 0.705 \times 3.523}{33.14\,(12 \times 0.497 + 1)} = 14{,}930 \text{ lb per sq in.}$$

The hoop stress p_{2s} due to steam pressure was found previously to be 5650 lb per sq in. The transverse compressive stress due to flattening, applying Equation [58], is

$$p_{2c} = \frac{2M_s r^2}{RI}\,\frac{(6\lambda^2 + 1)}{(12\lambda^2 + 1)} = \frac{2 \times 77000 \times 12.41}{36.3 \times 33.14} \times \frac{3.982}{6.964} = 910 \text{ lb per sq in.}$$

Hence, from Equation [59] the total transverse stress in the neutral axis is

$$p_2 = 14{,}930 + 5650 - 910 = 19{,}670 \text{ lb per sq in.}$$

At the section where $M_{s\max}$ is acting, there is a twisting moment as given by Equation [42]. With $\psi_{\max} = 42$ deg 3 min, we find after applying Equation [42] that $M_t = 26{,}520$ in-lb. Hence, a shearing stress is produced at the outer surface which, according to Equation [51], is found to be

$$q_t = \frac{M_t r_o}{2I} = \frac{(26{,}520 \times 3.644)}{(2 \times 33.14)} = 1460 \text{ lb per sq in.}$$

The equivalent stress in the neutral axis with $p_1 = 0$ and neglecting p_3, from Equation [66] is

$$p_{eq} = \sqrt{(p_2^2 + 3q^2)} = \sqrt{[19{,}670^2 + (3 \times 1460^2)]} = 19{,}900 \text{ lb per sq in.}$$

which is a little more than one-half the stress at the yield point.

The investigation of the stresses in the second bend are made in the same manner.

6—ACKNOWLEDGMENT

The author wishes to express his indebtedness to Henry C. E. Meyer, chief engineer of Gibbs and Cox Inc. for his advice in the preparation of this paper; in particular, the discussion of the permissible working stress. His thanks are due also to Horace L. Bickford of Gibbs and Cox Inc. for his assistance in the preparation of the diagrams as well as in working out the numerical example.

An Airfoil-Type Propeller Blower for Ventilating Electrical Machines

By C. E. PECK[1] and M. D. ROSS,[2] EAST PITTSBURGH, PA.

The authors discuss the development of an airfoil-type propeller blower, the design of which was influenced by space limitations in connection with the motor-driven blower equipment for a 165,000-kw turbine-generator unit installed at the Richmond station of the Philadelphia Electric Company. After the fan was designed, a model was built and tested before the full-size equipment was constructed. The authors present performance data for both the model and the full-size blower.

NEW types of apparatus of improved design and performance are frequently developed as a result of an effort to overcome certain difficulties in a particular commercial application. Space limitations in connection with the motor-driven blower equipment for a large turbine-generator unit brought about the development of the blower described in this paper. When the latest addition to Richmond station of the Philadelphia Electric Company was planned, it was decided to add a 165,000-kw turbine-generator unit. As the turbine-room structure was already in existence with space originally designed for four 50,000-kw units, it will be seen that the design of a 165,000-kw unit to go in the space originally occupied by a 50,000-kw unit involved a careful use of all the available space. The specifications called for all blowers, air coolers, and air ducts to be located below the generator. The height of the space available for this equipment was fixed by the height of the existing turbine-floor level which was 28 ft 6 in. above the condenser-floor level. The width of the space was limited on one side by a circulating pump and on the other by an elevated track. Space had to be provided to permit ready removal of air-cooler tubes. The specifications called for four motor-driven blowers, one of which was to be a spare unit. The generator required a maximum volume of 170,000 cfm. The calculated pressure drop in the generator air coolers and ducts was 12 in. of water. Therefore, the volume rating of each blower was to be 57,000 cfm.

A study of the space available under the generator soon indicated that the usual types of blowers available would occupy too much space and would not fit in with any reasonable system of air ducts. Having had considerable experience with propeller blowers built on the shafts of turbine generators, it was decided to undertake the development of a propeller-type blower of high speed and small dimensions to suit this application.

Another requirement to be met by these blowers was that they were to be suitable for full-voltage starting control for the induction motors driving them. As the generator would operate at all times with two or three blowers running in parallel, the blower characteristics had to be such that good parallel operation would be obtained. As the blowers were to be part of a closed ventilating system in which the air was returned to the generator after being cooled in finned-tube coolers, it was necessary to provide them with suitable intake ducts. Air leaving the generator and entering the blower may reach a temperature of 150 F and some means had to be provided in the design to avoid excessive heating of the blower bearings with this high-temperature air adjacent to them. The speeds available for the blowers corresponded to the standard speeds for 60-cycle induction motors. The speed adopted was 3550 rpm.

Description of Full-Size Blower

The design of the full-size blower is shown in section in Fig. 1. The fan runner consists of eight aluminum-alloy blades dovetailed into a forged-steel hub. The fan runner is overhung on two sleeve bearings mounted in a common rigid cast-iron pedestal as shown in Fig. 2. The shaft and bearings were designed on the premise that no serious damage would occur to these parts in the event of losing one blower blade at full speed. The provision of a rigid shaft and pedestal contributed largely to the smooth running of the unit as a whole as found by test. The thrust varies with the back pressures against which the blower is working. In order to take care of the highest thrust obtained, a two-shoe Kingsbury thrust bearing was incorporated in the same housing as the journal bearing adjacent to the motor coupling.

As the blower bearing losses amounted to something over 1 kw, a small centrifugal fan was mounted on the blower shaft to force a stream of air through passages under the oil chamber in the pedestal, thus cooling the lubricating oil. This cooling air discharges into the space between the pedestal and the blower casing and effectively prevents heat from the casing affecting the pedestal. To provide circulation of the lubricating oil, a small oil pump is mounted on the outboard-motor bracket and connected to the blower pedestal by brass piping.

Air from the generator enters the blower casing from the top through a hand-operated damper which can be closed when a blower is shut down with other blowers in operation. The blower casing is constructed of welded-steel plate sufficiently rigid to support the guide-vane and diffuser structure which is bolted with a flanged joint to the casing. Air enters the blower through a spun-aluminum intake nozzle, passes through the blower and then through the stationary guide vanes in the diffuser. The guide vanes are built up of aluminum castings.

Theoretical Design of the Fan

The fan without guide vanes and diffuser was designed on the basis of the theory outlined in a paper by Dr. O. G. Tietjens.[3] Within certain empirical limits the proportions of the fan itself,

[1] Engineer, Power Engineering Department, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. Jun. Mem. A.S.M.E. Mr. Peck was graduated from the Carnegie Institute of Technology in 1927 with an M.E. degree, and received his M.S. degree in 1928 from the University of Pittsburgh. Since 1928 he has held his present position, specializing in air flow, thermal problems, and blower developments in connection with large-size electrical machinery.

[2] Design Engineer, Westinghouse Electric & Manufacturing Company. Mr. Ross was graduated from the University of Toronto in 1922 and since has been employed in his present capacity.

Contributed by the Power Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers, to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "The Propeller-Type Fan," by O. G. Tietjens, Trans. A.S.M.E., vol. 54, 1932, paper APM-54-13, pp. 143–152.

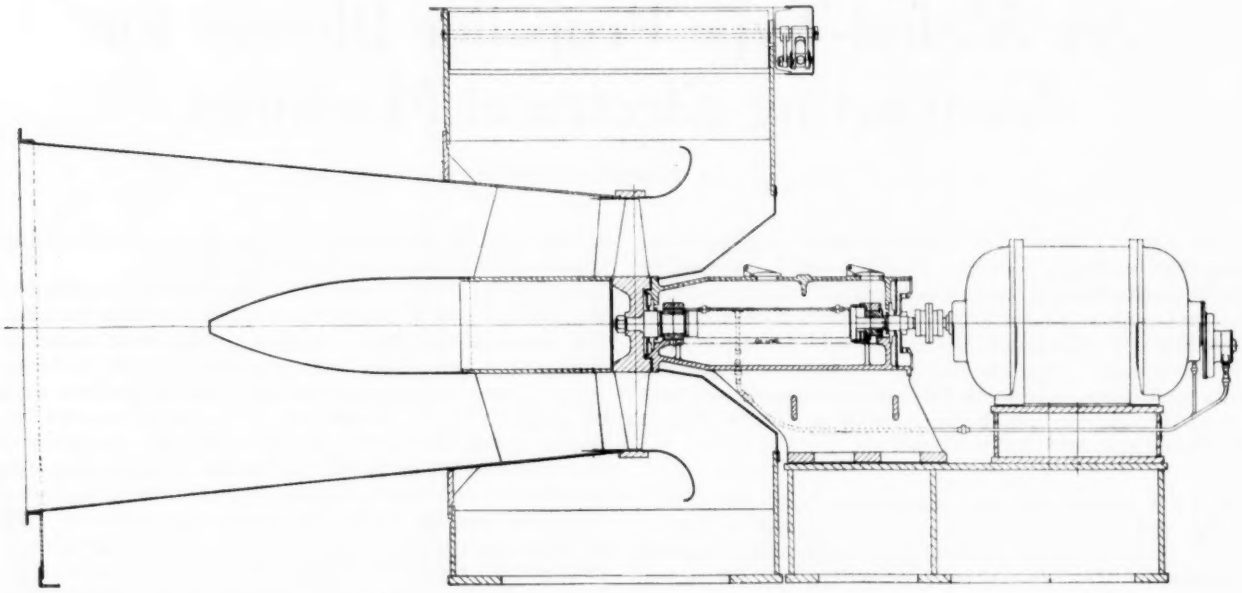


FIG. 1 CROSS-SECTION ASSEMBLY OF THE FULL-SIZE UNIT

without guide vanes or diffuser, can be accurately predetermined for a given pressure and volume. Certain empirical limits applied to the theoretical design had been found by means of considerable experimental investigation performed by H. C. Werner of the Westinghouse Research Laboratories on several different fans of the air-foil type. The guide vanes and diffuser add considerably to the capacity and efficiency of the fan. As a result, the theoretical design of the fan itself was based on 90 per cent of the required volume of 57,000 cfm per fan and 85 per cent of the required static pressure of 12 in. of water, which is approximately 25 per cent lower than the required capacity. It was assumed that the diffuser would recover between 70 and 80 per cent of the dynamic energy created by the fan in a direction parallel with the axis of rotation. In addition, it was assumed that the guide vanes would recover about 50 per cent of the rotational energy at the discharge side of the fan. How closely the tests verified these assumptions is discussed later.

The fan blade has an airfoil cross-section of varying profile from hub to tip. The general shape is that of a cross-section of an airplane wing having a flat side and well-rounded leading edge which is stable for large angles of attack. The blade width and angle with respect to a vertical plane gradually decrease from hub to tip. The fan contains eight blades so spaced at the hub that no overlapping exists. Mutual interference of one blade upon another was found to be very small with this spacing.

In line with the theory outlined in Tietjens' paper,[3] the following is a brief analysis of the factors to be considered in the theoretical design: In designing a propeller fan the total pressure generated by the fan multiplied by the volume rate of flow represents a certain power output. The volume rate of flow is equal to the fan area times the axial velocity through the fan. Hence, the power output is

$$Ap_t w = Tw \quad \ldots\ldots [1]$$

where A is the fan area, w is the axial velocity through the fan, p_t is the total pressure (static plus dynamic), and T is the total force acting on the fan area and is called the thrust. In general, the problem reduces to that of designing a fan which will produce the total thrust corresponding to the required total pressure.

From airfoil and propeller theory, the thrust depends on the magnitude and distribution of the aerodynamic circulation around the blade. This may be given by the relation

$$dT = (n\rho\ \Gamma\ r\omega)\ dr \quad \ldots\ldots [2]$$

where dT is the incremental thrust at a point corresponding to radius r on the blade, Γ is the circulation, ω is the angular velocity of the fan, n is the number of blades, and ρ is the air

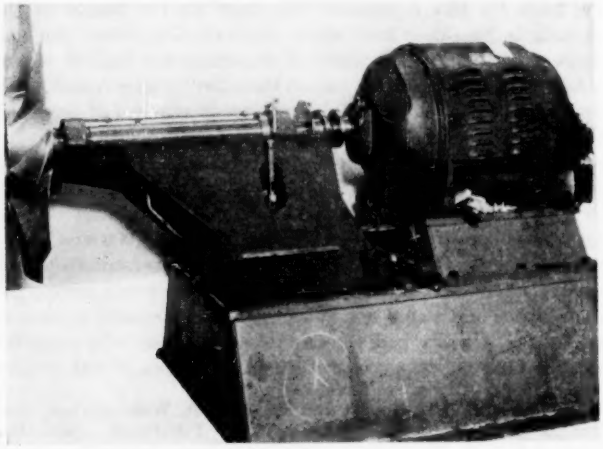


FIG. 2 FAN RUNNER AND BEARING OF THE AIRFOIL BLOWER

density. It has been shown theoretically that the pressure rise through a propeller fan and, hence, the thrust, can only be obtained by the creation of a rotational-velocity component which can be clearly observed at the discharge side of the fan. The circulation and rotation are related to the blade dimensions at radius r by the equation

$$\Gamma = C_L\ bv/2 \quad \ldots\ldots [3]$$

in which C_L is the coefficient of lift of the blade at the air-foil cross-section corresponding to the radius r; b is the width of the blade; and v is the velocity of air relative to the blade. The relation between v and the axial velocity through the fan and the peripheral

speed at the radius r and the rotational component at the radius r is shown geometrically in Fig. 3. The angle of attack α is necessary to produce the required lift at radius r, and $\alpha + \beta$ is the angle at which the blade is located on the hub.

The fundamental definition of the term circulation is given by the relation of the quantities defined in Equation [3]. Physically, the circulation is an idealized component of the actual flow around an airfoil which can be represented by the streamlines completely traversing the airfoil in a rotating manner. A detailed explanation of the meaning of circulation is given by Prandtl-Tietjens.[4]

If the circulation were assumed constant over the entire blade area in Equation [2], the thrust would vary directly with the radius only, and the total thrust could be obtained by analytical integration of Equation [2]. However, with constant circulation the portions of the blade at the smaller radii are worked at relatively high outputs and the rotation losses become much higher than if the circulation were decreased with decreasing blade radius. It is possible to have constant circulation over the blade area without large rotation losses if the hub diameter is made large relative to the outside diameter, which has the effect of increasing the outside diameter. For a given required total thrust, a compromise is reached between the size of hub relative to outside diameter.

The circulation distribution is decreased toward the hub in such a way that rotation losses are reduced to a value consistent with high efficiency. The fan considered in this paper would have had 45 per cent more rotation loss if the circulation had been constant over the blade area, which is equivalent to a reduction of 5 per cent in the fan efficiency. The ratio of hub diameter to outside diameter was 0.4. This rotation loss is the component of energy at the fan discharge which lies in a plane perpendicular to the axis of rotation of the fan. The dynamic energy is the axial component of the energy at the fan discharge.

Table 1 gives the important constants of the fan referred to in these tests. The volume and pressure are for the fan itself without guide vanes or diffuser at the calculated design point which is also the point of maximum efficiency.

TABLE 1 PRINCIPAL CONSTANTS OF THE AIRFOIL PROPELLER BLOWER

Air density, lb per cu ft	0.0749
Volume, cfm	52,500
Static pressure, in. of water	10.2
Speed, rpm	3600
Outside diameter, in.	38.31
Inside diameter, in.	14.50
Fan area, sq ft	6.87
Constant axial velocity through fan, cfm	7650
Dynamic pressure head, in. of water	3.55
Total head, in. of water	13.75
Ratio of dynamic to total head	0.258
Number of blades	8

The ratio of dynamic head to total head is an important constant for the propeller-type fan because if this value is made too low, such as 0.10 to 0.15, the application of airfoil theory to the fan design begins to become inaccurate because the nature of the flow around the blades is changing due to centrifugal effects caused by the air rotating much faster at the lower-volume points. Values of the ratio between 0.20 and 0.30 have been found satisfactory from the standpoint of performance and efficiency.

Description of Model-Test Set-Up

The model was built about one-quarter size and geometrically similar to the proposed full-size application in all the important details including the inlet box, inlet guides, and diffuser. The best proportions of such parts as the guide vanes and diffuser as determined from the model tests were incorporated in the final full-size unit with exact geometric similarity.

A schematic diagram of the model-test set-up is shown in Fig. 4. The fan draws air through a long gradually diverging inlet duct at the entrance of which is located a flow nozzle for measurement of air volumes. The inlet duct is fastened to the side of the inlet box of the fan. The fan discharges air directly to the room or into the diffuser and guide-vane arrangement. The fan shaft extends through the inlet box and is coupled to a $^1/_2$-hp electric dynamometer used to measure the power input to the fan and also drive the fan. The duct ahead of the flow nozzle was provided with a well-rounded entrance in the form of a one-piece smooth spun-copper ring. Between this ring and the duct

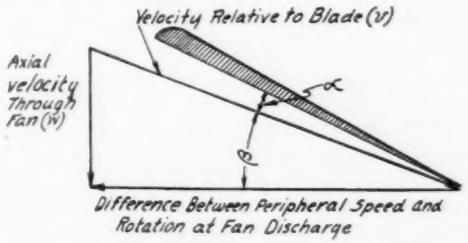


Fig. 3 Air Velocity Relative to the Blade

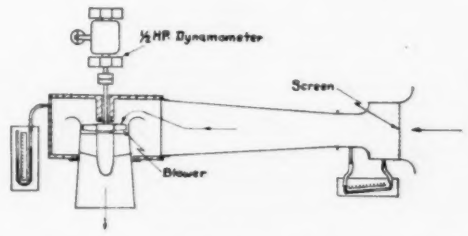


Fig. 4 Diagram of Test Arrangement for the Model Blower

flange, screens were placed in order to vary the pressure-volume delivery of the fan.

Measurements

The volume of air was measured by means of a flown ozzle made of smooth spun copper. The proportions of the nozzle were similar to one of those built by the Bureau of Standards and calibrated for use in calibrating orifices.[5] An average nozzle coefficient of 0.980 was used. The pressure drop across the nozzle was measured by means of eight accurately constructed static-pressure taps on the inlet and discharge sides of the flow nozzle. The pressure drops were measured on an inclined manometer the scale of which was such that errors were within $^1/_4$ per cent for normal fan delivery.

The static pressure developed by the fan was measured by means of taps in the top corners of the inlet box. The average velocity to the inlet box was about 500 fpm which is low enough to introduce negligible error in the measurement of static pressures from $^1/_2$ to 2.5 in. of water found in the model tests.

The power input was measured by means of a dynamometer arrangement consisting of a $^1/_2$-hp direct-current motor used to drive the fan. The stator of the dynamometer was suspended in ball bearings and equipped with a lever arm and scale pan. The torque reaction was measured by balancing it against weights

[4] "Applied Hydro- and Aeromechanics," by O. G. Tietjens, based on lectures of L. Prandtl, McGraw-Hill Book Company, Inc., New York, 1934.

[5] "Discharge Coefficients of Square Edged Orifice for Measuring the Flow of Air," by H. S. Bean, E. Buckingham, and P. S. Murphy, *Bureau of Standards Journal of Research*, March, 1929, pp. 568–652.

placed in the scale pan mounted on the lever arm. Speed was maintained at a given value by using current supplied from a separately excited direct-current generator driven by a single-phase alternating-current motor. The sensitivity of the suspension was such that the average input of $^1/_4$ to $^1/_3$ hp could be checked within 1 per cent. Extreme care was exercised to obtain accurate results. Checks were obtained by means of repeat runs after a considerable time interval. All data were corrected to the standard air density of 0.07488 lb per cu ft. On repeat runs, the pressure multiplied by the volume and divided by the power input which measures the fan efficiency could be checked within 2 per cent.

Model Tests and Test Results

Tests on Fan Itself. In all of the model tests made, the intake box was in place. All of the model data have been expressed in terms of the values for the full-size unit. The results of the tests of the fan itself without guide vanes or diffuser are shown in Fig. 5. The test-performance curve checks closely with the working point for which the fan was designed. The static efficiency of 56 per cent at the working point is the approximate maximum static efficiency. The total efficiency of the fan itself without guide vanes or diffuser is about 75.8 per cent and is based on the static pressure plus the dynamic pressure corresponding to the average axial-velocity component of the air at the fan-discharge area. The difference between 100 per cent and the total efficiency

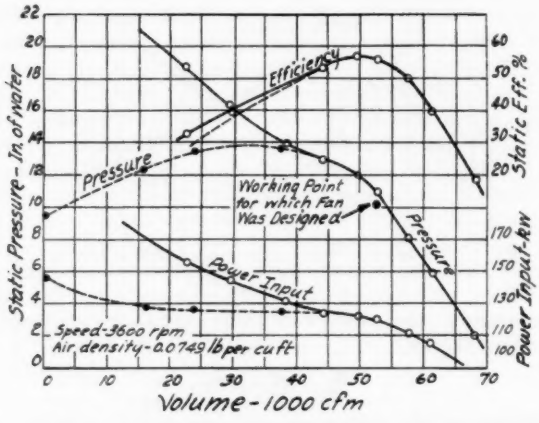


Fig. 5 Performance Curves of the Fan Alone

(Performance with inlet-box baffles removed is shown by dashed lines while performance with baffles in the inlet box is shown by solid lines.)

is a measure of the sum of the fan losses expressed in terms of per cent efficiency. A detail discussion of the magnitude and distribution of these losses is given later.

The solid curve in Fig. 5 represents the performance with the intake box in which were placed two vertical baffle plates diametrically opposite. These plates were fitted around the curved inlet guide which is part of the fan housing. With these plates in place, the pressure characteristic was rising with decreasing volume which is a necessary property for satisfactory operation of two or more fans in parallel. When the plates were omitted from the intake box, the pressure characteristic assumed the "hump" shape which is represented by the dashed line in Fig. 5. This shape is not suitable for parallel operation. The performance with and without the plates was the same at the higher-volume points and in the region of the desired working point. The "hump" shape is caused by the air whirl induced in the inlet box which becomes large enough in magnitude at the lower volumes to reduce the static pressure considerably below the values obtained when no such whirl is present.

The flat portion on the fan curve is characteristic of the propeller-type fan. From a theoretical standpoint, this sudden change in the slope of the pressure characteristic is caused by too large an angle of attack on the blade section. From airfoil theory, as the angle of attack increases, a point is reached at which the lift becomes a maximum and any further increase in angle of attack results in a sudden decrease in the lift, the flow conditions around the blade becoming unstable. In the propeller fan, as the axial velocity through the fan decreases the angle of attack at a

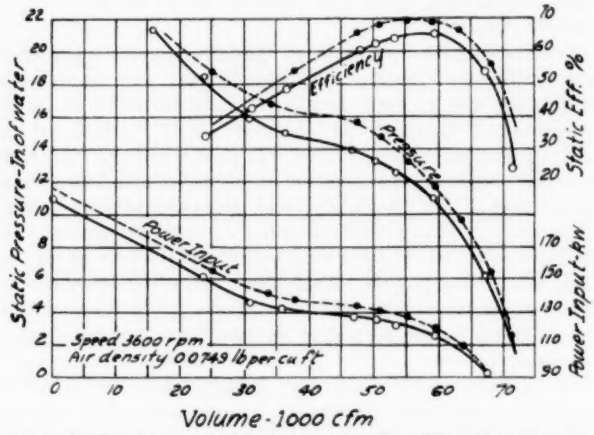


Fig. 6 Performance Curves of the Fan With Diffuser Only

(Performance with very close clearance between the housing and fan is shown by dashed lines while the performance with a clearance of 1.25 per cent of the fan housing is shown by solid lines.)

given point on the blade section increases until a point is reached where the angle of attack has reached a value corresponding to the maximum lift. Beyond this point the decrease in lift which takes place on a wing section is replaced in the propeller-fan blade by centrifugal action. Thus, at the point where the pressure characteristic flattens out, the airfoil theory no longer holds.

Tests With Diffuser and No Guide Vanes. The performance curves for the fan with diffuser and no guide vanes are shown in Fig. 6. A large gain in efficiency results when the diffuser alone is used. In this particular fan, a reasonably large gain using a diffuser alone may be expected because approximately 20 per cent of the fan power input is necessary to create the axial component of the dynamic energy present at the fan-discharge area. Of this 20 per cent, 3.2 per cent represents the axial component of the dynamic energy at the discharge area of the diffuser. The change in static efficiency of the fan from 56 to 66 per cent with the diffuser and same housing clearance in each case indicates that the diffuser recovered about 60 per cent of the available dynamic energy between the entrance and exit of the diffuser. Since no guide vanes were present to straighten the rotating air, the 80 per cent conversion of axial dynamic energy to static energy which is to be expected with a good diffuser could not be obtained. A test made with the center cone of the diffuser removed reduced the static efficiency from 66 per cent to 64 per cent.

Guide-Vane Model Tests. The proportions and angles of the guide vanes were determined by a set of tests. It was found that flat-plate vanes twisted so that the angle varied from the hub to tip gave but a small increase in static efficiency. The plate shape is much more sensitive to the direction at which the air leaving the fan strikes the vane than the blunt-nose airfoil-shaped vane. The air leaving the fan strikes the airfoil vane at a certain angle of attack. With the blunt-nose airfoil-shaped vane, the angle of attack may vary as much as 10 to 12 deg before the air flowing around the vane tends to "break" away from the vane

surface. At the "break" point the vane no longer acts as a guide to straighten the air flow from the rotational direction to the axial direction, but acts instead as an obstruction. The approximate angles at which the air was discharged from the fan were measured from hub to tip by means of a thread and protractor. From these

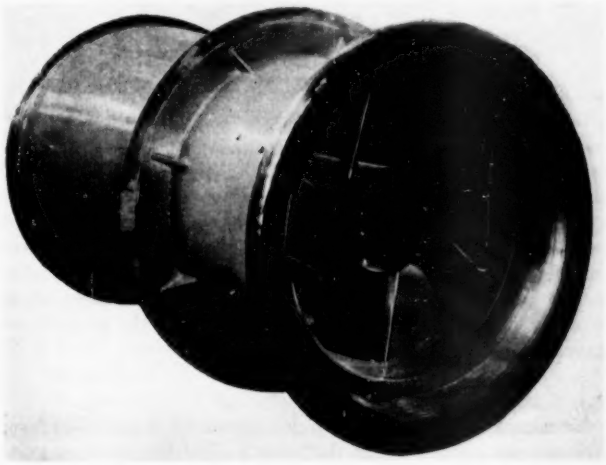


FIG. 7 THE DIFFUSER AND GUIDE VANES OF THE BLOWER

data several sets of airfoil-shaped vanes with various angles of twist were made and tested. It was found that the vane which gave the largest increase in static efficiency near the desired fan working point was set within about 5 to 8 deg of the theoretical discharge angle corresponding to the calculated rotation of the fan.

The number of guide vanes was varied and it was found that when less than six vanes were used the gains in efficiency were reduced, probably due to the lack of sufficiently close spacing between vanes to provide the required guiding action. Both the six-vane and eight-vane arrangements gave about the same efficiency. The six-vane arrangement was used on the final full-size unit. The guide vanes mounted in the diffuser are shown in Fig. 7.

FULL SIZE TESTS AND RESULTS

Description of Test Set-Up and Method of Test. The full-size unit assembled with inlet box and diffuser is shown in Fig. 8. A large discharge box was built at the discharge of the diffuser. A portion of this box can be seen in Fig. 8. On the roof of the discharge box, five stacks, each with a different discharge area, were constructed and used as flow nozzles. Various volume points on the performance curve of the fan were obtained by closing different combinations of the stack openings.

The air enters the inlet box from above and passes through the fan and diffuser to the discharge box. A large screen with a cheese-cloth covering is located in the discharge box just beyond the diffuser outlet. This serves to break up the velocity head of the air leaving the diffuser and thus provides uniform inlet conditions to the nozzles.

The air volume was measured by making a pitot-tube traverse at the exit of each nozzle. In order to investigate the air distribution in each nozzle and obtain an accurate average, a large number of pitot-tube readings were taken. The air distribution was found to be reasonably uniform and the direction of the air was essentially parallel with the axis of the nozzle. These conditions made it possible to measure the air volume very accurately. The use of the large discharge box and screens eliminated any errors which might be present due to pulsating flow conditions.

The static pressure generated by the fan was measured in the discharge box after the air had passed through the screen. At this point the air velocity was very low and thus an accurate measurement of static pressure could be obtained.

The fan is driven by a 3600-rpm induction motor. The power output of the motor was accurately obtained by means of a special calorimeter-test method which will not be described here. This method of test eliminated the uncertainty of the magnitude of the load loss which in this case was enough to affect the fan efficiency by approximately 2 per cent.

Test Results. The results of the test on the full-size fan with guide vanes and diffuser are given in Fig. 9. The performance includes the effect of the inlet box. The power input to the fan includes the loss in the thrust bearing which is of the order of 1.5 kw. The model tests with similar guide vanes and diffuser checked the full-size performance closely. The maximum static efficiency on the model was 77 per cent. The maximum static

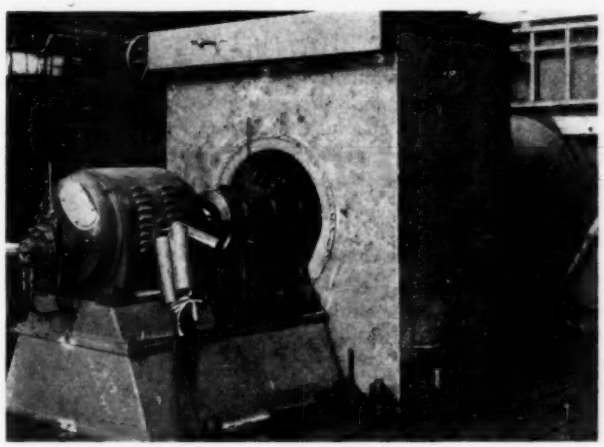


FIG. 8 TEST ARRANGEMENT OF THE ASSEMBLED FULL-SIZE BLOWER UNIT

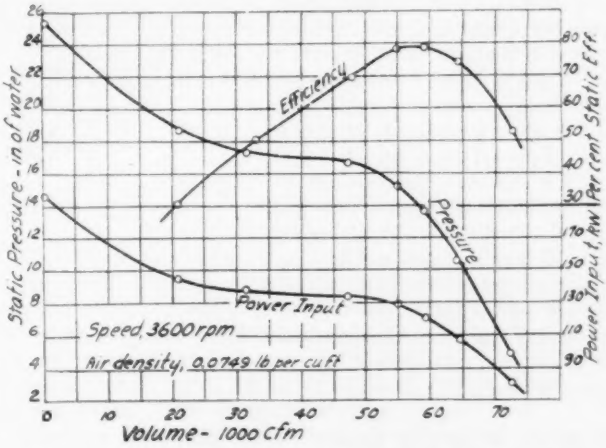


FIG. 9 PERFORMANCE CURVES OF THE FULL-SIZE UNIT WITH GUIDE VANES AND DIFFUSER

efficiency on the full-size unit was 78.5 per cent, including the loss due to the fan thrust bearing. The close check between the tests of the model and full-size unit indicated that the different methods of test used on the model and the full-size unit were satisfactory. The total efficiency of the unit with guide vanes and diffuser based on the static pressure plus the axial dynamic pressure at the diffuser discharge is 82.5 per cent, including the effect of the inlet-box and thrust-bearing loss.

Clearance Between Fan and Housing. The model tests demonstrated the importance of the clearance space between the outside fan diameter and the surrounding housing. Fig. 6 shows performances ascertained from two tests, one with a clearance between fan and housing of 1.25 per cent of fan radius, and the other with a very small clearance obtained on the model by molding a layer of modeling clay to the inside of the housing and scraping the excess off by running the fan in the housing. With this type of fan, an appreciable gain in efficiency results when clearances are close. The axial width of the eight-bladed fan at the blade tip is about 24 per cent of the radial height of the blade. The small axial width compared to the blade height may be linked with the sensitivity to clearance. The clearance between the blade tip and housing for the full-size unit is 0.163 per cent of the outside fan radius. In the test shown in Fig. 5, the efficiency of the fan itself was raised from 57 per cent to 61 per cent when an extremely small clearance was used.

Calculated Losses and Efficiencies Compared to Tests

The losses in a propeller fan may be classed under the two headings of recoverable and non-recoverable losses. The non-recoverable losses are those which originate in the fan itself and are called the drag losses. It is equivalent to the drag loss on an airfoil and is due to the fact that each blade element in producing a certain lift must overcome a certain drag due to friction and the nature of the flow around the airfoil.

The recoverable losses consist of the energy in the air at the discharge side of the fan. This energy is conveniently resolved into two components, namely, the rotational-velocity component which lies in a place perpendicular to the axis of rotation, and the axial-velocity component which is parallel with the axis of rotation of the fan.

Table 2 gives the magnitude of these various losses expressed as per cent of energy input to fan at working point of fan itself. (For point of maximum efficiency see Fig. 5.) Table 2 shows which losses limit the efficiency of the fan. The agreement between the calculated and test values of static efficiency indicates that the magnitude and relative apportionment of the total losses into their various parts is approximately correct.

A study of Table 2 makes the following points worthy of note.

TABLE 2 CALCULATED LOSSES AND EFFICIENCIES, PER CENT OF FAN INPUT

Item	Fan alone	Fan with guide vanes and diffuser
(*a*) Drag loss, not recoverable	13.3	13.3
(*b*) Rotational-loss component	9.3	3.7[a]
(*c*) Axial dynamic-energy component	20.0	3.4[b]
(*d*) Energy at diffuser exit, not recoverable	0	3.2
(*e*) Total losses, Items *a* + *d*, not recoverable	13.3	16.5
(*f*) Total losses, Items *b* + *c*, recoverable	29.3	7.1
(*g*) Total losses, Items *a* + *b* + *c* + *d*	42.6	23.6
Calculated static efficiency (100 — Item *g*)	57.4	76.4
Test static efficiency	56.5	78.5[c]

[a] Assumed 60 per cent recovered.
[b] Assumed 80 per cent recovered.
[c] Approximately 1 per cent low because of bearing loss.

If all of the recoverable losses were recovered, the maximum efficiency of the fan would be 86.7 per cent. The approximate percentage of losses recovered by means of the guide vanes and diffuser amounts to 75 per cent of the recoverable losses. The largest component of loss is the axial dynamic component which explains in part why the diffuser alone without the guide vanes gave a large increase in the static efficiency.

Noise

No complete investigations of noise levels were made but some comparisons indicated that the noise level of the fan would add little, if any, to the general noise level of the power station. These comparisons are based on the fan operating on the test floor where the intakes were open, but in the actual application the fan is part of a completely enclosed system.

Summary

The results of designing, building, and testing a fan of this type bring out the following possible advantages:

1 Within certain limits the performance and design proportions can be predetermined accurately.

2 From a mechanical standpoint, the propeller is particularly adaptable for high speeds.

3 The intake box has little effect on the fan performance.

4 For the pressures and volumes used in this application, the size of the unit is reasonably small.

5 The fan efficiency is high.

Progress Report on Cavitation Research at Massachusetts Institute of Technology

By J. C. HUNSAKER,[1] CAMBRIDGE, MASS.

This paper, a second progress report on the study of the nature of hydraulic cavitation being conducted at M.I.T., is continued along the lines reported in the first paper published in *Mechanical Engineering*, April, 1935, and discusses information obtained regarding the relation between frequency of vapor collapse and the controlling variables. The current work tends to confirm that previously reported. A new method for the study of resistance to cavitation attack is described.

IN THE present work on hydraulic cavitation it is found that the coefficient fL/V remains substantially constant, not only for geometrically similar venturi-type nozzles but also for all nozzles having the same exit flare. In the coefficient term fL/V, f is the frequency of collapse, L is the length of cavitation volume, and V is the velocity at the nozzle throat. Reynolds' number does not appear to control matters, possibly because the range for variation of Reynolds' number in the tests is not very great, but more probably because the flow is inherently turbulent and viscosity effect cannot be important. However, the relation fL/V can only be determined when cavitation is well developed and L is of substantial magnitude.

Further work has confirmed the opinion that the damage to surfaces caused by cavitation is severe only when the collapse is noisy, violent, and regular. For such fully developed cavitation the frequency of collapse is regular. When cavitation is not well developed and L is but one or two throat diameters, the frequency observed is highly irregular, severe shocks are infrequent or absent, and damage to walls is very slight.



FIG. 1 LEAD DISK $^5/_8$ IN. DIAMETER AFTER 0.5-MIN OSCILLATION IN CALCIUM-CHLORIDE SOLUTION

Wall damage also appears to depend on the abolute size of the nozzle used, or the energy available in the cavitation volume, since tests with small nozzles, geometrically similar to large ones produced little or no damage even when cavitation conditions appeared to be similar.

When the exit flare varies more or less from 21 deg, the frequency of cavitation collapse is irregular, severe shocks infrequent or absent, and damage slight regardless of the length of cavitation volume for equal throat velocities.

The conclusion from the current work is that cavitation of a type to cause severe damage requires, probably, some unique combination of variables, of which exit flare or profile shape appears to be important. Therefore, a technique for the quantitative measure of the severity of cavitation damage is required. In this connection exposure of test medallions of metal has proved of interest. The determination of weight loss against time of exposure, however, does not give a suitable numerical measure of the severity in a reasonable time. Lead, for example, may be severely damaged without showing appreciable loss of weight. However, it is found that properly cured cement will progressively lose weight under cavitation attack and it is planned to report tests of various cavitation conditions in terms of a scale based on loss of weight of cement.

The severity of cavitation not only depends on the type of cavitation but must also depend on the liquid, and especially its density and vapor pressure. However, it is impractical to repeat the hydraulic experiments with other liquids.

It is known, however, that high-frequency vibration of a submerged diaphram, such as is used in submarine signaling, can produce cavitation and damage. An apparatus, similar to that described by Gaines[2] for oscillating a nickel tube at high frequency by magnetostriction effect was used to oscillate $^5/_8$-in. disks of lead and cement with a frequency of about 8500 cycles per sec through an amplitude of about 0.01 mm.

Tests are being made with a number of common liquids of widely different physical properties. Tests of lead disks in water or in a calcium-chloride solution show vastly more damage than when tested in alcohol or in carbon tetrachloride. Presumably the high vapor pressure of the latter liquids accounts for this. The appearance of the damaged surface of the lead is identical

---

[1] Head of Mechanical Engineering Department, Massachusetts Institute of Technology. Mem. A.S.M.E. Dr. Hunsaker was graduated from the United States Naval Academy in 1908 and received his M.S. degree from M.I.T. in 1912 and D.Sc. degree in 1916. From 1909 to 1926 he was an officer in the U. S. Navy, but during this period served as instructor of aeronautic engineering at M.I.T. from 1912 to 1916. From 1916 to 1923 he was in the Navy Department, Washington, D. C., in charge of aircraft design. From 1923 to 1926 he was Assistant Naval Attaché at London, Paris, Berlin, and Rome. He was appointed assistant vice-president of the Bell Telephone Laboratories in 1923 which position he held until 1926 when he became vice-president of the Goodyear-Zeppelin Corporation. He assumed his present duties in 1933.

Contributed by the Hydraulic Division for presentation at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

[2] "A Magnetostriction Oscillator Producing Intense Audible Sound and Some Effects Obtained," by N. Gaines, *Physics*, vol. 3, no. 5, 1932, pp. 209–229.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.



FIG. 2 LEAD DISK 5/8 IN. DIAMETER AFTER 6 MIN OSCILLATION IN CALCIUM-CHLORIDE SOLUTION

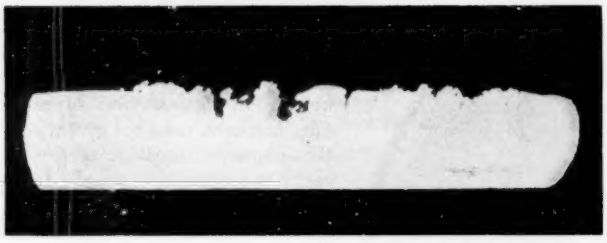


FIG. 3 CROSS-SECTION OF THE DISK SHOWN IN FIG. 2

with that observed on lead exposed to hydraulic cavitation in the original apparatus. However, the damage to lead is not consistently measured by loss of weight and the tests are being repeated with cement disks. The results of these tests will be reported later.

It is found that either with lead or with cement, appreciable damage is done in one minute and a test can be completed in less than 10 minutes. Fig. 1 shows the surface of a lead disk after oscillation of one-half minute duration in saturated calcium-chloride solution. Figs. 2 and 3, the latter a cross-section of the disk in Fig. 2, show a disk similar to the one in Fig. 1 after six-minutes exposure. It will be noted that material on the surface is displaced plastically and not lost. On the other hand, a cement surface is not displaced plastically and damaged particles are washed away. Fig. 4 shows the appearance of a cement disk after 1.2 min exposure to oscillation in water.



FIG. 4 CEMENT DISK 5/8 IN. DIAMETER AFTER 1.2 MIN OSCILLATION IN WATER

This new method of high-frequency oscillation permits a rapid estimation of the relative resistance to cavitation damage of various materials of construction. Likewise the method lends itself to a fundamental study of the influence of the properties of the liquid on cavitation damage. To oscillate a sample in a beaker of fluid for a few minutes is much cheaper and quicker than to run hydraulic cavitation tests.

Progress in Cavitation Research at Princeton University

By LEWIS F. MOODY[1] and ALFRED E. SORENSON,[2] PRINCETON, N. J.

The authors describe test apparatus and an experimental method used to determine the critical point of cavitation, or cavitation limit, of liquids flowing in a closed system as caused by the formation of vapor-filled cavities in the system. The experiments here described were directed to determine the cavitation limit, at which cavitation begins, and not to ascertain the consequences of maintained cavitation. The authors show that the experiments give results entirely consistent with the cavitation principle as now generally applied.

The Problem of Determining the Cavitation Limit

As THE effective head causing flow through a hydraulic apparatus is increased, the velocity head at any point in the system increases correspondingly and this increase in velocity head entails a compensating reduction in pressure head. This relation holds until the absolute pressure at some point in the system reaches the vapor pressure of the liquid, below which it cannot go while liquid remains present. Any further increase in the effective head merely produces boiling, the conversion of part of the liquid into vapor. When this occurs the principle of continuity of flow then fails to apply and a marked change in the performance of the apparatus occurs. The production of vapor-filled cavities in the flowing liquid is what is meant by the term *cavitation*. The consequences are an interconnected pressure pulsation and condensation and reevaporation of the vapor pockets constituting a series of explosions, or more strictly implosions, with resulting mechanical vibration and pitting of the conduit walls. The experiments here described were directed to determine the critical point or cavitation limit, at which cavitation begins, and not to ascertain the consequences of maintained cavitation.

The principles governing the incidence of cavitation in hydraulic machines such as turbines and pumps were analyzed and formulated by one of the authors at the Hydroelectric Conferences of 1922[3] and 1925[4] in Philadelphia and by D. Thoma at the First World Power Conference in 1924 at London,[5] and need not be repeated here. The use of Dr. Thoma's simple and convenient expression for the criterion of cavitation has become well standardized and will be adopted here. The relation is $\sigma = (H_b - H_s)/H$, in which H is the effective head producing flow through the system; H_b is the height of a water barometer or the atmospheric head minus the vapor-pressure head, that is, $H_b = H_a - H_{vp}$; H_s is the elevation of the point where cavitation begins above the free-water surface under static conditions such as the tail-water surface in the case of a turbine or pump; and σ is a coefficient which is a constant for a given apparatus. The relation is not limited in application to machines having moving parts, but is equally applicable to stationary conduits.

Questions have been raised as to the validity of this relation, and it has been stated by J. C. Hunsaker[6] in reporting on experiments on a special profile that "Experience with the profile shows that cavitation commences at pressures much higher than the true vapor pressure." (The authors suggest that the pressure here referred to may not have been that at the point of lowest pressure; also, that separation of air may be distinguished from true cavitation due to separation of condensable vapor.) It must be admitted that although the derivation appears to be based on logical deduction and its use in turbine practice seems to be satisfactory, nevertheless it is desirable to obtain direct experimental evidence of its validity or non-validity.

In the experiments here described, the formula was applied to a stationary conduit of simple form, namely a small venturi meter, in which the throat section can be relied upon to give good velocity distribution without curvature of the flow lines and to provide a piezometer location which may reasonably be expected to record the lowest pressure in the system. To avoid a varying elevation of the region of cavitation the conduit was kept horizontal. The entrance and discharge diameters were the same, so that the effective head producing flow was merely the difference in pressure heads, requiring no velocity-head corrections.

The Method Applied

As an indication of the performance of the apparatus, the authors used a ratio of heads representing the efficiency of conversion of velocity head into pressure head in the diffuser, or enlarging tapered section; that is

$$e = (h_{p3} - h_{p2})/(h_{p1} - h_{p2})$$

in which h_{p1}, h_{p2}, and h_{p3} are the pressure heads at entrance,

---

[1] Professor of Hydraulic Engineering, Princeton University, Princeton, N. J., and Consulting Engineer, Baldwin-Southwark Corp., Philadelphia, Pa. Mem. A.S.M.E. Professor Moody was graduated from the University of Pennsylvania in 1901 with a B.S. degree, and received his M.S. degree from the same University the following year. He has held positions as instructor of mechanical engineering at the University of Pennsylvania, 1902–1903; engineer in charge of tests, and designer and engineer in charge of construction for the I. P. Morris Company, 1904–1908; professor of hydraulic engineering at Rensselaer Polytechnic Institute, Troy, N. Y., 1908–1916; construction engineer, I. P. Morris Company and successors, 1911 to date; and professor of hydraulic engineering, Princeton University, 1930 to date.

[2] Assistant Professor of Engineering, Princeton University, Princeton, N. J. Jun. A.S.M.E. Professor Sorenson was graduated from Rensselaer Polytechnic Institute in 1924 with an M.E. degree and from that year until 1926 was engaged as instructor of mechanical engineering at the school from which he was graduated. Since 1926 he has been employed in his present position.

Contributed by the Hydraulic Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "The Hydraulic Turbine in Evolution," by H. B. Taylor and L. F. Moody, *Engineers and Engineering*, Engineers Club of Philadelphia, vol. 39, July, 1922, pp. 241–259.

[4] "Inter-Relation of Operation and Design of Hydraulic Turbines," by F. H. Rogers and L. F. Moody, *Engineers and Engineering*, Engineers Club of Philadelphia, vol. 42, July, 1925, pp. 169–187.

[5] "Experimental Research in the Field of Water Power," by D. Thoma, Transactions, First World Power Conference, London, 1924, vol. 2, pp. 536–551.

[6] "Cavitation Research at M.I.T.," by J. C. Hunsaker, *Mechanical Engineering*, vol. 57, April, 1935, pp. 211–216.

throat, and discharge, respectively, expressed as heights of water column above the centerline of the conduit, and e is the efficiency of the diffuser based on pressure heads (without introducing the small velocity heads at entrance and discharge). This ratio was used as an indicator because the deceleration process may be expected to be particularly sensitive to change of flow pattern. The tests were run to determine whether the point of

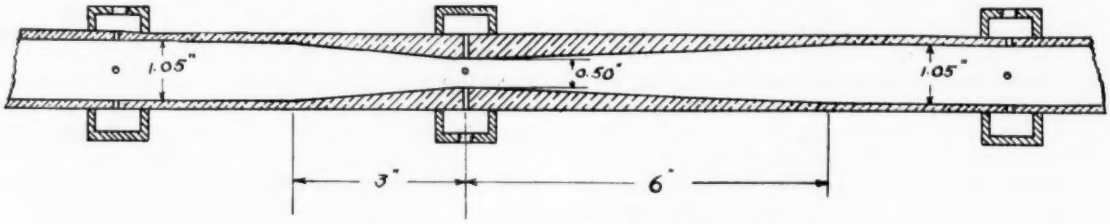


FIG. 1 CROSS-SECTION OF VENTURI METER

cavitation as evidenced by a sudden dropping-off in efficiency or discontinuity in the efficiency curve would coincide with the reaching of the vapor pressure at the throat of the conduit. The difference in pressure heads between discharge and throat is dependent on the velocity head and may be expressed as a coefficient σ multiplied by the effective head producing flow or

$$h_{p3} - h_{p2} = \sigma H$$

The absolute pressure at the throat is

$$H_{p2} = h_{p2} + H_a = h_{p3} + H_a - \sigma H$$

in which H_a is the atmospheric pressure head. As H is increased, H_{p2} decreases until it reaches the limit H_{vp}, the head corresponding to the vapor pressure. This according to the theory is the cavitation point, and the corresponding σ can be found from the equation

$$\sigma = (h_{p3} + H_a - H_{vp})/H = (H_b + h_{p3})/H$$

in which H_b is the barometric head $H_a - H_{vp}$.

If the pressure at discharge is lower than atmospheric, and the top of the water column is H_s ft below the elevation of the throat, then h_{p3} becomes $-H_s$, and $\sigma = (H_b - H_s)/H$, the form used for turbines or pumps.

By plotting the efficiency of the diffuser e versus σ, a point of discontinuity in the curve will indicate a change in the mode of flow such as would be caused by the formation of a vapor pocket. If the theory is correct this point should coincide with the point where the absolute throat pressure just reaches the vapor pressure of the water.

Finally, the results were plotted, as shown in Figs. 4 and 5, from which it will be seen that the break in the curve of efficiency versus σ, Fig. 5, coincides exactly with the reaching of the vapor pressure at throat as shown by Fig. 4. The corresponding values of the throat pressure are also shown in Fig. 5 for various values of σ, to show this coincidence more clearly. The slight slope of the efficiency curves in the region of no cavitation reflects the changing Reynolds number. The two curves for different ranges of back-pressure show agreement with the theory in each case.

APPARATUS AND PROCEDURE

The work described in this paper was conducted in the hydraulic laboratory of the Princeton University School of Engineering with the apparatus layout shown in Figs. 1 and 2. The venturi meter used was of brass, manufactured in the shop at the University. Before the tests were made, the instrument was carefully cleaned and the piezometer openings smoothed off with fine emery cloth. The throat diameter was 0.50 in. and the upstream and downstream sections measured 1.05 in. in diameter.

The water for these experiments was pumped from the laboratory sump to a tank on the top floor of the building from which it flowed by gravity through $1^1/_2$-in. piping to the venturi meter. The water was then discharged through $1^1/_2$-in. piping into a

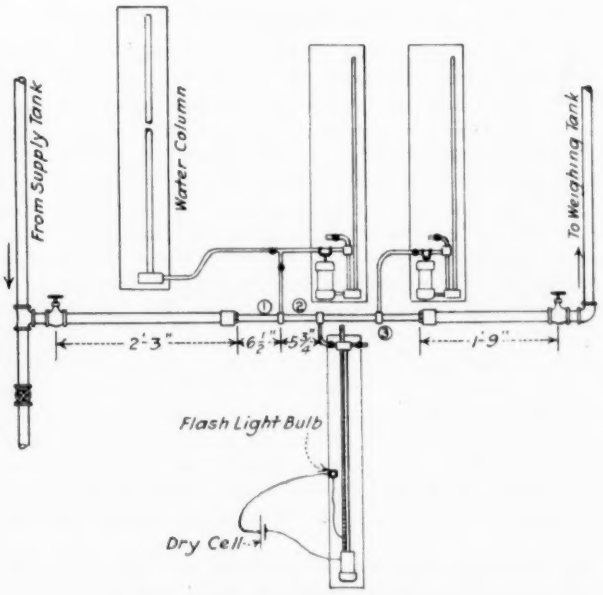


FIG. 2 GENERAL LAYOUT OF APPARATUS

pair of weighing tanks which were not used in this series of experiments.

Many types of gages were used before suitable instruments were obtained. There was so much oscillation in the plain U-tube manometer that it had to be modified for use in this connection. The gages for measuring the pressures at points 1 and 3 in Fig. 2 were made as shown by using a 2-in. pipe nipple capped on both ends, as a mercury pot. The pot was connected by a $^1/_8$-in. nipple to a cast-iron header which held the gage glasses. The gage glasses were $^3/_8$-in. outside diameter and $^1/_4$-in. inside diameter and were held in the header by means of rubber stoppers. Meter sticks graduated in millimeters were used as scales on all gages. The gage for recording the pressure at the throat of the meter had to be made differently because of the vacuum at this point. The gage which was finally successful was similar to a barometer but the scale was adjustable and was fitted with a strip of stainless steel set at its exact zero mark. The strip was connected through a dry cell to a flashlight bulb, so that when the point touched the mercury level in the pot, the circuit was completed and the bulb would light. It was found that this arrangement was very sensitive and gave excellent results.

The top of the gage glass was connected by means of a cast-iron header to the throat of the venturi. This header had to be made extremely tight and was boiled in paraffin to close all pores. As finally used, this gage was able to hold a 28-in. vacuum overnight. Connections to all gages were made with $^1/_4$-in. copper tubing and Dole compression fittings. Valves and vent valves on the gages were $^1/_8$-in. Dole needle valves. A water column 10 ft high made of $^1/_2$-in. glass tubing was used as a calibrating device for the gages.

The specific gravity of the mercury was first checked by finding the deflection of the mercury manometers as compared to a given height of water column. This varied slightly from day to day due to changes in temperature.

The downstream valve was opened up as wide as possible and the flow regulated by the valve upstream. It was found that as the quantity of water passing through the meter increased, the absolute pressure at the throat of the meter decreased to a certain point when it became constant no matter how much the velocity of the water at the throat was increased. This then should be the vapor pressure of the water. However, this pressure was always between 2 and 3 in. of water lower than the vapor pressure shown by the steam tables for water at the temperature used in the tests.

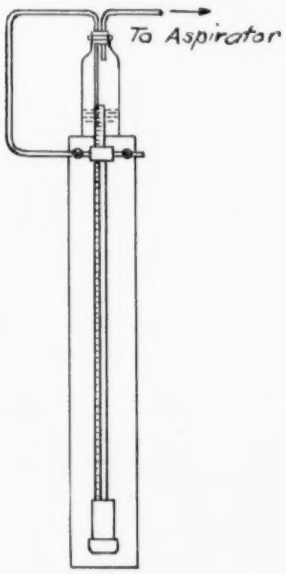


FIG. 3 APPARATUS FOR FINDING VAPOR PRESSURE

The water used in the tests was obtained from a well and it was found to be fairly hard. It was decided then to find the vapor pressure of the water. This was done by means of a static water column connected to a bottle and connected in turn to an aspirator, as shown in Fig. 3, forming a water barometer. This procedure gave a vapor pressure for the water which corresponded exactly with the minimum pressure found in the venturi meter. In order to check this method of determining vapor pressure, distilled water was placed in the bottle and gage connections, and the experiment repeated. The vapor pressure found in this way agreed with the value given in the steam tables for water at the temperature which was being used.

A series of runs was made with the discharge valve wide open and a second series with this valve partly throttled. For each series of runs, the pressure at the throat of the meter was progressively lowered by adjusting the inlet valve, runs being taken at suitable intervals. As many points as possible were taken just before cavitation took place and also just after it had occurred. Also a number of runs were taken beyond the cavitation point. For each run, the pressures at the entrance to the meter, the throat of the venturi, and the exit from the instrument were read as nearly as possible at the same time. Since cavitation should begin at the upper wall of the throat rather than at the centerline, the heads were corrected to correspond to this point as a datum elevation. The observed data taken during the runs and the results of the tests are given in the appendix of the paper. Before the meter was cleaned out, no noticeable noise was produced when cavitation occurred, but after the greasy deposit on the walls had been removed, cavitation caused a continual noise with periodic major disturbances causing a considerable variation in pressure at points 1 and 3, Fig. 2. This sound effect gives secondary evidence of the beginning of cavitation.

CONCLUSIONS

While the authors have no basis for asserting that the cavitation formula will satisfy all conceivable arrangements and conditions, these experiments appear to answer the question of its validity, for the apparatus and conditions adopted, by confirming it. This was true for two different values of back pressure, showing that σ remains constant and independent of the back pressure, in accordance with the theory. It was found that when dealing with hard waters, heavily charged with salts, the vapor pressure may be appreciably higher than that given for distilled water in the steam tables, and when applying the formula to unknown waters a small allowance should be provided in the value of H_b to cover this factor. At least when using ordinary run-of-laboratory water free from abnormal air content, the formula appears to be perfectly satisfactory, without modification.

In brief, these experiments give results entirely consistent with the cavitation principle as now generally applied.

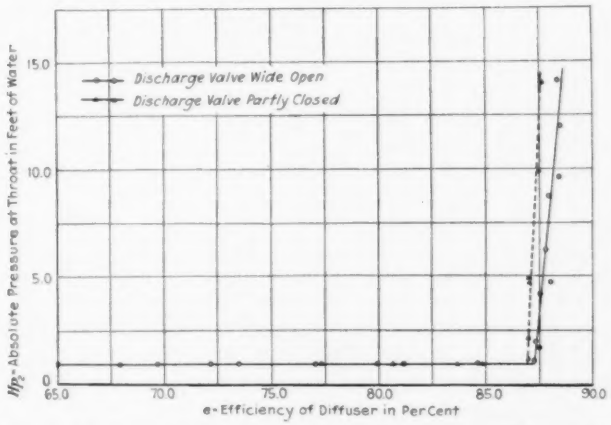


FIG. 4 CURVE OF ABSOLUTE PRESSURE AT THROAT OF METER VS EFFICIENCY OF DIFFUSER (H_{p_2} VS e)

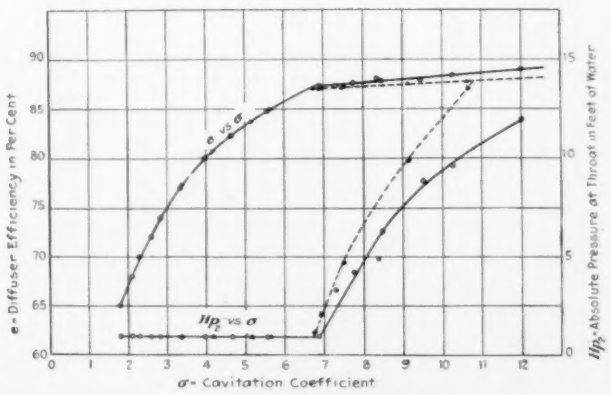


FIG. 5 CURVES OF EFFICIENCY OF DIFFUSER VS σ (e VS σ) ALSO ABSOLUTE PRESSURE AT THROAT OF METER VS σ (H_{p_2} VS σ)

Appendix

Observed Data Taken to Determine Cavitation Limits

Run No.	Gage No. 1 Col 1	Gage No. 1 Col 2	Gage No. 1 Net	Gage No. 2 Hg col	Gage No. 2 Hg ht below datum	Gage No. 3 Col 1	Gage No. 3 Col 2	Gage No. 3 Net
1	3.00	10.64	7.64	40.48	134.1	3.34	5.06	1.72
2	2.66	11.42	8.76	45.66	134.0	3.70	6.20	2.50
3	2.94	14.20	11.26	51.46	134.2	3.32	7.10	3.78
4	2.76	14.36	11.60	53.54	134.2	3.80	7.32	3.52
5	2.88	18.04	15.60	59.56	134.4	3.36	9.18	5.82
6	2.80	18.54	15.74	62.12	134.3	3.80	9.36	5.56
7	2.52	18.74	16.22	64.66	134.4	3.60	9.64	6.04
8	2.80	21.58	18.78	66.58	134.5	3.32	11.04	7.72
9	2.72	21.14	18.42	69.32	134.4	3.64	10.86	7.22
10	2.40	20.66	18.26	70.64	134.5	3.54	10.66	7.12
11	2.74	25.00	22.26	72.10	134.6	3.20	13.40	10.20
12	2.60	30.68	28.08	72.50	134.6	3.20	15.94	12.74
13	2.20	30.56	28.36	72.40	134.6	3.48	13.96	10.48
14	2.50	34.72	32.22	72.45	134.6	3.12	14.18	11.06
15	2.40	39.52	37.12	72.40	134.6	3.20	15.06	11.86
16	2.30	44.96	42.66	72.40	134.6	3.20	15.82	12.62
17	2.30	48.92	46.62	72.40	134.6	3.18	16.74	13.56
18	2.20	54.80	52.60	72.40	134.6	3.20	17.72	14.52
19	2.10	59.06	56.96	72.40	134.6	3.22	18.46	15.24
20	1.96	66.30	64.34	72.40	134.6	3.16	19.42	16.26
	Second Series—Discharge Valve Throttled							
1	2.90	22.10	19.20	42.00	134.1	3.50	15.04	11.54
2	2.76	27.00	24.24	50.62	134.2	3.44	18.34	14.90
3	2.16	27.88	25.72	62.86	134.3	3.44	17.52	14.08
4	2.56	34.88	32.32	63.34	134.4	3.36	23.48	20.12
5	2.50	40.32	37.82	68.08	134.4	3.30	26.92	23.62
6	2.14	31.56	29.42	69.42	134.5	3.38	19.88	16.50
7	2.46	42.26	39.80	71.20	134.9	3.30	28.68	25.38
8	2.36	45.88	43.52	72.40	134.6	3.30	29.40	26.10
9	2.30	49.50	47.20	72.40	134.6	3.30	31.18	27.88
10	2.10	59.82	57.72	72.40	134.6	3.20	35.90	32.70
11	2.00	67.78	65.78	72.40	134.6	3.18	37.74	34.56

Results of Tests to Determine Cavitation Limits

Run No.	H_{p1} ft gage	H_{p2} ft gage	H_{p3} ft abs	H_{p3} ft gage	$\frac{H_{p3} - H_{p2}}{H_{p1} - H_{p2}}$	$\frac{H_b - H_v}{H}$
1	3.122	—19.570	14.123	0.484	0.884	12.590
2	3.643	—21.716	11.993	0.843	0.890	12.000
3	4.730	—24.097	9.596	1.399	0.884	10.248
4	4.910	—24.971	8.738	1.299	0.879	9.431
5	6.460	—27.440	6.253	2.308	0.878	8.440
6	6.734	—28.520	4.970	2.534	0.881	8.355
7	6.959	—29.562	4.147	2.416	0.876	7.742
8	8.070	—30.335	3.358	3.152	0.872	7.297
9	7.925	—31.492	1.998	2.916	0.873	7.082
10	7.863	—32.031	1.678	2.894	0.875	7.174
11	9.616	—32.615	1.078	4.251	0.873	6.894
12	12.199	—32.777	0.916	5.336	0.847	5.547
13	12.351	—32.756	0.953	4.388	0.823	4.665
14	14.037	—32.756	0.937	4.630	0.799	3.972
15	16.213	—32.736	0.957	4.989	0.771	3.361
16	18.674	—32.736	0.957	5.327	0.740	2.852
17	20.435	—32.736	0.957	5.701	0.723	2.608
18	23.092	—32.736	0.957	6.190	0.697	2.303
19	25.027	—32.736	0.957	6.493	0.679	2.117
20	28.305	—32.736	0.957	6.945	0.650	1.858
	Second Series—Discharge Valve Partly Closed					
1	8.277	—20.196	13.497	4.770	0.877	10.695
2	10.515	—23.750	9.943	6.242	0.875	9.122
3	11.174	—28.800	4.909	5.989	0.870	7.473
4	14.102	—28.997	4.696	8.561	0.871	7.453
5	16.554	—30.981	2.509	10.207	0.866	6.738
6	12.821	—31.528	2.181	7.064	0.870	6.917
7	17.435	—32.268	1.222	10.990	0.870	6.757
8	19.077	—32.736	0.957	11.219	0.848	5.594
9	20.711	—32.736	0.957	12.010	0.837	5.143
10	25.383	—32.736	0.957	14.151	0.807	4.174
11	28.965	—32.736	0.957	14.977	0.773	3.411

Notes on First Table

Datum plane level = 45.87 cm above center line of venturi meter; gage 1 datum level = 61.35 cm; gage 2 datum level = variable; gage 3 datum level = 61.58 cm; all readings in this table are average values; all readings are given in centimeters.

The Measurement of Air Flow in Fan Inlet and Discharge Ducts

By LIONEL S. MARKS,[1] CAMBRIDGE, MASS.

This paper is devoted to a proposed method for determining the volume of air handled by any fan which is provided with either inlet or discharge ducts. The proposal is to use pitot tubes for capacities near the wide-open condition and square-edged inlet or discharge orifices for lower capacities down to the no-flow condition. To make the pitot-tube measurements accurate, a standard form of pitot tube is suggested, based on the investigations of Ower[2] and the unpublished researches of Merriam and Spaulding at Worcester Polytechnic Institute. The coefficients of Ebaugh and Whitfield[3] are suggested for use with inlet orifices, while those of Stach[4] are suggested for discharge orifices used with flange pressure taps on the approach side. The alternative procedure of using an impact tube with the discharge orifice is discussed, but coefficients are not as yet available.

IN MAKING tests of fans with inlet or discharge ducts, the volume of air to be measured will vary from a maximum to zero, the variation being obtained by changing restrictions on either the inlet or the discharge side of the fan. The restriction may either be separate from the volume-measuring device or may be an essential part of it.

The volume-measuring devices and restrictions which come under consideration are (1) the pitot tube, with independent restrictions; (2) a single nozzle in the duct (duct nozzle) with independent restrictions, or a series of nozzles used individually, acting as restrictions; (3) a series of inlet or discharge nozzles at the end of the duct, acting as restrictions; (4) an orifice in the duct (duct orifice) with independent restrictions, or a series of orifices acting as restrictions; and (5) a series of inlet or discharge orifices, acting also as restrictions.

No one of these devices is satisfactory for the complete range of operation of a fan. The pitot tube will not give accurate indications at low fan capacities; nozzles and orifices will not permit the investigation of a fan with wide-open discharge. When a large single nozzle or orifice is used (with supplementary restricting devices), it will not give accurate indications at low capacities; while if the single nozzle or orifice is small, it will not permit fan operation near the wide-open condition.

As between nozzles and orifices, assuming that the coefficients of both are equally definite for stated conditions and that they have been determined with equal accuracy, the choice must fall on the orifice. In fan testing, the ducts may be as large as 10 ft diameter. Well-rounded nozzles of large size are costly and heavy. Other nozzle forms, fabricated from sheet metal, have been proposed, but they cannot be made inexpensively of any desired exact form and dimensions, and consequently their coefficients cannot be ascertained with accuracy. Moreover, they are heavier and more costly than orifices. The one advantage of a well-rounded nozzle is that its coefficient can be determined with sufficient accuracy by means of a traverse with a small impact tube. Such a nozzle is more nearly a primary standard for the measurement of large quantities of air than any of the other devices proposed. As such it may be used for the calibration of other measuring devices such as the orifice. When the coefficient of an orifice has been determined by the use of a nozzle or other means, it becomes just as accurate as the nozzle and is much cheaper and more convenient.

A duct orifice is practicable only for a single orifice because of the difficulty involved, and the time consumed, in changing orifices in a large-sized duct. A single orifice presents the disadvantages stated previously of having either low accuracy at low capacities or of preventing fan operation near the wide-open condition. Moreover, this method requires a considerable total length of duct. The only advantage of duct orifices over discharge orifices is that the coefficients for duct orifices have been well established by a number of investigators.

There remains the use of a series of orifices either at the entrance to the inlet duct (inlet orifices) or at the end of the discharge duct (discharge orifices). This is the simplest and most convenient method of measuring air and is comparatively inexpensive. Its only disadvantage is that it is not applicable to measurements at or near the wide-open condition.

It would appear from this survey that two of the methods enumerated for measuring the flow of air in a fan duct have definite advantages over all the other methods. The pitot-tube method can be made quite satisfactory at higher fan discharges, while the square-edged inlet or discharge orifice is equally satisfactory at lower fan discharges. A combination of the two methods should permit accurate volume measurements throughout the whole range of fan operation. The author proposes, therefore, that in testing fans which are to be used with inlet or discharge ducts, the air measurements be made by a pitot-tube traverse at capacities approximately down to the capacity at maximum efficiency and from there down, by square-edged orifices. Pitot-tube traverses should also be made while the two largest orifices are

---

[1] Professor of Mechanical Engineering, Harvard University. Mem. A.S.M.E. Professor Marks was born in Birmingham, England. He received the degree of B.Sc. from the University of London in 1892 and M.M.E. from Cornell University in 1894. He was with the Ames Iron Works, Oswego, N. Y., in 1894 and then went to Harvard University as instructor in mechanical engineering. In 1900 he was made assistant professor and in 1909 was advanced to his present position. Professor Marks is author of "Steam Tables and Diagrams," "Gas and Oil Engines," "Mechanical Engineers' Handbook," "The Airplane Engine," and has contributed numerous articles to the technical press.

[2] "The Measurement of Air Flow," by E. Ower, Chapman and Hall, London, 1933; also, "The Design of Pitot-Static Tubes," by E. Ower and F. C. Johansen, Reports and Memoranda No. 981, Technical Reports of the Aeronautical Research Committee, August, 1925, London, England.

[3] "The Intake Orifice and a Proposed Method of Testing Exhaust Fans," by N. C. Ebaugh and R. Whitfield, Trans. A.S.M.E., vol. 56, December, 1934, paper PTC-56-3, pp. 903–912.

[4] "Die Beiwerte von Normdüsen und Normblenden im Einlauf und Auslauf," by E. Stach, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 78, no. 6, February 10, 1934, pp. 187–189.

Contributed by the Power Test Code Committee No. 10 on Centrifugal and Turbo-Compressors and Blowers for presentation at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

in use so as to determine the accordance of the two methods of measurement.

Before the proposed method can be used satisfactorily, it is necessary (1) to standardize the pitot tube and methods of using it, and to know its errors, and (2) to standardize the square-edged orifice and methods of using it, and to ascertain its coefficients under various conditions of use.

Standardization of the Pitot Tube

The desirable characteristics of a standardized pitot tube are (1) known coefficients, eliminating the necessity for calibration; (2) ease of duplication with ordinary machine-shop processes; (3) insensitivity to minor changes in form and dimensions; (4) static holes as near the tip as possible; (5) rigidity; and (6) small size.

The features to be considered while designing such a standardized pitot tube are (1) diameters, thicknesses, and materials of the outer and inner tubes; (2) the shape of the impact tube; (3) the size of the impact opening; (4) the distance of the static holes from the tip; (5) the distance of the static holes from the stem; and (6) the size, number, and arrangement of the static holes.

The researches of Ower and Johansen,[2] and the unpublished researches of Merriam and Spaulding at Worcester Polytechnic Institute give the necessary data for selecting a design. The two entirely independent investigations, covering to a large degree the same field, give results remarkably accordant and justify complete confidence in their reliability.

The selection of diameters and thicknesses of the inner and outer tubes is arbitary. Adequate stiffness can be obtained for traversing ducts 3 or 4 ft across by making the outer tube of 0.04-in. thick brass (No. 18 gage), $^5/_{16}$ in. outside diameter, and the inner tube preferably of 0.0285-in. thick copper (no. 21 gage), $^1/_8$ in. outside diameter. Other sizes may be used without affecting accuracy as long as geometrical similarity is maintained.

The shape of the impact tube should preferably be hemispherical. A conical tip of proper design will give equally satisfactory results, but it is more liable to damage, increases the total length of the tube, and locates the static openings at a greater distance from the impact opening.

The diameter of the impact opening is immaterial when the tube is directed accurately into the air stream, but has considerable influence on the impact readings under conditions of yaw. This is illustrated, for a hemispherical head, by the following tabulation taken from the work of Merriam and Spaulding at Worcester Polytechnic Institute:

Ratio of impact opening to tube diameter	0.5	0.4	0.3	0.2
Error in impact reading, per cent of velocity head, with 16-deg yaw	3.3	4.5	6.0	8.5

The velocity-head reading under conditions of yaw will vary with both the impact reading and the static reading. If the errors of both impact and static readings in yaw were of the same sign and magnitude, the errors would cancel and the velocity-head reading would be correct. This desirable condition cannot be realized by any known design of pitot tube. With an impact opening 0.2 times the diameter and with the static-hole design recommended later in this paper, the velocity-head readings vary less with yaw than when the impact openings are larger.

If it is desired to find only the velocity or volume of the air flowing, an impact opening of 0.2 times the tube diameter should be selected. But in fan testing it is also necessary to find the work done on the air and this is proportional to the product of (1) the volume of air delivered, and (2) the impact pressure. An impact opening 0.2 times the tube diameter gives satisfactory values of the difference between impact and static pressures up to 12-deg yaw but gives larger errors in the values of the impact pressure, and of the work done on the air, than result from the use of an impact opening 0.4 times the tube diameter. This latter dimension appears to be the best to select for fan testing. With an effective straightener to eliminate yaw, all sizes of impact opening would give the same indications. The depth of this opening has no influence on the impact reading.

The variation of velocity head with yaw, and the error in the calculated work done on the air, with zero static pressure, for an impact opening 0.4 times the tube diameter (with stem-effect error included) is found to be:

Yaw angle, deg	8.0	12.0	16.0	20.0
Error in velocity head, per cent	—0.8	—1.7	—0.5	1.7
Error in work done, per cent	0.1	0.5	4.3	8.2

The error in calculated work done on the air diminishes as the static pressure is increased, being halved when the static pressure is equal to the velocity head.

The distance from the static holes to the tip of the tube, is made up of the length of the tip proper (conical or hemispherical portion) and a length of straight tubing between the tip proper and the static holes. The latter alone is found to have an influence on the static readings. This influence is considerable. The effect of the tip is to make the static reading less than the true static reading by an amount which depends on the length of straight tubing between the static holes and the tip, and on the velocity head. The error diminishes rapidly as the length of straight tubing increases, and becomes practically constant when this length is eight tube diameters. At this location the static-pressure reading is low by about 0.2 per cent of the velocity head according to Ower;[2] it is still smaller according to Merriam and Spaulding. This distance of eight tube diameters is indicated as desirable for the location of the static holes.

The distance from the static holes to the stem has an important influence on the static reading. According to Ower,[2] the presence of the stem increases the upstream static pressure as in the following tabulation:

Distance of stem aft of static holes, in tube diameters	5	10	15
Excess pressure due to stem, in per cent of velocity head	1.75	0.8	0.5

The investigations of Merriam and Spaulding show a slightly greater pressure excess. As this error cannot be eliminated without removing the stem to an impracticably great distance, and as the error changes very slowly after a length of 16 diameters, it seems desirable to select this length of 16 diameters for the distance between the static holes and the stem axis.

With the stem extending across the duct (which may be desirable for very large ducts) the static error has twice the value just given in the tabulation and, therefore, for a 16-diameter length, may be as much as 1 per cent of the velocity head.

The number of static holes and their distribution around the tube is determined principally by the consideration that the degree and direction of yaw, for the air current being measured, will ordinarily be unknown. Consequently the static holes should be uniformly distributed around the tube and should be numerous. The logical development of this concept leads to a gap instead of a number of holes, as in the the Prandtl tube. The objection to the gap is that it reduces the rigidity of the tube and in practice may lead to slight flexure at the gap. This would have the same effect as a burr and, while difficult to see, would materially affect the static reading. Eight holes with even spacing are suggested as a satisfactory compromise. If these holes are 0.04 in. diameter, they will take away less than one-third of the tube material.

The size of the static holes influences the static-pressure error

under yaw. Small holes (0.02 in. diameter) have the advantage of giving comparatively small errors in yaw but are likely to clog and yield static readings which under yaw conditions are very sensitive to changes in diameter. With holes 0.04 in. diameter, the following results were obtained by Merriam and Spaulding:

Yaw angle, deg	8.0	12.0	16.0	20.0
Static error, per cent of velocity head	1.5	3.5	5.5	8.0
Static error, including stem effect, per cent	1.0	3.0	5.0	7.5

It is proposed for the purpose of achieving comparative insensitiveness to slight imperfections, that the static holes be made 0.04 in. in diameter. One other feature is recommended. The inner tube should be maintained coaxial with the outer tube in the vicinity of the static holes. A spacing ring soldered to the

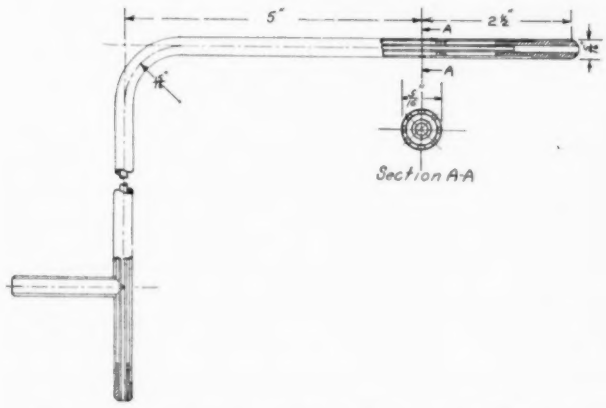


FIG. 1 PROPOSED STANDARD PITOT TUBE FOR FAN TESTING

copper tube will achieve this object. A pitot tube embodying the proposed features is shown in Fig. 1.

SQUARE-EDGED ORIFICES

The proposal to use square-edged orifices as a measuring device in fan testing is based upon the assumption that these orifices will always be comparatively large. If they are small, the condition of the edge of the orifice may influence appreciably the coefficient of discharge. Orifices above 8 in. diameter (approximately) will give predictable coefficients of discharge when they are machined with ordinary care and all burrs are removed. For smaller orifices, greater care is necessary; when the diameter is as small as 1 in., the orifice coefficient will be found to vary considerably with the condition of the edge of the orifice in which case the nozzle is preferable.

In addition to the minimization of the effect of edge sharpness when the orifice is large, there is the further fact that the coefficient varies but little with orifice size when the size is considerable. The elimination of these two disturbing factors leaves the orifice coefficient a function of two factors only: (1) the Reynolds number, and (2) the ratio of orifice area to duct cross-sectional area. These factors are considered later.

INLET ORIFICES

Square-edged inlet orifices have been investigated recently by Ebaugh and Whitfield[3] for a 22³/₄-in. duct with a 20 to 100 per cent orifice-area ratio. Some earlier work by Watson and Schofield[5] deals only with small orifices from ½ to 2 in. diameter and for very small orifice-area ratios and is not applicable to fan conditions. Both these investigations show discharge coefficients very close to 0.6. The volume of air flowing is based entirely upon the pressure drop at the orifice and this necessitates a choice as to the location of the pressure tap in the duct. The Ebaugh and Whitfield coefficients[3] are based on the location of the pressure tap in the plane of the vena contracta.

The recent investigations of Stach,[4] using the I.S.A. form of square-edged orifice with flange taps and carried out on a 10-in. duct with orifice-area ratios of 0.25, 0.36, 0.49, and 0.64 give a constant value of the coefficient of discharge of 0.60 for Reynolds' numbers in excess of 55,000. The accuracy of the coefficient is given by the following percentage tolerances:

Orifice-area ratio	0.25	0.36	0.49	0.64
Tolerance, per cent	+ 1.3	± 1.0	± 0.5	± 0.5

It would appear, using flange taps, that the coefficient of discharge is not quite constant and that the assumption of constancy leads to larger errors than are permissible at low orifice-area ratios. As a practical matter, it is desirable to have a definite location for the downstream pressure tap. Ebaugh and Whitfield[3] find that a location 40 per cent of the pipe diameter downstream will permit the use of a constant coefficient of 0.601 throughout a considerable range of orifice-area ratios with an error of less than 0.5 per cent for ordinary pressure drops.

The large amount of research on pipe orifices cannot be considered applicable to the conditions of an inlet or a discharge orifice. The bibliography of Ebaugh and Whitfield[3] does not include any investigations which actually apply to an inlet orifice of a size suitable for the testing of fans. While their work needs verification by others and extension to larger ducts, greater pressure drops, and smaller orifice-area ratios, it may be accepted for the present.

DISCHARGE ORIFICES

Square-edged discharge orifices have been investigated by Durley[6] up to 4½ in. diameter for very small orifice-area ratios. by Muller[7] up to 2½ in. diameter with large orifice-area ratios, and by Stach[4] for orifices from 5 to 8 in. diameter in a 10-in. duct.

The work of Durley[6] and Müller[7] does not cover the range of orifice sizes important in fan testing. The work of Stach[4] with I.S.A. orifices having flange pressure taps before the orifice, was for the same set of conditions as those stated previously, in the discussion of inlet orifices. The coefficients which Stach found are slightly lower than those for pipe orifices and vary with the orifice-area ratio m. The values are:

m	0.05	0.10	0.15	0.20	0.25	0.30	0.35
coefficient	0.598	0.602	0.608	0.615	0.624	0.636	0.651
m	0.40	0.45	0.50	0.55	0.60	0.65	0.70
coefficient	0.666	0.682	0.701	0.724	0.751	0.784	0.820

These values are found to hold for Reynolds' numbers R in excess of the following values:

m	0.25	0.36	0.49	0.64
R	45 000	55,000	75,000	183,000

The accuracy of these coefficients is given as ± 0.8 per cent. They apply to the simple flow equation $V^2 = 2gh$; and include the correction for velocity of approach.

The investigations of Stach[4] were made with a high degree of precision and his results apparently may be used with confidence.

The Reynolds numbers for inlet and discharge orifices in fan tests will ordinarily be in excess of the limit set by Stach.[4] The

[5] "On the Measurement of the Air Supply to Internal-Combustion Engines by Means of a Throttle Plate," by William Watson and H. Schofield, Proceedings of the Institution of Mechanical Engineers, May, 1912, pp. 517–552.

[6] "Measurement of Air Flowing Into the Atmosphere Through Circular Orifices in Thin Plates and Under Small Differences of Pressure," by R. J. Durley, Trans. A.S.M.E., vol. 27, 1906, pp. 193–231.

[7] "Messung von Gasmengen mit der Drosselscheibe," by A. O. Müller, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 52, no. 8, February, 1908, pp. 285–290.

absolute viscosity μ of air varies from 1.175×10^{-5} at 32 F, to 1.51×10^{-5} at 212 F, in lb-ft-sec units, the variation being proportional to the temperature change. The Reynolds number is $DV\rho/\mu$, where D is the orifice diameter, ft; V is the velocity of air in the orifice fps; and ρ is the density of the air in lb per cu ft. At 70 F with standard air density of 0.075 lb per cu ft, $\rho/\mu = 6.05 \times 10^3$. Assuming the lowest practical pressure drop through the orifice as 0.1 in. of water, which corresponds to an air velocity of approximately 21 fps, and assuming an orifice diameter of 0.5 ft, the Reynolds number $R = 0.5 \times 21 \times 6.05 \times 10^3 = 6.35 \times 10^4$. The conditions here assumed are those which would give the minimum value of R that is likely to occur in fan testing. It will be seen that Reynolds' numbers for actual fan-test condition generally will be in excess of Stach's minimum for constant-discharge coefficients for square-edged orifices. Any doubtful case should be calculated.

The weight or volume of air flowing through a discharge orifice may be determined either from observations of the drop of static pressure through the orifice or from impact-tube readings. In recorded investigations, only the pressure-drop method appears to have been used. It would seem, however, that the impact-tube method is as applicable here as it is for nozzles and that it has the same advantages.[8]

A traverse along the center line of the orifice shows that for orifices 8 in. diameter or larger, the impact reading is constant from the plane of the orifice to several inches out from that plane, and that it falls off slowly at increasing distances. Therefore, it is proposed that the impact tube be placed at a distance of 1 in. out from the plane of the orifice.

The weight of air discharging from a square-edged orifice is given by the equation[9]

$$w = 0.1145\, c\, D^2 \sqrt{(GBi/T_1)}$$

where w is the weight of air flowing, lb per sec; c is the coefficient of discharge; D is the orifice diameter, in.; B is the mean discharge pressure, in. of mercury; i is the impact pressure, in. of water; T_1 is the temperature of the air approaching the orifice, deg F abs; and G is the specific gravity of the air referred to normal dry air at the same pressure and temperature. The quantity B is preferably the mean of the static pressures on the two sides of the orifice, since this gives a somewhat greater constancy of coefficient of discharge than the use of the discharge (barometric) pressure. For small pressure drops it is immaterial which of these pressures is used; with a pressure drop of 1 in. of water the difference in weight of air flowing is only 0.125 per cent, and for a pressure drop, of 4-in. of water it will be 0.5 per cent. No coefficients are available at present for the determination of the volume of air flowing through a square-edged discharge orifice by means of an impact tube.

A desirable form and method of assembly of the square-edged orifices is shown in Fig. 2. The orifices are made from $^1/_4$-in.

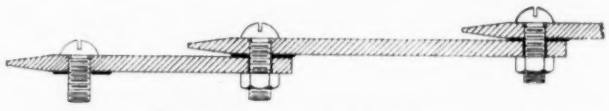


FIG. 2 SUGGESTED FORM AND METHOD OF ASSEMBLING SQUARE-EDGED ORIFICES

steel plate and are provided with studs to facilitate assembly. The studs are round-headed machine screws, $^3/_8$ in. diameter, threaded the whole length, and with the thread cut away in the immediate vicinity of the head. These screws are threaded into holes spaced 6 in. apart in the orifice ring; the holes are slightly countersunk on the entering side. If screwed up tight, the screws will act as studs. The studs are $1^1/_2$ in. from the orifice and are found to have no perceptible influence on the flow. A ring of hard packing such as vellumoid is shellacked on the back of each ring and the exposed face of the packing is graphited. A series of these orifices can be built from any desired maximum size, by steps of at least 4 in., down to any desired minimum size.

As shown in Fig. 2, the orifice plate is tapered on both sides for a distance of $^3/_4$ in. from the orifice, with a slope of 1 in. in 8 in., leaving a thickness of $^1/_{16}$ in. at the orifice. The purpose of this taper (which differs from the usual practice of having the approach side of the orifice flat and only the discharge side tapered) is to have an arrangement which will work equally well both for inlet orifices and discharge orifices. The justification is experimental. The author has investigated large orifices which were identical except that one was tapered, as in Fig. 2, and the other made in the usual manner with a flat approach side. The coefficients for orifices of this type are slightly higher than for the standard type.

[8] Discussion by S. A. Moss of "The Flow of Air and Steam Through Orifices," by H. B. Reynolds, Trans. A.S.M.E., vol. 38, 1916, p. 833.

[9] "Measurement of Flow of Air and Gas With Nozzles," by S. A. Moss, Trans. A.S.M.E., vol. 49–50, part 1, 1927–1928, paper APM-50-3.

The Definition of the Quantity-Head Characteristic of Fans

By M. C. STUART[1] and W. E. SOMERS,[2] BETHLEHEM, PA.

The purpose of a fan characteristic is to delineate usetul information regarding the performance of a fan in its installation. Specifically, the quantity-head characteristic shows the quantity of air delivered against a certain resistance in the intended installation. The authors discuss the set up of four basic arrangements of fan types and installations, and describe three currently used fan-head definitions. The definitions of fan head are evaluated, the fan head being defined as the discharge static head minus the inlet total head. A primary arrangement of each type of fan is described and a single primary quantity-head characteristic for each type of fan is defined. If a fan is installed in any arrangement other than its primary one, the primary characteristic cannot be assumed to apply, but a separate test must be conducted to determine its characteristic in its intended arrangement. Results of complete tests of a fan installed in its four possible arrangements are given to illustrate the principles which are set forth.

THE purpose of a fan characteristic is to delineate useful information regarding the performance of a fan in its installation. Specifically, the quantity-head characteristic shows the quantity of air which will be delivered against a certain resistance in the intended installation.

In general, it may be stated that for a given fan there does not exist a single easily defined relationship between the quantity of air discharged and the head produced by the fan. The two circumstances accounting for this situation are (1) the lack of agreement upon a definition of the term "head" as applied to fan performance, and (2) the performance of a given fan is influenced markedly by the different arrangements required in various applications.

[1] Professor of Experimental Mechanical Engineering, Lehigh University. Mem. A.S.M.E. Chairman, Subcommittee on Fans, Power Test Code Committee No. 10. Professor Stuart was graduated from the University of Pennsylvania in 1909 with a B.S. degree in mechanical engineering and received his mechanical-engineer's degree in 1926. Since graduation he served as instructor at Rensselaer Polytechnic Institute; assistant steam engineer with the Cambria Steel Company; mechanical engineer at the U. S. Naval Engineering Experiment Station, Annapolis, Md.; professor of mechanical engineering at the U. S. Naval Postgraduate School; and since 1926 has served in his present capacity at Lehigh University.

[2] Formerly Balwin Research Fellow, Lehigh University. Jun. A.S.M.E. Mr. Somers was graduated from Lehigh University with a B.S. degree in mechanical engineering in 1933, and received his master's degree in 1935. He was selected for fellowship on graduation and has continued in that position for the past two years. At present he is employed as a cadet engineer with the Babcock and Wilcox Company, Barberton, Ohio.

Contributed by the Power Test Code Committee No. 10 on Centrifugal and Turbo-Compressors and Blowers for presentation at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

DESCRIPTION OF BASIC FAN ARRANGEMENTS

An investigation of the different ways in which the arrangement of a fan in its application may affect the quantity-head characteristic reveals that four basic installation arrangements cover all cases. Before describing these basic arrangements the fan must be defined.

Prof. L. S. Marks in a series of papers[3] has shown that inlet boxes and other attachments at the inlets and outlets of fans have considerable effect on the performance of the fan in certain cases. Therefore, it is necessary in defining the fan to include not only the housing, but to have a definite understanding as to what attachments, such as inlet boxes, diffusers, mounting rings, flanges, dampers, etc., shall be considered as part of the fan. Having an understanding of what constitutes the fan, the four basic installation arrangements which are required to cover the various applications may now be developed. These basic installations are shown in Fig. 1 as arrangements *A*, *B*, *C*, and *D*.

Arrangement A—Bare Fan. In this arrangement there are no ducts attached to the fan; the intake is from one large space or room and the discharge is into another large space. This arrangement is that usually thought of in applications of propeller fans, although it is equally applicable to any housed fan. It may alternately be referred to as a wall installation.

Arrangement B—Blower. This arrangement consists of a fan installation with intake direct from a large space or room but discharging into a duct. This is the usual blower arrangement and is applicable whether the blower is of the centrifugal or the propeller type.

Arrangement C—Booster. This arrangement provides an intake through a length of duct and a discharge from the fan into a discharge duct. This arrangement is applicable to propeller as well as centrifugal-type fans thus installed.

Arrangement D—Exhauster. In this arrangement the intake is through an inlet duct but the air is discharged directly into a large space. It is applicable to all types of fans thus installed.

HEAD DEFINITIONS

Velocity Head h_v. The definitions of heads may be more readily understood if it is recognized that all heads represent essentially energy per pound of fluid. The velocity head at any point in the system, if expressed in feet of the fluid flowing, is the kinetic energy per pound of fluid at that point. This head expressed in inches of water is proportional to the kinetic energy by a factor involving the ratio of the density of water to that of the fluid flowing. In this investigation the velocity heads at the specified points in the inlet and discharge ducts were calculated from the quantity flowing, as measured by a nozzle installed

[3] "Influence of Inlet Boxes on the Performance of Induced-Draft Fans," by L. S. Marks and E. A. Winzenburger, Trans. A.S.M.E., vol. 54, 1932, paper FSP-54-16, pp. 213–220. "Influence of Bends in Inlet Ducts on the Performance of Induced-Draft Fans," by L. S. Marks, J. Lomax, and R. Ashton, Trans. A.S.M.E., vol. 55, 1933, paper FSP-55-9, pp. 133–143. "Influence of Bends or Obstructions at the Fan Discharge Outlet on the Performance of Centrifugal Fans," by L. S. Marks, J. H. Raub, and H. R. Pratt, Trans. A.S.M.E., vol. 56, 1934, paper FSP-56-12, pp. 767–771.

TABLE 1 QUANTITY-HEAD CHARACTERISTICS FOR THE INSTALLATION ARRANGEMENTS IN FIG. 1

Installation arrangement	Fan total head, H_T		Fan static head — 1st def., H_S		Fan static head — 2nd def., H'_S
A—Bare fan	H_{TA}	=	H_{SA}	=	H'_{SA}
B—Blower	H_{TB}		H_{SB}	=	H'_{SB}
C—Booster	H_{TC}		H_{SC}		H'_{SC}
D—Exhauster	H_{TD}	=	H_{SD}		H'_{SD}

NOTE: Characteristics H_{TA}, H_{SA}, and H'_{SA} can be represented by a single curve. Characteristics H_{SB} and H'_{SB} can be portrayed by a single curve. Characteristics H_{TD} and H_{SD} can be portrayed by a single curve. A separate curve is required for each of the following characteristics: H_{TB}, H_{TC}, H_{SC}, H'_{SC}, and H'_{SD}.

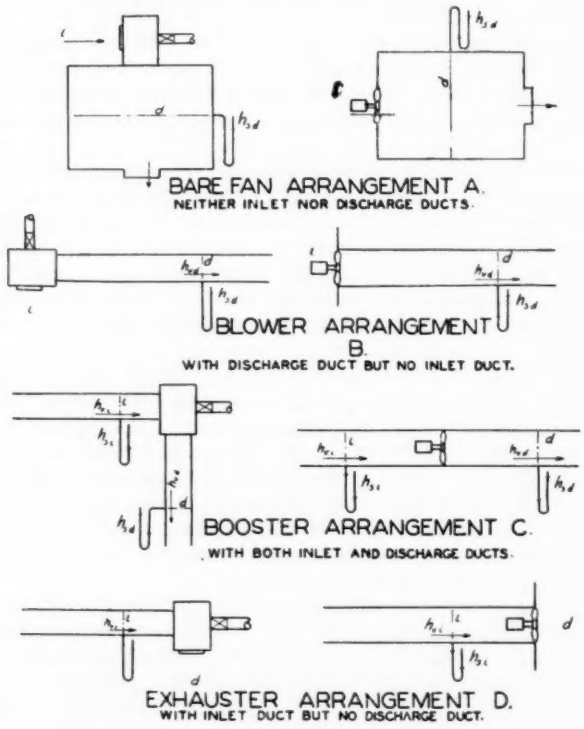


FIG. 1 BASIC ARRANGEMENTS OF FAN INSTALLATIONS

elsewhere in the system, and a knowledge of the duct area at the points in question.

Static Head h_s. Static head, if expressed in feet of the fluid flowing, is the work required per pound to cause the fluid to move against the pressure measured, in fan work, above atmosphere. This static head is equal to the product of the pressure above atmosphere and the specific volume of the air. It has been variously named pressure energy, flow work, displacement energy, and pressure head. This head, if expressed in inches of water, is proportional to the pressure energy by a factor involving the ratio of the density of water to that of the fluid flowing. In this work the static head was determined by observation of the static-pressure gages attached at the specified points.

Total Head h_t. The total head at a point in the system is the sum of the pressure energy and the kinetic energy, both measured at the point in question. If this head is expressed in inches of water, it will be proportional to this sum of energies.

In fan performance we are concerned with certain changes in energy that the fan produces for a given discharge. These increases in energy are the differences between the corresponding heads at the discharge and the inlet. These differences in head will be called the "fan heads," represented by the general symbol H, as distinguished from h, which is the symbol for the head at a certain point.

Of the several possible fan heads which may be obtained from various differences between the inlet and discharge heads, only three have been considered of value in fan work. These three are defined in the following paragraphs.

Fan Total Head. Fan total head is the total head at some specified plane on the discharge side of the fan minus the total head at some specified plane on the inlet side. It can be expressed as

$$H_T = h_{td} - h_{ti}$$

where H_T is the fan total head, h_{td} is the discharge total head, and h_{ti} is the inlet total head.

Fan Static Head—First Definition. The first definition of fan static head stipulates that this head is equal to the discharge static head minus the inlet total head, or

$$H_S = h_{sd} - h_{ti}$$

where H_S is the fan static head as given in this first definition, h_{sd} is the discharge static head, and h_{ti} is the inlet total head.

Fan Static Head—Second Definition. The second definition of fan static head stipulates that this head is equal to the discharge static head minus the inlet static head, or

$$H'_S = h_{sd} - h_{si}$$

where H'_S is the fan static head as given in this second definition, h_{sd} is the discharge static head, and h_{si} is the inlet static head.

With three definitions for fan head and four necessary arrangements to cover various applications, it appears that there are twelve possible quantity-head characteristic curves to completely cover the performance of a fan which has mounting flanges for inlet and discharge ducts, thus permitting it to be installed in any one of the four basic arrangements. However, twelve tests and as many corresponding curves are not required to determine and portray these characteristics.

THE FAN TESTS

In order to determine how the characteristics vary for the different arrangements, a fan was set up and tests conducted for all four basic arrangements, and the twelve characteristics computed. These characteristics are analyzed, both to evaluate their practical utility, and to determine the minimum number and character of the tests required to establish those characteristics which are considered to be of value. This leads to a definition of the quantity-head characteristic.

The tests were run on a single-inlet Clarage exhauster having forward-tilting blades. The inlet and discharge openings were both 9 in. in diameter. The fan was driven through a flexible coupling by a squirrel-cage induction motor of 1800 rpm synchronous speed.

The inlet and discharge ducts used were 9 in. in diameter and made of sheet metal, the joints being riveted and soldered to make them airtight. The inlet duct was 6 ft, or 8 diameters, long and the discharge duct was 10 ft, or about 13.5 diameters long. Brass bosses, drilled and tapped for $^1/_8$-in. pipe, were soldered on each duct for the static-pressure taps at points three-quarters of the duct length from the duct inlet. All burrs were removed from the edges of the static tap holes in the ducts.

All flow measurements were made with a 260-mm 1912 V.D.I. nozzle, using a coefficient of 0.97, placed at the outlet of a large test chamber 28 ft long and 33 in. in diameter. When the discharge duct was connected to the fan, the air was discharged from the duct into this test chamber, the control of the flow being made by means of a series of throttle plates inserted at the connection between the duct and the chamber as shown in arrangement *B* of Fig. 2. When no discharge duct was used the fan discharged directly into this test chamber as shown in ar-

rangement *A* of Fig. 2. Flow control was then effected by means of a multihole throttle plate inserted 10 ft downstream in this chamber. This plate also acted as a screen to straighten the flow for the nozzle approach.

In order to produce the four basic installation arrangements with the equipment described, it was set up as follows in the four test arrangements shown in Fig. 2.

Test Arrangement A—Bare Fan. The fan intake was directly from the room, and the discharge was directly into the test chamber.

Control of flow was effected by means of the multihole throttle plate 10 ft downstream in the test chamber.

Test Arrangement B—Blower. Intake was directly from the room, and the discharge was made through a discharge duct from which the air passed into the test chamber. The multihole throttle was removed, the chamber acting only as a device for smoothing the approach to the nozzle. Flow was controlled by means of the series of throttle plates at the point where the duct joins the test chamber.

Test Arrangement C—Booster. The intake was from the room through an inlet duct, and the discharge was through a discharge duct from which the air passed into the test chamber. Flow was again controlled by the series of throttle plates at the end of the discharge duct.

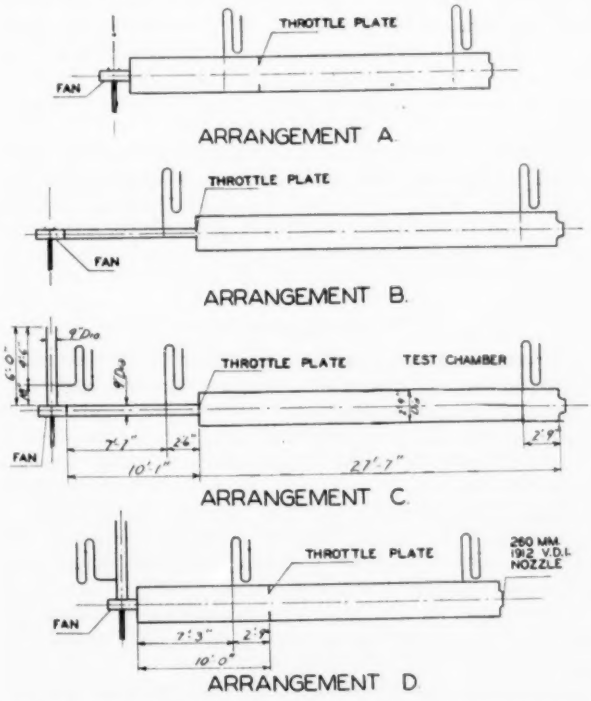


FIG. 2 FAN TEST INSTALLATIONS OF BASIC FAN ARRANGEMENTS

Test Arrangement D—Exhauster. The intake was again from the room through an inlet duct and the discharge was directly into the test chamber. The multihole throttle plate was used for flow control.

The inlet static head was measured, when required, by means of an inclined draft gage, read to 0.005 in. of water. The discharge static head was measured by a vertical U-tube water manometer. The velocity heads at the points at which the static heads were measured were calculated from the flow measured by the nozzle and the areas of the ducts at these points. When the fan discharged directly into the test chamber, the discharge pressure was read at a point 33 in. upstream from the throttle plate, or 2.7 chamber diameters after the discharge into the chamber. The nozzle differential was measured by means of an inclined-tube draft gage read to 0.001 in. of water. All draft gages were calibrated by means of a hook-gage manometer. Corrections were made to standard density, 0.07488 lb per cu ft, and to the fan speed of 1800 rpm. No measurements were made of power, since the scope of the investigation was limited to the study of the quantity-head characteristics.

Tests were conducted, giving six points, evenly distributed, on each characteristic curve for each of the four basic arrangements.

ANALYSIS OF DATA

Fig. 3 shows the 12 quantity-head characteristics obtained from this fan by applying the three definitions of fan head to the four

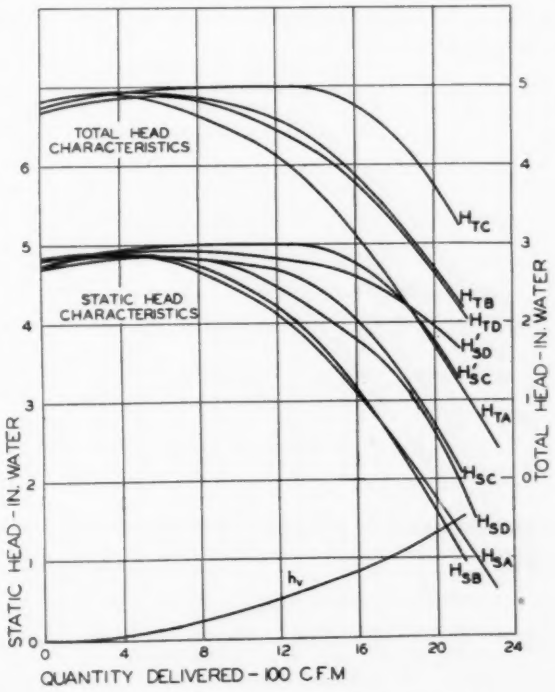


FIG. 3 QUANTITY-HEAD CHARACTERISTICS OF AN EXHAUSTER FAN

basic arrangements. The upper four curves are the total-head characteristics, while the lower group of curves are the static-head characteristics. These curves show the striking differences which exist between the characteristics for the different arrangements and for the various definitions of fan head. These differences show vividly the necessity for a very careful analysis of the definition of the quantity-head characteristic of fans. This is especially appreciated when it is recognized that these diverse curves represent results which may be obtained for a single fan by use only of definitions and arrangements which occur commonly in practice.

Before presenting a detailed analysis of the characteristics for each basic arrangement, a general principle will be stated relative to the places at which heads are measured or expressed. In any arrangement or definition, heads existing at the inlet must be measured at some specified plane on the inlet side, and heads at the discharge must be measured at some specified plane on the discharge side. On the discharge side of a blower for example, a procedure of adding heads measured at two different places on the discharge side is utterly untenable. The discharge total

head must be obtained by adding velocity and static heads, *both measured at the same plane.* This principle has been violated unfortunately, on occasion, in practice.

In arrangements where ducts are applied at the fan inlet or discharge, the heads are to be measured in the duct at a plane so located that the flow conditions have some measure of stability and uniformity. Where no duct is applied, the plane for head measurement may not be directly at the entrance to or exit from the rotor, because the uncertain flow conditions render impossible either the measurement or computation of heads at these places. The heads must be measured in these cases, at planes sufficiently removed from the fan inlet or discharge such that the kinetic energy at these planes is sensibly zero. In these circumstances the total head equals the static head.

Having established the places at which heads must be measured, indicated at i and d in Fig. 1, we may now proceed with the analysis of the characteristics as obtained for each basic arrangement.

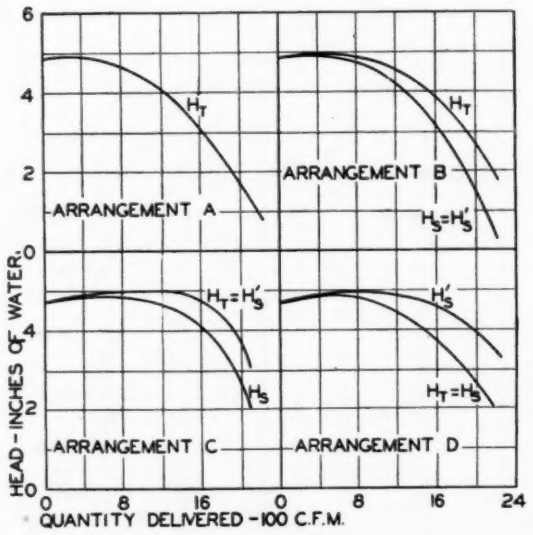


FIG. 4 QUANTITY-HEAD CHARACTERISTICS OF AN EXHAUSTER FAN SHOWN BY BASIC FAN ARRANGEMENTS

Arrangement A. It is recalled that arrangement A is for the bare fan, of either propeller or housed type, with no inlet or discharge ducts attached. The inlet total head, measured at i, Fig. 1, is equal to the inlet static head since the velocity head is sensibly zero at the plane of head measurement. The discharge total head, measured at d, is likewise equal to the discharge static head. Therefore, the single characteristic curve, shown in Fig. 4, represents the quantity-head characteristics of arrangement A for all three definitions of fan head.

Arrangement B. In arrangement B, which has a discharge duct but no inlet duct, the inlet heads are determined in open space at a plane i, Fig. 1, and the discharge heads are measured in the discharge duct at a plane d. The inlet static head will be equal to the total head. The discharge total head will be greater than the discharge static head by the amount of the discharge velocity head, all measured at plane d. This difference will cause the total-head characteristic to be higher than the static-head characteristic by the discharge velocity head, as shown in Fig. 4. However, the two definitions of fan static head give the same characteristic curve.

Arrangement C. As shown in Fig. 1, both inlet and discharge ducts are used in arrangement C. Therefore, the inlet heads are to be determined in the inlet duct at a plane i, and the discharge heads are to be determined in the discharge duct at a plane d. The inlet total head is greater than the inlet static head by the amount of the inlet velocity head. The discharge total head is greater than the discharge static head by the discharge velocity head. These circumstances cause the static-head (first definition) characteristic to be lower than the total-head characteristic by the discharge velocity head, as shown in Fig. 4. The second definition of fan static head is equivalent to adding the inlet velocity head to the fan head obtained by the first definition. This would, in general, make necessary three distinct characteristic curves applying to this arrangement. For this particular fan, however, the inlet velocity is the same as the discharge velocity, since the two ducts are of the same area. This makes the characteristic curve obtained by the second definition of fan static head coincide with the characteristic obtained by use of the fan total head.

Arrangement D. In arrangement D, it is recalled that there is an inlet duct but no discharge duct. The inlet heads are determined in the inlet duct at a plane i, while the discharge heads are measured at plane d, in open space at a distance sufficiently removed from the fan outlet that the velocity head is sensibly zero. The inlet total head and inlet static head, measured at plane i, differ by the inlet velocity head. The discharge total head is identical with the discharge static head, both measured at plane d, since the discharge velocity head is zero. Thus, the total-head characteristic and the static-head characteristic given by the first definition of fan static head will coincide, while the static-head characteristic given by the second definition of fan static head will be greater than these two by the inlet velocity head.

The various quantity-head characteristics for the four basic installation arrangements present the differences and equalities summarized in Table 1. It is seen from this table that a total of eight curves is required for the three head definitions and for the performance of fans arranged in the four basic installations shown in Fig. 1.

CHARACTERISTICS OF VARIOUS FAN TYPES

The foregoing analysis applies to fans provided with mounting flanges for inlet and discharge ducts, and thus adaptable to four possible installation arrangements. All fans do not have flanges to receive inlet and discharge ducts, however, and are thereby subject to certain limitations. The possible installation arrangements, the required test arrangements and the resulting characteristic curves for various fan types may now be established.

All fans may be classified under the four following types:

1 Fans with no mounting flanges for inlet or discharge ducts. Wall-type propeller or disk fans are frequently of this type.

2 Fans with no inlet-duct mounting flanges, but with discharge-duct mounting flanges. This is the common blower-type fan.

3 Fans with mounting flanges for both the inlet and discharge ducts.

4 Fans with inlet-duct mounting flanges, but no discharge-duct mounting flanges.

A wall-type propeller fan which is provided with no connections for ducts must be classified under type-1. It must be tested under arrangement A, and since this arrangement provides the only possible method for mounting such a fan, the single characteristic curve fully defines its performance.

A usual-type blower, having flanges for mounting only a discharge duct, is an example of type-2. The performance of this type fan for all its possible installation arrangements can be defined fully by three characteristic curves determined under test arrangements A and B.

Table 2 presents for each type of fan (a) all possible installa-

TABLE 2 NUMBER OF ARRANGEMENTS, CHARACTERISTICS, AND REQUIRED CURVES FOR VARIOUS FAN TYPES

	Fan type	Possible installation arrangements	Necessary test arrangements	Number of characteristics	Number of curves Per instal.	Number of curves Per type
1	No ducts	*A*	*A*	3	1	1
2	With discharge-duct flange only	*A*	*A*	3	1	3
		B	*B*	3	2	
3	With flanges for discharge and inlet ducts	*A*	*A*	3	1	8
		B	*B*	3	2	
		C	*C*	3	3	
		D	*D*	3	2	
4	With inlet-duct flange only	*A*	*A*	3	1	3
		D	*D*	3	2	

tion arrangements, (*b*) the corresponding necessary test arrangements, (*c*) the number of characteristics possible for each installation arrangement, (*d*) the number of curves required to portray the characteristics for each installation arrangement, and (*e*) the total number of curves required to portray all the characteristics possible for a given type of fan. The most significant deduction to be made from Table 2 is that from one to eight curves are required to portray the possible characteristics for the various fan types. It must be remembered that these characteristics are in strict accordance with the three currently used definitions of fan head as applied to the installation arrangements for which each type is adaptable.

Development of Fan-Head Definition

The purpose of a fan characteristic is to delineate useful and direct information regarding how the fan will perform in its installation. Specifically, the quantity-head characteristic shows the quantity of air which will be delivered against a certain resistance in the intended installation. It now becomes necessary to evaluate the three fan-head definitions to discover which of these truly contribute useful information toward this objective.

The fan-head definitions are each made up of various combinations of static and total heads at inlet and discharge. Let us first dispose of the heads on the inlet side. It will be shown that the only head on the inlet side which contributes a useful function in defining the fan characteristic is the total head.

Referring to the arrangement with no inlet duct, Fig. 5, let the inlet total head be measured at some point P at a distance from the fan inlet, the velocity being negligible at that point. Then let the static and velocity heads be measured at a point Q very close to the fan inlet, no inlet duct being used. At point P all of the energy is in the form of static-head or pressure energy. There are practically no losses in energy in the air in flowing from P to Q. The gain in kinetic energy at Q is at the expense of the pressure energy at Q. Expressed as a simple relation

$$h_{tP} = h_{sP} = h_{vQ} + h_{sQ}$$

Now let an ideal inlet duct (no inlet or wall-friction losses) be attached to the fan inlet. Let P be located outside the duct, again at a point of negligible velocity, and let Q be located inside the duct. Under these circumstances we again find that the total head remains constant from P to Q regardless of the kinetic-energy increase at Q. Thus, the fan, either without any inlet duct or with an ideal inlet duct, does not add any energy to the air before the air actually enters the fan rotor. This is counter to some current conceptions. For example, a fan draws air from the atmosphere. Then at point P, the total head and static head are both zero. At point Q the sum of the static and velocity heads are also zero. Any kinetic energy which the air possesses at fan inlet was not produced by the fan, but at the expense of a corresponding decrease in pressure energy in the air stream. The fact is, that any energies existing at the fan inlet are supplied *to* the fan and not *by* the fan.

The ideal inlet duct is now replaced by an actual inlet duct. Because of the inlet and wall-friction losses the total head at Q will now be less than at P. The energy which goes to the fan at the fan inlet is still exactly and only the total energy in the air at the inlet, regardless of losses or other conditions which exist up to the fan inlet. The conclusion is that in any definition of fan head, we must debit to the fan the energy supplied to the fan by the air at the fan inlet. This energy supply is the inlet total head and is equivalent to h_{sQ} for fans without inlet ducts, and to $(h_{sQ} + h_{vQ})$ for fans with inlet ducts.

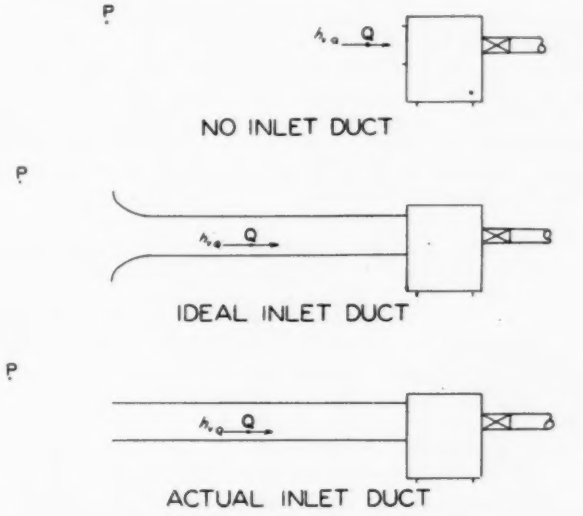


FIG. 5 HEADS AT INLET TO FAN

This eliminates the second definition of fan static head used throughout this paper, which erroneously credits the fan with the inlet velocity head.

Let us now consider the heads at the discharge side of the fan. In general, the resistance of the external system into which the fan discharges air is overcome by the energy of the discharge pressure. The only way by which the discharge kinetic energy of the air may have utility in overcoming resistance is by the introduction of a diffuser which converts kinetic energy into pressure energy. If the diffuser is applied at the fan discharge it should be considered as part of the fan and the discharge static head which is measured in this case would include the recovered kinetic energy at the fan outlet. If, on the other hand, the diffuser is installed at some point in the external system, it becomes a part of that system and any conversion of energy made by use of the diffuser cannot be properly credited to the fan. The authors, therefore, propose that in defining the fan head the discharge velocity head be entirely disregarded and the discharge static head only be used.

Consider two fans X and Y, having equal discharge total energy for equal quantity discharged, but fan X having a much larger portion of kinetic energy. These fans are not of equal utility for purposes of overcoming resistance. The purposes of factors such as good housing design and large outlet area are to convert kinetic energy to pressure energy. In most cases, if not in all cases, the fan should not be credited with unconverted kinetic energy.

We are now in a position to develop a definition of fan head. The fan head which gives the most useful information regarding fan performance in its installation is one which measures the

energy increase from the energy level of the air at the fan inlet to that energy level at the fan discharge which is directly usable, that is, without further conversions, in an external system. We have shown that at the fan inlet, the fan must be debited with both the pressure and kinetic energy existing there, while at the discharge of the fan, only the pressure energy may be properly credited to the fan.

The foregoing requirements for a fan-head definition are met precisely by the first definition of fan static head used throughout this paper. Specifically, *the fan head to be used in the quantity-head characteristic is defined as the discharge static head minus the inlet total head.* In arrangements which have discharge ducts, the discharge static head is measured in the duct at a specified plane so located that the flow conditions have some measure of uniformity. In arrangements without discharge ducts, the discharge static head is measured in the space into which the fan discharges. In arrangements which have inlet ducts, the inlet static and velocity heads are measured in the inlet duct at a specified plane so located that the flow conditions have some measure of uniformity. In arrangements without inlet ducts the inlet total head is the static head measured in the space from which air is supplied to the fan.

TABLE 3 NUMBER OF ARRANGEMENTS, CHARACTERISTICS AND CURVES FOR VARIOUS FAN TYPES, BASED ON STANDARD DEFINITION OF FAN HEAD

Fan type	Possible installation arrangements	Necessary test arrangements	Number of characteristics	Number of curves Per instal.	Number of curves Per type
1 No ducts.....	*A*	*A*	1	1	1
2 With discharge-duct flange only.....	*A*	*A*	1	1	2
	B	*B*	1	1	
3 With flanges for discharge and inlet ducts.....	*A*	*A*	1	1	4
	B	*B*	1	1	
	C	*C*	1	1	
	D	*D*	1	1	
4 With inlet-duct flange only.....	*A*	*A*	1	1	2
	D	*D*	1	1	

Using this definition of fan head, the complete schedule of fan types is given in Table 3, with all possible installation arrangements and corresponding necessary test arrangements, and the number of characteristics and curves required to completely portray the performance of each fan type.

From Table 3, it may be seen that for type-1, only one characteristic is possible, while for type-3, four characteristics and four curves are necessary to cover the four possible installation arrangements. The fan classified as type-3 is reported upon in this paper, the four characteristic curves being the four lower static-head characteristics shown in Fig. 3. Incidentally, the discharge velocity head, which is not included in the fan head, may be conveniently and usefully portrayed by plotting h_v vs Q, as shown in Fig. 3. This curve is identical for all possible arrangements which have discharge ducts, depending as it does only on the quantity discharged and the area of the discharge duct.

Quantity Measurement

A word need be said as to the measurement of the quantity term in the quantity-head characteristic. First, it should be recognized that by no means must the quantity be measured at the point at which a head is measured. The quantity may be measured at any convenient place in the system. Further, depending upon the test arrangement indicated for the particular cases, various acceptable methods of quantity measurement become more convenient and economical. The pitot tube, nozzle, and orifice are among the acceptable methods of measurement of air quantities in fan testing. Conditions of accuracy of these methods are under continual study by experimenters, and are eventually set forth in codes. In arrangements without inlet or discharge ducts, the use of nozzles or orifices in the walls of a large test chamber becomes an accurate, convenient, and economical method. In arrangements using small inlet or discharge ducts, nozzles or orifices at the entrance to inlet ducts or at the exit from discharge ducts are satisfactory. In very large ducts, nozzles, and orifices become more expensive, and the pitot tube is perhaps more satisfactory. In moderate-size ducts, the choice from among the acceptable methods of air measurement may be based upon considerations of convenience and availability of materials.

Definition of Quantity-Head Characteristic

The definition of fan head to be used in determining the quantity-head characteristic of a fan has been established. The quantity term is not subject to definition, but must be measured by some acceptable method. The quantity-head characteristic may now be defined as the relation between the quantity discharged and the defined fan head for a specified installation arrangement. Fan head is defined as the discharge static head minus the inlet total head.

It has been pointed out that for certain type fans there are possible more than one characteristic as defined by the established fan-head and quantity measurements, depending on the number of possible installation arrangements to which the fan may be adapted. Each particular fan type, however, is designed primarily for use in a particular installation arrangement which may be termed its *primary arrangement.* That characteristic which is obtained by use of the test arrangement corresponding to the primary installation arrangement of each type fan may be termed the *primary characteristic* for each type. For general use, then, this primary characteristic only need be determined, but with the understanding that this characteristic will not necessarily apply when the fan is installed in any arrangement other than its primary one. Thus, for fan type-1, with no mounting flanges for inlet or discharge ducts, and thus having only one possible arrangement, the only possible characteristic is the primary one. For the fan classified as type-2, the ordinary blower with only a discharge-duct mounting flange, and thus having two possible arrangements, the primary arrangement is considered to be that with a discharge duct attached. The primary characteristic will be that obtained using test arrangement *B*, the blower arrangement with the discharge duct attached. The primary arrangement of a fan classified as type-3 having mounting flanges for the inlet and discharge ducts, is that in which both inlet and discharge ducts are used. The primary characteristic is, therefore, that obtained by use of test arrangement *C*, which has both ducts attached. The primary arrangement for type-4 is that in which the inlet duct is attached. The primary characteristic is that obtained by test arrangement *D*. Thus, for any type of fan there may be established a primary arrangement and a single corresponding primary characteristic.

For the fan reported on in this paper, using the several fan-head definitions in common use and the four possible arrangements, 12 characteristics were originally determined and shown. By use of the established definition of fan head, the possible characteristics reduce to four. Of these four characteristics that one is considered the primary characteristic which shows the performance of the fan when installed in its primary arrangement which, for this fan, is the one with both inlet and discharge ducts in place. This characteristic is shown by the curve marked $H_{s,c}$ in Fig. 3. If the fan is to be installed in any other of the four possible arrangements it may not be taken for granted that this primary characteristic applies, but a separate test must be conducted to determine the characteristic for the intended arrangement.

Drying Problems of the Ceramic Industry

By JOHN L. CARRUTHERS,[1] COLUMBUS, OHIO

The purpose of this paper is to outline and discuss the various problems encountered in designing and operating ceramic-drying systems, and to indicate the several methods that may be used in attacking the more difficult problems that arise.

The process of drying clay wares may be a relatively simple one or it may be quite complex. This is due to (1) wide variations in production requirements in different industries, (2) extreme differences in the physical characteristics of clay bodies used, and (3) various sizes and shapes of different products. In many cases, the methods and equipment used may appear to be very slow and inefficient when considered in the light of modern drying practice. Their use is generally justified, however, when all factors are taken into consideration. However, this does not imply that all present systems are best suited for their particular task.

The problems discussed in the paper center around the development, maintenance, and determination of safe, rapid, and economical drying rates and the reduction or prevention of ware losses in order to increase drying and manufacturing efficiencies.

IN MANUFACTURING some wares, drying is immediately followed by a firing process, which in its early stages could function as a drying process, since there is a supply of heat and a suitable means for removing water vapor. Vitreous floor tile and small porcelain wares are commonly dried in the kiln, because the pieces are small and fragile and must not be subjected to unnecessary handling. However, a drying process is ordinarily required for the following reasons: (1) To give ware sufficient strength to be handled in loading kilns. (2) Kiln drying increases the time of the firing cycle thus requiring additional equipment. (3) There is danger of the ware being marred by kiln scum which forms if sulphur gases come in contact with water in the surface layers of the clay ware. (4) The ware must be dried before the glaze can be applied. (5) The temperature and humidity of the kiln gases cannot be readily controlled within the desired limits for satisfactory drying conditions.

Requirements of the Drying Process

The most important requirement is safe drying which means that the ware from the drier will be free of any defects that may be caused by improper drying conditions. Any ware spoiled during drying is practically worthless except in the few cases where cracked ware can be patched or repaired. Expenditures on spoiled ware are lost and the salvaged clay is of little value.

Clay wares should ordinarily be dried rapidly and uniformly and to the hygroscopic or regain water content (2 to 4 per cent). However, in some cases complete drying is not desired, because the ware may be too brittle for handling when bone dry, or it is too expensive to remove the last small percentage of water from the ware while in the drier. Also, several partial drying processes are used in the industry, e.g., drying of filtered clay for dry pressing and conditioning of clay blanks for turning, repressing, or "sticking."

Ware Forming Methods

A consideration of the methods used to form ceramic wares is necessary in the solution of drying problems, since some of the physical characteristics of the plastic and dry pieces will be affected by the method used. Following are brief descriptions of forming methods with lists of products indicating some of the conditions encountered:

1 *Dry Pressing.* A slightly moistened or damp clay body in dust form (water content 6 to 15 per cent) is placed in a steel mold and then compacted by the application of pressure to the die. The pressure may vary from several hundred to several tons per square inch. Products: Floor and wall tile, low-tension electrical porcelain, special refractories, and fire-clay bricks and shapes.

2 *Stiff-Mud Extrusion.* A semiplastic clay body (water content 15 to 25 per cent) is forced through an orifice or die which controls the dimensions of the cross-section of the piece. The extruded column is cut into required lengths by steel wires. Products: Common, face, glazed, paving, and fireclay brick; hollow building tile; telephone conduit; sewer pipe; drain tile; roofing tile; spark-plug cores; machine-made terra cotta; and blanks for electrical porcelain bushings.

3 *Soft-Mud Forming.* A soft plastic clay body (water content 20 to 30 per cent) is either modeled by hand or is pressed by hand or by mechanically operated mandrels into plaster, wood, or steel molds. Wares made in plaster molds are partially dried in the mold so that they may be easily removed and also to develop sufficient strength for handling and finishing. Products classified by name of process are: (*a*) Hand modeling—terra-cotta statuary, saggers, glass pots, and pottery. (*b*) Throwing on potter's wheel—art pottery and specialties. (*c*) Jiggering (plaster molds)—dinnerware, stoneware, chemical porcelain, and art ware. (*d*) Hot pressing (plaster molds)—electrical porcelain. (*e*) Machine pressing (plaster and steel molds)—roofing tile, flower pots, saggers, and crucibles. (*f*) Hand pressing (plaster molds)—terra cotta, faience tile, and chemical stoneware. (*g*) Hand and machine pressing (wood molds)—common and fireclay brick, and refractory shapes.

4 *Casting.* A clay-water mixture of the consistency of cream (water content 20 to 40 per cent) is poured into plaster molds. The clay is deposited on the side walls of the molds as the water is absorbed by the plaster. The use of electrolytes in the mixture makes it possible to use relatively small amounts of water. The ware must dry to a rigid condition by absorption while in the mold to permit removal. Products: Hollow shapes in dinnerware, art pottery, saggers, and sanitary ware.

5 *Additional Forming Methods.* Other methods may be required

[1] Associate Professor of Ceramic Engineering, Ohio State University. Fellow of the American Ceramic Society. Professor Carruthers was graduated from Ohio State University in 1921 and received the degree of Ceramic Engineer in 1934. He served in the Air Service during the War. He has held the following positions since 1921: Ceramic engineer with the Denver (Colo.) Terra Cotta Co. until Dec., 1922; construction, design, and service engineer for the Harrop Ceramic Service Company, Columbus, Ohio, from December, 1922, until September, 1925. This work was in connection with tunnel kiln installations in various sections of the U. S. From 1925 to 1930 he was assistant professor of ceramic engineering at Ohio State University and has held his present position since 1930. He is the author of several papers on kiln design, clay drying, and other phases of ceramic engineering.

Contributed by the Process Division and presented at the Semi-Annual Meeting of The American Society of Mechanical Engineers, held in Cincinnati, Ohio, June 19 to 21, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until December 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

to complete the piece only partially formed by one of the preceding operations. These methods either change the structure of the clay body or develop other conditions which may influence the drying operations. The methods are: (*a*) Repressing—applied to stiff- and soft-mud products, such as paving and fireclay brick. Preliminary drying is necessary with soft-mud products. (*b*) Finishing or tooling of surfaces—applied principally to wares formed in molds. (*c*) Turning on a lathe—spark-plug porcelains and electrical insulator bushings. (*d*) Mitering or "Sticking"—two or more pieces formed separately are combined by pressing the adjoining surfaces together, using a very soft plastic clay or clay slip as the bonding agent. This method is used for cup handles, sewer-pipe fittings, closet bowls, and terra-cotta corner pieces.

Drying Problems

The various problems of ceramic drying comprise: (1) Elimination of ware losses in drying; (2) developing faster safe rates of drying for clay bodies and shapes; and (3) developing methods for determining, experimentally, the safe drying time.

Elimination of Ware Losses. The seriousness of this problem depends upon local conditions. Ware losses may be due to cracking, checking of surfaces, warping, strains, slumping, efflorescence or "drier scum," blistering, or other troubles. The causes of these troubles may be in the drying system or may be attributed to raw materials or processing. There may be a lack of control in the system preventing the maintenance of uniform drying conditions, or there may be changes in the properties of the raw materials. Also, processing conditions are encountered that markedly influence the drying behavior of the ware. Unless such changes can be anticipated, it is impossible to know that drying conditions should be altered until after a considerable amount of ware has been spoiled.

Cracking, checking, warping, and strains in the ware are all due to drying at excessive rates, which set up too extreme variations in shrinkage throughout the piece and prevent an adjustment of the stresses. The actual fault developed will depend upon the degree of localized shrinkage, the shape and position of the piece, and the strength of the clay body. The factors influencing the rate of drying and the development of these troubles are humidity of the air; velocity of air; direction of air flow; temperature of the air and ware; size and shape of the piece; diffusion and shrinkage characteristics of the clay body; water content; and strength of the clay body.

Slumping is due to the absorption of water condensed from humid air on the surface of the piece. Also, plastic clay wares may slump if they are heated in a humid atmosphere that permits little, if any, drying. The increase in temperature lowers the viscosity of the water, and the clay-water mixture cannot support its own weight. Blisters are caused by air entrapped in surface layers of the piece. If the clay is softened by condensation and the air expanded by heating, then a blister will be formed. Efflorescence or "drier scum" is due to either soluble salts in the clay, brought to the surface during drying, or formed by the reaction between sulphur gases in the drier chamber, water and minerals in the clay. Other items that may cause a loss of ware are vibrations of conveying equipment, poor handling of the ware, or lack of provisions for shrinkage when setting the piece.

There are three general methods which may be used to solve the problem of ware losses. The choice of method will depend upon the results of an analysis of the prevailing conditions and the weight of the various factors. The mode of attack may be along any of the following lines:

(*a*) The use of a drier that may be closely controlled over a fairly wide range of drying conditions and changes in atmospheric conditions. This may require remodeling or replacing existing driers.

(*b*) The use of a control system which will eliminate changes in the drying system or the drying behavior of the clay ware. Ware of uniform characteristics should be delivered to the drier. However, possible variations in clays and processing conditions beyond the control of the drier or its operator often prevent satisfactory drying.

(*c*) Change the physical properties of the clay body, where possible, so that the ware can be safely dried under conditions that can be economically maintained in the existing equipment. This method is discussed in more detail in the next section.

2 *Development of Faster Safe Drying Rates.* A reduction in the excessive safe drying time required in many clay plants would have several obvious advantages. The solution of this problem is by improving drier designs and techniques or the application of ceramic technology. Research work in ceramics during the past ten years has made available new information on the nature of clays and methods of varying them, especially in respect to their drying behavior. Consequently any of these methods, or others, may be the most desirable one to use, providing the drier equipment is satisfactory.

Rapid drying is not always desirable from an economic standpoint, especially with wares such as dinnerware, stoneware, or art pottery. In some plants, a supply of partially fabricated ware is maintained at several stages of the manufacturing process in order to meet fluctuations in production orders. Consequently, storage rooms in which the ware is dried slowly are more economical than regular driers since this procedure eliminates an extra handling. Also, there are some types of ware, such as glass pots, which seem to have better service qualities if they are dried slowly.

Changes in drier design or operation quite often make it possible to dry ware safely at rates that would be disastrous with the existing system. Cut-and-try methods may be required to obtain the desired results due to conditions often difficult to explain. Several features of design or methods of operation, that might be investigated in the remodeling of old driers or the design of new ones, are as follows:

(*a*) Direction and velocity of air flow. This feature is quite vital with many products. Whenever possible, all surfaces should have uniform treatment.

(*b*) Location of heating units. Rapid safe drying has been obtained in some driers by placing the steam-radiator elements in the drying chamber. This arrangement eliminated the necessity for a large recirculating-air volume to deliver the required units to the ware.

(*c*) Heating the ware before placing it in the drier or heating it (with little drying) in a controlled humid atmosphere at the start of drying, will ordinarily make possible the use of increased drying rates.

(*d*) Provide a uniform air circulation throughout the ware setting, especially in continuous driers. If all ware does not receive the proper initial treatment due to poor circulation, then the rapid drying conditions later on will damage that ware not properly treated. In some cases the ware setting in the drier may require adjustment.

(*e*) Adjust air volumes and temperatures to prevent condensation of water vapor on the ware.

(*f*) Both continuous and intermittent drying systems have definite applications in the industry. A change from one type to another is sometimes indicated.

There are several methods, based upon application of ceramic technology that may be used to increase the safe drying rate of a clay or make it possible to dry certain clays satisfactorily for commercial use. These methods alter various physical properties that govern or influence the rate at which the clay may be safely dried. The properties are shrinkage, pore structure, and

strength which are dependent upon the grain size, the amount and state of colloids of the clay, the water content, and the nature of the minerals in the clay. Briefly, the methods are: (*a*) The addition of a non-plastic material or a less-plastic clay to decrease shrinkage in drying. (*b*) Preheating the clay at temperatures between 400 and 500 F to alter the character of the colloids and thus decrease shrinkage in drying. (*c*) Coarser grinding of materials to decrease shrinkage. (*d*) Weathering of raw clay, additional tempering, aging the plastic body, addition of an organic bond, or the addition of a more plastic clay may be used to increase the strength of the clay. (*e*) A small amount of sodium chloride (NaCl) added to a clay has been effective in several cases. (*f*) The addition of an electrolyte to flocculate the colloidal material. This increases the strength of the body and apparently increases the rate of water diffusion. (*g*) A chemical treatment to cause an exchange of bases in the colloidal material.

In some cases, drier troubles are due to the development of planes of weakness or strains in the pieces by faulty forming processes. The presence of auger, die, or hand laminations, pressure planes, etc., will require the use of slow drying rates to prevent heavy ware losses.

Determination of Safe Drying Rates. At the present time, there is no correlation between the drying behavior of trial pieces under controlled laboratory conditions and that of actual pieces of ware in commercial driers. A solution of this problem is needed for advancing the art of drying clay wares. The determination of required drying rates in commercial driers is a very expensive proposition.

Considerable work has been done both in this country and abroad to obtain more information in regard to the nature of clays and the forces or agencies that govern or influence phenomena such as shrinkage, diffusion, vapor pressures. Other studies have been made to determine what relationships there might be between humidity, temperature, pressure, water content, shrinkage, or clay characteristics and drying behavior. While much valuable information has been obtained, the methods now available for determining the safe drying time of a clay when formed into a definite shape are of a cut-and-try nature. The problem is exceedingly complex since there are many variable factors that must be considered.

Summary

The problems discussed center around the development, maintenance, and determination of safe, rapid and economical drying rates and the reduction or prevention of ware losses in order to increase drier and manufacturing efficiencies.

The principal factors involved in the solution of the problems are: (1) Requirements for drying systems; (2) manufacturing methods employed; (3) the physical characteristics of clays and product shapes; (4) ordinary phenomena of drying such as temperature, humidity, air flow (direction and velocity), heat transfer and vaporization; and (5) economic considerations.

The methods that may be used in solving the existing problems include the following:

(1) Provide correct drying conditions for the ware by remodeling or replacing existing driers so as to eliminate ware losses or increase safe drying rates. Also, provide suitable control methods and apparatus.

(2) Improve the drying behavior of the clay bodies by altering their physical properties so as to prevent ware losses and to increase the safe drying rate. This general method can only be used when the quality of the finished product is not adversely affected. The methods that may be employed are (*a*) the use of non-plastic or less-plastic materials, (*b*) chemical treatment, (*c*) the addition of stronger clays or organic bonds, (*d*) the use of additional processes such as preheating, weathering and aging, and (*e*) varying the methods used in grinding and tempering the clay.

There are plants that are continually encountering many of the difficulties described, while others seldom have drier troubles. However, the industry as a whole encounters these problems continually and only by recognizing that there are problems can improvements be made.

It is believed that the art of drying ceramic wares in the major portion of the industry is on a fairly high level when consideration is given to the complexity and wide variations in the composition and properties of clays, the variety of products made, the different processes used in manufacturing, and the science of drying.

Bibliography

Drying, Humidity, Fans, etc.

"Drying Clay Wares," by E. Lovejoy. Randall Publishing Company, Wellsville, N. Y., 1916.

"Science and Practice of Drying," by A. E. Brown. British Clayworker Press, London, England.

"Elements of Chemical Engineering," by W. L. Badger and W. L. McCabe. McGraw-Hill Book Company, New York, 1931.

"Principles of Chemical Engineering," by W. H. Walker, W. K. Lewis, and W. H. McAdams. McGraw-Hill Book Company, New York, 1934.

"Chemical Engineer's Handbook," by J. H. Perry. McGraw-Hill Book Company, New York, 1934.

"Fan Engineering," by R. D. Madison. Buffalo Forge Company, Buffalo, N. Y.

"Drying and Processing," by D. C. Lindsay. Carrier Engineering Corporation, Newark, N. J.

"Modern Drying Machinery," by E. Cronshaw. Ernest Benn Ltd., London, England, 1926.

"Ceramics—Clay Technology," by H. Wilson. McGraw-Hill Book Company, New York, N. Y., 1927.

Ceramic Encyclopedia Catalogs, Industrial Publications Co., Chicago, Illinois.

Clay Conditioning

"Altering Properties of Clays by Use of Electrolytes," by T. Husain. *Journal American Ceramic Society*, vol. 13, 1930, p. 805.

"Correcting a Tender Shale," by H. E. Simpson. *Journal American Ceramic Society*, vol. 13, 1930, p. 817.

"Correction of Cracking in Drying Brick," by H. Frechette and J. G. Phillips. *Journal American Ceramic Society*, vol. 12, 1929, p. 153.

"Fundamentals and Economies in the Clay Industries, Tough Plastic Clays," by E. Lovejoy. *Clayworker*, vol. 98, February, 1932.

"Conditioning of Clay in the Plastic State," by F. H. Clews. Trans. (Eng.) Ceramic Society, vol. 33, 1934, p. 1.

Mechanism of Drying

"Drying of Solids," by T. K. Sherwood. *Industrial and Engineering Chemistry*, vol. 21, January, 1929, p. 12; vol. 21, October, 1929, p. 976; vol. 22, February, 1930, p. 132; vol. 24, March, 1932, p. 307.

"Application of Theoretical Diffusion Equations to the Drying of Solids," by T. K. Sherwood. Trans. American Institute of Chemical Engineers, vol. 27, 1931, p. 190.

"The Drying of Porous Solids," by A. B. Newman. Trans. American Institute of Chemical Engineers, vol. 27, 1931, pp. 203–310.

"The Mechanism of Drying of Clays," by T. K. Sherwood. Trans. American Institute of Chemical Engineers, vol. 28, 1932, p. 118.

"Solving Drying Characteristics of Clay," by H. Spurrier. *Ceramic Industry*, vol. 16, 1931, p. 447.

"The Adiabatic Air Drying of Hygroscopic Solids," by D. W. McCready and W. L. McCabe. Trans. American Institute of Chemical Engineers, vol. 29, 1933, p. 131.

"Rate of Drying Due to Reduced Pressure," by L. Navias. *Journal American Ceramic Society*, vol. 12, 1929, p. 305.

"Laboratory Procedure for Determining Drying Characteristics," by E. J. Casselman. *Journal American Ceramic Society*, vol. 10, 1927, p. 59.

"Some Observations on Drying Properties of Clays," by D. C. Lindsay and W. H. Wadleigh. *Journal American Ceramic Society*, vol. 8, 1925, p. 677.

"Some Notes on Nature of Clay II," by F. H. Norton. *Journal American Ceramic Society*, vol. 16, 1933, p. 86.

"Water Movement in Drying Stiff-Mud Ware," by E. Lovejoy. *Journal American Ceramic Society*, vol. 16, 1933, p. 405.

"The Principles Underlying the Drying of Clay," by H. H. Macey. Trans. (Eng.) Ceramic Society, vol. 33, 1934, p. 42.

"The Drying of Clay Under Definite Conditions," by R. S. Troop and F. Wheeler. Parts 1, 2, and 3—Trans. (Eng.) Ceramic Society, vol. 26, 1927, pp. 231, 239, 261.

"Some Observations on the Dehydration and Firing Behavior of Clays," by R. F. Geller and W. H. Wadleigh. *Journal American Ceramic Society*, vol. 10, 1927, p. 925.

"Fundamentals and Economies in the Clay Industries," by E. Lovejoy. *Clayworker*, vol. 98, Sept., Oct., Dec., 1932.

Evaporation—Hygrometry

"Theory of Evaporation, Etc.," by A. E. Stacey. *Journal American Ceramic Society*, vol. 8, 1925, p. 457.

"Temperatures of Evaporation of Water Into Air," by W. H. Carrier and D. C. Lindsay. *Mechanical Engineering*, vol. 47, 1925, p. 327.

Drier Calculations—Humidity Charts

"Rational Psychrometric Formulas," by W. H. Carrier. Trans. A.S.M.E., vol. 33, 1911, p. 1005.

"Psychrometric Tables," by C. F. Marvin. Weather Bureau Bulletin No. 235. U. S. Government Printing Office, 1900.

"Humidity Charts," by W. Grosvenor. Trans. American Institute of Chemical Engineers, vol. 4, 1908.

"Waste Heat Drier Calculations and Charts," by A. E. R. Westman and Mills. *Journal American Ceramic Society*, vol. 12, 1929, p. 162.

"Drier Calculations," by R. K. Hursh. *Clayworker*, 1930.

"Drier Problems with Calculations," by J. L. Carruthers. *Journal American Ceramic Society*, vol. 14, 1931, p. 8.

General—Drying and Driers

"Notes on Drying and Drier Operation," by J. T. Robson. *Journal American Ceramic Society*, vol. 12, 1929, p. 709.

"Drying Heavy Clay Products in a Progressive Drier," by H. R. Straight. *Journal American Ceramic Society*, vol. 12, 1929, p. 175.

"Ceramic Drying and Driers," by A. E. Stacey, Jr., and H. B. Matzen. *Journal American Ceramic Society*, vol. 8, 1925, p. 525.

"Drier-Radiated Heat-Mechanical Circulation," by H. R. Straight. *Journal American Ceramic Society*, vol. 12, 1929, p. 175.

"Drying Problems of Terra Cotta," by F. A. Kirkpatrick. *Journal American Ceramic Society*, vol. 10, 1927, p. 487.

"Moderate Waste Heat Tunnel Driers," by T. W. Garve. *Journal American Ceramic Society*, vol. 16, 1933, p. 118.

"Improving Drying Practice," by E. W. Knapp. *Brick and Clay Record*, vol. 85, 1934–1935. (Series of articles.)

"Driers and Drying in Refractories Industry," by J. T. Robson. *Bulletin American Ceramic Society*, no. 12, 1933, p. 2.

"Greater Efficiency in Tunnel Driers," by W. D. Richardson. *Journal American Ceramic Society*, vol. 12, 1929, p. 761.

Discussion

A New Basis for the Rating of Roller-Chain Drives[1]

NORMAN ARNOLD.[2] Refinements in methods of design are handicapped in their adoption unless the mathematical work introduced by the change can be made simple. At the suggestion of Professor Bartlett, an effort has been made to devise a simple graphical method for performing the computations on the basis set forth in his paper.

The method here developed employs an alignment chart with a sliding scale, a device that is not as common among engineering computing diagrams as its convenience and simplicity warrant. Attention is called to the three mathematical conditions which must be satisfied. They are obtained from Equations [22] and [27] and Table 1 of the original paper:

$$x = \frac{1000 \text{ hp}}{N_1 N_2} \gtrless 23\ PB \quad \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

$$y = \frac{1000 \text{ hp}}{N_1 n_1} \gtrless 12.7\ PBd \quad \ldots\ldots\ldots\ldots\ldots\ldots [27]$$

$$z = n_1 \gtrless \text{rpm} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{Table 1}$$

The right-hand members of these inequalities involve pitch, bushing length, wt per ft, etc., all of which are constants for any particular standard chain. The author supplied tabulated values of the right-hand members for the various standard chains; and these values were used in preparing the chart.

It will be noted that 1000 hp/N_1 is contained in both Equations [22] and [27], therefore, in preparing the alignment chart, this division can be performed first, and the result used in solving both Equations [22] and [27]. On the accompanying chart, Fig. 1, this division is carried out by drawing a straight line through the points on the scales for N_1 and hp to the Q-axis, (dashed line A in the figure). The inequality 22 is satisfied by any chain whose number appears on the X-scale above the index line from the point on the Q-axis to the value of N_2 (line B in the figure). In general, the conditions limiting the rpm are satisfied by chains whose numbers are nearer the bottom of the chart than the given rpm, n_1; and inequality 27 is satisfied by chains whose numbers are on the Y-scale above the index line from the

[1] Published as paper MSP-57-1, by G. M. Bartlett, in the April, 1935, issue of the A.S.M.E. Transactions.

[2] Instructor, Engineering Drawing, Purdue University, West Lafayette, Ind.

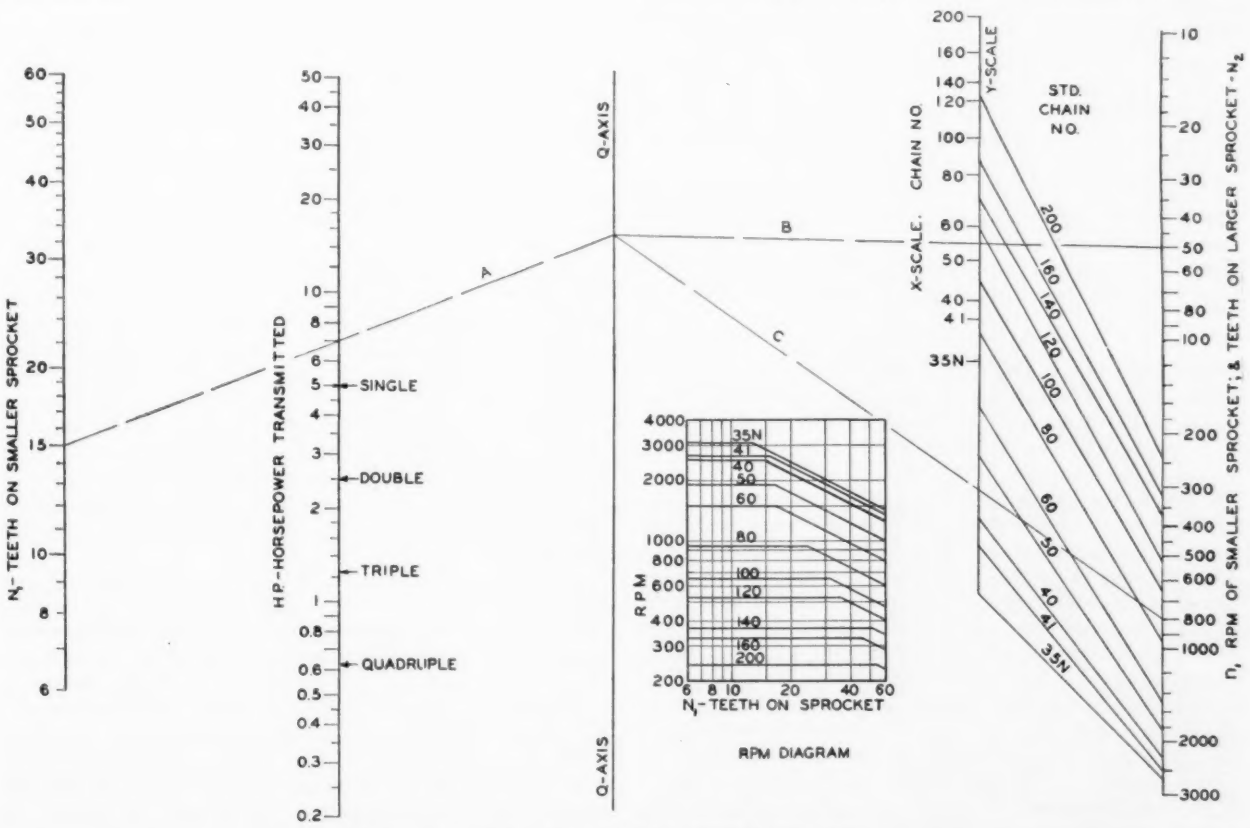


FIG. 1 ALIGNMENT CHART FOR SELECTING ROLLER CHAINS

Operation: (1) Draw index A through N_1 and hp to the Q-axis, (2) draw index B from the Q-axis to N_2, (3) draw index C from the Q-axis to n_1. A chain is satisfactory if index C cuts the inclined line with that number, and if index B passes below the number on the X-scale, and if the allowable rpm for the number of teeth, N_1, as obtained from the rpm diagram, is not exceeded.

point on the Q-axis to n_1. Therefore, both are satisfied by chains in the inclined lines cut by the index line from the Q-axis to n_1.

As an example, let it be required to select a chain to transmit 7 hp over sprockets having 15 and 50 teeth, where the smaller wheel turns 800 rpm. (1) Draw the index line, A, from 15 on N_1 through 7 on the horsepower scale to the Q-axis. (2) Through this point on the Q-axis draw an index line, C, to $n_1 = 800$, and note that chains Nos. 50 60, and 80 (cut by this line), satisfy two of the three conditional equations. (3) The index line, B, from the Q-axis to $N_2 = 50$, cuts the X-scale below chains Nos. 60, 80, 100, etc., only two of which satisfy the first two conditions. Hence, either chain No. 60 or chain No. 80 is satisfactory for this example.

This chart as shown is for single chains only. If it is found that a single chain will not satisfy the conditions, the chart may be easily modified to select multiple chains on the same basis. If the horsepower scale is made movable along its axis, then to select double chains, the arrow at 5 on the horsepower scale is matched with the arrow marked "double," and the chart is used in the same manner as just described. The procedure for triple and quadruple chains is obvious. Incidentally, shifting the horsepower scale in this way has the effect of dividing by 2 for double chain, by 3 for triple chain, and by 4 for quadruple chain. Therefore, one can use the chart as it is, if the horsepower is divided by 2, 3, or 4 as the case may be, when selecting multiple chains.

The conditions limiting the rpm are not completely fulfilled by the alignment chart, therefore, after tentatively selecting a chain by means of the chart one should refer to the rpm diagram shown here, or Table 1 in the author's paper, to make sure that the allowable speed is not exceeded. It will be observed that the No. 60 chain found in the example is satisfactory for 1500 rpm unless the number of teeth is greater than 17; above 17 teeth a gradual reduction in speed must be made. No. 80 chain is satisfactory for 940 rpm if the number of teeth is less than 24, and above 24 teeth a gradual reduction must be made as shown by the inclined portion of the curve marked 80.

The time required to select a chain by means of the chart is seldom greater than a minute if N_1, N_2, n_1, and hp are known. Obviously, the chart may be used inversely to find the maximum horsepower which may be transmitted or the limiting numbers of teeth that satisfy the two equations and the set of rpm curves.

Author's Closure

The closure can be used to no better advantage than to summarize briefly the conclusions of the mathematical research advanced in the paper and to add appropriate remarks relative to the possibilities and limitations of the theory, the formulas, and the tables developed.

Maximum Revolutions per Minute and Chain Velocities. Table 1 of the paper should have been provided with heavy lines drawn so as to divide it into three areas: An upper left-hand area showing figures calculated from formulas [6] and [7]; an upper right-hand area calculated from formulas [8] and [9]; and a lower left-hand area calculated from formulas [4] and [5]. Each of these formulas contains a constant the value of which can be determined properly only from practical tests. Arbitrary values have been used in this table only to show the general plan. They are subject to change so as to conform to facts as they may be discovered from a study of actual performance in practice.

Formulas [8] and [9] affecting the upper right-hand area of the table are built upon a theory the only excuse for the existence of which is that it seems to satisfy, to an approximate extent, the general belief that centrifugal force in some way limits the allowable chain velocity. Not only are the constants in formulas [8] and [9] subject to change, but also the structure of the formulas may be changed in case a more plausible theory is discovered.

Merely changing the constants in formulas [4] to [9], inclusive, would not alter the theory on which they are based. If, for example, it is found that the figures for a 17-tooth sprocket carrying a No. 40 chain should be 10 per cent higher than those given in Table 1, the constants in formulas [8] and [9] can be increased 10 per cent and this would increase most of the figures in the upper right-hand area and cause a shift in the location of the line dividing that area from the lower left-hand area.

Conditions for a Uniform Chain Elongation. This mathematical research on the rapidity of chain wear has been conducted on the assumption that it would be desirable to know just how the life of a roller chain is affected, in theory at least, by changes in pitch, width, pin diameter, chain length, and the number of sprocket teeth. Disregarding centrifugal pull, the effect of which is almost negligible, it is concluded from formulas [20] and [21] that if uniform conditions of lubrication and service are assumed, the rate of chain elongation due to wear is proportional to

$$H(N_1 + N_2)/BPN_1N_2L_p$$

If now it is assumed that all chain drives are to be designed so that, under uniform conditions of lubrication and service, the chains will reach a 3 per cent elongation at the end of a specified number of hours of service, as 10,000 hours for example, the horsepower to be transmitted would have to be calculated from the formula

$$H = KBPN_1N_2L_p/(N_1 + N_2) \ldots\ldots\ldots [22a]$$

where K is a constant to be determined by experiment. The constant K has been taken tentatively equal to 0.023 for the sake of illustrating the use of the formula and of tables computed from the formula. This formula and the ones given later in this closure are numbered as supplements to those given in the paper to facilitate reference to the paper.

In order to simplify the horsepower tables computed from formula [22a], it has been further assumed that the chain length L_p contains in every case as many links as there are teeth in the two sprockets. That is, the tabulated horsepowers in Table 3 in the paper are for chains containing exactly $(N_1 + N_2)$ links. This must not be overlooked since, when $L_p = N_1 + N_2$, the center distance is very much shorter than would be used in the average drive, and Table 3 is only designed to give horsepower ratings for drives having short center distances. In a drive using 16 and 25 teeth, $N_1N_2 = 400$ and the chain length is assumed as $16 + 25 = 41$. The center distance would then be about 10 pitches, and at 1200 fpm the rated horsepower is 7.3. If the chain length is doubled the center distance would be about 31 in., which is much closer to the average distance in actual practice. This makes the rated horsepower 14.6 instead of 7.3 as found in the Table 3.

If Tables 3 and 4 had been based upon an assumed minimum chain length of $2(N_1 + N_2)$ pitches, the constants in formulas [26], [28], and [30], inclusive, would be doubled, the horsepower ratings in the upper right-hand area would all be increased, and the heavy stepped line would be moved to a higher position.

Table for the Selection of a Suitable Chain. Table 5, given with this closure, illustrates an attempt to provide some means by which a proper chain can be quickly selected for a drive when the horsepower, the number of teeth N_1 and N_2, the revolutions per minute, and the approximate center distance (in pitches) are known. Disregarding the slight effect of centrifugal action, but taking account of the chain length, we have from formula [28]

$$H = 0.023PBN_1N_2\ L/(N_1 + N_2) \ldots\ldots\ldots [28a]$$

Substituting $2C + [(N_1 + N_2)/2]$ for L in formula [28*a*], this substitution being sufficiently accurate, we obtain

$$H = 0.023PBN_1N_2[2C/(N_1 + N_2) + 1/2] \quad \ldots\ldots [28b]$$

$$H = 0.0127PBdN_1n_1 \quad \ldots\ldots [29b]$$

The horsepower rating can now be said to be the lesser of the two values computed from formulas [28*b*] and [29*b*]. The effect of a change in center distance is shown in the following example.

Let $P = 1$ in., $N_1 = 15$, $N_2 = 21$, $C = 21$, and the speed = 547 rpm. Then the rated horsepower is 10.5.

If the center distance is changed to 42 pitches, the rated horsepower is one and two-thirds times as great, or 17.5 hp.

Now from formulas [28*b*] and [29*b*]

$$23PB = \frac{1000H}{N_1N_2[2C/(N_1 + N_2) + 1/2]} \quad \ldots\ldots [28c]$$

$$12.7PBd = 1000H/N_1n_1 \quad \ldots\ldots [29c]$$

The first member of formulas [28*c*] and [29*c*] has a fixed value for each standard *chain*, and the second member has a particular value for each chain *drive*.

In this closure the author gives a table, numbered Table 5 as a continuation of the table numbers given in the paper. In Table 5, column X shows values of $23PB$ for the several standard chains listed in Table 1 of the paper. The column headed Y shows values of $12.7PBd$ for the same chains, and the column headed Z gives the max rpm for a 17-tooth sprocket as listed in Table 1 of the paper.

If we compute

$$x = \frac{1000H}{N_1N_2[2C/(N_1 + N_2) + 1/2]}$$

and

$$y = 1000H/N_1n_1$$

for a proposed drive, and z is taken as the revolutions per minute of the smaller sprocket, it is then only necessary to refer to the table for a set of values of X, Y, and Z, each of which is greater than x, y, and z. The standard chain number is then found in column 4 of Table 5. In most cases there will be several possible selections, and the one highest in the list will usually be the most economical. A quick reference to the table of maximum revolutions per minute will give an adequate check on the selection.

As an example, let it be required to select a chain to transmit 15 hp over sprockets having 15 and 25 teeth, the smaller wheel turning at 900 rpm, and the center distance to be about 30 pitches. Then

$$x = \frac{1000 \times 15}{15 \times 25\{[(2 \times 30)/(15 + 25)] + 0.5\}} = 20$$

$$y = 1000 \times 15/15 \times 900 = 1.111, \quad \text{and } z = n_1 = 900$$

In Table 5, sixth row from top, the values of X, Y, and Z are 20.20, 3.470, and 940, respectively, all of which are greater than the required values. The standard chain number is 80 and is a 1-in. pitch chain. It will be noticed that double-, triple-, and quadruple-width chains are listed as well as the single-width chain. If a selection cannot be made from the list of single chains, reference is made to the double-width chains, then to triple-width or the quadruple-width chains. In this case chain number *D*60 could be used but it would not be as economical as the single-width chain No. 80.

TABLE 5 VALUES FOR THE SELECTION OF ROLLER CHAINS

Single-width chains					Triple-width chains				
X	Y	Z	Std. chain no.	Pitch and width	X	Y	Z	Std. chain no.	Pitch and width
2.46	0.185	2662	35N	3/8 × 3/16	7.41	0.555	2662	T35N	3/8 × 3/16
4.00	0.314	2500	41	1/2 × 1/4	12.05	0.942	2500	T41	1/2 × 1/4
4.96	0.427	2310	40	1/2 × 5/16	14.90	1.281	2310	T40	1/2 × 5/16
7.69	0.850	1830	50	5/8 × 3/8	23.10	2.550	1830	T50	5/8 × 3/8
11.90	1.534	1500	60	3/4 × 1/2	35.60	4.700	1500	T60	3/4 × 1/2
20.20	3.470	940	80	1 × 5/8	60.5	10.41	940	T80	1 × 5/8
30.50	6.320	645	100	1 1/4 × 3/4	91.5	18.96	645	T100	1 1/4 × 3/4
47.5	11.43	520	120	1 1/2 × 1	142.0	34.29	520	T120	1 1/2 × 1
57.8	16.00	370	140	1 3/4 × 1	172.0	48.00	370	T140	1 3/4 × 1
80.6	24.95	325	160	2 × 1 1/4	241.8	74.85	325	T160	2 × 1 1/4
122.0	51.26	240	200	2 1/2 × 1 1/2	366.0	153.78	240	T200	2 1/2 × 1 1/2
Double-width chains					Quadruple-width chains				
4.95	0.370	2662	D35N	3/8 × 3/16	9.90	0.740	2662	Q35N	3/8 × 3/16
8.06	0.628	2500	D41	1/2 × 1/4	16.10	1.256	2500	Q41	1/2 × 1/4
9.93	0.854	2310	D40	1/2 × 5/16	19.84	1.708	2310	Q40	1/2 × 5/16
15.40	1.700	1830	D50	5/8 × 3/8	30.80	3.400	1830	Q50	5/8 × 3/8
23.70	3.065	1500	D60	3/4 × 1/2	47.4	6.130	1500	Q60	3/4 × 1/2
40.30	6.940	940	D80	1 × 5/8	80.6	13.88	940	Q80	1 × 5/8
61.0	12.64	645	D100	1 1/4 × 3/4	121.8	25.28	645	Q100	1 1/4 × 3/4
95.0	22.86	520	D120	1 1/2 × 1	188.0	45.72	520	Q120	1 1/2 × 1
115.7	32.00	370	D140	1 3/4 × 1	231.5	64.00	370	Q140	1 3/4 × 1
161.0	49.90	325	D160	2 × 1 1/4	322.4	99.80	325	Q160	2 × 1 1/4
244.0	102.52	240	D200	2 1/2 × 1 1/2	488.0	205.04	240	Q200	2 1/2 × 1 1/2

Referring to J. N. Arnold's discussion, the difference in the results obtained from his chart and those obtained by the method just described is due to the fact that the author is here taking account of the effect of a change in center distances, whereas Mr. Arnold's chart was designed to give results corresponding to formulas [28] and [29] and to Table 3. Doubtless Mr. Arnold's chart could easily be modified to take care of varying center distances if desired.

Effect of an Excessive Number of Teeth. One more comment seems to be important. An inspection of Table 4 of the paper shows that in the case of drives having over 60 teeth in the larger sprocket the horsepower ratings seem to run higher than they should. The reason for this is that the present research has dealt only with the rate of chain elongation and has not taken into account the fact that, whereas chains operating over sprockets with less than 60 teeth can often stretch 3 per cent before being discarded, this is not true where the sprockets have considerably more than 60 teeth.

In the case of 90 teeth the chain will climb very nearly to the top of the teeth when the elongation is only 2 per cent; and this means that the allowable life of the chain on a 90-tooth sprocket is only two-thirds as great as one running on a 60-tooth sprocket. To take account of this a note may be attached to Table 5 stating that, where N_2 is more than 60, the values of x are to be multiplied by $N_2/60$ before being applied to the table.

The Leakage of Steam Through Labyrinth Seals[1]

R. L. Scorah.[2] The subject of flow through labyrinth seals has received rather limited study, even though it has been known for sometime that the older formulas do not always give reliable results. Mr. Egli's paper will no doubt form a welcome addition to the literature of this field.

It should be pointed out, however, that the static tests referred to in this paper do not simulate the real conditions of operation. In practice, the leakage steam flows in a different sort of channel, one part of which is stationary while another part, the shaft, rotates at high speed. It has not been demonstrated that the leakage is independent of the speed of rotation. Furthermore,

[1] Published as paper FSP-57-5, by Adolf Egli, in the April, 1935, issue of the A.S.M.E. Transactions.

[2] Mechanical Engineering Department, Stanford University, California.

the argument presented here is confined to the gas phase of the flowing fluid, that is, superheated steam. In some cases, the steam entering the seal will be wet or only slightly superheated. Under these conditions, the effect of supersaturation and water drops further complicate the problem. In view of the wide variety of designs used in practice and the absence of precise information regarding supersaturation and the behavior of entrained water drops, it would seem expedient at this time to investigate experimentally the leakage through various labyrinth constructions using a rotating-shaft element. Such experiments would not only provide reliable leakage data for the particular constructions tested but would also serve as a helpful check on the rational theoretical treatment of the problem.

H. G. Yates.[3] The writer is interested in knowing if Mr. Egli has studied the effect of inclining the baffle strips at an angle of 45 deg. Glands of this type in conjunction with a spring mounting have been found satisfactory. It is uncertain, however, how far their good sealing properties are due to the inclination and how far to the accurate maintenance of fine clearances by means of the spring mounting. Any gain in sealing properties would probably be proportionately smaller with very fine clearances (of the order of 0.01 in. or less) as the small radius, which necessarily exists on even a "sharp" baffle strip, would assume proportionately greater importance, and the constriction would cease to be a sharp-edged orifice and would tend toward one with well-rounded edges. It is presumably this effect which gives rise to the curves of Fig. 18 of the paper. An important advantage of the inclined baffle strip is that in the event of local heating due to a momentary rub, the strips tend to cockle up away from the shaft rather than to expand toward it.

Another gland which has been employed frequently is of the "straight-through" type but with the baffle strips adjacent to a number of thin closely pitched ribs turned on the shaft or gland sleeve, of different pitch from the strips themselves, so that there is no fear of a straight blow-through if the rotor should be moved in an axial direction. Besides having the property of rapid cooling in the event of a rub, it is believed that the "valleys" between the ribs or pips serve to break up the steam flow and give almost as good a seal as a gland of the well-staggered type. The writer would welcome Mr. Egli's opinion on this.

B. Hodkinson.[4] Mr. Egli refers more than once in his paper to the high velocity through the last throttling and points out that the Fanno line may at this stage be departed from.

A way of allowing for this in the calculation suggests itself. The pressure before the last throttling may be taken as twice the final pressure, and the calculation carried only to this point. In other words, the last throttling could be done away with, and the final pressure doubled. This would raise the calculated figure slightly, except in odd cases, and in the light of some tests at Trafford Park, Manchester, on a grooved valve spindle, would make the answer more reasonable, because we found a greater mass flow than appeared, according to calculation, to be possible.

The odd cases where the calculated discharge would not be increased occur when p_2/p_1 is higher than about 0.2, and then the last throttling probably does not carry sound velocity. Most practical labyrinths operate with p_2/p_1 a good deal less than 0.2. Thus, it would be well to use the above suggested way only when p_2/p_1 equals, say, 0.1 or less.

[3] English Electric Co., Ltd., Rugby, England.

[4] Turbine Experimental Section, Mechanical Engineering Department, Metropolitan Vickers Electrical Co., Ltd., Manchester, England.

Author's Closure

The author fully realizes the fact, mentioned by Prof. R. L. Scorah, that the actual flow conditions are not correctly simulated in the static tests referred to in the paper. Some of Friedrich's[5] tests, which were made with a rotating shaft, show the effect of peripheral speed to be rather small. At 200 ft per sec the decrease of leakage due to the speed of the shaft has been found to be 3.4 per cent. For moderate peripheral speeds, therefore, the static tests will not be much in error.

There is a further point with a view to simulating the actual flow conditions, which has been neglected in the static tests. It is the exact shape of the edges of the sealing strips after they have been rubbing in the running turbine. The strip generally becomes burred over and the sealing edge is no longer sharp, as assumed in the static tests, but is in cross-section shaped rather like a mushroom. The characteristic function ψ of such a mushroom-shaped throttling undoubtedly is somewhat different from the function ψ of a sharp-edged orifice shown in Fig. 2 of the paper. It would be principally possible to determine this function by testing a single "mushroom-shaped" strip. Curves φ versus pn/po similar to those of Fig. 7 of the paper could then be constructed with the methods there presented. For most practical purposes, however, the φ curves of Fig. 7 will be sufficiently accurate.

The author has had no experience with the sealing properties of the particular types of baffles described by H. G. Yates, and does not believe that by inclining the labyrinth strips the amount of leakage for a given clearance should descrease measurably. If such glands have proved successful in the turbine, it apparently is due to the maintenance of a fine clearance. The "straight-through" type seal with strips adjacent to a mumber of closely pitched ribs is probably somewhat tighter than the "straight-through" type gland referred to in the paper. It is his opinion, however, that this type of packing will not reach the good sealing properties of a well-staggered labyrinth. The leakage jet issuing from one clearance very easily "bridges" over the narrow "valleys" between the ribs by simply maintaining a series of stationary vortices in each valley.

The method of considering the great pressure drop across the last throttling, as suggested by H. B. Hodkinson may give fairly correct results, although the assumption of the pressure before the last throttling to be twice the final pressure is quite arbitrary. There is, however, no need of treating separately the last throttling when following the methods outlined in the paper. The curves of Fig. 7 and 7*a* automatically take into account the flow characteristics of each throttling in the labyrinth including the last one.

The physical nature of the leakage flow along a grooved valve stem, to which Mr. Hodkinson refers, is quite different from that in a labyrinth packing. Here the loss of kinetic energy due to friction in the narrow space between stem and bushing must be taken into consideration whereas in a labyrinth the friction in the short passage through the throttling gap plays a minor rôle only.

[5] "Untersuchungen ueber das Verhalten der Schaufelspaltdichtungen in Gegenlauf-Dampfturbinen," by H. Friedrich, *Mitt. Forsch. Anst. G. H. H.*, Oct., 1933.

Cause and Prevention of Turbine-Blade Deposits[1]

By FREDERICK G. STRAUB,[2] URBANA, ILL.

A survey was made of power plants encountering turbine-blade deposits and it was found that this trouble results from contamination of the steam by the boiler water. A study in the laboratory showed that sodium hydroxide is the material which causes the sticking to the blades. It was also shown that when the sodium hydroxide is neutralized and changed to a salt such as sodium carbonate, it will not adhere. The presence of sufficient amounts of inert salts, such as sodium sulphate, with the sodium hydroxide also will stop the deposit from forming.

The action of sodium sulphate in preventing the blade deposits was studied in a large central power plant, and the deposit was materially reduced. A small testing unit for detecting the presence of adhering salts in the steam was developed and used in the power-plant tests.

STEAM-electrical generating stations have encountered difficulty in the form of fouling of turbine blades. This difficulty has become of major importance in many large stations whereas it has only meant annoyance in other stations.

There are several types of deposits which form on the turbine blading and cause this fouling. One is that which is apparently caused by a deposition of solids carried in the steam from the boiler water and another is that caused by a chemical reaction between chemicals in the steam and the material in the turbine blade. The first type is most common and is readily distinguished because it is largely water-soluble and is washed off with comparative ease, whereas the other type adheres tenaciously to the blades.

The difficulty caused by the deposition of solids carried in the steam appears to be the one causing the major difficulty. The efforts of this research have been expended entirely toward the study of this type of deposit and no study has been made of the other type.

---

[1] Part of the research conducted in cooperation with the Utilities Research Commission, Chicago, Ill. Published by permission of Dean M. L. Enger, Director, Engineering Experiment Station, University of Illinois.

[2] Special Research Assistant Professor in Chemical Engineering, University of Illinois. Mr. Straub was graduated from the University of Illinois in 1920. After leaving the university, he was associated with Mellon Institute, Pittsburgh, Pa.; Semet Solvay, Syracuse, N. Y.; and Guggenheim Brothers Research Laboratories, New York, N. Y. He holds the degrees of Master of Science and Metallurgical Engineer from Pennsylvania State College. He has been conducting special research for the Utilities Research Commission, Inc., on boiler-feedwater treatment for the last eleven years at the University of Illinois. This has included work on determining the causes and methods of prevention of embrittlement in steam boilers and a study of the methods of preventing scale in high-pressure boilers.

Contributed by the Power Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication in a later issue of Transactions.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Industrial Experience

A survey was made of power plants encountering this trouble as well as those not having difficulty with blade deposits.

The summary of conclusions reached after assembling the available data in regard to turbine-blade deposits is as follows:

(1) The deposits formed independent of steam pressure or temperature.

(2) The deposits were not proportional to the total concentration of solids in the boiler water.

(3) The total solids in the steam were very low in many plants where deposits occurred.

(4) The deposits all contained sodium present as hydroxide, carbonate, or silicate.

(5) The deposits were practically all water-soluble.

LABORATORY EXPERIMENTS

Possible Cause of Deposits

The deposits which were formed in the plants under consideration all originated in the boiler water. A small amount of the boiler water may be easily carried into the steam. Actually, this amount does not have to be more than one-tenth of one per cent of moisture in the steam to cause appreciable trouble at the turbine. Thus if a boiler water should contain about 300 ppm total solids, which is a rather low concentration, and one-tenth of one per cent of the steam were present as boiler water, the steam would contain 0.3 ppm of total solids. This appears a negligible amount; however, with a turbine using one million pounds of steam per hour, 8 lb of solids would pass through the turbine in 24 hr. If only 10 per cent of this were to adhere to the turbine blades, the efficiency and capacity loss would increase at an alarming rate. Thus if there is present in the boiler water any material which would cause adherence to the turbine blades, the better quality steam might cause appreciable difficulty.

In order to determine whether there is present in the average boiler water material which would cause adherence or sticking to the turbine blade a study was made of the behavior of the various salts encountered in boiler waters as they pass from solution in wet steam to superheated steam. If only pure water were present in the boiler and a small amount of the boiler water were mechanically carried into the steam and to the superheater, the droplets of water would vaporize in the superheater and no free moisture would be present in the superheated steam. However, if sodium chloride, sodium carbonate, or sodium sulphate were present in the boiler water the droplet of water entering the superheater with the steam would contain a dilute solution of the salt or salts. As the steam became superheated the water would vaporize and leave a dry salt or powder of the salt or salts with the superheated steam. Consequently, they would pass through the steam pipes and turbine as a fine powder or dust and cause no appreciable difficulty. This is based on the assumption that there is only a small percentage of moisture in the steam. However, if sodium hydroxide were present this would not happen. This results from the fact that the behavior of sodium-hydroxide solutions is entirely different from that of the other salts discussed.

When a solution of sodium chloride, carbonate, or sulphate is boiled, the solution is concentrated as the steam is released and the solution concentrates until a saturated solution is obtained. Any further release of steam results in the precipitation of the salt. Eventually, if the boiling is continued, all the water is evaporated and dry salt or salts are left behind. If a solution of sodium hydroxide is boiled, the solution concentrates with a continual increase in temperature until a concentration is reached where the concentrated solution is in equilibrium with the vapor pressure of the surrounding atmosphere and further heating will cause a release of steam to the surrounding atmosphere with an increase in concentration of the caustic solution and an increase

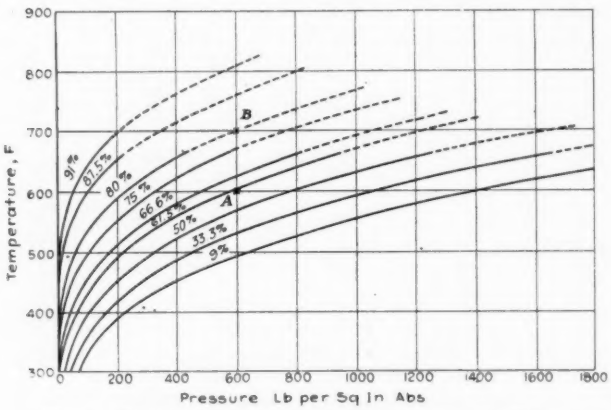


FIG. 1 CONCENTRATION OF NaOH IN RELATION TO STEAM PRESSURE AND TEMPERATURE

in the temperature of the solution. This is shown graphically in Fig. 1 and may be illustrated as follows:

If a drop of water containing sodium hydroxide in solution leaves a boiler at a pressure of 600 lb per sq in. abs and enters the superheater with the steam, the water will vaporize, thus concentrating the solution. When the temperature of the steam reaches 600 F the concentration of the caustic will represent about a 60 per cent solution, shown as *A* in Fig. 1. If the temperature and pressure remained constant, the droplet of caustic would remain 60 per cent sodium hydroxide and 40 per cent moisture, even in contact with superheated steam. As the temperature increases, the pressure remaining constant, the caustic concentration will increase. At a pressure of 600 lb and a temperature of 700 F, the concentration would be as shown at point *B*, an 80 per cent solution.

The concentration of caustic in equilibrium with superheated steam at various pressures and temperatures is shown in Fig. 1. This shows that the concentrations of sodium hydroxide reached in the average higher pressure plant is between 80 and 90 per cent. A solution of sodium hydroxide containing between 10 and 20 per cent moisture at these temperatures will be in a pasty or semi-fluid state and in going through the turbine will adhere to the blades. The other salts, present as a fine dust, will adhere with the sodium hydroxide, but not necessarily in the relative proportions in which they exist in the boiler water. When the temperature of the steam in the turbine is lowered until saturated steam exists these salts will be dissolved and thus wash off of the blades. The experience which turbine operators have had with washing of turbines confirms this.

In order to substantiate this theory tests were run in the laboratory. The procedure followed in these tests was to produce steam in a small boiler, contaminate the steam with a solution of a desired salt or combination of salts, superheat the contaminated steam, pass the steam through an orifice so that it impinged on a stationary blade, and then condense the steam at a predetermined pressure.

Fig. 2 shows the apparatus used for the tests. Condensed steam from the steam-heating system was passed through a small deaerating heater and then pumped to an electrically heated boiler to generate steam. The steam was then passed under pressure into the contaminator. The contaminator contained a dilute solution of the contaminating salt or salts and was so built that the steam entered the bottom, bubbled up through the solution, and was taken off at the top. The contaminator was heated by means of a hot plate to compensate for radiation loss and to maintain a constant level of solution. The steam leaving the contaminator was passed through an electrically heated superheater, then through an orifice so as to impinge on the removable blade, and then through a copper-coil condenser to the condensate storage. The steam pressure and the superheated-steam temperature were maintained constant by means of potentiometer temperature regulators. The temperature of the blade on the side opposite the point of steam impingement was recorded on a recording potentiometer temperature recorder. When a vacuum was maintained in the condenser this was done by evacuating the condensate storage to the desired pressure. The pressure in the container holding the blade was determined by means of a pressure gage.

If the temperature and pressure existing in the generating

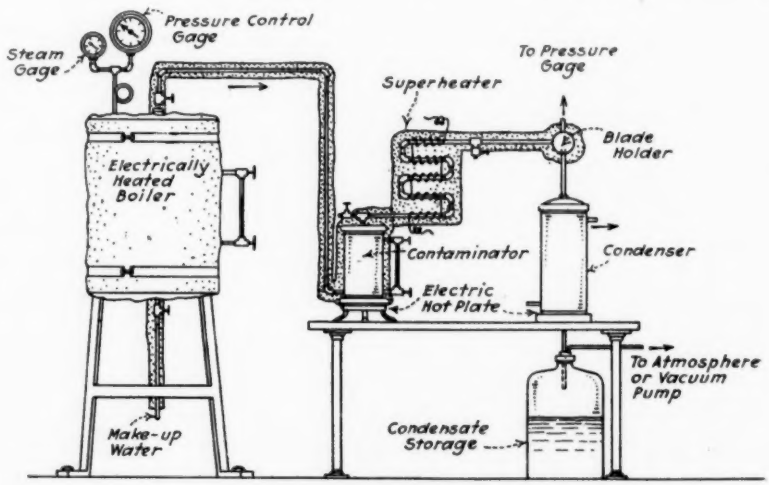


FIG. 2 ASSEMBLY OF APPARATUS

stations was in range of 600 lb and 700 F, a concentration of about 80 per cent sodium hydroxide would result. In order to simplify the construction of the test apparatus the tests were run at 45 lb per sq in. abs and 400 to 700 F, with a pressure of 0.8 lb in the blade chamber. Reference to the curve in Fig. 1 will show that under these conditions the sodium hydroxide should reach a concentration of between 85 and 90 per cent. This concentration is in the range of that reached in turbines. The orifice was $^1/_{16}$ in. in diameter and the blade was set at an angle of 30 deg to the line of steam flow leaving the orifice.

The first tests were run for 22 hr with 2 grams sodium hydroxide added to the contaminator and the temperature of the superheated steam varied between 400 and 700 F. Deposits formed on the blades. The theoretical composition of the sodium hy-

droxide in the steam at the time of contact with the blade during these tests varied from about 85 per cent at 400 F to about 98 per cent at 700 F. Tests run with distilled water alone in the contaminator gave clean blades. When sodium chloride was added to the contaminator and tests run at 500 F, no deposit formed on the blade in the area of high velocity. Similar tests with sodium sulphate gave like results. Trisodium phosphate and sodium silicate both formed deposits on the blade.

The sodium-hydroxide, silicate, and phosphate deposits were formed because hydroxide was present in all these salts. If this is true, any chemical added to the contaminated steam which would neutralize the hydroxide should prevent the hydroxide from forming. In order to prove this hypothesis the apparatus was modified so that carbon dioxide could be slowly added to the steam beyond the contaminator and ahead of the superheater. The carbon dioxide, if present in excess, should react with the hydroxide to form sodium carbonate. This salt should exist as a dry powder in the steam and no deposit should form. When carbon dioxide was added after contamination with sodium hydroxide, sodium silicate, and sodium phosphate, no deposits formed. This showed conclusively that the sodium hydroxide was the binding or adhering agent, and if it were changed to carbonate prior to reaching the blade, no deposit would form.

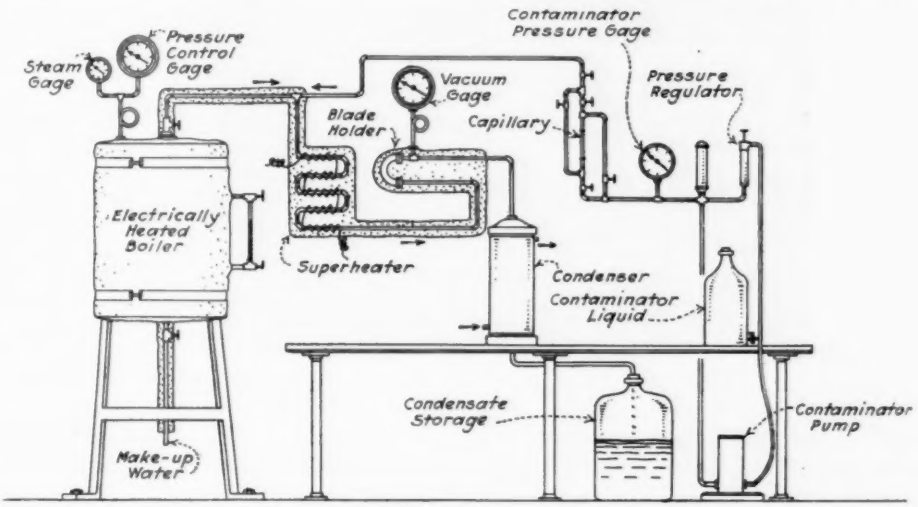


FIG. 3 ASSEMBLY OF APPARATUS, REDESIGNED

In order to study the possibility of the utilization of carbon-dioxide treatment the apparatus was modified so that the carbon dioxide could be added to the superheated steam instead of the wet steam. When tests were run using this method of treatment the deposits continued to form even when large excesses of carbon dioxide were present.

This change in the action of the carbon dioxide is easily explained. When the gas is added to the wet steam the sodium hydroxide in the droplets of water is in a dilute solution. In this state the carbon dioxide also is soluble in the water depending upon its partial pressure in the steam. As the two chemicals are in dilute solutions, they are strongly ionized and will react readily so that the sodium carbonate is formed before the steam is superheated. However, when the steam is superheated before adding the carbon dioxide the excess water present is converted to steam and the sodium hydroxide is present in the form of a highly concentrated solution, more than 80 per cent NaOH. In such a solution the hydroxide is only slightly ionized, the carbon dioxide is present in the steam as a dry gas, consequently there is practically no reaction between the gas and the sodium hydroxide. This appeared to make the carbon-dioxide method of treatment rather complicated.

CONCENTRATION OF CAUSTIC NECESSARY FOR DEPOSIT

After this substantiation of the hypothesis that sodium hydroxide is the basic adhering material in the turbine-blade deposits it seemed desirable to obtain further data on the limiting concentration of sodium hydroxide in the steam which would cause blade deposition and on other methods of prevention of this troublesome deposit.

In the apparatus already described the carry over or contamination of the steam was excessive in the early period of the test. This was shown by the fact that with a constant water level in the contaminator the concentration of the contaminating salts steadily decreased. At the same time it was almost impossible to know the amount of contamination in the steam at any particular time.

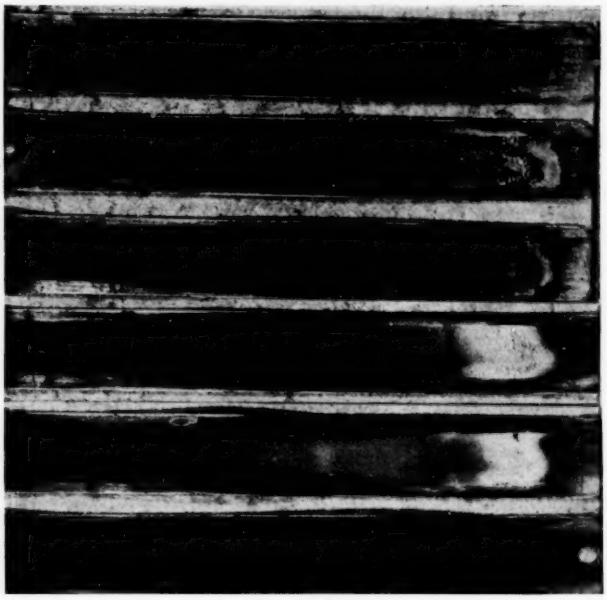
In order to overcome these objections the apparatus was redesigned as shown in Fig. 3. The blades were made from watch springs ($^3/_8$ in. by 0.020 in.) and were removed at the end of each test. The steam nozzle was made by slotting the end of the tube and this brought a sheet of steam in contact with a curved blade in a manner similar to that in which the steam strikes the blade of a turbine.

The contamination was brought about by pumping the solution containing the desired concentration of chemicals into a small reservoir where it was held at a definite pressure by means of a ball-seat pressure regulator. The excess liquid leaving the regulator was returned to the pump suction. The reservoir was connected through a small capillary orifice to the steam line ahead of the superheater. The steam pressure was maintained constant by means of a steam pressure regulator connected through relays to electrical heat controls on the boiler.

With a constant pressure drop across the orifice the amount of contaminating liquid entering the steam through the orifice was constant and depended upon the pressure drop across the orifice which was under definite control. About 0.55 ± 0.02 lb per lb per hour of contaminating liquid was fed into the steam. The total amount of steam—steam from boiler plus steam from contaminator liquid vaporized in the superheater—passing through the nozzle was about 10 lb per hr.

The water fed to the boiler was condensed steam from the steam-heating system which had passed through an electrically heated boiling deaerating heater prior to being pumped to the boiler. The boiler pressure was maintained at 40 lb gage. Several pressures in the chamber holding the blade were used.

Tests were then run using the new apparatus and holding the vacuum in the blade chamber at 20 in. of mercury. With the sodium hydroxide in the steam (calculated) at 48, 18, 5, 2, and 1.5 ppm deposits formed in 20-hr tests. Tests in which distilled water was added to the steam gave no deposit. When the steam was passed through the nozzle there was a marked change in velocity. The calculated velocity in the steam line ahead of the nozzle was 21.5 fps. The velocity through the nozzle was 1175



TEST NO.	CHEMICALS ADDED TO STEAM, PPM $NaOH$	Na_2SO_4	SiO_2
138	1.54	11.3	—
139	1.67	—	—
140	1.74	—	—
149	1.4	—	0.87
151	5.0	—	3.1
152	4.4	17.5	2.5

FIG. 4 PHOTOGRAPHS OF BLADES AFTER TESTS

fps, and the velocity past the blade was 950 fps. Thus the velocity at point of contact with the blade was approximately 1000 fps.

The maintenance of a vacuum in the blade holder complicated the operation of the test. Consequently, tests were run in which atmospheric pressure was maintained in the blade holder. With such a change in conditions the velocity of the steam in the holder dropped from a nozzle velocity of 1175 fps to 320 fps. Undoubtedly the velocity at the point of impact with the blade was still in the range of 1000 fps. Tests run under these conditions with 4, 5, 4.0, 3.2, and 2 ppm of sodium hydroxide added to the steam gave deposits in the 20-hr tests. This showed that a deposit would form with the sodium-hydroxide content of 2 ppm in 20 hr. The remainder of the tests were run at this pressure unless otherwise noted.

Sodium sulphate alone was added so as to have 10 ppm in the steam and no deposit formed. A mixture of sodium hydroxide and sulphate was then added so that the NaOH was 4.2 and the Na_2SO_4 8.6 ppm in the steam and small deposits formed, but not as much as that formed by the sodium hydroxide in the absence of the sodium sulphate. The sodium hydroxide was then increased to 5.8 and the sulphate to 25 ppm and no deposit formed. This test was followed by one with a sodium-hydroxide concentration of 2.9 ppm and no sulphate. A definite deposit formed. This appeared to indicate that the sulphate was preventing the hydroxide from adhering to the blade.

Tests were then run with the sodium-hydroxide concentration held constant at 3 ppm and the temperature at 350, 400, and 615 F. A deposit formed in all these tests, indicating that changing the temperature did not have much effect upon the tendency of the sodium hydroxide to adhere to the blade.

The theoretical composition of the sodium hydroxide in the steam leaving the nozzle and in the blade holder are given in Table 1 for these temperatures. This shows that the caustic soda is adhering over a range of approximately 50 to 93 per cent and possibly a greater range, and illustrates the fact that temperature does not materially affect the tendency to deposit if the steam is superheated.

The procedure followed in running these tests involved attaching the blade holder after the steam was superheated and removing it while the steam was still superheated. Modification of this procedure so as to attach the blade holder before the steam was superheated but while the caustic solution was being pumped into the steam gave a condensate having a distinct caustic alkalinity which corresponded with the theoretical contamination. However, when the steam was superheated the condensate was practically free from sodium hydroxide. This showed clearly that the sodium hydroxide was being removed prior to the condensing of the steam. The deposit on the blade showed that the last bit of sodium hydroxide was being removed there.

It was doubtful that all the sodium hydroxide was being removed at the blade since a large amount was undoubtedly being removed in the superheater and piping ahead of the nozzle. At the end of each run the wet steam was run through the superheater and pipe after the blade holder had been removed. This contained appreciable amounts of caustic soda and proved that it was depositing in the superheater.

In order to determine the actual amount of sodium hydroxide in the steam at the nozzle and to see how completely it was being removed at the blade the apparatus was redesigned so that the conductance and pH value of samples of the contaminated steam and condensed steam before and after the blade holder could be determined. The boiler, superheater, blade holder, and condenser were the same as those previously used. A steam sampler was installed just ahead of the nozzle. The $^3/_8$-in. steam pipe was expanded into a $1^1/_4$-in. pipe in which two $^1/_8$ in. tubes had been inserted. These were drilled with small holes. The $^1/_8$-in. tubes passed through condensers to a valve on the cool side of the condenser. In this manner a small amount of the contaminated superheated steam was removed continuously from the steam,

TABLE 1 THEORETICAL CONCENTRATION OF SODIUM HYDROXIDE IN STEAM BEFORE AND AFTER NOZZLE

Steam press. gage, lb per sq in.	Steam temp, F	Pressure in blade holder	—NaOH in steam— Before nozzle, per cent	In blade holder, per cent
40	350	atm	45	63
40	400	atm	62	75
40	500	atm	79	87
40	600	atm	90	93

condensed under pressure, cooled, removed, and passed, without exposure to the air, to a conductivity cell and a glass electrode. Thus the conductance and the pH value of the condensed contaminated steam were determined. The condensed steam leaving the condenser after the steam had passed over the blade was also passed through a conductivity cell and a glass electrode. A comparison could thus be made of the values obtained from the condensed steam.

Distilled water was used for make-up. It was passed through the electrically heated boiling deaerating heater to remove dissolved gases such as oxygen and nitrogen and was pumped to the boiler. With this type of boiler feedwater the condensed steam from the boiler had a conductance of 0.05×10^{-6} mho or less.

When feedwater of this better quality was used the test procedure was modified so that the larger blade was weighed before and after the tests. The time of the tests was reduced from 20 to $5^1/_2$ hr. Tests for carbon dioxide were also run on the condensed steam after the blade in some cases.

Tests were run under these new conditions. Fig. 4 shows photographs taken of the blades after the tests. The blades have been flattened out. These results are interesting because they show that when sodium hydroxide is added to the steam in amounts as low as 0.55 ppm appreciable deposits, 2.7 mg in $5^1/_2$ hr, formed. This concentration in the steam is a maximum value.

The results in Table 2 also show that sodium sulphate and disodium phosphate did not cause deposits to form, whereas trisodium phosphate and sodium chloride caused small deposits to form.

TABLE 2 RESULTS OF TESTS USING STEAM CONTAMINATED WITH SULPHATE, CHLORIDE, AND PHOSPHATE

(Steam pressure 40 lb gage; steam temperature 500 F; pressure in blade holder, atmospheric; conductance of boiler feedwater, between 0.33 and 0.50×10^{-6} mho)

Test no.	Chemical contamination in steam ahead of superheater, ppm	Specific conductance $\times 10^{-6}$ mho, condensed steam samples			Increase in weight of blade, mg	CO_2 after blade, ppm
		Before blade		After blade		
137*B*	None	1.00	0.91	0.83	0.0	...
156	5.9 Na_2SO_4	3.72	3.67	2.38	0.2	0.9
158	5.5 NaCl	2.15	2.21	1.42	1.5	0.4
160	1.3 NaH_2PO_4	1.68	1.38	1.33	0.0	0.4
159	5.4 Na_3PO_4	6.90	5.95	3.38	1.1	1.0

Prevention of Deposit

Table 3 shows the results of tests run with varying amounts of sodium sulphate when the sodium-hydroxide content was constant. When the ratio of the sodium sulphate to the sodium hydroxide was 4.4 and greater the deposit did not form. This checked with the previous results and indicated that the presence of a sufficient amount of sodium sulphate with the sodium hydroxide prevents the formation of a deposit on the blade.

The addition of sodium chloride to the sodium sulphate so that the ratio of sodium sulphate to sodium hydroxide was 3.3 and sodium chloride to sodium hydroxide was 2.5, or the ratio of the total of the sodium sulphate and chloride to the sodium hydroxide was 5.8, did not entirely prevent the deposit; however, the deposit was less than it would have been with the sulphate alone present with the hydroxide, indicating that sodium chloride when present with the sulphate aids in the prevention of the deposit.

The presence of silicate along with the hydroxide tended to decrease formation of the deposit. When the sodium sulphate was present in an amount four times the sodium hydroxide, only a small deposit formed. This indicated that the sulphate was also effective in the presence of the silicate.

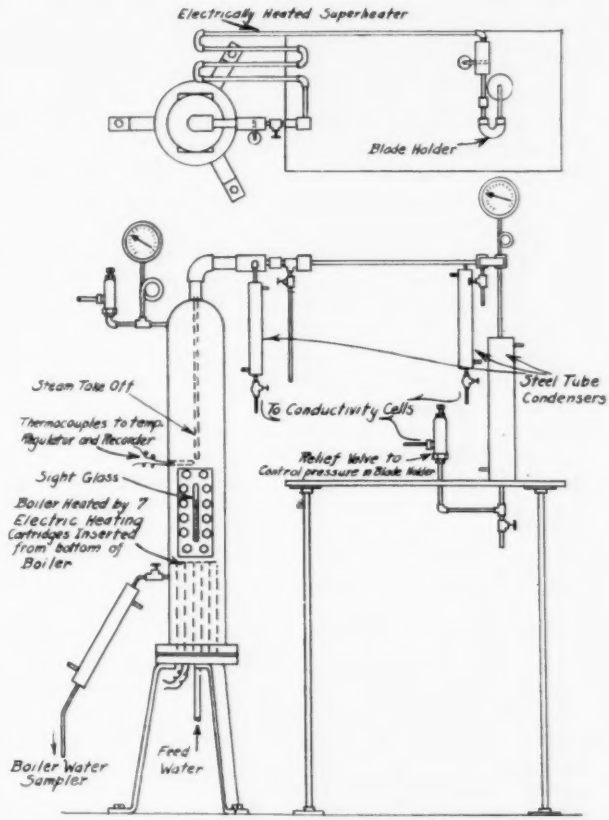


FIG. 5 APPARATUS USED FOR HIGHER-PRESSURE TESTS AND CONTAMINATION OF STEAM FROM BOILER WATER

Effect of Contaminating the Steam With Boiler Water

In the tests previously run the contamination was accomplished by adding the desired chemicals to the steam. This showed that if these chemicals were present in the steam the deposits could be formed or prevented. However, it seemed advisable to determine the actual ratios of the inhibiting chemicals necessary in

TABLE 3 RESULTS OF TESTS USING STEAM CONTAMINATED WITH SODIUM HYDROXIDE, SULPHATE, AND CHLORIDE

(Steam pressure 40 lb gage; steam temperature 500 F; pressure in blade holder, atmospheric; conductance of boiler feedwater, between 0.33 and 0.50×10^{-6} mho)

Test no.	Chemical contamination in steam ahead of superheater			Ratio		Specific conductance $\times 10^{-6}$ mho, condensed steam samples			Increase in weight of blade, mg	CO_2 after blade, ppm	pH	
	NaOH, ppm	Na_2SO_4, ppm	NaCl, ppm	Na_2SO_4 / NaOH	NaCl / NaOH	Before blade		After blade			Before blade	After blade
139	1.67	...	..	0.0	...	1.70	1.89	0.88	5.2	...	...	6.3
145	1.50	5.5	..	3.66	...	1.87	2.28	1.17	1.5	1.3	...	6.3
147	1.50	6.6	..	4.4	...	2.14	2.15	1.53	0.3	...	...	6.3
150	1.23	5.2	..	4.2	...	1.75	1.93	1.47	1.6	...	...	6.2
214	1.85	9.25	..	5.0	...	....	4.13	3.18	0.2	...	8.4[1]	8.5[1]
138	1.54	11.3	..	7.3	...	4.65	4.55	3.03	0.0	...	...	6.7
148	1.50	5.0	3.7	3.3	2.5	1.23	1.29	1.24	0.5	0.7	...	6.2
146	1.6	...	3.0	..	1.8	1.52	1.86	1.00	5.4	0.7	...	6.2
215	1.80	9.0[2]	1.80	5.0	1.0	....	5.30	4.03	0.1	...	8.3[1]	8.4[1]

[1] Using steel condenser, others using copper condenser.
[2] PO_4 also present 1.1 × NaOH.

TABLE 4 RESULTS OF TESTS USING STEAM CONTAMINATED WITH BOILER WATER

(Steam pressure 600 lb gage; steam temperature 700 F; pressure in blade holder 500 lb gage; conductance of boiler feedwater, between 0.33 and 0.50 $\times 10^{-6}$ mho)

Test no.	Concentration of salts in boiler water: NaOH, ppm	NaCl, ppm	Na_2SO_4, ppm	Ratio: Na_2SO_4/NaOH	Ratio: NaCl/NaOH	Specific conductance $\times 10^{-6}$ mho, condensed steam samples: Before blade	After blade	Increase in weight of blade, mg	pH[1]: Before blade	pH[1]: After blade
259	165	...	...	...	...	3.33	1.11	12.4	8.7	6.4
260	73	...	...	...	...	3.07	0.91	7.3	8.5	6.8
264	100	...	437	4.37	...	2.00	1.21	0.5	6.8	6.5
268	120	100	632	5.25	0.83	1.43	1.05	−0.1	6.6	6.7
269	260	72	1360	5.23	0.28	1.18	0.95	0.1	6.6	6.7
270	170	...	...	...	...	2.50	0.91	13.7	8.3	7.0

[1] Steel condensers.

the boiler water to prevent deposits. In such tests the contamination of the steam would result entirely from carry over from the boiler and the results would be more readily applied to actual operation.

The apparatus used for these tests is shown in Fig. 5. The superheater and the equipment beyond the boiler involved the same type of equipment as used before, except high-pressure fittings were used and stainless-steel pipe (18–8) was used for the superheater and subsequent piping. The boiler involved the use of seven 1.1-kw electric heating cartridges. These were inserted from the bottom into tubes welded into the bottom of the boiler. The water level and the behavior of the solution during boiling was observed by means of sight glasses placed on opposite sides of the boiler. The heat imput was controlled by means of potentiometer temperature regulators so as to maintain constant temperature and pressure in the boiler. The boiler feedwater was the same as previously described. The chemical contamination of the boiler water was accomplished by pumping the desired amount of chemical into the boiler at the beginning of the tests. Samples of the boiler water were taken for analysis at the beginning and end of each test. Condensed steam samples were taken ahead of the superheater, ahead of the nozzle and beyond the blade holder and the conductance and pH values were determined. These tests were run for $5^1/_2$ hr at a steam pressure of 600 lb per sq in. and a total steam temperature of 700 F. The orifice was changed to a round hole, No. 75 drill, instead of a slot. The pressure in the blade holder was changed from atmospheric to 500 lb per sq in. gage, thus giving a pressure drop of 100 lb across the nozzle. The amount of steam passing through the nozzle per hour was the same as in the previous lower-pressure tests.

When these tests were started the blades were found to be coated with a film of what appeared to be magnetic oxide. This coating formed all over the blade and appeared to be the result of a reaction between the superheated steam and the steel in the blade. When the pressure was dropped to 300 lb and the temperature to 500 F, this action stopped. Stainless-steel blades, 18 chromium, 8 nickel, were substituted for the steel blades. When the steam was of high quality and apparently contained no sodium hydroxide, the blade remained clean and was not attacked by the steam. When the boiler water contained sodium hydroxide the blades were attacked and became so brittle that they broke upon being removed. Substitution of monel-metal blades appeared to overcome these difficulties and a series of tests was run with this type of blade. The results of these tests are given in Table 4.

PLANT EXPERIMENTS

The results reported from many power plants indicate that the detection of the difficulty encountered with blade deposits is mainly in the form of loss of capacity. The efficiency loss becomes appreciable before capacity loss is detected. When capacity loss is noticed the deposits have formed, and they must be removed to obtain normal operation. Thus if the water treatment is to be studied or modified to prevent the deposits from forming, it is advisable to have a rapid method of studying the effect on the formation or prevention of deposits. Early in the research, attempts were made to devise a small testing unit which could be used to test samples of the steam in an operating plant and tell in a relatively short period of testing whether the steam would cause blade deposits or not. If such a unit could be devised, a definite study could be made on the effect of modification of water treatment on the deposits formed without waiting for the effect to be noticeable at the turbine.

A small test unit was made which would allow a small amount of the steam from the heater just ahead the turbine to flow through a nozzle, strike a section of regular turbine blades, and then exhaust through a regulating valve. With data relative to the temperature, pressure, and size of nozzle available, the velocity of the steam passing the blade and the quantity per unit of time could be calculated. The blade could be removed at any time desired and the amount of deposit formed determined by weighing.

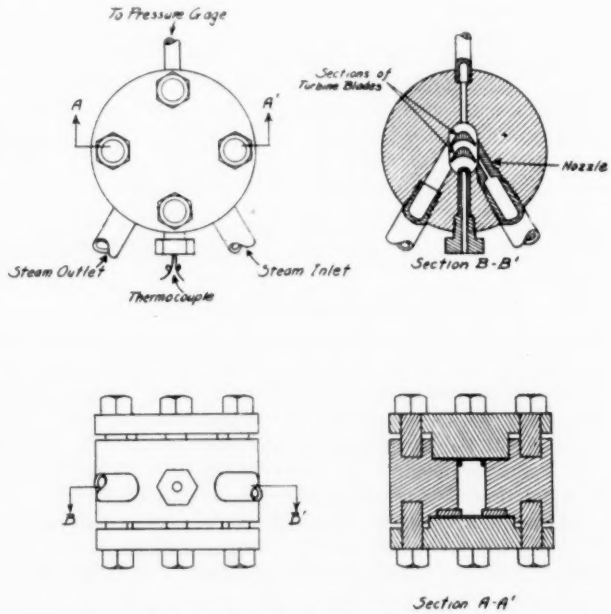


FIG. 6 TEST UNIT USED IN PLANT TESTS

Fig. 6 is a drawing of the test unit developed and used in the plant tests. The steam passed through a nozzle and over sections of regular turbine blades. Leaving the blades, the steam passed out of the unit and was piped through a valve to a desired point for disposal. By regulation of the exhaust valve the pressure in the chamber holding the blade could be held at any desired value. The temperature was determined by means of a thermocouple inserted in a well. The blade was cleaned, dried, weighed, and inserted into the holder. The cover was then bolted on and the steam pressure adjusted to the desired value. At th

TABLE 5 RESULTS OF TURBINE-BLADE DEPOSIT DETECTOR TESTS

Test no.	Starting date	Steam conditions: Flow, lb per hr	Nozzle: Press, lb per sq in.	Nozzle: Temp, F	Total flow, lb (thousands)	Duration of test, hr	Blade deposit: No. 1 blade g	No. 2 blade g	Mg per million lb steam	Boiler-water analysis: Dissolved solids	Total alkali as Na_2CO_3	PO_4	pH	Na_2SO_4	Ratio, Na_2SO_4 Total
1	4/11/34	870	346	668	45	53	0.0060	0.0053	251	170	74	52	10.2	26	0.1
2	4/16/34	870	345	666	205	234	0.0244	0.0211	221	182	70	50	10.1	28	0.3
3	4/26/34	430	415	686	112	260	0.0178	0.0175	310	198	83	44	10.2	30	0.4
4	5/ 8/34	430	415	675	22	52	0.0008	0.0013	...	203	86	46	10.1	31	0.4
5T	10/25/34	870	415	665	102	117	0.0014	0.0053	66	...	140	80	10.7	640	4.6
6T	10/30/34	870	415	660	119	137	0.0008	0.0016	20	...	150	75	10.8	655	4.3
7T	11/ 5/34	870	415	670	87	100	0.0006	0.0032	44	...	160	95	10.9	630	3.9
8T	11/12/34	870	415	675	65	75	0.0004	0.0009	20	...	180	95	10.9	625	3.5
9T	11/15/34	870	415	670	166	191	0.0014	0.0028	31	...	170	60	10.9	500	2.9
10T	11/23/34	870	415	660	143	164	0.0007	0.0011	13	...	120	60	10.8	470	3.9
11T	11/30/34	870 and 460	415	650	89	167	0.0156	0.0060	242	...	170	65	10.9	580	3.4
12T	12/ 7/34	460	415	640	75	162	0.0008	0.0011	25	...	190	60	11.0	420	2.3
13T	12/17/34	460	415	635	123	268	0.0008	0.0018	21	...	230	60	11.2	380	1.6
14T	12/28/34	460	415	640[1]	154	336	0.0045	0.0081	82	...	250	60	11.1	420	1.6
15T	1/11/35	460	415	640[1]	75	163	0.0003	0.0006	12	...	225	50	11.0	475	2.1
16T	1/18/35	550	415	640[1]	94	171	0.0017	0.0025	45	...	240	40	11.2	540	2.2
17T	1/25/35	550	415	640[1]	93	168	0.0064	0.0097	173	...	230	40	11.1	590	2.5
18T	2/ 4/35	550	415	640[1]	54	99	0.0002	0.0002	7	...	195	45	11.1	500	2.5
19T	2/ 8/35	460	415	640[1]	77	167	0.0014	0.0024	49	...	190	40	11.1	540	2.8
20T	2/15/35	460	415	640[1]	65	141	0.0010	0.0036	71	...	195	40	11.1	530	2.7
21T	2/21/35	460	415	640[1]	88	192	0.0044	0.0058	116	...	230	60	11.1	510	2.2

[1] Approximate.

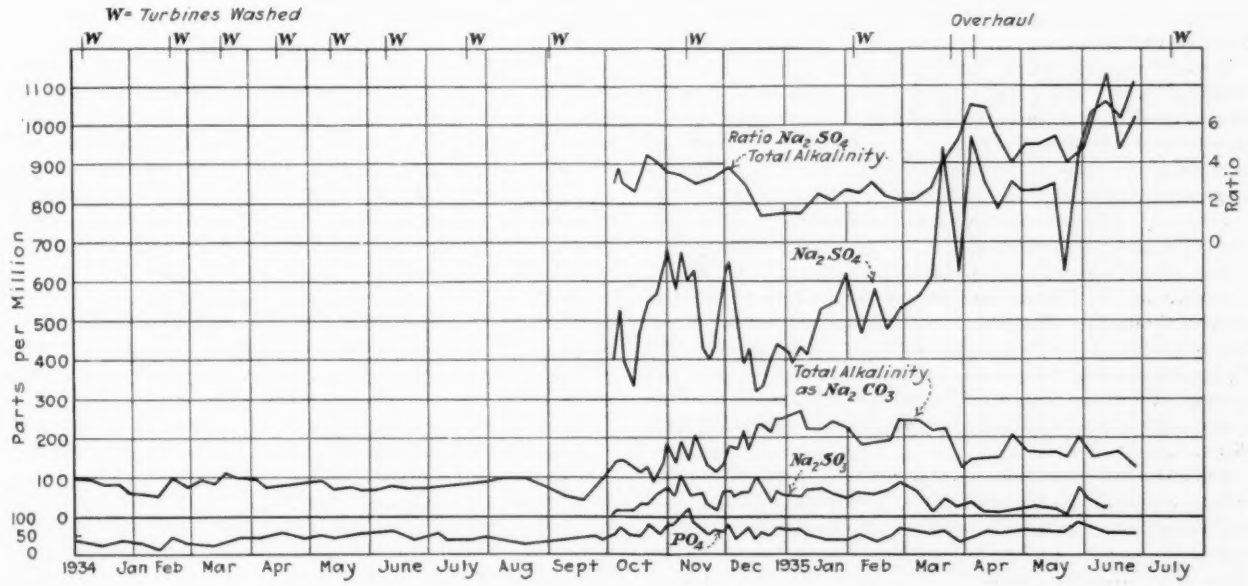


FIG. 7 AVERAGE ANALYSES OF ALL BOILERS IN SERVICE

end of the test the steam was shut off, the blade removed while still hot, cooled in a desiccator, and weighed. The amount of steam passing through the unit was calculated and the amount of deposit per unit of steam was determined. The deposit formed was reported in milligrams per million pounds of steam passing through the unit.

The unit was tested in a central plant operating at a pressure of 600 lb per sq in. and a temperature of 725 F. The steam pressure at the nozzle was between 350 and 520 lb per sq in. and the temperature between 635 and 725 F. The pressure drop through the unit was adjusted so as to obtain the desired rate of flow. This was held at 870 lb per hr in the majority of tests and was accomplished with a pressure drop of 100 lb. With this pressure drop the velocity of the steam passing the blades was about 460 fps. Table 5 gives the results of the tests run with the test unit.

The unit was installed on a line connected to the steam header just ahead of the turbine. The first test was run for 53 hr and gave a total deposit of 11.3 mg or at the rate of 251 mg per million pounds of steam. The next test was run under the same conditions for 234 hr and the amount of deposit was 45.5 mg and the rate of deposit was 221 mg per million pounds of steam. This indicated that the rate was almost constant and a test of 52 hr was sufficient to detect the amount of deposit forming.

In the next test the rate of flow was lowered from 870 to 430 lb per hr, which changed the velocity from 460 to 140 fps. The deposit increased forming at a rate of 310 mg per million pounds of steam. This third test showed that the deposit would form over a wide range of steam flow through the unit. During these tests the regular water treatment was in use and the plant was experiencing difficulty with blade deposits. The turbine was washed every four weeks.

Fig. 7 shows the average boiler-water conditions during these tests and the time of turbine washings. The boiler-water treatment was modified early in October, 1934, to increase the sulphate concentrations so that they would be similar to those found advisable in the laboratory tests. In order to obtain the desired sulphate, sodium sulphite was added. The sulphite would react with the small amount of oxygen present and reduce corrosion. The sulphate formed would aid in the prevention of blade deposits.

The sodium-sulphate content before the addition of the sul-

phite was between 15 and 30 ppm and is not shown. At the time of change in water treatment the sodium-hydroxide content was also increased with a corresponding change in pH value. All of these changes increased the total dissolved solids in the boiler water from a maximum of 260 ppm to about 1200 ppm. The ratio of the sodium sulphate to total alkalinity increased to a range between 3.5 and 4.6 for the period between October 1 and November 11.

The turbine was washed on July 22 and not washed until Sept. 2, a duration of six weeks instead of four weeks as usual. The loss due to blade deposits, calculated from increase in stage pressure over that of clean turbine, was greater at the end of these six weeks than for a regular four-week period, as was to be expected. The water treatment was changed about October 1, a period of four weeks after the turbine had been washed. At this time the tests of the turbine showed about the regular amount of loss due to deposits. The turbine was not washed. The deposit did not appear to increase as rapidly as before the changes in water treatment and the turbine was allowed to run until November 11 before washing. The deposit did not appear to be causing any more difficulty at the end of the ten weeks than at the end of the first four weeks.

Tests run on the small testing unit during the period after change in water treatment showed a deposit rate of between 20 and 66 mg per million pounds of steam as compared to 250 mg per million pounds of steam prior to the change, a reduction of about 90 per cent in the deposit. The amounts of deposits weighed were about 0.5 mg which was about the limit of accuracy of weighing. These results appeared to indicate that the deposit was being materially reduced.

The turbine was not washed again until February 3, a duration of twelve weeks. During this period the sodium sulphate to total-alkalinity ratio dropped from 3.5 to 1.6 and was 2.5 at the time of washing. The turbine was taken out of service on March 24, for overhauling and inspection, and was washed to aid in cooling. When the turbine was opened it was found free from deposits, indicating that if any deposits were forming they were all removed by washing.

While the turbine was out of service the method of adding the sodium sulphite was changed from intermittent dosage to a continuous feed. It was hoped that this would give a better control of chemical feed and thus show a more consistent sulphate concentration. However, the sulphite was added at a point where appreciable oxygen was present, ahead of the heaters, and this brought about an increase in the sulphate formed. The sulphate content increased to a much higher value than was previously maintained.

When the turbine went back into service blade-deposit formation appeared to be slow. The turbine was not washed again until July 14, a duration of 14 weeks. Thus the time of washing was changed from four-week periods prior to the change in water treatment to an average of 12 weeks; and this change resulted during the period when the control of the water treatment was in the experimental stage.

DISCUSSION OF RESULTS

The results of these studies indicate that the turbine-blade fouling is caused by chemicals carried into the steam from the boiler water. The amount of this contamination does not have to be large and steam which would normally be referred to as being of excellent quality might cause a great amount of deposit. The chemical causing the major portion of the difficulty appears to be sodium hydroxide. This is present in practically all boiler waters and when carried with steam that is superheated forms a concentrated sticky solution. This sticky material forms the basic binder and adheres to the turbine blades. Other salts present in the form of a dry powder would normally be swept past the blades. However, when caustic soda is present these salts adhere with the pasty caustic. This may cause a deposit in which the relative proportions of the constituents bear no relationship to the ratios in which they occur in the boiler water.

The amount of deposit is not proportional to the carry over from the boiler, but depends upon the relationship which exists between the various salts in the boiler water. Thus high hydroxide in respect to the total solids may cause an excessive deposition even with a low concentration of sodium hydroxide in the boiler water. However, a boiler water with a sodium-hydroxide content low with respect to the total solids present but high in concentration might not cause a deposit. This is because salts other than sodium hydroxide apparently aid in the prevention of the deposits.

The salts which form a dry powder in the superheated steam apparently adhere to the surface of the droplet of concentrated pasty sodium hydroxide, and if present in sufficient amount entirely coat the particle and allow it to pass through the turbine without sticking to the blades. Such a method of prevention naturally requires large amounts of the dry powder in proportion to the sticky material. The plant tests and laboratory experiments indicate that if sodium sulphate is to be used to prevent turbine fouling it must be present in amounts greater than four or five times the actual sodium hydroxide present. Power-plant operators hesitate to increase the total solids in the boiler water because they fear carry over. However, if this is properly controlled and the carry over does not become excessive, the deposits will be prevented.

The application of the inorganic-salt treatment to one large central power plant while still in the experimental stage showed that the period of turbine washings were reduced from four weeks to about twelve weeks, or from twelve times a year to four. It is quite possible that when the water treatment is better understood washing in this plant may be entirely eliminated. The cost of the treatment used has been low, less than $500 per year, and the savings have been materially high since corrosion has also been reduced.

The application of these data to power-plant operation should be made only after a careful study of the conditions existing at the particular plant and changes which are made should be under the control of a chemical engineer who thoroughly understands this field of water treatment.

CONCLUSIONS

A summary of conclusions drawn from the results of this work is as follows:

(1) The basic material causing turbine-blade fouling is sodium hydroxide.

(2) Inorganic salts, such as sulphate, chloride, and carbonates, if present in sufficient amounts, will prevent turbine-blade fouling.

Cavitation Testing of Model Hydraulic Turbines and Its Bearing on Design and Operation

By L. M. DAVIS,[1] HOLTWOOD, PA.

The author discusses cavitation tests made with model turbines at the Holtwood hydraulic laboratory of the Pennsylvania Water and Power Company and points out benefits which have been realized therefrom. In addition to presenting the results of these tests the author compares them with results obtained by other investigators in this field. The author presents his views regarding the further application of the equipment in the Holtwood laboratory for the purpose of investigating various elements in the design of turbines which may have some bearing on their cavitation characteristics. He points out the need of coordinating the results of cavitation investigations now being conducted in various laboratories so that the industry as a whole may be benefited thereby.

THE Holtwood hydraulic laboratory was put into operation early in the summer of 1930 and since that time has been in almost constant use. Although most of the work done in the laboratory has been in connection with the Safe Harbor development, turbine models of three large projects have been tested for manufacturers on a rental basis.

Since the initial Safe Harbor development is now complete, it seems desirable to review the accomplishments of the laboratory and to point out what benefits have been realized from the research which has been conducted.

The laboratory shown in Fig. 1 is laid out to accommodate normally a model 16 in. in diameter. The pressure chamber is 12 ft by 18 ft inside, so that scroll cases of the desired design may be constructed of wood or other material.

The wheel setting is vertical, and directly below the runner there is an observation chamber in which elbow draft tubes of the shape desired may be constructed. The discharge passes a diaphragm separating the observation chamber from a vacuum chamber, so called because it may be necessary to produce a vacuum in the space above the water in order to maintain the level sufficiently high to seal the draft tube. The set-up allows sufficient flexibility so that spreading tubes, or even straight conical tubes, may be tested, providing a collector is used to direct the discharge into the vacuum chamber. A more detailed description of the laboratory may be found in two previously published papers.[2,3]

In making complete tests of a model the procedure is first to determine the efficiency characteristics in the usual manner, except that with a movable-blade runner there is one more variable to be considered. Tests are made by adjusting the dynamometer to cover the range of speeds desired, observations being taken of the speed, the quantity of water used, the power output, and the head. The water quantity is measured by a 36-in. and 15-in. venturi meter, the output of the wheel by an electrical dynamometer capable of absorbing 300 hp, the speed by a synchronous timer driven by an a-c generator geared directly to the dynamometer shaft, the headwater elevation by a mercury manometer connected to piezometers in the pressure chamber, and the tailwater elevation by a float gage connected to pressure orifices in the vacuum chamber.

For each blade angle, curves are drawn as shown in Fig. 2 for appropriate gate openings, selecting these gate openings so that the most efficient combination of blade and gate will be bracketed. By adjusting the torque absorbed by the dynamometer, the speed may be varied over the desired range which is dictated by the need of including the extreme values of ϕ, which is the ratio between the peripheral speed of the runner and the theoretical spouting velocity of the water, which corresponds to the synchronous speed of the prototype and the maximum and minimum heads at which it may be operated. These tests are run at a very low, or even negative, draft head in order to create as great a pressure around the blading as possible, and thus preclude any likelihood of the results' being affected by separation between the water and the blades, which is cavitation.

After a series of such tests, one for each blade angle, the complete efficiency characteristics of the unit can be plotted. In this way the efficiency curves for the values of ϕ for the various heads in the prototype may be determined. More important, as far as subsequent cavitation tests are concerned, is the selection of the gate openings which will bracket the best combinations of blade and gate for each value of ϕ.

Tests must then be repeated for each value of blade and ϕ with individual series of tests for at least three gate openings; but while the foregoing are held constant, the headwater and tailwater levels are varied to change the ratio between draft head and total head. Any failure to maintain absolutely the right speed to correspond with the head actually used may be corrected on the basis of information obtained from the previous test in the same way that field tests are corrected to guaranteed head by means of the characteristics determined from model tests.

As the draft head becomes greater there will be a tendency for the water to pull away from the underside of the blades with consequent alteration of the efficiency and output characteristics as determined in the first series of tests. Therefore, the object of this second series is to measure to what extent the draft head can be increased without causing this so-called cavitation. It is consequently necessary to adopt some criterion which may be

---

[1] Hydraulic Test Engineer, Pennsylvania Water and Power Company. Mr. Davis received his C.E. degree at Cornell University in 1923. For five years following graduation he was in the employ of the Niagara Falls Power Company as engineer of hydraulics. The next two years he spent on hydro-power-plant design for the Aluminum Company of America. Since 1930 he has been associated with his present concern, during which period the Safe Harbor turbines were designed.

[2] "Safe Harbor Kaplan Turbines," by G. W. Spaulding and L. M. Davis, *Electrical Engineering*, October, 1932, vol. 51, pp. 728–733.

[3] "Model Testing at Holtwood Hydraulic Laboratory," by L. M. Davis, published by Pennsylvania Water & Power Company, Holtwood, Pa., May, 1934.

Contributed by the Hydraulic Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

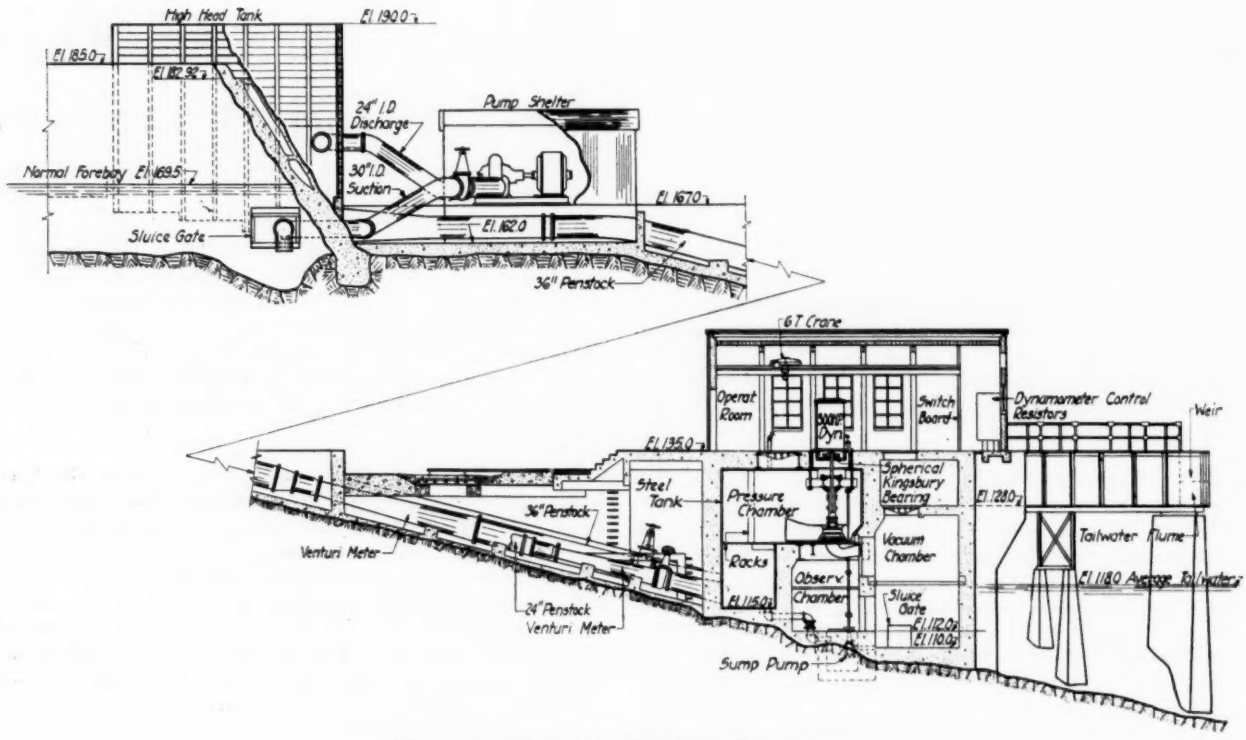


FIG. 1 THE HOLTWOOD HYDRAULIC LABORATORY

used for defining the conditions under which a given model will just begin to cavitate.

L. F. Moody and F. H. Rogers have offered[4] a cavitation coefficient K_c which is obtained from the equation

$$K_c = [h_a - h_m - h_s - (E_d C_2^2/2g)]/H$$

where h_a is the atmospheric-pressure head, ft; h_m is the absolute pressure at the point of minimum pressure, ft; h_s is the static draft head, ft; E_d is the draft-tube efficiency; C_2 is the absolute-discharge velocity from the runner, fps; g is the acceleration due to gravity, ft per sec per sec; and H is the total effective head on the turbine, ft.

Inasmuch as cavitation is assumed to take place only when the lowest pressure h_m becomes equal to the vapor pressure, this substitution may be made, and the value to be used is a function of water temperature alone. To obtain a numerical value of K_c it is necessary also to know the draft-tube efficiency and the absolute discharge velocity. It should be noted that since all the terms in the right-hand side of the equation are expressed in feet of water, K_c is dimensionless.

D. Thoma originated the cavitation coefficient σ through his work at Munich. The physical meaning of this coefficient can be visualized by imagining the following sequence of events. Suppose the model turbine to be operating without cavitation under a constant head and with constant discharge. Since the forebay and tailrace elevations are variable, the tailrace can be lowered while the total head is maintained constant. As the draft head is thus increased, the pressure on certain parts of the runner blade will decrease. When the minimum pressure at any point on the blades just reaches the vapor pressure H_v, cavitation is said to begin. Suppose the unit is shut down as soon as this condition is reached. After the turbine gates are closed, the absolute pressure on the blades is obviously the barometric height H_b minus the draft head H_s. In the operation of the unit, the pressure at some point on the blades was, therefore, lowered from

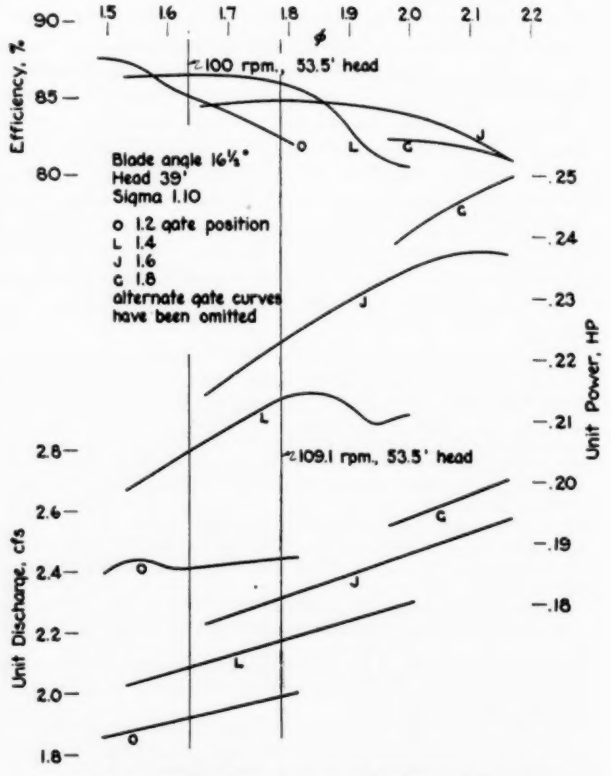


FIG. 2 TYPICAL RESULTS OF ϕ-EFFICIENCY TESTS

[4] "Inter-Relation of Operation and Design of Hydraulic Turbines," by L. F. Moody and F. H. Rogers, *Engineers and Engineering*, July, 1925, vol. 42, no. 7, pp. 169–187.

$H_b - H_s$ to the vapor pressure H_v. This decrease in pressure is a function of velocity and in fact is proportional to the square of the discharge of the unit. Since the square of the discharge is proportional to the head, it follows that the reduction in pressure, $H_b - H_s - H_v$, is itself proportional to the head. Therefore, it is possible to use in place of the pressure drop $H_b - H_s - H_v$, which applies only to the conditions of the test, the ratio $(H_b - H_s - H_v)/H$, which applies to all heads and which is identified as σ.

Comparison between the Moody and the Thoma formulas shows marked similarity. The h_a of the former is the same as H_b, or barometric pressure, in the latter. Likewise the static draft head h_s is the same as the suction head H_s. It has already been pointed out that h_m is equal to H_v. Consequently, the only difference lies in Professor Moody's term $E_d C_2^2/2g$ which is omitted by Dr. Thoma. The inclusion of this term simply means K_c applies to the runner alone since correction is therein made for the effectiveness of the draft tube, whereas σ applies to the wheel and setting as a whole.

The results of the cavitation or σ tests are shown in Fig. 3, where again efficiency, horsepower, and discharge are plotted, but this time as functions of σ. These curves represent the results of three gate openings corresponding to but a single combination of blade angle and ϕ, and similar tests must be conducted to cover all others in the range desired. Ordinarily, only the higher blade angles are of importance and from three to five values of ϕ are used generally to investigate fully the cavitation characteristics for the entire range of head used in the finished plant. From complete cavitation information at four or five values of ϕ

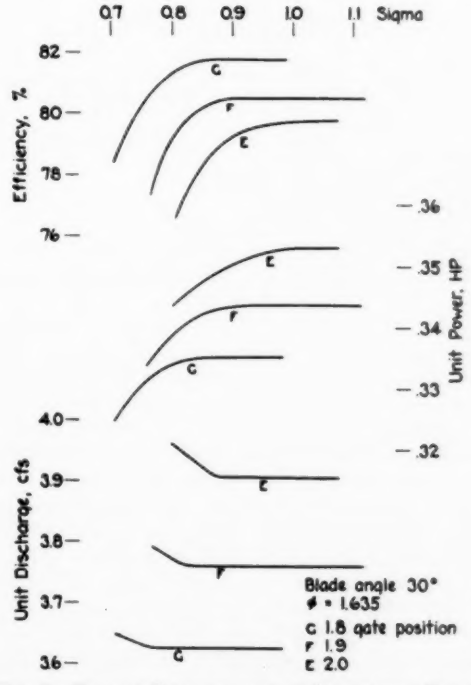


FIG. 3 TYPICAL RESULTS OF σ OR CAVITATION TESTS

it is possible to cross-plot and obtain data for any intermediate value.

It will be noted that on the right of Fig. 3 all of these curves become horizontal. The constant values correspond to those which have been observed in the preliminary ϕ-efficiency tests.

As σ becomes smaller with greater draft head, there occurs a change in the ordinates. The point at which this occurs is known as the break in the curve, or the beginning of cavitation. The exact determination of this point depends somewhat on judgment, and some variation will be found therein, but if several persons pick these points and results are plotted to form smooth curves, independent interpretations of the same data will agree satisfactorily.

One criterion of the break is the dropping off in efficiency. Another is the increase in discharge. These will not coincide precisely on account of the fact observed by Dr. Thoma that there is a slight tendency for efficiency to increase just as cavitation

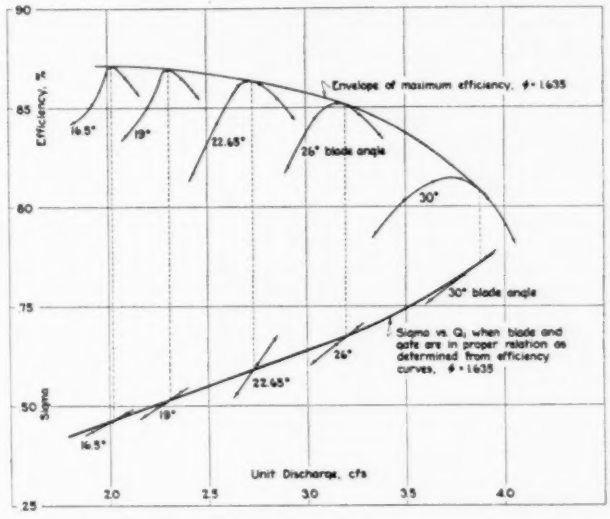


FIG. 4 RECAPITULATION OF σ-TEST RESULTS FOR $\phi = 1.635$

starts which, he explains, is probably due to the decrease in skin friction.

However, it has been observed that with some models the efficiency tends to decrease at a higher value of σ than the point where the unit discharge starts to increase. It is felt that this is due to faulty design resulting in local cavitation which takes place at one or more points on the blade at draft heads somewhat lower than the draft head which causes cavitation over a general area.

It has been the practice in the Holtwood hydraulic laboratory to use the break in the unit-discharge curve to determine critical σ. If there is any great discrepancy between the break in the efficiency curve and the break in the unit-discharge curve, the more conservative should be used, although it is felt that this is an indication of defective design, and possible alterations to correct this discrepancy should be investigated.

The question may be asked whether the break in the σ curve is affected by the head at which the test is conducted. Theoretically there should be no effect and tests made at one head should be applicable to others. This matter was investigated by Prof. Wilhelm Spannhake, of Karlsruhe, whose laboratory was adapted particularly for experiments covering a wide range of heads from about 33 ft down to 5 ft. It was found that especially at the very low heads there was a marked effect, but that for higher heads the break in σ reaches practically a constant value which is independent of further changes. Similar tests at much higher heads were performed at Holtwood without revealing any appreciable difference in σ at which the breaks occur. Therefore, it seems preferable, but not essential, that the cavitation test be made at a head closely approximating that of the prototype.

The next step is to recapitulate the results of the foregoing tests. This is done by plotting the value of σ corresponding to the break against unit discharge as shown in Fig. 4. In order to determine the value of σ at which cavitation begins when the unit is operated at the correct relation between gate and blade, the envelope of the individual efficiency curves is drawn, and

vertical lines are dropped from the points of tangency. The intersections of these verticals with the σ versus unit-discharge curves for the corresponding blade angles locate points on the final curve representing the relation between σ and discharge for a Kaplan unit when operating with the blade and gate in the proper relation to each other. This procedure is repeated for each value of ϕ, thus giving in the end the plot shown in Fig. 5. By cross-plotting, curves may be obtained for any intermediate values of ϕ.

The group of curves given in Fig. 5 gives the complete cavitation characteristics of the model for the range of ϕ indicated. From these curves the proper setting of a homologous runner for any installation with reference to tailwater for a given power output may be determined.

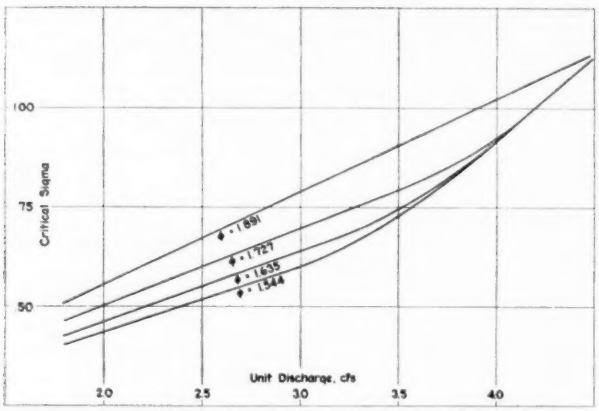


FIG. 5 σ VERSUS UNIT DISCHARGE FOR VARIOUS VALUES OF ϕ

One general characteristic of all tests so far has been the marked effect which the increasing of ϕ has had in aggravating the danger of cavitation. The practical application of knowledge gained through tests is illustrated by the adoption of a lower rotational speed for the most recent installation at Safe Harbor. This reduction proves to have resulted in a substantial increase in capacity in spite of the fact that judging only from the output-ϕ characteristics of the model, the capacity would have been adversely affected. Therefore, it is apparent that when cavitation becomes a limiting factor, the urge for higher speeds for the sake of greater capacity and lower generator cost must be tempered. Since it is obvious that, without cavitation, increasing speed gives greater physical capacity, while the effect of increasing speed on power output as limited by cavitation is just the opposite, it becomes exceedingly important to explore completely the characteristics of a model before making a decision regarding the speed of rotation.

The proper setting for the turbine with reference to tailwater elevation can be determined from Fig. 5 as soon as the runner diameter, speed, rated power, and total head are established. From the efficiency tests, the expected efficiency at the rated head and output may be determined and the discharge computed. The unit discharge may then be obtained, and from Fig. 5 the critical σ will be found. The values of all the terms in the formula for σ have now been obtained except the draft head H_D, which is readily obtained. Adding this value of the draft head to the tailwater elevation gives the runner elevation which will give the desired output without cavitation. Since cavitation would be expected to result if the maximum output as determined by test were slightly exceeded, it is customary, as a factor of safety, to lower arbitrarily the setting a small amount.

Not only is the determination of the critical σ, which is the value of σ at which cavitation begins, useful in fixing the proper height of setting, but conversely the laboratory results can be used to determine the allowable output under given conditions of forebay and tailwater level after the setting has been established.

At this point distinction should be made between critical σ as applied to the runner and plant σ which refers to the plant. Both are expressed by the same formula, but obviously plant σ must not be less than critical σ if avoidance of cavitation is to be expected. It is also clear that by assuming the critical value of σ equal to the plant σ for any given H_s and H_t, the maximum allowable unit discharge can be read directly from the scale of abscissas in Fig. 5.

Whereas it is probably correct, since cavitation is logically a function of water quantity, to derive the discharge for the prototype directly by stepping up from the model without allowance for improved efficiency, nevertheless hydraulic engineers are faced with a serious problem in determining the corresponding limiting power. In the investigations which have been made on the subject of the relation between maximum efficiency of the model and the prototype, no information has been made available as to whether the increased efficiency is obtained as a result of increased output from the same quantity of water, or decreased quantity for the same output. Perhaps there may be a combination of the two effects. It will be seen from Fig. 6, however, that as a practical matter it ordinarily makes very little difference whether better efficiency results from lowering the power-discharge curve or by shifting it to the right except for values near the extreme end of the curve. Thus, to find the prototype output corresponding to a discharge of Q', case 1, it makes no appreciable difference whether we use curve A derived by reduction in discharge, or curve B derived from increase in output. However, for a discharge of Q'', case 2, there may be some doubt as to whether cavitation will reduce the output from HP' to HP'' or whether HP'' would still be above the physical capacity HP''', and, therefore, not a limiting consideration.

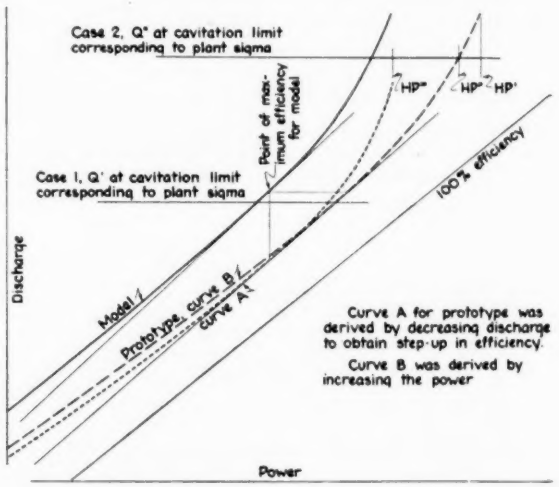


FIG. 6 EFFECT OF STEP-UP ON INTERPRETATION OF MODEL-TEST RESULTS

Therefore, it is sufficient in determining the output as limited by cavitation to assume increase in output in proportion to the expected rise in efficiency resulting from the computed reduction in hydraulic losses. On the other hand, there is great need for further investigation as to the exact nature of this shift by comparison of the discharge versus output curves for various units which have been tested in the field. It is no longer sufficient for our purpose to use a formula by which an increase in maximum efficiency alone can be estimated. There is obviously a need for

further investigation before it may be considered that the principles of stepping-up performance curves are well understood. Uncertainty in this respect is apparent from the fact that the application of the formulas of three separate authorities to the result of the test on one model which has come to our attention is reported to have given answers differing by as much as 4 per cent.

A convenient form for presenting the limiting power output for various head conditions is to draw constant limiting power-output lines using tailrace elevations as ordinates and forebay elevations as abscissas. To obtain this curve it is well first to develop a curve of limiting power output versus gross head, drawing lines of constant forebay elevation. Such a chart is useful in plant operation so that with varying conditions the risk involved in approaching the critical σ of the runner is always the same.

As cavitation data become available it is desirable to compare them so that progress in design may be noted and conclusions drawn as to the features which are or are not advantageous. Fig. 7 shows a plot of critical σ versus unit discharge at rated ϕ for several models which have been tested in the Holtwood laboratory, together with other tests which have come to the author's attention.

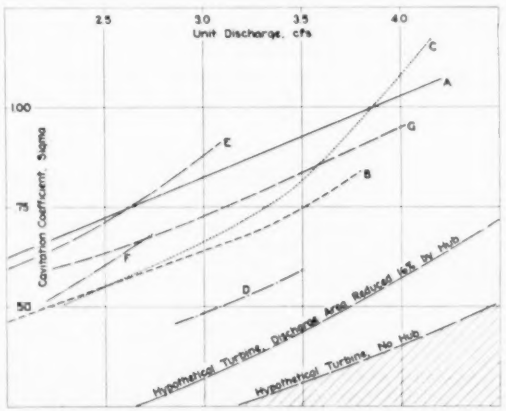


FIG. 7 COMPARISON OF CAVITATION TESTS ON SEVERAL MODELS ON THE BASIS OF UNIT DISCHARGE

While improvement can be expected with experience, this progress, like efficiency, cannot go on indefinitely. There must be some limit beyond which further improvement is impossible.

To illustrate this principle, there may be imagined a hypothetical turbine in which there would be no drop in pressure below the static pressure as the result of pressure difference on the two sides of the blade or irregularities of the pressure distribution along the blade; although this is clearly impossible, it is at least a limiting case. In spite of this, there would be a reduction in pressure simply by reason of the velocity through the throat. In other words the $(H_b - H_v - H_s)$, previously mentioned as the reduction of pressure caused by the operation of the turbine, now becomes only the velocity head of the water passing through the throat. Assuming that this throat may be perfect as an orifice, that the velocity is axial and equally distributed, and also that there is no reduction in area on account of the runner hub, it could then be shown that when H_t is equal to unity this drop in pressure would be equal to the velocity head $c^2/2g$.

Thus, since $Q_1 = ac$ and $a = \pi/4$

$$\sigma = (4Q_1)^2 / 2g\pi^2$$

The values of σ corresponding to various values of Q_1 are shown by the lowest curve in Fig. 7 which illustrates the ultimate limit of perfection. Should we assume, by way of example, a 16 per cent reduction in discharge area by reason of the runner hub, this limit obviously would be raised still further. Keeping this in mind, progress toward the ultimate goal may be judged. The economic importance of any move in this direction will be seen from a study of Fig. 7 where, at a value of 0.75 for plant σ, the gain in allowable power output for curve B over curve A is approximately 27 per cent. In a new installation for 35-ft rated head and a unit discharge of 2.6 cfs the turbine could be set 6 ft higher for curve B than for curve A.

F. H. Rogers and R. E. B. Sharp, in a paper[5] presented at the Cincinnati meeting of the A.S.M.E., June, 1935, discussed a relationship between plant σ and specific speed derived from a comparison of nine important propeller installations in America and Europe, as shown in their curve Fig. 7,[5] which is reproduced in Fig. 8 of this paper for comparison with the results of the models tested at Holtwood and other laboratories. In the discussion of the paper[5] by Rogers and Sharp, R. V. Terry, hydraulic engineer of the Newport News Shipbuilding & Dry Dock Company, suggested the relation $\sigma = N_s^2/25{,}600$ as a close approximation to the critical σ for normal design of turbines. For small installations where cavitation tests are not warranted, he suggests using a more conservative relation, $\sigma = N_s^2/22{,}500$, for determining the turbine setting with reference to tailwater elevations. It appears from test D, Fig. 8, that actually there are potentialities which may be realized as a result of further development and research and that a substantial improvement in design can be made over that which might be expected from the three relationships.

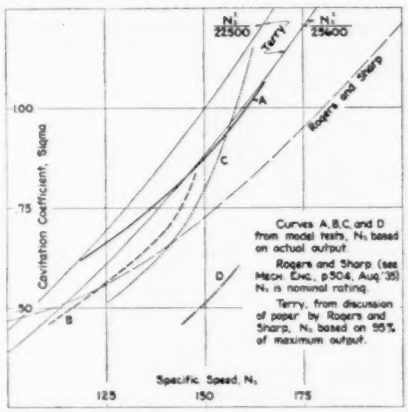


FIG. 8 COMPARISON OF CAVITATION TESTS ON THE BASIS OF SPECIFIC SPEED

BENEFITS DERIVED FROM TESTS

It would be well now to consider the use that has been made of the laboratory results and the benefits derived from the tests made in cooperation with the manufacturers in connection with the design and improvements on the Safe Harbor units.

As a preliminary step in cavitation testing, there is the derivation of complete ϕ-efficiency characteristics of the model. In important installations, strict accounting for the use of water is necessary for reasons of record, and for economy in the preparation of operating schedules. It is seldom possible, without unwarranted expense and loss of time in waiting for extraordinary circumstances, to conduct formal efficiency tests over a sufficient range of heads to gain complete knowledge of the operat-

[5] "45,000-Hp Propeller Turbine for Wheeler Dam," by F. H. Rogers and R. E. B. Sharp, *Mechanical Engineering*, August, 1935, vol. 57, no. 8, pp. 499–506.

ing characteristics of a unit. Consequently, model tests are relied upon in interpreting acceptance-test data to cover the desired variety of conditions to draw up those charts or tables which are necessary in daily operation and in planning the economic loading of units. So much has been written on this subject by other authors that the use of model tests for this purpose seems to be well understood.

An interesting application of model tests in conjunction with index tests in the field is described in a paper by E. T. Schuleen.[6] The reason for conducting the test described by Mr. Schuleen was that in the past there has been considerable uncertainty regarding the effect of runner size on the value of ϕ at which the peak efficiency occurs. Some hydraulic engineers contended that

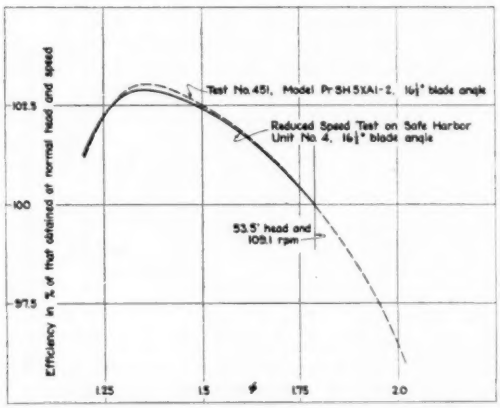


FIG. 9 THE ϕ-EFFICIENCY CHARACTERISTICS OF MODEL AND PROTOTYPE

the peak efficiency occurred at higher values of ϕ as the runner diameter was increased. This question became of vital importance in connection with establishing the speed of Safe Harbor unit No. 2 for 25-cycle railroad service since the model tests indicated an appreciable gain in efficiency at 100 rpm instead of 109.1 rpm, which is the speed of the 60-cycle units. It was consequently decided to test one of the Safe Harbor units at variable speed by means of an artificial load consisting of two other units operated as motors so that a direct comparison could be made between the model and prototype.

Fig. 9 shows a comparison between the envelope efficiencies obtained on this test and the envelope efficiencies obtained on a 16-in. model at the same blade angle of $16^1/_2$ deg. It is felt that this unique experiment proves that the peak efficiency of a given runner design occurs at the same value of ϕ regardless of runner diameter.

Operation of Kaplan units at Safe Harbor, the capacity of which is greater than other units installed so far in this country, has brought up many questions regarding the behavior of these units as affected by cavitation. In seeking an explanation for observed facts and in establishing principles and policies in operation, the laboratory has served as a guide in many cases. This is particularly true in understanding the need for correct setting of the cam for controlling the gates and blades. The importance of correct adjustment and the seriousness of departure from this relation have been brought out clearly by laboratory tests. It is evident from these that whereas error in adjustment on one side of the best blade position may be accompanied by rapid loss in efficiency, on the other side the drop in efficiency is more gradual, but the danger from cavitation is greatly increased.

[6] "Tests at Variable Speeds on a Safe Harbor Turbine," by E. T. Schuleen, published by Pennsylvania Water & Power Company, Holtwood, Pa., May, 1934.

Extensive tests have been made for determining what improvements in blade shape could be made in anticipation of installations of additional capacity. These tests were naturally confined to a narrow range of alterations, being restricted to some blades which were already cast with enough additional finish to allow fairly extensive revamping in machining, and on other blades which could only be chipped to the desired contour. The tests, however, have been sufficiently extensive in their scope to demonstrate the correctness of certain design principles.

Practically every hydraulic engineer who visits the Holtwood laboratory asks: "How do laboratory cavitation tests check actual cavitation conditions at Safe Harbor?" Fig. 10 shows this comparison. The solid line gives the results of the field efficiency tests in per cent of maximum efficiency. The dashed line shows the expected curve from laboratory tests which, as explained previously, are not affected by cavitation. It will be noted that the actual efficiency curve starts to depart from the expected curve at about 33,500 hp. From laboratory tests using the break in the unit-discharge versus σ curve as the criterion for the starting point of cavitation, a limiting output of 34,950 hp was obtained, thus showing a very reasonable agreement.

It should be borne in mind that the test from which Fig. 10 was drawn was conducted when the tailrace was particularly low in order to determine what effect there would be from the worst condition of cavitation. This does not represent ordinary operat-

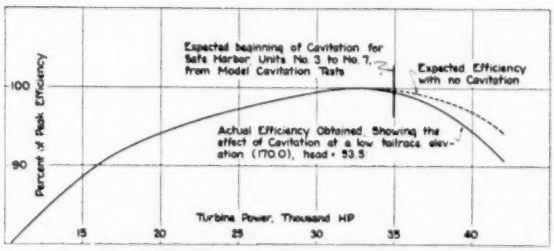


FIG. 10 BREAK IN EFFICIENCY OF PROTOTYPE COMPARED WITH THAT PREDICTED FROM MODEL TESTS

ing conditions and, therefore, the reduction in capacity is not indicative of failure to derive rated output continously from these units. With a normal forebay, the limitation apparently imposed by cavitation in Fig. 10 would be sufficiently raised so that the unit would deliver its full rated output.

It has been found from experience that it is possible to operate somewhat above the limit as determined by the laboratory without accruing enough damage from pitting to be considered out of proportion to the benefits derived from increased output. Inconvenience caused by vibration at these high ratings is successfully combated by the use of a moderate amount of air injected under the head cover.

During the three and one-half years that the Safe Harbor plant has been in operation a certain amount of pitting has occurred. It has been concluded after careful study of the pitting at the time of the numerous turbine inspections and also when the pitted areas were repaired by welding, that the major portion of this pitting resulted from local cavitation induced by irregularities in the blade surface. It is evident that water traveling at high velocities along the blade surfaces cannot follow sudden changes in curvature. When a hollow is present the water leaps over it, forming an unstable cavity. In making repairs great care was taken to grind the welded areas down to a smooth surface free from irregularities or reversals in curvature. Some of these welded portions have been in service two years and no pitting has occurred. This is partly due to the greater resistance to cavitation of the 18 per cent chrome and 8 per cent nickel stainless-steel welding rod which was used, but the improved surface, no

doubt, eliminated local cavitation on the areas which have been repaired and is largely responsible for the absence of pitting.

Further Application of the Holtwood Laboratory Tests

While the accomplishments in the past have proved the wisdom of establishing this laboratory for the purpose for which it was intended, consideration should be given to the future and to what possibilities lie ahead now that the laboratory equipment is available for further work. Up to the present time the experiments have been confined to the change of comparatively few variables, but there still remain for investigation many other elements in the design of a turbine which may have some bearing on its cavitation characteristics, such as the shape of guide vanes, arrangement of the scroll case, form of the head cover, design of the curb ring, influence of the hub diameter, and the runner clearance. Furthermore, there will eventually arise important problems in connection with redevelopment work necessary to secure the maximum amount of additional power from new wheels placed in existing equipment.

The desirability of undertaking further experimentation depends naturally upon what possibilities of improvement are to be expected. Such experimentation is important because it requires but a very slight betterment of either efficiency or cavitation characteristics of the large turbines being built today to justify considerable effort on the part of the designer. A few years ago it was the common impression that because of the high efficiencies attained by Francis turbines, the ultimate goal had been reached. However, with the introduction of the movable-blade propeller, a new field of possibilities was created because it became possible to widen the band of efficient loading. In a like manner cavitation has presented still another field of endeavor wherein the primary object is to make the σ-unit discharge curves, as shown in Fig. 7, approach nearer and nearer to the theoretical limit. It is clearly demonstrated by the admirable performance indicated by curve D, Fig. 7, that actually there are possibilities for a marked advance in this direction.

There is clearly needed some means of coordinating results of different laboratories so that progress may be measured. W. M. White, chief engineer of the Hydraulics Division of the Allis-Chalmers Manufacturing Company, has pointed out[7] his objection to the use of σ. He claims that there is too great a disparity between σ for a propeller turbine such as the Wheeler unit[5] and for a Francis turbine such as used at Boulder dam.[7] In place of σ he proposes the coefficient λ which is simply the expression for the Moody coefficient K_e with the term H omitted. As pointed out previously all of the terms in the formula for the Moody coefficient K_e are expressed in feet of water and therefore, by omitting the divisor H, the result is an expression which indicates simply the pressure available to hold the water to the low-pressure side of the blade. Although Dr. White secures in this way a figure which is more uniform in numerical value, the question may be raised whether or not extreme differences in σ do not express after all the essential difference between the very dissimilar conditions of operation.

Suggestions to relate σ to specific speed have been offered by Messrs. Rogers and Sharp[8] and also R. V. Terry, as mentioned previously. Their criteria have been shown already in comparison with actual tests in Fig. 8. By comparing Fig. 7 with Fig. 8 it is evident that there is a decided shifting of one curve with respect to another resulting from the element of rotational speed which enters into the determination of specific speed.

The ideal comparison would be on the basis of unit horsepower, but in this respect the engineer is still faced with the uncertainty in regard to the applicability of the set-up formula to values other than the point of peak efficiency. Until this problem is solved, the unit-discharge curve is probably the best basis of comparison and should be satisfactory at least for present needs.

A further suggestion comes from the fact that the σ curve is a parabola for the hypothetical turbine, with and without a hub as shown in Fig. 7. It seems logical then to attempt to plot σ against the square of the unit discharge as shown in Fig. 11. If this is done, the hypothetical curves obviously become straight lines.

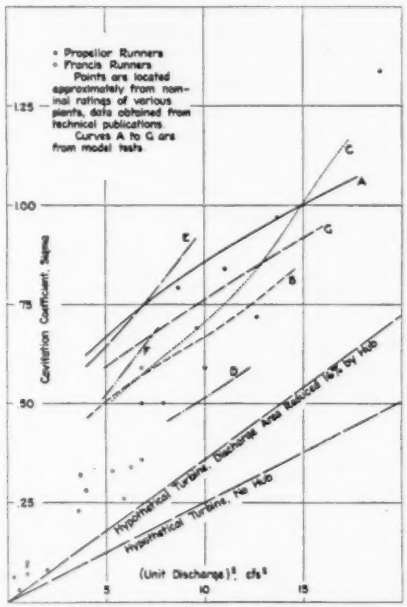


Fig. 11 Comparison of Cavitation Tests on the Basis of the Square of Unit Discharge With Addition of Plant σ's for a Number of Existing Installations

Many of the test curves are distinctly convex downward, particularly those which can be considered as the more reliable. Therefore, it would appear in general that if the curves were extended sufficiently in each case there would be an approach to a straight line passing through the origin followed by subsequent departure. Is it not possible that the point of tangency of a straight line through the origin with the curve in question indicates that value of unit discharge to which the blading is best adapted? Is it not possible also that as more data are collected and plotted in this way, considerable light may be thrown on the perplexing problem of correct shaping of the blades?

As further evidence of the hypothesis that σ is a function of the square of the discharge, there has been added to Fig. 11 points representing the approximate plant σ's and unit discharges for important installations of both Francis-type and propeller-type units. These points have been estimated as accurately as possible from data published in engineering literature, but without going back to original sources. Although this is but a rough comparison it is, nevertheless, striking that, over a wide range of conditions, the setting has been selected by experience (for as far as known no cavitation tests have been made), since for Francis runners there seems to be a distinct straight-line relationship between σ and the square of the unit discharge. Furthermore, if approach to the value of σ for the hypothetical turbine is the thing that is desired, the plant σ's for the propeller installations, most of which have been selected from laboratory tests, do not appear to be as advantageous from the point of excavation as those for the installations of Francis units. This raises the question whether or not in the past hydraulic engineers have been inclined to take risks

[7] "The Construction of the 115,000-Hp Boulder Dam Turbines," by W. M. White, *Mechanical Engineering*, September, 1935, vol. 57, no. 9, pp. 539–546.

that are too great in so far as the settings of the Francis wheels are concerned, or whether, on the other hand, there is some inherent reason why propellers would require more conservative values of σ. One element may be the loss of area taken up by the hub. On the other hand, to decrease the diameter of the hub involves, in the case of Kaplan turbines, mechanical problems and difficulties from clearance spaces, while for all propeller turbines the design of the blading near the hub is particularly difficult and can be mastered only by laboratory tests.

The apparent straight-line relation in Fig. 11 indicates that all that is necessary to reconcile the extremely low σ for the unit at Boulder dam with the high plant σ at the Wheeler dam is to divide each by the square of the unit discharge. No attempt is made here to estimate a normal value for this quotient, since the data for the points shown are but rough approximations. Furthermore, only the plants for which figures were most readily available are included, and no attempt has been made to differentiate between plants encountering serious pitting and those at which operation has been entirely satisfactory.

Inasmuch as the σ versus the unit-discharge curve does offer a reasonably satisfactory basis for comparison, it would seem desirable to have the results of the several laboratories in Europe and America coordinated in this manner. The accumulated data would serve a most useful purpose by informing each laboratory whether the results obtained in any one test were extraordinary in any respect, and thus aid in selecting hidden information that may be found from these tests. For example, curve *D* in Fig. 11 taken alone is simply a test result, but in comparison with the entire group of curves in Fig. 11 it is an achievement —an advance in hydraulic design.

Such a compilation might be undertaken by some central agency, such as the National Hydraulic Laboratory, although consideration should be given to whether or not laboratory errors might not render it worthless. Strange to say, the results are affected very little by laboratory errors because the measure of the break in the σ curve is a change of condition. Consequently, errors in power measurement may have any magnitude as long as they are consistent, while errors in discharge measurement would only shift the final curve slightly to the right or left. In this respect the situation differs essentially from the usual model efficiency tests since the results are not sensitive to ordinary errors.

Finally, it is the author's hope that efforts being made in the investigation of cavitation in various laboratories can be coordinated so that the industry as a whole may be benefited. The Holtwood laboratory offers its support to any movement of this kind.

PRO-57-4

Radiation From Nonluminous Flames

By H. C. HOTTEL,[1] CAMBRIDGE, MASS., and V. C. SMITH,[2] DETROIT, MICH.

Experimental results on the total radiation due to carbon dioxide, water vapor, and mixtures of the two that were recently reported were obtained in a chromel-wound furnace, the maximum operating temperature of which was 1850 F. The final plots were obtained by extrapolating to 3600 F the experimental data covering the range from room temperature to the furnace maximum.

A method is now described for determining these properties directly in the upper temperature range. It consists of measuring the emission of radiation, and the corresponding temperature, from overventilated flames of carbon monoxide, hydrogen, and mixtures of these in a Meker burner of varying length. The products in the flame are calculated from the mixture fed to the burner.

Results are presented for flames of carbon monoxide in mixtures of oxygen and nitrogen in which the burner grid length is varied from 2 to 16 in., the theoretical oxygen from 100 to 300 per cent, and the flame temperature from 2600 to 3800 F. The results establish the validity of the carbon-dioxide radiation chart in its high-temperature range. Less extensive measurements on illuminating-gas flames establish the validity of the water-vapor radiation chart at temperatures of 2500 to 2800 F when the thickness of gas layer is small.

THE problem of evaluating the expected radiant-heat transmission from a nonluminous flame has an obvious practical importance in the design of certain types of industrial furnaces, the evaluation of heat transmission in the internal-combustion engine, and studies in the mechanism of combustion of gases. The literature on the subject includes diverse opinions as to the nature of the phenomenon and does not contain data adequate for the substantiation of any proposed method of predicting the magnitude of the flame radiation. Accordingly the present work was undertaken to determine whether one of the proposed theories could be substantiated by experiment.

The radiation emitted from a nonluminous flame is due in part to radiation emitted consequent upon chemical changes of the molecules in the flame, called chemiluminescence; and in part to thermal radiation from the hot products of combustion, such as carbon dioxide and water vapor. The first kind of radiation depends upon the nature of the chemical reactions involved and is approximately proportional to the amount of fuel burned; the second depends on the temperature, size, and shape of the flame and its hot combustion products and on the concentration of radiating constituents in it, and is influenced by the amount of fuel burned only to the extent that the latter affects the temperature distribution in the flame.

The work of Garner and Johnson (1)[3] and of Johnson (2) has indicated that chemiluminescence, though of extreme importance in studying the mechanism of combustion reactions, accounts for a very small portion of the total radiation emitted from a flame. The calculation of total radiation from a nonluminous flame in an industrial furnace depends, consequently, on quantitative knowledge of the magnitude of thermal radiation from those hot products of combustion which are capable of radiating, namely, carbon dioxide and water vapor.

The earliest quantitative measurements of radiation from carbon dioxide and water vapor were made in 1894 by Paschen (3), who mapped the infrared emission and absorption spectra of hot carbon dioxide and water vapor and showed the interrelation between emission and absorption for those gases. Further work by him on the emission spectra of nonluminous flames led him to conclude that the radiation from them was purely thermal in origin and due solely to the carbon dioxide and water vapor in the flames. The work of H. Schmidt (4) in 1909 confirmed the validity of the assumption that radiation from the products of combustion in flames was thermal in character by proving that Kirchoff's law of the equivalence of absorptivity and emissivity (applicable only to thermal radiation), was applicable to radiations from an illuminating-gas flame. These fruitful suggestions of the physicists remained unused by engineers until Schack (5) in 1924 pointed out how data on the infrared absorption spectra of gases could be used to predict the magnitude of emission of radiation from them.

Moeller and Schmick (6) in the following year measured the radiation from small Bunsen flames, making the measurements in such a way as to permit comparison with the calculations of Schack. Although their experimental results were in agreement with the calculations within 5 to 25 per cent, the agreement was not conclusive since (*a*) the flame diameter was so small as to make edge effects large, (*b*) the effect of water vapor in the combustion products was hidden by the much greater effect of carbon dioxide, (*c*) the basis of comparison with theory (i.e., Schack's calculations based on infrared absorption data) was not established with adequate precision.

Recent experimental data have been presented by Hottel and Mangelsdorf (7), giving the total thermal radiation from hot carbon dioxide, water vapor, and mixtures of the two over a wide range of variation of concentrations, and at temperatures up to about 1900 F. By appropriate plotting, the data have been extrapolated to higher temperatures. Precise measurements of the right kind on nonluminous laboratory flames should serve as a basis for testing the numerical accuracy of the carbon-dioxide and water-vapor radiation charts at high temperatures; those charts could then be used for calculations of radiant-heat transmission from large industrial flames.

Measurements of total radiation of the type of Callendar (8), von Helmholtz (9), and Haslam, Lovell, and Hunneman (10) were made with the whole flame in view of the radiation receiving instrument. These are difficult to compare with the calculation of the expected radiation from hot carbon dioxide and water vapor, because knowledge of the variation of temperature and composition in the flames is not available, and because in some

[1] Associate Professor of Fuel Engineering, Department of Chemical Engineering, Massachusetts Institute of Technology. Professor Hottel received the degree of A.B. in chemistry from Indiana University in 1922, and that of M.Sc. in chemical engineering from the institution with which he is at present connected.

[2] Research Division, General Motors Corporation. Dr. Smith received the degree of B.S. in 1924, M.S. in 1926, and D.Sc. in 1930 from Massachusetts Institute of Technology. Until this year he was engaged as research associate with the Institute.

Contributed by the Process Industries Division and presented at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Numbers in parentheses refer to similarly numbered items in the bibliography at the end of the paper.

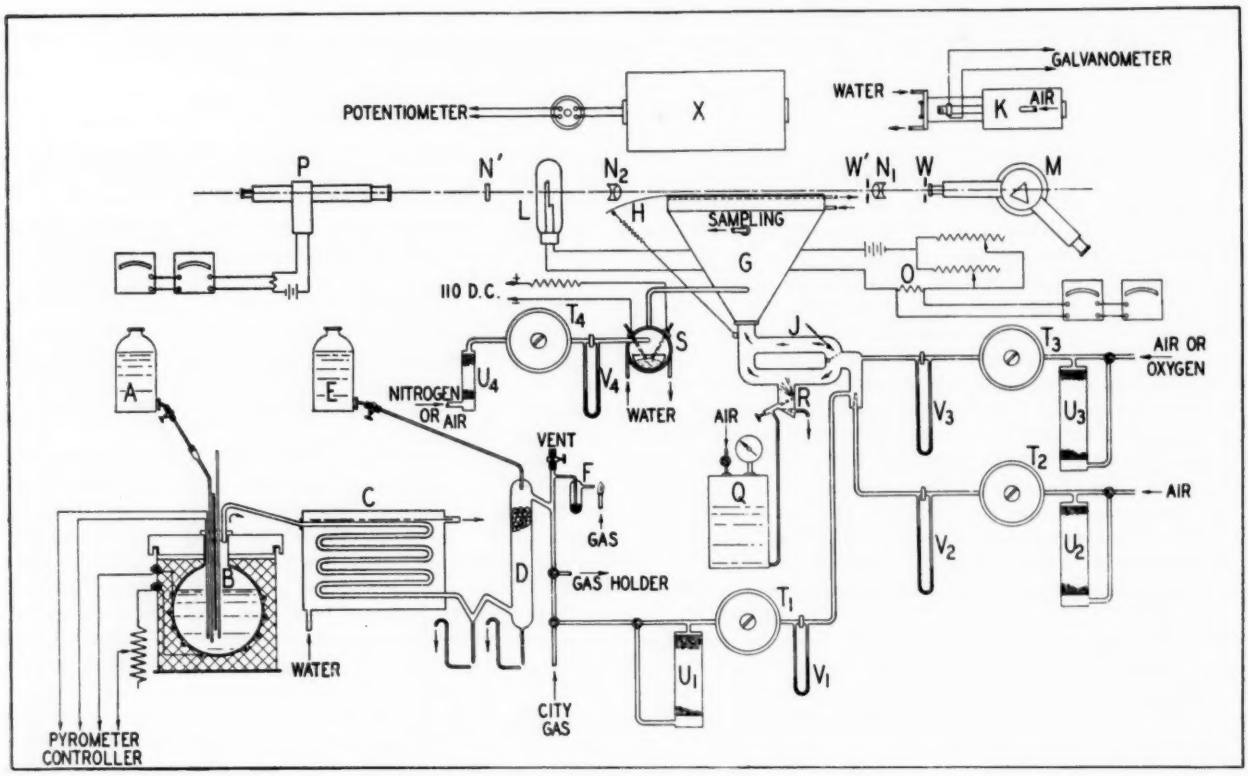


FIG. 1 APPARATUS ASSEMBLY

A = formic acid	*G* = burner	*N* = achromatic lenses	*T* = pressure regulators
B = carbon-monoxide generator	*H* = slide	*O* = standard resistance	*U* = calcium-chloride chambers
C = condenser	*J* = butterfly valve	*P* = optical pyrometer	*V* = orifice manometers
D = scrubbing tower	*K* = thermopile	*Q* = 25 per cent brine	*W* = diaphragms
E = caustic soda	*L* = tungsten band lamp	*R* = spray chamber	*X* = black-body furnace
F = safety trap	*M* = spectroscope	*S* = arc chamber	

cases a considerable portion of the thermal radiation was absorbed by a column of cold gas containing combustion products and occupying the space between the flame and the radiometer. Fishenden and Saunders (11), making a rough calculation of what to expect for one set of conditions studied by Callendar, found agreement with predictions based on thermal radiation inconclusive.

EXPERIMENTAL PROCEDURE

To permit a comparison of experimental results with calculations involving the temperature, composition, and depth of flame it is necessary to burn the gas in a burner of such type as will produce a flame uniform in composition and temperature along a line of sight normal to the direction of flow of the gases; to restrict the geometrical limits of vision of the radiation-measuring instrument to a narrow pencil of rays along such a line of sight; and to measure the temperature, composition, and depth of flame along that same line of sight. These experimental requirements are met in the apparatus shown in complete assembly in Fig. 1. For a description of apparatus and experimental procedure sufficiently detailed to permit judgment of the validity of the results, the reader is referred to the Appendix of this paper. Briefly, equipment is provided for generating, drying, and metering carbon monoxide, for drying and metering illuminating gas, air, and oxygen, and for burning the desired mixture of gases in a Méker-type burner *G* (top right center of Fig. 1) consisting of an approach chamber diverging upward to a rectangular water-cooled nickel grid 15.75 in. by 0.813 in., the open length of which can be changed by slide *H*. To measure the temperature of the flame the method of spectral reversal of the sodium-D line is used (see Appendix). The temperature as measured is an average value along the horizontal line of sight of the spectroscope, set just to clear the tips of the small Bunsen cones of the burner grid. During the temperature measurement sodium must be supplied to the flame. This is accomplished in either of two ways, (*a*) by using compressed air in tank *Q* to produce mechanical atomization of a salt spray at *R* in the base of the burner, or (*b*) by vaporizing molten salt with an electric arc in vessel *S* and sweeping the vapor into burner *G* with a stream of nitrogen. The latter method adds no moisture to the flame. Immediately following the temperature measurement, the spectroscope system *M-W-N-W'-N-L* is rocked out of position on its hinged base, and replaced by a thermopile *K*, which consists of a single thermoelement at the focus of a gold mirror. The thermopile, containing no lenses or windows between it and the flame, is suitably diaphragmed to receive a narrow pencil of rays from the same part of the flame in which temperature was previously determined. The thermopile is calibrated by allowing it to "see" into a black-body furnace kept at a fixed and measured temperature.

RESULTS ON CARBON-MONOXIDE FLAMES

The major portion of the experimental work was conducted on carbon-monoxide flames. Radiation was measured for various burner grid lengths, fuel-gas rates, and air-gas ratios, with dry and with moist gases. In addition, oxygen was added in about half the experiments to produce higher flame temperatures or to increase the velocity of flame propagation. The range of variation of the factors studied is shown in Table 1.

TABLE 1 RANGE OF VARIATION OF FACTORS STUDIED, CARBON-MONOXIDE FLAMES

Length of burner grid, L, ft	0.164–1.312
Partial pressure carbon dioxide in combustion products, P_c, atm	0.241–0.369
LP_c, ft atm	0.047–0.476
Flame temperature, t, F	2568–3778
Radiation from flame, Btu per sq ft per hr	8540–20,070
Flame emissivity, p_F	0.022–0.076
Theoretical oxygen, primary, per cent	95–309
Rate of burning carbon monoxide, cu ft per hr	7.1–40.1
Ratio, oxygen to air	0–0.79

TABLE 2

Run no.	Gas rates, cu ft per hr: Carbon monoxide	Air	Oxygen	Burner length, ft	Flame temp, K	Emissivity of flame	PL (atm) (ft)
1	20.0	49.0	....	0.655	2080	0.0478	0.220
2	17.8	42.7	....	0.655	2024	0.0474	0.226
3	17.7	43.0	....	0.984	1836	0.0686	0.335
4	26.4	62.5	....	1.312	1894	0.0761	0.456
5	25.5	62.2	....	1.312	1885	0.0712	0.446
6	31.7	71.5	....	1.312	1938	0.0732	0.476
7	31.7	74.4	....	1.312	1953	0.0702	0.458
8	29.2	88.8	....	1.312	1951	0.0612	0.370
9	29.3	89.0	....	1.312	1926	0.0585	0.370
10	29.3	89.0	....	1.312	1925	0.0632	0.370
11	29.3	89.0	....	1.312	1925	0.0652	0.370
12	29.3	70.7	....	1.312	1924	0.0717	0.450
13	29.3	70.3	....	1.312	1919	0.0728	0.453
14	33.3	76.9	....	1.312	1959	0.0710	0.454
15	33.3	80.0	....	1.312	1958	0.0704	0.452

Table 2 presents the data with measured radiation converted to "flame emissivity" by dividing by the radiation from a black body at the measured flame temperature. The data are difficult to present graphically because of the number of variables. However, if the radiation from a carbon-monoxide flame is due almost wholly to radiation from hot carbon dioxide, it should depend only on the flame temperature T and on the product term P_cL, where P_c is the partial pressure in atmospheres of carbon dioxide in the combustion products and L is the flame depth or burner grid length in feet. Unfortunately the data were not taken in such a way that either T or P_cL was constant during variation of the other. However, a large number of runs were made with a small variation in the product term P_cL and a large variation in temperature. Fig. 2 shows the results for all runs in which P_cL lies between 0.16 and 0.23, with flame emissivity plotted against temperature. The data points are seen to lie definitely in a band the width of which, within the limits of experimental error of the work, is probably due to the variation in P_cL. Fig. 3 is a carbon-dioxide gas-radiation chart from Hottel and Mangelsdorf (7) based on their measurements of radiation from an electrically heated furnace full of hot carbon dioxide. Although the maximum gas temperature attainable in their experiments was about 1900 F, their extrapolation was guided somewhat by measurements of absorption from a black body at 2400 F. Values from Fig. 3 for P_cL of 0.16 and 0.23 have been converted to emissivity and superimposed on the present experimental data of Fig. 2 as solid lines. Considering the fact that these lines were obtained by an extrapolation for which the authors could not claim a probable error less than 10 per cent or perhaps 20 per cent at 3600 F, the agreement represented by Fig. 2 is excellent. It is taken by the authors as a proof that the gas-radiation chart, Fig. 3, is substantially correct in its high temperature range, along the line $P_cL = 0.2$.

The relation between P_cL and flame emissivity, for a constant temperature, is presented in Fig. 4, in which all the data of Table 2 corresponding to flame temperatures between 1800 and 2200 K have been corrected to a common temperature of 2000 K (3140 F) by assuming that Fig. 2 establishes the relation between emissivity and temperature. In Fig. 4 a solid line is superimposed from the carbon-dioxide radiation chart, Fig. 3, corresponding to 3140 F. Although the line is possibly slightly low at low values of P_cL, the agreement with the data is considered satisfactory and within the errors of individual data points. Factors contributing to the imperfection of the correlation are (*a*) experimental

TABLE 2 (Continued)

Run no.	Gas rates, cu ft per hr: Carbon monoxide	Air	Oxygen	Burner length, ft	Flame temp, K	Emissivity of flame	PL (atm) (ft)
16	33.4	94.0	....	1.312	1983	0.0657	0.396
17	33.4	94.0	....	1.312	1989	0.0642	0.396
18	37.2	90.0	....	1.312	2003	0.0663	0.448
19	37.1	89.5	....	1.312	2002	0.0665	0.452
20	37.1	102.0	....	1.312	2021	0.0630	0.404
21	37.1	102.0	....	1.312	2021	0.0622	0.404
22	40.1	97.5	....	1.312	2030	0.0653	0.457
23	40.1	97.8	....	1.312	2031	0.0648	0.456
24	40.1	107.0	....	1.312	2052	0.0608	0.414
25	40.1	107.0	....	1.312	2055	0.0602	0.414
26	14.0	33.2	....	0.656	1857	0.0670	0.228
27	13.9	33.2	....	0.656	1898	0.0551	0.227
28	13.9	33.0	....	0.656	1873	0.0657	0.228
29	13.8	33.0	....	0.656	1900	0.0612	0.226
30	13.8	33.0	....	0.656	1905	0.0595	0.226
31	12.0	28.5	....	0.656	1864	0.0615	0.228
32	12.1	28.5	....	0.656	1860	0.0614	0.228
33	12.1	34.9	....	0.656	1884	0.0575	0.194
34	12.0	34.9	....	0.656	1887	0.0558	0.192
35	10.0	24.0	....	0.656	1799	0.0627	0.226
36	10.0	24.0	....	0.656	1798	0.0627	0.226
37	9.9	33.3	....	0.656	1820	0.0558	0.169
38	10.0	33.6	....	0.656	1842	0.0531	0.170
39	9.9	33.4	....	0.656	1843	0.0522	0.169
40	7.2	17.6	....	0.656	1718	0.0654	0.221
41	7.1	17.3	....	0.656	1696	0.0691	0.223
42	7.1	17.3	....	0.656	1682	0.0717	0.225
43	7.2	26.1	....	0.656	1730	0.0640	0.158
44	7.2	25.8	....	0.656	1727	0.0607	0.160
45	13.8	19.7	15.5	0.164	2244	0.0250	0.0435
46	14.0	27.1	15.3	0.164	2204	0.0218	0.0464
47	13.9	27.7	15.3	0.164	2182	0.0233	0.0456
48	13.8	27.7	15.5	0.164	2206	0.0229	0.0451
49	13.8	29.3	12.1	0.164	2022	0.0297	0.0467
50	13.8	29.3	12.1	0.164	2000	0.0311	0.0467
51	13.8	29.3	12.1	0.164	1994	0.0300	0.0467
52	13.8	29.3	12.1	0.262	2163	0.0295	0.0748
53	13.8	29.3	12.1	0.262	2167	0.0294	0.0748
54	13.8	29.3	12.1	0.262	2106	0.0311	0.0748
55	13.8	29.3	12.1	0.262	2104	0.0306	0.0748
56	16.7	29.3	12.1	0.262	2330	0.0270	0.0882
57	16.7	29.3	12.1	0.262	2330	0.0266	0.0882
58	16.7	29.3	12.1	0.262	2245	0.0288	0.0882
59*M*	16.7	29.3	12.1	0.263	2244	0.0291	0.0882
60	16.7	39.0	15.4	0.525	2019	0.045	0.1396
61	16.7	39.0	15.4	0.525	2008	0.0455	0.1396
62*M*	16.7	39.0	15.4	0.525	1940	0.0442	0.1396
63*M*	16.7	39.0	15.4	0.525	1940	0.0426	0.1396
64	18.0	30.4	18.8	0.525	2049	0.0502	0.162
65	18.0	30.4	18.8	0.525	2045	0.0503	0.162
66*M*	18.0	30.4	18.8	0.525	2004	0.0456	0.162
67*M*	18.0	30.4	18.8	0.525	2004	0.0455	0.162
68*M*	18.0	30.4	18.8	0.525	1964	0.0457	0.162
69*M*	17.9	30.4	18.8	0.525	1949	0.0457	0.162
70	18.0	30.4	21.5	0.525	2022	0.0502	0.155
71	18.0	30.4	21.5	0.525	2049	0.0503	0.155
72	18.0	30.4	20.2	0.525	2078	0.0490	0.159
73	18.0	30 4	20.2	0.525	2085	0.0477	0.159
74*M*	18.0	30.4	20.2	0.525	1993	0.0454	0.159
75*M*	18.0	30.4	20.2	0.525	1986	0.0452	0.159
76	19.2	35.2	21.5	0.656	2015	0.0546	0.190
77	19.2	35.2	21.5	0.656	2005	0.0559	0.190
78*M*	19.2	35.2	21.5	0.656	1903	0.0527	0.190
79*M*	19.2	35.2	21.5	0.656	1907	0.0526	0.190
80	18.0	30.4	20.2	0.525	2032	0.0496	0.159
81*M*	18.0	30.4	20.2	0.525	2032	0.0423	0.159
82*M*	18.0	30.4	20.2	0.525	2030	0.0416	0.159
83*M*	18.0	30.4	20.2	0.525	2043	0.0475	0.159
84*M*	18.0	30.4	20.2	0.525	1999	0.0435	0.159
85*M*	18.0	30.4	20.2	0.525	2003	0.0428	0.159
86*M*	18.0	30.4	20.2	0.525	1994	0.0442	0.159
87*M*	18.0	30.4	20.2	0.525	2003	0.0431	0.159
88	18.0	30.4	20.2	0.525	2048	0.0454	0.159
89*Mb*	18.0	30.4	20.2	0.525	2052	0.0417	0.159
90*Mb*	18.0	30.4	20.2	0.525	2054	0.0427	0.159
91	18.0	30.4	20.2	0.525	2054	0.0468	0.159
92*a*	18.0	31.4	16.3	0.525	2285	0.0339	0.163
93	18.0	31.4	16.3	0.525	2298	0.0330	0.163
94*b*	18.0	31.6	16.1	0.525	2283	0.0349	0.163
95*b*	18.0	31.6	16.1	0.525	2285	0.0348	0.163
96	18.0	31.5	16.1	0.525	2306	0.032	0.17
97	18.0	28.6	15.1	0.525	2301	0.0343	0.18
98	18.0	31.5	16.1	0.525	2306	0.0318	0.17
99	21.3	39.3	17.6	0.787	2213	0.0445	0.249
100	21.3	39.3	17.6	0.787	2213	0.0442	0.249
101	22.3	43.5	22.5	1.31	1957	0.0567	0.38
102	22.3	43.5	22.5	1.31	1961	0.0564	0.38
103	13.8	20.8	10.2	0.328	2343	0.0293	0.12
104	13.8	25.0	5.5	0.328	2343	0.0264	0.159
105	13.8	25.0	5.5	0.328	2341	0.0270	0.159
106	19.1	34.3	11.3	0.655	2280	0.0377	0.227
107	19.1	34.3	11.3	0.655	2280	0.0382	0.227
108	21.8	38.3	11.3	0.655	2354	0.0352	0.234
109	21.8	38.3	11.3	0.655	2343	0.0358	0.234
110	21.8	41.4	14.4	0.983	2131	0.0477	0.322
111	21.8	41.4	14.4	0.983	2131	0.0477	0.322
112	21.8	43.5	22.5	1.31	2022	0.0513	0.37
113	21.8	43.5	22.5	1.31	1978	0.0548	0.37
114	21.8	43.5	22.5	1.31	1978	0.0548	0.37

a = All runs preceding 92 with wet salt spray.
b = Sodium in flame during radiation measurement.
M = In all runs so labeled the combustible mixture contained 0.7 to 1.8 per cent moisture.

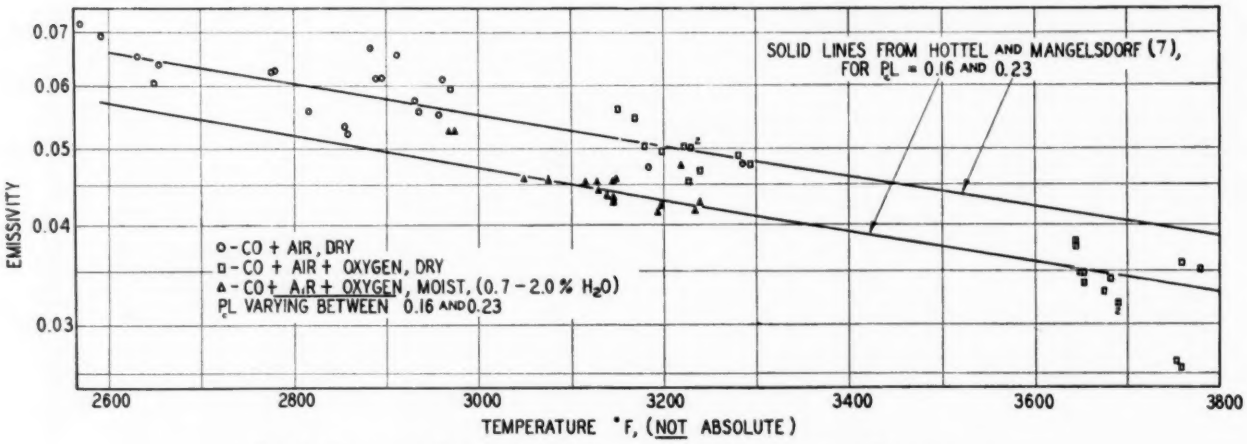


FIG. 2 RADIATION FROM CARBON-MONOXIDE FLAMES—EFFECT OF TEMPERATURE

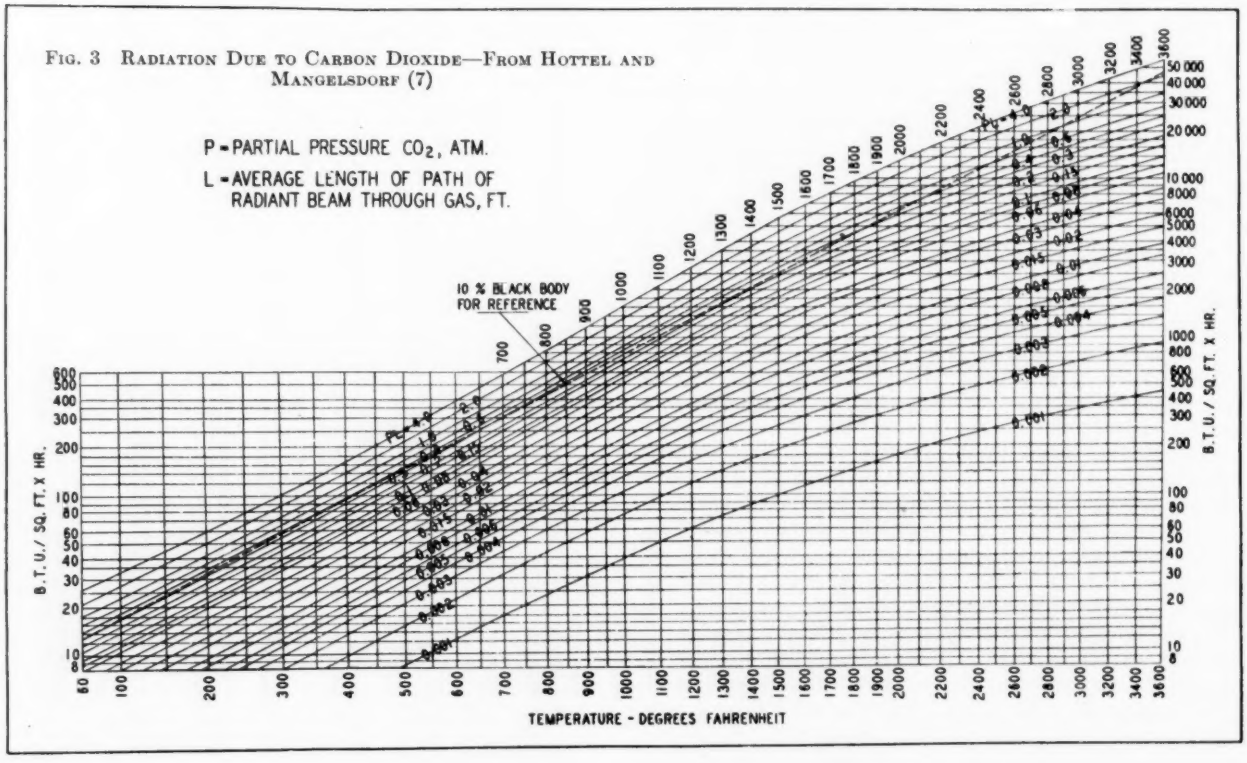


FIG. 3 RADIATION DUE TO CARBON DIOXIDE—FROM HOTTEL AND MANGELSDORF (7)

error, (*b*) difference between actual and assumed carbon-dioxide content of the combustion products due to dissociation (estimated error 3 per cent at highest temperatures), (*c*) infiltration of secondary air, producing a change in composition of combustion products, (*d*) edge effects due to absence of abrupt change from hot flame to cold air at the near flame face.

In some of the experiments (run numbers marked *M* in Table 2) the measurements of flame temperature and radiation were made with 0.7 to 1.8 per cent moisture in the flame. The effect of the moisture, as indicated in Fig. 2, was small. Because of the effect on flame speed of the salt spray used for temperature measurements in runs 1–91 and the consequent possibility that successive measurements (first of temperature with the flame moist and then of radiation with the flame dry) did not corre-

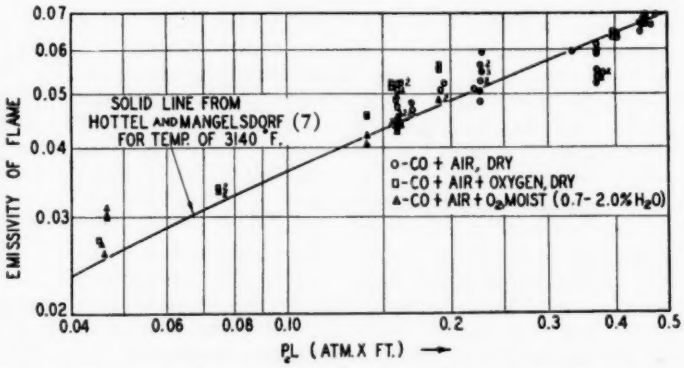


FIG. 4 RADIATION FROM CARBON-MONOXIDE FLAMES—EFFECT OF CONCENTRATION AND FLAME DEPTH

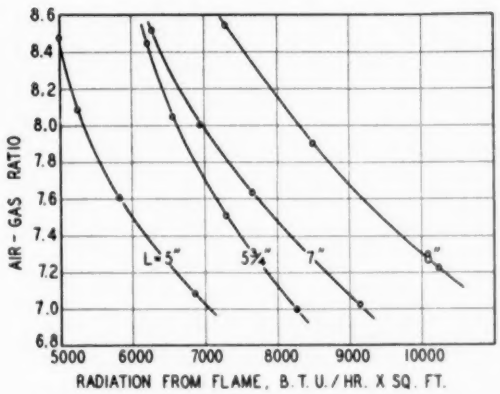


FIG. 5 RADIATION FROM ILLUMINATING-GAS FLAMES—VARIATION WITH AIR-GAS RATIO AND BURNER SIZE

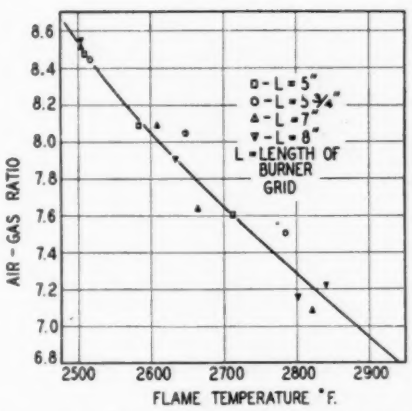


FIG. 6 VARIATION OF TEMPERATURE OF ILLUMINATING-GAS FLAMES WITH AIR-GAS RATIO

spond, the method of introducing salt was changed in all runs following No. 91. Addition of the salt in dry form had no appreciable effect on the correlation presented in Fig. 2.

RESULTS ON ILLUMINATING-GAS FLAMES

With the verification of the carbon-dioxide radiation charts at high temperatures established by combustion of carbon monoxide, the next logical step is a similar study on hydrogen flames. Experimental difficulties involved in the maintenance of proper flames of hydrogen have prevented such measurements thus far. However, some results have been obtained with illuminating gas-air flames (12). Although the experiments were somewhat less extensive than those on carbon-monoxide mixtures, the data are sufficiently reliable to warrant interpretation.

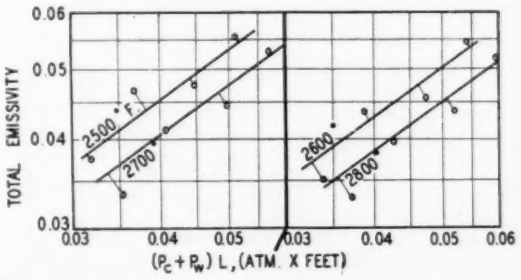


FIG. 7 RADIATION FROM ILLUMINATING-GAS FLAMES—COMPARISON OF EXPERIMENTAL RESULTS WITH CALCULATIONS BASED ON FIGS. 3 AND 8

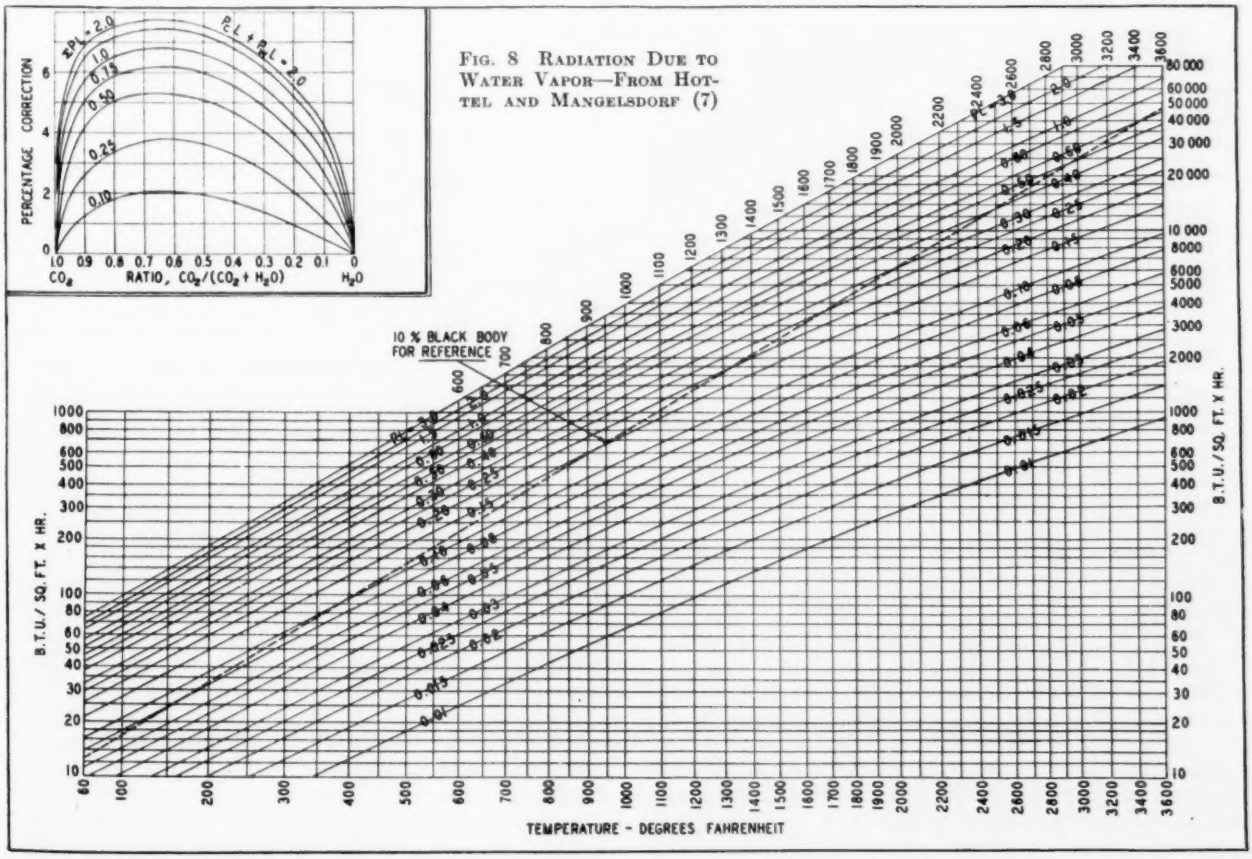


FIG. 8 RADIATION DUE TO WATER VAPOR—FROM HOTTEL AND MANGELSDORF (7)

Fig. 5 shows total radiation versus the air-gas ratio for four different lengths of burner grids and Fig. 6 flame temperature versus the air-gas ratio. The latter plot shows that, as expected, the measured flame temperature is independent of the length of the burner grid for the same air-gas ratio.

If as in the case with carbon monoxide, the radiation is dependent solely upon the combustion products, it may be expected that in the illuminating-gas flames the radiation will depend only upon the temperature and upon $(P_c + P_w)\,L$, where P_c and P_w are the partial pressures of carbon dioxide and water vapor, respectively, in the combustion products, and L is the length of the burner grid. Figs. 5 and 6 have accordingly been converted,

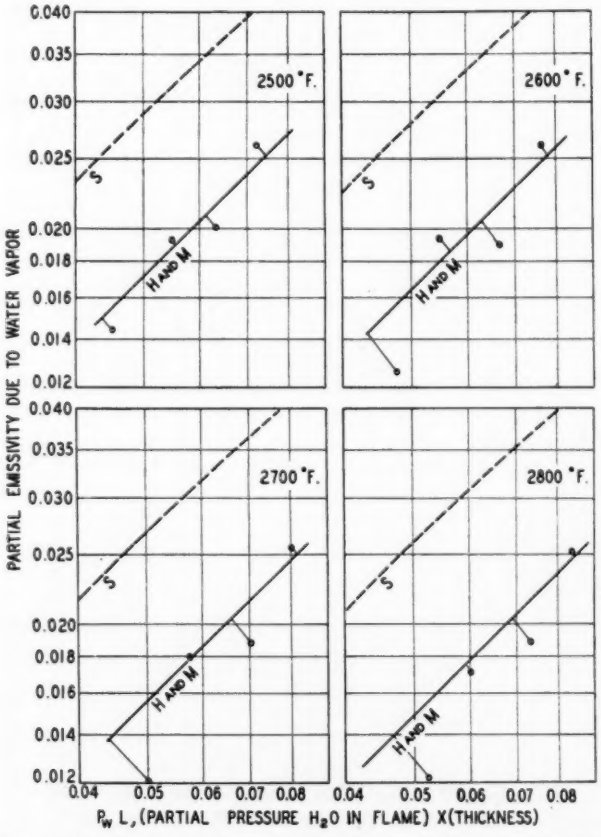


FIG. 9 RADIATION DUE TO WATER VAPOR IN ILLUMINATING-GAS FLAMES—COMPARISON WITH CALCULATIONS BASED ON FIG. 8, AND E. SCHMIDT (13)

by the use of the data of Table 3 on fuel-gas composition, to the form represented in Fig. 7; there total flame emissivity is plotted against $(P_c + P_w)L$ for each of four flame temperatures. By the use of Fig. 3 for carbon dioxide and the analogous Fig. 8 for water vapor (also from Hottel and Mangelsdorf, loc. cit.), the expected total radiation from the combustion products may be calculated. This has been done, the results appearing as solid lines in Fig. 7. It is seen that the gas-radiation charts for carbon dioxide and water vapor account satisfactorily for the measured radiation from an illuminating-gas flame.

TABLE 3 COMPOSITION OF ILLUMINATING GAS BURNED

CO_2	C_2H_4	H_2	CO	CH_4	O_2	N_2
2.2	4.2	48.0	9.4	22.0	1.2	13.0

At the time of publication of the water-vapor radiation chart, Fig. 8, it was pointed out (7) that the literature contained other data on water-vapor radiation by E. Schmidt (13), and that the new chart, though in agreement with Schmidt's results at high values of P_wL, called for considerably less radiation from water vapor at low values of P_wL. Since the first part of the present investigation has established the validity of the carbon-dioxide radiation chart at high temperatures, the data on illuminating gas may be used to obtain the radiation due to water vapor by subtracting from the experimentally measured total radiation that due to carbon dioxide. The residue, presumably due to water vapor, has been converted to an emissivity and plotted versus P_wL in Fig. 9, data points, for each of four temperatures. On that same plot appear solid lines representing water-vapor radiation, taken from Fig. 8 by Hottel and Mangelsdorf; and dotted lines representing water-vapor radiation, taken from the data of E. Schmidt. The agreement of the data points with the solid lines seems to indicate that Fig. 8 is not unduly conservative at low value of P_wL.

CONCLUSIONS

(1) The radiation from flames of carbon monoxide and illuminating gas is calculable from a knowledge of flame size, temperature, and composition.

(2) The accuracy of a chart giving radiation from carbon dioxide has been established over the temperature range 2600 to 3800 F, and the range of P_cL from 0.04 to 0.40 (atm × ft).

(3) The substantial accuracy of a chart giving radiation from water vapor has been established over the temperature range 2500 to 2800 F, at low values of P_wL where there had been some doubt as to whether the radiation chart was unduly conservative.

ACKNOWLEDGMENT

We wish to acknowledge our appreciation of the assistance of John E. Millard, who carried out the work on illuminating-gas flames as a part of a master's thesis at the Massachusetts Institute of Technology.

Appendix

Description of Apparatus and Procedure

CARBON-MONOXIDE GENERATOR

Methods of generating carbon monoxide of high purity in large quantities have been described by Hutton and Petavel (14), Burstall and Ellis (15), and Thompson (16). The principle involved depends upon the thermal decomposition of formic acid in the presence of a dehydrating agent, according to the reaction, $HCOOH = CO + H_2O$. The generator described herein is based on the method of Thompson with a few improvements to be mentioned later.

In Fig. 1, aspirator bottle *A* contains 85 per cent formic acid which is fed to the bottom of the rubber-stoppered, 5-liter Pyrex reaction flask *B*, the rate of feed being observable through the dropping tube sealed into the supply line. Flask *B* is about four-fifths full of 85 per cent phosphoric acid, and is maintained at 170 C. Heat is supplied electrically at a maximum rate of 1700 watts through a coil of chromel wire held between two layers of asbestos paper, the latter being molded while wet to a snug fit around the flask. The flask is supported by its ring neck and housed in a metal container filled with 85 per cent magnesia powder. A thermocouple immersed in the phosphoric acid is connected to a pyrometer controller, giving automatic regulation of the heat supplied to the generator. The gaseous products, carbon monoxide and steam, pass through a 16 mm Pyrex tube to the water-cooled glass condenser *C*, also of 16 mm Pyrex. The condensed liquid escapes through a trap and the cool gas then passes up through the scrubbing tower *D*, filled with short sections of glass tubing, while a 5 per cent sodium-hydroxide solution from aspirator bottle *E* trickles down and escapes through a

trap. Rubber tubing connected to the legs of the trap permits a variable pressure up to 12 in. of water in the generator. Since this type of trap is subject to blowoff under sudden rise of pressure, a mercury safety trap F with a pilot flame at the outlet is included to regulate the maximum pressure in the generator; the mercury trap is set to blow at a pressure less than the liquid heads in the drain traps or the water-sealed gasometer when the bell has risen to its upper limit. With the exception of bottles A and E, the entire generator and the gasometer are completely enclosed under a laboratory hood.

The output of the generator is 20 cu ft of carbon monoxide per hour, being limited not by the power input, as was the case with Thompson's generator, but by the increasing tendency of formic acid to pass through undecomposed if this rate is exceeded. Analysis of the carbon monoxide leaving the scrubber showed that it contained no impurity except water vapor.

The pyrometric control of heat input at a relatively high rate and the addition of an automatic trap have produced a generator of high output that is both easier and safer to operate than those previously reported.

Burner

The burner shown at G is of rectangular shape; it has a nickel grid of the Méker type $15^3/_4$ in. long and $^{13}/_{16}$ in. wide, giving a free port area of 7.7 sq in. The casing is brazed to the sides of the grid to increase the effectiveness of the water cooling. A metal slide H, for changing the burner length, operates in slots along the top of the casing. A flared manifold distributes the gas sufficiently to give flames of uniform height. The burner inlet has been divided into two sections, all of the gas being made to pass through one or the other by the butterfly valve J. For most of the experiments, the gas was passed through the lower section for the introduction of brine spray used in measuring the flame temperature, and through the upper section during the radiation measurements, this procedure being adopted to permit radiation measurements on flames containing no sodium chloride or water vapor.

Temperature and Radiation Measurements

Spectral reversal of the sodium resonance doublet is used to measure the temperature of the gas flames. Although the accuracy of this method has been questioned lately by David (17) the objections raised seem to have been answered satisfactorily by Lewis and von Elbe (18). The continuous source L, Fig. 1, is a G.E. 4-volt, 20-amp, No. T-20, SR-8 tungsten lamp with filament 0.080 in. wide and 0.0025 in. thick; a vee is cut in one side of the filament to insure use of the same portion in all measurements. The lamp is heated by storage batteries, the current input being controlled with variable resistances, and measured accurately with the deflection-potentiometer principle of Forsythe (19) across the fixed resistance O. Images of the filament are formed at the center of the burner and at the slit of the spectroscope M by the achromatic lenses N. The spectroscope is a Gartner L250 quartz spectrograph, in which the camera is replaced by a tube for holding the eyepiece, and the Cornu prism by one of flint glass. The red brightness temperature of the tungsten lamp is measured with the precision optical pyrometer P (20). The temperatures measured in these experiments were sufficiently low to cause no rapid aging of the band filament, conveniently permitting a transfer of the optical pyrometer calibration to a temperature-current plot for the band lamp.

Two methods are used to introduce sodium into the flames. The first consists of forming a spray in the gas inlet when a small, high-velocity jet of brine from tank Q impinges against the inside of the pipe at R. A better method, in that it eliminates the undesirable effect of water vapor on both the flame speed and total radiation from the flame, is the volatilization of the salt in an electric arc between carbon electrodes immersed in a crucible of salt S. A small stream of air or nitrogen passing through drying tower U, pressure regulator T, flowmeter V, and the enclosed, water-cooled arc chamber S assists in sweeping the volatilized salt into the burner manifold, as well as preventing flow of the gas mixture into the arc chamber with resulting ignition. If air is passed through the arc a correction is made for CO_2 formed at the electrodes.

The radiant energy emitted by the flames is measured with a reflecting-type, total-radiation thermopile K, focused on the center of the flame. The latter consists of a single spot-welded junction of copper and constantan wires, 0.001 in. in diameter, the free ends of which are fastened into two slotted copper leads cemented firmly in a celluloid block $^9/_{16}$ in. in diameter. These leads also form the cold junction. The target is 0.050 in. in diameter of tinfoil 0.001 in. thick, thoroughly coated with lampblack on the front, and cemented in close contact with the junction on the back surface. The mirror is a bright, opaque film of gold sputtered on the concave front surface of quartz; it is $^3/_4$ in. in diameter and has a focal length of 5 in. The thermopile, mounted in a telescope for focusing, is housed with the mirror in a wooden box K to which a water-cooled shield and shutter are fastened. Trays of soda-lime and calcium chloride are placed on the bottom of the box, and a slow stream of air, dry and free of CO_2, is passed through the box during measurements.

A Leeds and Northrup type-HS reflecting galvanometer, mounted on a Julius suspension and the whole enclosed in a compartment to protect it from air currents, is connected to the thermopile leads. The scale, placed at 3.5 m, is readable to 0.1 mm through a high-magnification telescope; the sensitivity of the galvanometer viewed at this distance is 101 cm deflection per microampere.

Two black bodies X of the type described by Coblentz (21), the temperatures of which are measured by carefully calibrated thermocouples, are used to calibrate the thermopile. These bodies are held at constant temperatures of about 730 F and 1820 F; over the thirteenfold range of variation of radiation intensity represented by these two radiation standards, the calibration constant of the thermopile-galvanometer system varied no more than 3 per cent, and the average was much less. A calibration is made before and after each group of flame experiments, the arithmetic mean of the constants so obtained being used for the calculations in the group.

In view of the marked dependence of radiation upon temperature, special experimental care is required to make the two properly related, particularly in a flame where temperature gradients may be great. Therefore, the band lamp, lenses, and spectroscope are mounted on an optical bench which is supported by two sturdy rods fastened into a steel base; the base is fastened to the desk top with hinges so that this system or the thermopile can be placed in line with the burner. Once in correct alignment and clamped, the separate parts undergo no relative motion and no further adjustment is required. The thermopile housing is mounted similarly on a solid base which slides back and forth transversely between fixed stops on a metal track. Once properly aligned and the stop fastened, the thermopile can also be moved and returned to the correct position.

In order to minimize the effect of temperature gradients, the flames are made as large as possible and the distance from the burner grid to the optical axis is chosen so as to place the latter in the most uniform temperature range. In addition, diaphragm W limits the cone of rays viewed during the temperature measurements, and is of such size that any ray drawn from the eyepiece image through the optical system falls well within lens N_2. The limitation of field of view of the thermopile system is accomplished

by two diaphragms of 0.125 and 0.030 in. in diameter, placed immediately in front of the mirror and the target, respectively, in the thermopile housing. The pencil of rays through the flame is thereby restricted to a diameter of 0.15 in. at the image plane near the center of the burner grid, and a maximum of 0.23 in. at the back edge of the grid.

Control of Gas Mixture

In order to avoid variation in mixture ratio while radiation and temperature measurements are being made, a $^3/_8$-in. type-*A* Reliance pressure regulator is placed in the carbon-monoxide line, and $^3/_8$-in. type-*H* regulators are placed in the other gas lines. The rates of flow are measured on calibrated orifice meters *V*. In almost all cases the total rate is such that the flame is just beginning to lift away from the grid. The gases, dried in the calcium-chloride towers if required, are tested for water vapor by aspirating a stream of gas from the manifold sampling tube through a dew-point apparatus.

Procedure

The thermopile is calibrated against the black bodies previously heated to constant temperatures of about 750 F and 1800 F; the carbon-monoxide generator, already up to operating temperature, is started; the mixture ratio is adjusted to give the kind of flame desired; the dew point of the mixture is determined; the thermopile is brought into position and galvanometer readings taken for about 15 successive 15-sec periods with the shutter alternately opened and closed; the thermopile is pushed aside; with the lamp already hot the optical bench is swung into position, the sodium generator is started, and three or four reversals made in the spectroscope, the potentiometer being read for each. This procedure, usually repeated as a check, is followed for each mixture ratio and burner length.

Calculations

The recorded brightness temperatures at $\lambda = 0.665\mu$, without the lens interposed, are corrected to those at $\lambda = 0.589\mu$, with the lens, by substituting appropriate values in Equation [1].

$$\frac{1}{T_R} - \frac{1}{T_{y\cdot L}} = \frac{0.589 \ln ABp_Y - 0.665 \ln Ap_R}{14{,}330} \quad \ldots\ldots [1]$$

where T_R = brightness temp, deg K, at $\lambda = 0.665\mu$, viewed through the bulb but without a lens

$T_{y\cdot L}$ = brightness temperature, deg K, at $\lambda = 0.589\mu$, viewed through the bulb and lens

A = transmissivity of the lamp bulb

B = transmissivity of the lens

p_R = emissivity of tungsten at $\lambda = 0.665\mu$ and true filament temperature, obtainable from T_R (22)

p_Y = emissivity of tungsten at $\lambda = 0.589\mu$ and true filament temperature.

The transmissivity of the lens is found experimentally by measuring the red-brightness temperature of the band lamp, with and without the lens between it and the optical pyrometer. For a fixed lamp temperature

$$\ln B = -\frac{14{,}330}{0.665}\frac{(T_1 - T_2)}{(T_1 T_2)} \quad \ldots\ldots [2]$$

where B = the transmissivity of the lens

T_1 = red brightness temperature of lamp, deg K, without lens

T_2 = red brightness temperature of lamp, deg K, with lens

Substitution into [2] of corresponding values obtained at several temperatures give an average value of 0.92 for *B*. This is adopted also for the transmissivity *A* of the lamp bulb and is used throughout for calculating the correction.

Values of p_R and p_y are taken from the data of Forsythe and Worthing (22).

The correction varies from +7 C at 1670 K to +14.6 C at 2200 K.

BIBLIOGRAPHY

(1) "The Effect of Catalysts on the Speed of Flame, Infra-Red Emission, and Ionization During the Combustion of Carbon Monoxide and Oxygen," by W. E. Garner and C. H. Johnson, *Journal of the Chemical Society of London*, February, 1928, p. 280.

(2) "A Method of Measuring the Radiant Heat Emitted During Gaseous Explosions," by C. H. Johnson and W. E. Garner, *Philosophical Magazine*, vol. 5, series 7, 1928, p. 301.

(3) "Die genauen Wellenlängen der Banden des ultrarothen Kohlensäure- und Wasserspectrums," by F. Paschen, *Annalen der Physik und Chemie*, vol. 53, 1894, p. 334.

(4) "Prüfung der Strahlungsgesetze der Bunsenflamme," by H. Schmidt, *Annalen der Physik*, vol. 29, 1909, p. 971.

(5) "Über die Strahlung der Feuergase und ihre praktische Berechnung," by A. Schack, *Zeitschrift für technische Physik*, vol. 5, 1925, p 267.

(6) "Über die Strahlung der Feuergase," by M. Moeller and H. Schmick, Wissenschaftliche Veröffentlichungen aus dem Siemens-Konzern, vol. 4, 1925, p. 239.

(7) "Heat Transmission by Radiation From Nonluminous Gases II. Experimental Study of Carbon Dioxide and Water Vapor," by H. C. Hottel and H. G. Mangelsdorf, Trans. American Institute of Chemical Engineers, vol. 31, 1935, p. 517.

(8) "Radiation From Flames," by H. L. Callendar, *Journal of Gas Lighting*, vol. 111, 1910, p. 644.

(9) Licht- und Wärmestrahlung Verbrennender Gase, by R. von Helmholtz, Verh. Ver. Beförderungdes Gewerbefleisses, Berlin, 1889.

(10) "Radiation From Nonluminous Flames," by R. T. Haslam, W. G. Lovell, and R. D. Hunneman, *Industrial and Engineering Chemistry*, vol. 17, 1925, p. 272.

(11) "The Calculation of Heat Transmission," by M. Fishenden and O. A. Saunders, His Majesty's Stationery Office, London, 1932.

(12) M.Sc. Thesis, Massachusetts Institute of Technology, by J. E. Millard, M.I.T. Library, 1935.

(13) "Messung der Gesamtstrahlung des Wasserdampfes bei Temperaturen bis 1000°C," by E. Schmidt, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 3, 1932, p. 57.

(14) "Preparation and Compression of Pure Gases for Experimental Work," by R. S. Hutton and J. E. Petavel, *Journal of the Society of Chemical Industry*, vol. 23, 1904, p. 90.

(15) "A Plant for the Production of Carbon Monoxide," by F. W. Burstall and S. J. Ellis, *Journal of the Society of Chemical Industry*, vol. 46, 1927, Transactions, p. 35T.

(16) "Pure Carbon Monoxide for Experimental Purposes," by J. G. Thompson, *Industrial and Engineering Chemistry*, vol. 21, 1929, p. 389.

(17) "The Sodium Line-Reversal Method of Determining Flame Temperatures," by W. T. David, *Engineering*, vol. 138, 1934, p. 475; Ibid., vol. 139, Feb. 22, 1935, p. 195; *Nature*, vol. 135, 1935, p. 470; *Philosophical Magazine*, vol. 20, no. 131, July, 1935, p. 65.

(18) "The Sodium Line-Reversal Method of Determining Flame Temperatures," by B. Lewis and G. von Elbe, *Engineering*, vol. 139, Feb. 15, 1935, p. 168.

"Remarks on the Experimental Determination and Theoretical Calculation of Flame Temperatures and Explosion Pressures," by B. Lewis and G. von Elbe, *Philosophical Magazine*, vol. 20, no. 131, July, 1935, p. 44.

(19) "Theory and Accuracy in Optical Pyrometry With Particular Reference to the Disappearing-Filament Type," by W. E. Forsythe, American Institute of Mining and Metallurgical Engineers, Pyrometry Symposium, 1920, p. 301.

(20) "Determination of True Temperature and Total Radiation From Luminous Gas Flames," by H. C. Hottel and F. P. Broughton, *Industrial and Engineering Chemistry*, Analytical Edition, vol. 4, 1932, p. 166.

(21) "Constants of Spectral Radiation of a Uniformly Heated Inclosure, or So-Called Black Body," by W. W. Coblentz, U. S. Bureau of Standards, Bulletin 204, vol. 10, 1914, p. 1.

(22) "The Properties of Tungsten and the Characteristics of Tungsten Lamps," by W. E. Forsythe and A. G. Worthing, *Astrophysical Journal*, vol. 61, April, 1925, p. 146.

Safe Operation of High-Speed Locomotives

By B. S. CAIN,[1] ERIE, PA.

In this paper are presented the basic theories, test information, and the chief conclusions of an investigation of factors entering into the mechanical design of locomotives for high-speed operation. The author points out that (1) recent trends in railroad operation have been toward higher speeds over longer distances and (2) the development of electric and oil-electric power, and lightweight equipment have changed many of the conditions under which these higher speeds are attained. In view of the radically new types of equipment available, the increased severity of requirements both as to performance and cost of equipment, and the necessity for preserving the railroad's record for safety, it has been necessary to make an extended theoretical and experimental analysis of high-speed locomotive operation. Such an analysis is given in this paper.

THE fundamental problem in regard to the safe operation of high-speed locomotives has been to explain, and to learn to control by design means, the oscillations of locomotives which develop at high speeds, and which place a definite limit on the safe speed at which any particular design of locomotive can be operated over a given section of track. These oscillations consist partly of nosing and rolling motions produced by the sidewise components of the wheel-friction and flange-impact forces when the wheels are out of exact line with the track. Instability occurs when these forces increase the energy of oscillation faster than the damping forces can reduce it. Other oscillations are due to rough track. It is the problem of the designer to control the design, the wheel arrangement, the spring constants, the weight distribution, and the clearances so as to minimize the tendency to instability and to obtain ample margins of safety under varying conditions of track.

Testing of locomotives has been improved during recent years. Stresses in the track can be accurately measured with electric strain gages. The author has used these gages extensively to make continuous records of lateral pressures between locomotive axles and frames on a number of locomotives up to very high speeds. The development of theories to determine the stresses and motions of locomotives at high speeds has proceeded before and coincidently with the test program. The theory as now developed logically explains the test results in so far as the fundamentals are concerned. Some of the principal results of this work are outlined in this paper, reference being made particularly to the high-speed electric locomotives on which the most complete experimental information exists.

General Theory

It is a matter of frequent observation that high-speed equipment may oscillate continuously of its own accord. The only source of energy available to keep these oscillations going is the driving energy which maintains the speed. In a balanced machine, the only way in which this energy can be tapped to produce a sustained oscillation is through friction at the tread of the wheels. Whenever the locomotive wheel base makes a small angle with the track, friction makes it run sideways until the wheel flanges hit the rail. As a result of the impact, the wheel base may become pointed toward the other rail. If this motion persists or increases there is an ever-present tendency to breakage or derailment.

A study of the friction at the head of the rail shows that for oscillations at high speed the frictional force is not constant, but actually approaches a creepage force, varying with the amount of slip. Theoretical calculations which have been made of this creepage force are fairly representative of test data.

The initial motion which starts an oscillation may be due either to inherent instability of the wheel base or to rough track. Both cases have been reduced to calculations by which the tendency of locomotives to start oscillation may be compared. The calculation of the oscillation itself has been divided into two parts.

First, assuming some reasonable type of oscillation, it is possible to calculate the speed at which the oscillation will just sustain itself because the losses and gains of energy are equal. Further details of these calculations are given in the Appendix. This speed is proportional to the frequency of the oscillation, which is still unknown. Other factors, such as the lag of the rear end behind the front end in side motions, can be calculated to give the lowest critical speed.

After this energy calculation is made, a study of the motion in more detail is made to determine the frequency and to ascertain whether the forces resulting from such a motion will approximate those required to produce it. The calculation is essentially one of successive approximations. It is practical because the frequency is not affected much by the friction and damping. Therefore, the calculation of frequency can, after some experience, be made quite simply in many cases.

These calculations apply chiefly to straight track. On curves, it is usually sufficient to draw a diagram of the locomotive in the curve showing the locomotive in equilibrium and to see whether, with large variations in friction, the position is stable. The author has found it most useful to use a diagram in which a reduced scale is used for the direction corresponding approximately to the length of the locomotive and to use a full scale at right angles to this direction. Then clearances are approximately full size and the diagram is accurate at all points.

These calculations, particularly the energy method of studying oscillations on straight track and the layout method for curves, generally allow a steady locomotive to be designed or indicate the reason for observed instability. If severe flange pressures cannot be avoided, their values can be estimated by following these methods further. The allowable limits of pressure have been investigated in an elementary way to bring out the important factors, and some methods of calculation have been outlined, relat-

---

[1] Locomotive Division, General Electric Co., Erie, Pa. Mem. A.S.M.E. Mr. Cain was born in Manchester, England. He received the degree of M.A. from the University of Cambridge and B.Sc. from the University of London. Since 1921 he has been employed by the British Thomson-Houston Co., the International General Electric Co., and the General Electric Co. He has held his present position since 1925 and has specialized on the mechanical design of electric locomotives. Since 1930 he has conducted numerous tests on high-speed locomotive operation.

Contributed by the Applied Mechanics and Railroad Divisions for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

ing to derailment and lateral shifting of track. These methods of calculation are given in the Appendix.

Test Methods

Most of the author's tests have been made to record lateral pressure between the axles and the locomotive frame. This is done by modifying the journal boxes so that the axles are restrained laterally by steel "weigh-bars" one on each side, these bars being adjustable to give the required clearances. Lateral pressure bends one of these bars, and an electromagnetic strain gage on the bar records the pressure on an oscillograph record. The bars are calibrated statically in a testing machine. Records have been obtained both on films and on paper rolls 100 ft long. Pressures on all axles of a locomotive may be recorded simultaneously.

Tests have also been made in which truck motion, acceleration, rail stress, and rail pressure have been recorded. The author has also had access to numerous tests of this nature made by other investigators in this field.

Conclusions

In order to keep a locomotive of conventional type from exerting high flange pressures on straight track when moving at high speed, the wheel base should be long and the clearances small to reduce angularity in the track. The independent swiveling of trucks on straight track should be prevented. Wheels near the center of the locomotive should be flanged and should preferably have lateral freedom in the frame, so that they guide themselves and do not reduce the stability of the wheel base.

Springs should be as soft as is practical to cushion blows and reduce the forces due to oscillation.

On high-speed curves, truck restraints and wheel bases should be studied to avoid instability. On sharp curves the long wheel base should be able to adjust itself to reduce angularity between flanges and rail, and to avoid high flange pressures.

The height of the center of gravity of the locomotive is a compromise. In general the height should be reduced where the locomotive is well guided and where the track is good.

Excessive axle weights should be avoided. In actual designs, these ideals can only be approached. Long rigid-frame locomotives tend to oscillate on high-speed curves and to derail on sharp curves. Therefore, designs must aim particularly at reducing these tendencies. Because articulated locomotives tend to oscillate on straight track at high speed, restraining devices are used to eliminate this tendency. Truck restraints with decreasing characteristics may also be necessary to avoid excessive flange pressures on sharp curves.

Good track maintenance and uniform construction and stiffness with the minimum practical flange clearance are always of great importance.

It is very desirable to obtain more measurements of forces produced by different locomotives and particularly to make further tests on the limiting forces which track can safely withstand.

An energy method of calculating locomotive oscillations has been developed by B. F. Langer and J. P. Shamberger,[2] but their calculations do not agree with those given in this paper.

Acknowledgment

The author is indebted to many engineers for information and suggestions. Particular reference should be made to E. G. Keller who, while with the General Electric Company in 1931 and 1932, cooperated with the author in devising methods for detailed calculation of locomotive oscillations. Dr. Keller worked out these methods in numerical detail, thereby providing clear representations of locomotive behavior and suggesting practical approximate methods. The author is indebted to F. M. Graham for information on track and for results of tests on which the theory of track movement is based. R. Eksergian and P. L. Alger have made valuable suggestions in regard to the subject matter and presentation of the paper.

[2] "Lateral Oscillations of Rail Vehicles," by B. F. Langer and J. P. Shamberger, A.S.M.E. Trans., vol. 57, 1935, paper RR-57-4.

Appendix

Friction and Creepage Between Wheels and Rails

According to a common theory, a wheel will roll on a rail without slipping until the tangential force at the point of contact is sufficient to overcome the friction, given by the load on the rail multiplied by the coefficient of friction. As long as the force retains this value, the wheel will continue to slip. The coefficient varies greatly with the conditions. Usual typical values are given in Table 1.

TABLE 1 COEFFICIENTS OF FRICTION FOR DIFFERENT RAIL CONDITIONS

Condition	Coefficient
Good dry conditions, no slipping	0.35
Wet surfaces, no slipping	0.25
Dry surfaces, slow speed and slight slipping	0.25
Dry surfaces, high speed and slight slipping	0.15
Dry surfaces, wheel spinning	0.05

If the wheel and rail were rubber, the rolling wheel could creep in the direction of any tangential force, due to the elasticity of the materials in contact. Steel being elastic, the same creep must occur to a reduced extent and theoretical studies indicate that the amount of the creep is sufficient to allow many of the

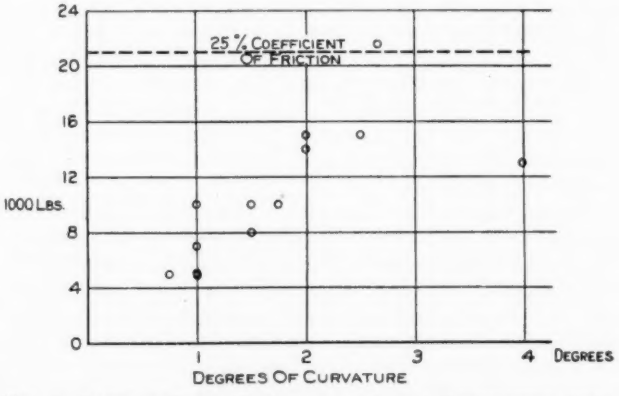


Fig. 1 Lateral Force Required to Slip a Locomotive Axle Laterally as Obtained From Tests at Speeds From 25 to 60 Mph

motions of actual locomotive wheels without requiring the whole area of contact to slip. The creepage is defined as the ratio of creeping to rolling displacement of the wheel. The creepage coefficient is the ratio of tangential force at the point of contact to the creepage.

The creepage coefficient has been calculated for a particular case of F. W. Carter,[3,4] who gives the formula

$$f = \text{creepage coefficient, lb} = 3500 \text{ (wheel diameter, in.} \times \text{axle load, lb)}^{0.5}$$

The present author has made about 50 measurements of the lateral force required to hold an axle with blind tires while going

[3] "On the Action of a Locomotive Driving Wheel," by F. W. Carter, Proceedings, Royal Society of London, series A, vol. 112, 1926, p. 151.

[4] "The Running of Locomotives With Reference to Their Tendencies to Derail," by F. W. Carter, Selected Engineering Paper No. 91, The Institution of Civil Engineers, 1930.

around various curves. The results are rough, due to such conditions as fluctuations in the force and uncertainty as to superelevation, but the results plotted in Fig. 1 for speeds of 25 to 60 mph and in Fig. 2 for speeds of 60 to 70 mph show a marked tendency for the lateral force to decrease to zero with the curvature, for curves less than about 3 deg. The angularity of the axle to the track radius can be estimated for the locomotive on which these tests were made. It increases roughly with the curvature. Calculations in the range from 1 to 2 deg give an approximate angularity (and therefore creepage) of from 0.0007 to 0.0017 and an average creepage coefficient of about 8,500,000 lb. Dr. Carter's[3] formula gives a creepage coefficient in this particular case of 8,600,000 lb. The difference is less than the probable error of either figure.

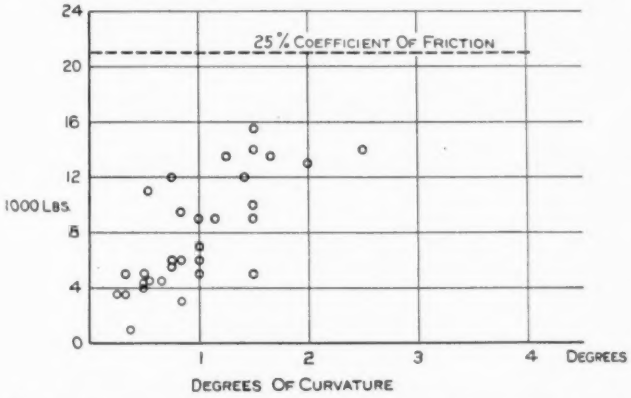


Fig. 2 Lateral Force Required to Slip a Locomotive Axle Laterally as Obtained From Tests at Speeds From 60 to 70 Mph

The creepages required to allow the oscillation of locomotives at high speeds on tangent track are within the range of these tests and the author therefore believes that the creepage theory is a legitimate one to use in such studies.

It should be noted that J. Buchli[5] made model tests and concluded that: "To cause lateral movement of a hauled wheel from its neutral position, a force of definite magnitude is necessary, the value of which is equal to the product of the load on the wheel multiplied by the coefficient of friction. . . . "

Applying Carter's formula to the loaded wheel used by Buchli (about 12-in. diameter with less than 1000 lb load) the maximum creep before complete slippage would be of the order of 0.04 per cent, which would be inappreciable in the test. Buchli's result applies to wheels running with considerable angularity on sharp curves and does not contradict the author's observations at high speed, where the angularity of wheels is very small.

Oscillations Started by Inherent Instability

There are some locomotives which are unstable on perfect track, even before the flange forces come into play. Such a locomotive will always start to deviate from steady motion down the track. When the flanges hit the rail, the resulting oscillation may sustain itself or may decrease, but if it decreases a new one will start, resulting in more flange impacts and so on indefinitely. This natural instability has been studied by F. W. Carter.[3,4] Broadly there are two causes of instability.

The first is the tendency of certain combinations of wheel bases to get out of line. A 4-4-0 locomotive with sufficient guiding force in the engine truck will follow that truck and tend to straighten itself. The same locomotive reversed, 0-4-4, will buckle because the rear truck tends to run in its own direction, which causes the drivers to become more and more out of line with the truck. A stable locomotive, from this point of view, will tend to straighten its wheel base and proceed in a straight line, but this line will not necessarily be the center line of the track. The angular deviation will be that produced by an accidental disturbance and will not be increased by buckling of the wheel base.

The second cause of instability is the coning of wheel treads. A rigid truck wheel base with all wheels equal and coned, moving slowly down uniform track will, if given a slight displacement, tend to move sinuously, the motion repeating after a certain distance. As the speed increases, the motion becomes unstable, due to the mass and moment of inertia of the truck. If the wheels are of different diameters, a truck with smaller wheels leading tends to run more stably and one with smaller wheels trailing tends to run more unstably.

In the author's opinion the effects of coning on large locomotives are generally small, because (1) the coning of new wheels is generally small, (2) the coning of new wheels is quickly worn down, and (3) the clearances between axles and truck frames usually tend to reduce its effect. On the trucks of light-weight high-speed cars, the effect of coning is considerable.

Oscillation Started by Rough Track

If track is poorly surfaced it will start a motion of the locomotive. A rolling motion results from variations in the level of the two rails. In general, this roll is reduced by giving the engine a long natural period of roll on its spring system and by increasing the locomotive length, so that the various axles produce their effects at different times and interfere with each other. If the surface irregularity is very short, the effect on each axle is to produce an angular velocity

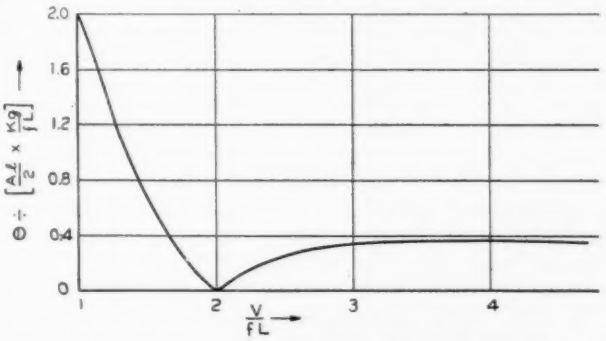


Fig. 3 Angle of Roll of a Vehicle After Passing Over a Track Irregularity

$$(Al/2) \times (kg/I) \times (1/V)$$

Where A = maximum cross-tilt of the track at the irregularity; l = the length of the irregularity which is assumed to have the shape, $A[1 - \cos(2\pi x/l)]$; k = moment acting on the body of the locomotive, due to the springs, per unit angular movement of the axle; g = acceleration due to gravity; I = the moment of inertia of the locomotive body in roll about an axis through the centers of the axles; and V = the speed of the locomotive. The angular velocity can be represented by a vector rotating with angular velocity equal to 2π times the natural frequency of the locomotive in roll.

If the next axle is a distance L behind the first one, it will receive its impulse after a time L/V and the subsequent motion is represented by adding to the first vector a second of a magnitude calculated for the second axle and lagging a time L/V

[5] "Behavior and Movement of Locomotive Wheels on the Track," by J. Buchli, Bulletin of the International Railway Congress, new series, vol. 6, no. 6, June, 1924, pp. 417–437.

behind the first. The effects of successive axles can be added in a similar way. If the locomotive springs are heavily damped, the resultant vector can have this damping applied to it during its rotation.

This vector diagram will show the rolling motion completely. It will be clear from inspection that if a locomotive has n similar axles which are equally spaced, it will leave the irregularity with no roll at all if the time taken to pass over it is just equal to $(n-1)/n$ times the natural period of the locomotive in roll (assuming undamped springs). Fig. 3 shows the relation between the final roll and the speed of a symmetrical two-axle truck plotted in dimensionless coordinates. As V/fL increases above 2, the roll

$$\theta/[(Al/2) \times (kg/fL)]$$

increases to about 0.36 and then decreases steadily toward zero.

Roll in itself will not always start any other oscillation, but it constitutes a reservoir of stored energy which produces nosing if any accidental angularity occurs.

Flange Impacts

When a wheel flange runs against the rail, its lateral motion is quickly stopped. The locomotive begins to roll and the flange tends to bounce back away from the rail. If the center of gravity of the locomotive continues to move in the same direction after the impact, the springs, in bringing the locomotive back from its roll, will pull the flange back again and a second impact will take place. On the other hand, if the center of gravity is low, so that the impact reverses the direction of its motion, the flange may not come in contact with the rail again until the whole locomotive has completed another oscillation. Actually, the impact is reduced by the effect of the increasing diameter in the throat of the wheel flange, which tends to prevent direct impact on the flange. The detail calculations of multiple flange impact are not difficult, but may be very laborious because of the repeated discontinuities in the forces. In many cases where the locomotive as a whole is being studied it will be found convenient to make the following simplifying assumptions:

With the flange hitting the rail, consider a point above the axle situated on the longitudinal axis through the center of gravity of the locomotive body. If, after impact, this point moves away from the impact point, then subsequent impacts are neglected. If this point moves toward the impact point, then the impact is recalculated, assuming that the flange is guided along the rail smoothly as the result of an inelastic impact. For the dividing line, where the point is stationary, the two assumptions do not give the same result, but the actual motion is considerably affected by small variations, so that it is not unreasonable to have a case where the solution is uncertain between two limits. When spring friction or damping is taken into account, the impacts are usually simplified, the approximations become more accurate and agree well among themselves. If necessary, the more exact calculations of multiple impact can always be made.

The theory of multiple impact is borne out by the nature of actual blows as recorded in high-speed oscillograms. Under varying conditions, particularly of spring friction, these show (1) single blows, (2) fluctuating blows, and (3) blows in which the rebound actually divides the blow into two parts, an initial peak, falling to zero and followed by a fluctuating pressure of longer duration. Calculation of impact for a single unit is made as follows:

If h = the height of the center of gravity above the centers of the axles, k_x = radius of gyration about a longitudinal axis, k_z = radius of gyration about a vertical axis, l = distance of impact ahead of the center of gravity, W = weight of the body, u = lateral velocity of the center of gravity, ω_z = angular velocity of the nosing action, and ω_x = angular velocity of roll, then the equivalent mass at the point of impact is

$$\frac{W}{g}\left[\frac{1}{1+(h^2/k_x^2)+(l^2/k_z^2)}\right]$$

and the impact is

$$\frac{W}{g}\left[\frac{2}{1+(h^2/k_x^2)+(l^2/k_z^2)}\right](u+l\omega_z-h\omega_x)$$

Therefore, after the impact, the velocity of the point $x = l$, $y = 0$, and $z = 0$ is

$$\left[u-\frac{2(u+l\omega_z-h\omega_x)}{1+(h^2/k_x^2)+(l^2/k_z^2)}\right] + l\left[\omega_z-\frac{2l}{k_z^2}\left\{\frac{u+l\omega_z-h\omega_x}{1+(h^2/k_x^2)+(l^2/k_z^2)}\right\}\right]$$

$$= u+l\omega_z-\left\{\frac{2[1+(l^2/k_z^2)]\,(u+l\omega_z-h\omega_x)}{1+(h^2/k_x^2)+(l^2/k_z^2)}\right\}$$

Critical Speed on Tangent Track

The energy method of calculating critical speed is similar to other energy methods used in engineering calculations. A reasonable sinuous motion is first assumed and the loss or gain of energy of the locomotive is calculated for this motion. The loss and gain depend on the speed of the locomotive, and the lowest speed at which they balance is called the critical speed. Subject to reasonable limitations the shape of the assumed path can be varied to obtain a minimum critical speed.

Assuming a constant forward speed V, the locomotive can dissipate energy (*a*) by lateral and angular oscillations which cause the wheel treads to slip or creep on the rail, (*b*) by rolling, due to friction and damping in the springs, (*c*) by the operation of such devices as internal friction plates and dampers, (*d*) by impacts between flanges and rails, and (*e*) by rough track where this reduces existing oscillations. It can gain energy by (*f*) the tendency of any angularity of the wheel base to give the engine a lateral velocity and (*g*) rough track where this starts or increases oscillations. It is easier to consider first (*a*) and (*f*), the forces at the head of the rail. It is clear that with perfect uniform track it is not possible to gain or lose energy in flange impacts because these are uniformly elastic and the only forces left are those of slip or creep between the head of the rail and the tread of the wheels.

The calculation is based on the fact that the rate of gain of energy at each point of contact is equal to the force acting on the wheel multiplied by the velocity of motion of the point of application in the direction of the force. If the motion is in a direction opposite to the force, the gain becomes a loss of energy. First assume that the wheels creep, with a creepage coefficient f.

Let an arbitrary reference point o shown in Fig. 4, be taken which moves down the center of the track with velocity V. Let the wheel base be deflected a distance y to o' and rotated an amount θ. Then the lateral deviation of an axle distant l ahead of the reference point is $y + l\theta$. The lateral velocity is

$$\dot{y} + l\dot{\theta}$$

Of this velocity, $V\theta$ is due to rolling displacement of the wheel, and the velocity of creep is

$$\dot{y} + l\dot{\theta} - V\theta$$

The creepage force is

$$-(f/V)(\dot{y} + l\dot{\theta} - V\theta)$$

The rate of gain of energy is creepage force times lateral velocity

$$= -(f/V)(\dot{y} + l\dot{\theta} - V\theta)(\dot{y} + l\dot{\theta})$$

Now assume that y and θ are sinusoidal, which is a reasonable first approximation, and let $\omega = 2\pi f$, where f is the frequency. Then

$$y = Y \sin \omega t$$

θ may be written $\theta \cos(\omega t + \epsilon)$, θ being now the maximum value, and the average rate of gain of energy

$$= -(f/V)[(Y^2\omega^2/2) + Yl\theta\omega^2 \sin \epsilon + (l^2\omega^2\theta^2/2)] + f(\theta Y\omega \cos \epsilon/2)$$

For the whole wheel base, summing over various axles

$$\text{Gain of energy} = -\Sigma(f\omega^2/2V)[Y^2 + 2lY\theta \sin \epsilon + l^2\theta^2 - (V/\omega)Y\theta \cos \epsilon]$$

For the critical speed, this must be zero, or

$$V_{cr} = \omega\left[\frac{\Sigma f(Y^2 + 2lY\theta \sin \epsilon + l^2\theta^2)}{\Sigma fY\theta \cos \epsilon}\right]$$

$$= \omega\left[\frac{Y}{\theta \cos \epsilon} + \frac{2\Sigma fl}{\Sigma f}\tan \epsilon + \frac{\Sigma fl^2}{\Sigma f}\frac{\theta}{Y \cos \epsilon}\right]$$

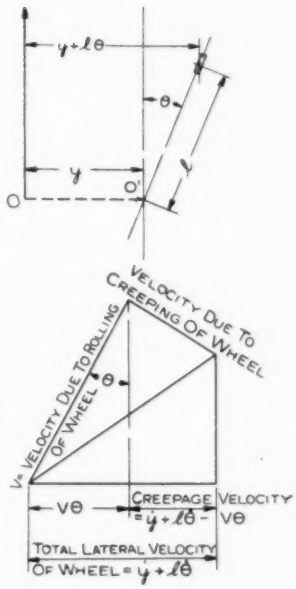


FIG. 4a POSITION DIAGRAM FOR A SIMPLIFIED TRUCK WHEEL

FIG. 4b VELOCITY DIAGRAM FOR A SIMPLIFIED TRUCK WHEEL

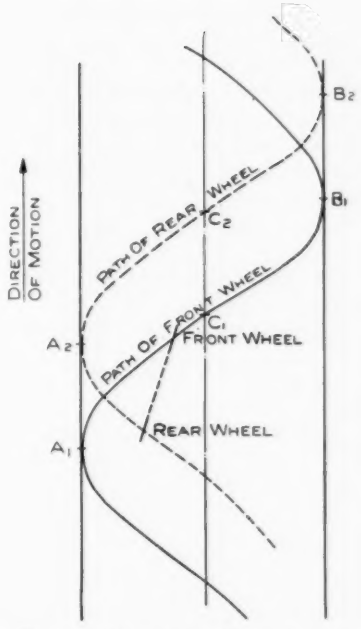


FIG. 5 DIAGRAM OF MOTION OF A SIMPLIFIED TRUCK

Choose the origin of the coordinates so the $\Sigma fl = 0$. This makes the second term vanish, making it obvious that the least value of critical velocity V_{cr} is given by the $\cos \epsilon = 1$, so that

$$V_{cr} = \omega[(Y/\theta) + (\Sigma fl^2/\Sigma f)(\theta/Y)]$$

which is a minimum when

$$Y/\theta = (\Sigma fl^2/\Sigma f)(\theta/Y)$$

Therefore

$$Y/\theta = \sqrt{(\Sigma fl^2/\Sigma f)}$$

giving

$$V_{cr} = 2\omega\sqrt{(\Sigma fl^2/\Sigma f)}$$

This calculation neglects the effect of the track gage, although it can be taken into account by adding the effect of longitudinal forces due to the different longitudinal speed of the two wheels on one axle.

The rate of gain of energy is increased by

$$-0.25(f/V)S^2\dot{\theta}^2$$

where S = the track gage measured between center lines of the tread contacts. Therefore, the speed

$$V = 2\omega\sqrt{\frac{\Sigma f[l^2 + (S^2/4)]}{\Sigma f}}$$

Finally, the effect of lateral clearances in the locomotive has to be taken into account. This theory is a first approximation, the motion of all parts being assumed sinusoidal. It follows that a proper assumption to make with regard to axles with clearance in the journal boxes is to take their motion as sinusoidal and to make the assumed positions coincide with the real positions at the extreme points of the cycle, where impacts take place. Therefore, assume that the motion of the frame laterally with respect to the axle is proportional to the motion of the axle laterally with respect to the track and that the internal clearance is taken up at the same time as the track clearance. In any oscillatory motion this will give very closely the same extreme conditions, the same track impacts and the same net energy transfer at the head of the rail as in the real motion.

As a second approximation, if necessary, the motion calculated in this way can be plotted and corrections made graphically.

Let C_i = the internal clearance between axle and frame, C_t = the track clearance between flanges and rail, and μ = the ratio of track clearance to total clearance, so that

$$\mu = C_t/(C_i + C_t)$$

First assume that μ is the same for all axles. Then

$$V_{cr} = 2\mu\omega\sqrt{\frac{\Sigma f[l^2 + (S^2/4\mu^2)]}{\Sigma f}} = 2\omega\sqrt{\frac{\Sigma f[\mu^2l^2 + (S^2/4)]}{\Sigma f}}$$

so that internal clearance is equivalent to a reduction in wheel base.

If different axles have different clearances, so that each axle has its own value of μ, then the origin of the coordinates is to be taken so that $\Sigma f\mu^2 l = 0$ and

$$V_{cr} = 2\omega\sqrt{\frac{\Sigma f\mu^2 \times \Sigma f[\mu^2l^2 + (S^2/4)]}{(\Sigma f\mu)^2}}$$

Losses due to spring friction and the effects of coning may be included similarly. These formulas depend on the creepage theory of wheel action. It is worth while to show that if slippage with constant friction is assumed, the results do not differ greatly.

Take the simple case of a truck with two equal axles and assume that the motions are sinusoidal. In Fig. 5 consider the motion of the front wheel across the track, along the path A_1B_1. The motion is uniformly to the right, so that if the friction force is also to the right, the front wheel will gain energy, but if it is to the left the front wheel will lose energy. In the position shown, both wheels are losing energy and will continue to do so until they become tangent to their paths or their directions of motion

change. An examination will show that the lowest critical speed is obtained by letting the rear wheel touch the left rail as the front wheel crosses the center line of the track tangentially to its path. Then, when the front wheel passes C_1 its loss will become gain. Similarly, the rear wheel will start to gain energy after it passes A_2. Each wheel gains energy over half its lateral path and loses it over the other half so that the gain and loss are balanced. This condition for minimum critical speed is expressed by the formula

$$2l = V_{cr}/\omega \text{ or } V_{cr} = 2l\omega$$

which is the same formula as was obtained by the creepage theory for this case.

More complicated wheel-base arrangements can be calculated

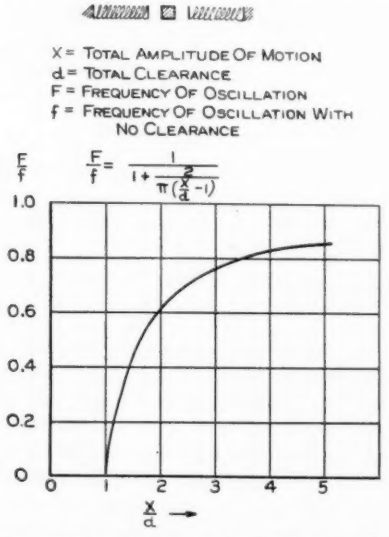


Fig. 6 Effect of Clearance and Amplitude on Frequency of Oscillation

analytically if required, using vector diagrams to portray phase relationships.

Frequency of Locomotive Oscillations

The frequency of roll is given by the well-known formula

$$f = (1/2\pi)\sqrt{[(M - Wh)g/Wk^2]}$$

where f = frequency, cycles per sec; M = the restoring moment, ft-lb per radian; W = the weight of the rolling part, lb; h = the height of the center of gravity of the rolling part above the center of rotation (center line of the axles), ft; k = the radius of gyration of the rolling part about a longitudinal axis through the center of rotation, ft; and g = acceleration due to gravity, 32 ft per sec per sec.

The ratio M/W is called the metacentric height h_m, and therefore the last given equation can be written

$$f = (1/2\pi)\sqrt{[(h_m - h)g/k^2]}$$

In some cases, the natural frequency of roll is also the frequency of nosing oscillations. This is true particularly of locomotives consisting of a single rigid frame, with a high center of gravity and a fairly low frequency of roll. When the flange of the rear driver of such a locomotive hits the rail, the rolling oscillation keeps this flange in contact until the return of the center of gravity allows the flange to leave the rail and the axle to move quickly across the track, where a roll in the opposite direction takes place.

If the frequency of roll is very high, the time taken by the roll will be short and the time taken by the axle to pass through the clearance will become important. To illustrate this, Fig. 6 shows the great effect which amplitude of motion has on frequency for the simplified case of a weight moving between springs, with clearance.

At the other extreme is a locomotive with an extremely low center of gravity, in which case the flange impacts do not produce any appreciable rolling and the frequency depends entirely on the speed with which the axles traverse the clearance. In the study of critical speed, it was shown that for a self-sustaining oscillation

$$V = Cf/\cos \epsilon$$

where V = speed, C = a constant depending on the type of locomotive, f = frequency of motion, and $(\pi/2) - \epsilon$ = phase difference between lateral and rotational motions. If $\cos \epsilon = 1$, then

$$f = (1/C)V$$

so that the frequency would increase with speed. The lateral blows on the flanges, which produce the necessary acceleration would be roughly proportional to the square of the speed. Actually, with extreme oscillation, the locomotive would tend to be thrown more nearly broadside onto the flanges, i.e., the lateral and rotational motions would come more nearly into phase. Then $\cos \epsilon$ would be less than unity and the frequency of sustained oscillations would no longer be proportional to speed. Nevertheless, the highest frequency oscillation which could be sustained is given by $f = (1/C)V$ and this will give an upper limit to the severity of the blows at any speed if rolling or other lateral motions in the locomotive are negligible. Detail calculations are necessary to proceed further. In starting the study of a locomotive, vector diagrams are very useful, as long as their limitations are recognized.

Another useful approximation is to consider the front end of the locomotive fixed against one rail and to calculate the period of swing of the wheel base as a damped pendulum, under the effect of the creepage forces. In some cases the motion can be divided into two types, to each of which an approximation applies.

Operation on Curves

Locomotives may oscillate at high speeds on curves of long radius. The energy methods of calculation are frequently useful but there are other ways of attacking the problem which are often more direct. The first stage in any study is to find the equilibrium position of the locomotive on the curve. For curves of very long radii the creepage theory is probably the most correct to use. For sharp curves, the theory of slipping is more correct. For preliminary studies it is generally very enlightening to take, for convenience, the theory of constant slipping force and to neglect the effect of the width of the gage. Then any wheel which slips is subject to a lateral force equal to the weight on the axle multiplied by the coefficient of friction. This coefficient may conventionally be taken as 25 per cent for slow speed on sharp curves (roughly up to 40 mph and on 6-deg curves and sharper), and 15 per cent for high speed on long-radius curves (roughly above 60 mph and up to 4-deg curves).

The locomotive wheel base is laid out in a possible position on the curve in question and the forces are evaluated. If the forces are in equilibrium the position has been correctly chosen. If the forces have a resultant, the locomotive is moved in the direction which this resultant indicates until finally an equilibrium position is located.[5]

When the equilibrium position is obtained it should be examined to see if it is stable. Stability is shown particularly by

two well-defined points at which the locomotive is held against the rail, these points being well separated to give a long base. If there is only one definite point and the locomotive is balanced by slipping wheels, guiding-truck restraints which are not locked, etc. the locomotive will be unstable at even slow speeds, because the frictional and other forces are not actually constant. Every slipping wheel will, if it can, roll and slip alternately, following a saw-tooth path. Therefore, the equilibrium position must be independent of large changes in the coefficient of friction. Also, wheels which make considerable angles with the rail should be able to execute small rapid oscillations without forcing the main mass of the locomotive to follow them, otherwise major oscillations will be started by small track irregularities.

A frequent cause of oscillation is due to excessive speed where the centrifugal force becomes sufficient to shift one or more axles from the inner to the outer rail, thus upsetting the balance and causing the locomotive to oscillate. More detailed calculations of any kind of motion which is indicated by this preliminary study can be made using energy methods as used for critical speed on tangent track.

Safety

Lateral pressure on an axle may (1) cause derailment, (2) spread or overturn the rail, (3) move the track bodily through the ballast, (4) break the axle, frame or other parts of the locomotive, or (5) may overturn the locomotive.

Derailment. The formula for the maximum lateral pressure on an axle which can be exerted before the wheel flange will climb the rail is,[6] for a positive angle of attack

$$H/W = [(\tan \alpha - f)/(1 + f \tan \alpha)] - (W'/W)f$$

For the flange tangent to the rail

$$H/W = \tan \alpha$$

The formula for the positive angle should be used in order to be safe. For a negative angle of attack

$$H/W = [(\tan \alpha + f)/(1 - f \tan \alpha)] + (W'/W)f$$

In these formulas H = the lateral force on the axle to produce derailment, W = the vertical load at the rail on the wheel which derails, W' = the vertical load at the rail on the other wheel of the same axle, α = the inclination to the horizontal of the plane of contact between the flange and the rail, and f = the coefficient of friction. The vertical loads are not the static loads but are those actually existing at the time of derailment.

The coefficient of friction is conventionally taken as 0.25. For high speed and small angles of attack it will probably be less although for slow speed on very sharp curves it may be more. It is suggested that for negative angles of attack, where decreased coefficient of friction decreases the allowable lateral load, 0.15 should be used for high speeds and small negative angles of attack.

The angle α, which is the inclination of the plane of contact between flange and rail, depends on the contour of the flange and of the rail, and on the angle of attack.

While the contour of the flange can be drawn, it is very laborious to determine graphically the effect of angularity between the plane of the wheel and the plane of the rail. The following semigraphical method can be used where the angularity is enough to make it worth while.

The contour of the tire is drawn. Any convenient point on the contour is chosen as the origin of the coordinates and the contour is then a curve connecting x and y as shown in Fig. 7.

[6] "Traité de Stabilité du Matériel des Chemins de Fer," by Georges Marié, Ch. Béranger, Paris, France, 1924, pp. 288–299.

If the wheel is now turned through an angle θ about a vertical axis, the projected contour becomes a new curve, determined by X and Y as before and also by Z, perpendicular to the plane of projection.

Then, corresponding to any original point (x, y) there is a new point on the projected contour (X, Y, Z) given by

$$(R - Y)^2 = (R - y)^2[1 - (y^1 \tan \theta)^2]$$
$$X = \cos \theta[x + (R - y)y^1 \tan^2 \theta]$$
$$Z = -\sin \theta[x - (R - y)y^1]$$

where R = the radius of the wheel measured to the point on the tire which is taken as the origin of the coordinates, and $y^1 = dy/dx$ in the original curve $y = f(x)$.

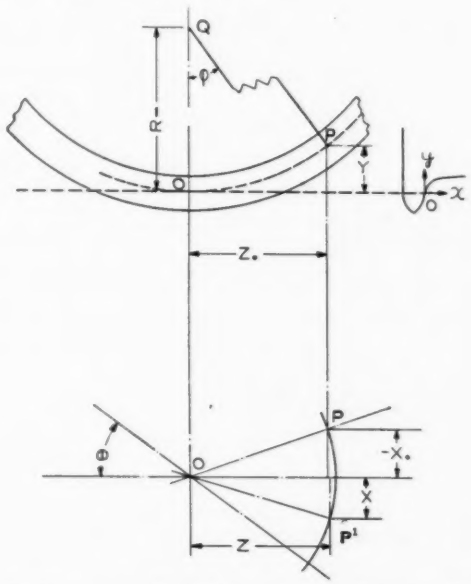


Fig. 7 Diagram of Tire Contour on an Inclined Wheel

Take a number of points (x, y) on the original contour, calculate X and Y by the latter three formulas, and plot Y against X. This will be the projected contour of the tire when turned through an angle θ. The projected contour can then be laid against a section of the rail to determine where they touch, and the slope of the tangent at the point of contact can be measured or calculated. If required, Z can also be calculated for this point to ascertain how far ahead of or behind the center of the wheel the point of flange contact is located. To prove the validity of these formulas refer to Fig. 7. Take the origin of coordinates at a point O on the tire. Let P be any point on the surface of the tire, with coordinates X_o, Y, Z_o. In the normal cross-section of the tire, P would have coordinates x, y.

Now rotate the wheel about a vertical axis OQ through an angle θ so that P moves to P^1. The coordinates of P^1 are X, Y, and Z. Then

$$X_o = x$$
$$QP^2 = (R - y)^2 = Z_o^2 + (R - Y)^2 \quad [1]$$
$$X_o^2 + Z_o^2 = X^2 + Z^2 \quad [2]$$
$$\tan \theta = \frac{(-X_o/Z_o) + (X/Z)}{1 + (X_oX/Z_oZ)} \quad [3]$$

A condition when X, Y, and Z are on the projected contour of the tire is expressed by

$$(\partial X/\partial Z)_{Y = \text{const}} = 0 \quad [4]$$

From Equations [1], [2], and [3]

$$(R - y)^2 - (R - Y)^2 = (Z \cos\theta + X \sin\theta)^2$$

and

$$x = X \cos\theta - Z \sin\theta$$

Differentiating with respect to Z, keeping Y constant and putting $(\partial X/\partial Z)_Y = 0$

$$-2(R - y)(\partial y/\partial Z)_Y = 2 \cos\theta(Z \cos\theta + X \sin\theta)$$

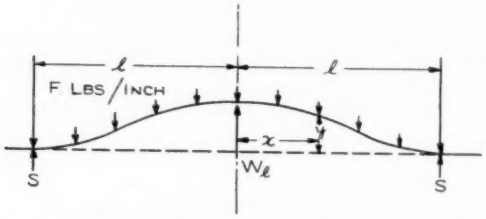


FIG. 8 TRACK DISPLACED BY A STATIONARY LOAD

and

$$(dx/dy)(\partial y/\partial Z)_Y = -\sin\theta$$

Eliminating $(\partial y/\partial Z)_Y$

$$(R - y)(dy/dx) \tan\theta = Z \cos\theta + X \sin\theta$$

Therefore

$$(R - Y)^2 = (R - y)^2[1 - (y' \tan\theta)^2] \quad [5]$$

$$X = \cos\theta[x + (R - y)y' \tan^2\theta] \quad [6]$$

$$Z = -\sin\theta[x - (R - y)y'] \quad [7]$$

If the contour can be expressed by a formula, the projection can be calculated directly. For example, if the contour is conical $y = x \tan\alpha$ and the slope of the projected cone is

$$\tan\alpha/\cos\theta \sqrt{(1 + \tan^2\alpha \tan^2\theta)}$$

MOVING TRACK

An excessive lateral load may move the track bodily through the ballast. The resistance to this motion is the friction between the ballast and the bottom of the ties.

A certain amount of side thrust can be exerted without any motion of the ties because the bending and twisting of the rail and compression of the ballast will distribute the thrust over a number of ties and the thrust on at least one tie must overcome the total friction on that tie. There is little data available as to the distribution of lateral pressure between ties up to the point where they slip. After slipping starts, the following simplified calculation is useful.

Assume the frictional force resisting side motion is constant and equal to F lb per in. Consider also, the case where deflection is all in one direction. In order for this to be possible, with a single applied load, reaction forces S are required at the points where deflection starts. Let the total length of deflected track be $2l$ in. Let the lateral load be W_l. Then $S = Fl - (W_l/2)$ as can be observed from Fig. 8.

For equilibrium and positive values of x, taking the origin at the point of application of the lateral load to the undeflected rail

$$y = \frac{9}{2048} \frac{W_l^4}{F^3 E I_l} \left[1 - 6\left(\frac{x}{l}\right)^2 + 8\left(\frac{x}{l}\right)^3 - 3\left(\frac{x}{l}\right)^4\right]$$

where E = the modulus of elasticity of rail steel, I_l = lateral moment of inertia of the track (two rails), $l = 0.75(W_l/F)$, and y = the lateral deflection of the track.

The maximum deflection at the load is

$$y_o = 0.0044(W_l^4/F^3 E I_l)$$

Field tests show that this equation represents the facts fairly accurately for deflections from 0.25 in. to 1 in. For small deflections, the loads are less than given by the equation. It appears that deflections up to approximately the order of 0.25 in. can be produced in heavily loaded track without causing permanent slip. In field tests, track was shifted considerably under a load and returned practically to the original position when the load was removed.

If the vertical load on an axle is W_v, the vertical force on the ballast per inch at a distance x is known to be

$$P = (\pi/8)(W_v/x_l)e^{-(\pi x/4x_l)}[\cos(\pi x/4x_l) + \sin(\pi x/4x_l)]$$

where $x_l = (\pi/4) \sqrt[4]{(4EI_v/U)}$, I_v = the vertical moment of inertia of one rail, and U = the modulus of foundation of one rail.

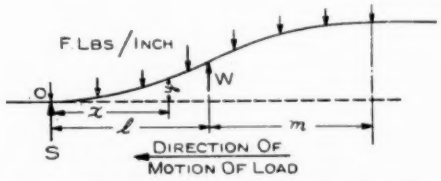


FIG. 9 TRACK DISPLACED BY A MOVING LOAD

The maximum pressure under load is

$$P_o = (\pi/8)(W_V/x_l)$$

Dividing the frictional force F by the vertical force P_o, an index of the nature of a coefficient of friction is obtained. Numerous tests on good track with wooden ties in stone ballast show values of F/P_o ranging from approximately 0.9 to 1.9 according to the condition of the track. The value increases with age up to the point where the ties decay if the track is undisturbed. If the track is raised and the ballast disturbed, the index is reduced. Observations on some metal ties indicate a materially lower resistance to lateral motion. Except on newly laid track or metal ties, it seems reasonable to rely on a ratio of at least 1.0.

Actually, the lateral friction force F will not be constant along the rail but will decrease as the vertical pressure on the ties decreases and will be affected by adjacent wheels. Except on the first wheel of a locomotive, however, the adjacent wheels tend to keep the track uniformly loaded and therefore the assumption of a constant force F per inch is reasonable. A leading wheel will move track a little more easily than these approximate formulas indicate. The previous discussion refers to static conditions. When the locomotive is moving, less lateral force is required to shift the track.

In Fig. 9, let O be the origin of the coordinates. Consider the track moving in one direction only and remaining shifted after the lateral load has passed. Let the coordinates move with the same speed as the load. Then, for the moving section

$$EI(d^2y/dx^2) = Sx + W\{x - l\} - (Fx^2/2)$$

where $\{\ldots\}$ are Macauley brackets, i.e., the bracket is taken as zero whenever the bracketed term is negative.

But

$$S = F(l + m) - W$$

Therefore

$$EI(d^2y/dx^2) = [F(l + m) - W]x + W\{x - l\} - (Fx^2/2)$$

and since $(dy/dx) = 0$ when $x = 0$

$$EI(dy/dx) = [F(l + m) - W](x^2/2) + (W/2)\{x - l\}^2 - (Fx^3/6)$$

Also putting $(dy/dx) = 0$ when $x = l + m$

$$(F/3)(l + m)^3 = (Wl/2)(l + 2m)$$

and taking moments about the origin

$$Wl = (F/2)(l + m)^2$$

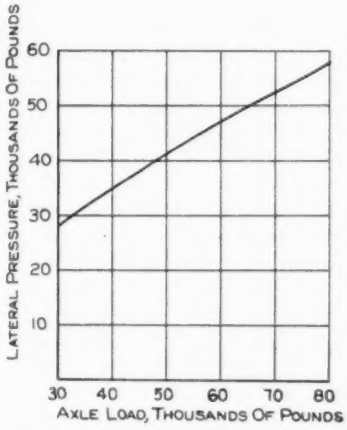


FIG. 10 LATERAL PRESSURE FROM A MOVING LOAD REQUIRED TO PRODUCE 0.25-IN. TRACK MOVEMENT
$(F/P_o = 1;\ Wl = 12.2W_v^{3/4}.)$

whence $l = 2m = (8/9)(W/F)$

Integrating again and putting $x = l + m$, the maximum deflection is found to be

$$y_{max} = (32/2187)(W^4/F^3) = 0.0146(W_l^4/F^3EI_l)$$

Otherwise, for the same maximum deflection, the ratio of moving force to static force will be

$$\sqrt[4]{(0.0044/0.0146)} = 0.74$$

The lateral force exerted by a moving locomotive axle to produce a 0.25-in. displacement of the track may be used as a convenient index. According to the previously given formulas

$$0.25 = (0.0146W_l^4/(F^3EI_l)$$

and

$$F = (F/P_o)(W_v/2)\sqrt[4]{(U/4EI_v)}$$

Therefore

$$W_l = (E^{1/16}/1.07)(F/P_o)^{\frac{3}{4}}(I_l^{\frac{1}{4}}/I_v^{3/16})U^{3/16}\,W_v^{\frac{3}{4}}$$

and if $E = 30{,}000{,}000$ lb per sq in. and $(F/P_o) = 1$

$$W_l = 2.74(I_l^{\frac{1}{4}}/I_v^{3/16})U^{3/16}\,W_v^{\frac{3}{4}}$$

The latter formula indicates that the weight of the rail has very little influence, that the modulus of foundation, i.e., the stiffness of the ballast, is quite important, and that, other things being equal, the allowable ratio of lateral to vertical load on an axle decreases as the axle load increases.

Taking typical values for good track with 131-lb rail, $I_l = 16.1 \times 2 = 32.2$, $I_v = 89$, and $U = 2500$. These values substituted in the last given equation give

$$W_l = 12.2W_v^{\frac{3}{4}}$$

the curve for which equation is plotted in Fig. 10.

In this discussion, the moment of inertia of the track I_l has been taken as being due to the rails alone, which is not strictly accurate inasmuch as other factors enter into it. Before these factors can be taken into consideration, additional experimental data is required, which is not available at the present time.

Lateral Oscillations of Rail Vehicles

By B. F. LANGER[1] and J. P. SHAMBERGER,[2] EAST PITTSBURGH, PA.

The authors point out in this paper that lateral oscillations, which either do not occur or are negligible at slow train speeds, are of vital importance at the high train speeds now demanded by the railroads in order for them to compete successfully with the swiftly moving automobile and the much swifter airplane. Aside from collisions and broken rails, practically all railroad accidents result from lateral derailments, since vertical derailments occur only on those rare occasions when a bridge or trestle collapses from flood or storm. Lateral derailments are caused by lateral pressure of wheel flanges against the rail. Therefore, the prevention of lateral derailments requires a knowledge of both the conditions which cause high lateral forces and of the conditions which must prevail in order to keep the lateral forces below some indicated safe value.

The literature on the subject of lateral oscillations of rail vehicles is rather meager. Nadal[3] and Marie[4] discuss the type of oscillations that are forced. Boedecker,[5] as well as the authors previously mentioned, discusses the weaving motion of the wheel base due to the conicality of the wheel treads. The weaving action which Boedecker discusses must be started by an initial disturbance and, in the absence of further periodic disturbances, can neither build up to a higher amplitude than that produced by the initial conditions nor be sustained, but must quickly die away. Carter[6] develops the stability condition for a rail vehicle as determined by the slippage at the treads.

The essential difference between the previous studies[3,4,5,6] and the present one is that the previous ones discussed forced oscillations the life of which depended upon the application of some periodic force, such as that from cylinder action or rail joints, whereas the present discussion describes and explains a type of oscillation which, after an initial disturbance, even though a minor one, may build up to dangerous proportions and sustain itself indefinitely on absolutely perfect track. This type of oscillation is frequently encountered and is commonly known as "nosing." It is only one phase of the whole problem, but it represents the most serious menace to the safe operation of rail vehicles at high speed.

1—FUNDAMENTAL CONSIDERATIONS

IN ORDER to investigate the possibilities of self-induced lateral oscillations in rail vehicles, the simple case of a symmetrical vehicle with two axles in a rigid frame supporting a spring-borne mass as shown in Fig. 1, will be considered first. It runs on a track the vertical flexibility of which is small compared to that of the springs of the vehicle, but which allows lateral motion of the wheels and axles, partly through flange clearance, partly through the riding-up of the wheel onto the throat of the flange, and partly through elastic distortion of the track itself. The springs of the vehicle allow vertical motion of the sprung mass relative to the unsprung mass. They also allow a sidewise rolling motion and a fore-and-aft pitching motion, but no relative rotation about a vertical axis. This system has six degrees of freedom, three of which are motions of sprung mass relative to unsprung mass, and three of which are motions of unsprung mass relative to the ground. The three motions of sprung mass relative to unsprung mass are (1) vertical motion, (2) rotational motion about a transverse axis which is defined as "pitching," and (3) rotational motion about a longitudinal axis which is defined as "rolling." (Since the height of this longitudinal axis above the rail is less than the height of the center of gravity, the roll results in a lateral displacement of the center of gravity of the sprung mass relative to the unsprung mass.) The three motions of the unsprung mass relative to the ground are (1) lateral motion, (2) longitudinal motion, and (3) rotational motion about a vertical axis. The system is not linear, the principal nonlinearity being in the lateral spring scale of the track.

As a first step in the investigation of the possibilities of self-induced oscillations, the system can be simplified considerably in order to determine which features of it are essential to the existence of the oscillations. Consider the case of a vehicle without any springs and no flange clearance. This eliminates the first three of the six degrees of freedom, which are the three motions of sprung mass relative to unsprung mass, and makes the lateral spring scale of the track linear. There are now just two possible types of lateral motion. One is a rotation about some vertical axis and the other is lateral translation of the wheel base. As far as lateral motions are concerned there are just two degrees of freedom. In order to have self-sustained oscillations in a system with two degrees of freedom there must be transfer of energy between the two degrees of freedom and also net energy input to the system. In other words, the following three conditions must all be satisfied: (1) The rotational motion must produce lateral translation; (2) the lateral translation must produce rotation; and (3) a periodic force must have a component in phase with the velocity of lateral translation and a periodic moment must have a component in phase with the velocity of rotation.

---

[1] Mechanics Division, Westinghouse Research Laboratories, East Pittsburgh, Pa. Mr. Langer was graduated from Stanford University, where he received the degree of B.A. in 1926 and of Engineer in 1928. He has been employed in his present position since 1928, on mechanical problems relating to railway engineering.

[2] Railway Engineering Department, Westinghouse Electric and Mfg. Co., East Pittsburgh, Pa. Mr. Shamberger was graduated from Cornell University in mechanical engineering in 1912. He was then employed as mechanical engineer for the Rock Island Railroad and later as engineer appraiser of railway mechanical equipment by the Interstate Commerce Commission. He has been employed in his present position since 1923, his work consisting of the design of electric-locomotive parts and locomotive testing.

[3] "Theorie de la Stabilite des Locomotives, Part 2; Mouvement de Lacet," by M. J. Nadal, Annales des Mines, vol. 10, 1896, pp. 232.

[4] "Traite de Stabilite du Materiel des Chemins de Fer," by Georges Marie, Librairie Polytechnique, Ch. Beranger, Paris, 1924.

[5] "Die Wirkungen zwischen Rad und Schiene," by Boedecker, Hahn'sche Buchhandlung, Hannover, 1887, p. 97.

Contributed by the Railroad Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers, to be held in New York, N. Y., December 2 to 6, 1935. Certain phases of this paper were presented at the A.S.M.E. National Applied Mechanics Meeting, University of Michigan, June 19, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] "On the Stability of Running of Locomotives," by F. W. Carter, Proceedings, Royal Society of London, vol. 121, 1928, p. 585.

As long as there is forward velocity and friction at the treads of the wheels, the first condition is satisfied. The vehicle tends to travel in the direction of its own center line rather than in the direction of the track center line, because of the friction at the treads. Thus, whenever there is an angular displacement between the track and vehicle center lines, the forward motion has a lateral component of which all points on the vehicle partake equally or, in other words, a lateral translation. Therefore condition (1) is satisfied.

In order to ascertain if the second condition is satisfied, see Fig. 2. Here the wheel base is in an angular position and the track is bowed out on one or both sides. There is a moment exerted by the track tending to restore the wheel base to its neutral position. Note, however, that this moment is a function of only the angular

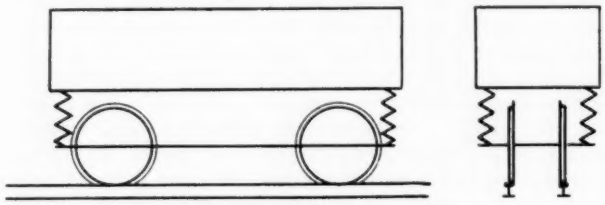


FIG. 1 SIMPLE RAIL VEHICLE

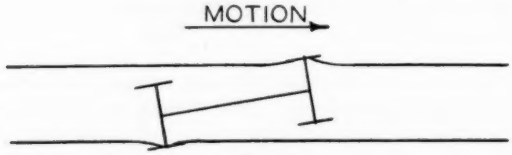


FIG. 2

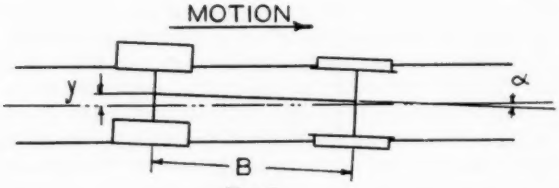


FIG. 3

position of the wheel base and lateral translation does not affect it. If the wheel base is translated laterally, say upward, in Fig. 2, the force at the leading axle increases but the force at the trailing axle decreases by the same amount and the moment remains the same. Inspection readily shows that this holds true for any position of the wheel base. Consequently, condition (2) is not satisfied. The lateral translation does not affect the angular motion and no self-induced oscillations are possible in this simplified system.

Now retrace one of these steps and replace the nonlinearity of the lateral resistance of the track. Actually, this nonlinearity is always present, except under very special conditions, because of the flange clearance. The spring scale may be considered as one which increases with displacement. Condition (1) is still satisfied. In Fig. 2, however, if the whole wheel base shifts upward, the leading axle moves in the direction of increasing spring scale, and the trailing axle moves in the direction of decreasing spring scale. Therefore, the force at the leading axle increases more than the force at the trailing axle decreases, and the moment is changed. This change in moment occurs for any positions of the wheel base except those in which its center line is parallel with that of the track.

Thus one condition is disclosed which is essential to the existence of self-induced lateral oscillations in rail vehicles. *The lateral resistance of the track must increase with the lateral displacement of the wheel from its neutral position. This condition is satisfied if flange clearance is present.*

It has been shown that for a conventional rail vehicle the nonlinearity of the track spring scale must be considered since it furnishes the coupling between lateral and rotational motion. The differential equations which exactly represent such a system are nonlinear and impossible to solve for the general case. As an approach to the problem it is interesting to consider a rigid two-axle rail vehicle with flanges on only the leading pair of wheels. Such a vehicle can sustain its oscillations on a linear track without clearance, and the equations which represent its motion are solvable.

In Fig. 3, let y = the lateral displacement between the center of the trailing axle and the center line of the track, α = the angular displacement between the track center line and the vehicle center line, V = the forward velocity of the vehicle, J = the polar moment of inertia of the vehicle about a vertical axis through the center of gravity, M = the mass of the vehicle, K = the lateral spring scale of the track, B = the wheel base of the vehicle, and c = the track damping constant.

In Fig. 3 let positive forces and displacements be upward and positive moments and angles counterclockwise. The lateral forces at the leading axle are $-K(y + B\alpha) - c(\dot{y} + B\dot{\alpha} - V\alpha)$. The lateral forces at the trailing axle are $-c(\dot{y} - V\alpha)$.

Note that the sliding velocity of a wheel is not its total lateral velocity, but the difference between the total lateral velocity and $V\alpha$. Longitudinal sliding has been disregarded.

The equilibrium of lateral forces gives

$$M[\ddot{y} + (B/2)\ddot{\alpha}] = -K(y + B\alpha) - c(2\dot{y} + B\dot{\alpha} - 2V\alpha) \quad [1]$$

The equilibrium of moments about the rear axle gives

$$\ddot{\alpha}[J + (MB^2/4)] + (MB/2)\ddot{y} = -BK(y + B\alpha) - cB(\dot{y} + B\dot{\alpha} - V\alpha) \quad [2]$$

These equations can be solved completely, but this is not necessary for our present purpose. By the application of Routh's criteria of stability[7] the conditions under which the motion is unstable can be determined. We find that for stability

$$V < \frac{B[2Jc^2B^2 + 2KJ^2 + (1/2)Mc^2B^4 + (1/8)M^2KB^4]}{4J^2c + 2cB^2JM + (1/4)B^4M^2c} \quad [3]$$

If $J = (B^2M)/4$, the case of the mass concentrated at the axles, then

$$V < B[(c/M) + (K/4c)] \quad [3a]$$

If $J = 0$, the case of a light swivel truck, then

$$V < 2B[(c/M) + (K/4c)] \quad [3b]$$

It is interesting to note that this critical speed, below which oscillations will die out and above which they will increase indefinitely, is directly proportional to the length of the wheel base. It also increases with increasing track stiffness and decreases with increasing mass of the vehicle. Combining these last two statements we may say that the higher the natural frequency of the vehicle on the track the higher the critical speed. The term "critical speed" as here used does not mean the speed at which the oscillations are a maximum, but merely a border-line value above which oscillations build up and below which they die out.

2—APPROXIMATE SOLUTION

The next step will be to develop an approximate solution for a more conventional type of vehicle. Take a two-axle vehicle the mass of which is supported on springs and the unsprung mass of

[7] "Mechanical Vibrations," by J. P. Den Hartog, McGraw Hill Book Co., New York, 1934, chap. 7.

which is negligible. It has flanges on all four wheels and has lateral flange clearance in the track. The lateral spring scale of the track is high compared to the stiffness of the springs in the vehicle. The friction between the wheel and the rail is constant, not proportional to velocity.

The difficulty in solving the problem lies with the nonuniformity of the track resistance, which changes from zero to a large value when a flange hits the rail. It is known, however, that whatever the nature of the track resistance, it is such as to transform lateral motion of the wheel base into angular motion. Therefore, if the wheel base has a periodic lateral motion, it will also have a periodic angular motion. Let it be assumed that this angular motion can be expressed by the function $\alpha = A\cos\omega t$. The effect on the following solution if α is some other function of t is discussed in Appendix 3.

Fig. 4 shows six successive positions of the wheel base and the body. Wherever the rotation is in such a direction as to make a pair of wheels move to the same side of the track as that to which the vehicle is pointed, that pair of wheels may be sliding in either direction, or else rolling. If the rotation is in the opposite direction, the wheels must be sliding in the direction of rotation. Thus the direction of sliding of the rear wheels is indeterminate in I and IV and that of the front wheels is indeterminate in III and VI.

Let Y_1, Y_2 = the flange pressure on the leading and trailing axles, respectively, G_1, G_2 = the friction forces at the leading and trailing axles, d = the lateral displacement of the body relative to the wheel base due to roll, Q = the vertical load on a wheel, μ = the coefficient of friction between a wheel and the rail, and K = the lateral spring scale of the body on the wheel base.

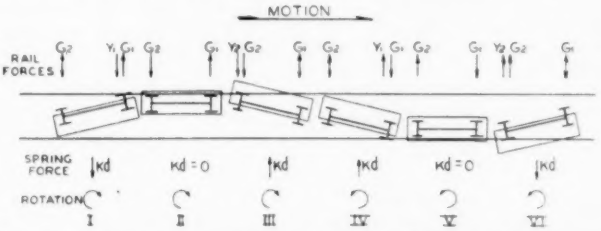


FIG. 4 SUCCESSIVE POSITIONS OF A VEHICLE DURING OSCILLATION, SHOWING DIRECTION OF ROTATION AND DIRECTION OF FORCES EXERTED BY THE TRACK OR BODY ON THE AXLES

The equilibrium equation for the forces on the wheel base is

$$Y_1 + Y_2 + G_1 + G_2 + Kd = 0 \quad [4]$$

For condition I, $Y_2 = 0$, $G_1 = 2\mu Q$. Then $Y_1 + 2\mu Q + G_2 + Kd = 0$, or $Y_1 = -2\mu Q - G_2 - Kd$.

Since Y_1 must be negative, and d is at this time negative, and G_2 cannot exceed $2\mu Q$, then Kd must, in absolute magnitude, be equal to or less than $4\mu Q$. If the amplitude of roll is such that Kd exceeds this value, then both pairs of wheels will slide to the right and $Y_1 = 0$. At the instant when Kd decreases to the value $4\mu Q$, the leading flange hits the left rail and period I can start. Thus, the wheel base cannot turn until a definite point in the cycle of the body's oscillation is reached. Therefore, the rolling of the body on its springs acts as an escapement, controlling the frequency of the oscillation of the wheel base, and in the expression $\alpha = A\cos\omega t$, ω is 2π times the natural frequency of body roll.

The mass of the body will tend, on the average, to rotate about its center of gravity. During I and IV the friction forces will tend to throw the center of rotation toward the rear, and during III and VI toward the front, but during the whole cycle the net effect will be that of the center of rotation in the middle of the wheel base. If y is called the lateral displacement of the center of the wheel base, then the assumption can be made that $\dot{y} = V\alpha$. This assumption is based on the idea that the lateral translation, y, is produced by forward motion, V at a small angle α. $V\alpha$ is probably not the correct value for $\dot{y}$, but it represents an upper limit and gives the lowest possible value for critical speed.

$$\text{If} \qquad \dot{y} = V\alpha, \; y = \int_0^t VA\cos\omega t = (VA/\omega)\sin\omega t$$

Since y is the purely translatory motion of the point under the center of gravity of the body, it is y which excites the rolling oscillation of the body. When a damped system is forced to oscillate at its natural frequency, the vibration lags the disturbing force by 90 deg. Therefore, $d = -d_{max}\cos\omega t$, and the relative positions shown in Fig. 4 are correct.

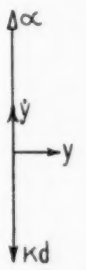


FIG. 5

The question of whether or not the oscillation can sustain itself is dependent upon the relation between energy input and energy loss. Fig. 5 shows the vector relationships between the various motions and forces which have been considered. The sum of all the rail forces, Y_1, Y_2, G_1, and G_2 is equal and opposite to Kd. Therefore, the rail forces are in phase with $\dot{y}$, the velocity of lateral translation, and they constitute the input force.

The energy input per cycle of a periodic force in phase with a velocity is equal to π times maximum force times amplitude.[8] Therefore

$$\text{input per cycle} = \pi Kd_{max}VA/\omega \quad [5]$$

If, during any part of the oscillation, $Kd > 4\mu Q$ then during such time the motion y is not taking place and there is an interim during which there is no input. Therefore, $4\mu Q$ can be substituted for Kd_{max} in calculating the true input, provided $Kd_{max} > 4\mu Q$, and

$$\text{input per cycle} = 4\pi\mu QVA/\omega \quad [5a]$$

The losses at the rail which drain energy from the oscillation consist of frictional losses due to sliding at the wheel treads. This sliding is due to (1) the angular oscillation about the center of the wheel base and (2) the lateral slippage of the whole wheel base when $Kd > 4\mu Q$. Considering (1) first, the force at each wheel is μQ and the distance through which it acts is $BA/2$ during each quarter cycle. Therefore

$$\text{loss per cycle} = 4(BA/2)\;4\mu Q = 8BA\mu Q$$

The loss due to lateral slippage may be calculated by the curves of Appendix 1. In using these curves it must be remembered that the amount of slipping may be limited by the flange clearances.

Let us now study what happens for various amplitudes of roll, that is, various values of Kd_{max}.

If $Kd_{max} = 4\mu Q$

$$\text{input per cycle} = 4\pi\mu QVA/\omega \quad [6]$$

$$\text{loss per cycle} = 8BA\mu Q \quad [7]$$

For stability, $8BA\mu Q > 4\pi\mu QVA/\omega$, or

$$V < 2\omega B/\pi \quad [8]$$

If $Kd_{max} < 4\mu Q$, the loss is the same as for $Kd_{max} = 4\mu Q$, but the input is smaller. Therefore, the critical speed is higher.

If $Kd_{max} > 4\mu Q$, the input is the same as for $Kd_{max} = 4\mu Q$,

[8] "Mechanical Vibrations," by J. P. Den Hartog, McGraw Hill Book Co., New York, 1934, p. 19.

but the loss is increased by the lateral slipping of the whole wheel base. Therefore, the critical speed is again higher.

This whole phenomenon may be illustrated by a curve such as that shown in Fig. 6, which was calculated for an actual locomotive. The curve ABC represents the critical speed for various initial amplitudes of roll. If the initial conditions for this locomotive are represented by either L_1 or L_3, the oscillation will die out. If they are represented by either L_2 or L_4, the conditions will change until the steady condition L_5 is reached, at which the input is equal to the loss.

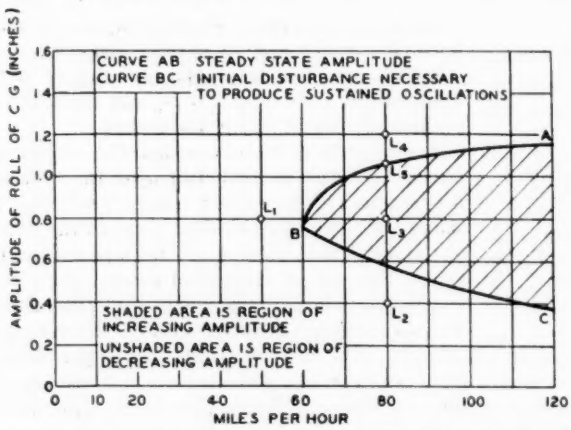


FIG. 6 AMPLITUDE OF ROLL OF AN ATLANTIC-TYPE LOCOMOTIVE

The most important critical speed is, of course, the lowest one, given in Equation [8]. Note that exactly the same conclusions regarding the effect of wheel base and natural frequency on critical speed can be drawn from Equation [8] as have already been drawn from Equations [3a] and [3b], which give the critical speed for the rigid vehicle with flanges on only the leading pair of wheels.

These same methods as were used can be applied to the case of a vehicle with more than two axles in a rigid wheel base. It is not necessary to follow the whole line of reasoning here. In Equation [9], the additional friction loss due to longitudinal sliding of the wheels is considered. The center of rotation should be taken at the center of the wheel base. Any other assumption would increase the friction loss and result in a higher critical speed.

If f = the natural frequency of the roll of the body, n = the number of axles in the rigid wheel base, x_i = the longitudinal distance from the center of rotation to an axle, s = the track gage, and the subscript i refers to the i-th axle in the wheel base, then

$$V_{cr} = [8f/n]\sum_{i=n}^{i=1}\sqrt{[x_i^2 + (s/2)^2]} \quad \ldots\ldots [9]$$

Values of $\sqrt{[x^2_i + (s/2)^2]}$ for various values of x_i are given in Table 1, Appendix 2.

In applying Equation [9] to a nonsymmetrical wheel base, the question of where to take the center of rotation arises. The safest assumption to make would be the one which gave the lowest value to $\Sigma\sqrt{[x^2_i + (s/2)^2]}$. As far as the oscillations discussed in this paper are concerned, there seems to be no inherent advantage or disadvantage in the use of a nonsymmetrical as compared to a symmetrical axle arrangement.

The symbol f is defined as the natural frequency of the roll of the body. This is approximately the correct frequency to use for most simple vehicles of the rigid-frame type and can be calculated according to the equation

$$f = [1/2\pi]\sqrt{[(K_s b^2 - 2Wh)/2I_r]} \quad \ldots\ldots [10]$$

where K_s = the spring scale for one side of the vehicle, neglecting cross-equalized springs, W = the sprung weight of the vehicle, I_r = the polar moment of inertia of the sprung weight about a longitudinal axis through the axle centers, h = the height of center of gravity of sprung mass above axle centers, and b = the lateral distance between springs.

Measurements have shown that the actual frequency at which the oscillation occurs is affected to a certain extent by the natural frequency of the locomotive in the track. The theoretical evaluation of this factor would require a much more detailed analysis than has been developed here; one which would take account of all possible degrees of freedom. It is highly advisable to use experimental data wherever possible in choosing a value of f for use in Equation [9].

Equation [9] was developed primarily for use on vehicles of the type of the rigid-frame locomotive. There is no reason why the same theory would not apply, however, to any individual rigid wheel base in a more complicated system, such as an articulated locomotive or a vehicle with swivel trucks. The only difference would be in the determination of the natural frequency which controls the oscillation. The authors' casual observations on street-railway cars with swivel trucks have led to the conclusion that the oscillations of the body are almost entirely angular about a vertical axis. Thus, the natural frequency would be determined not by the roll, but by the polar moment of inertia of the body about a vertical axis and the lateral restraint at the truck center pins.

If harmonic motion is assumed at all points of the vehicle, the center of rotation at a fixed point on the center line of the wheel base, and a known maximum lateral amplitude for a wheel, it is possible to solve the kinematics of the problem completely. This is done most easily by vector representation.

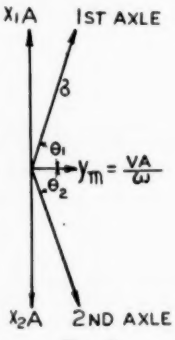


FIG. 7

Let 2δ = the maximum possible full amplitude for a wheel (flange clearance plus rail deflection); A = the maximum value of α during a cycle; θ_i = the phase angle of i-th axle referred to center of rotation; and the other symbols the same as given previously.

Fig. 7 shows the vector diagram for a two-axle wheel base with the center of rotation in the middle. The value of y_{max} has already been determined. It is

$$y_{max} = VA/\omega \quad \ldots\ldots [11]$$

The total amplitude of the i-th axle is seen to be

$$2VA/\omega \cos \theta_i \quad \ldots\ldots [12]$$

Also, if the leading axle has the full amplitude 2δ,

$$\delta = VA/\omega \cos \theta_1 \quad \ldots\ldots [13]$$

and hence

$$A = \delta\omega \cos \theta_1/V \quad \ldots\ldots [14]$$

However

$$\cos \theta_1 = \frac{VA/\omega}{\sqrt{[(V^2A^2/\omega^2) + x_1^2A^2]}} = \sqrt{[V^2/(x_1^2\omega^2 + V^2)]} \quad \ldots [15]$$

Substituting Equation [15] in Equation [14], we obtain

$$A = \delta\omega/\sqrt{(x_1^2\omega^2 + V^2)} \quad \ldots\ldots [16]$$

It is also of interest to know the expressions for energy input

and energy loss in terms of the known constants of the vehicle. These are found to be:

$$\text{Input per cycle} = \pi k d_{\max} V\delta / \sqrt{(x_1^2\omega^2 + V^2)} \quad [17]$$

or, at the critical speed, when $Kd_{\max} = 2n\mu Q$

$$\text{input per cycle} = 2\pi n\mu Q V\delta / \sqrt{(x_1^2\omega^2 + V^2)} \quad [18]$$

and due to rotation the

$$\text{loss per cycle} = \frac{8\mu Q\delta\omega}{\sqrt{(x_1^2\omega^2 + V^2)}} \sum_{i=n}^{i=1} \sqrt{[x_i^2 + (s/2)^2]} \quad [19]$$

3—DESIGN REQUIREMENTS FOR A HIGH-SPEED LOCOMOTIVE

The basic design requirement for a high-speed locomotive is that its critical speed be considerably above the highest speed at which the locomotive is expected to operate. The critical speed as given by Equation [9] is in error on the side of safety in that it assumes $\dot{y} = V\alpha$ and does not consider the energy loss from the friction in the equalization springs and at other rubbing surfaces that tend to damp the oscillation. The experience of the authors has been that the factor of safety introduced by these simplifying assumptions is sufficient for good practice, so that a locomotive can operate safely up to its calculated critical speed. The design features that influence steady performance at high speed are indicated in Equation [9], which is the equation for critical speed. For a given track gage s and a given number of axles n in the rigid wheel base, it is observed from Equation [9] that the inherent critical speed is a function of (1) the frequency of roll, and (2) the length of the rigid wheel base. A high frequency of roll and a long rigid wheel base produce a high critical speed. The critical speed of a locomotive may be raised or lowered to the extent that the designer can vary (1) the frequency of roll, and (2) the length of the rigid wheel base, these two factors being the ones which dominate the critical speed.

Equalization System

The equalization system is an important design feature as regards critical speed because of its influence on the frequency of roll. The equalization system influences the frequency of roll by (1) the scale of the spring, (2) the lateral distance between spring supports, and (3) the ratio between cross-equalization and side-equalization. The relative influence of these three factors on frequency is indicated by Equation [10] for natural frequency of roll. Other things remaining constant, the critical speed is raised by (1) increasing the stiffness of the springs, (2) by increasing the lateral distance between spring supports, and (3) by increasing the amount of side-equalization.

Stiff springs are desirable from a consideration of critical speed but are undesirable from a consideration of resulting forces on vehicle and track. Springs are employed in the equalization system in order to shield both the sprung weight and the track from forces in excess of those existing under static conditions. From the consideration of reducing forces on sprung weight and track, the softer the supporting springs the more effective they become. To stiffen the springs defeats the purpose for which they are used. The fact is that the designer can vary the stiffness of the equalization springs only within rather well-defined limits. To stiffen them defeats their purpose, to soften them beyond a certain limit introduces complications on account of the large vertical movement in the pedestal guides. It would seem, then, that the logical procedure to follow is to employ springs as soft as can be used from practical considerations without too great a sacrifice of frequency. From Equation [10], it is observed that the frequency of roll varies directly as the square root of spring stiffness. Because of this relationship between frequency and spring stiffness, it is possible to soften the springs of a locomotive considerably without producing more than a minor change in frequency.

The type of spring employed in the equalization system, that is, whether leaf-type or coil-type, is as important as the spring scale. Considerable frictional damping is introduced by leaf springs which is absent when coil springs are used. A satisfactory arrangement results from a combination of low-frequency leaf springs in series with high-frequency coil springs. In this arrangement the low-frequency leaf springs practically determine the period of roll and provide frictional damping while the high-frequency coil springs are placed directly over the journal boxes and absorb the high-frequency shocks from the road bed.

The lateral distance between supporting springs is fixed by the location of journal boxes. For a given angularity of roll, the greater the lateral distance between springs, the greater will be the variation in vertical load under the drivers, but as regards critical speed, the greater the lateral distance between springs, the higher will be the frequency of roll and the higher the critical speed. Other things remaining constant, outside journal boxes will produce a higher critical speed than will inside boxes. For steam locomotives, however, outside boxes are possible only with inside cranks, and in this country inside cranks are unknown except for three-cylinder locomotives. In Europe, inside cranks for steam locomotives are common. For electric locomotives, both in this country and elsewhere, outside boxes predominate.

With respect to the ratio between the amounts of cross-equalization and side-equalization, it is probable that the need for cross-equalization has been somewhat overemphasized in the past. The object of cross-equalization is to prevent a wide difference in the vertical loads under the two wheels of a given axle. But field tests have shown that cross-equalization of itself is unable to achieve this objective. Field tests indicate that for a locomotive with the customary amount of cross-equalization the vertical loads under the two wheels on the same axle vary the greatest under the condition of nosing. The prevention of nosing, then, should accomplish more toward equalizing vertical loads than does cross-equalization, and nosing is prevented not by cross-equalization but by side-equalization which, by influencing the frequency of roll, raises the critical speed.

From the standpoint, then, of critical speed and rail loads, the equalization system (1) should employ relatively soft leaf springs in series with stiff coil springs directly over the boxes, (2) the lateral distance between springs should be made large by placing the journal boxes outside the drivers if possible, and (3) the amount of side-equalization should be a maximum, and, if cross-equalization is insisted upon, restrict its amount to 25 per cent or less of the sprung weight.

Weight of Spring-Borne Parts

To determine the influence on critical speed, the absolute weight of the spring-borne parts must be considered in conjunction with the height of the center of gravity above the axle and the stiffness of the equalization springs. The weight of the spring-borne parts influences critical speed by influencing the frequency of roll. However, for a given center of gravity, the weight of the spring-borne parts may be varied at will without changing the critical speed provided the stiffness of the equalization springs is also varied. The spring-borne parts may thus be said to exert a neutral influence so far as critical speed is concerned. Furthermore, the weight of spring-borne parts is one of the design features that can be varied only within rather narrow limits when once the general characteristics of the locomotive have been determined. Hence, it becomes apparent that a varia-

tion of the sprung weight may as well be left out of consideration in the search for means of raising the critical speed.

Location of Center of Gravity of Spring-Borne Parts

The location of the center of gravity of the sprung mass is a most important factor in determining the critical speed, but like the absolute weight of the sprung mass, it is a design feature which can be varied only within exceedingly small limits for a locomotive with prescribed characteristics. Any consideration of the location of the center of gravity of the sprung mass is thus nonessential in the search for means for raising the critical speed. The higher the center of gravity of the spring-borne parts, the lower becomes the frequency of roll and the lower the critical speed. But, what can be done about it?

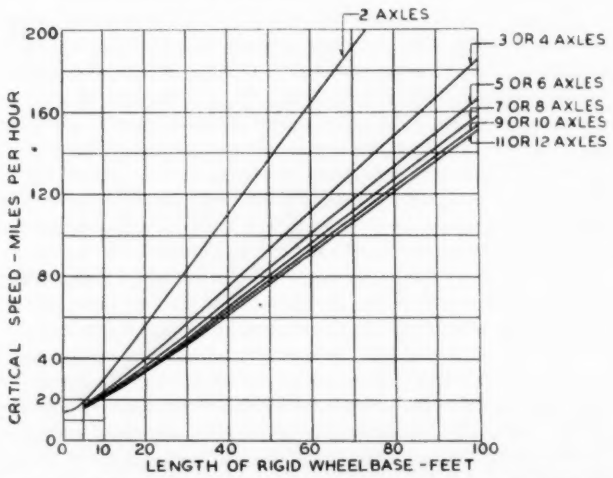


Fig. 8 Effect of the Length of a Rigid Wheel Base on Critical Speed for Various Numbers of Equally Spaced Axles Calculated From Equation [9], $f = 1.0$

The center of gravity of the sprung parts of an electric locomotive is usually somewhat lower than that for a steam locomotive. This lower center of gravity for the electric locomotive is frequently considered a disadvantage, but an analysis of the influence of the center of gravity on locomotive performance does not justify this belief. To determine the tendency from raising or lowering the center of gravity, let us assume extreme conditions. Assume first that the center of gravity is very high above the rail. For this condition the frequency of roll would be very low with a corresponding low critical speed, also a very small amount of roll would throw all of the weight on one rail and bring the center of gravity outside the gage, thus causing the locomotive to tip over. Now, assume that the center of gravity is on line with the center line of the axle. For this condition a lateral force would produce an unbalance of vertical loads at the rail, but this motion of the sprung mass would be purely rotational with no roll at all at the center of gravity. Hence, there would be no lateral translation of the center of gravity and, therefore, for this condition, lateral oscillations would be impossible because lateral translation of the sprung mass is absolutely necessary if the lateral oscillations are to be sustained. Finally, assume that the center of gravity is on line with the rail. For this condition, there is lateral rolling and lateral oscillations are possible but lateral forces now produce no change in the vertical wheel loads at the rail. Thus, the center of gravity of the sprung mass, from a consideration of lateral oscillations, should be located on the center line of the axle, but from a consideration of the unbalance of vertical loads at the rail, the center of gravity should be on line with the rail; that is, it should be in the plane in which the lateral force reacts on the locomotive. The conclusion is reached then that lateral oscillations would be impossible and there would be no unbalance in rail loads from lateral forces if the center of gravity of the sprung mass and the reaction on the locomotive from lateral forces both occurred at the center line of the axle.

For the conventional locomotive, the location of the center of gravity of the sprung mass is practically beyond the control of the designer. However, if possible to do so, move the center of gravity downward, not upward. Lowering the center of gravity raises the critical speed and decreases the amount of vertical-load unbalance at the rail. The lower center of gravity of the electric locomotive, as compared to the steam locomotive, is a distinct advantage.

Wheel Base

The length of the rigid wheel base is the most important design feature influencing critical speed. It has been stated by various writers in the past that a high-speed locomotive should have a long rigid wheel base, but the reason for this requirement has hitherto been unknown. This reason is revealed by an inspection of Equations [18] and [19] which are the equations for energy input and frictional losses during oscillations. These equations show that lengthening the rigid wheel base decreases the available energy input for sustaining oscillations and at the same time increases the frictional losses for damping them out. Hence, a long rigid wheel base becomes the principal requisite for every high-speed locomotive. The effect of increasing the length of the rigid wheel base is illustrated in Fig. 8.

Guiding Trucks

With the exception of the permanently rigid wheel base, the guiding trucks become the most important design feature determining critical speed. The guiding trucks determine critical speed because of their influence on the length of the rigid wheel base. The importance of guiding-truck design is best illustrated by considering two extreme values for the force necessary to produced initial lateral displacement of the truck, commonly known as truck crack-off.

Assume, first, that the crack-off is 10 per cent of the bolster load. For a truck with the usual amount of weight below the center pin, this would represent a lateral force at the rail of about 8.75 per cent of the vertical rail load. For truck wheels, with no tractive effort, the coefficient of friction against lateral slippage at the rail is considerably greater than 8.75 per cent for all normal operating conditions. This means it is possible to swing the truck bolster laterally without sliding the truck wheels. If the locomotive starts to oscillate laterally, the truck bolster will swing laterally within the truck frame without offering any damping force against the oscillations unless the guiding characteristic is so steep that the rail coefficient of friction against lateral slippage is exceeded during part of the oscillation cycle. The reason the lateral swing of the bolster offers no damping resistance is because the energy loss of the sprung weight in overcoming the truck resistance from central position to the extreme swing is immediately restored on the return swing so that the net effect is zero.

Assume now that the truck crack-off is 40 per cent of the bolster load. This represents a lateral force at the rail of about $35^1/_2$ per cent of the vertical rail load. A lateral force of this magnitude at the rail should be capable of sliding the truck wheels laterally across the head of the rail against the highest coefficient of friction that might be expected with the locomotive at speed. Experiments indicate that the coefficient of friction against the lateral sliding of a wheel in motion decreases with speed. Hence, a lateral force at the rail equal to $35^1/_2$ per cent of the vertical rail load should cause the wheels to slide laterally even at quite a

low speed. If the locomotive oscillates laterally, the truck wheels are slipped laterally across the rail head thus introducing an enormous damping force. So far as lateral oscillations are concerned, it is thus seen that a guiding truck with a crack-off high enough to slide the truck wheels laterally across the rail has the effect of lengthening the rigid wheel base to the truck center pin.

It is customary to design electric locomotives for two-way operation. Practically, though not necessarily, the specification for two-way operation demands a symmetrical-type locomotive. It is usual, then, for electric locomotives to have identical guiding trucks at both ends. From the standpoint of providing a turning moment on curves, this arrangement is more effective than might appear at first sight. The location of the center of rotation of a locomotive rigid wheel base is always such as to make the lever arm of the leading truck, with respect to the driver against the inner rail, longer than the corresponding lever arm of the trailing truck. This difference in lever arms produces a net guiding moment even for the condition in which both guiding trucks have the same amount of guiding, which is the case for trucks of the constant-resistance type. Therefore, it is evident that for electric locomotives it is possible to extend the rigid wheel base from truck center pin to truck center pin by employing trucks at both ends with a crack-off, with respect to the rail, greater than the lateral coefficient of friction without sacrificing adequate guiding for curves. If this initial crack-off resistance is too high for operation on sharp curves it is possible to give the truck restraint a drooping characteristic.

It is thus seen that a high-restraint truck possesses very great value as a means of extending the rigid wheel base, but this advantage of high restraint has been shown by experience to be virtually lost if the truck is free to swivel about its center pin, as is usually the case with four-wheel engine trucks. When free to swivel, the truck itself goes through an oscillation similar to that of the main wheel base and consequently runs back and forth through the track clearance without much sliding friction. This freedom of a four-wheel truck to run back and forth without sliding can be prevented by the application of an antiswiveling device such as a radius bar. To limit the flange pressures on extremely sharp curves and turnouts, the restraint of the radius bar on a four-wheel truck should be limited so that it becomes possible for the truck to swivel when its swiveling torque reaches some selected value.

In Equation [9] it is assumed that all axles have the same loading. If this is not the case, then the term $\sqrt{[x_i^2 + (s/2)^2]}$ for each axle must be given a weighting factor equal to the ratio between its actual load and the average load for all the axles being considered, and the symbol n instead of representing the actual number of axles represents the sum of the weighting factors. Letting k represent the weighting factor, Equation [9] may be written

$$V_{cr} = 8f\left[\frac{k_1\sqrt{[x_1^2 + (s/2)^2]} + k_2\sqrt{[x_2^2 + (s/2)^2]} + \ldots.}{k_1 + k_2 + \ldots\ldots\ldots}\right] \quad \ldots\ldots\ldots [20]$$

Since the expression $\sqrt{[x_i^2 + (s/2)^2]}$ increases in value with the increase in distance from the center of rotation, it follows that the heavier the axles at the ends of the locomotive, the higher becomes the critical speed. Since the end axles on a locomotive are usually truck axles, this means that the trucks should carry as much of the total load as is possible, considering wheel-load limitations and the weight requirements of the drivers for adhesion. The influence of changing the ratio between driver load and truck load is illustrated by curves A and B of Fig. 9. Another method for increasing the equivalent rigid wheel base of a locomotive is to give unrestrained lateral play to the intermediate axles of a rigid wheel base.[9] Unrestrained lateral play eliminates both the input and the loss from these axles. Since the input from an axle is fixed regardless of its position in the wheel base, and its loss is proportional to its distance from the center of rotation, this arrangement produces a considerable increase in the critical speed. The magnitude of the increase in critical speed, for a particular locomotive, by the use of unrestrained lateral is illustrated by a comparison between curves A and B of Fig. 9. In this figure, curve B represents the critical speed for a locomotive having the wheel arrangement as shown and for the condition of no free

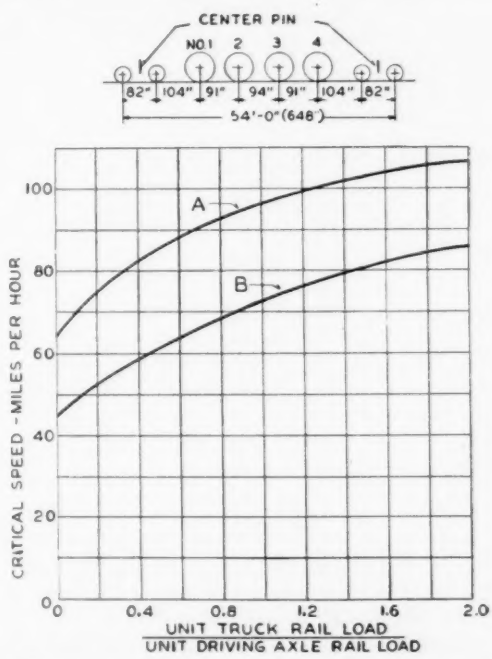


FIG. 9 EFFECT OF WEIGHT DISTRIBUTION ON CRITICAL SPEED CALCULATED FROM EQUATION [20], $f = 1.0$
(Curve A—Unrestrained lateral play at second and third drivers, Curve B—Normal minimum lateral play at all axles.)

lateral, while curve A represents the critical speed for the condition of unrestrained lateral in driving axles No. 2 and No. 3.

RAIL LOAD

The absolute value of the rail load is not a factor in determining critical speed. The axle loading influences critical speed only to the extent that the weight of the sprung mass influences the frequency of roll.

GYROSCOPIC ACTION

The revolving wheels and axles of a locomotive give rise to gyroscopic forces whenever their axes are subjected to angular rotation. On an electric locomotive, the motor armatures constitute additional rotating masses to those of the wheels and axles.

A rotating mass which is part of the deadweight does not have any gyroscopic effect whatever on the oscillations described in this report. The rotation of the wheel base α may cause a shifting of load from one side to the other, but there can be no rolling oscillation of the deadweight, so the shifting of load does not result in an oscillation.

Spring-borne motors in an electric locomotive, however, can affect the oscillation because the rotation α of the wheel base causes the motors to exert gyroscopic forces which aid the rolling of the cab. Calculations show the magnitude of this effect to be small.

[9] Suggested to the authors by B. S. Cain.

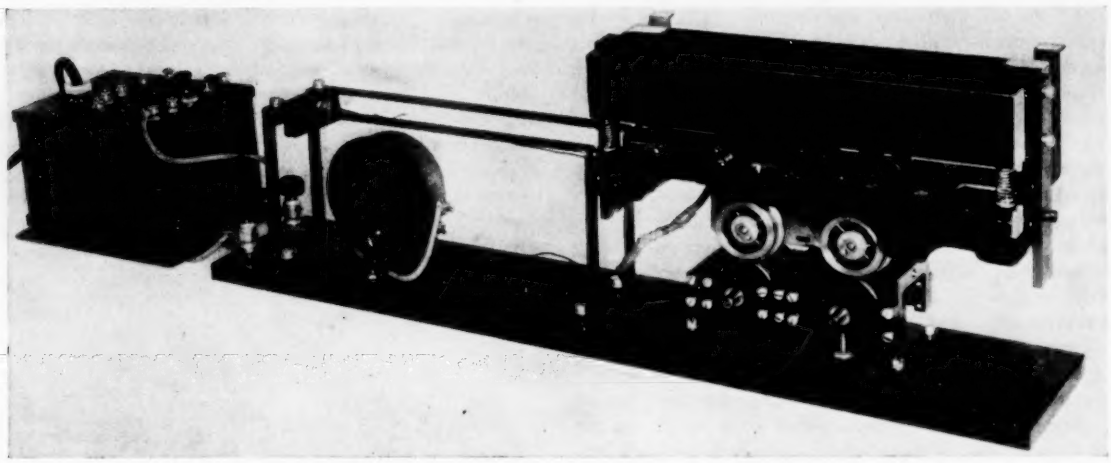


FIG. 10 MODEL FOR ILLUSTRATING LATERAL OSCILLATIONS

RAIL AND TIRE CONTOUR

There has been discussion in the past concerning the effect of conical wheel treads on locomotive and car performance. In this paper, the coning of the wheel tread is not considered and the wheel is treated as having a tread that is a true cylinder. No matter what the contour may be on leaving the shop, under service conditions, the tread surface quickly conforms to the contour of the rail head.

TRACK CLEARANCE

It is very important to keep the track clearance as small as possible. Although this quantity does not appear directly in the formula for critical speed, it does appear in the expressions for energy input and energy loss, both of which are directly proportional to track clearance δ. Therefore, at speeds above the critical, the larger the clearance the larger the amount of energy available to sustain and increase the oscillation. Thus, when the clearance is large there is less possibility of the spring friction being sufficient to control the oscillation.

Probably the most serious result of large track clearance is not shown by the equations, that is, its effect on the magnitude of the rail forces. The amplitude of the leading axle is equal to the track clearance. In any oscillation, velocity and acceleration increase proportionately to amplitude, and therefore the flange pressure increases in proportion to the track clearance, since it is the flange pressure that produces the lateral acceleration.

Therefore the smaller the track clearance the less the possibility of sustained oscillations and the less the damage they can do to the track if they do start. For the condition of zero track clearance, oscillations of the type discussed in this paper become impossible.

COEFFICIENT OF FRICTION

The effect of a low coefficient of friction is twofold. First, it lowers both the input and the loss energies and thus decreases the energy available to sustain oscillations. At the same time, however, it decreases the disturbance necessary to start sustained oscillation and thus makes the vehicle less stable. A low coefficient of friction against lateral slippage may be produced by wet rails or high tractive effort.[10] A very high coefficient of friction may be found in rubber-tired rail vehicles, which have recently been tried out.

[10] "Guiding and Running of Locomotive Wheels on Track," by J. Buchli, *Schweizerische Bauzeitung*, vol. 82, no. 10. September, 1923, pp. 119–125.

4—MODEL TESTS

In order to bring experimental evidence to bear upon the conclusions drawn from the theory without going at once to the expense of making tests on actual locomotives, a small model was constructed.

The critical speed of a model rail vehicle, according to Equation [9], is proportional to the natural frequency of roll of the sprung weight and the linear dimensions of the model. Therefore, if the natural frequency were made the same as that of a full-scale locomotive, the critical speed would be reduced in the same ratio as the scale of the model, and oscillations would occur at easily obtainable speeds.

An illustration of the model used is shown in Fig. 10. The first trials showed that it was not practical to run the model on a track. The track could not easily be made smooth enough nor the straight stretches long enough. Also, the forward motion of the model along the track made it difficult to study the lateral oscillations in detail. It was found that a much better arrangement was obtained by mounting the model on rollers. The model was restrained from longitudinal motion by a wire similar to a drawbar.

The model contained all the parts which, according to the theory, were necessary for the existence of lateral oscillations. It had two axles, both driven through gears by a single motor, and both mounted in the same rigid frame. It had a sprung weight supported from the frame by four helical springs, one at each corner. The center of roll of the sprung weight was made to coincide with the height of the axles by means of a pin and clevis arrangement at each end. The model had a wheel gage of $1^1/_4$ in., a wheel base of $2^5/_{16}$ in., a total length of 9 in., an unsprung weight of 2.6 lb, a sprung weight of 9.5 lb, a natural period of roll of 0.6 sec, and a critical speed of 1.9 fps.

The wheels had flanges and cylindrical treads. The rollers consisted of two cylinders, one under each axle. Each cylinder had two beads spaced $1^1/_4$ in. center to center for the wheel treads to run on and the flanges to bear against. These beads represented the rails. In order to equalize the weight on the four wheels, one roller was mounted on trunions.

The speed of rotation of the wheels, which represented the forward velocity of the model, could be varied by changing the voltage supplied to the motor. By the use of a rheostat any speed up to about 7 fps could be obtained.

OBSERVED OSCILLATIONS

At speeds above 3 fps a rolling oscillation started by hand be-

came self-sustaining and was accompanied by lateral oscillations of the running gear at the same frequency. At speeds below 3 fps the oscillation would die out. It died out quickly at very low speeds but more and more gradually as the speed was increased. Therefore, 3 fps was the actual critical speed. This was higher than the calculated value of 1.9 fps, but this discrepancy was to be expected due to the various frictional losses in the mechanism not considered in the theory.

It was observed that the oscillation of the model was of the same nature as that predicted by the theory, but no exact comparison could be made by mere visual observation. In order to study the motion quantitatively and in detail, a motion-picture camera was used.

There were four points where the motion had to be measured. They were at the front and the back of the sprung and the unsprung weights. The camera was placed so as to look directly down on the model. The four points in which the authors were interested were brought into the same focal plane by bringing up extensions from each end of the unsprung frame to the level of the top of the sprung weight. Spots of white paint located the four points to be observed. Pictures were taken at the rate of about 30 per second, which gave a sufficient number of readings during the 0.6 second period of the oscillation to enable the authors to plot the curves of the motion. The time scale was determined by placing a stop watch in the picture alongside the model. Fig. 11 shows a portion of a strip of pictures from which readings were taken.

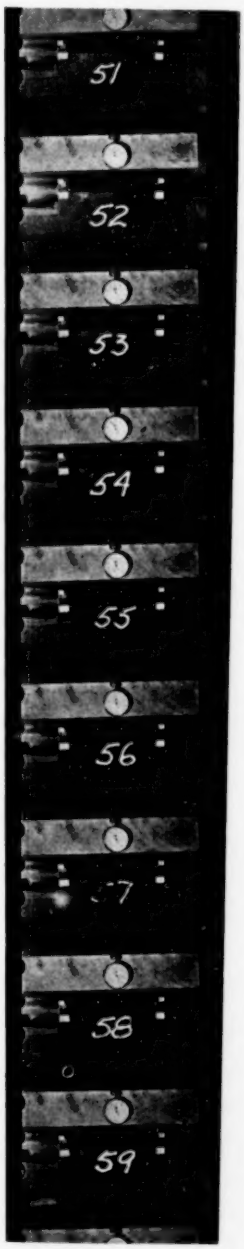


FIG. 11 SECTION OF MOTION-PICTURE FILM OF MODEL DURING OSCILLATION

The motions observed were (*a*) lateral motion of the front of the unsprung weight, (*b*) lateral motion of the rear of the unsprung weight, (*c*) lateral motion of the front of the sprung weight at the top of the model, and (*d*) lateral motion of the rear of the sprung weight at the top of the model. By means of these directly measured values, it was possible to derive (*e*) lateral motion of the top of the model due to roll by subtracting (*a*) from (*c*) or (*b*) from (*d*); (*f*) angularity of the wheel base α by subtracting (*b*) from (*a*) and dividing the result by the length of the model; (*g*) lateral motion of the center of rotation, which was approximately $\frac{1}{2}[(a) + (b)]$.

The primary motions are (*a*), (*b*), (*e*), (*f*), and (*g*). These five motions are plotted for one representative cycle in Fig. 12. It is interesting to note how well the experimental results check with the theory. The theoretical values of motions (*a*), (*b*), (*f*), and (*g*) are shown in Fig. 12 along with the experimental results. Motion (*e*), the roll, was not correctly represented by the model as far as its amplitude was concerned because the theoretical value greatly exceeded the solid position of the springs. The other motions, however, show a remarkably good check between theory and experiment. Note that the lateral slippage of the wheel base occurring at

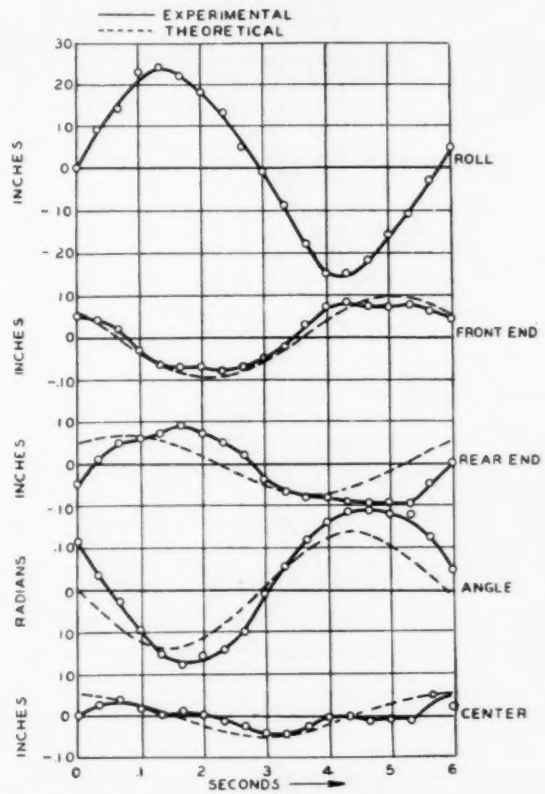


FIG. 12 LATERAL OSCILLATIONS OF LOCOMOTIVE MODEL

the instant of maximum roll shows up quite plainly in the experimental curves. This slippage was not considered in calculating the theoretical curves.

Other tests on the model showed that the critical speed was proportional to the frequency of the oscillation, as predicted by the theory.

5—CONCLUSIONS

In this paper the authors have offered a mathematical theory which explains the self-sustained lateral oscillations of rail vehicles. The formulas derived from this mathematical analysis check with all available experimental data.

The conclusions to which the theory leads may be summarized as follows:

1 For satisfactory operation at high speed, a rail vehicle should have a critical speed, as calculated from Equation [9], no lower than its highest operating speed.

2 Recommended tendencies in design are: (*a*) as high a natural frequency of roll as is possible without sacrificing adequate vertical resiliency; (*b*) a long rigid wheel base; (*c*) high-resistance guiding trucks with limited swivel restraint. This applies to four-wheel trucks; two-wheel trucks are inherently nonswiveling; (*d*) loads concentrated as much as feasible on the end axles; (*e*) unrestrained lateral motion in axles near the center of the wheel base.

3 Recommended tendencies in track maintenance include:

(*a*) the elimination of irregularities. (*b*) The narrowing of the gage to decrease the clearance between flange and rail.

There are three factors to be considered in determining whether or not the performance of a vehicle will be satisfactory in a given service. These factors are (1) the inherent critical speed of the vehicle, (2) the maximum speed at which it will operate, and (3) the condition of the track. For the most satisfactory operation, the calculated critical speed should be well above the operating range, say 30 per cent above the maximum speed. The vehicle should then operate safely on ordinary track. For track that is better than ordinary, the ratio between critical and operating speed may be reduced. On perfect track, any vehicle can operate at any speed. However, it is not safe to rely entirely on a good track condition to maintain satisfactory operation, since the best of track may develop an occasional rough spot.

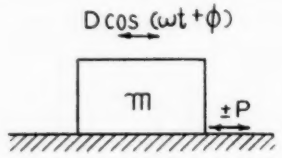


FIG. 13

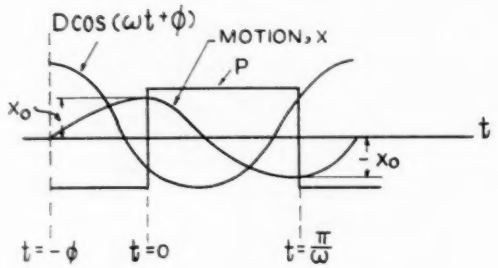


FIG. 14

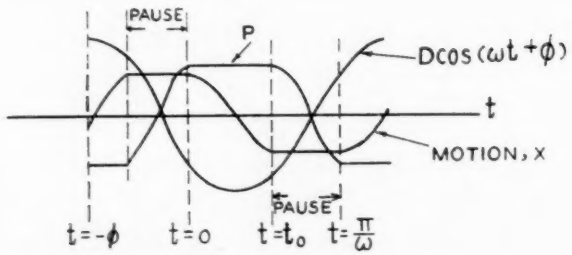


FIG. 15

The design of railway vehicles becomes of increasing importance with every increase in the speed of railway trains because the higher the train speed the greater becomes the effect on vehicle performance from track irregularities. However, in so far as self-sustained oscillations are concerned, it is believed that with proper vehicle design and with good track maintenance, it will be possible, with safety, to appreciably increase train speed above the general level now prevailing.

Appendix 1

Lateral Sliding of Wheels Due to Rolling of Body

The amount of lateral sliding of the wheel base which can be produced by the rolling of the cab may be calculated if the problem is simplified to that shown in Fig. 13. Here a mass m representing the total mass of the vehicle, is resting on a surface which resists any horizontal motion of the mass with a force P. A reciprocating force $D \cos(\omega t + \phi)$, representing the inertia force of the body, is applied to the mass. What is the motion of the mass m for various values of the ratio D/P?

Obviously, if $D/P < 1$ there is no motion at all. If D/P is slightly greater than unity there will be an intermittent motion with a pause during each half cycle. If D/P becomes quite large, the mass can be conceived as having a continuous reciprocating motion with no pauses.

First consider the case where the motion is continuous, Fig. 14. If x is the displacement of the mass m from its mean position, the equilibrium conditions for the mass during its motion in one direction can be expressed as

$$m(d^2x/dt^2) - P = D\cos(\omega t + \phi)$$
$$\text{or, } d^2x/dt^2 = P/m + (D/m)\cos(\omega t + \phi) \ldots [a]$$

Integrating

$$dx/dt = Ft/m + (D/m\omega) \sin(\omega t + \phi) + C' \ldots\ldots [b]$$

Integrating again

$$x = Ft^2/2m - (D/m\omega^2) \cos(\omega t + \phi) + C't + C'' \ldots [c]$$

The following conditions are known: When $t = 0$

$$x = x_0, \text{ and } dx/dt = 0 \ldots [d]$$

and when $t = \pi/\omega$

$$x = -x_0, \text{ and } dx/dt = 0 \ldots [e]$$

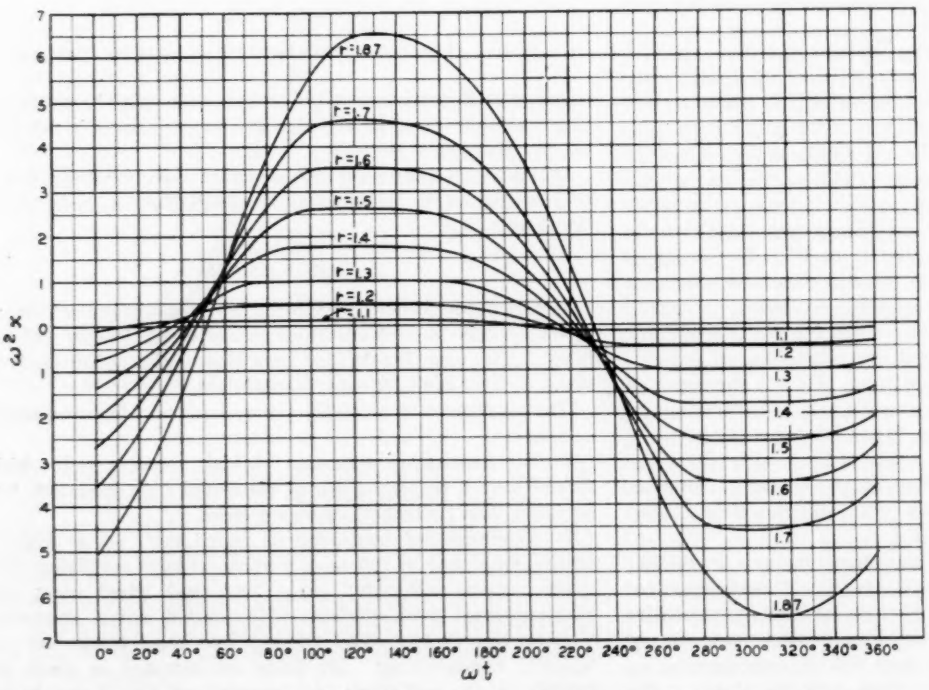


FIG. 16 LATERAL SLIDING OF A MASS (VEHICLE) ON A DRY SURFACE (RAIL) UNDER THE ACTION OF A HARMONIC FORCE (ROLL OF SPRUNG MASS)
(r = maximum exciting force/friction force.)

By using Equation [b] and condition [d], C' may be solved for, whence

$$C' = -(D/m\omega) \sin \phi \quad \ldots\ldots [f]$$

By using Equation [c] and condition [d], C'' may be solved for, whence

$$C'' = x_0 + (D/m\omega^2) \cos \phi \quad \ldots\ldots [g]$$

By using Equations [b] and [f] and condition [e] we can obtain

$$\sin \phi = P\pi/2D \quad \ldots\ldots [h]$$

By using Equations [c], [f], [g], and [h] and condition [e] we may solve for x and obtain the final solution

$$x = \frac{Pt}{2m}\left(t - \frac{\pi}{\omega}\right) + \frac{P}{2m\omega^2}\left(\sqrt{\frac{4D^2}{P^2} - \pi^2}\cos \omega t + \pi \sin \omega t\right) \quad \ldots\ldots [i]$$

Now consider the case where the motion is intermittent, Fig. 15. During motion of the mass in one direction Equations [a], [b], and [c] still hold. However, the end conditions now, when $t = 0$, are

$$x = x_0,\ dx/dt = 0, \text{ and } d^2x/dt^2 = 0 \quad \ldots\ldots [j]$$

and when $t = t_0$

$$x = -x_0, \text{ and } dx/dt = 0 \quad \ldots\ldots [k]$$

We find that our values of C' and C'' are the same as before and are given by Equations [f] and [g]. By using Equation [a] and condition [j] we find that

$$\cos \phi = -P/D \quad \ldots\ldots [l]$$

Now substitute condition [k] into Equation [b], using the known values of the constants for which we have already solved. We obtain

$$\omega t_0 - \sin \omega t_0 + \sqrt{(D^2/P^2) - 1}\,(\cos \omega t_0 - 1) = 0 \quad \ldots [m]$$

Using the same procedure with Equation [c] instead of [b] we obtain

$$x_0 = \frac{t_0}{2m\omega}\sqrt{D^2 - P^2} - \frac{Pt_0^2}{4m} - \frac{P}{2m\omega^2}\left(\cos \omega t_0 + \sqrt{\frac{D^2}{P^2} - 1}\sin \omega t_0 - 1\right) \quad \ldots\ldots [n]$$

The procedure should now be to solve [m] for t_0, substitute the value obtained into Equation [n], and assume that all of our unknown constants had been evaluated. Unfortunately, however, Equation [m] cannot be solved explicitly for t_0, so the best analytical solution we can give is that

$$x = \frac{Pt}{2m}\left(t - \frac{2}{\omega}\sqrt{\frac{D^2}{P^2} - 1}\right) + \frac{P}{m\omega^2}\left(\cos \omega t - 1 + \sqrt{\frac{D^2}{P^2} - 1}\sin \omega t\right) + x_0 \quad \ldots\ldots [o]$$

where x_0 is given by Equations [n] and [m].

For any practical problem, however, the solution can be completed graphically. This has been done for several values of the ratio D/P, and the results are shown in Fig. 16. This chart may be used to find the lateral sliding of the wheel base for any D/P ratio, and no use need be made of the equation given in this Appendix.

It is of interest to know the value of D/P at which the motion changes from intermittent to continuous. This may be found by putting $\omega t_0 = \pi$ in Equation [m]. Solving for D/P we find

$$D/P = (1/2)\sqrt{(\pi^2 + 4)} = 1.87 \quad \ldots\ldots [h]$$

Appendix 2

Values of $\sqrt{[x_i^2 + (s/2)^2]}$ for use in applying Equation [9]

x_i	$\sqrt{x_i^2+(s/2)^2}$	$\sqrt{x_i^2+(s/2)^2} - x_i$	x_i	$\sqrt{x_i^2+(s/2)^2}$	$\sqrt{x_i^2+(s/2)^2} - x_i$	x_i	$\sqrt{x_i^2+(s/2)^2}$	$\sqrt{x_i^2+(s/2)^2} - x_i$	x_i	$\sqrt{x_i^2+(s/2)^2}$	$\sqrt{x_i^2+(s/2)^2} - x_i$
0	29.5	29.5	40	49.7	9.7	80	85.3	5.3	120	123.5	3.5
1	29.5	28.5	41	50.5	9.5	81	86.2	5.2	121	124.5	3.5
2	29.6	27.6	42	51.3	9.3	82	87.1	5.1	122	125.5	3.5
3	29.7	26.7	43	52.2	9.2	83	88.1	5.1	123	126.5	3.5
4	29.8	25.8	44	53.0	9.0	84	89.0	5.0	124	127.5	3.5
5	29.9	24.9	45	53.8	8.8	85	90.0	5.0	125	128.4	3.4
6	30.1	24.1	46	54.7	8.7	86	90.9	4.9	126	129.4	3.4
7	30.3	23.3	47	55.5	8.5	87	91.9	4.9	127	130.4	3.4
8	30.6	22.6	48	56.3	8.3	88	92.8	4.8	128	131.4	3.4
9	30.8	21.8	49	57.2	8.2	89	93.7	4.7	129	132.3	3.3
10	31.2	21.2	50	58.1	8.1	90	94.7	4.7	130	133.3	3.3
11	31.5	20.5	51	58.9	7.9	91	95.7	4.7	131	134.3	3.3
12	31.9	19.9	52	59.8	7.8	92	96.6	4.6	132	135.3	3.3
13	32.2	19.2	53	60.7	7.7	93	97.6	4.6	133	136.2	3.2
14	32.7	18.7	54	61.5	7.5	94	98.5	4.5	134	137.2	3.2
15	33.1	18.1	55	62.4	7.4	95	99.5	4.5	135	138.2	3.2
16	33.6	17.6	56	63.3	7.3	96	100.4	4.4	136	139.1	3.1
17	34.1	17.1	57	64.2	7.2	97	101.4	4.4	137	140.1	3.1
18	34.6	16.6	58	65.1	7.1	98	102.3	4.3	138	141.1	3.1
19	35.1	16.1	59	66.0	7.0	99	103.3	4.3	139	142.1	3.1
20	35.6	15.6	60	66.9	6.9	100	104.3	4.3	140	143.1	3.1
21	36.2	15.2	61	67.8	6.8	101	105.2	4.2	141	144.1	3.1
22	36.8	14.8	62	68.7	6.7	102	106.1	4.1	142	145.0	3.0
23	37.4	14.4	63	69.6	6.6	103	107.1	4.1	143	146.0	3.0
24	38.0	14.0	64	70.5	6.5	104	108.1	4.1	144	147.0	3.0
25	38.7	13.7	65	71.4	6.4	105	109.0	4.0	145	148.0	3.0
26	39.3	13.3	66	72.3	6.3	106	110.0	4.0	146	149.0	3.0
27	40.0	13.0	67	73.2	6.2	107	111.0	4.0	147	149.9	2.9
28	40.7	12.7	68	74.1	6.1	108	112.0	4.0	148	150.9	2.9
29	41.4	12.4	69	75.0	6.0	109	112.9	3.9	149	151.9	2.9
30	42.0	12.0	70	76.0	6.0	110	113.9	3.9	150	152.9	2.9
31	42.8	11.8	71	76.9	5.9	111	114.9	3.9	151	153.9	2.9
32	43.5	11.5	72	77.8	5.8	112	115.8	3.8	152	154.8	2.8
33	44.3	11.3	73	78.7	5.7	113	116.8	3.8	153	155.8	2.8
34	45.0	11.0	74	79.7	5.7	114	117.7	3.7	154	156.8	2.8
35	45.8	10.8	75	80.6	5.6	115	118.6	3.6	155	157.8	2.8
36	46.5	10.5	76	81.5	5.5	116	119.6	3.6	156	158.8	2.8
37	47.3	10.3	77	82.5	5.5	117	120.5	3.5	157	159.7	2.7
38	48.1	10.1	78	83.4	5.4	118	121.5	3.5	158	160.7	2.7
39	48.9	9.9	79	84.3	5.3	119	122.5	3.5	159	161.7	2.7

x_i	$\sqrt{x^2_i + (s/2)^2}$	$\sqrt{x^2_i + (s/2)^2} - x_i$	x_i	$\sqrt{x^2_i + (s/2)^2}$	$\sqrt{x^2_i + (s/2)^2} - x_i$	x_i	$\sqrt{x^2_i + (s/2)^2}$	$\sqrt{x^2_i + (s/2)^2} - x_i$	x_i	$\sqrt{x^2_i + (s/2)^2}$	$\sqrt{x^2_i + (s/2)^2} - x_i$
160	162.7	2.7	212	214.0	2.0	264	265.6	1.7	316	317.4	1.4
161	163.7	2.7	213	215.0	2.0	265	266.6	1.7	317	318.4	1.4
162	164.7	2.7	214	216.0	2.0	266	267.6	1.7	318	319.4	1.4
163	165.6	2.6	215	217.0	2.0	267	268.6	1.7	319	320.4	1.4
164	166.6	2.6	216	218.0	2.0	268	269.6	1.6	320	321.4	1.4
165	167.6	2.6	217	219.0	2.0	269	270.6	1.6	321	322.4	1.4
166	168.6	2.6	218	220.0	2.0	270	271.6	1.6	322	323.3	1.3
167	169.6	2.6	219	221.0	2.0	271	272.6	1.6	323	324.3	1.3
168	170.6	2.6	220	222.0	2.0	272	273.6	1.6	324	325.3	1.3
169	171.6	2.6	221	223.0	2.0	273	274.6	1.6	325	326.3	1.3
170	172.5	2.5	222	224.0	2.0	274	275.6	1.6	326	327.3	1.3
171	173.5	2.5	223	224.9	1.9	275	276.6	1.6	327	328.3	1.3
172	174.5	2.5	224	225.9	1.9	276	277.6	1.6	328	329.3	1.3
173	175.5	2.5	225	226.9	1.9	277	278.6	1.6	329	330.3	1.3
174	176.5	2.5	226	227.9	1.9	278	279.6	1.6	330	331.3	1.3
175	177.5	2.5	227	228.9	1.9	279	280.6	1.6	331	332.3	1.3
176	178.5	2.5	228	229.9	1.9	280	281.6	1.6	332	333.3	1.3
177	179.4	2.4	229	230.9	1.9	281	282.5	1.5	333	334.3	1.3
178	180.4	2.4	230	231.9	1.9	282	283.5	1.5	334	335.3	1.3
179	181.4	2.4	231	232.9	1.9	283	284.5	1.5	335	336.3	1.3
180	182.4	2.4	232	233.9	1.9	284	285.5	1.5	336	337.3	1.3
181	183.4	2.4	233	234.9	1.9	285	286.5	1.5	337	338.3	1.3
182	184.4	2.4	234	235.9	1.9	286	287.5	1.5	338	339.3	1.3
183	185.4	2.4	235	236.8	1.8	287	288.5	1.5	339	340.3	1.3
184	186.4	2.4	236	237.8	1.8	288	289.5	1.5	340	341.3	1.3
185	187.3	2.3	237	238.8	1.8	289	290.5	1.5	341	342.3	1.3
186	188.3	2.3	238	239.8	1.8	290	291.5	1.5	342	343.3	1.3
187	189.3	2.3	239	240.8	1.8	291	292.5	1.5	343	344.3	1.3
188	190.3	2.3	240	241.8	1.8	292	293.5	1.5	344	345.3	1.3
189	191.3	2.3	241	242.8	1.8	293	294.5	1.5	345	346.3	1.3
190	192.3	2.3	242	243.8	1.8	294	295.5	1.5	346	347.3	1.3
191	193.3	2.3	243	244.8	1.8	295	296.5	1.5	347	348.3	1.3
192	194.3	2.3	244	245 8	1.8	296	297.5	1.5	348	349.3	1.2
193	195.2	2.2	245	246.8	1.8	297	298.5	1.5	349	350.2	1.2
194	196.2	2.2	246	247.8	1.8	298	299.5	1.5	350	351.2	1.2
195	197.2	2.2	247	248.8	1.8	299	300.5	1.5	360	361.2	1.2
196	198.2	2.2	248	249.8	1.8	300	301.5	1.5	370	371.1	1.1
197	199.2	2.2	249	250.7	1.7	301	302.4	1.4	380	380.1	1.1
198	200.2	2.2	250	251.7	1.7	302	303.4	1.4	390	391.1	1.1
199	201.2	2.2	251	252.7	1.7	303	304.4	1.4	400	401.0	1.0
200	202.2	2.2	252	253.7	1.7	304	305.4	1.4	425	426.0	1.0
201	203.2	2.2	253	254.7	1.7	305	306.4	1.4	450	450.9	0.9
202	204.1	2.1	254	255.7	1.7	306	307.4	1.4	475	475.9	0.9
203	205.1	2.1	255	256.7	1.7	307	308.4	1.4	500	500.8	0.8
204	206.1	2.1	256	257.7	1.7	308	309.4	1.4	550	550.8	0.8
205	207.1	2.1	257	258.7	1.7	309	310.4	1.4	600	600.7	0.7
206	208.1	2.1	258	259.7	1.7	310	311.4	1.4	700	700.6	0.6
207	209.1	2.1	259	260.7	1.7	311	312.4	1.4	800	800.5	0.5
208	210.1	2.1	260	261.7	1.7	312	313.4	1.4	900	900.4	0.4
209	211.1	2.1	261	262.7	1.7	313	314.4	1.4	1000	1000.4	0.4
210	212.1	2.1	262	263.7	1.7	314	315.4	1.4	....	.....	...
211	213.1	2.1	263	264.7	1.7	315	316.4	1.4	....	.....	...

Appendix 3

Effect of Nonsinusoidal Lateral Motion of Wheel Base

The theory described in this paper assumes that all displacements, velocities, and accelerations vary harmonically. This assumption is reasonable for the motions of the sprung weight on its supporting springs, since the scales of the springs are linear and the predominating forces are spring forces and inertia forces. Also, accelerometer records taken in the cabs of oscillating locomotives appear sinusoidal, which shows that the velocities and displacements must have been very close to following pure sine waves. The unsprung mass, however, has its lateral motion resisted partly by friction and partly by flange pressure, the latter coming into play only near the maximum displacement positions. Therefore, it cannot be assumed that the lateral motions of the unsprung mass will be harmonic. Let us determine the effect that nonharmonic motion will have on the theory.

At the critical speed, the energy input per cycle is

$$4\int_0^{\pi/2\omega} 4\mu QV\alpha dt = 16\mu QV\int_0^{\pi/2\omega} \alpha dt \ldots\ldots [a]$$

The energy loss per cycle is

$$8BA\mu Q \ldots\ldots [b]$$

Equating input to loss and solving for V, we obtain

$$V_{cr} = BA/2\int_0^{\pi/2\omega} \alpha dt \ldots\ldots [c]$$

The determination of the critical speed is thus dependent on the determination of an expression for α as a function of t. If, as a first approximation we take $\alpha = A\cos\omega t$, as done before, we obtain

$$V_{cr} = 2B\omega/\pi \ldots\ldots [d]$$

Consider that α is some function other than $A\cos\omega t$. The turning action on the vehicle comes from the flanges, which act during only a portion of each cycle. Therefore, the angularity may change rather quickly, remain constant, and then change back again. The angularity α plotted against the time t Fig. 17 will be a sine wave flattened at the peaks. The limit of such a tendency is the square wave form, Fig. 17, which could not exist, but is a limit which could be approached. It represents the most unfavorable condition because it produces the largest possible value of input.

If the wave form of α were square, then α would be constant and equal to $\pm A$ during each half cycle. Substituting A for α in Equation [c] of this Appendix, we obtain

$$V_{cr} = B\omega/2 \ldots\ldots [e]$$

If the wave form of the variation of α were sharper at the peaks than a sine wave, then the input would be smaller and the critical speed would be higher. However, this is too favorable an assumption. If the tendency exists at all it comes as a result of the speed and flange pressure being too small to turn the locomotive fast enough. High speeds and severe oscillations both tend to produce the square rather than the pointed wave form.

As a result of the foregoing analysis we find that the critical-speed theory advanced in this paper no longer depends on the

assumption of sinusoidal motion. If the motion of the wheel base is not sinusoidal, the critical speed may be reduced by the factor $\pi/4$ or 0.785 as the limit.

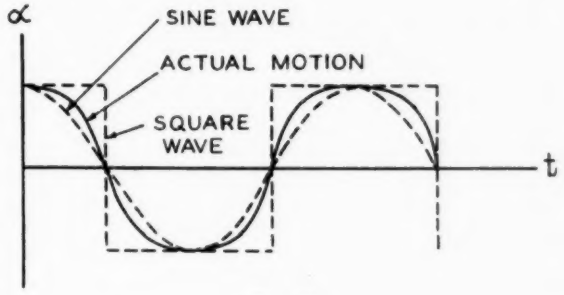


FIG. 17

Appendix 4

NOMENCLATURE AND IMPORTANT EQUATIONS

α = the variable angle between the center line of the vehicle and the center line of the track
A = the peak value of α during a cycle of the oscillation
B = the wheel base of a vehicle
b = the lateral distance between the springs supporting the sprung weight of a rail vehicle
C' and C'' = constants of integration
c = track damping constant
D = disturbing force (see Appendix 1)
d = lateral displacement of the center of gravity of the sprung weight relative to the unsprung weight due to roll
f = natural frequency of roll of a vehicle
G = the lateral component of the friction force at the tread which opposes rotation of the wheel base
h = height of the center of gravity of the sprung mass above the center line of the axle
I_r = the polar moment of inertia of the sprung weight about a longitudinal axis through the axle centers
J = the polar moment of inertia of a vehicle about a vertical axis through the center of gravity
i = a general subscript designating the i-th axle of a wheel base
K_s = the spring scale for one side of a vehicle, neglecting cross-equalized springs
K = the lateral spring scale of a vehicle
k = axle load/average axle load
M = the mass of the sprung weight of a vehicle
μ = the coefficient of friction
n = the number of axles in a wheel base
ϕ = the phase angle between the lateral slippage of the wheel base and the rolling of the sprung weight (see Appendix 1)
P = the frictional resistance to the lateral slippage of the wheel base (see Appendix 1)
Q = static wheel load
s = track gage
δ = half the maximum lateral amplitude of the wheel base allowed by the track and bearing clearances
θ_i = phase angle of motion of i-th axle referred to phase of center of rotation
t = time
V = the forward velocity of the vehicle
ω = 2π times the frequency of the oscillation
W = the sprung weight of a vehicle
x = lateral slippage of wheel base due to roll of body (see Appendix 1)
x_i = the longitudinal distance between the i-th axle and the center of rotation
x_1 = same for the first axle
Y = the flange pressure on an axle
y = lateral translation of a vehicle.

Natural frequency of roll

$$f = [1/2\pi]\ \sqrt{[(K_s b^2 - 2Wh)/2I_r]} \quad \ldots\ldots [10]$$

Angularity of wheel base

$$A = \delta\omega \cos\theta_1/V = \delta\omega/\sqrt{(x_1^2\omega^2 + V^2)} \quad \ldots [14] \text{ and } [16]$$

Amplitude of lateral translation of wheel base

$$2y = 2VA/\omega \quad \ldots\ldots [11]$$

Total amplitude of the i-th axle

$$= 2VA/\omega \cos\theta_i \quad \ldots\ldots [12]$$

Input per cycle

$$= 2\pi n\mu QV\delta/\sqrt{(x_1^2\omega^2 + V^2)} \quad \ldots\ldots [18]$$

Loss per cycle

$$= [8\mu Q\omega\delta/\sqrt{(x_1^2\omega^2 + V^2)}] \sum_{i=n}^{i=1} \sqrt{[x_i^2 + (s/2)^2]} \quad \ldots [19]$$

Critical speed

$$V_{cr} = [8f/n] \sum_{i=n}^{i=1} \sqrt{[x_{i1}^2 + (s/2)^2]} \quad \ldots\ldots [9]$$

Critical speed for nonuniform axle load

$$V_{cr} = 8f\left[\frac{k_1\sqrt{[x_1^2 + (s/2)^2]} + k_2\sqrt{[x_2^2 + (s/2)^2]} + \ldots}{k_1 + k_2 + \ldots\ldots}\right] \ldots [20]$$

Measurement of Steam Rate and Indicated Horsepower of Locomotives

By ARTHUR WILLIAMS,[1] EAST CHICAGO, IND.

It has been the customary procedure to determine the steam rate and indicated horsepower of locomotives and other reciprocating steam engines from indicator cards. This paper points out that particularly at high locomotive speeds, the obtainment of indicator cards is subject to gross errors, and that the expense of rigging up for such tests is hardly warranted by the results that can be expected. The paper discusses in detail another method of determining the steam rate and indicated horsepower of a locomotive, which other writers have referred to as the "heat-drop method." The paper proves both the simplicity of the method as well as its relative accuracy. The method is applicable to such locomotives only in which the exhaust steam is at least dry, and only in the absence of moisture, the Btu contents per pound of both admitted and exhausted steam can be accurately determined through pressure and temperature measurement. The paper contains detailed suggestions for successfully applying the method on the locomotive. Thus it contains data on the construction and location of the thermocouples, and the manner in which these thermocouples are applied and wired up to the instrument with which the pyroelectric force is determined. The results of the tests carried out with this method on two different locomotives, are presented in the paper and, in general, it is shown what advantages this simple method has for the test and analysis of all those design factors or devices which are liable to affect the steam rate of a locomotive.

Toward the conclusion of the paper, reference is made to a suggestion by L. K. Botteron, which appeared in the *Railway Mechanical Engineer* of July, 1930, to the effect that the exhaust nozzle of a locomotive could be considered a flowmeter nozzle for the determination of the quantity of steam that the locomotive exhausts. The paper proceeds to show how the specific volume and velocity of the steam, while passing through the mouth of the locomotive exhaust tip (which quantities are essential for the Botteron method), are derived from the exhaust-steam temperature and pressure measurements that are incidental to the carrying out of the heat-drop determination.

THE overall efficiency of a locomotive may be divided into three parts: The engine or cylinder efficiency, the boiler-combustion efficiency, and the machine efficiency. A measure of the cylinder efficiency is the steam rate, or steam consumption per indicated horsepower-hour. In the past this has been determined by measuring the indicated horsepower, and the steam to the engines, and dividing the second by the first. It is somewhat difficult on a road test to determine these values accurately. The indicated horsepower can be calculated when the mean effective pressure in the cylinders and the speed are known. The mean effective pressure in the cylinders is found by means of an indicator. The accuracy of the indicator depends in the first place upon the accuracy of the reducing motion from the crosshead to the indicator drum, the pencil motion on the indicator, the spring, and the area of the piston. With careful workmanship and calibration these errors can be made fairly small. It is more difficult, particularly at high speeds, to guard against errors due to the inertia of the moving parts and vibration of the apparatus.

Having obtained the mean effective pressure it is necessary to know the speed in order to calculate the indicated horsepower. A speed indicator is again a source of error, and if speeds are obtained by taking the time between mileposts the speed at the time at which the indicator card is taken may be different from the average time between mileposts.

The next step in the calculation of the steam rate is to measure the steam to the engines. This is usually done by measuring the water fed to the boiler from the tender tank and subtracting the steam to the auxiliaries, such as the air compressor, boiler feed pump, stoker, steam used for heating the train, or for other train services, and the steam wasted from the safety valve, injector overflow, and blower. Errors arise, of course, in the measurement or calculation of each of these quantities.

Since the accurate measurement of the steam rate depends upon the accurate measurement of the mean effective pressure, speed, boiler evaporation, steam to auxiliaries, and steam wasted, it can readily be seen that the correct determination of the steam rate on a road test, using the test procedure outlined, is difficult. The suggestion has been made[2] that the steam rate be determined by means of the observation of the difference in heat content of the steam in the steam pipe and the exhaust. It is the purpose of this paper to show that, with reasonable precautions this can be done on most locomotives with good accuracy and far less trouble than by using the indicator.

Theory of Measurement

The first law of thermodynamics states that "heat and mechanical energy are interconvertible and can neither be created nor destroyed. For steady-flow conditions, such as are obtained with an engine, it follows that "For any prime mover operating under these conditions the energy delivered by this apparatus in any unit of time is equal to the difference of the heat contents at entrance and exit from the apparatus, for the entire amount of

---

[1] Research Engineer, The Superheater Company. Mr. Williams was graduated from Queen's College, Taunton, England. He was with the Great Western Railway, England, for four years and for two years with the Franklin Railway Supply Company. He has been associated with The Superheater Company since 1928.

Contributed by the Railroad Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "Some Experimental Results From a Three-Cylinder Compound Locomotive," by Lawford H. Fry, Proceedings, Institution of Mechanical Engineers, 1927, vol. 2, pp. 923–1024.

"A Thermodynamic Analysis of the Steady Flow of Fluids," by C. H. Berry, *Mechanical Engineering*, vol. 51, 1929, p. 816.

working substance flowing in this unit of time, minus the radiation and conduction losses from the apparatus."[3]

With superheated steam, if the temperature and pressure are known, the heat content can be obtained from steam tables. With saturated steam it is necessary to know the percentage of moisture. To measure the temperature of the exhaust steam from locomotive cylinders, when it is superheated, is simple. To measure the amount of moisture in the exhaust steam accurately, when it is saturated, is difficult, if not impossible. Accordingly, the method described in this paper is limited to those cases where there is superheat in the exhaust steam. Since most locomotives in main-line service have some superheat in their exhaust, this limitation is not very important. It is one, however, that must always be borne in mind.

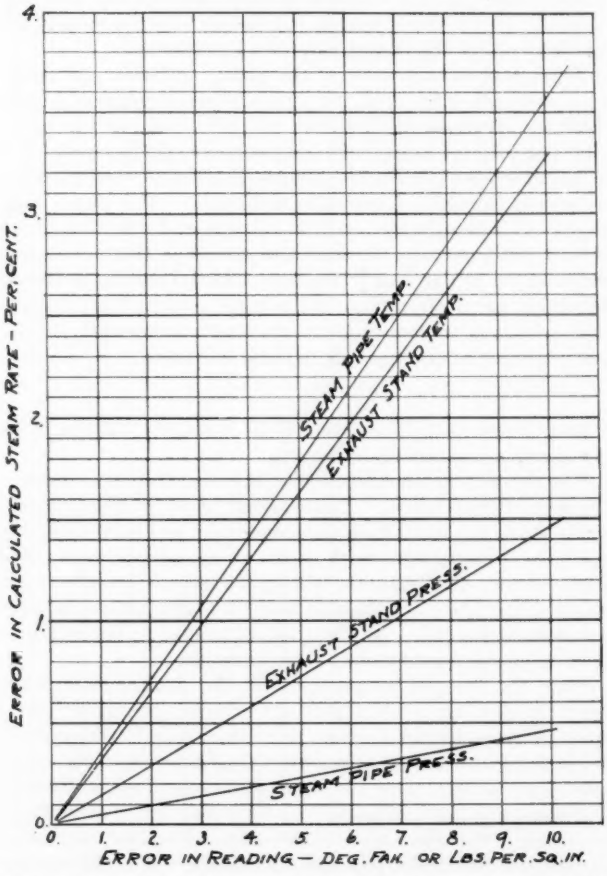


FIG. 1 EFFECT OF ERRORS IN READINGS ON CALCULATED STEAM RATE

Knowing the temperature and pressure of the admitted steam and of the exhaust steam, the two heat contents can be obtained from steam tables. The difference between the two gives the heat drop in Btu per pound of steam. This heat drop is equal to the work done in the cylinders per pound of steam, plus the heat lost through radiation. The radiation loss is relatively small and, if desired, can be allowed for. One horsepower-hour is equal to 2545 Btu. Dividing 2545 by the heat drop per pound of steam will give directly the pounds of steam per indicated horsepower hour. It makes no difference what processes are taking place in the cylinder, whether they are adiabatic, isothermal, or, as is actually the case, a turbulent process which does not follow any definite law. In the extreme case, where steam is leaking by the piston, with no work being done, the heat content would be the same in the steam pipe and in the exhaust, with due allowance for radiation. This case is the same as that of the well-known throttling calorimeter.

[3] "Heat-Power Engineering," Part 1, Thermodynamics and Prime Movers, by W. N. Barnard, F. O. Ellenwood, and C. F. Hirshfeld, John Wiley & Sons, Inc., New York, 1926, p. 46.

VALUE OF METHOD FOR LOCOMOTIVE TESTS

Since it is only necessary to measure the temperature and pressure in the steam pipe and in the exhaust of the locomotive cylinders, the test apparatus is relatively simple. It is possible for one man to ride in the cab of a locomotive and take all of the necessary readings. This may be contrasted with the test apparatus and test crew necessary for determining the steam rate using indicators.

It is obvious that where a complete test by taking indicator cards would not be justified, a test using the heat-drop method can be run with a low expenditure, and the various design features which influence cylinder performance can be studied as often as desired.

The following are the design features commonly in question:

(1) Boiler pressure, since it controls the steam pressure in the steam pipe.

(2) The throttle and dry pipe. The pressure drop through these affects the pressure in the steam pipe.

(3) The steam temperature in the steam pipe and superheater design.

(4) The size and design of the valves and cylinders.

(5) The valve setting.

(6) The exhaust nozzle, both size and shape.

The necessary readings for obtaining the steam rate can be taken at frequent intervals, so that a complete picture is easily obtained of the locomotive performance, at various speeds and rates of working. Even when tests are being made on the combustion efficiency of, and heat transfer in, the boiler, it is desirable to measure the steam rate and cylinder efficiency, so that the results may be more closely analyzed.

METHODS OF MEASUREMENT AND SOURCES OF ERROR

The instruments used in measuring the temperature and pressure should be as accurate as possible and suitable for use in road tests on locomotives. In order to obtain a better understanding of the relation between errors and accuracy, Fig. 1 has been prepared. In this figure the four curves show the effect of errors in measurement of the steam-pipe temperature, steam-pipe pressure, exhaust-steam temperature, and exhaust-steam pressure, on the steam rate. An error of 2 lb per sq in. in measuring the steam-pipe pressure will cause an error of 0.1 per cent in the calculated steam rate. An error of 1 lb per sq in. in the measurement of the exhaust pressure will cause an error of 0.14 per cent in the steam rate. To measure the steam-pipe pressure to within 2 lb per sq in. and the exhaust pressure to within 1 lb per sq in. only calls for reasonably accurate test gages. It may be necessary to make a correction for the hydrostatic head, due to the water in the pipe connecting the pressure gage and the point where it is tapped into the steam pipe or exhaust pipe.

The measurement of the steam pipe and exhaust-steam temperatures calls for more careful consideration. If they can be measured with a maximum error in each of 1 F, the resulting error in the steam rate will be 0.68 per cent. If they are measured with a maximum error in each of 2 F, the error in the steam rate will be 1.35 per cent. Adding to these figures the errors due to the steam-pressure measurements gives a total error of 0.92 per cent if the temperatures are measured to within 1 F and 1.59 per cent if the steam temperatures are measured to within 2 F.

It is possible to use instruments suitable for road tests on locomotives that will give results which will average somewhere between these two figures.

The apparatus described in this paper has been used by the author with good results. This statement is not meant to infer that this is the only apparatus suitable for measuring temperatures to within 1 or 2 F. Other means of temperature measurement will be mentioned later.

The temperatures are measured with thermocouples constructed as shown in Fig. 2. Wires used are iron and constantan, purchased in lengths. Samples of each length are calibrated, with reference to a thermometer which has been certificated by the Bureau of Standards. The wires are led through porcelain insulators to a steel plug welded in the end of a piece of $^1/_4$-in. pipe. The wires are brought through small holes drilled in the steel plug, bent over, and the ends then covered with silver solder. This gives a thermocouple which is tight against the steam pressure and will follow any change in steam temperature rapidly. The thermocouple is screwed into a socket, which, in turn, is screwed into the steam pipe. This gives a steam space surrounding the thermocouple all the way up to the thermocouple head, which serves to minimize any errors due to conduction along the thermocouple pipe. The steam-pipe thermocouple, illustrated in Fig. 2, is screwed into the steam pipe in any convenient place.

The exhaust-steam temperature is measured, not in the exhaust passage, but in the exhaust stand. It has been the common practice in the past to measure the exhaust-steam temperature in the exhaust passage close to the steam chest. This has been standard practice on the Pennsylvania Railroad in their tests, at Altoona, in their locomotive test plant. It is stated in sev-

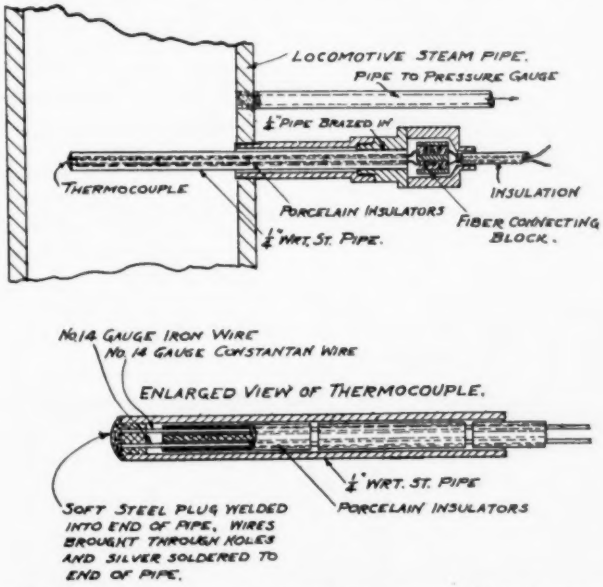


FIG. 2 STEAM-PIPE THERMOCOUPLE AND PRESSURE-GAGE CONNECTION

eral of the railroad's publications, that the exhaust-steam temperature measurements are believed to be higher than the true temperatures. The reason given is as follows:

> During admission of the steam into the cylinder, heat is transferred from the steam to the cylinder walls and head. As the steam expands and becomes cooler this heat transfer stops and then reverses, so that during the exhaust stroke heat is being transferred from the cylinder to the exhaust steam. After release, when the pressure in the cylinder is higher than the average exhaust pressure, there occurs a sudden rush of steam, which by reason of its high velocity does not have time to absorb very much heat from the cylinder walls. As the piston moves on the exhaust stroke the steam, which is now moving more slowly, is heated up by the cylinder walls, with a rise in temperature.

The Pennsylvania Railroad states that it believes the sudden rush of relatively cool steam at release does not register properly on the thermometer in the exhaust passage and that this thermometer is influenced more by the slow-velocity high-temperature steam during the exhaust stroke.

In measuring any fluctuating steam temperature it is desirable to provide for as much mixing of the steam as possible. Accord-

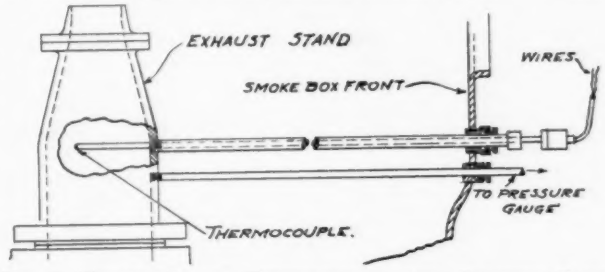


FIG. 3 EXHAUST-STAND THERMOCOUPLE AND PRESSURE-GAGE CONNECTION

ingly, the exhaust-steam thermocouple is located in the exhaust stand so that the steam passes through a chamber of some volume and with several bends before the temperature is measured. Also a thermocouple applied in one side of the exhaust stand will measure the temperature of the exhaust from both ends of one side of the engine, and thermocouples applied in both sides of the exhaust stand will measure the exhaust-steam temperatures from all four ends of the locomotive cylinders.

The accuracy of the assumption that better temperature measurements could be made in the exhaust stand than in the exhaust passage was tested on a locomotive in main-line service. On this particular test an observer was riding on the front of the locomotive and simultaneous readings were taken of the exhaust-steam temperature with a thermometer in the exhaust passage close to the steam chest, and a thermocouple in the exhaust stand. The average of a number of readings showed the observed exhaust-passage temperature, as indicated by the thermometer, to be 345 F and the observed exhaust-stand temperature, as indicated by the thermocouple, to be 325 F. In other words, if the exhaust-stand temperature is taken as being accurate the temperature measured in the exhaust passage close to the steam chest was reading 20 deg high. This figure would vary no doubt with the class of engine and conditions of working, but it is evident that measurements taken of the exhaust-steam temperature in the exhaust passage close to the steam chest are subject to considerable error.

The application of the thermocouple to the exhaust stand is shown in Fig. 3. The thermocouple is constructed in the same way as that shown in Fig. 2. A $^1/_2$-in. standard pipe is screwed in the exhaust stand and brought out through the smokebox front. The thermocouple is slid into this $^1/_2$-in. pipe, which provides an insulating steam jacket and also permits easy removal.

The thermocouple electromotive force is measured with a Leeds and Northrup potentiometer. The potentiometer is a well-known means of measuring temperatures in connection with thermocouples, and is suitable for locomotive testing. Because of severe vibration it is always possible to obtain a broken strand in one of the thermocouple wires, or a bad contact. With a direct-reading pyrometer this would throw the instrument off but with a potentiometer it would make no difference, as long as the wiring was good enough to provide a circuit.

With the galvanometer mounted in the potentiometer case as sent out by the makers, the vibration of the locomotive is sufficient to upset the galvanometer needle. In order to obviate this difficulty, the galvanometer is mounted on a board with a spring suspension as shown in Fig. 4. This board also serves for writing down the data and is held by the observer. The observer's body will absorb most of the shocks and vibration of the locomotive, and with the spring mounting of the galvanometer no trouble will be found in taking readings. The potentiometer is mounted on a bracket and the galvanometer wired to

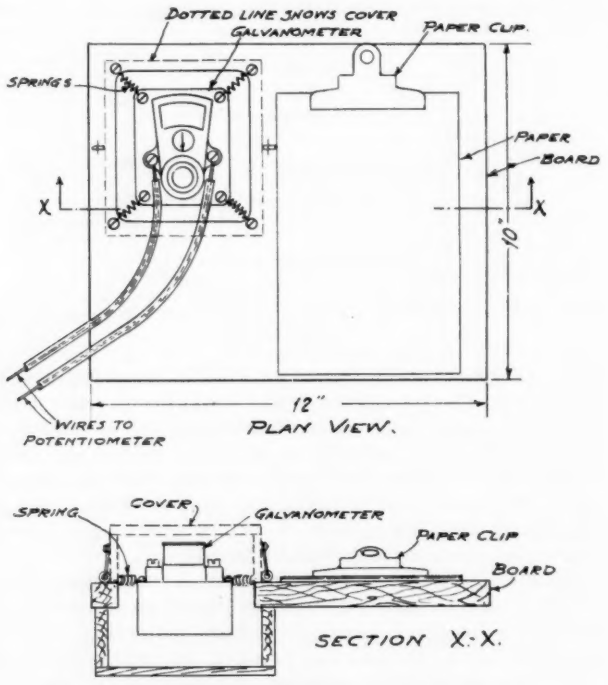


FIG. 4 GALVANOMETER MOUNTING OF DATA BOARD

the potentiometer by means of two copper wires of sufficient length to allow freedom of movement to the observer.

The thermocouples are connected to a rotary switch in the cab by means of which each one in turn may be put into the potentiometer circuit. The wiring diagram is shown in Fig. 5. A thermocouple measures the difference between the temperatures of the hot and cold junctions and consequently it is necessary to know the temperature of the cold junction and to connect the hot and cold junctions in such a way that no other electromotive force is set up. As in Fig. 5, the thermocouples are connected with special extension leads to a connecting strip which is located at some convenient spot near the front of the locomotive. These extension leads are of the same material as the thermocouple wire, so that no electromotive force is set up at their junction. This connecting strip is mounted in a closed metal box, so that all the junctions will be at the same temperature. From the connecting strip copper leads are run to the rotary switch in the cab. The common wires from the rotary switch, which are connected to each thermocouple in turn, are connected with copper leads forward to the connecting strip and from there connected with the special extension leads back to the cab. The ends of these leads are joined to copper wire, which is connected to the potentiometer. This junction of the extension lead and the copper wire forms the cold junction and is placed in a piece of $^3/_8$ in. pipe. The thermometer and cold junction will be at the same temperature and will not be affected by stray air currents. As many thermocouples as desired can be wired with extension leads to the connecting strip at the front of the engine and the single extension lead going back to the cold junction will automatically refer the cold junction from the connecting strip to the cab.

For best accuracy in measuring the steam rate it is desirable to have thermocouples in both steam pipes and both sides of the exhaust stand. The temperatures should not vary much from side to side, but with the two sets of thermocouples variations can be seen and a true average obtained.

Instead of using the thermocouples the temperatures can

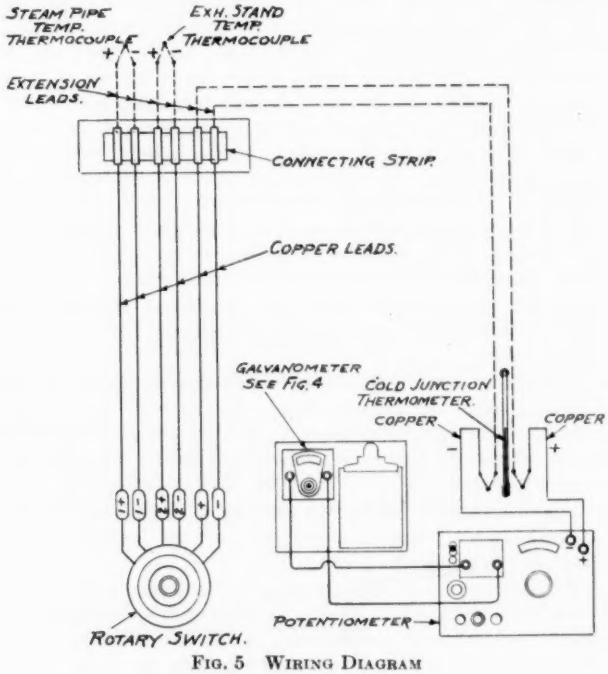


FIG. 5 WIRING DIAGRAM

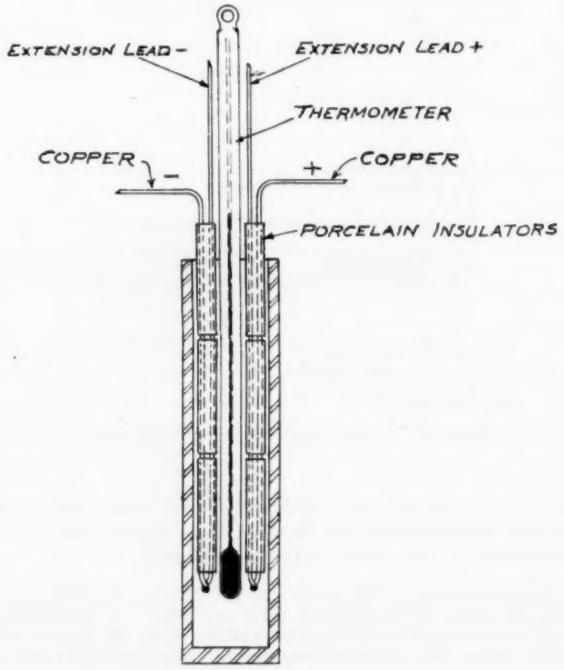


FIG. 6 COLD JUNCTION

be measured with even greater accuracy by using platinum-resistance thermometers.

It has already been pointed out that the heat drop from the steam pipe to the exhaust includes radiation from the steam pipes, steam chests, cylinders, and exhaust passages. For a typical Pacific-type locomotive, this radiation loss has been calculated to be approximately 0.42 per cent of the heat drop with the engine standing still and 0.83 per cent of the heat drop with the engine running at 100 mph. This radiation loss can be allowed for, but it is obvious that inaccuracies in its determination will be negligible in its effect on the calculation of the steam rate.

Road Tests of Locomotives

Table 1 shows the results obtained on road tests of a 2-8-2-type locomotive in fast freight service. The object of the test was to determine the effect on the engine performance of a special design of superheater unit. Tests were first run with the standard type-*A* superheater in place. The special superheater was then installed in place of the type-*A* and further tests were run. No attempt was made to obtain special conditions with reference to the running. The engine was in regular service, pulling whatever trains were assigned to it with various engine crews. The locomotive would run for half a division, then take on water, and then finish the run to the end of the division. Each test, shown in Table 1, is the average of readings taken every two miles for about 40 to 50 miles, representing about half a division. After starting, several miles were allowed before readings were commenced, in order that conditions might become reasonably constant. A caboose was carried back of the locomotive as a test car. The thermocouples were all wired to the caboose, where the temperatures were read by an observer with the potentiometer and rotary switch. The wiring connection between the caboose and tender was arranged to be easily connected and disconnected. Temperatures were measured on the left side of the engine only.

TABLE 1 TEST OF 2-8-2-TYPE LOCOMOTIVE

Test no.	Tonnage	Speed mph	Throttle opening	Cutoff, per cent	Steam pressure: Steam pipe, lb per sq in.	Steam pressure: Exhaust, lb per sq in.	Temperature: Steam pipe, F	Temperature: Exhaust steam, F	Steam rate, lb per ihp-hr
1	3779	30.7	Full	44	149.9	11.2	641	318	17.59
2	3879	24.0	Full	48	149.0	12.6	639	327	18.13
3	2016	42.4	Full	40	147.7	12.5	597	291	18.62
4	3000	28.8	Full	49	145.7	12.2	644	325	17.80
5	3532	27.8	90%	48	144.2	11.4	628	318	18.34
6	2690	32.0	Full	43	142.5	12.7	635	325	18.21
7	2629	34.3	Full	42	142.9	12.5	633	320	18.09
8	2699	30.6	Full	46	149.2	13.0	625	314	18.41
9	2819	29.3	Full	44	144.4	13.3	621	306	18.22
10	3760	27.6	Full	45	154.8	12.6	641	318	17.58
11	4200	27.3	Full	46	155.7	12.6	632	306	17.40
12	3476	33.3	90%	42	147.3	12.5	642	327	17.93
Avg	3207	30.7	Full	45	148	12.4	632	316	18.03
13	3637	29.8	Full	49	159.2	13.0	707	357	15.80
14	3718	28.5	Full	49	157.7	12.8	724	365	15.39
15	3680	33.4	Full	47	165.5	9.7	710	354	15.68
16	3701	30.8	Full	47	162.7	12.0	683	348	16.69
17	2966	28.1	Full	49	164.0	13.0	716	367	15.87
18	2623	31.9	Full	47	162.3	14.2	705	379	16.97
19	2660	38.0	Full	47	156.7	14.3	692	352	16.40
20	3067	37.2	Full	45	164.7	11.6	689	334	15.84
21	3789	31.4	Full	47	161.1	12.4	680	347	16.92
22	....	32.0	Full	45	160.4	11.2	677	357	17.53
23	....	36.9	Full	43	163.7	10.0	666	347	17.71
24	....	37.6	Full	47	159.8	14.5	718	395	17.21
25	....	29.8	Full	45	159.4	12.4	712	375	16.48
26	2580	36.6	Full	46	157.0	14.1	680	362	17.62
27	3670	26.3	Full	55	156.8	14.0	690	367	17.21
28	3400	31.6	Full	49	155.3	13.3	676	355	17.61
29	3382	27.7	Full	48	160.3	11.3	693	354	16.48
30	4112	30.8	Full	48	162.7	11.9	681	351	17.02
31	3354	31.7	Full	48	162.7	13.5	690	357	16.79
32	2754	37.4	Full	43	161.9	11.8	675	350	17.38
Avg	3318	32.4	Full	47	160.7	12.6	693	359	16.73

TABLE 2 TEST OF 4-6-2 TYPE LOCOMOTIVE

Test no.	No. of cars	Tonnage	Cutoff per cent	Pressure: Steam pipe lb per sq in.	Pressure: Exhaust, lb per sq in.	Temperature: Steam pipe, F	Temperature: Exhaust F	Steam rate, lb per ihp-hr
1	5	571	25	202	11.6	700	356	16.47
2	5	573	25	205	11.6	715	354	15.60
3	5	573	27	207	12.5	709	343	15.41
4	5	571	25	210	12.9	705	355	16.19
5	6	644	25	207	12.4	691	339	16.18
6	6	636	26	205	13.2	711	359	16.00
7	5	569	25	205	10.9	699	337	15.61
8	5	558	25	211	12.5	719	360	15.69
Avg	..	587	25.4	207	12.2	706	350	15.89
9	6	643	25	213	11.4	748	360	14.37
10	6	643	26	211	11.5	749	359	14.28
11	6	644	25	214	10	748	356	14.21
12	7	715	26	213	13.6	749	371	14.79
13	6	643	25	210	13.0	748	362	14.40
14	6	643	26	209	13.3	745	358	14.38
Avg	..	655	25.5	212	12.1	748	361	14.41

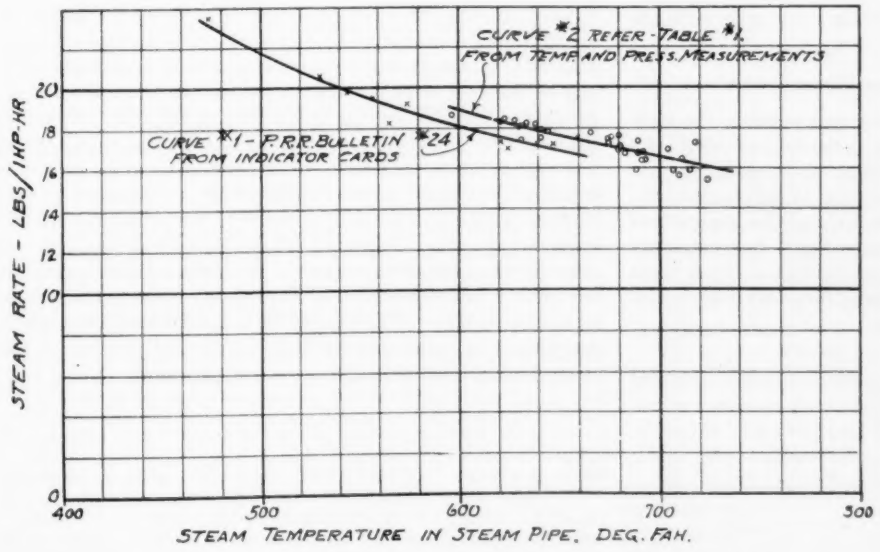


Fig. 7 Relation of Steam Rate and Steam Temperature

The method of testing was successful and the readings obtained were consistent and reasonable. The increase in steam temperature with the special type of superheater showed a decrease in steam rate, in line with other tests. This will be referred to later, when the accuracy of the heat-drop method is discussed.

Table 2 shows the results obtained in a similar test on a 4-6-2-type locomotive in fast passenger service. In this case temperatures were measured in both steam pipes and in both sides of the exhaust stand. The variation from side to side was not very great, but was enough to be significant. The potentiometer and rotary switch were mounted in the cab and even at the highest of speeds the galvanometer mounting shown in Fig. 4 was satisfactory. As in the test recorded in Table 1 the increase in steam-

pipe temperature showed a decrease in steam rate, as would be expected. The readings of the steam-pipe temperature, exhaust-stand temperature, and steam rate are more consistent than those shown in Table 1. This is because the weight of the train did not vary much. Only two crews were used for all of the tests, and the time-table schedule was kept more closely.

Accuracy of Heat-Drop Measurement of Steam Rate

It is difficult to estimate the absolute accuracy of the steam rate as determined by the proposed method. In a number of locomotive tests, particularly those made at the Altoona testing plant of the Pennsylvania Railroad, measurements were taken of the exhaust-steam temperature. This temperature was always measured in the exhaust-steam passage close to the steam chest, and, as already pointed out, the readings cannot be taken as being accurate. Consequently, it is not possible to compare the steam rate as obtained in these tests from the heat drop and, as obtained from the indicator cards, except in a general way.

Some idea of the possible accuracy of the heat-drop method can be obtained from Fig. 7. In this figure steam rate is plotted against temperature in the steam pipe. Curve 1 is taken from Pennsylvania Railroad bulletin No. 24, published in 1914, entitled "Superheater Tests." In this bulletin tests are described which were made on a Pacific-type locomotive. The only changes throughout the test were the length and arrangement of the superheater units. All points shown on curve 1 are for a constant cutoff of approximately 40 per cent and a consistent speed of approximately 240 rpm. The steam-pipe pressure varied from 170 to 195 lb. Curve 2 is plotted from the results given in Table 1. These tests were run at approximately 45 per cent cutoff, with a steam-pipe pressure of from 145 to 165 lb per sq in.

On account of the lower steam-pipe pressure it would be expected that curve 2 would lie above curve 1, showing a higher steam rate for the same steam temperature. The general trend of curve 2 is the same as that of curve 1. Another measure or indication of the accuracy of the results is the scattering of the points about the average line. It must be remembered that the tests for curve 1 were made on a stationary test plant, with the speed, cutoff, and horsepower being closely controlled. The tests for curve 2 were made in regular service, with a large variation in speed and tonnage, and with a number of different crews operating the locomotive. In view of this it is thought that the scattering of the points for curve 2 compares favorably with the scattering of the points for curve 1.

Measurement of Indicated Horsepower

It was pointed out by L. K. Botteron, in a paper in the *Railway Mechanical Engineer*, July, 1930, that the exhaust nozzle of a locomotive could be considered as a flowmeter. From the exhaust pressure and temperature the velocity of the steam in the exhaust-nozzle tip can be determined. Using the connections for pressure and temperature, as shown in Fig. 3, it is necessary to apply a correction for the velocity of the steam at the point where the pressure is measured. The usual formula for determining the velocity of steam in a nozzle is

$$V_2 = 223.7 \sqrt{H_1 - H_2} \quad \ldots\ldots [1]$$

where V_2 = outlet velocity, ft per sec
H_1 = inlet heat content, Btu per lb
H_2 = outlet heat content, Btu per lb.

In order to allow for the velocity of the steam at the point where the pressure is measured it is necessary to use the more exact formula

$$\frac{V_2^2 - V_1^2}{2g} = 778(H_1 - H_2) \quad \ldots\ldots [2]$$

where V_1 = inlet velocity, ft per sec

From a knowledge of the pressure and temperature in the exhaust pipe the quantity H_1 is determined. From a Mollier diagram the quantity H_2 is obtained, since the steam expands adiabatically and the entropy is the same at points 1 and 2. The pressure at point 2 is the pressure in the smokebox, and from the entropy and pressure the specific volume can be determined. Since the same weight of steam is flowing by points 1 and 2, the relation between V_1 and V_2 is

$$V_1 = V_2 \times \frac{A_2}{A_1} \times \frac{v_1}{v_2} \quad \ldots\ldots [3]$$

where A_1 = area of exhaust stand where pressure is measured, sq ft
A_2 = area of exhaust tip, sq ft
v_1 = specific volume at inlet, cu ft per lb
v_2 = specific volume at outlet, cu ft per lb.

Substituting this value of V_1 in Equation [2] gives

$$\frac{V_2^2\left[1 - \frac{A_2 V_1}{(A_1 v_2)^2}\right]}{2g} = 778\,(H_1 - H_2) \quad \ldots\ldots [4]$$

from which V_2, or the velocity in the exhaust tip, can be calculated. The flow of steam through the exhaust tip is equal to the velocity multiplied by the area, multiplied by the density, multiplied by a coefficient. It was assumed in the article by Botteron that this coefficient would be unity. Using this assumption the author has obtained some results which agreed fairly well with the measurement of water from the tender tank. It is felt that for better accuracy it would be desirable to calibrate the exhaust tip by means of a standing or blowdown test. It would be possible for any railroad to establish the coefficient for their design of exhaust tip and then to use the exhaust-stand pressure and temperature measurements to give accurate readings of the flow of steam through the exhaust tip. By making suitable corrections for the steam flowing from the exhaust to the feedwater heater or exhaust-steam injector and for the steam flowing from the auxiliaries to the exhaust passage the weight of steam to the engine can be calculated with fair accuracy. These readings would give the flow of steam to the engines at any instant during a run. When the steam rate is known, as obtained from the heat-drop measurements, the indicated horsepower, at any instant, can be obtained by dividing the steam to the engines by the steam rate. This enables the indicated horsepower to be determined by measuring the temperature and pressure of the steam in the steam pipe and in the exhaust stand.

The Correlation of Spring-Wire Bending and Torsion Fatigue Tests

By E. E. WEIBEL,[1] BERKELEY, CALIF.

The investigation reported in this paper was an attempt to correlate bending and torsion fatigue results for tempered Swedish valve-spring wire so that the torsion fatigue strength might be inferred within reasonable limits of accuracy from the fatigue values found in the simpler rotating-beam test. The original program called for (*a*) the development of suitable methods of gripping the wire for the torsion fatigue tests; (*b*) the determination of bending and torsion fatigue values for one size of wire in the as-received condition, and also after removal of the surface layer in a centerless grinder; (*c*) the determination of fatigue values for other sizes of wire between 1/8 in. and 1/4 in., as permitted by time limitations; and (*d*) a survey of the literature for data bearing on the correlation of bending and torsion results. The results of these experiments on straight specimens of tempered Swedish wire in torsion and bending fatigue may be summarized as follows:

1 Methods of gripping were developed which are satisfactory for both bending and torsion fatigue tests with the exception of torsion tests on shot-blasted wire.

2 Zero to maximum torsion fatigue values for straight specimens in the as-received condition were about 70 per cent higher than values estimated from previous tests on carefully coiled springs.

3 The smaller of two sizes of wire tested in the as-received condition had the higher torsion fatigue limit and lower ultimate torsional strength.

4 Removal of the natural surface by grinding increased the bending fatigue strength about 20 per cent, but decreased the torsion fatigue strength about 4 per cent.

5 Shot-blasting increased the bending fatigue strength about 27 per cent and increased the torsion fatigue strength an undetermined amount.

6 The zero-to-maximum torsion fatigue limits of two sizes of wire tested in the as-received condition were 1.5 and 1.6 times their respective bending fatigue limits.

7 For two sizes of wire in the as-received condition the values of 0.84 and 0.90 were obtained for the ratio of reversed torsion fatigue strength to reversed bending fatigue strength.

8 Approximate measurements of surface residual stress were made on the wire in different conditions before and after fatigue testing and before and after coiling, which may have a bearing on the fatigue properties of springs.

1—REVIEW OF THE LITERATURE

Methods of Gripping Wire in Torsion Fatigue Tests

A DESCRIPTION of gripping methods satisfactory for torsion fatigue tests on tempered carbon-steel wire could not be found.

F. C. Lea of Sheffield has developed methods of gripping which are satisfactory for cold-drawn wires. In his earlier work (1)[2] split bushings were used. These were not entirely satisfactory for tempered wires. In his later work (2) gripping shanks of an alloy with a low melting point (type metal) were cast on the ends of the wire specimen. It was found that 80 per cent of the cold-drawn-wire specimens failed at a distance from the grips.

Goodacre (3) reviews gripping methods in use in England for bending, tension, and torsion fatigue tests on ferrous and nonferrous wires. In the case of torsion Lea's casting method is mentioned.

A study of type-metal alloys by F. D. Weaver (4) was noted; also a paper by L. J. G. van Ewijk (5) on the penetration of steel by soft solder and other molten metals at temperatures up to 400 C. The latter's observations may explain why Lea's casting method, unmodified, was not successful with tempered Swedish wire.

In 1929, O. Föppl (6) obtained increased torsion and bending fatigue strengths of polished specimens by rolling the surface so as to produce local plastic flow.

A publication by P. Behrens (7) describes the rolling apparatus used by Föppl, and gives further results. An increase of 17 per cent in torsion fatigue strength of a mild steel was obtained, and increases of 20 to 50 per cent in bending fatigue strength of a low-carbon steel.

Thum and Wunderlich (8) determined the bending fatigue strength of shafts with and without a press-fitted collar. Heavy stress concentrations, microscopic rubbing with the formation of oxides and corrosion, lowered the fatigue strength considerably. By surface-rolling the shaft and shaping the collar to reduce stress concentration, an increase of 95 per cent in fatigue strength was obtained.

A paper by O. J. Horger (9) presents a considerable amount of data on the surface-rolling process. Increases in bending

---

[1] Instructor in Department of Mechanical Engineering, University of California. Dr. Weibel received the degree of B.Sc. in mechanical engineering at McGill University, Montreal, in 1918. During the summers, 1915 to 1917, he was employed as structural draftsman with the Dominion Bridge Company, Ltd., Montreal; and from 1918 to 1922 as mechanical designer and estimator on steam-turbine and electric-crane work. From 1922 to 1927 he was supervisor, under the mechanical engineer of the company, of the mechanical-design squad on cranes, hydroelectric regulating gates, etc. During 1928 and 1929 he studied mathematics at the University of Toronto and in 1929 returned to the Dominion Bridge Company where he was employed on mechanical standardization and as field engineer on the installation of hydroelectric equipment. In 1931 Dr. Weibel received the M.S.E. degree and in 1932 the degree of Ph.D. in engineering mechanics from the University of Michigan where he specialized in photoelastic stress determination. At the time these tests were carried out Dr. Weibel was research associate in the engineering research department of that University.

[2] Numbers in parentheses correspond to similarly numbered references given in the bibliography at the end of the paper.

Contributed by the Research Committee for presentation at the Annual Meeting of The American Society of Mechanical Engineers to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

fatigue strength around 30 and 40 per cent were obtained. Petersen and Wahl (10) also experimented with surface-rolling. Their paper includes a summary of Thum and Wunderlich's results.

Fatigue Results on Tempered Swedish Valve-Spring Wire

Swan, Sutton, and Douglas (11) tested straight specimens of a tempered Swedish valve-spring wire in torsion fatigue with maximum stress four times the minimum stress. When tested at full diameter of 0.128 in. the safe range was from 20,200 to 80,800 lb per sq in. When reduced to 0.100 in. diameter and polished, the safe range was from 31,400 to 125,500 lb per sq in., an increase of 56 per cent. These stress values were not measured directly but were assumed proportional to the angular travel of the gripping chucks, an assumption which will be shown later to be unjustified because of variations in the effective length of the specimen.

F. P. Zimmerli (12) tested a number of steels in the form of completed springs. The steels, numbers 4 and 7, were straight carbon pretempered Swedish wires, 0.148 in. diameter, with 0.75 and 0.65 per cent carbon content, respectively. The safe ranges decrease with increasing mean values of torsion stress in close agreement with the Goodman diagram. Probable values of zero to maximum torsion stress range extrapolated from the diagrams are 68,000 and 58,000 lb per sq in., respectively.

Bending fatigue tests were made on two sizes of Swedish valve-spring wire by Shelton and Swanger (13). The first wire, 0.162 in. diameter, 0.65 per cent carbon, had an ultimate tensile strength of 221,000 lb per sq in. and with the natural surface a fatigue strength 34 per cent of the ultimate, or ±76,000 lb per sq in. When reduced to 0.130 in. diameter and polished, the fatigue strength was 56 per cent of the ultimate or ±126,000 lb per sq in. The second wire, 0.148 in. diameter, 0.65 per cent carbon, had an ultimate strength of 217,000 lb per sq in. and a fatigue strength only 30 per cent of the ultimate or ±65,000 lb per sq in. It showed surface defects.

Other Fatigue Results

The torsion fatigue results of G. A. Hankins (14) on polished specimens of spring steel show safe stress ranges which are practically the same for alternating torsion and for zero-to-maximum torsion stress. For higher mean stresses there is a reduction in the safe range but values are always greater than would be given by the Goodman diagram.

The Correlation of Torsion and Bending Fatigue Results for Wire

No reports of experiments were found in which torsion and bending fatigue results were obtained for the same wire and compared. Considerable experimental work and some theoretical work having a bearing on the torsion fatigue strength and on the ratio of torsion to bending fatigue strengths was reviewed.

A criterion of fatigue failure of metals under any type of stress does not appear to have been found. Fahrenhorst and Schmid (15) review briefly the hypotheses as to the cause of fatigue failure of Kuntze, Haigh, Ljungberg, Ono, Ludwik, Gough, Hanson and Wright, and Schmid. These are many and various and will not be given here.

H. J. Gough (16) reported that behavior of metals was being studied under reversed bending, reversed torsion, and all possible combinations of these two types of fatigue loading. Mild steel, a ductile material, obeyed closely the distortion-energy theory of von Mises and Hencky and discussed in a paper by Nadai (17), according to which theory the safe torsion fatigue stress would be 0.577 times the safe bending fatigue stress. But an alloy steel and a cast iron exhibited other relations. He concludes that no general criterion of failure obtains under the stated stress conditions.

Fatigue experiments by A. F. Maier (18) on tubes under two types of reversed stressing obtained by means of internal pressure appeared to support the theory of maximum shear stress according to which the safe shear stress is 0.500 times the safe tensile stress. The results did not support the distortion-energy theory.

Moore and Kommers (19) list the reversed-torsion endurance limits and the reversed-bending endurance limits for polished specimens of a number of straight-carbon steels which had received widely different heat-treatments. The average of 19 values of the ratio of torsion endurance limit to bending endurance limit was 0.55. Extreme values were 0.48 and 0.64. The average of 49 values of the same ratio, from results of tests on polished specimens, listed by Gough (20), was 0.56. Extreme values were 0.44 and 0.80. These results are for presumably homogeneous and isotropic material, and surface effects have been eliminated as far as possible.

Gill and Goodacre (21) studied some aspects of the fatigue properties of patented wire. They found that wire with decarburized surface will take overloads as well as, or better than, wire not decarburized and conclude that there is no gain in removing the decarburized surface if wire will be overstressed much. The writers assume that the ratio of torsion fatigue strength to bending fatigue strength should have the value $^2/_3$, and using this value together with one of their bending fatigue results for polished wire, they calculate a hypothetical torsion fatigue strength which agrees closely with a previous experimental torsion fatigue value for polished wire obtained by Lea and Dick (22). They say, "It seems very probable therefore that the fatigue limit in bending will give a good indication of the torsional fatigue properties of polished spring wire." But they give no reasons why it should and offer only one sample calculated value, and do not check experimentally the torsion strength of the actual wire whose bending fatigue strength they used in their calculations. The arbitrary use of the ratio $^2/_3$ did not appear to be justified.

Values of stress concentration factor around spherical and cylindrical inclusions have been given by Southwell and Gough (23) and J. N. Goodier (24), and the stresses around an elliptical hole in a plate by C. E. Inglis (25).

Goodier shows that the elastic stress concentration factors at a small spherical cavity in regions of uniform pure shear and of uniform tension stress are 1.91 and 2.72, respectively. These values cannot be used in estimating the effect of such a cavity on fatigue strength in the two cases, since the first is in terms of maximum shear stress, and the second is in terms of maximum tensile stress. From Goodier's results it can be shown that for a cavity in a field of uniform tension stress, the maximum shear stress is increased just 1.91 times. If fatigue failure were known to be determined by maximum shear stress, this would lead to the practical conclusion that the ratio of torsion endurance limit to bending endurance limit is unaffected by the presence of small spherical inclusions which are not too close to the surface.

Since in wire, inclusions are probably lengthened out as a result of drawing, calculations for ellipsoidal cavities would be of interest, but none were found in the literature.[3]

For flaws at the surface of wire, the two-dimensional elastic stress concentrations for circular and for elliptical cylindrical holes might be applied assuming again an arbitrary criterion of fatigue failure. At a minute circular hole in the surface of the wire, for the case of torsion, the maximum shear stress is doubled, and for the case of bending, the maximum shear stress is trebled

[3] A paper by H. Neuber has since been noted, "Der Raümliche Spannungszustand in Umdrehungskerben," *Ingenieur Archiv*, vol. 6, no. 2, April, 1935, p. 133.

(24). Therefore, such a flaw is more effective in reducing the bending fatigue strength than in reducing the torsion fatigue strength.

Still assuming the maximum shear stress criterion, we may consider the elastic stress concentration factors for an elliptical flaw on the surface with major axis placed longitudinally and four times the length of the minor axis. For the case of bending, Inglis' formula gives a factor of 1.5. For the case of torsion, the maximum shear stress is increased by the factor $[1 + (a/2b) + (b/2a)]$, where a and b are the major and minor semiaxes, respectively. This formula is not given by Inglis, but was obtained from his results. The value of the factor in the present instance is 3.12. Hence, it is seen that a longitudinal elliptical surface flaw will reduce the torsion fatigue strength proportionately more than it will the bending fatigue strength. Numerical comparisons are limited by the lack of numerical values for notch sensitivity of the material, but it is apparent that any attempt to calculate the torsion fatigue strength from the bending fatigue strength must take account of the shape of flaws which are permitted by the specifications. A striking example of the formation of four fatigue cracks at the points of stress concentration around an approximately elliptical flaw of a torsion fatigue specimen is shown in Fig. 27.

Different materials under alternating loading react differently to stress concentration.

W. Buchmann (26) concludes that "Notch sensitivity is a utilizable property of the material, determinable by calculation." This is the same quantity which R. E. Peterson (27) calls "stress concentration index" q where

$$q = (k - 1)/(K - 1)$$

where k is the ordinary endurance limit without stress concentration divided by the endurance limit with stress concentration (fillet, hole, etc.), and K is the elastic stress concentration factor and is equal to the maximum stress at a fillet or hole divided by the average nominal stress.

W. Buchmann (26) made fatigue studies in bending and torsion on specimens about 0.3 in. diameter, with circumferential notches of 0.002 in. to 0.08 in. radii. Fatigue strength of annealed materials was practically unaffected by the presence of the notch (q practically equal to zero for very ductile materials), while for hardened steels the reduction in fatigue strength was practically proportional to the calculated increase of stress at the notch (q approximately unity for very brittle materials). For the same material the greatest difference between actual reduction of fatigue strength and reduction calculated from the elastic stress concentration factor occurred with the sharpest notches (q decreases with increased sharpness of the notch in a particular material).

Erich Scheil (28) published a paper on the origin of cracks in steel as a result of heat-treatment, and found a dependence on the internal-stress condition, the separation strength of the steel, and other factors.

The effect of internal stresses on fatigue strength were studied by Buehler and Buchholtz (29), who worked with specimens about 1 in. in diameter. They found that compressive surface stresses of about 30,000 to 50,000 lb per sq in. caused by quenching, raised the bending fatigue limit from 10 to 20 per cent. They quote results which show that large tensile surface stresses reduce the fatigue strength about 15 per cent, and that tensile stresses below 28,000 lb per sq in. have no appreciable effect on fatigue limit. They mention the gradual reduction of internal stresses as a result of fatigue stressing.

The same authors (30) studied the relation between residual temperature stresses and yield point of the material and found higher residual quenching stresses for materials with lower yield point due to the greater amount of plastic deformation which occurred during quenching.

Buehler and Scheil (31) studied the combined effect of temperature and transformation stresses in 2-in. diameter specimens of quenched steels. The nickel content was varied to give various transformation temperatures corresponding to different carbon content in straight-carbon steels. Lower transformation temperatures left tension stresses at the surface. Very high transformation temperatures left compressive stresses. Smaller-diameter specimens had smaller tension or compression surface stresses than large-diameter specimens.

Linicus and Sachs (32); in studying the effect of the die angle and degree of drawing on the internal stresses in wire, measured the internal stress by planing off half the width of the wire and noting the change of curvature. However, this gave only a qualitative measure of stress.

R. M. Brown (33), in measuring residual stresses in cold-drawn mild-steel rods, found that in general, zero stress is at an annulus 0.66 of the diameter, with tensile stress outside and compressive stress within.

General

H. J. French (34) discusses the phenomenon of delayed fractures. Hardened and slightly tempered steels which did not fail at 10 million repetitions, failed suddenly at 30 to 60 million repetitions. The steels were all nickel steels and delayed fractures were observed at hardnesses above the range 45 to 50 on the Rockwell C scale, but not at lower hardnesses. No mention of the same effect in straight-carbon steels was found in the literature.

Another delayed-fracture effect is mentioned by A. Jünger (35) who found that flat bending specimens of hardened chrome-molybdenum steel, in which stress concentration was produced by a small hole, were breaking even after 75 million repetitions, whereas unbored specimens did not break after 5 million repetitions.

2—MATERIALS INVESTIGATED

Three coils of straight-carbon tempered Swedish valve-spring wire were supplied by three different wire manufacturers for the purpose of these tests. The wire was specially selected in regard to freedom from inclusions and for as nearly a perfect surface as could be obtained.

The stress-strain curves for static tensile and torsional tests of these wires are plotted in Figs. 1, 2, and 3, while the dimensions, chemical analyses, Rockwell hardnesses, and strengths are given in Table 1. The ultimate torsion strength was calculated from the formula

$$\frac{5.1 \times \text{maximum twisting moment}}{\text{diameter}^3}$$

Carbon contents for the outer 0.001-in. thick layer were found to be 0.57, 0.59, and 0.59, respectively. Photomicrographs of the boundary material are discussed below.

Fatigue tests were made chiefly on the 0.225-in. and 0.187-in. diameter wires. Complete series of tests were not made on the 0.187-in. and 0.125-in. wires because of lack of time.

Polished longitudinal sections and cross-sections of the 0.187-in. and 0.225-in. diameter wires were examined for inclusions. At 100 magnifications, inclusions were smaller in general than about 0.015 in., and no large inclusions were observed.

The structure at 1000 magnifications of a cross-section of the 0.187-in. wire is shown in Fig. 4, and of a longitudinal section of the 0.225-in. wire in Fig. 5.

The boundary of the 0.187-in. wire at 1000 magnifications is

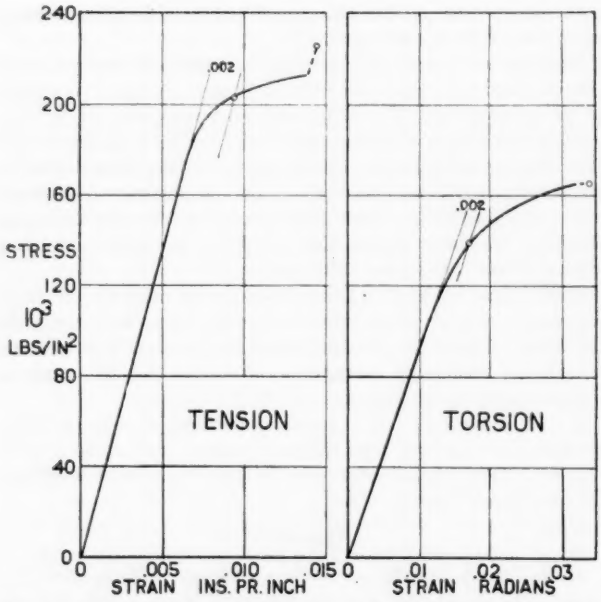


FIG. 1 CURVES FROM STATIC TESTS OF 0.125-IN. DIAMETER TEMPERED SWEDISH WIRE

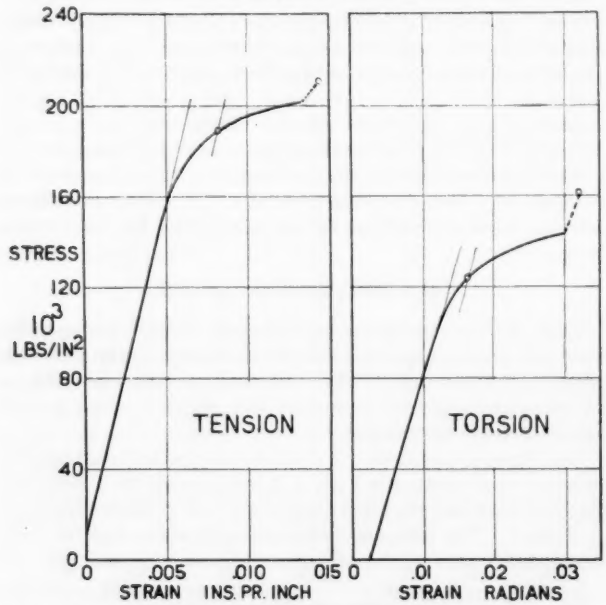


FIG. 2 CURVES FROM STATIC TESTS OF 0.187-IN. DIAMETER TEMPERED SWEDISH WIRE

shown in Fig. 6 in which decarburization to a depth of about 0.0005 in. is shown.

Fig. 7 shows decarburization at the boundary of the 0.225-in. wire, the oxide scale being in focus. Fig. 8 shows another portion of the boundary of the same wire, at which the distribution of ferrite is less disperse.

The photomicrographs show the extent of decarburization better than the carbon analyses previously given, as the latter gave the average carbon content for a layer twice as thick as the actual decarburized layer.

3—DESCRIPTION OF THE FATIGUE-TESTING MACHINES

Bending fatigue tests were made on a short-span rotating-beam-type machine, which was found to work satisfactorily for 0.225-

TABLE 1 RESULTS OF TESTS ON THREE SPECIMENS OF TEMPERED SWEDISH WIRE

Diameter of wire, in.	0.125	0.187	0.225
Coil diameters as received, in.	28	36	36
Chemical analyses:			
Carbon	0.60	0.71	0.65
Manganese	0.35	0.60	0.59
Silicon	0.18	0.21	0.15
Phosphorous	0.023	0.020	0.020
Sulphur	0.026	0.022	0.021
Tensile strength, lb per sq in.:			
Ultimate	226,000	211,000	204,000
Yield point	203,000	190,000	181,000
Torsional strength, lb per sq in.:			
Ultimate	166,100	162,000	166,000
Yield point	139,460	125,000	120,500
Elongation in 8 in., per cent	4.2	5.7	6.2
Elongation in 2 in., per cent	6.9	9.9	10.5
Reduction of area, per cent	51.8	51.7	43.7
Hardness, Rockwell *C*	45.8	42.8	42.2
Stress-strain curve	Fig. 1	Fig. 2	Fig. 3

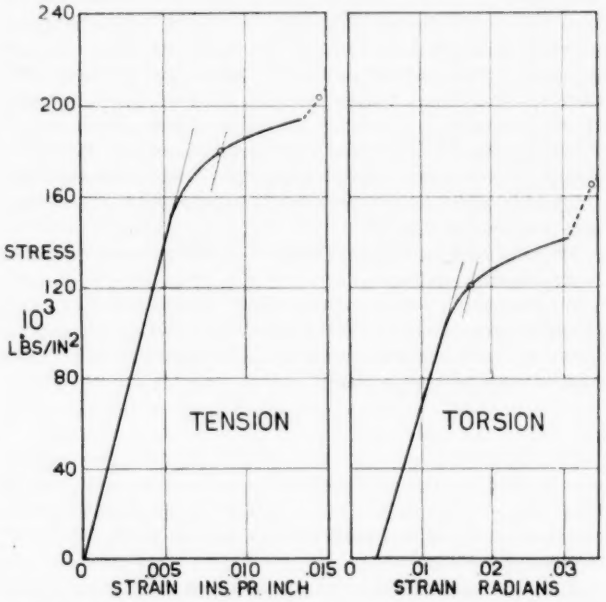


FIG. 3 CURVES FROM STATIC TESTS OF 0.225-IN. DIAMETER TEMPERED SWEDISH WIRE

in. and 0.187-in. diameter wires, but not for 0.125-in. wire. Clear lengths of specimens between $1^1/_2$ in. and $2^1/_2$ in. were used. Grips were cylindrically bored and fitted with an external cone and nut for clamping.

Torsion fatigue tests were conducted on three machines of the constant-strain type shown in Fig. 10. A $^1/_3$-hp driving motor running at 1725 rpm was coupled to a shaft with a variable eccentric. A connecting rod gives an oscillating motion to one of the gripping chucks through a rocker arm of variable radius, the maximum angular motion being about 12 deg.

The second gripping chuck was originally clamped solidly on these machines, but in order to provide a means of determining the stress range at all times while the machine was in operation, a torsion-measuring member was inserted in series with the specimen under test. Two dial indicators measured the relative rotation of the two ends of this member at the extremities of the cycle. This device was calibrated statically under known torsion moments. Inertia effects in it were negligible, the calculated error with the proportions used being 0.1 per cent. Readings of the dials for zero torque were made at the beginning of a test with the specimen in place by unclamping the dead end of the torsionmeter and giving the assembly a slight oscillatory motion to eliminate friction.

A revolution counter was connected to the eccentric shaft. The motor was controlled through a 6-volt relay circuit, a current of 0.125 amp passing through the specimen under test. A



FIG. 4 STRUCTURE IN A CROSS-SECTION OF THE 0.187-IN. DIAMETER WIRE (×1000)

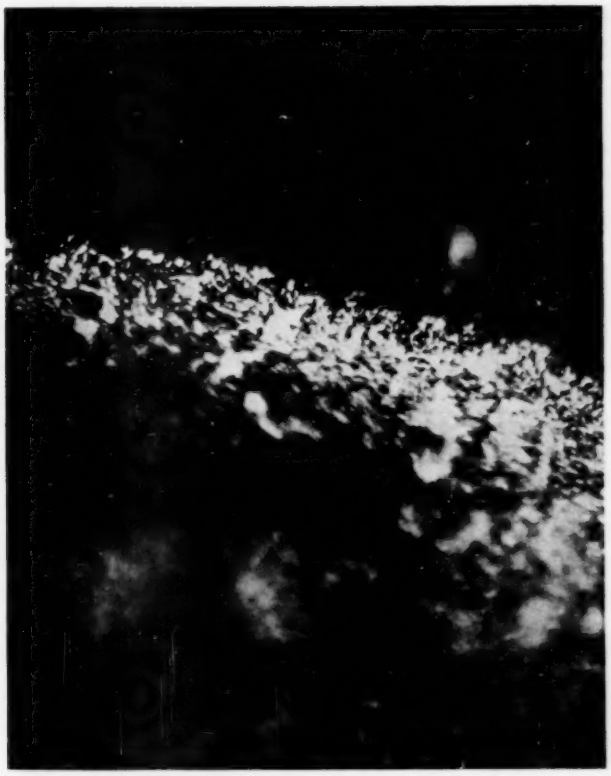


FIG. 6 DECARBURIZATION AT THE SURFACE OF THE 0.187-IN. DIAMETER WIRE (×1000)



FIG. 5 STRUCTURE IN A LONGITUDINAL SECTION OF THE 0.225-IN. DIAMETER WIRE (×1000)

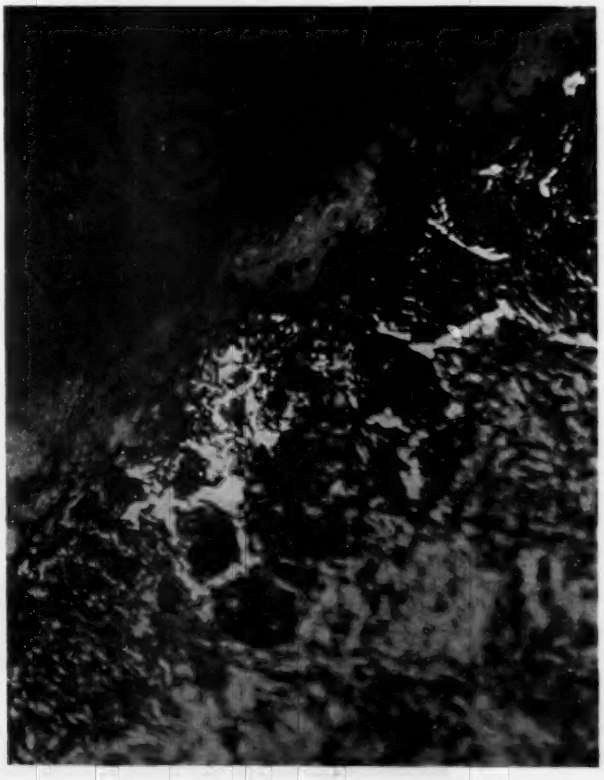


FIG. 7 DECARBURIZATION AT THE SURFACE OF THE 0.225-IN. DIAMETER WIRE, SHOWING ALSO THE OXIDE SCALE (×1000)

spring was provided to shift the oscillating grip a distance of about 0.0625 in., thus opening the circuit. Relays were adjusted so as to remain open after a momentary break in the circuit. This plan did not always stop the motor since contact was often maintained through the two parts of the fractured specimen. A phosphor-bronze leaf-spring contact in series with the specimen was therefore added at the end of the oscillating shaft which positively broke the circuit with a small endwise motion of the shaft and stopped the motor as desired.

Ball-bearings were used throughout, lubrication being maintained by oiling at the end of every one or two million cycles.

Vibration was reduced to a minimum by balancing the rotating eccentric parts and mounting the machines on thick sponge-rubber pads.

For tests at values of mean stress other than zero, a rotational adjustment was provided, which could be varied during operation.

The split chucks were cylindrically bored to 0.5 in. diameter and each was clamped by means of four $^5/_{16}$-in. heat-treated bolts.

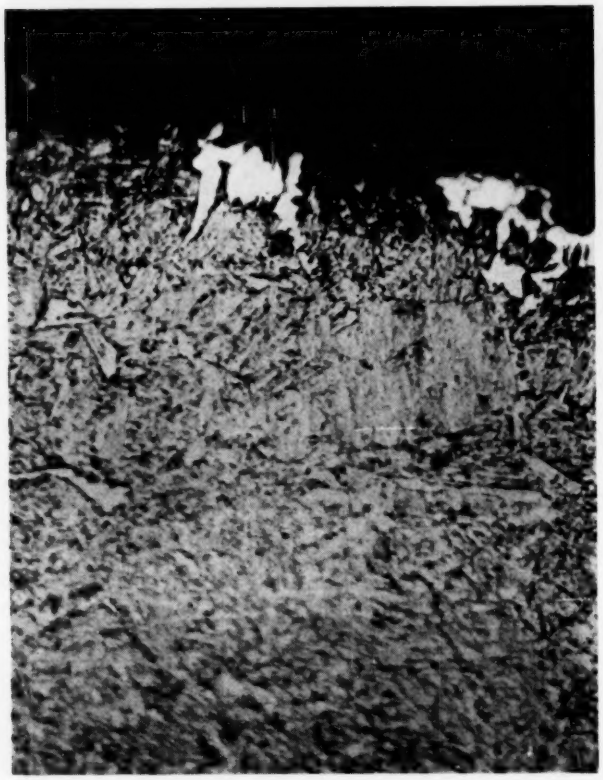


FIG. 8 ANOTHER PART OF THE SAME SECTION OF THE 0.225 IN-DIAMETER WIRE SHOWN IN FIG. 7 ILLUSTRATING A DIFFERENT TYPE OF DECARBURIZATION (×1000)

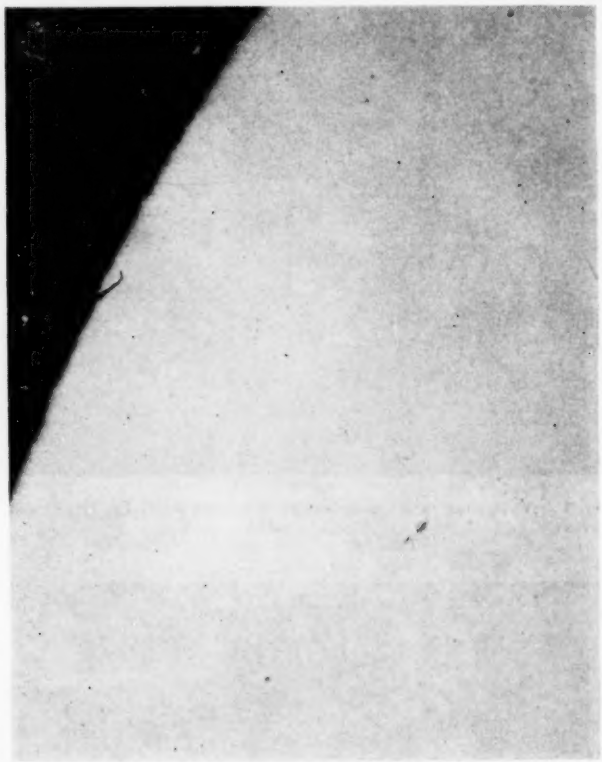


FIG. 9 SEAM ABOUT 0.001 IN. DEEP FOUND IN ONLY ONE SECTION OF THE 0.225-IN. DIAMETER WIRE (×100)

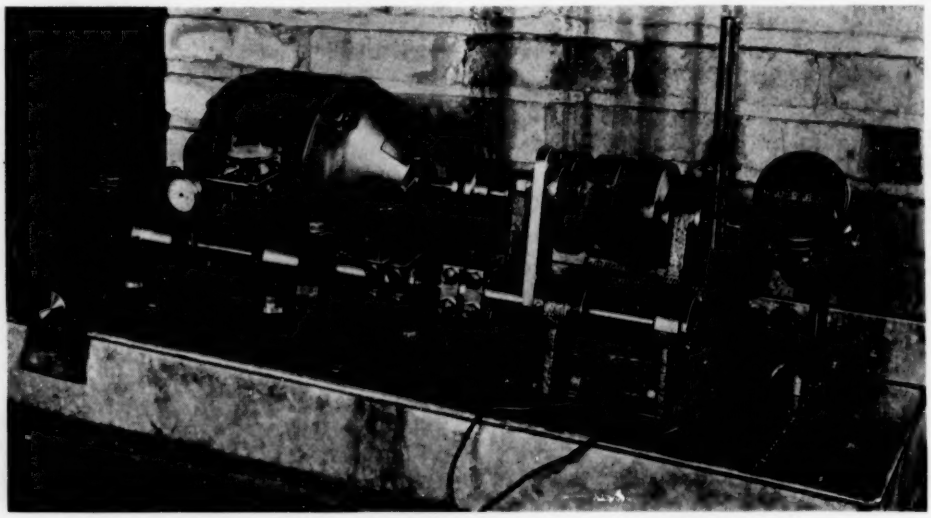


FIG. 10 TORSION FATIGUE MACHINE SHOWING MOTOR, ECCENTRIC SHAFT, OSCILLATING GRIP, SPECIMEN IN PLACE, TORSION MEASURING DIALS, AND CUT-OUT SWITCH AT THE RIGHT END OF THE OSCILLATING SHAFT



FIG. 11 SPLIT STEEL MOLDS; BENDING AND TORSION SPECIMENS WITH CAST TYPE-METAL ENDS; DIFFERENT TYPES OF STEEL, COPPER AND COPPER-LEAD BUSHINGS, AND SPECIMENS PREPARED IN DIFFERENT MANNERS

4—DEVELOPMENT OF GRIPPING METHODS FOR TORSION

CAST-END METHOD

At the inception of this project no satisfactory methods of gripping were known for the torsion fatigue testing of tempered valve-spring wire in the as-received condition. Failure of straight wire specimens invariably occurred at the grips where the phenomena of stress concentration, microscopic rubbing, oxide formation, and corrosion were all present. The difficulties may be illustrated by Swan, Sutton & Douglas' torsion fatigue tests on Swedish wire (11). Of 25 specimens tested only two failed away from the grips, and another two ran unbroken for ten million stress cycles.

For cold-drawn wires, the difficulties had been overcome by F. C. Lea (2) by the use of cast type-metal ends. Adhesion was obtained by first tinning the ends of the wire which was then gripped in a split steel mold and poured with type metal. Molds and specimens of this type are indicated in Fig. 11. The temperature reached by the central portion of the wire in the process was not above 100 C, and when such high temperatures were reached they were only of momentary duration.

Lea's method was taken as the starting point for the present work with tempered wire. The alloy used was described by Lea (2) as a white metal having a melting point as low as 107 C, and by Goodacre (3) as a type metal melting at 180 C. In the experiments reported in this paper, a matrix metal, a tin-lead solder, and a commercial monotype metal were tried first. The matrix metal contained 28.5 per cent lead, 14.5 per cent tin, 48 per cent bismuth, and 9 per cent antimony, and had a melting point of 105 C. The tin-lead solder had a melting point of approximately 180 C. The monotype metal analyzed 76 per cent lead, 8 per cent tin, and 16 per cent antimony, and had a melting point around 240 C.

The matrix alloy and the tin solder did not adhere to the wire in torsion fatigue trials, whereas the monotype metal always adhered, even for torsional stresses in the wire as high as 140,000 lb per sq in. Monotype metal is relatively inexpensive, and as it

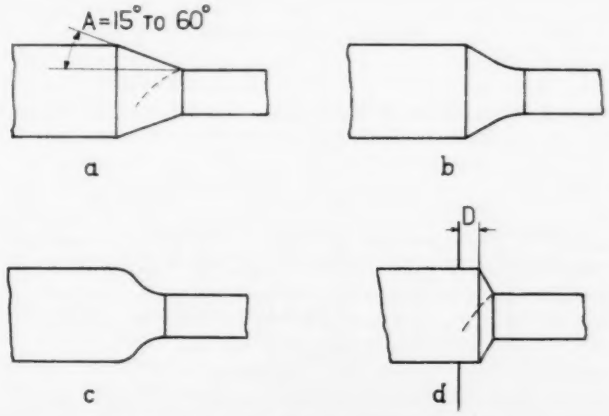


FIG. 12 SHAPE OF THE ENDS OF THE SPECIMENS USED IN TORSION FATIGUE TESTS

seemed to meet the initial requirements it was used in subsequent tests.

Specimens of 0.125-in. wire were prepared with cast monotype ends and tested in torsion fatigue. In all cases in the early experiments failure occurred at the inner end of the molded type-metal shank. To reduce stress concentration effects the ends were variously shaped as shown in Figs. 12*a*, 12*b*, and 12*c* and were gripped at varying values of *D* shown in Fig. 12*d*, but fractures were always at the junction with the white metal thus indicating that stress concentration was probably not the chief cause of failure. For small angles *A* and for large values of *D* shown in Figs. 12*a* and 12*d*, respectively, the white metal failed by cracking along a helix. In order to vary the hardness of the white metal, pouring temperatures were varied from 475 C to 300 C, and mold temperatures were reduced to as low as 5 C with no apparent effect on the position of wire fractures.

A few specimens were clamped directly in a split-copper bush

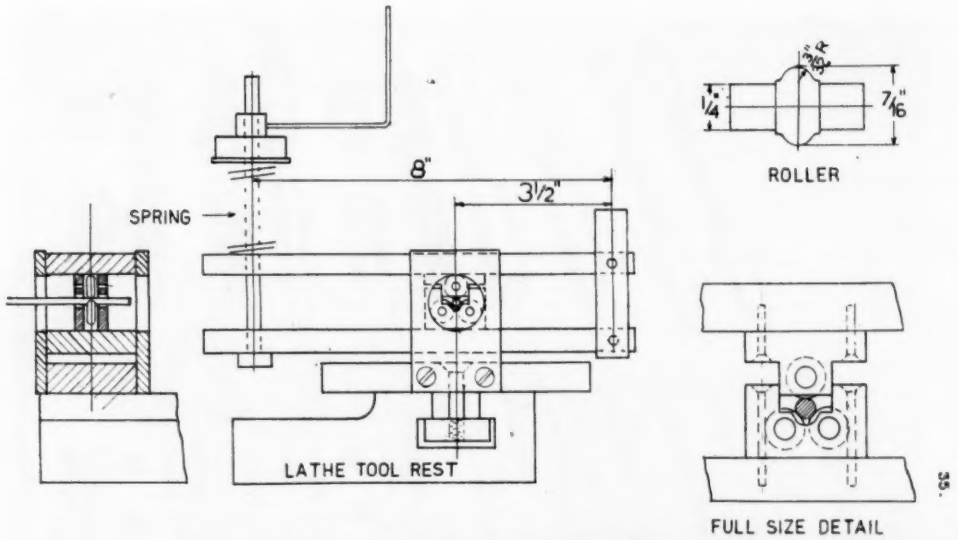


FIG. 13 DETAILS OF THE SURFACE-ROLLING DEVICE SHOWN IN FIG. 16

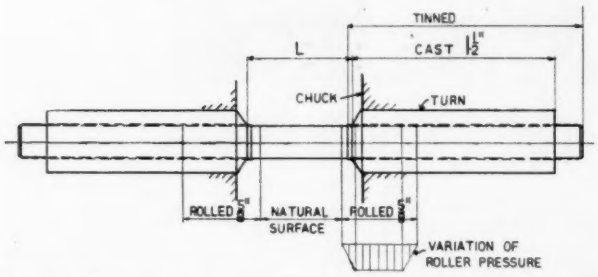


FIG. 14 DIMENSIONS OF THE ROLLED SECTIONS OF THE TORSION FATIGUE SPECIMENS

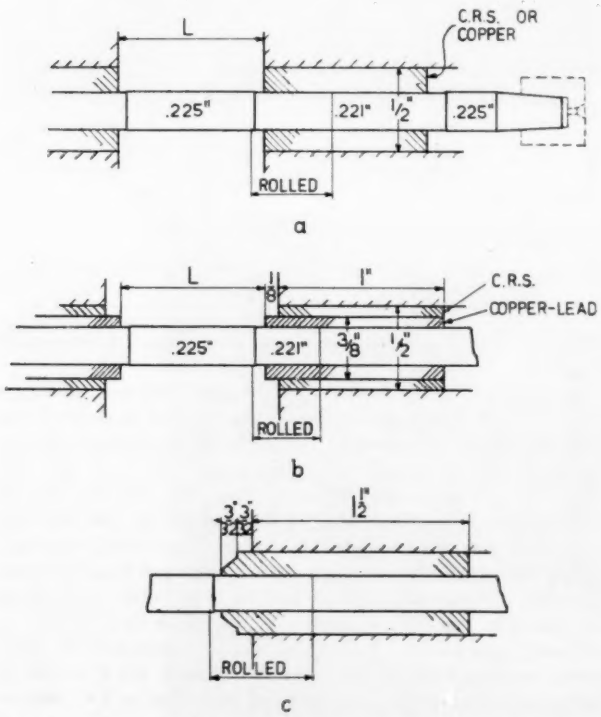


FIG. 15 BUSHING DETAILS FOR GRINDING, ROLLING, AND GRIPPING SPECIMENS

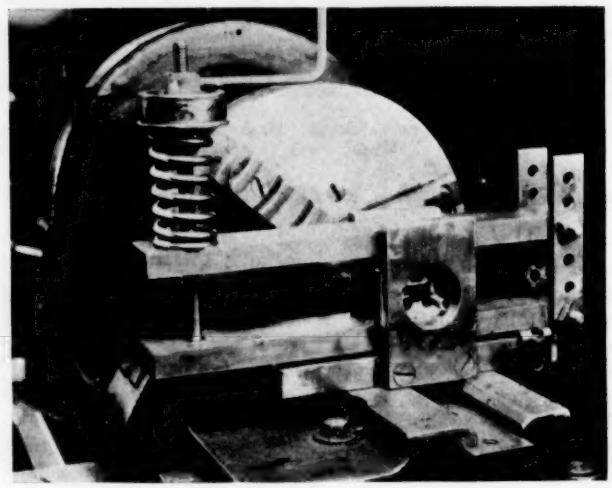


FIG. 16 DEVICE FOR ROLLING THE SURFACE OF THE WIRE. THE SPRING WAS CALIBRATED TO PRODUCE ACCURATE ROLLING PRESSURES

ing with a 0.009-in. layer of detail paper between the wire and bushing. These failed at the grips.

One specimen was clamped without paper in a split mild-steel bushing and failed at the grip.

Removal of the decarburized layer and polishing is known to greatly increase the fatigue strength of wire. When this was done and followed by tinning and casting on type-metal shanks, failure again, as with other methods, occurred at the grip.

Five specimens of 0.187-in. wire were prepared with cast monotype ends for testing in the rotating-beam machine, in the belief that conditions here would not be so drastic as for torsion and that bending fatigue experiments could be started. Although the different shapes of the ends of the specimens shown in Figs. 12*a*, 12*b*, 12*c*, and 12*d* were tried, failure always occurred at the grips. One bending specimen was not tinned, but cast directly on the wire which had been fluxed in zinc chloride. It failed at the grip, indicating that tinning was not an essential cause of failure.

The presence of a yellowish white oxide at the junction of wire with cast shank suggested a similarity in the gripping problem to that of the failure of axles at a press fit, where stress concentration

and corrosion fatigue due to the formation of oxides are present (8, 9, 10). In the experiments of Thum and Wunderlich remarkable increases of bending fatigue strength were obtained due to surface-rolling. This process was next tried and since it met with success it will be described in detail.

Surface-Rolling of Wire Specimens

The apparatus detailed in Fig. 13 and shown in Fig. 16 was assembled and used to roll the surface of wire specimens at the points where failure had previously been found to occur. The rollers were made of $^{7}/_{16}$-in. diameter hardened steel with the face crowned to $^{3}/_{32}$-in. radius. Roller dimensions were taken from the dissertation of O. J. Horger (9). A load of approximately 100 lb was used at the spring, giving roller loads of about 230 lb each. The approximately straight wire specimens were gripped in a lathe chuck and rotated at various speeds. The 0.225-in. wire was rotated at 25 rpm and the 0.187-in. wire at 35 rpm, giving a surface speed of 1.5 fpm. A feed of 0.0055 in. per revolution was used.

With the roller dimensions and loading as given, the maximum shear stress in a perfectly elastic solid calculated by the Herz theory for contact stresses was in the neighborhood of 500,000 lb per sq in. The yield point in shear of the tempered wire is from 120,000 to 140,000 lb per sq in. Consequently, this extremely high stress is not reached but plastic flow takes place to a depth depending upon the roller pressure. The slow rolling speed allows time for plastic flow to occur. As a result of rolling, the surface material is work-hardened, is made more dense, and is left with heavy compressive residual stresses, all of which probably contribute to its increased fatigue strength.

Two portions of the wire specimen, each about $^{5}/_{8}$ in. long, were rolled as shown in Fig. 14. The roller pressure was increased linearly from zero to a maximum in a length of $^{1}/_{16}$ in. to $^{1}/_{8}$ in. and similarly reduced to zero at the other end of the rolled portion. The tinning was not permitted to touch the natural surface of the wire in the center portion because of possible penetration and weakening (5). The cast end was kept clear of the end of tinned portion so that perfect adhesion was obtained. The beveled ends of the wire specimen were centered in female lathe centers for turning the type-metal ends to fit the chucks of the torsion machine. It was obviously not necessary for the wire to be straight with this method of gripping.

Before tinning, the ends of the wire were rubbed lightly with emery cloth to remove scale, and fluxed in zinc-chloride solution. They were tinned by dipping into molten 50/50 lead-tin solder at 325 C. The momentary temperature reached at a point in the wire just clear of the tinning did not in general exceed 75 C. This was measured by inserting a thermocouple in small holes drilled in pieces of $^{1}/_{8}$-in. and $^{1}/_{4}$-in. drill rod, and tinning these as for the wire specimens.

The monotype metal was poured at 325 C and the molds were usually at or slightly below room temperature. The split molds weighed 0.60 and 1.25 lb for the 0.187-in. and 0.225-in. wires, respectively. The metal cooled quickly and wire temperatures as measured by thermocouples did not exceed 75 C.

The combination of cast ends with surface rolling of the wire was successful for alternating torsion fatigue tests on as-received wire and on wire with the surface ground. Practically every specimen failed away from the gripping points. The method was not successful for shot-blasted wire, since the surface material of this wire is strengthened greatly by work-hardening. The method was successful for bending fatigue tests in a short-span rotating-beam machine on wire in the as-received, ground, shot-blasted, and shot-blasted-and-blued conditions.

To obtain torsion fatigue results for shot-blasted wire, a surface layer of approximately 0.0015 in. was removed from the two end portions of the specimens after which they were polished and rolled. Since the wire specimens were not perfectly straight it was necessary to rotate them in a lathe and clean them with emery cloth. All scratches had to be removed to eliminate the danger of the specimen breaking at the scratches. The few specimens prepared in this way did not fracture as desired, probably due to scratches from the emery, but the double strengthening due to removal of decarburized layer and to surface rolling should more than counterbalance the strengthening in the center portion of specimen due to shot-blasting. Further work with this method should yield results.

When the cast-end type of specimen was tested in a zero-to-maximum torsion-stress cycle, there was a continuous angular creep in the type-metal ends. The effect of this creep was to shift the stress range to a position with a lower mean stress value so that it was necessary to adjust the machines at frequent intervals in order to maintain the intended stress cycle. The cast-end method was consequently abandoned for zero-to-maximum stress cycles, although it is probable that an automatic method of maintaining the proper stress cycle can be worked out. This may be a desirable development as the cast-end method has the great advantage of reducing stress concentration to a minimum.

Split-Bushing Methods

For tests in a zero-to-maximum torsion-stress cycle, split bushings were used to grip the wire. In conjunction with the bushings surface rolling was always used to strengthen the wire in the critical region. Bushing materials experimented with were ordinary cold-rolled steel, hard-drawn copper, and a copper-lead alloy with about 25 per cent lead. Steel was used because of its high coefficient of friction, requiring lower gripping pressures, while copper-lead was used because of its successful application in the pulsating tensile tests of Shelton and Swanger (13). The friction coefficient of brass was too low for satisfactory gripping. In some cases the ends of bushings were faced square with no attempt at relieving stress concentration, while in other cases they were beveled in an attempt to reduce the pressure peak occurring at the ends. When set in the machine they were chucked sometimes flush at the ends, and sometimes protruding a short distance, which was believed to reduce the pressure peak due to gripping. In some cases a thickness of paper was used between the wire surface and the bushing bore. Rolling and subsequent gripping of the wire were done either directly on the natural surface or on a portion which was polished after being reduced about 0.004 in. in diameter to remove the decarburized layer.

The wire diameter was reduced either by grinding between centers which could only be done on samples specially selected for straightness, or more practically by hand in a lathe using strips of successively finer emery cloth. Bushings were drilled solid with a twist drill and sometimes finish-bored by a gun reamer with which it was attempted to bore them 0.0005 in. smaller than the neat-wire size. They were split into halves by a $^{1}/_{64}$-in. milling cutter, and in one case each half was slotted to provide greater flexibility in clamping around the surface of the wire.

Five specimens of 0.225 in. wire in the as-received condition were selected for straightness and ground between centers to 0.221 in. diameter using the tapered-end fittings shown in Fig. 15*a*. They were rolled and gripped by the methods of either Fig. 15*a* or Fig. 15*b*. Specimens Nos. 350 and 351, shown in Fig. 25, were gripped in the copper-lead bushing detailed in Fig. 15*b*. Both of these specimens fractured in the center as shown in Fig. 25. However, the fracture of specimen No. 350 was at a surface flaw as is evidenced from the stress-repetition or S-N curve in Fig. 18. Specimen No. 352 was gripped in a copper

bushing 1 in. long, as shown in Fig. 15*a*, and was run 10.1 million cycles of 0 to 97,000 lb per sq in. stress without breaking. Specimen No. 353 shown in Fig. 25 was gripped in a cold-rolled steel bushing 1 in. long and broke in the center portion. Specimen No. 354 similarly gripped in a steel bushing was removed unbroken after 4.4 million cycles. Very little galling of the wire surface was evidenced. Copper-lead bushings could not be used more than once because of plastic deformation. The method may be considered successful but somewhat elaborate.

In an effort to simplify the procedure, the reduction in diameter of the gripped portions of specimens was omitted. The natural surface of the wire, or 0.187 in. diameter in this case, was rolled as required, the scale was removed with fine emery paper, and the wire was gripped in split copper bushings. Although the wire had been straightened by hand bending none of the specimens were actually straight so that gripping in the torsion machine would introduce small static bending stresses which were neglected. Specimens Nos. 201 to 207 were prepared in the manner described and gripped in copper bushings having an end detail as shown in Fig. 15*c*. Specimens Nos. 201, 205, and 206 fractured clear of the grips while specimens Nos. 202, 203, and 204 broke at the grips. Specimen No. 207 ran 10.1 million cycles at 0 to 105,000 lb per sq in. stress. There was noticeable galling and probably penetration of the copper into the steel wire at points near the inner ends of the bushings where the pressure was greatest. Fatigue cracks started at such points. It was believed that the decarburized surface was particularly susceptible to this welding or seizing action and the next group of specimens Nos. 208 to 210, were prepared with the gripped portions reduced in a lathe to 0.183 in. by emery cloth previous to the surface-rolling operation. Fig. 15*c* indicates the tapered end of bushing and projection from the chuck as used for all three specimens. Specimen No. 208 was gripped in a steel bushing and broke at the grip. Specimens Nos. 209 and 210 were gripped in split copper bushings but were first wrapped with a 0.003-in. layer of a high-grade linen bond paper. The bushings had been bored to 0.185 in. diameter and therefore had to be forced onto the paper-covered specimen. Both specimens broke clear of the bushings. The paper was somewhat disintegrated at the inner ends of bushings, as is usual, due to small relative motion. There was also some fusing of copper to steel.

It is believed that with better distribution of gripping pressure most of the failures at the grips in these experiments might have been avoided. A good bond-paper filler will compensate for irregularities in the surface of the bushing bore, but it is also necessary that the gripping pressure be distributed radially around the wire surface rather than along two lines which are diametrically opposite to one another. To obtain this distribution, an undersized bushing bore was tried as mentioned, and in some cases a paper filler was placed between the outside of the bushing and the bored hole of the torsion-machine chucks. This had the effect of forcing-in the opposite edges of the bushing halves and thus improving the distribution of pressure.

Welding of the bushing material to the wire surface had troublesome consequences in the tests. Apart from providing points of weakness, it reduced the effective length of specimen between grips and thus, since the torsion machines are constant-strain machines, increased the range of stress applied to the specimen. The action was progressive and called for frequent strain adjustments to keep the stress range within reasonable limits. This effect was observed on the 0.187-in. wire gripped on the natural surface and when reduced to 0.183 in. by emery cloth and paper. It was also observed on the 0.225-in. wire ground between centers to a 0.221-in. diameter gripping seat when gripped in copper-lead bushings (specimen No. 351), but was not observed when gripped in steel or copper bushings (specimens Nos. 354 and 352). These

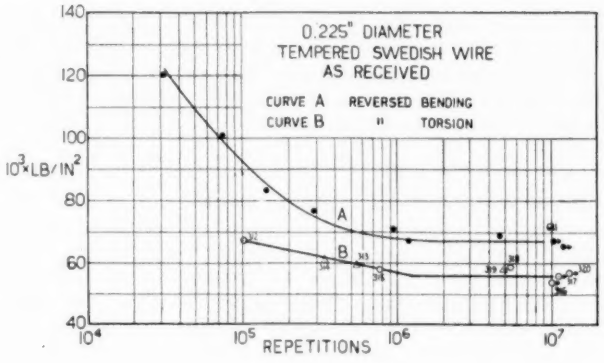


Fig. 17

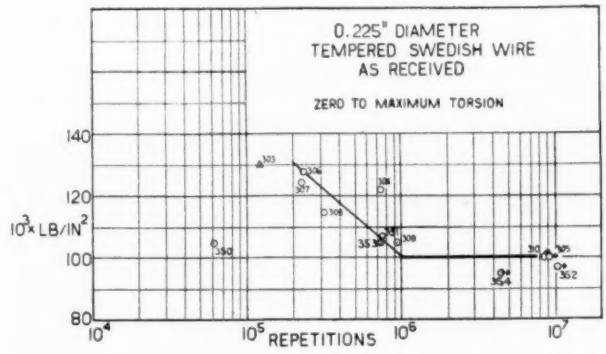


Fig. 18

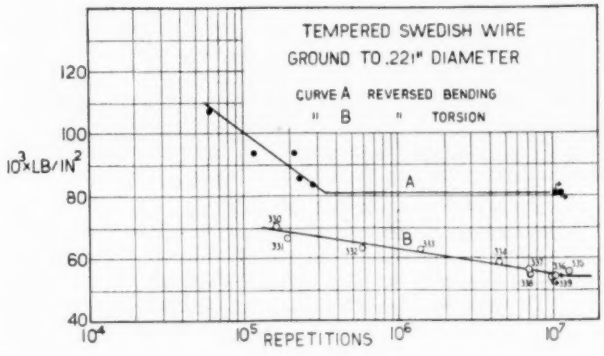


Fig. 19

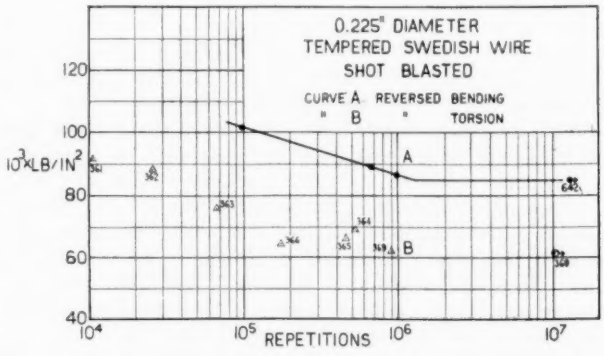


Fig. 20

TABLE 2 VARIATIONS IN STRESS RANGE IN SEVERAL OF THE TESTS

Specimen no.	Type grip	Variation in range
207	Copper bushing	Increase 6% in 2 million cycles
208	Steel bushing	Increase 3 to 5% in 0.2 million cycles
224	Monotype metal	± 7% in 3.6 million cycles
226	Monotype metal	Decrease 1.5% in 2.5 million cycles
364	Monotype metal	Decrease 8% in 0.5 million cycles
354	Steel bushing	Decrease 1% in 4.5 million cycles
351	Copper-lead metal	Increase 7.5% in 0.7 million cycles
352	Copper bushing	Decrease 4.5% in 2.2 million cycles
338	Copper bushing	Decrease 1.0% every 2.5 million cycles

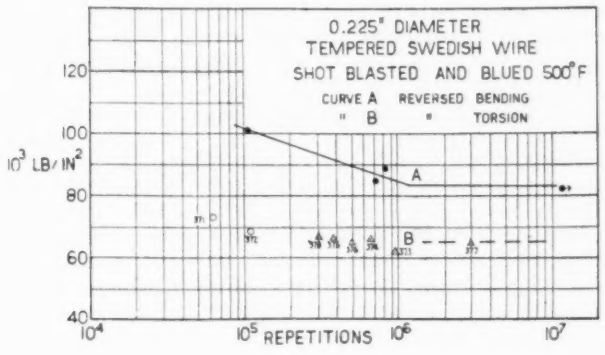


FIG. 21

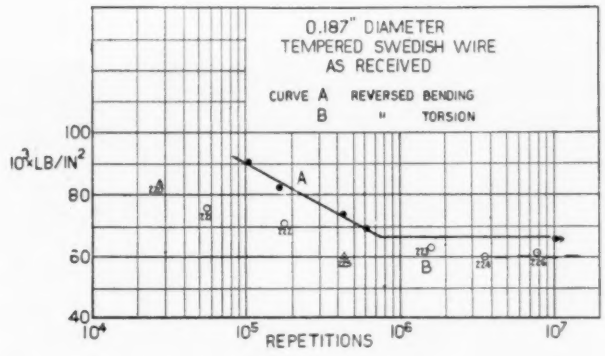


FIG. 22

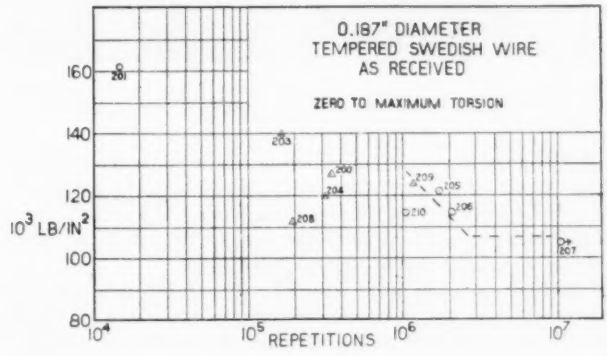


FIG. 23

latter results do not agree with Shelton and Swanger's (13) conclusion that copper-lead causes less galling of the surface than copper, and it may be, as mentioned previously, that the distribution of gripping pressure was the determining factor.

The stress range did not remain constant throughout the endurance test of any one specimen, although in some instances the variation was very small. In contrast with the phenomenon mentioned previously, some specimens gripped in split bushings showed a gradual reduction in stress range, probably due to a loosening of the wire at the inner ends of the bushings. The specimens with cast type-metal ends always showed a reduction in range which seemed to be due to relaxation under the gripping pressure, and consequent loosening in the chuck. This was corrected easily by tightening the gripping bolts. The effect was greatest at high stress ranges when specimens heated up considerably, but was quite small in tests near the fatigue limit. Regular attention and readjustment was thus necessary in all torsion tests to maintain stress ranges within satisfactory limits. The observed variations in stress range for some individual specimens are given in Table 2.

5—TEST PROCEDURE

The wire surface was examined by etching in a solution of equal amounts of concentrated hydrochloric acid and water maintained at 80 C. The wire was reduced in diameter in steps of about 0.0005 in. and examined at each stage until a reduction of about 0.004 in. had been made. The surface of all three sizes was exceptionally homogeneous and free from defects. With much deeper etching longitudinal streaks appeared which were probably not related to the original surface.

A seam about 0.001 in. deep, found in a photomicrograph of the 0.225-in. wire, shown in Fig. 9, could not be located in other sections and therefore probably did not extend a great distance in the wire.

Fractures were examined directly and at about 10 magnifications. In general, inclusions, if present at the point of origin of the fatigue crack, were too small to be seen, but their presence was often indicated by a slight distortion of the fractured surface at this region, as it is known that the value of stress and its direction in the region of a cavity are largely determined by the shape of the cavity. Some individual specimens which did not show up well in the fatigue tests showed failure to have begun at an inclusion or seam as, for example, specimens Nos. 350 and 221, shown in Figs. 25 and 27, respectively. A few cross-sections of failed specimens were polished and examined at 100 magnifications for cracks without success.

The clear length between grips of torsion specimens was determined by the maximum angular travel of the oscillating chuck, and the stress range desired for the individual specimen. Values of length L used in the tests were approximately as follows:

Wire diameter, in.	L
0.125	$^5/_8 \pm {}^1/_8$
0.187	$^3/_4 \pm {}^1/_8$
0.225	$^7/_8 \pm {}^1/_4$

Specimens were cut to a length of about $4^1/_2$ in. and prepared with either cast ends or split bushings as previously described.

In the early tests on 0.225 in. wire, it was used just as it came from the coil, having a slight natural curvature. In all later tests, for convenience, the wire was hand-straightened by being bent in opposition to this natural curvature. Residual stresses were no doubt present in both cases.

Care was taken not to scratch the wire in preparing specimens, as it was found that scratches were often, though not always, the points at which the deciding fatigue crack started.

Specimens were mounted in the testing machine which was started at approximately the desired stress range. By adjustment of the angular travel of the oscillating grip the desired stress range would be obtained in a few thousand stress cycles. Adjustments were made as required to maintain a fairly constant range.

In general, a point on the fatigue diagrams of Figs. 17 to 23 represent a stress cycle which did not vary more than ±5 per

cent. Error in reading torque was about 0.5 per cent, and the error in initial calibration is estimated at 0.5 per cent.

Torsion stresses were calculated by the formula

$$\text{Maximum shear stress} = \frac{5.1 \times (\text{maximum twisting moment})}{\text{Diameter}^3}$$

Bending stresses were calculated by the formula

$$\text{Maximum bending stress} = \frac{10.2 \times (\text{maximum bending moment})}{\text{Diameter}^3}$$

Because of its possible effect on the relative strength in torsion and bending fatigue, the presence of residual stress in the wire was verified experimentally. The method neglects any circumferential stress which may be present and, therefore, at best is only approximate. A flat surface about $^1/_{32}$ in. wide was ground on one side of a length of wire and the change in curvature of the wire noted. From the change in curvature and the calculated segmented area of the portion removed the longitudinal stress in the surface material can be found. A change of curvature in one direction indicates a tension stress, while a change in the opposite direction indicates a compression stress.

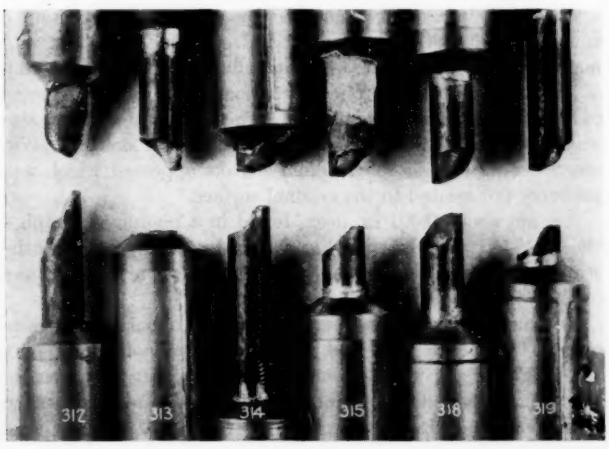


FIG. 24 TYPICAL FATIGUE FRACTURES OF 0.225-IN. WIRE TESTED IN ALTERNATING TORSION. WIRE USED IN AS-RECEIVED CONDITION

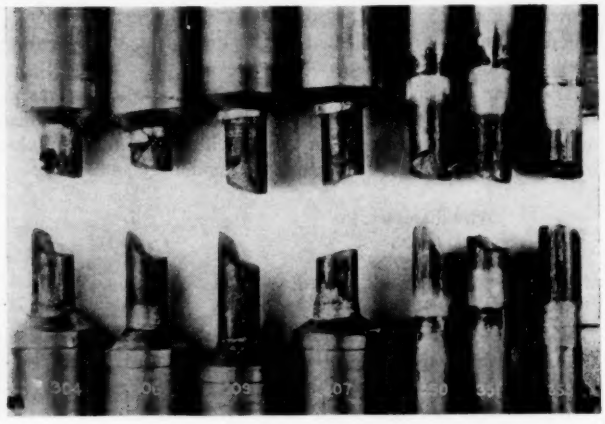


FIG. 25 TYPICAL FATIGUE FRACTURES OF 0.225-IN. WIRE TESTED IN ZERO-TO-MAXIMUM TORSION. WIRE USED IN AS-RECEIVED CONDITION

Measurements were made on straight pieces of wire in different conditions and on half-coils of wire specially prepared for the measurement of coiling stresses. Fig. 30 shows the device for measuring the coiling stresses with a half coil clamped in position and an ordinary micrometer arranged to measure deflections. Contact was determined by the use of ear phones and a battery circuit.

Where residual stress values are given, the possible error might easily be ±50 per cent although the figures given are averages of 2 to 4 separate determinations which usually agreed within ±25 per cent. Measurements of coiling stresses were made in a limited time and only relative values of stresses are available.

6—RESULTS OF TESTS

The results of tests on 0.225-in. and 0.187-in. wire are plotted as stress against the logarithm of the number of repetitions to failures on the S-N diagrams of Figs. 17 to 23. Fractures which occurred away from the ends are indicated by circles while specimens which did not fracture are indicated by a circle and an arrow. In some cases, since they provide useful information, fractures which occurred at or near the grips are included, and are indicated by triangles.

RESULTS OF TESTS ON 0.125-IN. WIRE AS RECEIVED

A complete series of tests was not made on 0.125-in. wire as received, but the results of tests on two specimens give some idea of the fatigue strength in a zero-to-maximum torsion stress cycle. One specimen without failure withstood 15 million repetitions of a stress cycle which at the beginning of the test was from zero to 125,000 lb per sq in., at 10 million zero to 107,000 lb per sq in., and at 15 million zero to 106,000 lb per sq in.

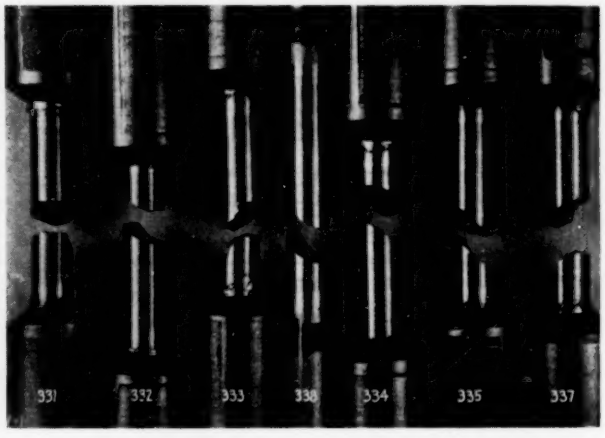


FIG. 26 TYPICAL FATIGUE FRACTURES OF 0.221-IN. CENTERLESS-GROUND WIRE TESTED IN ALTERNATING TORSION

Another specimen broke after 5.7 million repetitions of a stress cycle which at the beginning of test was from zero to 124,000 lb per sq in., at 0.26 million cycles it was from zero to 116,000 lb per sq in., and at the end of the test it was from zero to 113,000 lb per sq in.

From these two tests the zero-to-maximum torsion endurance limit for 0.125 in. wire as received may be estimated at something over 110,000 lb per sq in.

RESULTS OF TESTS ON 0.225-IN. WIRE AS RECEIVED

The bending fatigue results show little scatter, as seen from the curves in Fig. 17, and give a smooth curve, with the endurance limit at 67,000 lb per sq in. which is 33 per cent of the ultimate

strength of 204,000 lb per sq in. Reversed torsion tests show an endurance limit of 56,000 lb per sq in. which is 34 per cent of the ultimate torsional strength of 166,000 lb per sq in. Bending fractures shown in Fig. 29*a* were smooth and showed clean material. Torsion fractures also were all in homogeneous material, the crack starting at about 45 deg to the axis forming two roughly symmetrical fractured surfaces as shown in Fig. 24. Reversed torsion endurance limit is 0.84 times the reversed bending endurance limit.

Zero-to-maximum torsion tests showed an endurance limit of 100,000 lb per sq in. as observed from Fig. 18. Fractures of this specimen, shown in Fig. 25 were unsymmetrical and of a different type. The fatigue crack started at about 45 deg and curved around the specimen in circumferential and longitudinal directions. Specimen No. 350 gave a point well off the curve and the position of the surface flaw responsible for failure can be seen in Fig. 25. Fractures otherwise showed clean material.

Tests of 0.225-In. Wire Reduced to 0.221 In. in a Centerless Grinder

This series of tests was run to determine the effect of a surface-grinding treatment which might be commercially practicable. A single pass across a 60-grit wheel was used. The surface though smooth in appearance was considerably coarser than a surface polished with No. 1 emery paper. It was found that due to slight curvature of the wire before grinding, a variable thickness

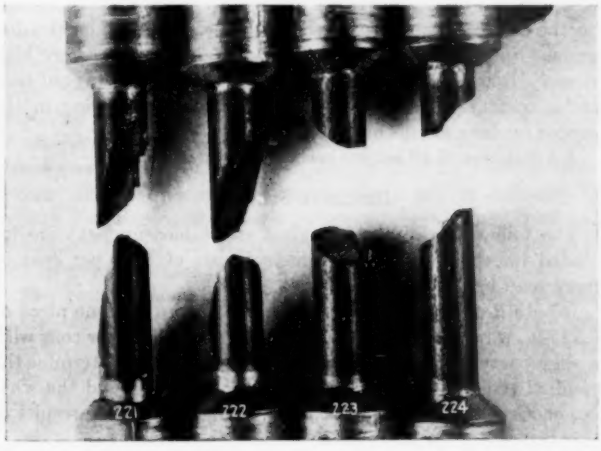


Fig. 27 Typical Fatigue Fractures of 0.187-In. Wire Tested in Alternating Torsion. Wire Used in As-Received Condition

of layer was removed. Only specimens which had "cleaned up" all over were tested.

Fatigue curves for this test are shown in Fig. 19. The bending endurance limit is 81,000 lb per sq in., an increase of 21 per cent over the value for wire as received, in spite of the fact that grinding marks are circumferential. Bending fractures of the specimens used in this test, shown in Fig. 29*b*, are seen to be slightly stepped and ragged, and are not as clean cut as for the as-received wire.

The endurance limit in reversed torsion from the curve in Fig. 19 is 54,000 lb per sq in., which is 4 per cent lower than the value for as-received wire and 0.67 times the reversed bending endurance limit of ground wire. Fractures occur at 4½, 6, and 13 million cycles, and the curve does not become horizontal until about 10 million cycles are reached.

The "delayed fractures" of French (34) and Jünger (35) are suggested by these results. French found that fractures occurred after 10 million cycles in nickel steels harder than about 45–50 Rockwell *C*, but not in steels with lower Rockwell values. Removal of the soft outer surface may leave a surface which is in the hardness range where delayed fractures occur, and the usual test to 10 million cycles is not sufficient. Also the grinding marks might provide stress concentrations and cause delayed fractures such as Jünger observed. These effects are not present in the bending fatigue curve, however.

An examination of the fractured torsion specimens showed that a cross-section of specimen No. 330 revealed no cracks or seams. The fractured surface was stepped and a crack extended at about 45 deg into the metal, suggesting the presence of an inclusion. Specimens Nos. 332, 333, 334, 335, and 337, Fig. 26, showed slight irregularities on the fractured surface at the region of crack formation which might have been flaws, whose effect was accentuated in some manner by the grinding.

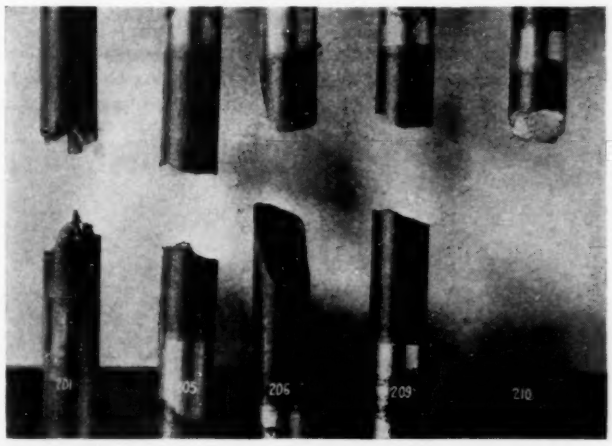


Fig. 28 Fatigue Fractures of 0.187-In. Wire Tested in Zero-to-Maximum Torsion. Wire Used in As-Received Condition

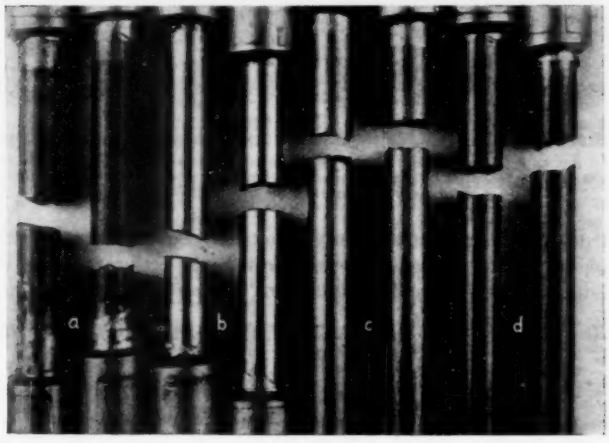


Fig. 29 Typical Bending Fatigue Fractures of 0.225-In. Wire. (*a*) As-Received Condition; (*b*) Ground to 0.221 In. Diameter; (*c*) Shot-Blasted Condition; (*d*) Shot-Blasted and Blued Condition

Results of Tests on 0.225-In. Shot-Blasted Wire

Wire in the as-received condition was shot-blasted in a standard revolving drum for 25 min with air at a pressure between 70–75 lb per sq in.

The reversed bending endurance limit as shown by the curve in Fig. 20 was 85,000 lb per sq in. which is 27 per cent higher than for wire as received. Bending fractures of the specimens are shown in Fig. 29*c*.

TABLE 3 BENDING AND TORSION ENDURANCE VALUES FOR TEMPERED SWEDISH VALVE-SPRING WIRE[b]

Diameter, in.	0.125[b]	0.225[b]	0.221[c]	0.225[d]	0.225[e]	0.187[a]
Reversed bending endurance limit, lb per sq in.	.....	67,000	81,000	85,000	83,000	67,000
Per cent of ultimate tensile strength	.....	32.8	.....	.....	.....	31.8
Reversed torsional endurance limit, lb per sq in.	.....	56,000	54,000	61,500+	65,000+	60,000
Per cent of ultimate torsional strength	.....	33.7	.....	.....	.....	37.0
Ratio of reversed torsional stress to reversed bending stress	.....	0.84	0.67	0.72+	0.78+	0.90
Zero-to-max. torsional stress endurance limit, lb per sq in.	111,000+	100,000	.....	.....	.....	107,000
Ratio of zero-to-max. torsional endurance limit to reversed bending stress	.....	1.49	.....	.....	.....	1.60
Ultimate tensile strength, lb per sq in.	226,000	204,000	.....	.....	.....	211,000
Ultimate torsional strength, lb per sq in.	166,100	166,000	.....	.....	.....	162,000

[a] Based upon 10 million stress reversals.
[b] Wire tested in as-received condition.
[c] Wire ground to size from 0.225 in. diameter with No. 60 grit.
[d] Wire shot-blasted before testing.
[e] Wire shot-blasted and blued before testing.

All torsion fractures occurred at or near the grips and these are shown on the diagram in Fig. 20 by triangles. One specimen ran 10 million cycles at 61,500 lb per sq in. without fracture. The

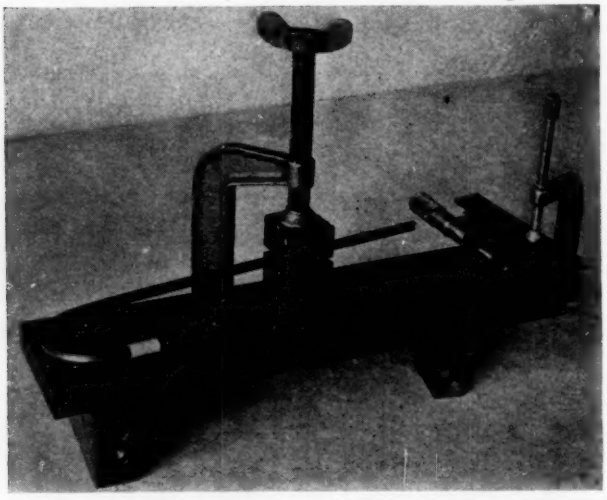


FIG. 30 DEVICE FOR MEASURING COILING STRESSES IN A HALF-COIL
(A flat spot is ground on the inner surface of the coil and the deflection of the arm is measured by a micrometer. Earphones and a battery circuit are used to record contact at the micrometer.)

true fatigue curve will lie above the triangular points in Fig. 20 representing fracture at the grips so that the endurance limit is estimated to be an unknown amount above 61,500 lb per sq in.

RESULTS OF TESTS ON 0.225-IN. SHOT-BLASTED-AND-BLUED WIRE

After shot-blasting the specimens were blued at 500 F for 30 minutes in a Homo furnace and cooled in the furnace.

The bending-endurance limit shown by the curve in Fig. 21 is 83,000 lb per sq in. which is about 24 per cent greater than for wire as received. Fractures were slightly irregular but clean, and are shown by the specimens in Fig. 29*d*.

Only two reversed-torsion specimens fractured clear of the grips although some of the other fractures started clear of the grips and extended to them. None of the specimens ran 10 million cycles without fracture, but the endurance limit is probably above 65,000 lb per sq in.

RESULTS OF TESTS ON 0.187-IN. WIRE AS RECEIVED

The bending fatigue limit for 0.187-in. wire as-received is shown on the curve in Fig. 22 to be 67,000 lb per sq in., which is about 32 per cent of the ultimate strength of 211,000 lb per sq in.

Seven points are shown for the reversed-torsion endurance curve, two of which represent failure at the grips. No specimen ran through 10 million cycles without failure, up to the time of making the diagram, but the endurance limit is very probably at 60,000 lb per sq in., which is 0.90 times the reversed bending endurance limit, and 37 per cent of the ultimate torsional strength.

Some torsion fractures of the 0.187-in. specimens are shown in Fig. 27. Specimen No. 221 failed at a longitudinal surface flaw or inclusion about 0.04 in. long. Two diagonal cracks are seen extending from the points of stress concentration at each end of the flaw.

The curve for zero-to-maximum torsion tests on 0.187 in. wire is shown in Fig. 23. One specimen was unbroken after 10 million reversals at 105,000 lb per sq in. A greater number of significant points was not obtained because of difficulties encountered with seizing of the copper-bushing to the wire thereby increasing the stress range in an erratic manner. The endurance limit is estimated at 107,000 lb per sq in. Specimen No. 210 failed clear of the grips but a radial crack about 0.03 in. long could be seen on the fractured surface, which explains the position on the diagram. Specimen No. 201 was accidentally run at a very high stress. Fracture started at many points forming a ragged cone and a conical hollow. There was considerable slipping in the copper bushing at this high stress.

A tabulation of all fatigue results is given in Table 3.

RESIDUAL STRESSES

The following results of residual stress measurement are included for their suggestive value. Errors of ±50 per cent or more may be present.

(*a*) *As Received.* One piece of 0.187-in. wire and one piece of 0.225-in. wire, just as they came off the 36-in. diameter coils with a slight permanent set or curvature, were used to determine the residual stress along four edges spaced 90 deg around the wire. No check measurements were made in this case. The results of this test are given in Table 4.

TABLE 4 LONGITUDINAL RESIDUAL STRESSES IN WIRE AS RECEIVED[a]

Diameter of wire, in.	0.187	0.225
Kind and location of stress:		
Inside edge of coil, tension	180,000	90,000
Outside edge of coil, compression	70,000	120,000
Flat side (1), tension	40,000	30,000
Flat side (2), tension	15,000	20,000

[a] All values in lb per sq in.

The results in Table 4 show very well that even coiling to 36 in. diameter leaves large tensile stresses at the inside of the coil and corresponding compression stresses at the outside. The averages of the values measured along the two flat sides are approximately 25,000 lb per sq in. in both cases, which is probably not sufficient to materially affect fatigue strength (29).

(*b*) *Ground 0.221-In. Diameter Wire.* Measurements on three samples of this wire were erratic and gave compressive longitudinal stresses from 0 to 8000 lb per sq in. The fact that an uneven thickness of layer was removed will explain the variation in stress measurements.

(*c*) *Shot-Blasted 0.225-In. Diameter Wire.* Measurements on three samples gave values of longitudinal compressive stress which were within ±12 per cent of 165,000 lb per sq in.

(*d*) *Shot-Blasted and Blued 0.225-In. Diameter Wire.* Three measurements gave values of longitudinal compression within ±14 per cent of 27,600 lb per sq in. The bluing treatment reduced shot-blasting stresses to one-sixth their value.

(*e*) *Measurements on Fatigue Specimens.* Bending specimen No. 611, made of 0.225-in. diameter as-received wire, after 10 million reversals at 67,000 lb per sq in., showed an average value of longitudinal tension stress, for four sides of the wire of 73,000 lb per sq in. No other specimens were available to check this result which would indicate that residual tension stresses from tempering do not disappear under fatigue loading, and may, therefore, have an effect on the fatigue strength.

Bending specimen No. 642 of shot-blasted wire had run 13 million reversals at 85,000 lb per sq in. The only two measurements obtained on this specimen were zero and 56,000 lb per sq in. longitudinal compressive stress. No other specimens were available for check measurements. It would appear that the heavy compressive stress due to shot-blasting does not remain under fatigue stressing and consequently tests to 20 or 30 million cycles might give lower fatigue values than those obtained.

(*f*) *Coiling Stresses.* After coiling as-received wire in a 2-in. diameter coil, heavy residual longitudinal tensile and compressive stresses were present on the inside and outside of the coil, respectively. Bluing reduced these stresses to about one-third their original value, but they were still appreciable in magnitude.

7—CONCLUSION

The zero-to-maximum torsion endurance limits of 110,000, 107,000, and 100,000 lb per sq in. for as-received wire are about 70 per cent higher than the values of 68,000 and 58,000 lb per sq in. estimated from the results of experiments on completed springs of similar material by F. P. Zimmerli (12). Residual tension stresses at the inside of the coils, even if they were shown to be present after the bluing treatments, would not be sufficient to explain such large differences in fatigue strengths, as Buehler and Buchholtz' results (29) show variations of only 15 or 20 per cent due to internal stress.

The results given in this paper may be considered as ideal values as it is possible to attain if no serious flaws are present. The twelve 0.225-in. specimens tested in zero-to-maximum torsion, the results of which are plotted in Fig. 18, represent a total length of not over 12 in. of wire actually submitted to the repeated stressing. In a single complete spring, possibly 40 in. of wire would be under test and if there were only one flaw in each spring such as that in specimen No. 350, indicated in Fig. 18, it is obvious that lower fatigue values would be obtained in the tests on complete springs. The only remedy for such a condition would be the elimination of all serious flaws in the material.

Comparison may also be made with Swan, Sutton, and Douglas torsion results (11) on straight specimens of 0.182-in. diameter tempered Swedish wire with natural surface. The present zero-to-maximum value of 110,000 lb per sq in. for 0.125 in. wire is about 70 per cent higher than the results of their tests which were made at a ratio of minimum-to-maximum stress of 1:4.

The consistently high values of torsion endurance limit obtained for three sizes of wire in the experiments reported in this paper might suggest the presence of some consistent error in the work. However, the tests were made on three machines which were separately calibrated statically; calibration factors agreed closely and test results for the same wire on different machines were consistent among themselves.

Although ratios of reversed-torsion fatigue strength and of zero-to-maximum torsion fatigue strength to reversed-bending fatigue strength have been obtained for 0.187 in. and 0.225 in. wires and given in Table 3, it has not been possible to correlate these ratios with the factors which largely determine them, such as the shape of inclusions and of surface flaws. The extent and relative effect on bending and torsion fatigue strengths of anisotropy due to the drawing process has not been investigated. The effect of residual stress, if present, on the ratio of bending and torsion fatigue strengths would appear to call for investigation. Until information is available in the directions indicated it is not apparent that the bending fatigue strength of a wire will be a certain measure of the torsion fatigue strength of that wire.

Acknowledgment

The tests reported in this paper were sponsored by the A.S.M.E. Special Research Committee on Mechanical Springs, and were carried out through the Engineering Research Department of the University of Michigan, Ann Arbor. Acknowledgment is here made to members of the subcommittee, in particular to F. P. Zimmerli, to Profs. A. E. White and E. L. Eriksen for assistance of various kind, and to Prof. S. Timoshenko for helpful criticism and suggestions. Full responsibility for results or statements, however, is taken by the author.

Bibliography

1 "Properties of Cold-Drawn Wires With Particular Reference to Repeated Torsional Stresses," by F. C. Lea and R. A. Batey, Proceedings of the Institution of Mechanical Engineers, vol. 2, 1928, pp. 865–899.

2 "Torsional Fatigue Tests of Cold-Drawn Wires," by F. C. Lea and J. Dick, Proceedings of the Institution of Mechanical Engineers, vol. 120, 1931, pp. 661–667.

3 "The Mounting of Wire for Fatigue Testing," by R. Goodacre, *Engineering*, vol. 137, May 4, 1934, p. 503.

4 "Type-Metal Alloys," by F. D. Weaver, *Journal of the Institute of Metals*, paper no. 691, vol. LVI, 1934 (54) (advance copy).

5 "Penetration of Steel by Soft Solder and Other Molten Metals at Temperatures Up to 400° C," by L. J. G. van Ewijk, *Journal of the British Institute of Metals*, vol. 56, 1934. (Presented March 6, 1935.)

6 "Dās Drücken der Oberfläche von Bauteilen aus Stahl" (Rolling of the Surface of Steel Construction Elements), by O. Föppl, *Stahl und Eisen*, vol. 49, April 25, 1929, p. 575.

7 "Surface Rolling for Raising the Torsion Fatigue Strength," by P. Behrens, Braunschweig, Technische Hochschule, Wöhler Institute, heft 6, 1930.

8 "Der Einflub von Einspann- und Kraftangriffsstellen auf die Dauerhaltbarkeit der Konstruktionen" (The Effect of Press Fits on Fatigue Strength of Parts), by H. Thum and F. Wunderlich, *Zeit. V.D.I.*, vol. 77, no. 31, 1933, pp. 851–853.

9 "Fatigue Tests of Specimens With Cold-Worked Surface," by O. J. Horger, presented at the National Applied Mechanics Meeting of The American Society of Mechanical Engineers held at Ann Arbor, Mich., June 18–35, 1935.

10 "Fatigue of Shafts at Fitted Members," by R. E. Peterson and A. M. Wahl, Trans. A.S.M.E., *Journal of Applied Mechanics*, vol. 2, no. 1, March, 1935, p. A1.

11 "An Investigation of Steels for Aircraft-Engine Valve Springs," by A. Swan, H. Sutton, and W. D. Douglas, Proceedings of the Institution of Mechanical Engineers, vol. 120, 1931, pp. 261–299.

12 "Permissible Stress Range for Small Helical Springs," by F. P. Zimmerli, Department of Engineering Research, University of Michigan, Bulletin No. 26, 1934.

13 "Fatigue Properties of Steel Wire," by S. M. Shelton and W. H. Swanger, U. S. Bureau of Standards, *Journal of Research*, vol. 14, 1935, research paper RP 754, pp. 17–32.

14 "Torsional Fatigue Tests on Spring Steels," by G. A. Hankins, Great Britain Department of Scientific and Industrial Research, engineering research special report no. 9, 1928.

15 "Wechseltorsionsversuche an Zinkkristallan," by W. Fahrenhorst and E. Schmid, *Mitteilungen der deutschen Materialprüfungsanstalten*, sonderheft 19, Julius Springer, Berlin, 1932, pp. 36–41.

16 "Notes on Some Problems of Fatigue of Metals," by H. J. Gough, Proceedings of the Fourth International Congress for Applied Mechanics, Cambridge, Eng., 1934, p. 279.

17 "Theories of Strength," by A. Nadai, Trans. A.S.M.E., vol. 55, 1933, paper APM-55-15, p. 111.

18 "Wechselbeanspruchung von Rohren unter Innendruck" (Fatigue Loading of Tubes Under Internal Pressure), by A. F. Maier, *Stahl und Eisen*, vol. 54, 1934, p. 1289.

19 "The Fatigue of Metals," by H. F. Moore and J. B. Kommers," McGraw-Hill Book Company, New York, N. Y., 1927, p. 147.

20 "The Fatigue of Metals," by H. J. Gough, D. Van Nostrand Company, 1924, p. 298.

21 "Some Aspects of the Fatigue Properties of Patented Steel Wire," by E. T. Gill and R. Goodacre, *Engineering*, vol. 138, 1934, p. 692.

22 "Torsional Fatigue Tests of Cold-Drawn Wires," by F. C. Lea and J. Dick, Proceedings of the Institution of Mechanical Engineers, vol. 120, 1931, p. 661.

23 "On the Concentration of Stress in the Neighborhood of a Spherical Flaw," by R. V. Southwell and H. J. Gough, *Philosophical Magazine*, vol. 1, series 7, 1926, p. 71.

24 "Concentration of Stress Around Spherical and Cylindrical Inclusions and Flaws," by J. N. Goodier, Trans. A.S.M.E., vol. 55, 1933, paper APM-55-7, pp. 39–44.

25 "Stresses in a Plate Due to the Presence of Cracks and Sharp Corners," by C. E. Inglis, Transactions of the Institution of Naval Architects, vol. 55, part 1, 1913, p. 219.

26 "Die Kerbempfindlichkeit der Workstoffe" (Notch Sensitivity of Materials), by W. Buchmann, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 5, no. 1, 1934, p. 36.

27 "Stress Concentration Phenomena in Fatigue of Metals," by R. E. Peterson, Trans. A.S.M.E., vol. 55, 1933, paper APM-55-19, p. 157.

28 "Entstehung von Rissen in Stahl bei der Wärmebehandlung" (Origin of Cracks in Steel in Heat-Treatment), by E. Scheil, *Archiv für das Eisenhüttenwesens*, vol. 8, no. 7, January, 1935, pp. 309–314.

29 "Die Wirkung von Eigenspannungen auf die Biegeschwingungsfestigkeit" (The Effect of Internal Stresses on Bending Fatigue Strength), by H. Buehler and H. Buchholtz, *Stahl und Eisen*, vol. 53, no. 51, 1933, p. 1330.

30 "Zusammenhang Zwischen Wärmespannungen und Festigkeitseigenschaften von Stahl" (Relation Between Temperature Stresses and Strength Properties of Steel), *Archiv für das Eisenhüttenwesen*, vol. 6, no. 8, 1932–1933, p. 335.

31 "Zusammenwirken von Wärme- und Umwandlungsspannungen in abgeschreckten Stählen" (Combined Effect of Temperature and Transformation Stresses in Quenched Steels), by H. Buehler and E. Scheil, *Archiv für das Eisenhüttenwesen*, vol. 6, no. 7, 1932–1933, p. 283.

32 "Versuche über die Eigenschaften gezogener Drähte und den Krafbedarf beine Drahtziehen" (Research on the Properties of Drawn Wire), by W. Linicus and G. Sachs, *Mitteilungen der deutschen Materialprüfungsanstalten*, sonderheft 16, Julius Springer, Berlin, 1932, pp. 38–66.

33 "An Investigation Into Some Effects of Cold Drawing on the Strength and Endurance of Mild Steel," by R. M. Brown, Transactions of the Institution of Engineers and Shipbuilders of Scotland, vol. 71, 1928, p. 495.

34 "Fatigue and Hardening of Steels," by H. J. French, Trans. A.S.S.T., vol. 21, 1933, pp. 899–946.

35 "Einflub von Querbohrungen auf die Dauerfestigkeit eines vergüteten Chrom-Molybdänstahles" (Effect of Lateral Holes on the Fatigue Strength of a Hardened Chrome-Molybdenum Steel), by A. Jünger, *Mitteilungen aus den Forschungsanstalten* G. H. H. Konzern, vol. 3, no. 2, June, 1934, pp. 29–32.

Evaluation of Effective Radiant Heating Surface and Application of the Stefan-Boltzman Law to Heat Absorption in Boiler Furnaces

By H. F. MULLIKIN,[1] NEW YORK, N. Y.

In the first section of this paper a method has been developed for converting well-known types of furnace heat-absorbing surfaces into an equivalent effective radiant heating surface. It concerns the evaluation of this effective radiant heating surface as controlled by (1) various spacings of waterwall tubes, (2) various kinds of blocks on the waterwall tubes, (3) various amounts of slag on the waterwall tubes or blocks, and (4) the emissivity of the waterwall surface exposed to the flame. The second section deals with the fraction of heat supplied to the furnace that is given up by the gases in the furnace which is solved most simply and practically by the use of the Stefan-Boltzman radiation law.

Results computed in this manner have been compared with results reported in a paper entitled, "An Experimental Investigation of Heat Absorption in Boiler Furnaces."[2] It is found that for large pulverized coal, and oil-fired and gas-fired furnaces, the assumption of an effective furnace emissivity of unity leads to results which are in substantial agreement with experimental data. For stoker-fired furnaces in general, the effective furnace emissivity appears to be less than unity.

1—EFFECTIVE RADIANT HEATING SURFACE

OF THE number of methods offered for the evaluation of effective radiant heating surface in steam-boiler furnaces, the most logical method appears to be that involving in part the work of Hottel.[3,4] Briefly this method is based on the fact that radiant-heat transmission is governed by the laws of absorption and reflection. Thus, Lambert's cosine law is applicable, that is, a rough or corrugated "black" surface will not receive any more radiant heat than a similar plane or flat surface. Although this principle has been contested by some investigators it is generally accepted.

In this work we are interested in the areas in so far as they effect the radiant-heat transfer. Therefore, we may adopt as a standard unit of area, a square foot of effective radiant heating surface (water- or steam-cooled surface) which absorbs a maximum amount of radiant heat. If our premises are based on the Stefan-Boltzman law this unit square foot of radiant heating surface must necessarily be defined as a square foot of plane surface, perfectly clean, having an emissivity of unity, and so cooled by water or steam that its temperature is not greater than that of the water or steam which is cooling it and absorbing heat.

Radiation to this unit surface from a black body or unity emissive surface at various temperatures will be as shown in Fig. 1 which was derived from the Stefan-Boltzman law. This curve does not change appreciably for temperatures of the absorbing radiant heating surface varying from 0 to 1000 F abs. Hence, the standard unit square foot of effective radiant heating surface may be further defined as that square foot of surface which would absorb an amount of heat as indicated by Fig. 1 when receiving radiation from a black-body source at the given temperature.

Using this latter definition, any area having less radiant-heat absorption per square foot may be converted into an equivalent smaller area having the defined unit radiant-heat absorption per square foot by the use of multiplying terms which take into account the divergence of the conditions of the actual surface from those of the theoretically defined surface.

In practice, radiant-heat surfaces differ from the defined unit surface in that they are formed of metal tubes, often spaced at intervals, separated by refractory, and perhaps are covered by steel, cast-iron, or refractory-faced blocks. Various amounts of slag will form on the surfaces thus causing them to be less effective in absorbing heat. In addition the surfaces will have an emissivity less than unity.

If A is the projected overall waterwall area under these latter conditions and A_{rhs} is the effective radiant heating surface as defined, then

$$A_{rhs} = AF_AF_CF_SF_E \quad [1]$$

where F_A is an area factor, F_C is an average conductivity factor, F_S is a slag factor, and F_E is an emissivity factor.

The area factor F_A is used to reduce overall surface to an equivalent effective radiant surface, assuming the surfaces clean and of perfect heat conductivity. The average conductivity factor F_C is used to take into account the fact that for refractory-

[1] Research Fellow, Superheater Company—Yale University Fellowship. Jun. A.S.M.E. Dr. Mullikin was graduated from Baltimore Polytechnic Institute and Johns Hopkins University, and is the recipient of graduate degrees, including Ph.D. at Yale. He spent a year in the test department on boiler development at the General Electric Company.

[2] "An Experimental Investigation of Heat Absorption in Boiler Furnaces," by W. J. Wohlenberg, W. A. Armacost, C. W. Gordon, and H. F. Mullikin, Trans. A.S.M.E., vol. 57, 1935, paper RP-57-4.

[3] "Radiant Heat Transmission Between Surfaces Separated by Non-Absorbing Media," by H. C. Hottel, Trans. A.S.M.E., vol. 53, 1931, paper FSP-53-19b, p. 265.

[4] "Radiant Heat Transmission," by H. C. Hottel, *Mechanical Engineering*, vol. 52, 1930, p. 699.

Contributed by the Special Research Committee on Absorption of Radiant Heat in Boiler Furnaces for presentation at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS to be held in New York, N. Y., December 2 to 6, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

faced blocks the heat absorbed by the waterwall is less than for blocks consisting of a material of perfect heat conductivity. That is, if a refractory face on a 1-sq ft block diminishes the heat transfer one-half, this surface is equivalent to 0.5 sq ft of defined effective radiant heating surface. It can be shown that this F_C-factor varies with furnace temperature. In this paper, however, a mean value for the average furnace temperature has been assumed. The value will be sufficiently correct for the

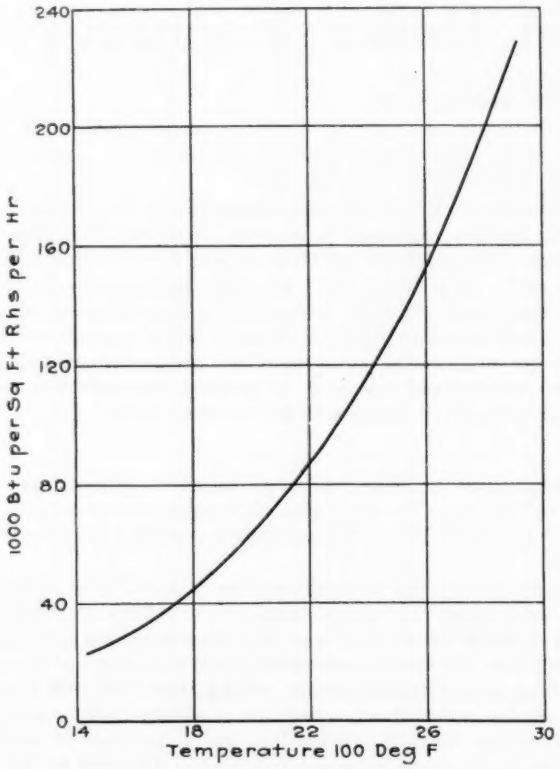


FIG. 1 RADIATION BASED ON THE STEFAN-BOLTZMAN LAW. EMISSIVITY = 1

present until more data becomes available. The slag factor F_S is similar to F_C, but is dependent for its value on the thickness of slag present. The emissivity factor F_E is used to take into account the fact that the emissivity of the surface is less than unity.

Thus, effective radiant heating surfaces are computed on a basis of radiant-heat-absorbing ability, and all furnace waterwalls, no matter what their construction, may be reduced to a number of defined unit effective radiant heating square feet of surface by suitable selection of correction factors.

AREA FACTOR F_A

The selection of these factors must be based upon existing data. Values of F_A may be mathematically calculated for all shapes and constructions of waterwalls. Fig. 2 shows curves taken from Hottel[3] for various bare-tube arrangements. Ordinarily curve 2, Fig. 2, is the only curve of those given that is used to determine F_A in boiler furnaces. The curves hold without significant error for all values of the dimension T for one row of tubes from zero to infinity.

The foregoing is applicable to completely bare tubes. Occasionally it is necessary to obtain F_A-values for bare tubes half recessed in refractory. It is necessary to solve for this separately using the method of Hottel as given in Fig. 4 of his first paper,[3] which will not be dealt with here. However, it is readily possible to draw up a curve similar to Fig. 2 for this special case.

Fin tubes may be handled in the same way when recessed in refractory and when the fins do not overlap or touch each other as is sometimes the case. Here the edges of the fins are to be considered as the heat-absorbing surface extremities. If these fin tubes are located at a distance from a refractory wall, Fig. 2 would be used. The heat-transfer coefficient of the metal of the fins is assumed to be sufficiently great so that the fins absorb as much heat per unit of exposed area as the tube wall does. This assumption is open to question but is adopted in view of the lack of data indicating otherwise. On this basis a fin tube has an effective diameter equal to the distance between the extreme edges of the fins. Where the tubes or fins touch each other, F_A must equal unity as L/D in Fig. 2 will equal unity.

For reasons similar to those stated for the fins of fin tubes, bolted-on or shrunk-on metal blocks with either plain or refractory faces have been assumed to have an effective area equal to the area of the face of the block. Where the blocks are adjacent to each other, F_A is equal to unity. The reduction in heat transfer caused by the joint between the block and the tube is taken into account in the F_C-factor.

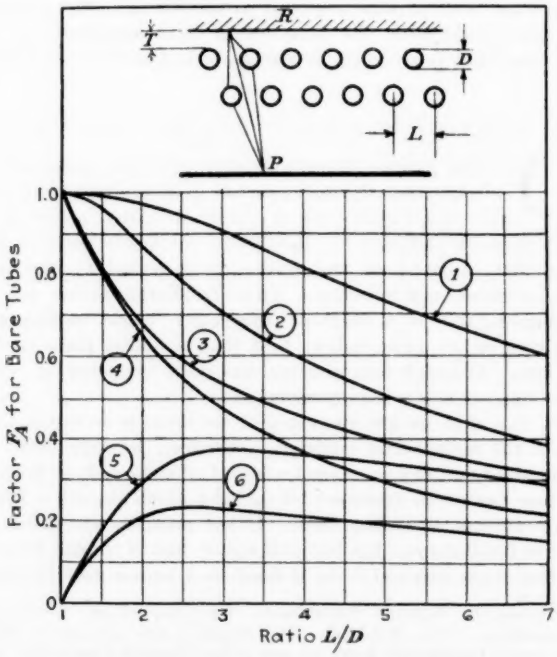


FIG. 2 VALUES OF THE F_A-FACTOR FOR BARE TUBES. THESE VALUES HOLD FOR ALL VALUES OF T WITHOUT APPRECIABLE ERROR

(Curve 1—total to two rows when two are present. Curve 2—total to one row when only one is present. Curve 3—total to first row when two are present. Curve 4—direct to first row. Curve 5—total to second row when two are present. Curve 6—direct to second row. R denotes refractory. P denotes radiating plane. When two rows are present they are spaced so that the tubes form equilateral triangles. The tubes are considered to be perfect absorbers. The curves give the fraction (F_A) of total plane incident radiation absorbed either by the total tube, including reradiation, or by the tube directly, excluding reradiation by the refractory. For ordinary boiler-furnace calculations curve 2 only is used.)

The factor F_A is most simply defined as that factor by which it is necessary to multiply projected overall clean waterwall surface (including perfect conducting tubes and exposed refractory between the tubes) in order to obtain the effective exposed radiant surface. This effective surface is not appreciably different from the projected tube surface except where there are reradiating refractory walls in back of the waterwall tubes.

Conductivity Factor F_C

The conductivity factor F_C is introduced to take care of the heat resistivity of the wall. This factor is necessary because, although exposed areas of a bare metal block and a refractory-faced metal block may be equal, i.e., F_A is identical, these areas will absorb different amounts of heat by radiation from the furnace. Therefore, F_C is a ratio between the actual heat conducted and the maximum amount of heat that could be conducted, other conditions remaining constant.

E. G. Bailey[5] has given some figures indicating that the comparative heat-transfer rates of type-*BAG* refractory-faced blocks and type-*C* bare-faced blocks are 50,000 and 70,000 Btu per sq ft per hr. Thus F_C would be equal to 50,000/70,000 or 0.71. Other published material[6] and experimental data which cannot be included here would indicate that this value of the F_C-factor is somewhat high. After careful consideration of the meager data available the values of F_C in Table 1 were adopted.

TABLE 1 CONDUCTIVITY FACTORS

Type of surface	F_C
Bare tubes	1.00
Fin tubes	1.00
Bare-faced metal blocks on tubes	0.70
Refractory-faced metal blocks on tubes, type-*BAG*	0.35

Theoretically, F_C varies with furnace temperature but this correction is omitted as the present knowledge of the effect of refractory on waterwalls is so scanty as to render small corrections unimportant. There is urgent need of laboratory experiments to establish the F_C-values more correctly than the approximations given in this paper.

Slag Factor F_S

The slag factor F_S is introduced to take care of the reduced heat absorption of waterwalls due to accumulated slag. It is similar to the conductivity factor in its variation with furnace temperature, but in view of the meager data this correction seems superfluous. When the boiler furnace is first operated, F_S is equal to unity, it being assumed that the surfaces are uncoated with slag. During the course of time slag will build up on the waterwall surface until a certain average equilibrium thickness is reached. Therefore, it is possible to compute by the use of F_S the difference in heat-absorbing capacity of a boiler furnace when initially started and when operated under known conditions of wall cleanness.

The literature is almost devoid of data on this phase of the subject. Some information was obtained from two boiler tests[7] run under the direction of the A.S.M.E. Radiation Committee. In a stoker-fired boiler, in which the waterwall tubes comprise a separate screen boiler, records were available to show the amount by which the proportional evaporation of the screen boiler had been reduced from five days after initial operation up to 18 months after initial operation. While reliable records over such a period of time were not available for a pulverized-fuel-fired boiler having a similar separate screen boiler, a separate test was run on this boiler for which the slag was completely cleaned off of the side-wall tubes and the tubes wire-brushed to assure cleanness. Fig. 3 has been plotted from the results and is used to determine this slag factor. The derivation of these curves and the special case where bare tubes are located entirely in front of refractory are discussed in the Appendix to this paper.

[5] "Some Factors in Furnace Design for High Capacity," by E. G. Bailey, Trans. A.S.M.E., vol. 49–50, 1927–1928, paper FSP-50-78, p. 253.

[6] "Water-Cooled Furnaces for Underfeed Stokers," by O. De Lorenzi, *Combustion*, vol. 3 (new notation), May, 1932, p. 15.

Emissivity Factor F_E

There are very few data upon which the value of the emissivity factor F_E may be based for conditions as they exist within furnaces. This factor, as used by some investigators, was not

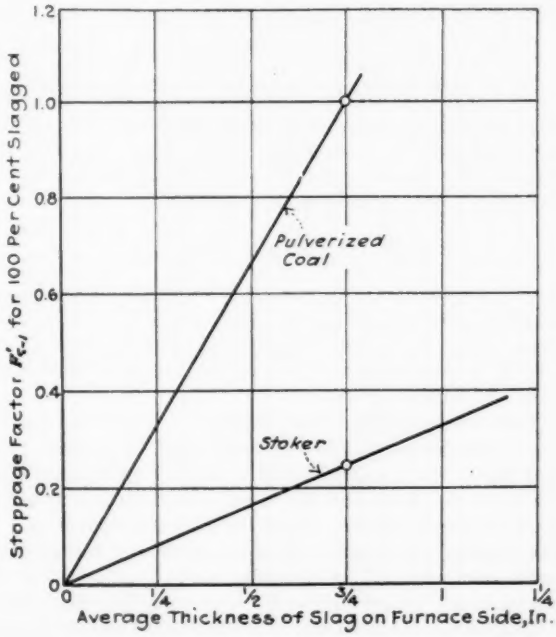


Fig. 3 Determinations of Slag Factor F_S
(For tubes half embedded in refractory, fin tubes, or waterwall block tubes, $F_S = (1 - xF'_{S-1})$ where x = fraction of the front of the tube that is slagged.)

clearly differentiated from the F_A-, F_C-, and F_S-factors, hence their data cannot be used. Wohlenberg has used a value of 0.95. There is every reason to believe that this factor is not far from unity.

Furnace Exit

Any radiant-heat ray from the furnace toward the exposed bottom boiler tubes at the furnace exit must strike tube surface, hence F_A will be unity. When these tubes are of the ordinary bare type, F_C is unity. The factor F_S may also be taken as unity when the tubes are relatively free of slag. Under these conditions the effective radiant heating surface A_{rhs} of the furnace exit will be equal to the product of the width and length of the tube bank.

In the special case of the double-ended boiler, it is obvious that the above will not hold. In this case it is suggested that the apex be considered nonexistent, that is, the area of the furnace exit be taken as the product of the length of the bottom side of the triangle of which the boiler tubes form two sides, and the width of the tube bank. The furnace volume used is then computed only up to this furnace exit.

Total Refractory Surface in the Furnace

Having obtained the effective radiant heating surface A_{rhs} by use of Equation [1], there will be an equivalent summation of areas A_{ref} defined by

$$A_{ref} = A - A_{rhs} \quad [2]$$

The area A_{ref} is not refractory surface in the ordinary sense, no effective radiant heating surface; but rather one which may be considered to be the surface equivalent of a summation of non-absorbing particles of refractory located among the waterwall tubes.

Here a new concept is offered. For calculation of the total exposed refractory surface in the furnace in order to determine the fraction cold, it would appear reasonable to add this refractory area to the pure refractory occurring in the furnace as such. Thus, the total refractory area in a furnace will be the sum of the areas of any purely refractory walls plus the value $(A - A_{rhs})$.

The Stefan-Boltzman Law and Furnace Emissivity

It is known that radiation from an absolutely "black" body follows the Stefan-Boltzman law as shown in Fig. 1 which is comparatively simple and, for a hot surface completely surrounded by a cold surface, is expressed by

$$R = ECS[(T_1/100)^4 - (T_2/100)^4] \quad [3]$$

where R = radiation, Btu per hr; C = black-body constant, 0.1723 Btu per sq ft per hr per (deg F abs)^4; S = area of hot surface, sq ft; E = effective emissivity, 1.00 for both surfaces black; T_1 = temperature of hot surface, F abs; T_2 = temperature of cold surface, F abs.

A furnace approximates these conditions only if the mass of hot gases or flames acts like a black body. To exactly what extent this is true is problematical. Small or microscopic black particles in the flame and the effect of reradiating refractory walls will cause the effective emissivity E to approach unity.

For purposes of simplification of the problem of furnace-heat-transfer calculation, T_1 of Equation [3] is taken equal to the furnace-exit gas temperature rather than a possibly higher mean radiating furnace temperature. Mathematically speaking, this is simply stating that the radiation of a body at a given temperature and given emissivity may be equal to that of the same body at a lower temperature and higher emissivity. Therefore, it is believed that the emissivity based on furnace-exit temperature may not be far from unity.

TABLE 2 EMISSIVITY DATA

Station no.	Location	Avg flame emissivity[a]	Remarks
1	Grate	1.00	250,000 lb per hr rating
1	Furnace	0.75	250,000 lb per hr rating
2	Middle furnace	1.00	Same at all loads
2	Top of furnace	0.80	Same at all loads
7	Middle furnace	1.00	Same at all loads
8	Middle furnace	0.68	Increases with load
10	Middle furnace	0.72	Same at all loads

[a] Calculated.

TABLE 3 CALCULATED EMISSIVITY OF PULVERIZED ILLINOIS COAL BURNED IN SPECIAL EXPERIMENTAL FURNACE

Distance from burner, ft	3.5	7.5	11.5
Emissivity, 10-ft flame depth	0.96	0.85	0.83
Emissivity, 20-ft flame depth	1.00	0.98	0.97

In regard to this particular matter the experimental data is not conclusive. Table 2 gives some results of calculation based on radiation pyrometer data taken in the course of the A.S.M.E. Radiation-Committee investigation.[2]

Table 3 contains some data given by Sherman.[7] Koessler[8]

[7] "Burning Characteristics of Pulverized Coal and Radiation From Their Flames," by R. A. Sherman, *Combustion*, vol. 5 (new notation), December, 1933, p. 30.

[8] "Messungen der Flammenstrahlung in Dampfkesselfeuerungen," *Zeitschrift der Bayerischens Revisions-Vereins*, vol. 34, 1930; no. 6, March 31, pp. 75–78; no. 7, April 15, pp. 95–98; no. 10, May 31, pp. 146–149; no. 12, June 30, pp. 175–178; no. 13, July 15, pp. 184–187.

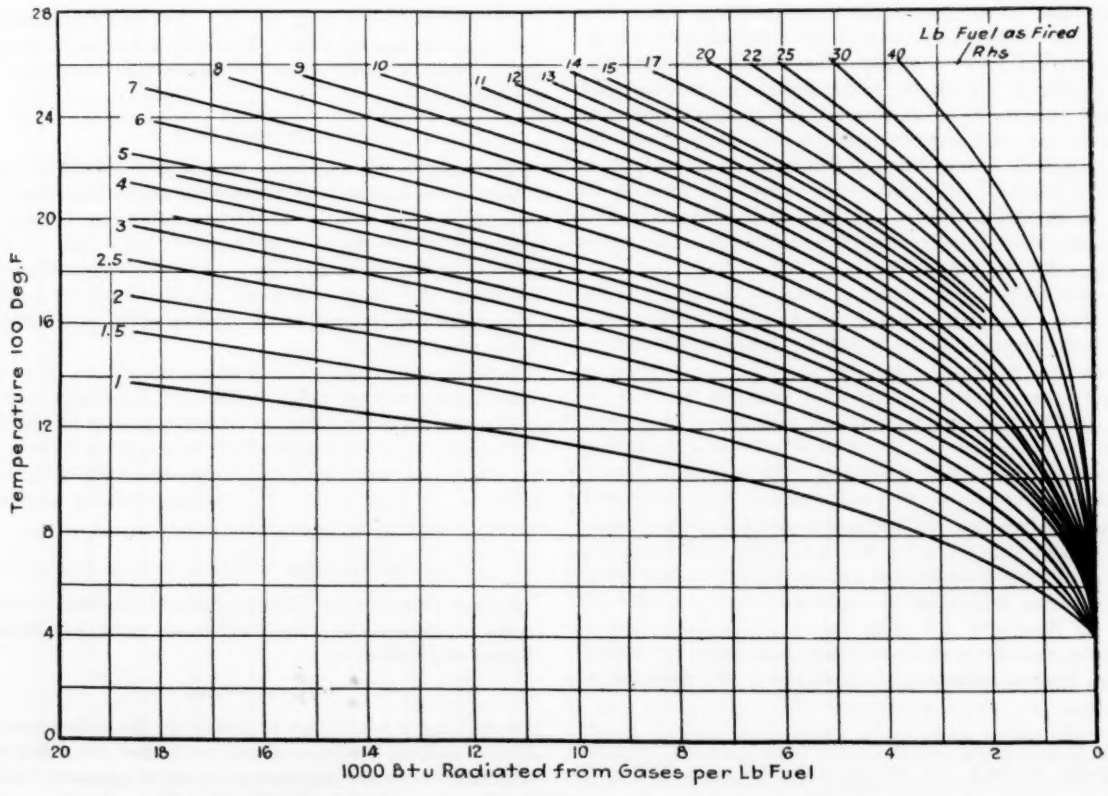


FIG. 4 HEAT RADIATED FROM GASES PER LB OF FUEL BURNED BASED ON THE STEFAN-BOLTZMAN LAW
(Lb fuel burned equals lb fuel as fired if consumption is assumed to be complete.)

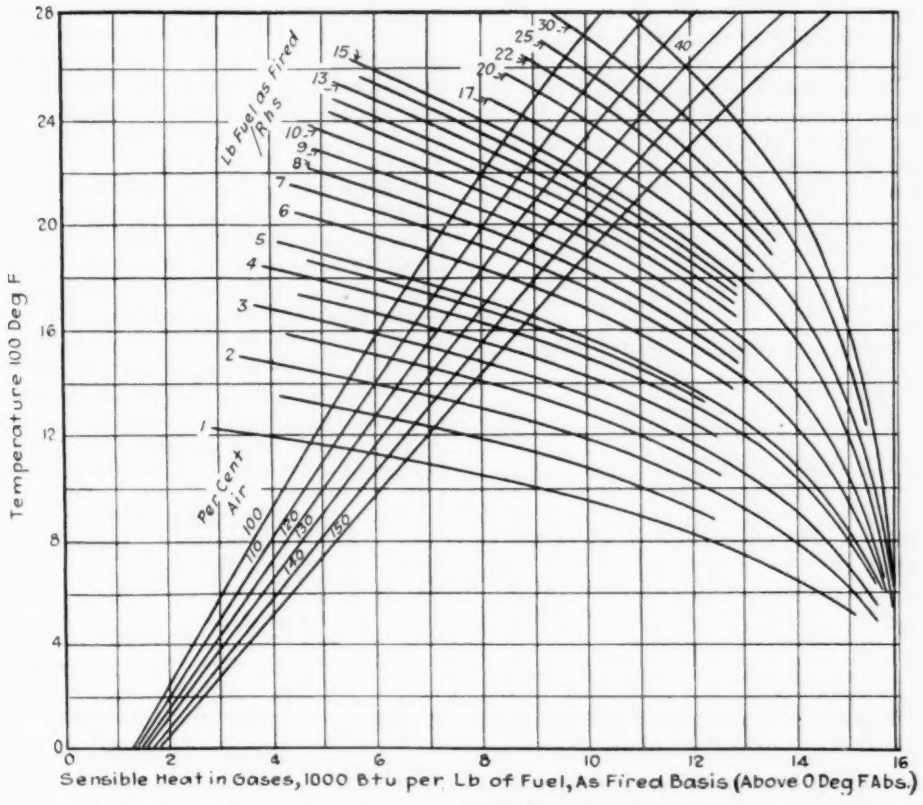


FIG. 6 CALCULATION OF μ_L BY MEANS OF THE METHOD ADVANCED IN THIS PAPER
(Example for test No. 1 on boiler No. 1 given in the A.S.M.E. Radiation-Committee report.[2])

has also given considerable data on several different types of furnace in which the values of emissivity very often reach unity. It is not possible to give his data here.

2—APPLICATION OF THE STEFAN-BOLTZMAN LAW TO FURNACE-HEAT-TRANSFER CALCULATIONS

Utilizing the preceding information, a method was developed for the solution of the problem of furnace radiation on the basis that each square foot of effective radiant heating surface will absorb heat according to the Stefan-Boltzman law using an emissivity of unity, and that the furnace-exit temperature may be used as the mean radiating furnace temperature.

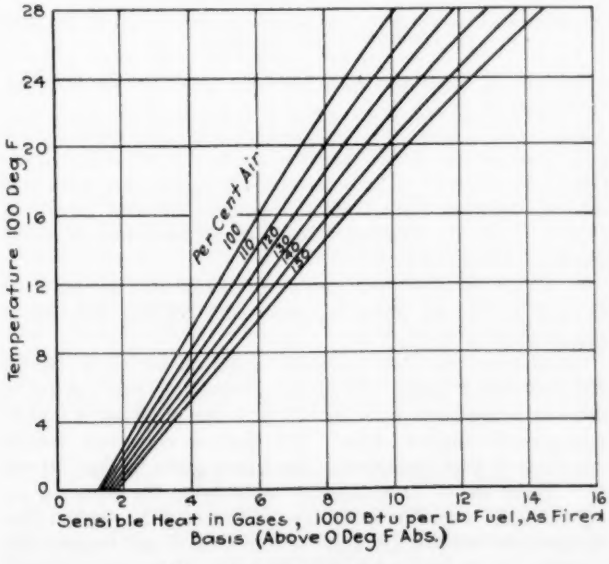


FIG. 5 HEAT-CAPACITY CURVES

Fundamentally, the relation between the radiation to the furnace walls, furnace temperature, and heat in the gases leaving the furnace, is based upon a furnace heat balance which may be expressed by

$$Q_U + Q_A = Q_R + (Q_{GF} - Q_G) \quad [4]$$

or

$$Q_U + Q_A + Q_G - Q_{GF} = Q_R \quad [5]$$

where Q_A = heat of preheated air, Q_G = heat of gases at 80 F, Q_{GF} = heat of gases at furnace temperature, Q_R = heat radiated, Q_U = low heat value of fuel. To make use of Equation [5], the effective radiant heating surface in the furnace must be determined according to the principles given in Part 1 of this paper.

It will be observed that the principal differences between the proposed method and older schemes of applying the Stefan-Boltzman law to furnace-heat-transfer calculations lie in the manner of evaluating effective radiant heating surfaces and in the use of an emissivity of unity. A third difference lies in the practical method of graphical calculation now to be explained.

In Fig. 1 furnace temperature is plotted against heat received by radiation per square foot of effective radiant heating surface. It is desirable to introduce the combustion rate expressed as pounds of fuel burned per square foot of effective radiant heating surface. Now

$$\frac{\text{(Heat radiated per hr/rhs)}}{\text{(Lb fuel burned per hr/rhs)}} = \frac{\text{heat radiated}}{\text{lb of fuel burned}} \quad [6]$$

By use of Fig. 1, Equation [6] and various values of pounds of fuel burned per square foot of effective radiant heating surface, the curves of Fig. 4 may be obtained. Since the effective radiant heating surfaces will absorb only a given amount of heat at any furnace temperature, this merely states that if the combustion rate is doubled only half as much heat can be absorbed per pound of fuel supplied. This will hold only if the furnace temperature remains the same. Actually the smaller quantity

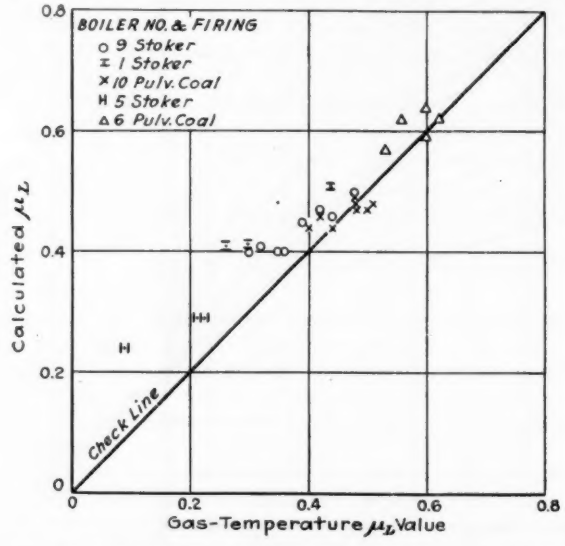


FIG. 7 COMPARISON BETWEEN RESULTS OBTAINED BY THE METHOD ADVANCED IN THIS PAPER AND THOSE OBTAINED FROM EXPERIMENTAL DATA

(Experimental data and boiler numbers are those given in the A.S.M.E. Radiation-Committee report.[2])

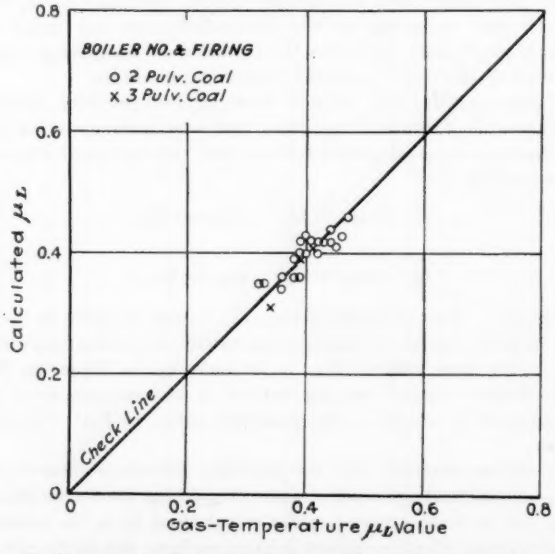


FIG. 8 COMPARISON BETWEEN RESULTS OBTAINED BY THE METHOD ADVANCED IN THIS PAPER AND THOSE OBTAINED FROM EXPERIMENTAL DATA

(Experimental data and boiler numbers are those given in the A.S.M.E. Radiation-Committee report.[2])

of heat absorbed by radiation causes the furnace temperature to rise and hence more heat will be radiated until a balance is reached.

To use Fig. 4 in calculations the procedure is as follows: Using a sensible-heat curve for the flue gases of the particular fuel, such as Fig. 5, the furnace temperature is found on both Figs. 4 and 5 such that the effective or low heat value per pound of wet fuel, plus the heat of preheat in the air, plus the heat in the gases of combustion at 80 F, all being the heat input per pound of wet fuel, minus the heat in the gases at the furnace temperature from Fig. 5, is equal to the heat radiated per pound of wet fuel in Fig. 4.

In order to eliminate the time involved in a trial-and-error solution, Fig. 4 should be drawn on tracing paper. It can then be superimposed on Fig. 5 as shown in Fig. 6, the origin on the heat-radiated curve being placed on the heat-capacity-curve base line at a value equal to the total heat input (using low heating value) to the furnace per pound of fuel as fired. At the intersection of the curve for per cent supplied air in the furnace and the curve for pounds of fuel per square foot of effective radiant heating surface, the furnace temperature, the heat in the gases at the furnace temperature, and the heat radiated may be obtained simultaneously. The value μ_L is then a ratio of the heat radiated to the heat input per pound of wet fuel, the

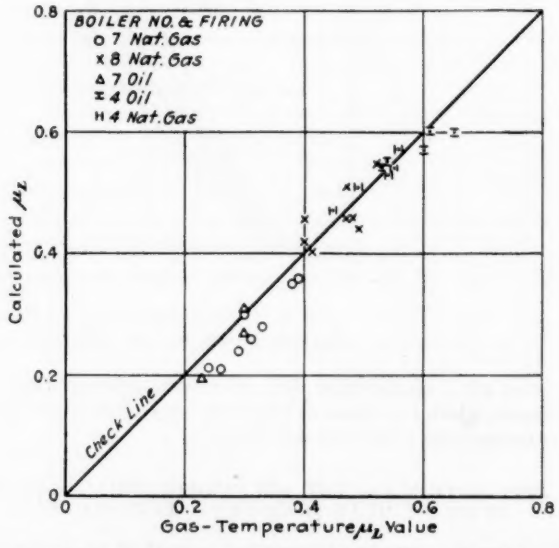


FIG. 9 COMPARISON BETWEEN RESULTS OBTAINED BY THE METHOD ADVANCED IN THIS PAPER AND THOSE OBTAINED FROM EXPERIMENTAL DATA

(Experimental data and boiler numbers are those given in the A.S.M.E. Radiation-Committee report.[2])

subscript $(\ )_L$ denoting that μ has been based on the low heat of the fuel.

Values of μ_L using this method have been computed for the tests given on ten boiler furnaces in the A.S.M.E. Radiation-Committee report.[2] These computed μ_L-values are compared with gas-temperature μ_L-values determined in that report[2] in Figs. 7, 8, and 9. The boiler numbers correspond to the boiler numbers of the report.[2]

It will be observed that results are satisfactory, for all types of fuels and firing, excepting the results for the stoker-fired furnaces which are high.

An attempt was made to derive empirical correction factors K_X that would modify the results obtained by this method in order to bring them more in line with the observed A.S.M.E. Radiation-Committee data.[2] The derived correction factors appear to be best embodied in the curves given in Figs. 10 and 11. These are to be applied by multiplying the obtained μ_L-values by the correction factor K_X. The modified μ_L-values thus obtained are compared in Figs. 12 and 13 with gas temperature μ_L-values given in the A.S.M.E. Radiation-Committee report.[2]

It will be seen that the modified results represent the data rather closely considering the simple correction used. The multiplying correction factor is unity for oil and gas firing except for all-refractory furnaces, in which case the mean furnace temperature is probably somewhat higher than the furnace-exit temperature due to the lack of waterwall surfaces available for absorbing heat in the lower portion of the flame. For this same reason the correction factor for pulverized coal approaches unity for furnaces in which the waterwall surface is not a large fraction of the total surface. The correction factor for stoker firing depends upon fraction cold in very much the same manner as has been shown by Wohlenberg in his method.

The new method given is not difficult. The greatest amount of time required in obtaining solutions by its use will be used in plotting a heat-capacity curve and in computing the effective

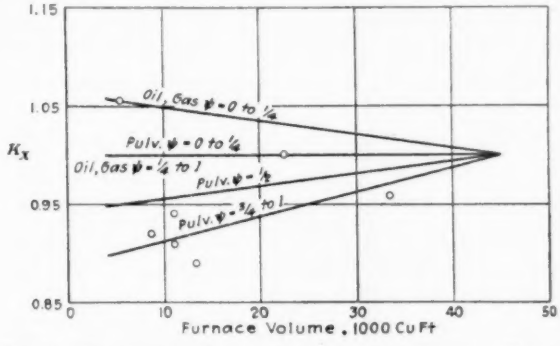


FIG. 10 CORRECTION FACTOR K_X FOR μ_L — SUSPENSION FIRING
(Fraction cold ψ = effective radiant heating surface/total furnace surface less grate area.)

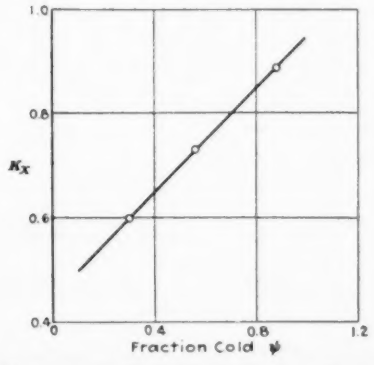


FIG. 11 CORRECTION FACTOR K_X FOR μ_L — STOKER FIRING
(Fraction cold ψ = effective radiant heating surface/total furnace surface less grate area.)

radiant heating surface. For extended use the former may be simplified by plotting heat-capacity curves for typical commercial fuels, which can also be used for boiler-design purposes. In routine work, a heat-capacity curve corresponding to the nearest fuel analysis may be used for calculations. There has been considerable agitation for a more rational method for evaluating effective radiant heating surface and it is possible that a method on the order of that which is given in this paper may be used in the future by boiler manufacturers to rate their product.

COMPARISON WITH THE BROIDO AND ORROK METHODS

It can be shown that the Broido and Orrok methods are derivable from the preceding method of calculation. Using the Stefan-Boltzman law curve of Fig. 1 and a heat-capacity curve as in Fig. 5, other types of curves may be plotted. One type is shown in Fig. 14. This curve was obtained by assuming a furnace temperature from which the heat radiated per square foot of effective furnace surface was found in Fig. 1 and the value of μ_L for a particular per cent air and heat input to the furnace per pound of fuel was found from Fig. 5. Dividing the heat radiated by μ_L gives the heat liberated per square foot of radiant heating surface for the particular conditions. This was

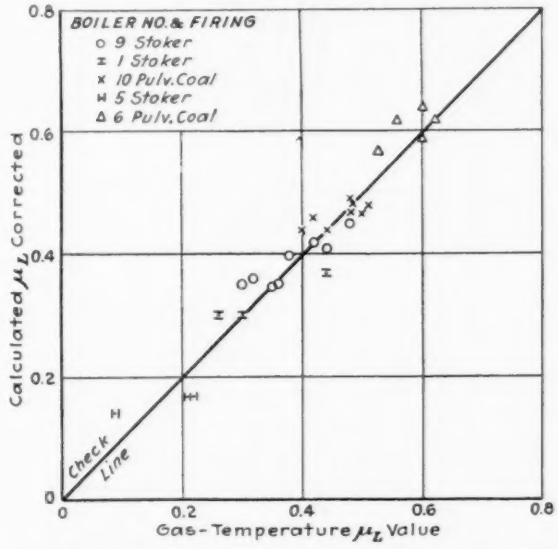


FIG. 12 COMPARISON BETWEEN CORRECTED RESULTS OBTAINED BY THE METHOD ADVANCED IN THIS PAPER AND THOSE OBTAINED FROM EXPERIMENTAL DATA
(Experimental data and boiler numbers are those given in the A.S.M.E. Radiation-Committee report.[2])

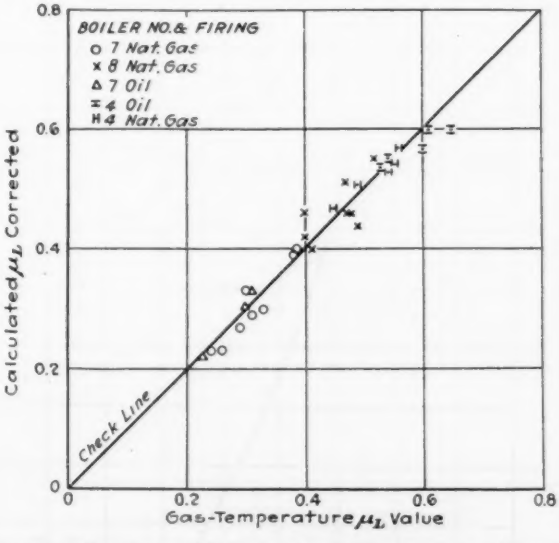


FIG. 13 COMPARISON BETWEEN CORRECTED RESULTS OBTAINED BY THE METHOD ADVANCED IN THIS PAPER AND THOSE OBTAINED FROM EXPERIMENTAL DATA
(Experimental data and boiler numbers are those given in the A.S.M.E. Radiation-Committee report.[2])

done for a number of values of furnace temperature and the results were plotted as in Fig. 14. Thus, this curve might have been used to determine μ_L according to the new method derived. However, it was not used because it is not as flexible as the curves previously given which were plotted on a slightly different basis.

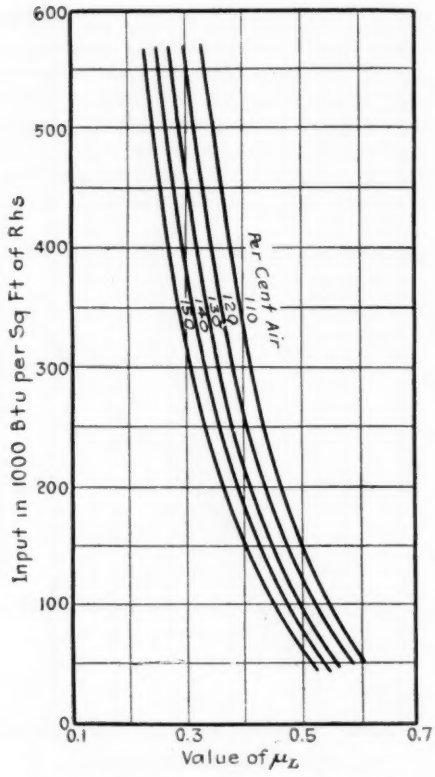


FIG. 14 SPECIAL TYPE OF μ_L CHART

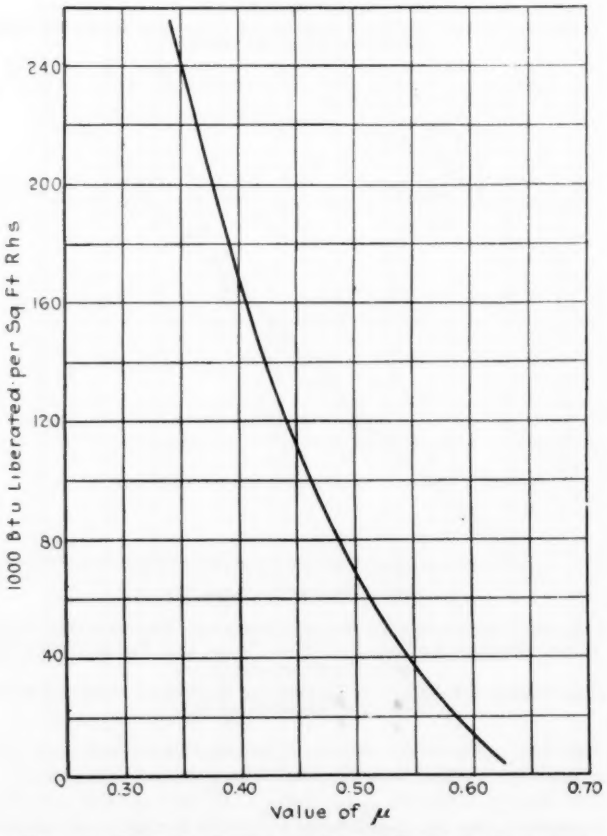


FIG. 15 BROIDO'S CURVE OF BOILER-FURNACE RADIATION

An interesting fact is revealed by this type of curve. If it is compared with Fig. 15, which is a plot of the Broido curve, it will be seen that the Broido curve might be derived from the curves in Fig. 14 assuming a certain relation between the per cent of theoretical air for combustion and the rate of driving. A similar comparison for Fig. 16, which is a plot of the Orrok equation, shows that the Orrok equation might be similarly derived by assuming a given heat value input per pound of fuel.

Therefore, we might conclude that the Broido curve should be accurate for a furnace having a certain excess-air characteristic for various boiler loads and the Orrok equation is applicable to a fuel having a given heat value.

ACCURACY OF THE PROPOSED METHOD

To the engineer who frequently has used empirical formulas for the calculation of furnace-heat transfer, the present attack based on the Stefan-Boltzman law may appear as a rational attack in that the empirical corrections, considering the diversity

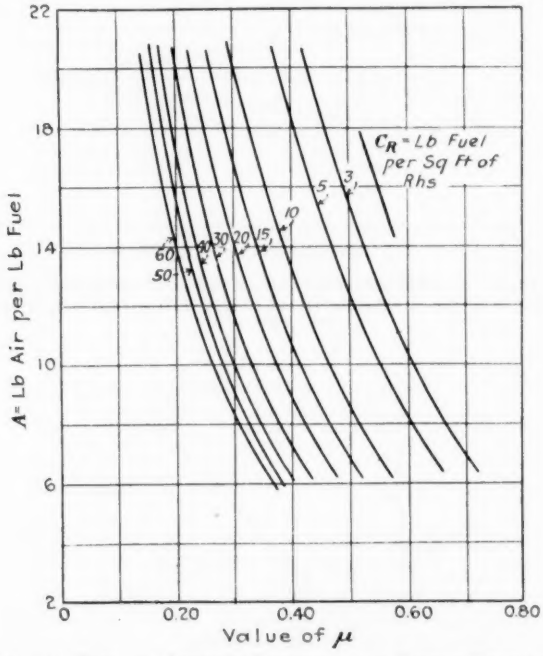


FIG. 16 PLOT OF ORROK'S EQUATION FOR BOILER-FURNACE RADIATION

in types of fuels and firing arrangements, are not great. From the strictly theoretical viewpoint of the physicist it will appear to a large extent empirical. Whether the method be considered to be based upon a firm theoretical foundation or upon an empirical basis it is submitted as being in agreement with such experimental data as is available, as having sufficient simplicity for use in practical calculations, and as showing clearly from the graphical standpoint the influence of various operating factors on the result desired. It is probable that the method will give an average furnace-exit temperature within 75 F. It should be noted however that the data by means of which the method advanced in this paper has been checked were taken almost entirely from large furnaces. Therefore, application to small furnaces must be made with caution.

STUDY OF THE EFFECT OF COMBUSTION CONDITIONS OF SUSPENSION FIRING ON FURNACE-OUTLET TEMPERATURE

A special study was made in order to determine the effect of fixed furnace characteristics such as fraction cold, furnace

volume, and furnace length, as well as such operating conditions as per cent air, temperature of preheated air, and energy-release rate. Using the method developed in this paper, the furnace-outlet temperature was computed for various arrangements

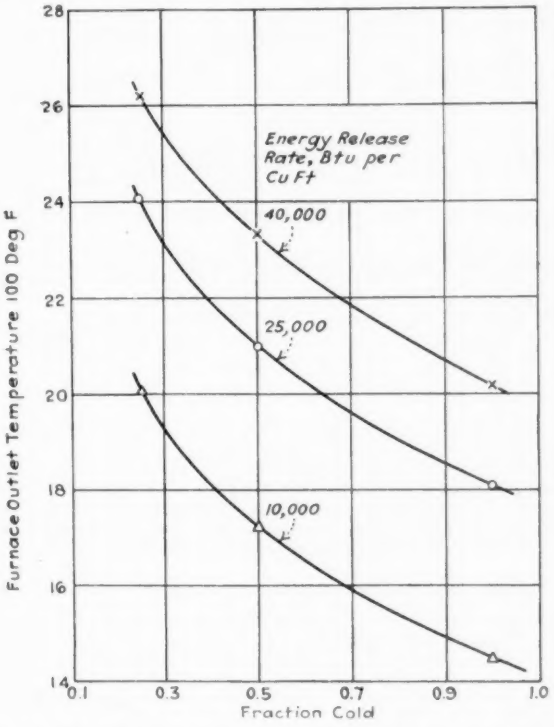


FIG. 17 EFFECT OF VARIATION OF FRACTION COLD ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF ENERGY-RELEASE RATE

(Air = 120 per cent, air temperature = 80 F, furnace volume = 8000 cu ft, and furnace length = 20 ft.)

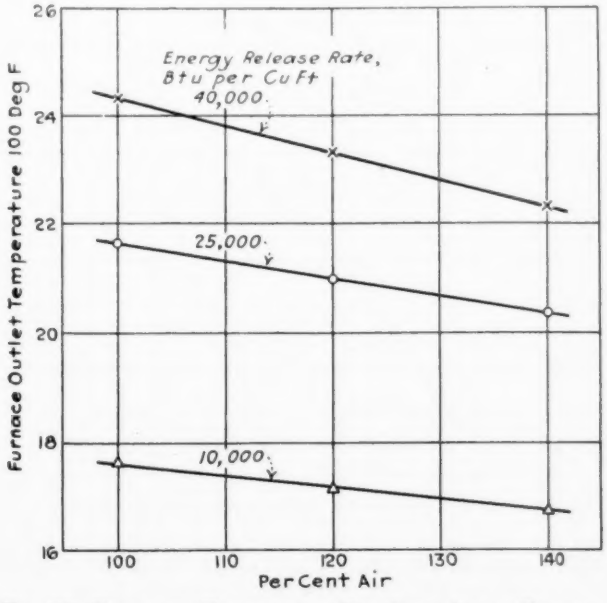


FIG. 18 EFFECT OF VARIATION OF PER CENT AIR ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF ENERGY-RELEASE RATE

(Fraction cold = 0.50, air temperature = 80 F, furnace volume = 8000 cu ft, and furnace length = 20 ft.)

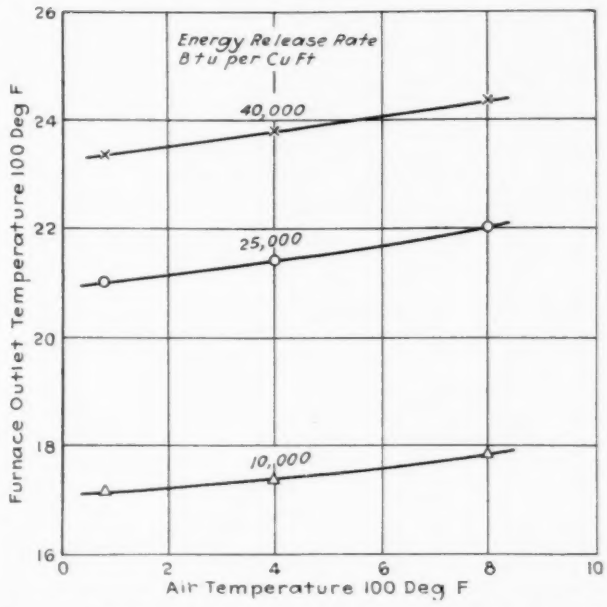


FIG. 19 EFFECT OF VARIATION OF AIR TEMPERATURE ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF ENERGY-RELEASE RATE

(Fraction cold = 0.50, air = 120 per cent, furnace volume = 8000 cu ft, and furnace length = 20 ft.)

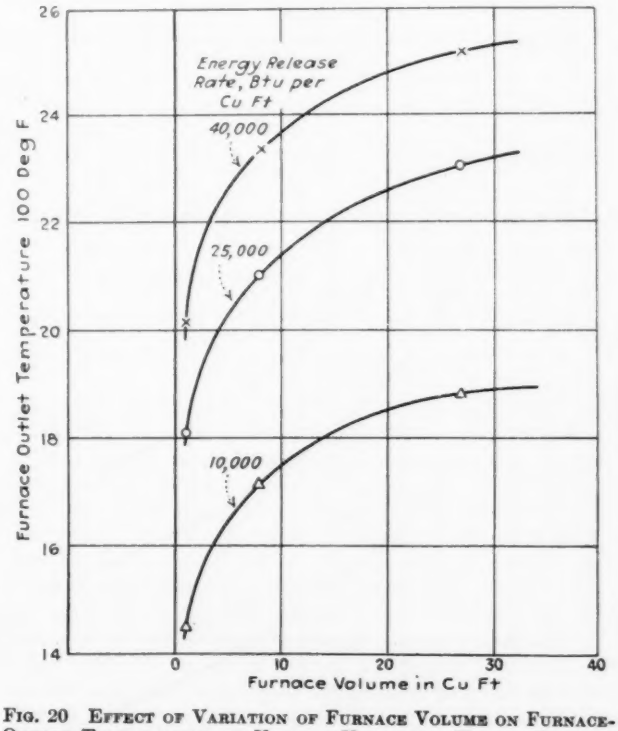


FIG. 20 EFFECT OF VARIATION OF FURNACE VOLUME ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF ENERGY-RELEASE RATE

(Fraction cold = 0.50, air = 120 per cent, air temperature = 80 F, furnace length = 10, 20, and 30 ft.)

and conditions of the six variables mentioned previously. Three values were chosen for each variable as shown in Table 4. Fraction cold is defined as the effective radiant heating surface in the furnace divided by the area of the entire furnace envelope.

The fuel chosen for these calculations was a representative

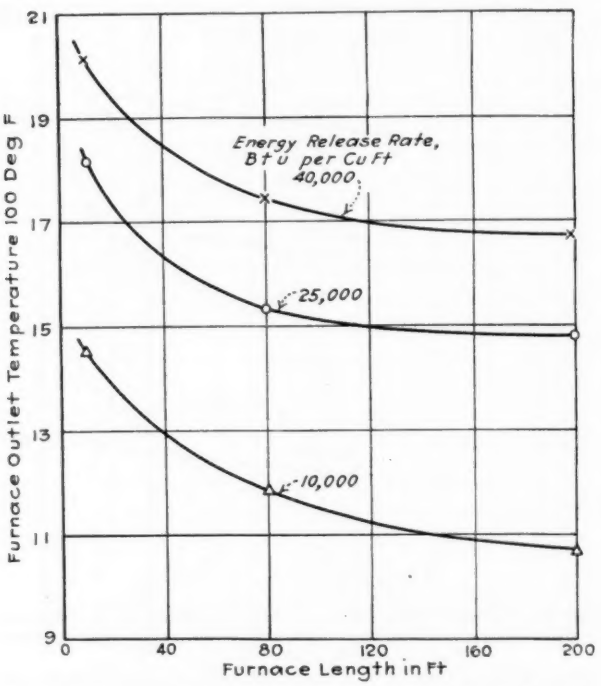


FIG. 21 EFFECT OF VARIATION OF FURNACE LENGTH ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF ENERGY-RELEASE RATE

(Fraction cold = 0.50, air = 120 per cent, air temperature = 80 F, furnace volume = 1000 cu ft.)

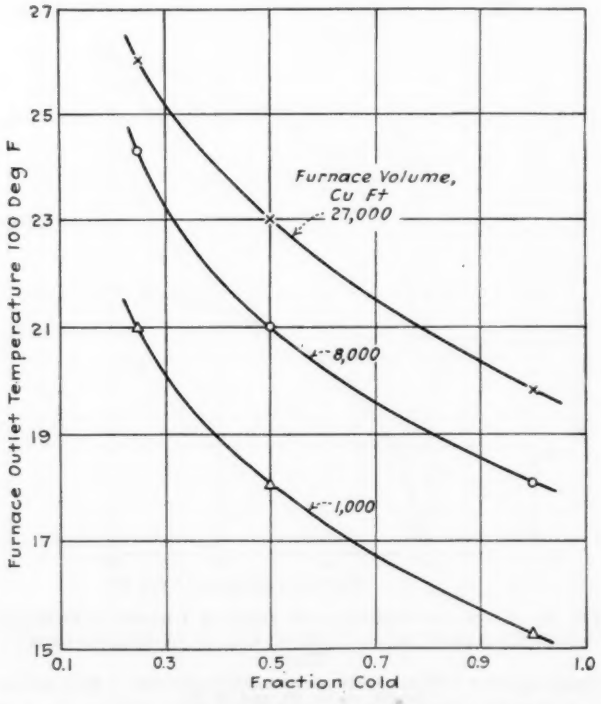


FIG. 22 EFFECT OF VARIATION OF FRACTION COLD ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF FURNACE VOLUME

(Air = 120 per cent, air temperature = 80 F, energy release rate = 25,000 Btu per cu ft, and furnace length = 10, 20, and 30 ft.)

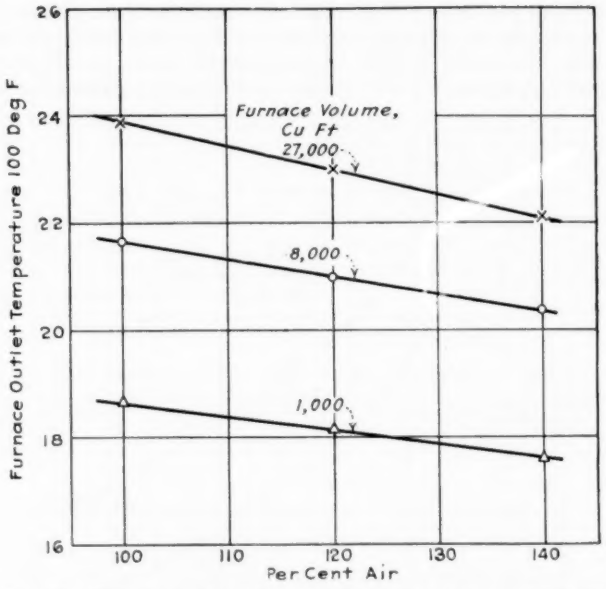


FIG. 23 EFFECT OF VARIATION OF PER CENT AIR ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF FURNACE VOLUME

(Fraction cold = 0.50, air temperature = 80 F, energy-release rate = 25,000 Btu per cu ft, and furnace length = 10, 20, and 30 ft.)

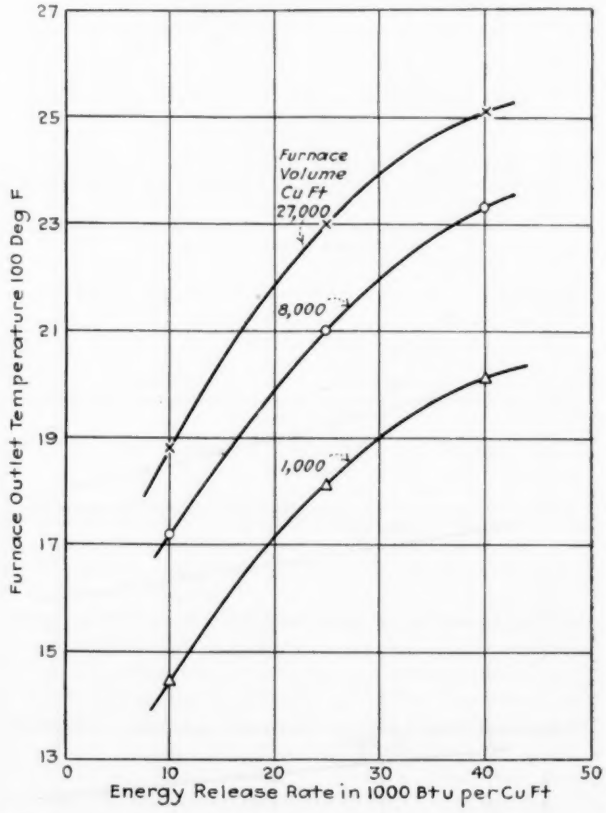


FIG. 24 EFFECT OF VARIATION OF ENERGY-RELEASE RATE ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF FURNACE VOLUME

(Fraction cold = 0.50, air = 120 per cent, air temperature = 80 F, furnace length = 10, 20, and 30 ft.)

Eastern bituminous coal, the heat-capacity curve of which is shown in Fig. 5.

All of the six groups of variables given in Table 4 were varied one at a time with respect to any other five held constant and

TABLE 4 VALUES CHOSEN FOR SIX VARIABLES GOVERNING FURNACE-OUTLET TEMPERATURES

	Variable	Values chosen: Low	Medium	High
1	Fraction cold	0.25	0.50	1.00
2	Per cent combustion air	100	120	140
3	Temperature combustion air, F	80	400	800
4	Energy-release rate, Btu per hr	10000	25000	40000
5	Furnace volume, cu ft	1000	8000	27000
6	Furnace length, ft	10, 20, or 30[a]	80	200

[a] Minimum furnace length is based on volume taken as a cube.

TABLE 5 QUALITATIVE EFFECT OF VARYING COMBUSTION CONDITIONS ON FURNACE-OUTLET TEMPERATURES

	Increasing this factor	Effect on furnace-outlet temperature
1	Fraction cold	Decrease
2	Per cent combustion air	Decrease
3	Temperature of combustion air	Increase
4	Energy-release rate	Increase
5	Furnace volume	Increase
6	Furnace length	Decrease

furnace-outlet temperatures calculated by the method developed, except that the K_x correction factors were not applied. Therefore, the results, as far as the indication of absolute values is concerned, are slightly inaccurate, although the trends are correct.

Part of the results are presented in Figs. 17 to 26. From the shape of the curves, it is possible to draw certain general conclusions which are summarized in Table 5. These results agree

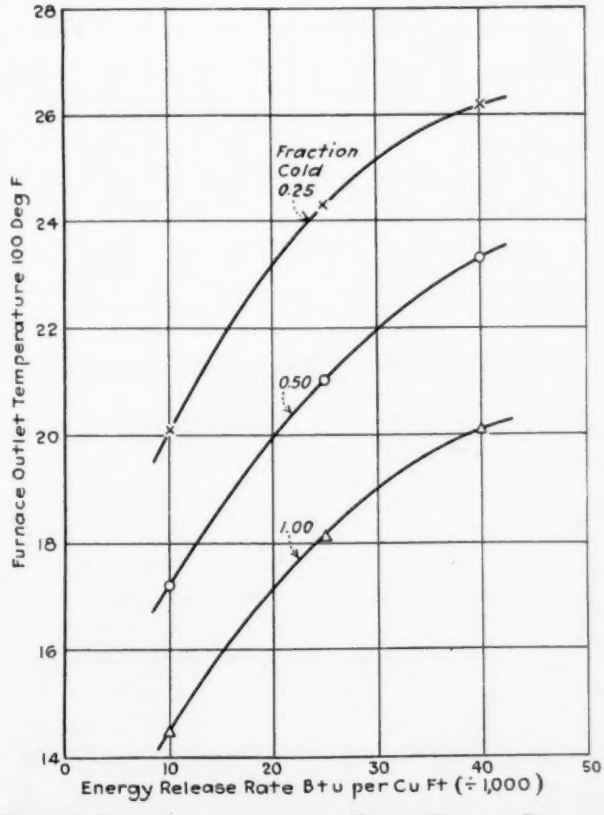


FIG. 25 EFFECT OF VARIATION OF ENERGY-RELEASE RATE ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF FRACTION COLD

(Air = 120 per cent, air temperature = 80 F, furnace volume = 8000 cu ft, furnace length = 20 ft.)

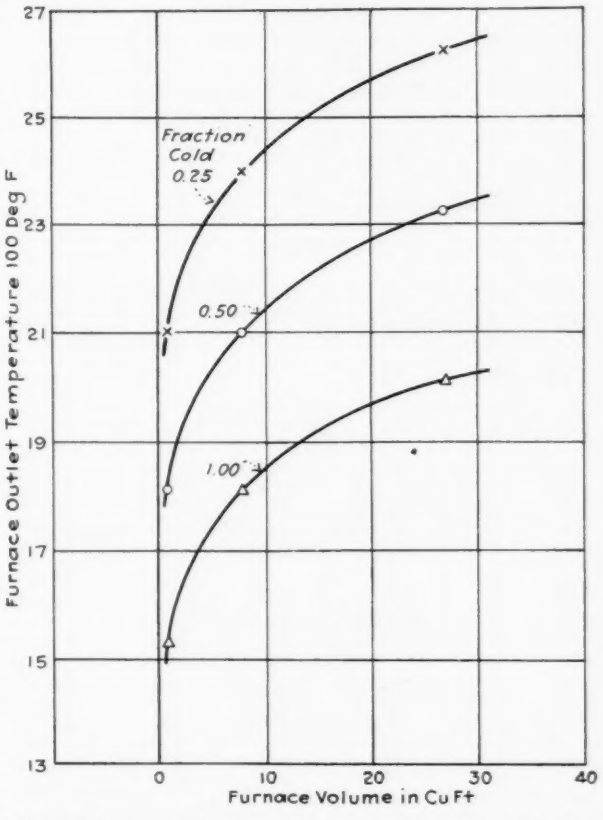


FIG. 26 EFFECT OF VARIATION OF FURNACE VOLUME ON FURNACE-OUTLET TEMPERATURE AT VARIOUS VALUES OF FRACTION COLD

(Air = 120 per cent, air temperature = 80 F, energy-release rate = 25,000 Btu per cu ft, furnace length = 10, 20, and 30 ft.)

with those which would be expected from qualitative theoretical considerations.

Appendix

CALCULATION OF SLAG FACTORS FOR TUBES BY USE OF EXPERIMENTAL DATA

From the A.S.M.E. Radiation-Committee report,[2] the test of boiler No. 1, which is a stoker-fired unit, makes available data on a separate screen or waterwall boiler composed of sidewall tubes and a slag screen at the furnace exit. Data for the evaporation of the screen boiler, compared to the total evaporation, for clean tubes and for slag-covered tubes after 18 months' operation, are given in Table 6. The average per cent of slag on the furnace effective radiant-heating-surface area of 2027 sq ft of this boiler has been computed to be 55 per cent. In addition there are 394 sq ft of radiant heating surface at the furnace exit. It is assumed that the furnace-exit surfaces evaporate the same quantity of steam after long operation as

TABLE 6 EVAPORATION OF SCREEN BOILER NO. 1[a]

Total boiler evap, 1000 per hr	Screen-boiler evap, 1000 lb per hr: Clean tubes	Slagged tubes	Evap ratio, slagged to clean tubes
120	67	58	0.865
140	75	67	0.895
180	93	84	0.903
200	102	92	0.900
240	120	108	0.900
260	129	114	0.885
300	147	124	0.845

[a] Data taken from a test of boiler No. 1 as given in the A.S.M.E. Radiation-Committee report.[2]

they did at initial operation, that is, they do not accumulate slag, the slag being accumulated only on the side waterwalls. From this an equation may be set up in which x = the slag factor and E = the original screen-boiler evaporation. Then, for initial operation

$$394 + 2027 = f(E) \quad [7]$$

and for slagged operation

$$394 + 2027x = 0.9f(E) \quad [8]$$

The value of 0.9 was taken as the approximate average reduction in evaporation of the screen boiler from Table 6. Solving the equation, $x = 0.88$. Thus, although the screen boiler evaporates

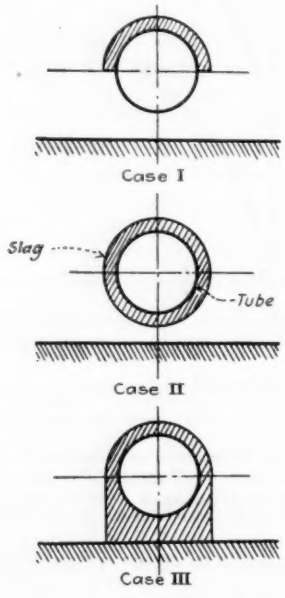


FIG. 27 DETERMINATION OF F_S-FACTOR FOR VARIOUS CONDITIONS

0.90 of its initial evaporation, because of the constant evaporation of the furnace-exit surface the screen waterwalls only evaporate 0.88 of their initial evaporation. If the wall is 55 per cent slagged with slag estimated $^3/_4$ in. thick, it has a stoppage factor of 1.00 — 0.88 = 0.12. A wall 100 per cent slagged would have a factor of 0.12 (100/55) = 0.22. This value has been plotted as F'_{S-1} in Fig. 3. It is realized that the above solution is not correct inasmuch as it does not take into account convection at the furnace exit. More involved calculations would not appear to be justified by the data.

The second boiler from which tests are available in the A.S.M.E. Radiation-Committee report[2] is a pulverized-fuel-fired boiler with a bottom screen and side waterwalls on separate circulation. Here the effect of slagged waterwall tubes was obtained by comparing the evaporation of the screen or radiant boiler when slagged and when the side-wall tubes had been

TABLE 7 EVAPORATION OF SCREEN BOILER NO. 2[a]

Total boiler evap, 1000 lb per hr	Screen boiler evap, 1000 lb per hr — Clean side of wall tubes	Screen boiler evap, 1000 lb per hr — Slagged side of wall tubes	Evap ratio, slagged to clean tubes
150	72	64	0.890
200	91	82.5	0.906
250	107	98	0.906
280	114	103.5	0.908

[a] Data taken from a test of boiler No. 2 as given in the A.S.M.E. Radiation-Committee report.[2]

cleaned of slag and wire-brushed. Results are summarized in Table 7.

Boiler No. 2 is different from the stoker-fired boiler No. 1 in that the horizontal tubes of the screen boiler were not appreciably slagged and were therefore not cleaned. Their evaporation should remain constant.

It is estimated that in boiler No. 2 there were 340 sq ft of side-wall surface 30 per cent slagged with $^3/_4$-in. thick slag. The horizontal bottom-screen tubes and the furnace exit together comprise 1083 sq ft of effective radiant heating surface. From Table 7, the heat transfer of the boiler was reduced 90.5 per cent because of the slag. For clean operation using the same method as for Equations [7] and [8]

$$1083 - 340 = f(E) \quad [9]$$

and for slagged operation

$$1083 - 340x = 0.905f(E) \quad [10]$$

from which $x = 0.60$.

Since this was for tubes 30 per cent slagged, tubes 100 per cent slagged would have a stoppage factor of (1.00–0.60) (100/30) = 1.33 which of course is impossible as 1.00 would be the highest value. Due to lack of data the factor has been taken as equal to 1.0 and is plotted as F'_{S-1} in Fig. 3.

CALCULATION OF SLAG FACTORS FOR SINGLE ROW OF TUBES IN FRONT OF REFRACTORY

The estimated per cent of slag is usually given as a percentage of the surface of the waterwall tubes on the furnace side. On this account a distinction must be made between tubes placed with the rear half imbedded in refractory and tubes placed en-

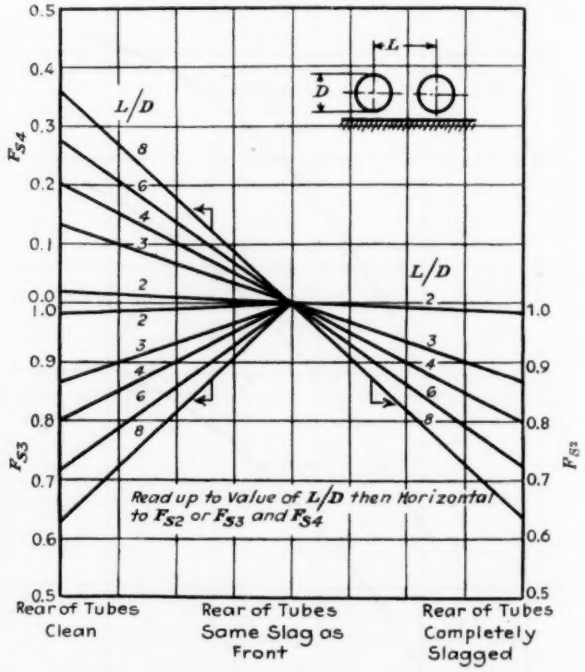


FIG. 28 CALCULATION OF SLAG FACTORS FOR TUBES IN FRONT OF REFRACTORY

tirely in front of refractory. The first of these will be taken as the standard condition. The second case will necessarily be calculated somewhat differently, but will be related to the first case. The second case may be divided into three additional

divisions. The first is when the front of the tube is slagged and the rear of the tube is clean. The second is when the whole circumference of the tube is uniformly slagged. The third occurs when the front is slagged and the rear is completely slagged up. These cases are illustrated in Fig. 27.

TABLE 8 TABULATION OF RESULTS FROM HOTTEL'S CURVES

L/D	2	3	4	6	8
Heat direct to furnace side	0.66	0.47	0.37	0.25	0.08
Total heat to one tube	0.68	0.54	0.45	0.35	0.18
Diff. = heat to rear of tube	0.02	0.07	0.09	0.10	0.10
Fraction to front of tube	0.97	0.87	0.80	0.72	0.64
Fraction to rear of tube	0.03	0.13	0.20	0.28	0.36

It is necessary to set up a proportion between the amount of heat received by the front of the tube and the amount of heat received by the back of the tube. This has been done by the use of Hottel's curves of Fig. 2. The results are tabulated in Table 8.

Let F_S = the true slag factor, F'_{S-1} = the stoppage factor = (1.00 — slag factor), and x = the estimated fraction of front of tube slagged.

Consider the specific case where $L/D = 6$. Using Table 8 for the values of the heat to the front and the rear of the tubes and Fig. 3 for F'_{S-1} values, then for case 1

$$F_S = 0.28 - 0.72(1.00 - xF'_{S-1}) \quad [11]$$

and for case 2

$$F_S = 1.00 - xF'_{S-1} \quad [12]$$

and for case 3

$$F_S = 0.72(1.00 - xF'_{S-1}) \quad [13]$$

This last case assumes that no heat is transferred to the back of the tube when it is completely slagged.

By letting $0.28 = F_{S4}$ and $0.72 = F_{S3}$ in case 1 and $0.72 = F_{S2}$ in case 3 a curve may be plotted to show the relation for various values of L/D as in Fig. 28. Equations [11], [12], and [13] then become

$$F_S = F_{S2}(1 - xF'_{S-1}) \text{ or } F_{S4} + F_{S3}(1 - xF'_{S-1}) \quad [14]$$

which is the relation desired to define slag factors for tubes in front of refractory. The values for insertion in Equation [14] are obtained from Figs. 3 and 28.

Review of Methods of Computing Heat Absorption in Boiler Furnaces

By W. J. WOHLENBERG,[1] NEW HAVEN, CONN., and H. F. MULLIKIN,[2] NEW YORK, N. Y.

This paper contains results obtained by each of a number of methods of computing heat absorption in boiler furnaces. These are compared with the results reported by the A.S.M.E. Committee on Absorption of Radiant Heat in Boiler Furnaces.[3] Particular emphasis has been placed on the Broido, Orrok, and Wohlenberg methods. It is shown that the influence of various conditions as found by experiment are in substantial agreement with results expected from a theoretical consideration of the problem and those arrived at by certain well-known methods now in use.

Introduction

THE purpose of this paper is to endeavor to check, by such experimental data as may be available, the various formulas and methods for the calculation of boiler-furnace heat transfer as have been advanced by various investigators. It is not proposed to discuss either the derivation or the methods themselves to any extent as the original articles may be readily inspected.

These methods have never been checked quantitatively, with the exception of a few limited calculations on individual furnaces, principally because of the lack of experimental data which could be used for that purpose.

The report of the A.S.M.E. Committee on Radiation in Boiler Furnaces entitled "An Experimental Investigation of Heat Absorption in Boiler Furnaces,"[3] and hereafter referred to in this paper as the Radiation-Committee Report, furnished such data based on various sizes and types of furnaces fired in different manners. This report also has a bibliography which contains the titles and locations of the different papers describing the various methods of calculation to be utilized in this paper, as well as a short discussion of each.[4]

The form in which the original investigators have stated their results has in some cases required revision in order to enable comparisons to be made in a reasonable amount of time, such revision generally consisting of putting formulas in chart form. These revisions are described, in the first part of this paper, after which the calculations made on the basis of the experimental data are considered.

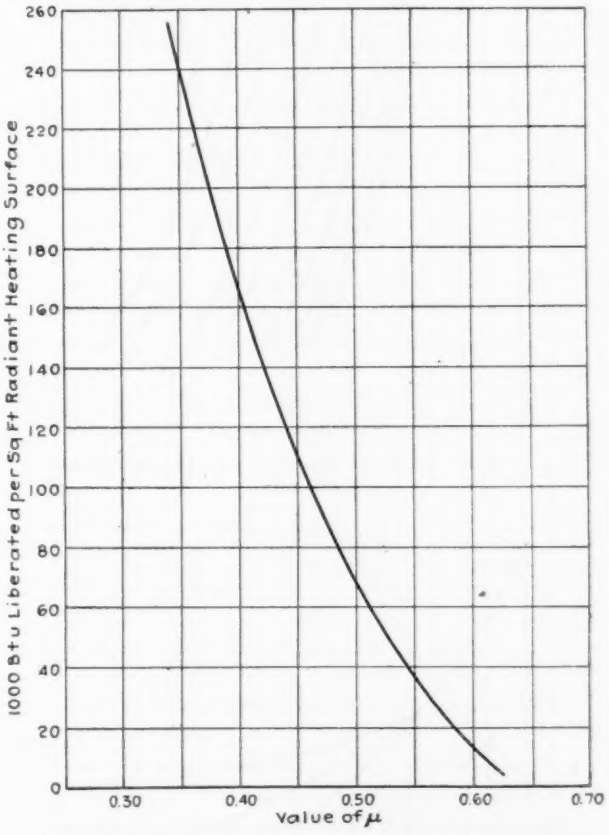


FIG. 1 BROIDO'S CURVE OF BOILER-FURNACE RADIATION

Five methods of calculation have been investigated. These are (*a*) actual measurement (7, 12, 22);[4] (*b*) Broido method (7); (*c*) Orrok formula (20); (*d*) Wohlenberg method (1, 2, 3, 4, 5); and (*e*) Stefan-Boltzman law method. Method (*e*), for reasons stated later, is not included in the present paper.

Broido Method

Broido (7) has given a curve which represents the first important contribution to the subject. This curve, given in Fig. 1,

[1] Professor of Mechanical Engineering, Sheffield Scientific School, Yale University. Mem. A.S.M.E. Professor Wohlenberg received the degree of B.Sc. in mechanical engineering from the University of Nebraska in 1910, and the degree of M.S. from the University of Illinois in 1916. From 1910 to 1918 he served in the engineering department of the Westinghouse Machine Company as Research Fellow at the University of Illinois, and on the engineering faculties of the Universities of Oklahoma and Montana. He has been a member of the engineering faculty of Yale University, and engaged in consulting work in the field of power and steam generation, since 1918. He is the author of numerous papers dealing with power generation and in particular with radiation in furnaces.

[2] Research Fellow, Superheater Company-Yale University Fellowship. Jun. Member A.S.M.E. Dr. Mullikin is a graduate of Baltimore Polytechnic Institute and Johns Hopkins University, and the recipient of graduate degrees, including Ph.D. from Yale. He spent a year in the test department on boiler development at the General Electric Company.

[3] "An Experimental Investigation of Heat Absorption in Boiler Furnaces," by W. J. Wohlenberg, W. H. Armacost, C. W. Gordon, and H. F. Mullikin, Trans. A.S.M.E., vol. 57, paper RP-57-4.

Contributed by the Special Research Committee on Absorption of Radiant Heat in Boiler Furnaces for presentation at the Annual Meeting, December 2 to 6, 1935, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] Numbers in parentheses correspond to similarly numbered references given in the bibliography of the A.S.M.E. Radiation-Committee report "Experimental Investigation of Heat Absorption in Boiler Furnaces."[3]

shows the relation between μ and heat input to the furnace per square foot of exposed projected radiant surface, where μ is defined as the ratio of heat given up from the furnace gases by radiation to the heat input to the furnace.

Orrok Formula

Orrok has given the following empirical formula derived from Hudson (39)

$$\mu = \frac{1}{1 + (A\sqrt{C_R/27})} \quad \ldots\ldots\ldots\ldots [1]$$

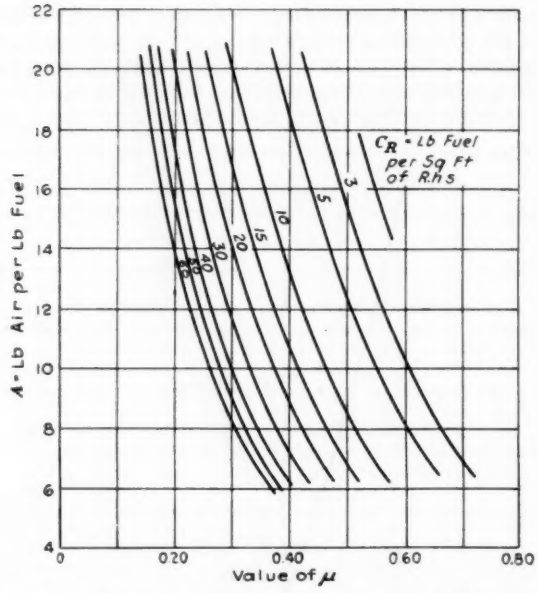


Fig. 2 Plot of Orrok's Equation for Boiler-Furnace Radiation

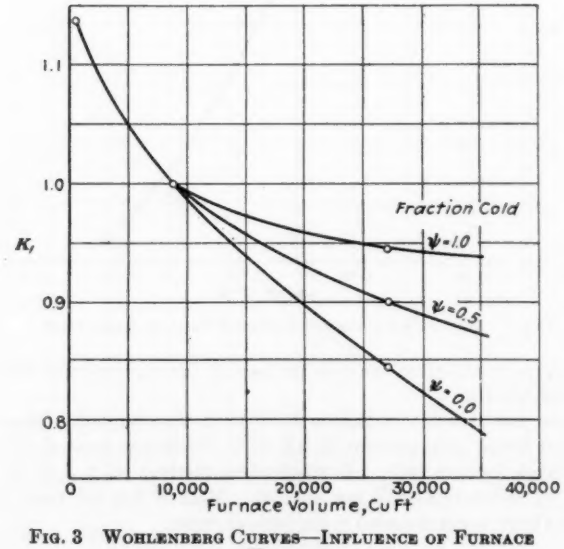


Fig. 3 Wohlenberg Curves—Influence of Furnace Volume

$\left(\psi = \frac{\text{effective radiant heating surface}}{\text{total furnace surface less grate}}\right)$

where μ = the fraction of liberated energy given up by gases in the furnace, A = lb of air per lb of fuel, and C_R = lb of fuel per sq ft of projected radiant heating surface. Fig. 2 shows Equation [1] plotted in the form of a series of curves for ready use. The Wilson, Lobo, and Hottel modification (45) is apparently not valid for the furnaces of steam boilers.

TABLE 1 CONDITIONS OF BASE SOLUTION FOR WOHLENBERG CURVES

1	Furnace volume	8,000 cu ft
2	Release rate	25,000 Btu per cu ft for pulverized-coal firing 40,000 Btu per cu ft for stoker firing
3	Excess air	120 per cent for pulverized-coal firing 140 per cent for stoker firing
4	Fraction cold[a]	unity
5	Coal	Illinois bituminous
6	Fineness	75 per cent through 200 mesh

[a] Fraction cold is defined as the effective exposed radiant heating surface in the furnace divided by the total exposed furnace surface (exclusive of the grate in stoker firing).

Wohlenberg Method

This is an extensive theoretical attack based upon the use of various physical constants rather than an empirical solution as given in the two previous methods. The method is quite complex in that it attempts to evaluate, as far as possible, all the factors that enter into furnace design, in contrast to the simplicity of the empirical attacks which depend upon one or two variables. In applying this method use is made of a furnace heat balance and values of μ for various given conditions of furnace volume, energy-release rate, etc., which are given in several of the papers previously presented on this method.

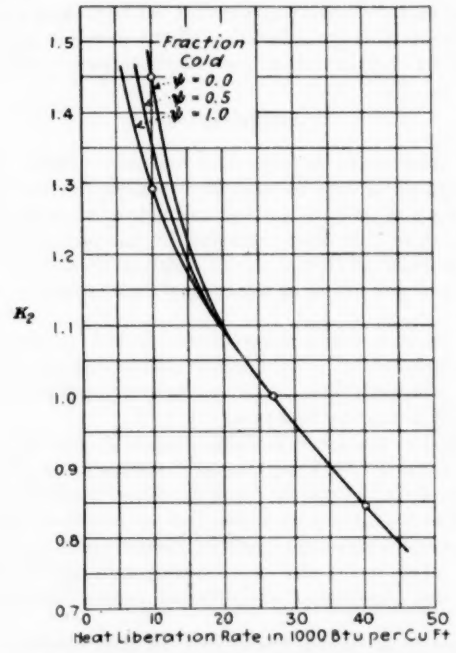


Fig. 4 Wohlenberg Curves—Influence of Liberation Rate. Liberation Rate Based on High Heat Value of Fuel + Air Heat

$\left(\psi = \frac{\text{effective radiant heating surface}}{\text{total furnace surface less grate}}\right)$

It is possible, by use of this material to arrive at results by interpolation. For convenience in calculation, however, these data, together with some additional information, have been graphically restated in the present article in such a form as to enable a more rapid solution of the problem.

After trying several schemes of presentation it was decided to take as a base, the results for a furnace whose physical operating conditions were about in the middle of the range of physical characteristics selected for solution in the original papers. These conditions are given in Table 1.

Since the original publication of the Wohlenberg method,

better physical data have become available, especially in regard to the Stefan-Boltzman and gas-radiation coefficients. Thus, a Stefan-Boltzman coefficient of $\alpha = 0.1723$ Btu per sq ft per hr per (deg F abs)[4] (6), and the radiation values of Schmidt[5] are to be preferred.

The values of μ for the base furnace are henceforth designated as the F-factors. Under conditions of the base furnace defined in Table 1, the F-factors (μ-values) may be calculated using the

TABLE 2 VALUE OF F-FACTOR

Firing method	Method of calculation—With old coef	Method of calculation—With new coef	Coef ratio new/old
Pulverized	0.418	0.452	1.08
Stoker	0.258	0.311	1.20

old coefficients as given in the original papers (1, 2, 3, 4, 5) and also using the new coefficients mentioned in this paper. A comparison of results is shown in Table 2. It will be noted that the

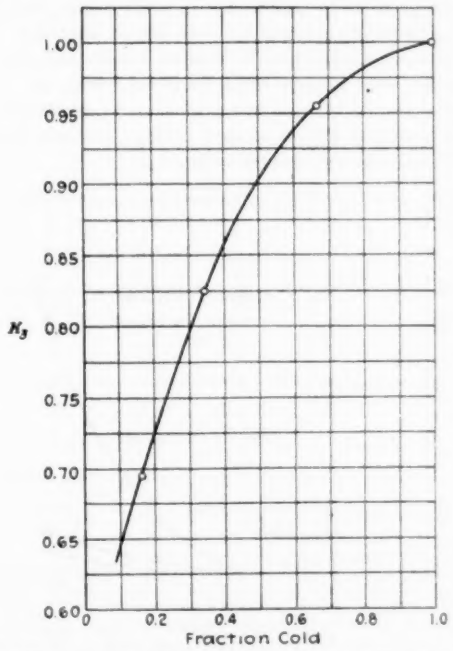


FIG. 5 WOHLENBERG CURVES—INFLUENCE OF FRACTION COLD

$$\left(\text{Fraction cold} = \frac{\text{effective radiant heating surface}}{\text{total furnace surface less grate}}.\right)$$

values of the new coefficients are as much as 20 per cent greater than the old coefficients. The old values are valuable as a basis of comparison but the new F-factors should be used in computations.

Returning for the moment to the data tabulated by Wohlenberg and Lindseth (2) for pulverized coal, each of the factors of Table 1 was varied separately and the resultant ratios of the μ-values, termed K-factors, were plotted as shown in Figs. 3 to 7, inclusive. In order to obtain results within the desired degree of error, it was necessary to modify the curves somewhat as may be noted in the figures. The method of introducing this modification was determined by trial and error.

Using the curves obtained for the pulverized-coal furnace, a ratio was obtained, as shown in Figs. 8 and 9, between the results for pulverized-coal firing and for stoker firing.

The effect of preheated air was next noted in Fig. 10, where C is an additive increase in the value of μ.

[5] "Messung der Gessamtstrahlung des Wasserdampfes bei Temperaturen bis 1000 deg C," by E. Schmidt, *Foschung auf dem Gebiete des Ingenieurwesens*, vol. 3, 1932, p. 57.

Using some calculations made by the 1931 graduate power class at Yale University, the curve of Fig. 11 showing the effects of variation of particle size has been computed.

The curves give multiplying or additive correction factors to apply to the F-factor (μ-value) for the base or standard furnace. The procedure may be embodied in the equation

$$\mu = F\,K_1K_2K_3K_4K_5K_6K_7K_8 + C \qquad [2]$$

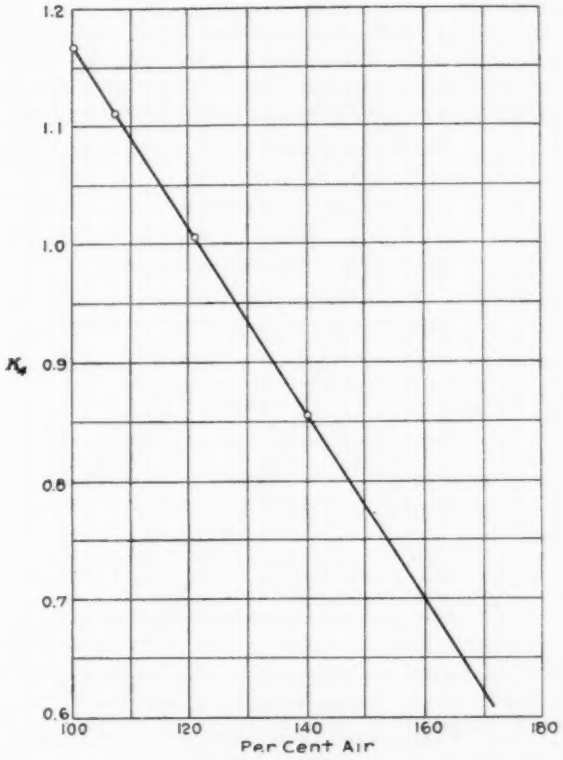


FIG. 6 WOHLENBERG CURVES—INFLUENCE OF EXCESS AIR

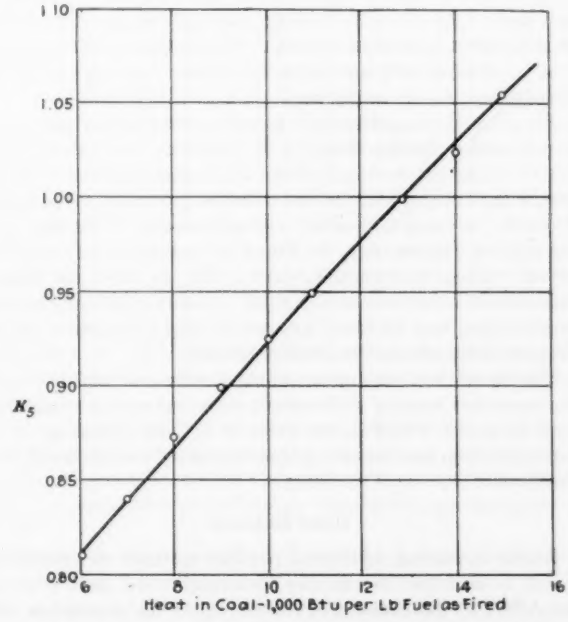


FIG. 7 WOHLENBERG CURVES—INFLUENCE OF FUEL HEAT VALUE

where $F = 0.452$ for pulverized-fuel firing and 0.311 for stoker firing. The K- and C-values are obtained from Figs. 3 to 11, inclusive, for the given conditions.

Thus, these figures give a series of correction curves for pulverized-coal and stoker furnaces which serve to determine μ for any condition of: (*a*) furnace volume, (*b*) heat-liberation rate, (*c*) amount of water-cooled surface in the furnace, (*d*) per cent air used for combustion, (*e*) heat value of fuel, (*f*) temperature of preheated air, and (*g*) coal fineness through 200 mesh.

One of the present authors has made a sample calculation (7), given in Table 3, for the value of μ for a gas-fired furnace as compared with pulverized-coal-fired and stoker-fired furnaces.

It will be seen that, according to the Wohlenberg method, μ

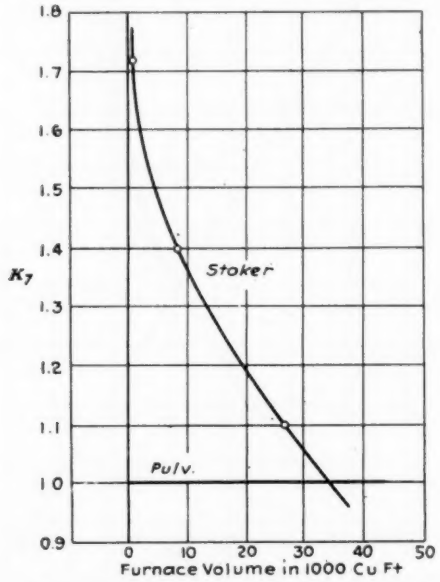


FIG. 8 WOHLENBERG CURVES—ADDITIONAL INFLUENCE OF FURNACE VOLUME

is much less for gas firing than for either stoker or pulverized-coal firing. As will be later shown, the experimental data at hand would seem to indicate otherwise. This results in the conclusion that the Wohlenberg method in its present form and as applied does not hold for gas or oil firing.

Equation [2] thus developed may be solved for any pulverized-coal or stoker furnace within 2 or 3 minutes, the only work involved being the selection of the appropriate K- and C-factors from Figs. 3 to 11, inclusive, and a simple slide-rule multiplication. From the value of μ obtained, the temperature of the flue gases leaving the furnace may be found by means of heat-capacity curves. These heat-capacity curves, Fig. 12, show the relation between the sensible heat of a given quantity of gas and its temperature and may be found by conventional combustion calculations using the ultimate analysis of the fuel.

The curves just developed will check values obtained by using the somewhat lengthy Wohlenberg equations within a maximum error of about ± 0.03 in the value of μ. An advantage of the curves given in this manner is that they show clearly the effect of variation of any one of the factors.

HEAT RELEASE

Before discussing additional possible methods of calculation, it will be desirable to examine the experimental data given by the A.S.M.E. Radiation-Committee report[3] in connection with the Broido, Orrok, and Wohlenberg methods. In making such an examination, it is necessary to consider two quantities which

TABLE 3 COMPARISON OF μ FOR DIFFERENT FIRING METHODS AS COMPUTED BY THE WOHLENBERG METHOD

Firing method	Air, per cent	Furnace temp, F	μ
Pulv. coal	120	2230	0.40
Stoker	120	2386	0.36
Gas	120	2720	0.25

enter into the combustion process, namely, heat released in the furnace and the radiant heating surface in the furnace. There has been considerable diversity in the method of calculating these items in the literature.

In regard to the heat liberated in the furnace, some of the attacks on the problem have used an energy-release rate based on the calorific or high heat value of the fuel plus any sensible heat in the combustion air due to preheating.

From a theoretical point of view this is not thoroughly logical in that the latent heat required to vaporize the moisture due to hydrogen and water in the fuel is invaluable for raising the temperature of the gases or for radiation. In other words, instead of the high heat value, the low heat value should be used as in European practice, even though it may be somewhat indeterminate.

Thus, as far as furnace radiation is concerned, the low or net heat value should be used in calculations to determine the heat released per cubic foot of furnace volume, the heat released per pound of fuel supplied, and the value of μ.

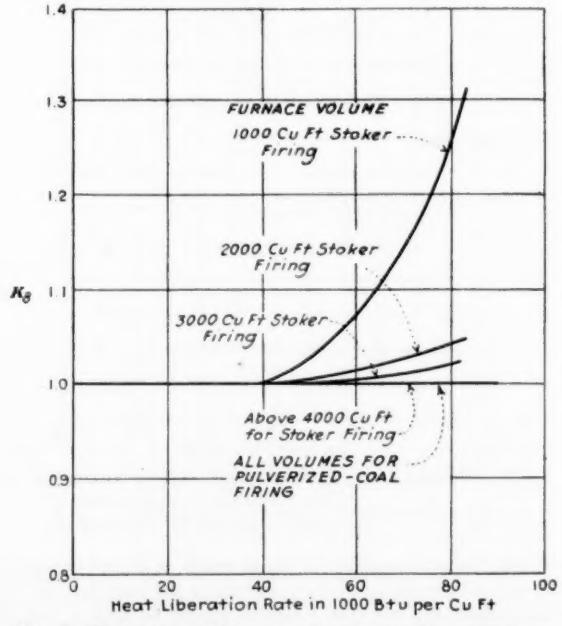


FIG. 9 WOHLENBERG CURVES—ADDITIONAL INFLUENCE OF LIBERATION RATE. LIBERATION RATE BASED ON HIGH HEAT VALUE OF FUEL + AIR HEAT

Various investigators have used the high or low heat value of the fuel rather indiscriminately. The Wohlenberg method made use of the high value and the Orrok method has made use of the low value although Orrok has compared his μ-values directly with values given in the Wohlenberg papers, thereby introducing confusion. Broido's procedure is not clear. The A.S.M.E. Radiation-Committee Report[3] gives experimental values of μ computed on both the high- and low-heat-value basis.

The heat supplied per pound of fuel fired should take into account incomplete combustion in any form.

In order to relate the various furnace-heat-transfer methods to each other, they have been reduced to a determination of μ, from which the furnace-exit temperature may be found through the use of heat-capacity curves. This μ in the case of the Wohlenberg

method has been based on the high heat value of the fuel, denoted as μ_H, hence the comparison with the experimental data will be made on this basis. The Orrok and Broido methods will be compared by a μ based on the low heat value of the fuel, denoted as μ_L. The μ values calculated in either manner do not differ greatly, the maximum differences occurring, of course, for natural-gas firing.

Since the degree of completion of combustion in the various

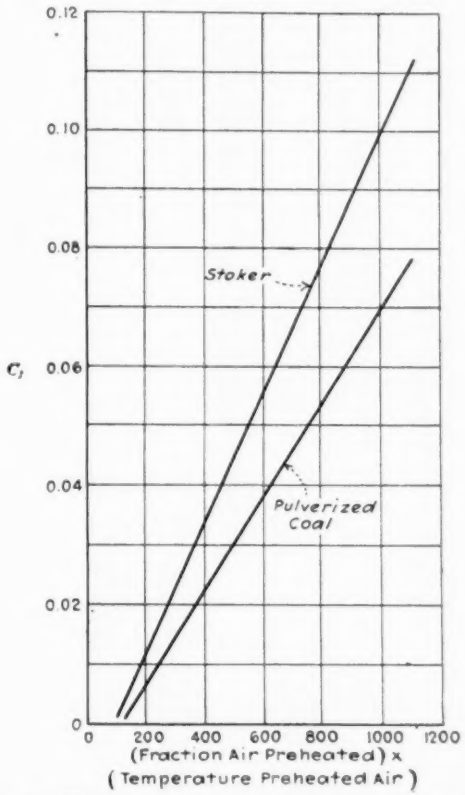


FIG. 10 WOHLENBERG CURVES—INFLUENCE OF PREHEAT

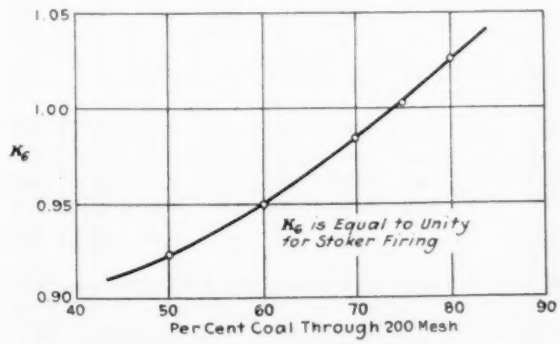


FIG. 11 WOHLENBERG CURVES—INFLUENCE OF PULVERIZATION FINENESS

furnaces listed in the A.S.M.E. Radiation-Committee report[3] is not accurately known, it has seemed advisable to compare μ-values, computed by the various methods, on the basis of complete combustion. The gas-temperature μ-values of the report[3] were likewise computed on this basis and therefore the error in making this comparison should not be large.

EFFECTIVE FURNACE RADIANT HEATING SURFACE

The radiant heating surfaces in the furnace are those portions of the waterwalls and lower surface of the tubes of the boiler proper which "see" the gases in the furnace. Considerable difference of opinion exists on the exact method of evaluating this surface, due to different arrangements of bare tubes in front of refractory, bare tubes recessed in refractory, fin tubes, cast-iron or steel block covered tubes, and refractory-faced block-covered tubes.

For bare tubes the projected surface, that is, the diameter

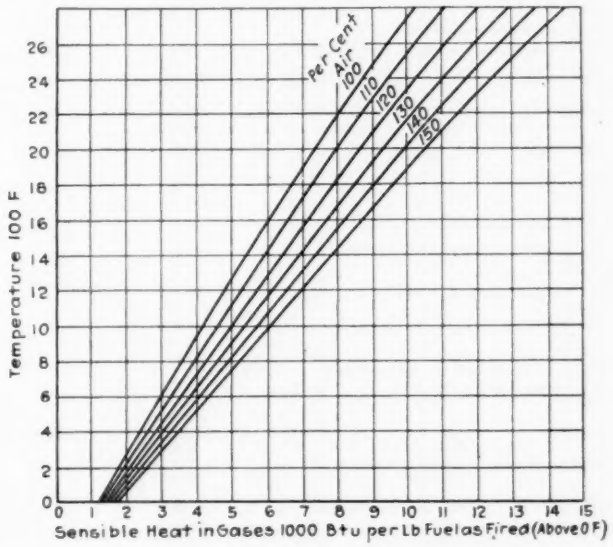


FIG. 12 HEAT-CAPACITY CURVES FOR BITUMINOUS COAL USING GOODENOUGH SPECIFIC HEATS

times the length is often taken. Many use the surface based on half the circumference, the surface based on the whole circumference, and other possible combinations. In comparing or using any method, the manner of measuring the surface must be noted.

Obviously, the effective radiant heating surface should have a value independent of any method that may be used for the calculation of heat absorption. It is usually thought of as the equivalent plane surface which absorbs the same amount of heat as the cold surface or waterwall under consideration.

Most of the methods have given the procedure for the calculation of heat absorption, that is, the μ-value, without stating how to reduce the furnace walls actually found in practice to this plane-surface base.

In order to establish the values of the effective furnace radiant heating surfaces of the different boilers tested, usually denoted by rhs, it was necessary to decide upon a procedure which would be applicable to all types of furnace surfaces such as bare block, refractory-faced block, bare tube, and fin tube. This meant that certain assumptions had to be made, based upon such scanty information as could be discovered concerning the relative heat-absorbing capacities of such surfaces as refractory-faced blocks and fin tubes.

It is appreciated that a procedure so selected may not be acceptable to the proposers of the various formulas and methods, on the ground that they either recommend other procedures or had other procedures in mind when presenting their method.

However, the impracticability of calculating different effective radiant heating surfaces for each of the various furnaces is obvious and the results given in this paper must be accepted with this in mind.

For the purpose of this paper, the radiant heating surface has

TABLE 4 DETERMINATION OF FURNACE EFFECTIVE RADIANT HEATING SURFACES[a]

		1	2	3	4	5	6	7	8	9	10	11	12	13
1	Boiler no.[b,c]													
2	Total refractory surface	100	2985	3439	900	1136	142	1503	1224	180	184	2071	10.8	1940
Refractory-faced waterwall block:														
3	Overall surface	616	..	..	..	162	417	..	..	..	..	..	0 42	..
4	F_A	1.00	..	..	..	1.00	1.00	..	..	..	..	..	1.00	..
5	F_C	0.35	..	..	..	0.35	0.35	..	..	..	..	..	0.35	..
6	F_S	0.83	..	..	..	0.89	0.84	..	..	..	..	..	0.98	..
7	Effective surface	180	..	..	..	52	122	..	..	..	..	..	0 14	..
8	Additional refractory surface	436	..	..	..	110	295	..	..	..	..	..	0 28	..
Bare-faced waterwall block:														
9	Overall surface, front and rear	278	..	..	..	..	..	..	..	166	706	..	...	..
10	F_A	1.00	..	..	..	..	..	..	..	1.00	1.00	..	...	..
11	F_C	0.70	..	..	..	..	..	..	..	0.70	0.70	..	...	..
12	F_S	0.82	..	..	..	..	..	..	..	0.96	0.92	..	...	..
13	Effective surface	160	..	..	..	..	..	..	..	112	454	..	...	..
14	Additional refractory surface	118	..	..	..	..	..	..	..	54	252	..	...	..
15	Overall surfaces, sides	488	..	..	..	234	..	..	..	78	..	..	125.5 83.5	..
16	F_A	1.00	..	..	..	1.00	..	..	..	1.00	..	..	1.00	..
17	F_C	0.70	..	..	..	0.70	..	..	..	0.70	..	..	1.00	..
18	F_S	0.95	..	..	..	0.90	..	..	..	0.96	..	..	1.00	..
19	Effective surface	324	..	..	..	148	..	..	..	52	..	..	125.5 83.5	..
20	Additional refractory surface	164	..	..	..	86	..	..	..	26	..	..	0	..
Bare-tube waterwall:														
21	Overall surface, front	..	..	..	..	..	407	..	..	417	..	..	...	..
22	F_A	..	..	..	..	..	1.00	..	..	0.97	..	..	...	..
23	F_C	..	..	..	..	..	1.00	..	..	1.00	..	..	...	..
24	F_S	..	..	..	..	..	0.84	..	..	0.96	..	..	...	..
25	Effective surface	..	..	..	..	..	343	..	..	388	..	..	...	..
26	Additional refractory surface	..	..	..	..	..	64	..	..	29	..	..	...	..
27	Overall surfaces, sides	580	454	..	360	..	898	..	..	574	..	..	...	660
28	F_A	0.67	0.75	..	0.66	..	1.00	..	..	1.00	..	..	...	0.62
29	F_C	1.00	1.00	..	1.00	..	1.00	..	..	1.00	..	..	...	1.00
30	F_S	0.88	0.63	..	1.00	..	0.84	..	..	0.96	..	..	...	0.96
31	Effective surface	342	214	..	238	..	754	..	..	552	..	..	...	392
32	Additional refractory surface	238	240	..	122	..	144	..	..	22	..	..	...	268
33	Overall surface, rear	384	..	..	364	..	294	..	..	333	..	228	...	..
34	F_A	0.67	..	..	0.93	..	1.00	..	..	1.00	..	0.76	...	..
35	F_C	1.00	..	..	1.00	..	1.00	..	..	1.00	..	1.00	...	..
36	F_S	0.89	..	..	1.00	..	0.84	..	..	0.96	..	0.96	...	..
37	Effective surface	229	..	..	338	..	247	..	..	320	..	167	...	..
38	Additional refractory surface	155	..	..	26	..	47	..	..	13	..	61	...	..
39	Overall surface, bottom	..	860	860	..	..	57	..	..	..	945	314	...	700
40	F_A	..	0.75	0.75	..	..	1.00	..	..	..	0.77	0.76	...	0.65
41	F_C	..	1.00	1.00	..	..	1.00	..	..	..	1.00	1.00	...	1.00
42	F_S	..	1.00	1.00	..	..	1.00	..	..	..	0.92	0.96	...	1.00
43	Effective surface	..	645	645	..	..	57	..	..	..	670	229	...	455
44	Additional refractory surface	..	215	215	..	..	0	..	..	..	275	85	...	245
Fin-tube waterwall:														
45	Overall surface, front and rear	..	..	..	355	..	..	..	792	..	1618	..	...	..
46	F_A	..	..	..	1.00	..	..	..	1.00	..	0.63	..	...	..
47	F_C	..	..	..	1.00	..	..	..	1.00	..	1.00	..	...	..
48	F_S	..	..	..	1.00	..	..	..	1.00	..	0.92	..	...	..
49	Effective surface	..	..	..	355	..	..	..	792	..	938	..	...	..
50	Additional refractory surface	..	..	..	0	..	..	..	0	..	680	..	...	..
51	Overall surface, sides, top	..	..	..	800	..	..	..	800	..	2894	..	...	1060
52	F_A	..	..	..	1.00	..	..	..	1.00	..	1.00	..	...	1.00
53	F_C	..	..	..	1.00	..	..	..	1.00	..	1.00	..	...	1.00
54	F_S	..	..	..	1.00	..	..	..	1.00	..	0.92	..	...	0.96
55	Effective surface	..	..	..	800	..	..	..	800	..	2660	..	...	1016
56	Additional refractory surface	..	..	..	0	..	..	..	0	..	234	..	...	44
Furnace exit:														
57	Overall effective surface	394	438	438	364	408	360	357	475	555	432	360	...	330
Summation:														
58	Total refractory	1211	3440	3654	1048	1332	692	1503	1224	324	1625	2217	10.8 38.8	2497
59	Total effective waterwall surface	1629	1297	1083	2095	608	1883	357	2067	1979	5154	756	125.5 97.5	2193
60	Total furnace surface, excluding grate	2840	4737	4737	3143	1940	2575	1860	3291	2303	6779	2973	136.3	4630
61	Fraction cold (item 59/item 60)	0.57	0.27	0.23	0.67	0.31	0.73	0.19	0.63	0.86	0.76	0.25	0.92 0.71	0.47
62	Effective surface, screen boiler	1629	859	645	..	..	..	..	..	..	..	396	125.5 97.5	1471

[a] All surfaces are given in square feet.
[b] Boiler numbers 1 to 10, inclusive, are the same as listed in the Report of the Committee on Absorption of Radiant Heat in Boiler Furnaces, "An Experimental Investigation of Heat Absorption in Boiler Furnaces."[5]
[c] Data for boilers 11, 12, and 13 are taken from the literature.
NOTE: F_E taken equal to 1.00.

been computed on the basis given in an article by one of the present authors.[6]

This procedure makes use of the formula

$$A_{rhs} = A\,F_A F_C F_S F_E \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

where A_{rhs} = equivalent effective plane radiant heating surface; A = the total overall plane area of the wall in question, regardless of the tube spacing, whether bare or block covered, etc.; F_A = an area factor to reduce overall surface to equivalent effective radiant heating surface, assuming all surfaces clean and of perfect conductivity; F_C = a conductivity factor to take into account the heat conductivity of various types of block-covered walls; F_S = a slag factor to account for reduction in heat transfer caused by slag coatings; and F_E = an emissivity factor.

The value of the factors for use in Equation [3] under various conditions are given by H. F. Mullikin[6] and not repeated here.

[6] "Evaluation of Effective Radiant Heating Surface and Application of the Stefan-Boltzman Law to Heat Absorption in Boiler Furnaces," by H. F. Mullikin, Trans. A.S.M.E., vol. 57, 1935, RP-57-2.

TABLE 5 DETERMINATION OF μ-VALUES BY DIFFERENT METHODS

2	39	40	41	42	43	44	45	46	47
Test	Heat absorbed by screen boiler, 10^6 Btu per hr	Heat to furnace surfaces based on screen boiler, 10^6 Btu per hr	μ_L based on screen boiler evap	Heat (high) input per unit r_{hs}[a] 1000 Btu per hr	μ calculated by Broido method	Air-fuel rate R, lb air per lb fuel	Fuel-r_{hs} rate C_R, lb wet fuel per unit r_{hs}	μ calculated by Orrok method	μ calculated by Wohlenberg method
1-1	105	105	0.26	252	0.34	12.1	17.3	0.35	0.31
1-2	84	84	0.30	179	0.39	14.5	12.2	0.35	0.29
1-3	49	49	0.35	89	0.47	13.6	6.1	0.45	0.39
2-1	52.1	78.8	0.41	154	0.41	13.3	11.2	0.38	0.36
2-2	56.6	85.6	0.44	154	0.41	13.6	11.2	0.37	0.36
2-3	58.7	88.7	0.46	150	0.41	12.1	10.9	0.40	0.40
2-4	54.5	82.3	0.43	155	0.41	14.6	11.3	0.36	0.33
2-5	61.7	93.2	0.48	154	0.41	13.6	11.2	0.38	0.36
2-6	57.8	87.4	0.46	153	0.41	12.8	11.2	0.39	0.38
2-7	56.0	84.7	0.44	154	0.41	13.8	11.2	0.37	0.35
2-8	54.2	82.0	0.43	154	0.41	13.8	11.2	0.38	0.35
2-9	56.0	84.6	0.44	153	0.41	12.9	11.2	0.38	0.38
2-10	59.0	89.1	0.30	237	0.35	12.6	17.3	0.34	0.35
2-11	64.3	97.1	0.34	230	0.36	11.5	16.8	0.36	0.38
2-12	67.7	102	0.35	234	0.36	11.8	17.0	0.35	0.37
2-13	75.5	114	0.39	234	0.36	12.0	17.0	0.35	0.36
2-14	81.7	123	0.40	244	0.36	11.5	17.8	0.36	0.38
2-15	84.5	128	0.43	240	0.36	12.3	17.5	0.34	0.36
2-16	75.5	114	0.38	239	0.36	14.6	17.5	0.30	0.29
2-17	79.5	120	0.41	233	0.35	12.6	17.0	0.33	0.35
2-18	85.5	129	0.45	230	0.35	11.4	16.8	0.36	0.38
2-19	65.4	98.8	0.28	287	...	12.9	20.8	0.30	0.31
2-20	68.4	103	0.28	293	...	12.8	21.4	0.30	0.30
2-22	72.0	109	0.29	299	...	12.5	21.8	0.31	0.31
2-23	81.3	123	0.34	294	...	11.9	21.4	0.32	0.33
2-24	93.5	141	0.39	292	...	10.8	21.3	0.34	0.33
2-25	84.3	127	0.34	298	...	12.7	21.7	0.30	0.30
2-26	90.3	136	0.36	299	...	12.5	21.8	0.31	0.31
2-27	95.7	144	0.39	295	...	11.0	21.5	0.33	0.35
3-8	49.4	83.0	0.44	183	0.38	13.2	13.3	0.37	0.33
3-6	57.7	97.0	0.34	273	0.33	12.5	19.9	0.33	0.33
3-13	63.7	107	0.28	364	...	13.3	26.6	0.28	0.28
4-1	...	...	...	45.0	0.54	17.8	2.2	0.49	0.58
4-2	...	...	...	49.2	0.54	19.9	2.4	0.46	0.50
4-3	...	...	...	74.0	0.49	14.7	3.7	0.48	0.58
4-4	...	...	...	80.2	0.48	16.6	4.0	0.45	0.52
4-5	...	...	...	59.7	0.52	15.4	3.0	0.50	0.60
4-6	...	...	...	61.6	0.52	18.6	3.0	0.45	0.50
4-7	...	...	...	79.2	0.48	8.3	3.2	0.64	0.52
4-8	...	...	...	84.5	0.48	13.0	3.4	0.53	0.39
4-9	...	...	...	61.1	0.51	10.2	2.5	0.64	0.51
4-10	...	...	...	61.6	0.51	13.8	2.4	0.55	0.42
4-11	...	...	...	48.2	0.53	10.0	2.0	0.65	0.54
5-1	...	...	...	115	0.44	21.9	13.2	0.30	0.17
5-2	...	...	...	230	0.35	18.9	26.2	0.22	0.17
5-3	...	...	...	319	...	13.0	36.6	0.26	0.21
6-1	...	...	...	47.6	0.54	9.9	4.4	0.58	0.39
6-2	...	...	...	44.9	0.54	8.7	4.2	0.61	0.45
6-3	...	...	...	43.0	0.55	7.6	4.0	0.65	0.51
6-4	...	...	...	72.2	0.50	7.7	6.8	0.59	0.44
6-5	...	...	...	70.0	0.50	6.8	6.6	0.62	0.47
7-1	...	...	...	257	0.33	11.1	12.5	0.42	0.31
7-2	...	...	...	250	0.33	8.2	12.1	0.48	0.37
7-3	...	...	...	249	0.34	6.9	12.0	0.53	0.40
7-4	...	...	...	532	...	9.2	25.8	0.37	0.27
7-5	...	...	...	523	...	8.2	25.4	0.40	0.28
7-6	...	...	...	523	...	6.9	25.4	0.43	0.31
7-8	...	...	...	815	...	8.2	39.5	0.35	0.23
7-9	...	...	...	845	...	7.3	41.0	0.37	0.25
7-10	...	...	...	276	0.32	21.3	14.5	0.26	0.29
7-11	...	...	...	480	...	19.5	25.2	0.22	0.27
7-12	...	...	...	706	...	20.7	37.2	0.18	0.21
8-1	...	...	...	64.7	0.51	5.3	3.2	0.74	0.55
8-2	...	...	...	68.7	0.50	6.9	3.4	0.68	0.51
8-3	...	...	...	73.5	0.50	8.9	3.7	0.60	0.45
8-4	...	...	...	141	0.42	5.5	7.0	0.65	0.43
8-5	...	...	...	101	0.46	7.3	5.0	0.63	0.43
8-10	...	...	...	145	0.42	5.5	7.3	0.65	0.42
8-11	...	...	...	108	0.46	5.5	5.4	0.70	0.47
8-12	...	...	...	158	0.41	5.5	7.9	0.64	0.42
8-13	...	...	...	213	0.36	5.5	10.6	0.61	0.38
9-6	...	...	...	150	0.41	14.0	9.8	0.39	0.32
9-7	...	...	...	166	0.40	13.0	10.9	0.39	0.32
9-14	...	...	...	86.0	0.48	17.0	5.6	0.40	0.29
9-15	...	...	...	81.8	0.48	15.0	5.4	0.40	0.35
9-16	...	...	...	158	0.47	16.7	10.3	0.40	0.25
9-17	...	...	...	158	0.41	17.0	10.3	0.40	0.25
9-18	...	...	...	194	0.38	15.0	12.7	0.33	0.26
9-19	..	...	...	196	0.38	15.3	12.7	0.33	0.26
10-12	...	...	...	238	0.36	11.5	15.2	0.37	0.37
10-13	...	...	...	182	0.39	12.8	11.6	0.38	0.37
10-14	...	...	...	120	0.45	14.5	7.6	0.41	0.39
10-15	...	...	...	111	0.45	14.9	7.0	0.41	0.38
10-16	...	...	...	105	0.46	18.0	6.5	0.37	0.29
10-17	...	...	...	166	0.41	13.9	10.5	0.37	0.36
10-18	...	...	...	168	0.41	14.9	10.5	0.36	0.33
10-19	...	...	...	170	0.40	13.3	11.4	0.38	0.38
10-20	...	...	...	218	0.37	12.2	13.7	0.38	0.37

[a] Effective furnace radiant heating surface.

Equation [3] has been used, as shown in Table 4, to determine the effective radiant heating surface of the boilers enumerated in the A.S.M.E. Radiation-Committee report.[3] It was deemed best to take F_E equal to unity at the time the calculations were made although a value of 0.95 would perhaps have been preferable.

Data taken from the literature for three additional boiler tests is also included.

Check of Experimental Data by Waterwall Evaporation

Ordinarily the check of experimental data by waterwall evaporation is accomplished by segregation of a part of the furnace waterwalls or, as in the case of the first three units tested in the A.S.M.E. Radiation-Committee report,[3] by constructing the waterwalls so as to have their own feed and steam drum, thus acting as a separate steam-generating unit. Some of the boilers of the Ford Motor Company at Fordson, Mich., are also constructed in this manner. There are several difficulties encountered in the use of this method.

In the first place, results will depend upon the estimated relative heat-absorption rates at different kinds of surfaces. These rates are needed to determine the values of the effective radiant heating surfaces. Two such values are involved. The first is the total effective radiant heating surface in the furnace and the second is that part of the surface for which the absorption was measured. The ratio of the first surface to the second is a factor used in the calculation.

In the second place, certain boilers, as in the case of the first unit of the A.S.M.E. Radiation-Committee report,[3] are constructed so that the furnace-exit slag screen is part of the waterwall boiler. Under these conditions some heat will be transferred by convection through this slag screen into this waterwall boiler.

It is difficult to allow for this heat transfer. In addition, some radiant heat will pass between these slag-screen tubes, be absorbed by the boiler tubes proper and not be measured. In the calculation given in the present paper, these last two quantities of heat have been assumed equal, a very rough approximation.

TABLE 6 DETERMINATION OF μ_L BY HEAT ABSORPTION OF PART OF FURNACE WALLS

1	3		5	39	40	41
Name of boiler	Firing and fuel	Fraction cold	Boiler evap	Heat absorbed by screen boiler	Heat to furnace based on screen boiler	μ_L based on screen boiler evap
			—10^6 Btu per hr—			
Cahokia	Pulv. coal	0.25	90	29.4	56.2	0.59
Cahokia	Pulv. coal	0.25	120	29.4	56.2	0.43
Cahokia	Pulv. coal	0.25	144	44.0	84.0	0.52
			—1000 kcal per hr—			
Sweden	Oil	0.92	727	727	727	0.68
Sweden	Oil	0.92	1257	1257	1257	0.66
Sweden	Oil	0.71	1036	1036	1036	0.61
Sweden	Pulv. coal	0.71	711	711	711	0.57
Sweden	Pulv. peat	0.92	713	713	713	0.61
Sweden	Wood pulp	0.92	446	446	446	.51
			1000 lb per hr	Boiler evap, per cent		
Fordson	Pulv. coal	0.47	150	33	49	0.40
Fordson	Pulv. coal	0.47	300	22	33	0.27
Fordson	Pulv. coal	0.47	450	18	27	0.21

TABLE 7 INDICATED CALCULATIONS FOR ITEMS IN TABLE 5[a]

Item 39	[(heat of steam) — (heat of feedwater)] × item 6
Item 40	[(rhs[b] in furnace)/(rhs in screen boiler)] × item 39
Item 41	(item 40 × item 25)/(item 33 × item 26)
Item 42	item 33/rhs
Item 43	Fig. 1 using item 42
Item 44	calculated using item 20
Item 45	item 42/item 25
Item 46	Fig. 2 using items 44 and 45
Item 47	Equation [2] and Figs. 3 to 11, inclusive.

[a] Items previous to item 39 will be found in Table 3 of the report "An Experimental Investigation of Heat Absorption in Boiler Furnaces."[3]
[b] Effective furnace radiant heating surface.

The plane of the furnace exit (side of boiler tubes adjacent to furnace) is always an effective exposed furnace surface which cannot be segregated for radiant-heat measurement without inducing the problem of convection-heat transfer.

With this in mind, calculations have been made for six boiler furnaces for which test data are available to show the heat absorbed by a portion of the furnace surfaces. Three of these units are boilers Nos. 1, 2, and 3 of the A.S.M.E. Radiation-Committee report[3] for which the computations are given as items 39, 40 and 41 of Table 5 and indicated in Table 7. Table 5 is, in effect, a continuation of Table 3 of the A.S.M.E. Radiation-Committee report.[3] Tests were also available from the literature on boilers for which the construction data of the furnaces are given in Table 4. In this table, boiler No. 11 is a unit at Cahokia Station, St. Louis, Mo. (27); boiler No. 12 is a special test boiler at the Tekniska Hogskolan, Stockholm, Sweden (15, 16); and boiler No. 13 is a Fordson, Mich., unit (10, 19). The μ-determinations, based on segregated furnace-wall surfaces for boilers Nos. 11, 12, and 13 are given in Table 6. The full calculations

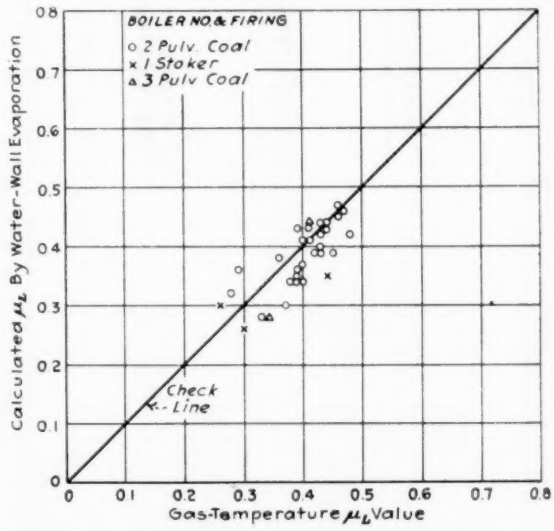


FIG. 13 COMPARISON OF RESULTS BY MEASUREMENT OF WATERWALL EVAPORATION

have been omitted to save space. The μ-values calculated in this manner for boilers Nos. 11, 12, and 13 are compared in Fig. 13 with the values as determined by furnace-exit temperature in the A.S.M.E. Radiation-Committee report[3] for boilers Nos. 1, 2, and 3. Reliable μ-values were not available for the units taken from the literature. The results check approximately. The μ-values, calculated by means of the waterwall-boiler evaporation, are seen to vary somewhat with the flame length, as effected by changes in the primary and tertiary air pressures. Long flames produce a greater heat absorption in the screen boiler than short flames. This causes a variation in the resulting μ-determination.

The gas-temperature μ-values do not exhibit as much variation. It is evident that the increased heat transfer of the surfaces of the waterwall boiler in the case of the long flame would have to be compensated for by decreased heat absorption in the furnace-exit surfaces in order that the gas-temperature μ-values might remain nearly constant.

The foregoing appears logical since long flames occupy more of the bottom portion of the furnace, thus causing increased radiation to the bottom tubes and decreased radiation to the top of the furnace. They also cause considerable convection-heat transfer to the bottom screen-boiler tubes.

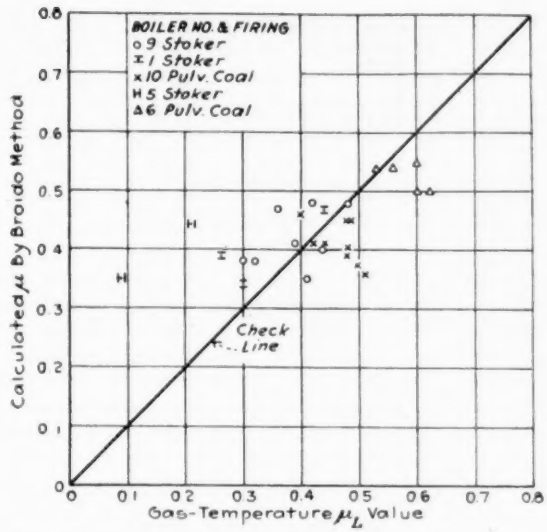


FIG. 14 COMPARISON OF RESULTS BY BROIDO METHOD FOR BOILERS NOS. 1, 5, 6, 9, AND 10[3]

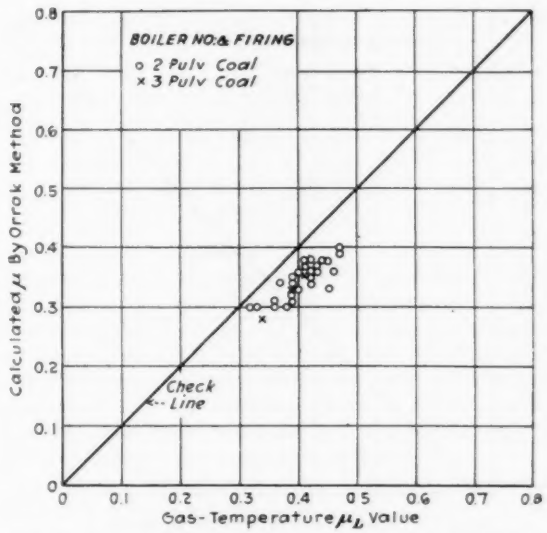


FIG. 15 COMPARISON OF RESULTS BY BROIDO METHOD FOR BOILERS NOS. 2 AND 3[3]

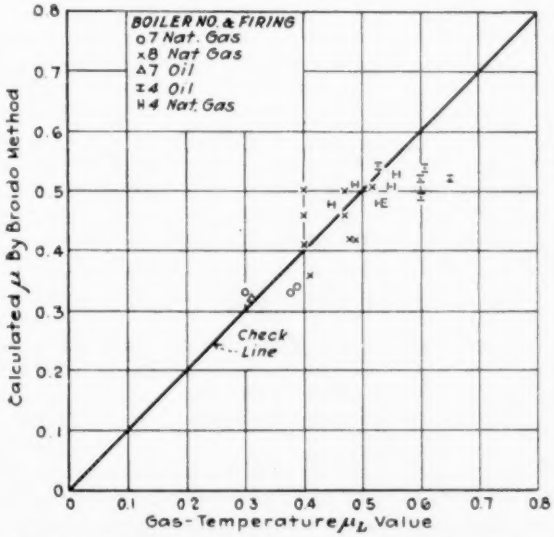


FIG. 16 COMPARISON OF RESULTS BY BROIDO METHOD FOR BOILERS NOS. 4, 7, AND 8[3]

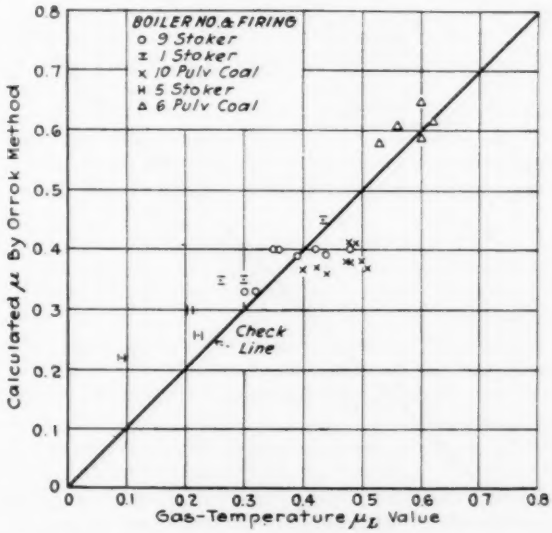


FIG. 17 COMPARISON OF RESULTS BY ORROK METHOD FOR BOILERS NOS. 1, 5, 6, 9 AND 10[3]

It is interesting to note that the values of μ check as closely as they do, but since the method employing the evaporation of the waterwall boiler is limited to special cases, and the gas-temperature μ-values determined by the method of furnace heat balance are more accurate, those determined by the evaporation method appear to be of no particular importance.

CHECK OF BROIDO, ORROK, AND WOHLENBERG METHODS

In Table 5 are given calculations for the tests on the units for which data is given in the A.S.M.E. Radiation-Committee report.[3] Item 43 in the table gives the μ-values obtained by the Broido method, item 46 the μ-values obtained by the Orrok method, and item 47 the μ-values obtained by the Wohlenberg method.

The results obtained by these three methods are compared in Figs. 14 to 21, inclusive, to the gas-temperature μ-values as given by A.S.M.E. Radiation-Committee report.[3] The line marked "check line" in these figures shows where the points lie if the two values check each other. The boiler numbers given in the various charts and tables correspond with those of the A.S.M.E. Radiation-Committee report.[3]

It should be again emphasized that in any usage of the Wohlenberg, Broido, or Orrok formulas, the results are dependent on the method of calculating the effective furnace radiant heating surface and that the surface calculated in a manner different from that used in this paper would yield different results.

It will be noted that the Broido determinations are surprisingly close considering the fact that only a single curve is used in the method.

The Orrok results are fairly representative except for the natural-gas tests for which the formula yields rather high values. The Orrok formula was probably not intended for natural-gas fuels. Further comment on these findings does not seem necessary, as the relative value of the methods, for the units for which tests are available, is quite apparent from the curves.

In view of the foregoing results it appears on the surface as though the values for gas radiation given by Schack (59) and used in the Wohlenberg method are somewhat low.

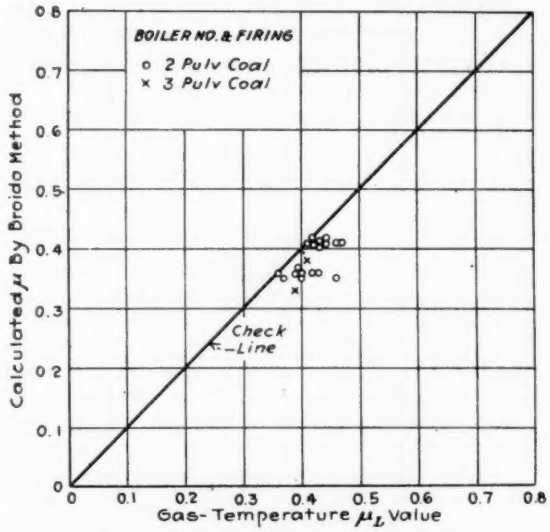


FIG. 18 COMPARISON OF RESULTS BY ORROK METHOD FOR BOILERS NOS. 2 AND 3[3]

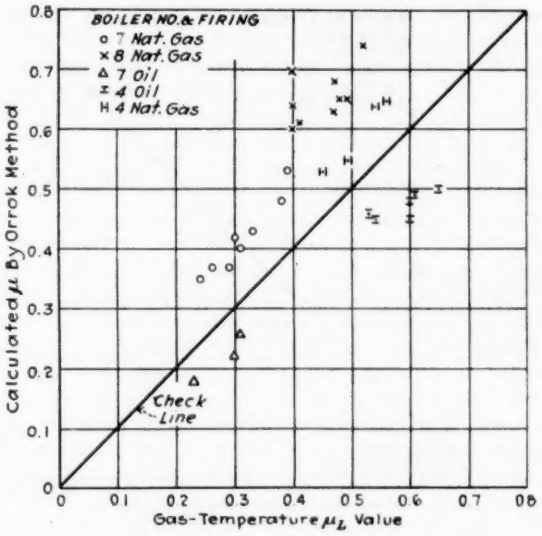


FIG. 19 COMPARISON OF RESULTS BY ORROK METHOD FOR BOILERS NOS. 4, 7, AND 8[3]

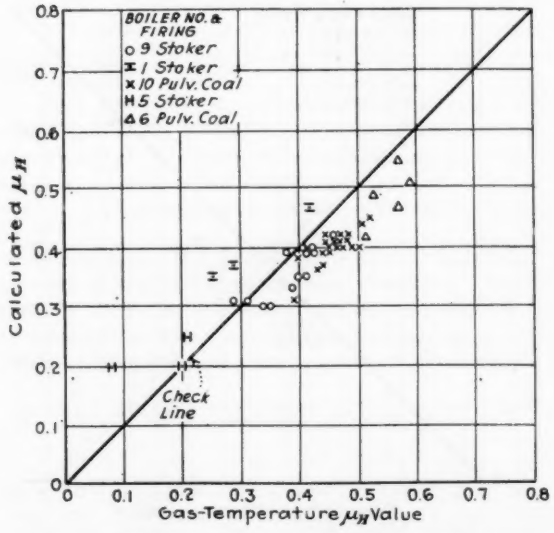


FIG. 20 CHECK OF WOHLENBERG METHOD FOR BOILERS NOS. 1, 5, 6, 9, AND 10[3], USING THE NEW COEFFICIENTS (*F*-FACTORS)

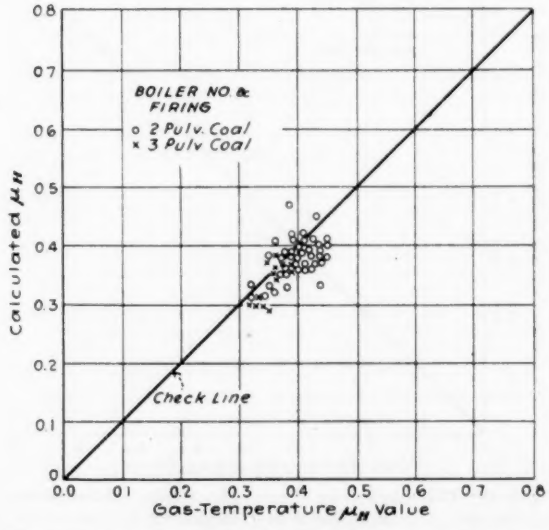


FIG. 21 CHECK OF WOHLENBERG METHOD FOR BOILERS NOS. 2 AND 3[3], USING THE NEW COEFFICIENTS (*F*-FACTORS)

This apparent discrepancy may, however, be due to the fundamental assumption of the Wohlenberg method that the furnace-exit-gas temperature is also equal to the mean radiating furnace temperature.

The so-called straight Stefan-Boltzman law method has had so many different values given for the constants therein contained, and is in such a crude form as ordinarily used, that no determinations have been made by this method for presentation in this report. While the Stefan-Boltzman law is used in the Wohlenberg derivation, it is only one of the factors used to obtain a solution, and therefore the method is not classified under the foregoing heading. An attack using the Stefan-Boltzman law is included in a paper by one of the present authors.[6]

An Experimental Investigation of Heat Absorption in Boiler Furnaces

By W. J. WOHLENBERG,[1] NEW HAVEN, CONN., H. F. MULLIKIN,[2] NEW YORK, N. Y., W. H. ARMACOST,[1] NEW YORK, N. Y., AND C. W. GORDON,[2] CHICAGO, ILL.

This paper is a report of the A.S.M.E. Committee on Absorption of Radiant Heat in Boiler Furnaces and includes test data obtained on ten boilers in different power plants in the United States. It involves stoker-fired, pulverized-fuel, and oil- and gas-fired units with various combinations of refractory and water-cooled furnaces. The data include furnace-exit gas temperatures as measured by means of the high-velocity thermocouple. These temperatures are used to determine the fraction of heat input to the furnace that was absorbed in its walls.

AT THE time the committee[4] submitting this report was formed it was suggested that radiation tests on large commercial boiler furnaces be made to supplement work of a more fundamental nature which was being conducted by various agencies. As a result a fellowship, sponsored by the Superheater Company, was established under the joint direction of Professor Wohlenberg of Yale University and Messrs. R. M. Gates, R. M. Osterman, and C. W. Gordon to study the problem and make such tests as might be necessary to put the subject on a more practical basis. H. F. Mullikin was appointed research fellow under this arrangement and had charge of the tests.[5] These tests were made on ten boilers in different power plants in the United States and involve stoker, pulverized-fuel, oil, and gas firing with various combinations of refractory and water-cooled furnaces.

Before describing these tests in detail, it seems advisable to discuss briefly the more important papers which have appeared on the subject. It is not proposed to outline in detail the previous material printed on this subject but merely to refer to the works of past authors.

Since the general adoption of the waterwall furnace for large steam-generating units beginning about 1924, many articles have been written offering various methods of obtaining a solution to the furnace heat-transfer problem. The majority of these have been technical, and are based on physical constants and laws, and on observed general conditions in furnaces. Several have been based on empirical test data.

The principal objection to any type of solution based on theory is that it is difficult to take into account a large number of design variables such as type of fuel; fineness of fuel; location and type of burners; location, type and surface of waterwalls and exposed boiler tubes; volume and shape of furnace; combustion characteristics of different fuels or of the same fuel under slightly different conditions; and such operating factors as slag on surfaces, flame length, draft, flame impingement, flame volume, excess air, method of air supply, and temperature of air.

Because of the difficulties involved in making heat-transfer tests, very few of them have been made, and it is upon the data from this insufficient number of tests that most empirical methods for solving heat-transfer problems are based. Therefore, it is questionable as to what extent these results can be applied. The empirical formulas generally neglect most of the factors and depend only upon one or two of the principal variables. Very few of the methods have not been checked by any one other than the original investigator.

With these facts in mind, attention is called in the bibliography to some of the more notable attacks on the problem. The list is not claimed to be complete but it is believed that most of the methods yielding fairly reasonable quantitative results are included.

In 1925 Broido (7)[6] presented a paper giving an excellent general discussion of the problem with a single curve to summarize several tests. Wohlenberg and various associates have presented numerous papers (1, 2, 3, 4, 5) containing an attack on the problem from a fundamental theoretical basis. Modified forms of this method, some of which are better adapted to practical application, are given by Haslam and Hottel (13, 60), and Münzinger (21, 41, 55). Even so, it is difficult to evaluate certain practical conditions in the process of solution. Orrok (20) has presented a formula empirically derived from Hudson (39). Wilson, Lobo, and Hottel (45) have adapted the Orrok equation in oil-still design. In most of these methods the Stefan-Boltzman law has been adapted.

Others making use of the Stefan-Boltzman law are: Schack (6, 59), Roszak and Veron (23), Gerbel (40), Ritchie (42), Forssblad (9), Seibert (48), and Müller (46). Rosin and Fehling (47) have developed theoretical equations to give maximum furnace heat liberation rates. DeBaufre (17) has reported tests on several units and has used a furnace heat balance to correlate his results. Kuhn (14) and Friedrich (54) have reported tests similar to those of DeBaufre.

A large number of tests have been run on boiler furnaces in which one or more tubes were isolated and the actual heat absorbed was measured by water circulated through the tubes. Results have in most cases been reported, but no attempt was made to derive an equation or formula therefrom. A number of these tests have been reported by Broido and Orrok (7). Others may be found in some of the N.E.L.A. serials (27, 28, 29, 33), transactions of various societies (10, 25, 34, 36, 37) and other sources (19, 26). A number were reported in papers by Artsay (12) and Ramzin (22).

---

[1] Mem. A.S.M.E.

[2] Jun. A.S.M.E.

[3] Assoc-Mem. A.S.M.E.

[4] The Committee on Absorption of Radiant Heat in Boiler Furnaces consists of W. J. Wohlenberg, Chairman, E. G. Bailey, R. M. Gates, C. W. Gordon, E. L. Lindseth, Geo. A. Orrok, R. J. S. Pigott, and John Van Brunt.

[5] This report abstracts in part a dissertation presented by Mr. Mullikin for the degree of Doctor of Philosophy at Yale University.

Contributed by the Special Research Committee on Absorption of Radiant Heat in Boiler Furnaces for presentation at the Annual Meeting December 2 to 6, 1935, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1936, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] Numbers in parentheses correspond to similarly numbered references given in the bibliography at the end of the paper.

TABLE 1 GENERAL BOILER DESCRIPTION AND DATA

General data:											
1	Boiler number	1	2	3	4	5	6	7	8	9	10
2	Rated surface, sq ft	24,600	27,548	27,548	10,830	15,000	10,760	15,150	15,930	22,900	60,706
3	Type boiler	S[a]	DS[b]	DS	S	HX[c]	HX	HX	HX	HX	DS
4	Make boiler	B&W[d]	W&W[e]	B&W	Heine	CE[f]	Springe	W&W	W&W	Spring	CE
5	Min steam rating, 1000 lb per hr	100	....	....	....	....	40	....	60	....	....
6	Normal steam rating, 1000 lb per hr	225	225	225	120	....	....	120	175	....	600
7	Max steam rating, 1000 lb per hr	340	400	400	160	....	200	230	325	....	1250
8	Date in service	1929	1929	1924	1931	1931	1929	1930	1929	1928	1929
9	Rated preheater surface, sq ft	26,930	8225	2425	....	13,660	11,550	none	none	....	82,721
10	Make preheater	B&W	F-W	Special	Ljun[g]	F-W	Thermix	....	....	Ljun.	CE[f]
11	Calc. furnace volume to exit, cu ft	14,000	22,410	22,410	11,000	5420	8740	5550	13,200	9040	33,400
12	Add. furnace volume above exit, cu ft	....	1930	1930	....	....	....	....	....	....	4040
13	Furnace width, ft	26.270	20.062	23.062	16.00	24.166	18.917	23.00	23.791	27.75	23.166
14	Make furnace	FL[i]	CE[f]	B&W	CE	FL	Springe	....	CE	TS[k]	CE
15	Burners per side	....	8	8	8	....	2	14	14	....	20
16	No. sides fired	....	2	2	1	....	1	1	2	....	2
17	Type burner	....	Fish[l]	Fish	....	....	HT[m]	....	....	....	Fish
18	Make burners	....	Lop[n]	Lop	....	....	P[o]	F[p]	F	....	Lop
19	Primary air, %	100[q]	15	15	100[q]	100	30[q]	100	100	100[q]	25[q]
20	Secondary air, %	....	50	50	....	....	70[q]	....	....	....	75[q]
21	Tertiary air, %	....	35[q]	35[q]	....	....	....	....	....	....	....
22	Type stoker	UF[r]	....	....	....	CG[s]	....	....	....	UF	....
23	Make stoker	Taylor	....	....	....	Ill.	....	....	....	Riley	....
24	Area of stoker, sq ft	605	....	....	....	528	....	....	....	360	....
25	Retorts, stoker	15	....	....	....	....	....	....	....	15	....
26	Tuyères, stoker	57	....	....	....	....	....	....	....	22	....
27	Length, stoker	19.2	....	....	....	22	....	....	....	13	....
Coal data:											
28	Moisture, %	2.70	2.45	2.45	....	21.50	9.11	....	....	4.10	0.25
29	Volatile, %	34.18	35.55	35.55	....	26.50	37.63	....	....	19.43	19.85
30	Fixed carbon, %	56.40	54.73	54.73	....	32.00	34.75	....	....	68.70	72.42
31	Ash, %	6.72	7.27	7.27	....	20.00	18.51	....	....	7.77	7.48
32	Ash fusion, deg F	....	....	....	....	....	....	....	....	2230	....
33	Oxygen, %	5.93	8.65	8.65	....	5.88	15.88	....	....	2.68	4.38
34	Sulphur, %	0.92	0.86	0.86	....	6.07	5.98	....	....	1.80	1.36
35	Ash, %	6.90	7.45	7.45	....	25.40	17.90	....	....	8.07	8.02
36	Nitrogen, %	1.23	1.49	1.49	....	0.92	1.10	....	....	1.41	1.25
37	Hydrogen, %	5.09	5.20	5.20	....	3.75	4.57	....	....	4.53	4.29
38	Carbon, %	79.93	76.35	76.35	....	58.00	54.57	....	....	81.51	80.70
39	High heat. value, dry, Btu per lb	14,230	13,800	13,800	....	10,500	11,020	....	....	14,509	14,580
40	High heat. value, as fired, Btu per lb	13,850	13,450	13,450	....	8250	10,050	....	....	14,150	14,550
41	Through 200 mesh, %	....	55.7	55.7	....	....	47.2	....	....	....	....
42	Through 100 mesh, %	....	75.6	75.6	....	....	62.4	....	....	....	....
43	Through 50 mesh, %	....	91.6	91.6	....	....	....	....	....	....	....
44	Through 30 mesh, %	....	98.2	98.2	....	....	92.6	....	....	....	....
Oil data:											
45	Carbon, %	....	....	....	89.8	....	....	....	....	....	....
46	Hydrogen, %	....	....	....	8.95	....	....	....	....	....	....
47	Oxygen, %	....	....	....	0	....	....	....	....	....	....
48	Sulphur, %	....	....	....	0.93	....	....	0.75	....	....	....
49	Nitrogen, %	....	....	....	0	....	....	....	....	....	....
50	Specific gr at 60 F	....	....	....	1.036	....	....	0.96	....	....	....
51	High heat. value, as fired, Btu per lb	....	....	....	18,424	....	....	19,000	....	....	....
52	Low heat. value, as fired, Btu per lb	....	....	....	17,400	....	....	17,970	....	....	....
53	Flash point, F	....	....	....	355	....	....	275	....	....	....
Natural-gas data											
54	Ethane, %	....	....	....	1.70	....	....	1.28	7.93	....	....
55	Methane, %	....	....	....	95.50	....	....	90.38	81.90	....	....
56	Oxygen, %	....	....	....	0.20	....	....	0.11	0.34	....	....
57	Carbon dioxide, %	....	....	....	2.00	....	....	0.22	....	....	....
58	Nitrogen, %	....	....	....	....	....	....	8.01	9.77	....	....
59	Specific gravity	....	....	....	0.636	....	....	....	....	....	....
60	High heat. value at 30-in. hg, and 60 F, Btu per cu ft	....	....	....	993	....	....	....	970	....	....
61	Low heat. value at 30-in. hg, and 60 F, Btu per cu ft	....	....	....	903	....	....	....	....	....	....
Calculated refractory surface:											
62	Front or uptake, sq ft	....	776	776	340	480	62	347	265	....	92
63	Each side, sq ft	....	523	750	85	28	....	241	157	....	....
64	Rear or downtake, sq ft	....	776	776	76	600	80	310	193	83	92
65	Top, sq ft	....	387	387	....	....	....	....	....	....	....
66	Bottom, sq ft	100	....	....	314	....	....	364	452	97	....
Refractory-faced waterwall block:											
67	Front, surface, sq ft	446	....	....	....	....	....	....	....	....	....
68	Front, amt slagged, %	75	....	....	....	....	....	....	....	....	....
69	Front, slag thickness, in.	0.75	....	....	....	....	....	....	....	....	....
70	Each side, surface, sq ft	....	....	....	....	81	....	....	....	....	....
71	Each side, amt slagged, %	....	....	....	....	50	....	....	....	....	....
72	Each side, slag, thickness, in.	....	....	....	....	0.75	....	....	....	....	....
73	Top, surface, sq ft	170	....	....	....	....	....	....	....	....	....
74	Top, amt slagged, %	75	....	....	....	....	....	....	....	....	....
75	Top, slag thickness, in.	0.75	....	....	....	....	....	....	....	....	....
76	Bottom, surface, sq ft	....	....	....	....	....	417	....	....	....	....
77	Bottom, amt slagged, %	....	....	....	....	....	100	....	....	....	....
78	Bottom, slag thickness, in.	....	....	....	....	....	0.125	....	....	....	....
Bare-faced waterwall block:											
79	Front surface, sq ft	....	....	....	....	....	....	....	....	83	353
80	Front, amt slagged, %	....	....	....	....	....	....	....	....	100	25
81	Front, slag thickness, in.	....	....	....	....	....	....	....	....	0.125	0.25
82	Each side, surface, sq ft	244	....	....	....	117	....	....	....	39	....
83	Each side, amt slagged, %	20	....	....	....	50	....	....	....	100	..
84	Each side, slag thick, in.	0.75	....	....	....	0.75	....	....	....	0.125	....
85	Rear, surface, sq ft	278	....	....	....	....	....	....	....	83	353
86	Rear, amt slagged, %	80	....	....	....	....	....	....	....	100	25
87	Rear, slag thick., in.	0.75	....	....	....	....	....	....	....	0.125	0.25

TABLE 1 (*Continued*)

Bare-tube waterwall:											
88	Front, diam tubes, in.	....	....	....	....	....	3.0	....	....	3.5	....
89	Front, distance center to center, in.	....	....	....	....	....	3.5	....	....	10 1/8/2	....
90	Front, total surface, sq ft	....	....	....	....	....	407	....	....	417	....
91	Front, amt slagged, %	....	....	....	....	....	100	....	....	100	....
92	Front, slag thickness, in.	....	....	....	....	....	0.125	....	....	0.125	....
93	Each side, diam tubes, in.	3.25	3.25	....	3.00	....	3.00	....	....	3.50	....
94	Each side, distance center to center, in.	6.00	9.00	....	10.25	....	3.50	....	....	8.5/2	....
95	Each side, total surface, sq ft	290	227	....	180	....	449	....	....	287	....
96	Each side, amt slagged, %	55	30	....	....	....	100	....	....	100	....
97	Each side, slag thickness, in.	0.75	0.75	....	....	....	0.125	....	....	0.125	....
98	Rear, diam tubes, in.	3.25	....	....	3.00	....	3.00	....	....	3.50	....
99	Rear, distance center to center, in.	6.0	....	....	5.125	....	3.50	....	....	8.5/2	....
100	Rear, total surface, sq ft	384	....	....	361	....	294	....	....	333	....
101	Rear, amt slagged, %	50	....	....	....	....	100	....	....	100	....
102	Rear, slag thickness, in.	0.75	....	....	....	....	0.125	....	....	0.125	....
103	Bottom, diam tubes, in.	....	3.25	3.25	....	....	....	....	....	....	4.0
104	Bottom, distance center to center, in.	....	9.0	9.0	....	....	....	....	....	....	10.5
105	Bottom, total surface, sq ft	....	860	860	....	....	57	....	....	....	945
106	Bottom, amt slagged, %	....	....	....	....	....	....	....	....	....	25
107	Bottom, slag thickness, in.	....	....	....	....	....	....	....	....	....	0.25
Fin-tube waterwall:											
108	Front, diam tubes, in.	....	....	....	....	....	....	....	4.0	....	4.0
109	Front, distance center to center, in.	....	....	....	....	....	....	....	7.0	....	10.5
110	Front, total surface, sq ft	....	....	....	....	....	....	....	486	....	555
111	Front, amt slagged, %	....	....	....	....	....	....	....	....	....	25
112	Front, slag thickness, in.	....	....	....	....	....	....	....	....	....	0.25
113	Each side, diam tubes, in.	....	....	....	3.0	....	....	....	4.0	....	4.0
114	Each side, distance center to center, in.	....	....	....	5.125	....	....	....	7.0	....	6.0
115	Each side, total surface, sq ft	....	....	....	400	....	....	....	400	....	1447
116	Each side, amt slagged, %	....	....	....	....	....	....	....	....	....	25
117	Each side, slag thickness, in.	....	....	....	....	....	....	....	....	....	0.25
118	Rear, diam tubes, in.	....	....	....	3.0	....	....	....	4.0	....	4.0
119	Rear, distance center to center, in.	....	....	....	5.125	....	....	....	7.0	....	10.5
120	Rear, total surface, sq ft	....	....	....	355	....	....	....	306	....	555
121	Rear, amt slagged, %	....	....	....	....	....	....	....	....	....	25
122	Rear, slag thickness, in.	....	....	....	....	....	....	....	....	....	0.25
123	Top, diam tubes, in.	....	....	....	....	....	....	....	....	....	4.0
124	Top, distance center to center, in.	....	....	....	....	....	....	....	....	....	10.5
125	Top, total surface, sq ft	....	....	....	....	....	....	....	....	....	508
126	Top, amt slagged, %	....	....	....	....	....	....	....	....	....	25
127	Top, slag thickness, in.	....	....	....	....	....	....	....	....	....	0.25
Furnace exit:											
128	Furnace exit, diam tubes, in.	3.25	3.25	3.25	3.0	4.0	3.0	4.0	4.0	3.0	3.0
129	Furnace exit, effective length, ft	15.00	17.75	....	22.0	17.0	19.0	15.5	20.0	20.0	18.66
130	Effective length of tubes, ft.	....	20.00	20.0	....	....	....	....	....	....	29.0
131	Furnace exit, effective width, ft.	26.25	24.75	....	16.50	24.00	18.92	23.00	23.75	27.75	23.10
132	Distance center to center, first row, in.	12.0	....	6.0	....	14.0	10.625	16.25	7.0	10.75	16.0
133	Second row tubes	sg[t]	sg	sg	p11[u]	sg	p11	sg	sg	p11	sg
134	Distance between first and second rows, in.	9.0	6.0	6.0	6.5	12.0	4.0	8.125	7.0	4.0	7.0
135	Distance, center to center second row, in.	12.0	6.0	6.0	6.0	14.0	10.625	16.25	7.0	10.75	16.0

[a] Stirling; [b]Double Stirling; [c] Horizontal cross-drum; [d] Babcock & Wilcox; [e] Walsh & Weidner; [f] Combustion Engineering; [g] Springfield; [h] Foster-Wheeler; [i] Ljüngstrom; [j] Fuller-Lehigh; [k] Superheater Company; [l] Fishtail; [m] horizontal turbulent; [n] Lopulco; [o] Peabody; [p] Forney; [q] preheated air; [r] underfeed; [s] chain grate; [t] staggered; [u] parallel.

Bailey (24) has developed a method of measuring heat absorption in furnaces having waterwalls covered with refractory facings. Schack has suggested the use of the radiation pyrometer for such measurements (6, 13). Koessler has used the thermopile (32). Croft and Schmarje (56) have used an absorption calorimeter to measure incident radiation to the walls of a furnace.

In Sweden, Lindmark and his associates have reported results (15, 16, 43) of tests involving the use of oil and pulverized fuel in an experimental furnace. In this country Sherman has published (51, 52, 53) considerable material on an experimental installation burning pulverized coal and gas. Brown advanced an empirical curve (8) which Marshall has discussed (44). Bottomley has presented a paper (49) based on data reported by Orrok.

Sherman has reported data on furnaces (31) taken during the course of an investigation on refractories. Artsay has presented two papers (11, 12) giving an analysis based on hydraulic similitude. Data on gas radiation have been reported by numerous investigators who will not be mentioned here.

Methods for Determining Radiant-Heat Absorption

Consideration of the previously described articles and papers reveals that five principal methods have been used in attempts to determine radiant-heat absorption in boiler furnaces. They are (*a*) determination of the actual evaporation in one or more tubes exposed to radiant heat, (*b*) use of physical methods such as the thermopile or radiation calorimeter, (*c*) the establishment of a temperature gradient through conducting material (only applicable to refractory-faced blocks), (*d*) a determination of the furnace temperature and subtraction of the heat leaving the furnace in the gases from the heat input to the furnace, and (*e*) the use of known physical data such as the Stefan-Boltzman law, and values of gas radiation.

The first four of these methods are empirical whereas the last is primarily a theoretical attack. Obviously the ideal procedure would be to develop first an attack which would give theoretical formulas to cover all cases and conditions, and then second to determine with the aid of data supplied from tests, the values of constants which appear in the theoretical formulas.

Unfortunately, the data given in the literature for each individual test usually are so meager that it is impossible to coordinate data from various sources in order to show the effect of such dimensions as furnace volume, and radiant heating surface. Thus, the committee submitting this report decided to conduct tests on a number of existing furnaces in order to obtain data which could be used as a reference for checking theoretical or empirical formulas.

The first of the four previously mentioned practical methods for determining radiant-heat absorption (segregating portions of the waterwall) is not ordinarily feasible in the case of the usual boiler furnace because of the expense and time involved. There are perhaps half a dozen boiler installations in the United States where the waterwall circulation is in some manner separated entirely from the boiler proper. Tests on three of these installations are given in this paper. Another notable example is the boiler installation at the Ford Motor Co., Fordson, Michigan, (10, 19), in which the side-wall tubes constitute an entirely separate boiler. Even here the top tubes of the furnace and the lower

boiler-tube row belong to the boiler proper and hence their heat absorption cannot be experimentally determined in this manner.

During the preliminary work on tests conducted by the committee, attempts were made to use a radiation calorimeter or "thermaprobe" as it has been termed. It was made of a flat-faced hollow steel block through which water was circulated. The heat absorbed by the circulating water was found by measuring its weight per unit of time, and its initial and final temperature. This method was discarded as being unsatisfactory. The objections are: (1) Limitation as to number and location of openings which may be made in the furnace walls, (2) difficulty in controlling the water flow at a constant value, (3) difficulty of operating more than one thermaprobe at the same time, (4) lag in responsivity during time intervals in which furnace conditions change, (5) difficulty in obtaining the same amount of slag on the thermaprobe that existed on the furnace tubes, or of allowing for it by correction and (6) possibility of considerable error due to peculiar local conditions.

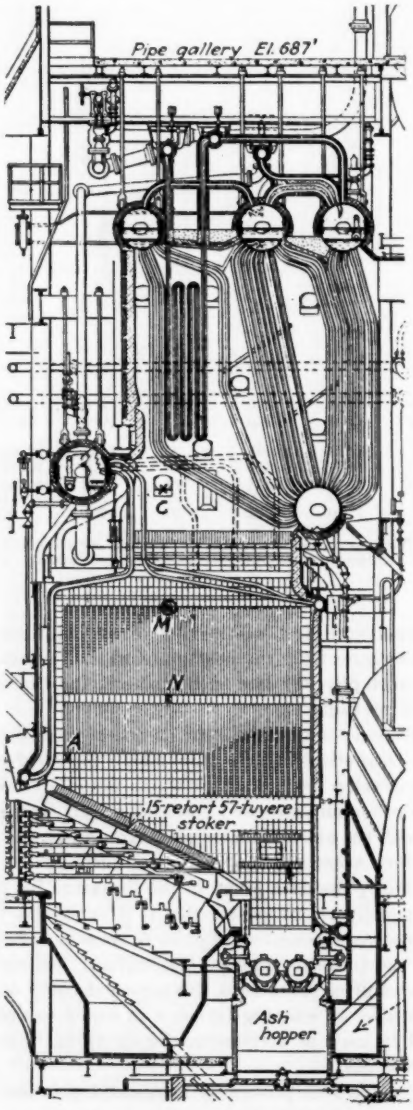


FIG. 1 UNDERFEED STOKER-FIRED BOILER WITH WATERWALLS ON ALL FOUR SIDES AND SEPARATE WATERWALL BOILER CIRCULATION

TABLE 2 DATA FROM SPECIAL TEST ON UNIT SHOWN IN FIG. 1

Boiler evap, lb per hr	Temperatures, deg F				
	Point *A*	Point *N*	Point *M*	Point *C*[a]	Point *C*[b]
115,000	2455	2125	2150	1940	1985
215,000	2920	2525	2630		2215
300,000	2780	2810	2810	2490	2505

[a] North side of boiler.
[b] South side of boiler.

The third method for determining radiant-heat absorption given by Bailey (24), is not applicable to bare-tube or fin-tube construction.

It appeared that the fourth method, which involves measuring the furnace-outlet gas temperature and setting up a furnace heat balance along the lines suggested by Kuhn (14) and DeBaufre (17), furnishes the most feasible method for determining heat transfer in boiler furnaces. The difference between the test procedures given by Kuhn and DeBaufre lay in the method of measuring gas temperatures. Kuhn used a high-velocity thermocouple, which is at present generally conceded to be the most accurate method of temperature measurement. DeBaufre, it is understood, used ordinary bare thermocouples.

TEST APPARATUS

The high-velocity thermocouple used in making the tests discussed in this report is shown in Fig. 11. The instrument consists of a water-cooled brass tube which can be inserted into the gas stream, the usual distance being 92 in. minus the wall thickness. The gases are aspirated past the hot junction of the thermocouple at a very high velocity by means of the air ejector. This increases the heat transfer from the gas to the thermocouple, thereby causing the thermocouple to read close to the true gas temperature. A more complete theory and description has been given elsewhere (18).

For some of the tests a radiation pyrometer was available which was used to take readings, where possible, for use according to Schack's (6) or Koessler's (32) method. These readings were later discarded, as early experiments showed that only indefinable mean results could be obtained by use of the radiation pyrometer. However, they did show that under certain conditions the emissivity of the furnaces, as indicated by a comparison of the high-velocity thermocouple and radiation-pyrometer determinations, may approach close to the value of unity.

DESCRIPTION OF THE BOILERS USED IN THE TESTS

In choosing boilers for test purposes, the committee endeavored to select representative boilers of various types burning various fuels in different manners. The following boilers were selected for testing:

1 Semivertical underfeed stoker-fired boiler, Fig. 1, with waterwalls on all four sides and 24,600 sq ft of heating surface.

2 Double-ended pulverized-coal-fired boiler, Fig. 2, with waterwalls on two sides, a slag screen on the bottom, and 27,548 sq ft of heating surface.

3 Double-ended pulverized-coal-fired boiler, Fig. 3, with a slag screen on the bottom and 27,548 sq ft of heating surface.

4 Semivertical combination gas- and oil-fired boiler, Fig. 4, with water walls on four sides and 10,830 sq ft of heating surface.

5 Horizontal cross-drum boiler, Fig. 5, with waterwalls on two sides, 15,000 sq ft of heating surface, and equipped with a chain-grate stoker.

6 Horizontal cross-drum boiler, Fig. 6, water cooled on four sides and the bottom. This boiler has 10,760 sq ft of heating surface and burns pulverized fuel.

7 Horizontal cross-drum combination gas- and oil-fired boiler, Fig. 7, with a refractory furnace and 15,150 sq ft of heating surface.

8 Horizontal cross-drum gas-fired boiler, Fig. 8, with waterwalls on four sides and 15,930 sq ft of heating surface.

9 Horizontal cross-drum boiler, Fig. 9, with waterwalls on four sides and 22,900 sq ft of heating surface. This boiler is equipped with an underfeed stoker.

10 Double-ended boiler, Fig. 10, with waterwalls on all sides and 60,706 sq ft of heating surface. This unit burns pulverized fuel.

Thus, data in this report include those from tests on three stoker-fired boilers, three gas-fired boilers, two oil-fired boilers, and four pulverized-coal-fired boilers.

A detailed description of these boilers is given in Table 1. This table also includes the various carefully measured exposed radiant surfaces, as well as construction and firing data. A considerable amount of data are given in order to furnish complete information concerning the tests, as it is not known exactly what factors influence the results obtained.

The surface areas given in Table 1 are overall values measured on the particular furnace walls taken as planes. The proximate analyses given are usually based on the fuel used during the tests. It was not always possible to take an ultimate analysis, and therefore, in some cases the values for the ultimate analyses are for the average station fuel rather than the specific values for the coal used during the tests. An estimate of the slag on the actual waterwall surfaces has been made. It was impossible to judge flame volumes to any degree of accuracy.

Preliminary Tests

Since the temperature measured at the furnace outlet (denoted by *M* in the figures) was the most important determination of the tests it is advisable to note some of the preliminary work.

During the tests on the first unit, Fig. 1, it was decided to measure temperatures by high-velocity thermocouples on one side of the boiler at points *A*, *M*, *N*, and *C*, and on the other side of the boiler at point *C*. Some of the data obtained at these points on the boiler shown in Fig. 1 are given in Table 2.

The temperature differences at *C* between the two sides of the boiler are less than 50 F at low loads and less than 20 F at high loads. The high-velocity thermocouples could be inserted up to a maximum distance of 7 ft into the boiler pass. Since this boiler, Fig. 1, is 24 ft wide, this left a space of 12 ft in the middle (less in the other boilers) which could not be reached by the high-velocity thermocouples. However, it was noted early and confirmed repeatedly that the gas temperature within reach of the tip of the high-velocity thermocouple showed no discernible temperature gradient to within 20 in. of the wall. On this account it is believed that the temperatures in the middle of the furnace width are not appreciably higher than at a point 6 or 7 ft from the wall.

Moreover, the furnace-gas temperatures are subject to minute-to-minute variations in the neighborhood of 100 F in most of the boilers tested, especially in the stoker-fired units.

Thus, it would seem that one reading on one side of the furnace width, while not as desirable as an average of temperatures taken across the pass and weighted according to mass flow, is nevertheless, over a period of time quite indicative of the average furnace-exit temperature within the accuracy of the other factors governing the tests.

The maximum error caused by this procedure of measuring temperatures will obtain at low loads where stratification of gas is more apt to occur. At high loads the gases will fill the furnace more uniformly and will promote temperature conditions which are more nearly uniform. Likewise, the furnace temperatures will be higher and since radiation is roughly proportional to the fourth power of the temperature, this radiation will also tend to equalize gas temperatures in various parts of the furnace cavity. This last statement is confirmed by the data of Table 2 where it will be seen that the temperatures at *A*, *N*, and *M* are in close agreement at the high load. Possibly it might be considered that high-load tests yield more accurate results than tests at lower loads.

It is possible that the temperature gradient frequently observed across boiler passes in the neighborhood of the superheater is more pronounced than in the hotter zones. This is due to unequal convection-heat absorption and the fact that radiation is insufficient to permit temperature equalization.

Description of Tests and Test Data

The boilers tested are shown in Figs. 1 to 10, inclusive, and are numbered from 1 to 10, respectively, in the tables. The furnace-exit temperature is taken at the point *M* as shown in the figures. This point was, whenever possible, chosen so as to be in the main gas stream approximately 1 ft underneath the boiler tubes at the furnace exit and about 6 ft toward the center of the furnace, measured from the inside furnace wall. Measurements were taken on one side of the furnace. The steam flows were measured

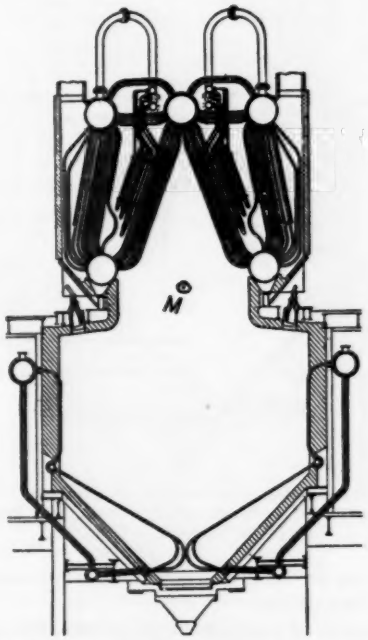


Fig. 2 Double-Ended Pulverized-Coal-Fired Boiler With Separate Waterwall-Boiler Circulation and Two Side Walls of Furnace Water Cooled

by flowmeters which were usually "water-columned" before the tests. The flowmeter readings are believed to be within 5 per cent of the true values, which is within the accuracy desired. Flowmeter corrections due to pressure and temperature were not made as they were small. The boiler-outlet temperatures were averages determined by a number of bare thermocouples or by calibrated recorders. Steam pressures were obtained by checked station gages. Feedwater and steam temperatures were obtained by thermometers or thermocouples.

Various auxiliary data which may be of interest were also included. The gas analysis is an average for samples taken across the top of the first pass or at the boiler outlet as noted in item 16 of Table 3.

The thermoelements of the high-velocity thermocouples were calibrated before a number of the tests and were always found to be within 10 F of their original calibration. The gas temperatures within the furnace, within the reach of the end of a high-velocity thermocouple located at any opening, were nearly always found to be constant at all points up to within about 2 ft of the

inside wall of the furnace for any of the tests, neglecting momentary total furnace-temperature variations of as much as 100 F.

A number of the readings of the high-velocity thermocouple were taken in various portions of the furnace and in the boiler passes but, as this paper is confined to the calculation of furnace radiation, they have not been included.

The furnace-temperature readings are usually averages of separate readings taken 10 minutes apart, the high-velocity thermocouple being removed between readings to avoid slagging difficulties. Duration of tests varied from 30 minutes to $3^1/_2$

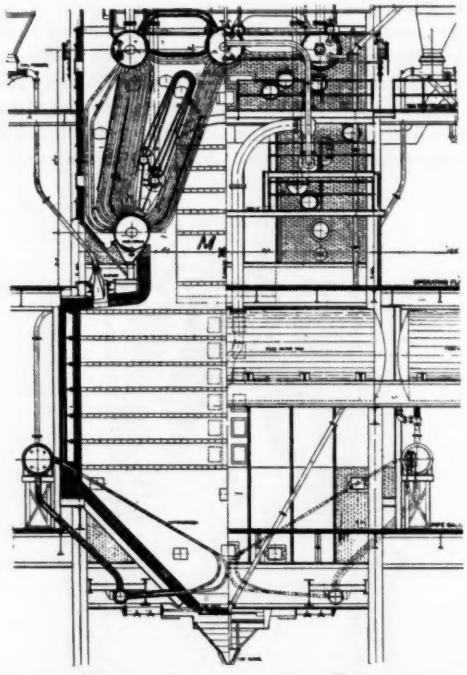


FIG. 3 DOUBLE-ENDED PULVERIZED-COAL-FIRED BOILER WITH SEPARATE WATERWALL-BOILER CIRCULATION AND WATER COOLING ONLY ON BOTTOM OF FURNACE

hours according to conditions governing boiler operation at the individual power plants.

The percentage of moisture in the steam entering the superheater was measured in almost all of the tests and was found to be less than 0.5 per cent.

The data of a number of specific tests are given in Table 3 in some detail. Due to space limitations a large number of the less important tests are not included.

The first boiler which was tested is fired by an underfeed stoker and has a separate screen or waterwall boiler. The greater portion of the front wall is covered with refractory block. The other walls are covered with bare-faced blocks in the lower part and bare tubes in the upper part of the furnace.

The furnace water-walls, constituting the screen boiler, have a separate drum, the steam from this drum mixing with the steam from the boiler proper before entering the convection superheater.

Flames passing up into the boiler as far as the superheater indicated somewhat delayed combustion. An attempt was made to take gas analyses at points in the furnace in order to estimate the amount of this delayed combustion. Results obtained were extremely erratic, carbon monoxide in the middle of the furnace ranging anywhere from zero to 10 per cent. An analysis for hydrogen was not made. After numerous trials this gas sampling was given up as being too difficult for practical accomplishment because the time required to obtain representative samples would be longer than the boiler could be held at the given condition. This matter of delayed combustion was found to be characteristic of stoker-fired furnaces. No such extreme condition was evident in the furnaces fired by fuel in suspension.

Due to the difficulty encountered in the testing, tests were run only at high, medium, and low loads, the actual time of the tests being but a few minutes although sufficient time was allowed between tests to obtain fairly steady conditions.

The second boiler tested, although fired by pulverized coal, was similar to the first in that the bottom screen and side-wall tubes constituted an entirely separate boiler as may be seen by reference to Fig. 2. In this type of firing, the fuel is conveyed from the burners by a small amount of primary air. Secondary air is drawn in through ports in the side wall adjacent to the burners, located in this case, among the side-wall tubes. In order to assure a sufficiently long flame at the higher combustion rates, a small amount of preheated forced-draft air is introduced under

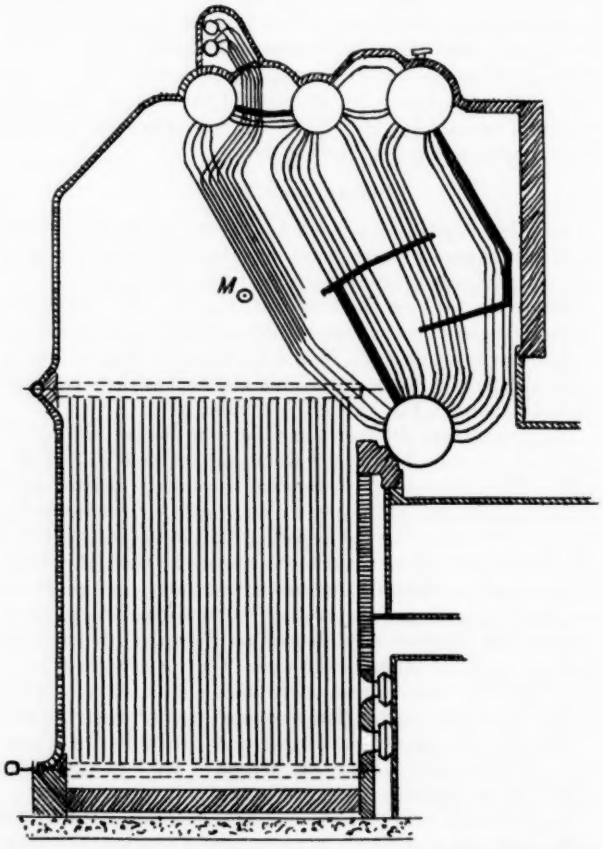


FIG. 4 ALL WATER COOLED COMBINATION GAS- AND OIL-FIRED BOILER

pressure at the burners. Because of this, the length of the flame is a function of both the primary and forced-draft air pressures.

Due to the operating and control conditions on this boiler it was feasible to make more extensive tests than were made on the first boiler. On this account each test was repeated for three excess-air values in order to determine the resulting effect.

Included in the first group of tests are results for three loads at three flame lengths and three excess-air values each, thus making a total of 27 tests. These tests were run half an hour apart, two readings being taken for each test. Gas analyses were also taken in the furnace during these tests. Although no carbon monoxide could be found, the gas-analysis results were so varied

that they were discarded and reliance was again placed on boiler-outlet gas-analysis values.

It should be noted that the terms "short flame," "normal flame," and "long flame" are only relative and do not have exactly the same magnitude for different tests.

As some of the results obtained in this first group of tests did not agree, it was thought that perhaps the duration of the tests was too short. It was then decided to run a second series of tests during which readings were taken every 10 or 15 minutes for a test period of 3 hours.

Comparison of the data obtained from this series of tests with the test data from boiler No. 1, showed that, in spite of the additional length of the tests, the results were not much better, and therefore the data have been omitted. The reason for this is probably the fact that the increased accuracy of the readings due to the longer time of the tests, is more than balanced by changed furnace conditions. Hence no more long-time tests were attempted on this boiler.

During the previous tests the side-wall tubes were covered with a certain amount of slag, the 2-in. space between the tubes and

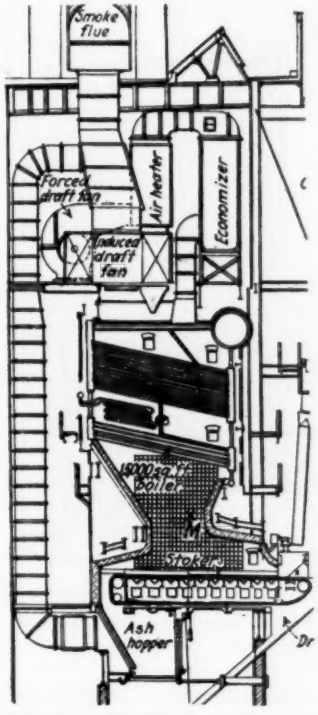


FIG. 5 BOILER COOLED ON TWO SIDES OF FURNACE AND FIRED BY CHAIN-GRATE STOKER

the wall being completely filled with slag. In order to ascertain the effect of this slag on the screen-boiler evaporation, the boiler was taken off the line, the slag removed, and the tubes wire-brushed. Tests 2–8*C*, 2–14*C*, and 2–23*C*, were made under these new conditions.

The third boiler tested, Fig. 3, is exactly similar to boiler No. 2 except that there are no side-wall tubes and the superheater and baffling, with which we are not concerned, are slightly different. Comments applying to the boiler No. 2 apply also to boiler No. 3. Fewer tests were run on this boiler. From among the first group of tests, 3–8, 3–6, and 3–13 are given for three steam ratings.

As was done in the case of boiler No. 2, a second series of tests was run using longer test periods. The results were susceptible to the same conclusion as was arrived at in the case of the boiler No. 2, and are likewise omitted.

The fourth boiler tested was the oil- and gas-fired unit shown in Fig. 4. Oil was the fuel ordinarily used, natural gas being used as a stand-by fuel. The gases within the furnace were almost transparent. Due to operating conditions, a high load could not be carried on this boiler. Tests were made at three loads with two excess-air values each, for both oil- and gas-firing, except that test 4–12, which would have been with a high per cent air, was omitted. Duration of tests was approximately 1 hour.

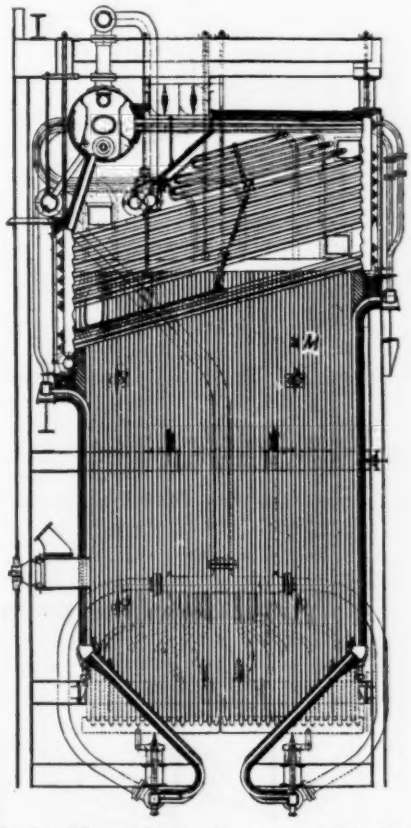


FIG. 6 ALL-WATER-COOLED PULVERIZED-COAL UNIT

The fifth boiler tested, shown in Fig. 5, was fired by a chain-grate stoker having two overhung combustion arches. The tests on this boiler were not as satisfactory as on the other boilers. Delayed combustion was again evident. Combustion was uneven on the grate and excess air was high. Instead of measuring the furnace-exit temperature just under the boiler tubes it was necessary to measure this temperature in the middle of the furnace as no other opening was available or could be made. It was not possible to vary the excess air, hence only three short tests were made at three boiler evaporations. These tests should not be considered as reliable as the other tests.

The sixth boiler tested, shown in Fig. 6, was an almost completely water-cooled furnace, burning pulverized coal and fired by two turbulent burners in the front wall. Due to temporary difficulty with the induced-draft fan, a high load could not be carried on the boiler. Similarly, a high excess-air medium load could not be carried. Hence, but five tests were run; three at different excess-air values at low load and two at different excess-air values at medium load. Duration of tests was from 1 to 2 hours.

The seventh boiler tested was the refractory-furnace gas- and oil-fired boiler shown in Fig. 7. Tests were made at three loads, when burning natural gas, operating with three excess-air values

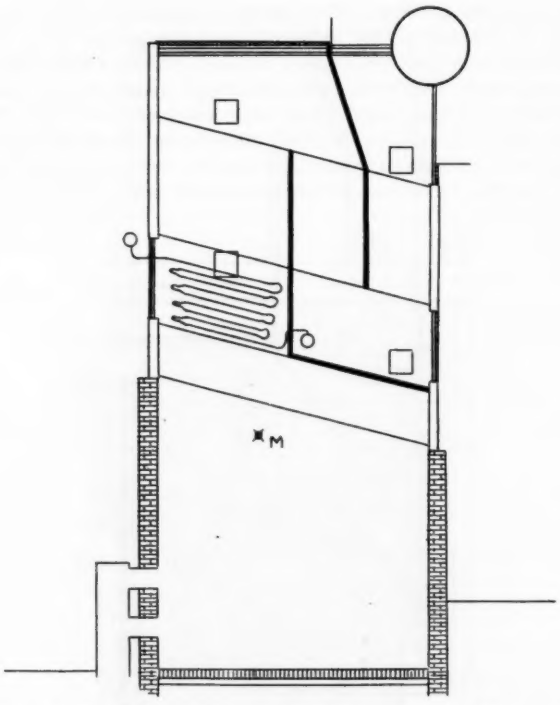


FIG. 7 COMBINATION GAS- AND OIL-FIRED BOILER WITH ALL-REFRACTORY FURNACE

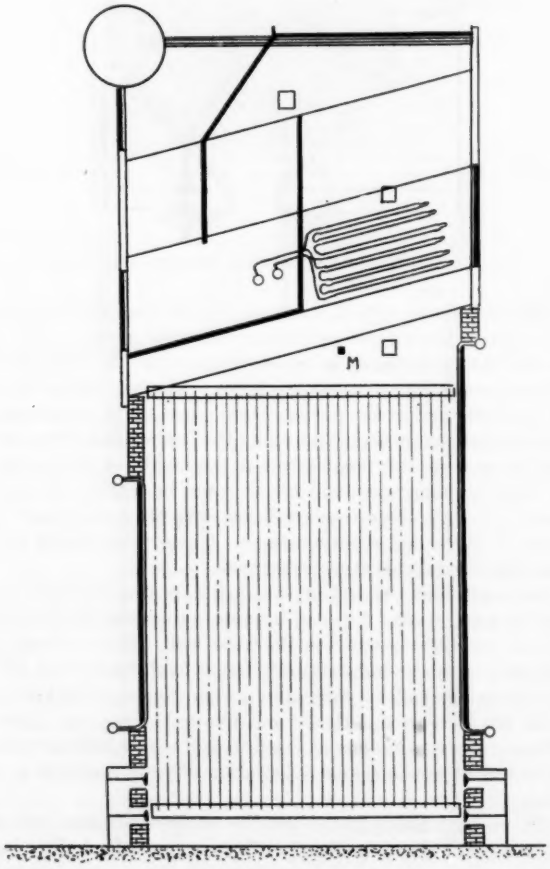


FIG. 8 GAS-FIRED BOILER WITH FURNACE COOLED ON FOUR SIDES

at each load except for the higher load where one test was omitted. Duration of tests was approximately half an hour with considerable time between tests.

The tests using oil (7–10, 7–11, and 7–12) were made at three boiler loads and were of limited duration because of the desire not to use oil, inasmuch as it was only an emergency or stand-by fuel. The furnace gases were transparent at all ratings.

The eighth boiler tested was the water-cooled natural-gas-fired boiler shown in Fig. 8. There was no provision for oil firing. Due to trouble in the turbine room, it was difficult to put high evaporative loads on this boiler. Tests 8–1 to 8–5 were run with low and medium loads and various air values. Tests 8–10, 8–11 and 8–12 were run at a medium boiler evaporation with different feedwater temperatures. Test 8–13 represents the one high-load test that was run. The furnace gases were perfectly transparent at all ratings. Each test required about 1 hour.

The ninth boiler was the underfeed-stoker-fired boiler with bare-tube waterwalls shown in Fig. 9. Tests 9–6 and 9–7 show

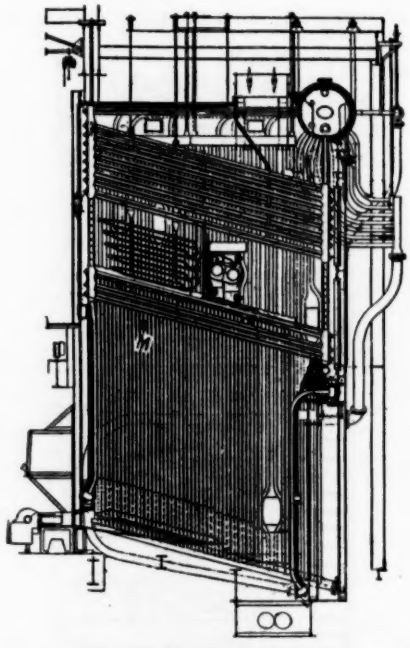


FIG. 9 UNDERFEED-STOKER-FIRED UNIT WITH ALL WATER-COOLED FURNACE

the effect of varying the feedwater temperature while holding the boiler at a constant output. Tests 9–6 and 9–7 were made on the boiler just before it was taken off the line for repairs. The tubes were dirty and the grates burned. Tests 9–14 to 9–19 were made after the grates had been reconditioned, the boiler having been in operation two days in order to obtain representative conditions. These tests were made at three evaporative ratings with two values of excess air each and each test lasted one-half hour.

The tenth boiler test was made on the large double-ended pulverized-coal-fired boiler shown in Fig. 10. This was the only boiler furnace in which any attempt was made to keep the side waterwall tubes clean during normal operation. The tubes were dusted twice daily by means of a compressed-air lance through four openings in each side wall, thus tending to retard slag formation on a portion of the waterwall surface.

Tests 10–12, 10–13, and 10–14 were run at three boiler loads with a low feedwater temperature. The feedwater temperature was increased and tests 10–15 to 10–20 followed at three loads with various excess-air values at each load. Each of these tests

were of 15-minutes duration. The data of a series of longer tests lasting perhaps three hours were discarded as being inconsistent, undoubtedly because boiler conditions change from day to day.

Discussion of the Tests

Several important points may be noted by examining the data obtained. Unless all of the tests are run in one day it is difficult to reconcile the data taken on two different days because of varying furnace conditions which are not a function of the operation of the boiler control instruments. In regard to future tests, it is believed that the influence of excess air has been amply demonstrated. Also, except for extremely accurate work, the importance of flame length in regard to furnace-exit temperature appears to be insufficient to warrant extended tests to show its effect.

As the object of these tests was to determine actual results which might be checked against theoretical calculations and existing formulas, it was advisable to determine values of the ratio of heat absorbed by radiation to the heat input to the furnace. These calculations are included in Table 3. The method of making the calculation is outlined in Table 4.

In calculations it is necessary to distinguish between the amount of excess air in the furnace and the amount at the boiler outlet. Using data obtained from a considerable number of tests, an average value of 7 per cent air leakage between the top of the first pass and the boiler outlet has been assumed. For calculation purposes the percentage of air at the top of the first pass has been assumed to be the same as that in the furnace. The percentages of air used as given in items 20 or 21 in Table 3, were obtained from calculated CO_2 curves for the fuels in question.

In order to determine the heat in the gases at the boiler outlet and at the furnace exit it was necessary to plot heat-capacity curves for the different fuels burned. A sample curve is given in Fig. 12. These curves are based on the values for the specific heat of gases recommended by Goodenough (58). Eastman's specific heats (57) are perhaps to be preferred and would result in an increase of about 2 per cent in the sensible heats. This difference in sensible-heat values affects the μ-values less than 2 per cent.

The initial temperature of the air, as well as the temperature at which the fuel is assumed to enter the furnace, has been taken as 80 F. The calculations are made as if the fuel entered the furnace at this temperature, the heat derived from combustion plus any preheat in the air being used to raise the products of combustion from 80 F to the actual furnace temperature. The sensible heat in the gases at any temperature therefore contains, not only any heat added by combustion, but also the heat in the gases at 80 F, i.e., the heat-capacity curves are based upon 0 F abs.

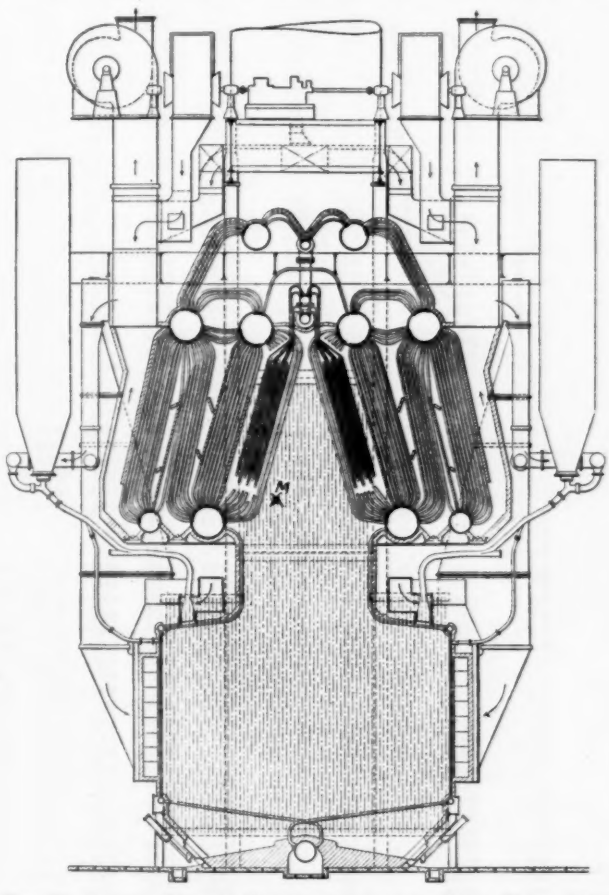


Fig. 10 Large Double-Ended Pulverized-Fuel Unit With Waterwalls on All Sides of Furnace

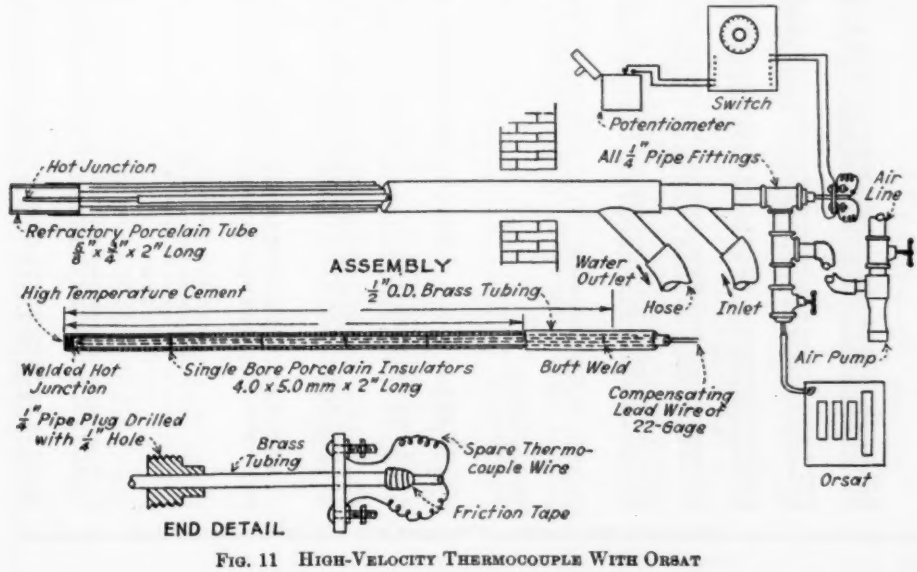


Fig. 11 High-Velocity Thermocouple With Orsat

The heat in the preheated air and the latent-heat losses have been calculated by conventional methods. The moisture is from (*a*) moisture in the air, (*b*) moisture in the coal, and (*c*) moisture resulting from the combustion of hydrogen and hydrocarbons in the fuel. The first does not occasion a latent-heat loss. The latter two require heat from the coal to vaporize the moisture.

It is to be noted that the energy as heat taken from the combustion heat to evaporate hydrogen and moisture into water vapor does not appear as British thermal units in the gases in the heat-capacity curves. Since the

TABLE 3 DATA OF TESTS INCLUDING FURNACE-EXIT

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
No. of boiler	Test number	Fuel	Flame conditions	Actual total boiler evap, 1000 lb per hr	Actual screen boiler evap, 1000 lb per hr	Furnace exit temperature, F	Boiler outlet temperature, F	Steam pressure, lb per sq in., abs	Feedwater temperature, F	Steam temperature, F	Forced-draft pressure, in. water	Primary air pressure, in. water	Room temp, F	Air temperature at burner, F	Gas analysis at first pass or outlet	Carbon dioxide, %	Oxygen, %	Carbon monoxide, %	Furnace air, %	Boiler outlet air calculated, %
1[a]	1-1	coal	..	300	124	2800	788	413	380	688	3.9	...	...	362	outlet	15.0	...	0	116	123
	1-2	coal	..	215	98	2575	688	412	372	683	2.9	...	...	325	outlet	12.6	...	0	139	146
	1-3	coal	..	115	55	2140	568	415	350	654	0.5	...	...	310	outlet	13.5	...	0	130	137
2[b]	2-1	coal	S	150	58	2150	560	395	335	715	0.1	6.0	65	435	outlet	13.1	6.0	0	133	140
	2-2	coal	S	150	63	2150	555	393	335	707	0.1	4.0	66	428	outlet	12.8	6.2	0	136	143
	2-3	coal	S	150	65	2155	550	398	330	705	0.1	4.0	63	...	outlet	14.5	4.4	0	121	127
	2-4	coal	N	150	60	1970	550	390	325	710	1.4	4.0	63	...	outlet	12.0	6.5	0	146	153
	2-5	coal	N	150	68	2080	550	388	325	710	0.4	6.0	69	...	outlet	12.8	6.1	0	136	143
	2-6	coal	N	150	64	2100	550	397	330	703	0.1	4.0	69	...	outlet	13.5	5.2	0	128	135
	2-7	coal	L	150	62	2035	550	397	330	710	0.4	9.0	69	...	outlet	12.7	6.2	0	138	145
	2-8	coal	L	150	60	2005	550	394	330	710	0.1	8.0	69	423	outlet	12.7	6.3	0	138	145
	2-9	coal	L	150	62	2100	550	396	330	703	0.1	7.0	73	435	outlet	13.4	5.5	0	129	136
	2-10	coal	S	225	66	2390	610	393	340	750	0.1	5.0	74	475	outlet	13.8	5.2	0	126	133
	2-11	coal	S	228	72	2435	610	395	340	745	0.1	5.0	74	480	outlet	15.1	3.7	0	115	122
	2-12	coal	S	225	75	2460	600	405	330	738	0.1	5.0	74	460	outlet	14.7	4.3	0	118	125
	2-13	coal	N	225	84	2310	606	410	335	735	1.4	11.0	69	490	outlet	14.5	4.5	0	120	127
	2-14	coal	N	235	90	2325	590	385	325	723	1.4	9.0	67	424	outlet	15.2	3.5	0	115	122
	2-15	coal	N	230	93	2350	580	391	325	723	0.7	7.0	70	447	outlet	14.1	4.5	0	123	130
	2-16	coal	L	225	84	2175	570	380	335	732	2.6	13.0	72	418	outlet	12.0	6.8	0	146	153
	2-17	coal	L	225	88	2275	570	405	330	720	2.5	14.0	73	400	outlet	13.8	5.1	0	126	133
	2-18	coal	L	225	94	2245	560	391	325	720	2.0	12.0	73	410	outlet	15.3	3.5	0	114	121
	2-19	coal	S	270	75	2485	630	387	360	763	0.1	9.0	75	490	outlet	13.5	5.7	0	129	136
	2-20	coal	S	275	78	2525	620	390	355	767	0.2	9.0	76	495	outlet	13.8	5.4	0	128	125
	2-22	coal	N	280	82	2390	630	394	355	762	1.4	9.0	77	482	outlet	13.9	5.0	0	125	132
	2-23	coal	N	280	92	2425	615	392	350	752	1.4	9.0	77	442	outlet	14.6	4.3	0	119	126
	2-24	coal	N	280	104	2480	600	398	325	747	1.4	9.0	78	435	outlet	16.0	2.9	0	108	105
	2-25	coal	L	280	95	2350	615	402	345	750	3.0	11.0	78	448	outlet	13.7	5.3	0	127	134
	2-26	coal	L	280	101	2305	610	390	340	744	2.7	11.0	77	422	outlet	14.0	5.0	0	125	132
	2-27	coal	L	285	106	2360	600	398	330	737	2.8	11.0	78	418	outlet	15.7	3.1	0	110	117
	2-8*C*	coal	..	147	70.4	2000	528	...	...	...	0.1	8.8	88	...	...	...	...	...	...	...
	2-14*C*	coal	..	229	100.0	2265	576	...	...	...	1.0	8.5	86	...	...	...	...	...	...	...
	2-23*C*	coal	..	281	113.0	2375	630	...	...	...	1.4	10.0	82	...	...	...	...	...	...	...
3[b]	3-8	coal	N	150	53.7	2170	520	400	315	691	0.1	5.0	74	...	first	13.9	5.1	0	132	139
	3-6	coal	N	225	71.0	2320	570	407	330	687	0.8	11.0	72	...	first	14.7	4.1	0	125	132
	3-13	coal	N	285	70.5	2360	645	411	350	726	1.2	12.5	74	...	first	13.8	5.2	0	133	140
4[e]	4-1	oil	..	70	...	1600	575	440	274	632	...	...	105	500	outlet	12.0	6.3	0	133	140
	4-2	oil	..	73	...	1600	585	440	276	640	...	...	105	510	outlet	10.8	7.7	0	148	155
	4-3	oil	..	114	...	1880	630	445	270	657	...	...	97	535	outlet	14.1	3.3	0	112	119
	4-4	oil	..	122	...	1985	665	445	270	680	...	...	97	557	outlet	12.8	4.1	0	124	131
	4-5	oil	..	95	...	1610	595	440	270	618	...	...	97	514	outlet	13.8	4.0	0	115	122
	4-6	oil	..	94	...	1615	632	440	270	637	...	...	97	530	outlet	11.5	7.0	0	139	146
	4-7	gas	..	114	...	1910	638	442	274	670	...	...	100	545	outlet	9.7	4.0	0.2	114	121
	4-8	gas	..	115	...	1930	680	442	274	684	...	...	100	573	outlet	7.4	8.3	0.1	149	156
	4-9	gas	..	89	...	1785	598	445	274	650	...	...	100	531	outlet	8.7	5.7	0.1	127	134
	4-10	gas	..	87	...	1780	622	445	276	670	...	...	100	540	outlet	7.4	7.9	0.1	149	156
	4-11	gas	..	71	...	1730	575	440	276	650	...	...	100	513	outlet	8.8	5.5	0.1	125	132
5[d]	5-1	coal	..	49.5	...	1905	420	380	295	662	...	...	113	230	outlet	9.0	10.2	0	198	205
	5-2	coal	..	91	...	2325	554	380	315	717	...	...	110	275	outlet	9.8	9.5	0	182	189
	5-3	coal	..	128	...	2415	625	390	320	742	...	...	109	302	outlet	12.4	6.5	0.2	144	151
6[b]	6-1	coal	..	65	...	1790	557	456	217	577	2.17	0.75	67	405	outlet	12.4	...	...	145	152
	6-2	coal	..	63	...	1885	540	456	216	565	15.0	0.60	62	412	outlet	14.0	...	...	128	135
	6-3	coal	..	61	...	1845	526	456	216	556	10.0	0.13	62	416	outlet	15.8	...	...	113	120
	6-4	coal	..	99	...	1885	610	463	218	587	6.5	0.80	79	430	outlet	15.6	...	...	115	122
	6-5	coal	..	99	...	1910	575	463	221	559	4.1	0.75	79	403	outlet	17.2	...	...	103	110
7[e]	7-1	gas	..	60	...	2175	500	360	233	678	5.0	...	96	96	first	7.8	...	...	145	152
	7-2	gas	..	60	...	2205	490	359	230	664	5.3	...	96	96	first	9.1	...	...	125	132
	7-3	gas	..	60	...	2255	490	360	233	651	5.2	...	96	96	first	9.9	...	...	116	123
	7-4	gas	..	120	...	2400	600	363	234	725	4.0	...	102	102	first	8.6	...	...	132	139
	7-5	gas	..	121	...	2430	590	365	243	720	4.0	...	94	94	first	9.1	...	...	125	132
	7-6	gas	..	121	...	2490	580	364	242	715	4.4	...	94	94	first	9.9	...	...	116	123
	7-8	gas	..	176	...	2640	710	367	230	750	7.0	...	101	101	first	9.1	...	...	125	132
	7-9	gas	..	184	...	2700	710	367	238	750	6.5	...	101	101	first	9.6	...	...	118	125
	7-10	oil	..	68	...	2190	505	363	232	682	7.2	...	87	87	first	10.6	...	...	159	166
	7-11	oil	..	116	...	2410	615	366	233	712	6.8	...	87	87	first	11.5	...	...	146	153
	7-12	oil	..	160	...	2475	680	368	227	739	7.8	...	87	87	first	10.9	...	...	154	161
8[e]	8-1	gas	..	100	...	1970	495	395	284	637	...	...	67	67	first	10.7	1.5	...	109	116
	8-2	gas	..	102	..	1955	510	397	285	652	...	...	67	67	first	9.7	3.4	...	120	127
	8-3	gas	..	101	..	2010	625	399	286	669	...	...	68	68	first	8.7	5.2	...	134	141
	8-4	gas	..	201	...	2095	630	407	289	710	...	...	69	69	first	10.6	1.9	...	110	117
	8-5	gas	..	198	...	1920	646	407	289	740	...	...	69	69	first	9.4	4.2	...	123	130
	8-10	gas	..	204	...	2060	630	407	302	708	...	...	69	69	first	10.6	1.9	...	110	117
	8-11	gas	..	203	...	2345	640	405	246	740	...	...	69	69	first	10.6	1.8	...	110	117
	8-12	gas	..	206	...	2335	640	404	218	750	...	...	69	69	first	10.6	1.8	...	110	117
	8-13	gas	..	297	...	2315	714	407	309	740	...	...	69	69	first	10.6	2.3	...	110	117
9[a]	9-6	coal	..	225	...	2330	557	383	336	698	3.9	...	...	404	first	14.0	5.4	0	134	141
	9-7	coal	..	225	...	2310	551	383	221	721	4.5	...	...	403	first	15.0	3.1	0	125	132
	9-14	coal	..	127	...	2005	472	377	332	747	2.0	...	...	336	first	11.5	7.5	0	163	170
	9-15	coal	..	125	...	1920	462	375	331	713	1.9	...	...	328	first	13.0	6.0	0	144	151
	9-16	coal	..	224	...	2200	554	379	332	774	3.2	...	...	371	first	11.6	8.0	0	161	168
	9-17	coal	..	230	...	2200	538	379	341	742	3.3	...	...	375	first	11.5	8.3	0	163	170
	9-18	coal	..	275	...	2460	578	382	332	763	3.7	...	...	410	first	13.0	6.2	0	144	151
	9-19	coal	..	275	...	2495	575	384	334	775	3.8	...	...	407	first	12.7	5.3	0	147	154
10[c]	10-12	coal	..	800	...	2345	685	407	206	771	...	15.2	78	495	outlet	16.2	...	...	109	116
	10-13	coal	..	615	...	2215	630	407	203	740	...	12.7	78	465	outlet	14.7	...	...	121	128
	10-14	coal	..	415	...	2040	570	407	207	716	...	11.1	78	440	outlet	13.1	...	...	137	144
	10-15	coal	..	415	...	1980	570	407	287	714	...	10.7	78	450	outlet	12.7	...	...	141	148
	10-16	coal	..	415	...	1950	585	408	289	723	...	11.5	78	450	outlet	10.6	...	...	170	177
	10-17	coal	..	590	...	2305	640	407	291	770	...	13.7	78	475	outlet	13.6	...	...	132	139
	10-18	coal	..	600	...	2095	645	407	291	737	...	14.5	78	480	outlet	12.6	...	...	142	149
	10-19	coal	..	620	...	2135	640	407	293	733	...	14.3	78	480	outlet	14.2	...	...	126	133
	10-20	coal	..	730	...	2270	690	407	292	743	...	17.0	78	500	outlet	15.3	...	...	116	123

[a] Equipped with underfeed stoker. [b] Burns pulverized coal. [c] Equipped to fire oil or gas. [d] Equipped with chain-grate stoker. [e] Burns natural gas.

TEMPERATURES AND CALCULATED μ-VALUES

22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
Sensible heat in furnace gases at boiler outlet, 1000 Btu per lb wf	Sensible heat in gases at 80 F, 1000 Btu per lb wf	Heat in preheated air, 1000 Btu per lb wf	(High heat fuel + air heat), 1000 Btu per lb wf	(High heat fuel + air heat + latent heat), 1000 Btu per lb wf	(High heat fuel + air heat + gas heat at 80 F − latent heat), 1000 Btu per lb wf	Combustible + boiler radiation loss, %	Heat loss in dry gases at boiler outlet, 1000 Btu per lb wf	Summation of losses	Boiler efficiency based on furnace input	Heat added by boiler to total steam, 10^6 Btu per hr	Heat input to furnace, 10^6 Btu per hr	Heat in gases at furnace temp, 1000 Btu per lb wf	Heat to furnace surface, 1000 Btu per lb wf	Gas temp μ based on + [(high heat fuel) (air heat)]	Gas temp μ based on + [(low heat fuel) (air heat)]	Furnace heat input per unit volume, 1000 Btu per cu ft
4.35	1.70	0.83	14.68	14.18	15.90	5.0	2.65	0.265	0.735	301	410	11.7	4.20	0.29	0.30	29.3
4.65	2.05	0.83	14.68	14.18	16.25	4.2	2.60	0.254	0.746	216	290	12.5	3.75	0.25	0.26	20.7
3.90	2.00	0.83	14.61	14.11	16.10	3.5	1.90	0.194	0.806	117	145	9.9	6.20	0.42	0.44	10.3
3.80	1.90	0.25	13.70	13.19	15.10	2.5	1.90	0.201	0.799	160	200	9.7	5.40	0.39	0.41	8.9
3.85	2.00	0.25	13.70	13.19	15.20	2.5	1.85	0.197	0.802	160	200	9.8	5.40	0.39	0.41	8.9
4.40	1.80	0.25	13.70	13.19	15.00	2.5	1.60	0.179	0.821	160	195	8.8	6.20	0.45	0.47	8.7
4.00	2.10	0.25	13.70	13.19	15.30	2.5	1.90	0.200	0.800	161	201	9.7	5.60	0.41	0.42	8.9
3.85	2.00	0.25	13.70	13.19	15.20	2.5	1.85	0.197	0.803	161	200	9.6	5.60	0.41	0.42	8.9
3.60	1.85	0.25	13.70	13.19	15.25	2.5	1.75	0.190	0.810	160	198	9.0	6.25	0.45	0.47	8.8
3.85	2.00	0.25	13.70	13.19	15.20	2.5	1.85	0.197	0.803	160	199	9.5	5.70	0.42	0.43	8.9
3.85	2.00	0.25	13.70	13.19	15.20	2.5	1.85	0.197	0.803	160	199	9.4	5.80	0.42	0.44	8.9
3.65	1.85	0.25	13.70	13.19	15.05	2.5	0.80	0.193	0.807	160	198	9.2	5.85	0.43	0.44	8.8
3.80	1.85	0.25	13.70	13.19	15.05	3.2	1.95	0.212	0.788	242	307	10.2	4.85	0.35	0.37	13.7
3.50	1.70	0.25	13.70	13.19	14.90	3.2	1.80	0.200	0.800	238	298	9.6	5.30	0.39	0.40	13.3
3.50	1.75	0.25	13.70	13.19	14.95	3.2	1.75	0.196	0.804	244	304	9.8	5.15	0.38	0.39	13.6
3.60	1.75	0.25	13.70	13.19	14.95	3.2	1.85	0.204	0.796	242	304	9.4	5.55	0.41	0.42	13.6
3.45	1.70	0.25	13.70	13.19	14.90	3.2	1.75	0.197	0.803	254	316	9.2	5.70	0.42	0.43	14.1
3.60	1.80	0.25	13.70	13.19	15.00	3.2	1.80	0.201	0.799	249	312	9.8	5.20	0.38	0.39	13.9
4.15	2.10	0.25	13.70	13.19	15.30	3.2	2.05	0.219	0.781	242	310	10.6	4.70	0.34	0.36	13.8
3.65	1.85	0.25	13.70	13.19	15.05	3.2	1.80	0.201	0.799	241	302	9.7	5.35	0.39	0.40	13.5
3.30	1.70	0.25	13.70	13.19	14.90	3.2	1.60	0.186	0.814	242	298	8.8	6.10	0.45	0.46	13.3
3.90	1.85	0.25	13.70	13.19	15.05	4.0	2.05	0.226	0.774	288	372	10.7	4.35	0.32	0.33	16.6
3.85	1.85	0.25	13.70	13.19	15.05	4.0	2.00	0.223	0.777	295	380	10.8	4.25	0.31	0.32	17.0
3.85	1.80	0.25	13.70	13.19	15.00	4.0	2.05	0.226	0.774	300	388	10.2	4.80	0.35	0.36	17.3
3.65	1.75	0.25	13.70	13.19	14.95	4.0	1.90	0.215	0.785	300	382	9.8	5.15	0.38	0.39	17.1
3.30	1.60	0.25	13.70	13.19	14.80	4.0	1.70	0.201	0.799	303	379	9.3	5.50	0.40	0.42	16.9
3.85	1.85	0.25	13.70	13.19	15.00	4.0	2.00	0.222	0.778	301	387	10.0	5.00	0.36	0.38	17.3
3.80	1.80	0.25	13.70	13.19	15.00	4.0	2.00	0.222	0.778	302	388	9.8	5.20	0.38	0.39	17.3
3.30	1.70	0.25	13.70	13.19	14.90	4.0	1.60	0.193	0.807	309	383	9.0	5.90	0.43	0.45	17.1
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3.60	1.90	0.25	13.70	13.19	15.10	2.5	1.70	0.186	0.814	161	198	9.7	5.40	0.39	0.41	8.9
3.60	1.80	0.25	13.70	13.19	15.00	3.2	1.80	0.201	0.799	236	296	9.8	5.20	0.38	0.39	13.2
4.10	1.90	0.25	13.70	13.19	15.10	4.0	2.20	0.237	0.763	301	394	10.6	4.50	0.33	0.34	17.6
5.10	2.60	1.73	20.15	19.30	21.85	3.0	2.50	0.196	0.804	75.5	94	10.1	11.70	0.58	0.61	8.5
5.60	2.00	2.04	20.46	19.61	21.60	3.0	2.70	0.235	0.765	79.0	103	11.1	10.50	0.51	0.53	9.3
4.65	2.25	1.62	20.04	19.19	21.45	3.0	2.40	0.192	0.808	125	155	10.0	11.45	0.57	0.60	14.1
5.15	2.40	1.90	20.32	19.47	21.85	3.0	2.75	0.208	0.792	133	168	11.4	10.45	0.51	0.54	15.3
4.50	2.30	1.25	19.67	18.82	21.10	3.0	2.20	0.185	0.815	102	125	8.9	12.20	0.62	0.65	11.4
5.55	2.70	2.00	20.42	19.57	22.25	3.0	2.85	0.211	0.789	102	129	10.6	11.65	0.57	0.60	11.7
5.90	2.80	2.00	24.50	22.32	25.00	3.0	3.10	0.246	0.754	125	166	12.9	12.10	0.50	0.54	15.1
7.70	3.50	2.76	25.16	22.98	26.50	3.0	4.20	0.284	0.716	127	177	16.3	10.20	0.41	0.45	16.1
6.30	3.20	2.15	24.65	22.47	25.60	3.0	3.10	0.244	0.756	96.8	128	13.1	12.50	0.51	0.55	11.6
7.20	3.50	2.60	25.10	22.92	26.40	3.0	3.70	0.264	0.736	95.5	129	15.2	11.20	0.45	0.49	11.7
6.00	3.10	2.04	24.54	22.36	25.40	3.0	2.90	0.238	0.762	77.2	101	12.8	12.60	0.52	0.56	9.2
2.90	1.70	0.46	8.71	8.21	9.90	4.0	1.20	0.235	0.765	53.4	70	8.2	1.70	0.20	0.21	12.9
3.20	1.50	0.52	8.77	8.27	9.75	4.5	1.70	0.295	0.705	99.0	140	9.0	0.75	0.08	0.09	25.8
2.80	1.30	0.48	8.75	8.23	9.50	5.0	1.50	0.280	0.720	140	194	7.7	1.80	0.21	0.22	35.8
2.75	1.40	0.77	10.82	10.34	11.70	4.0	1.25	0.200	0.800	71.7	89.6	6.2	5.50	0.51	0.53	10.3
2.40	1.35	0.65	10.70	10.22	11.55	4.0	1.05	0.183	0.817	69.0	84.5	5.8	5.70	0.53	0.56	9.7
2.15	1.15	0.60	10.65	10.17	11.30	4.0	1.00	0.179	0.821	66.5	81	5.2	6.10	0.57	0.60	9.3
2.35	1.20	0.63	10.68	10.20	11.40	4.0	1.15	0.202	0.798	109	136	5.3	6.10	0.57	0.60	15.6
2.10	1.05	0.56	10.61	10.13	11.25	5.0	1.05	0.194	0.806	107	132	5.0	6.25	0.59	0.62	15.1
5.70	3.10	0	20.60	18.60	22.40	3.0	2.60	0.245	0.755	69.2	91.7	16.1	5.60	0.27	0.30	16.5
5.00	2.80	0	20.60	18.60	22.10	3.0	2.20	0.226	0.774	68.8	89.8	14.4	7.00	0.34	0.38	16.0
4.90	2.60	0	20.60	18.60	21.90	3.0	2.30	0.231	0.769	68.2	88.8	13.9	7.30	0.35	0.39	16.0
6.00	3.00	0	20.60	18.60	22.30	2.5	3.00	0.259	0.741	141	190	16.3	5.30	0.26	0.29	34.2
5.60	2.90	0	20.60	18.60	22.20	2.5	2.70	0.245	0.755	141	187	15.7	5.80	0.28	0.31	33.7
5.30	2.60	0	20.60	18.60	21.90	2.5	2.70	0.245	0.755	141	187	15.0	6.20	0.30	0.33	33.7
6.40	2.90	0	20.60	18.60	22.20	2.0	3.50	0.278	0.722	210	291	17.0	4.50	0.22	0.24	52.4
6.10	2.60	0	20.60	18.60	21.90	2.0	3.50	0.278	0.722	218	302	16.3	4.90	0.24	0.26	54.3
5.50	3.00	0	19.00	18.15	21.15	3.0	2.50	0.205	0.795	78.5	98.8	15.5	5.65	0.30	0.31	7.8
5.55	2.85	0	19.00	18.15	21.15	2.5	2.70	0.210	0.790	135	171	15.7	5.45	0.29	0.30	30.8
6.40	2.90	0	19.00	18.15	21.20	2.0	3.50	0.247	0.753	190	252	17.1	4.10	0.22	0.23	45.4
4.20	2.50	0	20.00	18.11	20.60	2.5	1.70	0.204	0.796	117	147	11.1	9.50	0.48	0.52	11.1
4.20	2.60	0	20.00	18.11	20.70	2.5	2.10	0.224	0.776	120	154	12.4	8.60	0.43	0.47	11.7
5.90	2.80	0	20.00	18.11	20.90	2.5	3.10	0.274	0.726	120	165	13.7	7.20	0.36	0.40	12.5
5.00	2.50	0	20.00	18.11	20.60	2.0	2.50	0.239	0.761	223	293	11.9	8.70	0.43	0.48	22.2
5.60	2.70	0	20.00	18.11	20.80	2.0	2.90	0.259	0.741	156	210	12.3	8.50	0.43	0.47	15.9
5.20	2.40	0	20.00	18.11	20.50	2.0	2.80	0.254	0.746	224	300	11.7	8.80	0.44	0.49	23.7
5.00	2.40	0	20.00	18.11	20.50	2.0	2.60	0.244	0.756	162	214	13.2	7.50	0.36	0.40	16.2
5.00	2.40	0	20.00	18.11	20.50	2.0	2.60	0.244	0.756	248	328	13.2	7.50	0.36	0.40	24.9
5.30	2.40	0	20.00	18.11	20.50	2.0	2.90	0.259	0.741	328	442	13.1	7.40	0.37	0.41	33.7
3.90	2.00	1.08	15.23	14.79	16.80	4.2	1.90	0.195	0.805	238	296	11.0	5.80	0.38	0.39	32.7
3.65	1.90	0.97	15.12	14.68	16.60	4.2	1.75	0.187	0.813	267	328	10.2	6.40	0.42	0.44	36.3
4.30	2.40	1.05	15.20	14.76	17.20	3.5	1.90	0.189	0.811	138	170	11.0	6.20	0.41	0.42	18.8
3.80	2.15	0.87	15.02	14.58	16.80	3.5	1.65	0.174	0.826	134	162	9.8	7.00	0.46	0.48	17.9
4.60	2.40	1.19	15.34	14.90	17.30	4.2	2.20	0.214	0.786	247	314	12.0	5.30	0.34	0.36	34.7
4.60	2.40	1.15	15.30	14.86	17.30	4.2	2.20	0.215	0.785	247	314	12.1	5.20	0.34	0.35	34.7
4.30	2.15	1.15	15.30	14.86	17.05	5.0	2.15	0.219	0.781	301	385	12.3	4.75	0.31	0.32	42.6
4.40	2.20	1.19	15.34	14.90	17.15	5.0	2.20	0.222	0.778	302	388	12.7	4.45	0.29	0.30	42.9
3.70	1.70	1.14	15.69	15.28	17.10	5.0	2.00	0.204	0.796	980	1230	9.3	7.80	0.50	0.51	36.8
4.00	1.80	1.10	15.65	15.24	17.05	4.0	2.20	0.207	0.793	747	942	9.7	7.35	0.47	0.48	28.2
4.10	2.00	1.14	15.69	15.28	17.30	3.0	2.10	0.190	0.800	496	620	10.0	7.30	0.46	0.48	18.6
4.20	2.00	1.32	15.87	15.45	17.50	3.0	2.20	0.194	0.806	463	574	10.1	7.40	0.47	0.48	17.2
4.80	2.30	1.01	16.16	15.75	18.05	3.0	1.50	0.148	0.832	464	544	11.7	6.35	0.39	0.40	16.3
4.20	1.90	1.24	15.79	15.38	17.30	4.0	2.30	0.212	0.788	674	855	10.8	6.50	0.41	0.42	25.6
4.50	2.00	1.43	15.98	15.57	17.60	4.0	2.50	0.222	0.778	674	866	10.7	6.90	0.43	0.44	26.0
4.10	1.80	1.17	15.72	15.31	17.10	4.0	2.30	0.212	0.788	693	887	9.7	7.40	0.47	0.48	26.4
4.00	1.70	1.28	15.83	15.42	17.10	5.0	2.30	0.221	0.779	877	1125	9.4	7.70	0.49	0.50	33.7

/ Short-flame conditions. # Normal-flame conditions. ♭ Long-flame conditions. ♦ Wet fuel.

TABLE 4 INDICATED CALCULATIONS FOR TABLE 3

Items 1 to 19	data
Item 20	calculated from item 17, using fuel analysis
Item 21	assumed equal to item 20 plus 7 per cent
Item 22	heat capacity curves using items 8 and 21
Item 23	heat capacity curves using item 20
Item 24	(fraction of air heated) × [(heat in air at burner temp) — (heat in air at 80 F)]
Item 25	high heat value of fuel + item 24
Item 26	item 25 — latent heat
Item 27	item 26 + item 23
Item 28	assumed
Item 29	item 22 — item 23
Item 30	$\frac{\text{latent-heat loss in per cent + item 28}}{100} + \frac{\text{item 29}}{\text{item 25}}$
Item 31	1.00 — item 30
Item 32	item 5 × [(heat of superheated steam — heat of feed-water)]
Item 33	item 32/item 31
Item 34	heat-capacity curves using items 7 and 20
Item 35	item 27 — item 34
Item 36	item 35/item 25
Item 37	item 35/item 26
Item 38	item 33/furnace volume

TABLE 5 ASSUMED RADIATION LOSSES

Boiler surface, sq ft	Per cent loss
2,500 to 7,500	3.00
7,500 to 12,500	2.00
12,500 to 20,000	1.50
20,000 to 50,000	1.25
Over 50,000	1.00

heat is lost in so far as any heat transfer is concerned, unless the dewpoint of the gases is reached, it must be subtracted from the high heat value of the fuel.

It was impossible to conduct a weighed-fuel and water test on the boilers, thus necessitating the use of the heat-balance method (38) for the calculation of boiler efficiency. Results should be sufficiently close for the purposes of this report. The carbon, hydrogen, hydrocarbon, and overall boiler radiation loss have been combined and estimated in item 28 of Table 3.

The radiation loss through the furnace walls in per cent of the furnace heat input was taken as in Table 5.

The other losses were, judging by some experimental data (30), estimated as about 1 per cent for oil- and gas-fired boilers. For coal firing, these losses were estimated at values ranging from 2 per cent at normal ratings to 6 per cent at the higher ratings, these values being based on available data.

The calculations made in Table 3 to obtain the heat input to the furnace are evident. Using the heat-capacity curves in order to determine the heat in the gases at the furnace exit, the difference between this value and the heat of combustion plus the heat in the gases at 80 F plus the heat in the preheated air, will be the heat radiated or convected to the furnace surfaces. Complete combustion is assumed for this calculation. The ratio of the heat radiated to the heat input is readily calculated, and is termed the gas-temperature μ-value.

It will be seen that the heat in the preheated air used for combustion has been assumed to act exactly as if it were part of the calorific value of the fuel. This assumption is open to question in view of the possibility of chemiluminescence but no data exist to form a more exact conclusion. Any error from this source cannot be large.

There are two possible methods of computing the gas-temperature μ-value. In item 36 it is given as a ratio of the heat given up from the furnace gases to the calorific heat of the fuel plus sensible heat in air supplied from combustion.

Since the energy as heat required to evaporate the moisture resulting from hydrogen and moisture in the fuel is not available for radiation or to increase the sensible heat of the gases, a second determination of μ as a ratio of the heat given up from the furnace gases to the low heat value of the fuel plus any air preheat has been given as item 37.

In the cases where the boilers tested have separate waterwall circulation and drums, it is possible to estimate the heat absorbed in the furnace by use of a proportion between the total exposed effective radiant heating surface in the screen boiler and that part of the waterwall boiler on separate circulation.

This has not been done in this report since it involves an estimation of the relative heat-absorbing abilities of various kinds of waterwall surfaces which, in the present state of our knowledge of this subject, is to some extent dependent on individual opinion.

It should be noted that in calculating the gas-temperature μ-values, it was necessary to determine the boiler efficiency by the heat-balance method. This involves some element of personal judgment and it is realized that others would perhaps obtain different values for the boiler efficiency, thus getting slightly different μ-values. However, the differences should not be large and it has been deemed advisable to calculate and give these μ-values so as to present the data more effectively.

The data which are included will also serve, it is believed, as a criterion for checking results obtained by the various methods of calculation advanced by different investigators.

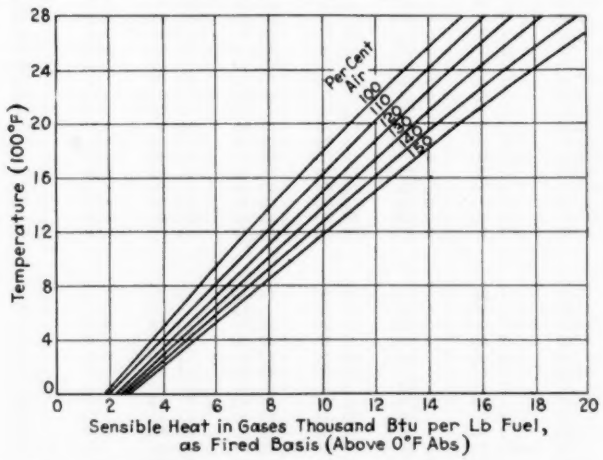


FIG. 12 TYPICAL HEAT-CAPACITY CURVES FOR NATURAL GAS (Goodenough specific heats used.)

With this in view the values of μ as computed from gas-temperature data have been included in Table 3, as stated previously. It is felt that the results included in a report by the committee should not be carried beyond this point because major assumptions would be involved about which there is no general agreement.

However, an investigation has been conducted in which the results of each of several well-known methods of arriving at μ-values are compared to the gas-temperature values of μ included in this report. This information is presented in the two papers, immediately preceding this paper, RP-57-2 and RP-57-3.

A study of the method of taking the experimental data included in this report will reveal the probable magnitude of any errors involved. The readings of the high-velocity thermocouples, by far the most important determination, are believed to indicate the true gas temperature within 50 F.

Furthermore, inspection of the data (of which a considerable amount is not given in this paper) will show that even under identical conditions of boiler-operating control, the furnace temperature may vary as much as 100 F from day to day. Theoretically, it would be possible to determine the cause of this variation which is probably due to such factors as irregular coal feeding, shifting combustion conditions, and different amounts of slag on the walls. Practically, the causes are indeterminate, hence, it would seem that a method yielding the greatest accuracy desirable should give results, in regard to the furnace temperature, of about ±50 F.

It is appreciated that many faults may be found with the

results. However, it is believed that a contribution has been made which will help in advancing the subject but which will require much more work, both practical and theoretical, before it can be said that it is on a thoroughly sound basis.

In the literature, the increasing tendency to measure furnace-exit-gas temperatures as a part of the complete boiler test is noted with gratification (30, 35, 50), although it is regrettable that high-velocity thermocouples are not always used for this determination.

It is impossible to mention individually all those who have cooperated in this investigation. Grateful acknowledgment is particularly due to the operating companies, who by lending the use of their facilities and personnel, have made this investigation possible. The names of the various power plants have been deleted by request.

Bibliography

1 "Radiation in the Pulverized-Fuel Furnace," by W. J. Wohlenberg and D. G. Morrow, Trans. A.S.M.E., vol. 47, 1925, pp. 127–176.

2 "The Influence of Radiation in Coal-Fired Furnaces on Boiler Surface Requirements, and a Simplified Method for Its Calculation," by W. J. Wohlenberg and E. L. Lindseth, Trans. A.S.M.E., vol. 48, 1926, pp. 849–937.

3 "Some Fundamental Considerations in the Design of Boiler Furnaces," by W. J. Wohlenberg and F. W. Brooks, Trans. A.S.M.E., vol. 49–50, 1927-1928, paper FSP-50-39, pp. 141–158.

4 "Influence of Coal Type on Radiation in Boiler Furnaces," by W. J. Wohlenberg and R. L. Anthony, Trans. A.S.M.E., vol. 51, part 1, 1929, paper FSP-51-36, pp. 235–246.

5 "Heat Reaction in the Boiler Furnace," by W. J. Wohlenberg, Trans. A.S.M.E., vol. 52, part 1, 1930, paper FSP-52-26, pp. 177–183.

6 "Der Industrielle Warmeubergang," by A. Schack, Verlag Stahleisen, Dusseldorf, 1929.

7 "Radiation in Boiler Furnaces," by B. N. Broido, Trans. A.S.M.E., vol. 47, 1925, pp. 1123–1178.

8 "Some Notes on the Performance of Boiler Furnaces," by A. T. Brown, *Combustion*, vol. 3 (new notation), April, 1932, p. 38.

9 "Calculation of the Temperature of a Boiler Furnace," by N. Forssblad, *Combustion*, vol. 15 (old notation), July, 1926, p. 33.

10 "Some Operating Data of Large Steam Generating Units," by H. Kreisinger and T. E. Purcell, Trans. A.S.M.E., vol. 49–50, 1927–1928, paper FSP-50-34, pp. 83–101.

11 "Analysis of Heat Absorption in Boilers and Superheaters," by N. Artsay, Trans. A.S.M.E., vol. 51, part 1, 1929, paper FSP-51-37, pp. 247–258.

12 "Radiation in Boiler Furnaces," by N. Artsay, presented at the Chicago Meeting of the A.S.M.E., February, 1931 (not published).

13 "Combustion and Heat Transfer," by R. T. Haslam and H. C. Hottel, Trans. A.S.M.E., vol. 49–50, 1927–1928, paper FSP-50-3, pp. 9–26.

14 "Versuche über Temperaturverteilung, Wärmeabgabe und Verbrennungsverlauf," by E. Kuhn, Berichtfolge des Kohlenstaubausschusses des Reichskohlenrates, March, 1930, V.D.I. Verlag, Berlin.

15 "Flame Radiation in Water-Cooled Boiler Furnaces," by T. Lindmark and H. Edenholm, part 1, Ingeniors Vetenskaps Akademien, Handlingar No. 66, 1927, Stockholm, Sweden.

16 "Flame Radiation in Water-Cooled Boiler Furnaces," by T Lindmark and E. Kignell, part 2, Ingeniörs Vetenskaps Akademien Handlingar No. 109, 1931, Stockholm, Sweden.

17 "Heat Absorption in Water-Cooled Furnaces," by W. L. DeBaufre, Trans. A.S.M.E., vol. 53, 1931, paper FSP-53-19a, pp. 253–264.

18 "The Accurate Measurement of High Gas Temperature," by H. F. Mullikin, *Power*, vol. 78, 1934, p. 565.

19 "..., 500,000 Pounds of Steam From One Unit," Combustion Engineering Corp., publication IC-2, 1926.

20 "Radiation in Boiler Furnaces," by G. A. Orrok, *Mechanical Engineering*, vol. 48, 1926, pp. 218–220.

21 "Berechnung und Verhalten von Wasserrohrkesseln," by F. Münzinger, Julius Springer, Berlin, 1929.

22 "Radiation in Boiler Plants," by L. K. Ramzin, Trans. Second World Power Conference, Berlin, vol. 7, 1930, pp. 215–256.

23 "Le Rayonment Calorifique Envisage du Point de vue des Applications Industrielles," by C. Roszak and M. Veron, *Revue de Metallurgie*, vol. 28, August, September, and October, 1924, pp. 435–449 in the August issue, pp. 549–564 in the September issue, and pp. 600–609 in the October issue.

24 "Some Factors in Furnace Design for High Capacity," by E. G. Bailey, Trans. A.S.M.E., vol. 50, 1927–1928, paper FSP-50-78, pp. 253–283.

25 "Recent development in Boiler Design," by T. E. Murray, paper presented before the Association of Edison Illuminating Companies, October 19, 1925 (not published).

26 "An Investigation of Powdered Coal as a Fuel for Power-Plant Boilers," by H. Kreisinger, J. Blizard, C. E. Augustine, and B. J. Cross, U. S. Bureau of Mines Bulletin No. 223, 1923.

27 "Pulverized Fuel," National Electric Light Association, 1925, statement by the Union Electric Light and Power Company.

28 "Boilers, Superheaters, Economizers," National Electric Light Association, 1927, statement by the Union Electric Light and Power Company, p. 24.

29 "Boilers, Superheaters, Economizers," National Electric Light Association, 1927, statement by the Milwaukee Electric Railway and Light Company, p. 20.

30 "Comparative Performance of a Large Boiler Using Oil and Natural-Gas Fuels," by F. G. Philo, Trans. A.S.M.E., vol. 54, 1932, paper FSP-54-11, pp. 99–114.

31 "A Study of Refractories Service Conditions in Boiler Furnaces," by R. A. Sherman, U. S. Bureau of Mines Bulletin No. 334, 1931.

32 "Messungen der Flammenstrahlung in Dampfkesselfeuerungen," by P. Koessler, *Zeitschrift des Bayerischen Revisions-Vereins*, vol. 34, March-July, 1930; no. 6, March 31, pp. 75–78; no. 7, April 15, pp. 95–98; no. 10, May 31, pp. 146–149; no. 12, June 31, pp. 175–178; and no. 13, July 15, pp. 184–187.

33 "Stokers and Furnaces," National Electric Light Association, August, 1924, statement by the New York Edison Company, p. 14.

34 "An Outline of the Theory of Heat Transmission and Absorption in Application to Ordinary Engineering Problems," by J. E. Bell, Proceedings of the Engineers Society of Western Pennsylvania, vol. 32, July, 1916–January, 1917, pp. 797–846.

35 "Tests of a Mikado-Type Locomotive Equipped With Nicholson Thermic Syphons," by E. C. Schmidt, E. G. Young, and H. J. Schrader, University of Illinois, Bulletin No. 220, February, 1930, Urbana, Ill.

36 "High Combustion Densities in Restricted Furnace Space," by H. Nielson, Proceedings of the International Conference on Bituminous Coal, vol. 2, 1931, p. 271.

37 "Comparative Performance of a Pulverized-Coal-Fired Boiler Using Bin System and Unit System of Firing," by A. E. Grunert, Trans. A.S.M.E., vol. 53, 1931, paper FSP-53-4, pp. 35–50.

38 "The Heat Balance Test as a Means of Obtaining Evaporative Efficiencies of Steam-Generating Equipment," by B. J. Cross, Trans. A.S.M.E., vol. 52, part 1, 1930, paper FSP-53-33, pp. 255–264.

39 "Heat Transmission in Boilers," by J. G. Hudson, *Engineer*, vol. 70, 1890, pp. 449, 483, and 523.

40 "Die Grundgesetzte der Warmestrahlung," by M. Gerbel, Julius Springer, Berlin, 1917.

41 "Die Leistungssteigerung von Grossdampfkesseln," by F. Münzinger, Julius Springer, Berlin, 1933.

42 "Radiant Heat," by E. G. Ritchie, *Combustion*, vol. 14 (old notation), March, 1926, pp. 170–175.

43 "Study of Heat Transmission in Boiler Furnaces," by T. Lindmark, Ingeniörs Vetenskaps Akademien Handlingar. No. 91, 1929, Stockholm, Sweden.

44 "Discussion of A. T. Brown's Paper 'Some Notes on the Performance of Boiler Furnaces,'" by L. J. Marshall, *Combustion*, vol. 4 (new notation), June, 1935, p. 35.

45 "Heat Transmission in Radiant Sections of Tube Stills," by D. W. Wilson, W. E. Lobo, and H. C. Hottel, *Industrial and Engineering Chemistry*, vol. 24, 1932, p. 486.

46 "Die Strahlung des Rostes in Einem Wasserrohrkessel," by R. W. Müller, *Feuerungstechnik*, vol. 20, no. 3, March 15, 1932, p. 37.

47 "Loads on Pulverized Coal Furnaces," by P. Rosin and R. Fehling, *Die Wärme*, vol. 54, 1931, pp. 42–50. Abstracted in *Mechanical Engineering*, vol. 53, 1931, p. 382.

48 "Einfluss der Gasstrahlung auf die Wärmeaufnahme der bestrahlten Kesselheizbläche," by O. Seibert, *Die Wärme*, vol. 53, 1930, pp. 537–543.

49 "Radiation in Boiler Furnaces," by W. T. Bottomley, N. E. Coast Institution of Engineers and Shipbuilders, vol. 49, 1933, p. 115.

50 "Performance of a Pulverized-Coal-Fuel Boiler," by J. J. Grob and J. Gershberg, *Combustion*, vol. 2 (new notation) June, 1931, p. 21.

51 "Burning Characteristics of Pulverized Coals and the Radiation From Their Flames," by R. A. Sherman, Trans. A.S.M.E., vol. 56, 1934, paper FSP-56-6, pp. 401–410.

52 "Radiation From Luminous and Nonluminous Flames," by R. A. Sherman, Trans. A.S.M.E., vol. 56, 1934, paper IS-56-1, pp. 177–192.

53 "An Experimental Study of the Burning Characteristics of Pulverized Fuels," by R. A. Sherman, Proceedings of the International Conference on Bituminous Coal, vol. 2, 1931, p. 510.

54 "Warmeübertragung durch Strahlung im Feuerraum," by H. Friedrich, *Mitteilungen aus den Forschungsanstalten*, vol. 1, 1930–1932, pp. 227–243.

55 "Dampfkraft," by F. Münzinger, Julius Springer, Berlin, 1933.

56 "Radiation Intensities and Heat Transfer by Radiation in Boiler Furnaces," by H. O. Croft and C. F. Schmarje, Trans. A.S.M.E., vol. 57, no. 3, 1935, paper FSP-57-4, pp. 105–113.

57 "Specific Heats of Gases at High Temperatures," by E. D. Eastman, United States Bureau of Mines Technical Paper No. 445, 1929.

58 "Mechanical Engineers Handbook," L. S. Marks, editor, McGraw-Hill Book Company, New York, third edition, 1930, p. 385.

59 "Industrial Heat Transfer," by A. Schack, John Wiley and Sons, New York, N. Y., 1933.

60 "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Company, New York, 1933.

Discussion

Steam-Turbine Leaving Losses and Vacuum Corrections[1]

ERNEST L. ROBINSON.[2] Professor Helander has presented a detailed set of computations of the effect of leaving velocity at the wheel annulus on vacuum corrections for steam-turbine performance. The corrections are based, as they should be, on a particular known performance and the author wisely qualifies his methods as suitable for use in the field where manufacturers' corrections are not available, and his basic theoretical analysis of the steam action in the last-stage bucket exit seems to be correct.

There are, however, a number of influences that must be taken into account in preparing the vacuum corrections for steam-turbine performance. The most important of these is the leaving-velocity loss from the last-stage wheel and this he has considered. Another important element is the pressure drop through the exhaust hood from the wheel annulus to the exhaust flange, which occurs despite considerable excess area. These two influences are closely inter-related and only an approximate analysis of the actual leaving loss can be made without taking account of the variation of pressure due to drop through the hood. Omitting the exhaust-hood loss, however, does give a first approximation to the vacuum corrections since when the exhaust-hood loss is larger, the leaving loss is less. Certain upstream effects are also to be reckoned with for a stage or two preceding the last, particularly at light loads. We understand from the author that these latter effects, while not evaluated, are allowed for in Appendix B if certain factors are known.

Manufacturers' vacuum corrections attempt to evaluate all these influences with the best available knowledge as to the performance of the several elements, supplemented with empirical corrections to the theoretical calculations following analyses of many actual tests. It would be our recommendation that where important decisions are involved, actual tests at various vacuums be carried out or the manufacturers' vacuum corrections, which are based on a wide knowledge of such tests, be used.

C. HAROLD BERRY.[3] The determination of vacuum corrections is an important undertaking in connection with an acceptance test of a steam turbine. The A.S.M.E. Test Code for Steam Turbines now requires that vacuum corrections be established by prior agreement between the manufacturer and the purchaser, either in the purchase contract or in a separate agreement, or else that the corrections be found by auxiliary tests. The first alternative is tantamount to accepting the manufacturer's correction curves; if the purchaser is unwilling to do this, he must conduct a costly series of tests.

Some readers may be of the opinion that Professor Helander has developed another method for securing the desired corrections that will avoid the difficulties of the foregoing alternatives. That this is not the case seems evident from the fact that items 2 to 6 and 28 in the table on page 154 of the original paper are stated to rest upon information furnished by the manufacturer. In the case of a test, item 5 would be known from the test results, and some of the other items might be obtained by sufficiently skillful measurement; but in the end the computed results will rest upon data furnished by the manufacturer and accepted by the purchaser. If the purchaser is willing to accept basic data, he will probably be equally willing to accept the final result. Thus it appears to me that this method should not find a place in the A.S.M.E. Test Codes.

The writer speaks as chairman of Power Test Codes Individual Committee No. 6, which is now engaged in the revision of the Test Code for Steam Turbines. He does not, however, speak for the Committee, since there has been insufficient time for a poll of its membership.

JOHN A. DENT.[4] Both designers and users of turbines will find Professor Helander's paper useful in the estimation of leaving losses and vacuum correction. The influence of critical speed and the limitations of possible vacuum at different loads are clearly brought out. While at first glance the procedure seems rather complicated the development is entirely logical and with a little study and practice can be applied with a reasonable expenditure of time and work.

The problem of determining leaving losses is exceedingly complex, and many variables enter its solution. The author is careful to point out that his method gives only a reasonable approximation and calls attention to some of the neglected variables which may influence the result, namely, leakage, moisture, and variation in nozzle efficiency. Other items which might be mentioned are:

(*a*) Variation in leaving velocity across section of jet
(*b*) Variation in leaving velocity from root to tip of blade
(*c*) Variation in gaging from root to tip.

Under (*a*) it should be remembered that the discharge varies as the mean velocity across the jet, while the kinetic energy varies with the "root mean square" of the velocity. Ordinarily the error in using mean velocity for computing kinetic energy is quite small.

The error in computing kinetic energy from the leaving velocity at mean-blade speed may be considerable in some cases, and entirely negligible in others. For example if the tangential component of the relative velocity is equal to the mean-blade speed, the absolute velocity leaving is a minimum at the middle of the blade, and increases both toward root and tip. If the kinetic energy is calculated on the basis of the velocity at the middle of the blade the result will obviously be too low. In the accompanying diagram, Fig. 1, *OA* is the relative velocity, *AB* the mean-blade speed, *AC* the root-blade speed, and *AD* the tip-blade speed. Then *OB* is the absolute velocity at the middle of the blade, *OC* at the root, and *OD* at the tip. Plainly, the mean leaving velocity is greater than *OB*.

The writer made a few calculations on the magnitude of this error using as an example a two-foot blade mounted on a six-foot drum revolving at 1800 rpm. A straight blade was assumed with

[1] Published as paper FSP-57-8, by Linn Helander, in the May, 1935, issue of the A.S.M.E. Transactions.

[2] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[3] Gordon McKay Professor of Mechanical Engineering, The Graduate School of Engineering, Harvard University, Cambridge, Mass. Chairman of A.S.M.E. Power Test Codes Individual Committee No. 6, Steam Turbines.

[4] Professor and Head of Department of Mechanical Engineering, University of Pittsburgh, Pittsburgh, Pa. Mem. A.S.M.E.

a constant gaging giving sin α = 0.45. The figures chosen correspond to a mean-blade speed of 754 ft per sec and a tip speed of 942 ft per sec.

At a relative velocity of 850 ft per sec the kinetic energy was 108 per cent of that found by assuming the leaving velocity corresponding to the mean-blade speed. At 1000 ft per sec the ratio dropped to 103.4 per cent and at 1270 ft per sec became 100 per cent. A maximum of 115 per cent was found at 582 ft per sec.

No attempt has been made to evaluate the effect of change of gaging from root to tip. Such an investigation would require a

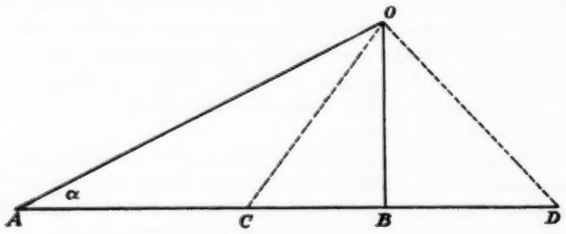


FIG. 1

detailed study of the design of blading used in any individual turbine. Such procedure is far beyond the intended scope of the author's paper. The writer wishes merely to call attention to this factor as one of the variables whose influence has been neglected.

The determination of the critical conditions is one of the most interesting and important contributions in the paper. The curves and approximate formulas developed by the author will do much to save time and labor in this calculation. The data supplied are almost the exact reverse of those given in impulse-turbine nozzles. There a nozzle of given dimensions is supplied with steam of known pressure and quality, and the discharge is easily computed. In the present instance a given discharge is delivered at a known pressure, but unknown quality, and the critical conditions have to be found. A method of successive approximations is used. The author shows that a correct solution is quickly reached. In fact the first approximation appears to be well within the limits of accuracy of the method, but the author properly refuses to take this for granted and carries through the second approximation, showing that his first assumptions were close to the truth.

The writer wishes to express appreciation of the paper and to welcome it as a valuable contribution to our knowledge of the turbine.

AUTHOR'S CLOSURE

The procedure for evaluating steam-turbine vacuum corrections proposed in the paper was prepared principally for use in making economic and heat-balance studies of the kind usually undertaken before the purchase of turbines and condensers to determine among other data, the vacuum which probably would be the most economical and, therefore, the approximate vacuum for which the turbines and their condensers should be designed.

In view of the uncertainty of data other than vacuum corrections, such as cost data and load curves, which must be estimated or assumed for these preliminary studies, it was felt inadvisable to introduce refinements that might add much to the complexity of the problem of evaluating approximate vacuum corrections and yet have but a small influence on final conclusions. For this reason, and also because precise data on the design of blades and exhaust hoods are not available usually for preliminary studies, the factors to which Professor Dent and Mr. Robinson appropriately call attention were ignored.

Where the necessary data are available and precise results are desired, exhaust-hood losses should be calculated and leaving losses should be more accurately evaluated taking into account, as Professor Dent suggests, variations in gaging from root to tip as well as variations in blade speed from root to tip. When variations in gaging and in blade speed from root to tip are taken into account, the equation for leaving losses takes the form of an integral which can be evaluated graphically if it cannot conveniently be evaluated analytically.

In this connection, it may be well to point out that in evaluating vacuum corrections, two leaving losses calculated for different vacua are subtracted and their errors may be compensatory to some extent. Table 1 of this discussion shows this to be the

TABLE 1 ERROR IN CHANGE IN LEAVING LOSSES ACCOMPANYING A CHANGE IN RELATIVE VELOCITY BASED UPON DATA PRESENTED BY PROFESSOR DENT[a]

Relative velocity, fps	582	850	1000	1100
Approx. leaving losses, Btu per lb	2.48	2.95	4.46	6.00
Error in leaving losses, per cent	15.50	8.00	3.40	1.60
Absolute error in leaving losses, Btu per lb	0.383	0.235	0.151	0.097
Absolute error in change in leaving losses, Btu per lb	Base	0.148	0.232	0.286
	...	Base	0.084	0.138
	...	...	Base	0.054
Error in change in leaving losses, per cent of increment of relative kinetic energy[a]	Base	1.95	1.75	1.65
	...	Base	1.50	1.40
	...	...	Base	1.30

[a] Increment of relative kinetic energy = [(Terminal rel. vel.)^2 — (Base rel. vel.)^2]/50,000.

case for the conditions assumed by Professor Dent. It shows also, for these conditions, that when leaving losses for two different relative velocities are evaluated at the mean diameter, the error in their difference is but a small fraction of the conversion of energy required to accelerate the steam from the lower to the higher velocity, and therefore may be ignored where approximate data are satisfactory.

In the writer's sample calculations, if leaving losses had been evaluated by taking into account variations in blade speed from root to tip on the assumption that straight blades of constant gaging are employed, then the calculated vacuum correction factor, Item 58 on page 155 of the paper, would have changed from 4.60 per cent to 4.67 per cent under Case 1 and from 1.33 per cent to 1.38 per cent under Case 2. For the purposes for which these estimated vacuum corrections are likely to be used, these differences are negligible.

The foregoing considerations do not in any sense detract from the force and value of Professor Dent's criticisms and suggestions. For conditions other than those here investigated the error introduced by evaluating leaving losses at the mean diameter may have a greater effect on the vacuum correction; and as previously stated, where precise values of leaving losses and vacuum corrections are desired, the factors Professor Dent discusses should be considered. The author believes, however, that where the data needed for evaluating leaving losses precisely are not available, his procedure will give approximations of vacuum corrections satisfactory for use in preliminary studies.

The author wishes to thank Professor Dent, Mr. Robinson, and Professor Berry for the discussions they have contributed. Their points of view and criticisms have helped to clarify the intent of the paper and the limitations of the procedure proposed. He agrees with Professor Berry that the procedure outlined for evaluating vacuum corrections should not be made a part of the A.S.M.E. test code.

The Psychrograph[1]

WARREN VIESSMAN.[2] The Norris-Westinghouse psychrograph chart is ingenious, and its invention is a material contribution to the design of air-conditioning installations.

The recognition and use of the heat-ratio lines of sensible heat to total heat through a point of known condition, as a locus of all changes of psychrometric conditions of the air under constant-heat ratio (as represented by a point on the chart, and as stated in the first theorem in the paper) offers an accurate and convenient method of determining the saturation temperature of a mixture of air leaving the washer, and greatly simplifies the work of the air-conditioning engineer.

The second theorem given in the paper is useful in establishing the condition of the air entering the washer, and also the condition of the air entering the room, when a by-pass washer with make-up air, as shown in Fig. 1 of this discussion is used. The straight line connecting the two points covering the conditions of the two quantities to be mixed is the locus of all conditions of the mixed air.

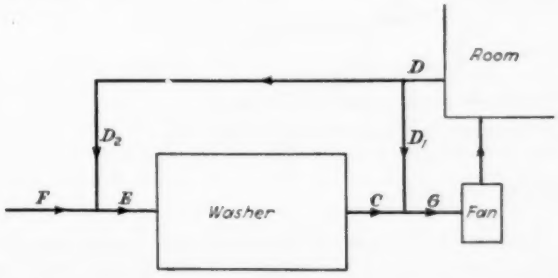


FIG. 1 DIAGRAM OF CONNECTIONS FOR A BY-PASS WASHER WITH LETTERS CORRESPONDING TO THOSE GIVEN IN EXAMPLE 5. FIG. 5, OF THE PAPER

The chart as constructed is easily read and of much larger scale for the same overall size than the usual chart. The values as read from the chart check to within three significant figures with those obtained from tables and by calculation.

In his Fig. 5, the author has solved a typical by-pass air-washer problem. The letters on the chart in Fig. 5 of the paper, representing conditions at various points, correspond with those on Fig. 1 of this discussion.

In this discussion let C = quantity of air leaving washer, lb per min; D_1 = quantity of room air of condition D recirculating and by-passing the washer, lb per min; D_2 = quantity of room air of condition D recirculating and entering the washer, lb per min; E = quantity of air entering washer to be conditioned, lb per min; F = quantity of outside ventilation air entering washer, lb per min; G = quantity of conditioned and recirculated air entering the room, lb per min; t_e = temperature of air leaving washer, deg F, dry bulb; t_d = temperature of room air recirculated, deg F, dry bulb; t_s = temperature of air entering the washer, deg F, dry bulb; t_f = temperature of outside air, deg F, dry bulb; and t_g = temperature of air entering room, deg F, dry bulb.

The quantity of conditioned and recirculated air G entering the room is determined by the temperature differential desired between the entering air and the room air. The quantity of outside ventilation air F entering the washer is usually expressed as a percentage of G.

The quantity of air E entering the washer to be conditioned is

$$E = \frac{\text{Total heat to be removed from room, Btu per hr}}{0.24 \times 60 \text{ (room wet-bulb temp} - t_e)}$$

which is equal to the quantity of air C leaving the air washer, or $C = E$.

The quantity of room air D_1 of condition D recirculating and by-passing the washer is equal to the quantity of conditioned and recirculated air G entering the room minus the quantity of air C leaving the washer, or $D_1 = G - C$.

The quantity of room air D_2 of condition D recirculating and entering the washer is equal to the quantity of air E entering the washer to be conditioned minus the quantity of outside ventilation air F entering the washer, or $D_2 = E - F$. Since $D_1 = G - D$, and $D_2 = E - F$, it is seen that $D_1 + D_2 = G - F$.

The temperature of the air t_e leaving the washer is obtained on the Norris Psychrograph by the intersection of the heat-ratio line through the room condition with the saturation curve, and is 52 F in example 5, Fig. 5, of the paper.

The temperature t_d of the room air recirculated, the temperature t_f of the outside air, and the temperature t_g of the entering air, are selected design temperatures. The temperature t_s of the air entering the washer is equal to $(D_2 t_d + t_f F)/E$.

From the foregoing relationships it is seen that the determination of t_e, the temperature of the air leaving the washer, from the Norris Psychrograph readily simplifies a series of relationships which otherwise would be more difficult to solve. The location of t_s and t_g on the locus lines of the chart immediately establishes the condition of the air entering the washer and the air entering the room.

In example 5 given by the author, which is solved in Fig. 5 of the paper, the total heat load of the room was 100,000 Btu for which 59.1 lb of ventilation air was required. The air recirculated through the washer was 118.3 lb per min.

The total heat of outside air F entering the washer at 78 F wet bulb = 40.64 Btu per lb, and the total heat of outside air F leaving the room at t_d deg and 50 per cent relative humidity = 30.80 Btu per lb. Therefore, the heat removed from the ventilation air above room temperature = 40.64 — 30.80 = 9.84 Btu per lb. Hence, the heat removed per hour = 59.1 × 60 × 9.84 = 35,000 Btu per hr. The refrigeration required would then be (100,000 + 35,000)/12,000 = 11.3 tons, which checks with 11.44 obtained by the author in his example 5.

Again, the total heat of recirculated air D_2 to the washer = 30.80 Btu per lb, and the total heat of recirculated air D_2 from the washer at 52 F wet bulb = 21.30 Btu per lb. Therefore, the heat removed from the recirculated air D_2 = 30.80 — 21.30 = 9.50 Btu per lb, and since 118.3 lb per min is circulated, the heat removed per min = 118.3 × 9.50 = 1130 Btu per min. The total heat of outside air F to the washer at 78 F wet bulb = 40.64 Btu per lb, and the total heat of outside air F from the washer at 52 F wet bulb = 21.30 Btu per lb. Therefore, the heat removed from the outside air F = 40.64 — 21.30 = 19.34 Btu per lb, and since 59.1 lb per min pass through the washer, the total heat removed per minute = 59.1 × 19.34 = 1150 Btu per min. The refrigeration required would then be (1130 + 1150)/200 = 11.4, which again checks with 11.44 obtained by the author in his example 5, Fig. 5.

The scale on the Norris Psychrograph giving cubic feet per pound of dry air and vapor present is useful in changing pounds of air to cubic feet for the various conditions when determining fan and duct sizes. The determination of air leakage by use of the Psychrograph will undoubtedly be of service to operating and construction personnel in the locating and the correcting of faults in equipment.

---

[1] Published as paper PRO-57-2, by A. M. Norris, in July, 1935, issue of the A.S.M.E. Transaction.

[2] Associate Mechanical Engineer, Procurement division, U. S. Treasury Department, Washington, D. C. Mem. A.S.M.E.

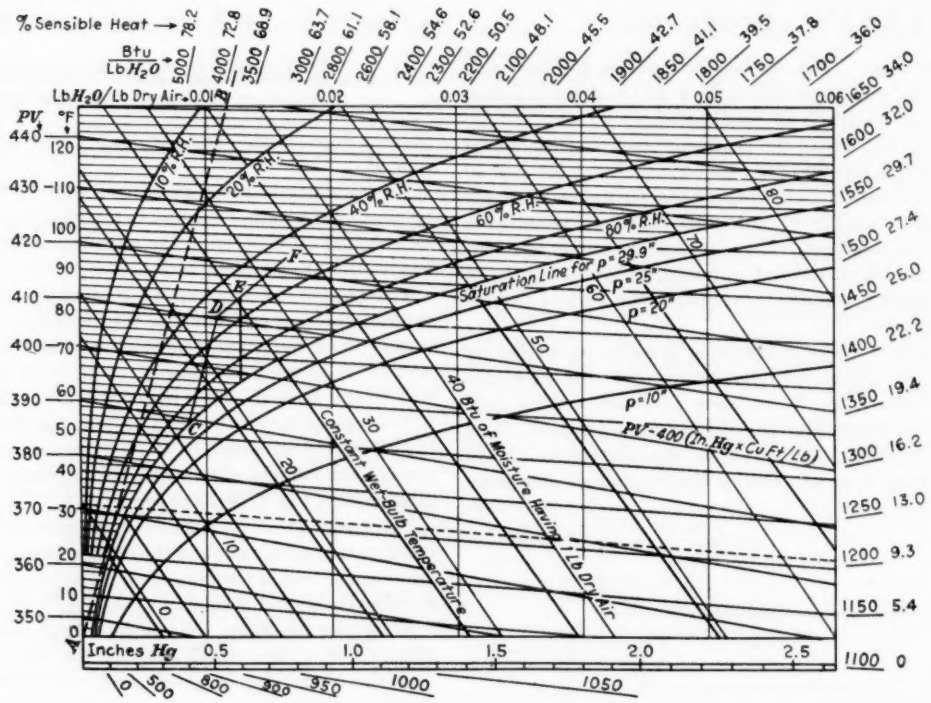


FIG. 2 MOLLIER WATER-MIXTURE CHART

ARNOLD WEISSELBERG.[3] Graphical solutions of air-conditioning problems as recommended by the author are so well established that the merit of such procedure need not be stressed further.

The need for the development of a new chart, such as the author's Psychrograph, for this purpose is open to question in view of the existence of the Mollier water-air-mixture chart which accomplishes the same results in just as simple a manner and is perhaps more convenient to use because it has only one temperature scale compared to three such scales on the author's Psychrograph.

In the Mollier chart the marginal scale furnishes the amount of total heat per pound of moisture removed, which is another way of expressing the ratio of sensible heat to total heat in the author's paper. To make this scale conform exactly to that of the Psychrograph, it is only necessary to divide the heat content of one pound of water at the average temperature of 70 F or 1090 Btu by the scale values and deduct the value of the ratio from unity in order to express the marginal scale in the percentages of the Psychrograph.

[3] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

The procedure is now the same with both charts, except that greater accuracy may be claimed for the Mollier chart, since therein the sensible heat of the vapor in the air is also taken into consideration. This is shown on Fig. 2 of this discussion wherein the writer solves the author's example 5, which in the paper is solved in Fig. 5.

In the second paragraph of theorem 2 on page 229 of the paper, the author states that in case the line joining two points crosses the saturation curve, the locus of possible mixtures will follow the saturation curve instead of crossing it. The word "may" should be used rather than "will" inasmuch as the author himself indicates in the same paragraph that the quality of the mixture will be somewhat uncertain.

In the writer's opinion the chief value of the paper lies in the interesting graphical solution to a variety of problems which are especially adapted to demonstrate the advantages of graphical treatment of an air-conditioning problem. In that respect the author has done an excellent job and the fact that he could have used other charts, which, as has been stated in this discussion, are at least as good for the purpose, appears to be of secondary importance.

Fatigue of Shafts at Fitted Members, With a Related Photoelastic Analysis

By R. E. PETERSON[1] and A. M. WAHL,[2] EAST PITTSBURGH, PA.

This paper deals primarily with the fatigue of shafts with press-fitted members, a problem frequently arising in connection with rotating machinery involving gears, wheels, couplings, rotors, etc. Failures at the edge of fitted members are not uncommon in practice. A survey is made of existing data, after which a report of fatigue tests on the press-fit problem carried out at the Westinghouse Research Laboratories is given. A related photoelastic investigation and stress analysis is also reported. Practical expedients for increasing the strength of shafts with fitted members are given. Suggestions for future research along the foregoing lines are made throughout the paper.

ALTHOUGH nearly all shafting carries such machine elements as gears, pulleys, wheels, couplings, rotors, etc., but few data have been available on which to estimate the effect of these members on the endurance strength of the shaft, with the result that sometimes rather surprising failures occur.

A report by Dr. Kühnel of the German State Railways lists, in Table 1, twenty-three fatigue failures of axles at the wheel fit. From tests of specimens taken from the broken axles, it was concluded that the physical properties were such as to correspond to normal material of that type. Recent researches have shown that in the case of a shaft subjected to alternating stresses a press fit introduces a combined stress-concentration and rubbing-corrosion effect of such magnitude as to weaken the shaft to a degree not generally suspected. Hence the surprising failures in perfectly normal material. It is proposed in this paper to review the available data, contained chiefly in two German publications, and then to supplement this by a report of similar work carried out intermittently over a period of years at the Westinghouse Research Laboratories. The photoelastic study and stress analysis were made quite recently in an attempt to explain some of the observed effects.

Review of Published Data

Fatigue tests by Kühnel[3] as given in Table 2, showed that a shaft with a press-fitted collar had but 56 per cent of the endurance strength of a shaft with a large contour radius, i.e., practically no stress concentration in the latter case corresponding approximately to the ordinary "endurance limit." Specimens having the same contour as the press-fit specimens, but cut from a solid bar, Table 2, gave the same endurance limit as the press-fit specimens.

In Table 3, results are shown of tests made at the Darmstadt Materials Testing Laboratory.[4] Here again it will be noticed that a shaft with a press-fit member had only about half the endurance strength of the shaft material without stress concentration. It is of interest to note that the "obvious" expedient of rounding the sharp corner, shown at 7 and 8, Table 3, was not very effective in raising the strength, particularly as compared with a groove turned in the face of the collar, 5. It is further apparent, from 4, that rolling of the shaft surface increased the press-fit strength to a surprising degree, and when this expedient was combined with grooving the hub face, 6, full shaft strength was obtained. It should also be mentioned that other methods of increasing surface strength, such as case hardening and nitriding, were also very effective in increasing the press-fit endurance strength (not shown in Table 3 since complete endurance-limit data were not available).

Westinghouse Fatigue Tests of Fitted Members

An investigation of the fatigue of shafts with press-fitted collars was started at the Westinghouse Research Laboratories in January 1931 in connection with the design of shafts for motors and gears. It was felt at that time that three of the main effects present in the press-fit fatigue problem were: stress concentration, a three-dimensional stress system, and rubbing corrosion,[5] and that the tests should include some study of separate variables if possible. Three series were decided on as follows: (*a*) Heavy press fit; (*b*) very light press fit; (*c*) solid specimen of same outline dimension as press-fit specimen. In (*b*) the pressure variable is practically eliminated, or at least reduced to small magnitude, although bending may introduce some increase of stress at the corners in this case, and in (*c*) pressure and rubbing-corrosion effects are not present. The aforementioned program included series 1–5 in Table 5, while series 6 and 7 were added quite re-

---

[1] Manager, Mechanics Division, Westinghouse Research Laboratories. Assoc-Mem. A.S.M.E. Mr. Peterson received his B.S. in mechanical engineering from the University of Illinois in 1925, and one year later his M.S. in theoretical and applied mechanics from the same institution. In September, 1926, he entered the employ of the Westinghouse Company in the railway-motor engineering department, and in January, 1928, was transferred to the research department. During 1926–1929 he was also part-time instructor in analytical mechanics and strength of materials at Carnegie Institute of Technology. In 1930 he was made section engineer in charge of strength of materials, Westinghouse Research Laboratories, and in 1931 was appointed to his present position.

[2] Westinghouse Research Laboratories. Assoc-Mem. A.S.M.E. Dr. Wahl received his education at Grinnell College and Iowa State College, being graduated from the mechanical engineering course of the latter in 1925. He then entered the employ of the Westinghouse Electric & Manufacturing Company as a graduate student. In 1927 he received his M.S. and in 1932 his Ph.D. degree from the University of Pittsburgh. Since 1926 he has been engaged at the research laboratories of the company on problems of applied mechanics, particularly in the field of stress analysis. He was the recipient of the 1929 A.S.M.E. Junior Award.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until May 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "Achsbrüche bei Eisenbahnfahrzeugen und ihre Ursachen," by R. Kühnel, *Stahl und Eisen*, Oct. 6, 1932, vol. 40, p. 965.

[4] "Der Einfluss von Einspann- und Kraftangriffsstellen auf die Dauerhaltbarkeit der Konstruktionen," by A. Thum and F. Wunderlich, *Zeit. V.D.I.*, Aug. 5, 1933, vol. 77, no. 31.

[5] By rubbing corrosion is meant the oxidation of steel surfaces in contact due to a microscopic oscillating sliding relative motion. See "The Rusting of Steel Surfaces in Contact," by Tomlinson, Proc. Royal Soc., A., 1922, vol. 115, p. 472.

TABLE 1 TEST RESULTS FROM AXLES BROKEN AT WHEEL FIT IN SERVICE

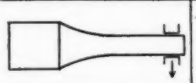
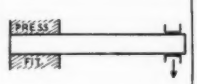
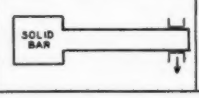
(Kühnel—German State Railways)

	Type of service	Year supplied	Year of failure	Chemical composition per cent C	Si	Mn	P	S	Physical properties: Elongation per cent	Yield str. lb/sq in.	Ultimate strength, lb/sq in.	Endurance limit, lb/sq in.	Impact value,[1] m-kg/sq in.	No. of repeated impacts[2]	Fatigue portion of cross-section
Older Material															
1	Freight car	1884	1928	0.14	Trace	0.48	0.10	0.05	3–32	44000–48000	53000–68000	.....	0.7–9.0	..	1/10
2	Freight car	1888	1927	0.28	Trace	1.15	0.11	0.08	21–26	43000–47000	78000–82000	37000	0.8–3.2	5300	1/5
3	Beer car	1882	1929	0.26	0.19	0.48	0.10	0.06	25–27	36000–37000	63000–64000	30000	1.7–3.7	4500	1/5
4	Freight car	1900	1929	0.40	0.15	0.49	0.09	0.04	15–22	43000–45000	84000–85000	33000	1.4–1.8	6000	1/4
5	Baggage car	1886	1929	0.37	0.17	0.56	0.10	0.07	25–27	41000–43000	77000–78000	34000	4.0–4.3	6200	1/20
6	Freight car	1882	1929	0.13	Trace	0.46	0.15	0.05	2–28	41000	50000–64000	37000	0.9–3.7	3800	1/50
7	Freight car	1898	1930	0.29	0.06	0.20	0.09	0.04	24–28	45000–48000	81000–85000	34000	0.9–2.4	8000	1/20
Newer Material															
8	Sleeping car	?	..	..	..	..	..	..	21	38000–43000	80000	....	.....	..	1/2
9	Freight car	?	1925	..	..	..	..	..	25–27	50000–53000	81000–84000	....	.....	..	..
10	Freight car	1910	1927	0.26	0.17	0.76	0.02	0.08	26–27	37000–38000	72000–75000	30000	4.7–5.6	8000	1/20[3]
11	Locomotive	1910	1927	0.44	0.31	0.44	0.01	0.03	25–31	36000–37000	78000	30000	4.2–5.3	6700	3/4
12	Freight car	?	1927	..	..	..	..	..	28–29	38000	75000–77000	....	.....	..	1/4
13	Freight car	1904	1927	0.27	0.26	1.15	0.04	0.03	29–30	44000–45000	75000–78000	36000	4.0–6.9	7700	1/10
14	Passenger car	?	1928	0.40	0.20	0.05	0.04	0.03	27–34	40000	73000–74000	31000	6.2–6.8	53000	1/20[3]
15	Freight car	1903	1928	..	..	..	..	..	24–27	45000–48000	84000–85000	...	.....	..	1/20[4]
16	Freight car	1901	1928	..	..	..	..	..	..	.....	97000	....	3.0–5.8	..	1/3
17	Freight car	1902	1928	0.51	0.36	0.60	0.01	0.05	28–29	37000–40000	74000–77000	31000	5.3–5.6	5000	1/10 & 1/3[5]
18	Freight car	1916	1928	..	..	..	..	..	28–30	40000	71000–72000	....	.....	..	1/20
19	Freight car	1902	1929	0.41	0.23	0.64	0.04	0.03	27–29	40000–43000	74000–77000	30000	5.4–5.9	8900	4/5 & 1/10[5]
20	Sleeping car	1904	1929	0.42	0.35	0.60	0.03	0.03	28–29	40000–41000	74000–77000	31000	4.8–5.7	7200	3/4
21	Freight car	?	..	..	..	..	..	..	27–30	40000–41000	71000–73000	....	.....	..	4/5
22	Freight car	?	1930	0.46	0.24	0.62	0.05	0.04	29–30	36000–37000	74000–77000	30000	1.5–2.3	7200	1/10
23	Driving car	?	1930	..	..	..	..	..	24–27	41000–44000	78000–81000	...	1.8–4.1	..	4/5

[1] Specimen 5 × 10 mm cross-section, 70 mm support distance.
[2] Krupp repeated impact machine with 15 mm diameter notched specimen.
[3] Crack discovered in shop.
[4] Crack.
[5] Cracks on opposite sides.

TABLE 2 SELECTED FATIGUE-TEST DATA CONCERNING PRESS-FIT PROBLEMS

(Kühnel—German State Railways)

NO.	TYPE OF SPECIMEN	ENDURANCE STRENGTH LBS/SQ IN.	% ORDINARY ENDURANCE LIMIT	STRESS CONCENTRATION FACTOR[a] K
1		33000	≈100	
2		18500	56	1.8
3		18500	56	1.8

a—STRESS CONCENTRATION FACTOR $K = \frac{\text{ORDINARY ENDURANCE LIMIT OF SPECIMEN WITH LARGE CONTOUR RAD.}}{\text{ENDURANCE LIMIT WITH STRESS CONCENTRATION PRESENT}}$

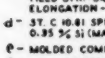
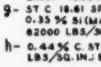
cently as a result of the publication of the interesting results obtained at Darmstadt[4] on grooving and rolling. In planning the tests cited it was decided to use as large specimens as feasible in order to obtain as accurate a fit allowance as possible.[6] The equipment available[7] permitted the use of a $1^5/_8$ in. shaft diameter, allowing for the weakening effect of the press fit. The material used for both shafts and collars was medium carbon axle steel, 0.42 per cent C, the properties of which are given in Table 5.

[6] More recent tests have shown the desirability of using as large specimens as possible for any fatigue tests involving stress concentration, since small specimens show relatively low stress-concentration effect in some cases while large specimens tend to show higher stress-concentration effect approaching theoretical values as a seemingly general tendency. Data are given in "Model Testing as Applied to Strength of Materials," by R. E. Peterson, Trans. A.S.M.E., 1933, vol. 55, paper APM-55-11, p. 79.

[7] For description of large testing machine used, see "Fatigue Tests of Large Specimens," by R. E. Peterson, Proc. Am. Soc. for Testing Materials, 1929, vol. 29, part II, p. 371.

TABLE 3 ENDURANCE STRENGTH OF FITTED SPECIMENS

(Darmstadt Materials Testing Laboratory—Thum and Wunderlich)

NO.	TYPE OF FIT	MATERIAL SPECIMEN[a]	MATERIAL COLLAR	FIT PRESSURE LBS/SQ IN	REMARKS	ENDURANCE STRENGTH LBS PER SQ. IN.	ENDURANCE STRENGTH % ORDINARY ENDURANCE LIMIT OF SPECIMEN MATERIAL (SEE FOOTNOTES)	STRESS CONCENTRATION FACTOR[b] K
1		MEDIUM C. STEEL[c]	LOW C. STEEL[d]	9000		23000	85	1.8
2		"	"	17000		23000	85	1.8
3		"	"	30000		22000	53	1.9
4		"	"	"	ROLLED SURFACE	38000	83	1.1
5		"	"	"		30000	72	1.4
6		"	"	"	ROLLED SURFACE	43000	103	1.0
7		"	"	"	r = 0.04 IN.	23000	55	1.8
8		"	"	"	r = 0.12 IN.	26000	62	1.8
9		"	"	"		24000	59	1.7
10		"	HARDENED BUSHING	NOT KNOWN		19000	47	2.2
11		"	PERTINAX[e] BUSHING	1300		36000	86	1.2
12		Cr-Ni STEEL[f]	LOW C. STEEL[d]	13000		28000	39	2.6
13		LOW C. STEEL[g]	"	17000		20000	57	1.8
14		MEDIUM C. STEEL[h]	"	13000		25000	60	1.7
15		"				19000–20000	≈ 48	≈ 2.1

a—SPECIMEN 0.55" DIA. - SURFACE GROUND

b—STRESS CONCENTRATION FACTOR $K = \frac{\text{ORDINARY ENDURANCE LIMIT WITH LARGE CONTOUR RADIUS}}{\text{ENDURANCE LIMIT WITH STRESS CONCENTRATION AT CRITICAL SEC.}}$

c—0.57 % C; 0.29 % Si; 0.55 % MN; 0.02 % P; 0.03 % S
YIELD STR.-54000 LBS/SQ IN; ULT. STR.-95000 LBS/SQ.IN.; ENDURANCE LIMIT-41000 LBS/SQ. IN.
ELONGATION - 22 %

d—ST. C 10.61 SPECIFICATION (VDE WERKSTOFF HANDBUCH DIN 1611): 0.08-0.18 % C; 0.5 % MN (MAX); 0.35 % Si (MAX); ULT. STR.-54000 LBS/SQ.IN.; ELONGATION - 21 % (MIN.)

e—MOLDED COMPOSITION MATERIAL (NON-METALLIC)

f—VCN 35 SPECIFICATION (VDE WERKSTOFF HANDBUCH H11-45): 0.25-0.40 % C; 0.35 % Si (MAX); 0.4-0.8 % MN; 0.55-0.85 % Cr; 3.25-3.75 % Ni. QUENCHED AND TEMPERED. TEST DATA: YIELD STR.-136000 LBS/SQ. IN.; ULT. STR.-150000 LBS/SQ. IN.; ENDURANCE LIMIT-73000 LBS/SQ. IN.

g—ST. C 18.61 SPECIFICATION (VDE WERKSTOFF HANDBUCH DIN 1611): 0.11-0.18 % C; 0.4 % MN (MAX); 0.35 % Si (MAX). ANNEALED 650°C. TEST DATA: YIELD STR.-45000 LBS/SQ. IN.; ULT. STR.-62000 LBS/SQ. IN.; ENDURANCE LIMIT-25000 LBS/SQ. IN.

h—0.44 % C. STEEL AS RECEIVED. TEST DATA: YIELD STR.-35000 LBS/SQ. IN.; ULT. STR.-87000 LBS/SQ. IN.; ENDURANCE LIMIT-41000 LBS/SQ. IN.

The specimen dimensions are given in Fig. 1. Fig. 2 is a view of the test set-up.

The collar bore and the critical shaft diameter were accurately machined to give the desired press-fit allowance. In connection with series 6 and 7, careful measurements of the diameters were made. With exception of series 1 the pressing was done in a hydraulic testing machine, so that accurate data on the axial load during pressing were obtained as given in Table 4. From these two sets of data it has been possible to estimate the average radial pressure by two independent methods. Making use of the axial pressing load

$$p = \frac{P}{\pi d l \mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where p = radial pressure, lb per sq in.
P = pressing load, lb
d = shaft diameter, in.
l = length of collar, in.
μ = coefficient of friction

The only doubtful factor is the coefficient of friction. As a result of a literature survey together with extensive data of his own, Baugher[8] concluded that, "a general average of coefficient of friction realized when pressing in shafts would be probably in the neighborhood of 0.15. . ." This value was used in calculating the pressures given in column 5 of Table 4.

TABLE 4 DATA ON PRESS-FIT PRESSURES

Series	Specimen No.	Press load, lb	Difference in diam, in.	Avg. radial pressure, lb/sq in. Caled. from Eq. [1]	Caled. from Eq. [2]
2	2–1	16000	....	14000	...
Heavy	2–2	21300	....	18000	...
fit	2–5	17000	....	15000	...
plain	2–6	19500	....	17000	...
3	3–1	100	....	90	...
Light fit	3–2	1000	....	900	...
plain	3–4	100	....	90	...
6	6–1	14000	0.0018	13000	13000
Heavy fit	6–2	13000	0.0017	12000	12000
grooved	6–3[a]	14000	0.0014	13000	10000
7	4[b]	14000	0.0015	12000	11000
Heavy fit	5[b]	8500	0.0017	7400	12000
grooved	6[c]	9000	0.0006	8000	4200
(shaft	7[c]	5500	0.0011	6000	8000
surface	8[c]	4500	0.0009	4000	6500
rolled)	9[d]	6000	0.0004	5000	3000

[a] Shaft galled in pressing (19000).
[b] Flat roller P = 800 lb.
[c] 1 1/2 in. contour radius roller P = 400 lb.
[d] 1 1/2 in. contour radius roller P = 800 lb.

TABLE 5 ENDURANCE STRENGTH OF FITTED SPECIMENS

(Westinghouse Research Laboratories)

NO	TYPE OF FIT	MATERIAL: SPECIMEN[a]	MATERIAL: COLLAR	FIT PRESSURE LBS/SQ IN	REMARKS	ENDURANCE STRENGTH: LBS PER SQ. IN.	ENDURANCE STRENGTH: % ORDINARY ENDURANCE LIMIT (SEE FOOTNOTE[c])	ENDURANCE STRENGTH: STRESS CONCENTRATION FACTOR[b] K
1		MEDIUM C. STEEL[c]	MEDIUM C. STEEL[c]	NOT KNOWN[d]	d=2 1/2" D=3 3/16" l=1 1/2	21600	68	1.5
2				≅16000	d=2 5/8" D=3 1/2" l=1 1/2	15800	50	2.0
3		"	"	≅ 90	" " "	22000	69	1.4
4		"			" " "	14000	44	2.3
5		"			" D=3 5/16" "	16500	52	1.9
6		"	"	≅12000	" D=3 1/2" "	18700	59	1.7
7		"	"	≅8000	SURFACE ROLLED ROLL FORCE-400LB	23800	75	1.3

a-1 3/8" DIA. (SEE FIG. 1)
b-STRESS CONCENTRATION FACTOR K = ORDINARY ENDURANCE LIMIT OF SPECIMEN WITH LARGE CONTOUR RAD. / ENDURANCE LIMIT WITH STRESS CONCENTRATION PRESENT
c-0.42 % C., 0.62% Mn., 0.15 % Si., 0.028 % S., 0.017 % P., YIELD STR.-37200 LBS/SQ. IN. ULT. STR.-73800 LBS/SQ. IN., ELONGATION-34 %, RED.-61 %
d-PROBABLY LESS THAN 5000 LBS/SQ. IN. (SEE COMMENTS IN PAPER)

(*Endurance Limit = 31,700 lb per sq in.*)

The other method is to make use of the difference of measured diameters in connection with the Lamé equation[9]

$$p = \frac{Ef}{2dD^2}(D^2 - d^2) \quad \ldots\ldots\ldots\ldots\ldots [2]$$

where f = difference in diameters, in.
D = diameter of collar, in.
E = modulus of elasticity, lb per sq in.
Other notation as for Equation [1].

The calculated values for the cases where measurements were made are given in column 6 of Table 4. It will be seen that while the agreement between the results of the two methods is not by any means exact, yet if one considers the assumptions made and the inherent inaccuracies involved, the agreement seems quite reasonable. The widest discrepancies occur in connection with the rolled shafts and this may be due to the fact that some of these surfaces were slightly wavy due to plastic flow, so that the coefficient of friction was probably quite different from the assumed value. It should also be mentioned that some difficulty was encountered in pressing the plain collars on to the shafts due to the relatively sharp collar edge, resulting in higher pressing loads and in severe load fluctuations. This also rendered the subsequent radial-pressure calculations of doubtful accuracy. The grooved collars, however, pressed on to the shafts smoothly. In all cases a light mineral oil[10] was used as a lubricant in the pressing operation.

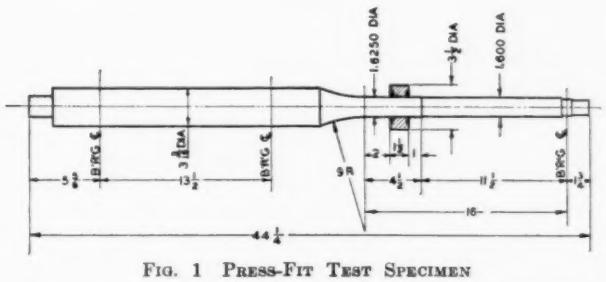


FIG. 1 PRESS-FIT TEST SPECIMEN

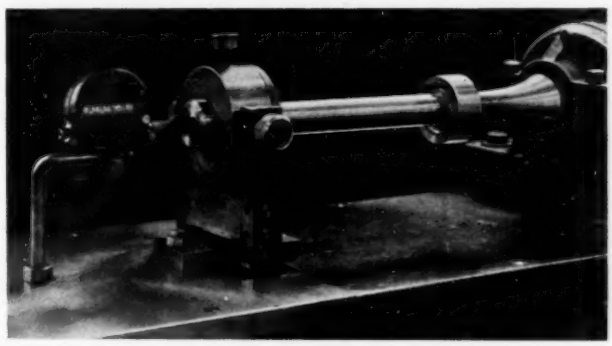


FIG. 2 FATIGUE TEST SET-UP (GROOVED COLLAR)

With regard to the surface rolling of fatigue specimens, considerable work done in German laboratories[11] has shown that the ordinary endurance limit can be increased 5 to 30 per cent, depending on material, by rolling under sufficient pressure to deform the surface plastically. This expedient, as mentioned previously,[4] was also found to be particularly effective in raising the endurance strength of press fits.

Fig. 3 is a drawing of the rolling apparatus used in connection with the Westinghouse tests. Fig. 4 is a view of the rolling apparatus in position. The specimen to be rolled is set up in a lathe and rotated slowly while the roll frame, being in equilibrium with respect to normal forces, is easily held against friction torque from rotating. By exerting sufficient lateral pressure against the frame it can also be caused to travel slowly in an

[8] "Transmission of Torque by Means of Press and Shrink Fits," by J. W. Baugher, Trans. A.S.M.E., 1931, vol. 53, paper MSP-53-10, p. 85.

[9] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., vol. 2, p. 535.

[10] Viscosity, 200 sec (Saybolt) at 100 F.

[11] "Das Drücken der Oberfläche von Bauteilen aus Stahl," O. Föppl, *Stahl und Eisen*, vol. 49, 1929, p. 575. See also publications of Wöhler Institut, Braunschweig, for later work, especially no. 6, "Das Oberflächendrücken zur Erhöhung der Drehschwingungsfestigkeit," by P. Behrens. For an English article covering part of the Wöhler Institut work see "Cold Rolling Raises Endurance Limit," by G. S. Heydekampf, *Iron Age*, Sept. 18, 1930, p. 775.

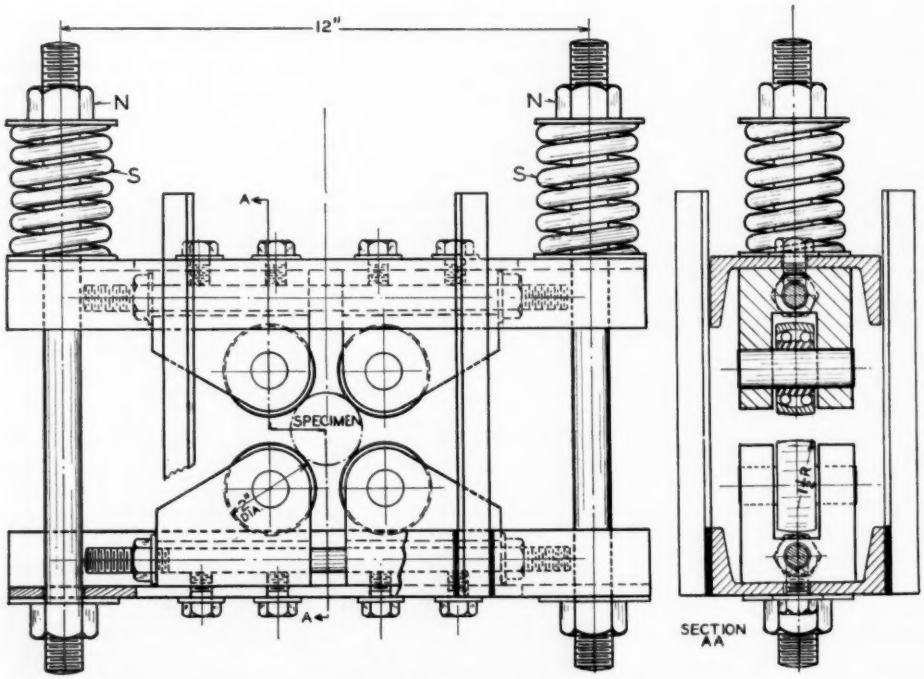


FIG. 3 APPARATUS FOR ROLLING FATIGUE SPECIMENS

axial direction along the specimen; for high rolling pressures, a power feed for traversing is desirable. The rollers are made from ball bearings, the outer surface being ground to a $1^1/_2$-in. contour radius[12] to obviate a "digging in" action which previously

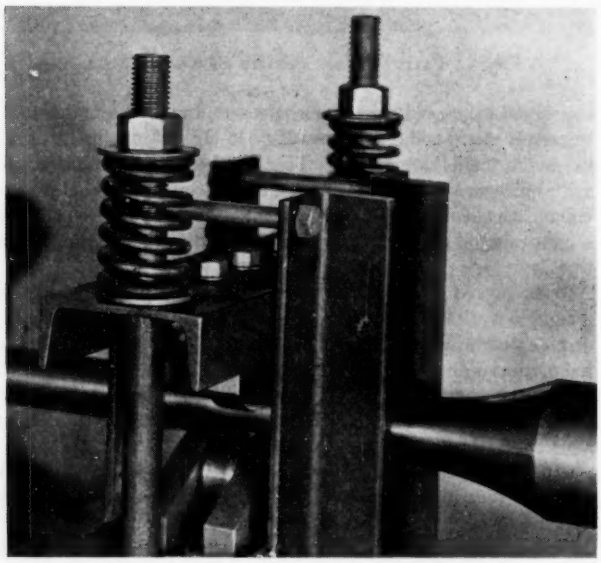


FIG. 4 ROLLING APPARATUS IN POSITION

tended to occur during axial traverse. The desired roller pressure is readily obtained by turning nuts N, Fig. 3, until the calibrated springs S are the proper length.

The roller load used in preparing series 7 was 400 lb. Two additional specimens were rolled with 800-lb load. If one calculates stresses according to the elasticity equations of Thomas and Hoersch[13] then, for example, for a load of 400 lb a maximum stress of 320,000 lb per sq in. is obtained. It is of course obvious that no such stress is reached, but that plastic deformation occurs, so that increasing the load results mainly in increasing the depth of the plastic region. Measurements of diameter before and after rolling showed that for a load of 400 lb the decrease was approximately 0.0008–0.0010 in.

With regard to the reason for the increase of fatigue strength due to rolling, considerable difference of opinion exists.[14] Prof. O. Föppl, of Wöhler Institut, Braunschweig, attributes the increase of fatigue strength to a fundamental change in the material in the outer layer of the specimen due to rolling, resulting in a denser material having a closer and differently oriented molecular structure, a changed microstructure, and a somewhat higher specific weight. Prof. A. Thum, of Darmstadt, believes that the increase is due to favorable residual compressive stresses at the surface as a result of rolling. It is not inconceivable that both effects are present. Further research with an attempt to isolate the above variables would be interesting and illuminating.

DISCUSSION OF FATIGUE RESULTS

The results of the Westinghouse fatigue tests are shown in Fig. 5 and summarized in Table 5. It will be noticed that all results are compared with the ordinary endurance limit of 0.3-inch-diameter beam specimens of the shaft material, 31,700 lb per sq in. Previous size-effect tests without stress-concentration effect of steel purchased under the same specification showed that the effect, if any, was not large.[6]

In Table 5 as well as Tables 2 and 3, fatigue-stress-concentration factors K, as ordinarily defined and as used in design formulas, are given.

$$K = \frac{\text{Ordinary endurance limit of specimen with large contour radius}}{\text{Endurance limit with stress concentration at critical section}}$$

In the press-fit tests where not only stress concentration is involved, but also a rubbing-corrosion effect, the term "fatigue-stress-concentration factor" is not an accurate one, but since we could not find a better term it was decided that for this paper we would adhere to "fatigue-stress-concentration factor," defined as follows in the press-fit case:

$$K = \frac{\text{Ordinary endurance limit of specimen with large contour radius}}{\text{Endurance limit of press-fit specimen } \left(S = \frac{M}{\pi d^3/32}\right)}$$

[12] One test was made of a specimen rolled with flat rollers, Fig. 5; all others were rolled with $1^1/_2$-in. contour radius roller.

[13] "Stresses Due to the Pressure of One Elastic Solid Upon Another," by H. R. Thomas and V. A. Hoersch, Univ. of Illinois Exp. Sta. Bulletin No. 212.

[14] See for example the published letters of O. Föppl and A. Thum under the heading "Steigerung der Dauerhaltbarkeit durch Oberflächendrucken" in *Zeit. V.D.I.*, vol. 77, 1933, p. 1335.

Comparing series 2 of Table 5 with the German results given in Tables 2 and 3, it will be noticed that the decrease in endurance strength is of the same order for all cases, roughly about half. One might expect that the difference between 50 per cent, Westinghouse tests, and 55 per cent, Darmstadt tests, might be greater because of size effect,[6] the Westinghouse specimens being $1^5/_8$ in. in diameter and the Darmstadt specimens 0.55 in. in diameter. This is probably due to the fact that the 0.42 per cent C steel used in the Westinghouse tests is less sensitive to stress-concentration effects in fatigue than the 0.57 per cent C steel used in the Darmstadt tests.

Comparing series 2 and 3, it is apparent that radial pressure is an important variable. As was pointed out in discussing the original test plans, the heavy press fit involves: stress concentration, a three-dimensional stress system at the point of failure, and rubbing corrosion.[5] The very light press fit involves *mainly rubbing corrosion*, with possibly some stress increase at the corners due to bending. It would therefore appear that the decrease in endurance strength of an ordinary heavy press fit is due to a localized stress system acting on a material weakened by rubbing corrosion.

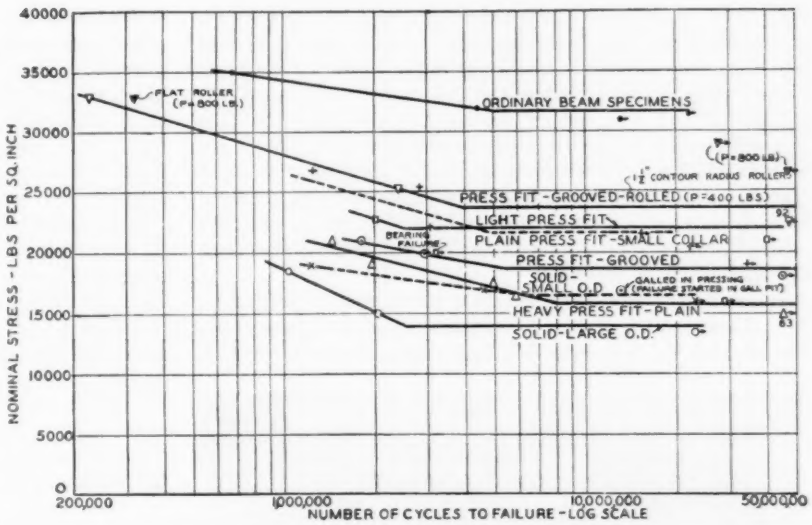


FIG. 5 FATIGUE-TEST DATA OF PRESS-FIT INVESTIGATION

In connection with rubbing corrosion, a fatigue test was made with the press-fitted collar and adjacent shaft portions completely packed in heavy grease, the idea being to exclude air in an attempt to prevent rubbing corrosion. It was found that the life was not effectively prolonged and that rubbing corrosion had occurred. It would be interesting to run small press-fit fatigue specimens in a neutral atmosphere to see how rubbing corrosion is modified. In this connection the tests of wear of metals in various atmospheres being carried out at the National Bureau of Standards will be of interest.

In Fig. 6 a plot of all available radial-pressure data as affecting endurance strength is shown. In the case of the Darmstadt tests, it appears that over a wide range the endurance strength is independent of pressure. This is probably because a condition of yielding has been reached at the surface of the collar, in which case the maximum stress peak no longer increases with increase of allowance. In Fig. 6 dotted lines have been drawn toward zero pressure to connect up with the ordinary endurance limits. With the limited data available the form of these curves is not definitely known, but it seems reasonable to assume that their general shape would be such as indicated.

Returning to Table 5, one might expect that series 1 would show an even lower endurance limit than series 2, because of size effect. The difference may be due to two reasons. The pressure in the case of series 1, although not determined, was probably less than 5000 lb per sq in. (estimated from Equation [2]) because the collar is a relatively thin one for the customary maximum allowance of 0.001 inch per inch of diameter. The length of fit in series 1 is much shorter in proportion to the shaft diameter than in series 2, thus decreasing considerably the stress concentration due to bending.

The solid specimens of the same contour as the press-fit specimens were included in the original program in order to throw some light on the relative stress-concentration effects. The endurance strength of the solid comparison specimens, series 4, Table 5, was found to be 14,000 lb per sq in., somewhat lower than the corresponding value of the heavy press fit, 15,800 lb per sq in. Some comments will be made on the relation of these

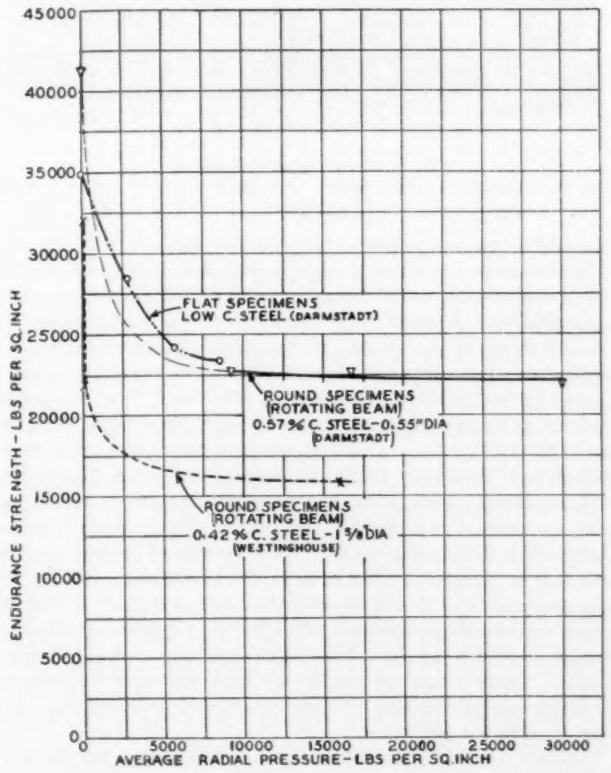


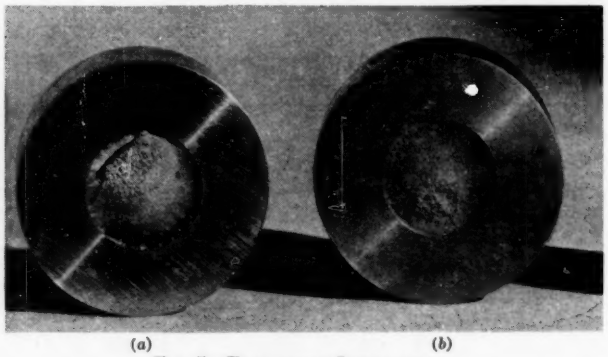
FIG. 6 EFFECT OF PRESS-FIT PRESSURE ON ENDURANCE STRENGTH

values in connection with the photoelastic analysis described later in the paper. Due to a mistake a series of "solid" specimens was made with a small outside diameter, $1^{15}/_{16}$ in., series 5, but the results are interesting so the data are included in Fig. 5 and Table 5. The endurance strength was found to be 16,500 lb per sq in. as compared with 14,000 for the larger outside diameter specimens. The difference is apparently due only to difference in stress concentration, and should be readily shown photoelastically.

With regard to the grooved construction, the Darmstadt tests, Table 3, series 5, showed a 31 per cent increase over the

plain press fit, which is the usual construction of this type encountered in practice; while our tests, Table 5, series 6, showed an 18 per cent increase. The comparative figures do not seem unreasonable if one considers the difference in materials and specimens, and furthermore demonstrate definitely that the grooved construction is worth considering for certain design applications. A study of the form and size of the groove lip might result in some further increase in strength although practical limitations must be considered in this connection also; for example, one cannot make the lip so thin that special precautions in handling will be necessary.

By combining grooving and rolling the German investigators were able to obtain full shaft strength, failure occurring away from the press fit, Table 3, series 6. Their rolling pressures were not



(a) (b)

FIG. 7 FRACTURED SPECIMENS

[(a) Plain press fit (heavy pressure); (b) "Solid" specimen of same contour as (a).]

stated,[4] so an independent study of this variable was started. This would be advisable in any case since the larger specimens to be comparable, would require heavier rolling loads to produce greater depth of plastic flow. The first rolling load selected, P = 400 lb, resulted in a 50 per cent improvement of endurance strength as compared with the plain press fit, Table 5, series 7, compared with series 2. A specimen rolled with P = 800 lb did not fail at a stress of 26,500 lb per sq in., the test being stopped at 41,000,000 cycles. A second specimen also rolled with P = 800 lb but stressed at 29,000 lb per sq in. had run 29,000,000 cycles without failure at the time this paper was sent in, indicating that an increase of at least 84 per cent can be obtained as compared with the plain press fit. Another way of looking at this result is to compare 29,000 with 31,800 (the endurance limit of the material) and it will be seen that rolling with P = 800 lb raises the endurance strength of a shaft with a grooved collar to within at least 9 per cent of the endurance limit of the shaft material. There seems no doubt but that full shaft endurance strength can be obtained by a combination of grooving and rolling of sufficient intensity.

Strictly speaking, the stress-concentration factor for series 7 should have been obtained by comparison with the endurance strength of a rolled-beam specimen, without collar. Such tests were not made, however, but it would be interesting to know if the stress-concentration factor agrees with that of series 6. The same comment applies to series 4 and 6 of Table 3.

The typical appearance of the fractured surfaces of a plain press-fit specimen, series 2, and a "solid" specimen, series 4, is shown in Fig. 7. The "solid" specimens had totally enclosed well-defined rupture areas displaced from the axis, as is customary in specimens of this kind.[16] The press-fit specimens, however, showed irregular rupture areas, some of which were totally enclosed, some segmental in form, but as a whole not following any clear-cut order. This is to be expected since due to small surface irregularities the pressure actually is not perfectly uniform and consequently the edge stresses must vary somewhat around the periphery. "Beach marks"[15] were quite pronounced on the fracture surfaces of the press-fit specimens, showing that cracks originate at a number of places on the periphery.

Failure generally started on the shaft surface inside the press fit, varying from 0.02 in. to 0.25 in. from the edge of the collar, except in the case of two highly stressed rolled specimens, breaking at 224,000 and 313,000 cycles, where failure started right at the edge. The explanation may be as follows. In press-fit tests, rubbing corrosion causes considerable reddish brown oxide powder to accumulate at the edge of the collar fit after the test has been run for some time. This may be considered equivalent to the wearing away of some of the material for a distance in from the edge, with a consequent shift of peak stress. In the case of the highly stressed specimens, the total life was only 3 to 4½ hr

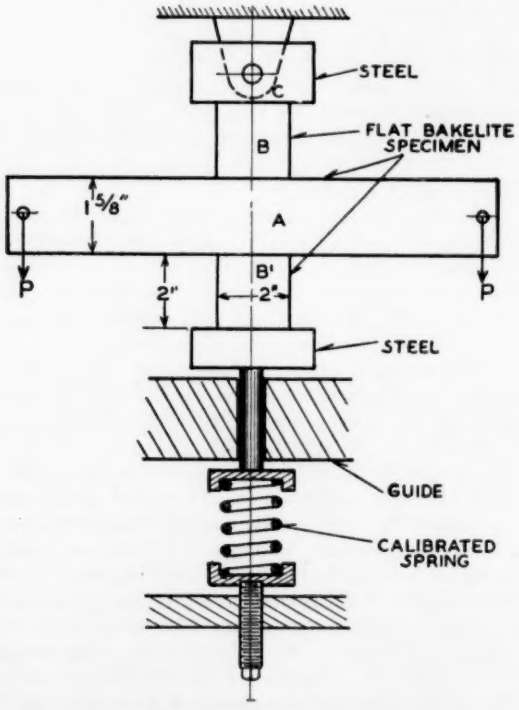


FIG. 8 PRESS-FIT TEST ARRANGEMENT

so that the opportunity for corrosion and wear was not the same as in the other specimens which lasted over 1,000,000 cycles.

PHOTOELASTIC TESTS AND LATERAL-STRAIN MEASUREMENTS

In order to simulate the stress conditions occurring in press fits, some tests were made using an arrangement shown schematically in Fig. 8. Essentially it consists of a flat bar of bakelite A representing the shaft, which is squeezed between two flat bakelite pieces B, B', representing the hub or collar to simulate the press-fit pressures produced by the fit allowance. Loads on these blocks are produced by the calibrated spring as indicated, while a pin C attached to the frame allows the two blocks to adjust themselves to compensate for slight inaccuracies in machining. Bending moments are produced by the loads shown to simulate the conditions of a shaft subjected to the combined action of a press-fit pressure produced by a collar and an alternating

[15] The radial displacement depends on the stress. See "Stress Concentration Phenomena in Fatigue of Metals," R. E. Peterson, Trans. A.S.M.E., 1933, vol. 55, paper APM-55-19, p. 157.

bending stress. Fig. 9 is a view of the complete apparatus in the testing machine.

To determine the principal stress differences, photoelastic tests were made by means of this apparatus in conjunction with the well-known fringe method[16] (monochromatic light being used). The photoelastic specimens were made of C-25 bakelite plates approximately 0.34 in. thick. In machining these specimens, light cuts were taken in a milling machine and the final 0.001 in. or so was ground off using a surface grinder. This was done in order to eliminate, as far as possible, irregularities along the contact surfaces, since tests using the lateral extensometer described later showed that very slight irregularities, of the order of 0.0005 in. in 2-in. length, had a marked effect on the stress distribution.

Because of the relatively large size of the photoelastic model it was found advisable to take four fringe photographs and fit them together as indicated, for example, in Fig. 11. The following different cases were tested:

Case 1. Plain Press Fit. In this case, shown in Fig. 10*a* a nominal press-fit pressure of 1480 lb per sq in. and a nominal bending stress of 2020 lb per sq in. were used. The stress conditions in this test correspond approximately to the fatigue tests wherein the nominal bending stress at the endurance limit was about one-third greater than the nominal compressive stress due to the press-fit pressure. Fig. 11 comprises the combined fringe photographs for this case.

Case 2. Plain Press Fit With Edge Beveled Off, the Nominal Stresses Being as in Case 1. In this case, shown in Fig. 10*c*, the bevel was made by taking a very light cut, as shown, to simulate the wear which occurs at the edge due to rubbing corrosion under continuous fatigue stressing. Fig. 12 is a photograph for this case.

Case 3. Press Fit With Grooved Construction, the Nominal Stresses as in Case 1. This piece, Fig. 10*b*, was tested to determine the effect of the groove in reducing the stress concentration; Fig. 13 is a fringe photograph representing this case.

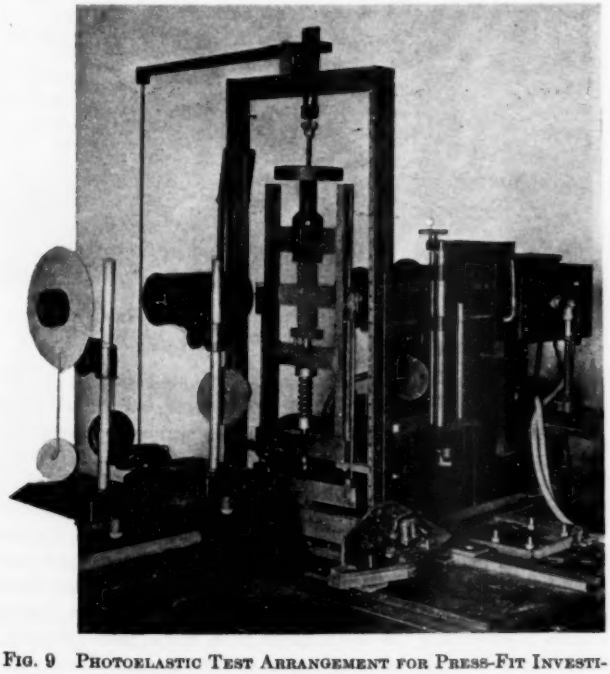


FIG. 9 PHOTOELASTIC TEST ARRANGEMENT FOR PRESS-FIT INVESTIGATION

[16] Z. Tuzi, Sci. Papers, Inst. Phys. and Chem. Research, Tokyo, vol. 8, p. 247, and vol. 12, p. 21. M. M. Frocht, Trans. A.S.M.E., 1931, vol. 53, paper APM-53-11.

To determine the sum of the principal stresses, a recently developed lateral extensometer[17] was used. Referring to Fig. 14, the extensometer consists essentially of a frame *D* and a movable bar *B*, which is constrained to move in a direction parallel with its length by the spring steel strips *C*, *C'*. These springs exert pressure, causing the points *P* and *P'* to bear firmly against the specimen, the friction produced by this pressure being sufficient

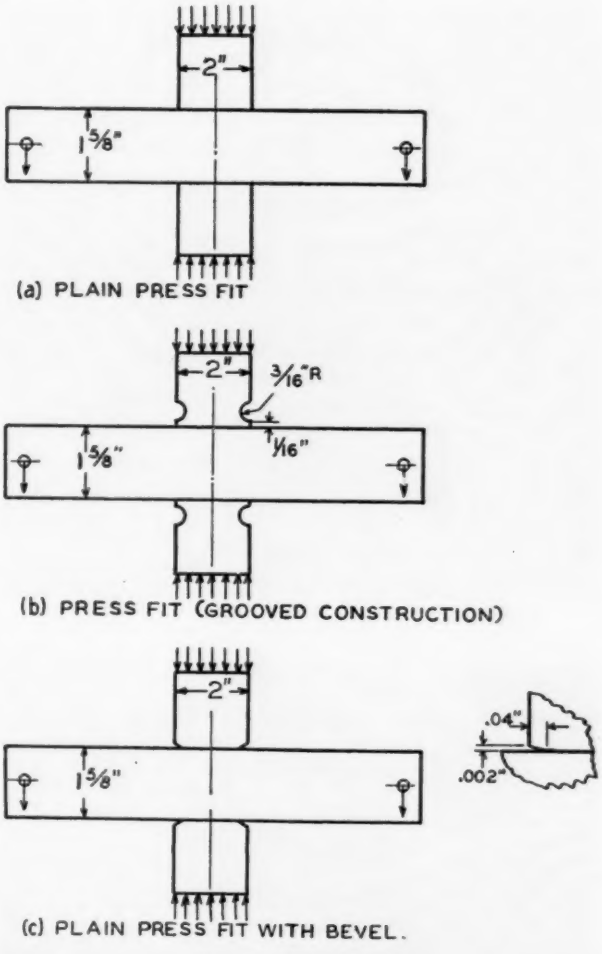


FIG. 10 CASES TESTED

to hold the points in one position during the test. Any lateral extension or contraction of the specimen will cause a relative movement of points *P* and *P'*, and hence of the bar *B*, with respect to the frame *D*. This movement is indicated by a Huggenberger extensometer which is clamped as shown. The screw *A* is used mainly for convenience in holding the points in one position while shifting the instrument from one location to another on the test specimen. Fig. 15 is a photograph of the extensometer while Fig. 16 is another view of the instrument in position on the test specimen.

By means of the balancing weight *E* shown in Figs. 14 to 16, the center of gravity of the instrument may be adjusted to coincide with the line of action of the resultant force produced by the springs, Fig. 16. Since the points *P* and *P'* thus carry practically no weight the friction is sufficient to hold them firmly in one position during the loading of the specimen. This is of advantage since if even a small amount of slippage of the points should

[17] A complete discussion of this extensometer will be published later.

occur, the resulting reading would probably be considerably in error due to slight variations in the thickness. If, for example, due to this cause, a change of only 0.00005 inch in the distance between points P and P' should occur, a considerable error in the extensometer readings would result. It is, of course, necessary to load the specimen once for each point at which a lateral-strain measurement is made.



FIG. 11 FRINGE PHOTOGRAPH OF PLAIN PRESS FIT

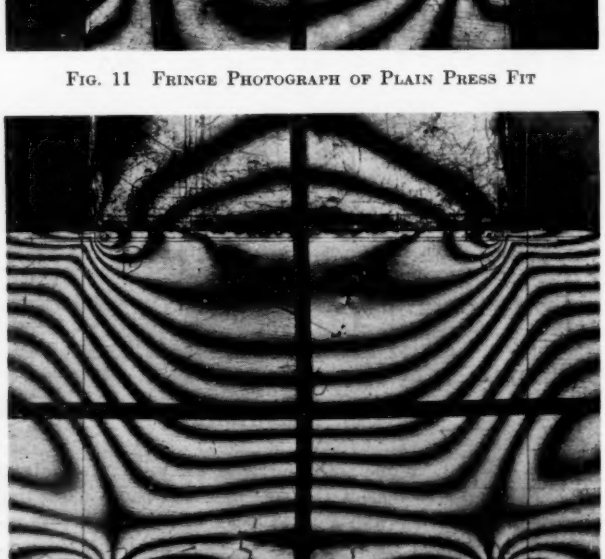


FIG. 12 FRINGE PHOTOGRAPH OF PLAIN PRESS FIT WITH BEVEL

To calibrate the instrument, it was clamped to a bar of bakelite subjected to a known bending moment, from which the stress could be computed. As a check it was also clamped to a bar in tension. Both of these methods gave practically identical results. For bakelite specimens of about 0.34 in. thickness the estimated possible error in measuring the lateral strains corresponded to an error of less than 100 lb per sq in. in the sum of the principal stresses $\sigma_1 + \sigma_2$, which is small compared with the peak value of approximately 5000 lb per sq in. registered at certain points.

Tests were made on specimens corresponding with cases 1 and 3, the principal stress sum being measured along the lines of contact between the hub and the shaft, and also somewhat outside these, along the edges. Because of the small size of the extensometer points, it was found possible to take measurements within 0.03 in. of the edge, and these were considered to be at the edge, although this involves some error. The measurements along the edge outside the hub were corrected to take this into account (the correction amounting to about 4 per cent). The taking of several measurements near the edge and then extrapolating to the edge was tried in a few cases, but it was found that the difference amounted to but a few per cent in these cases, and it was felt that this small difference did not warrant the extra time and labor involved in making a complete set of such measurements.

STRESS ANALYSIS AND COMPARISON WITH FATIGUE DATA

The distribution of the principal stress sums $\sigma_1 + \sigma_2$ as found by the lateral extensometer along the edge of the bar A, Fig. 8, for cases 1 and 3 together with the corresponding stress differences $\sigma_1 - \sigma_2$ as found by photoelastic tests are shown in Fig. 17 for the plain press fit, and in Fig. 18 for the press fit with the grooved construction. All points on the curves represent the averages between values found on the two opposite sides of the specimen to eliminate the effect of slight eccentricities of loading of the compression blocks B and B', Fig. 8. In determining the peak stresses on the compression side for cases 1 and 2 the extrapolation method described in a previous publication[18] was used because of the rapid change in stress difference near these points. From the $(\sigma_1 + \sigma_2)$ and $(\sigma_1 - \sigma_2)$ curves, the dotted curves representing the distribution of σ_1 and σ_2 were obtained. The σ_1 curves near the corner were drawn to connect with the triangular points representing the photoelastic-test results since the latter are considered more accurate at these locations than are the points representing $\sigma_1 + \sigma_2$ obtained by means of the

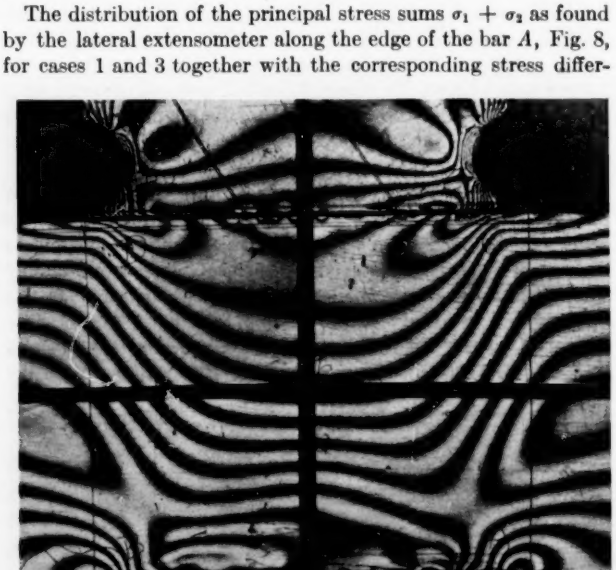


FIG. 13 FRINGE PHOTOGRAPH OF PRESS FIT, GROOVED CONSTRUCTION

[18] "Stress Concentration Produced by Holes and Notches," by A. M. Wahl and R. Beeuwkes, Trans. A.S.M.E., 1934, vol. 56, paper APM-56-11.

lateral extensometer. This is because the fairly rapid change in the values of $\sigma_1 + \sigma_2$ at such points is not completely followed by the change in lateral strain since the finite thickness of the specimen tends to hinder lateral contraction or expansion for a short distance from the corner. The directions of the stresses σ_1 and σ_2 near the corner are indicated approximately in Figs. 17 and 18. It may be seen that at a short distance from the corner the $(\sigma_1 + \sigma_2)$ curves approach the $(\sigma_1 - \sigma_2)$ curves within the limits of experimental error, as they should, since $\sigma_2 = 0$ at these points. This is a further check on the accuracy of the lateral-strain measurements. The complete distribution of the stresses σ_1 and σ_2 is shown to a smaller scale for cases 1 and 3 in Figs. 19 and 20. On the tension side these curves may be considered to give a rough indication of the variation of axial stresses and normal

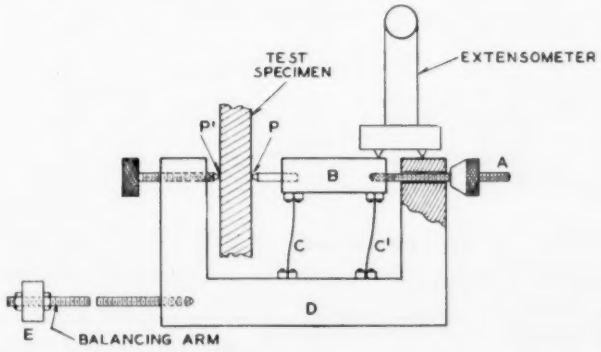


FIG. 14 DRAWING SHOWING CONSTRUCTION OF LATERAL EXTENSOMETER

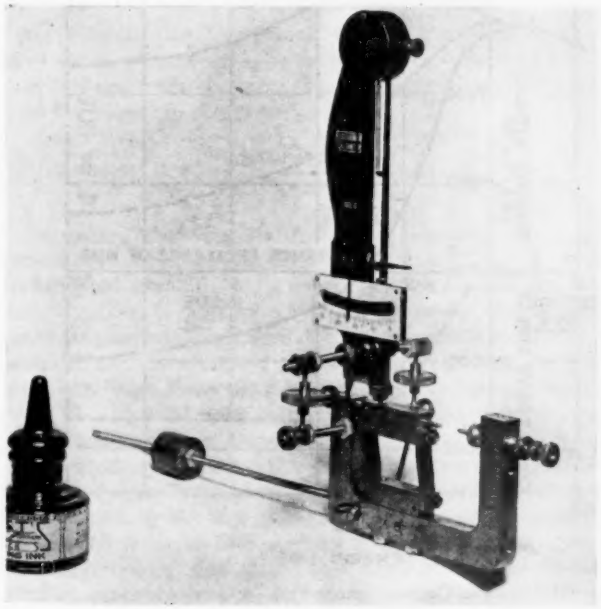


FIG. 15 LATERAL EXTENSOMETER

pressures, respectively, along the contact surfaces; on the compression side, because of the rotation of the principal stress directions, this correspondence does not exist except near the corners.

The tests made on the specimen of case 2, which represents a plain press fit with wear, showed that the stress distribution and also the values of peak stresses were not greatly different from the corresponding results obtained for the tests on the plain press fit without wear, case 1. Consequently the photoelastic results can be considered to represent approximately the fatigue specimens even after some rubbing corrosion has taken place.

In determining the reduction in strength due to the stress concentration produced by the press fit, the *maximum-shear theory* of strength will be taken as a basis,[19] i.e., the highest range in shearing stress among several planes during a complete cycle of bending will be compared with the nominal range of shearing

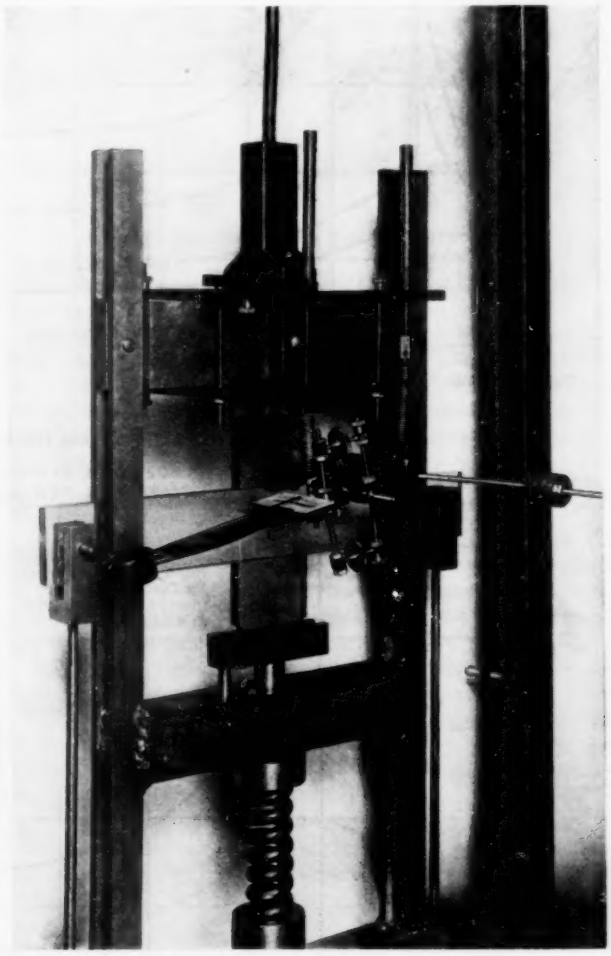


FIG. 16 LATERAL EXTENSOMETER IN POSITION ON TEST SPECIMEN

stress as computed from the known moment acting on the test specimen. The reason for considering several planes is that there is, in general, a rotation of the plane of maximum shearing stress during bending. An examination of the isoclinic lines in the photoelastic apparatus, however, indicated that for points near the edge of the collar where the fatigue failures occur, it would be sufficiently accurate to assume the directions of maximum shear for the worst conditions at 45 deg to either the radial and axial, or to the axial and circumferential directions. (This was checked up by calculating the shear-stress range on several planes, taking into account the rotation of the principal stress

[19] Dr. H. J. Gough has found from his work on specimens of single crystals and aggregates of a few crystals that fatigue fractures are initiated along the plane of maximum resolved shearing stress, but that failure may proceed across the specimen in another direction, depending on the stress distribution. See his Edgar Marburg Lecture, "Crystalline Structure in Relation to Failure of Metals, Especially by Fatigue," Proc. A.S.T.M., 1933, part 2, p. 3.

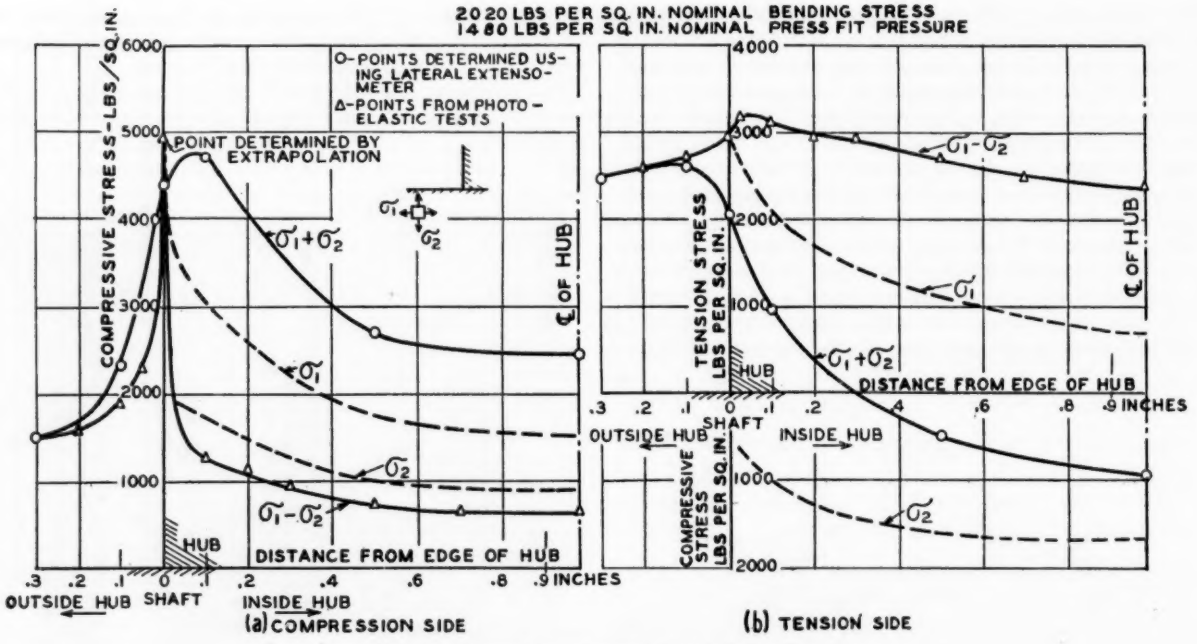


FIG. 17 STRESS DISTRIBUTION, PLAIN PRESS FIT

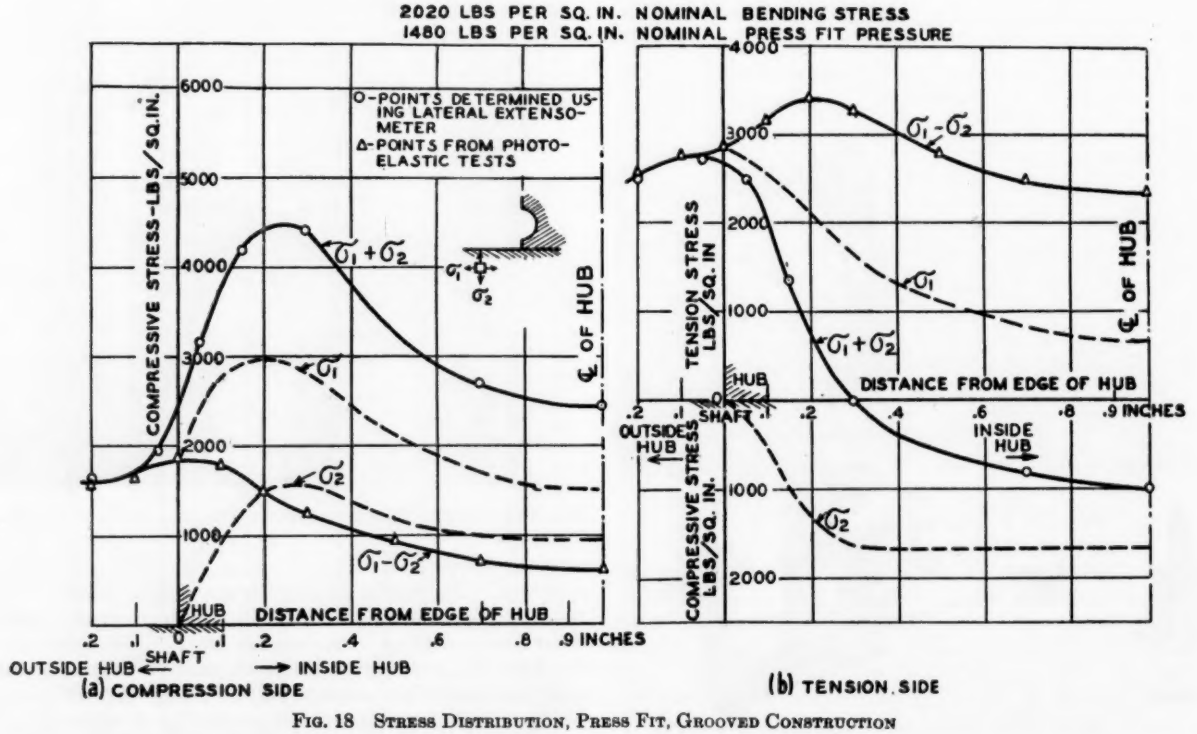


FIG. 18 STRESS DISTRIBUTION, PRESS FIT, GROOVED CONSTRUCTION

directions as shown by the isoclinic lines.) Hence, considering an actual shaft and assuming σ_3 as the stress in the circumferential direction, the principal stresses σ_1, σ_2, and σ_3 near the edge of the collar may be taken in the axial, radial, and circumferential directions, respectively.[20] In addition the further assumption will be made that the circumferential stress σ_3 does not change appreciably during bending, this assumption being supported by some rough estimates of the deflection produced by the change in normal pressure, during bending, on the shaft surface.[21] Hence the ranges in shearing stress to be considered are those in $\sigma_1 - \sigma_3$ and in σ_1 only.

[20] The von Mises-Hencky distortion-energy theory was also applied to this case but the final results did not differ greatly from those obtained using the maximum-shear theory. Since the directions of the principal stresses do not enter into the calculations in the former case this indicates that the above assumptions regarding the planes of maximum shear are not greatly in error.

[21] The other extreme assumption, i.e., that $\sigma_3 = \sigma_2$ during bending, will give a slightly lower stress-concentration factor in the case of the notched collar, the difference being only around 8 per cent. It is believed that the assumption σ_3 = constant is nearer the truth for an actual shaft.

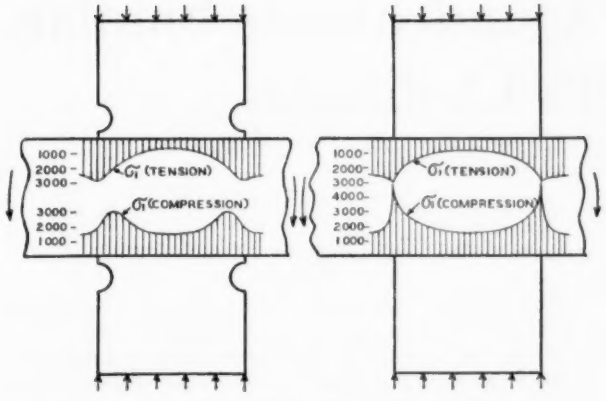


FIG. 19 DISTRIBUTION OF PRINCIPAL STRESSES, σ_1

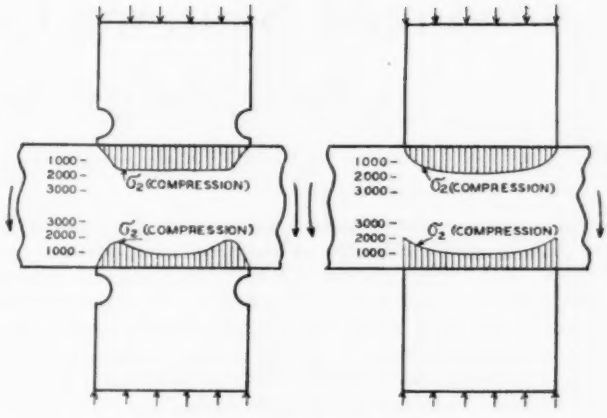


FIG. 20 DISTRIBUTION OF PRINCIPAL STRESSES, σ_2

We thus define the photoelastic stress-concentration factor as

$$K_p = \frac{(\sigma_1 - \sigma_2)_{max} - (\sigma_1 - \sigma_2)_{min}}{2\sigma} \quad \text{or} \quad \frac{(\sigma_1)_{max} - (\sigma_1)_{min}}{2\sigma}$$

depending on which is larger. In these expressions, σ is the nominal stress due to bending. Thus for the plain press fit it may be shown, as in Fig. 17, that the point giving the highest stress-concentration factor is right at the edge of the collar. For this case, where $\sigma = 2020$ lb per sq in.

$$K_p = \frac{4940 + 2940}{4040} = 1.95$$

Likewise for the case of the press fit with the grooved construction the worst case was found to be the range in $\sigma_1 - \sigma_3$ which is that in σ_1 (σ_3 being taken constant) at a point approximately 0.1 inch from the edge. In this case

$$K_p = \frac{2600 + 2800}{4040} = 1.34$$

Comparing these figures with the fatigue-stress-concentration factors obtained in the Darmstadt tests, Table 3, i.e., $K = 1.8$ for the plain press fit and 1.4 for the grooved construction, it is seen that there is fair agreement. In the case of the Westinghouse fatigue tests, the values of stress-concentration factors K as found, i.e., $K = 2.0$ and 1.7 for the plain and grooved constructions, respectively, show agreement for the case of the plain press fit while the fatigue-test value for the notched construction is somewhat higher than the photoelastic test value.

In general, it can be said that the fatigue-stress-concentration factors are closer to the photoelastic stress-concentration factors than is ordinarily the case. (Usually the former are considerably less than the latter.) This may be due to the fact that in a press-fit fatigue test, the shaft material at the point of failure is weakened by rubbing corrosion, and consequently the fatigue-stress-concentration factor, which would otherwise be lower, is increased.

The reason for the superiority of the grooved construction in the fatigue tests, is clearly shown by Figs. 19 and 20 to be due to the elimination of stress peaks at the corners.

It will be recalled that fatigue tests on a specimen cut out of a solid bar and having the same contour as the press-fit specimens showed a higher fatigue-stress-concentration factor. A photoelastic test was made to check this and it was found that the solid piece with a sharp corner had higher stress concentration than did the corresponding press-fit specimen.

CONCLUSIONS

(1) An ordinary press-fit member was found to decrease the endurance strength of a shaft to roughly half the endurance limit of the shaft material (as determined from specimens without stress concentration). This has also been independently found at two German laboratories.

(2) For very light press-fit pressures the endurance strength of a shaft is somewhat higher than for heavy pressures.

(3) German tests indicate that beyond a certain pressure, the endurance strength is independent of pressure. It appears that rubbing corrosion is an important factor in reducing endurance strength.

(4) Two expedients for increasing the endurance strength of shafts with press-fitted members, i.e., using a grooved construction, and rolling the shaft surface, were found to be very effective.

(5) Photoelastic tests combined with a stress analysis showed interesting results of fundamental value. The reason for the effectiveness of the grooved construction was shown by photoelastic tests to be due to reduction of stress peaks at the corners.

ACKNOWLEDGMENTS

The authors appreciate the interest of Prof. S. Timoshenko of the University of Michigan and Dr. L. W. Chubb, director of Westinghouse Research Laboratories. Acknowledgment is also made of the assistance of F. Cassel in the fatigue tests and R. Beeuwkes in some of the preliminary photoelastic tests. The authors also wish to thank Dr. S. Way for information on the theoretical aspects of the rolling process, and for his reading of the manuscript. Some discussions have been had with O. G. Horger, of Ann Arbor, and his comments have been of value.

Ordinate Systems for Quadratures Similar to Tchebycheff's Ordinates

By GEORGE B. KARELITZ,[1] NEW YORK, N. Y.

The author presents a geometrical interpretation of Tchebycheff's rule for calculating quadratures and shows that other systems of ordinates can be chosen which will be as valid as Tchebycheff's. A system of ordinates is proposed, the virtue of which lies in the fact that the longitudinal location of the center of gravity of the ship's lines can be obtained with good accuracy without increasing the errors of area calculations.

NAVAL architects continually use rules of approximate quadrature for computing the displacement and stability of ships. The determination of the various elements of hull geometry resolves itself into finding values of curvilinear areas such as shown in Fig. 1*a*. The principal rules for calculating these ordinates are (*a*) the trapezoidal, (*b*) Simpson's rule, and (*c*) Tchebycheff's rule.

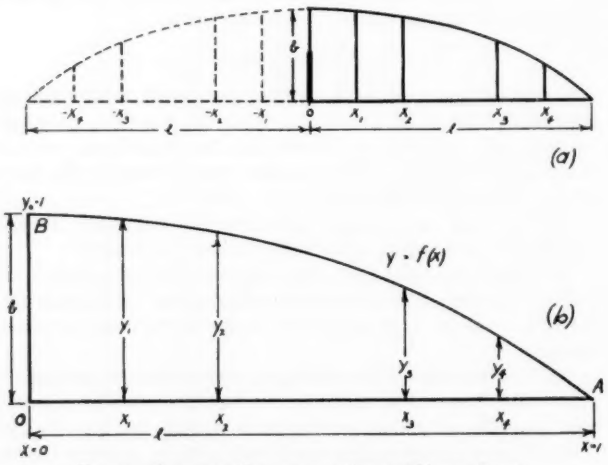


FIG. 1 GENERAL SHAPE OF A SHIP'S WATER LINE

The trapezoidal rule requires the base $2l$ to be divided into n equal parts after which the perpendicular ordinates, $y_0, y_1 \ldots y_{n+1}$, are scaled off at the division points and the segments of the curvilinear boundary are replaced by straight lines. The area then becomes

$$S = \frac{2l}{n}\left(\tfrac{1}{2}y_0 + y_1 + y_2 \ldots + y_n + \tfrac{1}{2}y_{n+1}\right)$$

Simpson's "first" rule requires the base $2l$ to be divided into an *even* number of n equal parts, after which perpendicular ordinates are erected and scaled off at the division points, and each pair of adjacent segments is replaced by a parabolic segment through the three points on the curve. The area can then be expressed as:

$$S = \frac{2l}{3n}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-1} + 4y_n + y_{n+1}\right)$$

The trapezoidal rule is accurate to $^1/_3$ per cent with 21 ordinates (l divided into ten parts), and to 0.2 per cent with 41 ordinates (l divided into 20 parts). Simpson's rule is accurate to within 0.1 per cent with 21 ordinates, but the rule requires the application of multipliers 2 and 4. Therefore, much arithmetical work is involved for completely computing the displacement and stability of a hull.

Tchebycheff's rule employs fewer ordinates while giving the same high accuracy and because of this advantage is gaining favor with naval architects,[2] especially for stability calculations where its usefulness is particularly striking. The ordinates are not, however, equidistant, but are taken as listed in Table 1. A simple geometrical derivation of these coefficients, comparable to the geometrical demonstration of the trapezoidal rule and Simpson's rule, has not been given to date.

TABLE 1 TCHEBYCHEFF'S ORDINATES

Number of ordinate	Distance from middle of base in fractions of half-length				
$n = 2$		0.57735			
$n = 3$	0	0.70711			
$n = 4$		0.18759	0.79465		
$n = 5$	0	0.37455	0.83250		
$n = 6$		0.26664	0.42252	0.86625	
$n = 7$	0	0.32391	0.52966	0.88386	
$n = 9$	0	0.16791	0.52876	0.60102	0.91159

The area is $S = \frac{2l}{n}(y_1 + y_2 + \ldots + y_n)$

TCHEBYCHEFF'S ORDINATES

Tchebycheff's derivation[3] can be interpreted, as shown by Kryloff,[4] in the following manner.

Find $C_n, x_1, x_2, x_3 \ldots x_n$ such that

$$\int_{-1}^{+1} f(x)\,dx = C_n\left[f(x_1) + f(x_2) + \ldots + f(x_n)\right] \ldots [1]$$

will be exact for any function $f(x)$, where

$$f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} + a_nx^n$$

[1] Associate Professor, Department of Mechanical Engineering, Columbia University. Mem. A.S.M.E. Mr. Karelitz was born in Russia in 1895 and received degrees in naval architecture and mechanical engineering from the Imperial Polytechnic Institute at Petrograd in 1918. For three years he was special engineer with the Waterways of Communication in Russia. He came to the United States in 1922 and has since been connected with the Westinghouse Electric & Manufacturing Company in the capacities of research engineer, manager of the mechanics division of the Pittsburgh Research Laboratories, manager of the transportation division of the South Philadelphia Works engineering department, and consulting engineer. He began his work at Columbia in 1930.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until May 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "The Naval Constructor," by George Simpson, D. Van Nostrand Co., New York, 1914.

[3] "Sur les Quadratures," by P. L. Tchebycheff, *Journal de Mathematiques*, Liouville, vol. 19, 1874.

[4] "Approximate Calculations," by A. N. Kryloff, Institute of Ways and Communications, Petrograd, 1911.

From Equation [1]

$$\int_{-1}^{+1} f(x)\,dx = \int_{-1}^{+1} (a_0 + a_1 x + \ldots + a_{n-1} x^{n-1} + a_n x^n)\,dx$$

$$= 2\left(a_0 + \frac{a_2}{3} + \frac{a_4}{5} + \frac{a_6}{7} + \ldots\right)$$

$$= C_n[f(x_1) + f(x_2) + \ldots + f(x_n)] \ldots\ldots\ldots [2]$$

$$= C_n\left\{n a_0 + a_1 \sum_1^n x_i + a_2 \sum_1^n x_i^2 + \ldots + a_{n-1} \sum_1^n x_i^{n-1} + a_n \sum_1^n x_i^n\right\}$$

This holding for any value of $a_0, a_1 \ldots a_{n-1}, a_n$, $(n+1)$ equalities can be written:

$$\left.\begin{aligned} & nC_n = 2 \text{ or } C_n = \frac{2}{n} \\ & x_1 + x_2 + x_3 + \ldots + x_n = 0 \\ & x_1^2 + x_2^2 + x_3^2 + \ldots x_n^2 = \frac{n}{3} \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ & x_1^n + x_2^n + x_3^n + \ldots x_n^n = \frac{n}{n+1} \text{ or } 0 \end{aligned}\right\} \ldots\ldots [3]$$

It will be seen that the system contains n equations, which yield the values of $x_1, x_2, \ldots x_n$.

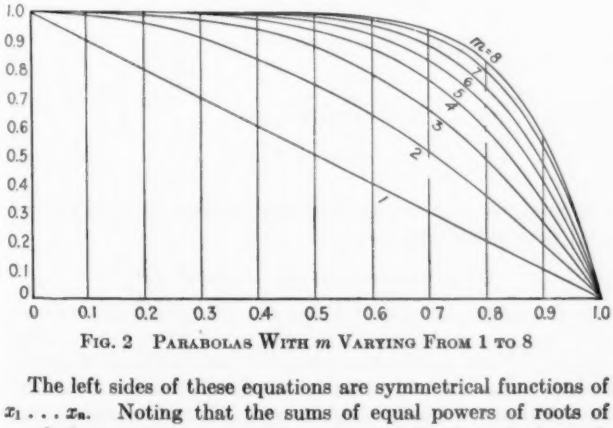


FIG. 2 PARABOLAS WITH m VARYING FROM 1 TO 8

The left sides of these equations are symmetrical functions of $x_1 \ldots x_n$. Noting that the sums of equal powers of roots of an algebraic equation are expressed rationally through the coefficients of the equation, we can write an equation

$$F(x) = x^n + A_1 x^{n-1} + A_2 x^{n-2} + \ldots + A_{n-1} x + A_n = 0 \ldots [4]$$

such that $x_1, \ldots, x_n$ will be its roots.

To find the coefficients $A_1 \ldots A_n$, we use the well-known Newton's formulas[5] for like powers of the roots of Equation [4].

Denoting $\sum_1^n x_i = s_1$; $\sum_1^n x_i^2 = s_2$; $\ldots$ $\sum_1^n x_i^{n-1} = s_{n-1}$

we can write:

$$A_1 + s_1 = 0$$
$$2A_2 + s_1 A_1 + s_2 = 0$$
$$3A_3 + s_1 A_2 + s_2 A_1 + s_3 = 0$$
$$(n-1)A_{n-1} + s_1 A_{n-2} + s_2 A_{n-3} + \ldots + s_{n-1} = 0$$

Substituting in these formulas the values of $s_1 \ldots s_{n-1}$ from Equations [3]

$$A_1 = 0$$
$$2A_2 + \frac{n}{3} = 0$$
$$A_3 = 0$$
$$4A_4 + \frac{n}{3} A_2 + \frac{n}{5} = 0$$
$$A_5 = 0$$
$$6A_6 + \frac{n}{3} A_4 + \frac{n}{5} A_2 + \frac{n}{7} = 0, \text{ etc.}$$

For each value of n we are able now to set up a function

$$F(x) = x^n + A_1 x_1^{n-1} + \ldots + A_{n-1} x + A_n = 0$$

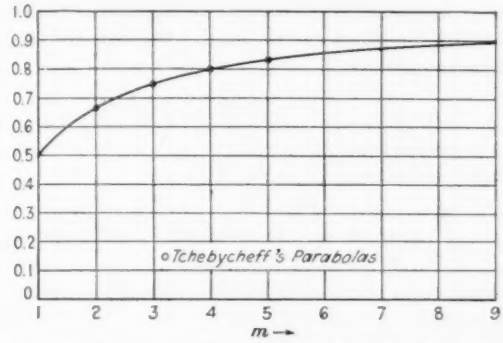


FIG. 3 MEAN ORDINATES OF PARABOLAS PLOTTED AGAINST VALUE OF m

For various ordinate numbers, this function is given in Table 2. The equation for $n = 8$ gives imaginary roots. The other equations yield the roots listed in Table 3, which determine the so-called Tchebycheff's ordinates.

TABLE 2 VALUES OF THE FUNCTION $F(x)$ FOR GIVEN ORDINATE NUMBERS

Ordinate numbers	Values of the function $F(x)$
$n = 2$	$F(x) = x^2 - 1/3 = 0$
$n = 3$	$x^3 - 1/2\,x = 0$
$n = 4$	$x^4 - 2/3\,x^2 + 1/45 = 0$
$n = 5$	$x^5 - 5/6\,x^3 + 7/12\,x = 0$
$n = 6$	$x^6 - x^4 + 1/5\,x^2 - 1/105 = 0$
$n = 7$	$x^7 - 7/6\,x^5 + 119/360\,x^3 - 149/6480\,x = 0$
$n = 8$	$x^8 - 4/3\,x^6 + 22/45\,x^4 - 148/2835\,x^2 - 43/42525 = 0$
$n = 9$	$x^9 - 3/2\,x^7 + 27/40\,x^5 - 57/560\,x^3 + 53/22400\,x = 0$

TABLE 3 VALUES OF TCHEBYCHEFF'S ORDINATES

Ordinate numbers	Values of abscissas
$n = 2$	$x_1 = -x_2 = 0.57735$
$n = 3$	$x_1 = -x_3 = 0.70711$; $x_2 = 0$
$n = 4$	$x_1 = -x_4 = 0.79465$; $x_2 = -x_3 = 0.18759$
$n = 5$	$x_1 = -x_5 = 0.83250$; $x_2 = -x_4 = 0.37454$; $x_3 = 0$
$n = 6$	$x_1 = -x_6 = 0.86625$; $x_2 = -x_5 = 0.42252$; $x_3 = -x_4 = 0.26664$
$n = 7$	$x_1 = -x_7 = 0.88386$; $x_2 = -x_6 = 0.52966$; $x_3 = -x_5 = 0.32391$; $x_4 = 0$
$n = 9$	$x_1 = -x_9 = 0.91159$; $x_2 = -x_8 = 0.60102$; $x_3 = -x_7 = 0.52876$; $x_4 = -x_6 = 0.16791$; $x_5 = 0$

STANDARD PARABOLAS

A ship's water line has the general appearance of that of Fig. 1*a*. Denoting the ratio of the abscissas to the half length l as x

[5] "Elementary Theory of Equations," by L. E. Dickson, John Wiley & Sons, 1914, p. 69, or other algebra manual.

$\left(\frac{\text{abscissa}}{l} = x\right)$ and the ratio of the ordinate to the maximum half width b as y $\left(\frac{\text{ordinate}}{b} = y\right)$, the area S of AOB, Fig. 1*b*, is equal to

$$S = bl \int_0^1 y\, dx$$

where $y = 1$ at $x = 0$; $y = 0$ at $x = 1$; and $\frac{dy}{dx} = 0$ at $x = 0$.

Tchebycheff's problem of finding the mean ordinate can now be stated as follows:

$$\int_0^1 y\, dx = \frac{1}{k}(y_1 + y_2 + y_3 + \ldots + y_k) \ldots\ldots [5]$$

or, if the ordinate at $y_0 = 1$ at $x = 0$, is to be taken into account

$$\int_0^1 y\, dx = \frac{1}{k + 1/2}\left(\frac{1}{2} + y_1 + y_2 + y_3 + \ldots + y_k\right) \ldots [6]$$

We will take k curves $y = f(x)$, which would have the general character as shown in Fig. 1*b* and find such values of $x_1, \ldots x_k$ that the conditions of Equation [5] and Equation [6] would be fulfilled exactly for the k curves taken. The simplest functions of this type are parabolas

$$y = 1 - x^m \ldots\ldots [7]$$

where m is greater than 1.

The mean ordinate of [7] is equal to

$$\int_0^1 y\, dx = \int_0^1 (1 - x^m) dx = \frac{m}{m+1} \ldots\ldots [8]$$

Substituting the result of Equation [8] in Equations [5] and [6], we obtain, respectively,

$$\frac{m}{m+1} = \frac{1}{k}(y_1 + \ldots + y_k) \ldots\ldots [9]$$

$$\frac{m}{m+1} = \frac{1}{k + 1/2}(1/2 + y_1 + \ldots + y_k) \ldots\ldots [10]$$

Equations [9] and [10] can be rewritten, respectively, as

$$\frac{m}{m+1} = \frac{1}{k}(1 - x_1^m + 1 - x_2^m + 1 - x_3^m + \ldots + 1 - x_k^m)$$

or

$$x_1^m + x_2^m + \ldots + x_k^m = \frac{k}{m+1} \ldots\ldots [11]$$

and

$$\frac{m}{m+1} = \frac{1}{k + 1/2}\left(\frac{1}{2} + 1 - x_1^m + 1 - x_2^m + \ldots + 1 - x_k^m\right)$$

or

$$x_1^m + x_2^m + \ldots + x_k^m = \frac{2k+1}{2(m+1)} \ldots\ldots [12]$$

In both instances, i.e., whether middle ordinate y_0 be used or not, k standard parabolas (k values of m) must be chosen to provide k equations for $x_1 \ldots x_k$.

Now, assuming that we choose the standard parabolas as listed in Table 4, the values of the roots as determined from Equations [11] and [12] will be those listed in Table 5.

TABLE 4 ORDINATES FOR STANDARD PARABOLAS AS CHOSEN

Number of ordinates in Fig. 1*a*	Value of k: Ordinate y_0 not taken, Equation [11]	Ordinate y_0 taken, Equation [12]	Standard parabolas chosen	Mean Ordinates
2	1		$m = 2$	0.6667
3		1	2	
4	2		$m = 2,3$	0.6667
5		2	2,3	0.7500
6	3		$m = 2,3,4$	0.6667
				0.7500
7		3	2,3,4	0.8000
8[a]	4		$m = 2,3,4,5$	0.6667
				0.7500
9		4	2,3,4,5	0.8000
				0.8333

[a] Equation [11] gives imaginary roots for $k = 4$.

TABLE 5 VALUES OF ROOTS AS DETERMINED FROM EQUATIONS [11] AND [12]

Number of ordinates	Equations [11] or [12]	Roots
2	$x_1^2 = \frac{1}{3}$	$x_1 = 0.5774$
3	$x_0 = 0;\ x_1^2 = \frac{3}{6} = \frac{1}{2}$	$x_1 = 0.7071$
4	$x_1^2 + x_2^2 = \frac{2}{3}$; $x_1^3 + x_2^3 = \frac{2}{4}$	$x_1 = 0.1876$; $x_2 = 0.7947$
5	$x_0 = 0;\ x_1^2 + x_2^2 = \frac{5}{6}$; $x_1^3 + x_2^3 = \frac{5}{8}$	$x_1 = 0.3745$; $x_2 = 0.8325$
6	$x_1^2 + x_2^2 + x_3^2 = \frac{3}{3}$; $x_1^3 + x_2^3 + x_3^3 = \frac{3}{4}$; $x_1^4 + x_2^4 + x_3^4 = \frac{3}{5}$	$x_1 = 0.2666$; $x_2 = 0.4225$; $x_3 = 0.8663$
7	$x_0 = 0;\ x_1^2 + x_2^2 + x_3^2 = \frac{7}{6}$; $x_1^3 + x_2^3 + x_3^3 = \frac{7}{8}$; $x_1^4 + x_2^4 + x_3^4 = \frac{7}{10}$	$x_1 = 0.3239$; $x_2 = 0.5297$; $x_3 = 0.8839$
9	$x_0 = 0;\ x_1^2 + x_2^2 + x_3^2 + x_4^2 = \frac{9}{6}$; $x_1^3 + x_2^3 + x_3^3 + x_4^3 = \frac{9}{8}$; $x_1^4 + x_2^4 + x_3^4 + x_4^4 = \frac{9}{10}$; $x_1^5 + x_2^5 + x_3^5 + x_4^5 = \frac{9}{12}$	$x_1 = 0.1679$; $x_2 = 0.5288$; $x_3 = 0.6010$; $x_4 = 0.9116$

It is evident that the standard parabolas chosen as in Table 4 yield Tchebycheff ordinates.

The Proposed System of Ordinates

It is evident that the foregoing choice of standard parabolas is by no means unique and that an infinite number of other combinations of values of m_i (not necessarily integral) can be chosen for each value of k. It would be reasonable for naval architects, for instance, to choose such standard parabolas which would cover a wider range of mean ordinates, and furthermore, parabolas with areas varying by equal increments. (See Fig. 3.)

Moreover, the longitudinal location of the center of gravity of the area included by the lines is required in naval architecture, or the integral

$$M = \int_0^1 xy\, dx$$

It has been noticed that Tchebycheff ordinates do not give this moment with satisfactory accuracy. The function $y = f(x)$ can be written

$$y = 1 - a_2 x^2 - a_3 x^3 - a_4 x^4 - a_5 x^5 - \ldots$$

provided that $a_2 + a_3 + a_4 + \ldots = 1$

then

$$\int_0^1 xy\,dx = \int_0^1 (1 - a_2x^2 - a_3x^4 - a_4x^5 - \ldots)dx - \int^1 (1 - x)\,dx$$

Now, Tchebycheff's ordinates are well suited to evaluate the first term of this integral, but will give the value of the second term with considerable error. It is, therefore, imperative to include the parabola with $m = 1$, or $y = (1 - x)$ as a standard parabola.

TABLE 6 SUGGESTED STANDARD PARABOLAS

Number[a] of ordinates Fig. 1a	Value of k: Ordinate y_0 not taken, Equation [11]	Value of k: Ordinate y_0 taken, Equation [12]	Standard parabolas chosen	Mean ordinate of standard parabolas
2	1		$m_i = 2$	0.6667
3		1	2	
4	2		$m_i = 1;3$	0.5000
5		2	1;3	0.7500
6	3		$m_i = 1;2;6$	0.5000, 0.6667
7		3	1;2;6	0.8571
8	4		$m_i = 1;2;4;8$	0.5000, 0.6667, 0.8000
9		4	1;2;4;8	0.8889

[a] 2,3,4 ordinates have little practical value and are given only for record.

TABLE 7 EQUATIONS FOR DETERMINING THE ORDINATES OF THE SUGGESTED PARABOLAS

Number of Ordinates Fig. 1a	Equations [11] and [12]
2	$x_1^2 = 1/3$
3	$x_2 = 0$; $x_1^2 = 3/6$
4	$x_1 + x_2 = 1$; $x_1^3 + x_2^3 = 2/4$
5	$x_3 = 0$; $x_1 + x_2 = 5/4$; $x_1^3 + x_2^3 = 5/8$
6	$x_1 + x_2 + x_3 = 3/2$; $x_1^3 + x_2^3 + x_3^3 = 3/3$; $x_1^5 + x_2^5 + x_3^5 = 3/7$
7	$x_4 = 0$; $x_1 + x_2 + x_3 = 7/4$; $x_1^3 + x_2^3 + x_3^3 = 7/6$; $x_1^5 + x_2^5 + x_3^5 = 7/14$
8	$x_1 + x_2 + x_3 + x_4 = 2$; $x_1^3 + x_2^3 + x_3^3 + x_4^3 = 4/3$; $x_1^4 + x_2^4 + x_3^4 + x_4^4 = 4/5$; $x_1^5 + x_2^5 + x_3^5 + x_4^5 = 4/9$
9	$x_5 = 0$; $x_1 + x_2 + x_3 + x_4 = 9/4$; $x_1^3 + x_2^3 + x_3^3 + x_4^3 = 9/6$; $x_1^4 + x_2^4 + x_3^4 + x_4^4 = 9/10$; $x_1^5 + x_2^5 + x_3^5 + x_4^5 = 9/18$

TABLE 8 RECOMMENDED ORDINATES FOR SHIPBUILDING COMPUTATIONS

Number of ordinates	Abscissas as determined from equations in Table 7
$n = 2$	$x_1 = -x_2 = 0.57735$
$n = 3$	$x_1 = -x_3 = 0.70711$; $x_2 = 0$
$n = 4$	$x_1 = -x_4 = 0.21133$; $x_2 = -3 = 0.78868$
$n = 5$	$x_1 = -x_5 = 0.43406$; $x_2 = -x_4 = 0.81594$; $x_3 = 0$
$n = 6$	$x_1 = -x_6 = 0.15821$; $x_2 = -x_5 = 0.47755$; $x_3 = -x_4 = 0.86424$
$n = 7$	$x_1 = -x_7 = 0.36574$; $x_2 = -x_6 = 0.49872$; $x_3 = -x_5 = 0.88554$; $x_4 = 0$
$n = 8$	$x_1 = -x_8 = 0.09780$; $x_2 = -x_7 = 0.42543$; $x_3 = -x_6 = 0.57659$; $x_4 = -x_5 = 0.90018$
$n = 9$	$x_1 = -x_9 = 0.27047$; $x_2 = -x_8 = 0.41791$; $x_3 = -x_7 = 0.65263$; $x_4 = -x_6 = 0.90900$; $x_5 = 0$

On the basis of the foregoing observations, the standard parabolas listed in Table 6 are suggested. Substituting in Equations [11] and [12] the values of k and m as given in Table 6, we obtain equations for $x_1 \ldots x_k$ that are listed in Table 7. The roots of these equations were found by successive approximation to the sixth decimal place and are tabulated in Table 8. It is recommended that these abscissas be used in shipbuilding computations instead of Tchebycheff's ordinates. These suggested ordinates are compared graphically with Tchebycheff's ordinates in Fig. 4.

Empirical Verification

The two curves shown in Fig. 5 have the general character of waterlines. Curve a has the equation $y = 0.7\,(1 - x^3) + 0.3\,(1 - x^6)^2$, while curve b has the equation $y = \left(0.935 \cos \frac{\pi}{2} x\right) + \left(0.065 \cos \frac{3\pi}{2} x\right)$. The areas and moments of these curves, when

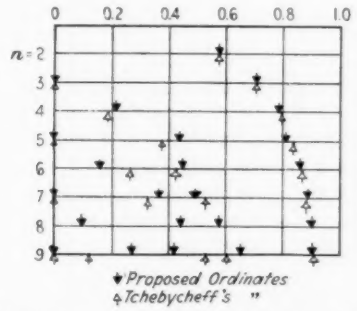
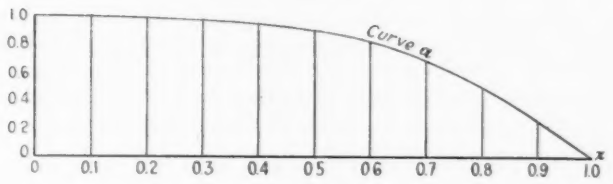


FIG. 4 COMPARISON OF TCHEBYCHEFF'S AND THE PROPOSED ORDINATES



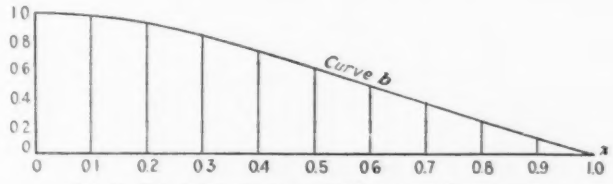


FIG. 5 TEST CURVES FOR THE PROPOSED ORDINATES

calculated analytically and by approximate quadrature rules, are given in Table 9.

Application of the Proposed Ordinates

Referring to Fig. 1*a* and taking ordinates at abscissas indicated in Table 8

$$\int_{-1}^{+1} y\,dx = 2/n \sum_1^n y_i \ldots\ldots\ldots\ldots [13]$$

where $n = 2k$ or $(2k + 1)$.

This equality is exact not only for a line composed of two symmetrically disposed standard parabolas, but is always exact for

(1) $y = a_0$, since in this case $\int_{-1}^{+1} y\,dx = 2a_0$ and $\sum_1^n y_i = n\,a_0$

(2) $y = 1 - x^{2m+1}$ with $m = 0, 1, 2 \ldots$ since in this case $\int_{-1}^{+1} (1 - x^{2m+1})\,dx = 2$ and $\sum_1^n y_i = n$ in view of the fact that

$$y_{-k} + y_{+k} = (1 + x_k{}^{2m+1}) + (1 - x_k{}^{2m+1}) = 2$$

It is, therefore, evident that a function having $y = a$ at $x = -1$ and $y = b$ at $x = +1$ can be integrated by the proposed rule. Indeed, taking $y = c$ at $x = 0$, we can write

$$y = b - \frac{b-a}{2}\left[d_1(1-x) + d_3(1-x^3) + d_5(1-x^5) + \ldots\right] + \left(c - \frac{b+a}{2}\right)\left[d_2(1-x^2) + d_4(1-x^4) + d_6(1-x^6) + \ldots\right]$$

where $d_1 + d_3 + d_5 + \ldots = 1$

and $d_2 + d_4 + d_6 + \ldots = 1$

and $$\frac{b-a}{2} d_1 = \left|\frac{dy}{dx}\right|_{x=0}$$

The integrals of the first and second terms are expressed exactly by Equation [13] while the third term is integrated with an approximation demonstrated above.

The proposed ordinates are, therefore, as universal in their application as the ordinates of Tchebycheff.

TABLE 9 COMPARISON OF VALUES COMPUTED BY THE VARIOUS QUADRATURE RULES

	Curve *a*				Curve *b*			
	Area		Moment		Area		Moment	
	Value	Per cent error	Value	Per cent error	Value	Per cent error	Value	Per cent error
Analytical value—exact	0.7624	...	0.3064	...	0.584	...	0.200	...
Trapezoidal rule with 10 ordinates in half-length l (n = 21)	0.7606	−0.3	0.3038	−0.8	0.580	−0.7	0.202	+1.0
4½ Tchebycheff's ordinates. (k = 4; n = 9)	0.761	−0.2	0.296	−3.2	0.582	−0.3	0.189	−5.5
4½ Ordinates as proposed and listed in Table 8 (k = 4; n = 9)	0.761	−0.2	0.3045	−0.7	0.581	−0.5	0.201	+0.5
3½ Tchebycheff's ordinates. (k = 3; n = 7)	0.759	−0.4	0.297	−2.9	0.581	−0.5	0.198	−1.0
3½ ordinates as proposed and listed in Table 8. (k = 3; n = 7)	0.761	−0.2	0.304	−0.8	0.580	−0.7	0.203	+1.5

Conclusion

It should be noted that the choice of standard parabolas for determination of $x_1, x_2 \ldots x_k$ in the foregoing was dictated primarily by the requirements of the geometry of a hull. Other applications may dictate the choice of different standard parabolas.

The author is indebted to Prof. L. Siceloff for his criticism and suggestions and to Miss Vera Mintz for her assistance with the numerical computations.

Buckling of Flat Curved Bars and Slightly Curved Plates

By S. TIMOSHENKO,[1] ANN ARBOR, MICH.

This paper considers the case of the flat arch and the slightly curved plate, with uniformly distributed load, which may fail not as a result of high stresses and yielding of the material but because of instability, and it is shown how the critical load at which buckling occurs may be calculated. The case of the bimetallic strips used as part of thermostats is also considered.

IF A flat arch is submitted to the action of a uniformly distributed load, Fig. 1, the failure of the structure may occur not as a result of high stresses and yielding of the material, but because of instability. The compressed arch may buckle and take the position indicated in the figure by the dotted line. In the following discussion it is shown how the critical load, at which buckling occurs, can be calculated.

To simplify the calculations, assume that the center line of the arch is initially given by the equation

$$y_1 = a \sin \pi x/l \ldots\ldots\ldots\ldots\ldots\ldots [a]$$

and that the ends of the arch are hinged. The following notation is used:

q = the intensity of the uniform load
EI = the flexural rigidity of the arch

$$u = \frac{5}{384} \frac{ql^4}{EI} \frac{1}{a} \ldots\ldots\ldots\ldots\ldots\ldots [b]$$

is the ratio of the deflection, calculated as for a beam, to the rise of the arch
H = the thrust of the arch

$$\alpha = Hl^2/\pi^2 EI \ldots\ldots\ldots\ldots\ldots\ldots [c]$$

is the ratio of the thrust to Euler's load. Then the center line of the arch after loading, assuming that it is symmetrical with respect to the middle, can be represented with sufficient accuracy by the following equation[2]

$$y_2 = \frac{a(1-u)}{1-\alpha} \sin \frac{\pi x}{l} \ldots\ldots\ldots\ldots\ldots\ldots [d]$$

This equation can be used not only for $u < 1$, but also for $u > 1$, i.e., in the cases where the deflection of the curved bar, calculated as for a simple beam, is larger than the initial rise of the arch. The quantity α can also be larger than one, but it must be smaller than four since for $\alpha = 4$, buckling of the arch with an inflection point at the middle occurs and the assumption of Equation [d] that the arch remains symmetrical with respect to the middle is no longer fulfilled.

Assuming first that $u < 1$, we find, from Equation [d], that y_2 is positive if $\alpha < 1$, and negative when $\alpha > 1$. This means that the arch has the form shown in Fig. 1 by the full line if the thrust is smaller than the Euler load for a bar with hinged ends, and the same arch can be kept deflected downward, as shown in the figure by the dotted line, if a thrust larger than the Euler load is applied. When $u > 1$, y_2 is positive for $\alpha > 1$, and it becomes negative for $\alpha < 1$.

The actual shape of the arch, after loading, can be determined only when the quantity α, i.e., the thrust H, is known. The equation for calculating the thrust H is obtained by putting the change in length of the span due to deflection equal to the compression of the bar due to thrust. Assuming for flat curves that the compressive force along the length of the bar is constant and equal to H, we obtain the equation

$$\frac{Hl}{AE} = \frac{1}{2}\int_0^l \left(\frac{dy_1}{dx}\right)^2 dx - \frac{1}{2}\int_0^l \left(\frac{dy_2}{dx}\right)^2 dx \ldots\ldots [e]$$

in which A is the cross-sectional area of the bar. Substituting

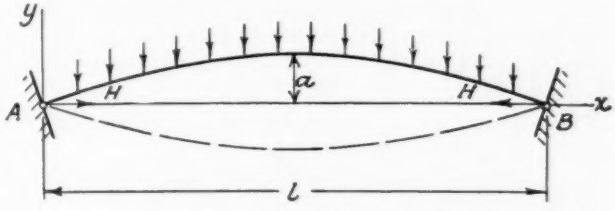


FIG. 1

for y_1 and y_2 their expressions [a] and [d] and integrating, we obtain

$$(1-u)^2 = (1-m\alpha)(1-\alpha)^2 \ldots\ldots\ldots\ldots\ldots [f]$$

in which

$$m = \frac{4I}{Aa^2} \ldots\ldots\ldots\ldots\ldots\ldots [g]$$

For a given arch the quantity m is easily calculated and if the load q is given, the quantity u can be determined from Equation [b]. Then, the corresponding value of α and the thrust H are obtained from Equation [f]. Since this equation is not linear, more than one real root for α can be obtained under certain conditions, which indicates that there are several possible forms of equilibrium, and the stability of these forms must be investigated.

Considering the right side of Equation [f] as a function of α, we find that for $m < 1$ this function has a minimum equal to 0 for $\alpha = 1$ and a maximum for $\alpha = (2+m)/3m$. The magnitude of this maximum is

$$\frac{4}{27} \frac{(1-m)^3}{m^2} \ldots\ldots\ldots\ldots\ldots\ldots [h]$$

In Fig. 2 the right side of Equation [f] is represented graphically for the case where $m = 1/2$. For this value of m the maximum

---

[1] Professor of Engineering Mechanics, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E. Dr. Timoshenko was educated in Russia and was called to the Polytechnic Institute of Kiev to occupy the chair of applied mathematics. For five years he was professor of applied mechanics in various engineering schools of St. Petersburg, and professor of applied mechanics at the Polytechnic Institute in Zagreb, Jugoslavia. In 1923 he entered the research department of the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., leaving in 1927 to assume his present position.

[2] See author's "Strength of Materials," vol. 2, p. 422.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until May 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

occurs at $\alpha = {}^5/_3$ and the magnitude of this maximum, from Equation [h], is ${}^2/_{27}$. If the load q is of such magnitude that the left side of Equation [f] is larger than this maximum, we obtain only one real solution for α, which indicates that only one form of equilibrium is possible, hence the equilibrium is stable. If the left side of Equation [f] is smaller than the quantity [h], three solutions for α are obtained as shown by intersection points s,

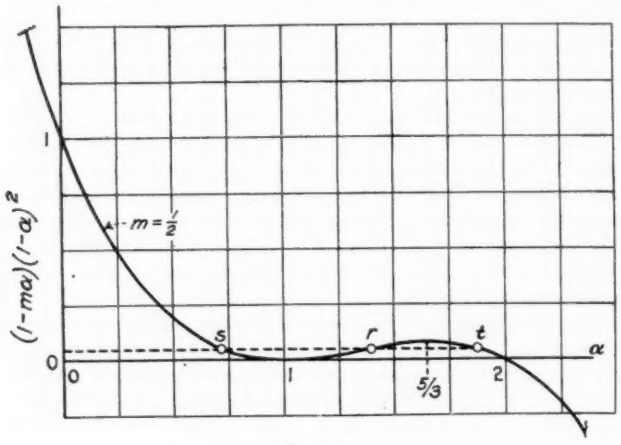


FIG. 2

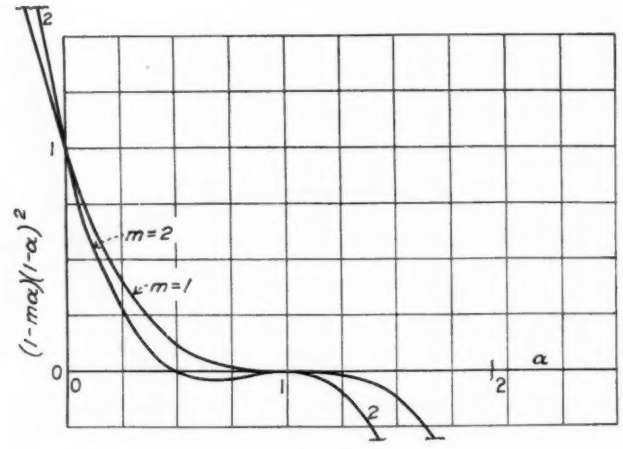


FIG. 3

r, and t in Fig. 2 and the question of stability of the corresponding forms of equilibrium must be considered. Applying these conclusions in the numerical example just cited we find that the equilibrium is always stable if

$$(1 - u)^2 > 2/27$$

which is equivalent to the conditions

$$\left.\begin{aligned} u &< \left(1 - \sqrt{\frac{2}{27}}\right) \\ u &> \left(1 + \sqrt{\frac{2}{27}}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [i]$$

The first of these conditions corresponds to the form of equilibrium convex upward, as shown in Fig. 1 by the full line, and the second to the form convex downward, as indicated in the figure by the dotted line.

For any value of m smaller than 1 the conditions of stability, equivalent to conditions [i], are

$$\left.\begin{aligned} u &< \left(1 - \sqrt{\frac{4}{27}\,\frac{(1-m)^2}{m^2}}\right) \\ u &> \left(1 + \sqrt{\frac{4}{27}\,\frac{(1-m)^2}{m^2}}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots [j]$$

Hence, conditions that make possible more than one form of equilibrium and where an investigation of stability is necessary are

$$\left.\begin{aligned} u &> \left(1 - \sqrt{\frac{4}{27}\,\frac{(1-m)^2}{m^2}}\right) \\ u &< \left(1 + \sqrt{\frac{4}{27}\,\frac{(1-m)^2}{m^2}}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots [k]$$

If $m \geqq 1$, Equation [f] has only one real root, as can be seen from Fig. 3, in which the right side of Equation [f] is represented by curves for $m = 2$ and $m = 1$. It is seen that for any positive value for $(1 - u)^2$ we obtain only one value for α, which is less than unity. Hence for $m \geqq 1$ there is only one possible form of equilibrium which will be stable. The question of instability will arise only if $m < 1$, and the load is within the limits indicated by conditions [k].

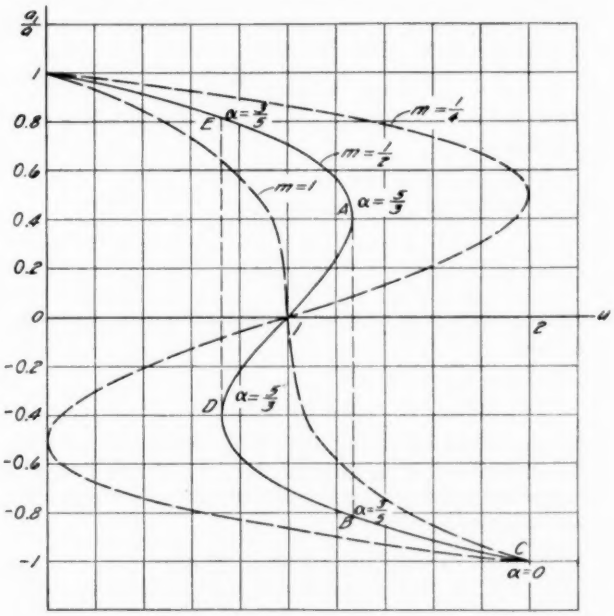


FIG. 4

In the investigation of the stability it is advantageous to represent graphically the rise of the arch as a function of the load or as a function of the quantity u. By using Equation [d] this rise is

$$a_1 = \frac{a(1-u)}{1-\alpha} \quad \ldots\ldots\ldots\ldots\ldots [l]$$

By taking in each particular case a series of values of α the corresponding values of u can be calculated from Equation [f] and the rise a_1 from Equation [l]. In Fig. 4 the values of a_1/a are plotted against u. The full line represents the case where $m = {}^1/_2$, and the two dotted lines the cases $m = {}^1/_4$ and $m = 1$. Considering the case $m = {}^1/_2$, it is seen from the curve that the deflection gradually increases with increasing of the load up to the point A which corresponds to the maximum (at $\alpha = {}^5/_3$) on the curve shown in Fig. 2. Beginning at this point further de-

flection continues with the decreasing of u, i.e., with the decreasing of the load q. This fact indicates that the point A, i.e., for

$$(1 - u)^2 = 2/27$$

the form of equilibrium of the arch shown in Fig. 1 by the full line becomes unstable and the arch buckles downward as shown by the dotted line. The sag of this new form of equilibrium is given in Fig. 4 by the position of the point B. This new form of equilibrium is stable and any further increase of the load produces a gradual increase in deflection as is shown by the portion BC of the curve in Fig. 4. The thrust H during this loading decreases. It becomes zero at the point C and becomes negative with a further increase of the load. If, starting from the point B, we begin to diminish the load the deflection of the arch gradually diminishes up to the point D. At this point the load becomes insufficient to keep the arch deflected convex downward and it buckles upward to the position given in Fig. 4 by the point E. From this discussion we see that there exists a possibility of having more than one form of equilibrium in the region limited by the vertical lines ED and AB which corresponds to the conditions [i]. In the general case this region is defined by the conditions [k] and we conclude that the critical load at which the arch will buckle convex downward is determined from the equation

$$u = 1 + \sqrt{\frac{4}{27}\frac{(1 - m)^2}{m^2}} \quad \ldots\ldots\ldots\ldots [1]$$

From the curves in Fig. 4 it may be concluded that the region in which more than one form of equilibrium is possible becomes smaller and smaller with the increase of m, and when m becomes equal to one the two limits given by [k] coincide, hence, beginning from this value of m only one form of equilibrium is possible.

In the foregoing discussion it was assumed that a uniformly distributed load is acting on the arch, but the results obtained can be used in all cases in which the deflection of the arch, considered as a beam, can be represented with sufficient accuracy by the sine curve. Taking, for instance, a concentrated vertical load P applied at the middle of the arch, the critical value of this load is obtained from Equation [1],[3] it being only necessary to substitute in this equation

$$u = \frac{Pl^3}{48EI}\frac{1}{a} \quad \ldots\ldots\ldots\ldots [m]$$

In an analogous manner the problem can be solved also if the load P is applied at some point other than the middle. It is only necessary to substitute in Equation [m] the corresponding deflection at the middle of the beam for $Pl^3/48EI$.

The results of the foregoing discussion can be used in investigating buckling of a bimetallic strip submitted to a variation of temperature. Such strips are used as a part of instruments (thermostats) serving for regulating temperature. Simple considerations show that due to difference in coefficients of expansion of the two metals a straight bimetallic strip becomes curved when temperature changes. The curvature can be calculated from the equation[4]

$$\frac{1}{\rho} = \frac{3}{2}\frac{(\epsilon_2 - \epsilon_1)(t - t_0)}{h} \quad \ldots\ldots\ldots\ldots [2]$$

in which ϵ_1 and ϵ_2 are coefficients of thermal expansion (it is assumed $\epsilon_2 > \epsilon_1$), h is the thickness of the strip, and $t - t_0$ is the rise of temperature. Thus the curvature of a bimetallic strip produced by heating is proportional to the rise in temperature and to the difference of the two coefficients of expansion and inversely proportional to the thickness of the strip. Equation [2] derived for two metals with equal moduli can also be used with sufficient accuracy in the case in which the materials have somewhat different moduli of elasticity as, for instance, in the case of monel metal and nickel steel.[5]

Having the curvature, the deflection at the middle of the strip, supported at both ends, due to increase of temperature $(t - t_0)$ is obtained as for a flat arc of a circle, and is equal to

$$\delta = \frac{l^2}{8\rho} \quad \ldots\ldots\ldots\ldots [n]$$

in which l is the length of the strip and ρ the radius of curvature determined from [2].

Let us now consider a bimetallic strip which has a slight initial curvature and assume that the ends of the strip are hinged as shown in Fig. 1 by the full line. If the metal on the concave side has a larger coefficient of thermal expansion than the metal on the convex side, the strip will deflect downward during heating and a thrust H will be produced as a result of this deflection. At a certain temperature, depending on the proportions of the strip and on the difference of the coefficients of expansion of the two metals, the form of the strip shown in Fig. 1 by the full line becomes unstable and the strip buckles suddenly downward as indicated by the dotted line. In a thermostat the source of heating is cut off at the moment of buckling of the strip and cooling begins which results finally in buckling of the strip in the upward direction, thus turning on the source of heat again. This phenomena is analogous to that of buckling of a flat arch during loading and unloading, just discussed. To determine the temperature at which buckling occurs we need only to use the deflection [n] instead of the deflection produced by loading, and Equation [1] becomes

$$\frac{l^2}{8\rho a} = 1 + \sqrt{\frac{4}{27}\frac{(1 - m)^2}{m^2}}$$

Using for the curvature expression [2], the temperature t_1, at which buckling of the strip in a downward direction occurs, is obtained from the equation

$$\frac{3}{16}\frac{l^2}{ah}(\epsilon_2 - \epsilon_1)(t_1 - t_0) = 1 + \sqrt{\frac{4}{27}\frac{(1 - m)^2}{m^2}} \quad \ldots\ldots [3]$$

The temperature t_2, at which the strip buckles in an upward direction after cooling, is obtained from the first of the inequality [k]. Hence

$$\frac{3}{16}\frac{l^2}{ah}(\epsilon_2 - \epsilon_1)(t_2 - t_0) = 1 - \sqrt{\frac{4}{27}\frac{(1 - m)^2}{m^2}} \quad \ldots\ldots [4]$$

Taking, for instance, $l/h = 100$, $\epsilon_2 - \epsilon_1 = 4 \times 10^{-6}$, and the values of $m = h^2/3a^2$, such as is given in the first line of Table 1, the ratios of the initial rise a of the strip to its thickness are such as are given in the second line of the table. The values of $(t_1 - t_0)$, calculated from Equation [3], and $(t_2 - t_0)$, calculated from Equation [4], are given in the third and fourth lines of the table, respectively.

It is seen from the table that the temperature of buckling t_1 in-

[3] This particular problem has been discussed by Navier, "Résumé des lecons sur l'application de la mécanique," second edition, p. 273, Paris, 1833. See also C. B. Biezeno, Koninklijke Akademie van Wetenschappen te Amsterdam, Proceedings, vol. 32, 1929, p. 990. The case when the center line of the arch is not a flat curve has been considered by A. Nadai, Technische Blättern, nos. 3 and 4, Prag, 1915.

[4] See author's "Strength of Materials," vol. 1, p. 245.

[5] A more general investigation of bending and buckling of bimetallic strips is given in the author's paper, see *Journal Optical Society of America*, vol. 11, 1925, p. 233.

TABLE 1 NUMERICAL DATA ON BUCKLING OF BIMETALLIC STRIPS

m =	$^2/_3$	$^1/_2$	$^1/_3$	$^1/_4$
a/h =	0.707	0.814	1.000	1.154
$t_1 - t_0$ =	104	137	217	307
$t_2 - t_0$ =	83	79	50	0

creases with the increase of the initial curvature and at the same time the temperature t_2 decreases, so that the sensitivity of the thermostat, given by the difference $(t_1 - t_2)$, decreases rapidly with the increase of the temperature action t_1. The conditions can be improved by introducing elastic supports for the bimetallic strip.

If, under the action of the thrust H, the distance between the supports increases by some quantity βH proportional to H, the equation for calculating the thrust becomes

$$\frac{Hl}{AE} + \beta H = \frac{1}{2}\int_0^l \left(\frac{dy_1}{dx}\right)^2 dx - \frac{1}{2}\int_0^l \left(\frac{dy_2}{dx}\right)^2 dx$$

and, instead of Equation [f] we obtain

$$(1 - u)^2 = (1 - m_1\alpha)(1 - \alpha)^2 \ldots\ldots\ldots\ldots [o]$$

in which

$$m_1 = m\left(1 + \frac{\beta AE}{l}\right) \ldots\ldots\ldots\ldots [p]$$

It is seen that the operation of a bimetallic strip with elastic supports can be investigated in the same manner as it was before.

It is only necessary to introduce, instead of the quantity m, a new quantity m_1 depending on the elasticity of supports. By a suitable adjustment of this elasticity the desired sensitivity of the thermostat can be obtained.[6]

[6] See author's paper, loc. cit., p. 233.

A Theoretical Investigation of the Maximum-Lift Coefficient

By Th. von Kármán[1] and Clark B. Millikan[2]

In the present paper the application of a laminar boundary-layer theory, previously developed by the authors, to the problem of the maximum-lift coefficient of airfoils is discussed. The calculations are carried through in detail for a first approximation, called a single-roof profile, to the potential velocity distribution over the upper surface of an airfoil. The results indicate a large variation in $C_{L\max}$ with turbulence but the quantitative dependence on Reynolds' number and turbulence is not satisfactory. The calculations are then repeated for a so-called double-roof profile which approximates to the flow over the upper surface of an N.A.C.A. 2412 airfoil. These results are compared with those obtained from an experimental investigation on the same airfoil. The agreement is considered to indicate that for moderate values of R and $C_{L\max}$ the phenomenon of the maximum-lift coefficient is controlled by a contest between the separation and transition points of the laminar boundary layer over the nose of the airfoil. The difficulties involved in extending the theory to larger values of R, or to airfoils whose C_L vs. α curves are not approximately linear up to the stall, are mentioned.

1—Introduction

THE PROBLEM of the maximum lift of airfoils has concerned the authors greatly since there were first discovered in the spring of 1932 serious discrepancies in this characteristic between results obtained in the wind tunnel of the Guggenheim Aeronautics Laboratory at the California Institute of Technology (GALCIT) and those reported from certain other wind tunnels. An elaborate experimental investigation by the junior author and A. L. Klein[3] indicated that the value of $C_{L\max}$ for a given airfoil was strongly affected both by Reynolds' number and by the degree of turbulence in the tunnel wind stream. It had long been known that the maximum-lift behavior was intimately connected with the separation of the boundary layer from the upper surface of an airfoil, but it had in general been supposed that for practically important Reynolds' numbers this phenomenon involved only the turbulent state of the boundary layer. The great dependence of $C_{L\max}$ on the degree of free-stream turbulence, as revealed by the aforementioned experiments, made the authors suspect that the boundary layer at a separation point may be sometimes turbulent and sometimes laminar. The free-stream turbulence would then have a great influence in determining which of the two régimes was in effect for a particular case, and it is easy to see that the separation phenomenon might be importantly influenced by the possible existence of these alternative types of flow in the boundary layer. It is well known that a turbulent boundary layer resists separation much more effectively than does a laminar one. Hence an early transition of the boundary layer from the laminar to the turbulent state, induced by large external turbulence, might delay the separation and increase the maximum lift. The quantitative discussion of such an hypothesis requires an analysis of the laminar boundary layer, and in particular a determination of its separation point and its thickness at that point.

The existing methods for the calculation of a laminar boundary layer with a more or less arbitrary pressure distribution were felt to be unsatisfactory for this purpose. Accordingly, an entirely new procedure was developed which has been presented in detail in a previous paper by the authors.[4] In the present paper the application of this boundary-layer analysis to the problem of the maximum-lift coefficient will be discussed. The explicit aim of this paper is, then, to investigate the dependence of $C_{L\max}$ on Reynolds' number and free-stream turbulence over the regions available for normal wind-tunnel tests.

2—Outline of the Basic Procedure[5]

The boundary-layer theory is based on the assumption that the influence of friction is restricted to the neighborhood of the walls and does not essentially change the outside flow. Its application is, therefore, restricted to those cases in which the outside flow is known. Hence, we are able to investigate whether or not a given flow pattern about an airfoil, involving a certain magnitude of the lift and a certain pressure distribution, is possible without separation of the flow from the surface, but we are not able to determine flow patterns with separation.

In view of the restricted possibilities of the theory we are led to a kind of inverse method to determine the maximum value of the lift coefficient C_L as a function of Reynolds' number and the degree

---

[1] Director of Guggenheim Aeronautics Laboratory, California Institute of Technology. Mem. A.S.M.E. Dr-Ing. von Kármán received his M.E. at Budapest in 1902 and Ph.D. at Göttingen in 1908; honorary degree of Doctor of Engineering, University of Berlin, 1929. He was Privat-Docent, Göttingen, 1910–1913; Professor of Mechanics and Aerodynamics, Director of the Aerodynamical Institute, University of Aachen, 1913; member of Gesellschaft der Wissenschaften zu Göttingen, 1925; foreign member of the Royal Academy of Sciences, Turin, 1928; Director of the Graduate School of Aeronautics, California Institute of Technology, 1928.

[2] Associate Professor of Aeronautics, Daniel Guggenheim Graduate School of Aeronautics, California Institute of Technology. Assoc. Mem. A.S.M.E. Professor Millikan was born in Chicago in 1903, and received his Ph.B. from Yale University in 1924 and Ph.D. in Physics and Mathematics from California Institute of Technology in 1928. He was a Teaching Fellow at the California Institute of Technology from 1924 to 1928, and since then has been on the staff of the Aeronautics Department of that Institute.

[3] "The Effect of Turbulence: An Investigation of Maximum-Lift Coefficient and Turbulence in Wind Tunnels and in Flight," by C. B. Millikan and A. L. Klein, *Aircraft Engineering*, August, 1933 (London).

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until May 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] "On the Theory of Laminar Boundary Layers Involving Separation," by Th. von Kármán and C. B. Millikan, N.A.C.A. Technical Report No. 504.

[5] Condensed accounts of portions of the discussion of this section have already been presented by the authors; cf. "Quelques Problemes Actuels de l'Aerodynamique," by Th. von Kármán, Journees Techniques Internationales de l'Aeronautique, Paris (1932). "The Effect of Turbulence: An Investigation of Maximum-Lift Coefficient and Turbulence in Wind Tunnels and in Flight," by C. B. Millikan and A. L. Klein, loc. cit. "The Use of the Wind Tunnel in Connection With Aircraft-Design Problems," by Th. von Kármán and C. B. Millikan, Trans. A.S.M.E., 1934, pp. 151–166, paper AER-56-4.

of turbulence. We start with a given value of C_L and inquire whether or not the potential flow which would produce the assumed lift in a non-viscous fluid can be maintained in a viscous fluid without separation, i.e., whether or not the influence of the friction will be restricted to the boundary layer.

It is easily seen that the problem put in this form is not quite identical with the original problem of the maximum lift. Let us neglect for a moment the possibility of turbulence in the boundary layer. Then the possible values of C_L would be given by flow patterns which do not lead to separation of the laminar boundary layer, and the limiting values of C_L would correspond to the flow with separation just at the trailing edge. Now it is a well-known result of the laminar boundary-layer theory that for geometrically similar bodies the location of the separation point is independent of the Reynolds number. Thus we should conclude that $C_{L\max}$ determined in this way gives only an inferior limit, because flow patterns with separation may give higher lifts than those with separation at the trailing edge, but it is certain that any value below this calculated $C_{L\max}$ can be reached without separation. For this reason the actual values of $C_{L\max}$ may be higher than those determined from the "no separation" criterion, and may be variable with Reynolds' number.

Fortunately, the case in which the boundary layer is laminar along the whole or a considerable part of the airfoil is without practical importance. In the practical range of Reynolds' numbers, both in the case of full-scale airfoils in flight and in that of normal wind-tunnel tests, the laminar régime is restricted to a small portion of the chord. Hence, if the laminar boundary layer separates from the airfoil before transition to the turbulent régime occurs, the separation occurs not far from the leading edge. Such a separation must clearly lead to an abrupt drop in the lift coefficient, such as experiments actually reveal. As a matter of fact, for most modern "efficient" airfoil sections, the experimental $C_L = f(\alpha)$ curves (α = angle of attack) are almost straight up to the stalling point, as would be the case for the pure potential flow, and show a more or less sharp break at the stalling point. Thus in the practically important range we can assume that C_L values which lead to separation of the laminar layer prior to the change to the turbulent state are impossible for such airfoils. Our considerations may, however, not be valid for airfoils in which the C_L vs. α curves have a considerable curvature well before the stall is reached.

Before discussing the second alternative, that the laminar layer becomes turbulent before it reaches the separation point, let us review briefly the pertinent experimental facts for the boundary layer along a flat plate placed parallel with a uniform flow. It is convenient to introduce two Reynolds' numbers for such a boundary-layer flow. The basic-flow Reynolds number R is defined in terms of the characteristic length of the problem, i.e., the length of the plate or airfoil in the direction of flow, and of the free-stream velocity far from the airfoil or plate. The boundary-layer Reynolds number R_δ is defined in terms of the boundary-layer thickness δ at any point along the surface, and of the velocity just outside the boundary layer at this point. The two definitions are $R = U_0 l/\nu$ and $R_\delta = U\delta/\nu$ where U_0 is the undisturbed velocity, l the chord of airfoil or plate, U the potential velocity just outside the boundary layer, δ the boundary-layer thickness at the same point along the surface, and ν coefficient of kinematic viscosity. The definition of the boundary-layer thickness is somewhat arbitrary, but using the same definition throughout the discussion, the results are independent of the definition.

For the flow along a flat plate which we are discussing for the moment, the boundary layer starting from the upstream edge of the plate is laminar in character even though the outside flow is highly turbulent or fluctuating. The boundary-layer thickness increases continuously as one goes downstream from the leading edge, so that the boundary-layer Reynolds number R_δ varies in the same way. For any given type of turbulence or fluctuations in the outside flow there exists a certain value of R_δ at which the laminar régime in the boundary layer becomes unstable and a transition occurs to a turbulent state of flow. Actually there is a region of finite length along the plate in which this transition from the laminar to the so-called "fully developed" turbulent

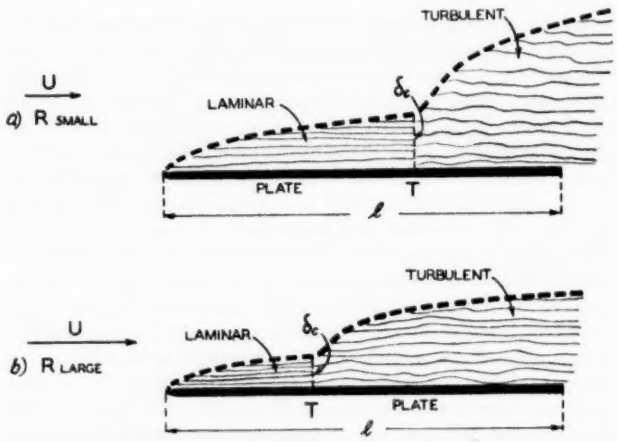


FIG. 1 THE BOUNDARY LAYER ALONG A FLAT PLATE AT TWO REYNOLDS' NUMBERS

régime takes place. For simplicity in both discussion and calculation it is, however, customary to assume that the transition is instantaneous and occurs at a definite point which we shall refer to as the transition point, and which will be indicated by a subscript $(\,)_t$. The boundary-layer Reynolds number at which the transition occurs is called the critical Reynolds number and will be denoted by R_{δ_c}. The variation of R_δ with the basic Reynolds number of the flow, at a given point along the plate, is given by the relation $R_\delta \sim \sqrt{R}$. Assuming the state of turbulence in the outer flow to be independent of the value of R, it follows that for a given air stream R_{δ_c} is also independent of R. Consequently, the transition point, i.e., the point at which R_δ reaches the value R_{δ_c}, moves upstream along the plate as R increases. The conditions for two values of R are indicated schematically in Fig. 1. R_{δ_c} is experimentally found to be independent of R for a given air stream, but to vary with the degree of outside turbulence in such a way that it decreases as the turbulence increases. The observed values of R_{δ_c} lie between about 10,000 for very smooth flows and 1500 for very turbulent flows.

We assume, with considerable experimental justification, that very similar phenomena occur for the boundary layer over the upper surface of an airfoil. In other words, for any given external air flow there is a definite value of R_{δ_c} such that, when the boundary-layer Reynolds number reaches this value, there is a transition point downstream from which the airfoil boundary layer is turbulent, while upstream it is laminar.

We have already discussed the case in which the laminar layer separates before the change to the turbulent régime. This will obviously take place if, for the calculated Reynolds number of the laminar layer at the separation point R_{δ_s}, we have the inequality $R_{\delta_s} < R_{\delta_c}$. In the opposite case when $R_{\delta_s} > R_{\delta_c}$, the layer becomes turbulent before separation and whether or not the value of C_L considered can be reached, obviously depends on the behavior of the turbulent layer. Unfortunately, the theory of the turbulent boundary layer is not as yet so far advanced as to furnish a definite answer to this question. However, it is well known that the turbulent layer exhibits a considerably higher re-

sistance to separation than does the laminar layer. Hence we consider first the idealized case that the turbulent layer clings under all circumstances to the airfoil surface, i.e., we neglect the possibility of separation for the turbulent layer. This neglecting of the possibility of separation of a turbulent boundary layer implies that as $R \to \infty$ indefinitely large values of C_L could be reached, and obviously imposes limitations on the range of validity of the theory. We discuss this in more detail in a later section of this paper entitled "Comparison With Experiment and Region of Validity of the Theory."

With this assumption (no separation of a turbulent boundary layer) the following procedure leads to a family of $C_{L\max}$-curves giving $C_{L\max}$ as a function of Reynolds' number for different degrees of turbulence in the outside stream: We start with an assumed potential-velocity distribution corresponding to a definite value of C_L. From this we calculate the laminar separation point, which will occur at a fixed point on the upper surface of the airfoil. The corresponding value of R_{δ_s} will be proportional to $\sqrt{R}$. We now consider an external flow with a definite degree of turbulence so that R_{δ_c} has a definite value independent of R. If we start with small enough values of R then R_{δ_s} will be less than R_{δ_c}, separation will occur, and the boundary layer will behave as in (*a*) of Fig. 2. The assumed potential flow can obviously not occur and the assumed C_L will not be obtainable. If, now, we increase R, R_{δ_s} increases continuously until it finally equals and then surpasses the constant value of R_{δ_c}.

We can alternatively describe this process in the following way: For small values of R, the transition point is downstream from the separation point (case *a*); with increasing R, the transition point moves upstream, while the location of the separation point is independent of R. When $R_{\delta_s} = R_{\delta_c}$, the two points coincide. A still further increase of R leads now to case (*b*), i.e., the boundary layer at the calculated separation point is turbulent and so clings to the airfoil surface, the assumed potential flow can occur and the assumed C_L is reached. If we repeat the calculations for a series of potential-velocity distributions (i.e., lift coefficients), we get a definite value, of R for each C_L, above which the C_L value is attained and below which it is not. Plotting C_L against R we

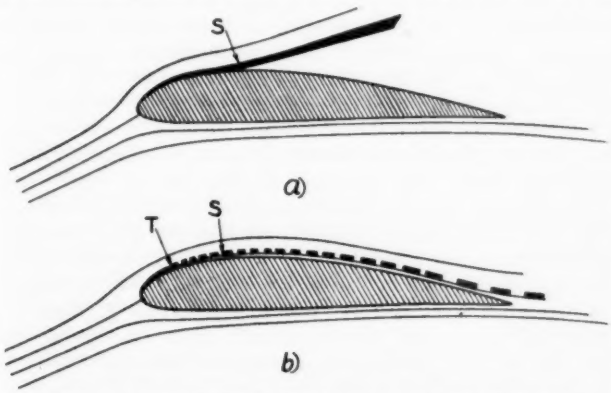


FIG. 2 ALTERNATIVE FLOWS ABOUT AN AIRFOIL SECTION

therefore obtain a curve giving the maximum C_L which can be obtained at any R for a given type of air flow. Again repeating for other values of R_{δ_c} we finally get a family of curves giving $C_{L\max} = f(R)$ for a series of flows characterized by different amounts of external turbulence.

3—BOUNDARY-LAYER NOTATION EMPLOYED

In the following sections we consider the application of the procedure just described to certain special cases. The laminar boundary-layer characteristics are determined using the boundary-layer theory given in the preceding paper by the authors (loc. cit.). The conventions adopted in the preceding paper will also be followed here and are the following: All quantities are dimensionless, lengths being divided by the characteristic length which is taken as the airfoil chord, and velocities being divided by the characteristic velocity taken as the free-stream velocity far from the airfoil. The basic Reynolds number R is then defined in terms of these characteristic quantities. The dimensionless potential velocity just outside of the boundary layer is denoted by U, the dimensionless distance measured along the upper surface of the airfoil from the stagnation point by x, and the dimensionless boundary-layer thickness by δ. R_δ, the boundary-layer Reynolds number, is defined in terms of the actual boundary-layer thickness and the actual potential velocity just outside of the boundary layer at an arbitrary point along the surface. The subscripts $(\ )_s$ and $(\ )_t$ denote, respectively, separation point (of the laminar layer) and transition point (from the laminar to the turbulent régime). R_{δ_c} is the value of R_δ at which the transition occurs, and is a constant for a particular external flow.

In the boundary-layer analysis mentioned above it is found convenient to introduce an alternative variable, in addition to x, to give the distance downstream from the stagnation point along the airfoil upper surface. This variable is defined by $\varphi = \int_0^x U dx$ and is identical with the value of the external-flow potential function at the point x.

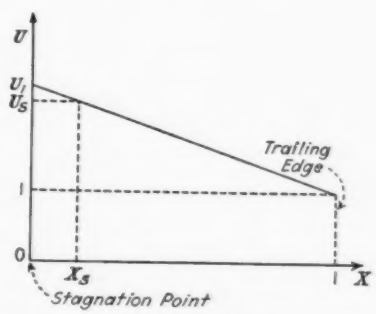


FIG. 3 THE SINGLE-ROOF PROFILE $U = U(x)$

In the paper mentioned, formulas are given for determining the separation point and the value of R_{δ_s} for the laminar boundary layer corresponding to any function $U^2(\varphi)$ which may be represented by one or two polynomials in φ. In the succeeding sections two general types of such flows are considered in detail.

4—THE "SINGLE-ROOF" PROFILE

The "single-roof" profile, represented in Fig. 3, is a first, crude approximation to the potential-flow distribution over the upper surface of an airfoil. It is developed by assuming that the velocity is a linear function of the distance downstream from the stagnation point, while the rapid increase in the velocity from zero to its maximum value U_1 is replaced by a discontinuous jump. It is also assumed that the stagnation point coincides with the leading edge; so that $x = 1$ corresponds to the trailing edge. It will be noticed that the velocity at the trailing edge has been taken equal to the undisturbed free-stream velocity, so that U(trailing edge) = 1. The latter restriction was imposed after a study of the calculated potential-velocity distributions around several airfoils, and is in good accordance with the corresponding experimental velocity distributions.

The single-roof profile has two advantages, first the boundary-layer calculations are especially simple, and second, plausible assumptions lead to a very easy determination of C_L as a function of the maximum velocity U_1. The assumption which was made in the latter connection was that the upper surface contributes all of the lift, i.e., the average pressure over the bottom surface is the same as that at the upper-surface trailing edge, which in turn equals the static pressure far from the airfoil. From this assumption, and using the Bernoulli theorem, we easily obtain the relation

$$C_L = \frac{1}{3}(U_1 - 1)(U_1 + 2) \quad [1]$$

connecting C_L and the maximum velocity U_1.

Turning now to the application of the boundary-layer analysis to this single-roof profile, we shall make use of results presented in the previous paper by the authors (loc. cit.), in which the laminar boundary-layer theory is worked out in some detail for this example. Equations [38] and [41] in the paper cited, furnish the result that the laminar separation point occurs at the position along the surface where $U_1^2/U^2 = 1.241$, i.e.,

$$U_s = 0.897\, U_1 \quad [2]$$

In addition, Equations [44] give the value of the boundary-layer Reynolds number at this separation point as

$$R_{\delta_s} = 2.18 \frac{U_1}{\sqrt{U_1 - 1}} \sqrt{R} \quad [3]$$

These are the only results of the laminar boundary-layer analysis which we shall need in this section.

If U_s as given by Equation [2] is less than 1 there is obviously no separation point on the airfoil surface. Hence $U_1 = 1/0.897 = 1.115$ is a limiting value below which the flow always clings to the airfoil without separation. The corresponding value of C_L from Equation [1] is $C_L = 0.120$. Hence $C_{L\max} = 0.120$ is a lower limit which can always be reached for a flow without separation, no matter how small the Reynolds number. (The Reynolds number must, of course, be large enough so that the basic assumptions of the boundary-layer theory remain valid.) If the possibility of a turbulent boundary layer were excluded this would also be the largest possible value of $C_{L\max}$ according to our simplified assumptions for the phenomenon as they were outlined in section 2, "Outline of the Basic Procedure." However, the assumptions already discussed regarding the transition to and behavior of a turbulent boundary layer alter the matter completely, and require a more elaborate analysis to obtain values of $C_{L\max}$. In the following paragraph we shall outline the calculations leading to the determination of these values.

We remember that for a given degree of turbulence in the external flow there exists a definite value of R_{δ_c}. For a fixed value of U_1, (i.e., of C_L) $R_{\delta_s} = R_{\delta_c}$ is a limiting case such that this C_L is just achieved. If R is decreased R_{δ_s} is decreased and the assumed value of C_L is not reached. Conversely, if R is increased the assumed C_L will always be realized. In order to determine this limiting case we actually calculate $R_{\delta_s}/\sqrt{R}$ as a function of C_L from Equations [1] and [3]. Then putting R_{δ_s} equal to a series of numerical values R_{δ_c}, we obtain a corresponding series of functions $R(C_L)$ or $C_L(R)$. The curves of C_L vs. R determined in this way are really curves of $C_{L\max}$ vs. R for various degrees of external turbulence, since each curve gives a definite value of R above which a given value of C_L will always be reached and below which it can never be reached. The results of such a calculation are plotted in Fig. 4, the regions in which a given C_L can be attained being indicated by the cross-hatching at one side of the curves. The values of R_{δ_c} are given by the numbers without subscripts and represent various degrees of external turbulence. Thus 1000 corresponds to a high degree of turbulence and 10,000 to a low degree.

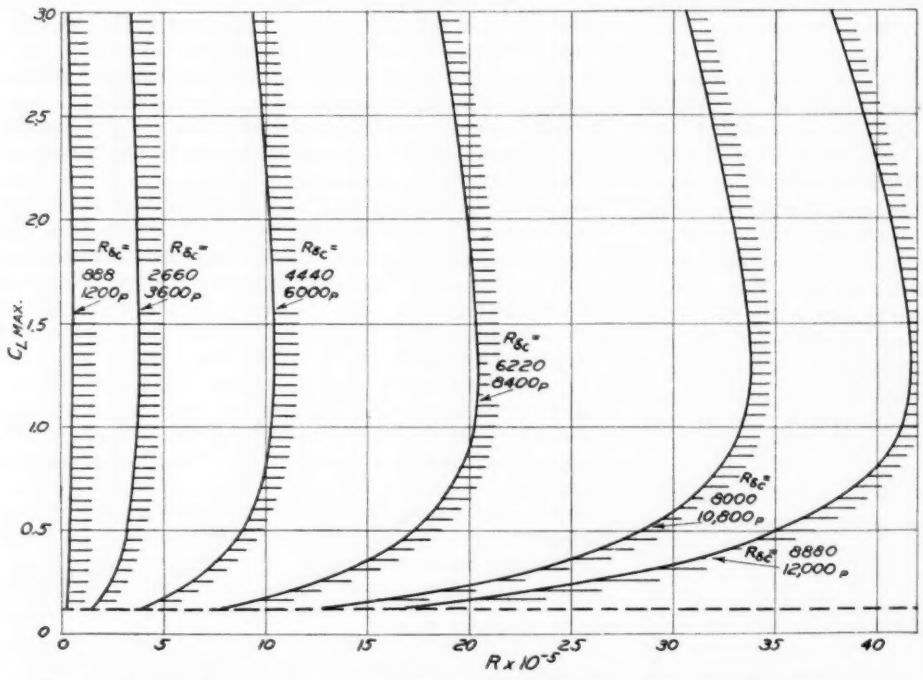


FIG. 4 $C_{L\max}$ AS FUNCTION OF R AND EXTERNAL TURBULENCE FOR THE SINGLE-ROOF PROFILE

The results are very interesting in that they reveal an extremely large dependence of $C_{L\max}$ on external turbulence, so that our basic conception of the phenomenon is, to some extent, justified. However, the dependence on Reynolds' number is entirely unsatisfactory and indicates that the single-roof profile is too crude an approximation to yield any quantitative results.

Before passing on to the refinements required to improve the results, it might be mentioned that the familiar Pohlhausen boundary-layer method may be simply applied to this case. The boundary-layer analysis has already been given by the authors (loc. cit.), and the application of the results to the $C_{L\max}$ problem follows exactly the procedure just discussed. It is easy to see that identically the same curves are obtained as followed from our boundary-layer theory. The only difference is that the values of R_{δ_c} associated with a particular curve are different for the two methods. The Pohlhausen values have been included in Fig. 4 with a subscript $(\ )_p$.

5—THE "DOUBLE-ROOF" PROFILE AND ITS APPLICATION TO THE MAXIMUM-LIFT ANALYSIS OF THE N.A.C.A. 2412 AIRFOIL

It has been mentioned above that the authors' boundary-layer analysis has been developed for external flows such that $U^2(\varphi)$ may be represented by one or two polynomials in φ. The single-roof profile corresponds to one of the simplest cases of such a flow, i.e., U^2 is given by a single linear function of φ. In the present section we consider a somewhat more complicated case in which U^2 is given by two linear functions of φ, and there is an

actual stagnation point at the origin of x. Such a flow may be described analytically as follows:

$$\left.\begin{aligned} &\text{For } 0 \leqslant \varphi \leqslant \varphi_1; \quad U^2 = b\varphi \\ &\text{For } \varphi_1 \leqslant \varphi; \quad U^2 = (b+\beta)\,\varphi_1 - \beta\varphi \end{aligned}\right\} \ldots\ldots [4]$$

where φ_1 is a fixed value of φ, and b and β are positive constants. We call such a velocity distribution a "double-roof" profile, the reason being obvious from Fig. 5 where the profile has been represented schematically by the full lines.

For our present purposes we need only two results of the boundary-layer analysis, which, just as in the case of the single-roof profile, are the position of the laminar separation point φ_s, and the value of the boundary-layer Reynolds number at this point, R_{δ_s}. From section 14 of the earlier paper we obtain

$$\left.\begin{aligned} \varphi_s &= \varphi_1\,\theta_s \\ R_{\delta_s}/\sqrt{R} &= \sqrt{\varphi_1}\,\Delta^*_s \end{aligned}\right\} \ldots\ldots [5]$$

where θ_s and Δ^*_s are given in Fig. 6 as functions of $r = \beta/b$. The necessary laminar boundary-layer characteristics are accordingly given in terms of the double-roof parameters φ_1 and r.

The experimental investigation of the maximum-lift phenomenon mentioned at the beginning of the paper dealt with wind-tunnel measurements on a rectangular airfoil with the N.A.C.A.

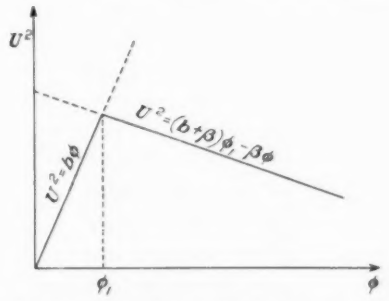


FIG. 5 THE DOUBLE-ROOF PROFILE, $U^2 = U^2(\varphi)$

2412 profile whose ordinates and general characteristics have previously been published.[6] It was natural, therefore, to base the theoretical considerations upon the same airfoil section, which is a very efficient one of moderate thickness and small camber. Accordingly the potential-velocity distribution over the 2412 profile for a two-dimensional flow was calculated by Theodorsen's method for a series of values of airfoil-lift coefficient C_L. Since Theodorsen's method has been described in extenso[7] no details of the calculation will be repeated here. For the problem in hand we require only the velocity distribution over the upper surface of the airfoil for some distance downstream of the separation point. The functions $U^2(\varphi)$ determined for this region are given by the full lines of Fig. 7, corresponding to calculated values of C_L from 0.8 to 1.8. For the sake of clearness the curves are not extended quite down to the stagnation point origin.

It appears that all of the functions $U^2(\varphi)$ given in Fig. 7 can be fairly satisfactorily approximated by double-roof profiles for the region upstream from the separation point. Such an approximation seems to be especially justified by the fact that the main purpose of the investigation is to discover the physical bases for the variation of $C_{L\max}$ with turbulence and Reynolds' number rather than to deduce the precise quantitative dependence of

[6] "Tests of N.A.C.A. Airfoils in the Variable Density Wind Tunnel, Series 24," by E. N. Jacobs and K. E. Ward. N.A.C.A. Technical Note No. 404, January, 1932.

[7] "Theory of Wing Sections of Arbitrary Shape," by Theodore Theodorsen. N.A.C.A. Technical Report No. 411 (1932).

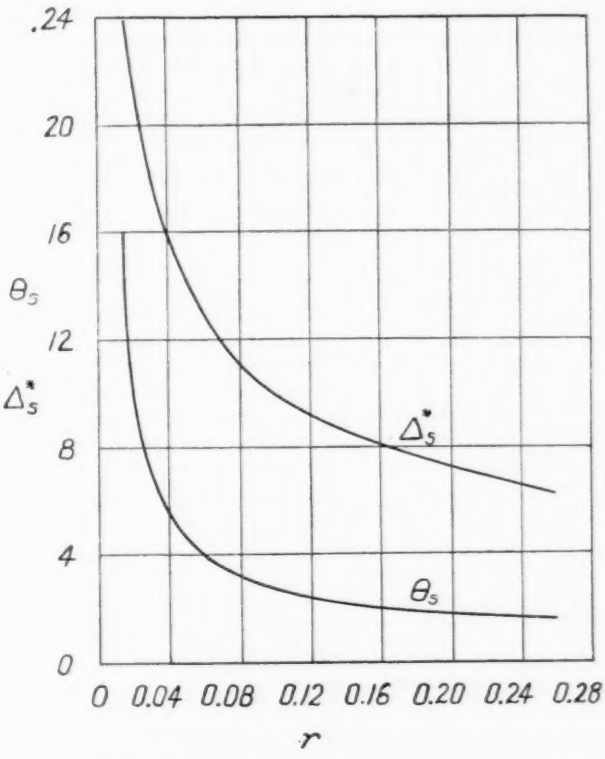


FIG. 6 GENERAL FUNCTIONS GIVING SEPARATION-POINT CHARACTERISTICS FOR THE DOUBLE-ROOF PROFILE

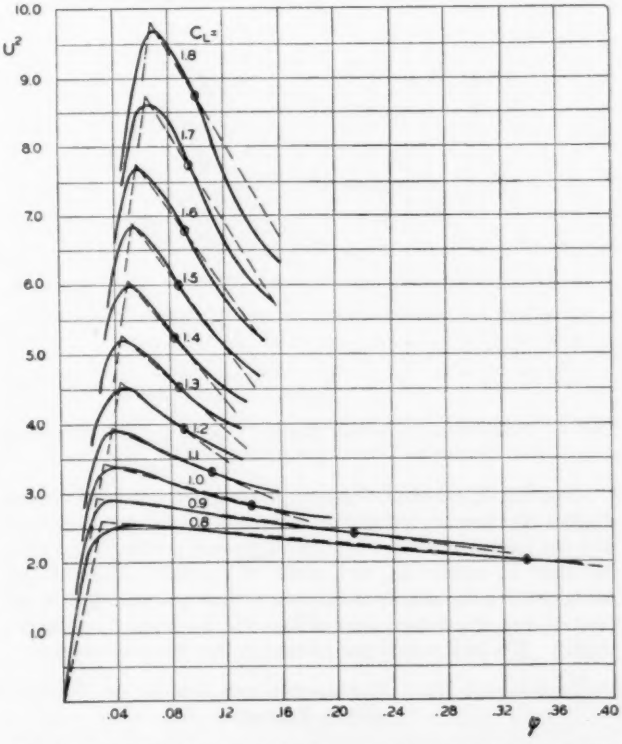


FIG. 7 POTENTIAL-VELOCITY DISTRIBUTIONS FOR THE N.A.C.A. 2412 AIRFOIL, AND THE APPROXIMATING DOUBLE-ROOF PROFILES

$C_{L\max}$ on these two variables. All the further calculations are, therefore, made with double-roof profiles chosen so as to approximate as closely as possible to the solid curves of Fig. 7 between the stagnation and separation points. The double-roof profiles are indicated by the dotted lines in Fig. 7, and the oval symbols give the values of φ_s as determined from Equations [5] and Fig. 6. $R_{\delta_s}/\sqrt{R}$, obtained in the same way, is plotted as a function of C_L in Fig. 8.

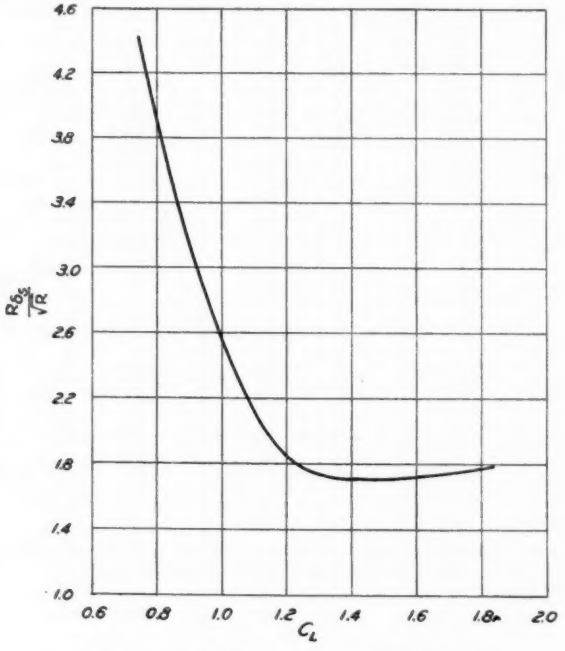


FIG. 8 $R_{\delta_s}/\sqrt{R}$ VS. C_L FOR 2412 AIRFOIL

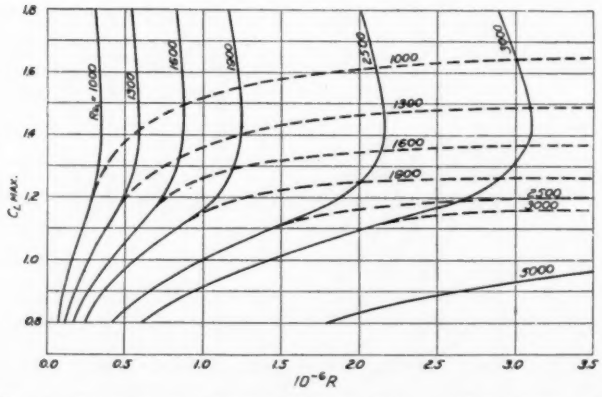


FIG. 9 THEORETICAL CURVES GIVING $C_{L\max}$ AS FUNCTION OF R AND EXTERNAL TURBULENCE FOR THE 2412 AIRFOIL

Using Fig. 8, curves of $C_{L\max}$ vs. R are determined for various degrees of external turbulence in exactly the same manner as was outlined in section 4 for the single-roof profile. It is only necessary to equate R_{δ_s} to a series of constant values of R_{δ_c}, characterizing the degree of external turbulence, and then to determine from Fig. 8 the values of R at which a given C_L is just attained. The final results are plotted in Fig. 9 as solid curves.

6—COMPARISON WITH EXPERIMENT AND REGION OF VALIDITY OF THE THEORY

Before discussing the theoretical results, it will be desirable to consider the results of the experimental investigation of the same problem, to which reference has already been made. As has been stated, the experiments were performed in the Guggenheim Aeronautics Laboratory at the California Institute of Technology wind tunnel on a model of the N.A.C.A. 2412 airfoil. In order to obtain a series of degrees of turbulence a wire grid or screen was placed at a number of distances upstream from the model. With the grid close to the model the degree of turbulence at the airfoil is high, with the grid far away the turbulence is low. At each position of the grid measurements were taken at from six to fifteen different Reynolds' numbers. The results as far as $C_{L\max}$ is concerned are presented in Fig. 10. Each curve corresponds to a grid position, i.e., to a given degree of turbulence and therefore

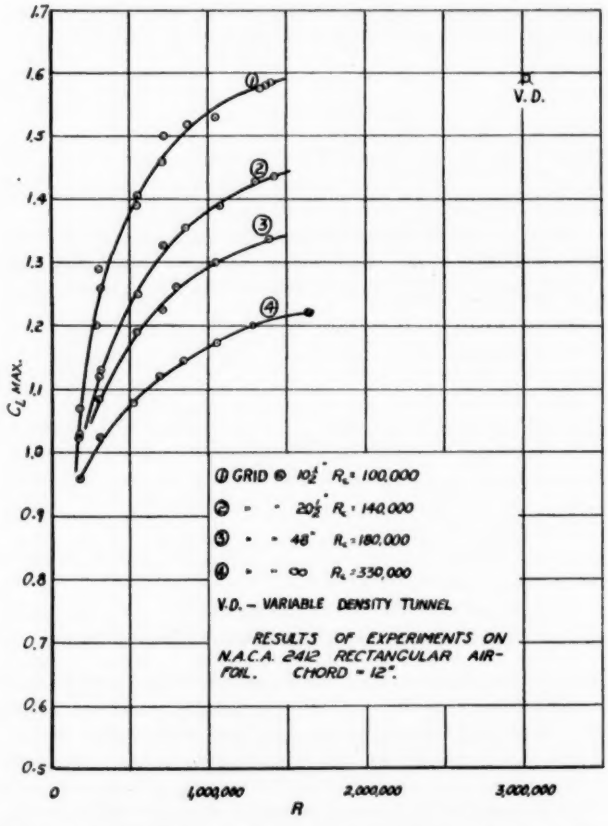


FIG. 10 EXPERIMENTAL CURVES CORRESPONDING TO FIG. 9, TAKEN FROM WIND-TUNNEL TESTS ON A 2412 AIRFOIL

to a definite value of R_{δ_c}. In other words, the curves are directly comparable with those of Fig. 9. It should be noted that the degree of turbulence for each of the curves of Fig. 10 has been indicated by giving the critical Reynolds number R_c, at which a sphere has a drag coefficient[8] $C_D = 0.3$. Just as in the case of R_{δ_c}, a low value of R_c corresponds to a high degree of turbulence and vice versa.

Comparing the two figures we see that for $C_{L\max} \leq 1.2$ the theoretical curves for $1000 \leq R_{\delta_c} \leq 1900$ exhibit a very similar character to the four experimental curves. In this region then the theory gives a qualitatively satisfactory prediction of the dependence of $C_{L\max}$ on R and free-stream turbulence. Above $C_L = 1.2$ the theory diverges more and more from experiment. In section 2 it was pointed out that one of the basic assumptions of the theory, namely, that of the non-existence of a separation

[8] "Effect of Turbulence in Wind Tunnel Measurements," by H. L. Dryden and A. M. Kuethe. N.A.C.A. Technical Report No. 342, 1929.

point for a turbulent boundary layer, implied that infinitely large values of $C_{L_{max}}$ could theoretically be reached. It is obvious that this assumption represents a considerable oversimplification, since it is quite certain that somewhat similar laws actually govern the separation of laminar and turbulent boundary layers. The portions of the theoretical curves which are concave upward or have negative slopes are believed to be in error because of this oversimplification in the basic assumptions of the theory. The dotted curves of Fig. 9 represent the authors' judgment as to the nature of the curves which would be obtained if it were possible to determine the separation-point characteristics of a turbulent boundary layer. The relatively low values of R_{δ_e} for the theoretical curves which furnish the best approximation to the experimental results are not at all surprising when the unstable nature of boundary-layer profiles near a separation point is remembered.

The comparatively good agreement between the theoretical and experimental curves for moderate values of $C_{L_{max}}$ and R, as well as the quite understandable nature of the discrepancies at higher values of $C_{L_{max}}$ or R, seems to demonstrate conclusively that the physical basis for the large dependence of $C_{L_{max}}$ on these two parameters, in the normal wind-tunnel range, has been correctly determined as residing in a kind of contest between the separation point of the laminar boundary layer over the upper surface of an airfoil, and the transition point to a turbulent boundary-layer régime. This is physically the same picture as was used by Prandtl many years ago to explain the phenomenon of the sudden drop in the drag coefficient of a sphere and its dependence on external turbulence and Reynolds' number.

The extension of the present theory so as to obtain valid results for large values of $C_{L_{max}}$ and of R depends upon the development and application of a satisfactory theory of the separation characteristics of a turbulent boundary layer, and such a theory has, unfortunately, not yet been developed. The discussion of airfoils whose lift does not increase approximately linearly with angle of attack up to the stall would appear to present still further difficulties, but such airfoils are not widely used in practice and consequently are not of great immediate interest.[9]

7—Conclusions

In the present paper the application of a laminar boundary-layer theory previously developed by the authors to the problem of the maximum-lift coefficient of airfoils is discussed. The calculations are carried through in detail for a first approximation, called a single-roof profile, to the potential velocity distribution over the upper surface of an airfoil. The results indicate a large variation in $C_{L_{max}}$ with turbulence but the quantitative dependence on Reynolds' number and turbulence is not satisfactory. The calculations are then repeated for a so-called double-roof profile which approximates to the flow over the upper surface of an N.A.C.A. 2412 airfoil. These results are compared with those obtained from an experimental investigation on the same airfoil. The agreement is considered to indicate that for moderate values of R and $C_{L_{max}}$ the phenomenon of the maximum-lift coefficient is controlled by a contest between the separation and transition points of the laminar boundary layer over the nose of the airfoil. Finally, the difficulties involved in extending the theory to larger values of R, or to airfoils whose $C_{L_{max}}$ vs. α curves are not approximately linear up to the stall, are mentioned.

[9] It might be mentioned that, since the present paper was written, additional experiments have been made and reported by one of the present authors: ("Further Experiments on the Variation of the Maximum-Lift Coefficient With Turbulence and Reynolds' Number," by Clark B. Millikan, A.S.M.E. Transactions, November, 1934.) These experiments were made with rather unconventional airfoil profiles, whose lift curves had considerable curvature below the stall. As might have been expected, the dependence of $C_{L_{max}}$ on Reynolds' number and turbulence was quite different from that for conventional airfoils like the 2412. The desirability of an extension of the present theory in order to explain the maximum-lift phenomenon for some unconventional profiles has, therefore, been experimentally demonstrated.

Review of Recent Research Work in Fluid Mechanics

By Th. von KÁRMÁN,[1] W. BOLLAY,[2] and A. J. IPPEN[3]

I—General

DURING the period reviewed, the first volume of the "Aerodynamic Theory" edited by W. F. Durand (1)[4] was published. This contains a section on the mathematical aids necessary for hydrodynamics and aerodynamics by W. F. Durand, a presentation of general fluid dynamics by W. F. Durand and M. Munk, and an historical sketch of the development of aerodynamics by R. Giacomelli.

In the magazine *Science* W. F. Durand (2) gave an article on the development of our knowledge of fluid dynamics. A. Eula (3) discussed the present state and aims of the experimental research in aerodynamics. J. Bukowski (4) published a book on the experimental technique used in aerodynamics. The book of H. Richter (5) deals with the fundamental laws and the results of experimental research concerning flow in pipes. R. Spronck (6) discussed in a review the general ideas of hydraulics.

II—Potential Flow

W. G. Bickley (7) calculated the two-dimensional potential flow around obstacles of rectangular shape. L. Dufour (8) investigated the flow produced in a perfect fluid by movement of an elliptic cylinder. B. Caldonazzo (9) computed the non-stationary potential flow produced by movement, especially by oscillation of a plane airfoil and found some generalized form of the well-known Kutta-Joukowski theorem. B. Demtchenko (10) gave a proof of the validity of the latter theorem for compressible fluids; in his publication he referred to an old and practically unknown paper of M. A. Tchapliguine, in which this author successfully extended the Helmholtz-Kirchhoff method of free jets to compressible fluids. C. Jakob (11) applied Tchapliguine's method to calculation of the pressures acting on an obstacle in a compressible field.

III—Laminar Flow in Viscous Fluids

J. N. Goodier (12) pointed out the analogous character of the differential equation for the stream function in the case of the two-dimensional flow of a viscous fluid and of the equation for Airy's function and showed that this analogy can be used for practical solution of some problems. W. Müller (13) investigated the analogy between heat and vorticity transfer in the case of laminar motion. In a second paper W. Müller (14) investigated some problems of non-stationary flow in viscous fluids. H. Schlichting (15) calculated the expansion of a fluid jet in the case of laminar motion. M. Lippke (16) gave a contribution to the theory of capillar-tube viscosimeters, computing the necessary corrections (Hagenbach's correction). The case of the laminar flow of a viscous fluid entering into a pipe was treated by G. Vogelpohl (17). S. Goldstein and A. Fage (18) investigated theoretically and experimentally the case of two-dimensional steady flow of viscous fluid behind a solid body. They compared the results of the boundary-layer theory with those obtained by Filon's method and compared both calculations with the experiment. L. Loitzansky (19) gave a new method for calculation of the separation point in the case of a laminar boundary layer. H. J. Luckert (20) gave a method for integration of the equations of the laminar boundary layer, using the transformation of the equations by von Mises. N. A. V. Piercy and H. F. Winny (21) calculated the case of the laminar boundary layer along a flat plate comparing the results obtained by Oseen's method with those obtained by Prandtl and Blasius. C. W. Oseen (22) developed further his well-known method for integration of the equations of viscous fluids. T. E. Garstang (23) applied Oseen's approximate equations to the flow of a viscous fluid past spinning bodies. T. Gustafson (24) applied the theory of Oseen and Zeilon to the flow around a rotating cylinder, especially in the case of vanishing viscosity. H. B. Squire (25) investigated the problem of the flow of a fluid with vanishing viscosity ($\lim \nu \to 0$) from a general point of view, especially the location of the point of separation in this limiting case.

The stability of laminar flow in viscous fluid was the subject of a second paper by H. B. Squire (26). He investigated the stability of the flow between two parallel walls for three-dimensional disturbances and showed that this case can be reduced to that of two-dimensional disturbances.

IV—Turbulence and Skin Friction

Research on turbulence made considerable progress during the last year, the practical importance of the turbulence becoming more fully realized. As to publications giving a review of the state of knowledge of the subject and recent advances, the articles of von Kármán (27), Dryden (28), Fage (29), and Loitzansky (30) are to be mentioned. J. M. Burgers (31) continued his researches on the application of statistical methods to the turbulence problem. W. Tollmien (32) made a critical investigation on Burger's methods. G. D. Mattioli (33) advanced a new theory of the turbulent interchange with several applications; a paper by G. Dedebant, Ph. Schereschewsky, and Ph. Wehrle (34) deals with statistical similitude of the mechanism of turbulence.

Papers dealing with special problems in the field of turbulence: H. Kurzweg (35) and L. Schiller (36) investigated the origin of turbulent flow experimentally. H. Schlichting (37) continued his theoretical investigations of the development of turbulence in a boundary layer along a flat plate. J. L. Synge (38) investigated the statistics of heterogeneous fluids rotating between two cylinders. The articles of A. M. Kuethe and F. L. Wattendorf (39), H. Reichardt (40), A. Fage (41), L. F. G. Simmons and C. Salter (42), and H. C. H. Townend (43) contain chiefly measurements on the magnitude and distribution of turbulent velocity fluctuation. E. Förthmann (44) investigated the intermingling of turbulent flows with different velocities (theory of jets).

F. Wendt (45) published measurements on turbulent flow between rotating cylinders. A. Thom and S. R. Sengupta (46) measured the torque acting on a cylinder which rotates in an air stream. R. J. Cornish's (47) article deals with flow through fine eccentric clearances. R. Gregorig (48) measured the pressure drop in straight and curved pipes in the case of large Reynolds

---

[1] Director, Guggenheim Aeronautics Laboratory, California Institute of Technology. Mem. A.S.M.E.

[2] Research Fellow in Aeronautical Engineering, California Institute of Technology.

[3] Research Fellow in Mechanical Engineering, California Institute of Technology.

[4] This and similar numbers in parentheses refer to the bibliography at the end of the paper.

number. O. Fritzsche and H. Richter (49) measured fluid resistance in curved pipes. Chr. Kentner (50) investigated losses, pressure and velocity distribution in a vertical bend. Also a paper of O. Miyagi (51) deals with flow in curved pipes. J. Allen (52) and P. Vuille (53) made investigations on laminar and turbulent flow in open channels. The latter investigated especially the influence of roughness. B. Randisi's paper (54) deals with the influence of turbulence on aerodynamic characteristics of airfoils; E. Ower and R. Warden (55) criticized the methods of producing artificial turbulence in wind tunnels. H. Bouasse (56) investigated the behavior of turbulent fluid jets in several cases, especially their reaction on obstacles. J. A. Kiebel (57) gave a review of the problem of atmospheric turbulence; H. Köhler (58) published special investigations on atmospheric turbulence; H. Ertel (59) investigated magnitude and distribution of the turbulent exchange coefficient in layers near the ground.

K. E. Schoenherr (60) reviewed the experimental material concerning the frictional resistance of flat surfaces moving through a fluid, gave a new formula for the skin-friction based on von Kármán's turbulence theory and contributed interesting experimental facts to the problem of the transition between laminar and turbulent boundary layer. A. Fage's paper (61) deals with the skin-friction of flat plates below the critical Reynolds number. L. Prandtl and H. Schlichting (62) published calculations on the skin friction of rough plates. F. Gebers (63) investigated the influence of the shape of plates on their frictional resistance. H. F. Nordström (64) discussed the problem of frictional resistance of ships.

V—Vortex Motion

Quite a number of articles published in the last year dealt with the generation and motion of vortex systems. F. Tricomi (65) tried to give a new illustrative interpretation of the curl in fluid motion. A. Masotti's paper (66) deals with the general theory of vortex-source combinations. W. B. Morton (67) and A. Kneschke (68) discussed the points of relative rest of parallel vortices. E. Laura (69), W. W. Golubeff (70), and L. S. Da Rios (71) discussed the stability of systems of vortex filaments in different arrangements. E. Paul (72) and A. Miyadzu (73) calculated the motion of a vortex near different boundaries. V. Garten's paper (74) deals with the motion of a vortex column with a cross-section of finite area. A. Ratib (75) calculated the effect of viscosity on eddies. N. W. Akimoff (76), O. Nomoto, Y. Sakagami, and Y. Fujii (77), and finally H. F. Winny (78) published experimental investigations on vortex systems; the latter investigated the interesting case of a vortex system generated behind a sphere moving in a fluid. J. Letzmann (79) made interesting theoretical and experimental investigations regarding atmospheric vortices.

VI—Waves and Ship Resistance

Miss M. L. Dubreil-Jacotin's paper (80) deals with the exact determination of the shape of waves with finite height. M. Schuler (81) investigated breaking of waterwaves on the ocean. A. Dimpker (82) investigated the waves produced by the oscillation of a plate on the water surface. E. G. Barrillon (83) gave an introduction to the theory of wave resistance which is worth mentioning. C. Wigley (84) gave a résumé of the advance made since 1930 in the calculation of the resistance and shape of waves generated by the motion of a ship. Th. von Kármán (85) reported at the Cambridge Congress on (*a*) the minimum wave-resistance of cylindrical bodies, (*b*) the resistance of a gliding plane surface of infinite span and (*c*) the resistance of a gliding plane surface with finite aspect ratio. T. H. Havelock (86) made new contributions to the calculation of wave resistance. L. Sretenskij's paper (87) deals with the problem of a flat plate gliding on the water surface. H. Wagner (88) gave a general review of the problems of gliding of bodies over a water surface. G. Weinblum (89) investigated the wave resistance of boats with rounded shapes. W. Graff (90) investigated the wake resistance of such shapes. S. Truscott (91) gave a description of the new high-speed towing basin for testing of seaplane float models. L. J. Coombes (92) presented data concerning wall interference, depth effect, and scale effect in tank tests on sea-plane models. H. E. Saunders and A. S. Pitoe (93) reported on full-scale trials of a destroyer, H. E. Saunders (94) on tests with three geometrically similar ship models, K. Helm (95) on model test of a river boat in flowing water. O. Schlichting (96) gave a method to compute resistance of boats moving in shallow water. W. G. A. Perring (97) made a theoretical study of the stability of sea-planes and compared his results with those from model tests. S. Watanabe (98) continued his investigations on impact of a plate on the water surface; this problem has some importance for the correct landing of floats.

VII—Cavitation

Interesting experimental investigations on the mechanism of cavitation have been carried out under Hunsaker (99) at M.I.T., especially the flow through a diffuser being investigated. D. Riabouchinsky (100) published some considerations on the possible types of fluid motion with cavitation. E. F. Eggert (101) reported at the meeting of the Society of Naval Architects and Marine Engineers on experiments carried out in the U. S. Experimental Model Basin on propeller cavitation. P. de Haller (102) and H. Schröter (103) dealt with the problem of corrosion by cavitation.

VIII—Lift and Drag

Streamlining, Wing Theory. E. V. Huntington (104) investigated the air resistance of falling spheres. S. Nikitine (105) made some observations on possible reduction of the drag of a body in a liquid by producing a layer of air at the surface of the body. F. Westendorp (106) published investigations on resistance of railway train models. R. H. Heald (107) investigated the aerodynamic characteristics of automobile models. Other papers about streamlining of automobiles by W. E. Lay (108), M. C. Horine, P. Altman, H. G. Winter, E. G. Reid, R. H. Upson (109), and M. Platt (110) are to be mentioned.

J. E. Garrick (111) calculated the theoretical pressure distribution for a large number of airfoils, using Theodorsen's method. F. Weinig (112) calculated the influence of the shape of the leading edge on the pressure distribution in the case of symmetrical profiles. A. Betz (113) and J. Stüper (114) tried to develop methods which would facilitate the computation of the real pressure distribution on airfoils, i.e., the correction of the theoretical pressure distribution due to viscosity and boundary layer. M. Biot (115) calculated the induced drag of a wing with dihedral. L. Poggi (116) improved the present method of calculation of auto-rotative moments, taking into account the velocities induced by the trailing vortices. He found that the deviations from the results obtained by the usual "strip-method" are in many cases quite considerable. A. Dufaure de Lajarte (117) investigated critically the usual applications of the first order wing theory to the calculation of the down-wash behind a wing. W. S. Diehl (118) gave semi-empirical formulas for the distribution of the lift of biplanes. Two papers of C. Ferrari (119) deal with the theory of the biplane with infinite aspect ratio. Another paper of the same author deals with the interference between wing and propeller (120); the same subject has been treated by C. Wieselsberger (121). J. Vladea (122) investigated the interference between wing and fuselage.

Considerable work has been done on the subject of wind-tunnel

interference. H. Glauert (123), and C. N. H. Lock and F. C. Johansen (124) investigated the wind-tunnel interference on the drag of symmetrical, and especially streamlined bodies. S. Tomotika (125) and others calculated the influence of walls on the lift of a flat plate. L. Sackmann (126) investigated experimentally the flow past an obstacle between parallel plates. E. Pistolesi (127) solved the problem of a wing traversing a free jet; the same problem has been treated according to a different method by Stüper (128). The interference of the wind tunnel with rectangular cross-section was the subject of several papers—various authors obtaining partly conflicting results. The results of Th. Theodorsen (129) and L. Rosenhead (130) are partly erroneous; Toussaint (131) gave the correct calculations. The cause of the discrepancy was revealed by P. de Haller (132) in a paper presented to the International Congress for Applied Mechanics in Cambridge. A critical summary of wind-tunnel interference on wings, bodies, and air-screws was given by H. Glauert (133). C. B. Millikan (134) and E. F. Relf (135) reported on the effects of turbulence on variations in maximum-lift coefficient. E. Weinel (136), F. Numachi and J. Masuko (137), F. Weinig (138), L. Lazzarino (139), and I. Lotz (140) published theoretical investigations on the flow through lattices. K. Frey (141) investigated experimentally the reduction of energy losses in channels by a system of vanes.

IX—Compressible Fluids

F. Frankl (142) investigated theoretically the flow of compressible fluids in the case in which the velocity of the flow is of the same order as the velocity of the sound. A. Huber (143) calculated the oscillations in pressure and velocity of an ideal compressible fluid moving in an elastic pipe. The mathematical investigations of J. A. Kiebel (144) deal with the theory of the motion of compressible fluids taking into account the influence of gravity. A. F. Zahm (145) discussed the dynamics of very rare fluids with applications to the conditions in the stratosphere.

X—Experimental Methods in Fluid Mechanics

In this section some papers are collected in which the authors presented more or less new methods for flow-investigations or discussed rules for model investigations.

A. Favre (146) suggested a new method for the exploration of the velocity around obstacles, using colored fluid filaments oscillating with a given frequency; if the velocity of the fluid is variable along a streamline the wave length of the filament will be variable and the velocity can be estimated by measuring the wave length.

Several papers discussed the problem of model rules based on considerations of mechanical similarity. The papers of V. Joukovsky and P. Wolkoff (147), F. Gutsche (148), E. Engel (149), M. Lelli (150), and G. De Marchi (151) are to be mentioned.

T. B. Abell (152) used the Hele-Shaw method for photographical study of a ship's wake, U. Schmieschek (153) suggested some improvements for illuminating liquid currents for flow pictures. A. Fage (154) obtained a new kind of flow picture by means of the ultra-microscope. D. Riabouchinsky and V. Volkovisky (155) reported on their new experimental technique for visualizing air flow especially close to solid boundaries and for high, partly supersonic velocities. C. Sadron (156) showed that the velocity distribution in optically active viscous fluids can be measured by observing the double refraction of light due to the shearing stress in the liquid.

Several papers deal with improvements of the hot-wire anemometers for measuring magnitude and directions of the air velocities. M. Ziegler (157), J. Ulsamer (158), and T. C. Sen (159) tried to use the hot-wire for measuring the velocity of air and of water and found a method which is feasible at least for low velocities. L. S. Marks (160) suggested a new kind of pitot-tube suitable for the determination of the magnitude and direction of the velocity of a fluid. A. Égal (161) devised a thermoelectric method of measuring liquid flow by having one end of the thermocouple near the inside walls of a tube, the other end on the outside under a controlled temperature. When there is a fluid flow, a difference of potential arises which can be calibrated against the velocity. T. Sasaki and T. Yosida (162) designed speedometers for airplanes which indicate true air-speed independent of the altitude. A. H. Fette (163) reported on the use of a rotating chamber for a study of pressure and velocity distribution in rotating channels.

XI—Some Special Problems

In this section some special problems of technical interest are mentioned. A paper of M. Muskat (164) deals with the flow of a compressible fluid through porous media with application to the motion of natural gas through the earth. The paper of W. Hurst (165) deals with a similar problem. C. G. J. Vreedenburgh and O. Stevens (166) studied the flow of fluids through porous media by an electrodynamic analogy.

The paper of E. Tyler (167) deals with the instability of liquid jets taking into account the capillarity of the fluid. A. Weinstein (168) extended the theory of discontinuous potential motion introducing the effect of capillarity in the boundary connection of the free surface.

Several papers deal with the problem of motion of sediment in water and air. W. Barth (169) studied the motion of dust particles in pumps and ventilators. E. Kilb (170) calculated the upward speed of an air current necessary to support a hailstone. M. Biot (172) gave the equations of movement of a fluid holding particles in suspension. A. Schoklitsch (173) and M. P. O'Bryan (174) deal with the problem of bedload carried by water in channels and rivers.

BIBLIOGRAPHY

1 W. F. Durand, "Aerodynamic Theory," Berlin, Springer, 1934, 398 pp.

2 W. F. Durand, *Science*, 1933, vol. 78, pp. 343–351.

3 A. Eula, *Aerotecnica*, 1933, vol. 13, pp. 1021–1037, 1181–1192.

4 J. Bukowski, "Laboratoriumstechnik der aerodynamischen Messungen," Warszawa, Wydawnictwo Inst. Aerodyn., 1933, 290 pp.

5 H. Richter, "Rohrhydraulik," Berlin, Springer, 1934, 256 pp.

6 R. Spronck, Société royale des Sciences de Liège, Mémoires, 1933, vol. III, s. 18, no. 2, pp. 1–126.

7 W. G. Bickley, Proceedings, London Mathematical Society, II, 1934, vol. 37, pp. 82–105.

8 L. Dufour, Mathesis, recueil mathématique, 1933, vol. 47, pp. 353–358; 1934, vol. 48, pp. 212–217.

9 B. Caldonazzo, Atti della Pontificia Accademia delle Scienze nuovi Lincei, 1934, vol. 87, pp. 273–322.

10 B. Demtchenko, Publication Mathematique de l'Universitee, Belgrade, 1933, vol. 2, pp. 85–105.

11 C. Jakob, Comptes Rendus, Institute de France, Académie des Sciences, 1933, vol. 197, pp. 125–126.

12 J. N. Goodier, *Philosophical Magazine*, 1934, vol. 17, pp. 554–576, pp. 800–803.

13 W. Müller, *Annalen der Physik*, V. F., 1934, vol. 19, pp. 809–828.

14 W. Müller, *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 395–408.

15 H. Schlichting, *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 260–263.

16 M. Lippke, *Zeitschrift für technische Physik*, 1933, vol. 14, no. 10, pp. 412–418.

17 G. Vogelpohl, *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 446–447.

18 S. Goldstein and A. Fage, Proceedings, Royal Society of London, 1933, vol. 142, pp. 545–573.

19 L. Loitzansky, Applied Mathematics and Mechanics, 1933, vol. 1, pp. 38–46 (Russian, with German summary).

20 H. J. Luckert, Schriften des mathematischen Seminars und Instituts für angewandte Mathematik der Universität Berlin, 1933, vol. 1, pp. 245–274.

21 N. A. V. Piercy and H. F. Winny, Proceedings, Royal Society of London, 1933, vol. 140, pp. 543–561.

22 C. W. Oseen, *Arkiv för Mathematik Astronomi och Fysik*, 1934, vol. 24A, no. 20, pp. 1–23.

23 T. E. Garstang, Proceedings, Royal Society of London, 1933, vol. 142, pp. 491–508.

24 T. Gustafson, Lund, Dissertation, 1933, 89 pages.

25 H. B. Squire, *Philosophical Magazine*, 1934, vol. 17, pp. 1150–1160.

26 H. B. Squire, Proceedings, Royal Society of London, 1933, vol. 142, pp. 621–628.

27 Th. von Kármán, *Journal of the Aeronautical Sciences*, 1934, vol. 1, no. 1, pp. 1–20. Also paper presented at Cambridge Congress for Applied Mechanics, 1934, to be published shortly.

28 Dryden, National Advisory Committee for Aeronautics Report 497, 1934.

29 A. Fage, *Journal Royal Aeronautical Society*, 1933, vol. 37, pp. 573–600.

30 L. G. Loitzansky, Transactions of the Scientific Research Institute of Hydrotechnics, Moscow, 1933, vol. 9, pp. 1–30 (Russian, with English summary).

31 J. M. Burgers, Proceedings, Koninklijke Akademie van Wetenschappen te Amsterdam, 1933, vol. 36, no. 3, pp. 276–284; vol. 4, pp. 390–399; vol. 5, pp. 487–496; vol. 6, pp. 620–628.

32 W. Tollmien, *Physics*, 1933, vol. 4, pp. 289–290; *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 331–347.

33 G. D. Mattioli, Comptes Rendus, Institute de France, Académie des Sciences, 1933, vol. 196, pp. 1, 282; pp. 1866–1868; Rendiconti del Seminario Matematico della R. Universita' di Padova, 1933, vol. 4, pp. 67–91; Atti della R. Accademia Nazionale dei Lincei, Rendiconti, 1933, vol. VI, s. 17, pp. 932–934.

34 G. Dedebant, Ph. Schereschewsky, and Ph. Wehrle, Comptes Rendus, Institut de France Académie des Sciences, 1934, vol. 198, pp. 1571–1573.

35 H. Kurzweg, *Annalen der Physik*, 1933, vol. 18, pp. 193–216.

36 L. Schiller, *Zeitschrift für angewandte Mathematik und Mechanik*, 1934, vol. 14, pp. 36–42.

37 H. Schlichting, *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, 1933, vol. II, no. 38, pp. 181–208.

38 J. L. Synge, Transactions, Royal Society of Canada, 1933, vol. III, no. 27, pp. 1–18.

39 F. L. Wattendorf and A. M. Kuethe, *Physics*, 1934, vol. 5, pp. 153–164.

40 H. Reichardt, *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 177–180.

41 A. Fage, *Journal Royal Aeronautical Society*, 1933, vol. 37, pp. 573–600.

42 L. F. G. Simmons and C. Salter, Proceedings Royal Society of London, 1934, vol. 145, pp. 212–234.

43 H. C. H. Townend, Proceedings, Royal Society of London, 1934, vol. 145, pp. 180–211.

44 E. Förthmann, *Ingenieur-Archiv.*, 1934, vol. 5, pp. 42–54.

45 F. Wendt, *Ingenieur-Archiv.*, 1933, vol. 4, pp. 577–595.

46 A. Thom and S. R. Sengupta, Great Britain Air Ministry, Aeronautical Research Committee, Reports and Memoranda 1520, 1933, pp. 1–7.

47 R. J. Cornish, *Philosophical Magazine*, 1933, vol. 16, pp. 897–912.

48 R. Gregorig, Zürich, Dissertation, 1933, 71 pp.

49 O. Fritzsche and H. Richter, *Forschung auf dem Gebiete des Ingenieurwesens*, 1934, vol. 4, pp. 307–314.

50 Chr. Kentner, *Zeit. V.D.I.*, 1933, vol. 77, pp. 1205–1209.

51 O. Miyagi, Tohoku Imperial University, 1933, vol. 11, pp. 1–15.

52 J. Allen, *Philosophical Magazine*, 1934, vol. VII, s. 17, pp. 1081–1112.

53 P. Vuille, Berlin, Dissertation, 1933, 49 pp.

54 B. Randisi, *Aerotecnica*, 1933, vol. 13, pp. 867–889.

55 E. Ower and R. Warden, Great Britain Air Ministry, Aeronautical Research Committee, Reports and Memoranda 1559, 1934, pp. 1–8.

56 H. Bouasse, *Annales de la Faculté des Sciences*, University of Toulouse, 1933, vol. III, s. 25, pp. 1–39, *Annales de Physique*, 1933, vol. 19, pp. 89–228.

57 J. A. Kiebel, Transactions of the Scientific Research Institute of Hydrotechnics, Moscow, 1933, vol. 10, pp. 50–59 (Russian).

58 H. Köhler, *Svenska Detensk*, Akad. Hdl., 1933, pp. 1–54.

59 H. Ertel, *Meteorologische Zeitschrift*, 1933, vol. 50, pp. 386–388.

60 K. E. Schoenherr, Society Naval Architects and Marine Engineers. Transactions, 1933, vol. 40, pp. 279–313.

61 A. Fage, Great Britain Air Ministry, Aeronautical Research Committee, Reports and Memoranda 1580, 1934, pp. 1–7.

62 L. Prandtl and H. Schlichting, *Werft, Reederei, Hafen*, 1934, vol. 15, pp. 1–4.

63 F. Gebers, *Schiffbau, Schiffahrt und Hafenbau*, 1933, vol. 34, pp. 252–256.

64 H. F. Nordström, *Teknisk, Tidskrift*, 1933, A. 63, Heft 42, Skepps byggnads Konst, 1933, no. 10, pp. 71–86.

65 F. Tricomi, Atti della R. Accademia Nazionale dei Lincei, Rendiconti, 1934, vol. VI, s. 19, pp. 399–401.

66 A. Masotti, Rendiconti R. Istituto di Scienze e Lettre di Lombardo, 1933, vol. II, s. 66, pp. 386–398.

67 W. B. Morton, Proceedings, Royal Irish Academy, Dublin, 1933, 41 A, vol. 9, pp. 94–101.

68 A. Kneschke, *Zeitschrift für angewandte Mathematik und Mechanik*, 1934, vol. 14, pp. 178–183.

69 E. Laura, Atti R. Istituto Veneto di Scienze, Lettere e Arti, 1933, vol. 92, pp. 387–398.

70 W. W. Golubeff, Recueil Mathematique de Moscow, 1933, vol. 40, no. 1, pp. 73–85.

71 L. S. Da Rios, Atti della R. Accademia Nazionale dei Lincei, Rendiconti, 1933, vol. VI, s. 17, pp. 924–926.

72 E. Paul, *Zeitschrift für angewandte Mathematik und Mechanik*, 1934, vol. 14, pp. 105–116.

73 A. Miyadzu, *Philosophical Magazine*, 1933, vol. 16, pp. 553–562.

74 V. Garten, Mathematische Annalen, 1934, vol. 109, pp. 445–464.

75 A. Ratib, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 575–576.

76 N. W. Akimoff, *Schiffsbautechnische Gesellschaft*, Berlin, Jahrbuch, 1934, vol. 35, pp. 149–163.

77 O. Nomoto, Y. Sakagami and Y. Fujii, Proceedings of the Physico-Mathematical Society of Japan, 1934, vol. 16, pp. 16–34.

78 H. F. Winny, Great Britain Air Ministry, Aeronautical Research Committee, Reports and Memoranda 1531, 1933, pp. 1–14.

79 J. Letzmann, *Gerlands Beiträge zur Geophysik*, 1933, vol. 39, pp. 167–205.

80 Miss M. L. Dubreil-Jacotin, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 818–821.

81 M. Schuler, *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 443–446.

82 A. Dimpker, *Annalen der Physik*, 1934, vol. 19, no. 3, pp. 225–251.

83 E. G. Barrillon, *Science et Industr., Métallurg.*, etc., 1933, vol. 17, pp. 136–140, 200–203, 292–294.

84 C. Wigley, Paper read at Cambridge Congress. To be published shortly. 1934.

85 Th. von Kármán, Paper read at Cambridge Congress. To be published shortly. 1934.

86 T. H. Havelock, Proceedings, Royal Society of London, 1934, vol. 144, pp. 514–521.

87 L. Sretenskij, Bulletin de l'Academie des Sciences de l'U.S.S.R., 1934, vol. 7, no. 6, pp. 817–834.

88 H. Wagner, Paper read at Cambridge Congress. To be published shortly. 1934.

89 G. Weinblum, *Schiffbautechnische Gesellschaft*, Berlin, Jahrbuch, 1934, vol. 35, pp. 164–192.

90 W. Graff, *Schiffbautechnische Gesellschaft*, Berlin, Jahrbuch, 1934, vol. 35, pp. 193–234.

91 S. Truscott, National Advisory Committee for Aeronautics Report, 470, 1933, pp. 1–23.

92 L. J. Coombes, Paper read at Cambridge Congress. To be published shortly. 1934.

93 H. E. Saunders and A. S. Pitoe, Society Naval Architects and Marine Engineers, Trans., 1934, vol. 41, pp. 243–295.

94 H. E. Saunders, Society Naval Architects and Marine Engineers, Transactions, 40, 1933, pp. 75–148.

95 K. Helm, *Werft, Reederei, Hafen*, 1934, vol. 15, pp. 146–148.

96 O. Schlichting, *Schiffsbautechnische Gesellschaft*, Berlin, Jahrbuch, 1934, vol. 35, pp. 127–148.

97 W. G. A. Perring, Paper read at Cambridge Congress. To be published shortly. 1934.

98 S. Watanabe, Institute of Physical and Chemical Research, Tokyo, 1933, Science papers no. 477, pp. 118–135; 1934 Science papers no. 483, part V, pp. 202–208.

99 Hunsaker, Paper read at Cambridge Congress. To be published shortly. 1934.

100 D. Riabouchinsky, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 390–392.

101 E. F. Eggert, Society Naval Architects and Marine Engineers, Transactions, 1933, vol. 40, pp. 58–74.
102 P. de Haller, *Schweizerische Bauzeitung*, 1933, pp. 243–246, 260–264.
103 H. Schröter, *Zeit. V.D.I.*, 1933, vol. 77, pp. 865–869; 1934, vol. 78, pp. 349–351.
104 E. V. Huntington, *American Mathematical Monthly*, 1933, vol. 40, pp. 451–455.
105 S. Nikitine, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 896–898.
106 F. Westendorp, *Ingenieur* 48 (Dutch), 1933, W 115–120.
107 R. H. Heald, Bureau of Standards, Journal of Research, 1933, vol. 11, pp. 285–291. Society Automotive Engineers Journal, 1933, vol. 33, no. 5, pp. 903–906.
108 W. E. Lay, Society Automotive Engineers Journal, April, 1933, pp. 144–156, 177–186.
109 M. C. Horine, P. Altman, H. G. Winter, E. G. Reid, and R. H. Upson, Society Automotive Engineers Journal, August, 1933, pp. 261–267.
110 M. Platt, *Motor* (London), 1933, vol. 63, no. 1645, pp. 903–906.
111 J. E. Garrick, National Advisory Committee for Aeronautics, Report 465, 1933, pp. 1–15.
112 F. Weinig, *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 224–235.
113 A. Betz, *Zeitschrift für Motorluftschiffahrt*, 1933, vol. 24, pp. 437–439.
114 J. Stüper, *Zeitschrift für Motorluftschiffahrt*, 1933, vol. 24, pp. 439–441.
115 M. Biot, Annales Soc. Sci. de Bruxelles, 1933, B 53, pp. 192–194.
116 L. Poggi, *Aerotecnica*, 1933, vol. 13, pp. 1255–1293.
117 A. Dufaure de Lajarte, Bulletin de l'Association technique maritime (et aéronautique), Paris, 1933, no. 37, pp. 491–551.
118 W. S. Diehl, National Advisory Committee for Aeronautics, Report 458, 1933, pp. 1–17; Report 501, 1934, pp. 1–8.
119 C. Ferrari, Memorie della R. Accademia delle Scienze di Torino II, 1933, vol. 67, no. 3, pp. 1–20.
120 C. Ferrari, *Aerotecnica*, 1933, vol. 13, pp. 989–996.
121 C. Wieselsberger, Aachen, Technische Hochschule Aerodynamisches Institut, Abhandlungen, 1933, vol. H 13, pp. 3–11.
122 J. Vladea, *Zeitschrift für Motorluftschiffahrt*, 1933, vol. 24, pp. 555–558.
123 H. Glauert, Great Britain Air Ministry, Aeronautical Research Committee, Reports and Memoranda 1544, 1933, pp. 1–8.
124 C. N. H. Lock and F. C. Johansen, Great Britain Air Ministry, Aeronautical Research Committee, Reports and Memoranda 1451, 1933, pp. 1–21; 1452, 1933, pp. 1–19.
125 S. Tomotika, Tokio Imperial University, Aeronautical Research Institute, Report 7, 1933, pp. 357–393. S. Tomotika and others; Ibid., 1933, 8, pp. 1–60.
126 L. Sackmann, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 898–900.
127 E. Pistolesi, Atti della Pontificia Accademia delle Scienze nuovi Lincei, 1933, vol. 86, pp. 405–417.
128 J. Stüper, *Ingenieur-Archiv.*, 1932, p. 338.
129 Th. Theodorsen, National Advisory Committee for Aeronautics Report 461, 1933.
130 L. Rosenhead, Proceedings, Royal Society of London, 1933, vol. 142, pp. 308–320.
131 Toussaint, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 1291–1293.
132 P. de Haller, Paper read at Cambridge Congress. To be published shortly. 1934.
133 H. Glauert, Great Britain Air Ministry, Aeronautical Research Committee, Reports and Memoranda, 1566, 1933, pp. 1–75.
134 C. B. Millikan, Paper read at Cambridge Congress. To be published shortly. 1934.
135 E. F. Relf, Paper read at Cambridge Congress. To be published shortly. 1934.
136 E. Weinel, *Ingenieur-Archiv*, 1934, vol. 5, pp. 91–105.
137 F. Numachi and J. Masuko, *Ingenieur-Archiv*, 1934, vol. 4, pp. 470–480.
138 F. Weinig, *Zeit. V.D.I.*, 1934, vol. 78, pp. 742–746.
139 L. Lazzarino, *Aerotecnica*, 1934, vol. 14, pp. 245–254.
140 I. Lotz, Paper read at Cambridge Congress. To be published shortly. 1934.
141 K. Frey, *Forschung auf dem Gebiete des Ingenieurwesens*, 1934, vol. 5, pp. 105–117.
142 F. Frankl, Recueil Mathematique de Moscow, 1933, vol. 40, no. 1, pp. 59–71 (Russian, with German summary).
143 A. Huber, *Zeitschrift für angewandte Mathematik und Mechanik*, 1933, vol. 13, pp. 239–240.
144 J. A. Kiebel, Recueil Mathematique de Moscow, 1932, vol. 39, H 4, pp. 141–151 (Russian, with French summary); Applied Mathematics and Mechanics, 1933, vol. 1, pp. 51–55 (Russian, with French summary).
145 A. F. Zahm, *Franklin Institute Journal*, 1934, vol. 217, pp. 153–166.
146 A. Favre, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 196, pp. 1, 958.
147 V. Joukovsky and P. Wolkoff, Appl. Math. a Mech., 1934, vol. 1, 179–208 (Russian, with German summary).
148 F. Gutsche, *Zeit. V.D.I.*, 1933, vol. 77, pp. 1255–1260.
149 E. Engel, *Schiffbau, Schiffahrt und Hafenbau*, 1934, vol. 35, pp. 1–4.
150 M. Lelli, Rendiconti della R. Accademia Nazionale dei Lincei, 1933, vol. VI, s. 17, pp. 718–724.
151 G. De Marchi, Rendiconti del Seminario Matematico e Fisico di Milano, 1933, vol. 7, 27 pp.
152 T. B. Abell, Institution of Naval Architects, 1933, no. 9, pp. 1–5; North-East Coast Institute Engineers and Shipbuilders Trans., 50, 1933, pp. 20–24.
153 U. Schmieschek, *Zeitschrift für technische Physik*, 1933, vol. 14, no. 12, pp. 566–567.
154 A. Fage, Proceedings, Royal Society of London, 1934, vol. 144, pp. 381–386.
155 D. Riabouchinsky and V. Volkovisky, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 900–905, 1589–1591.
156 C. Sadron, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 1293–1296.
157 M. Ziegler, Proceedings, Koninklijke, Akademie van Wetenschappen te Amsterdam, 1933, vol. 36, pp. 426–431.
158 J. Ulsamer, *Forschung auf dem Gebiete des Ingenieurwesens*, 1933, vol. 4, pp. 121–128.
159 T. C. Sen, Mitteilungen des hydrodynamischen Instituts, München, 1933, no. 7, pp. 74–86.
160 L. S. Marks, *Franklin Institute Journal*, 1934, vol. 217, pp. 201–212.
161 A. Égal, Comptes Rendus, Institut de France, Académie des Sciences, 1933, vol. 197, pp. 910–913; 1934, vol. 198, pp. 1134–1137.
162 T. Sasaki and T. Yosida, Tokio Imperial University, Aeronautical Research Institute Report 7, 1933, pp. 395–415.
163 A. H. Fette, *Zeitschrift für technische Physik*, 1933, 14, 7, pp. 257–266.
164 M. Muskat, *Physics*, 1934, vol. 5, pp. 71–94.
165 W. Hurst, *Physics*, 1934, vol. 5, pp. 20–30.
166 C. G. J. Vreedenburgh and O. Stevens, 1. Congrès des grands barrages, 1934, vol. 4, pp. 217–228; *Ingenieur* 48, B (Dutch), pp. 187–196.
167 E. Tyler, *Philosophical Magazine*, 1933, vol. VII, s. 16, pp. 504–518.
168 A. Weinstein, Comptes Rendus, Soc. physique Genève, 1933, vol. 50, pp. 196–199.
169 W. Barth, *Zeitschrift für Mathematik und Mechanik*, 1933, vol. 13, pp. 422–424.
170 E. Kilb, *Forschung auf dem Gebiete des Ingenieurwesens*, 1934, vol. 5, pp. 6–13.
171 G. Grimminger, Monthly Weather Rev., 1933, vol. 61, pp. 198–200.
172 M. Biot, Annales Soc. Sci. de Bruxelles, 1934, vol. 54, pp. 22–24.
173 A. Schoklitsch, *Wasserkraft u. Wasserwirtschaft*, 1934, vol. 29, pp. 27–43.
174 M. P. O'Bryan, Paper presented at Berkeley conference, 1934.

A Review of Recent Papers on Plasticity

By A. NADAI,[1] EAST PITTSBURGH, PA.

AT THE International Congress for Applied Mechanics, held in Cambridge, England, in July, 1934, G. I. Taylor, Yarrow Professor of the Royal Society, reported on some further remarkable results of his own investigations on the mechanism of plastic deformation of crystals.[2] While the geometrical manner in which the permanent deformations in a single crystal occur, particularly in the metals having crystals of the regular system, is well understood and has been carefully analyzed in a number of recent investigations by Elam and Taylor, Gough, E. Schmid, Polanyi, and others, there remained much to be learned concerning the mechanical laws on which are dependent some of the fundamental strength properties of the metal crystals. It is known that certain important strength properties, such as the yield stress or the stress at rupture, could not, until recently, be quantitatively predicted from the data of the lattices. Other characteristic phenomena, such as the "strain-hardening" of a soft single crystal, could not be understood; that is, it was not clear why the stress required to produce further plastic distortion must increase quite considerably while the lattice structure apparently remains intact. Only during the last few years has more light been thrown on some of these interesting properties of metal crystals and it is only since giving up the concept of the perfect crystal that the consequences of the various effects of certain deviations from the regular arrangements of the lattice elements, and of other "imperfections" of the crystal lattice structure, have been considered.

From the investigations aforementioned, it has been found that if one plots the component of the shearing stress in the direction of slip (τ) as a function of the shear strain (γ), measured as the ratio of the relative slip of two parallel planes of slip to the distance between them, the stress-strain curve [$\tau = f(\gamma)$] thus obtained at a given temperature is a unique curve for single crystals of a given metal. The first yield stress at which the plastic deformations start is practically negligible when compared to the shearing stresses (τ) in later stages of the deformation of a single crystal. For aluminum, copper, gold, and iron single crystals, these curves resemble ordinary parabolas.

In his recent papers,[2] G. I. Taylor makes some very remarkable suggestions about the mechanism of slip, describing here for the first time a comparatively simple mechanism which not only enables him to obtain strain-hardening in a lattice structure but also enables him, from a few simple assumptions, to predict quantitatively the parabolic shape of the strain-hardening curve of a single crystal.

Until recently the phenomenon of gliding was generally regarded as a simple shift of one portion of the crystal relative to the other by the multiple of the atomic distances. It seems that the great theoretical difficulties which arise from this assumption are completely eliminated when, according to Taylor, the slipping is considered to occur not simultaneously over all the atoms of a slip plane but over a limited region. Thus he is led to introduce certain "dislocations." These are "traveling" disturbances in the regular arrangement of the elements of the lattice (of the regular spacing of the atoms) starting from a center and moving along lattice planes. It is assumed that in two adjacent layers of atoms, one layer contains, over a certain given length, one more unit cell of the lattice than the adjoining layer. Such a disturbance produces "potential hills and valleys" in the interatomic force fields and makes it possible for one atom whose connections are the weakest to jump over its potential barrier into a vacant place, its movement being actuated by the thermal agitation of the lattice structure. Thus the effect of the temperature is brought into play by allowing for "Platzwechsel" ("jumps" from one position of equilibrium of an atom to a neighboring and more stable position). This effect has also been suggested by Richard Becker and L. Prandtl in an investigation of the plasticity of amorphous materials and an investigation of elastic hysteresis, respectively. The stress distribution near a single dislocation is compared with a known case of inherent stress in an elastic body, slotted along a plane. A length L, the mean width of small subdivisions (blocks) of the crystal, is introduced. The boundaries of the crystal are supposed to be impervious to the movement of these dislocations. If τ designates the shearing-stress component, γ the plastic shear strain, G the modulus of rigidity, λ the lattice constant (distance of two atoms in the slip plane along the direction of slip), the mechanism just indicated leads to the following relation:

$$\tau = \text{const.}\, G \sqrt{\gamma} \sqrt{\frac{\lambda}{L}}$$

This expression is remarkable because of its extreme simplicity. Thus, the stress-strain curve [$\tau = f(\gamma)$] for a single crystal is a parabola.

Professor Taylor compares this formula with results of tests conducted by himself, and also by Schmid and Boas, on single aluminum crystals at various temperatures and finds that various conclusions agree very well. He also applies the theory to a non-metallic substance, namely, rock salt. For the length L, which corresponds somewhat to the free path of a dislocation, his estimates are furnishing values of the order of 10^{-4} cm for rock salt. This is the same order of magnitude as the mean dimensions of the "blocks" of the mosaic structure assumed by various recent investigators of the strength properties of crystals.

Tensile Tests Under High Velocities of Deformation[3]

The effect of the speed of deformation on the yield stress when ductile metals are stretched is becoming more and more important, especially in tests where either the temperatures are raised or where the stretching velocities increase rapidly, such as in impact tests. Brinkmann investigates the effect of the speed of deformation for copper at normal temperatures. Using a piezo-electric strain gage and an oscillograph, he compares the yield stresses in tensile impact tests made with copper bars with slow tests made under ordinary speeds of deformation. For the same amounts of elongation, the yield stresses increase as the logarithms of the speeds of deformation, when tests made under varying velocities are compared. The range of these velocities varied as 1 to 10^6.

---

[1] Research Laboratories, Westinghouse Electric & Manufacturing Co. Research Professor, University of Pittsburgh, Pa. Mem. A.S.M.E.

[2] "The Mechanism of Plastic Deformation of Crystals," by Prof. G. I. Taylor, Proceedings, Royal Society of London, vol. 145, no. A-855, 1934. See part I: "Theoretical," p. 362; and part II: "Comparison With Observations," p. 388. In the same issue of these proceedings see also, "The Strength of Rock Salt," p. 405.

[3] "Zerreissversuche mit hohen Geschwindigkeiten," by Hans Brinkmann, Doctor's thesis, Techn. Hochschule Hannover, 1933.

Recent Progress in the Design of Helical Springs[4]

The law of elasticity is utilized by engineers daily in all kinds of force-measuring devices and instruments. Spring scales have been built for 300 years, since the time of Robert Hooke. It is only recently, however, that efforts to perfect the spring scales have succeeded to an extent that they become instruments with a known range of accuracy. As spring scales and helical springs are universally used as the most convenient and simplest force-measuring devices in testing materials and in scientific laboratory practice, it was thought that it is perhaps of interest to engineers to know that recent important advances have been made by mechanical analysis which have improved the accuracy of the springs and reduced their inherent defects to a negligible minimum.

M. F. Sayre,[5] A. V. de Forest,[6] R. B. Wasson,[7] in cooperation with George E. Chatillon & Sons, have developed an improved spring material which, within limits, is not sensitive to temperature changes. Inasmuch as the modulus of elasticity and of rigidity is a function of the temperature, the calibration constant of a spring depends on the temperature to which it is exposed. With the use of this spring material, complete elimination or reduction to very small amounts has been effected in defects such as elastic after-effects, hysteresis, creep, differential variation in length due to difference of the thermal-expansion coefficients of the spring and the other parts of the balance, untwisting of the helical spring causing a deviation of the straight-line characteristics, etc.

Alloys such as Invar and Elinvar (36 per cent nickel, 12 per cent chromium, 1 per cent silicon, and the rest iron) which are known to have a low thermal-expansion coefficient have other disadvantages which seem to have been overcome in the new "isoelastic alloy" recently suggested. A number of formerly disturbing secondary defects also have been eliminated successfully by a clever arrangement of the ends of the helical coils, by using flat rectangles as wire cross-sections, by combining two helical springs of equal length but wound in opposite senses, by attaching the springs to flexible arms which will correct for the non-linear characteristics of coiled springs, etc.

The details and comparative test results about the elastic properties of various materials may be found in various recent papers[8] by M. F. Sayre and in the paper by M. F. Sayre and A. V. de Forest just mentioned.

On Spiral Cracks of Fuji Volcano[9]

An interesting observation has been made by geophysicists in Japan. S. Fujiwhara finds when mapping the locations of the secondary cones situated on the slope of the great volcanic cone of the Fujiyama, that they are arranged along regular curves of spiral shape. It is known that around the impression of a stamp pressed into soft steel, logarithmical spirals become visible as slip lines on the polished surface of the specimen. The theory of the equilibrium of loose materials (sand) can be based on the properties of these surfaces of slip, which in the case where solid friction is present, will intersect each other at given angles. Systems of logarithmical spirals, as slip lines around a cavity with pressure acting within it in a radial direction, are known also to occur in loose materials. Although the mechanical conditions in the interior of a large volcano consisting partly of ejected ashes and partly of the solidified and slowly cooling magmatic materials cannot be supposed to be of a simple nature, it is remarkable enough that the secondary craters on the slope of the main cone are situated along such regular spiral lines as described by Fujiwhara, indicating that the stress field in the interior of a volcano is radially symmetrical.

---

[4] "New Spring Formulas and New Materials in Precision Spring-Scale Design," by M. F. Sayre and A. V. de Forest. Paper presented at A.S.M.E. Annual Meeting, December 3–7, 1934, New York, N. Y.

[5] Associate Professor of Applied Mechanics, Union College, Schenectady, N. Y. Mem. A.S.M.E.

[6] Associate Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[7] George E. Chatillon & Sons, New York, N. Y.

[8] "Laws of Elastic Behavior in Metals," by M. F. Sayre, Trans. A.S.M.E., July, 1934, paper RP-56-7; "Plastic Behavior in the Light of Creep and Elastic Recovery Phenomena," by M. F. Sayre, Trans. A.S.M.E., July, 1934, paper RP-56-8.

[9] "On Spiral Cracks of Fuji Volcano," by S. Fujiwhara, *Geophysical Magazine*, vol. 7, no. 2, 1933, published by the Central Meteorological Observatory, Tokyo, Japan.

Design Data

It is important that the data contained in technical papers be made readily available to designing engineers. In order to satisfy these needs of industry, this section of the Journal will include a concise presentation of data and information drawn chiefly from papers previously published by the Applied Mechanics Division of The American Society of Mechanical Engineers.

The data are prepared by a subcommittee of the Applied Mechanics Division on strength of materials (R. E. Peterson, chairman), under the general guidance of S. Timoshenko.

The first contribution, which follows, is by A. M. Wahl and treats of the subject of spring design. Future articles will cover various phases of mechanics, dealing in the first instance with questions on the strength of materials.

Helical Compression and Tension Springs

By A. M. WAHL,[1] EAST PITTSBURGH, PA.

Formulas

FORMULAS for calculating load, deflection, and stress in helical compression and tension springs of round or square wire are given in Table 1. Based on recent research,[2] the stress formulas differ from those ordinarily used in that they are multiplied by a factor K (Fig. 1) dependent on the ratio c of mean coil diameter to wire diameter (or side of square cross-section). This factor takes into account the stress increase due to curvature and direct shear, and, though derived primarily for round-wire springs, also gives sufficiently accurate results for most practical purposes when applied to square-wire springs,[3] the formulas for which are based on St. Venant's theory.[4]

The formulas given in Table 1 do not apply accurately to open-wound springs when the pitch angle is in excess of 12 deg but these are not often used in practice.[5]

Inactive Turns

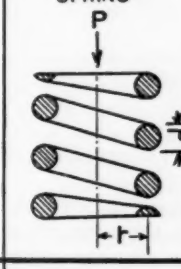
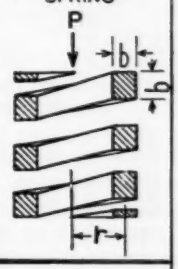
The number of inactive turns in springs depends on the design of the end coils. For the usual design of compression spring (Fig. 2) where the end turns are set up and ground as indicated, an approximation will be had by assuming one inactive turn at each end of the springs. The number of active turns then will equal the total number of turns (including the tapered end turns), minus two. For the shape of tension spring shown in Fig. 3 the number of active turns will be slightly greater than the nominal number in the body of the spring, due to the additional deflection of the hooks at the ends.

Unless supported laterally, compression springs should not have too great a length as otherwise undesirable buckling effects may occur.[6]

Charts for Helical Tension or Compression Springs

The charts shown in Figs. 4 and 5 are given to enable the designer to make a rapid computation of load and deflection in

TABLE 1 FORMULAS—HELICAL TENSION AND COMPRESSION SPRINGS

	ROUND WIRE SPRING	SQUARE WIRE SPRING
		
f = TOTAL DEFLECTION OF SPRING	$f = \frac{64Pr^3n}{Gd^4}$	$f = \frac{44.5Pr^3n}{Gb^4}$
S = MAXIMUM SHEARING STRESS	$S = \frac{16PrK}{\pi d^3}$	$S = \frac{4.80PrK}{b^3}$
P = LOAD ON SPRING	$P = \frac{\pi d^3 S}{16rK}$	$P = \frac{Sb^3}{4.80rK}$

IN ALL CASES $K = \frac{4C-1}{4C-4} + \frac{.615}{C}$

WHERE $C = \frac{2r}{d}$ FOR ROUND WIRE, $C = \frac{2r}{b}$ FOR SQUARE WIRE

VALUES OF K MAY BE TAKEN FROM THE CURVE OF FIG. 1
G = TORSIONAL MODULUS OF ELASTICITY IN LBS/SQ. IN.
n = NUMBER OF ACTIVE TURNS

[1] Research Engineer, Westinghouse Research Laboratories, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa. Assoc-Mem. A.S.M.E.

[2] "Stresses in Heavy Closely Coiled Helical Springs," by A. M. Wahl, Trans. A.S.M.E., 1929, paper APM-51-17. This factor K agrees closely (within 2 per cent for round-wire springs) with a formula derived using more exact theory by O. Göhner. See "Die Berechnung Zylindrische Schraubenfedern," *Zeit. V.D.I.*, Mar. 12, 1932. More exact formulas for square- or rectangular-wire springs are also given in the latter article.

[3] "Further Research on Helical Springs of Round and Square Wire," by A. M. Wahl, Trans. A.S.M.E., 1930, paper APM-52-18.

[4] "Strength of Materials," by S. Timoshenko, part I, p. 77, D. Van Nostrand Co., New York. See also article mentioned in footnote 3.

[5] For calculating such springs, see article by Göhner, loc. cit.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until May 10, 1935, for publication at a later date.

[6] For calculating buckling loads, see article "Die Knickung von Schraubenfedern," by R. Grammel, Proc. First International Congress for Applied Mechanics.

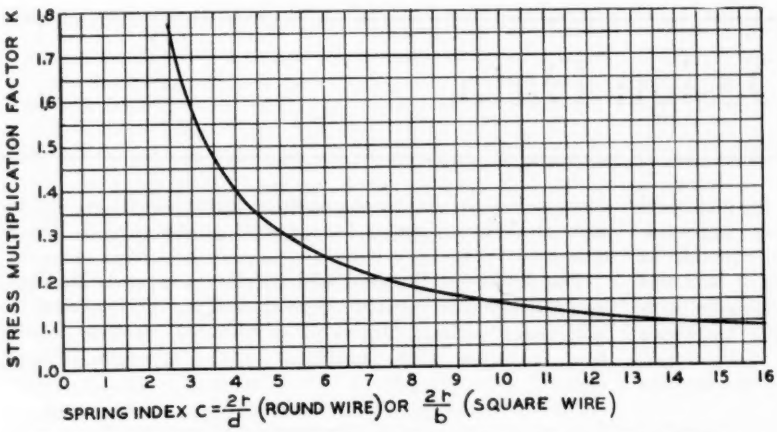


FIG. 1 CURVE FOR COMPUTING STRESS-MULTIPLICATION FACTOR K—HELICAL TENSION OR COMPRESSION SPRINGS

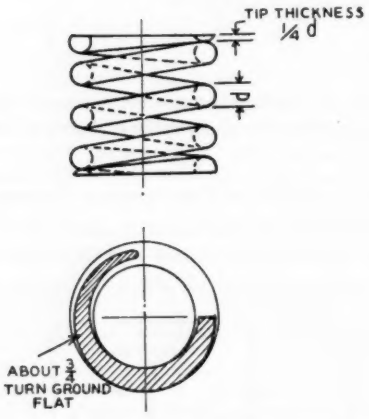


FIG. 2 COMPRESSION SPRING—ENDS SET UP AND GROUND



FIG. 3 TENSION SPRING

helical tension or compression springs of round wire when the working stress is known.

To compute the *load* on the spring, the factor C_1 is first obtained from Fig. 4 by finding the intersection of the vertical ordinate representing the wire diameter and the curve representing the outside coil diameter OD (see example and dotted line on chart). Interpolation may be necessary for intermediate dimensions. The permissible load P on the spring is found by multiplying C_1 by the ratio $\frac{S}{10{,}000}$ where S is the allowable working stress (in shear). (The load P may also be considered as that which will produce a maximum shearing stress S in the spring.)

To compute the total *deflection* of the spring, the factor C_2 is likewise obtained from Fig. 5, from the wire diameter and outside coil diameter (see example). The curves shown are based on a torsion modulus G = 11,400,000 lb per sq in., which approximates the actual value for ordinary carbon-steel springs. Hence, for such springs, to find the total deflection f, the value of C_2 thus found is multiplied by the ratio $\frac{S}{10{,}000}$ and by the number of active turns n (S being the working stress in the spring at a deflection f). The deflection *per active turn* is given by $C_2 \times \frac{S}{10{,}000}$. In cases where the torsion modulus G is different from 11,400,000 (as for alloy-steel, phosphor-bronze, or severely cold-worked springs), the value of f thus computed must be multiplied by the ratio $\frac{11{,}400{,}000}{G}$. Thus in the example given in the chart where f = 0.094 in., if the torsion modulus G is actually 10,000,000 lb per sq in., the total deflection f will be $\frac{11{,}400{,}000}{10{,}000{,}000} \times 0.094 = 0.107$ in.

It should be noted that there will always be a small inaccuracy in reading the charts, particularly if interpolations are made. Hence,

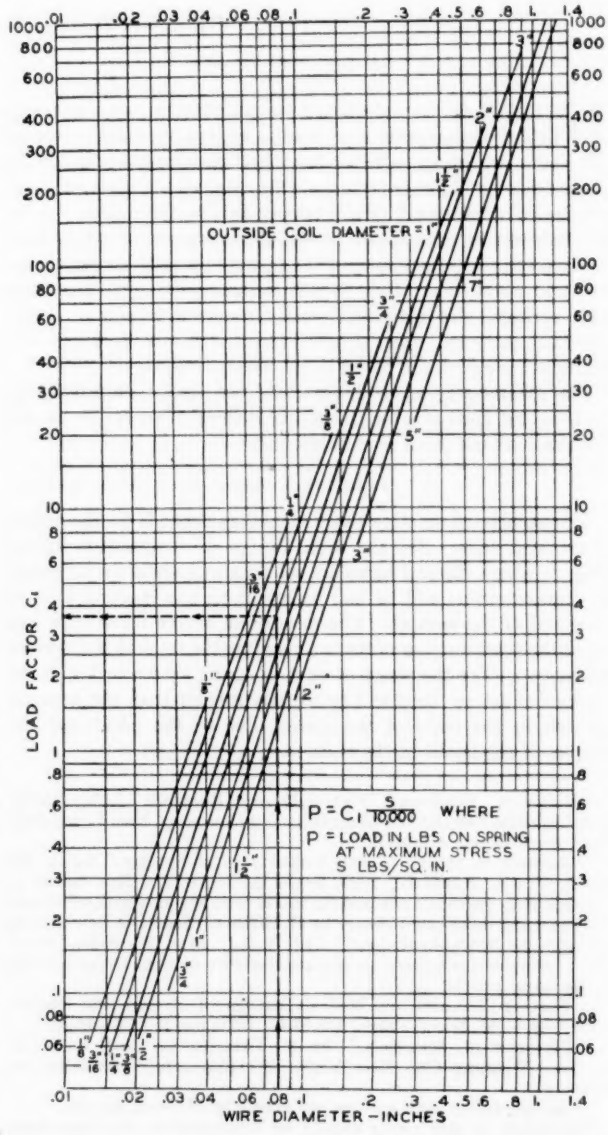


FIG. 4 (*Right*) CHART FOR VALUES OF LOAD FACTOR C_1—HELICAL COMPRESSION OR TENSION SPRINGS OF ROUND WIRE

(Example: Wire diam = 0.080 in.; S = 50,000 lb per sq in.; outside coil diam = ½ in. From chart C_1 = 3.6—see dotted line—$P = 3.6 \frac{50{,}000}{10{,}000}$ = 18 lb.)

if high accuracy is necessary, springs should be calculated by using the formulas of Table 1. It should be noted, however, that because of unavoidable commercial variations in coil diameter and wire size, the actual deflection of the spring will usually deviate by several per cent from that calculated, except where special precautions in manufacturing have been taken.

For calculating *square-wire springs*, the charts of Figs. 4 and 5 may also be used. It is merely necessary to calculate the load and deflection at the given working stress for the corresponding round-wire spring, i.e., one having the same outside coil diameter, number of turns, and a wire diameter equal to the side of the square cross-section. The loads thus found are multiplied by the factor 1.06 and the deflections by 0.738 to find those for the square-wire spring at the given stress.

Working Stresses

The choice of working stresses for springs depends on a great many factors, among which are: (1) kind and quality of material used, (2) decarburization of the surface layers which may act to reduce the fatigue strength of the material to less than one-half that with the surface layers ground off,[7] (3) condition of surface, i.e., whether rough or smooth, (4) stress range to which the spring is subjected, a smaller stress range permitting a higher working stress,[8] (5) number of repetitions of load, a considerably higher working stress being allowable if the load is applied only a few times, and (6) corrosion effects, which operate to greatly reduce spring life.

If failure of the spring is a serious matter, and the spring is under severe operating conditions, the working stress should be lower than is otherwise advisable, while special precautions in manufacturing and inspection may be necessary. In such cases it is well to consult the spring manufacturer, particularly since the fatigue strength of springs may be considerably increased by the use of recent developments in spring materials and manufacturing technique.

TABLE 2 ALLOWABLE WORKING STRESSES[6] IN SHEAR—HELICAL COMPRESSION OR TENSION SPRINGS (STEEL)

Diameter of wire, inches	Severe service	Average service	Light service
Up to 0.085	60,000	75,000	93,000
Above 0.085 to 0.185	55,000	69,000	85,000
Above 0.185 to 0.320	48,000	60,000	74,000
Above 0.320 to 0.530	42,000	52,000	65,000
Above 0.530 to 0.970	36,000	45,000	56,000
Above 0.970 to 1.5	32,000	40,000	50,000

[6] Used as a basis for design by Westinghouse Elec. & Mfg. Co.

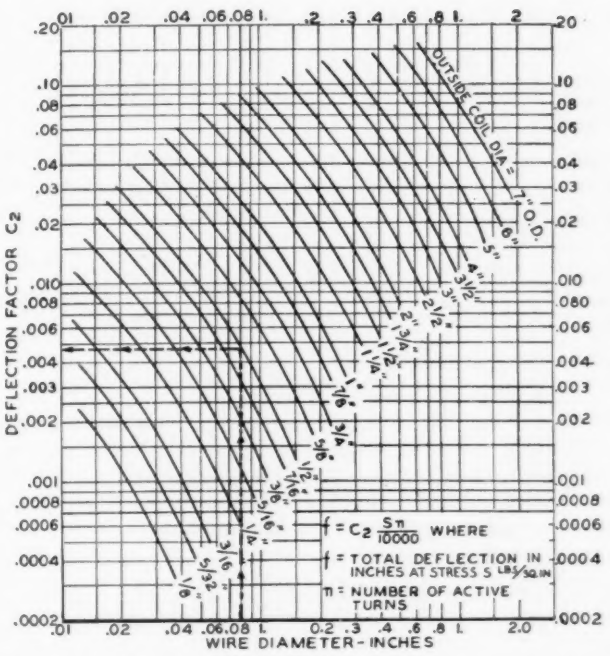


FIG. 5 CHART FOR VALUES OF DEFLECTION FACTOR C_2—HELICAL COMPRESSION OR TENSION SPRINGS OF ROUND WIRE

(Example: Wire diam = 0.080 in.; S = 50,000 lb per sq in.; G = 11,400,000 lb per sq in.; outside coil diam = ½ in.; n = 4. From chart—see dotted line—C_2 = 0.0047; $f = 0.0047 \left(\frac{50,000}{10,000}\right)$; 4 = 0.094 in. Values given are for a torsion modulus G = 11,400,000 lb per sq in. For values of G different from 11,400,000 lb per sq in., multiply f thus calculated by 11,400,000 ÷ G.)

For phosphor-bronze springs 50 per cent of these values are used. It will be noted that lower stresses are used for the larger wire sizes and for severe service, in accordance with practical experience. In most cases, these stresses will be found to be conservative, and often may be increased at the discretion of the engineer, particularly if higher grade materials and manufacturing technique are employed.

A more rational method of determining working stress in general,[9] which may be applied to spring design, will appear later.

[7] See, for example, Special Report No. 5, Department of Scientific and Industrial Research, "Endurance of Spring Steel Plates Under Repetition of Reversed Bending Stress," by G. A. Hankins, published by H. M. Stationery Office, London.

[8] Some fatigue-test data are given in "Permissible Stress Range for Small Helical Springs," by F. P. Zimmerli, Univ. of Mich., Eng. Res. Bulletin No. 26.

[9] A discussion of this is given in "General Considerations in Designing Mechanical Springs," by A. M. Wahl, *Machine Design*, May, 1930.

Book Reviews

Thermodynamics

TECHNISCHE THERMODYNAMIK. By W. Nusselt. V.D.I. Sammlung Göschen. No. 1084. Berlin/Leipzig, 1934. Walter de Gruyter and Co., G. E. Stechert and Co. Cloth, 4 × 6 in., 144 pages, 65 figs., $0.60.

REVIEWED BY JOHN A. GOFF[1]

THE author of this small book must be commended on a presentation of the elements of engineering thermodynamics which is at once singularly brief and comprehensive. A short introduction sketches the historical background of the subject. There follows a development of the First Law which is characterized by careful distinction between external work of expansion and what the author calls "technical work," that is, external work derived from steady fluid flow. An appreciation of the fundamental difference between reversible and irreversible processes is essential to a clear understanding of the laws of thermodynamics and especially of its application to such technically important processes as heat transfer, combustion, and fluid friction. The author pays careful attention to this difference throughout the book. His development of the Second Law is particularly clear and understandable.

Of special interest is the chapter on properties of vapors with, of course, special reference to the properties of steam. The more important "characteristic equations" for superheated steam that have been proposed, from that of Zeuner (1867) to that of Keyes and Smith (1934), are listed chronologically with brief comments on their salient features. In this way, the application of thermodynamics to the study of higher and higher pressures and temperatures is emphasized. The book recommends itself to readers of German—practicing engineers and students alike.

Theory of Elasticity

THEORY OF ELASTICITY. By S. Timoshenko. Engineering Societies Monograph Series. McGraw-Hill Book Company, Inc., New York, 1934. Cloth, 5¾ × 9 in., 416 pp., 203 figs., $5.

REVIEWED BY D. L. HOLL[2]

THIS is one of the series known as "Engineering Societies Monographs," published under the joint auspices of four national engineering societies whose aim ". . . is to make accessible to many users of engineering textbooks information which otherwise would be long delayed in reaching more than a few in the wide domains of engineering." In this respect the service rendered by the societies is greatly appreciated and the timely appearance of a book on elasticity by an eminent and productive author is being welcomed by an increasing number of interested readers. It provides a mine of important methods and applications of general theory to technical problems for which the elementary methods of strength of materials is inadequate. It helps bridge the gap between the common run of elementary texts on strength of materials and the more comprehensive treatises.

By introducing elementary but fundamental solutions of two-dimensional problems, the author inductively builds more complex solutions by the principle of superposition of stress systems and arrives at the disk problem under general loads (p. 107) and to the problem of a finite force or couple in an infinite plate (p. 111). Many readers would have been pleased to have the author introduce the Weighardt-Westergaard "slab-slice" analogy of Airy's stress function, and others would prefer that additional treatment be given the Fourier integral solutions of the differential equation of stress. The latter method is briefly mentioned in the von Kármán-Seewald problem where the new and interesting result is found that the deflection of the central line of a long beam is wave shaped outside of a region of high concentration of loading. An even greater technique in employing the superposition principle is given in chapter 11 where the problems of three-dimensional axially symmetrical systems furnish the solutions of Boussinesq and of thick circular plates in terms of rational integral functions of the coordinates (Legendre polynomials).

Extended expositions of photoelastic examination of stresses (with an adequate treatment of multiply connected regions, p. 115 et seq.), of soap-film determinations for the torsion and flexure problems, and for the invariant sum of principal stresses $\sigma_1 + \sigma_2$, are given in considerable detail. Important place is given to the minimizing method of Ritz to two-dimensional stress problems, to the torsion and flexure of prismatical bars. Complex function theory, curvilinear coordinates, geometrical inversion of stress systems, solid zonal harmonics, elliptic integrals (Hertz's contact pressures), Bessel's functions (circular cylinder stresses on p. 352), and the solution of differential equations by the method of successive approximations (twist of thick circular ring problem) indicate the type of analytical tools employed in the analysis. Such a formidable array of mathematics causes one to question the author's statement in the preface that . . . "The mathematical derivations . . . do not require more mathematical knowledge than is given in engineering schools." In justice to the author the reviewer hastens to add that much of the foregoing, although of practical importance, is relegated to small type and may be omitted during the first reading.

Apart from the general theory of stress and strain and the compatibility equations, the chapters on two-dimensional problems, torsion, flexure of prisms, and axially symmetrical stress problems receive the greater share of attention. The chapters on strain-energy methods, complex variable (fine print), and wave propagation are treated less extensively.

The symbolism used follows largely that of the German writers. There is an occasional use of Lame's constant λ. The author uses ν for Poisson's ratio. Among other symbols in use for this constant are μ, σ, and m. Some standard list of symbols ought to be urged upon writers of forthcoming articles to this journal and perhaps the list of notations in this book with some changes might serve the purpose. It might be well to adopt a single symbol for stresses instead of σ and τ. On pages 64 and 86 appears $\epsilon_{r\vartheta}$ for $\gamma_{r\vartheta}$. This may indicate a natural tendency to be consistent in the symbols used for strains. While it is not incorrect to use τ_{xy} and τ_{yx} interchangeably, many of the figures (e.g.,

[1] Professor of Thermodynamics, University of Illinois, Urbana, Ill., Assoc-Mem. A.S.M.E.

[2] Professor of Mathematics, Iowa State College, Ames, Iowa.

113 and 129) and equations would satisfy the meticulous reader if the defining convention of page 4 were followed more closely.

Apart from a few minor errors, such as a misspelled name, p. 134, the editing of the context material, the statement of the problems with the hypotheses, the importance given significant results (260 numbered equations), the listing of nearly 400 references, half of which are in foreign languages, and the general tenor of the book, all invoke our high esteem for the courage of the author in so ably accomplishing the purpose in presenting this difficult subject.

Mechanical Vibrations

MECHANICAL VIBRATIONS. By J. P. Den Hartog. McGraw Hill Book Co., Inc., New York, 1934. Cloth, $5^3/_4 \times 8^1/_2$ in., 390 pp., 282 figs., $5.

TORSIONAL VIBRATION. By W. A. Tuplin, M.Sc. John Wiley & Sons, Inc., New York, N. Y., 1934. Cloth, $5^1/_2 \times 8^1/_2$ in., 320 pp., 89 figs., $5.50.

REVIEWED BY C. R. SODERBERG[3]

THE literature dealing with the problem of vibrations in engineering has grown rapidly in recent years and a new contribution to the field can no longer claim the unique position it might have found a few years ago.

Professor Den Hartog's book is apparently written with a definite object of creating a textbook for a specific course on the subject. This object has somewhat limited the scope of the book but it has also furnished the incentive for an unusually clear and lucid presentation, free from obscure mathematical complexity.

As pointed out by the author in the preface, much of the subject material of this book was collected in connection with lectures presented to the Design School of the Westinghouse Company in East Pittsburgh, Pa. This has given the subject material a relevance to practical engineering problems which is not often found in textbooks of this type.

Chapter 1 deals with certain kinematic and mathematical preliminaries. In chapter 2 a discussion of problems of one degree of freedom is presented. This presentation includes many useful examples of mechanical systems and practical applications to vibration problems in engineering. A section of particular interest is the one on "vibration isolation." Chapter 3 continues with a discussion of problems with two degrees of freedom and again the application to the problem of vibration absorption forms an important part of the subject matter. Chapter 4 covers problems with many degrees of freedom and problems of vibrations of continuous bodies. Chapter 5 is devoted to problems encountered in reciprocating machinery. Chapter 6 gives a discussion of problems peculiar to rotating machinery. It covers the more common problems, such as balancing of rigid rotors and critical speeds in rotors. It also includes many special problems of outstanding interest such as the vibrations in frames of electric machines, vibrations in propeller blades, vibrations in turbine blades, and similar subjects. Chapter 7 is devoted to the subject of self-excited vibrations. This class of vibration phenomena has assumed a very important place in practical engineering work during recent years. The treatment given in this chapter is more complete than most of those available in the engineering literature, and it will be warmly welcomed by many engineers who have encountered this type of problem in their practical work. The simple and direct treatment of this rather complicated problem given in this chapter represents one of the outstanding features of the book. The last chapter is devoted to the problem of non-linear vibrations.

The book illustrates an admirable style of engineering literature and it deserves to be ranked among the valuable contributions within its field.

Mr. Tuplin's book also belongs to a field in which the literature has received many valuable contributions in recent years. However, this book cannot honestly be referred to in the superlative terms which are natural in the case of Professor Den Hartog's book. The subject is well covered, but the organization of the material and its presentation do not evoke the same praise. The book is free from the more obstruse mathematics so difficult to avoid in this subject, but in many instances the ingenuity of treatment required to compensate for this omission is lacking.

Chapter 1 covers the usual kinematical and dynamical preliminaries. Chapter 2 is headed "Equivalent Dynamical Systems" and describes the process by which actual systems with their more complicated details are reduced to abstract systems ready for the mathematical treatment. In this chapter the author has brought together a considerable amount of practical information which will be welcomed by designers working in this field. Chapters 3 and 4 present this mathematical treatment for two- and three-mass systems. Chapter 5 is inserted at this point to present the problem of the solution of high-order algebraic equations. In chapters 6, 7, and 8 the treatment is extended to problems of multi-mass systems where trial-and-error methods of computation are required. Certain aspects of the treatment in these three chapters, particularly those cases found under the heading "Special Cases of Multi-Mass Systems" are certain to be welcomed as clearing up many points which are not usually clear to the average designer. Chapter 9 discusses the subject of harmonic analysis, while chapter 10 covers the determination of vibration amplitudes and torques, without damping, for systems treated in previous chapters from the point of view of natural frequencies. Chapter 11 covers the subject of frequencies of impressed torques occurring in the usual combinations of reciprocating machinery. Chapter 12 presents a discussion of the problem of vibration amplitudes and torques at resonance under the influence of damping, while chapter 13 presents a discussion of vibration dampers and vibration absorbers. In chapter 14 a discussion is given of the problems introduced by torque fluctuations in geared systems. Chapter 15 presents a specialized treatment for cases involving vibrations of continuous bodies. Chapter 16 is devoted to a discussion of couplings. Chapter 17 presents a discussion of impact problems in torsion systems. This is a subject treated very briefly in earlier literature and embraces many problems of real importance. It is to be regretted that the author did not find it possible to enlarge further on this important subject. Chapter 18 is a brief discussion of the torsiograph and the general problem of measurement of torsional vibrations.

It is difficult to avoid a comparison of the book under review with some of the earlier contributions in this field which, more or less justifiably, may be referred to as classics. "Die Berechnung der Drehschwingungen" by Heinrich Holzer, published in 1921, which incidentally was not included in the bibliography of Mr. Tuplin's book, is perhaps the most important of these classics. The reviewer has been able to find very little material of fundamental significance in the present contribution which cannot be found in the book by Holzer. In many instances, the question may be raised whether Holzer's treatment is not to be preferred. It has been found difficult, for example, to improve upon the method for trial-and-error numerical computations given by Holzer. The method presented in the book under review does not appear to qualify as an exception to this rule.

[3] Manager, Turbine Engineering Dept., Westinghouse Electric & Manufacturing Co., South Philadelphia, Pa. Mem. A.S.M.E.

It would be unfair, on the other hand, to present Mr. Tuplin's book as entirely devoid of merit. The above criticism concerns chiefly the formal aspect of the arrangement of the material, which is after all to a large extent a matter of personal taste. The English literature on this subject is somewhat meager, and the book assumes a more important place in existing literature if books in German are excluded. The reader will find practically all of the relevant material collected in English for the first time in one treatise. It will serve as an excellent practical guide to the subject. With this purpose in mind, the book is well worthy of recommendation.

General Solution for Rectangular Plates

Eine Lösung für die Berechnung der biegsamen rechteckigen Platten. (A General Solution for Rectangular Plates.) By S. Iguchi. Hokkaido Imperial University, Sapporo, Japan. Julius Springer, Berlin, 1933. Paper, $6^1/_2 \times 9^1/_2$ in., 56 pp., 3 charts, 13 figs., 5 rm.

Reviewed by A. Nadai[4]

THIS recent contribution to literature on the bending theory of flat elastic plates deals with the case of a rectangular plate of a non-isotropic material. Investigations with regard to such plates by engineers are rather recent, although a few previous attempts have been made to find similar solutions for bodies having certain types of crystal symmetry by physicists (by Voldemar Voigt and by others). Probably the first papers devoted to the bending theory of a non-isotropic rectangular plate are those by M. T. Huber (1923), who introduced for the elastic constants a certain symmetry condition which he calls "orthotropy." This leads to the assumption that the bending rigidity of the rectangular plate is different in the two directions parallel to the sides of the rectangle. The case has some interest because of the bending of reinforced-concrete slabs when the steel bars are inserted parallel to both sides of the rectangle. The particular solutions used by Iguchi for the rectangle do not completely satisfy the partial differential equation of the anisotropic plate (this is a linear equation of the fourth order with constant coefficients); however, they are constructed so that the boundary conditions on all four sides of the rectangle can be satisfied exactly in at least nine different cases, among which are contained the practically important cases of straight-line support without constraint and fixed (built-in) edges. Mixed cases of boundary conditions also are included. The terms contain a product of a function of x with a function of y. Each function contains an algebraic portion consisting of four power terms $C_0 + C_1x + C_2x^2 + C_3x^3$ and a sine term $C_4 \sin nx$. The coefficients are first adjusted to satisfy the boundary conditions and the remaining undetermined constants are chosen so that the double sum in which they appear expresses the distribution of the given load $p = f(x,y)$ in its totality. Iguchi can thus solve practically all important cases of loading and of boundary conditions, including also those of an isotropic rectangular plate.

The detail work of determining the constants in the double series seems considerable, although Iguchi has worked out a few cases, such as those of the rectangular plate with the conditions

(w = deflection) $w = 0$ and $\frac{\partial^2 w}{\partial y^2} = 0$ along an edge x = const.

and also the case of a plate with clamped edges, and he is able to reduce the double sums by adding up one portion of the double sums into a simple series. An example of a rectangular plate on elastic foundation is also worked out. The idea of combining a few power terms in x or y with simple sine terms seems attractive, as it enables Iguchi to use one and the same general form of a certain double series in practically all cases of boundary conditions. Also partially loaded plates are treated. Tables and a few graphs are given.

[4] Research Laboratories, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa. Mem. A.S.M.E.

Hydromechanics

Hydromechanik—Volume 2. By W. Kaufmann. Julius Springer, Berlin, 1934. Cloth, 6 × 9 in., 293 pp., 210 figs., 18 rm.

Reviewed by J. P. Den Hartog[5]

THE first volume of this work appeared some three years ago and dealt with the theoretical basis of hydro- and aerodynamics in the usual manner. The second volume is much more useful and interesting, as it gives a review of practically all applications that have been made of the theory so far, and at the same time is understandable in itself, without reference to the first volume. The topics dealt with include ship resistance, ground-water flow, and lubrication, as well as the more usual applications of hydro- and aerodynamics, so that necessarily the treatment is not exhaustive. However, the important elements of each of these subjects are discussed fully, and a large number of quotations from the literature are given which makes the book very valuable as a reference.

Volume 2 is divided into nine chapters, of which the first deals with weir and nozzle coefficients. For most of these a theoretical calculation is given by means of Helmholtz' two-dimensional free-jet method. In this manner the streamline picture as well as some of the coefficients can be found. The second chapter is on pipe flow, including the latest results on rough pipes, and the new theories of Prandtl and von Kármán on the turbulent velocity distribution. A rather complete discussion is given on the flow and pressure losses in pipe bends, elbows, and sudden expansions. Also the surges in pipe lines and penstocks are treated. The third chapter is on open channels, especially on St. Venant's theory of the distinction between "torrents" and "rivers" with the related phenomena of hydraulic jump. In the next chapter wave motion and the phenomenon of tides are discussed briefly, while the fifth chapter takes up viscous flow and bearing lubrication. The following chapter is the largest in the book, and deals with the usual applications of such a treatise as lift and drag of airplanes, including biplanes, airfoil theory and conformal mapping, propellers, fans, and hydraulic turbines. Chapter 8 gives the elements of ship resistance in a few pages, while the concluding chapter, written by O. Flachsbart, contains a discussion of wind pressure on buildings with as much experimental data as is available at the present time.

[5] Assistant Professor of Applied Mechanics, Harvard University, Cambridge, Mass.

Distributed Gravity and Temperature Loading in Two-Dimensional Elasticity Replaced by Boundary Pressures and Dislocations

By M. A. BIOT,[1] CAMBRIDGE, MASS.

This paper deals with two-dimensional stresses due to an important class of body forces. Gravity stresses and temperature stresses are produced by body forces of the type investigated.

In the case of gravity stresses it is proved that these stresses may be calculated by forgetting about the action of gravity on the body and applying instead only external normal loads that are identical with a hydrostatic pressure.

This opens the possibility of applying photoelastic tests to the measurement of gravity stresses; a model loaded by these external pressures would show an isochromatic fringe pattern identical with the one that would appear in a model submitted to an intense gravity field. In Fig. 5 is shown the type of external loading to be applied to find the gravity stresses in a dam.

In the case of two-dimensional temperature stress in cylindrical bodies where the temperature distribution is stationary, it is shown that a distinction is to be made between solid and hollow cylinders. For a solid cylinder the only thermal stress component appearing is an axial stress acting perpendicularly to cross-sectional planes. In other words, the solid cylinder expands freely along the cross-sectional planes. For a hollow cylinder a general stress condition may arise which is identical with the stresses produced by cutting a longitudinal slit of a certain small width in the cylinder wall and sticking together the two sides of the slit (dislocation).

GENERAL THEORY

THE EQUATIONS of equilibrium of a rectangular element in two-dimensional elasticity with body forces are

$$\left.\begin{aligned}\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+X=0\\ \frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+Y=0\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [1]$$

If the body forces are derived from a function V such that

$$\left.\begin{aligned}X=-\frac{\partial V}{\partial x}\\ Y=-\frac{\partial V}{\partial y}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [2]$$

the equations of equilibrium become

$$\left.\begin{aligned}\frac{\partial}{\partial x}(\sigma_x-V)+\frac{\partial \tau_{xy}}{\partial y}=0\\ \frac{\partial}{\partial y}(\sigma_y-V)+\frac{\partial \tau_{xy}}{\partial x}=0\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [3]$$

They are satisfied by introducing a "stress function" ϕ

$$\left.\begin{aligned}\sigma_x-V=\frac{\partial^2\phi}{\partial y^2}\\ \sigma_y-V=\frac{\partial^2\phi}{\partial x^2}\\ \tau_{xy}=-\frac{\partial^2\phi}{\partial x\,\partial y}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [4]$$

By using Hooke's law for either plane stress or plane strain distribution and the compatibility equation of the strain components, we find an equation of the type[2]

$$\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}=C\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right) \quad \ldots\ldots [5]$$

In case the function V satisfies the Laplace equation (potential function)

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0 \quad \ldots\ldots\ldots\ldots [6]$$

Equation [5] reduces to

$$\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}=0 \quad \ldots\ldots\ldots\ldots [7]$$

In this case the stress function ϕ satisfies the same equation as if there were no body forces. How will the boundary conditions be expressed?

Let l, m be the cosines of the normal direction to the boundary with the x and y directions and $\bar{X}$, $\bar{Y}$ the components of the force applied per unit length at the boundary. We have

$$\bar{X}=l\sigma_x+m\tau_{xy}$$

$$\bar{Y}=m\sigma_y+l\tau_{xy}$$

---

[1] Instructor in Applied Mechanics, Graduate School of Engineering, Harvard University. Dr. Biot received at the University of Louvain, Belgium, the degrees of Mining Engineer, 1929, Electrical Engineer, 1930, and Sc.D. in physics and mathematics, 1931. From 1931 to 1933 he was research fellow of the C.R.B. Educational Foundation at the California Institute of Technology under Prof. Th. von Kármán and at the University of Michigan under Prof. S. Timoshenko. In 1932 he received a Ph.D. from the California Institute of Technology. During 1933–1934, as a research associate of the National Research Council of Belgium, he visited the technical schools of Delft and Zürich and the Universities of Göttingen and Cambridge. He has held his present position at Harvard since September, 1934.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Equation [5] is derived in "Theory of Elasticity," by S. Timoshenko, McGraw-Hill, 1934, pp. 25–26.

or from Equations [4]

$$\left.\begin{aligned}\overline{X} - lV &= l\frac{\partial^2\phi}{\partial y^2} - m\frac{\partial^2\phi}{\partial x\,\partial y}\\ \overline{Y} - mV &= m\frac{\partial^2\phi}{\partial x^2} - l\frac{\partial^2\phi}{\partial x\,\partial y}\end{aligned}\right\} \quad \ldots\ldots\ldots [8]$$

Hence the problem of finding the stresses produced by the body force loading X, Y just considered and the boundary loading

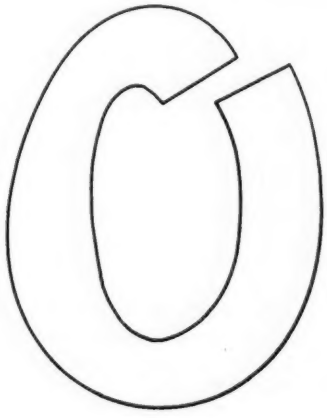


FIG. 1

$\overline{X}$ $\overline{Y}$ reduces to the solution of Equation [7] with the boundary conditions [8]. This is equivalent to finding a stress distribution

$$\left.\begin{aligned}\sigma'_x &= \frac{\partial^2\phi}{\partial y^2}\\ \sigma'_y &= \frac{\partial^2\phi}{\partial x^2}\\ \tau'_{xy} &= -\frac{\partial^2\phi}{\partial x\,\partial y}\end{aligned}\right\} \quad \ldots\ldots\ldots [9]$$

produced in the same body free from body forces but submitted to boundary forces $\overline{X} - lV$ and $\overline{Y} - mV$. This boundary load is the actual load ($\overline{X}$, $\overline{Y}$) plus a normal hydrostatic pressure equal to V.

Apparently, according to Equations [4], the actual stress in the body is the difference of the stresses σ'_x, σ'_y, τ'_{xy}, and the hydrostatic-pressure distribution represented by the potential V.

However, a very important distinction is to be made; *the statement is true only for body forces such that no singularity of the potential V occurs within the body or in any holes if the latter is hollow.*

This distinction arises from the fact that the displacements and rotations of the medium, corresponding to the hydrostatic-stress condition $p = V$ might not be single-valued, i.e., they might not take the same value when we come back to the same point after following a closed path around a singularity of the function V. In order to get single-valued displacements and rotations, we must superpose to the hydrostatic-stress condition a so-called "dislocation"-stress distribution. The meaning of a dislocation is illustrated by the following example. Take a hollow circular cylinder and cut it open by a longitudinal slit of small width (Fig. 8). We shall produce stresses in this cylinder by sticking together the two sides of the slit. This type of deformation is called a dislocation. The stress condition thus generated is not derived from the application of external forces. In our case, for instance, if the body is of hollow shape, let us cut it open so as to connect the inside with the outside. If we now assume that a stress equal to a hydrostatic pressure $p = V$ is distributed in the body, the two edges of the cut may or may not stick together (Fig. 1). If they do separate, we produce dislocation stresses by sticking them together again. As we shall see, the amount of dislocation is determined by two displacements u, v, of one face of the slit with respect to the other and by relative rotation. It can be easily calculated from the value of the potential V.

Let us call ϵ the two-dimensional linear extension of the body produced by the hydrostatic-stress distribution $p = V$. We have

$$\epsilon = -kp = -kV \quad \ldots\ldots\ldots [10]$$

The corresponding displacements of the material satisfy the equations:

$$\left.\begin{aligned}\frac{\partial u}{\partial x} &= \epsilon\\ \frac{\partial v}{\partial y} &= \epsilon\\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} &= 0\\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} &= 2\omega\end{aligned}\right\} \quad \ldots\ldots\ldots [11]$$

where ω is the rotation.

We deduce:

$$\left.\begin{aligned}\frac{\partial u}{\partial y} &= -\omega\\ \frac{\partial v}{\partial x} &= \omega\\ \frac{\partial \epsilon}{\partial y} &= -\frac{\partial \omega}{\partial x}\\ \frac{\partial \epsilon}{\partial x} &= \frac{\partial \omega}{\partial y}\end{aligned}\right\} \quad \ldots\ldots\ldots [12]$$

From these equations we see that ϵ and ω are conjugate harmonic functions so that

$$Z(z) = \epsilon + i\omega$$

is an analytic function of $z = x + yi$. The increase of rotation on a closed circuit is

$$\omega_1 - \omega_2 = \oint -\frac{\partial \epsilon}{\partial y}\,dx + \frac{\partial \epsilon}{\partial x}\,dy \quad \ldots\ldots\ldots [13]$$

It is a quantity proportional to the flux of the body force through this closed path.

The displacement is given by

$$u + iv = \int Z dz \quad \ldots\ldots\ldots [14]$$

so that the increase in displacement on a closed circuit is given by the contour integral

$$\oint Z dz$$

This proves that the two edges of the slit show a relative displacement only when the function Z or, what amounts to the same thing, the potential V has a singularity inside the contour. Yet if the singularity is such that the flux of body force produced by it is zero and if $\oint Z dz = 0$ no dislocation stress has to be introduced.

DISTRIBUTED GRAVITY LOADING

One of the most important cases of potential field body force is the case of the gravity field acting upon a body of uniform density.

Body Resting Under Its Own Weight

Let us consider a two-dimensional elastic body B (Fig. 2) resting under its own weight on a plane surface S. Taking the vertical y axis as positive and calling δ the specific weight of the material, the body force is expressed as $X = 0$, and $Y = -\delta$, and the corresponding potential is $V = \delta y$. This potential presents no singularity whatever, hence no dislocation stress will have to be considered.

The stress condition $\sigma'_x, \sigma'_y, \tau'_{xy}$ of the above theory is produced by considering a hydrostatic boundary loading due to a pressure

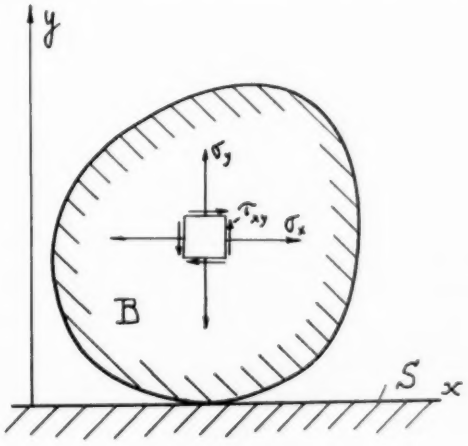


Fig. 2

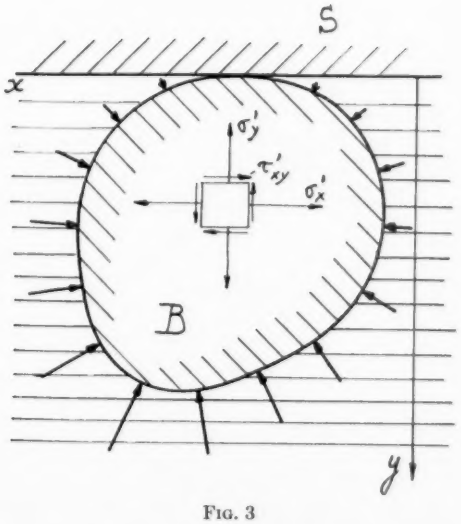


Fig. 3

$p = V = \delta y$. This loading may be applied by turning the body upside down, free from gravity, into a liquid of specific mass δ which is under the action of gravity (Fig. 3). The hydrostatic pressure $p = \delta y$ pushes it upward against the surface S with the same force as would its own weight in Fig. 2 and the contact pressures will be approximately realized. Due to this hydrostatic boundary loading a stress distribution $\sigma'_x, \sigma'_y, \tau'_{xy}$ is produced in the body B. Since no dislocation stresses have to be introduced, the value of the actual stress $\sigma_x, \sigma_y, \tau_{xy}$ due to distributed gravity loading in Fig. 2 is the difference of the stress $\sigma'_x, \sigma'_y, \tau'_{xy}$, and a pressure $p = y\delta$. Hence

$$\sigma_x = \sigma'_x + y\delta$$
$$\sigma_y = \sigma'_y + y\delta$$
$$\tau_{xy} = \tau'_{xy}$$

Gravity Stress in a Dam

Another example as shown in Fig. 4 is given by the case of a dam which is stressed under its own weight, the specific mass of which is δ.

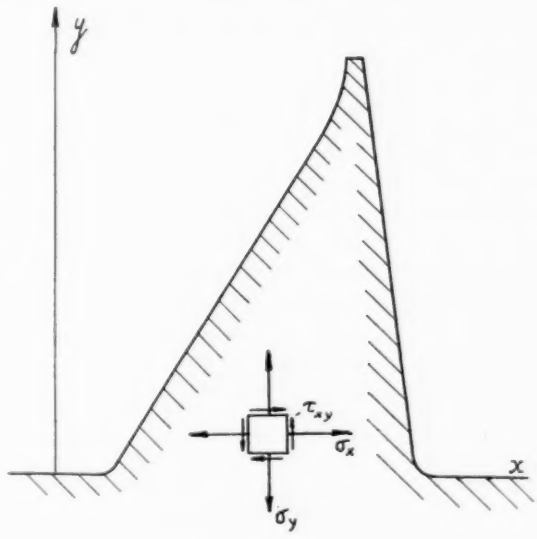


Fig. 4

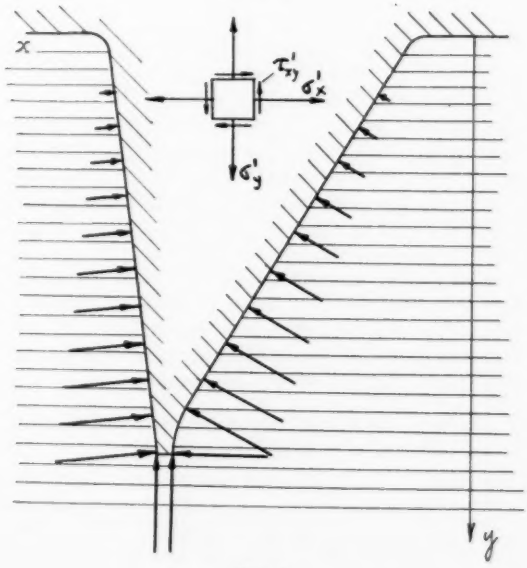


Fig. 5

The stress distribution $\sigma'_x, \sigma'_y, \tau'_{xy}$ is produced in a dam having no gravity and immersed upside down in a liquid of specific mass δ which is under the action of gravity, Fig. 5. This liquid exerts upon the dam hydrostatic boundary pressure as indicated in this figure. The actual stress $\sigma_x, \sigma_y, \tau_{xy}$ in the dam resting under its own weight as in Fig. 4, is given by the difference of the stresses $\sigma'_x, \sigma'_y, \tau'_{xy}$ and a pressure distribution $p = \delta y$. Therefore

$$\sigma_x = \sigma'_x + \delta y$$
$$\sigma_y = \sigma'_y + \delta y$$
$$\tau = \tau'_{xy}$$

Photoelastic Measurement of Gravity Stresses

The present theory shows that gravity stresses may be measured by photoelasticity by applying the proper boundary loading. The fringe pattern and the value of the maximum shear will be the same as if a magnified gravity were acting on the model.

Body With Holes

An interesting property is introduced by considering a body, with a hole, under the action of gravity. This equivalence of hydrostatic boundary loading applies provided we fill the

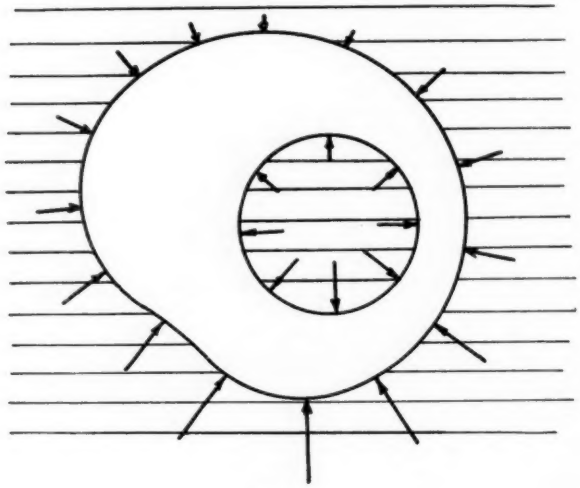


Fig. 6

hole with the weighing liquid and establish a connection between the liquid inside of the hole and the liquid outside such that both are in hydrostatic equilibrium (Fig. 6).

A theorem developed by J. H. Michell[3] shows that for a hollow two-dimensional body the stresses depend on the elasticity constants if the forces applied to the hole have a resultant different from zero. In this case the forces applied to the boundary of the hole have a resultant equal to the weight of the fluid inside. From this we conclude the following theorem:

In a solid two-dimensional homogeneous body the stresses due to gravity do not depend on the elasticity constants of the material; they do, however, in general for a body with holes.

It is important to keep this in mind when investigating gravity stresses by photoelasticity or with gelatine models.

TWO-DIMENSIONAL TEMPERATURE STRESSES RESULTING FROM STEADY HEAT FLOW

We consider a two-dimensional temperature distribution in an arbitrary cylindrical body; the temperature is supposed to be the same along any straight line parallel with the boundary. We also assume that the temperature has reached a state of equilibrium, i.e., the temperature may be different from point to point but remains constant at a given point and has any given arbitrary distribution along the boundary of the cross-section S (Fig. 7). As we know, in such a case the temperature θ in a cross-sectional plane (x, y) must satisfy the potential equation,

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0 \qquad [15]$$

[3] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill, 1934, pp. 113–117.

Let

$$\epsilon = k\theta \qquad [16]$$

where k is the coefficient of linear thermal expansion in three dimensions. It is known that this problem of two-dimensional strain is the same as that of the deformation of the same body under the body forces

$$\left.\begin{aligned} X &= -\frac{2G(1+\nu)}{1-2\nu}\frac{\partial\epsilon}{\partial x} = -\frac{\partial V}{\partial x} \\ Y &= -\frac{2G(1+\nu)}{1-2\nu}\frac{\partial\epsilon}{\partial y} = -\frac{\partial V}{\partial y} \end{aligned}\right\} \qquad [17]$$

and a normal boundary tension

$$\frac{2G(1+\nu)}{1-2\nu}\epsilon = V \qquad [18]$$

where G is the modulus of elasticity by shear, and ν is the Poisson ratio.

Since we have a steady-state heat problem, we have, according to Equations [15], [16], and [18]

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

We are thus exactly in the general case considered by the previous theory. The theory is easily applied to this case; we shall state only the conclusions.[4]

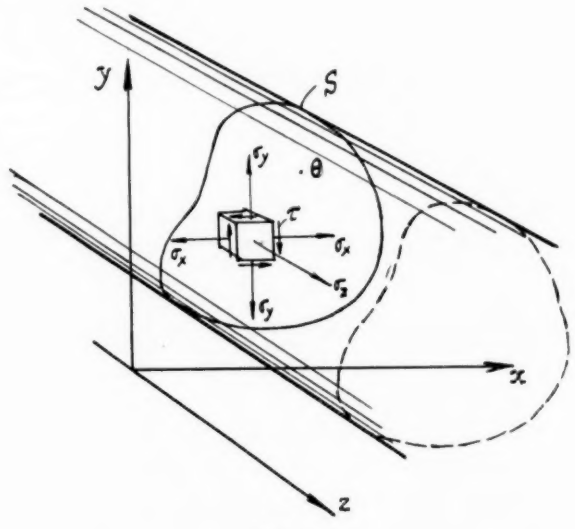


Fig. 7

We have to distinguish between the case where the cylinder is (1) solid or (2) hollow.

(1) If a solid cylinder is heated, either uniformly or not, but in such a way that a steady-state temperature distribution exists the same in every cross-section, the only stress produced is a tension or compression σ_z acting normally to the cross-section and equal to $\sigma_z = -E\epsilon = -Ek\theta$, where E is the Young modulus of the cylinder, k its coefficient of thermal expansion, and θ the

[4] It is expected that the complete analysis of this case by an independent method will be published later in the *Philosophical Magazine*: "A General Property of Two-Dimensional Thermal Stress Distribution," by M. A. Biot. See also, "Propriété generale des tensions thermiques en régime stationnaire dans les corps cylindriques," by M. A. Biot, Annales de la Sociète Scientifique de Bruxelles, vol. 54, series B, 1934.

temperature. We may say that the cylinder expands freely in the plane of its cross-section. This value $\sigma_z = -E\epsilon$ of the longitudinal stress is due to the assumption that longitudinal extension is prevented.

(2) If the cylinder is hollow, we make a longitudinal slit so as to connect the interior with the outside. When heated the two edges will in general separate. The cylinder becomes then a simply connected body without holes and we fall back on the previous case. In this slitted cylinder the temperature distribution will only produce axial stresses σ_z. But if we stick the sides of the slit together again, dislocation stresses σ'_x, σ'_y, τ'_{xy} will appear. Hence in a hollow cylinder stationary temperature

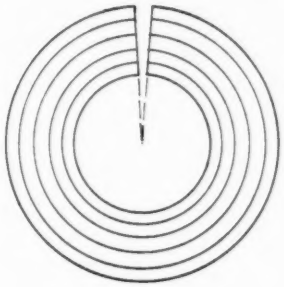


FIG. 8

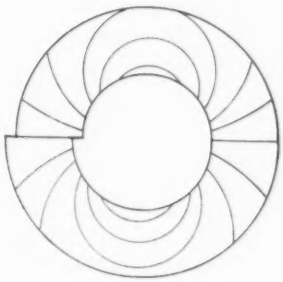


FIG. 9

distribution will generally give rise both to an axial stress σ_z and a dislocation stress system σ'_x, σ'_y, τ'_{xy}. They are the thermal stresses. In order that there should be no relative motion of the edges of the slitted cylinders, hence no stress except σ_z, the temperature distribution must satisfy three conditions, one of them being that the total flow of heat out of the hole is zero. These conditions are automatically satisfied if the temperature distribution has no singularity inside the hole (no source or sink or source-sink doublet).

Photoelastic Thermal-Stress Analysis

Thermal stresses may be measured by photoelasticity. A transient thermal-stress condition is equivalent to a boundary load and a dislocation. As previously shown, a steady-state thermal-stress condition is equivalent to a dislocation only and appears only in a hollow body. We shall have to calculate the amount of dislocation or relative motion of the two edges of a slit connecting the inside with the outside, make a model having that gap, and stick the two edges of this gap together.

Hollow Cylinder With No Heat Source Inside

In steady-state temperature distribution stresses other than σ_z may exist if the cylinder is heated from the outside. Those stresses disappear, however, if we fill the cylinder with a liquid having the same thermal conductivity as the cylinder because then all the singularities of the temperature distribution inside the hole disappear.

Circular Cylinder

For a radial heat flow we get concentric circular isothermal lines. A radial slit opens like that indicated in Fig. 8; no stress appears in the slitted cylinder. The thermal stresses in the non-slitted cylinder are those produced when bringing together the two edges of the slit. This amounts to the bending of a ring. For a doublet heat singularity (source and sink at the center) the isothermal lines are circles (Fig. 9). A radial slit as in the figure does not open up; the edges slide along each other.

Any heat-source singularity of an order higher than a doublet does not produce any heat stress in the cylinder.

On the Hydrodynamic Analogy of Torsion

By J. P. DEN HARTOG,[1] CAMBRIDGE, MASS., AND J. G. McGIVERN,[2] PULLMAN, WASH.

The authors herein describe a means for carrying out experimentally Kelvin's hydrodynamic analogy. A real liquid is used instead of the "ideal fluid" required by the analogy, the eddying flow caused thereby which violates the mathematical identity between the two cases being eliminated by taking photographs of the real liquid before eddying flow begins. The apparatus makes it possible to obtain photographs representing stress lines in shafts and also pictures of warped cross-sections.

THE first rational theory of torsion of prismatical bars, given by St. Venant in 1855, solved the problem completely for a limited number of simple cross-sections. However, a general method was lacking for the determination of the stress in any given practical section. St. Venant's solution was discussed in the first volume of Thomson and Tait's famous "Treatise of Natural Philosophy," in 1867, and it was there illustrated by a hydrodynamic interpretation, thus making it possible to visualize the general form of the solution for any kind of bar.[3]

In 1903, Prandtl disclosed an analogy employing a stretched membrane that was actually realized in experiment by Taylor and Griffith in 1916.[4] Prandtl's membrane analogy has since become so widely known that at present a large number of universities have such equipment available for instruction purposes.

Kelvin's hydrodynamic analogy is very much older than Prandtl's but we are not aware that it has ever been carried out in actual experiment. Undoubtedly the reasons for this are that an "ideal fluid" as required by the analogy cannot be obtained, and that any real liquid will show an eddying flow which violates the mathematical identity between the two cases. However, it is possible to succeed in the experiment if proper precautions are taken, and it is our purpose to describe how that may be done. Moreover, a slight modification of the experiment will give us a picture of the shape of the warped cross-section.

Kelvin's Analogy

As Thomson and Tait originally stated, the analogy employs a weightless prismatic box filled completely with "ideal fluid." This box must have the same shape as the cross-section of the elastic prism that is to be twisted and must be of unit height. If a couple is applied to the box in a plane perpendicular to its height, then the shear stress of the twisted solid will be proportional to, and have the same direction as, the velocity of the ideal fluid relative to the containing box.

Moreover, the couple or torque required to give the box a unit angular acceleration is equal to the "correction by which the torsional rigidity of the elastic prism, calculated by the false extension of Coulomb's law, must be diminished to give the true torsional rigidity."

Thus, if a flat tank having the cross-section of the twisted bar is filled with ideal fluid and rotated, a photograph of dust particles on the surface of the fluid, taken by a camera rotating with the tank, will show a set of closed curves coinciding with the stress lines in torsion.

It can also be deduced from the torsion equations that a photograph taken with the camera at rest and with the tank rotating about the "center of twist" gives a set of curves which are perpendicular to the lines of equal elevation of the warped cross-section in torsion.[5]

If a purely circular tank is rotated about its center, the ideal fluid inside will stand still, if it was initially at rest, because the ideal fluid has no friction against the tank walls. Photographed by a camera rotating with the tank, a set of concentric circles (the stress lines) will appear. A still camera, however, records no motion, expressing the well-known fact that a circular cross-section does not warp in twist. When the cross-section of the tank is not circular but roughly resembles a circle (shaft with keyway or square shaft), again the bulk of the fluid will hardly rotate, although a small part of the fluid near the corners of the edge will take part in the motion. Photographed with the rotating camera, this will give a set of closed curves (the stress lines) which are no longer circles but resemble circles at some distance from the boundary irregularities. The picture taken with the non-rotating camera will show considerable motion of the fluid near the protruding irregularities of the boundary but very little motion at some distance away.

Description of Apparatus

Difficulties from eddy formation and other frictional effects, which will occur in the non-ideal fluid used in the actual experiment, were avoided by taking the photograph immediately after starting the vessel from rest. It is known that any hydrodynamic system starting from rest exhibits a potential motion at the beginning and it is only after some time that eddies develop as a result of the friction in the fluid layers near the boundaries. To be sure, the time during which this potential motion occurs is very short, so that special precautions have to be taken in making the photograph. The apparatus consisted of a heavy horizontal disk rotatable on a large conical bearing about a vertical axis. The camera, facing downward, was solidly mounted at some height above the disk by means of vertical posts attached to the disk. The vessel containing water could be either clamped to the disk or mounted on it by means of another bearing so that it was rotatable with respect to the disk. Small aluminum particles were scattered on the water as is usual in hydrodynamic experiments. The ordinary precautions were taken of mixing india

[1] Assistant Professor of Applied Mechanics, Harvard Graduate School of Engineering. Mem. A.S.M.E. Professor Den Hartog was graduated as an electrical engineer from the University of Delft, Holland, in 1924 and received the degree of Ph.D. in 1929 from the University of Pittsburgh. From 1925 to 1932 he was connected with the Westinghouse Research Laboratories, with the exception of one year (1930–1931) when he was sent to the University of Göttingen for special studies.

[2] Instructor in Mechanical Engineering, State College of Washington. Mr. McGivern was graduated from Northeastern University with the degree of B.M.E. in 1928 and served as an instructor there for the three following years. In 1931 he received the degree of Ed.M. from Boston University and in 1932 an M.S. from Harvard University.

[3] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., New York, 1934, p. 263.

[4] Technical Report of the Advisory Committee for Aeronautics, London, vol. 3, 1917, p. 310.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] "Theory of Elasticity," by E. A. H. Love. Fourth edition. Cambridge University Press, 1927, p. 400.

ink or potassium permanganese with the water in order to make the bottom invisible and of waxing the sides of the vessel to insure a proper meniscus.[6]

The vessel was photographed during the first 10 deg of its rotation, approximately, while a tripper mechanism opened and closed the lens of the camera at the right instants. In this manner pictures could be obtained of the ideal flow before the boundary layer had time to upset its pattern.

Discussion of Results

Figs. 1*a* and 1*b* pertain to a square shaft and represent the stress conditions and cross-sectional warping, respectively. Fig. 1*a*

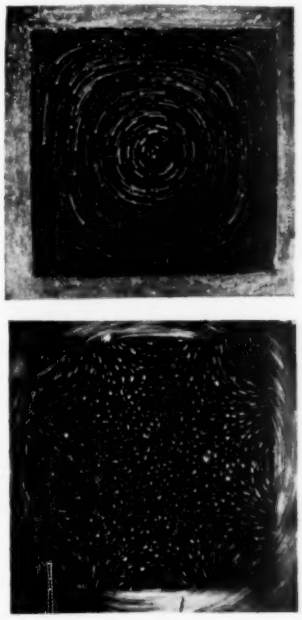


Fig. 1*a* (top) Stress Lines of a Square Shaft

Fig. 1*b* Warped Cross-Section of a Square Shaft

was taken with a camera rotating with the tank, resulting in a picture in which a sharp outline of the tank appears. The lines follow the direction of the shear stresses while the lengths of the individual streaks are proportional to these stresses. When photographing Fig. 1*b*, the camera was held still while the tank was rotated, the resulting picture being one in which a sharp outline of the tank does not appear. The lines photographed follow the direction of maximum slope of the warped cross-section and are perpendicular to the lines of equal height of that section. The lengths of the individual streaks are proportional to the slope. Since the square is a symmetrical cross-section, the center of twist coincides with its center of symmetry.

Figs. 2*a* and 2*b* depict a shaft with a keyway. Fig. 2*a*, illustrating a sharp keyway, is self-explanatory. Because of the sharp edge of the keyway, it is particularly difficult to avoid eddy formation. On the right or upstream side of the illustration no trouble occurs, but on the left or downstream side of the keyway the beginning of any eddy can be seen. This, of course, has to be discounted in interpreting the picture.

The non-symmetry of the keyway shaft is not very pronounced so that its center of twist is not far removed from the geometrical center. When photographing Fig. 2*b*, the tank was rotated about its geometrical center with the camera at rest and, therefore, shows an approximation of the lines of maximum slope of the cross-section. It is seen that the distortion is limited to the neighborhood of the keyway.

Fig. 3 illustrates the stress lines of an elongated rectangular section.

For sections made up of very thin parts, such as structural beams and split tubes, it is necessary to turn the tank through a very large angle (30 or 45 deg) in order to obtain sufficient relative motion. This, however, sets up a wave motion in the tank and makes it very difficult to secure a good picture. However, even in such cases the hydrodynamic analogy is very useful as it enables us to visualize the shape of the warped cross-section.

As an example, consider the split thin-walled tube shown in Fig. 4*a*. The center of twist T of this section lies at a distance R to the left if the wall is very thin. Imagine a tank of this shape rotated about T through an angle α. It can readily be seen that the displacement of any point of the tank walls is very great in comparison to the relative motion of the liquid with respect to the walls. The absolute motion of the liquid is non-rotational,



Fig. 2*a* (top) Stress Lines of a Shaft With a Keyway

Fig. 2*b* Warped Cross-Section of a Shaft With a Keyway



Fig. 3 Stress Lines of a Rectangular Shaft

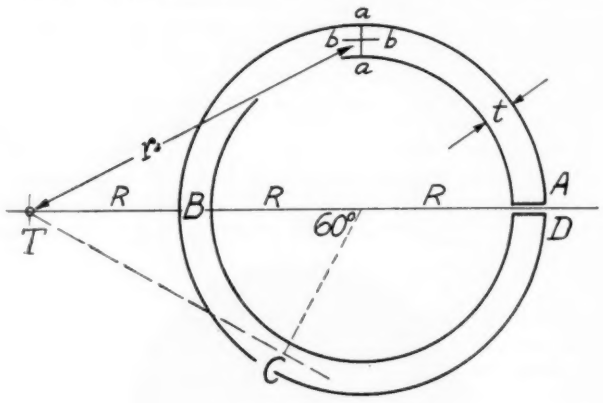
and since the line *bb* in Fig. 4*a* is at rest relative to the tank, *bb* turns through an angle α with the tank. For non-rotational (potential) flow it is then necessary for the perpendicular *aa* to turn through an angle $-\alpha$, so that the 45-deg dashed cross in Fig. 4*b*[7] retains its direction in space. With a wall-thickness t,

[6] "Applied Hydro- and Aeromechanics," by Prandtl-Tietjens, McGraw-Hill Book Co., New York, N. Y., 1934, p. 270.

[7] "Fundamentals of Hydro- and Aeromechanics," by Prandtl-Tietjens, McGraw-Hill Book Co., New York, N. Y., 1934, p. 154.

this leads to a relative displacement of the liquid at the walls of αt. The displacement of that point of the tank wall is αr, so that the ratio of the relative to the tank displacements is t/r, which is negligible for thin walls.

Thus, the warped cross-section picture is practically the same as that of the tank motion alone, or in other words, the water may be considered frozen. *The lines of maximum slope of the warped shape of a thin-walled section are circles about the center of twist, and the lines of equal elevation are radii through the center of twist,*



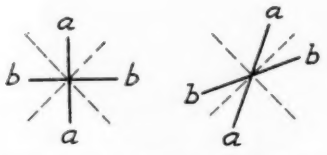


FIG. 4a (ABOVE) SPLIT THIN TUBE WITH ITS CENTER OF TWIST T

FIG. 4b (BELOW) THE DASHED CROSS RETAINS ITS DIRECTION IN SPACE WHEN THE PERPENDICULAR aa IN FIG. 4a TURNS THROUGH AN ANGLE HAVING A VALUE OF $-\alpha$

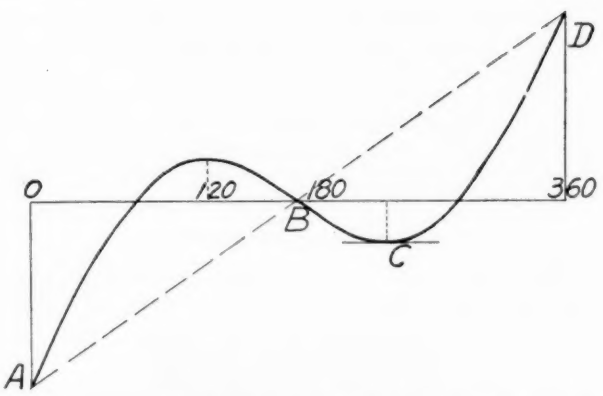


FIG. 5 DISTORTED CROSS-SECTION OF THE TUBE OF FIG. 4a DEVELOPED ON THE CIRCUMFERENCE

provided we consider only regions which are many wall thicknesses away from the center of twist.

For the split tube in particular it can be verified that the tank velocity at the slit is three times greater than at a point 180 deg away from the slit, so that the slope of the warped section is also three times as great. From the fact that the velocity is proportional to the slope, the shape of the warped section can easily be found to be

$$h = \theta - \pi + 2 \sin \theta$$

where h is the elevation and θ the angular distance from the slit. Fig. 5 shows this shape developed on the circumference, the points A, B, C, and D corresponding to those in Fig. 4a. This result so easily obtained shows the value of the warping analogy, because from casual observation of split tubes one is inclined to believe that Fig. 5 should have the shape of an inclined straight line, and the warped section that of a helix. In reality, however, the slope of the warped surface 180 deg from the slit is opposite in sign to what it would be in the case of a helix.

Pitting Due to Rolling Contact

By STEWART WAY,[1] EAST PITTSBURGH, PA.

Pitting, as the term is used in this paper, means a form of failure which occurs on gear teeth near the pitch circle, on rollers, in anti-friction bearings, and on other machine parts where cylindrical surfaces come in rolling contact under heavy loads. The purpose of the investigation described in this paper is to study the manner in which pits develop, to discover the reason for their development, and to find what conditions must be fulfilled to prevent their occurrence.

A testing machine was used in which steel rollers approximately 1.5 in. in diameter, rolling together and loaded as shown in Fig. 2, could be tested. More than eighty tests have been made with rollers of different materials, with different lubricants, and with various loads.

Pits are found to have a definitely characteristic shape and orientation with respect to the direction of rolling. They were observed to develop from minute cracks in the roller surfaces which appear after about 500,000 revolutions. Photographs were taken tracing the growth of these pitting cracks from their earliest observed stage. The cracks from the very beginning have a characteristic shape and orientation.

The principal findings in the tests on the steel rollers were: (1) That a lubricant must be present if pitting is to take place; (2) that if the lubricant is of a viscosity above a certain critical value which depends on the load, pitting can be prevented; (3) that the nature of the surface finish on the rollers greatly influences the tendency to pit, pitting being prevented on a highly polished surface and accelerated on a rough-machined surface; (4) that nitrided rollers will not pit under conditions that would result in severe pitting of quenched and tempered mild carbon-steel rollers, also that pits are smaller on harder surfaces, in general.

In the theoretical section of the paper, the load-carrying capacity for the oil film between two rollers having a common peripheral velocity is calculated, and it is shown that the critical viscosity necessary to prevent pitting is far below that necessary to prevent metal-to-metal contact, a fact also checked experimentally. The normal and tangential oil forces on the rollers after metal-to-metal contact are also calculated, and the tangential component is found to be small compared with the normal component. Calculation showed that no tensile stress would exist at the surface, where pitting cracks appear to start, due to contact pressure or oil forces. However, it is found by analysis that a small crack in the surface, if filled with oil, will tend to grow, provided it has a certain initial direction. This direction is the same as that of the cracks which precede pitting. This strongly suggests that oil penetration of very small surface cracks with a certain initial direction is the reason for the growth of these cracks until a particle is separated from the main body of material, leaving a pit. The experimental results harmonize, also, with this theory.

WHEN two cylindrical metallic surfaces engage in rolling contact, a form of failure known as "pitting" often occurs. This mode of failure is particularly troublesome on gear teeth in the vicinity of the pitch circle, where the teeth roll on each other without sliding. To give a clear idea as to what is meant by "pitting," a photograph, Fig. 1, of pits produced in the laboratory on steel rollers is shown.

Several features will be noted in Fig. 1: (*a*) The pit is bounded roughly by two straight lines and a curved line, and may be said to be fan-shaped; (*b*) at the vertex, there is a nipple-shaped extension to the cavity; (*c*) the pit is symmetrical about a line *AB* parallel to the direction of rolling; (*d*) the bottom of the pit is characterized by a series of curved steps, indicating a progressive fracture. Though it is not apparent in the figure, the direction of rolling, by which is meant the direction of motion of the contact area, is from *A* toward *B*. All pits that have been observed exhibit, with minor deviations, the four features just mentioned, and also have the previously mentioned orientation with respect to the direction of rolling.

In spite of the economic importance of preventing pitting, practically nothing has been done to discover the fundamental cause of this type of failure. Several theories have been proposed, however. The one most commonly advanced is that a fatigue crack begins as a result of repetition of the contact stresses, and that it probably begins near the region of maximum

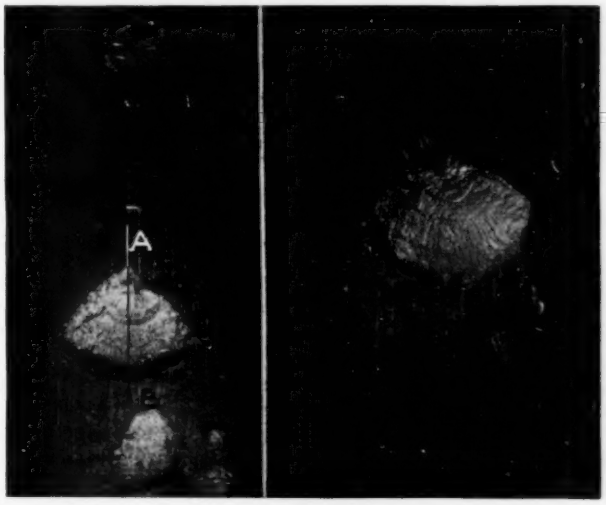


FIG. 1 TYPICAL PITS, ORIENTATION INDICATED BY LINE *AB*

---

[1] Research laboratories, Westinghouse Electric & Manufacturing Company. Mr. Way was graduated from Stanford University, Department of Mathematics, in 1929. He was employed six months as exchange work-student in the A.S.W. Grosskraftwerk in Böhlen bei Leipzig, Germany, and attended Göttingen University the summer semester of 1930. He was associated then with the Douglas Aircraft Company engineering department in Santa Monica, Calif., for one year and from there went to the University of Michigan for graduate study, receiving a doctor's degree in engineering mechanics in February, 1934.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

shearing stress a short distance below the surface.[2] Lanchester[3] suggests that pitting is due to the welding of material of one body to that of the other in a small region, which results in a particle's being torn away. Still another theory, obviously incomplete, is that pitting is due to initial inaccuracies in machining, leading to stress concentration at high spots and destruction of the surface at those places. To evaluate these several theories, to arrive at a better theory of the cause of pitting, and, if possible, to find a way to prevent such failure, the following experiments with steel rollers, were carried out.

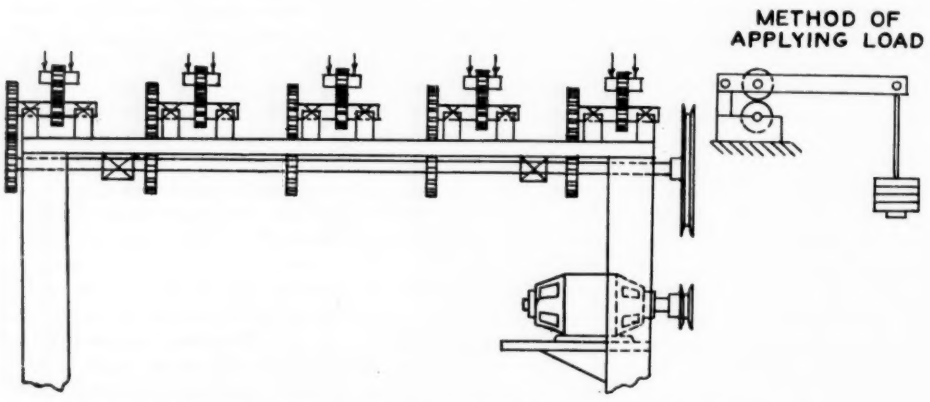


FIG. 2 SCHEMATIC DIAGRAM OF MACHINE FOR TESTING FIVE PAIRS OF ROLLERS



FIG. 3 PHOTOGRAPH OF THE TESTING MACHINE SHOWN IN FIG. 2

TESTS OF STEEL ROLLERS

The testing apparatus is shown schematically in Fig. 2, and by photograph, Fig. 3. The rollers have a face width of 1/2 in.; and the lower one is 1.576 in. in diameter while the upper has a diameter of 1.500 in. The difference in diameters insures that any point on one roller shall come in contact with a different point on the other roller with each revolution. The lower roller is driven at 400 rpm from a lineshaft by gears, and it in turn drives the upper by traction in the contact area.

There is good reason to believe that there is no relative sliding between the rollers.[4] The lower roller dips into an oil bath, the same lubricant being used for the bearings as for the rollers. Shock loads are minimized by spring suspension for the weights on the lever arms.

The main variables which can be controlled and studied with the present equipment are material, lubricant, and load. Speed may also be varied by changing gears or pulleys. The material variable must be understood to include not only all manner of metals and alloys, but also various treatments of these materials and various types of surface finish. In this paper the results of tests on three materials are discussed. The composition and properties of these materials are given in Table 1.

Steels Nos. 1 and 2 are of the same composition but have different treatments. Except where otherwise noted, all tests were made with rollers having a ground-surface finish.

About 80 pairs of rollers have been tested so far. Space does not permit a description of each test. The outstanding results are as follows:

Pitting Will Not Occur With-

TABLE 1 COMPOSITION AND PROPERTIES OF MATERIALS TESTED

	Steel No. 1 (Rolled)	Steel No. 2 (Forged)	Steel No. 3
Carbon, per cent.....	0.40–0.50	0.40–0.50	0.25–0.35 (Al = 0.90–1.40%)
Manganese, per cent...	0.50–0.80	0.50–0.80	0.40–0.70 (Mo = 0.60–1.00%)
Treatment.....	Quenched and tempered	Quenched	25-hr nitriding treatment to give 0.018 in. case
Tensile strength, lb per sq in.....	108,000	220,000–240,000	.....
Yield point, lb per sq in.	78,000	....	.....
Hardness (VPN).....	250	470	.....

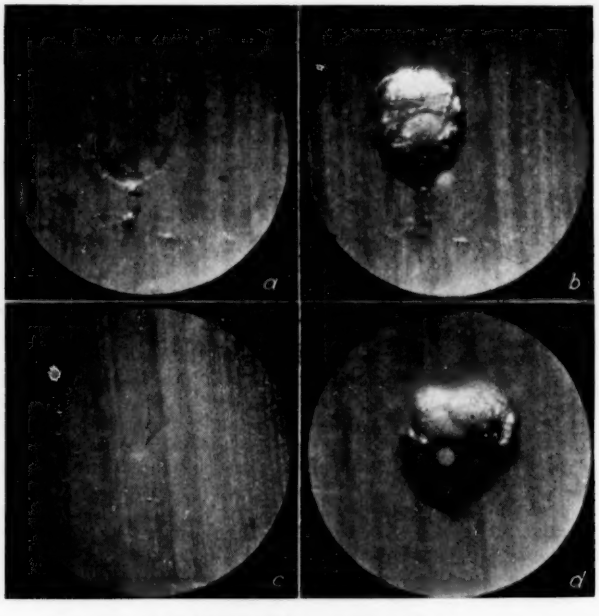


FIG. 4 *a–d* FORMATION OF PITS
(*a* Pre-pitting crack and *b* the pit which formed shortly after; *c* another crack, and *d* the eventual pit.)

[2] "Determining Capacity of Helical and Herringbone Gearing," W. P. Schmitter, *Machine Design*, July, 1934.

[3] "Spur Gear Erosion," by F. W. Lanchester, *Engineering*, June 7, 1921, p. 733.

[4] Dents caused on one roller by an irregularity on the surface of the other are spaced a distance equal to the difference in circumferences.

out Lubricant. In all investigations of the wear and pitting of gear teeth that have come to the author's attention, the tests were carried out with lubrication. A. H. Burr,[5] however, made a large number of tests on steel rollers similar to those tested by the author, but running without lubricant. In no case was pitting obtained. Rollers running dry have also been tested by the author, with no pitting resulting. When oil is used to lubricate the rollers, pitting sometimes results and sometimes does not, depending on the load used, the viscosity of the oil, and other factors. One fact, however, was consistently borne out in tests by both Mr. Burr and the author, i.e., that pitting could not be produced without lubrication, regardless of the load used. The duration of these tests was, in most cases, in excess of five million cycles, and in one case twelve million and in another eighteen million. In 54 tests with lubricated rollers which pitted, pitting occurred in 50 per cent of the cases between 670,000 and 1,450,000 cycles, while the longest run was 4,867,000 cycles.

When steel rollers were run without lubricant, oxides formed on the surfaces. These oxides form very soon after the start of the test, and flake off. The composition of the oxide flakes was found to be 1.1 per cent Fe, 44.6 per cent FeO, and 54.2 per cent Fe_2O_3.

One pair of rollers was run without lubricant for 5,400,000 cycles. Oil was then added. At 5,560,000 cycles several pits had formed.

Pits Develop From Cracks That Start at the Surface. It was concluded at an early stage that the most satisfactory way to find the cause of pitting was to study pits while they were being formed. It was discovered, first, that a pit does not result from the gradual enlargement of a small cavity, but results from the growth of a crack which eventually separates a particle from the main body of material. In Fig. 4, *a* and *b* are the top view of a pre-pitting crack and the pit which formed shortly after. In *c* and *d* another crack and its eventual pit are shown. It will be noted that there is as definite a characteristic shape and orientation for the cracks as for the pits.

In Fig. 4 *e*, *f*, *g*, *h*, and *i* are shown various stages in the growth of a particular crack, at 13 × magnification. When first observed, cracks usually appear as in Fig. 4*e*. The shape is that of a V with the point toward the contact region as the crack approaches this area.

The cracks have also been studied by cutting the rollers perpendicularly to the axis. A crack in the stage of development of 4*e*, or perhaps slightly earlier, is shown in section at *a* in Fig. 5, at 100 × magnification. A crack a little further along is shown in top view and in vertical section in Figs. 5*b* and 5*c*. The top view was photographed after the sectional cut had been made.

[5] Formerly at Westinghouse research laboratories, East Pittsburgh, and now at Rice Institute, Houston, Texas.

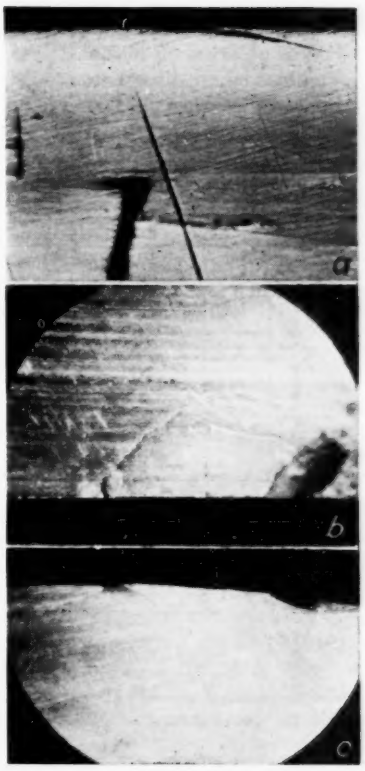


FIG. 5 ENLARGEMENTS OF PITTING CRACKS

[*a*, Section cut perpendicularly to the axis showing crack (×100); *b*, top view of a crack (×13); *c*, section through the roller and crack shown in *b* (×13).]

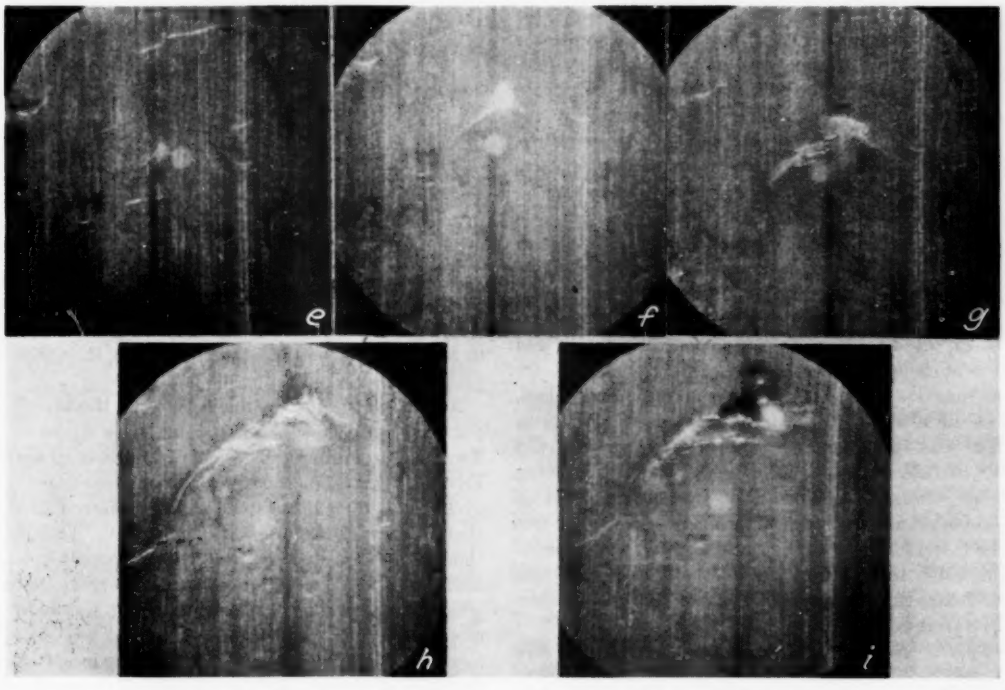


FIG. 4 *e–i* VARIOUS STAGES IN THE GROWTH OF A CRACK (×13)

The cracks show every evidence of having started at the surface. No cracks bearing any resemblance to pre-pitting cracks could be observed on rollers not yet tested. These cracks were found to start usually between 500,000 and 1,500,000 cycles. In most cases less than 100,000 cycles were required from the time a crack first appeared to the time when a pit was formed. The cracks extend into the roller at an acute angle to the direction of motion of the contact region along the surface.

M. Ulrich[6] says that in case-hardened gear teeth, cracks were observed starting beneath the surface. The author questions

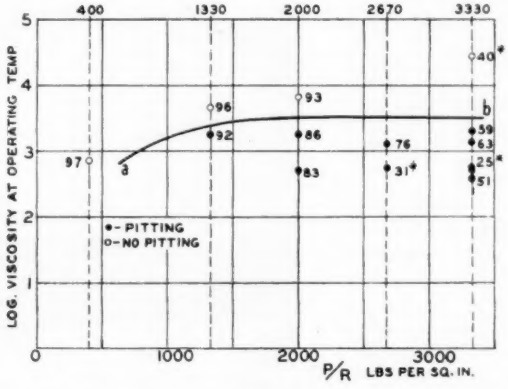


FIG. 6 LOAD-VISCOSITY RELATION IN PITTING TESTS
(0.40 per cent carbon steel rollers of 250 VPN hardness; speed 400 rpm; *speed 700 rpm.)

whether these cracks are related to the cracks which precede pitting.

Pitting Can Be Prevented by Using Oil of Sufficiently High Viscosity. It would seem reasonable to believe that if an oil of viscosity sufficiently high to prevent metal-to-metal contact of the rollers were used, no pitting would result. The interesting fact is, however, that even with metal-to-metal contact, pitting can be prevented by increasing the viscosity of the oil above a certain critical value. This critical value will depend on the load carried by the rollers. In Fig. 6, points are plotted for a number of pitting tests in a viscosity-load coordinate system. The load is measured by P/R, the pounds per inch of roller face width per inch of radius, and the viscosity is measured by the logarithm of the Saybolt viscosity at the operating temperature. Points indicated by dots represent tests where pitting took place, while points indicated by small circles represent tests where no pitting took place. All tests represented in this figure were made with steel No. 1. A curve *ab* of indefinite shape and position can be drawn so that the no-pitting points lie above it and the pitting points lie below it. Additional tests would more definitely determine the position of this curve. The location of the curve would be expected to depend on the speed of rotation and on the properties of the material.

To settle the question of whether or not the oil film is broken, in tests with an oil of viscosity high enough to prevent pitting, the upper roller was insulated from the lower except for the contact region. An electromotive force was introduced between upper and lower rollers. Running at no-load, no measurable current was conducted across the oil film, but as soon as the test load was applied, the film was broken and considerable current passed from one roller to the other. That the oil film is broken at much lower loads than necessary to cause pitting is shown theoretically in a later portion of this paper.

Hard and Smooth Surfaces Better Resist Pitting. In five tests with steel No. 1; a mineral oil of viscosity 87 sec Saybolt at 210 F and 900 sec at 100 F; and a load P/R = 3330 lb per sq in., the average number of cycles that elapsed until pitting was first observed was 1,555,000. In three tests with steel No. 2, the same oil, and the same load, the average number of cycles until pitting began was 1,416,000. It seems that the number of cycles necessary for pitting is practically independent of the hardness. However, an extremely hard surface, as that on the nitrided rollers of steel No. 3, was found not pitted at all after 6,000,000 cycles. In the test made with nitrided rollers, the load was 2670 lb per sq in. and the oil was the same as that already mentioned. Referring to Fig. 6, at P/R = 2670 the viscosity necessary to prevent pitting would have to be about 2000 sec Saybolt for steel No. 1. The oil used had a viscosity at the operating temperature of about 700 sec. Both steel No. 1 and steel No. 2 were found to pit badly with oils even heavier and a load of 2670 lb per sq in.

In the case of steel No. 2, although pitting began as soon as it did on rollers of steel No. 1, the pits were much smaller. The average diameter of the ten largest pits on a roller of steel No. 1 was 0.070 in., while that of the ten largest pits on a roller of steel No. 2 was only 0.036 in.

The degree of roughness of the surface is another factor that influences pitting. The rollers of steel No. 1 in the five tests previously mentioned had ground finishes. Two tests were made similar to these five except that the surfaces were polished. In one case, a test made by Mr. Burr, the rollers ran 12,600,000 cycles without pitting, and in the other case pitting began after 4,267,000 cycles. It is believed that the polish on the latter rollers was not as fine as on the former. But the point that should be emphasized is that polishing decidedly reduces the likelihood of pitting. In three tests with rough-machined surfaces, pitting began on an average at 430,000 cycles.

Significance of Experimental Results. A satisfactory explanation of pitting must take account of the part played by the lubricant. It must explain also why lubricants of a sufficiently high viscosity prevent pitting, even though they do not prevent metal-to-metal contact. It must account for the effects of surface roughness. Finally, it must explain why pits and the cracks which precede them have a certain characteristic shape and orientation.

The theory that pitting comes as a result of repetitions of the Hertzian system of contact stresses, discussed later, fails to explain why pitting does not take place without oil and why an extremely high polish on the surface will greatly retard or even prevent such failure. The theory that there is a welding together of the rollers in a small region, resulting in the tearing away of a particle, does not seem to be substantiated by observations of the pitting cracks.

It was found that the pits tend to form in planes perpendicular to the roller axis. This implies that the rollers do not have a truly cylindrical surface, and that the diameter is slightly larger in some planes than in others, resulting in higher loads in those planes.

A distinction must be made between the primary cause of the pitting cracks shown in Fig. 4 and the cause of their growth. Two results of importance should be mentioned in this connection. First, in the test of the rollers which were run dry for 5,400,000 cycles, and then lubricated, pitting occurred in the remarkably short span of 160,000 additional cycles. This implies that, although oil is necessary for pitting, some form of destruction leading to the formation of pitting cracks takes place independently of the oil. Secondly, a test was made in the usual manner with oil until several pits and ten pitting cracks had formed. The rollers were then cleaned and dried. On continuation of the test without oil none of the cracks developed into a pit. This indicates that oil is necessary for crack growth.

[6] "Zur Frage der Grübchenbildung bei Zahnrädern," M. Ulrich, *Zeit. V.D.I.*, 1934, vol. 28, no. 2.

The theory was advanced that steel particles, torn from the roller surfaces, would adhere to the rollers because of the oil, and would be carried into the contact region, causing high tensile stresses at the surface and possibly starting cracks. But it was found that oil was not essential to bring particles into the contact region; dry rollers were seen to be dented by particles, as were those tested with lubricant. Also, the fact pointed out in the last paragraph, that oil is probably not responsible for the beginning of the destructive process, leads to the conclusion that though particles may play an important part in this stage of the process, the oil is not a necessary aid in their action.

In support of the assumption that particles are a powerful contributing factor in the initial formation of pitting cracks stands the result of a test with polished rollers and oil filled with steel particles. In this test pitting began at 537,000 cycles. Also in tests in which felt wipers or a compressed-air jet were used in an effort to remove all steel particles, pitting was decidedly retarded. In opposition to the particle-action theory are numerous observations of pitting cracks at a very early stage, though perhaps not early enough, which showed no signs that a particle had been responsible for their origin. Also, the introduction of steel particles, MgO powder, or rouge powder into the oil did not materially accelerate pitting on rollers with ground surfaces.

Before going further into the discussion of the cause of pitting, some conclusions arrived at theoretically will be reviewed.

Theoretical Considerations[7]

Two problems present themselves, one to determine the loading on the rollers and the other to find the stresses caused by this loading. The loading consists of the forces due to metallic contact and the forces applied by the oil.

If the oil were not present, it would probably be justifiable to assume that the loading would be an elliptical distribution of contact pressure over a narrow rectangular contact area, in accordance with the Hertz theory of the contact of elastic bodies. If the cylinders are lubricated with oil, but are not in contact, the oil pressure can be calculated in the oil film in the same manner that the pressure in oil films in bearings is calculated. If the cylinders are in contact and also are lubricated with oil, the problem becomes extremely difficult unless the assumption is made that the deformation of the surface in front of the contact area is that due to contact pressures alone. The oil pressure at any point in front of the contact area can then be calculated. Also the tangential oil forces can be found. Within the contact area the loading will be the elliptically distributed contact pressure.

In calculating the oil forces, the amount of oil that passes between the rollers must be considered. This quantity will depend on the height of irregularities on the roller surfaces. For the pitting-test rollers with ground surfaces, the irregularities were found to have a maximum height of about $2\delta = 0.0002$ in.[8]

When loads are given, the problem of finding the stresses in the rollers at any particular point presents little analytical difficulty. The contact stress system has to be superimposed on the stress system due to the oil forces.

Without going into the calculations, the results of the theoretical study of loads and stresses will now be briefly reviewed.

A study of the contact stresses for parallel elastic cylinders as determined by the Hertz theory and further calculations by Belajef[9] show definitely that there is no tensile stress near the region of contact. Analysis of the oil forces before the rollers are in contact shows that at loads high enough to produce contact the oil pressure is so low, for usual viscosities and speeds, that the effect of pressure upon viscosity can be neglected. Equation [18], Appendix 2, arrived at for the load-carrying capacity of the oil film, indicates that in the pitting tests the oil film is broken at loads very much lower than those necessary for pitting. Investigation of oil forces after the rollers are in contact shows that as long as the influence of pressure on viscosity can be neglected the oil tangential traction will be quite small compared with the normal oil pressure. Normal oil pressures are found to be quite small compared with contact pressures that exist in the pitting tests, which tends to justify the assumption that the shape of the roller surfaces in front of the contact area is determined mainly by the contact forces. The tensile stresses at the surface due to the tangential oil forces are found to be quite small compared with the compressive stresses due to normal oil pressure, so that the net effect of the oil forces is to produce only compressive stresses.

Neglecting the stresses due to tangential oil forces, the resultant stress at any point will consist of the stress due to the distributed oil pressure as shown at p_2, Fig. 13, and the distributed contact pressure p_1. Referring to Fig. 14, the stress due to these loads at a point, A, a short distance below the surface and in front of the contact area can be calculated. The stress at a point, B, at the surface but closer to the contact area than A can also be calculated. A plane passing through A and B will then make an acute angle with the roller surfaces. If this acute angle falls in a certain range, it can be shown that the normal stress on plane AB at A is less than at B. If the angle θ, Fig. 14, is negative, it can be shown that the normal stress on plane AB is greater at A than at B, for A sufficiently close to the surface. This has a certain significance. For if we imagine that a very small crack exists, extending along the plane BA, and if BA is so oriented that the normal stress at A is less than at B, oil penetration of that crack would cause a high tensile stress at A. The oil that penetrates the crack will be under a pressure equal to the value of p_2 at B, which is also the value of the compressive stress on plane AB at B. The entrance of the oil into the crack would raise the pressure on the walls of the crack from some value less than the value of p_2 at B to this value. The stress system resulting from these added forces on the walls of the crack would give a concentrated tensile stress at the end of the crack. Thus, if a fine crack exists at the surface, inclined at an acute angle to the surface in the direction of rolling, and if the oil is of a viscosity sufficiently low to permit it to enter the crack, it is probable that the crack will grow.

Since we find that contact pressure and oil forces do not produce any tensile stress when the tangential oil forces are small compared with the normal oil forces, it would seem very improbable that repetitions of these loads could result in a fatigue crack.

General Conclusions

The two questions regarding pitting that puzzle the engineer are: What causes pitting? and How can pitting be prevented?

Although the author is not prepared to say definitely what causes pitting, he has arrived at some conclusions which can guide future investigation. Explanations of pitting advanced in the past are shown by the test results to be unsatisfactory or incomplete. The development of pitting cracks from a minute crack at the surface to a pit has repeatedly been carefully traced. The reason for the growth of these cracks, no matter how small they are to start with, seems to be adequately explained by the theory of oil penetration for the following reasons:

(1) The theory requires that oil be present for pitting.

[7] A more detailed theoretical treatment is given in the appendixes.

[8] A gelatine cast was made of the surface of the roller and examined in section under high magnification. The technique of such observations is described by W. Norman Thomas, *Engineering*, October 12, 1923.

[9] Institute of Ways and Communications, Leningrad, U.S.S.R.

(2) The theory requires that the oil be of viscosity low enough to permit penetration of fine cracks, if the cracks are to grow.

(3) The theory shows that cracks will grow only if they have a certain initial direction. This direction is the same as that of pitting cracks observed at a very early stage, as shown in Fig. 5.

(4) The theory explains why the critical viscosity for pitting does not increase appreciably with the load at high loads; for the oil pressure p_2 is practically independent of the total load carried, once there is metal-to-metal contact. p_2 will depend on the load only as δ is reduced by increasing the load.

(5) The theory is consistent with the finding that the rougher the surface finish the greater the likelihood of pitting. It is natural to expect more and larger cracks initially in the surface of a rough-machined roller than in that of a polished roller.

Possible methods of preventing pitting found so far are as follows: (1) Using no oil; (2) using highly polished surfaces; (3) increasing the viscosity of the oil; (4) using a very hard surface and tough interior as are obtained by nitriding.

The first method is obviously not practical and the second is quite expensive. The third method in many cases can be successfully applied. The fourth method deserves much further study.

Future investigation should be mainly along two lines, one aiming definitely to determine the primary cause of pitting cracks starting (as distinguished from the cause of their growth) at the surface, and the other aiming to find materials and heat treatments which show superior resistance to the formation and growth of pitting cracks. The effects of different grain sizes deserve special attention.

Acknowledgment

The author wishes to emphasize the importance of the work done by A. H. Burr in the Westinghouse Laboratories, who, to the author's knowledge, was the first to find substantial evidence that rollers run dry will never pit. Mr. Burr also succeeded in preventing pitting by the use of polished rollers. He suggested that pitting might be due to oil penetration of fine cracks, which theory is substantiated by the experimental and theoretical work of the author.

The work could not have been carried out without the cooperation of R. S. Marthens of the Nuttall Works, Westinghouse Electric and Manufacturing Company, which concern supplied the test rollers. The author is also indebted to Howard Scott, Miss M. Ferguson, and J. G. Hoop for assistance and advice on the metallurgical side of the investigation, and to J. G. Ford for his cooperation in the lubrication field.

Appendix 1

Contact Stresses

The Hertz[10] theory is based on the assumptions that the contacting bodies are perfectly elastic, that there are no shearing stresses in the contact area, and that the radii of curvature are large in comparison with the dimensions of the area of contact. The results of this theory are well verified by experiment[11] if reasonable precautions are taken to fulfil the assumptions, but if the assumptions are not fulfilled, as is the case if there is a large tangential stress between the bodies, the results of the Hertz theory may be considerably in error.[12]

[10] "Miscellaneous Papers," by H. Hertz, 1896, p. 146. "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Co., 1934, p. 344.

[11] A. Lafay, *Ann. Chim. Phys.*, vol. 23 (7), 1901; R. Striebeck, *Zeit. V.D.I.*, 1901, vol. 45, pp. 73, 118, and 1421; Schwinning, Ibid., 1901, vol. 45, p. 332; F. Auerbach, Handbuch d. Ph. u. Tech. Mechanik, vol. IV–1, p. 271; Thomas and Hoersch, Univ. of Ill. Eng. Exp. Station Bull. No. 212.

[12] Hans Fromm, *Zeit. f. Tech. Physik*, 1928, pp. 229–311.

For cylinders of radii R_1 and R_2, the Hertz theory gives the following expressions for width of contact area and load distribution

$$2b = \sqrt{\frac{32\,P\,(1-\mu^2)}{\pi E\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}} \quad \ldots\ldots [1]$$

$$p = p_{\max}\sqrt{1-\frac{y^2}{b^2}} \quad \ldots\ldots [2]$$

$$p_{\max} = \frac{2P}{\pi b} \quad \ldots\ldots [3]$$

The maximum shearing stress occurs at the point $y = 0$, $z = 0.786b$, axes being located as shown in Fig. 7. The magnitude of

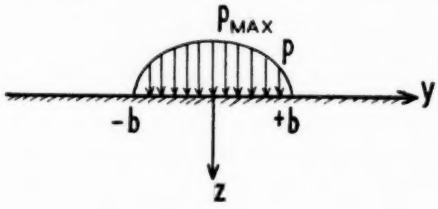


Fig. 7

the maximum shearing stress is 0.300 $p_{\max}$. The general expressions of Hertz for the stresses have been evaluated for this case by Belajef,[9] who finds the principal stresses are given by

$$\sigma_{1,2} = -\,p_{\max}\,e^{-\alpha}\sin\beta\left\{1 \mp \frac{\sin h\,\alpha}{\sqrt{\sin^2\beta+\sin h^2\,\alpha}}\right\} \quad \ldots [4]$$

where α and β are elliptical coordinates which specify the position

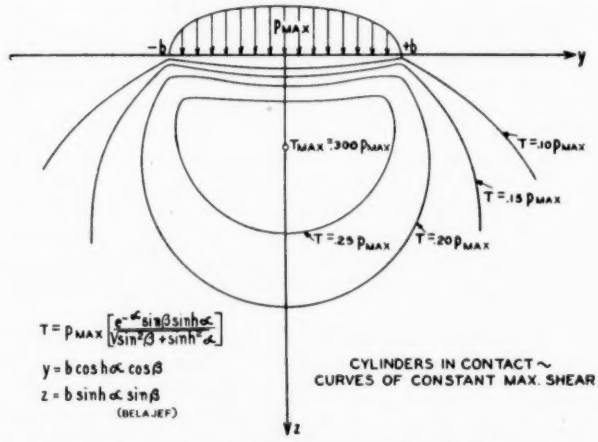


Fig. 8*a* Curves of Constant Principal Shearing Stress

of any point in the body. We see that σ_1 and σ_2 are always compressive stresses, since α and β are positive. Belajef gives for the principal shearing stress at any point

$$T = p_{\max}\left\{\frac{e^{-\alpha}\sin\beta\,\sin h\,\alpha}{\sqrt{\sin^2\beta+\sin h^2\,\alpha}}\right\} \quad \ldots\ldots [5]$$

Curves of constant principal shearing stress based on Equation [5] are plotted in Fig. 8*a*. In Fig. 8*b* the photoelastic picture of bakelite cylinders in contact shows good confirmation, qualitatively, of the theory.

For the rollers used in the pitting tests we have the following data:

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.75} + \frac{1}{0.788} = 2.603$$

$$E = 30 \times 10^6 \text{ (lb per sq in.)}$$

$$\mu = 0.3$$

$$\frac{1 - \mu^2}{\pi E} = 0.965 \times 10^{-8}$$

The width of contact area, maximum compressive stress, depth of the point of maximum shearing stress, and magnitude of the maximum shearing stress are found to be, respectively

$$2b = 3.44 \times 10^{-4} \sqrt{P} \text{ (in.)}$$

$$p_{max} = 3695 \sqrt{P} \text{ (lb per sq in.)}$$

$$0.786b = 1.352 \times 10^{-4} \sqrt{P} \text{ (in.)}$$

$$Y_{z\,max} = 1110 \sqrt{P} \text{ (lb per sq in.)}$$

where P is the load per inch of roller-face width. Values of these quantities for a number of loads are given in Table 2.

TABLE 2

Load	500	1000	1500	2000	2500
P/R[a]	677	1333	2000	2670	3330
$2b$	0.0077	0.0109	0.0133	0.0154	0.0172
p_{max}	82,600	116,800	143,100	165,300	184,700
$0.786b$	0.0030	0.0043	0.0052	0.0060	0.0068
$Y_{z\,max}$	24,800	35,100	43,000	49,600	55,500

[a] $R = 0.75$ in.



FIG. 8b PHOTOELASTIC TEST OF CYLINDERS IN CONTACT
(Compare shape of constant-shear curve with Fig. 8a.)

Appendix 2

OIL FORCES BEFORE METALLIC CONTACT

The rollers are assumed to be cylindrical and separated a distance 2δ as shown in Fig. 9. The y-axis is chosen mid-way between the rollers. Neglecting inertia forces and considering only the region where the distance between the rollers is small compared with the roller radii, the equation of equilibrium in the y-direction takes the form[13]

$$-\frac{dp}{dy} + \lambda \frac{\partial^2 v}{\partial z^2} = 0 \quad \ldots\ldots [6]$$

where p is the pressure, v the y-component of velocity and λ the viscosity. If the velocity is $-\alpha$ at the bounding surfaces $z = z_1$, $z = -z_2$, Equation [6] gives, after integration

$$v = \frac{1}{2}(z + z_2)(z - z_1)\frac{1}{\lambda}\frac{dp}{dy} - \alpha \quad \ldots\ldots [7]$$

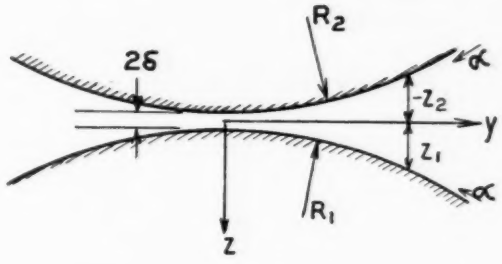


FIG. 9

The total quantity of oil passing between the rollers per unit time is

$$Q = -\int_{-z_2}^{z_1} v\,dz \quad \ldots\ldots [8]$$

and if we substitute Equation [7], integrate, and rearrange the terms, we obtain

$$\frac{1}{\lambda}\frac{dp}{dy} = \frac{3Q - 6\alpha\zeta}{2\zeta^3}; \quad \zeta = \frac{z_1 + z_2}{2} \quad \ldots\ldots [9]$$

If we assume for ζ and λ the expressions

$$\zeta = \delta + \frac{y^2}{2R}; \quad \frac{1}{R} = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \ldots\ldots [10]$$

$$\lambda = \lambda_0 (1 + \gamma p^2) \quad \ldots\ldots [11]$$

where λ_0 is the viscosity at zero pressure and γ is a constant to be determined experimentally, Equation [8] may be integrated, with the result

$$\frac{1}{\sqrt{\gamma}} \tan^{-1} p\sqrt{\gamma} = \frac{3\lambda_0'}{\delta'^{3/2}} f(y', \beta) \quad \ldots\ldots [12]$$

in which

$$\lambda_0' = \lambda_0\alpha/R;\ \delta' = \delta/R;\ y' = y/\sqrt{R\delta};\ \beta = Q/2\alpha\delta$$

$$f(y',\beta) = \frac{\beta}{4}\left\{\frac{y'}{\left(1 + \frac{y'^2}{2}\right)^2} + \frac{^3/_2\, y'}{1 + \frac{y'^2}{2}} + \frac{3}{\sqrt{2}}\left(\tan^{-1}\frac{y'}{\sqrt{2}} - \frac{\pi}{2}\right)\right\} - \frac{y'}{1 + \frac{y'^2}{2}} - \frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{y'}{\sqrt{2}} - \frac{\pi}{2}\right) \quad \ldots\ldots [13]$$

The condition that $p = 0$ when $y = \infty$ has been used. To determine β, it is assumed that on the out-flow side p and dp/dy become zero together, or that p is zero when the velocity is uniform and equal to $-\alpha$. Equations [9] and [12] then give the value

[13] See "Mechanical Properties of Fluids," Blackie & Son, Ltd., London, p. 115.

1.225 for β, and we find that dp/dy is zero at $y' = \pm 0.671$. When $y' = -0.671$ p is zero, and when $y' = 0.671$ p is maximum, p_{max} is given by

$$\frac{1}{\sqrt{\gamma}} \tan^{-1} p_{max} \sqrt{\gamma} = \frac{3\lambda_0'}{\delta'^{3/2}} (0.1793) \quad [14]$$

The total load carried by the oil film is

$$\frac{P}{R} = \sqrt{\delta'} \int_{-0.671}^{\infty} p dy' \quad [15]$$

Assuming that γp^2_{max} is small compared with unity we shall have, instead of Equations [12], [14], and [15] for $\beta = 1.225$

$$p = \frac{3\lambda'_0}{\delta'^{3/2}} f(y', \beta)_{\beta=1.225} \quad [16]$$

$$p_{max} = \frac{3\lambda_0'}{\delta'^{3/2}} (0.1793) \quad [17]$$

$$\frac{P}{R} = 1.227 \frac{\lambda'_0}{\delta'} \quad [18]$$

To show the distribution of oil pressure, p has been plotted against y' in Fig. 10.

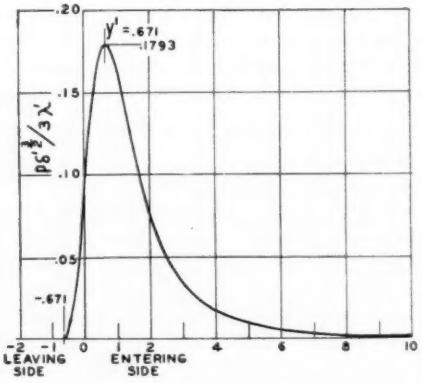


FIG. 10 PRESSURE VARIATION IN OIL FILM BETWEEN ROLLERS SEPARATED BY A DISTANCE 2δ; $\left(\delta' = \frac{\delta}{R}\right)$

Since 2δ is the average thickness of the oil film when $y = 0$, the condition for the rollers to come into contact is that 2δ shall equal the height of the surface irregularities. As previously mentioned, the irregularities on the surfaces of the pitting-test rollers had a height $2\delta = 0.0002$ in., so that $\delta' = 0.000133$. The maximum oil pressures have been computed for this value of δ' and for various values of λ_0', by Equations [14] and [17], with the results as given in Table 3.

TABLE 3 VALUES OF p_{max} IN LB PER SQ IN.

λ'_0, dynes per sq cm.....	20	50	100	200	400
p_{max} Eq. [14].....	101	254	509	1030	2150
p_{max} Eq. [17].....	101	254	509	1015	2030

In Equation [14] the value 0.285×10^{-8} cm²/dyne was taken[14] for $\sqrt{\gamma}$. Values for λ_0' correspond to the range of viscosities of the oils used in the pitting tests and the mean angular velocity of 42 radians per sec. We see that p_{max} by Equation [17] is only slightly less than p_{max} by Equation [14], indicating that the influence of pressure on viscosity can usually be neglected. From Equation [18], if $\lambda_0' = 400$ and $\delta' = 0.000133$, $P/R = 53.5$ lb per sq in. We conclude that the load-carrying capacity of the oil film will in general be much less than the loads applied to the rollers in the pitting tests.

[14] Viscosity pressure tables for several mineral oils are given by J. H. Hyde, Proc. Royal Society of London, 1920, 97A, p. 240.

Appendix 3

OIL FORCES AFTER METALLIC CONTACT

When the rollers are in contact there will be a flattened area of width $2b$, as shown in Fig. 11. We shall assume that the

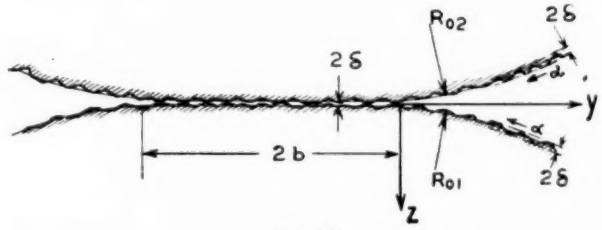


FIG. 11

deformation of the surface is that due to the contact pressure alone. This leads to the conclusion that the contact pressure distribution is elliptical, in accordance with Hertz's law, Equation [2]. The contact pressure will be denoted by p_1 and the oil pressure by p_2, shown in Fig. 13, and the y and z axes will be located as shown in Fig. 11.

The radii of the roller surfaces for small values of y will be reduced because of elastic deformation by the p_1 forces to R_{01}, R_{02}. Equation [12] for the pressure distribution holds true for the case at hand when $1/R$ is replaced by $1/R_0$, where $\frac{1}{R_0} = \frac{1}{2}\left(\frac{1}{R_{01}} + \frac{1}{R_{02}}\right)$. In this case we make β unity, since the discharge Q is $2\alpha\delta$.

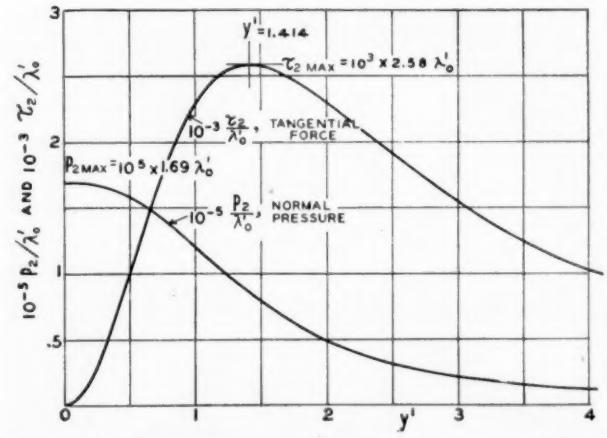


FIG. 12 NORMAL AND TANGENTIAL OIL FORCES FOR $\delta' = 0.00029$ (Rollers in contact.)

In addition to the normal oil pressure p_2, we can say something about the tangential oil traction τ_2. Since the shearing stress in the oil is $\lambda \frac{dv}{dz}$, we have, for $z = z_1$, by Equation [7]

$$\tau_2 = \lambda \left(\frac{z_1 + z_2}{2}\right) \frac{1}{\lambda} \frac{dp}{dy} \quad [19]$$

Using the relations $\frac{z_1 + z_2}{2} = \zeta$, $\zeta = \delta + \frac{y^2}{2R_0}$, and $y' = y/\sqrt{R_0\delta}$

and also Equation [9], the tangential oil traction becomes, for $\beta = 1$

$$\tau_2 = -\frac{3\lambda_0'}{\delta'}(1+\gamma p^2)\frac{y'^2/2}{\left(1+\frac{y'^2}{2}\right)^2} \quad \ldots\ldots [20]$$

For small values of γp^2, the pressure- and tangential-stress distributions due to the oil are given by

$$p_2 = \frac{3\lambda_0'}{4\delta'^{3/2}}\left\{\frac{y'}{\left(1+\frac{y'^2}{2}\right)^2} - \frac{y'/2}{1+\frac{y'^2}{2}} - \frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{y'}{\sqrt{2}} - \frac{\pi}{2}\right)\right\} \quad \ldots\ldots [21]$$

$$\tau_2 = -\frac{3\lambda_0'}{\delta'}\frac{\frac{y'^2}{2}}{\left(1+\frac{y'^2}{2}\right)^2} \quad \ldots\ldots [22]$$

From these expressions curves have been plotted in Fig. 12, for unit λ'_0 and for $\delta' = 0.00029$;[15] p_2 and τ_2 have maximum values at different points

$$p_{2\max} = (p_2)_{y'=0} = \frac{3\lambda'_0\pi}{8\sqrt{2}\,\delta'^{3/2}} \quad \ldots\ldots [23]$$

$$\tau_{2\max} = (\tau_2)_{y'=\sqrt{2}} = -\frac{3\lambda_0'}{4\delta'} \quad \ldots\ldots [24]$$

Stresses Due to Oil and Contact Forces. Axes are chosen in the same position as in the discussion of contact stresses. We

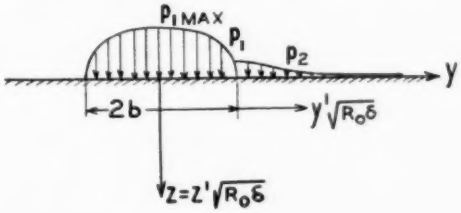


FIG. 13

use the additional coordinates y' and z' to aid in defining the oil forces. y' and z' are defined by

$$y' = \frac{y-b}{\sqrt{R_0\delta}}; \quad z' = \frac{z}{\sqrt{R_0\delta}}$$

The loads applied to the roller surfaces are the oil forces given by Equations [21] and [22] and the contact pressure

$$p_1 = p_{1\max}\sqrt{1-\frac{y^2}{b^2}} \quad \ldots\ldots [25]$$

The stress at some point in the interior may be expressed by

$$Y_y = Y_{y1} + Y_{y2};\ Z_z = Z_{z1} + Z_{z2};\ Y_z = Y_{z1} + Y_{z2} \ldots [26]$$

where Y_{y1}, Z_{z1}, and Y_{z1} are contact stresses and Y_{y2}, Z_{z2}, and Y_{z2} are stresses due to loads p_2 and τ_2, as follows:

$$\left.\begin{aligned} Y_{y2} &= -\frac{2}{\pi}\int_v^\infty \frac{z'(y'-v)^2 p_2 dv}{((y'-v)^2+z'^2)^2} - \frac{2}{\pi}\int_0^\infty \frac{(y'-v)^2\tau_2 dv}{((y'-v)^2+z'^2)^2} \\ Z_{z2} &= -\frac{2}{\pi}\int_0^\infty \frac{z'^3 p_2 dv}{((y'-v)^2+z'^2)^2} - \frac{2}{\pi}\int_0^\infty \frac{(y'-v)z'^2\tau_2 dv}{((y'-v)^2+z'^2)^2} \\ Y_{z2} &= -\frac{2}{\pi}\int_0^\infty \frac{z'^2(y'-v)p_2 dv}{((y'-v)^2+z'^2)^2} - \frac{2}{\pi}\int_0^\infty \frac{(y'-v)^2 z'\tau_2 dv}{((y'-v)^2+z'^2)^2} \end{aligned}\right\} . [27]$$

$$\left.\begin{aligned} Y_{y1} &= p_{\max}\left\{-e^{-\alpha}\sin\beta + \sin\beta\sinh\alpha\left(1-\frac{\sin h\,2\alpha}{\cos h\,2\alpha - \cos 2\beta}\right)\right\} \\ Z_{z1} &= p_{\max}\left\{-e^{-\alpha}\sin\beta - \sin\beta\sinh\alpha\left(1-\frac{\sin h\,2\alpha}{\cos h\,2\alpha - \cos 2\beta}\right)\right\} \\ Y_{z1} &= p_{\max}\left\{-\sin h\,\alpha\sin\beta\,\frac{\sin 2\beta}{\cos h\,2\alpha - \cos 2\beta}\right\} \end{aligned}\right\} . [28]$$

$$y = b\cos h\,\alpha\cos\beta;\ z = b\sin h\,\alpha\sin\beta \ldots\ldots [29]$$

in Equation [27] p_2 and τ_2 are load intensities at the point $y' = v$.

In Equation [4] it is clear that Y_{y1}, Z_{z1}, and Y_{z1} cannot give rise to any tensile stress. If Y_{y2}, Z_{z2}, and Y_{z2} are to give rise to a tensile stress, it is natural to look for that tensile stress at the surface. At the surface, the contact pressure p_1 contributes nothing to the stress system outside the contact area. The p_2 forces contribute the compressive stress $-p_2$ in both the y' and z' directions, and the τ_2 loading contributes the second integrals in Equation [27]. These latter stresses, due to τ_2 are

$$\left.\begin{aligned} (Y_y\tau_2)_{z'=0} &= \frac{3\lambda_0'}{\pi\delta'}\left\{\frac{\frac{y'^2}{2}}{\left(1+\frac{y'^2}{2}\right)^2}\log\frac{y'^2}{2} + \frac{\left(\frac{y'^2}{2}-1\right)\left(\frac{\pi\sqrt{2}}{4}y'-1\right)}{\left(1+\frac{y'^2}{2}\right)^2}\right\} \\ (Z_z\tau_2)_{z'=0} &= 0 \\ (Y_z\tau_2)_{z'=0} &= \tau_2 \end{aligned}\right\} .. [30]$$

The stress $(Y_y\tau_2)_{z'=0}$ is tension for small values of y', but with increasing y' changes to compression. For there to be a tensile stress at the surface, the tension $(Y_y\tau_2)_{z'=0}$ must be greater at some point than p_2. The ratio of $(Y_y\tau_2)_{z'=0}$ to p_2 is greatest at $y' = 0$ and has the value $8\sqrt{2\delta'}/\pi^2$. Unless $\sqrt{\delta'}$ is greater than $\pi^2/8\sqrt{2}$, $(Y_y\tau_2)_{z'=0}$ cannot be greater than p_2 and there can be no tension at the surface. But for $\sqrt{\delta'}$ to be of the order of $\pi^2/8\sqrt{2}$ is absurd. We conclude that the oil forces p_2 and τ_2 together cannot produce tension at the surface.

The stresses will now be calculated at a point under the surface and in front of the contact area. Referring to Fig. 12, the tan-

[15] A consideration of the elastic deformation of the roller surfaces leads to the approximate value $0.46R$ for R_0. If δ/R is 0.000133, then δ/R_0 is 0.00029.

gential forces are quite small compared with the normal oil forces. In the present calculation, therefore, the stresses due to τ_2 will be neglected. We assume the following numerical data:

$$R_0 = 0.46R;\ b/R = 0.00588;\ \delta/R_0 = \delta' = 0.0001$$

$$E = 30 \times 10^6 \text{ lb per sq in.};\ \mu = 0.3$$

and calculate stresses at the point $y' = 0.8; z' = 0.1$

Since

$$\frac{y}{b} = 1 + \frac{R_0}{b} y'\sqrt{\delta'};\quad \frac{z}{b} = \frac{R_0}{b} z'\sqrt{\delta'}$$

the coordinates of the selected point are $y/b = 1.626$; $z/b = 0.0783$.

Evaluating the first integrals in Equation [27], making use of

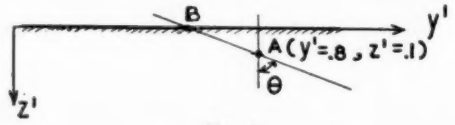


FIG. 14

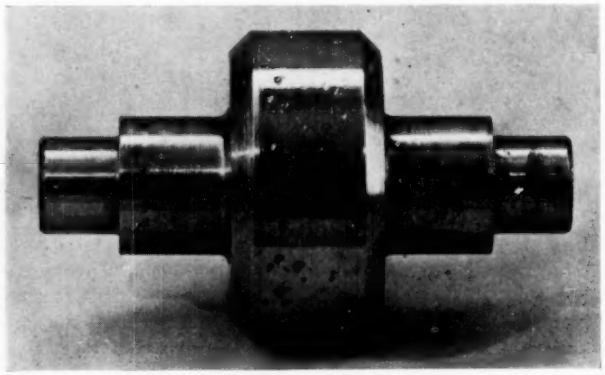


FIG. 15 PITTED TEST ROLLER

Equation [21], the stresses due to the oil pressure p_2 at the selected point are

$$Y_{y2} = -0.570 \times 10^6\lambda'_0;\quad Z_{z2} = -0.703 \times 10^6\lambda'_0;$$

$$Y_{z2} = -0.032 \times 10^6\lambda'_0$$

The contact stresses in pounds per square inch at the point $y = 1.626b$, $z = 0.0783b$ which is the same as $\alpha = 1.0705$, $\beta = 0.0609$ are found from Equation [28] to be

$$Y_{y1} = -4050;\ Z_{z1} = -9.7;\ Y_{z1} = -278$$

Assuming values of 100, 200, and 300 dynes per sq cm for λ'_0 (0.145×10^{-2}, 0.290×10^{-2}, 0.435×10^{-2} lb per sq in.) the stresses at $y' = 0.8$, $z' = 0.1$ due to the p_2 loading are found to be as given in Table 4.

TABLE 4 STRESSES DUE TO OIL PRESSURE

(Lb per sq in.)

Viscosity × ω	Y_{y2}	Z_{z2}	Y_{z2}
$\lambda_0' = 0.00145$	−826	−1020	−46
$\lambda_0' = 0.00290$	−1650	−2040	−93
$\lambda_0' = 0.00435$	−2480	−3060	−139

Combination of oil pressure stresses and contact stresses gives resultant values as in Table 5.

TABLE 5 RESULTANT STRESSES

(Lb per sq in.)

Viscosity × ω	Y_y	Z_z	Y_z
$\lambda_0' = 0.00145$	−4870	−1030	−324
$\lambda_0' = 0.00290$	−5700	−2050	−371
$\lambda_0' = 0.00435$	−6530	−3070	−417

For $\delta' = 0.0001$ and the above values for λ_0' the intensity of load p_2 at several points on the surface between $y' = 0$ and $y' = 0.8$ is given in Table 6.

TABLE 6 p_2 AT VARIOUS POINTS

(Lb per sq in.)

Viscosity × ω	$y' = 0.4$	$y' = 0.5$	$y' = 0.6$	$y' = 0.7$
$\lambda_0' = 0.00145$	1167	1135	1094	1043
$\lambda_0' = 0.00290$	2330	2270	2190	2090
$\lambda_0' = 0.00435$	3500	3400	3280	3130

Let A be the point $y' = 0.8, z' = 0.1$ and let B be a point on the surface between $y' = 0$ and $y' = 0.8$, as shown in Fig. 14. Let a plane pass through A and B perpendicular to the $y' = z'$ plane. Let θ be the angle between the z'-axis and plane AB. We shall calculate the normal stresses on plane AB at points A and B.

The normal stress at B is the compressive stress $-p_2$. The normal stress at A is calculated from the values in Table 5 and the relation

$$N = Y_y \cos^2\theta + Z_z \sin^2\theta - 2Y_z \sin\theta\cos\theta$$

In Table 7 the normal stresses on plane AB at A and B are given for various values of θ and for λ_0' equal to 300 dynes per sq cm (0.00435 lb per sq in.).

TABLE 7 NORMAL STRESSES AT A AND B

Angle θ	B	Stress at A lb per sq in.	Stress at B lb per sq in.
$\tan^{-1} 4$	$y' = 0.4$	−3070	−3500
$\tan^{-1} 3$	$y' = 0.5$	−3160	−3400
$\tan^{-1} 2$	$y' = 0.6$	−3430	−3280
$\tan^{-1} 1$	$y' = 0.7$	−4370	−3130

In this particular case, if θ is between 66 and 83 deg the normal stress at A is less than at B. If other data had been assumed for λ_0', δ', and b/R it would be possible by taking the point A close enough to the surface to again find a range of values of θ between 0 and 90 deg for which the normal stress at A would be less than that at B. When θ is negative and when A is very close to the surface, the normal stress at A will in general be greater than at B.

Film Lubrication in Sleeve Bearings

By M. STONE,[1] PITTSBURGH, PA.

The author discusses the design of bearings as based on fluid friction in hydrodynamic films and investigates the limiting design factors in such bearings. He points out that the thickness of the oil film is dependent on the viscosity of the lubricant alone and that this thickness may be controlled, thus making it possible to base the design of a bearing on a minimum thickness of the oil film. Two appendixes conclude the report, one which presents simplified theoretical considerations for the design of film-lubricated bearings while the other explains the highly accurate electromagnetic micrometer used for determining shaft positions in the bearing.

IN THE early days of rotating shafts, when an iron shaft was rotated in a wooden bearing shell, a reduction in friction was obtained by rubbing the surface with suet or smearing it with asphalt. Such a bearing, from the point of view of modern engineering, is hardly a bearing, the design now being approached from the background of fluid friction in hydrodynamic films. Sleeve bearings, designed on this principle, vary widely in their efficiencies, the extent of their friction losses and length of life depending on the magnitude of the load being carried, the speed of rotation, the viscosity and oiliness of the lubricant, temperature, running clearance, bearing metals, and fineness of machine finish. The problem is to determine the proper design of a bearing for a given load and speed requirements. The purpose of this paper, therefore, is to present briefly the limiting factors which must be considered in the design of a fluid-film bearing and to show how the thickness of the oil film, which in the final analysis determines the success of the bearing, may be controlled in design.

A ready means of differentiating between the various types of sleeve bearings is given in Fig. 1, in which ZN/P is plotted against μ, where Z is absolute viscosity in centipoises, N is speed of the shaft in rpm, P is the projected-area pressure in lb per sq in., and μ is the equivalent coefficient of friction. The justification for drawing such a curve goes back to the theory of film lubrication which is presented briefly in Appendix 1. As seen from the curve, Fig. 1, two, and perhaps three, regions of bearing friction are recognized. In the region to the right of the point marked "breakdown," the friction in the bearing is entirely made up of fluid friction within the lubricant itself, and the friction coefficient is very small, becoming as little as 0.001, and under, for carefully machined surfaces. The curve as shown is for a fixed value of bearing clearance, i.e., a fixed ratio of sleeve to shaft diameter. The region referred to already may be termed the region of perfect lubrication. To the left of the breakdown point, the friction coefficient increases very rapidly which, as will be discussed, is due to a combination of fluid and metal-to-metal friction. In this region, where μ has a value of between 0.010 and 0.100, there is an imperfect hydrodynamic film and it is therefore termed the region of partial lubrication. For values of μ between 0.100 and 0.300, a condition of practically solid friction exists and we have no bearing in the ordinary sense.

On the basis of Tower's (1)[2] investigation of the pressure distribution in sleeve bearings, Reynolds (2) developed the hydrodynamic theory of film lubrication which has formed the basis of modern sleeve-bearing design. In Reynolds' considerations, the viscosity at the operating temperature of the bearing was the only property of the lubricant involved. Since the promulgation of the theory, there has been considerable experimentation to substantiate or disprove the original contentions, confusion resulting when it was found that there is considerable difference in bearing losses and temperatures in two cases of exactly similar bearings using two oils of exactly the same vis-

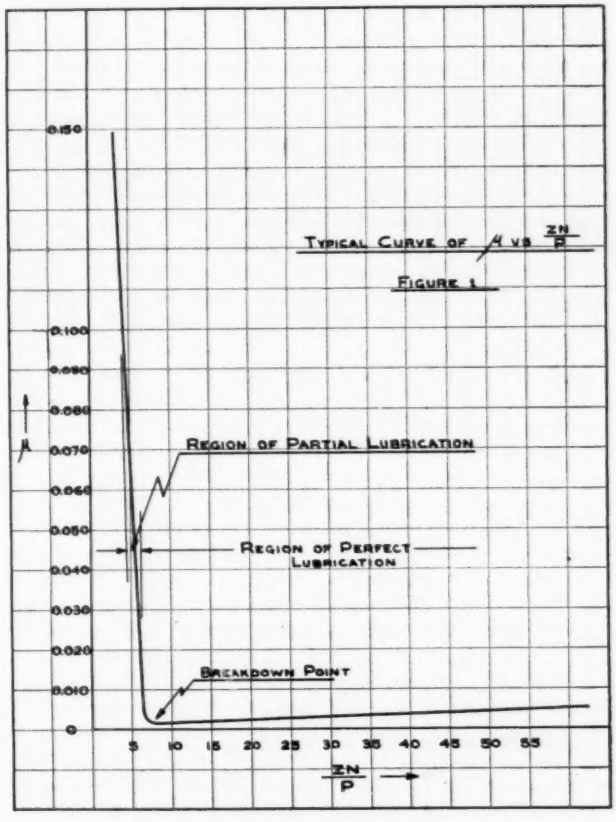


FIG. 1 TYPICAL CURVE OF μ VS. ZN/P

[1] United Engineering and Foundry Company, Pittsburgh, Pa. Mr. Stone was graduated from the Harvard Engineering School in mechanical engineering in 1923, and received the degree M.S. in 1925. He later, in 1933, obtained the doctor's degree from the University of Pittsburgh for advanced engineering work. The years 1924–1925 were spent in work on the A.S.M.E. high-pressure steam program at Harvard. From 1925 to 1934, Mr. Stone was associated with the Westinghouse Electric & Manufacturing Company, first on mechanical research, and later as supervisory design engineer on large power apparatus, petroleum equipment, air-conditioning apparatus, etc. During 1930–1931, he was in Germany and England as Lamme Fellow on advanced study and liaison engineering work. His present work is on the design of steel-mill machinery, particularly large strip mills.

Presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 3 to 7, 1934.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to similarly numbered references listed at the end of the paper.

cosity. As a result, a question was raised as to the limitations of the theory, or more exactly, as to the nature of the physical conditions within the bearing. It was found, in general, that as the speed or viscosity decreased, or as the pressure increased, the friction torque decreased for a time, and then suddenly increased, being accompanied by bearing wipe. The thought occurred that the thickness of the film was influenced by the changing conditions, and that some quantitative ideas concerning the latter would clarify the picture. It was soon appreciated that as ZN/P decreased, a breakdown point was reached. Considerable work has been done of late, principally by the McKees (3), to determine the lowest safe value of ZN/P.

The work of Karelitz (4), as referred to in Appendix 1, resulted in a curve relation between h_{min} and ZN/P which is shown in Fig. 3. Thus, from the curves in Fig. 3, we see that as ZN/P decreases, both h_{min} and μ decrease down to the breakdown point. In these curves, m represents the ratio between the mean radial clearance and the journal radius. This forcefully presents the argument that as the bearing friction decreases, the minimum oil film also decreases until a sudden collapse of conditions takes place.

Satisfactory results have been obtained from tests on bearings where 5 is the assigned value of ZN/P. From the upper graph in Fig. 3, the value of h_{min} corresponding to ZN/P is 0.0002 in. for a 10-in. diam bearing with a 120-deg bearing arc. For usual turbine-bearing practice, a value of 0.0006 in. for h_{min} is usually used, since load and speed conditions do not demand a smaller figure. However, for heavy loads and low speeds, considerably lower values are necessary if successful bearings are to be made. This, of course, emphasizes the necessity for accurate grinding of shafts and absence of bearing-sleeve distortion. We come then to the ultimate criterion of a satisfactory bearing, i.e., a lower limit on minimum oil-film thickness. In this connection, reference should be made to G. B. Karelitz' discussion of oil-ring bearings (5).

The tests herein discussed were made to measure these oil-film thicknesses, in order to test the validity and accuracy of curves such as shown in the upper graph in Fig. 3. Using an oil with a viscosity of 110 S.S.U. (16 centipoises, absolute), tests were made at speeds varying from 0 to 900 rpm on a full-size power bearing with 90-deg support angle, 9 in. diam, 0.012 in. diametral clearance, and a projected-area pressure of 100 lb per sq in. A very accurate micrometric electromagnetic measuring arrangement, as described in Appendix 2 and illustrated in Fig. 4, was employed, which allowed the simultaneous recording of the vertical and horizontal instantaneous positions of the shaft with respect to the bearing. A typical oscillograph record is shown in Fig. 7, where the modulations of the carrier frequency are the measures of the shaft position. Tests were conducted for a series of speeds, and the results obtained are shown in Fig. 8, where the instantaneous positions of the shaft are shown plotted within the clearance circle, i.e., a circle of radius δ within which the center is physically confined. If, for the moment, we pass over the fact that the shaft executes cyclical motions at rpm frequency about a mean position, and plot h_{min} against ZN/P as obtained from both these results and calculations based on Karelitz' work, we get the comparison between test results and calculated values as shown in Fig. 9. The agreement between the two is indeed good, especially when we realize that such effects as side and leakage, variable viscosity within the film necessarily are introduced incompletely into an already involved calculation. This, then, can be taken as definite support for the general correctness of values based on pure calculation. Although earlier measurements of oil-film thickness were made by Vieweg (6), Wolff (7), and Hummel (8), it is felt that the aforementioned tests on full-size bearings of commercial design, employing recording apparatus of requisite accuracy, provide a final touch to this phase of fluid-film bearing design. The work of Wolff, particularly, utilizing interferometric means, must have been seriously interfered with by the cyclical motion of the shaft, and probably accounts for its failure to confirm the present results.

The fact that the shaft may not necessarily assume a fixed equilibrium position relative to the sleeve was first investigated analytically and experimentally by Stodola (9) and Hummel (8) in small test bearings, and it was their contention that the film provided a definite elastic force field, which could under circumstances be made to vibrate violently at resonance. While it is not within the scope of this paper to develop this phase of shaft behavior, suffice it to say that simultaneous pedestal vibration readings were taken by means of a Geiger vibrograph, and that whereas a definite critical speed was observed at 700 rpm, the amplitude of motion of the shaft relative to the bearing was still increasing up to 900 rpm and over. Tests were repeated with the machine perfectly balanced and with the rotor as badly unbalanced as personal safety would permit, with no remarkable change in the film records; the pedestal vibration, however, was proportional to the unbalance. All these facts point to a much more complex condition than Stodola assumed, and might well merit a more thorough investigation. The fact, however, that a sustained cyclical motion may take place, suggests a mechanism for supplying energy for the vibration along the general lines of self-induced phenomena, which have been discussed in recent literature. The fact, however, that such shaft motions may exist, presents another possible limitation on the permissible minimum oil-film thickness.

As ZN/P decreases, the rate of lubricant shear becomes higher and higher, and it might well be that the force necessary to tear the oil free from the metal surfaces is reached at a critical shear rate, even if perfect shaft and sleeve surfaces were attainable. As is generally the case, however, the film thins down to the dimensions of the machining irregularities, and actual metal-to-metal contact begins to take place in spots, resulting in an increase of friction. In this region of lubrication, where true film conditions no longer exist, new properties of the lubricant begin to assume greater importance, and oiliness, as it is conveniently termed, begins to be a factor. For such bearings, and they still inevitably exist, as the water-lubricated metal, wood, and fiber types of roll-neck bearings, plane-slide bearings, and rocker bearings, there are different contentions as to what are the proper and best types of lubricant. For this class of bearings, wear is inevitable, while for the fluid-film bearing, bearing life should be indefinite, as long as sufficient lubricant is provided, and frequent stopping and starting is not necessary.

In summary, then, it has been demonstrated that it is tenable and further that a quantitative basis exists, to base bearing design on a minimum oil-film thickness. Further, that for carefully constructed bearings a value of 0.0001 in. may well form the lower limit for h_{min}, compatible with fine machining and absence of distortion. Finally, that viscosity alone, as concerns the lubricant, determines the film thicknesses and bearing losses.

Appendix 1

Simplified Theoretical Considerations in Film-Lubricated Bearings

As a result of Tower's early experiments, and at the suggestion of Lord Rayleigh, Osborne Reynolds developed the modern hydrodynamic theory of film lubrication, and established the important fundamental fact that real bearing capacity is only

developed when a wedge-shaped film can be formed, the wedge narrowing in the direction of the sliding surface.

Modern expositions of journal-bearing theory have been given by Howarth (10), Kingsbury (11), and Karelitz (4), but the following presentation gives the salient relations in a simple general form. Beginning with Reynolds' simplified fluid-flow equations

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right) = -6ZU\frac{dh}{dx} \quad \ldots\ldots [1]$$

$$f = \frac{ZU}{h} + \frac{h}{2}\frac{dp}{dx} \quad \ldots\ldots [2]$$

where h is the film thickness, p is the oil-film pressure, and f is the shearing stress in the oil.

Integrating Equation [1], and calling $h = h'$ at the point of maximum pressure, i.e., at $dp/dx = 0$ or $p = p_{max}$, we get

$$\frac{dp}{dx} = -6ZU\frac{h - h'}{h^3} \quad \ldots\ldots [3]$$

Now for the case of a clearance type of sleeve bearing, such as shown in Fig. 2, let

$$x = r\theta$$
$$e = c\delta, \text{ where } \delta = mr(c < 1)$$
$$h = mr(1 + c\cos\theta), \text{ and}$$
$$h_{min} = mr(1 - c);\quad h' = mr(1 + c\cos\theta')$$

By substituting these values in Equation [3]

$$\frac{dp}{d\theta} = \frac{6ZU}{m^2r}f(\theta, \theta', c) \quad \ldots\ldots [4]$$

where $f(\theta, \theta', c)$ is a definite function, the form of which we are not concerned with for the time being.

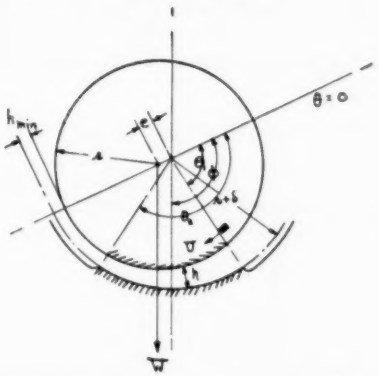


FIG. 2 CLEARANCE TYPE OF SLEEVE BEARING USED IN THE TESTS

Integrating Equation [4]

$$p = \frac{6ZU}{m^2r}F(\theta, \theta', \theta_1, c) \quad \ldots\ldots [5]$$

Now since $p = 0$ for $\theta = \theta_1$, and $\theta = \theta_2$, we see that the second of the two conditions fixes a relation between θ and c for a given bearing arc (θ_1, θ_2), i.e., Equation [5] may be modified to

$$p = \frac{6ZU}{m^2r}F'(\theta, \theta_1, \theta_2, c) \quad \ldots\ldots [6]$$

Now since the vertical component of the bearing pressure must equal the load, W, then, too, the component of W along and perpendicular to the line $\theta = 0$ must equal the sum of the p components along the same directions, i.e.,

$$\left.\begin{aligned} W\cos\phi &= r\int_{\theta_1}^{\theta_2} p\cos\theta\, d\theta \\ W\sin\phi &= r\int_{\theta_1}^{\theta_2} p\sin\theta\, d\theta \end{aligned}\right\} \quad \ldots\ldots [7]$$

Integrating by parts:

$$\left.\begin{aligned} W\cos\phi &= r(p\sin\theta)\Big|_{\theta_1}^{\theta_2} - r\int_{\theta_1}^{\theta_2}\frac{dp}{d\theta}\sin\theta\, d\theta \\ &\quad\Big\|\ 0 \\ W\sin\phi &= -r(p\cos\theta)\Big|_{\theta_1}^{\theta_2} + r\int_{\theta_1}^{\theta_2}\frac{dp}{d\theta}\cos\theta\, d\theta \end{aligned}\right\} \quad \ldots\ldots [8]$$

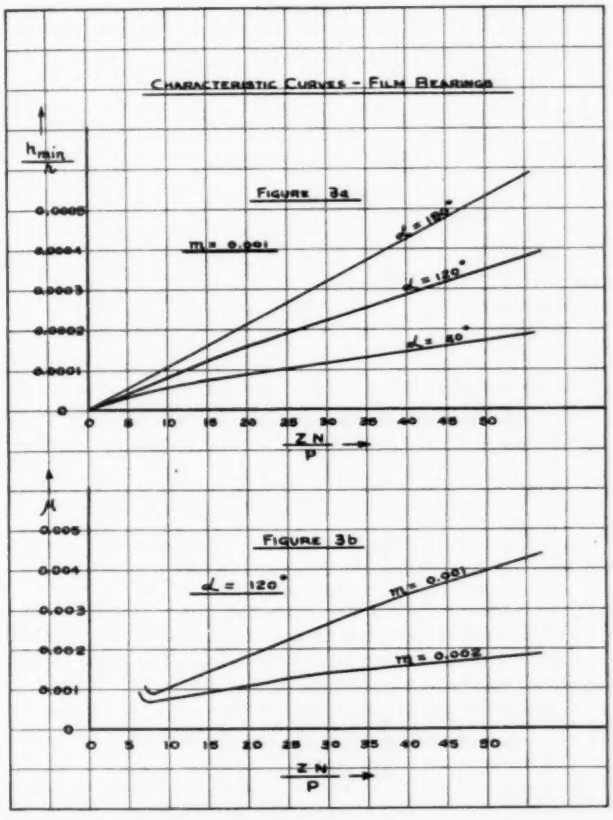


FIG. 3 CHARACTERISTIC CURVES OF FLUID-FILM BEARINGS

Combining Equations [4] and [8]

$$W\cos\phi = \frac{6ZU}{m^2}g_1(\theta_1, \theta_2, c)$$

$$W\sin\phi = \frac{6ZU}{m^2}g_2(\theta_1, \theta_2, c)$$

or $$W = \frac{6ZU}{m^2}\sqrt{g_1^2 + g_2^2} = \frac{6ZU}{m^2}G(\theta_1, \theta_2, c) \quad \ldots\ldots [9]$$

Since $U = 2\pi rN$ and $W = 2rP$, where P is the projected area pressure, we may write

$$c = G'\left(\theta_1, \theta_2, m, \frac{ZN}{P}\right) \quad \ldots\ldots [10]$$

and since $h_{min} = mr(1 - c)$, then finally

$$\frac{h_{min}}{r} = H\left(\theta_1, \theta_2, m, \frac{ZN}{P}\right) \quad \ldots\ldots [11]$$

Thus for a given bearing design—θ_1, θ_2, m, and r—h_{min} and ZN/P are definitely related, as worked out numerically by Karelitz. The quantitative relations are given in the upper graph in Fig. 3, for $m = 0.001$ and $\alpha = \theta_2 - \theta_1$, symmetrical with respect to the vertical.

Proceeding further, from Equations [2] and [4], we get directly

$$f = \frac{ZU}{mr} k(\theta, \theta', c) \quad \ldots\ldots\ldots\ldots\ldots\ldots [12]$$

Without going too far into all the considerations involved, the equivalent force, F, which multiplied by the radius produces the frictional torque, may be defined as

$$F = \int_{\theta_1}^{\theta_2} f\, r\, d\theta = \frac{ZU}{m} K(\theta_1, \theta_2, c) \ldots\ldots\ldots [13]$$

and the equivalent coefficient of friction defined as $\mu = F/W$ is obtained from Equations [9] and [13] to be

$$\mu = \frac{m}{6} \frac{K(\theta_1, \theta_2, c)}{G(\theta_1, \theta_2, c)}$$

But since c is a function of ZN/P as seen from Equation [10], we may also write

$$\mu = Q(\theta_1, \theta_2, m, ZN/P) \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

Relations between μ and ZN/P have been experimentally determined and checked analytically by several men, including McKee, and are of the general nature of the lower graph in Fig. 3.

Appendix 2

Electromagnetic Micrometer for Determining Shaft Positions

The experimental apparatus employed in this investigation is extremely accurate, accuracy to less than $^1/_{100,000}$ in. being readily obtained. The apparatus also provided at the same time a means of recording the motion of the shaft. The schematic arrangement of the apparatus is shown in Fig. 4. Its fundamental principle is illustrated in Figs. 6*a* and 6*b*. An oscillographic record obtained by its use is shown in Fig. 7.

Referring to Fig. 6*a*, it will be seen that the electromagnetic micrometer is in reality two coupled variable-reluctance transformers, indicated by the two flux paths ϕ_1 and ϕ_2. The primaries

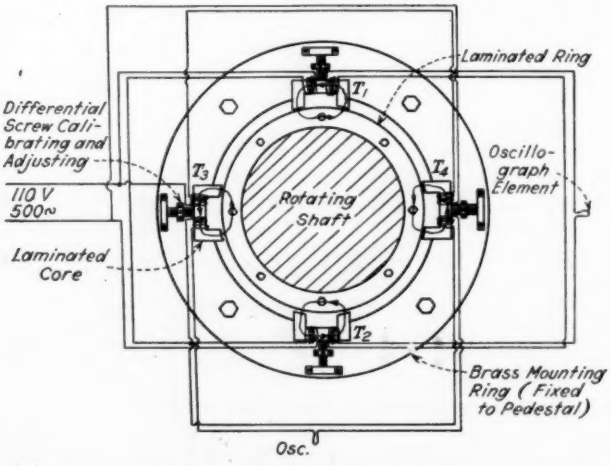


Fig. 4 Principle of the Electromagnetic Micrometer

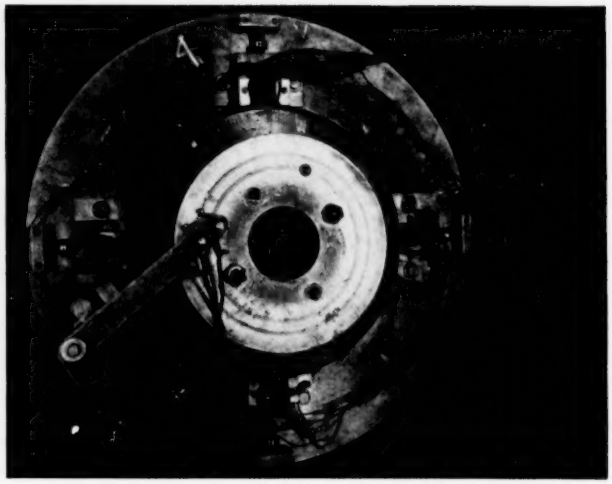


Fig. 5 Experimental Set-Up for Determining Thickness of the Oil Film

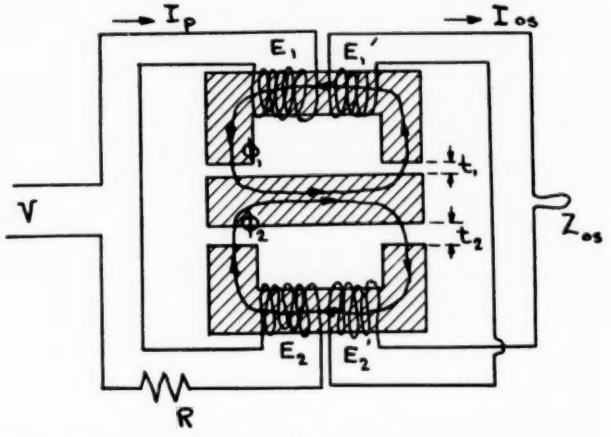


Fig. 6*a* Circuit Diagram of Electromagnetic Micrometer

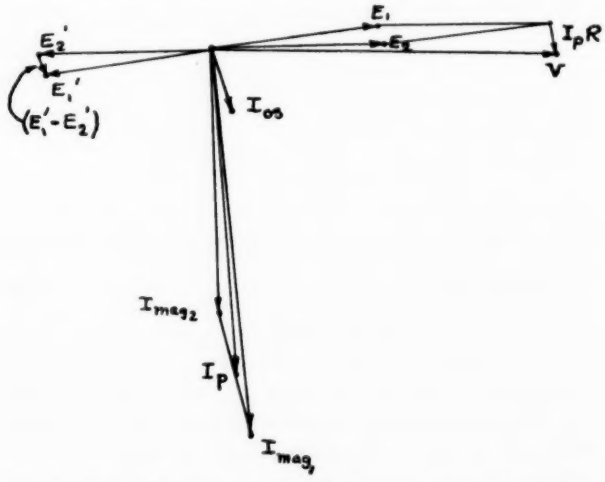


Fig. 6*b* Vector Diagram of the Electromagnetic Micrometer

are connected in series, and the secondaries in series opposed. For a central position of the armature ($t_1 = t_2$), $E_1 = E_2$ and so $E'_2 - E'_1 = 0$. For a slight movement of the armature, an appreciable value of $E'_2 - E'_1$ is obtained which is a direct measure of the movement.

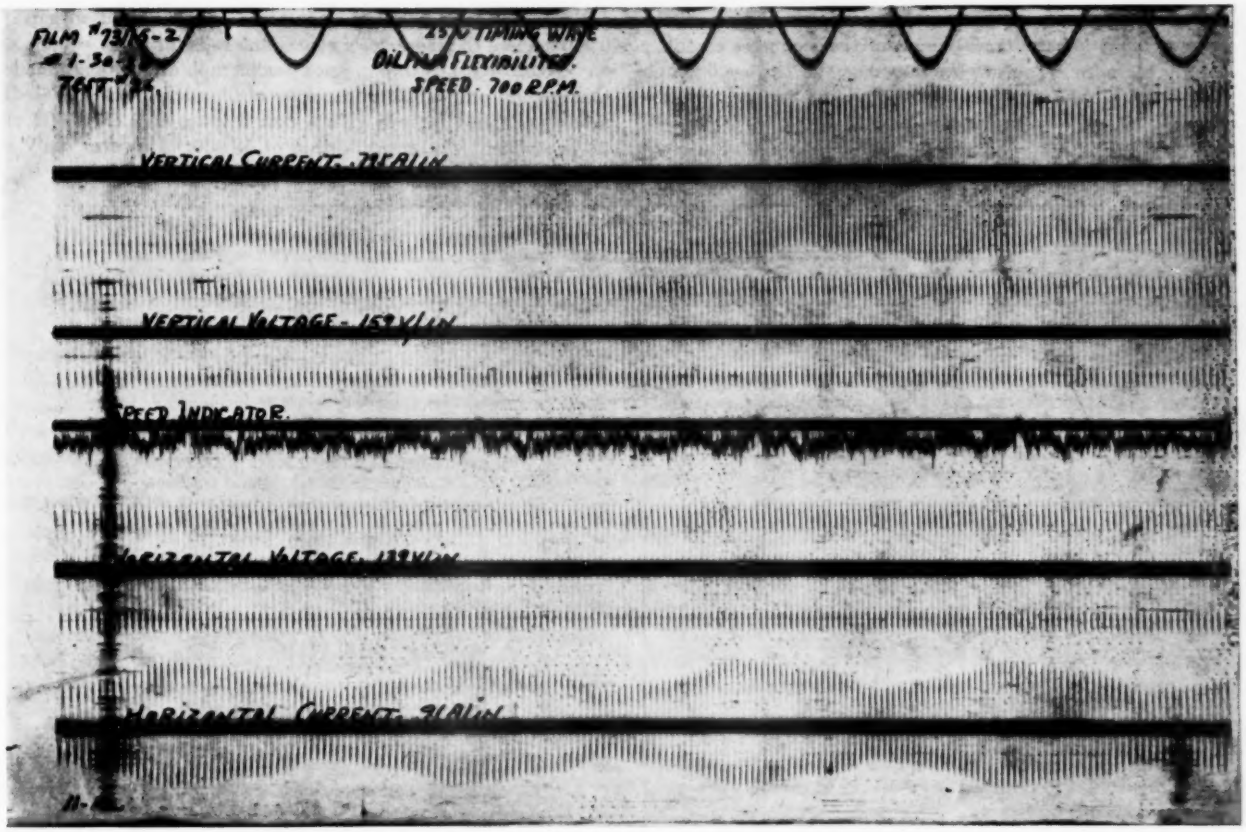


FIG. 7 OSCILLOGRAPHIC RECORD OBTAINED WITH THE ELECTROMAGNETIC MEASURING ARRANGEMENT. THE MODULATIONS OF THE CARRIER FREQUENCY ARE THE MEASURES OF SHAFT POSITION WITH RESPECT TO THE BEARING SURFACE

To determine the sensitivity of the instruments quantitatively, the simple vectorial-circuit equations can be written

$$E'_1 - E'_2 = I_{os}Z_{os}$$

or

$$\left.\begin{aligned} E_2 - E_1 &= I_{os}Z_{os} \\ V &= E_1 + E_2 + I_pR \\ I_{mag1} &= K_1E_1 \\ I_{mag2} &= K_2E_2 \\ I_p &= K_1E_1 - I_{os} \\ I_p &= K_2E_2 + I_{os} \end{aligned}\right\} \quad \ldots\ldots [15]$$

By elimination and combination among Equations [15], the final expression for the oscillograph current becomes

$$I_{os} = V\left[\frac{K_1 - K_2}{(K_1 + K_2)Z_{os} + 4 + R(K_1 + K_2 + K_1K_2Z_{os})}\right] \ldots [16]$$

Now K is the reciprocal of the inductive reactance of the coils. For the central position of the armature, $K_1 = K_2 = K_0$ and for a displaced position of the armature, the increase and decrease in the K's is the same, i.e.

$$\left.\begin{aligned} K_1 &= K_0(1 + \epsilon) \\ K_2 &= K_0(1 - \epsilon) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots [17]$$

which influences Equation [16] to become

$$I_{os} = V\left[\frac{\epsilon}{Z_{os} + \frac{2}{K_0} + R\left(1 + \frac{K_0}{2}Z_{os} - \frac{K_0}{2}Z_{os}\,\epsilon^2\right)}\right] \ldots [18]$$

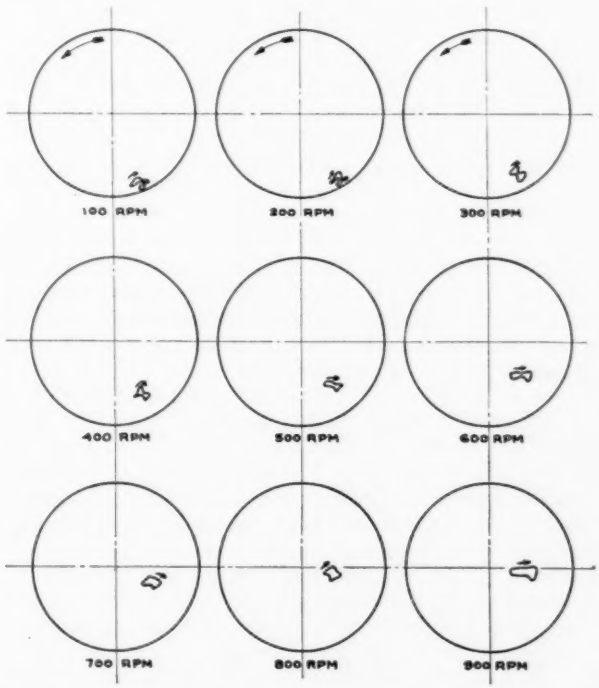


FIG. 8 SHAFT MOTIONS AND POSITIONS PLOTTED WITHIN THE CLEARANCE CIRCLE

For the instrument as built, there are 150 turns per coil, an air gap of 0.050 in., the cross-section of the magnetic circuit is $^1/_2$ in. by $^1/_2$ in. or $^1/_4$ sq in., the impedance of the oscillograph circuit is 1.5 ohms (pure resistance), and the primary coil resistance is 0.90 ohm. Thus $K_0 = 0.176$, where the carrier frequency was picked equal to 500 cycles, and hence vectorially:

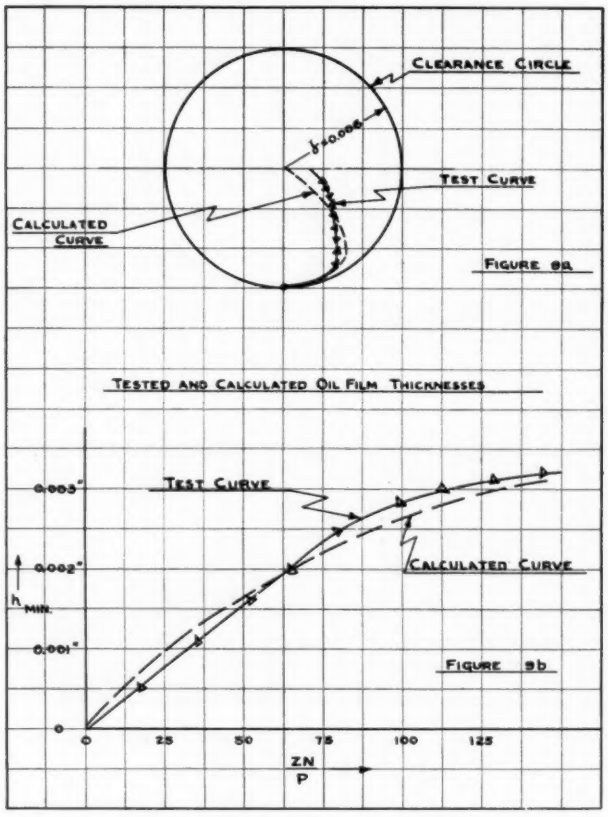


FIG. 9 A COMPARISON BETWEEN TEST RESULTS AND CALCULATED VALUES OF OIL-FILM THICKNESS

$$\left.\begin{aligned} V &= 65 + j\,0 \\ R &= 0.9 + j\,0 \\ Z_{os} &= 1.5 + j\,0 \\ K_0 &= 0 - j\,0.176 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots [19]$$

Substituting Equation [19] in the various expressions for E_1, E_2, I_{mag1}, I_{mag2}, I_p, I_{os}, etc., we get the vector diagram as shown in Fig. 6*b*, which illustrates the theory of the micrometer. For the purpose of rapid calculation, however, a simplified interpretation of Equation [19] shows that

$$I_{os} = \frac{K_0 V \epsilon}{2} = 0.09\, V\epsilon$$

For a motion of 0.001 in., $\epsilon = 0.02$ since the initial air gap is 0.050 in., and therefore, for a value of 65 volts for V, $I_{os} = 0.09 \times 65 \times 0.02 = 0.117$ amp. Since oscillograph elements are made that require only 0.002 amp per in. of film deflection, this shaft motion of 0.001 in. will give a film reading of 0.117/0.002 or 58 in., which is a magnification of 58,000. However, only a magnification of 1000/1 was actually utilized during the tests.

LIST OF SYMBOLS

Z = absolute viscosity (centipoises)
N = rpm of shaft
P = projected area pressure (lb per sq in.)
μ = equivalent coefficient of friction
U = rubbing speed (ft per min)
h = film thickness
h_{min} = minimum oil-film thickness
h' = film thickness at point of maximum pressure
p = oil-film pressure (lb per sq in.)
f = shearing stress in oil
x, θ = general linear and angular coordinates
θ_1, θ_2 = angles defining bearing arc with respect to location of minimum oil-film thickness (radians)
ϕ = angle defining position of load with respect to location of minimum oil-film thickness (radians)
r = journal radius (inches)
δ = radial clearance (inches)
e = eccentricity of bearing (inches)
m = ratio = δ/r
c = ratio = e/δ
V = applied potential (volts)
E_1, E_2 = back emf in primary windings (volts)
E'_1, E'_2 = induced voltages in secondary windings (volts)
I_p = primary current (amperes)
I_{os} = oscillograph current (amperes)
I_{mag1}, I_{mag2} = magnetizing current for the two transformers (amperes)
R = primary resistance (ohms)
Z_{os} = oscillograph impedance (ohms)
K_0, K_1, K_2 = permittance values (ohms)
ϵ = ratio of armature movement, with reference to normal air gap

REFERENCES

1 B. Tower, Proceedings Institute of Mechanical Engineers, 1884.
2 "Theory of Lubrication," by O. Reynolds, Phil. Trans. Royal Society, 1886.
3 "Friction of Journal Bearings as Influenced by Clearance and Length," by S. A. and T. R. McKee, Trans. A.S.M.E., vol. 51, 1929, paper APM-51-15, p. 161.
4 "Charts for Studying Oil Films in Bearings," by G. B. Karelitz, Trans. A.S.M.E., 1925.
5 "Performance of Oil Ring Bearings," by G. B. Karelitz, Trans. A.S.M.E., vol. 52, 1930, paper APM-51-15, p. 161.
6 "Optische Messgeräte zur Bestimmung der Dicke der Oelschicht in Lagern," by V. Vieweg, *Petroleum*, 1922.
7 "Über die Schmierschicht Messung durch Interferenz," by R. Wolff, Forschungsarbeiten No. 308, V.D.I., 1928.
8 "Kritische Drehzahlen als Folge der Nachgiebigkeit des Schmiermittels im Lager," by C. Hummel, Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, 1926.
9 A. Stodola, same reference as 8.
10 "Graphical Study of Journal Lubrication," by H. A. S. Howarth, Trans. A.S.M.E., vol. 45, 1923, p. 421; vol. 46, 1924, p. 809; vol. 47, 1925, p. 1073.
11 "Optimum Conditions in Journal Bearings," by A. Kingsbury, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-7, p. 123.

Review of Recent Research in Thermodynamics

By JOHN A. GOFF,[1] URBANA, ILL.

ACTIVITY in steam research still deserves prominence in a report of progress in thermodynamics. In the United States, Keyes, Smith, and Gerry (1)[2] present an interesting formulation of p-v-t data for superheated steam obtained at Massachusetts Institute of Technology. In the first place, it is noted that extrapolation to the saturation limit appears to give reliable values of the saturation volumes. From the formulation, three different relations are deduced by application of the general equations of thermodynamics: The first gives $c_{p=0}$, the isobaric specific heat of the limiting zero-pressure or perfect-gas state which is a function of temperature only, in terms of h'', the enthalpy at saturation; the second gives $c_{p=0}$ in terms of c_p; the third gives $c_{p=0}$ in terms of μ, the Joule-Thomson coefficient. Using the experimental data of Osborne, Stimson, and Fiock for h'', those of Koch for c_p, and those of Davis and Kleinschmidt for μ, an ensemble of $c_{p=0}$ values is obtained from which an empirical equation is deduced by the method of least squares after excluding c_p values of Koch near saturation. The reason for excluding these values is that they exhibit a sharply increasing trend becoming quite pronounced at high pressures, and, in the opinion of the authors, indicate that the Koch c_p data near saturation are in error.

Since this report was written, Keyes, Smith, and Gerry (13) have revised their formulation, taking account of new low-pressure data by S. C. Collins obtained with the new calorimeter which measures $(dh/dp)_T$ and $(dT/dp)_h$ in the same apparatus and correcting an error of computation of the p^{12} term in the p-v-t formulation previously referred to. Moreover, the indirect determination of $c_{p=0}$ as already described has been abandoned and values taken directly from measurements of the band spectrum of water vapor have been used instead. The results of these changes are better agreement with the Joule-Thomson data of Davis and Kleinschmidt and satisfactory accord with the c_p values of Koch near saturation.

Smith and Keyes (2) present a formulation of the M.I.T. liquid-volume data which also appears to give reliable values of saturation volumes by extrapolation. Using experimental data of Osborne, Stimson, and Fiock on h', the enthalpy of saturated liquid, values of the isobaric specific heat of the liquid at saturation are deduced by application of the general thermodynamic equations. Osborne, Stimson, and Ginnings (3) present a report of progress in calorimetric determinations of the enthalpies of saturated liquid and vapor at the United States Bureau of Standards. To date, the results for the liquid thoroughly cover the range up to 370 C (3050 lb per sq in.); for the vapor, the results are not so complete, but include measurements at 325 C (1750 lb per sq in.) and 350 C (2400 lb per sq in.). Osborne, Stimson, and Ginnings (14) have, since this report was written, communicated additional measurements on the enthalpies of saturated liquid and vapor as prepared for presentation at the International Steam Tables Conference of 1934. Completion of a detailed report has been postponed pending further experiments near the critical region. A valuable contribution to the knowledge of the pressure-temperature relation at saturation is a formula constructed by Osborne and Meyers (4) based on all available trustworthy data and covering the whole range from 0 to 374 C. The formula gives what are considered best values not only of the pressure-temperature relation itself, but also of the pressure-temperature derivative. The authors offer their formula as suitable for adoption in the International Skeleton Steam Table at its next revision.

In Germany, Jakob and Fritz (5) have extended their measurements of heat of vaporization r from 310 C (101 kg per sq cm) to 340 C (149 kg per sq cm). Koch (6) presents measurements of the isobaric specific heat c_p of liquid water from 0 to 350 C and from saturation to pressures as high as 260 kg per sq cm; each isobar on the c_p t-plane exhibits the well-known minimum value above 0 C and a rapid rise at higher temperatures.

Of importance especially to the thermodynamics of combustion, is a critical study by E. Justi (7) of available high-temperature specific-heat data for water vapor and other so-called technical gases. The author judges it proper to disregard values obtained by the explosion method as being generally too high, also those obtained by the sound-velocity method as being generally too low; only values obtained by the Lummer-Pringsheim and flow methods are considered reliable. These latter are shown to be in good agreement with values deduced from spectroscopic measurements which are therefore used as a basis for the construction of $c_{p=0}$ t-tables extending up to 3000 C. Equilibrium constants and their variation with temperature as deduced from the specific-heat data are also given for use in calculation of dissociation.

Under the heading of dynamical properties, may be mentioned determinations of the viscosity of superheated steam presented by Schiller (8). The flow-velocity at which the discharge coefficient of a suitable nozzle executes an abrupt rise was observed for water as a reference medium and for steam. Equating the two Reynolds numbers gives a relation from which the viscosity of steam in terms of that of water can be calculated. The results show a characteristic dependence of viscosity upon pressure as well as temperature which appears to admit of reasonable extrapolation to the critical point. Of importance to the subject of heat transfer are measurements by E. Schmidt (9) of the relative radiation constants of water and ice forming wetted and frosted surfaces. Thin layers of frost appear to emit as much as 98.5 per cent of black-body radiation, this figure being the highest yet measured.

Supersaturation of steam is the subject of a paper by Yellott (10). By means of a search tube, the variation of static pressure along the axis of a nozzle of rectangular cross-section with one glass side plate was measured. Under illumination along the nozzle axis, condensation in very small drops—as evidenced by the fact that the incident light was scattered rather than reflected and that the scattered light was blue in color—was seen to occur suddenly at a point where the static pressure executed an abrupt rise. In a written discussion of the paper, Keenan (11) presented an interesting analysis of the phenomena involved.

An experimental investigation of the changes of state occurring in the uniflow engine has been reported by Ulsamer (12). Temperature measurements in the clearance space and at various points on the cylinder walls disclose the fact that the main portion

---

[1] Associate Professor of Thermodynamics, University of Illinois, Assoc-Mem. A.S.M.E., Chairman, Applied Mechanics Division.

[2] Numbers in parentheses refer to bibliography at the end of the paper.

of the cylinder steam never crosses the saturation curve but remains superheated even though the initial superheat is quite small. On the basis of this fact, simultaneous measurements of temperature and pressure suffice to follow the course of evaporation and condensation on the cylinder walls.

BIBLIOGRAPHY

1 "A Comparison of Derived and Observed Quantities by Means of an Equation of State for Steam Based on the New M.I.T. Volume Data," by F. G. Keyes, L. B. Smith, and H. T. Gerry, *Mechanical Engineering*, vol. 56, no 2, Feb., 1934, p. 87.

2 "An Equation for the Volume of Liquid Water," by L. B. Smith and F. G. Keyes, *Mechanical Engineering*, vol. 56, no. 2, Feb., 1934, p. 92.

3 "Progress of Work at the U. S. Bureau of Standards," by N. S. Osborne, H. F. Stimson and D. C. Ginnings, *Mechanical Engineering*, vol. 56, no. 2, Feb., 1934, p. 94.

4 "A Formula for the Saturation Pressure of Steam in the Range 0 to 374 C," by N. S. Osborne and C. H. Meyers, *Mechanical Engineering*, vol. 56, no. 4, April, 1934, p. 207.

5 "Die Verdampfungswärme des Wassers und das spezifische Volumen von Sattdampf im Bereich von 100 bis 150 atm," by M. Jakob and W. Fritz, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 4, no. 6, 1933, page 295.

6 "Die spezifische Wärme des Wassers von 0° bis 350° C und vom jeweiligen Sättigungsdruck bis kg/cm², " by W. Koch, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 5, no. 3, 1934, p. 130.

7 "Spezifische Wärme technischer Gase und Dämpfe bei höheren Temperaturen," by E. Justi, *Forschung auf dem Gebiete des Ingenieurwesens*, Berlin, vol. 5, no. 3, 1934, p. 130.

8 "Bestimmung der Zähigkeit von Wasserdampf," by W. Schiller, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 5, no. 2, 1934, p. 71.

9 "Die Wärmestrahlung von Wasser und Eis von bereiften und benetzten Oberflächen," E. Schmidt, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 5, no. 1, 1934, p. 1.

10 "Supersaturated Steam," by John I. Yellott, Jr., Trans. A.S.M.E., vol. 56, 1934, paper FSP-56-7, p. 411.

11 Ibid., p. 427.

12 "Dampfzustand im Zylinder einer Gleichstrom-Dampfmaschine," by J. Ulsamer, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 5, no. 2, 1934, p. 80.

13 "The Equation of State for Superheated Steam," by F. G. Keyes, L. B. Smith, and H. T. Gerry, *Mechanical Engineering*, vol. 57, no. 3, March, 1935, p. 164.

14 "Steam Research at the Bureau of Standards," by N. S. Osborne, H. F. Stimson, and D. C. Ginnings, *Mechanical Engineering*, vol. 57, no. 3, March, 1935, p. 162.

Design Data

It is important that the data contained in technical papers be made readily available to designing engineers. In order to satisfy these needs of industry, this section of the Journal will include a concise presentation of data and information drawn chiefly from papers previously published by the Applied Mechanics Division of The American Society of Mechanical Engineers.

The data are prepared by a subcommittee of the Applied Mechanics Division on strength of materials (R. E. Peterson, chairman), under the guidance of S. Timoshenko.

Factors of Stress Concentration Photoelastically Determined[1]

By MAX M. FROCHT,[2] PITTSBURGH, PA.

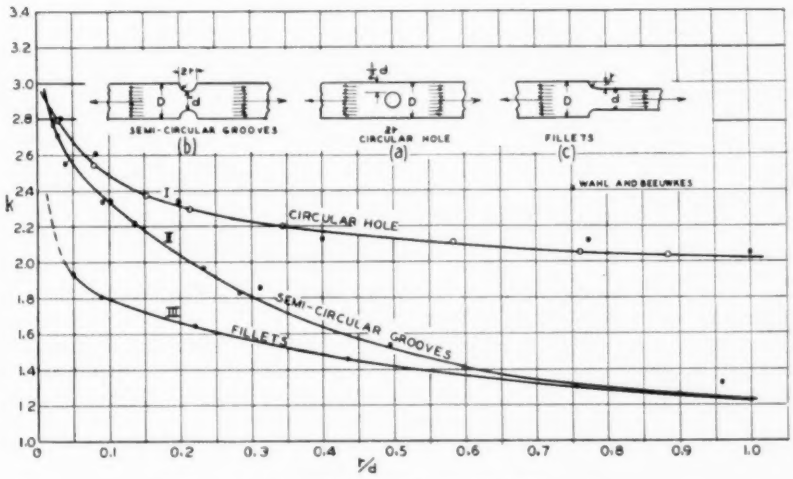


FIG. 1 INVARIANT CASES IN TENSION OR COMPRESSION

THE factor of stress concentration is defined as the ratio of the maximum to the minimum stress in the section, or $k = s_m/s$, where

k = factor of stress concentration
s_m = maximum stress in section
s = average stress in section = P/A
P = axial load
A = cross-sectional area of minimum section

Experiments show that the factor of stress concentration is a function of the ratio r/d, Fig. 1, and of the stress distribution on both sides of the discontinuity. The special cases in which the factors of stress concentration can be expressed as a function of r/d only are here defined as invariant cases. Three basic invariant cases are shown in Fig. 1. Curves I, II, and III, Fig. 1, give k as a function of r/d, respectively, for circular holes, semi-circular grooves, and fillets between fields of uniform tension or compression.[3]

Pure bending as used here refers to that in which the bending moment is constant and the longitudinal stresses are linear on each side of the discontinuity, Fig. 2.

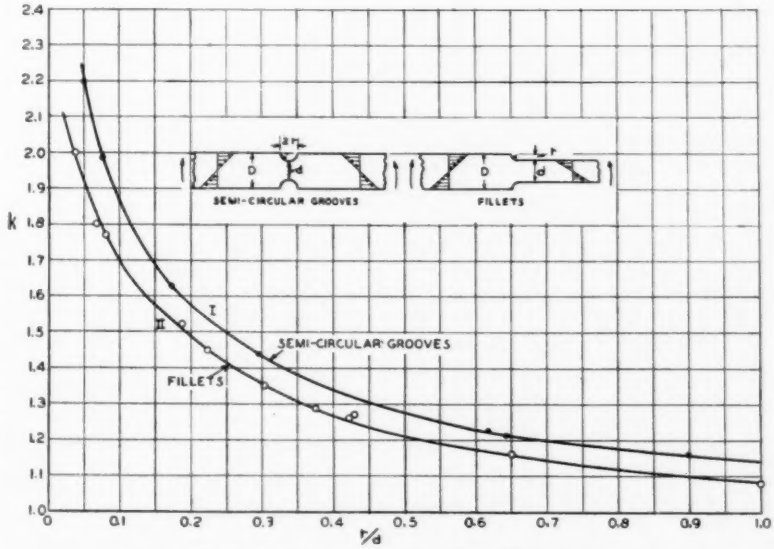


FIG. 2 INVARIANT CASES IN PURE BENDING

[1] Special cases from a paper by the author entitled "Photoelastic Studies in Stress Concentrations," to be presented at the National Meeting, Ann Arbor, Mich., June 18 and 19, 1935, of the Applied Mechanics Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Other nonvariant cases and related aspects of stress concentrations are included in the paper.

[2] Associate Professor of Mechanics, Carnegie Institute of Technology.

[3] Present results are in good agreement with those from comparable tests by other investigators.

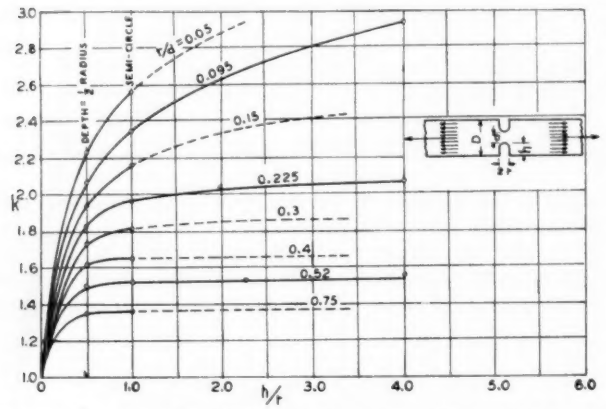


FIG. 3 FACTORS OF STRESS CONCENTRATION FOR GROOVES OF VARYING DEPTHS IN TENSION OR COMPRESSION

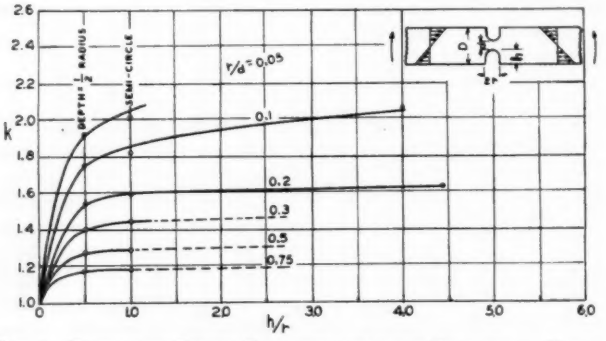
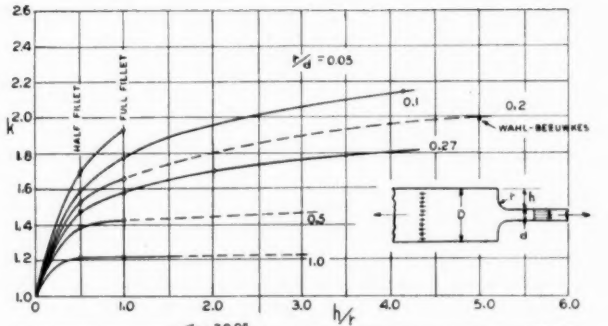


FIG. 4 FACTORS OF STRESS CONCENTRATION FOR GROOVES OF VARYING DEPTHS IN PURE BENDING

Curves I and II, Fig. 2, give k as a function of r/d for two basic invariant cases—semi-circular grooves and fillets.

Referring to Fig. 3 a deep groove is one in which h/r is greater than unity and a shallow groove one in which h/r is less than unity.

The effect of deep grooves depends on the ratio r/d. For small values of r/d such as 0.2 or less there is a pronounced increase



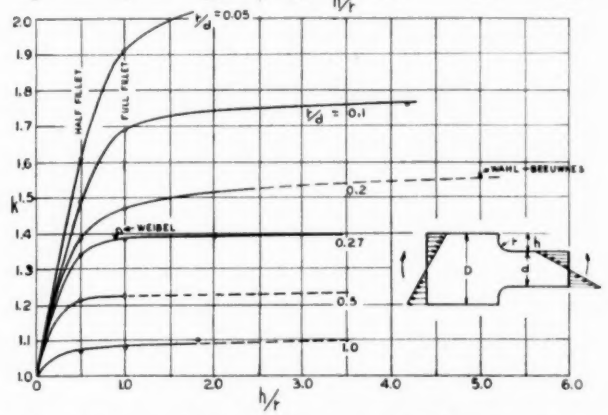


FIG. 5 FACTORS OF STRESS CONCENTRATION FOR FILLETS OF VARYING DEPTHS IN TENSION OR COMPRESSION

FIG. 6 FACTORS OF STRESS CONCENTRATION FOR FILLETS OF VARYING DEPTHS IN PURE BENDING

in the value of k as h/r increases. This increase is more noticeable in tension than in bending.

Shallow grooves reduce the factors of stress concentration, k approaching unity as h/r approaches zero. The curves in Figs. 3 and 4 show k as a function of h/r in pure tension and pure bending, respectively.

The curves in Figs. 5 and 6 show k as a function of h/r for fillets in pure tension, compression, and bending.

Discussion

Fatigue of Shafts at Fitted Members, With a Related Photoelastic Analysis[1]

CHARLES S. BARRETT.[2] The surface-rolling treatment that is so effectively used by the authors offers some worth-while problems to those interested in the practical applications of X-rays. We should like to be able to settle by X-rays whether or not the rolling is effective through the action of the internal stresses it produces. The fact that the diameter of the specimen is always decreased by the rolling seems to me to point to such stresses. There could scarcely be enough voids in the metal, closed up by the rolling, to account for the decrease in diameter; it more probably occurs through an increase in length, which would put the core of the specimens in tension and the outer fibers in compression. Perhaps the decrease in density of the outer layer, which results from the cold-working process, is an important factor in building up these stresses. It has recently been shown by Buchholtz and Buehler that compressive stresses in the outer fibers can materially increase the endurance limit, and it remains only to prove their existence in surface-rolled specimens. While X-rays have been used for stress measurement, they are at a disadvantage when dealing with cold-worked metal such as this, and the problem can better be attacked by the usual mechanical methods.

There is another item in the rolling technique, however, that can be studied effectively by X-rays; namely, the depth to which the metal is cold worked. There is an X-ray method available for measuring this depth simply and directly which I have used on one of the authors' specimens. It involves the principle that the Debye rings in a diffraction pattern from a metal are made up of spots which lose their sharpness and distinctness when the metal is cold worked. The penetration of the cold-worked layer may be determined by taking a series of photograms at different depths and observing the amount of fuzziness and overlapping of the diffraction spots on each photogram.

This method was applied to one of the authors' cantilever specimens of 0.42 per cent carbon whose surface had been cold worked by rollers of 2-in. diam and $1^1/_2$-in. contour radius under a radial force of 400 lb. The specimen was studied at a point along its length where its diameter was $1^5/_8$ in. It was mounted in a back-reflection X-ray camera in which cobalt radiation was used so that the penetration of the X-rays into the specimen was negligible. The diffraction pattern of the surface of the specimen showed a continuous ring of completely overlapping spots. A layer 0.014 in. thick was etched away with 50 per cent nitric acid and another photogram prepared of the metal thus exposed. The effects of the cold work was less severe at this depth. Photograms after etching to depths of 0.035 in., 0.040 in., 0.044 in., and 0.049 in. showed a steadily decreasing amount of cold work, and when a depth of 0.059 in. was reached, the photograms consisted of spots as sharp as those from the center of the specimen. From the evidence of these photograms we may conclude that to a depth of perhaps 0.030 in. all the grains were plastically deformed, that between 0.030 in. and 0.055 in. certain grains were deformed and others were not, and that below 0.055 in., ± 0.005 in., there was no plastic deformation. Mr. Peterson tells me that if one calculates the stress intensities under the rolls by the formula of Thomas and Hoersch, the penetration can be estimated, and that for this specimen it is in the neighborhood of 0.067 in., slightly greater than was found by the X-rays.

S. WAY.[3] In this paper the subject of increasing fatigue strength by surface rolling is introduced. In this connection, it is desirable to know approximately the depth of cold worked material when a shaft is rolled with rollers of a certain form under a certain load.

If an elastic body has another brought into contact with it, the greatest principal shearing stress along a line normal to the surface and extending inward from the center of the contact area will vary as shown in Fig. 1 of this discussion. The shear

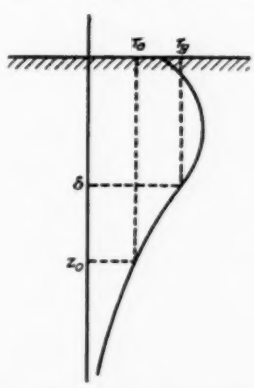


FIG. 1 VARIATION OF PRINCIPAL SHEARING STRESS ALONG A LINE NORMAL TO THE SURFACE OF AN ELASTIC BODY WHEN ANOTHER BODY IS BROUGHT INTO CONTACT WITH IT

stress τ_0 occurs at depth z_0. When a shaft is rolled, a layer of material of thickness δ will have experienced plastic deformation. Assume that the stress system under the roller at a depth greater than δ is negligibly different from that which would exist were the body perfectly elastic. The depth δ can then be determined as the place where the shear stress is equal to the yield strength, τ_y. The yield strength in shear should be taken equal to one-half that in pure tension. This can be demonstrated by the energy of distortion theory of strength when two of the principal stresses are equal, as is very nearly the case in the problem at hand.

In calculating the depth, z, of the point where the shear stress is τ, three relations will be of interest. The first is the Boussinesq formula for the case of a load applied at a point:

$$\tau = \frac{P}{8\pi}\left[\frac{7-2\mu}{z^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

For a circular contact area we have

$$\tau = \frac{aE}{2R(1-\mu^2)}\left[\frac{3}{\pi}\frac{a^2}{a^2+z^2} + \frac{2}{\pi}(1+\mu)\left(\frac{z}{a}\cot^{-1}\frac{z}{a} - 1\right)\right] \quad \ldots\ldots\ldots [2]$$

[1] By R. E. Peterson and A. M. Wahl. Published in the March, 1935, issue of the JOURNAL OF APPLIED MECHANICS (A.S.M.E. Transactions).

[2] Metals Research Laboratory, Carnegie Institute of Technology, Pittsburgh, Pa.

[3] Westinghouse Research Laboratories, East Pittsburgh, Pa.

where a is the radius of the circle of contact and $\frac{1}{R}$ is the average of the four principal curvatures of the two bodies. For the general case[4] of an elliptical contact area of major semi-axis, a, and minor semi-axis, b

$$\tau = \frac{aE}{R(1-\mu^2)} \cdot f\left(k, \mu, \frac{z}{a}\right); \quad k = \frac{b}{a} \ldots\ldots\ldots [3]$$

The semi-axis, a, is given by

$$a = \sqrt[3]{\left(\frac{3PR(1-\mu^2)E(k')}{2\pi k^2 E}\right)}; \quad k'^2 = 1 - k^2 \ldots\ldots [4]$$

which, for the case of the circular contact area, reduces to

$$a = \sqrt[3]{\left(\frac{3PR(1-\mu^2)}{4E}\right)} \ldots\ldots\ldots\ldots [5]$$

Using relations expressed in Equations [3] and [4], τ was calculated as a function of z for $R = 1$ in., $P = 1000$ lb, $\mu = {}^3/_{10}$ and $E = 30 \times 10^6$ lb per sq in., and for various values of k. The results are plotted in Fig. 2, showing the effect of the manner

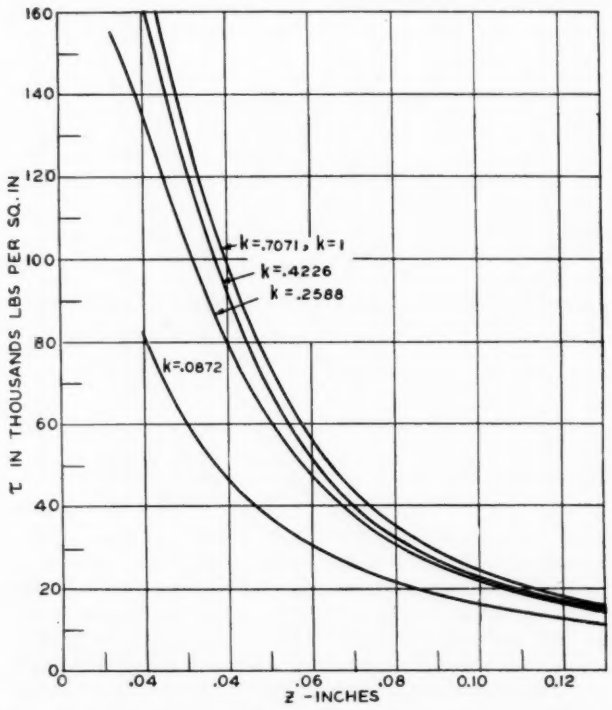


FIG. 2 SHEAR STRESS AT VARIOUS DEPTHS WHEN $P = 1000$ LB, $R = 1$, AND $K = b/a$

of distribution of the contact pressure on the shearing stress beneath the surface. For values of k greater than 0.5 we see it is justifiable to assume we have a circular contact area.

Using relations [2] and [5], τ was plotted in Fig. 3 as a function of z for $P = 1000$ lb, $\mu = {}^3/_{10}$, $E = 30 \times 10^6$ lb per sq in. and for various values of R. These curves can be used when k is greater than 0.5 as well as for the special case $k = 1$. The curve $R = 0$ is obtained from the Boussinesq solution, Equation [1]. If we have a load $P_1 = 1000\,m$, the stress $\tau_1 = \tau\sqrt[3]{m}$ will exist at depth $z_1 = z\sqrt[3]{m}$, where τ is the stress at depth z with 1000 lb load. This can easily be verified by inspection of Equations [2] and [5].

Example. Radius of shaft = 1.5 in.; radius of roller = 1.02 in.; radius of roller in plane of shaft axis = 1.5 in.; $\tau_y = 30{,}000$ lb per sq in. Required the load on the rollers necessary to cause cold working to a depth of 0.2 in.

$$\frac{1}{R} = \frac{1}{4}\left(\frac{1}{1.5} + \frac{1}{1.5} + \frac{1}{1.02} + \frac{1}{\infty}\right) = \frac{1}{1.728}$$

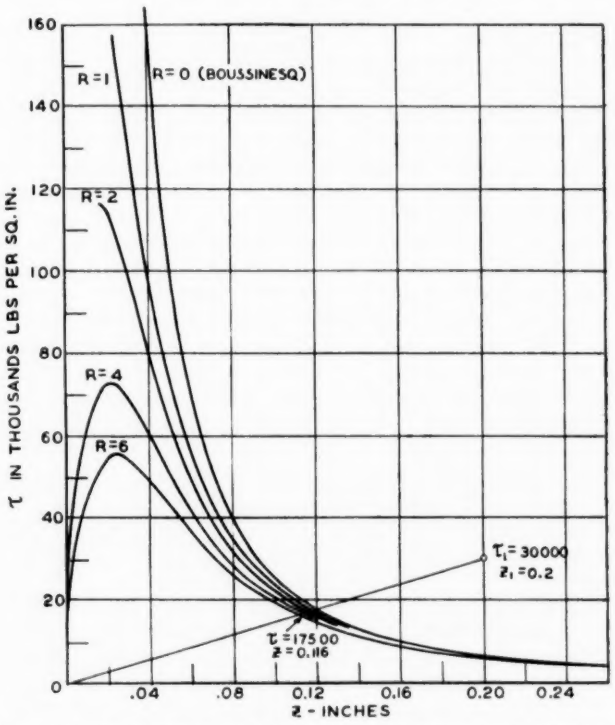


FIG. 3 SHEAR STRESS AT VARIOUS DEPTHS WHEN $P = 1000$ LB, AND $k = 1$

Calculation[4] shows that $k = 0.55$, so we use the curves in Fig. 3. In Fig. 3 let a radial line be drawn from the origin to the point $\tau_1 = 30{,}000$, $z_1 = 0.2$. This line would meet the curve for $R = 1.728$ at the point $\tau = 17{,}400$, $z = 0.116$. m is now determined, for

$$\sqrt[3]{m} = \frac{2}{0.116} = \frac{30{,}000}{17{,}400} = 1.724$$

$$m = 5.12$$

The required load is therefore 5120 lb. The Boussinesq relation, Equation [1] gives $P_1 = 4730$ lb. If a load of 4730 lb. were used, the penetration by the relation expressed in Equation [2] would be 0.194 in., instead of 0.2 in., so the Boussinesq formula is only slightly in error. The range of values over which the Boussinesq solution should be applied can be estimated from Fig. 3.

For k less than 0.5 the curves in Fig. 2 should be used. If $P_1 = 1000\,m$, $R_1 = n \times 1 = n$ the stress $\tau_1 = \tau\sqrt[3]{(m/n^2)}$ will be at the point $z_1 = z\sqrt[3]{(mn)}$, where τ is the stress at the point z with $P = 1000$ lb and $R = 1$.

The condition[4] for k to be greater than 0.5 is that

$$\frac{(1/R_1' + 1/R_2')}{(1/R_1 + 1/R_2)} < 2.85 \ldots\ldots\ldots\ldots [6]$$

[4] See Thomas and Hoersch, Bulletin 212, Engineering Exp. Station, University of Illinois.

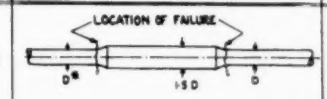
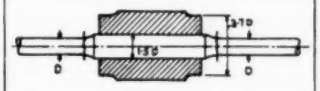
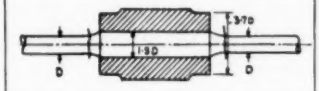
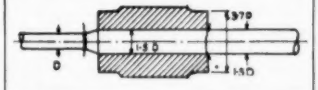
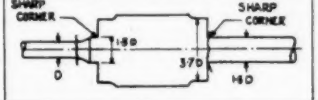
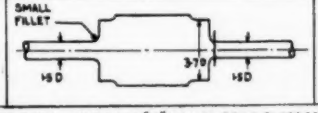
where R_1 and R_1' are the principal radii of one body and R_2 and R_2' principal radii of the other. R_1 and R_2 sweep arcs in the same plane, as do also R_1' and R_2'. The radii of the bodies should be assigned symbols R_1, R_2, R_1', R_2' in such a manner that the fraction in the expression [6] is greater than unity.

O. J. Horger[5] and J. L. Maulbetsch.[6] Although engineers are accustomed to using stress-concentration factors where fillets are involted in designed parts, too little consideration has been given to the greatly reduced strength of press-fitted members.

Kühnel was only partially quoted in the authors' Table 2, but it is believed that the importance of this problem warrants the submission of his further findings. They would appear to have considerable bearing on a possible means of improving the strength of press-fit assemblies. For this reason the entire tests of Kühnel are quoted in Table 1.

TABLE 1 FATIGUE-TEST DATA ON THE PRESS-FIT PROBLEM

(Kühnel-German State Railways)

	TYPE OF SPECIMEN		ENDURANCE STRENGTH LBS./SQ. IN.	% ORDINARY ENDURANCE LIMIT	STRESS CONCENTRATION FACTOR
1			33000	≈100	1.00
2			34000	≈100	1.00
3			31000	94	1.06
4 5		L R	30000 19000	91 61	1.10 1.74
6 7		L R	30000 19000	91 61	1.10 1.74
8			23000	70	1.44

*NOTE THAT ALL RATIOS OF "D" SCALED FROM PHOTOGRAPHS OF TEST SPECIMENS FROM KUHNEL

An examination of Table 1 indicates the decided reduction (from 2.0 to 1.1) in stress concentration when the axle is designed with a raised wheel seat or pad for the outer press-fitted member. While no dimensions of the test specimen are given by Kühnel it would appear from scaling the photographs that the pad diameter is about 1.5 times that of the adjoining section. Such a pad-diameter ratio is difficult to attain in practice, but the effect is promising enough to justify further tests on ratios more feasible in the design of machine parts.

In this connection the writers have made a photoelastic study of the pad construction just outlined, as shown in Fig. 4. The determination of σ_1 and σ_2 have not yet been made, so that the principal stresses are unknown. It would appear, however, that the $\sigma_1 - \sigma_2$ fringes indicate less stress concentration. This may be expected because a more gradual transition in section from the axle into the sleeve is obtained, the ends of the pad provide less lateral restraint with a consequent reduction in peak

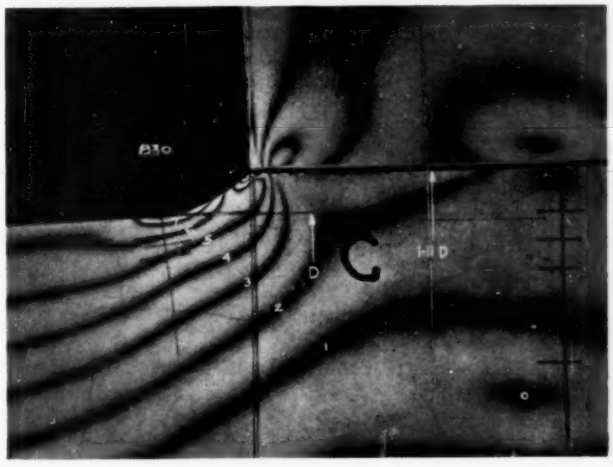


Fig. 4 Stress-Concentration at the Raised Wheel Seat of an Axle

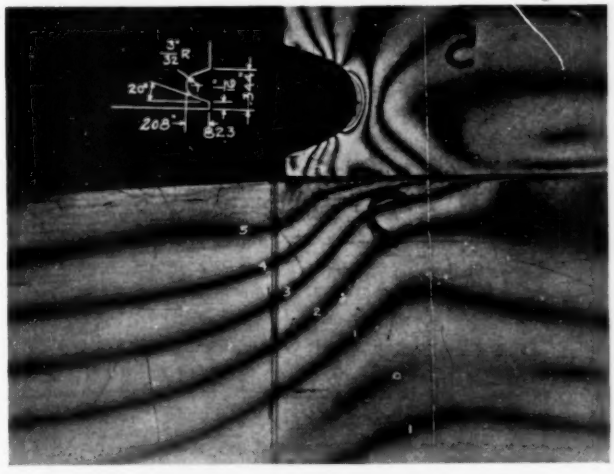


Fig. 5

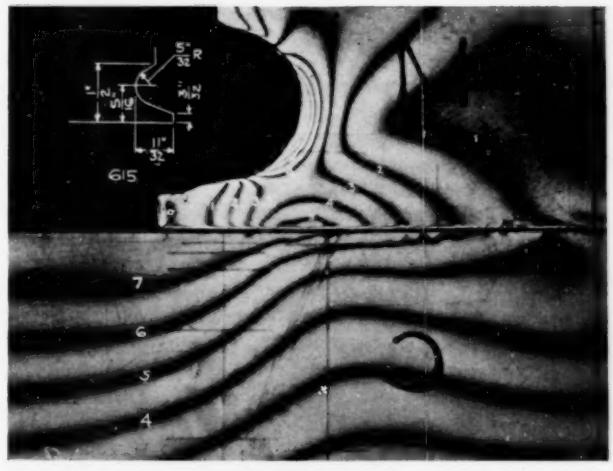


Fig. 6

Figs. 5 and 6 Stress Concentration in Grooved Sleeves

[5] Timken Roller Bearing Company, Canton, Ohio. Jun. A.S.M.E.
[6] Timken Roller Bearing Company, Canton, Ohio.

press-fit pressure at the end of the sleeve, and the bending stresses finally disappear at the ends of the pad.

In connection with the testing of grooved sleeves, the authors mention the possible effect of groove shape on stress concentration. The writers have investigated different types of grooves by photoelastic studies, a number of the better types of which are shown in Figs. 5 and 6. Here again no separate values of σ_1 and σ_2 are yet available, but the point is being made that groove shape appears to have some influence on the final strength of the press-fit assembly. Only fatigue tests of press-fit assemblies with different types of groove designs can tell what improvement in strength is to be expected.

Regarding the stress-concentration factors, the authors state that the fatigue stress-concentration factors are in general closer to the photoelastic stress-concentration factors than is ordinarily the case, and a comparison is made between the numerical values of stress-concentration factors obtained by photoelasticity and from fatigue tests.

The writers believe that a direct comparison of these numerical values is misleading. As long as only a comparison between two different designs is desired, as for instance between the plain press fit and the grooved construction, it is reasonable to assume if one construction shows in the photoelastic tests a smaller stress-concentration factor than some other design, that the superiority of the first design over the other will also be found by fatigue tests, but not necessarily by the same difference in stress concentration.

The numerical values cited by the authors seem to indicate a close agreement between the fatigue and photoelastic stress-concentration factors. However, in the Darmstadt tests 11 and 12 (Table 3) the stress-concentration factors are 2.2 and 2.6 although the photoelastic test gives 1.95 for the plain press fit. The objection may be raised that in these two tests the material of the shaft does not have the same physical properties as the material of the collar while in the photoelastic study the same material is used for both parts. In test 12 the shaft is of Cr-Ni steel, and the collar is of low-carbon steel; this combination gives the stress-concentration factor of 2.6. If tests were made with a collar of the same material as the one of the shaft, i.e., Cr-Ni steel, it would be found that the stress-concentration factor would be still higher, probably near 3 or higher. This increase in stress concentration when a material of higher physical properties is used for the collar is shown by tests 2, 10, and 11 (Table 3).

The photoelastic studies show only the stress concentration due to the shape of the construction. In fatigue tests, as mentioned by the authors, the stress concentration is due to the weakening action of the rubbing corrosion superimposed on the stress concentration due to shape, and as known will be a function of the sensitivity of the material and size of the specimens. For a material with low sensitivity to stress concentration it may happen that the fatigue stress concentration due to shape plus the effect of rubbing corrosion will give an apparent fatigue stress-concentration factor of the same magnitude as the photoelastic factor. If the material has a high sensitivity this equivalence of numerical values will not hold unless the effect of rubbing corrosion should be small for this material.

The reduction of the size effect noted by the authors may be due to the fact that only the stress concentration due to shape is subject to the size effect, while the weakening action due to rubbing corrosion may be little affected by the size of the specimen.

It is noted that no reaction is taken through the pressed-on sleeve in the fatigue tests, while in the photoelastic tests the reaction of the loads used to produce bending is taken through the lower sleeve which acts as a support. For this reason the resultant of the pressure forces acting on the tension side of the Bakelite beam will not equal the resultant of the pressure forces acting on the compression side. In the fatigue tests described by the authors the resultants for the tension and the compression sides must be equal. In practical applications both cases may be found and it would be of interest to know the opinion of the authors on the effect of the reaction taken through the press fit on the stress-concentration factor.

The authors indicate the use of the term "fatigue stress concentration" in variance with the strict interpretation of the term, for lack of a better one. Eventually the factors which introduce an increase in stresses and those which decrease the strength of the material should be segregated.

Authors' Closure

The X-ray reflection method of determining the depth of the plastic layer due to rolling as presented by Dr. Barrett in his discussion seems to offer considerable promise. It should be mentioned that microscopic examination did not prove effective in this connection for the material used (0.42 per cent C steel). This is in agreement with German experience with medium carbon steel, although for 0.10 per cent C iron, micrographs show considerable grain flattening near the surface.

Dr. Way's discussion forms a valuable contribution to our knowledge of the rolling process. For the 1⅝-in.-diameter fatigue specimen and 800-lb rolling load, the computed depth of plastic penetration is approximately 0.1 in. Since practically full endurance strength of such a shaft with a press-fitted collar was obtained, it was decided to attempt to roll larger shafts of the same material, keeping the ratio of depth of plastic penetration to shaft diameter constant. From the results shown in Table 2, it will be noticed that good agreement was obtained between the computed depths of penetration and the X-ray determinations of Dr. Barrett. Vickers hardness measurements also gave good agreement for the larger shaft diameters.

TABLE 2 COMPARISON OF METHODS FOR OBTAINING PLASTIC PENETRATION ON VARIOUS-DIAMETER SHAFTS

Shaft diam, in.	Rolling load, lb	Roller diam, in.	Roller contour radius, in.	Depth of plastic layer, in.		
				Computed	X-Ray	Hardness
1⅝	400	2	1½	0.067	0.055	.....
1⅝	800	2	1½	0.100	.....	.....
3 3/16	3000	2	1½	0.200	.....	0.21–0.23
5	7000	2	1½	0.300	.....	.....
7¼	12000	6	3	0.450	0.41–0.47	0.42–0.52

Messrs. Horger and Maulbetsch present Kühnel's complete results as having considerable bearing on the development of the pad construction which appears to have possibilities as a method of strengthening press fits. It seems to the writers that the ideas connected with the pad construction as presented by the discussers are essentially new and not to be derived from Kühnel's test results. From Table 1 it would appear that the fillet end (4) carrying 30,000 lb per sq in. nominal stress is better than the plain press fit (5) carrying 19,000 lb per sq in. in nominal stress. However, the section modulus of (5) is over three times that of (4) so that (5) can actually carry twice as much moment as (4). If the smaller diameter is limited and the press-fit diameter can be increased considerably, the press-fit strength can obviously be increased by virtue of increased section modulus. The new idea introduced by the discussers is that the pad construction provides more gradual transition of section, less lateral restraint and consequently reduced peak pressure. This means that with *a given press-fit diameter* (d_2), it is possible by decreasing the diameter a *small* amount outside the fit and using a generous transition radius to actually increase the *moment carrying capacity*, as compared to a shaft of constant diameter (d_2). The discussers' Fig. 4 shows that the above possibility exists. It is hoped that

the discussers will extend this interesting possibility to find optimum conditions of variables d_2/d_1 and fillet radius.

As mentioned in the paper, the authors believe that the press-fit fatigue problem involves essentially a high stress concentration acting on a material weakened by rubbing corrosion. Some fatigue tests have been made with the idea of isolating the rubbing-corrosion effect by use of special clamps. Although this work is not completed, it appears that a reduction due to corrosion alone of about 25–30 per cent will be obtained for the same material as used in the press-fit tests. In Table 2, column (5) is obtained by dividing the values of column (4) by 1.4 which is intended to give a fatigue value due to stress alone for comparison with photoelastic values.

Additional photoelastic tests were also made on plain press-fit models to determine the effect (on the stress-concentration factor) of the ratio l/d of the length of collar to diameter of shaft, as well as that of the ratio p/σ of nominal press-fit pressure to nominal bending stress. These tests were made by using an arrangement to give a pure bending moment on the bar representing the shaft, since the cantilever apparatus described in the paper (Fig. 8) did not lend itself to the study of light press fits because of the compressive load on the lower block representing the collar. The photoelastic stress-concentration factors K_p thus obtained are represented by the experimental points of Fig. 7 from which it may be seen that within limits the relation

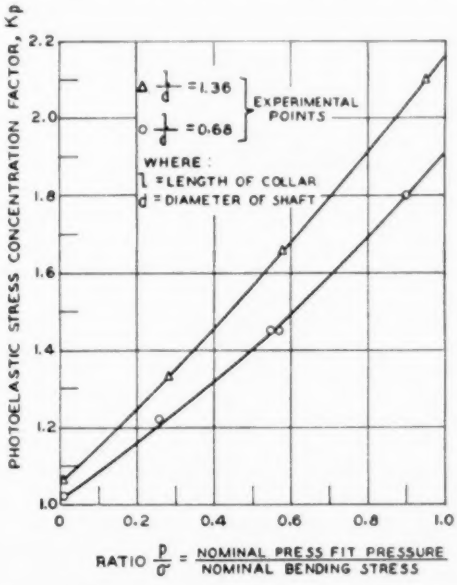


FIG. 7 PHOTOELASTIC STRESS-CONCENTRATION FACTORS
(Press-fit tests.)

between the factor K_p and the ratio p/σ is roughly linear for given values of l/d. Using the curves of Fig. 7, a plot showing the relation between K_p and l/d for various values of p/σ was drawn (Fig. 8). Since it is reasonable to assume that the stress-concentration factor will approach unity for small values of l/d, the curves have been so drawn as to approach this value. Because of the small number of experimental points available, these curves should be considered as rough approximations only. The photoelastic factors K_p shown on Table 3 were taken from Fig. 8.

An examination of Table 3 shows that the photoelastic stress-concentration factors are greater than the fatigue stress-concentration factors (corrected for corrosion). This is in agreement with experience, as in most cases photoelastic factors are lower than the corresponding fatigue factors, and in no case does the latter exceed the former.

In order to explain the results of series 4 and 5 (Table 5) obtained on solid specimens having the same contour as the

TABLE 3 COMPARISON OF FATIGUE AND PHOTOELASTIC STRESS-CONCENTRATION FACTORS
(PRESS FITS)

Fatigue series No. (Table 5)	Ratio l/d	Ratio p/σ at endurance limit	Fatigue stress-concentration factor K	Fatigue stress-concentration factor corrected for corrosion K_c	Photoelastic stress-concentration factor K_p
1 (Plain press-fit)	0.60	0.23[a]	1.47	1.05	1.18
2 (Plain press-fit)	0.92	1.01	2.00	1.43	2.02
3 (Plain press-fit)	0.92	0.01	1.44	1.03	1.10
6 (Grooved construction)	0.92	0.64	1.70	1.21	1.34[b]

[a] Estimated value of p taken as 5000 lb per sq in.
[b] In this case, because of lack of test data no correction was made to take into account the difference in l/d obtaining in the fatigue test and in the corresponding photoelastic test. This correction should be small, however.

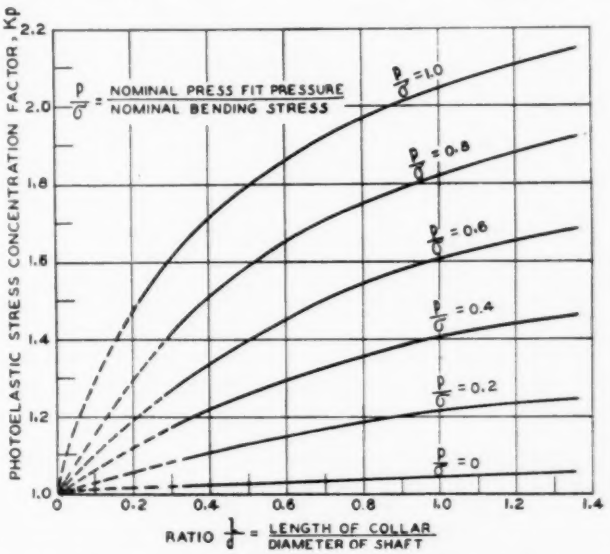


FIG. 8 PLOT OF PHOTOELASTIC STRESS-CONCENTRATION FACTORS—PRESS-FIT TESTS
(Curves obtained from data of Fig. 7.)

press-fit specimens, some photoelastic tests were also made on flat bars with enlargements having sharp corners, to simulate the conditions occurring in the fatigue test. When the ratio of the width of the enlarged portion to the width of the bar proper was 3.7 (corresponding to the ratio D/d) a value of $K_p = 2.62$ (average of four tests) was obtained. Although the stress-concentration factor is theoretically infinite for a sharp corner, yet actually a small radius always exists and in the photoelastic tests when the fringe order curve was extrapolated to the edge, a finite value of stress was found. In a similar test where the ratio of the widths of the enlarged part of the bar to that of the narrow part was 1.2, corresponding to the proportions obtaining in test 5 (Table 5), an average photoelastic factor $K_p = 2.07$ was found. These photoelastic values compare with corresponding fatigue factors of 2.3 and 1.9, respectively, and thus explain why the results of series 5 show lower stress-concentration values than those of series 4.

The discussers state that by using a high-grade alloy-steel shaft and collar, a fatigue stress-concentration factor of 3 may be found (referring to Thum's tests). High factors may be due to the following: (1) High ratio l/d of collar length to shaft

diameter. (2) A high-strength steel collar would allow the use of higher press-fit pressures without yielding at the collar edge. (3) The rubbing-corrosion effect may be greater with a higher-strength shaft material. If all these factors are taken into account it is believed that the fatigue values will not exceed the photoelastic stress-concentration factors.

The question concerning the size effect, mentioned by the discussers in connection with the fatigue tests of series 1 and 2, is answered by the subsequent photoelastic tests which show a considerable decrease in K_p with decreasing values of the ratios l/d and p/σ. These tests thus confirm the opinion expressed in the paper that the lower value of K found in series 1 is caused primarily by the relatively short collar and low press-fit pressures used.

Regarding the question on the effect of the collar reaction raised by Messrs. Horger and Maulbetsch, a comparison of the results obtained with the bending-moment apparatus and those obtained with the cantilever apparatus (Fig. 8) shows only a small difference (of the order of 5 per cent at the most) for ratios of p/σ around 0.7 to 0.8. This would be expected since, in the case of the present tests, the reaction increases the total press-fit pressure by only about 7 per cent on the compression side.

Assuming a like increase in peak stress on the compression side, the effect on the stress range (which determines the factor K_p) would only be around $3^1/_2$ per cent. For these reasons the authors believe that the effect of the reaction is of minor importance, except possibly for extremely short shafts.

Book Reviews

Steel Structures Research

SECOND REPORT OF THE STEEL STRUCTURES RESEARCH COMMITTEE. Department of Scientific and Industrial Research. His Majesty's Stationery Office, London, 1934. Cloth, 6 × 9½ in., 369 pp., 164 figs., 7s 6d.

REVIEWED BY L. C. MAUGH[1]

THE Steel Structures Research Committee is composed of a distinguished group of engineers who represent the British Steelwork Association, the Institution of Civil Engineers, and the Government of Great Britain. Its establishment was primarily due to the desire of the members of the British Steelwork Association to encourage a more efficient, economic, and extended use of steel, and a willingness on their part to contribute financially toward such an undertaking. In collaboration with the Institution of Civil Engineers, a committee was formed in August, 1929, under the direction of the Department of Scientific and Industrial Research. The aims and purposes of this Committee were clearly stated in the first report, published in 1931, namely:

(a) "To review present methods and regulations for the design of steel structures, including bridges.

(b) To investigate the application of modern theory of structures to the design of structures, including bridges, and to make recommendations for the translation to practice of such of the results as would appear to lead to more efficient and economical design."

In accordance with these objectives the first report was devoted largely to a compilation and comparison of representative specifications and research data relating to structural-steel design. Much attention was also given to the development of more accurate methods of determining the allowable capacities of riveted, bolted, and welded connections. A recommended code of practice, which has been widely used in Great Britain, was included for temporary use.

The second and latest report presents the progress of the committee for the period March, 1931, to March, 1933. The contents of this bulletin treats, in general, the problem of stress analysis of building frames. More specifically, information and results of analytical and experimental research on the following subjects has been recorded:

(a) The effective length of continuous columns in structural-steel frames.

(b) Observed stresses in a number of structural-steel frames, mainly with seat-angle connections.

(c) Experimental investigation of the elastic properties of various types of beam and column connections, particularly the relation between moment and angular deformation in riveted and bolted connections.

(d) The analysis of the moments in the members of a frame having rigid or semi-rigid connections under vertical load by means of modified slope-deflection equations and the method of moment distribution.

(e) Welding of steel structures.

The results presented in this report have considerable value in showing what bending stresses are produced in columns by ordinary riveted beam connections, when the beams are subjected to vertical loads. Various methods of analysis are suggested in which the elastic properties of the various types of connections can be introduced. Although these methods are not likely to be of immediate practical use, nevertheless they give a start toward removing that glaring inconsistency which is so common in the design of steel buildings, namely, that of assuming no restraint at the ends of beams when subjected to vertical loads and full restraint at the connections when the building is subjected to horizontal loads. There seems to be opportunity for more economical design if this problem can be satisfactorily solved.

[1] Assistant Professor in the Department of Civil Engineering, University of Michigan, Ann Arbor, Mich.

Welded Structures

FABRICATED WELDED STEEL CONSTRUCTION. By J. Collinson, M.I.W.E. Draughtsman Publishing Co. Ltd., London, 1934. Cloth, 51 pp., 15 photographs, 3s.

RIVETING AND ARC WELDING IN SHIP CONSTRUCTION. By Commander H. E. Rossell, U.S.N. Professor of Naval Construction, Massachusetts Institute of Technology. Simmons-Boardman Publishing Co., New York, N. Y., 1934. Cloth, 5 × 7½ in., 210 pp., 46 figs., $2.25.

REVIEWED BY C. H. JENNINGS[2]

MR. ROSSELL'S book is essentially a text and reference book dealing with the design and construction of riveted and welded joints as applied to ship construction. The fact that the book deals specifically with ship construction does not detract from its value, however, because in nearly all cases the joints and problems discussed are directly applicable to structural, machine, and tank construction.

The book is divided into two major sections, riveting and welding. The joint designs considered in each section are carefully analyzed and developed by taking into consideration both the practical and theoretical phases of the problem. Numerous specific examples are worked out as a means of illustrating the method of applying the various formulas that are derived.

Special consideration is given to the practical phases of both welded and riveted construction such as the production of oil-tight and watertight joints and the reduction of warping and residual stresses in welded joints. A critical comparison is made between welds made with bare and "shielded-arc" electrodes, and applications, where each are suitable, are discussed.

The designer of welded constructions, whether the constructions are ships, buildings, bridges, or machines, will find in this book a wealth of valuable data from the standpoint of selecting the proper joint and the design of the same.

Mr. Collinson's article contains a discussion of the important factors he has found to have an influence in the economic design of arc-welded structures. The factors discussed cover both drafting-room and shop practices and although some are not in accordance with the American practice they are worthy of consideration. In some cases the procedures employed in obtaining certain conclusions are rather incomplete and difficult to follow, thereby detracting from their value.

The draftsman and designer of welded machinery will be particularly interested in the author's discussions dealing with the proper developed lengths of formed plates, specification of welds,

[2] Research Engineer, Westinghouse Research Laboratories, East Pittsburgh, Pa.

on drawings, design notes, and cost analysis. The cost analysis of welded structures is of particular importance because the author has attempted to follow the general practice used in estimating the cost of castings, i.e., reduce the castings to a system of classification and weight.

Broadly speaking, Mr. Collinson's article is a worth-while contribution to the art of welding design, a field greatly in need of careful study and analysis if the advantages and savings that welding affords are to be obtained.

Rayleigh's Principle

RAYLEIGH'S PRINCIPLE AND ITS APPLICATIONS TO ENGINEERING. By G. Temple and W. G. Bickley. Oxford University Press, London, 1933, Cloth, 5½ × 8½ in., 156 pp., 22 figs., $4.75.

REVIEWED BY J. N. GOODIER[3]

RAYLEIGH'S PRINCIPLE, the central subject of this book is of great importance in engineering. It provides valuable simple methods for estimating the natural frequencies of vibration of elastic structures and machine elements, especially those which come under the headings of thin bars and plates. A knowledge of the natural frequency is, of course, essential where resonant vibration is to be avoided. The principle is equally useful as a means of investigating stability. It can be made to give good approximate formulas for whirling speeds and buckling loads. This book puts the principle on a sound footing and exhibits its application in a variety of examples of great interest to engineers, including the whirling of non-uniform shafts under the influence of torque and tension, the tippling or sidewise buckling of deep beams, vibrations of rotating disks, and the buckling of non-uniform struts. The simplicity of the method as applied to some of these otherwise rather forbidding problems is often very striking.

In a freely vibrating system the energy is all potential when the system is at the extremity of its swing, and all kinetic when it is passing through its equilibrium position. These two energies must be equal when there is no damping, since each is equal to the total energy, which is necessarily constant. If we know the configurations, or amplitudes, of the system as it vibrates, we can calculate the potential energy. In any simple harmonic motion the kinetic energy of each particle is given by its mass, amplitude, and frequency. It is proportional to the square of the frequency. Consequently, the equality of the maximum kinetic and potential energies becomes a formula for the frequency. Now, even if the configuration of the system at the extremity of its swing is not known *accurately*, it can, nevertheless, often be represented *approximately*, and simply, from observation or intuition, and with this approximation the calculation outlined will give an approximation to the natural frequency. Rayleigh's principle may be expressed by the statement that whatever the approximation chosen for the configuration, or deflection in the vibration, the resulting approximation to the frequency will always be greater than the true value, which would be obtained if the true deflection were used. As a matter of fact it has turned out that any reasonable representation of the deflection gives a frequency surprisingly close to the true value.

It has been customary to judge the accuracy and scope of the method by applying it to systems for which an exact formula for the frequency is known, such as uniform bars, and making a comparison between the Rayleigh approximation and the exact value. The same applies, of course, to the stability problems. There are, however, other simple approximate methods. The use of these, as well as of Rayleigh's principle, provides both upper and lower limits to the value required. These methods are discussed in the book. When the limits so obtained are sufficiently close, nothing more is required. If it is otherwise, the Rayleigh approximation can be repeatedly improved and the error can be estimated by methods given in the book.

The book can be read with an eye to results of engineering value without a detailed study of the mathematical argument. The results of the analysis are conveniently summarized in the introduction and in an appendix to chapters 3 and 4. The introduction also gives a general account of the principle and its applications, including a method for improving the approximation. Chapters 1 and 2 are devoted to stability problems. Chapter 3 contains a rigorous proof of the principle. Chapter 4 describes a second method for improving the approximation, and develops from it a means of estimating the accuracy at any stage. The estimation of the second natural frequency is also discussed. Three of the remaining four chapters are devoted to applications of the results, mainly to engineering problems of the types already mentioned. The other chapter deals with the methods which are supplementary to Rayleigh's. They are based on a group of "comparison and synthetic theorems," which are proved with the help of Rayleigh's principle.

[3] Ontario Research Foundation, Toronto, Canada.

Atomic Structure

ATOMIC STRUCTURE AND SPECTRAL LINES. By Arnold Sommerfeld. Translated from the Fifth German Edition by Henry L. Brose. E. P. Dutton and Company, Inc., New York, 1933. Cloth, 670 pp., 151 diagrams and photographs, $10.80.

REVIEWED BY J. ORMONDROYD[4]

THE Fifth German edition of Professor Sommerfeld's classic work on atomic physics appeared in two parts. This volume, which is a translation of the first of these, contains the older quantum theory. A second volume, to appear shortly in translation, will contain the more recent wave-mechanics theories.

Only ten years ago Bohr was charming American audiences with accounts of his microscopic solar system in which the planets (electrons) repelled each other (although perturbations played no important rôle in the theory of the simple hydrogen and helium atoms) and jumped from orbit to orbit as they gained or lost energy according to Planck's famous quantum theory of radiant energy. This little system explained much in spectral analysis; but not enough. Wave mechanics now rules in the field of atomic theories. But the latest theories can only be understood when the theories which grew around the Rutherford-Bohr atom are completely assimilated. This volume gives that necessary background.

While "Atomic Structure and Spectral Lines" was written for mathematical physicists there are, undoubtedly, some members of the engineering profession who may have an occasional need for the technical information contained in this book. There are many who have a strong cultural interest in the ideas, theories, and results which form the core of present-day physics. To both types of readers this work can be recommended as being authoritative as far as the older quantum mechanics is concerned. It is difficult to imagine a more definitive work than the present volume.

The work is mathematical in form, using vector analysis and Hamiltonian mechanics to arrive at its conclusions. The dynamics of the "special theory of relativity" is used in describing the fine structure of the spectral lines. The mathematical devices are introduced in some extension and are presented persuasively to the reader. If, in spite of this, the engineer is

[4] Experimental Division, Westinghouse Electric and Manufacturing Company, South Philadelphia, Pa. Mem. A.S.M.E.

terrified by the mathematical language, much of the text can be read with a high degree of understanding independent of the formulas—the numerous results being stated, as often as it is possible to do so, in words as well as in symbols.

Chapter 1 contains a critical historical résumé of the important physical developments which bear on the subject, and also fundamental definitions and concepts. Here, as throughout the volume, footnotes give complete references to the original papers which form the basis of the science. Chapter 11 outlines the older theory of the hydrogen spectrum which stands as a foundation for all later developments. The orbital theories of Bohr are given in extenso and an introduction to Hamiltonian mechanics is outlined. After this the natural system of elements, X-ray spectra, the theory of fine structure, the intensity of spectral lines, the series-laws of spectral lines, the complex structure of the series terms, and band spectra are treated in the order named.

Prof. Arnold Sommerfeld, holder of the chair of Theoretical Physics at the University of Munich, has himself contributed much to the subject which he treats in this volume. He has earned the gratitude of physicists throughout the world for carrying out so successfully the monumental task of giving a comprehensive and lucid exposition of modern atomic physics. Professor Brose, of University College, Nottingham, has achieved a smooth and, one feels, a completely authentic translation of the original work.

Seaplane Design

SEAPLANE DESIGN. By Lieut.-Comm. W. Nelson, Lecturer, New York University. McGraw-Hill Book Company, New York, 1934. Cloth, 6 × 9 in., 274 pp., $3.50.

REVIEWED BY SCHUYLER KLEINHANS[5]

THE belief that the seaplane is destined to become increasingly important in the field of transportation has provided sufficient motive for the writing of this book, both as a textbook for the student and a source of information for the practicing engineer. The author's experience as an officer in the Construction Corps of the United States Navy and various tours of duty as both shop superintendent and chief engineer of the Naval Aircraft Factory has well fitted him to write on this subject, so long neglected by engineers and educators because of the scant amount of information available.

In the words of the author, this volume "is a compilation of information and data attained to a great extent from the references." As the references include practically all of the published information on the subject of seaplane design, the book is fairly complete although a bit brief in sections. The subject matter of the book passes successively through the topics of displacement and buoyancy, stability afloat, water resistance, float forms and dimensions, float arrangements, model tests of hulls and floats, take-off of the seaplane, seaplane stresses, watertight subdivision, weight, construction features and details, loft practice, and finally, mooring, towing, and beaching of seaplanes.

The author passes quickly over those branches of aeronautical and naval architecture not specifically connected with the seaplane problem by referring the reader to the many excellent textbooks on these two subjects, and confines himself to the particular questions associated with the design of the seaplane itself. The book is remarkably free from prejudice and causes one to regret that the author's natural modesty prevented his expressing more of his personal ideas. After a summary of the history of seaplanes, the author discusses some applications of seaplanes and then launches into the heart of his subject beginning with the statics of flotation. He here discusses the properties of liquids, Archimedes' principle, Simpson's rules, displacement, coefficients of fineness, permeability, and center of buoyancy location.

The next chapter, dealing with stability afloat, discusses conditions of equilibrium, the metacenter, location of the center of gravity, inclining experiments, free water and its effects on the stability, flooded compartments, righting moments, dynamic stability and metacentric height of seaplanes.

Under water resistance, the author discusses again properties of liquids, waves produced by a vessel, natural sea waves with a brief résumé of the natural-wave theory, and the components of water resistance.

Float forms and dimensions are considered along with the traditional features of seaplane and hydroplane hulls, steps, and general shape of the under-water body. Decks, wing-tip float forms and air resistance are also under this section of the book.

The general topic of float arrangements is expanded to include water stability in detail using the principles of naval architecture developed in earlier chapters, at the same time going over the various arrangements of floats as seen in actual practice.

Model-basin tests are given their fair share of space, and rightly so, for they represent the backbone of seaplane knowledge to date. After discussing the purpose of model tests, there is a section devoted to similitude with paragraphs on Froude's law and Reynolds' number, as well as a comprehensive discussion of model limitations and results. A review of model-basin procedure and apparatus is followed by a description of the hydrovane and the complete float test with some remarks about flight tests of large models on a small inexpensive airplane.

The important problem of the seaplane takeoff occupies a chapter in itself with illustration of take-off calculations including float resistance, propeller thrust and effects of the trimming angle.

Seaplanes, which are subjected to hogging and sagging stresses as well as the stresses caused by landing and take-off, and sometimes special cases such as catapulting, must be designed to withstand these loads. The author has grouped these various stress-producing conditions under "Seaplane Load Determinations," where he also gives a general description of the theories of impact landing as well as a review of the impact-landing requirements of several governments.

The necessity of watertight subdivision is expounded and the questions of stability and change of trim due to flooded compartments are given in detail in a chapter devoted to this subject along with a description of subdividing methods.

After some remarks on the traditional subject of weight, which includes a tabulation of the detail weights of a typical patrol-type flying boat, the author considers various features of construction and discusses some of the materials available for float and hull structures. There are paragraphs on framing, bulkheading, and shell plating. Construction details include types of rivets, efficiency of riveted joints, watertight construction, and closures.

Under line drawings and loft practice, the author gives only a glimpse into this fascinating and most rapidly growing department of a modern aircraft factory. There is, however, sufficient explanation to awaken the reader to the tremendous possibilities of the mold loft as a means of improving aircraft manufacturing technique and materially reducing the cost of the finished article.

Under mooring, towing, and beaching seaplanes the author describes anchors and towing lines as well as hoisting, docking, and beaching procedures.

The material and composition of "Seaplane Design" is such that, except for some twenty-six pages of appendixes devoted to sketches of typical seaplanes which are bound to become obsolete

[5] Douglas Aircraft Corporation, Santa Monica, Calif.

almost as soon as the book is off the press, it should stand the test of time and be as useful some years hence as it is today. The excellent index is a detail that will be appreciated both by the casual reader and the more serious student. The book is a distinct contribution to the small but growing amount of literature on seaplane design and it is to be hoped that the appearance of this volume will be an inspiration to other writers to take up the work where space limitation has caused Mr. Nelson to leave off.

Aerodynamic Theory

AERODYNAMIC THEORY. Dr. William Frederick Durand, Editor-in-Chief. Vol. 1. Julius Springer, Berlin, 1934. Cloth, 6 × 10 in., 398 pp., 151 figs., 20 rm.

REVIEWED BY J. C. HUNSAKER[6]

The Guggenheim Fund for the Promotion of Aeronautics, before closing its active and most useful life, made provision for the preparation of a series of monographs on aerodynamic theory by the outstanding international authorities in this field, to be contained in six volumes of which this is the first. The plan of the editor-in-chief is to present knowledge of aerodynamic theory with especial reference to progress in the past twenty-five years, the period since flight became an element of our civilization. Such a work is most welcome, as the original material is generally unavailable except to those having access to the great libraries; and even here an extensive search through many publications in many languages would be required.

The first volume appropriately furnishes a general introduction, giving in some detail the special mathematical methods of fluid mechanics and a critical review of the development of modern aerodynamic theory from the classical hydrodynamics of the last century. The complete series will present the theory of perfect, viscous, and compressible fluids, with applications to airfoils, bodies, propellers, and cooling, and to the dynamics and performance of airships and airplanes. Aircraft design itself, being an art rather than a science, and hence changed rapidly by economic, political, metallurgical, and other conditions, as well as by unpredictable inventions, is not to be included.

The first volume begins with an elegant yet concise treatment of the mathematical aids used in the later development of the mechanics of fluids. Here the grateful reader will find that, without further preparation than that given by ordinary college mathematics, he may acquire an understanding of the complex variable, vector algebra, vector fields, and the motion of a fluid described by potential functions. Gauss's, Green's, and Stokes's theorems are developed and the method of conformal transformation explained and illustrated. A really adequate and rigorous treatment of the theory of dimensions and of kinematic similitude is set forth by Dr. Durand, whose genius as a teacher is to be noted throughout.

Dr. Durand contributes ten chapters on fluid mechanics, following directly after his development of the necessary mathematical technique. These chapters begin with the fundamental equations of motion and of equilibrium for the perfect fluid and the engineering reader is easily carried through the solution of the elementary problems of parallel flow, sources and sinks, doublets, vortices, and their combinations, as well as the application of conformal transformation to flow fields. This portion of the work is intended to serve as an introduction to more advanced analysis. Dr. Munk then follows with a more comprehensive treatment of somewhat the same ground, using the methods of space-vector analysis, and extending to the solution of problems of motion of solids in a perfect fluid. The generalized three-dimensional method introduces the conceptions of apparent mass, polar and elliptic coordinates, and leads to consideration of the elongated ellipsoid resembling an airship. Here, Munk discusses generalized potential flow and the forces which are of interest in design. These solutions by Munk are frankly for the advanced reader and may be compared with those by Horace Lamb in his "Hydrodynamics" for a somewhat fuller treatment.

The last ninety pages of the book are devoted to a conscientious review of the history of fluid mechanics. This section is the first adequate survey of the contributions of the various workers in the field during the present century. A critical appraisal is badly needed because of the interruption to the free exchange of scientific thought by the war and the inevitable duplication of research effort.

The historical review is divided into three parts: The first covers the period of early thought to the end of the seventeenth century, marked by Leonardo's correct statement of relative motion as the basis for flight. The second period, that of classical hydrodynamics, begins with Newton and ends with the nineteenth century. It is marked by a veritable flowering of applied mathematics in the hands of such giants as Newton, Bernoulli, D'Alembert, Euler, Lagrange, Helmholtz, Kirchoff, Rankine, and Rayleigh. The classical theory, in order to permit mathematical solutions, postulated a perfect frictionless fluid. While useful principles were obtained, the problem of actual flight could not be solved until experiments with real fluids were undertaken by Reynolds, Froude, Wenham, Philips, Mouillard, Lilienthal, Dines, Langley, and the Wrights. The classical theory could not account for the results of experiment.

At the end of the century there was a wide gap between theory and experiment, and aerodynamic theory played but a small part in the early development of the airplane. But with the stimulus given by the first actual flight, scientific attention was sharply focused on the underlying fluid mechanics. Theoretical advances came rapidly thereafter.

In the third part of their review, Giacomelli and Pistolesi trace the remarkable renaissance of hydrodynamics in the hands of Lanchester, Kutta, Joukowski, Prandtl, and their followers. Today aerodynamic theory determines the correct shape and proportions of wings, propellers, etc., and predicts the practical performance of the aircraft built to conform to theoretical considerations. That this can be true, results from the application of the methods of classical hydrodynamics to the mechanics of real fluids (Lanchester-Prandtl wing theory), the theory of dynamic similitude (Reynolds-Rayleigh), and the theory of the boundary layer (Prandtl).

The development of the modern wing theory for finite spans by Prandtl and his collaborators, from the qualitative and intuitional conception of Lanchester and the classical two-dimensional solutions of Kutta and Joukowski, is a fascinating example of the international character of research. It is to be regretted that in the history of theoretical aerodynamics no American contributions are worthy of note, although on the experimental side we might suggest D. W. Taylor, Langley, Zahm, and the Wrights.

The historical survey closes with the wing theory and does not attempt to appraise recent contributions toward the unsolved problem of resistance in real fluids, although the early work of Mallock (1907), Bénard (1908), and von Kármán (1911) is carefully set forth.

In Volume 2, von Kármán and Burgers will deal with the wing theory. In Volume 3, Prandtl will treat the mechanics of viscous fluids, and G. I. Taylor the mechanics of compressible fluids. There has been significant work on turbulence from each of these four, and one may hope that upon completion of the six volumes planned for the Guggenheim series on aerodynamic

[6] Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

theory, there may be found means to publish a seventh volume to include advances in the theory of turbulence and boundary-layer mechanics, made since the series was planned. The reviewer ventures the prediction that the next important step in aerodynamic theory will give an understanding of the boundary layer by which its apparently whimsical tendency to separate from a wing or other solid body may be predicted and dealt with by practical designers. The problems of boundary-layer separation and of the consequent turbulent wake resistance are fundamental and must be solved before aerodynamic theory can have full engineering application.

From Galileo to Cosmic Rays

"From Galileo to Cosmic Rays," Harvey Brace Lemon, Ph.D., Professor of Physics, the University of Chicago. The University of Chicago Press, 1934. Cloth, 6 × 9 in., 450 pp., 100 photographs, 325 line drawings. Educational edition $3.75, stereoscope $0.75 extra; trade edition $5, includes stereoscope.

Reviewed by J. Ormondroyd[7]

THIS book, with sections on mechanics, heat, electricity and magnetism, electricity and matter, and waves and radiation, is offered as a textbook to be used in an introductory survey course of the physical sciences. Professor Lemon's well-instrumented demonstration lectures at the University of Chicago form a secure basis for this work. While it is difficult to put one's self into the frame of mind of an undergraduate again, those who read this book will wish that the subject had been presented to them in the style and manner used by the author.

The textbook writer usually attempts to satisfy self-imposed demands as to a preconceived quantity of material compressed into a logical order of presentation. With this ideal he succeeds in presenting a number of perfectly correct and desiccated formulas properly cataloged and card-indexed. But he often forgets that it takes two to make a textbook—a reader as well as a writer. Professor Lemon avoids this common pitfall by thinking of his readers first, last, and always.

To the student the material usually presented seems to be arbitrarily selected; unconsciously his reaction is "Yes—What of it?" That contact between his familiar life and the new material is often woefully absent and the professor seems to have little sympathetic intuition of his unvoiced questioning. As far as he can, Professor Lemon ties his physics to every-day objects in the undergraduate's life, and elicits a feeling of comradeship by clearly indicating that he understands the student's psychological reactions. He even talks in the student's slangy idiom at times. Only mathematics of the simplest form is used—a concession filled with wisdom in view of the fact that the book has as its essential purpose the creation of an interest which can be satisfied by subsequent deeper and more detailed delving.

"One picture is better than a thousand words." The reader is never out of sight of one or more of the hundreds of photographs and line drawings. The photographs show views of demonstration apparatus in action adding concreteness to the text; while the line drawings are frequently humorous and always instructive. As a crown to the pedagogical strategy the book is provided with many double-exposure pictures and a stereoscope through which to view them. This not only adds the element of play to reading the book; but also insures close attention and greater revealed detail to the pictures so treated.

The title indicates that the subject matter is treated historically. Living glimpses of the master creative minds as given in this book add another humanizing touch to the text. The arrangement of the text in the order as given is also historically correct and prepares the student for that change from the mechanical point of view to the electromagnetic outlook which dominates physical thought today. Besides technical instruction, the book affords that generalizing view-point which gives it high cultural value.

Why should a book primarily designed for the instruction of youngsters be brought to the attention of practicing engineers? First of all, any engineer who reads this book will be highly amused and delighted. Secondly, most of us really do not know too much about the fundamental physics of the things we create and operate. Perhaps we could learn more than an undergraduate would be prepared to learn from this most human attempt at scientific exposition.

[7] Experimental Division, Westinghouse Electric and Manufacturing Company, South Philadelphia, Pa. Mem. A.S.M.E.

Statics and Dynamics of Shells

Statik und Dynamik der Schalen. By W. Flügge. Julius Springer, Berlin, 1934. Cloth, 6 × 9 in., 240 pp., 98 figs., 22.50 rm.

Reviewed by J. P. Den Hartog[8]

THIS book deals with the subject of stress, deformation, stability, and vibration of *curved* thin shells in a clear and complete manner. Written by a member of the Göttingen school, it forms a worthy counterpart to the excellent book on *flat* plates produced some ten years ago in the same laboratory by Professor Nadai under the title "Die Elastische Platten."

Curved thin plates or "shells" are employed in many important structures, e.g., domes or vaulted roofs of reinforced concrete, pressure vessels, boilers, oil tanks, pipe lines, and airplane fuselages or struts. An extensive literature on the subject has been built up, especially during the last fifteen years, and of this the book gives a complete account. The mathematical methods employed necessarily include a great deal of partial differentiation, but beyond that do not require any further knowledge on the part of the reader. A number of the most important results are illustrated by graphs.

The book is divided in ten chapters, of which the detailed contents are as follows:

The first is an introductory chapter giving the relations between stresses and bending moments at an element and also the relations between the stresses at a point by means of Mohr's circles. This latter method is worked out rather completely for shells of reinforced concrete.

The second chapter deals with the "membrane theory" of shells of revolution, especially with domes of various shapes and liquid reservoirs. By "membrane theory" is meant a theory which neglects bending stresses and only takes account of tensions. It is shown that this approximate theory is quite satisfactory in many cases, and that bending only comes in near the boundaries. The situation is somewhat analogous to structures built up of bars, in which the "primary" stresses are all pure tension, whereas the bending comes in only as a "secondary" effect.

The problem of a dome of equal strength (variable thickness) loaded by its own weight is solved. An interesting discussion and a good photograph is given of an oil-storage tank of equal strength having the shape of a drop of mercury reposing on a horizontal plane. Though the membrane stresses in the shell itself can be calculated from equilibrium conditions only, some statically indeterminates are usually introduced by the supports. For the purpose of calculating these, some methods for finding

[8] Assistant Professor of Applied Mechanics, Harvard University, Cambridge, Massachusetts. Assoc-Mem. A.S.M.E.

the deformations are discussed rather fully. Finally, the energy method for calculating the indeterminates directly is given.

The third and fourth chapters treat the same topics for cylindrical shells and for shells generated by the parallel translation of some curve. The principal applications here are pipe lines on many supports, slightly vaulted cylindrical reinforced-concrete roofs, and also domes made up of intersecting cylinders.

The fifth chapter shows how all previous results can be extended to shells generated from those discussed before by stretching them in one or more directions. For instance, the stresses and deformations of ellipsoidal shells can be deduced from those of the sphere.

The chapters six to eight which form the backbone of the book, discuss the more complete bending theory, taking full account of the literature which has been accumulating on this subject.

Chapter nine deals with the stability or buckling problem of a cylinder under external or endwise pressure with and without stiffened ends, as well as of cylinders under torsion. The stability of a thin-walled sphere under external pressure is also discussed. In the final chapter a brief treatment is given of the vibrations of cylindrical and spherical shells.

A very complete bibliography is appended, taking the place of footnotes in the text.

Through Space and Time

THROUGH SPACE AND TIME. By Sir James Jeans, D.Sc., F.R.S. The Macmillan Company, New York, N. Y. University Press, Cambridge, 1934. Cloth, 6 × 9 in., 218 pp., 106 figs., $3.

REVIEWED BY J. ORMONDROYD[9]

THIS little book, based on the Christmas Lectures of the Royal Institution delivered in 1933, is designed, according to the author, for an eager and critical audience, ranging in age from under eight to over eighty and in scientific knowledge from the aforesaid child to the staid professor of science. In other words it is offered to any one who can read and who is interested in the phenomena that are presented to his senses. It is a notable example, among numerous similar efforts, of the attempt which is being made increasingly by scientists and scholars to interpret and expose to the world at large the results of science and scholarship which are the dominating influences in our lives today, but which seem so remote and so incomprehensible to the major part of the human race. The scientist here stoops to conquer the non-scientific mind.

The objective and general outline of the book are announced by the author somewhat in the following fashion:

> We shall travel—or pretend to travel—so far through space that our earth will look like less than the tiniest of motes in a sunbeam, and so far through time that the whole of human history will shrink to a tick of the clock, and a man's whole life to something less than the twinkling of an eye.
>
> As we travel through space, we shall try to draw a picture of the universe as it is—vast spaces of unthinkable extent and terrifying desolation.
>
> As we travel through time we will show not only the present, but also the past and the future of the universe. We shall see the sky as it was a million-million years ago—vast colonies of stars being born, living their lives and finally dying. We shall watch our sun broken up in great turmoil producing a family of planets. We shall watch our earth, a globe of hot gas, which gradually cools and becomes a suitable abode for life. We shall see, finally, man arriving, taking possession of his tiny speck of dust in space and surveying with astonishment the strange universe in which his life is cast.

It can be seen that the author does not shrink from putting his material into the most popular of forms. If the reader has never been astonished at his universe before, he will be after he reads this little treatise. One inevitably thinks of P. T. Barnum or Mr. Brisbane on reading the above announcement. Because of its avowed intent, no mathematics or involved trains of reasoning are used in the text. The matter is purely descriptive statement of fact which requires merely passive absorption. "One picture is worth ten thousand words," some one has said, and following this psychological hint the book is liberally illustrated with interesting photographs. The book might be justly described as a collection of scientific "Just-So Stories" and is certain to appeal to all engineers who enjoy reading.

This geocentric survey of the universe starts on the earth, glances at the atmosphere which beneficiently surrounds the earth and arrives, in the third chapter, at the phenomena in the sky which form the most extensive part of the matter treated. The moon, the planets, the sun, the stars, and the nebulae are discussed in great detail. Geology, astronomy, the various branches of physics, are touched upon as they are needed to describe the phenomena selected for exposition.

How do we know the earth is round and rotates? How was its size first measured? How far is it from the other bodies swimming in space? Why is the sky blue? Why do radio signals fade? What about this stratosphere every one is attempting to reach? How can pictures be taken in the dark? Would mirrors reflect if their surfaces did not conduct electricity? What would we find if we could visit the other members of our solar system? How big, how hot, how far are the stars? How do nascent universes condense out of nebulae? All these questions and a thousand more are treated in building up this integrated picture of our universe.

To the constant reader of popular scientific outlines or histories some of the facts given will be inevitably familiar; but never have they been joined together so logically into such a pleasing, entertaining and instructive whole.

Sir James Jeans has such an ease of delivery, both in speaking and writing, that his utterances always find a receptive audience. His background of creative work in the subject matter of this little book as represented in his mathematical treatises "Astronomy and Cosmogony" and "Problems of Cosmogony and Stellar Dynamics" renders his simplest statement rich in content and authority. It is rare, indeed, that the mind capable of distilling such scientific output can bend itself to the more lowly task of putting the interesting results into elementary form.

---

[9] Experimental Division, Westinghouse Electric and Manufacturing Company, South Philadelphia, Pa. Mem. A.S.M.E.

A Method of Balancing Reciprocating Machines

By W. E. JOHNSON,[1] SCHENECTADY, N. Y.

General types of balancing problems are classified into three groups: Those requiring perfect balance, those in which the machine inherently is balanced sufficiently well for practical purposes, and those requiring a distinct compromise, such as single-cylinder engines. It is a method of securing an optimum condition for the last class that is presented.

The method consists, first, of the selection of a suitable criterion for the particular problem, such as minimizing the average potential energy of vibration in a spring mounting, which tends to secure the least possible transmission of vibration. Another criterion may reduce the vibration of the machine itself to a minimum. The selection of a suitable criterion is a matter for the judgment of the designer. When the selection is made, the attainment of the optimum sought is carried out by mathematical analysis. This general method is particularly useful for balancing spring-mounted machines. An illustrative problem is given for this case, showing the effects of working to several different criteria.

THE problem of balancing a machine is not one that offers a definite solution in each case, for often the problem has an economic side that makes the expense of a perfect balance prohibitive. It assists one's comprehension of the general situation if the various types of balancing problems are classified as follows:

1 There are occasional problems which require a perfect balance, regardless of cost. This makes the problem perfectly definite. Moreover, if the machine is self-contained, it is theoretically possible to obtain a perfect balance both of the dynamic forces and couples, and of couples derived from gas forces. As a proof of this statement, consider a conventional gasoline motor. If two identical motors were fastened head to head, the crankshafts being geared together properly to give correct timing, and the corresponding cylinders being connected with gas passages to equalize the pressures, it is obvious that the resulting composite machine would be perfectly balanced, provided it does not have to transmit torque to an external machine. However, if the external machine which absorbs the power is perfectly balanced and made to absorb the power equally from each crankshaft, it becomes part of a larger system which is in perfect balance, both with respect to dynamic and gas forces. (The term "force" is often generalized to include couples.) So this type of problem, while it may not be simple, is certainly straightforward, and one may well add, rather unusual in engineering practice.

2 One of the most frequent problems is that of obtaining a reasonably good balance in a machine that is not designed primarily for perfect balance but which, nevertheless, is inherently capable of being sufficiently well balanced for practical purposes by the addition of relatively simple counterweights. In such machines it is usually possible to balance out all the first harmonic forces and couples perfectly. The second and higher harmonics may be small enough in magnitude to be neglected. If they are not, it may be necessary to use a special device for balancing one or more of the higher harmonics. A machine falling into this general class, for example, is the multicylinder automobile engine. Other examples may be found in certain rotary compressors, where all parts are balanced except, perhaps, a small blade whose inertia force is negligible. The balancing problems presented by these machines are definite, and can all be solved by the fundamental method of balancing forces and couples. It may not be amiss, before leaving this general problem, to point out that occasionally a machine can be thrown off balance dynamically partially to balance torques introduced by gas forces, with a net reduction in the vibration. The author is aware of one case where this was done in a high-speed compressor in order to improve the performance at working pressures. Of course, the compressor was then not as well balanced under no-load conditions as it could have been.

3 In the third group may be placed those problems which require a distinct compromise as far as balancing is concerned. Single-cylinder reciprocating engines or compressors furnish examples of such problems, for usually cost is all important, the expense of perfect balance being prohibitive. These machines, when balanced as well as possible with a simple counterweight will still be appreciably out of balance. For this reason, and also to isolate noise, small compressors of this type are often spring mounted. The real heart of the balancing problem in this case lies in the realization of the best balance attainable. It is this problem that is to be treated in this article.

The Selection of Suitable Criteria

Perhaps it is not obvious at first sight, but it is a quite difficult task in many cases to define the "best" balance, or optimum condition. Yet this must be done before the problem of balancing can be solved. Some of the criteria that may furnish optimum conditions in various instances will, therefore, be discussed.

1 The maximum forces may be kept to a minimum. This, in effect, is what a designer does when he balances all of the rotating plus half of the reciprocating weight in a single-cylinder machine. The weakness of the method is that couples may be neglected, and the response of the system to the forces is disregarded. Fundamentally, there is no basic assurance that the minimum of human response is obtained, and, therefore, no philosophical basis can be claimed for the method. Actually, it has some practical value in cases where the machine and its mounting are symmetrical and uniformly responsive.

2 The average kinetic energy of vibration of the machine produced by the unbalanced forces may be minimized, considering the frame of the machine to be a free body in space. This method will produce an optimum condition for the machine itself provided the mounting is extremely flexible. It does not, however, assure attainment of the least possible transmission of

[1] Designing Engineer, Refrigeration Engineering Department, General Electric Company. Assoc-Mem. A.S.M.E.

Presented at the National Applied Mechanics Meeting, University of Michigan, Ann Arbor, Mich., June 18–19, 1935, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until November 11, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

vibration to the foundation. In any case, it will produce a reasonably good balance at the expense of very little effort in the computations.

3 The average kinetic energy may be minimized, taking account of the effect of flexibility in the mounting. This will result in an optimum state of vibration of the machine, whatever the mounting, but will not necessarily produce the least transmission of vibration that is possible.

4 The average potential energy in the mounting may be minimized, thus obtaining directly the least transmission of vibration to the foundation. This method is very sound from the philosophical viewpoint, and practically is very useful in the treatment of spring-mounted machines. One difficulty, though, will arise occasionally in its application, for mathematically there is no regard whatever for the balance of the machine itself, and occasionally the optimum condition is obtained in the mounting at the expense of excessive vibration of the machine itself. When this occurs, it is advisable to use the next method, mentioned under (5).

5 In order to prevent a disagreeable vibration of the machine itself, and at the same time have as little transmission as possible, it may be desirable to minimize the total average energy of vibration, which is the sum of the average kinetic and average potential energies. This will undoubtedly produce good results, although the method in (4) may surpass it in certain cases.

6 Occasionally it is desirable to apply a modification of the method discussed in (4). In a domestic refrigerator, for example, if the compressor unit is mounted on top of the cabinet, the cabinet will be much more susceptible to horizontal vibration than to vertical. In this case it is advantageous to minimize a modified potential or force function which recognizes only horizontal displacements of the mounting springs. Generally speaking, either the kinetic or potential functions or both may be minimized in modified forms to suit the needs of a particular problem, or a particular force function may be minimized.

After surveying these available criteria, it is evident that the particular one to be used in any instance will depend upon the circumstances and upon the results desired. For example, there should be a different treatment for a spring-mounted machine than for one that is rigidly mounted, and it must be decided whether the vibration of the machine is more important than the vibration transmitted by the machine. These things the designer must decide for himself, using his own judgment as a guide. The method of obtaining, mathematically, the optimum conditions sought will now be treated.

Method of Obtaining Optimum Conditions

Fundamentally, the method of obtaining these optimum conditions is simple. Suppose that the condition sought is that of minimum potential. Then it is only necessary to determine the average potential energy V_a as a function of the counterweight force ϕ. Briefly

$$V_a = V_a(\phi) \quad [1]$$

If two counterweights are to be used, then

$$V_a = V_a(\phi_1, \phi_2) \quad [2]$$

etc. The value of the counterweight force required for minimum V_a is determined from the equation

$$\frac{dV_a}{d\phi} = 0 \quad [3]$$

for one counterweight, or from the equations

$$\frac{\partial V_a}{\partial \phi_1} = 0 \text{ and } \frac{\partial V_a}{\partial \phi_2} = 0 \quad [4]$$

for two counterweights. The method may be generalized to include n variables, of course, except that the resulting simultaneous equations must be independent. The treatment of the problem will now be further elucidated.

The frame of a machine, together with all stationary parts attached to it, may be considered as a rigid body. When the machine is operating at constant speed, this body is acted upon by the unbalanced forces, and will, as a result, have a certain kinetic energy of vibration. The moving links of the machine will also vibrate with the frame except, theoretically, in the coordinate for which their motion is independent. For example, the piston of a reciprocating engine will not follow the vibration of the frame in the direction of its motion except when it is at the two dead-center positions. Actually, the friction coupling between the links and the frame may cause it to follow the vibration very closely. Again, a heavy rotating member, with a relatively small friction coupling, will not follow angular vibration of the frame about the axis of the rotating member. When determining the mass and moments of inertia to be used,

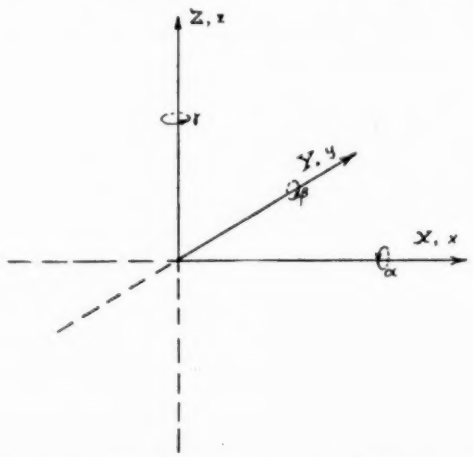


Fig. 1 Reference Axes Used in Writing an Expression for the Kinetic Energy of a Machine in Terms of Its Velocities

these facts must be considered. For instance, if the axis of a heavy rotating member is parallel to one of the coordinate axes, its effective moment of inertia is the product of its mass by the square of the distance between the two axes, the moment of inertia about its own axis being neglected.

Let it be assumed, for the sake of clarity, that the machine is spring mounted. Obviously, the vibration of the machine will produce oscillating displacements of the springs, and because of this fact there will reside in the springs a certain potential energy as a result.

Both the kinetic and potential energies will be fluctuating in time, but if the cyclic speed of the links is constant, each will have a constant average value. Moreover, the kinetic and potential functions, and also their sum, are positive definite forms, so that when the average value of one of these functions is reduced to zero, its value at any instant is also zero. The significance of this lies in the fact that when the average potential energy is reduced to zero, there can be no vibration transmitted through the mounting springs. Similarly, if the average kinetic energy is reduced to zero, there can be no vibration of the machine itself. By minimizing these functions, we, therefore, approach the ideal condition as closely as possible.

Now, the first mathematical step is to write an expression for the kinetic energy of the machine in terms of its velocities. To accomplish this, consider a set of Cartesian axes, *OXYZ*, fixed

in space, with the origin at the center of gravity of the machine when it is at rest. Any suitable orientation may be used. Consider also that the frame of the machine has fixed within it a set of axes, $oxyz$, which coincide exactly with the axes, $OXYZ$, when the machine is at rest in equilibrium. The relation of the axes is shown in Fig. 1.

Let the displacement of o with respect to O be denoted by x, y, z. Also, let α = the rotation of ox with respect to OX, β = the rotation of oy with respect to OY, and γ = the rotation of oz with respect to OZ, all rotations being positive when counterclockwise, looking toward the origin.

Obviously, simultaneous rotations will produce inaccuracies in these definitions, but for small vibration, the errors introduced are negligible.

Regarding the dynamic specifications of the machine, let M = the total mass, and A, B, and C = the moments of inertia about ox, oy, and oz, respectively. Also, let F = the product of inertia about oy and oz; let G = the product of inertia about ox and oz; and let H = the product of inertia about ox and oy.

The definitions of the products of inertia, incidentally, are

$$F = \Sigma myz; \quad G = \Sigma mxz; \quad \text{and } H = \Sigma mxy$$

the summations extending over all particles of mass m in the body.

With these definitions, it may be shown that the kinetic energy T of the machine frame is

$$T = \frac{1}{2} M (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2} (A\dot{\alpha}^2 + B\dot{\beta}^2 + C\dot{\gamma}^2 - 2F\dot{\beta}\dot{\gamma} - 2G\dot{\alpha}\dot{\gamma} - 2H\dot{\alpha}\dot{\beta}) \quad [5]$$

where the dots denote differentiation with respect to time.

Very often, a coordinate plane will be a plane of symmetry, in which case two of the products of inertia are zero. If two planes of symmetry coinciding with coordinate planes exist, all the products of inertia are zero. In cases where a machine is very nearly symmetrical about two of the chosen coordinate planes, it may be permissible to neglect the products of inertia.

The potential function V will in general involve the squares of all coordinates and all possible products. In any particular case it may be difficult to derive, but the form of the function for a helical-spring mounting, where the springs are "built in" at each end, has been obtained and will be given here. It is first necessary to define the spring constants and the coordinates, which are as follows:[2]

K_1 = the spring constant (lb per in., or similar units) for a displacement of one end of the spring along the axis of the coil

K_2 = the spring constant for a displacement of one end at a right angle to the axis of the coil, both ends remaining parallel with the axis of the unstrained coil

K_3 = the spring constant (lb-in. per radian) for an angular displacement of one end about an axis at a right angle to the axis of the coil, with no translation permitted

K_4 = the spring constant for an angular displacement of one end about the axis of the coil

h = the active length of the coil.

The spring is assumed to be placed with its axis vertical, the lower end being fixed to the foundation, and the upper end fixed to the machine at a point whose x, y, and z coordinates are, respectively, a, b, and c. Using these definitions, it can be shown that the potential energy stored in a single spring because of a general displacement of the machine is

$$V = \frac{1}{2} [K_2x^2 + K_2y^2 + K_1z^2 + (K_2c^2 + K_1b^2 + K_3 - hcK_2)\alpha^2 + (K_2c^2 + K_1a^2 + K_3 - hcK_2)\beta^2 + (K_2b^2 + K_2a^2 + K_4)\gamma^2 + K_2(2c - h)\beta x - K_2b(2c - h)\beta\gamma - K_2(2c - h)\alpha y - K_2a(2c - h)\alpha\gamma - 2bK_2\gamma x + 2aK_2\gamma y + 2K_1ba z - 2K_1a\beta z - 2K_1ab\alpha\beta] \quad [6]$$

The total potential, of course, is the sum of the potentials of all the springs. If the mounting is symmetrical about the oz axis, a large number of the terms in Equation [6] will vanish.

The next important step is to evaluate x, y, z, α, β, and γ in terms of the forces, and to substitute these values in either T or V, depending on which is to be minimized. To accomplish this, the equations of motion must be derived and solved. This is most conveniently done by using Lagrange's equations, which are in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_i}\right) + \frac{\partial V}{\partial q_i} = Q_i \quad [7]$$

where q_i = one of the coordinates, x, y, etc.

Q_i = the force acting in this coordinate.

There will, in general, be six equations which must be solved simultaneously. The number may, however, be reduced to two identical sets of two simultaneous equations in special cases of symmetry, and where certain of the forces are zero. When the solutions of Equation [7] are obtained, they will be in the form

$$x = f(Q_x, Q_y, Q_z, Q_\alpha, Q_\beta, Q_\gamma) \quad [8]$$

with similar solutions for y, z, etc. Now, the Q_i's will contain the counterweight forces ϕ_1, ϕ_2, etc. Hence, when Equation [8] is substituted in Equation [6], there is obtained the explicit function

$$V = V(\phi_1, \phi_2, \ldots) \quad [9]$$

From this, the time is eliminated by taking the average value V_a, which is minimized by equating its differential to zero. A similar method is followed if it is desired to minimize T_a, or any modifications of T_a and V_a. It will be noted that the solution is much simpler if $V = 0$ and T_a only is minimized. This corresponds to a body floating in space, and is only approached practically with extremely flexible mountings.

Before leaving this technique, it may be well to touch upon the matter of modified, or restricted, potential functions. As explained in the discussion of criteria, it may be desired to obtain the least possible disturbance in, say, the horizontal plane only. One may, in such a case, without a serious violation of logic, select from the potential function only those terms from which horizontal forces may be obtained, calling this collection of terms a "modified" potential function.

One disconcerting result of this procedure, however, is that the new function, being only a part of some true potential function, may sometimes be negative in value, and a negative potential function in this case *is* disconcerting because one loses his concept of what is being accomplished. This difficulty can be obviated in a straightforward manner by minimizing the average of the square of the force acting in the horizontal (or other) plane.

In order to illustrate this method it will be applied to the case in which the mounting consists of a plurality of vertical springs, and translatory disturbances in the horizontal plane are to be minimized. The notation used previously is continued, and in addition the displacements of the upper end of a single spring are denoted as follows:

δ_1, δ_2, and δ_3 = displacements of translation in the x, y, and z directions, respectively

[2] Expressions for these spring constants will be found in the appendix.

ζ_1, ζ_2, and ζ_3 = respective angular displacements about axes ox', oy', and oz', parallel to ox, oy, and oz, but with the origin o at the top, and on the center line of the spring.

The relations between these coordinates and those of the machine are

$$\left.\begin{aligned} \delta_1 &= x + c\beta - b\gamma, \\ \delta_2 &= y - c\alpha + a\gamma, \\ \delta_3 &= z + b\alpha - a\beta, \\ \zeta_1 &= \alpha, \quad \zeta_2 = \beta \quad \text{and} \quad \zeta_3 = \gamma \end{aligned}\right\} \ldots\ldots\ldots [10]$$

Using this notation, it may be shown that the expression for the potential energy stored in a single spring is

$$V_n = \frac{1}{2} K_2\delta_1^2 + \frac{1}{2} K_2\delta_2^2 + \frac{1}{2} K_1\delta_3^2 + \frac{1}{2} K_3\zeta_1^2 + \frac{1}{2} K_3\zeta_2^2 + \frac{1}{2} K_4\zeta_3^2 + \frac{h}{2} K_2\delta_2\zeta_1 - \frac{h}{2} K_2\delta_1\zeta_2 \ldots\ldots [11]$$

The subscript n is used to relate the function V to a particular spring, "n."

Denoting by i and j the unit vectors in the ox and oy directions, respectively, the horizontal force acting is

$$i\frac{\partial V_n}{\partial \delta_1} + j\frac{\partial V_n}{\partial \delta_2} \ldots\ldots\ldots\ldots\ldots [12]$$

The net force acting in the horizontal plane because of simultaneous displacements of "m" springs is

$$\psi = i\sum_1^m \frac{\partial V_n}{\partial \delta_1} + j\sum_1^m \frac{\partial V_n}{\partial \delta_2} \ldots\ldots\ldots [13]$$

The absolute value is

$$|\psi| = \sqrt{\left[\left(\sum_1^m \frac{\partial V_n}{\partial \delta_1}\right)^2 + \left(\sum_1^m \frac{\partial V_n}{\partial \delta_2}\right)^2\right]} \ldots\ldots [14]$$

and the function to be minimized is the average value of

$$\psi^2 = \left(\sum_1^m \frac{\partial V_n}{\partial \delta_1}\right)^2 + \left(\sum_1^m \frac{\partial V_n}{\partial \delta_2}\right)^2 \ldots\ldots\ldots [15]$$

By using Equations [10], Equation [15] may be written in terms of the coordinates of the frame thus

$$\psi^2 = K_2^2\left[mx + \left(\sum_1^m c_n - \frac{1}{2}mh\right)\beta - \sum_1^m b_n\gamma\right]^2 + K_2^2\left[my - \left(\sum_1^m c_n - \frac{1}{2}mh\right)\alpha + \sum_1^m a_n\gamma\right]^2 \ldots\ldots [16]$$

It will be found that this expression is very similar to a potential function. This force function, as it may be called, is essentially positive, and in using it (or functions of its type) for such problems as that cited here, one has the advantage of clearly visualizing what is being accomplished. Such functions cannot, of course, be derived in this manner when the general disturbance is to be minimized. The energy functions, as pointed out previously, should be used in the general case. Incidentally, the greatest weakness of the "force function" lies in the fact that couples acting in the plane must be neglected if the forces are minimized and vice versa. In the elimination of this weak point lies the power of the energy method.

An Illustrative Problem

The problem to be solved here is that of obtaining an optimum balance for the machine that is shown in Fig. 2. For purposes of illustration, the optimum will be obtained for each of several criteria.

As shown in Fig. 2, the machine, a simple compressor, has a vertical, overhung crankshaft. The machine itself is symmetrical about two vertical planes through the center line of the crankshaft, on which the center of gravity lies, and the mounting springs are symmetrically placed about the vertical axis.

The first necessary step is that of writing the potential-energy and kinetic-energy functions, V and T. Equation [6] is used for obtaining V, the evaluation requiring substitution of the values of the spring coordinates a, b, and c for each spring. For the present it will be convenient to leave these values in algebraic form, the substitutions being made according to the following table:

VALUES OF a, b, AND c

Spring number	a	b	c
1	0	R	$-d$
2	$0.866R$	$-0.5R$	$-d$
3	$-0.866R$	$-0.5R$	$-d$

Making these substitutions, the function V becomes, for this case

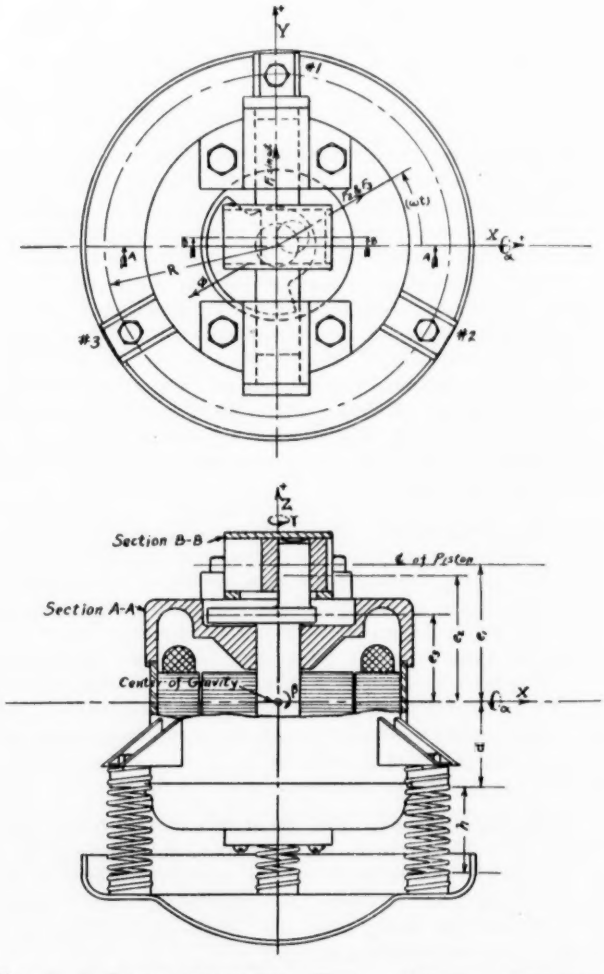


FIG. 2 A SIMPLE COMPRESSOR WITH A VERTICAL, OVERHUNG CRANKSHAFT USED IN THE PROBLEM FOR OBTAINING OPTIMUM BALANCE

$$V = \frac{1}{2}[3K_2x^2 + 3K_2y^2 + 3K_1z^2 + 3(K_2d^2 + \frac{1}{2}K_1R^2 + K_3 + dhK_2)\alpha^2 + 3(K_2d^2 + \frac{1}{2}K_1R^2 + K_3 + dhK_2)\beta^2 + 3(K_2R^2 + K_4)\gamma^2 - 3K_2(2d + h)\beta x + 3K_2(2d + h)\alpha y] \quad [17]$$

Similarly, from Equation [5], noting that F, G, and H are zero because of the symmetry

$$T = \frac{1}{2}M(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}(A\dot{\alpha}^2 + B\dot{\beta}^2 + C\dot{\gamma}^2) \quad [18]$$

In this problem, the numerical data are: ω = the shaft speed = 182 radians per sec, K_1 = 75 lb per in., K_2 = 42 lb per in., K_3 = 58 lb-in. per radian, K_4 = 17 lb-in. per radian, M = 0.086 lb sec^2 per in., $A = B$ = 0.60 lb-in.-sec^2, d = 1.92 in., h = 2 in., R = 4 in., e_1 = 3.2 in., e_2 = 2.98 in., and e_3 = 2.2 in.

The dimensions e_1, e_2, and e_3, locate, respectively, the piston, crank pin and counterweight from the center of gravity, as shown in Fig. 2. It is usually desirable, when a numerical solution is sought, to start numerical work immediately by substitution of numerical coefficients in V and T. In the present instance, this is delayed for other reasons, and V written with abridged coefficients is

$$V = \frac{1}{2}(a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + a_{44}\alpha^2 + a_{55}\beta^2 + a_{66}\gamma^2 + 2a_{51}\beta x + 2a_{42}\alpha y) \quad [19]$$

Substituting Equations [18] and [19] in Equation [7], the equations of motion are obtained. Thus

$$\left.\begin{aligned} M\ddot{x} + a_{11}x + a_{51}\beta &= Q_x \\ M\ddot{y} + a_{22}y + a_{42}\alpha &= Q_y \\ M\ddot{z} + a_{33}z &= Q_z \\ A\ddot{\alpha} + a_{44}\alpha + a_{42}y &= Q_\alpha \\ B\ddot{\beta} + a_{55}\beta + a_{51}x &= Q_\beta \\ C\ddot{\gamma} + a_{66}\gamma &= Q_\gamma \end{aligned}\right\} \quad [20]$$

Of these six equations, the two in z and γ are independent. Moreover, it will be seen that the forces Q_z and Q_γ, acting in these coordinates, are zero, so that z and γ are both zero and may be neglected in the rest of the problem. Of the remaining four equations, the two in x and β are independent of the two in y and α. So the system reduces to two pairs of simultaneous equations, a very simple system to solve. The simplicity, of course, is a result of the symmetry.

Before proceeding further it may be well to consider the nature of the forces Q_x, etc. The mechanism illustrated is the simple Scotch yoke (see Fig. 2), so that all inertia forces will be simple-harmonic in form. Denoting component forces by q_y, q_α, etc., the forces due to the piston are

$$q_y = F_1 \sin \omega t; \quad \text{and} \quad q_\alpha = -e_1F_1 \sin \omega t$$

The slide block contributes the components

$$q_y = F_2 \sin \omega t; \quad \text{and} \quad q_x = F_2 \cos \omega t$$
$$q_\alpha = -e_1F_2 \sin \omega t; \quad \text{and} \quad q_\beta = e_1F_2 \cos \omega t$$

The crankpin gives

$$q_y = F_3 \sin \omega t; \quad \text{and} \quad q_x = F_3 \cos \omega t$$
$$q_\alpha = -e_2F_3 \sin \omega t; \quad \text{and} \quad q_\beta = e_2F_3 \cos \omega t$$

Finally, the counterweight force, considered variable, gives

$$q_y = -\phi \sin \omega t; \quad \text{and} \quad q_x = -\phi \cos \omega t$$
$$q_\alpha = e_3\phi \sin \omega t; \quad \text{and} \quad q_\beta = -e_3\phi \cos \omega t$$

Adding the components in each coordinate, the results are

$$\left.\begin{aligned} Q_x &= (F_2 + F_3 - \phi)\cos \omega t \\ Q_y &= (F_1 + F_2 + F_3 - \phi)\sin \omega t \\ Q_\alpha &= (-e_1F_1 - e_1F_2 - e_2F_3 + e_3\phi)\sin \omega t \\ Q_\beta &= (e_1F_2 + e_2F_3 - e_3\phi)\cos \omega t \end{aligned}\right\} \quad [21]$$

Considering the sinusoidal nature of these forces, and making the numerical substitutions, the steady-state solutions of Equations [20] are

$$\left.\begin{aligned} x &= (0.0805Q_\beta - 3.7Q_x) \times 10^{-4} \\ \beta &= (0.0805Q_x - 0.595Q_\beta) \times 10^{-4} \\ y &= -(0.0805Q_\alpha + 3.7Q_y) \times 10^{-4} \\ \alpha &= -(0.0805Q_y + 0.595Q_\alpha) \times 10^{-4} \end{aligned}\right\} \quad [22]$$

Now, in Equations [21], let

$$\left.\begin{aligned} \theta_1 &= F_2 + F_3 \\ \theta_2 &= F_1 + F_2 + F_3 \\ \theta_3 &= e_1F_1 + e_1F_2 + e_2F_3 \\ \theta_4 &= e_1F_2 + e_2F_3 \end{aligned}\right\} \quad [23]$$

Then substituting Equations [21] in [22]

$$\left.\begin{aligned} x &= (0.00805\theta_4 - 0.37\theta_1 + 0.352\phi)\cos \omega t \times 10^{-3} \\ y &= (0.00805\theta_3 - 0.37\theta_2 + 0.352\phi)\sin \omega t \times 10^{-3} \\ \alpha &= (0.0595\theta_3 - 0.00805\theta_2 - 0.123\phi)\sin \omega t \times 10^{-3} \\ \beta &= (-0.0595\theta_4 + 0.00805\theta_1 + 0.123\phi)\cos \omega t \times 10^{-3} \end{aligned}\right\} \quad [24]$$

Also, from Equations [17] and [18]

$$V = 63x^2 + 63y^2 + 1462\alpha^2 + 1462\beta^2 - 367.53x + 367.5\alpha y \quad [25]$$

$$T = 0.043(\dot{x}^2 + \dot{y}^2) + 0.3(\dot{\alpha}^2 + \dot{\beta}^2) \quad [26]$$

Substituting the Equations [24] in [25], it will be observed that the factors involving the time are all of the form $\sin^2 \omega t$ or $\cos^2 \omega t$, the average values of which are 0.5. Denoting differentiation with respect to ϕ by primes

$$\frac{dV}{d\phi} = (126x' - 367.5\beta')x + (126y' + 367.5\alpha')y + (2924\alpha' + 367.5y')\alpha + (2924\beta' - 367.5x')\beta \quad [27]$$

Making numerical substitutions in this expression, taking its average, and setting the result equal to zero, there is obtained the value of the counterweight force for minimum average potential energy. That is

$$\phi = 0.72F_1 + 1.48F_2 + 1.378F_3 \quad [28]$$

It is interesting to note that in this case, which results in a minimum of transmission, the counterweight is called upon to balance considerably more than "all of the rotating plus half of the reciprocating mass."

The method of computation has been indicated in sufficient detail to make further work unwarranted. However, it will be of interest to tabulate the results of working to other criteria in this same machine. For instance, if the average kinetic energy is minimized, considering the effect of the mounting, the result is

$$\phi = 0.605F_1 + 1.210F_2 + 1.164F_3 \quad [29]$$

which is considerably less than in Equation [28]. On the other hand, if the total energy $T_a + V_a$ is minimized

$$\phi = 0.611F_1 + 1.222F_2 + 1.173F_3 \quad [30]$$

which value is found to fall between the two previous results.

If the machine is considered to be floating in space, and T_a is minimized, then

$$\phi = 0.560F_1 + 1.119F_2 + 1.049F_3 \quad [31]$$

requiring the smallest counterweight of all.

One other particular case is important. Suppose that it is possible to use two counterweights rather than one. Such a system can be represented by a rotating force at the center of gravity and a rotating couple acting in the plane of the force and the shaft center-line. Let the force be ϕ_1, and the couple ϕ_2. Then it may be shown by a general proof that for a symmetrical machine with a symmetrical mounting, and with $dV_a = 0$

$$\phi_1 = \frac{1}{2} F_1 \qquad \text{and} \qquad \phi_2 = \frac{1}{2} e_1 F_1 \quad [32]$$

where F_1 is the maximum piston force, and e_1 the distance of the center of gravity of the piston from the center of gravity of the machine, as before. This solution is obtained by minimizing V_a, and is independent of either spring characteristics or location, as long as symmetry is observed. This does not mean, of course, that V_a cannot be reduced by shifting the plane of attachment of the springs. It does mean that for any given location the minimum V_a is obtained by the relations expressed in Equations [32].

Conclusion

The method of balancing explained here has been used by the author for two or three years with a marked degree of success. The results obtained have been checked experimentally, and have shown a definite improvement over those obtained by less scientific methods.

That there are distinct advantages in using such a method can scarcely be questioned, for there is sound philosophy behind reasoning that goes back as nearly as possible to human response for its measuring stick. Aside from this, there is definite power in the method of attack. Even the so-called "rigid mounted" machines may be treated by simply realizing that in this case the kinetic energy of the machine tends to vanish and the energy of the mounting is all that remains to be minimized. There is also the possibility of studying other variables than counterweights, such as spring positions, and adjusting these to obtain optimum conditions by the same general procedure.

Designers, by acquainting themselves with this method, will be able to approach their vibration problems with a new viewpoint and a powerful technique. It is reasonable to expect that this will, in time, bear its fruit—less vibration in machinery.

Appendix

The Constants for Helical-Spring Deflections

The expressions which follow are approximate, and apply only to helical springs of round wire. The approximations involved require that:

(*a*) If ϕ is the angle of lead (radians), then ϕ^2 is negligible compared to unity.

(*b*) There is either an integral number of turns or else a sufficient number of turns so that a fractional number may be used in the formulas without appreciable error. Ordinarily, the formulas will be sufficiently accurate if the spring has eight or more active turns.

(*c*) In the case of transverse constants, the displacement must be small.

Notation:

- G = modulus of rigidity
- E = Young's modulus
- J = polar moment of inertia of the wire cross-section
- I = plane moment of inertia of the wire cross-section about a diameter
- l = total length of wire in the active part of the spring
- R = radius of cylinder in which lies the helix formed by the center line of the wire
- h = active height of the spring
- n = number of active turns

Formulas:

$$K_1 = \frac{GJ}{lR^2} \quad [33]$$

$$K_2 = \frac{2EI}{l\left[\left(\frac{3}{16} + \frac{41}{48\pi^2 n^2}\right) h^2 + R^2\right]} \quad [34]$$

$$K_3 = \left[\frac{1}{3} h^2 + \frac{GR^2}{2G + E}\right] K_2 \quad [35]$$

$$K_4 = \frac{E}{2G} R^2 K_1 \quad [36]$$

where definitions of these constants are the same as given previously.

Investigations of the Turbulent Mixing Regions Formed by Jets

By ARNOLD M. KUETHE,[1] AKRON, OHIO

The turbulent mixing of two jets of fluid of the same density is investigated theoretically by means of Prandtl's theory of turbulence and the results compared with experiment.

A solution obtained by Tollmien for the mixing region of a plane jet, in which he considers the fluid bounding the jet at rest, is extended to the general case of the mixing of two parallel streams of different velocities. Distributions of the tangential and normal velocities in the mixing region are given. The solution is worked out for two different conditions imposed on the normal velocity at the boundary of the mixing region.

Next, the turbulent mixing region surrounding an axially symmetrical jet issuing into fluid at rest is considered. The problem is solved, as far as the core of potential flow extends, by the application of a new method of approximation. The velocity field, as determined by a second approximation to the velocity profiles, is given. An extension of the method of solution, applicable to problems in which the velocity profiles differ widely, is mentioned.

In the theory applied in this paper, the mixing length is assumed proportional to the breadth of the mixing region. The proportionality factor which is the only empirical quantity in the theory was experimentally evaluated. Velocity profiles in the mixing region of an axially symmetrical jet were measured by means of a pitot tube and good agreement is found between theory and experiment.

Measurements of velocity fluctuations in the core of potential flow near the jet mouth and in the mixing region at a distance of 9.3 diameters were made by means of the hot-wire anemometer.

THIS investigation deals with the turbulent mixing of two parallel streams of fluid of the same density moving at different velocities. Such cases occur often in practical applications of such streams, and it is of some importance to know at which rate the discontinuity between the two streams is "washed out," and what is the magnitude of the thickness of the layer which is affected by the mixing. The boundaries of the open-jet wind tunnel and of the slipstream of a propeller are two examples to which the results of the present investigation can be applied.

Considering first a perfect fluid, it is well known that a surface of discontinuity in the velocity, though not in the pressure, may exist at the boundary separating the two streams. Helmholtz[2] first pointed out the possibility of the existence of this surface, but proved it to be unstable,[3] i.e., a slight disturbance will build up into a regular succession of vortices traveling along the surface. The vena contracta and the boundaries of the dead-water region behind a flat plate are examples which have been treated by Kirchhoff, Rayleigh, and others.

The regular succession of vortices which are given off at the rim can be observed only below a certain velocity limit. As the velocity is increased the periodic vortex system is replaced by a turbulent mixing region.

Turbulence, as distinct from those cases in which a regular succession of vortices is formed, is characterized by an irregular fluctuating motion superposed on a mean flow. These fluctuations give rise to a momentum transport between neighboring fluid regions and, as a result, "apparent stresses" will act between these regions. If we consider a flow, the main component of which is in the x-direction, its magnitude being a function of y, the momentum transport will depend upon the velocity gradient and the most important apparent stress will be τ_{xy}. Prandtl[4] gives the expression

$$\tau_{xy} = \rho l^2 \left(\frac{\partial u}{\partial y}\right) \left|\frac{\partial u}{\partial y}\right| \quad \ldots\ldots\ldots\ldots\ldots [1]$$

where ρ is the density of the fluid, u is the mean velocity, and l the "mixing length." The length l is analogous to the mean free path in the kinetic theory of gases and is proportional to the mean distance which a fluid mass will travel in the y-direction while retaining its initial momentum. Briefly, the mechanism is assumed to be the following: Fluid masses entering a constant velocity layer from opposite sides have x velocities $u + l\frac{\partial u}{\partial y}$ and $u - l\frac{\partial u}{\partial y}$, respectively. Then, at this place, fluctuations in the x-direction about the mean velocity will be $\pm l\frac{\partial u}{\partial y}$. The resulting y-component is assumed to be proportional[5] to $l\left|\frac{\partial u}{\partial y}\right|$; the proportionality constant may be included in l since this quantity is not defined exactly. The mean value over the time of the product of these two velocity components and the density of the fluid gives the magnitude of the turbulent shearing stress. The sign of the product is chosen as in Equation [1] since the shearing stress must have the same sign as the velocity gradient. In cases of free turbulence, i.e., cases in which there are no confining walls, the very simple assumption that l is constant over any

---

[1] Research Fellow in Meteorology, Daniel Guggenheim Airship Institute. This work was carried out at the Guggenheim Aeronautical Laboratories, California Institute of Technology, Pasadena.

[2] "Hydrodynamics," by H. Lamb, Macmillan Co., New York. Cambridge University Press, London, Article 232, fifth edition, 1930.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until November 11, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] On the other hand, Ackeret (Helvetica Physica Acta I:V, p. 301, 1928, published by E. Birkhauser & Co., Basilae, Switzerland) has shown that the surface is stable for hypersonic velocities.

[4] "Bericht über Untersuchungen zur ausgebildeten Turbulenz," by L. Prandtl, *Zeit. für angewandte Mathematik und Mechanik*, vol. 5, 1925, p. 136.

[5] Th. von Kármán (Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, 1930) pointed out that this assumption implies a constant correlation between the fluctuation of the two considered velocity components over the whole turbulent region. He showed that, assuming constant correlation, Equation [1] can be deduced without the assumption of a special mechanism for the turbulent interchange.

cross-section and proportional to the breadth of the mixing region gives results in good agreement with practice. The proportionality constant must be evaluated empirically for each case.

Tollmien[6] with the aid of Equation [1] solved the problem of the plane jet issuing into a medium at rest and also obtained a solution valid at great distances from an axially symmetrical jet. In these cases the mean motion is not exactly parallel; however, the shearing-stress component τ_{xy} is large in comparison with the other stress components and Equation [1] represents a good approximation for τ_{xy}. These solutions hold for a wide range of velocities and this is not surprising when we consider the strong discontinuity in the velocity at the rim of the jet. Very large velocity disturbances result and the completely turbulent state is realized at a comparatively low velocity.

The object of the present work was to extend Tollmien's results to other cases of practical importance. We treat, first, the two-dimensional problem and, second, that of the axially symmetric jet.

2 The Plane Jet

The arrangement, as shown in Fig. 1, in which η_1 and η_2 are the boundaries of the turbulent mixing region, finds practical

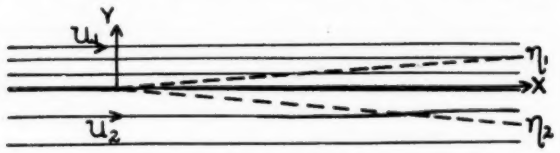


Fig. 1 Schematic Diagram of the Plane Jet

application both in the case of the open-jet wind tunnel and the propeller slipstream. The limit of its applicability, because of the finite diameter of the wind tunnel or the propeller, will be discussed in part 3.

Tollmien's solution for $u_2 = 0$ is here extended to the case in which u_2 has any value from 0 to u_1. In the boundary-layer theory we assume that no appreciable pressure difference exists normal to the main-flow direction, and, since the pressure in the jets outside the mixing region is constant, the pressure gradients can be completely neglected. Hence, we need consider only the first equation of motion, and this reduces to

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\tau_{xy} \quad [2]$$

where v is the y component of the mean velocity and τ_{xy} is given by Equation [1]. This equation, combined with the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad [3]$$

and suitable boundary conditions then describes the field of flow in the mixing region.

We assume

$$u = f(y/x)$$

i.e., that the breadth of the mixing region is proportional to x. Hence, the assumption that the mixing length is proportional to the breadth of the mixing region can be expressed as

$$l = cx \quad [4]$$

where c is an empirical constant.

We obtain, from Equation [3], the stream function

$$\psi = xF(y/x)$$

and, consequently

$$\left.\begin{aligned} u &= F' \\ v &= -F + \frac{y}{x}F' \end{aligned}\right\} \quad [5]$$

Substituting Equations [1], [4], and [5] in Equation [2] leads to the ordinary differential equation

$$F(\eta) + F'''(\eta) = 0 \quad [6]$$

where

$$\eta = \frac{y}{\sqrt[3]{(2c^2)x}}$$

The solution to Equation [6] can be expressed in the following form:

$$F(\eta') = d_1e^{-\eta'} + d_2e^{\eta'/2}\cos\frac{\sqrt{3}}{2}\eta' + d_3e^{\eta'/2}\sin\frac{\sqrt{3}}{2}\eta' \quad [7]$$

where $\eta' = \eta - \eta_1$, and d_1, d_2, and d_3 are constants.

Four of the five boundary conditions necessary to determine the constants d_1, d_2, d_3, η_1, and η_2 as functions of the ratio of the two jet velocities are

$$\text{at } \eta = \eta_1(\eta' = 0)\begin{cases} u = u_1 \\ \dfrac{\partial u}{\partial y} = \dfrac{\partial u}{\partial \eta'} = 0 \end{cases}$$

$$\text{at } \eta = \eta_2(\eta' = \eta_2')\begin{cases} u = u_2 \\ \dfrac{\partial u}{\partial y} = \dfrac{\partial u}{\partial \eta'} = 0 \end{cases} \quad [8]$$

For the fifth boundary condition, Tollmien (for the case $u_2 = 0$) chose $v_1 = 0$ at $\eta' = 0$ and the present calculation was at first carried out retaining this condition. $v_1 = 0$ was chosen with a view to its applicability to the mixing region near the mouth of a closed, say a circular, jet. It was later pointed out by von Kármán that for the plane boundary between two jets the condition

$$u_1v_1 = -u_2v_2 \quad [9]$$

is preferable. This corresponds to the assumption that no external forces are acting on the total fluid system perpendicular to the main flow. For $x < 0$, $v_1 = 0$, $v_2 = 0$, i.e., the two jets are exactly parallel. In the region $x > 0$, both are deflected. Equation [9] expresses the condition that the reactions of the two jets, corresponding to their deflection, perpendicular to x, are equal and opposite.

Since all of the quantities in Equation [8] are functions of η_2', d_1, d_2, d_3, i.e., neither η_1 nor η_2 enters explicitly, the four resulting equations determine these four quantities independent of the remaining boundary condition. Hence, neither the width of the wedge-shaped mixing region nor the u-velocity profiles are affected by the use of the condition $u_1v_1 = -u_2v_2$ instead of $v_1 = 0$. The v-velocity profiles and η_1 and η_2 are affected an appreciable amount.

In terms of the function $F(\eta')$, Equations [8] and [9] reduce to

$$F'(0) = u_1 \qquad F'(\eta_2') = u_2$$

$$F''(0) = 0 \qquad F''(\eta_2') = 0$$

$$m[-F(\eta_2') + \eta_2F'(\eta_2')] = F(0) - \eta_1F'(0)$$

where $m = \dfrac{u_2}{u_1}$. For the boundary condition $v_1 = 0$, it was possible to solve for d_1, d_2, d_3, η_1, and η_2. Then, since only η_1

[6] "Berechnung Turbulenter Ausbreitungsvorgänge," by W. Tollmien, *Zeit. für angewandte Mathematik und Mechanik*, vol. 6, 1926, pp. 468–478.

TABLE 1

					$v_1 = 0$		$u_1v_1 = -u_2v_2$	
m	$\frac{d_1}{u_1}$	$\frac{d_2}{u_1}$	$\frac{d_3}{u_1}$	η'_2	η_1	η_2	η_1	η_2
0	−0.0062	0.987	0.577	−3.02	0.981	−2.040	0.981	−2.040
0.1	−0.012	0.976	0.577	−2.773	0.962	−1.811	0.934	−1.839
0.2	−0.020	0.960	0.577	−2.565	0.940	−1.625	0.905	−1.660
0.4	−0.038	0.924	0.577	−2.190	0.885	−1.305	0.850	−1.340
0.6	−0.067	0.866	0.577	−1.821	0.797	−1.024	0.783	−1.038
0.8	−0.117	0.766	0.577	−1.380	0.647	−0.733	0.645	−0.735
0.9	−0.159	0.682	0.577	−1.080	0.522	−0.558	0.523	−0.557
1.0	−0.333	0.333	0.577	0	0	0	0	0

and η_2 change when $u_1v_1 = -u_2v_2$ is used instead of $v_1 = 0$, we can substitute in the last equation given, using the previously determined values for d_1, d_2, d_3, and η_2'. This equation and the relation $\eta_2' = \eta_2 - \eta_1$ allow the determination of η_1 and η_2 as functions of m.

Table 1 gives the quantities d_1, d_2, d_3, η_1, η_2, and η_2' for both choices of the v-boundary condition as functions of m. In Fig. 2 η_1 and η_2 are plotted as functions of m.

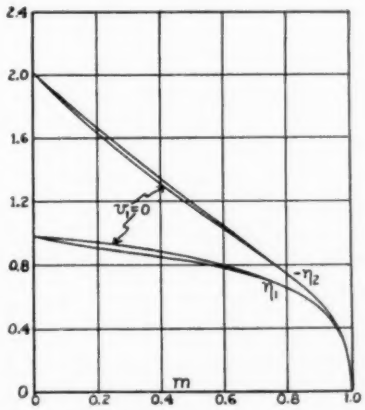


FIG. 2 BOUNDARIES OF THE TURBULENT MIXING REGION AS FUNCTIONS OF $m = u_2/u_1$

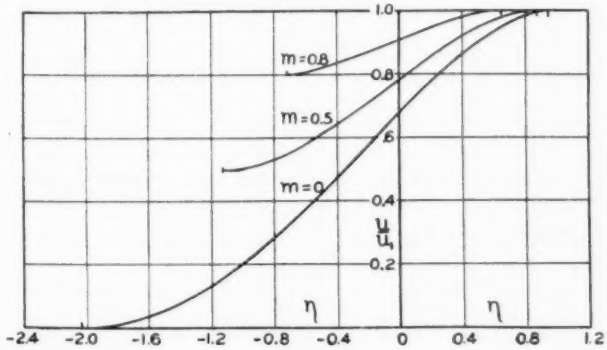


FIG. 3 DISTRIBUTION OF u/u_1 FOR THREE VALUES OF $m = u_2/u_1$

It is seen that for $m = 0$ the mixing region is not symmetrical about the extension of the boundary initially separating the two jets, but, as m increases, the boundaries gradually approach a position symmetrical about this line. Fig. 3, in which u-velocity profiles for three values of m are plotted, shows clearly this transition.

Velocity profiles can be plotted for any given m by the use of Table 1 and Equations [5] and [7]. It must be remembered, however, that the scale of y/x has been reduced by the factor $\sqrt[3]{(2c^2)}$.

To show the change in shape resulting from a change in m, two of the profiles of Fig. 3 are replotted, along with the limiting expression, in Fig. 4.

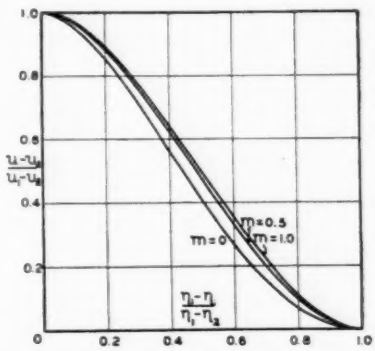


FIG. 4 DISTRIBUTION OF $u - u_2/u_1 - u_2$ FOR THREE VALUES OF $m = u_2/u_1$

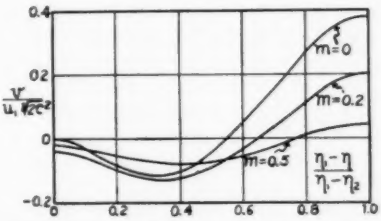


FIG. 5 DISTRIBUTION OF THE NORMAL VELOCITY FOR THREE VALUES OF $m = u_2/u_1$

The limiting expression is found by substituting the series expansions for the transcendental terms in the equation $\frac{u}{u_1} = F'(\eta')$, suppressing all terms of order higher than η_1^3. As is to be expected from the fact that the boundaries of the mixing region become symmetrical near $m = 1$, the limiting curve for $m = 1$ is anti-symmetrical about the center line. This calculation was carried out for the condition $v_1 = 0$, since in this case d_1, d_2, d_3 were found as analytical functions of η_1. However, as m approaches unity, η_1 and η_2 for the two choices of the v-condition are nearly identical. Hence, we would expect the limiting profile to be independent of this choice.

Measurements of the mean velocities in the mixing region at the boundary of the open-jet wind tunnel at Göttingen show good agreement with theory if the value $c = 0.0174$ or $\sqrt[3]{(2c^2)} = 0.0845$ is used.

Tollmien carried out an investigation of the pressure difference existing between the center of the mixing region and the undisturbed flow by substituting his results for u, v, $\tau_{xy}(u_2 = 0)$ in the second equation of motion

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial x}$$

and determining p by integrating with respect to y. He found this difference in pressure to be $0.00482\,\frac{\rho}{2}\,u_1^2$, which, if taken into account in determining a second approximation, would not appreciably alter the stream lines.

3 The Axially Symmetrical Jet

The case of the discharge of an axially symmetrical jet into fluid at rest occurs very frequently, especially in experimental arrangements. Tollmien's solution for this case in which he considers the jet as issuing from a point holds very well for values of x greater than about eight diameters. However, if we wish to obtain a solution valid near the mouth of the jet, it appears to be impossible to transform the equation for the mean motion into a total differential equation. Accordingly, a method for obtaining an approximate solution was applied directly to the partial-differential equation.

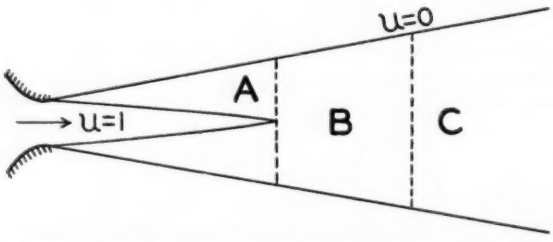


Fig. 6 Schematic Section of the Flow From a Circular Nozzle

We can form a qualitative picture of the flow from a consideration of the building up of the turbulent mixing region. This region, originating at the rim of the jet mouth, will form an annular ring enclosing a core of potential flow in which the velocity is constant and equal to the outflow velocity. At a point $x = x_1$ this core will vanish and the entire jet then becomes a turbulent mixing region. A schematic longitudinal section of the flow is shown in Fig. 6. Region A consists of the annular mixing region surrounding the core of potential flow. In region B the entire jet is a mixing region and the central velocity u_0 decreases as x increases. The velocity profiles and the x gradient of u_0 finally reach an asymptotic state as the fluid enters region C. In C, the central velocity is inversely proportional to the distance from some point near the jet mouth and all profiles are similar, i.e., plotting u/u_0 against y/b where y is the radial distance of a point from the axis and b is the breadth of the jet, all points fall on a single curve. It is in this last region that Tollmien's solution is valid.

At the point $x = x_1$, i.e., where the potential core vanishes, we can, for physical and mathematical reasons, expect considerable difficulty in finding a solution reasonably close to practice. Consequently, we will consider at first only region A.

For the same reasons as in part 2, the pressure gradients are neglected and the equation for the mean motion is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho y}\frac{\partial}{\partial y}(\tau y) \quad \ldots\ldots [10]$$

The continuity equation is

$$\frac{\partial}{\partial x}(uy) + \frac{\partial}{\partial y}(vy) = 0 \quad \ldots\ldots [11]$$

For region A we have the following boundary conditions:

$$\text{at } y = d \begin{Bmatrix} u = 1 \\ \dfrac{\partial u}{\partial y} = 0 \\ v = 0 \end{Bmatrix}; \quad \text{at } y = d + b \begin{Bmatrix} u = 0 \\ \dfrac{\partial u}{\partial y} = 0 \end{Bmatrix} \ldots [12]$$

where d is the radius of the potential core, b is the breadth of the mixing region, and $v = 0$ at $y = d$ is necessary, as for the plane jet for $u_2 = 0$, if the fundamental assumption that no pressure difference exists between the jet and the surrounding air is to be satisfied.

At $x = 0$, $y = 1$ (taking the jet mouth to be of unit radius), Equation [10] reduces to the two-dimensional Equation [2], so that the limiting profile is that for $m = 0$ in Fig. 4. Then, as x increases, there must be a gradual transition from this profile to that obtained by Tollmien for large values of x. These two solutions are shown in Fig. 7.

In both cases, the mixing length l was taken proportional to the breadth of the mixing region, but this is justified only when the profiles are similar. However, since the profiles do not differ markedly, we can, to a good approximation, again set

$$l = cb \quad \ldots\ldots [13]$$

where the value of c is not the same as it was in part 2. Substituting Equations [1] and [13] in Equation [10] we obtain

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{c^2b^2}{y}\frac{\partial}{\partial y}\left\{y\left(\frac{\partial u}{\partial y}\right)\left|\frac{\partial u}{\partial y}\right|\right\} \quad \ldots\ldots [14]$$

To solve this differential equation approximately, we assume, as a first approximation, a reasonable velocity distribution in the mixing region. Then, by means of the differential equation we obtain a second approximation to the solution.

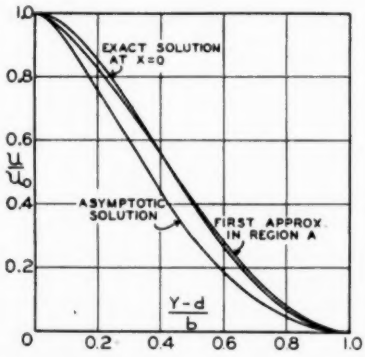


Fig. 7 Comparison Between Solutions for $x = 0$ and for x Large. First Approximation to All Profiles in Region A

We can consider u as a function of y, d, and b, where d and b are as yet undetermined. We assume, for simplicity, that u_1, where here as in subsequent expressions the subscript refers to the order of the approximation, is a function only of θ, defined by the formula

$$u_1 = f(\theta), \quad \text{where} \quad \theta = \frac{y - d}{b} \quad \ldots\ldots [15]$$

This assumption expresses the fact that the first approximation is made up of similar profiles. The dependence expressed by Equation [15] is in complete agreement with the results of part 2 at the rim of the jet mouth when the origin of coordinates is transformed to that point.

The simple expression[7]

$$u_1 = (1 - \theta^{3/2})^2 \quad \ldots\ldots [16]$$

satisfies the boundary conditions of Equation [12] (3.3) and, as is

[7] The corresponding expression,

$$u = \left\{1 - \left(\frac{y}{b}\right)^{3/2}\right\}^2$$

was found by Miss Swain, Proceedings, Royal Society of London, 105A, 1929, pp. 647–659; and Schlichting, *Ingenieur Archiv*, vol. 1, 1930, p. 533, as a first approximation to the velocity profiles for the axially symmetric and the plane wind shadows, respectively.

shown in Fig. 7, is reasonably close to the limiting profile at $x = 0$. It was, therefore, chosen as a first approximation to the profiles in region A. The corresponding value of v_1, as determined from Equation [11] is

$$v_1 = -\frac{1}{y}\int_d^y \frac{\partial u_1}{\partial x}\, y\, dy \quad \ldots\ldots\ldots\ldots [17]$$

In the first approximation, Equation [16], the quantities d and b are still unknown. The first attempt to find these parameters was made by means of the well-known momentum theorem. Multiplying both sides of Equation [14] by y and integrating with respect to y, we get

$$uvy - \frac{\partial}{\partial x}\int_y^{d+b} u^2 y dy = c^2 b^2 \left(\frac{\partial u}{\partial y}\right)^2 y \quad \ldots\ldots [18]$$

This relation is specialized for $y = 0$, corresponding to the axis of the jet. Then

$$\int_0^{d+b} u^2 y dy = \text{constant} = 0.5 \ldots\ldots\ldots [19]$$

where the integral represents, to a factor of 2π, the total momentum passing any section per second. This is set equal to the momentum issuing at the mouth of the jet. Secondly, we place y in Equation [18] equal to unity, corresponding to the point in the mixing region on the cylindrical extension of the jet mouth. We now substitute $u = u_1$ and $v = v_1$ from Equations [16] and [17] in Equation [19]. There result two relations between b, d, $\frac{db}{dx}$, and $\frac{dd}{dx}$ from which we can easily determine b and d as functions of x. This method of building up a first approximation is similar to one suggested by von Kármán and applied by Pohlhausen[8] in the theory of the laminar boundary layer.

To obtain a second approximation u_2, we substitute the first approximation u_1, v_1 in the left side of Equation [18] and identify the right side with the second approximation. One integration then gives

$$u_2 = \int_y^{d+b} \sqrt{\frac{1}{c^2 b^2 y}\left(u_1 v_1 y - \frac{\partial}{\partial x}\int_y^{d+b} u_1^2 y dy\right)}\, dy \quad \ldots\ldots [20]$$

This expression is so constructed that the boundary conditions $u_2 = \frac{\partial u_2}{\partial y} = 0$ at $y = d + b$ are satisfied. Unfortunately, the second approximation had a strong deviation, amounting to from 5 to 10 per cent, from u_1 at the inner boundary $y = d$ of the mixing region of the first approximation. This behavior would have been very difficult to correct since it signifies an overlapping of the mixing regions for the two approximations. Accordingly a method, suggested by Tollmien, was applied by which the coincidence of the boundaries of the two approximations was immediately insured.

We shall delay, for the moment, the determination of the parameters b and d in the first approximation in order to develop a different and more convenient method for proceeding from u_1 to u_2. For this purpose we write Equation [14], the equation of motion, in the form

$$2b^2c^2 \frac{\partial^2 u}{\partial y^2} = -\frac{u\dfrac{\partial u}{\partial x}}{\dfrac{\partial u}{\partial y}} - v - \frac{b^2c^2}{y}\frac{\partial u}{\partial y} \quad \ldots\ldots\ldots [21]$$

The denominator $\frac{\partial u}{\partial y}$ causes no singularity at its vanishing points, since at these points $\frac{\partial u}{\partial x}$ vanishes with at least the same order. We now place u_1 in the right and u_2 in the left side of Equation [21] and get for the second approximation

$$\frac{\partial u_2}{\partial y} = \int_y^{d+b} \frac{1}{2b^2c^2}\left\{-\frac{u_1\dfrac{\partial u_1}{\partial x}}{\dfrac{\partial u_1}{\partial y}} - v_1 - \frac{b^2c^2}{y}\frac{\partial u_1}{\partial y}\right\} dy \ldots [22]$$

$$u_2 = \int_y^{b+d}\int_d^y \frac{1}{2b^2c^2}\left\{-\frac{u_1\dfrac{\partial u_1}{\partial x}}{\dfrac{\partial u_1}{\partial y}} - v_1 - \frac{b^2c^2}{y}\frac{\partial u_1}{\partial y}\right\} dydy \quad \ldots\ldots [23]$$

where the integration limits are chosen so that $\frac{\partial u_2}{\partial y} = 0$ at $y = d$, $u_2 = 0$ at $y = d + b$. According to the idea already outlined the boundaries of both approximations shall coincide, i.e., the boundary conditions $u_1 = u_2 = 1$ at $y = d$ and $\frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial y} = 0$ at $y = d + b$ shall be satisfied. Then, from Equations [22] and [23]

$$\int_d^{b+d} \frac{1}{2b^2c^2}\left\{-\frac{u_1\dfrac{\partial u_1}{\partial x}}{\dfrac{\partial u_1}{\partial y}} - v_1 - \frac{b^2c^2}{y}\frac{\partial u_1}{\partial y}\right\} dy = 0$$

$$\int_d^{b+d}\int_d^y \frac{1}{2b^2c^2}\left\{-\frac{u_1\dfrac{\partial u_1}{\partial x}}{\dfrac{\partial u_1}{\partial y}} - v_1 - \frac{b^2c^2}{y}\frac{\partial u_1}{\partial y}\right\} dydy = 1$$

We again assume that

$$u_1 = f(\theta)$$

Then the above conditions are transformed into

$$\frac{1}{2}\int_0^1 \left\{u_1(d' + b'\theta) - \frac{v_1}{c^2} - \frac{\partial u_1}{\partial \theta}\frac{1}{\theta + \delta}\right\} d\theta = 0 \ldots [24]$$

$$\frac{1}{2}\int_0^1\int_0^\theta \left\{u_1(d' + b'\theta) - \frac{v_1}{c^2} - \frac{\partial u_1}{\partial \theta}\frac{1}{\theta + \delta}\right\} d\theta d\theta = 1 \ldots [25]$$

where

$$\delta = \frac{d}{b}; \qquad d' = \frac{1}{c^2}\frac{dd}{dx}; \qquad b' = \frac{1}{c^2}\frac{db}{dx}$$

Equations [16] and [17] are again used as a first approximation. If these are substituted in Equations [24] and [25] the integrations can be carried out analytically and result in equations of the type

$$\left.\begin{aligned} b'f_1(\delta) + d'f_2(\delta) + f_3(\delta) &= 0 \\ b'f_4(\delta) + d'f_5(\delta) + f_6(\delta) &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots [26]$$

Eliminating b' and d' successively gives

$$\left.\begin{aligned} b' &= g_1(\delta) \\ d' &= g_2(\delta) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [27]$$

[8] "Zur Integration der differentialgleichung der laminaren Grenzschicht," by K. Pohlhausen, *Zeit. für angewandte Mathematik und Mechanik*, vol. 1, 1921, pp. 252–268.

or

$$\frac{dd}{db} = \frac{g_2(\delta)}{g_1(\delta)} = b\frac{d\delta}{db} + \delta$$

Integration of this differential equation gives

$$\log \frac{b}{b_1} = \int_0^\delta \frac{g_1 d\delta}{g_2 - g_1\delta}$$

where b_1 is an integration constant. The indicated integration can be carried out graphically and we have

$$\frac{b}{b_1} = h(\delta) \quad \ldots\ldots\ldots\ldots\ldots\ldots [28]$$

In order to determine b as a function of x we first plot the first equation of [27] and Equation [28] and eliminate δ. Then, plotting $c^2\frac{dx}{db} = \frac{1}{b'}$ against b/b_1 and integrating, we obtain

$$\int_0^{b/b_1} \frac{d(b/b_1)}{b'} = \frac{c^2x}{b_1} = F\left(\frac{b}{b_1}\right) \quad \ldots\ldots\ldots\ldots [29]$$

where the condition that $b = 0$ at $x = 0$ has been used. Further, from Equation [28] we get δ as a function of $\frac{c^2x}{b_1}$. Then, from the relation $d = \delta b$

$$\frac{c^2x}{b_1} = G\left(\frac{d}{b_1}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots [30]$$

Finally, by the condition $d = 1$ at $x = 0$, the integration constant b_1 is determined and we have both parameters in the form

$$\left.\begin{aligned} b &= b(c^2x) \\ d &= d(c^2x) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots [31]$$

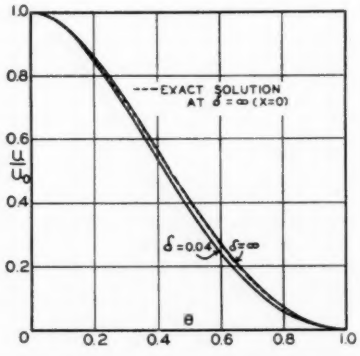


FIG. 8 SECOND APPROXIMATION TO PROFILES IN REGION A. COMPARISON BETWEEN SECOND APPROXIMATION AND EXACT SOLUTION FOR $x = 0$

The boundaries of the mixing region are now fixed and u_2 is calculated from Equation [23]. Fig. 8 shows two profiles ($\delta = 0.04$ and $\delta = \infty$) calculated in this manner. We know the limiting profile for $\delta = \infty$ to be that for the plane jet for $m = 0$, where m is again the ratio of the outside velocity to the jet velocity. This profile is also plotted in Fig. 8 and we see that the agreement between the second approximation and the exact solution is very good indeed.

Fig. 9 shows the boundaries of the mixing region as given by Equation [31] and also the lines of constant velocity as derived from the profiles for various values of δ. At $\delta = 0$ the main correction term in u_2 vanished, having an infinite derivative at that point. The calculated profile was obviously considerably in error and consequently is not given here.

Since the momentum integral Equation [19] has not yet been used in this process, we can, by its use, determine how far we may consider the approximation good. The momentum for the profile at $\delta = 0$ was 12 per cent high while that for $\delta = 0.04$ was still within 2 per cent of the correct value. This indicates that, in spite of the unfortunate behavior near $\delta = 0$ or $c^2x = 0.0473$, we can still describe, to a good approximation, the velocity field as far as $\delta = 0.04$ or $c^2x = 0.044$, which is quite near to the end of the core of potential flow.

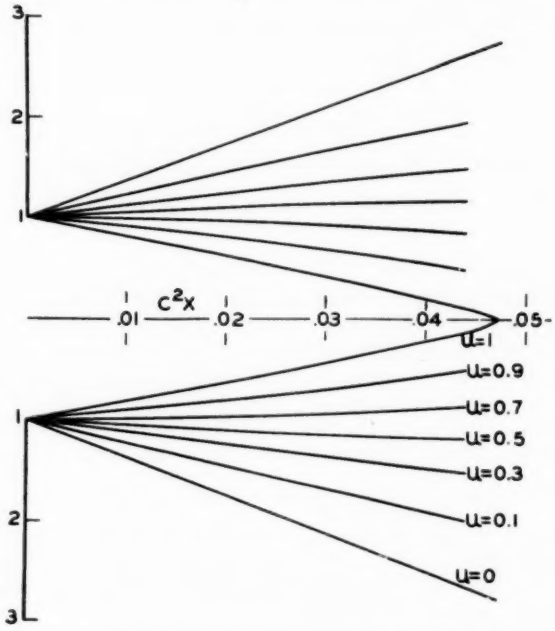


FIG. 9 SECOND APPROXIMATION TO THE VELOCITY FIELD IN REGION A

Fig. 9 shows that the lines of equal velocity are very nearly straight and we know that, for small values of x, they agree with those for the plane case. Furthermore, Fig. 8 shows that the divergence between profiles in region A is not large.

We can then easily predict the approximate flow picture if the jet is issuing into a moving fluid, which is the problem analogous to that considered in part 2 for the plane jet. We know that the limiting conditions at the jet mouth will be those already found for the plane jet. For $m = 0$, the results of the foregoing analysis show that the change, both in the boundaries of the mixing region and the velocity profiles, is not large. This indicates that, for any m, the results of part 2 for the plane jet can be used to a first approximation as far as the core of potential flow extends.

A refinement of the foregoing method was applied in region B. However, near the point where the core of potential flow vanishes, the results were not in accord with experiment. Since, at this point, the physical behavior is not clear, making the assumption for l very doubtful, no further attempt was made to describe the field of flow in this region.

The refined method consisted essentially of introducing dissimilar profiles for the first approximation. Hence, u_1 must contain a "shape factor." For region A this proposal would lead, to an assumption

$$u_1 = \left[1 - \left(\frac{y-d}{b}\right)^p\right]^3$$

In addition to b and d we must now determine p, the shape factor,

as a function of x. To carry out this determination we must have three conditions, one more than was required by the preceding method. The momentum theorem Equation [19] applied to u_1 gives us this third condition. This is differentiated with respect to x and we have three simultaneous differential equations to solve for b, d, and p. The method of solution is similar to that outlined for the preceding method.

The utility of these methods is by no means restricted to the foregoing problem. The methods are equally useful for the solution of various problems in laminar as well as turbulent flow. It is necessary, of course, to know the approximate shape of the velocity profiles. Higher approximations can be found by repeating the processes, though a considerable amount of calculation is involved.

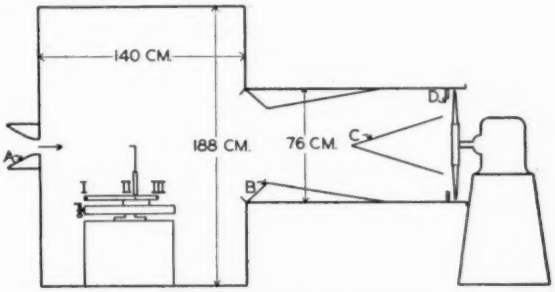


FIG. 10 LONGITUDINAL SECTION OF EXPERIMENTAL ARRANGEMENT SHOWING THE MEASURING DEVICE IN POSITION

Tollmien, by the method described in part 2, carried out an investigation of the pressure difference between the center of the mixing region and the undisturbed fluid for x large. This difference was found to be $-0.00295 \frac{\rho}{2} u_0^2$ where u_0 is the central velocity in the jet at the point considered. This quantity is again negligible in so far as its consideration would alter the mean stream lines. Since the case considered is intermediate between those considered by Tollmien, no investigation of the pressure difference was made.

4 EXPERIMENTAL DETERMINATION OF c

For the purpose of evaluating the empirical constant in the calculation of part 3, measurements were made of mean-velocity distributions in an axially symmetrical jet.

The arrangement, a longitudinal section of which is shown in Fig. 10, was essentially an Eiffel-type wind tunnel, the observation room being 140 × 188 × 188 cm and the outlet tube 76 cm in diameter. This part of the apparatus with the motor and fan already existed, so it was necessary to use an entrance small enough to permit an exploration of the velocity field as far as the asymptotic region.

A 10-cm nozzle with fairing as shown in Fig. 10 was used. Air was drawn through the arrangement by a two-bladed fan driven by a 7.5-hp a-c motor, the speed of which was varied by varying the frequency of the current. With the outlet unobstructed, strong pulsations, backflow, and rotation of the stream were observed. The bell mouth B eliminated the pulsations and the cone C, supported by three symmetrically placed guide-vanes, and the annular ring D eliminated the greater part of the backflow and rotation. In order to obtain a uniform velocity distribution across the inlet, it was necessary that the nozzle have a cylindrical section at least 0.3 diameter in length.

A two-way traversing mechanism carried the measuring device. By bolting the device in positions I, II, and III successively it was possible to obtain a longitudinal traverse of 9.3 diameters, which, along with a lateral motion of 27 cm, was sufficient for the investigation.

Lateral traverses were made with a pitot tube at distances from 3 to 9.3 diameters from the jet mouth, the inflow velocity being maintained at 26.5 meters per sec. Because of the high velocity gradient and large fluctuations about the mean velocity, measurements at distances less than three diameters could not be relied upon. However, Tollmien checked his results for the plane jet by comparison with measurements[9] made in Göttingen at the boundary of the jet, one-half diameter from the mouth of an open-jet wind tunnel 2.24 m in diameter. At that dis-

[9] Göttingen Ergebnisse, II Lief., 1923, pp. 69–73, R. Oldenbourg, Munich.

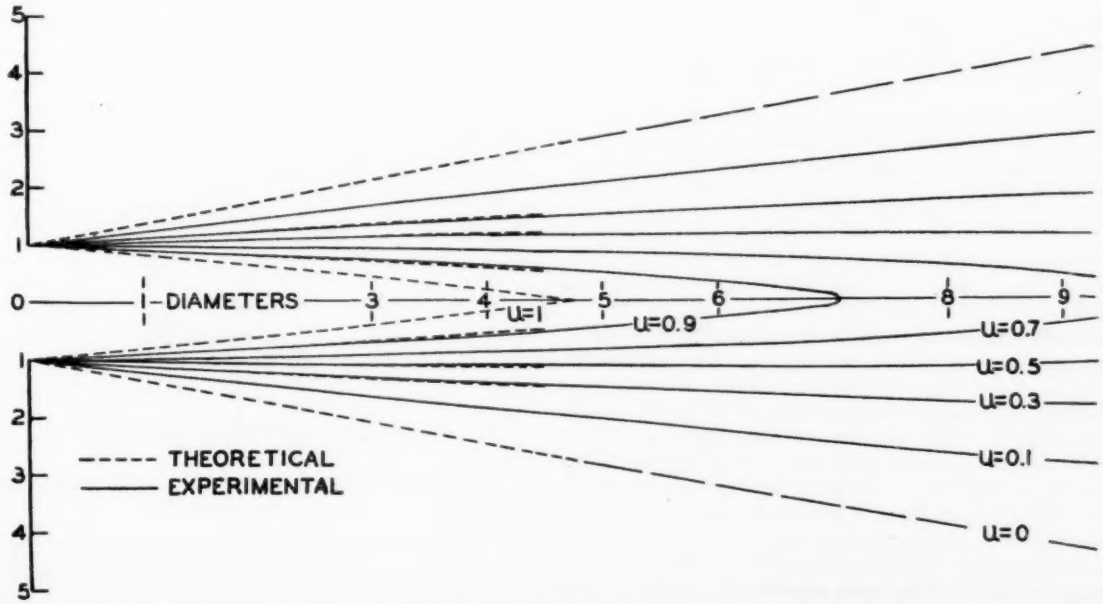


FIG. 11 EXPERIMENTALLY DETERMINED VELOCITY FIELD OF THE JET AND COMPARISON WITH THEORY FOR $c^2 = 0.00496$. THE BOUNDARIES OF THE JET AT 9.3 DIAMETERS ARE CALCULATED FROM TOLLMIEN'S SOLUTION FOR x LARGE. BETWEEN 4.76 AND 9.3 DIAMETERS THE OUTER BOUNDARIES ARE INTERPOLATED

tance the mixing region was wide enough so that this difficulty was not encountered. Those measurements showed very good agreement between theory and experiment for a value of c very close to the one used here. Hence, for the present purpose, the velocity field for distances up to three diameters from the mouth could be interpolated very closely. The lines of equal velocity are shown in Fig. 11, in which, for convenience in carrying over the theoretical results, the radial distance is measured in radii while the longitudinal is measured in diameters.

The dotted lines in Fig. 11 show the agreement between the theory of part 2 and experiment for $c^2 = 0.00496$, i.e., $c = 0.0705$.

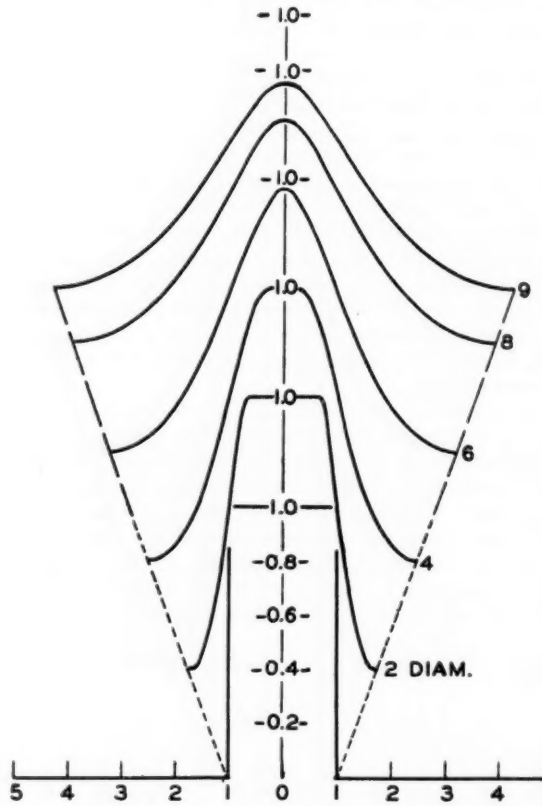


FIG. 12 EXPERIMENTAL PROFILES AT VARIOUS DISTANCES FROM JET MOUTH. SLOPES OF THE OUTER BOUNDARIES ARE MAGNIFIED IN THE RATIO 2:1. THE ORDINATE IS DISPLACED 0.2 D, WHERE D IS THE NUMBER OF DIAMETERS FROM THE JET MOUTH ALONG THE VELOCITY AXIS

Fig. 12 shows profiles at 0, 2, 4, 6, 8, and 9 diameters. The ordinates in this figure are displaced 0.2 along the velocity axis for each diameter from the mouth.

Fig. 13 shows the scatter of the experimental points and their agreement with the theoretical curve at $x = 4.44$ diameters, which is the last point for which the theory is given. Aside from a small systematic difference near the center of the jet the agreement is quite good. This difference is in the direction indicated by the fact that the momentum calculated from the theoretical profile was 2 per cent low. In Fig. 14 the experimental points are plotted along with Tollmien's solution for large values of x. There is again a very small systematic difference near the center.

Since the completion of this work an experimental investigation similar to that cited has been reported by Ruden.[10] The agreement between the two, in so far as the data given allow, is as good as can be expected. The two arrangements may not be quite comparable and this would preclude the possibility of accurate agreement.

[10] Naturwissenschaften, 21:23, pp. 373–378, May, 1933, Julius Springer, Berlin.

5 VELOCITY FLUCTUATION MEASUREMENTS

The foundations of the theory applied to this problem contain

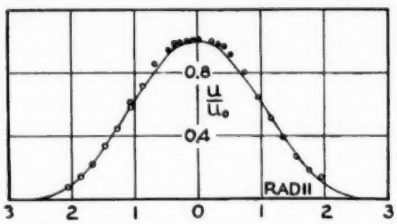


FIG. 13 COMPARISON BETWEEN THEORETICAL PROFILE AND EXPERIMENTAL POINTS AT $x = 4.44$ DIAMETERS, WHICH IS THE LAST POINT FOR WHICH THE THEORY IS GIVEN. VERTICAL LINES REPRESENT BOUNDARIES OF THE MIXING REGION

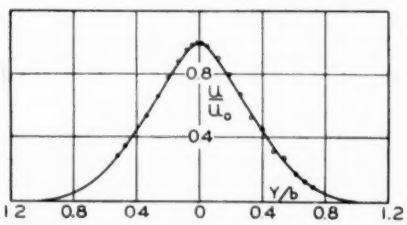


FIG. 14 COMPARISON BETWEEN TOLLMIEN'S THEORETICAL PROFILE FOR x LARGE AND THE EXPERIMENTAL POINTS AT $x = 9.3$ DIAMETERS

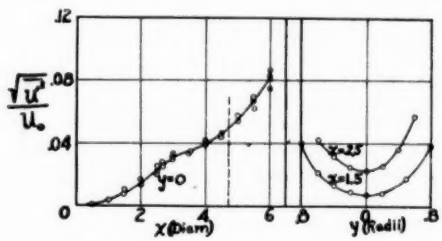


FIG. 15 (*a*) $\frac{\sqrt{(\bar{u}'^2)}}{u_0}$ ALONG THE AXIS OF THE JET. THE BROKEN LINE INDICATES THE END OF THE CORE OF POTENTIAL FLOW. (*b*) $\frac{\sqrt{(\bar{u}'^2)}}{u_0}$ ACROSS THE POTENTIAL CORE AT TWO STATIONS

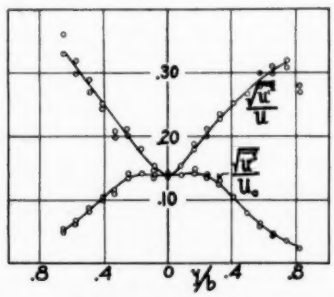


FIG. 16 LATERAL TRAVERSE IN THE MIXING REGION OF THE JET AT $x = 9.3$ DIAMETERS

the assumption that there are no velocity fluctuations in the core of potential flow. This was investigated[11] by means of the hot-

[11] F. L. Wattendorf and A. M. Kuethe, *Physics*, vol. 5, no. 6, pp. 153–165, June, 1934. For details of the apparatus, the reader is referred to this paper.

wire anemometer and auxiliary apparatus developed by F. L. Wattendorf and the author.

The fluctuations in the velocity of air passing an electrically heated platinum wire (the wire used was 0.0005 in. diam, and 3 mm long) cause a fluctuation in the temperature of the wire. The consequent fluctuations in voltage drop across the wire (the current is kept at an approximately constant value) are impressed across the input of a five-stage amplifier containing a circuit which compensates for the lag in amplitude of the response of the hot wire due to the relatively high frequency (frequencies from 40 to 500 cycles per sec are reproduced with small relative error) of the air-speed fluctuations.

By means of calibration curves of the hot wire and the amplifier plus compensation, the mean-square value of the output, as measured on a thermogalvanometer, is converted into percentage fluctuations in velocity where u' is the x component of the fluctuations.

Fig. 15 shows $\frac{\sqrt{(\bar{u}'^2)}}{u_0}$ at the center of the jet as a function of the distance from the jet mouth. Two lateral traverses across the potential core are also shown. In the mixing region surrounding the potential core the velocity fluctuations were very high (on the order of 50 to 100 per cent of the mean velocity) and the results were erratic. The mean jet velocity was again 26.5 meters per sec. The hump in the longitudinal traverse at $x = 3$ diameters was also present at a velocity of 20 meters per sec.

The relatively high fluctuations in the region of potential flow probably originate in the surrounding mixing region. In the experiments described in reference 11, an analogous phenomenon was observed in the region of potential flow between two boundary layers in a channel. Since the velocity gradient is zero, precluding any possibility of the existence of a shearing stress, the fluctuations in both cases must be purely random in character.

Measurements of the velocity fluctuations were made at $x = 9.3$ diameters. Fig. 16 shows the distribution of $\frac{\sqrt{(\bar{u}'^2)}}{u}$ and $\frac{\sqrt{(\bar{u}'^2)}}{u_0}$ where the latter is derived from the former by means of the velocity profile (Fig. 15).

A complete study of the fluctuations in the jet was not made. The magnitude of the fluctuations is so high that the response of the hot wire may be in error by a considerable amount. The problem is further complicated by the fact that the velocity fluctuations calculated from hot-wire results represent $\frac{\sqrt{(\bar{u}'^2)}}{u}$ only when u' is small compared with u.[11]

When the fluctuations become large, the measured value may be seriously affected by the v' and w' components.

Acknowledgment

The author wishes to express his sincere appreciation to Dr. W. Tollmien and to Dr. Th. von Kármán. The investigation was undertaken at the suggestion of Dr. von Kármán; Dr. Tollmien made many helpful suggestions in the course of the work.

The Temperature History During the Constant-Rate Heating of a Solid Circular Cylinder

By ALBERT B. NEWMAN[1] and AUSTIN H. CHURCH,[2] NEW YORK, N. Y.

The authors present a solution of the partial-differential equation for heat conduction in a solid circular cylinder for the condition where the heat is forced into the cylindrical surface at a constant rate. A mathematical analysis is made of the temperature-history curve at the heated face and at the axis of the cylinder and the results are reduced to tabular and graphical form for practical application.

SOLUTIONS of the partial-differential equation for heat conduction are available for cases of solid circular cylinders in which the cylinder is initially at a known uniform temperature and the cylindrical surface is either suddenly heated to a higher known temperature and maintained at that temperature or is placed in a fluid of known temperature under conditions such that the fluid temperature remains constant. These solutions enable one to trace the temperature-history curve for the axis or for any cylindrical distance from the axis; or the temperature distribution at any instant can be calculated and plotted. The tabulated values of these solutions have been plotted by Gurney and Lurie[3] and have been reproduced in several books.[4]

An important case for which no solution has been published, as far as the authors are aware, is that in which heat is forced into the cylindrical surface at a constant rate, such as might be approximately accomplished by electrical heating or by the application of a constant frictional force. Frequently, it is desirable to predict the temperature-history curve at the heated face and at the axis, and it is the purpose of this paper to present a mathematical analysis of this case and to reduce the results to tables and curves which can be used with ease in practical work. The analogous case for slab heating has been published recently by Newman.[5]

Nomenclature

K = thermal conductivity, Btu per ft per sq ft per hr per deg
C = specific heat, Btu per lb per deg
ρ = density, lb per cu ft
$k = K/C\rho$ = thermal diffusivity, sq ft per hr
θ = time from the beginning of heating, hr
θ_1 = time at end of heating period
q = constant rate of heat input, Btu per sq ft per hr
r = distance from axis, ft
a = total radius of cylinder, ft
t = temperature in deg F, varying with r and θ
t_0 = initial uniform temperature
$P = k\theta/a^2$ = a dimensionless ratio; $P_1 = k\theta_1/a^2$

Bessel functions of x

$$J_0(x) = \left[1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \frac{x^6}{2^2 4^2 6^2} + \dots\right]$$

and

$$-J_1(x) = \frac{d[J_0(x)]}{dx} = \left[\frac{x}{2} - \frac{x^3}{2^2 4} + \frac{x^5}{2^2 4^2 6} - \dots\right]$$

The B-function of r/a and P

$$B(r/a;\ P) = 2\sum_{n=1}^{\infty} \frac{1}{\beta_n^2}\left(\frac{J_0(\beta_n r/a)}{J_0(\beta_n)}\right) e^{-\beta_n^2 P}$$

in which β_n is defined by $J_1(\beta_n) = 0$.

General Conditions of the Problem

It is assumed that the cylinder is homogeneous and isotropic and that heat is being actually driven into the surface of the cylinder at the constant rate specified, no allowance being made for the heat capacity of the heating element, for heat leakage outward from the heating element or toward the ends of the cylinder. This latter condition implies that the ends of the cylinder are heat-insulated or that the cylinder is so long in comparison with its radius that axial heat flow is negligible. It is assumed that there is no surface resistance between the heating element and the surface of the cylinder.

The Integrated Equation

The integrated equation must satisfy all of the following five conditions:

(*a*) $$\frac{\partial t}{\partial \theta} = k\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}$$

This is the partial-differential equation of heat conduction in cylindrical coordinates, for strictly radial heat flow.

(*b*) $t - t_0 = 0$ when $\theta = 0$. This equation merely states that the initial temperature at all points in the cylinder is t_0.

(*c*) $\partial t/\partial r = q/K$ at $r = a$. At the heated surface, a constant rate of heating results in a constant temperature gradient, and the gradient is proportional to the heating rate but inversely proportional to the thermal conductivity.

(*d*) $\partial t/\partial r = 0$ at $r = 0$. At the axis, heat is coming in equally from all radial directions; therefore, no heat crosses the axis and there is no temperature gradient at the axis.

---

[1] Professor of Chemical Engineering, Cooper Union.
[2] Instructor, Cooper Union. Assoc-Mem. A.S.M.E.

[3] "Charts for Estimating Temperature Distribution in Heating or Cooling Solid Shapes," by Gurney and Lurie, *Industrial and Engineering Chemistry*, vol. 15, 1923, p. 1170.

[4] "Industrial Furnaces," by W. Trinks, vol. 1. Second edition, John Wiley and Sons, New York, 1926, p. 322.
"Fuels and Their Combustion," by R. T. Haslam and R. P. Russel, McGraw-Hill Book Company, New York, 1926, p. 768.
"Heat Transmission," by W. H. McAdams, McGraw-Hill Book Company, New York, 1933, p. 33.
"Chemical Engineers' Handbook," McGraw-Hill Book Company, New York, 1934, p. 840.

[5] "The Temperature-Time Relations Resulting From the Electric Heating of the Face of a Slab," by Albert B. Newman, Trans. A.I.Ch.E., vol. 30, 1934, p. 598.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, N. Y., and will be accepted until November 11, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

(*e*) $\frac{1}{a}\int_0^a (t - t_0)C\rho r dr = q\theta$. The integrated value of the heat content of the cylinder above t_0 at any time must equal the total heat input up to that time.

An equation, all of the terms of which are dimensionless, and which satisfies all of these requirements is

$$\frac{(t - t_0K)}{qa} = 2P + \frac{1}{2}\left(\frac{r}{a}\right)^2 - \frac{1}{4} - B(r/a;\ P) \quad [1]$$

In this equation, β_n is defined by $J_1(\beta) = 0$. In explanation, it may be said that if one solves the $J_1(x)$ equation by substituting various values of x, and then plots $J_1(x)$ as ordinate against x as abscissa, a curve is obtained which resembles a sine curve, crossing the x-axis periodically. The value of x at the first, second, and third crossings, etc., are β_1, β_2, and β_3, etc. The first fifty values of β with the corresponding values of $J_0(\beta)$ are tabulated by Gray, Mathews, and MacRobert.[6]

Any one with a sufficient knowledge of the calculus to differentiate and integrate can test the validity of Equation [1] by solving it for $t - t_0$, differentiating once with respect to θ and twice with respect to r, and substituting the values so obtained in the equations expressing the first four conditions. Condition (*e*) is verified by substituting the value of $t - t_0$ under the integral and performing the integration. It is necessary to know the following properties of these Bessel functions in order to carry out the above operations.

For condition (*a*)

$$\frac{\partial}{\partial r}\left[J_0(\beta_n r/a)\right] = -\frac{\beta_n}{a} J_1(\beta_n r/a) \quad [2]$$

$$\frac{\partial}{\partial r}\left[J_1(\beta_n r/a)\right] = \frac{\beta_n}{a} J_0(\beta_n r/a) - \frac{1}{r} J_1(\beta_n r/a) \quad [3]$$

For condition (*b*)

$$B(r/a;\ 0) = 2\sum_{n=1}^{\infty} \frac{1}{\beta_n^2}\left(\frac{J_0(\beta_n r/a)}{J_0(\beta_n)}\right) = \frac{1}{2}\left(\frac{r}{a}\right)^2 - \frac{1}{4} \quad [4]$$

For condition (*e*)

$$\frac{\partial}{\partial x}\left[xJ_1(x)\right] = xJ_0(x), \quad \text{therefore,}$$

$$\int_0^a rJ_0(\beta_n r/a)dr = \frac{ar}{\beta_n}\left[J_1(\beta_n) - J_1(0)\right] = 0 \quad [5]$$

Integrated Equation for the Equalizing Period

If the constant-rate heating is suddenly discontinued, at $\theta = \theta_1$, and if no heat is allowed to escape from the cylindrical surface, the temperature at the axis will continue to rise and that at the surface will fall. Eventually, the temperature will become equalized throughout the cylinder. The temperature history at the surface and at the axis during this period may be traced by means of the equation

$$(t - t_0)K/qa = 2P_1 + B(r/a;\ P - P_1) - B(r/a;\ P) \quad [6]$$

When $\theta = \theta_1$, $P = P_1$ and Equation [6] must reduce to Equation [1]. This can be shown to be true by the aid of Equation [4]. When $\theta = \infty$

$$(t - t_0)K/qa = 2P_1$$

showing the final equalizing of temperature, independent of r.

[6] "A Treatise on Bessel Functions," by Gray, Mathews, and MacRobert. Second edition, Macmillan & Co., London, 1922, p. 301.

Numerical Solution of the Integrated Equation

All of the variables in the integrated equation are contained in the three dimensionless groups: $(t - t_0)K/qa$; r/a; and P. Each one of these groups may be treated as a variable and the equation may be handled as one of three variables. If one is interested in the temperature history of the heated surface ($r = a$), he may substitute $r/a = 1$ and obtain an equation in two variables. Obviously a curve may be plotted for such a case. If interested in the temperature history of the axis, the value $r/a = 0$ can be substituted, and another curve may be plotted for the other two variables. Table 1 gives the values of $B(r/a; P)$ for $r/a = 0$ and 1. The values of $(t - t_0)K/qa$ for a series of values of P for the cases $r/a = 0$ and $r/a = 1$ have been calculated by using Table 1 and are given in Table 2. For certain terminal values, it is convenient to know that

$$B(1;\ 0) = 2\sum_{n=1}^{\infty}\frac{1}{\beta_n^2} = \frac{1}{4};\quad B(0;\ 0) = 2\sum_{n=1}^{\infty}\frac{1}{\beta_n^2 J_0(\beta_n)} = -\frac{1}{4}.$$

For values of P equal to or greater than unity, the value of the series in the equation becomes negligible, and $(t - t_0)K/qa = 2P - 1/4$ for $r/a = 0$ or 1. The authors have not computed values for r/a other than 0 and 1. If desired, however, values for intermediate radii can be computed by the same methods.

TABLE 1 VALUES OF $B(r/a;P)$ FOR $r/a = 0$ AND 1

P	$-B(0\ ;P)$	$B(1\ ;P)$
0.0	0.25000	0.25000
0.005	0.24028	0.17760
0.01	0.23000	0.15055
0.02	0.20999	0.11954
0.03	0.19003	0.09790
0.04	0.17028	0.08171
0.05	0.15120	0.06896
0.06	0.13325	0.05866
0.07	0.11670	0.05005
0.08	0.10188	0.04288
0.09	0.08862	0.03683
0.10	0.07691	0.03167
0.12	0.05771	0.02350
0.14	0.04317	0.01748
0.16	0.03223	0.01302
0.18	0.02405	0.00970
0.20	0.01794	0.00723
0.25	0.00861	0.00347
0.30	0.00413	0.00166
0.35	0.00198	0.00080
0.40	0.00095	0.00038
0.45	0.00046	0.00018
0.50	0.00022	0.00009
1.00	0.00000	0.00000

TABLE 2 VALUES OF $(t - t_0)K/qa$ FOR EQUATION [1]

P	$r/a = 0$	$r/a = 1$
0.000	0.000000	0.000000
0.005	0.000000	0.082397
0.010	0.000000	0.118140
0.020	0.000001	0.170459
0.030	0.000027	0.212103
0.040	0.000281	0.248292
0.050	0.001200	0.281042
0.060	0.003250	0.311340
0.070	0.006704	0.339947
0.080	0.011876	0.367116
0.090	0.018621	0.393174
0.100	0.026913	0.418326
0.120	0.047715	0.466496
0.140	0.073166	0.512518
0.160	0.102234	0.556981
0.180	0.134051	0.600300
0.200	0.167938	0.642770
0.250	0.258612	0.746531
0.300	0.354133	0.848336
0.350	0.451984	0.949201
0.400	0.550952	1.049617
0.450	0.650457	1.149816
0.500	0.750219	1.249923
1.000	1.750000	2.250000

In order to construct a table for the values of $(t - t_0)K/qa$ for Equation [6], one must first select a value for P_1. Ordinarily this value of P_1 is fixed approximately by the conditions of the practical problem, which will dictate a maximum value for the ratio of maximum temperature rise at the surface to the accom-

plished temperature rise at the axis. This ratio is plotted against P_1 in Fig. 1. In constructing this curve, the accomplished temperature rise at the axis is taken as the value of $t - t_0$ in $(t - t_0) K/qa = 2P_1$, implying the final temperature rise attained at complete equalization. Theoretically, this condition is not attained at finite times and in any practical case the value of P_1 selected will be based upon a somewhat lower temperature-rise ratio than the allowable maximum.

Numerical Example

A long solid cylinder of composition material $^5/_{16}$ in. diam, initially at the uniform temperature 70 F, is to be heated electrically all along its surface using a constant rate of heat input. The axis must be heated to 155 F, but the surface temperature must not exceed 255 F.

It is required to estimate the rate of heat input, to plot temperature-history curves for surface and axis, and to estimate the time of heating and the time elapsed before the axis reaches the required temperature.

Data:

K = 0.092 Btu per sq ft per hr per deg F
k = 0.0029 sq ft per hr
a = 0.013 ft

Solution:

Maximum rise at surface = 255 F — 70 F = 185 F
Required rise at axis = 155 F — 70 F = 85 F
Ratio = 185/85 = 2.18
Selecting a somewhat lower ratio = 2.10

Fig. 1 gives $P_1 = 0.1 = k\theta_1/a^2$. Substituting known values of k and a^2, $\theta_1 = (0.1)(0.013)^2/0.0029 = 0.00583$ hr or 21 sec.

At the heated surface $(t - t_0)K/qa = 0.418$ at the end of the heating period ($P_1 = 0.1$), from Table 2. Substituting known values, q = (185)(0.092)/(0.013)(0.418) = 3130 Btu per sq ft per hr = heating rate.

Fig. 2 was constructed from the data of this example, by using Tables 1 and 2. The upper and lower curves represent the temperature rise at the surface and at the axis, respectively. It is seen that the required rise at the axis (85 deg) is reached at θ = 78 sec.

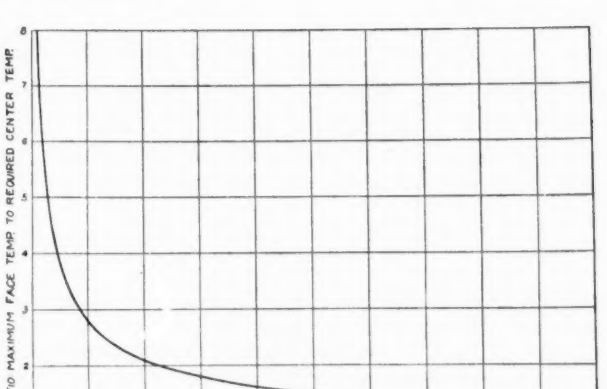


Fig. 1 Values of P_1 or $k\theta_1/a^2$ for Various Ratios of Maximum Temperature Rise at the Surface to Accomplished Temperature Rise at the Axis of Solid Circular Cylinders

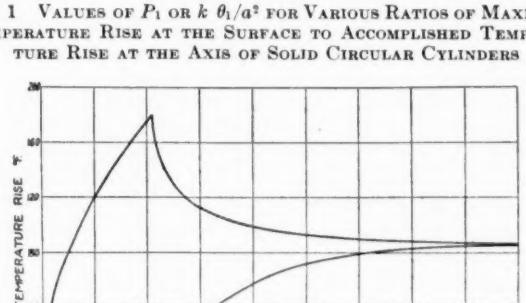


Fig. 2 Temperature Rise at the Surface (Upper Curve) and at the Axis (Lower Curve) of a Solid Circular Cylinder During Constant-Rate Heating

Summary:

Rate of heat input = 3130 Btu per sq ft per hr, or 75 watts per running ft.

Heating time = 21 sec.

Total elapsed time for center to reach required temperature = 78 sec.

An Application of the Interferometer Strain Gage in Photoelasticity

By R. W. VOSE,[1] CAMBRIDGE, MASS.

This paper was written at the suggestion of Mr. Mieth Maeser,[2] in response to numerous inquiries concerning the methods of photoelastic analysis in use at the Massachusetts Institute of Technology.

By the use of any of the usual photoelastic methods the difference of the principal stresses and their direction at any point in a suitable loaded specimen are determined, and through a knowledge of Poisson's ratio their sum is obtained (and a solution made possible) by a measurement of the lateral deformation of the specimen by means of an interferometer strain gage. This instrument, together with its accessories and their use, is illustrated and described in the paper. Examples of the problems solved by the use of the instrument show its accuracy and the consistency of the results obtained by the method.

UNDER the general name of photoelasticity the change in the index of refraction with change of the stress in certain transparent materials is utilized in determining the distribution of the stress in loaded specimens. A number of different methods of carrying out this type of determination are in use, each with its advantages and limitations.

The most direct method consists of determining the actual change of the index of refraction of the specimen (or the accompanying retardation of plane polarized light) along the two planes of principal stress at any point. For this determination Favre[3] uses a Mach-Zehnder interferometer, while Solakian[4] utilizes the two surfaces of the specimen itself for an interferometer as suggested by Fabry.

A second general method determines only the relative retardation of the light along the two principal planes, and hence the difference of the two principal stresses, by means of the interference pattern produced when the beams of plane polarized light from the two principal planes are analyzed in a common plane by means of a Nicol prism. Except at a free physical boundary of the specimen where one of the principal stresses is necessarily zero, additional means must be employed to determine the individual stresses. For this there are available the graphical-integration method of Filon,[5] an improved graphical method recently presented by Neuber,[6] the membrane analogy of Den Hartog[7] (all of which depend on free-boundary conditions), and the lateral-deformation methods of Frocht[8] and of Coker,[9] which are independent of boundary or other surrounding conditions.

For several years Filon's graphical-integration method has been used at the Massachusetts Institute of Technology, but the results obtained apparently have not been of the desired order of accuracy, and in addition the method could only be applied to cases in which good free-boundary conditions prevailed. These limitations led Mr. Maeser,[2] then in charge of photoelasticity, to suggest in the spring of 1933 that a sensitive lateral-strain gage should be devised for carrying out Coker's method. Accordingly, in October of 1933 the author built the interferometer strain gage described in the following paragraphs, and assisted in its experimental use on several specimens.

Interferometer Strain Gage

The strain gage is shown in Fig. 1 and the method of applying it in Fig. 2. At *A* are two rounded contact points held in contact with the specimen by the spring *B*. Any relative motion of the contacts is transmitted through the levers *C* which are pivoted at *D* by a crossed-plate fulcrum to a pair of small optical flats which form the interferometer at *E*. Three spring-opposed screws carry the mounting of one of the flats so that the parallelism and separation of the pair may be adjusted.

Mounting the strain gage is accomplished by supporting its weight on a string tied around the plate fulcrum and depending on the friction of the contacts with the specimen to retain its position, as shown in Fig. 2. This seemingly crude mounting has the advantage of easy adjustment and absolute freedom from the restraining forces accompanying a rigid mounting.

Since the interferometer flats measure their change of separation in terms of the wave length of light, which is in itself a standard of length, the only calibration of the instrument is that of its lever ratio. This is easily determined by direct measurement of the lever arms, and since these are 3 in. long an accuracy of better than 1 per cent may be obtained by use of a steel rule, or closer results by a measuring microscope. The fiducial point of the interferometer system is a set of cross "hairs" etched on the surface of one of the flats, and the measurement of the levers is taken to this point. A multiplication of unity has been found satisfactory for use on specimens from $^1/_4$ in. to $^1/_2$ in. in thickness.

Optical System

Interference fringes or bands may be observed very simply by placing two optical flats nearly in contact, as is done in optics shop practice. This type of fringe, similar to Newton's rings, becomes useful for accurate quantitative work only when observed with monochromatic light through a low-power microscope or telescope.

The green line of the mercury arc as isolated by a Wratten

---

[1] Instructor in Materials Testing and Photoelasticity, Massachusetts Institute of Technology, Cambridge, Mass. Jun. A.S.M.E.

[2] Formerly instructor in charge of photoelasticity, Massachusetts Institute of Technology, and now with the United Shoe Machinery Corp., Beverly, Mass.

[3] "New Developments in Photoelasticity," by A. G. Solakian, *Journal of the Optical Society of America*, vol. 22, no. 5, May, 1932.

[4] Discussion by A. G. Solakian, of paper on "Studies in Photoelastic Stress Determination," by E. E. Weibel, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-13, p. 652.

[5] "On the Graphical Determination of Stress From Photoelastic Observations," by L. N. G. Filon, *Engineering*, London, Oct., 1923.

[6] "New Method of Deriving Stresses Graphically From Photoelastic Observation," by A. Neuber, Proceedings Royal Society of London, A, vol. 141, 1933, p. 314.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until November 11, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expression of their authors, and not those of the Society.

[7] "A Membrane Analogy Supplementing Photoelasticity," by J. G. McGivern and H. L. Supper, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-9, p. 601.

[8] "On the Application of Interference Fringes to Stress Analysis," by M. M. Frocht, *Journal of the Franklin Institute*, July, 1933.

[9] "Photoelasticity," by E. G. Coker and L. N. G. Filon, Cambridge University Press, 1931.

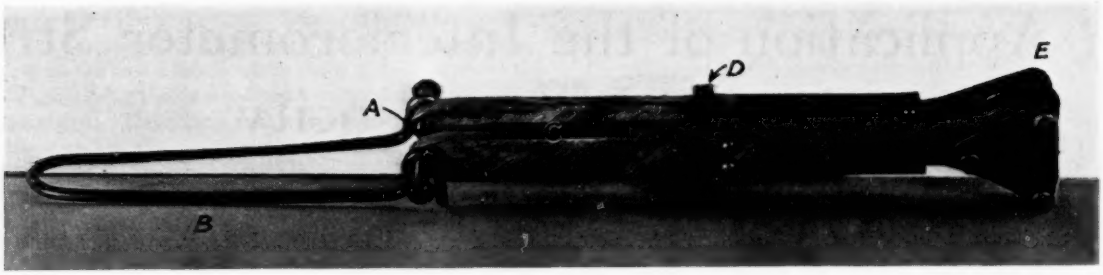


FIG. 1 INTERFEROMETER STRAIN GAGE

No. 62 filter gives a convenient and brilliant source of monochromatic light with a wave length of 5461 Å, and a corresponding band spacing of 0.00001075 in. This line is near the point of maximum sensitivity of the human eye, and an observer can count several thousand fringes per day without undue fatigue.

If the flats of the interferometer be illuminated by reflection from a plain glass mirror and observed through the mirror, as is shown in Fig. 2, a pattern of dark and light bands will be seen on the flats if the latter be adjusted so that there is a small air gap between them. These bands are due to the periodic changes in phase relationship between the light reflected from the front flat and that from the back flat as the air gap changes. This latter change in the air gap may be due either to a change in position over the flats, resulting in bands showing a contour map of the air-gap thickness, or to a change in separation of the flats, resulting in a movement of the set of contour bands across the flats. A count of the number of bands which pass a given point on the flats will thus indicate the change in the air-gap thickness at this point produced by a relative change in separation of the flats. Numerically this change will be equal to one-half the product of the number of bands counted, by the wave length of the light used.

Even with unfavorable conditions, fractions can be estimated to one-fifth of a band, or two millionths of an inch, and the author has made determinations with a similar strain gage in which the calculated probable error of a single setting on the center of a band was 0.009 band, or less than one ten-millionth of an inch. It will be noticed that the measurements are obtained without physical contact of the moving members at the measuring point, and hence the only possibility of friction or hysteresis comes in the flexing of the contact spring *B* and the plate fulcrum *D*, and is entirely negligible. With the usual photoelastic materials, however, this high sensitivity cannot be fully utilized because of plastic and thermal creep in the material.

METHOD OF USE

The difference of the principal stresses at any point in a loaded specimen may be determined by any of the usual photoelastic methods, such as the use of Coker's compensator,[9] or the comparison of fringes with those of a specimen of known stress.

To determine the sum of the principal stresses the strain gage is attached to the specimen at the point in question and the lateral deformation measured by counting the interferometer bands as the specimen is slowly loaded or unloaded. The solution is made from these readings by the use of Poisson's ratio and Young's modulus for the material. These latter properties, or more conveniently their product as a single constant, are usually determined on the compensator or comparator member used in the photoelastic work.

With the foregoing determinations of the sums and differences at points throughout the specimen the individual stresses may be separated by algebraic solution and the results plotted or tabulated.

ACCURACY

The accuracy of the method may be best illustrated by actual figures obtained in the calibration of a tension specimen. Here one photoelastic fringe used in determining the difference of the principal stresses was found to be equivalent to 90.4 lb per in., while the corresponding strain-gage interference fringe used in determining the sum of the principal stresses was equivalent to 18.46 lb per in. Thus the sensitivity of the strain gage is five times that of the photoelasticity apparatus, and in use is better adapted to the estimation of tenths. In addition, this calibration of 18.46 lb per in. was obtained with a calculated probable error of 0.06 per cent, which illustrates the accuracy with which the strain gage will repeat its indications.

In addition to the foregoing illustrations of instrumental accuracy, it is to be emphasized that the stress determination at any

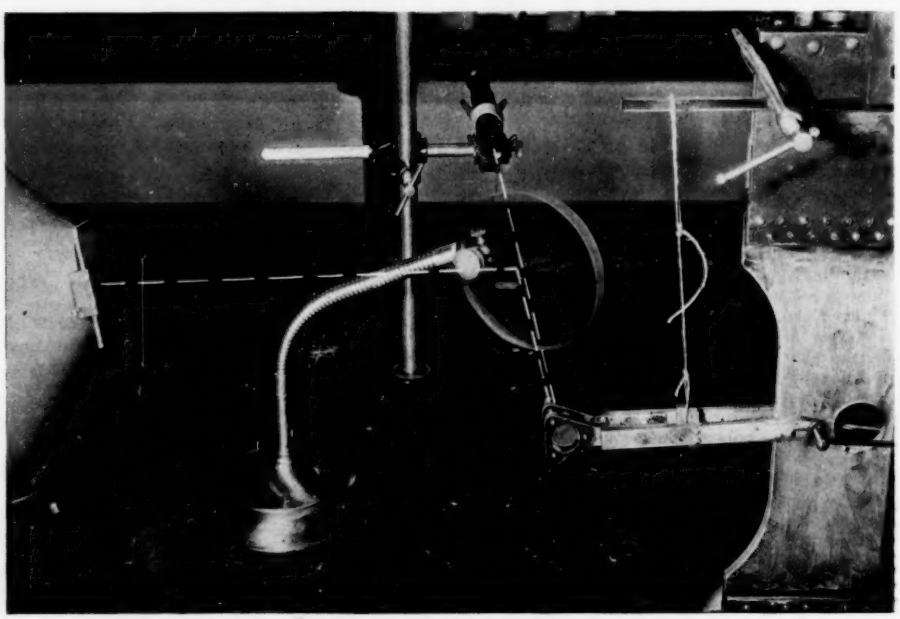


FIG. 2 THE APPLICATION AND METHOD OF READING THE STRAIN GAGE

given point in a specimen is independent of that made at any other point, so that small errors are not cumulative from point to point. A final plot of independent observations on consecutive points across a specimen gives an opportunity for "fairing" the curve and eliminating any accidental errors if such are found.

Limitations

To date the method outlined has been found applicable to all cases of plane stress. However, most plane models are constructed of material of considerable thickness in order to amplify the photoelastic effect, and in many cases the stress gradients are

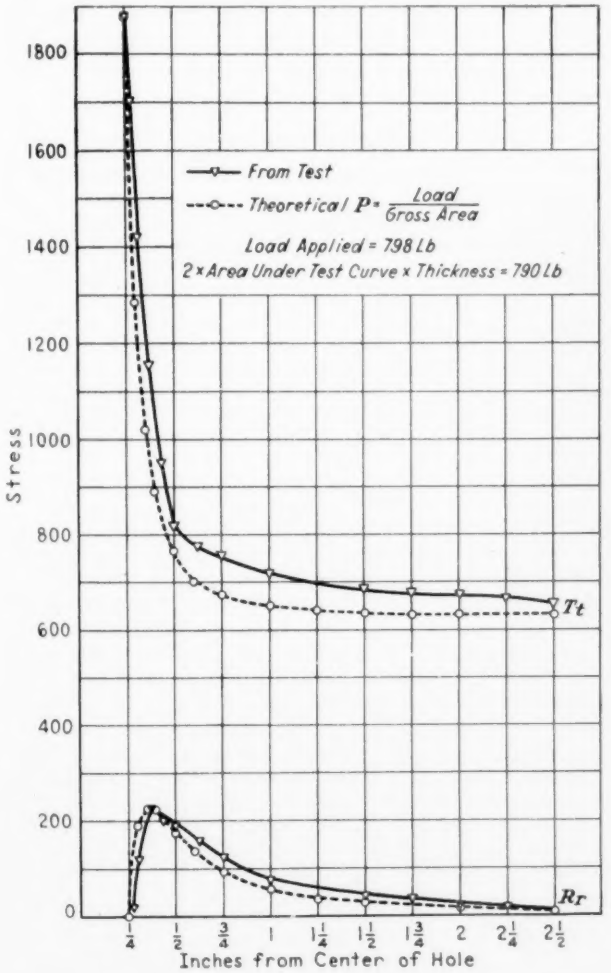


Fig. 3 Stress Distribution Across a Plate With a Central Hole

so large that a third dimensional restriction of deformation is apparent. Furthermore, if a boundary of the specimen is loaded (as in example 2, following) there is a further restriction of lateral deformation by the loading piece which is of detectable magnitude for a distance from the edge equal to one-half the thickness of the specimen, but is of importance for only a small fraction of this distance. While the lateral-deformation method particularly shows up these departures from the theoretical case because it measures them fairly directly, all other methods of photoelastic analysis are vitiated to an equal degree.

The errors here outlined are particularly prevalent at the boundaries of a specimen and, hence, any method of analysis which is dependent upon boundary stresses carries serious errors

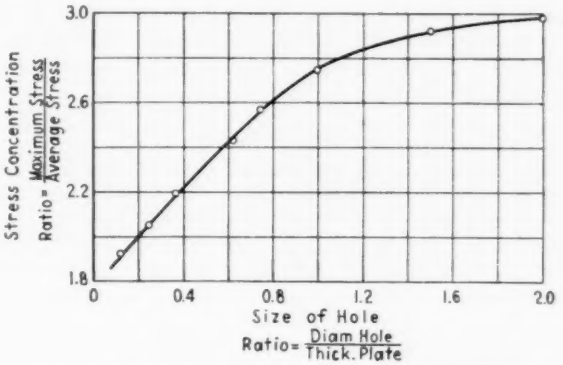


Fig. 4 Variation in Stress Concentration With Size of Central Hole in the Plate

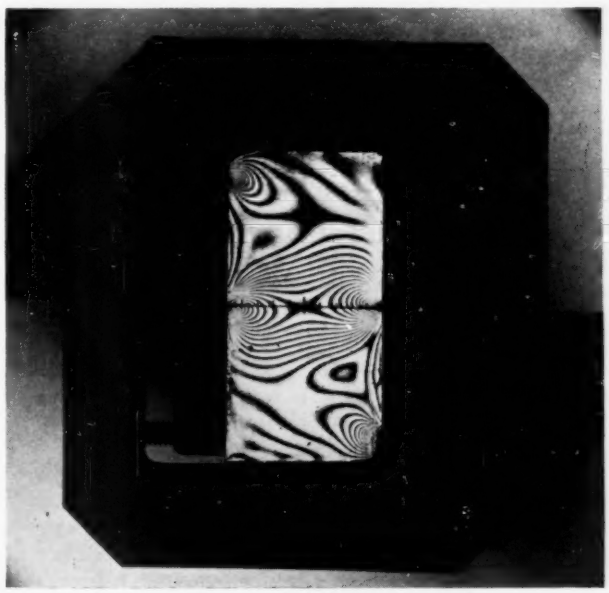


Fig. 5 Block in Direct Shear

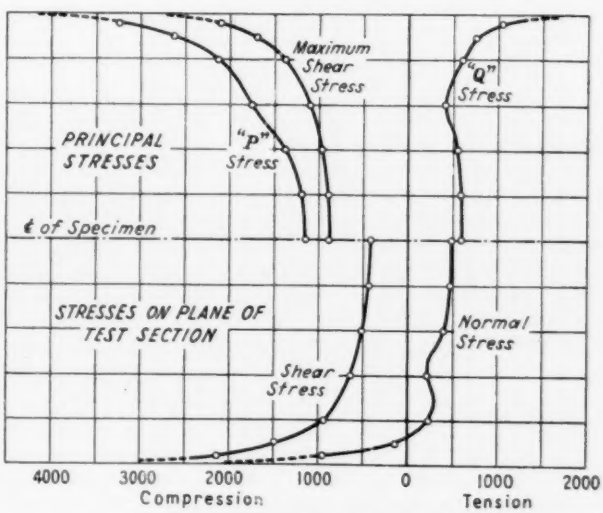


Fig. 6 Stresses in Shear Block

through the entire work. In contrast with this the lateral-deformation method gives accurate, independent results through the body of a specimen, and can be extrapolated to the extreme edge with fair accuracy.

Ease of Solution

While the amount of experimental work required in the application of this method may seem large, the desired results follow almost immediately from the experimental values without laborious calculations, graphical constructions, or auxiliary model making. In example 2, following, the entire problem was handled in about one day by a single observer. Attention may be concentrated on a few points of interest, in particular problems, and readings taken only at critical locations without covering areas of low stress.

Examples

Example 1. In Fig. 2 is shown a plate specimen with a central hole, which was investigated by the present method. The results were furnished by Mr. Maeser, and include the distribution of horizontal and vertical stresses on the cross-section at the hole as compared with theoretical values (Fig. 3) and the variation of the maximum stress with variations in the size of the hole (Fig. 4). This latter variation is due to a third-dimensional effect, since with a very small hole the stress distribution departs from that due to plane stress.

Example 2. Fig. 5 shows the photoelastic fringe distribution in a block of bakelite $\frac{1}{2}$ in. $\times$ $1\frac{1}{2}$ in. subjected to shear by the holders which are shown in silhouette. The stress distribution in this specimen is particularly difficult to determine by any method dependent on boundary stresses, since most of the boundary is restrained in an unknown manner. By the lateral-deformation method the principal stresses across the section of shear were determined, and from these and the usual isoclinics the normal and shearing stresses on the plane of the section were calculated. The results are shown in Fig. 6, and illustrate the uncertainty of the boundary conditions. As a check on the work the total shear on the section was found to differ from the applied load by less than 5 per cent.

Acknowledgments

The author is indebted to the staff of the Materials Testing Laboratory of the Massachusetts Institute of Technology, and particularly to Prof. A. F. Holmes for his support and advice; to Prof. I. H. Cowdrey for the photography; and to M. Maeser for the use of his results.

Recent Research Work in Vibration and Sound

By J. ORMONDROYD[1]

I—General

THE general field of vibrations, including the physical aspects of acoustics, was covered by some twelve books and about two hundred papers during 1934. Only those papers readily accessible to American engineers in publications usually available have been included in the following outline. The titles of all the papers listed have been translated into English for convenience in reference while the titles of the books are given in the language in which they were printed.

II—General Vibration Studies

Two general textbooks in English on all phases of vibration theory have appeared, a mathematical treatise built around actual technical problems, an advanced text by J. P. Den Hartog (1)[2] and an elementary textbook by E. B. Cole (2). The second volume of "Schwingungstechnik" by Ernst Lehr (3) appeared in 1934; the first volume of this book was published in 1930. The subject is also covered in the first part of volume 17 of the "Handbuch der Experimental Physik" (4), which was published in 1934, and (5) and (6) represent additional references.

III—Vibration Theory and Analysis

Special studies in the mathematical treatment of vibrations have appeared in numerous papers during 1934. On the energy method for critical speeds, a book on Rayleigh's principle by Temple and Bickley (7) gives a general résumé of the energy method of determining natural frequencies in engineering structures. An important paper bearing on the calculation of natural frequencies in undamped systems of many degrees of freedom by the direct use of matrices was published by Collar and Duncan (8). K. Klotter (9) gives a generalized outlook on the various physical forms of the matrix elements for characteristic equations of diversely coupled vibrating systems. Graphical methods of solving vibration problems received attention in three papers (10), (11), (12). Additional references are given under (13), (14), (15), (16), (17).

IV—Vibration Measurement

On the subject of vibration measurement only two papers are noted, (18) and (19).

V—Special Technical Problems

(*a*) *Vibration of Beams and Shafts, Rotor Balancing.* "Dynamik der Stabwerke" by Hohenemser and Prager (20), a book published in 1933, should be noted here since it gives a good generalized view of the modes of vibration in various combinations of beams common in structural engineering. See also (21). The old problem of shaft whirling still attracts attention in the technical literature (22), (23), (24), (25). The balancing of rotating machinery receives attention in papers describing a new automatic balancing machine (26), a new method of vector analysis from vibration amplitudes (27), and a new apparatus and associated method for field balancing (28). A paper by Galmiche (29) deals with the vibration of electrical machinery.

(*b*) *Torsional Vibrations.* Two books on torsional vibration have recently appeared, both in English and both published by John Wiley & Sons. A book containing practical methods for calculating vibration frequencies and amplitudes by Tuplin (30) gives a complete theoretical account of critical-speed calculations in systems of many degrees of freedom in unbranched and branched forms which are commonly met in practice. "Practical Solution of Torsional Vibration Problems" by W. Ker Wilson (31) as its name implies does not aim at the theoretical completeness displayed in (30). Four papers (32), (33), (34), (35) are devoted to torsional oscillations.

(*c*) *Damping of Vibrations and Spring Mounting.* The problem of vibrating systems operating at a critical speed gives great importance to the subject of damping and other means to limit the amplitudes of motion. Papers on damping in general appeared under the authorship of Spaeth (36), Miller (37), and Klein (38). Dynamic vibration absorbers received attention in four papers, respectively, by Inglis (39), Föppl and Knackstedt (40), Geislinger (41), and Koch (42). The paper by Koch (42) on "Mechanical Wave Filters" gives a generalized outlook on dynamic vibration absorption. The effect of transient conditions of vibration on spring-mounting design for electrical and reciprocating machinery was treated by Johnson (43) and the characteristics of rubber spring mountings was discussed by Thum and Oeser (44). Ghosh (45) discusses the impact effect on a damped string.

(*d*) *Maintained Oscillations.* The forced oscillations of simple systems at a natural frequency have been put to many uses. The pendulum clock is the oldest device which utilizes this principle. A book entitled "Pendules Électriques" by Granier (46) gives details of various electrical methods for maintaining and regulating the oscillations of clock pendulums. This little book contains information on the possible errors in frequency which can occur in pendulum designs. In (47) Sell discusses a mechanically controlled bolometer. Testing machines based on forced vibration at resonance and mechanical oscillators for other purposes are discussed by Spaeth in two papers (48) and (49). Stretched-wire vibrators, which have often been used as strain gages and thermometers by using the relationship between tension and natural frequency, are discussed by Stuart (50). Self-exciting tuning forks in radio circuits used as a standard of frequency form the subject of a paper by Dye and Essen (51). The natural vibrations of metallic reeds have been used frequently to determine the modulus of elasticity at room temperature and elevated temperatures. Davies and James discuss "An Electrically Maintained Vibrating Reed and Its Application to the Determination of Young's Modulus" (52). The paper is divided into two parts, theoretical and experimental.

(*e*) *The Vibration of Structures.* Two books appeared on this subject in 1934. C. E. Inglis discussed with great completeness the old problem of railway-bridge vibration in a book entitled "A Mathematical Treatise on Vibration in Railway Bridges" (53). Creskoff discusses the design of buildings to resist the destructive effects of earthquakes (54). Vibrations

---

[1] Experimental Division, Westinghouse Electric & Manufacturing Co., South Philadelphia, Pa. Mem. A.S.M.E.

[2] Numbers in parentheses refer to the bibliography at the end of the paper.

caused by moving vehicles are discussed in papers by Ramspeak (55) and Lürenbaum (56).

(*f*) *Transmission-Line Vibrations.* This subject was considered important enough in 1934 to give rise to seven papers (57), (58), (59), (60), (61), (62), and (63).

(*g*) *Ship Stabilization.* Although the Frahm tanks for stabilizing ships are almost as old as the present century, the theory underlying their action has been the subject of very recent controversy. O. Föppl clears up some of the questions in a paper on "The Theory of Frahm Ship Stabilizers" (64). The year 1934 saw the installation by a German firm of an activated Frahm tank in which the water in the stabilizing system does not create its own dynamical phase relationship to the ship's motion, but is artificially forced by pumps to take a more advantageous phase relationship. N. Minorsky, the inventor of an activated tank control, published a paper on the subject (65) which was the subject of editorial comment in the same issue. H. Hört gave a description of actual stabilizing installations (66) and E. G. Barillon prepared a new résumé of the whole theory of ship rolling (67).

(*h*) *Turbine-Blade Vibration.* When turbine blades break it is usually caused by "fatigue." The subject of turbine-blade vibration receives continuous attention from turbine designers. Papers by Peter (68) and Karas (69) appeared on this subject in 1934.

(*i*) *Hydrodynamic Oscillations.* This subject was treated in two papers by Hooker (70), (71).

(*j*) *Sound.* Since sound is produced by vibrating material bodies, books and papers on the physical aspects of sound are usually treatises on vibration and should be listed in this place.

Loud speakers, mechanical devices for pumping energy into the air, have been treated mathematically and structurally in two books by McLachlan (72) and (73). General treatises, in book form, on acoustic theory appeared under the authorship of Davis (74) and Olson and Massa (75). An article on acoustical measurements by Myer (76) appeared in 1934. Two other articles appear in the same issue, one on the application of sound waves in free media and the other on the theory of electro-acoustic transformers (telephones, microphones, loud speakers) (76).

Several papers on noise in electrical machines also appeared in 1934 (77), (78), (79). The noise meter as a shop tool for inspection is described by Johnson (80); and (81), (82), and (83) represent additional references on this subject.

BIBLIOGRAPHY

1 "Mechanical Vibrations," by J. P. Den Hartog, McGraw-Hill Book Company, New York, 1934.

2 "Theory of Vibration for Engineers," by E. B. Cole, Crosby Lockwood, London, 1935.

3 "Schwingungstechnik," vol. 2, by Ernst Lehr, Julius Springer, Berlin, 1934.

4 "Schwingungs und Wellen lehre, Ultraschallwellen," by E. Grossman, H. Martin, and H. Schmidt, Handbuch der Experimental Physik; vol. 17, part 1, Akad. Verlags gesellschaft, Leipzig, 1934.

5 Vibration Problems Discussed at the Physical Congress at Bad-Pyrmont, September, 1934. "Vibration of Elastic Cables," by R. Höger; and "Influence of Earth Tremors on Columns," by K. Ludwig, *Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 14, December, 1934, p. 361.

6 "Three Dimensional Vibration Problems," by B. Schlippe, *Luftfahrtforschung*, vol. 11, no. 7, Jan., 1935, p. 192.

7 "Rayleigh's Principle and Its Application to Engineering," by G. Temple and W. G. Bickley, Oxford University Press, London, 1933.

8 "A Method of the Solution of Oscillation Problems by Matrices," by A. R. Collar and W. J. Duncan, *Philosophical Magazine*, vol. 17, 1934, p. 865.

9 "The Coupling of Mechanical Vibration," by K. Klotter, *Ingenieur-Archiv*, vol. 5, 1934, p. 157.

10 "Graphical Evaluation of Resonance Curves," by H. Lorenz, *Ingenieur-Archiv*, vol 5, 1934, p. 376.

11 "Graphical Method for Representation of Unsymmetrical Dyads and for the Determination of Natural Frequencies and Amplitude Ratios of Two Bodied Vibration Systems," by G. Gerber and K. Klotter, *Ingenieur-Archiv*, vol. 5, 1934, p. 470.

12 "Locus Curves in Graphical Vibration Mechanics," by F. Reinhardt, *Ingenieur-Archiv*, vol. 5, 1934, p. 110.

13 "Harmonic Analyzer," by J. Harvey, *Engineering*, vol. 138, December 12, 1934, p. 667.

14 "Properties of Simple Electrical and Mechanical Vibrating Systems," by J. Fischer, *Archiv für Electrotechnik*, vol. 28, December 12, 1934, p. 774.

15 "Harmonic Analysis of Sound-Frequency Oscillations With a Stroboscopic Disc," by T. D. Nemes, *Philosophical Magazine*, vol. 18, 1934, p. 303.

16 "Operational Treatment of Certain Mechanical and Electrical Problems," by A. N. Lowan, *Philosophical Magazine*, vol. 17, 1934, p. 1134.

17 "Natural Frequencies and Vibration Forms of Beams Formed From Circular Arcs," by F. W. Waltking, *Ingenieur-Archiv*, vol. 5, 1934, p. 429.

18 "Measurement of Vibrations," by H. Sell and G. Turetschek, *Zeit. für technische Physik*, vol. 15, no. 12, 1934, p. 644.

19 "Indicator for Rapid Pressure Oscillations," by S. Berg, *Zeit. V.D.I.*, vol. 78, no. 44, November 3, 1934, p. 1295.

20 "Dynamik der Stabwerke," by K. Hohenemser and W. Prager, Julius Springer, Berlin, 1933.

21 "Motion of Beams With Moving Loads," by H. Steuding, *Ingenieur-Archiv*, vol. 5, 1934, p. 275.

22 "Whirling of Shafts," by D. Robertson, *Engineer*, vol. 158, August 31, 1934, p. 216; and September 7, 1934, p. 228.

23 "Vibration of Beams and Whirling Shafts," by H. H. Jeffcott. Proceedings, Royal Dublin Society, vol. 21, no. 10, 1934, p. 87.

24 "Oil-Film Whirl—A Non-Whirling Bearing," by B. L. Newkirk and L. P. Grobel, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-10, p. 607.

25 "Practical Methods for the Calculation of the Critical Velocities of Rotors," by G. Zerkowitz, *Ricerche di Ingegneria*, vol. 2, November, December, 1934, p. 245.

26 "The Measurement of Dynamic Unbalance on a Balancing Machine With Electromagnetic Control," by H. Hoppe, Mitteilungen des Wöhler-Institute, Braunschweig, H., part 17, Nem-Verlag, Berlin, 1933.

27 "Rotor Balancing," by J. Bromberg, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-14, p. 707.

28 "Dynamic Balancing of Rotating Machinery in the Field," by E. L. Thearle, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-19, p. 745.

29 "Vibration of Electrical Machinery," by C. Galmiche, *Revue Generale de l'Électricité*, vol. 35, June 2, 1934, p. 739.

30 "Torsional Vibration," by W. A. Tuplin, John Wiley & Sons, New York, 1934.

31 "Practical Solution of Torsional Vibration Problems," by W. Ker Wilson, John Wiley & Sons, New York, 1935.

32 "Torsional Vibration in Block Engines," by R. Grammel, *Ingenieur-Archiv*, vol. 5, February, 1934, p. 83.

33 "Calculation of Torsional Vibration Stress of Marine Oil-Engine Installation," by W. K. Wilson, *Engineering*, vol. 137, May 25, 1934, p. 582 and p. 611.

34 "Graphical Solutions for the Torsional Oscillations of Shafting," by K. Waimann, *Zeit. V.D.I.*, vol. 78, September 15, 1934, p. 1083.

35 "Damping of Engine Torsional Oscillations," by J. Geiger, *Zeit. V.D.I.*, vol. 78, no. 46, November 17, 1934, p. 1353.

36 "Damping of Oscillating Systems," by W. Spaeth, *Archiv für Electrotechnik*, vol. 28, no. 4, April 5, 1934, p. 257.

37 "Calculation of Vibration With Velocity Squared Damping," W. Miller, *Ingenieur-Archiv*, vol. 5, 1934, p. 306.

38 "The Measurement of Damping in Building Materials by Recording Their Resonance Curves," by W. Klein, *Ingenieur-Archiv*, vol. 5, 1934, p. 1.

39 "Vibration Neutralizer for Ships," by C. E. Inglis, *Engineering*, vol. 157, 1934, p. 380.

40 "Dampers for Pipe Line Vibrations," by O. Föppl and W. Knackstedt, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 4, no. 4, July-August, 1933, p. 173.

41 "Theory of Resonance Vibration Dampers," by L. Geislinger, *Ingenieur Archiv*, vol. 5, 1934, p. 146.

42 "Mechanical Wave Filters," by H. W. Koch, *Zeit. für Angewandte Mathematik und Mechanik*, vol. 14, June, 1934, page 173.

43 "Transient Vibrations of Machines," by W. E. Johnson, *General Electrical Review*, vol. 37, September, 1934, p. 423.

44 "Machines on Rubber Springs," by A. Thum and K. Oeser, *Zeit. V.D.I.*, vol. 78, no. 19, 1934, p. 587.

45 "Hertzian Impact of Elastic Hammer on a Damped Pianoforte String," by M. Ghosh, *Philosophical Magazine*, vol. 17, 1934, p. 521.

46 "Pendules Électriques," by J. Granier, Dunod, Paris, 1934.

47 "Mechanically Controlled Bolometer," by H. Sell, *Zeit. für technische Physik*, vol. 15, 1934, no. 3, p. 112; and no. 4, p. 129.

48 "Mechanical Oscillators," by W. Spaeth, *Electrotechnische Zeit.*, May 10, 1934, p. 465.

49 "Theory and Practice in Vibration Testing Machines," by W. Spaeth, *Zeit. V.D.I.*, vol. 78, no. 51, 1934, p. 1487.

50 "Valve Maintained Stretched Wire Vibrator," by W. S. Stuart, *Philosophical Magazine*, vol. 18, October, 1934, p. 566.

51 "The Valve Maintained Tuning Fork as a Primary Standard of Frequency," by D. W. Dye and L. Essen, Proceedings Royal Society of London, A, vol. 143, 1934, p. 285.

52 "An Electrically Maintained Vibrating Reed and Its Application to the Determination of Young's Modulus," by R. M. Davies and E. G. James, *Philosophical Magazine*, vol. 18, 1934, p. 1023.

53 "A Mathematical Treatise on Vibration in Railway Bridges," by C. E. Inglis, Cambridge University Press, 1934.

54 "Dynamics of Earthquake Resistant Structures," by J. J. Creskoff, McGraw-Hill Book Company, New York, 1934.

55 "Building Vibrations Caused by Earthquakes and Transportation," by A. Ramspeak, *Zeit. V.D.I.*, vol. 78, no. 22, 1934, p. 669.

56 "Vibration Theory in Transportation," by K. Lürenbaum, *Zeit. V.D.I.*, vol. 78, no. 39, September 29, 1934, p. 1131.

57 "Observations of the Causes and Limitation of Conductor Vibration on Transmission Lines," by H. Maass, *Electrotechnik und Maschinenbau*, vol. 52, January 14, 1934, p. 13.

58 "Method of Damping the Vibration of Overhead Cables," by M. Preiswerk, Association Suisse des Électriciens, *Bulletin*, vol. 25, May 11, 1934, p. 252.

59 "Effect of Vibration on Steel Cored Aluminum Conductors," by L. L. Ene, *Electrician*, vol. 112, p. 656. Discussion, p. 565, April 27, 1934, p. 565; and *World Power*, vol. 21, May, 1934, p. 247.

60 "Overhead Line Vibration," by E. M. Wright and J. Miné, Jr., *Electrical Engineering*, vol. 53, July, 1934, p. 1123.

61 "Transmission Line Breakage by Vibration," by W. Schwinning and E. Dorgesloh, *Zeit. für Metallkunde*, vol. 26, July, 1934, p. 162.

62 "Vibration Analysis of Transmission Line Conductors," by W. C. Buchanan, *Electrical Engineering*, vol. 53, November, 1934, p. 1478.

63 "Non-Swinging Transmission Line Conductors," by M. Preiswerk, *Electrotechnische Zeit.*, vol. 55, December 13, 1934, p. 1225.

64 "Contribution to the Theory of Frahm Ship Stabilizers" (Schlingertanks), by O. Föppl, *Ingenieur-Archiv*, vol. 5, 1934, p. 35.

65 "Ship Stabilization by Activated Tanks," by N. Minorsky, *Engineering*, vol. 158, 1934, p. 154.

66 "Description and Test Results of Constructed Ship Stabilization Installations," by H. Hört, *Schiffsbautechnische Gesellschaft*, Jahrbuch, vol. 35, 1934, p. 292.

67 "On the Theory of Double Systems of Rolling of Ships Among Waves," by E. G. Barillon, Trans. Institution of Naval Architects, vol. 76, 1934, p. 420.

68 "Calculation of Blade Vibrations in Steam Turbines," by W. Peter, *Brown Boveri Review*, vol. 21, May, 1934, pp. 78–82; July, 1934, pp. 123–127.

69 "Vibration of Steam Turbine Blades," by K. Karas, *Ingenieur-Archiv*, vol. 5, 1934, p. 325.

70 "Oscillations in High Speed Jets of Compressible Fluids," S. G. Hooker, *Philosophical Magazine*, 17, 1934, p. 651.

71 "Two Dimensional Oscillations in Divergent Jets of Compressible Fluid," by S. G. Hooker, Proceedings, Royal Society of London, A, vol. 145, 1934, p. 52.

72 "Loud Speakers," by N. W. McLachlan, Clarendon Press, Oxford, Eng., 1934.

73 "Elements of Loud Speaker Design," by N. W. McLachlan, Oxford University Press, London, 1935.

74 "Modern Acoustics," by A. H. Davis, Macmillan Co., New York, 1934.

75 "Applied Acoustics," by H. F. Olson and Frank Massa, P. Blakiston's Son & Company, Philadelphia, 1934.

76 "Akutische Messtechnik," by E. Myer; "Anwendungen der Ausbreitung des Schalles in Freien Medien," H. Hecht and F. A. Fischer; "Analytische Theorie der Elektro-aukstischen Wandler," H. Hecht and F. A. Fisher, vol. 17, part 2, Handbuch der Experimental Physik, 1934.

77 "Mechanical-Acoustic Behavior of Electrical Machines," by E. Lubske, *Zeit. für technische Physik*, vol. 15, no. 12, 1934, p. 652.

78 "Magnetic Noise of Electrical Machines," by M. DeGavre, Société Francaise des Électriciens *Bulletin*, vol. 4, December, 1934, p. 1211.

79 "Measurement of Noise From Small Motors," by C. G. Veinott, *Electrical Engineering*, vol. 53, December, 1934, p. 1624.

80 "Utility of the Noise Meter in Quantity Production," by W. E. Johnson, *General Electric Review*, vol. 37, 1934, p. 504.

81 "Acoustic Radiation Pressure on Spheres," by L. V. King, Proceedings, Royal Society of London, A, vol. 147, 1934, p. 212.

82 "Sound Radiation From a Condenser Discharge," by W. McFarland, *Philosophical Magazine*, vol. 18, 1934, p. 24.

83 "Damping Measurements on Sound Waves in Solid Bodies," by R. Schmidt, *Ingenieur-Archiv*, vol. 5, 1934, p. 353.

Design Data

It is important that the data contained in technical papers be made readily available to designing engineers. In order to satisfy these needs of industry, this section of the Journal will include a concise presentation of data and information drawn chiefly from papers previously published by the Applied Mechanics Division of The American Society of Mechanical Engineers.

The data are prepared by a subcommittee of the Applied Mechanics Division on strength of materials (R. E. Peterson, chairman), under the guidance of S. Timoshenko.

Working Stresses[1]

By C. RICHARD SODERBERG,[2] PHILADELPHIA, PA.

I INTRODUCTION

THERE is a considerable number of variables, outside of magnitude of stress, involved in the problem of strength, but the following have been found to be of fundamental significance.

1 *Temperature.* While it is impossible to speak of any well-defined boundaries between the different regions, it is permissible, for practical purposes, to speak of (*a*) normal temperatures, where the problem of strength is based on elastic behavior, and (*b*) elevated temperatures, where the problem of strength must be based on plastic behavior. For steel and its alloys, the limit between these two regions is approximately 250 C.

2 *Ductility.* A definite classification of the materials with regard to ductility is equally difficult, because the term has meaning only when the state of stress is specified. Nevertheless, if tensile tests are made of soft steel and cast iron, the results differ so fundamentally that we can regard the former as ductile and the latter as brittle. It is on the basis of such a difference that the materials are classified as (*a*) ductile materials, giving a standard tensile test with more than 5 per cent elongation, and (*b*) brittle materials, giving less than 5 per cent elongation.

3 *Time-Mode of Stress Application.* The time element has been found to be of particular significance. It is possible to distinguish between (*a*) steady stress and (*b*) variable stress, for which the problem of failure must be considered separately.

At the present time only the following combinations of these variables can be treated with any degree of certainty: (*a*) ductile materials under steady stress at normal temperatures; (*b*) ductile materials under variable stress at normal temperatures; (*c*) brittle materials; and (*d*) ductile materials under steady stress at elevated temperatures.

[1] This paper was prepared for the Design Data section of the JOURNAL OF APPLIED MECHANICS and contains material published in "Factor of Safety and Working Stresses," by C. Richard Soderberg, Trans. A.S.M.E., vol. 52, part 1, 1930, paper APM-52-2, pp. 13–28.

[2] Manager, Turbine Apparatus Division, South Philadelphia Works, Westinghouse Electric & Manufacturing Company, Philadelphia, Pa.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until Nov. 11, 1935, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

II DUCTILE MATERIALS UNDER STEADY STRESS AT NORMAL TEMPERATURES

THE TENSILE TEST

There are many physical tests of importance, but for purposes of predicting failure the tensile test is practically the only source of information. Of the quantities furnished by the tensile test, only the yield strength σ_s has received general sanction as a measure of the strength. This is based on more than an arbitrary selection, because the usefulness of most engineering structures is impaired when the plastic deformation reaches a value which is large in comparison with the elastic deformation. For materials having no well-defined yield point, the stress corresponding to 0.2 per cent permanent deformation is often used as the yield strength.

Under the conditions of the tensile test, the working stress is now defined by the relation

$$\sigma = \sigma_s/n \quad [1]$$

where n is the factor of safety. (It is more convenient mathematically and also somewhat more logical to use a factor of utilization $u = 1/n$, however, because of the wide usage of the factor of safety, it was considered advisable to retain its use in these articles.[1])

The general treatment of the problem of strength is postulated on the assumption that it is always possible to find a certain limiting stress which for more complicated cases plays the role of the yield strength of the tensile test.

The object of the following rules is to define the factor of safety, rather than to give specific instructions as to its absolute value. This must remain the prerogative of the responsible designer for whom these rules furnish accepted methods of interpreting his experience. It has become customary to regard $n = 1.25$ to 1.33 as minimum values to be used under most favorable circumstances with reliable materials subjected to accurately known stresses. For the opposite extreme, maximum values of $n = 3$ to 4 are regarded appropriate. If practical experience indicates a need for still higher factors of safety, this should call for a more thorough analysis of the problem, either in the evaluation of the stresses or in interpreting the circumstances of failure. Most applications fall in the region where $n = 1.5$ to 3.

THEORIES OF STRENGTH

In order to extend the treatment to other forms of stress application, it is necessary to refer to a *theory of strength*, estab-

lishing in more detail the actual mechanism of failure. A considerable amount of experimental and theoretical material is now available and the practical choice has narrowed down to two theories: (*a*) the maximum-shear theory, and (*b*) the theory of constant energy of distortion.

The Maximum-Shear Theory. The condition for yielding, according to the maximum-shear theory, is based on the assumption that yielding starts in planes of maximum shearing stress, when the shearing stress in these planes reaches a certain limiting value. If the principal stresses are σ_1, σ_2, and σ_3 in algebraic order of magnitude, the maximum shearing stress occurs in the diagonal planes parallel with σ_2, Fig. 1, and has the value $(\sigma_1 - \sigma_3)/2$. The intermediate stress is assumed to be without influence, a fact, therefore, which has thrown doubt on the absolute correctness of this theory. When $\sigma_2 = \sigma_3 = 0$, the maximum shearing stress is $\sigma_1/2$, occurring in planes at 45 deg to σ_1. In the tensile test, therefore, failure takes place when the maximum shearing stress reaches the value $\sigma_s/2$, where σ_s is the yield strength in tension.

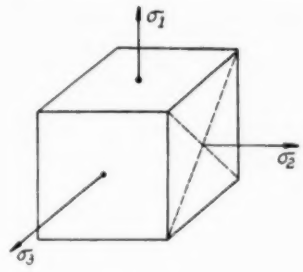


Fig 1

The Theory of Constant Energy of Distortion. The yielding of ductile materials has been found by careful experiments to obey the law

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_s^2 \ldots\ldots [2]$$

where σ_s again is the yield strength in straight tension. As far as the tensile test alone is concerned, the theory gives values identical with those obtained by the maximum-shear theory.

The greatest discrepancy occurs for the case of pure shear. This is obtained by making $\sigma_3 = -\sigma_1 = \tau$, $\sigma_2 = 0$, when Equation [2] becomes $\tau = (1/\sqrt{3})\sigma_s = 0.57\sigma_s$, as against $0.5\sigma_s$ on the basis of the maximum-shear theory.

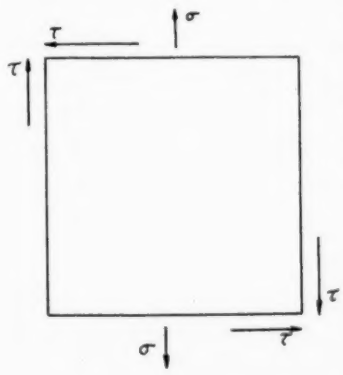


Fig 2

In the case of combined tensile stress σ and shearing stress τ, Fig. 2, the principal stresses are, in the order of magnitude:

$$\sigma_1 = 1/2[\sigma + \sqrt{(\sigma^2 + 4\tau^2)}]$$
$$\sigma_2 = 0$$
$$\sigma_3 = 1/2[\sigma - \sqrt{(\sigma^2 + 4\tau^2)}]$$

The condition of yielding then becomes $\sqrt{(\sigma^2 + 3\tau^2)} = \sigma_s$ instead of $\sqrt{(\sigma^2 + 4\tau^2)} = \sigma_s$ for the maximum-shear theory.

The experimental evidence is definitely in favor of the distortion energy theory, and if accuracy were the only guiding motive, the choice would certainly be in favor of this theory. The maximum-shear theory is so outstandingly simple, however, that it was considered advisable to use it as a basis for determination of working stresses.

Stress Concentration

Changes in shape of members, such as those due to fillets, holes, grooves, keyways, etc., cause local concentrations of stress. The stress-concentration factor k is defined as the ratio of maximum stress to nominal stress and is computed as $\sigma = P/a$ for tension-compression or $\sigma = M/(I/c)$ for bending. Values of k for various cases were given in the preceding article of the Design-Data series, in the June, 1935, issue of the Journal of Applied Mechanics, page A-67. For ductile materials, local concentrations of stress do not contribute to failure under steady stress, so that for this case stress concentration may be neglected.

Rules for Working Stresses for Ductile Materials Under Steady Stress at Normal Temperatures

1 Tension, compression, or bending, $n = \sigma_s/\sigma$ ..... [3]

2 Pure shear, $n = \sigma_s/2\tau$ ..... [4]

3 General case of combined stress, $n = \sigma_s/2\tau_{max}$ ..... [5]

4 Tension σ and shear τ, $n = \sigma_s/\sqrt{(\sigma^2 + 4\tau^2)}$ ..... [6]

The stresses are evaluated in each case without reference to local concentrations of stress.

III DUCTILE MATERIALS UNDER VARIABLE STRESS AT NORMAL TEMPERATURES

The Simple Fatigue Test

The general case of variable stress involves the combination of a steady stress σ_0 and a variable stress σ_v, Fig. 3. The strength properties under the influence of the steady component have been discussed in the preceding section. The property of strength is embodied, for practical purposes, in the yield strength σ_s. When the stress is alternating with the amplitude σ_v, the criterion of failure is the endurance limit,[3] denoted as σ_e.

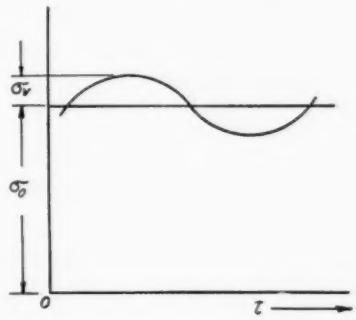


Fig 3

The most significant difference from the conditions for steady stress concerns the influence of stress concentration. Under variable stress both the average stress and the local maximum of the stress at any point influence the failure. The actual conditions depend on the sharpness of the stress concentration and the nature of the material.

Of equal significance is the dependence of the endurance limit upon other factors such as corrosive conditions, temperature of the atmosphere surrounding the specimen, and finish and surface treatment of the specimen. These create points of utmost importance which must be taken care of in each individual case. It is possible in many cases to conduct the tests so that the results include the effect of stress concentration. For the present purpose, it will be assumed that the true value of the endurance limit σ_e and the appropriate factor of stress concentration $k > 1$ have been established for the conditions of the problem in question.

Superposition of Steady and Variable Stress

Assuming the conditions for the separately acting stresses σ_0 and $k\sigma_v$ to be fully determinate, the case of simultaneous application of these stresses still raises an important problem. The experimental material is fairly extensive and does permit certain conclusions. Plotted in the manner shown in Fig. 4, the relation

[3] "The Fatigue of Metals," by H. F. Moore and J. B. Kommers, McGraw-Hill Book Company, New York, first edition 1927, p. 119.

between σ_0 and σ_v, corresponding to failure, is generally a curve going through (O, σ_e) but almost always falling outside the straight line connecting (O, σ_e) and (σ_s, O). In view of the uncertainty of the available material, it appears reasonable to treat the problem on the basis that this straight line is the actual line of failure, particularly since this is almost always on the safe side. The equation of this line is $\sigma_0/\sigma_s + k\sigma_v/\sigma_e = 1$, which thus expresses the combinations of σ_v and σ_0 which will cause failure. In order to obtain a factor of safety n, the stresses must follow the relation

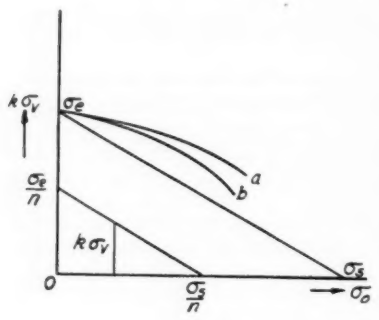


Fig 4

$$\sigma_0/\sigma_s + k\sigma_v/\sigma_e = 1/n \quad [7]$$

representing a straight line connecting the points $(O, \sigma_e/n)$ and $(\sigma_s/n, O)$. It is evident that $\sigma_e/k\sigma_v = n_v$ and $\sigma_s/\sigma_0 = n_0$ represent the factors of safety when the stresses are acting alone and hence this result may be written

$$1/n_x + 1/n_0 = 1/n \quad [8]$$

Equation [8] may also be written

$$1/n = [\sigma_0 + k\sigma_v(\sigma_s/\sigma_e)]/\sigma_s \quad [9]$$

in which form it expresses the fact that the variable stress can be added to the steady stress by "weighting" it with the factor $k(\sigma_s/\sigma_e)$.

Combined Stresses

The difficulties become greater when cases of combined stresses are considered, because very little in the way of experimental material is available on this point. Gough's work[4] on the fatigue of single crystals shows that fatigue failure is generally preceded by slip on planes having maximum-resolved-shear stress. Tests of low-carbon steel subjected to combined alternating torsion and alternating bending[4] have produced results agreeing closely with the distortion-energy theory. The ratio of endurance limits obtained from torsion tests and from bending-test averages somewhat greater than one-half for steels.[5,6] Taking the foregoing results into consideration it seems reasonable, for the present, to use the maximum-shear theory for variable stresses also.

Failure is thus defined by the relation

$$2\tau_0/\sigma_s + 2k_\tau\tau_v/\sigma_e = 1 \quad [10]$$

between the constant component τ_0 and the variable component $k_\tau\tau_v$ of the shearing stresses in the plane of failure. The factor of safety is found from the relation

$$1/n = 2\tau_0/\sigma_s + 2k_\tau\tau_v/\sigma_e \quad [11]$$

The method may be criticized on the basis that great care has to be exercised in selecting the appropriate combination. The steady components may cause failure in one plane, while the variable components may cause failure in another plane. Equation [10] should apply to the plane which makes n a minimum.

The conditions are best illustrated by applying this result to the common case of combined tension $\sigma_0 \pm k\sigma_v$ and shear $\tau_0 \pm k\tau_v$, Fig. 5. The combined shearing stress in the plane PP is made up of the constant component $(\sigma_0/2)\sin 2\alpha - \tau_0 \cos 2\alpha$ and a variable component $(k_\sigma\sigma_v/2)\sin 2\alpha - k_\tau\tau_v \cos 2\alpha$.

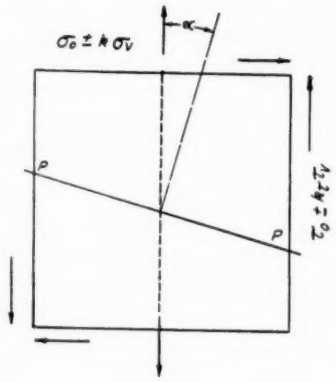


Fig 5

The value $1/n$ in this plane may thus be obtained by applying Equation [11] to

$$\frac{1}{n} = \frac{\sigma_0 \sin 2\alpha + 2\tau_0 \cos 2\alpha}{\sigma_s} + \frac{k_\sigma\sigma_v \sin 2\alpha + 2k_\tau\tau_v \cos 2\alpha}{\sigma_e} \quad [12]$$

The minimum value of n occurs when

$$\tan 2\alpha = -[\sigma_0 - k_\sigma\sigma_v(\sigma_s/\sigma_e)]/2[\tau_0 - k_\tau\tau_v(\sigma_s/\sigma_e)] \quad [13]$$

When $\sigma_s = \sigma_e$ this is the plane of maximum shear. The absolute value of the factor of safety in the plane is

$$n = \frac{\sigma_s}{\sqrt{[\{\sigma_0 + k_\sigma\sigma_v(\sigma_s/\sigma_e)\}^2 + 4\{\tau_0 + k_\tau\tau_v(\sigma_s/\sigma e)\}^2]}} \quad [14]$$

which thus establishes the working stresses for this case.

When the variable components of the stresses exist as a small addition to the steady stresses, it is sometimes convenient to introduce the "weighting" factors

$$k_\sigma = 1 + k_\sigma(\sigma_v/\sigma_0)(\sigma_s/\sigma_e) \text{ and } k_\tau = 1 + k_\tau(\tau_v/\tau_0)(\sigma_s/\sigma_e) \quad [15]$$

whence the expression for maximum factor of safety

$$n = \sigma_s/\sqrt{[(k_\sigma\sigma_0)^2 + 4(k_\tau\tau_0)^2]} \quad [16]$$

This method was in practical use before the derivation was known.

Rules for Working Stresses for Ductile Materials Under Variable Stress at Normal Temperature

The following rules may be formulated for working stresses:

1 Alternating stress in tension, compression or bending

$$n = \sigma_e/k\sigma_v \quad [17]$$

2 Alternating stress in shear

$$n = \sigma_e/2k\tau_{v\,\max} \quad [18]$$

3 Combinations of steady and alternating stress

$$1/n = \sigma_0/\sigma_s + k\sigma_v/\sigma_e$$

$$n = \sigma_s/[\sigma_0 + k(\sigma_s/\sigma_e)\sigma_v] \quad [19]$$

4 General case of combined variable stress. Evaluate the constant component τ_0 and the variable component $k\tau_v$ of the shearing stress in the plane of minimum n, where

$$1/n = 2[(\tau_0/\sigma_s) + (k\tau_v/\sigma_e)] \quad [20]$$

5 Special case of combined variable tension stress and variable shearing stress

$$n = \frac{\sigma_s}{\sqrt{[\{\sigma_0 + k_\sigma\sigma_v(\sigma_s/\sigma_e)\}^2 + 4\{\tau_0 + k_\tau\tau_v(\sigma_s/\sigma_e)\}^2]}} \quad [21]$$

[4] "Crystalline Structure in Relation to Failure of Metals—Especially by Fatigue," by H. J. Gough, Proceedings A.S.T.M., vol. 33, 1933, p. 3.

[5] "The Fatigue of Metals," by H. F. Moore and J. B. Kommers, McGraw-Hill Book Company, New York, first edition 1927, p. 147.

[6] "Kerb- und Korrosionsdauefestigkeit," by P. Ludwik, *Metallwirtschaft, Metallwissenschaft, Metalltechnik*, vol. 10, Sept. 11, 1930, pp. 705–710.

Discussion

Ordinate Systems for Quadratures Similar to Tchebycheff's Ordinates[1]

L. T. Fleming.[2] Professor Karelitz' paper is an interesting application of approximate-quadrature theory to a specific type of problem. The author compares the method he has developed with that of Tchebycheff, but not with Gauss's formula nor with those using a rational integral polynomial of high-degree and equidistant ordinates.

The accuracies of approximate quadrature formulas depend not only on the number of ordinates used, but also on the goodness of fit of the function that is substituted for the given curve or function. Good practice ordinarily requires that the error in computation should, at most, not exceed that due to scaling measurements or the inaccuracies of fabrication. It is this writer's opinion that, in a majority of cases, formulas employing equidistant ordinates will give a better fit than those in which the ordinates are not equidistant.

As an example, consider, for shapes such as these water lines, the tenth-degree polynomial with intervals between ordinates of $(2L/10)$. The coefficients for computing the moments and the whole and half areas above the x-axis of such a curve follow. As the formulas are equally applicable when the end ordinates are not zero, coefficients for y_5 and y_{-5} are given. The author's notation is followed except that the symbol L, for the half length, replaces the letter l.

Whole area, from $x = -L$ to $x = L$:

$$A_1 = (2L/10)\ (0.268341\,(y_{-5} + y_5) + 1.775359\ (y_{-4} + y_4) - 0.810436\ (y_{-3} + y_3) + 4.549463\ (y_{-2} + y_2) - 4.351551\ (y_{-1} + y_1) + 7.137646\ y_0)$$

Half area, from $x = 0$ to $x = L$ (reverse the ordinates for area from $x = -L$ to $x = 0$):

$$A_2 = (2L/10)\ (-0.006001\ y_{-5} + 0.066988\ y_{-4} - 0.341276\ y_{-3} + 1.050153\ y_{-2} - 2.184818\ y_{-1} + 3.568823\ y_0 - 2.166733\ y_1 + 3.499310\ y_2 - 0.469159\ y_3 + 1.708371\ y_4 + 0.274343\ y_5)$$

Moment about y_0 of whole area, from $x = -L$ to $x = L$:

$$M_1 = (2L/10)^2\ [1.341707\ (y_5 - y_{-5}) + 7.101438\ (y_4 - y_{-4}) - 2.431307\ (y_3 - y_{-3}) + 9.098926\ (y_2 - y_{-2}) - 4.351551\ (y_1 - y_{-1})]$$

Moment about y_0 of half area, from $x = 0$ to $x = L$ (reverse the ordinates and multiply by (-1) for moment about y_0 of area from $x = -L$ to $x = 0$):

$$M_2 = (2L/10)^2\ [-0.027986\ y_{-5} + 0.311971\ y_{-4} - 1.585828\ y_{-3} + 4.858778\ y_{-2} - 9.993580\ y_{-1} + 14.614977\ y_0 - 14.345131\ y_1 + 13.957704\ y_2 - 4.017135\ y_3 + 7.413408\ y_4 + 1.313721\ y_5]$$

The areas and moments of the curves a and b, Fig. 5 of the paper, used by the author as examples, computed by the above formulas, differ by the following per cents from the exact values: curve a, area, —0.13 per cent, moment, —0.30 per cent; curve b, area, +0.0017 per cent, moment, +0.0041 per cent. It will be noted that these errors are appreciably smaller than those reported by the author in his Table 9.

The statement in the text that "...Tchebycheff ordinates do not give... (the) moment with satisfactory accuracy," might be misleading; the moment being the area under a curve, can of course be computed by these ordinates to any reasonable degree of accuracy. The Tchebycheff ordinates used for the area computation, multiplied by their respective abscissas, should give the moment of the whole figure, from $x = -L$ to $x = L$, with, usually, but slightly less accuracy for an unsymmetrical figure than that obtained for the area. But from geometrical considerations rather than for the reason cited in the text, we cannot expect to obtain a satisfactory moment of the half area by use of ordinates taken at the same points as used for the area of the whole figure, as the moment curve then has a sharp break at $x = 0$ and is not the smooth curve required by the Tchebycheff polynomial.

This difficulty is admirably overcome by the ordinates proposed by the author, but it should be noted that they do not give an exact moment for curves which are exactly integrated as to area, a possible defect not inherent in the equidistant-ordinate formulas. It will require considerable experience with these ordinates to give convincing proof that the polynomial which they integrate gives as good a fit to the general run of curves as a polynomial with no terms missing and no restrictions on the parameters.

S. J. Mikina.[3] The appearance of Professor Karelitz' paper with its elaboration on Tchebycheff's work of 70 years ago, prompts the writer in turn to resurrect a little-known quadrature formula of even more ancient antecedents. A good and sufficient reason for this is that the formula in question is superior in point of simplicity of application and accuracy of results to anything of the kind in present-day literature on mensuration and well deserves to be brought to light and given recognition. Reference is made to the formula in the book, "The Calculus of Finite Differences," by George Boole (the first edition of which appeared in 1860), who attributes it to an English mathematician of an earlier date named Weddle. Weddle's rule for obtaining an approximate expression for the area under a curve requires that the given area be divided into six portions bounded by seven equidistant ordinates, as shown in Fig. 1 of this discussion. If the common distance between these ordinates is h, the area is given by

$$S = \frac{3h}{10}\left[y_0 + y_2 + y_4 + y_6 + 5(y_1 + y_3) + 6y_3\right]$$

This practical rule, which ought to find a place in every engineer's note-book, may be conveniently kept in mind by the following mnemonic:

The proposed area being divided into six portions by seven equidistant ordinates, add into one sum the even ordinates, 5 times the odd ordinates, and the middle ordinate, and multiply the result by $^3/_{10}$ of the common distance of the ordinates.

The theoretical basis for the rule was laid by evaluating the area integral of a sixth-degree polynomial passing through the

[1] By G. B. Karelitz. Published in the March, 1935, issue of the Journal of Applied Mechanics (A.S.M.E. Transactions).

[2] Santa Cruz, Calif. Assoc-Mem. A.S.M.E.

[3] Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. Jun. A.S.M.E.

TABLE 1 APPLICATION OF WEDDLE'S RULE TO KARELITZ' CURVES *a* AND *b* OF FIG. 5

	Curve *a*				Curve *b*			
	Area		Moment		Area		Moment	
	Value	% error	Value	% error	Value	% error	Value	% error
Analytical value—exact.....	0.7624	....	0.3064	....	0.581	...	0.200	....
Weddle's rule.....	0.7628	+0.05	0.3071	+0.20	0.581	0	0.200	0
Karelitz' rule with 4½ ordinates...	0.7610	−0.20	0.3045	−0.70	0.581	0	0.201	+0.50

seven ordinates. The remarkable simplicity of the numerical constants in the rule is due to the application of the calculus of finite differences to the evaluation of the area integral in question. By expressing the polynomial in terms of successive finite

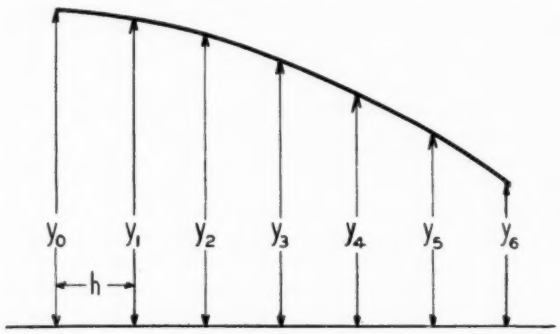


FIG. 1

differences in a manner analogous to the representation of a function by Taylor's series in terms of its successive differential coefficients, it was found possible to gage accurately the error committed in neglecting certain terms and in this way to simplify the final expression by purging it of all arithmetically complex, but demonstrably negligible, factors.

The application of Weddle's rule to Professor Karelitz' clinical curves $y = 0.7\ (1 - X^3) + 0.3\ (1 - X^6)^2$, designated by him as curve *a* in Fig. 5 of the paper, and $y = 0.935 \cos\ (\pi\ x/2) + 0.065 \cos\ (3\pi x/2)$, designated as curve *b* in Fig. 5 of the paper, gives the results shown in Table 1 of this discussion for the region $X = 0$ to $X = 1$.

The value of the area under curve *b* was given by Weddle's rule as being equal to 0.5814, correct to four places. The somewhat unfavorable comparison of this value with the value of 0.584, given as exact in Professor Karelitz' paper, provided the incentive for a recalculation of the exact value. This was found to be 0.5814, thus checking Weddle's value. For both curves, Weddle's-rule values are consistently better than those determined with the formulas considered in the paper.

The differences in error between Weddle's rule and Professor Karelitz' proposed rules are hardly great enough to argue about, however, and any preference for one or the other on the basis of minimum error must be a matter of personal taste. From the point of view of ease of application, however, Weddle's rule with its equidistant ordinates has the advantage over the peculiar ordinate spacing required by Professor Karelitz' rules. Just as a matter of simple drafting it is apparent that it is much easier, and more conducive to accuracy in the final result, to divide a line into six equal parts by the usual geometrical construction than to lay off such abscissas as 0.27047, 0.41791, or 0.65263 times the length of the line.

Weddle's rule will be found particularly useful in such applications as the determination of moments of inertia of irregular shapes, such as propeller- or turbine-blade sections, in the evaluation of indicator-card areas, and in the evaluation of definite integrals which cannot be integrated in terms of elementary functions. In the case of indicator-card areas it is suggested that the diagram be divided up so as to avoid applying the formula over regions of too sharp a variation in ordinate heights. This may be done in any case where extreme accuracy is desired.

AUTHOR'S CLOSURE

The principal value of Tchebycheff's coefficients, or those proposed in the paper, is the possibility of obtaining the area and moment of curves with a small number of ordinates and without multipliers. As mentioned, it is of primary use for practical shipbuilders when they compute the stability elements of a ship's hull. Certainly, there are numerous other formulas which give the area with a satisfactory accuracy, but the Tchebycheff method involves the least amount of labor.

The ship's lines are, with our present knowledge, empirical lines, and it is hardly possible to determine which member of the polynomial series would be the most important. In fact, there is no reason why the powers of the polynom should be integral numbers, or why the water-lines would not be represented better by a sum of trigonometric functions. Experience has shown that Tchebycheff's formula with 9 ordinates (four sections fore and four sections aft) gives results of sufficient accuracy for practical ship calculations.

The author takes this opportunity to thank Mr. Mikina for calling attention to the arithmetical error in the example *b*. It seems that the half-length of the "clinical" lines was divided into six parts for the application of Weddle's formula. This would require 13 ordinates for the application to a ship's water line instead of 9, as with Tchebycheff's or the author's ordinates.

Pitting Due to Rolling Contact[1]

GEORGE W. ALLAN.[2] The author is to be congratulated for his interesting paper which clears up many of the obscurities regarding the surface failure of metals under rolling stresses. He has been successful in obtaining experimental results free from the troubles encountered by other experimenters in the same line of work. He has also most ingeniously analyzed the effect of lubricant on the rolling stresses.

There are a few minor matters on which the writer would like to have Mr. Way's opinion. In the pictures of the damaged rolls, the pitting or fine scoring, depending on the running conditions, appears to be heaviest at the center of the surface, there being a narrow band of surface at the ends which shows practically no damage. This may be due, in the writer's opinion, to either of two causes: The metal in the immediate vicinity of the edge, being less constrained than that in the middle, may not experience as much contact pressure. The second cause might arise from a type of error which the writer thinks would not be easy to adjust or measure in the author's machine. If the axes of the two rolls while lying in parallel horizontal planes were not parallel with each other so that those lines on the two rolls, which under correct conditions when the rolls were just making contact formed the common line of contact, crossed each other instead at a slight angle, we would get point contact. Under load, this would become an elliptical contact area so that the pressure at the center would be greater than that at the ends.

The author mentions that no slip took place between his rolls. His method of proving this seems to have been to turn the rolls around once and to ascertain that the spacing of two dents on one roll caused by one protuberance on the other was equal to the circumference of the other. This was hardly running conditions and it would have been interesting to attach counters on each

[1] By Stewart Way. Published in the June, 1935, issue of the JOURNAL OF APPLIED MECHANICS (A.S.M.E. Transactions).

[2] Department of Engineering Mechanics, University of Michigan, Ann Arbor, Mich.

roll shaft to ascertain whether any slow creep took place over a period of running time.

The writer would like to know whether any oxide of iron such as the author found after a dry test was present in the oil after a test with lubricated surfaces. Manufacturers of friction drives ap-

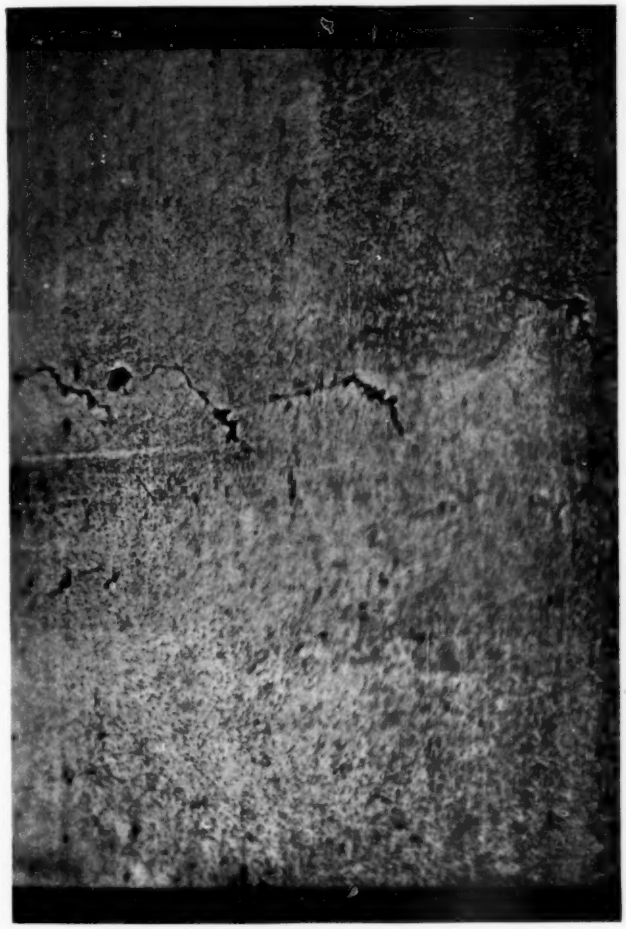


FIG. 1 (× 100)

pear to have found that this oxide formed even with lubricant present. Of course this may be due to the fact that a friction drive using steel disks experiences sliding abrasion to a great extent whereas in the author's tests, this was practically absent.

The writer notes that in the photographs showing the development of a pit that the crack, which was the first visible stage of its development was a short isolated and comparatively straight one. In the tests on which the writer is at present engaged and in which he uses smaller and harder rolls, the cracks are spread over the surface generally before the failure becomes general. The accompanying photomicrograph, Fig. 1 (100 magnifications), shows the type of crack which the writer has found.

The author's analysis of pressure forces due to the oil affords an interesting picture of the forces produced by oil in very high-speed gear teeth and other fast moving machinery. Has the author considered the boundary film properties of his lubricant in determining the point at which metal-to-metal contact takes place? With boundary lubrication, a heavier load would have to be applied to make metal-to-metal contact than if there were none.

Finally the writer would like to know whether the surface temperature of the rolls rose much while running.

E. LEONARD MARMONT.[3] Many theories have been advanced in connection with pitting of cylindrical surfaces when in rolling contact under load. Mr. Way's paper shows that he has devoted much thought to the subject and as far as his experimenting goes his conclusions are justified.

Those men engaged in the rolling of sheet and tin plate who constantly give their attention to pitting (or spalling) of mill rolls may first turn to the theory that spalling is the result of creeping. For the sake of explanation we may study the action of a plain (unthreaded) rubber tire when subjected to load and in rolling motion. As shown in Fig. 2 of this discussion, the rubber

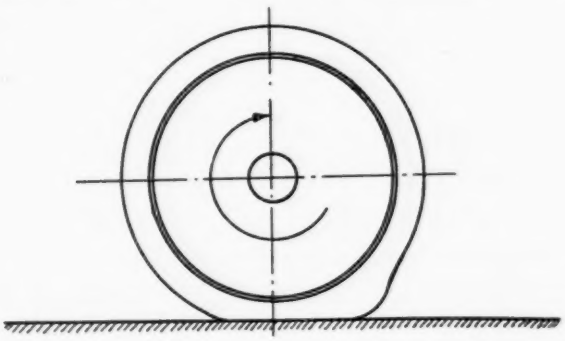


FIG. 2 CREEPING OF TIRE TREAD DUE TO LOAD AND ROLLING MOTION

is being compressed with the result that creeping takes place in front of the vertical center line in the direction of rolling. This creeping is continuous except when load or road conditions cause that part to snap back with the result that excessive wear takes place at this point. It is for this reason a smooth rubber tire does not wear evenly but develops cavities. For the purpose of relieving this creeping action and to provide better traction, tires are made with a thread design which allows a free displacement. According to the theory, a similar creeping action takes place in a mill roll and eventually causes spalling.

A second theory maintains that spalling is caused by an unequal stress distribution set up in the roll at the time of its manufacture combined with the stress produced when rolling under heavy load. There seem to be certain similarities between this theory of spalling and our equally uncertain knowledge concerning the cause of earthquakes. It is a well known fact that in the cooling of a solid, non-homogeneous body such as the earth, unequal stresses must necessarily be set up in the outer regions. To these stresses present in the crust of the earth must be added frictional stresses caused by the earth's contact with the surrounding atmosphere. Some scientists say that because of the earth's high velocity of over 1000 mph, this friction is influential in connection with earthquakes. In the manufacture of mill rolls high stresses are similarly set up in the outer parts of the rolls due to cooling. According to the theory, we add to these stresses those resulting from more or less impure rolling contact. Even though a comparatively long period may elapse before spalling occurs the roll will finally spall because the stresses are accumulative. It is thought that if the mill rolls were heat-treated correctly to relieve the accumulated stresses, the spalling problem would be overcome. The correctness of these or any other theories can be settled only by scientific research.

It seems appropriate to mention a mill incident, of recent date. It was observed that the strip on a four-high cold mill was being marked by a spalled roll. The mill was stopped immediately, the spalled roll exchanged for another and placed on the floor.

[3] Engineer-Draftsman, American Sheet and Tin Plate Co., Gary, Ind. Jun. A.S.M.E.

Examination of the roll showed a spalled part approximately equal to the cross-sectional area of an ordinary pencil. A few minutes elapsed when suddenly a 35- or 40-sq in. piece disengaged itself from the surface of the spalled roll and shot 20 ft into the air. Luckily, no one was hurt. Such occurrences are, fortunately very infrequent. But some mills provide a pit in which rolls are put immediately upon removal from the mills. The rolls remain in the pit until they have acquired room temperature.

Cold-rolling of tin plate is being done both with and without the use of oil on the rolls and spalling occurs in both cases. Oil is used as a cooling medium on mills working under screw-down pressures of about 70,000 lb per in. of roll face but it is not used on so-called skin-pass mills where the highest pressures are about 20,000 to 25,000 lb per in. of roll face. To say that the presence or absence of oil has no effect on spalling would be a hasty statement since the fact remains that mills using oil suffer more lost time due to spalled rolls than mills operating dry; although that could be attributed to higher screw down pressures.

Opinions as to the cause of spalling are more diversified than this discussion shows. Thus, in one article which appeared in *Blast Furnace and Steel Plant*, August, 1925, Harold Harris makes the following statement: "To avoid spalling, a plate-mill roller should carry water on the working surfaces of his rolls at all times." Evidently this implies that the temperature of the roll is a factor in spalling.

The elimination of the present confusion in regard to spalling would be of immense value. It is, therefore, hoped that more experimentation will be undertaken with full understanding of the complexity of the problem.

HAAKON STYRI.[4] The results obtained by Dr. Way are very interesting but it seems that the conclusions he has drawn are much too sweeping. It is possible that publication of more detailed data will show a better foundation for his conclusions but objections can be raised to most of them, even if applied only to the test conditions used by Dr. Way. Still greater objection can be raised against applying the conclusions to antifriction bearings.

The authors experiments consistently showed that there was no pitting when the rollers were run without lubricant. It would be desirable to learn how much the rollers lost in diameter during these tests due to oxidation of the contact surfaces and also if there was any measurable small slip between the rollers due to friction in the journal bearings. Since only one roller was driven, it seems that precautions should have been taken to mount the idle roller in ball bearings to reduce the friction. It is then possible that a different result would have been obtained. At least one fact observed in practical operation suggests this, namely, that locomotive tires which run without lubrication on the rail can pit or spall as shown in Fig. 3 of this discussion, in which case the spalling has progressed further than recorded by Dr. Way. His experiments have shown that under the conditions used in the tests, apparently all pitting starts from the surface and that it occurs only when a lubricant is used. It is not permissible to conclude that this also will be the case in other applications. If the initial crack starts at a point below the surface of cylindrical rollers and progresses to the surface along the direction of an obtuse cone it may break through and appear on the surface as a lip extending in the direction of the rolling. Fig. 4 of this discussion shows the beginning of a crack below the surface of a ball bearing where no crack was visible in the ball path even by etching. Fig. 5 shows the results of plastic deformation in a ball-bearing race due to running under heavy load. The structure is developed by etching in nitric acid and alcohol and is magnified 1000 times. Ball and roller bearings have, of course, highly polished rolling surfaces, and a hardness over 60 Rockwell, recorded on the *C*-scale.

In all kinds of endurance tests on ball and roller bearings, the dispersion in life before failure of an adequate number of specimens at any given load is very great, frequently of the order of 1 to 10. The dispersion in life before flaking or pitting in ball and roller bearings is of this order or even greater under all test conditions, including test conditions for roller bearings similar in nature to those used by Dr. Way for rollers. It seems reasonable, therefore, to expect that Dr. Way should obtain very



FIG. 3 SPALLING OF A LOCOMOTIVE TIRE



FIG. 4 BEGINNING OF A CRACK BELOW THE SURFACE OF A BALL BEARING

[4] Director of Research, SKF Research Laboratories, SKF Industries Inc., Philadelphia, Pa. Mem. A.S.M.E.

great dispersion in life before pitting is observed under any given test condition. He states that the number of cycles necessary for the beginning of pitting is practically independent of the hardness, although he observes that a harder surface resists pitting better. In roller-bearing practice, the relationship between load and average life before failure due to pitting or flaking has been quite well established and can be expressed mathematically by an empirical formula

$$P = C/n^{0.3}$$

where P is load, n is the life in cycles, and C is a constant depending on the material and construction of the bearing. In



FIG. 5 RESULTS OF PLASTIC DEFORMATION IN A BALL-BEARING RACE

this connection it would be of interest to record the lives before pitting for the loads used in Dr. Way's Fig. 6. The life under any given load condition does depend greatly on the hardness of the material but there is no direct relationship. There is a distinct and sharp drop in life with decrease in hardness value from about 59 Rockwell, C-scale, down to lower values. Very few test data are available for ball or roller bearings with a hardness below 55 Rockwell, C-scale, because the life is so much reduced that the bearings would be considered defective. In order to obtain fair average values at least 20 tests must be run for each load condition.

We have never been able to find any relationship between failure due to pitting and viscosity of lubricant, although a great number of tests have been run with widely different oils, under conditions varying from flood lubrication to insufficient quantities of oil to operate the bearings properly.

It is possible that Dr. Way has been successful in identifying a special phenomenon entering into the complex mechanism of fatigue failure. However, it is not safe to generalize results which were obtained under a particular set of conditions. As stated previously by the writer, established facts relating to the development of flaking or fatigue in antifriction bearings are at variance with the results revealed by Dr. Way's tests. The fatigue failure due to surface flaking in antifriction bearings is a matter of great practical interest, and we are forced to conclude that the action described by Dr. Way is in this instance entirely subordinated to other influences which are of controlling importance.

AUTHOR'S CLOSURE

Considering Mr. Styri's question on the necessity of lubrication for pitting, the pitting of the locomotive tire shown in Fig. 3 of the discussion may have been promoted by water on the rails acting as the lubricant. As to tangential traction in the pitting tests affecting the results, it can be stated that this effect was minimized by mounting all rollers in antifriction bearings. The rate of wear of the dry tested rollers was sometimes small and sometimes large. In one case, a roller of 250-VPN hardness run dry for 12 million cycles at a load of 2500 lb per in. decreased only 0.0003 in. in diameter.

Another open question is, "Exactly where do the pitting cracks start?" Figs. 6 and 7 of this discussion show pitting cracks in

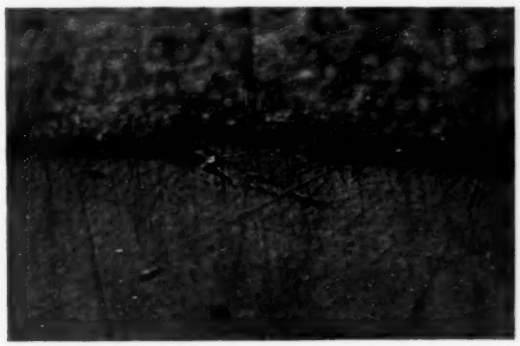


FIG. 6 CROSS-SECTION OF PITTING CRACK (× 300)



FIG. 7 CROSS-SECTION OF PITTING CRACK (× 300)

cross-section which are not more than 0.0008 and 0.002 in. deep, respectively. The depth of the point of Hertz' maximum-shear stress is 0.007 in. Other minute cracks have also been observed in section. Therefore, we can say that in the case of rolling cylinders (the stress system is different for ball bearings) pitting cracks start somewhere in a surface layer of material which is thin compared to the depth of the point of Hertz' maximum shear. It is very interesting that Mr. Styri has obtained illustrations of cracks in ball bearings beneath the surface. Can we be sure these cracks are prepitting cracks?

The antifriction-bearing manufacturer is interested in relations of average life to load and hardness. The gear manufacturer, on the other hand, wants to be sure that his gear will not fail by pitting. The present pitting studies have been made more from the standpoint of the needs of the gear industry. Therefore, it has not been the author's object to conduct an investigation of the average life of the test rollers at different loads and hardnesses, but rather to find conditions of load, hardness, viscosity and other factors which will insure against pitting.

Since preparing the paper, the author has had an opportunity

to study the effects of hardness variation on load-carrying capacity. One lubricant was used in these tests, and the same steel was used with the same treatment in all cases, except that drawing temperatures were varied to secure different hardnesses. If different types of heat-treatment are used giving different structures, there is evidence that the relation of load and hardness will vary. Fig. 8 of this discussion shows the results of a group of tests. From this series it appears that if the maximum

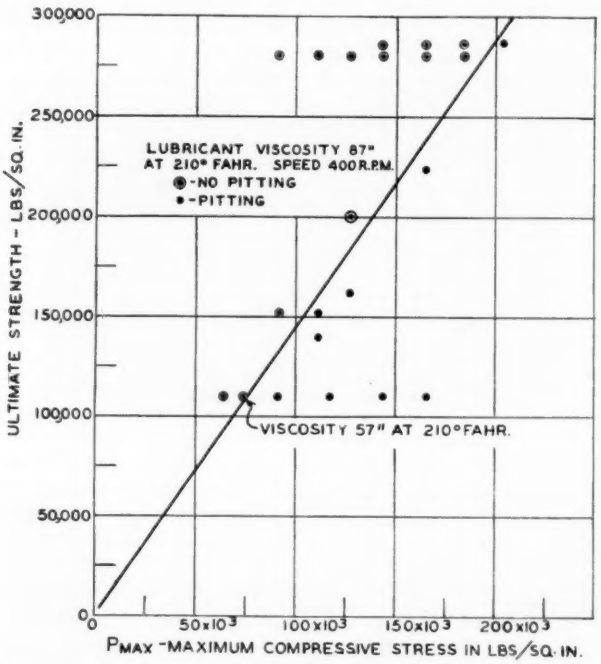


FIG. 8 LOADS THAT CAUSE PITTING OF QUENCHED AND TEMPERED 0.40-C STEEL ROLLERS

compressive stress is less than 0.7 of the ultimate strength, pitting will not occur.

If p_{max} is 0.7 of the ultimate strength, the maximum range of shear stress is only ± 0.175 of the ultimate strength, whereas the shear endurance limit is of the order of ± 0.25 to ± 0.28 of the ultimate. (The maximum range of shear stress rather than the maximum shear stress should be considered when determining the cause of fatigue cracks.) Furthermore, the point where the range of the shear stress is a maximum is 0.005 in. below the surface for a load of 2500 lb per in. Thus, it is apparent that the formation of pitting cracks in the very beginning cannot be due to repetitions of the theoretical Hertz stress system for rollers that are perfectly cylindrical. It is quite likely that these cracks are produced by the highly localized stresses near the surface caused by surface irregularities.

Mr. Styri remarks that variations in oil viscosity have little effect on bearing life even at a given load. It is possible that if the viscosity were increased far above any practical value, the effect would be noticed. It seems that only in the case of relatively soft materials such as gear steels, it is practical to attempt to eliminate pitting by increasing the oil viscosity. To get a clear conception of this situation, one must look at a three-dimensional diagram of the critical values of load, hardness and viscosity. Such a diagram is at present being constructed as rapidly as data can be obtained.

The pit on the locomotive tire shown in Fig. 3 of this discussion resembles those the writer has seen on ball bearings, one of which is shown in Fig. 9 of this discussion. There is a summit in the middle of the pit, the surface of which is the original surface of the ball. The pit appears to have grown radially from this central summit. Such a radial structure is also suggested in the pit in Fig. 3 of this discussion. There is evidently a characteristic structure for pits on spherical or ellipsoidal surfaces of contact



FIG. 9 PIT ON A BALL BEARING (× 8)

different from that for pits on cylinders. This presents a field for further study. A study of the pit in Fig. 9 strongly suggests that the crack which precedes the pit results from tensile stresses at the boundary of the contact area.

Mr. Allan asks about several details of the tests. The wear is confined to the middle portion of the upper roller because the face width of the upper roller is greater than that of the lower. Counting the revolutions shows that there is no slip between the rollers. A chemical analysis was not made of the steel dust which collects in the oil, but all of it was found to be magnetic. The tests Mr. Allan made were conducted on rollers that were quite hard, inasmuch as the pitting cracks obtained by him were quite irregular. This irregularity has also been found to exist by the author to a more marked degree on harder rollers.

The question of "oiliness" is one about which very little is known. We can be quite sure of one thing, however, and that is that the thickness of the film of molecules associated with the term "oiliness" is very much smaller than the height of surface irregularities on the test rollers or on gear teeth in general. I believe it is justifiable to neglect the adhesive forces binding the oil molecules to the surface in calculating the load necessary to bring the surface irregularities into contact.

The temperature of the lubricant was found to increase from 20 C to 40 C above room temperature, depending on the viscosity.

Mr. Marmont raises the important subject of the spalling of mill rolls. It is perhaps in order to rule out of the discussion failures of a more or less explosive character due in whole or in part to thermal causes and consider only surface failures due to the rolling forces. It is questionable whether spalling due to rolling forces is a failure of the same nature as pitting on gear teeth or on test rollers, so different are the conditions of operation. The main differences that might be mentioned are (1) the plastic condition of the rolled material, (2) the presence of high tangential traction, (3) high temperatures in the case of hot mills. The "creeping" explanation of spalling would not account for pitting under conditions where there is negligible tangential traction. Also, one feels periodic slipping or "creeping" would produce gradual wear rather than pitting cracks. Residual stresses cannot play an important part in mechanical pitting, as is shown by the occurrence of pitting on untreated gears and rollers.

Book Reviews

Plasticity and Solid State of Matter

DISTORTION OF METAL CRYSTALS. By C. F. Elam (Mrs. G. H. Tipper). Clarendon Press, Oxford, 1935. (The Oxford Engineering Science Series) Oxford University Press, London, 1935. Cloth, $6^1/_2 \times 9^1/_2$ in., 195 pp., 5 plates, 94 figs., $5.

KRISSTALLPLASTIZITÄT MIT BESONDERER BERÜCHSICHTIGUNG DER METALLE. (Plasticity of Crystals With Particular Consideration of the Metals.) By E. Schmid and W. Boas. Julius Springer, Berlin, 1935. Cloth, $5^3/_4 \times 8^1/_2$ in., 373 pp., 222 figs., 33.80 rm. G. E. Stechert, New York, N. Y., $13.20.

INTERNATIONAL CONFERENCE ON PHYSICS, London, 1934, Vol. 2. The Solid State of Matter. (A joint conference organized by the International Union of Pure and Applied Physics and the Physical Society.) Papers and discussions in two volumes. Published by the Physical Society, London. University Press, Cambridge, Eng., 1935. Cloth, $7 \times 10^1/_2$ in., 191 pp., 69 figs., 12s 6d, in paper cover, 10s.

REVIEWED BY A. NADAI[1]

DISTORTION OF METAL CRYSTALS

IT is now about 15 years since a small group of young scientists, among whom the names of M. Polanyi, Miss C. F. Elam, and E. Schmid may be mentioned as well as those of H. C. H. Carpenter, G. I. Taylor, T. Czochralski, and others, became interested in the plastic deformation of large crystals. Until that time the subject was known to only a few mineralogists. Among these D. Mügge was probably the first to discover that rock-salt crystals, which at ordinary temperature fracture as a brittle material, may easily be bent permanently to a considerable extent without fracturing if heated, and that under a polarizing microscope beautiful curved lines (slip lines) become visible after permanent bending. The salt crystals had been permanently deformed along numerous regularly curved packages of layers. Similar but straight "slip bands" were subsequently described by W. Rosenhain and G. Tammann in most of the polycrystalline ductile metals after these metals had been plastically deformed. It is probable that comparatively large metallic crystals have been produced accidentally in many cases prior to systematic studies of these crystals. The enormous softness and ductility which became apparent as soon as the first single-crystal wires of metals were stretched and the most regular manner according to which these crystals deform were some of the astonishing facts which led to further investigation. X-ray analysis of these materials contributed much to the structural discussion of deformed metals and crystals.

Mrs. G. H. Tipper, well known for her pioneer testing work on the distortion of metallic single crystals and for her joint investigations with H. C. H. Carpenter and G. I. Taylor in Cambridge, England, has undertaken to present briefly some of the most important general results of the extended investigations by herself and by a few physicists and engineers. During the last 15 years this work has greatly helped to establish the laws of the plastic deformation of single crystals of the ductile metals. Mrs. Tipper states in the preface of the volume that it was decided to limit the scope of the book to an account of the changes in structure and in physical and mechanical properties accompanying plastic deformation of metallic single crystals and to the effect of temperature. The corresponding phenomena in polycrystalline metals are not treated in detail.

Fifteen chapters are devoted to the following subjects: Preparation of metal crystals, deformation of metal crystals, calculation of forces required to produce deformation, deformation by twinning, deformation of polycrystalline metal, deformation of alloys, deformation under repeated stresses, fracture, effect of temperature, duration of test, rate of application of load, alteration in the X-ray reflections accompanying deformation, relation between work done and energy absorbed during deformation, changes in physical properties, plasticity, effect of heat on deformed metal, and a summary and theoretical discussion.

The aim of the book is to avoid unnecessary details, and to lay stress on the experimental methods and to furnish an account of their principal results as well as of theoretical considerations without entering into a mathematical discussion. The little book accomplishes these aims satisfactorily and will be appreciated by many engineers and by those who wish to learn about the numerous interesting and practically important phenomena relating to the permanent deformation of crystals. The book is lucidly written, and the simplest language is used to describe physical or geometrical facts that are not simple. Reading is therefore delightful.

CRYSTAL PLASTICITY

The circle o freaders to whom this book will appeal is perhaps larger than that for which Mrs. Tipper's was probably written. E. Schmid is a professor at the University of Freiburg (Switzerland) and is also one of the pioneers in this field who has contributed considerably through his own exact and careful experimental investigations to a knowledge of the laws of the plastic deformation of single crystals. The book has chapters devoted to the following subjects: Crystallographic fundamentals crystal-elasticity, production of single crystals, methods of finding the orientation of axes in crystals, geometry of the mechanism of plastic deformation, plasticity and strength of metal crystals and of ionic crystals (rock salt), theories and behavior of polycrystalline metals. In the two short introductory chapters the notation by indexes used by the mineralogists is explained, the crystal systems are discussed and the linear equations expressing Hooke's law of elasticity between the six stress and strain components for crystals treated. Among the various methods for production of single crystals the methods by recrystallization after a critical strain hardening, by local crystallization of a powdered metal (tungsten), by growing from the molten state, by pulling out of the liquid metal, and by sublimation are described. A long chapter is devoted to crystal lattices and to the various methods of structural analysis by means of X-rays. The mechanism of simple and double slip and of twinning in single crystals is then described in detail, illustrated by a large number of beautiful photographs of slip bands and of the strange local necking phenomena characteristic for the last stage of deformation of certain metal crystals. An instructive addition consists in the numerous photographic illustrations of model surfaces made of plaster of Paris representing the shearing stress at the yield point (*Fliessgefahrkoerper*) or the modulus of elasticity or the tensile strength of single crystals in a kind of a vector diagram (surface) in space. Even for the regular system of crystals some of these surfaces show unexpectedly large variations in the corresponding variable

[1] Research Laboratories, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

quantity. The effect of temperature and time is studied. Space is devoted to phenomena related to recrystallization and to heat-treatment. A chapter is devoted to the theories of the solid state of matter and of strain-hardening and one deals with the so-called *fiber* structure in polycrystalline materials due to severe cold-working. The book is well illustrated and contains an accurate description of the most important phenomena related to the deformations of single crystals. For a better understanding of the fundamental properties of metals upon which their ductility, strength, and other mechanical properties depend, this book will be a welcome source of useful information.

Solid State of Matter

Some 200 distinguished physicists attended the international conference on physics, held in London in 1934 at the Royal Society and at the Royal Institution of Great Britain.

The deviations of the numerical values for certain mechanical and strength properties actually determined by tests on real crystals from the values predicted by the theory of lattices of ideal crystals occupy today the center of discussion in connection with the theory of the solid state of matter. It was therefore thought that it might be of some interest to engineers to have a brief summary of some of the papers and discussions, which were presented at the London conference in 1934 and are now available in print in the second volume devoted to this conference. It has been said, and it does not appear improbable, that engineers will some day utilize one or the other of the remarkable observations which seem to indicate that the natural strength of crystalline or amorphous solid bodies under certain conditions can be substantially increased. For the writer of this review, as one of those who can observe these recent advances only from a distance, it seems after glancing over the conference reports that the conceptions upon which the computations of strength of crystals rest are at the present time in a highly controversial and unsettled state.

The following papers were presented at the conference: "The Mosaic Texture of Rock Salt," by P. P. Ewald and M. Renninger; "Experimental Evidences of Group Phenomena in the Solid Metallic State," by A. Goetz; "On the Cause of the Low Value of Mechanical Strength," by A. Joffe; "On the Mechanism of Brittle Rupture," by A. Joffe; "The Rupture of Plastic Crystals," by E. Orowan; "The Structure-Sensitive Properties of Salt Crystals," by A. Smekal; "Shear Hardening and Recrystallization of Aluminum Single Crystals," by W. G. Burgers; "On Plasticity, Crystallographic and Non-crystallographic," by E. Schmid. The conference report is introduced by a short article by Sir W. H. Bragg in which he points out rightly, that this was by no means the first conference to discuss the problem of the solid state. One of the most striking facts revealed by the X-ray analysis is the prevalence of the regularity of arrangement of the elements in the solid state of bodies. On the other hand there are important phenomena which are not revealed in X-ray patterns or are not dependent on the few atoms of the unit cell in a crystal; strength, stresses at rupture and at plastic limit, and strain-hardening, are only a few of them. The papers are devoted to these important and unknown conditions that affect the properties of solids above the regular arrangement of the ideal lattices.

P. P. Ewald and M. Renninger (Stuttgart) report that when investigating rock-salt crystals they could not find traces of a mosaic structure (that is, of blocks at most 10^{-4} cm) in carefully grown artificial samples, while natural salt crystals, freshly cleaved or polished were not free of such structural disturbances of this magnitude. They conclude that in perfect artificial crystals no trace of the mosaic structure of the order of 10^{-4} cm can be detected by X-ray reflections contrary to the predictions by Darwin, Zwicky, and others, according to which a mosaic or block structure should exist in every real crystal. The experiments of A. Goetz (Pasadena), however, point in favor of the secondary structure and to the fact that the orientation of a crystal persists even beyond the process of fusion, so that on subsequent solidification the same structure reappears.

A. Joffe summarizes his well-known experiments with rock salt in the conclusion that the practical weakness (against rupture) consists principally of sharp discontinuities along *the surface of the body* and cannot be ascribed to small *internal* faults (Griffith), *Lockerstellen* (Smekal), mosaic structure, or the like. His experiments with Jurkov furnished them new evidence that the weakness of glass or quartz should be due to irregularities on the surface of filaments, while the same kind of small faults when distributed throughout the bulk would not act so. From his experiments with *mica* E. Orawan believes that trivial cracks in cut edges are the cause of the lowering of the strength of this material, while if precautions are taken to avoid cut surfaces so that no free edges are present in the region where stresses are high, the tensile strength is considerably raised. According to him the assumption of a secondary (mosaic) structure in crystals is also unnecessary and it leads to impossible conclusions.

A. Smekal (Halle) critically reviewed the following subjects: Stability of lattices, instability of the ideal crystal, laws of crystal growth, primary and secondary flaws, and experiments with rock salt. Space does not permit the abstracting of the various discussions which strengthen further the impression, already given, that agreement with regard to the existence or the absence of a mosaic, block, or secondary group structure within the lattice of a crystal could not be reached by the speakers of the conference. Two very instructive papers by W. G. Burgers (Eindhoven) and E. Schmid (Freiburg, Switzerland) on shear-hardening and recrystallization of aluminum single crystals and on crystal plasticity terminate the volume.

New Pathways in Science

New Pathways in Science. By Sir Arthur Eddington, M.A., D.Sc., LL.D., F.R.S. Plumian Professor of Astronomy and Experimental Philosophy in the University of Cambridge. Messenger Lectures, 1934. The Macmillan Company, New York, N. Y. University Press, Cambridge, 1935. Cloth, $5^3/_4 \times 8^1/_2$ in., 326 pp., and index, 4 plates, $3.

Reviewed by J. Ormondroyd[2]

THE Messenger lectures delivered in the spring of 1934 by Sir Arthur Eddington at Cornell University are collected and published in this book. The author considers this collection as a disjointed addendum to his previously published popular scientific treatises, "Stars and Atoms" (1927), "The Nature of the Physical World" (1928), and "Science and the Unseen World" (1929). Reference to, and comparisons with, previous statements made in these books are given frequently in this volume. Under these circumstances no attempt is made in "New Pathways in Science" to provide a systematic introduction to modern scientific ideas. Atomic physics provides the background almost exclusively and, except for inevitable allusions, the theory of relativity is not discussed. Enough on this subject is given to indicate to the reader that the two branches of modern physics, atomic theory and relativity, have developed along independent paths which are just beginning to converge to a unified road.

Chapter 1 gives the philosophical outlook of modern science, and attempts to relate the scientific picture of the world to the "familiar story" in our minds. Chapter 2 is a summary of the present outlook on atomic physics. Chapters 3 to 6 deal mainly

[2] Experimental Division, Westinghouse Electric and Manufacturing Company, South Philadelphia, Pa. Mem. A.S.M.E.

with laws of the statistical type, with a complete discussion of the position of determinacy and casual connections in the new physics. The last chapter in this group treats the subject of probability which is naturally the mathematical foundation of all statistical law making. Chapters 7 to 10 outline the latest knowledge in astro-physics. The discussion on the "Cosmical Constant" in Chapter 10 leads naturally to a consideration of the most fundamental conceptions of physics which are fully treated in Chapter 11 under the heading of "The Constants of Nature." Eddington himself feels that this chapter is a little too severe and difficult for the surroundings in which it finds itself. However, its inclusion was inevitable since this subject forms the core of Eddington's intellectual activity of the last five years. Chapter 12 introduces the ideas of the Theory of Groups, a subject which touches the very keynote of scientific philosophy.

An active thinker who happens to have a pungent tongue is certain to stir up discussion. Chapter 13, on "Criticisms and Controversies," gives Eddington's answers to some of the philosophical obstacles which have been put in the path of his reasoning in the last few years.

Chapter 14, the "Epilogue" which closes the spirited tale, is written in a religious and penitent mood. The ordinary man in Eddington concludes, with evident relief, that physical science has at last so definitely delimited itself that the esthetic and religious aspects of life may emerge from the threatening shadows and blossom in any direction which satisfies the human soul. The last words of the book constitute an "amend honorable" to Newton, who must have turned over in his grave under the scathing and heartless satire heaped upon his famous three axioms in "The Nature of the Physical World." After all, the present needed a past to provide its roots and Newton still stands, a Titanic figure, looming up in that misty darkness which is swiftly enveloping the older gods.

To sum up the book with perfect honesty the reviewer usually does not know what Eddington is talking about but he always likes the way in which it is said.

Engineers who read this book will enjoy it much in the same manner as they enjoy a performance by any clever prestidigitator. The logical sequence of events which bring about the effects remains hidden but the effects themselves are more than marvelous and Eddington provides that continuous stream of epigrammatic, brilliantly humorous patter so necessary to the business of public entertainment.

Many philosophical physicists, Planck, Bohr, Jeans, as well as Eddington, have attempted to stoop low enough to lift the layman up to a level where he can have a bowing acquaintance with modern physics. They succeed only in making a formal introduction followed by interpretative eulogies without ever producing the "thing itself." They cannot produce it for us because it consists only of "symbols connected by mathematical equations." It is the product of mathematicians for the consumption of mathematicians. As at the Academy in ancient Athens only those who know mathematics are admitted to the actual presence. Pictures are no longer possible and scientific model making has become worse than a crime—a meaningless blunder. But by the very nature of our profession we are hopelessly model-minded. Perhaps Lord Kelvin was the last great physicist whom we could expect to understand. Eddington, somewhere in his book, has called the classical physicists engineers, simply because he could imagine nothing worse to fling at them.

"Mass is equivalent to energy." "Mass (energy) is a curvature in space-time." "Force is the difference between an assumed geometry and the real geometry of a space." "Electrons are waves of probability." "The position and velocity of an electron cannot both be measured with the same high accuracy at the same instant, therefore, the principle of causality vanishes." These, and many other phrases, undoubtedly stick in the minds of readers of popular scientific treatises, mainly because they strike the sense of the bizarre. But, after all, they are meaningless phrases waiting to receive solid content.

Perhaps the reviewer paints the picture too darkly. Many of the ideas of modern physics find our minds prepared for acquiescence. That all we know physically reduces to "pointer readings" is a familiar fact of engineering. That only "intervals" can be measured seems natural enough, although our stubborn belief that there is something absolute about rotation indicates that we really miss the point here. That the process of measurement can profoundly influence the magnitude of the quantity measured is a commonplace in engineering experimentation. Under these circumstances functional relationships between the measured quantities cease to be useful. Indeterminacy and its philosophical consequences are almost within the grasp of our usual mental outlook.

The popular treatises do hint at the road to complete understanding. All that we require in order to turn modern physics from a series of empty phrases to something functionally real is a knowledge of partial-differential equations, transformation of coordinates, calculus of variations, probability, non-Euclidean geometry, matrix theory, tensor analysis, and the theory of groups. Here time alone stands in the way, not lack of intellectual ability. One wonders why mathematics has escaped attempts at popularization in these days when all the other sancta have been thrown open to the public.

From the philosophical standpoint we are somewhat handicapped in our consideration of this third revolution in physical outlook by having remained blissfully ignorant of the philosophies of the first two modern scientific upheavals. When Galileo, Newton, and their contemporaries were turning Greek mechanics from a mass of meaningless words into a body of effective results they used and expressed an underlying philosophy which would be understood by the modern engineer if he ever took the trouble to become aware of its existence. Similarly the "age of coordination" which produced the generalized principle of the conservation of energy and the theory of thermodynamics gave rise to philosophical interpretations which remain a closed book to most of us.

As long as engineering is defined as a technical trade, and not as an intellectual profession, those who practice it will be doomed to read such books as "New Pathways in Science" merely for mystified amusement.

Theory of Functions

THEORY OF FUNCTIONS AS APPLIED TO ENGINEERING PROBLEMS. By R. Rothe, F. Ollendorff and K. Pohlhausen. Technology Press, Cambridge, Mass. 1933. Cloth, 6 × 9½ in., 184 pp., 108 figs., $3.50.

REVIEWED BY J. C. HUNSAKER[3]

THIS little book of 184 pages is the authorized translation by Alfred Herzenberg of "Funktonien theorie und Ihre Anwendung inder Technik," published in Germany in 1931 by Julius Springer. The material was first presented by the Ausseninstitute of the Berlin Institute of Technology in 1929–1930, jointly with the Berlin Electrical Engineering Society, in an effort to present together the mathematical theory of functions and its applications. R. Rothe, in the first 83 pages, develops the general mathematical theory and the principal theorems and principles necessary for the practical use of function theory. This is followed by five sections on applications: (*a*) the con-

[3] Head of Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

struction of electric and magnetic fields by means of source-line potentials; (*b*) two-dimensional fields of flow; (*c*) field distribution in the neighborhood of edges; (*d*) complex treatment of electric and thermal transient phenomena; (*e*) spreading of electric waves along the earth.

The material on applications is naturally very largely in the field of electrical engineering, with the exception of Pohlhausen's chapter on fluid mechanics. However, the great value of the book lies in its compact and straightforward development of the mathematical tools obtained from function theory with their use illustrated by examples. The mechanical engineer might prefer to see the applications made to problems in elasticity, vibration, acoustics, and heat flow. However, the applications that are given cover a wide range of problems from the electric, magnetic, thermal, hydrodynamic and aerodynamic fields.

The exposition, in the opinion of a mathematical colleague, is "simple, sufficiently thorough, and should be followed without undue difficulty by any one familiar with calculus." An engineering colleague, on the other hand, finds the treatment highly compressed and demanding a comprehensive familiarity, if not facility, with advanced calculus. The fundamental conceptions of line and hook integrals, Heaviside operational calculus, conformal mapping of Riemann surfaces, etc., are developed in a few paragraphs each, and in W. Schottky's chapter a vectorial notation is employed in discussing the properties of potential fields.

The book may be considered both as an introduction to function theory and as a reference text for engineers concerned with the theoretical analysis of fluid flow, wave propagation, and the configuration of fields of force. As such, it should prove stimulating and useful to mechanical engineers interested in learning the use of some of the tools and methods very generally employed in physics.

Hydro- and Aeromechanics

FUNDAMENTALS OF HYDRO- AND AEROMECHANICS, VOL. 1 AND APPLIED HYDRO- AND AEROMECHANICS, VOL. 2. Based on lectures by L. Prandtl of Göttingen. By O. G. Tietjens with English translation of vol. 1 by L. Rosenhead and vol. 2 by J. P. Den Hartog. Engineering Monograph Series. McGraw-Hill, New York, 1934. Cloth, $5^3/_4 \times 9$ in., vol. 1, 270 pp., 186 figs., $4; vol. 2, 275 pp., 311 figs. including 226 figs. in text and 68 on supp. plates, $4.

REVIEWED BY F. WATTENDORF[4]

THIS book, one of the Engineering Societies Monograph Series, is an English translation of the book "Hydro- und Aeromechanik," based on lectures by Prof. L. Prandtl of Göttingen, by Dr. O. G. Tietjens, one of his former assistants.

The book appears in two volumes. Volume 1, translated from the German by Dr. L. Rosenhead, is entitled "Fundamentals of Hydro- and Aeromechanics," and presents the basic theories of fluid statics, as well as perfect and viscous fluid motion. Volume 2, translated by Dr. J. P. Den Hartog, is entitled "Applied Hydro- and Aeromechanics," and deals with the mechanics of real fluids with technical applications. The material is presented in such a way that each volume can be used independently of the other.

In keeping with the spirit of Professor Prandtl's lectures, the book gives the reader a clear physical picture of the fundamental principles underlying the various theories, unobscured by masses of detail. In most cases one or two practical examples are given for the purpose of illustrating each principle, information of a statistical nature being avoided as much as possible.

[4] Research Fellow in Aeronautics, California Institute of Technology, Pasadena, Calif.

The first volume is divided into three parts. Part 1, "The Statics of Liquids and Gases," presents classical hydrostatic theory, with application to the equilibrium and stability of the atmosphere, and to the static equilibrium and stability of balloons. A short section on surface tension is also included in Part 1. Part 2, "Kinematics of Liquids and Gases," deals largely with the Eulerian and Lagrangian methods of describing perfect fluid flow and includes a section on vectorial representation of fields of flow. Part 3, "The Dynamics of Non-Viscous Fluids, gives the fundamental concepts and theorems of potential flow, including conformal transformation and vortex motion. One short section is devoted to the influence of compressibility on potential flow and another to energy and momentum theorems, with a discussion of the impulse propeller. The final chapter of the volume treats the derivation of the Navier-Stokes equation for viscous flow. In this one case the American edition differs from the German in that this chapter was a part of Volume 2 in the German edition.

Volume 2 deals with the application of hydrodynamic theory to the flow of real fluids. It begins with a physical interpretation of viscosity, and the relationship between viscous and inertia forces, on which the law of similarity is based. After this introduction there follows a section on flow in pipes and channels. This is a well-organized treatment of the subject, beginning with the early experiment of Hagen and Poiseuille, the classical work of Osborne Reynolds on the transition between laminar and turbulent flow, and extending through the work of Nikuradse and Dönch on convergent and divergent channels.

There follows a section on boundary layers, which is in close accordance with Prandtl's well-known lectures on the subject.

The next topic considered is the resistance of bodies moving through fluids. A clear picture is given of the various physical components of drag, and their relative importance for various bodies and flows.

This is followed by a well-presented treatment of airfoil theory, including both two-dimensional and three-dimensional theory, with application to multiplanes, and a short discussion of wind-tunnel interference.

The concluding chapter, a well-written summary of experimental methods and apparatus as used in modern aerodynamic laboratories, is not based on Prandtl's lectures, but has been added by Dr. Tietjens. The book closes in a fitting manner with a series of interesting flow photographs.

The translations into English have been made in an excellent manner by Dr. Rosenhead and Dr. Den Hartog. Much credit is due Dr. Den Hartog for his choice of suitable words for difficult and hitherto untranslated technical expressions of recent origin.

Photoelasticity

FESTIGKEITSLEHRE MITTELS SPANNUNGSOPTIK. By Ludwig Föppl and Heinz Neuber. R. Oldenbourg, Munich and Berlin, 1935. Board, $5^1/_2 \times 8^1/_2$ in., 115 pp., 80 figs., 6.60 rm.

REVIEWED BY E. E. WEIBEL[5]

THE possibilities of the photoelastic method are being increasingly realized both as an aid to the practical designer and for the more detailed study of stress distribution in problems for which mathematical methods are insufficient. Until now the "Treatise on Photo-Elasticity" by E. G. Coker and L. N. G. Filon, published by the Cambridge University Press, 1931, has been the only textbook for reference on the subject. Although not to be compared in size with the English work, the present volume contains the essentials of the photoelastic theory as well

[5] Engineering Mechanics, University of Michigan, Ann Arbor, Mich.

as a few sections of heretofore unpublished material. The authors who are capable exponents of the theory of elasticity have drawn upon the results of their photoelastic experiments extending over a period of six years in the Mechanical-Technical Laboratory at Munich.

The book is divided into four chapters, the first of which covers fundamentals of the photoelastic method. The use of glass models and of compensation methods of measuring maximum shear stress at many separate points are current practice in Germany and are described. Some reference is also made to the use of more active optical materials such as bakelite and phenolite as used in this country.

The second chapter, on the complete determination of stresses, describes the lateral-extensometer method and Filon's graphical-integration method of determining principal stress values, and then presents the theory underlying Neuber's new graphical method which was discussed in his recent paper. (See Trans. A.S.M.E., 1934, paper APM-56-17, p. 733.) The membrane analogy also is presented as a supplement to the photoelastic method for the complete determination of stresses.

In the third chapter three special problems in photoelasticity are given mathematical treatment. The course of principal stress trajectories is derived for the region around a singular point or points at which the principal stresses are equal. The second problem is that of the effect of Poisson's ratio in the interpretation of photoelastic results. It is known that for most cases the stresses are independent of the elastic constants, but that for certain types of loading of multiply connected bodies a correction is necessary. The authors show that in problems which are important enough to warrant the extra work the correction stresses can be found by making supplementary photoelastic experiments on models which have been slit from outer to inner boundary and which have had known displacements applied at the slit. The third study considers those stress cases for which the stress trajectories satisfy the equations for stream lines in fluid motion. A limited number of simple problems would appear to meet these conditions.

In the fourth and last chapter a number of applications of the photoelastic method are discussed and illustrated. The authors include a few fringe photographs, the models for which were made of trolon, a synthetic resin available in Germany, with satisfactory optical properties but mechanically inferior to bakelite and phenolite.

Conjugate Functions for Engineers

CONJUGATE FUNCTIONS FOR ENGINEERS. By Miles Walker, M.A. (Cantab.), D.Sc. Eng. (London), F.R.S. Sometime Professor of Electrical Engineering in the Faculty of Technology, University of Manchester. Oxford University Press, London, 1933. Cloth, 116 pp., 92 figs., $4.75.

REVIEWED BY J. C. HUNSAKER[5]

DR. WALKER is an electrical engineer who has undertaken to develop in this little book the details of conformal transformation, and to give what is essentially a mathematical manipulation an appearance of physical reality. In this, the author has shown great ingenuity and patience in following through a series of simple steps to establish a procedure which is then applied to the solution of a series of problems.

The first 31 pages consist of a consideration of a two-dimensional field of force (magnetic, electric, pressure, thermal) simplified to a pattern of lines of force and of flux which are continuous and without vortices. Consequently, only a steady-state potential field is involved. The fundamental conceptions of potential are developed by use of the line integral, and it is shown that the equipotential lines are at right angles to the stream lines and that the force at any point is the space gradient of the potential.

This orthogonal system of lines is then shown to be identical in pattern with the system of lines that results from plotting certain functions of a complex variable known as conjugate functions. This analogy is very properly taken as merely a fortunate property of the mathematical tool available. A brief résumé is then given of complex algebra and the plane of numbers in which the imaginary square root of minus one, or j, is treated, by definition, as an operator, which rotates a vector 90 deg. De Moivre's theorem is quoted in order to permit the use of complex exponents. The conditions that a complex function may be "analytic" and its real and imaginary parts "conjugate," are deduced mathematically and illustrated graphically. The point is made that where $w = f(z) = \phi + j\psi$, the transformation of a pattern on a z-plane (coordinates x and $j\,y$) to a truly corresponding pattern in the w-plane (coordinates ϕ and $j\,\psi$) requires that $\frac{dw}{dz}$ be independent of the slope of dz or of $\frac{dy}{dx}$. This is precisely the requirement that ϕ and ψ must satisfy to be classed as "conjugate functions." Consequently a plot of conjugate functions is shown to represent on the w-plane the pattern of potential and flux on the z-plane.

Up to this point, the reader has followed a clear path of reasoning but now he must make a leap in the dark and accept as an historical event (Schwarz in 1869 and Christoffel in 1867) a differential equation stating the nature of the change in a function of z, bounded by any straight-sided figure, in order that this boundary may be transformed into one straight line in the w-plane. It is possible that this acceptance of the Schwarz-Christoffel theorem saves the engineering reader time and difficulty if his only object is to learn a practical procedure for working out flow patterns, but he may resent the publishers statement: "This book explains the method in such simple language that even the author can understand it." However, the "method" to be used is very clearly illustrated in a series of thirteen problems whose detailed solutions make up the balance of the book.

These problems are all in the field of electrical engineering, such as: Current fed into one point of a thin conducting sheet; current between cylinders standing in a solution; current flow around corners; magnetic flux across an air gap; dynamo slots, etc. Similar solutions can be worked out for corresponding ideal two-dimensional problems in elasticity, hydromechanics and heat flow, but the mechanical engineer rarely has reason to attack such idealized problems. Had the worked examples been chosen more liberally and included the classical fields of force and flow in applied mechanics, interest in a very skillfully contrived exposition would be more general.

Practical Solution of Torsional Vibration Problems

PRACTICAL SOLUTION OF TORSIONAL VIBRATION PROBLEMS. By W. Ker Wilson, M.Sc. John Wiley & Sons, Inc. New York, 1935. Cloth, 5 1/2 × 8 1/2 in., 438 pp., 106 figs., $7.

REVIEWED BY R. EKSERGIAN[6]

THE increasing applications of internal-combustion engines and their peculiar susceptibility to critical torsional vibrations has brought forward the analysis of torsional vibrations as a major problem in design. The theory of torsional vibrations for engine installations is by no means a simple problem, and for

[6] Edward G. Budd Manufacturing Co., Philadelphia, Pa. Mem. A.S.M.E.

this reason approximations and simple computation methods assume importance in practical design.

Mr. Wilson's book is particularly commendable from this latter point of view. Chapters 1 and 2 are devoted to natural-frequency calculations of typical engine installations as well as their approximations to two- and three-mass systems. In the tabulation method for multi-mass systems for one- and two-mode vibrations, the author introduces a column for stresses per degree of deflection at the end mass. These data are later used in the determination of forced oscillation stresses.

In Chapter 4, Mr. Wilson gives a method for determining stresses at non-resonance conditions on the basis of "equilibrium amplitudes" and amplified by a "dynamic-magnifier factor." While it is difficult to accept his proof for a multi-mass system by the simple system he uses, the subsequent formula and applications, using the equilibrium factor and dynamic magnifier, really suggest the theory of normal coordinates to practical applications in the calculation of non-resonance stresses in torsional-vibration problems.

In estimating the effect of the exciting forces, the author apparently deals singly with individual modes; whereas, actually in estimating the resultant amplitudes and total stresses it is necessary to consider the relative excitation of the several modes of oscillation of the system and their phase relations. Due to the proximity of the forced frequency with the primary natural frequency for a system with heavy fly-wheel and distributed crank masses, Mr. Wilson's numerical calculation shows good agreement with the tabulated method for forced oscillations.

Frequently in multi-mass systems for good approximations it is only necessary to consider the first and second modes due to the rapid convergence of the higher modes, and sometimes only the first mode alone. This is true because of the relative importance of this mode, particularly when the excitation is powerful with respect to it and the higher modes are far removed from it.

In Chapter 5, resonance stresses are obtained by the usual method of equating the work of the exciting forces to those of the damping forces. Where these stresses are at resonance with a given mode, simplification is effected due to the quadrant phase relation between the excitation forces and displacement amplitudes. This chapter offers much interesting detail in various damping components.

The remaining chapters are devoted to the measurement of torsional-vibration amplitudes and stresses with various types of torsiographs, and a discussion of torsional-vibration damping devices. A welcome feature is the interpretation and investigation of actual torsiograph records and some detail discussion on flexible couplings.

In the final chapter which is of an elementary nature a brief discussion is given on electrical generating sets direct coupled to internal-combustion engines.

On the whole, the work is a valuable contribution to the literature. Not only are clear-cut and well-balanced discussions given to the various factors on torsional-vibration analysis with illuminating examples and worked-out data, but the author introduces an ingenious method of analysis as well.

The Story of Energy

THE STORY OF ENERGY. By Morton Mott-Smith, Ph.D. Appleton New World of Science Series. D. Appleton-Century Company, Inc., New York, 1934. Cloth, 5 × 7½ in., 290 pp., 33 figs., $2.

REVIEWED BY J. ORMONDROYD[2]

THIS little book should hold great interest for engineers since it tells a half-familiar tale. It outlines the history of prime-mover development including the practical, theoretical, and philosophical aspects of the subject. The subject matter divides itself into three sections.

Chapters 1 to 6 consider the development of steam engines from Hero to the work of James Watt and includes the advances made by some of the lesser contemporaneous successors of that great inventor. This section ends with a detailed account of the analysis of the motive power of heat which Sadi Carnot published in 1824. Carnot saw the problem so clearly that he prophesied correctly the future steps needed to improve the performance of heat engines. But he was limited in his achievements by the caloric theory of heat and by early death. The complete theoretical outlook used today was fashioned by other and later hands as is outlined in the next section.

Chapters 7 to 17 trace the rise of the generalized doctrine of the conservation of energy and consequent formation of the theory of thermodynamics. These chapters give the tragic story of Mayer's labors in this field and outline the happier parts which Joule and Helmholtz played in the same drama. Chapter 15 shows the idea of energy conservation growing inevitably from human experience; slowly gathered first in the guise of animism, then in religious form and finally through magic to the modern scientific outlook. Chapter 16 describes the reconciliation of the apparently contradictory principles of Carnot and Joule by Clausius, and the creation of modern thermodynamics from this by Clausius and Thompson. The last chapter of this section is dedicated to that mysterious and exciting "arrow of time," entropy.

Chapters 18 to 20 describe various cycles and engines which have actually been developed to transform heat into mechanical work in the last seventy years. Air engines, steam turbines, and internal-combustion engines are reviewed historically. The book ends with a description of mechanical refrigeration.

In carrying out the program outlined in the title, the author gives an understandable statement of the philosophy of classical physics interwoven with the evolution of thermodynamic theory and practice. The historical parts are unusually complete. Important passages from the papers of Carnot, Mayer, and others are quoted in extenso. A rather complete bibliography of the original publications in the field treated is given in the footnotes and at the end of the book.

A few factual errors must be noted. Heat engineers will be surprised to find the critical temperature and pressure of water noted respectively as 689 F and 2947 lb per sq in. on page 40 and in Fig. 8. This surprise may turn to amazement as they read that impulse and Rateau turbines are now practically obsolete having been completely displaced by Parson's reaction turbine (page 256) and that the maximum inlet pressure for reaction turbines is 250 lb per sq in. (page 257). Isolated facts, however, are only incidental to the purposes of the book, which does not pretend to be a technical text, and these slips would seem to arise from the use of somewhat obsolete reference material on the part of the author.

Historians of science certainly would not connect Descartes' theory that the quantity of motion in the universal remains constant with the impact experiments of Wallis and Huygens (page 81) who published their findings on this subject some nineteen years after the great philosopher's death. Descartes, according to his own lights, found a more satisfactory basis for this theory in the infinite wisdom of God rather than in mere experimental results.

In a popular work these defects have minor importance. Engineers who read this book will agree with the reviewer that it is entertaining and highly instructive.

Journal Bearing Performance

By R. BAUDRY[1] and L. M. TICHVINSKY,[2] EAST PITTSBURGH, PA.

This paper contains an application of the hydrodynamical theory of lubrication to the practical design of journal bearings. In a simplified method satisfactory for most engineering applications the authors consider the journal bearing as a plane bearing. Then correction factors make allowance for (1) the type of the journal bearing, (2) the shape of the oil film and width-length ratio of the bearing, and (3) the losses in the nonactive parts of the bearing.

In a more accurate method, the same correction factors are applied to the rather complete data published on journal bearings of infinite width. It is also shown that the clearance of the bearing can vary between rather large values and still maintain closely optimum conditions of performance of the bearing. Results of tests are shown to be in close agreement with these methods of calculation.

Introduction

ALTHOUGH the hydrodynamical theory of lubrication was developed a long time ago, the design of modern bearings is still based to a great extent upon empirical data. Theoretical analysis considers only an ideal bearing which is never realized in practice. Even for this ideal bearing the analysis is very complicated, particularly for the case of journal bearings.

Furthermore, deformation of the bearing produced by load and nonuniform temperature distribution, accuracy of machining, and degree of finish are important factors which cannot be well introduced in any theoretical analysis.

Although many laboratory tests of bearings verify satisfactorily the hydrodynamical theory, these tests are only of small help in the design of large bearings. Improvements in the design of bearings can only be made by means of tests duplicating as much as possible the normal conditions of operation and manufacturing. Much useful information may also be gathered from tests on bearings of machines in normal operation in order to verify the losses and temperature distribution.

Bearing Operation

The operation of a journal bearing is better understood by considering first the operation of the plane bearing such as shown in Fig. 3 (1).[3] The simplest theory usually assumes that (*a*) the bearing is of infinite width, (*b*) the lubricant is of uniform viscosity throughout the oil film, and (*c*) the inlet and outlet pressures are both equal to atmospheric pressure. In Fig. 4 are given the coefficient of friction, oil-film thickness, oil flow and temperature rise of the oil going through the oil film, for various ratios of h_i/h_o. In this ideal case, the line of action of the load can be located at a certain distance back of the center of the bearing which will give either the maximum oil-film thickness or the minimum coefficient of friction.

The assumption of infinite width is obviously never met in practice. Michell (2) has given a theoretical solution of the rectangular-plane bearing for uniform viscosity through the oil film and uniform pressure at the edges. In this case the flow of oil is two-dimensional; the effect of the side leakage increases the coefficient of friction and decreases the oil-film thickness as shown in Fig. 5. The increase in temperature rise of the oil and the change in flow of oil at the inlet and outlet are given in Fig. 6.

The viscosity of the oil decreases with the increase in temperature as shown in Fig. 7. R. O. Boswall (1) has shown by assuming a simple law of viscosity variation with temperature that it was permissible to consider only the average temperature of the oil through the film.

Bearings are made also with a rounded edge or sometimes with a small chamfer at the inlet so that due to the momentum (4) of the oil, the pressure at the beginning of the film is not equal to the pressure at the outlet as assumed. This, together with the variation of viscosity, produces a displacement of the line of action of the load toward the center as shown by tests (3, 5). Experience also shows that a centrally loaded plane bearing operates satisfactorily. This simplifies manufacturing and permits operation of a machine in any direction of rotation.

A. Kingsbury (6), also assuming a journal bearing of infinite width, uniform viscosity and boundary pressure, has given a complete analysis of the optimum conditions of operation of journal bearings. He shows how the different elements of a bearing can be determined so as to have either a minimum coefficient of friction or a maximum load which can be carried on the basis of a given minimum oil-film thickness. The curves shown in Fig. 8 give the minimum coefficient of friction and corresponding oil-film thickness which can be obtained from a journal bearing of correct proportions as compared with a plane bearing operating under the same conditions.

S. J. Needs (7), by the use of an electrical analogy, has made a theoretical analysis of the 120-deg central partial bearing of finite width. In a discussion of his paper (8) it was shown that the values of leakage factors so obtained were in close agreement with the assumptions previously made, thus showing that the leakage factors calculated for plane bearings can be applied with sufficient accuracy to journal bearings. Many authors are applying to the journal bearing the leakage factors calculated by Michell, Boswall, and others for the plane bearing. In this case it is assumed, as is usually done for the plane bearing, that the inlet-outlet ratio of the oil film is constant and equal to 2.5, and the length of the oil film is equal to the developed length of the bearing arc.

In Fig. 8 it is seen that particularly for small angles the operation of a journal and of a plane bearing are nearly the same. Boswall (3) has also made tests showing that journal bearings behave in the same way as plane bearings. For example, a fitted bearing (smaller than 90 deg and with round, beveled, inlet edge) can be operated satisfactorily with a central load contrary to the simplified theory. No complete theoretical analysis of the side leakage for the journal bearings has yet been made on account of its complexity as compared with a plane bearing. Actually, the shape of the oil film, i.e., the inlet-outlet ratio and the active

---

[1] Power Engineering Department, Westinghouse Electric & Manufacturing Co.

[2] Research Laboratories, Westinghouse Electric & Manufacturing Co.

[3] Numbers in parentheses refer to bibliography given at the end of the paper.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., December 3 to 7, 1934, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

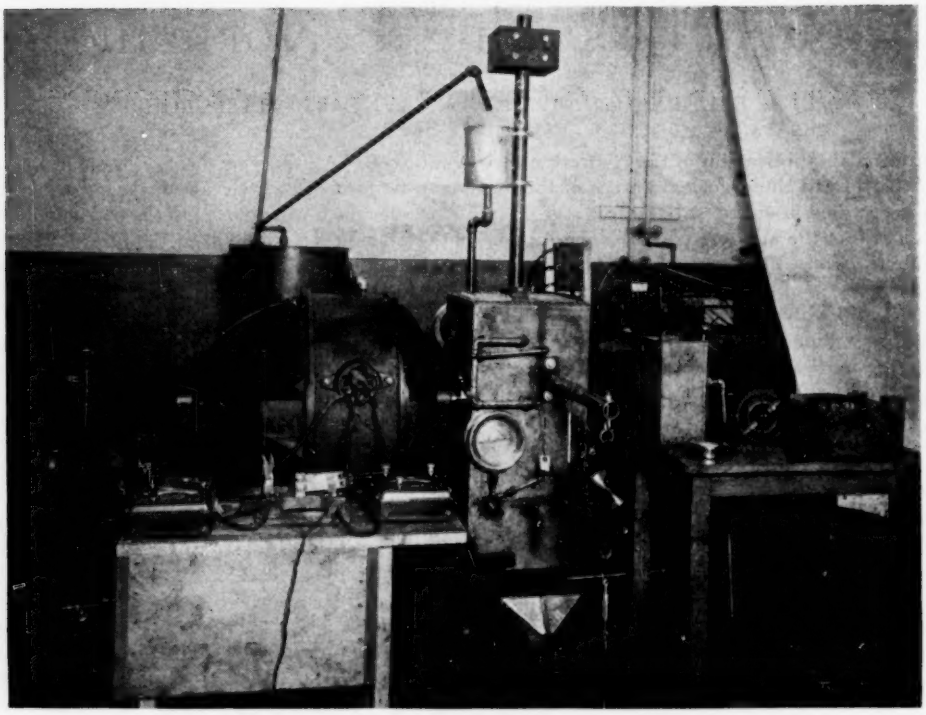


FIG. 1 THE TEST SET-UP OF THE 6-IN. × 12-IN. JOURNAL BEARING

(Left to right are the high-pressure pump for loading the bearing, the driving motor, oil water cooler, and electrical meters. The bearing shell and housing are overhung on the journal. The lever arm and scale for torque measurement, thermocouple distributor, and potentiometer are seen on the right.)

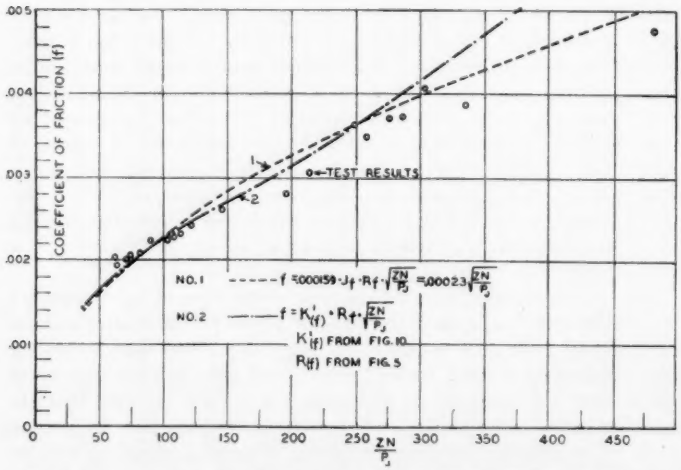


FIG. 2 COEFFICIENT OF FRICTION OF 6 IN. × 12 IN. 90-DEG CENTRAL PARTIAL JOURNAL BEARING, $\eta/r = 0.0015$

(For curve No. 1 it is assumed that the journal bearing operates similarly to a plane bearing. Allowance is made for the J_f type of journal bearing, curves for which are shown in Fig. 8. The constant leakage factor is the same as for a plane bearing having a length equal to the developed bearing arc and a ratio $h_i/h_o = 2.5$. The coefficient of friction is proportional to $\sqrt{(ZN/P_j)}$. Curve No. 2 is obtained from theoretical analysis for a journal bearing of infinite width given in Fig. 10. The active length of the oil film, the shape of the oil film h_i/h_o, and the corresponding value of leakage factor R_f change with load, speed, and viscosity ZN/P_j.)

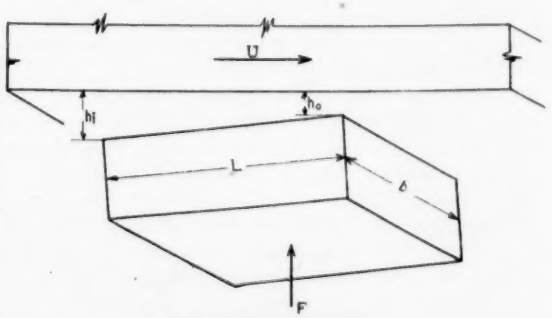


FIG. 3 PLANE BEARING

length, change with the speed, load, and viscosity (ZN/P_j). The position of the journal relative to the bearing is determined by the eccentricity and the angle between the line of centers and the direction of the load as shown in Fig. 9.

Rather complete data are available (6, 9, 10) for different kinds of journal bearings of infinite width.

In Figs. 10, 11 and 12 are given characteristics of central clearance partial bearings 90-deg, 120-deg, and 180-deg angle. In these figures the coefficient $K'(\eta/r) = (\eta/r)/\sqrt{(ZN/P_j)}$ determines the eccentricity of a bearing having a clearance ratio η/r and a characteristic number ZN/P_j. From this value of the eccentricity can be found:

1 The coefficient of friction $f = K'_f \sqrt{ZN/P_j}$
2 The minimum oil-film thickness $h_o = K'_{h_o} \sqrt{ZN/P_j}$
3 The position of the center of the journal α
4 The ratio h_i/h_o

With this knowledge of the shape of the oil film (α and h_i/h_o), it is possible to determine more accurately the leakage factor for friction and oil-film thickness.

The length of the bearing should be taken equal to the zone of positive pressure of the oil film. In partial bearings of a small angle, this zone extends from the beginning of the bearing arc in the direction of rotation to a point approximately half way between the point of nearest approach and the end of the bearing arc.

The curves in Figs. 10, 11, and 12 show very clearly the optimum condition for operation of a bearing. They show also

that the coefficient of friction and the minimum oil-film thickness do not change appreciably with a rather large variation of the clearance ratio, showing that a bearing with a given clearance ratio will operate near the optimum condition over a wide range of loads and speeds. For example, for a range of ZN/P_j be-

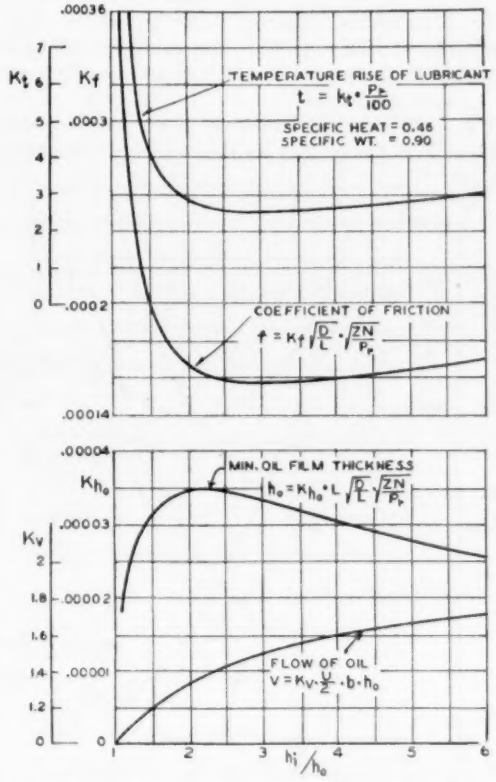


FIG. 4 CHARACTERISTICS OF A PLANE BEARING OF INFINITE WIDTH

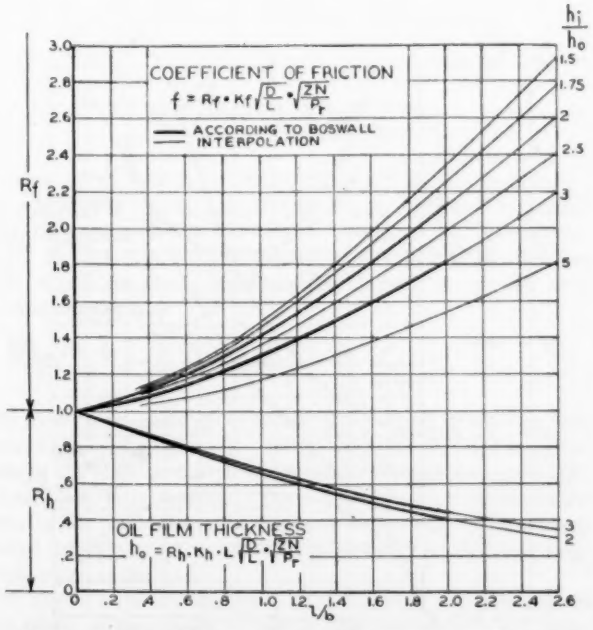


FIG. 5 LEAKAGE FACTORS FOR THE COEFFICIENT OF FRICTION AND OIL-FILM THICKNESS FOR A RECTANGULAR PLANE BEARING

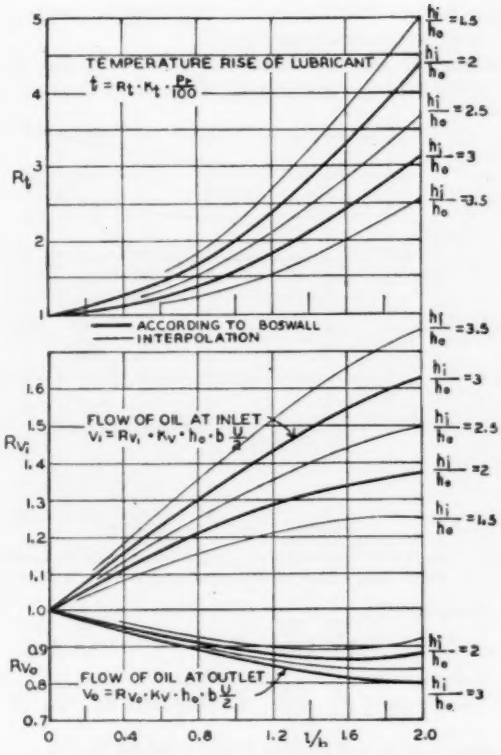


FIG. 6 LEAKAGE FACTORS FOR THE FLOW OF LUBRICANT AND TEMPERATURE RISE FOR RECTANGULAR PLANE BEARINGS

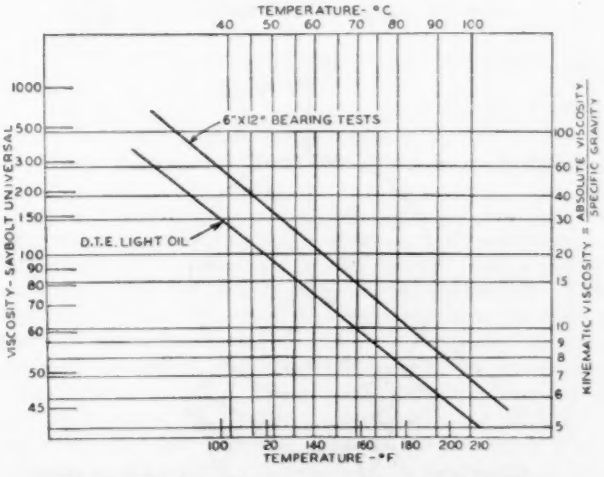


FIG. 7 VISCOSITY-TEMPERATURE CHART OF OILS USED DURING TESTS

tween 80 and 300, the coefficient of friction of a 90-deg bearing will not vary more than 2 per cent and the oil-film thickness more than 1 per cent from the possible theoretical minimum. This small variation of the coefficient of friction and minimum oil-film thickness with a rather large change of the clearance ratio has a very practical aspect from the manufacturing and operation standpoint. It makes possible the use of a constant clearance ratio over a large range of operation while still remaining near the optimum condition of friction and oil-film thickness. In Fig. 13 are shown the optimum clearance ratios for 90-deg, 120-deg and 180-deg central partial clearance bearings. They agree very well with existing practice for electrical machines ($ZN/P_j <$ 100) and for high-speed steam turbines ($ZN/P_j >$ 100) where

clearance ratios of 0.001 in. per in. and 0.002 in. per in., respectively, have been used for many years.

A sufficiently accurate value of the losses in the nonactive parts of the bearings (upper half, oil throwers, etc.) can be obtained by assuming that the clearance of these points is uniform and filled with oil. The losses can then be calculated by considering a uniform shear as shown in Table 1. It was shown by the authors when this paper was presented in more complete form at the Annual Meeting, 1934, of The American Society of Mechanical Engineers, that in a 30-in. × 60-in. journal bearing, the losses in the nonactive parts amounted to approximately 10 per cent of the total bearing losses.

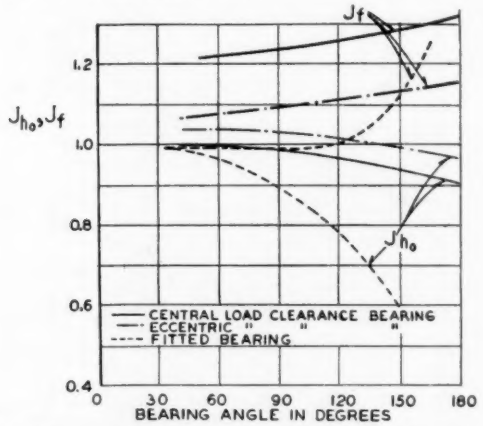


FIG. 8 MINIMUM COEFFICIENT OF FRICTION AND CORRESPONDING OIL-FILM THICKNESS IN BEARINGS OF INFINITE WIDTH (6)

(1) Plane bearing

$$f = 0.000159\sqrt{\frac{D}{L}}\sqrt{\frac{ZN}{P_r}};\quad h_0 = 0.0000345\,L\sqrt{\frac{D}{L}}\sqrt{\frac{ZN}{P_r}}$$

(2) Journal bearing

$$f = 0.000159\,J_f\sqrt{\frac{d}{l}}\sqrt{\frac{ZN}{P_r}};\quad h_0 = 0.0000345\,J_h l\sqrt{\frac{d}{l}}\sqrt{\frac{ZN}{P_r}}$$

where P_r is the unit pressure on the developed bearing surface. Usually for journal bearings the unit pressure is referred to the projected area on the diameter, in that case the following formulas should be used:

$$f = 0.000159\,J_f\sqrt{\frac{ZN}{P_j}};\quad h_0 = 0.0000345\,J_h\,\frac{\pi r}{180}\,\theta\sqrt{\frac{ZN}{P_j}}$$

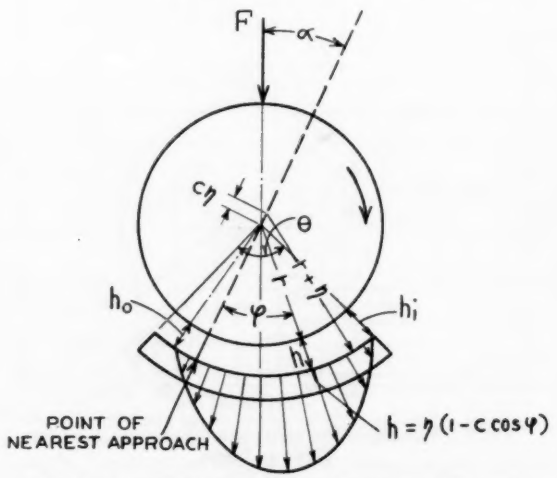


FIG. 9 CENTRAL LOAD PARTIAL CLEARANCE BEARING

TEMPERATURE CONDITIONS

The satisfactory operation of a bearing is usually determined by its maximum temperature. In connection with electrical apparatus it is limited to 80 C in the oil well or a 40-deg rise above 40 C ambient.

Due to friction, the temperature of the oil film increases between the inlet and outlet. Boswall (1) has shown that in the calculation of the bearing performance the average temperature of the oil film can be used with a fair degree of accuracy.

In Fig. 4, t gives the temperature rise through the oil film for a plane bearing of infinite width. This curve has been calculated for an oil having a specific heat of 0.46 and specific weight γ of 0.9.

In Fig. 6, R_t gives the correction factor for the temperature rise of the oil in the oil film for the case of a rectangular plane

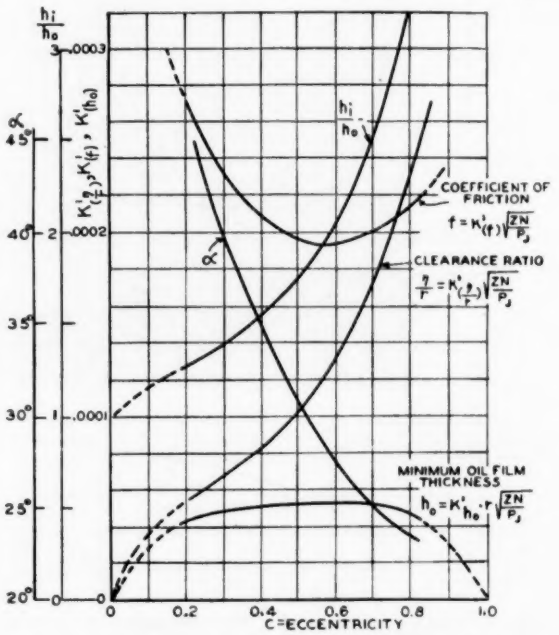


FIG. 10 CHARACTERISTICS OF A 90-DEG CENTRAL PARTIAL BEARING OF INFINITE WIDTH

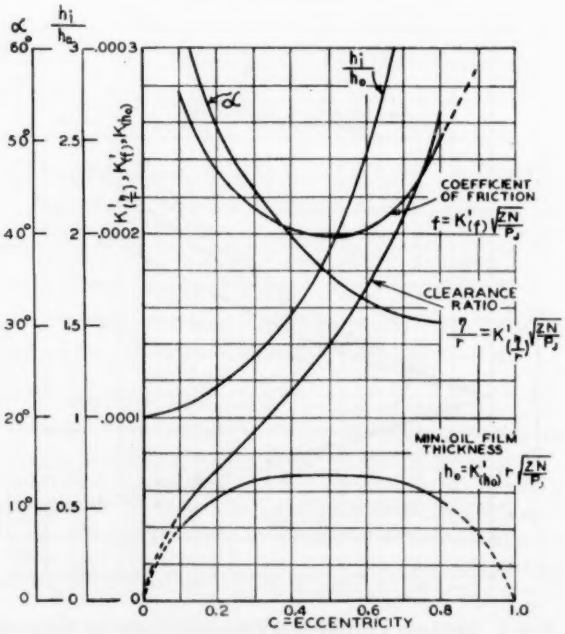


FIG. 11 CHARACTERISTICS OF 120-DEG CENTRAL PARTIAL BEARING OF INFINITE WIDTH

TABLE 1 CALCULATION OF BEARING PERFORMANCE

	Plane bearings		Journal bearings	
	Infinite width $h_i/h_o = 2.5$	Rectangular $h_i/h_o = 2.5$	Characteristic corresponding to min coef of friction	General characteristic
Coef of friction f	$0.000159\sqrt{\frac{D}{L}\frac{ZN}{P_r}}$	$0.000159R_f\sqrt{\frac{D}{L}\frac{ZN}{P_r}}$	$0.000159R_fJ_f\sqrt{\frac{ZN}{P_j}}$	$R_fK'_f\sqrt{\frac{ZN}{P_j}}$
Min oil-film thickness h_o, in.	$0.0000345L\sqrt{\frac{D}{L}\frac{ZN}{P_r}}$	$0.0000345R_hL\sqrt{\frac{D}{L}\frac{ZN}{P_r}}$	$0.0000345R_hJ_hL\sqrt{\frac{ZN}{P_j}}$	$R_{ho}K'_{ho}\sqrt{\frac{ZN}{P_j}}$
Temp rise of oil film t, C	$0.025P_r$	$0.025R_tP_r$	$0.025R_t\frac{J_f}{J_h}P_j$	$0.00658\frac{K'_fR_f}{K'_{ho}R_{ho}}\frac{P_j}{K_v\gamma\rho}$
Flow of oil:				
Inlet V_i, cu in. per sec	$1.43\frac{U}{2}bh_o$	$1.43\frac{U}{2}R_{vi}bh_o$	$1.43\frac{U}{2}R_{vi}J_hbh_o$	$R_{vi}K_vh_ob\frac{U}{2}$
Leakage V_L, cu in. per sec	0	$1.43\frac{U}{2}(R_{vi}-R_{vo})bh_o$	$1.43\frac{U}{2}(R_{vi}-R_{vo})J_hbh_o$	$(R_{vi}-R_{vo})K_vh_ob\frac{U}{2}$

Shear force on oil films, lb $= 1.45 \times 10^{-7}(ZAU/h)$. This force is that necessary to move at a velocity U, a plane of area A, separated from a stationary larger plane by an oil film of uniform thickness h and uniform viscosity Z.

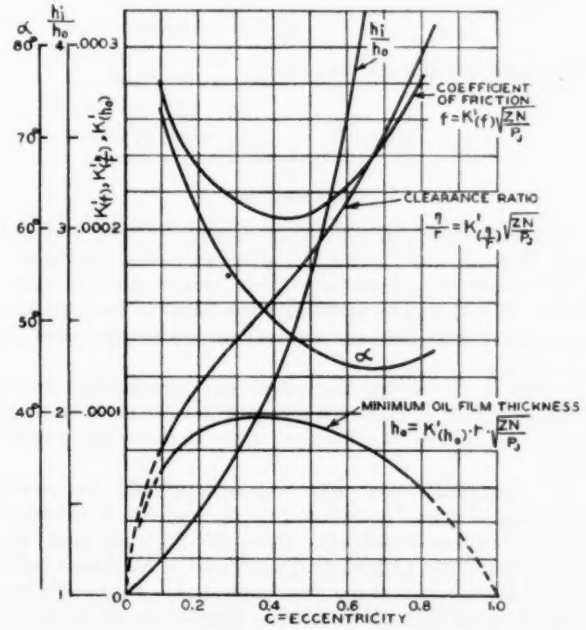


FIG. 12 CHARACTERISTICS OF 180-DEG CENTRAL PARTIAL BEARING OF INFINITE WIDTH

BEARING ANGLE	90°	120°	180°
COEFFICIENT OF FRICTION f	$.00097\sqrt{\frac{ZN}{P_j}}$	$.00200\sqrt{\frac{ZN}{P_j}}$	$.0021\sqrt{\frac{ZN}{P_j}}$
MIN. OIL FILM THICKNESS h_o	$.00052r\sqrt{\frac{ZN}{P_j}}$	$.00067r\sqrt{\frac{ZN}{P_j}}$	$.00097r\sqrt{\frac{ZN}{P_j}}$

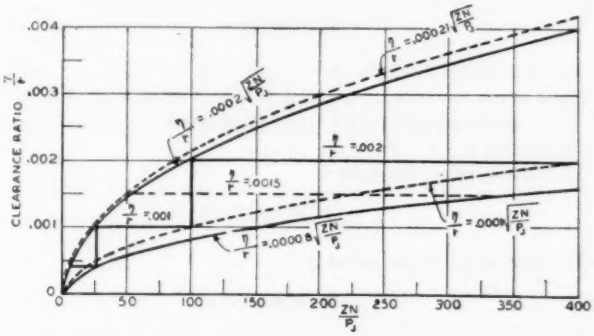


FIG. 13 CLEARANCE RATIO FOR OPTIMUM CONDITION OF CENTRAL PARTIAL CLEARANCE BEARINGS
(Solid line for 90-deg bearings. Dash line for 120-deg and 180-deg bearings.)

bearing. For journal bearings calculated as plane bearings, allowance must also be made for the higher coefficient of friction and the smaller oil-film thickness as compared with plane bearings. In this case, with additional correction factors used (see Figs. 4 and 10) the temperature rise through the oil film of a journal bearing is

$$t_1 = 0.00658\,\frac{K'_f}{K'_{h_o}}\,\frac{R_f}{R_{h_o}}\,\frac{P_j}{K_v\gamma\rho}$$

The average temperature of the oil film is equal to the temperature of the inlet oil plus $t/2$. It is to be noted that the temperature of the inlet oil film is not equal to the temperature of the oil supplied to the bearing. There is a film of hot oil which adheres to the shaft and is continuously recirculated. This film is cooled outside the active oil film, the make-up for the oil leakage being added at the inlet.

It should be noted that for a given bearing the temperature rise of the oil is a function only of the unit pressure and independent of speed and of viscosity of the oil.

The average temperature of the oil film in bearings cooled by means of circulating oil is usually only slightly higher than the temperature of the bearing shell. It can be easily seen that the heat-transfer rate between the oil film and the bearing shell is much higher than between the latter and the ambient air. This is particularly true when the area of contact between the bearing shell and bearing housing is very small. A. Kingsbury has given an exact method of calculating the temperature distribution through the oil film (11). A simpler approximate method can be used to determine the average temperature of the oil film by considering the flow of heat through the oil film which is assumed to be of a uniform thickness equal to the average as in a guide bearing. In a guide bearing where there is a uniform clearance, the shear of the oil is uniform throughout (assuming uniform viscosity which is approximately true for small temperature variation). The same quantity of heat Q flows from the oil film to the bearing shell and from the latter to the ambient air by convection or radiation. Knowing the temperatures of the shell and ambient air, the quantity of heat Q flowing from the shell can be calculated with the use of known heat-transfer data (12, 13). In most bearings the flow of heat through the shaft is negligible (12) and the latter's temperature usually can be taken equal to that of the oil film. For such cases, the temperature distribution through the oil-film thickness can be calculated from the known equation of the steady flow of heat through a body such as the oil film where heat is being uniformly generated. This temperature distribution is parabolic and the

average value of the temperature rise of the oil film above the bearing shell is

$$t_f = Qh/3K = 80\,Qh$$

for ordinary lubricating oils.[4]

This method of calculation will still hold when heat is carried away by means of circulating oil since t_f applies only to the flow of heat through the bearing shell which is a function only of its temperature rise above ambient air.

This method has been applied to determine the average temperature rise of the oil film above the measured temperature of the bearing shell in the described test of 6-in. × 12-in. bearing.

Oil Flow

The supply of oil in any bearing must always be sufficient to maintain a perfect oil film, otherwise, a much higher temperature of operation is to be expected. In some properly designed bearings it is sufficient to supply an amount of oil equal to the side leakage, but it is usually much safer to have a supply equal to the flow of oil at the inlet.

The flow of oil in a plane bearing of infinite width is

$$V = K_v h_o b(U/2)$$

where K_v is as given in Fig. 4.

For a plane rectangular bearing, due to side leakage, the flow of oil is increased at the inlet and decreased at the outlet, as shown in Fig. 6.

The flow of oil at the inlet is

$$V_i = R_{vi} K_v h_o b(U/2)$$

The flow of oil at the outlet is

$$V_o = R_{vo} K_v h_o b(U/2)$$

The side leakage is

$$V_L = V_i - V_o = (R_{vi} - R_{vo}) K_v h_o b(U/2)$$

The flow of oil in a journal bearing of finite width is nearly the same as for a plane rectangular bearing. This method gives results very close to the ones obtained by S. J. Needs (7) by means of an electrical analogy, and also with tests made at the Research Laboratories of the Westinghouse Electric and Manufacturing Company (14).

Application of Data

The application of curves given in this paper is illustrated by an example. The bearing, 6 in. in diameter and 12 in. long, tested in the laboratories (Figs. 1 and 2) was taken for this purpose. The active length of the bearing = 9¾ in., the radial clearance = 0.0045 in., the clearance ratio = 0.0015 in. per in., the load = 10,000 lb, the speed = 1600 rpm, the oil-film temperature = 70 C, and the peripheral velocity = 500 in. per sec. The unit pressure on the projected area P_j = 171 lb per sq in., and the characteristic number of the bearing ZN/P_j = 10.67.

Referring to Fig. 10, the characteristics of the bearing are: $K'(\eta/r)$ = 0.00014, K'_f = 0.000195, K'_{h_o} = 0.000052, α = 27 deg, and h_i/h_o = 2.15. The active length l of the 81-deg oil film = 4.25 in., and the length-width ratio l/b = 0.435.

Leakage factors for the coefficient of friction, oil-film thickness, oil flow, and temperature rise from Figs. 4, 5, and 6 are: R_f = 1.12, R_{h_o} = 0.86, R_{vi} = 1.13, R_{vo} = 0.95, K_v = 1.36, R_i = 1.25, and K_i = 2.7.

[4] Taking K = 0.0042 watt per sq in. per deg C per in. from W. J. King (13).

Coefficient of friction

$$f = R_f K'_f \sqrt{(ZN/P_j)} = 1.12 \times 0.00195 \times 10.67 = 0.00234$$

Minimum oil-film thickness

$$h_o = R_{h_o} K'_{h_o} r \sqrt{(ZN/P_j)} = 0.86 \times 0.000052 \times 3 \times 10.67 = 0.00143 \text{ in.}$$

Inlet oil flow

$$V_i = R_{vi} K_v h_o b(U/2) = 1.13 \times 1.36 \times 0.00143 \times 9.75 \times 250 = 5.36 \text{ cu in. per sec}$$

Outlet oil flow

$$V_o = R_{vo} K_v h_o b(U/2) = 0.95 \times 1.36 \times 0.00143 \times 9.75 \times 250 = 4.5 \text{ cu in. per sec}$$

Oil leakage

$$V_L = V_i - V_o = 0.86 \text{ cu in. per sec}$$

Temperature rise through the oil film

$$t_1 = 0.00658 \frac{K'_f R_f}{K'_{h_o} R_{h_o}} \frac{P_j}{K_v \gamma \rho} = 9.75 \text{ C}$$

Conclusions

Tests were made recently on the apparatus shown in Fig. 1 by the Westinghouse Electric and Manufacturing Company on a journal bearing 6-in. diameter × 12-in. long of standard manufacture. A rather close agreement was found to exist between the results of these tests and theoretical analysis made available in recent years.

In Fig. 2 are shown theoretical and experimental values of the coefficient of friction for varying conditions of load and speeds. It is given as a function of the characteristic number ZN/P_j.

The theoretical curves agree very well with the test results in the range of normal operation of the bearing, that is, for values of ZN/P_j between 80 and 300. Below ZN/P_j = 80, conditions are such that the hydrodynamical theory is no longer valid since this is the region of semifluid lubrication. For large values of ZN/P_j above 300, the momentum effect of the oil at the oil-film inlet becomes of great importance; this effect is not considered in the theoretical analysis in common use and some discrepancy with tests can, therefore, be expected.

In Table 1 the data are summarized for the calculation of the performance of bearings of various types under conditions described in the paper.

Nomenclature

- A = area of a guide bearing, sq in.
- b = width of the bearing surface (axial length of bearing perpendicular to direction of motion), in.
- c = eccentricity of journal bearing
- D = mean diameter of plane bearing, in.
- d = journal bearing diameter, in.
- F = total load acting on bearing, lb
- f = coefficient of friction
- h = oil-film thickness, in.
- h_o = oil-film thickness at outlet, in.
- h_i = oil-film thickness at inlet, in.
- J_f, J_h = type factors for journal bearings
- K_f, K_h, K_t, K_v = coefficients for plane bearing of infinite width
- K = heat-transfer coefficient, watts per sq in. per deg C per in.
- L = length of plane bearing in direction of motion, in.

l = length of the active oil film in journal bearing, in.
N = speed, rpm
P_j = F/bd = unit bearing pressure projected on journal diameter, lb per sq in.
P_r = F/bL = actual unit bearing pressure for rectangular plane bearing, lb per sq in.
Q = flow of heat through bearing surface, watts per sq in.
R_f, R_h, R_v, R_v = leakage factors for plane rectangular bearing
r = $d/2$ = radius of journal bearing, in.
S = shearing force, lb
t_1 = average temperature rise through the oil film, C
t_f = average temperature rise of oil film above bearing shell, C
U = peripheral velocity of journal, in. per sec
V_i = flow of oil at the inlet of the oil film, cu in. per sec
V_o = flow of oil at the outlet of the oil film, cu in. per sec
V_L = volume of oil leaking on the sides of the bearing = $V_i - V_o$, cu in. per sec
Z = oil viscosity, centipoise
α = angle for location of minimum oil-film thickness in journal bearing, deg
γ = specific weight, lb per cu in.
η = radial clearance, in.
ρ = specific weight of oil.

BIBLIOGRAPHY

1 "The Theory of Film Lubrication," by R. O. Boswall, Longmans, Green, and Company, London and New York, N. Y., 1928.

2 "The Lubrication of Plane Surfaces," by A. G. M. Michell, *Zeitschrift für Mathematik und Physik*, vol. 52, 1905, pp. 123–137.

3 "The Film Lubrication of Journal Bearings," by R. O. Boswall, Transactions of the Institution of Mechanical Engineers, vol. 122, 1932, p. 423.

4 "On Problems in the Theory of Fluid-Film Lubrication, With an Experimental Method of Solution," by A. Kingsbury, Trans. A.S.M.E., vol. 53, 1931, paper APM-53-5, pp. 59–75.

5 "Paliers a butée modernes," *Revue Brown, Boveri & Cie*, January, February, and March, vol. 4, 1917. Also, "Regles générales pour la construction des paliers," *Revue Brown, Boveri & Cie*, January, February, and May, vol. 6, 1919.

6 "Optimum Conditions in Journal Bearings," by A. Kingsbury, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-7, pp. 123–148.

7 "Effects of Side Leakage in 120-Deg Centrally Supported Journal Bearings," by S. J. Needs, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-16, pp. 721–732.

8 Discussion by R. Baudry of "Effects of Side Leakage in 120-Deg Centrally Supported Journal Bearings," by S. J. Needs, Trans. A.S.M.E., vol. 57, no. 3, April, 1935, p. 136.

9 "Graphical Analysis of Journal Bearing Lubrication," by H. A. S. Howarth, Trans. A.S.M.E., vols. 45, 46, and 47, 1923, 1924, and 1925, respectively. Combined reprint 1926, 35 pp.

10 "Charts for Studying the Oil Film in Bearings," by G. B. Karelitz, Trans. A.S.M.E., vol. 47, 1925, p. 1101.

11 Heat Effects in Lubricating Oils," by A. Kingsbury, *Mechanical Engineering*, vol. 55, 1933, pp. 685–688.

12 "Performance of Oil-Ring Bearings," by G. B. Karelitz, Trans. A.S.M.E., vol. 52, 1930, paper APM-52-15, p. 161.

13 "The Basic Laws and Data of Heat Transmission," by W. J. King, *Mechanical Engineering*, vol. 55, nos. 3 to 8, inclusive, 1932, pp. 190, 275, 347, 410, 492, and 560.

14 "Side Oil Flow in a 120-Deg Bearing," by G. B. Karelitz, *Mechanical Engineering*, vol. 57, no. 5, 1935, p. 292.

Effect of Surface Rolling on the Fatigue Strength of Steel[1]

By O. J. HORGER,[2] CANTON, OHIO

The investigation discussed in this paper was made to determine the effect of cold-working on the endurance limit and other related physical properties of steel. This cold-working was of a surface nature resulting in an almost negligible reduction in area. The preliminary work conducted in this field, for the most part, has been a laboratory development in so far as the technical and mechanical details of the surface-rolling process is concerned, but it has been sufficient to show the advantageous effect of this process on the endurance limit of steel, thereby justifying further study. Therefore, the investigation discussed in this paper was inaugurated to determine the various factors involved in the immediate application of this surface-rolling process in practice. As a result this paper is in the nature of a progress report covering principally tests made on 0.3-in. and 1-in. diameter cold-worked specimens without stress concentration. Further fatigue tests are being conducted on 1-in. and 2-in. diameter surface-rolled specimens with and without stress concentration and on 0.080-in. diameter fatigue specimens machined from various layers of 2-in. diameter surface-rolled specimens.

Introduction

THE failure of machine parts in fatigue which suggested the investigation reported in this paper usually occurs at points of local weakness. A few of the typical failures found in practice occur at the wheel or gear seat of locomotive and railroad-car axles, at the wheel or bearing seats of automobile axles, at automobile rear-axle housings adjoining the differential, at the pressed or fitted bits of long drill rods used in oil-well and rock-drilling operations, and at grooves and holes in axles. In general, many cases of fitted members subjected to reversed bending action have been a constant source of failure. Some means of reducing the effect of the local stress concentrations causing such failures is very desirable.

As a result of a development made in German laboratories it was shown that cold-working of the surface of the steel in the region of the stress concentration improved its fatigue strength. On the basis of such promising preliminary investigations as have already been made both in Germany and the United States, it appeared worth while to learn more about the phenomena of this surface-rolling process.

Development of the Surface-Rolling Process. At the Wöhler-Institute in Braunschweig, Germany (1),[3] the importance of this method was first recognized in 1928. It was observed in studies on surface conditions, that the effect of hammer and chisel marks did not decrease fatigue strength as did filing or grinding. The fact that failure did not occur in these compressed areas led to further tests.

O. Föppl (2) made fatigue-bending tests on specimens having surfaces prepared under certain rolling pressures and found that the endurance limit was increased 5 to 30 per cent. P. Behrens made torsion-fatigue tests and found an increase in fatigue strength between 6 and 29 per cent. An increase in fatigue strength was indicated by torsion tests conducted by Heydekampf (4) and also by bending tests conducted by Döring (5).

More recently, surface-rolling investigations have been devoted to practical problems of design where severe stress concentration exists. Kühnel (6) studied railroad-axle failures occurring at the wheel fit and found the fatigue strength was reduced one-half due to the press fit of the wheel, and Thum (7) continued this study by surface-rolling the axle wheel seat. He found that rolling the axle surface gave about the same strength as the axle without the press fit or an improvement of 69 per cent.

The effect of compressing plastically the surface area surrounding various forms of stress-concentration systems such as holes and grooves in machine parts were studied at the Darmstadt materials-testing laboratory (8, 9, 10) and an improvement of as much as 38 per cent was obtained. Staedel (11) extended the rolling process to cut threads on bolts, reporting an increased fatigue strength of as much as 100 per cent. Also, Koch (12) studied keyways and reported a small improvement in fatigue strength. A synopsis[4] of various forms of cold-work investigations was given by Thum and Wunderlich (13) in 1934.

In the United States, Peterson and Wahl (14) and Buckwalter (15) have published papers contributing substantiating data on the beneficial effect of cold-working $1^{5}/_{8}$-in. and 2-in. diameter axles at wheel and bearing seats. While the German laboratories have confined their tests to specimens with a maximum diameter of about $^{9}/_{16}$ in., it was indicated that this same improvement may be extended to the larger diameters more frequently met in practice.

While there is no difference of opinion as to the favorable effect of cold-working, there is no uniformity of opinion as to the explanation of why the fatigue strength is increased. There are two schools of thought, and the controversy arising therefrom is represented by Thum and Föppl (16).[4] Briefly, Föppl contends that the change which takes place in the metal structure is largely responsible for this improvement in fatigue strength, while Thum insists that residual stresses is the contributing factor. At the present time there is no direct evidence available to support conclusively either opinion.

Materials, Tests, Testing Equipment, and Test Specimens

Materials Tested. An S.A.E. 1045 open-hearth steel was

[1] From a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Science at the University of Michigan.

[2] Timken Roller Bearing Co. Jun. A.S.M.E.

[3] Numbers in parentheses correspond to similarly numbered references given in the bibliography at the end of the paper.

Presented at the National Applied Mechanics Meeting of The American Society of Mechanical Engineers, Ann Arbor, Mich., June 18–19, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] After this paper was written, Thum and Bautz published an article in *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 6, May–June, 1935, p. 121, in which it was stated that four-fifths of the increased fatigue strength in reversed bending was due to residual stresses and one-fifth was due to hardening, while increased torsional-fatigue strength was all due to strain-hardening.

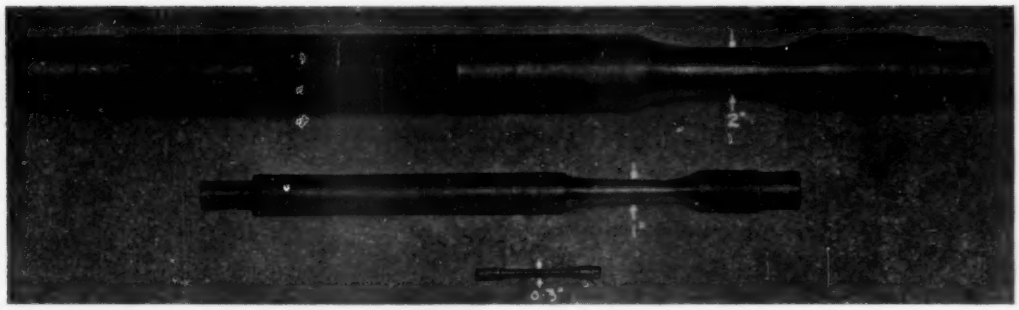


FIG. 1 FATIGUE SPECIMENS TESTED

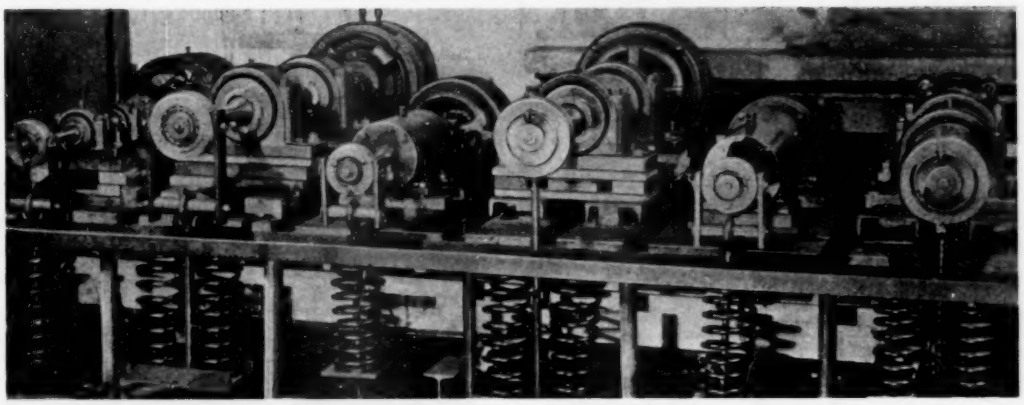


FIG. 2 CANTILEVER-TYPE FATIGUE MACHINES USED TO TEST THE 1-IN. AND 2-IN. DIAMETER SPECIMENS

used in all the tests discussed in this paper. The chemical analysis of each bar tested is shown in Table 1. The physical properties of the bars are given in Table 2. All specimens were normalized at 1620 F and drawn at 1115 F before machining.

Fatigue Tests. The three different diameters of fatigue specimens tested in rotating bending are shown in Figs. 1 and 3. The fatigue machines used to test the 0.3-in. diameter specimens were of the conventional Farmer type, but the 1-in. and 2-in. diameter specimens were tested in cantilever-type machines shown in Fig. 2. Pillow blocks of the selfaligning type with Timken bearings were fitted to the specimen by means of a taper sleeve between the bearing cone and specimen. The load was applied on the end of the 1-in. and 2-in. specimens by the action of compression springs, which had a low constant of about 300 to 400 lb per in. of travel, permitting very close adjustment to the proper load.

Tensile Tests. The 0.3-in. diameter specimen shown in Fig. 4 was used so that the tensile data obtained on surface-rolled specimens would be comparable with fatigue-test data obtained on surface-rolled specimens of the same diameter.

Two Martens mirror extensometers with 100-mm gage length (3.937 in.) were used on all the tensile-test specimens.

Hardness Tests. The Vickers pyramidal diamond test was used to investigate the hardness of the surface-rolled specimens shown in Fig. 5. The Bierbaum scratch-hardness test was used on a transverse section.

Surface Finish of Specimens. The machined surface of the test section of all fatigue specimens was polished with 000 emery cloth previous to the surface-rolling operation. A profilograph (17, 18) was used to measure the surface irregularities along the length of the specimens before rolling and after rolling under various conditions of roller shapes, feeds, and pressures. Briefly, the principle of this instrument consists of (*a*) tracing over the surface with a very sharp diamond point (*b*) magnifying the movements of this tracer point through an optical system, and (*c*) recording them on sensitized paper.

Description of Rolling Equipment. A diagrammatic arrangement of the device used to roll the surface of the specimens is shown in Fig. 6. An engine lathe was adapted for this rolling operation by supporting the specimen in lathe centers and the rolling device in a transverse slide fixed to the lathe carriage. This setup is shown in Figs. 7, 8, and 9, while the various proportions of rollers which were used are shown in Fig. 10. The roller load was applied through calibrated springs with long travel to permit

TABLE 1 CHEMICAL ANALYSIS OF STEELS TESTED

	1/2-in. diameter cold-rolled bar stock	1 3/4-in. diameter hot-rolled bar stock	3 1/4-in. diameter hot-rolled bar stock
Carbon	0.48	0.45 –0.48	0.45 –0.49
Manganese	0.81 –0.82	0.67 –0.68	0.67 –0.77
Phosphorous	0.012–0.018	0.013–0.016	0.013–0.017
Sulphur	0.030–0.039	0.027	0.031–0.037
Silicon	.....	0.22 –0.23	0.20 –0.26
Chromium	.....	0.03	0.03
Nickel	.....	0.12	0.04 –0.06
Molybdenum	.....	.....	.....

TABLE 2 PHYSICAL PROPERTIES OF THE STEELS TESTED

	Cold-rolled[a]		Hot-rolled[b]	
	As rec'd	After heat-treat	1 3/4-in. diam stock	3 1/4-in. diam stock
Yield point, lower, lb per sq in.	....	58600	46200	47800
Ultimate strength, lb per sq in.	115000	88600	89900	88800
Elongation,[c] per cent	16.0	33.2	30.6	32.0
Reduction of area, per cent	33.8	56.9	47.2	48.5
Izod impact,[d] ft-lb	9	35	...	...
Brinell hardness	220	183	174	177
Diameter of specimen, in.	0.3	0.3	0.750	0.750

[a] Specimens cut from 1/2-in. cold-rolled bar stock.
[b] Specimens cut from hot-rolled bar stock after normalising and drawing.
[c] Elongation measured over a gage length L, where $L = 4.51\sqrt{A}$, and A is the cross-sectional area of the specimen.
[d] Standard round specimen.

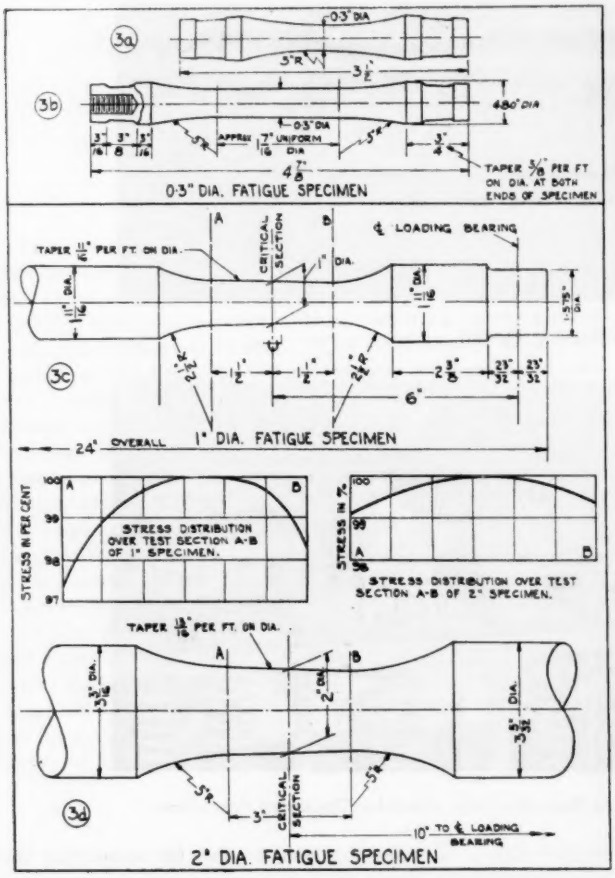


FIG. 3 DETAILS OF THE FATIGUE SPECIMENS

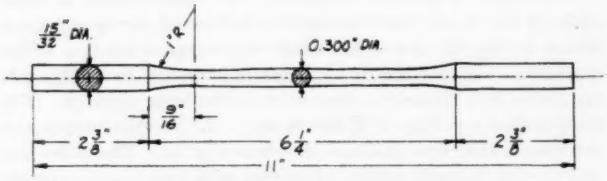


FIG. 4 TENSILE-TEST SPECIMEN

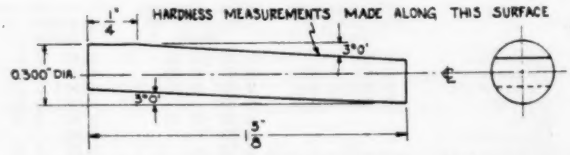


FIG. 5 SPECIMEN FOR VICKERS HARDNESS TEST

accurate load adjustment. The 0.3-in. diameter specimens were rotated in lathe centers at 20 rpm while the 1-in. and 2-in. diameter specimens were rotated at 22 rpm.

Preparation of Cold-Stretched Specimens. Fatigue specimens were machined from 1/2-in. diameter cold-rolled 0.48 carbon steel, normalized and drawn after it had been stretched cold in tension and given a two-hour heat-treatment at 490 F with a furnace cool. Cold-stretching was

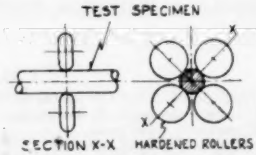


FIG. 6 DIAGRAMMATIC ARRANGEMENT OF DEVICE FOR ROLLING THE SPECIMENS

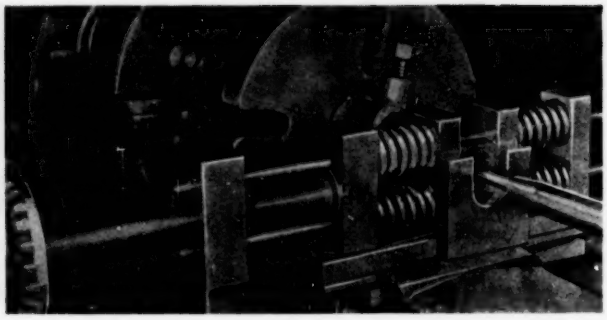


FIG. 7 ROLLING DEVICE IN POSITION ON LATHE CARRIAGE. FATIGUE SPECIMEN MOUNTED IN HOLDERS SUPPORTED IN LATHE CENTERS

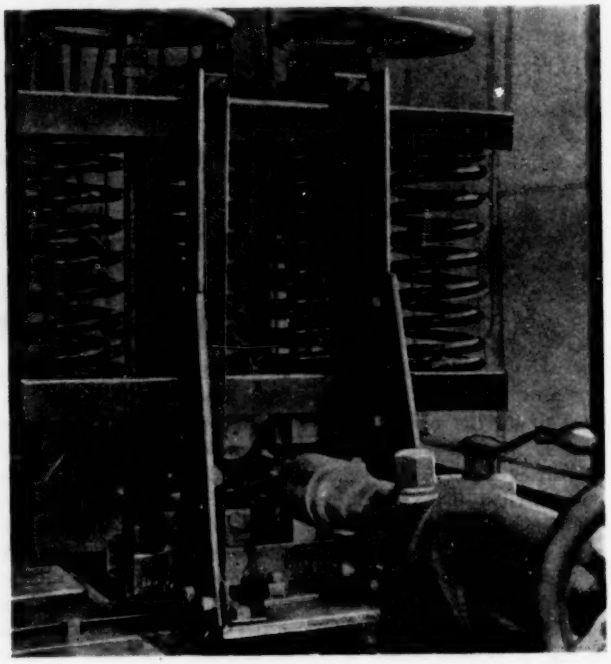


FIG. 8 METHOD OF ROLLING THE 2-IN. DIAMETER SPECIMENS

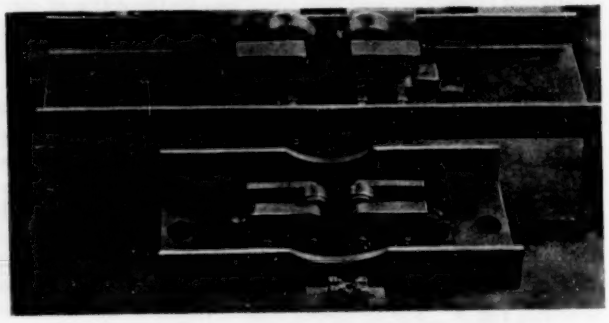


FIG. 9 METHOD OF MOUNTING ROLLERS IN SUPPORTS

investigated at reductions in area of 8 per cent and 17 per cent. In the latter, necking started to take place and the center of the specimen was selected at this minimum diameter.

DISCUSSION OF TEST RESULTS

Endurance Limit of Specimens Without Cold-Working. The

TABLE 3 ENDURANCE LIMITS OF VARIOUS DIAMETER SPECIMENS WITHOUT COLD-WORKING

Line	Diam, in.	Material[a]	Specimen design	Endurance limit, lb per sq in.
1	0.3	1/4-in. diam cold-rolled as received from the mill	Fig. 3b	51600
2	0.3	1/4-in. diam cold-rolled, normalised and drawn	Fig. 3b	39800
3	0.3	1 3/4-in. diam hot-rolled, normalised and drawn	Fig. 3b	39800
4	0.3	Same as line 3	A.S.T.M.[b]	39800
5	0.3	Same as line 3	Fig. 3a	39800
6	1.0	Same as line 3	Fig. 3c	36000
7	2.0	3 1/4-in. diam hot-rolled, normalised and drawn	Fig. 3d	34400

[a] All material S.A.E. 1045 steel.
[b] A.S.T.M. specification "Report of the Committee on Fatigue of Metals," 1935 preprint, Fig. 5. This specimen has a radius of 9 7/8 in.
NOTE: Specimens in lines 3, 4, 5, and 6 were machined from the same bar.

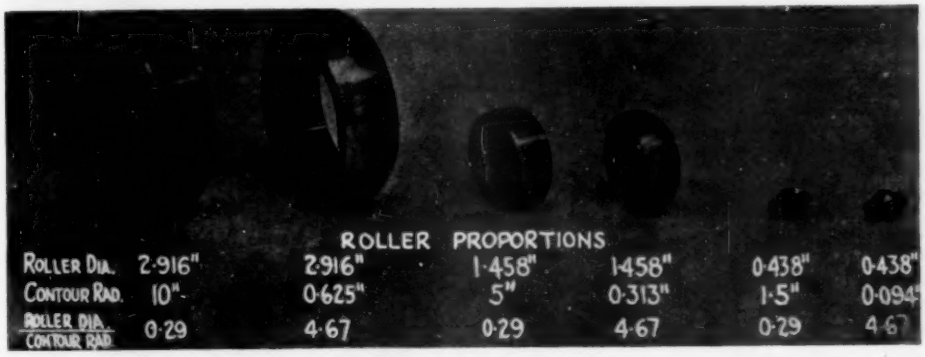


FIG. 10 ROLLER PROPORTIONS

endurance limits[5] established on the various diameter specimens are summarized in Table 3. The values of endurance limit are used as a base from which any improvement due to cold working is calculated.

Lines 3 and 6 of Table 3 indicate that there is a change in fatigue strength in going from the 0.3-in. to 1-in. diameter. This effect of variation of fatigue strength with size of specimen is in accordance with that reported in the literature (19, 20) for specimens without stress concentration.

Effect on Endurance Limit of Surface Rolling. The results of the tests on 0.3-in. diameter surface-rolled specimens are given in Table 4 and shown graphically in Figs. 11 and 12.

TABLE 4 ENDURANCE LIMIT OF 0.3-IN. SURFACE-ROLLED SPECIMENS

S.A.E. 1045 steel, normalised and drawn	Normal pressure per roller, lb	Endurance limit 7/16 × 3/32-in. roller[a] Lb per sq in.	Per cent	Endurance limit 7/16 × 1 1/2-in. roller[b] Lb per sq in.	Per cent
Specimens made from 1/4-in. diam cold-rolled bar	0	39800	100	39800	100
	70.5[c]	46400	116	...	...
	130	49000	123	46500	117
	183	51400	129	49000	123
	210	52200	131	50000	126
	250	52600	132	50000	126
Specimens made from 1 3/4-in. diam hot-rolled bar	0	36000	100	...	...
	130	45200	125	...	...
	183	49500	138	...	...
	210	52000	144	...	...

[a] Roller has a 7/16 in. diameter and a 3/32-in. contour radius.
[b] Roller has a 7/16 in. diameter and a 1 1/2-in. contour radius.
[c] Rolled with a feed equivalent to 320 threads per in. All other specimens rolled with a feed equivalent to 224 threads per in.
NOTE: The endurance limit of the 1/4-in. diameter cold-rolled steel as received from the mill is 51,600 lb per sq in.

From Fig. 11 it will be seen that the middle region of the curves indicates that the increase in endurance limit is directly proportional to the roller pressure for both shapes of rollers. The two

[5] Space does not permit the presentation of S-N diagrams. The endurance limits given are based on a minimum of 10 million cycles. The 0.3-in. diameter specimens were subjected to 1750 stress cycles per min while the 1-in. diameter specimens were subjected to 1150 stress cycles per min.

TABLE 5 EFFECT OF ROLLER SHAPE ON THE ENDURANCE LIMIT OF 0.3-IN. SPECIMENS

Normal pressure per roller, lb	Endurance limit 7/16 × 3/32-in. roller, (1)[a]	Endurance limit 7/16 × 1 1/2-in. roller, (2)[b]	Ratio (1)/(2)
70.5	46400	43800	1.06
130	49000	46500	1.05
183	51400	49000	1.05
210	52200	50000	1.04
250	52600	50000	1.05

[a] Roller has a diameter of 7/16 in. and a contour radius of 3/32 in.
[b] Roller has a diameter of 7/16 in. and a contour radius of 1 1/2 in.

shapes of rollers differ only in contour radius, their diameters being the same. The effect of roller shape is given in Table 5, where it is observed that under constant roller pressure there results a change of only 5 per cent in endurance limit. Considering the same endurance limit with both types of rollers, it is seen in Table 6 that the flat roller of 1 1/2-in. contour radius requires considerable more pressure than the sharp roller of 3/32-in. contour radius to obtain this differential of 5 per cent. The effect of roller shape is discussed later in the paper.

The fatigue studies discussed, on 0.3-in. diameter specimens, have been extended to include 1-in. and 2-in. diameter specimens. A portion of this program has been completed and

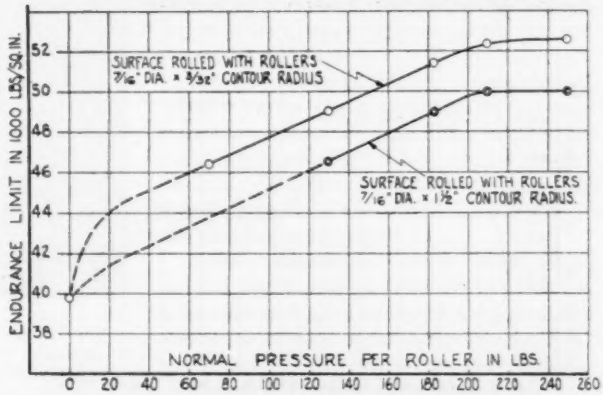


FIG. 11 EFFECT OF ROLLER PRESSURE ON ENDURANCE LIMIT OF SPECIMENS MACHINED FROM NORMALIZED AND DRAWN 1/4-IN. DIAMETER COLD-ROLLED S.A.E. 1045 STEEL

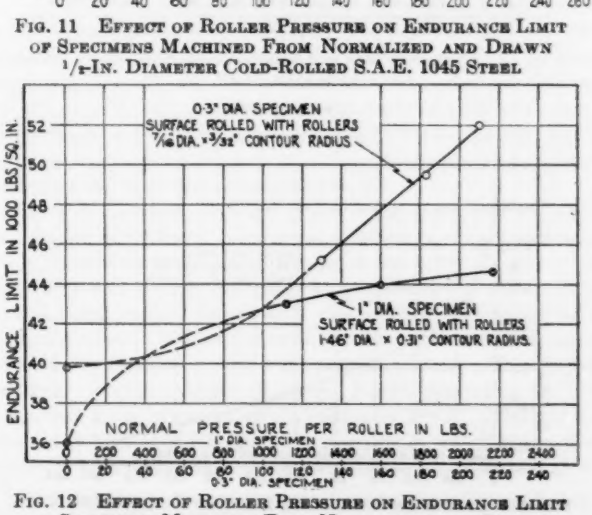


FIG. 12 EFFECT OF ROLLER PRESSURE ON ENDURANCE LIMIT OF SPECIMENS MACHINED FROM NORMALIZED AND DRAWN 1 3/4-IN. DIAMETER HOT-ROLLED S.A.E. 1045 STEEL

TABLE 6 ROLLER PRESSURES REQUIRED FOR SIMILAR ENDURANCE LIMITS OF 0.3-IN. SPECIMENS

Endurance limit, lb per sq in.	Normal pressure per roller, lb $^7/_{16} \times {}^3/_{32}$-in. roller, (1)[a]	$^7/_{16} \times 1^1/_2$-in. roller, (2)[b]	Ratio (2)/(1)
46400	78.5	128	1.82
48000	107	162	1.51
49500	140	194	1.39

[a] Roller has a diameter of $^7/_{16}$ in. and a contour radius of $^3/_{32}$ in.
[b] Roller has a diameter of $^7/_{16}$ in. and a contour radius of $1^1/_2$ in

TABLE 7 ENDURANCE LIMITS OF 1-IN. DIAMETER SURFACE-ROLLED SPECIMENS

Normal pressure per roller, lb	Endurance limit — 1.458 × $^5/_{16}$-in. roller[a] — Lb per sq in.	Per cent	1.458 × 5-in. roller[b] — Lb per sq in.	Per cent	Ratio (1)/(2)
0	36000	100	36000	100	...
1125	43000	119	39700	110	1.08
1600	44000	112	...	...	...
2170	44600	124	...	...	..

[a] Roller has a diameter of 1.458 in. and a contour radius of $^5/_{16}$ in.
[b] Roller has a diameter of 1.458 in. and a contour radius of 5 in.
NOTE: The material from which the specimens were made was $1^3/_4$-in. diameter hot-rolled S.A.E. 1045 steel, normalized and drawn. All specimens were rolled with a feed equivalent to 96 threads per in.

TABLE 8 COMPARISON OF ENDURANCE LIMITS OF 0.3-IN. AND 1-IN. DIAMETER SPECIMENS

0.3-in. specimen[a] — Normal roller pressure, lb	Endurance limit — Lb per sq in.	Per cent	1-in. diameter specimen[b] — Normal roller pressure, lb	Endurance limit — Lb per sq in.	Per cent
0	39800	100	0	36000	100
70.5	41500	113	780	42100	117
130	45200	125	1440	43700	121
183	49500	138	2030	44600	124

[a] Roller used had a diameter of $^7/_{16}$ in. and a contour radius of $^3/_{32}$ in.
[b] Roller used had a diameter of 1.46 in. and a contour radius of 0.31 in.

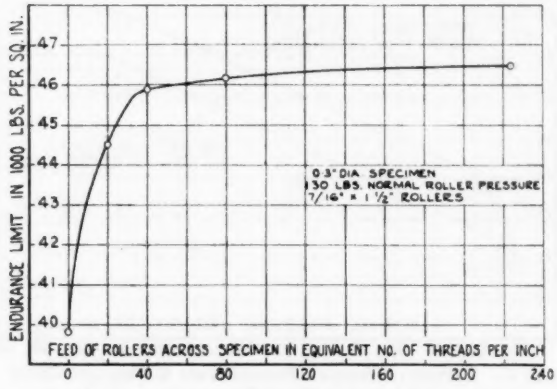


FIG. 13 EFFECT OF ROLLER FEED ACROSS SPECIMEN ON THE ENDURANCE LIMIT

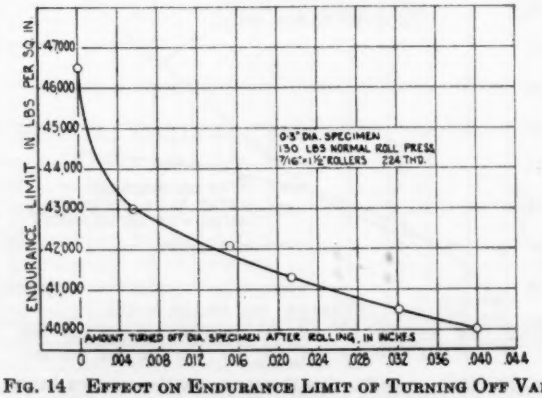


FIG. 14 EFFECT ON ENDURANCE LIMIT OF TURNING OFF VARIOUS AMOUNTS ON THE DIAMETER OF THE SPECIMENS AFTER ROLLING

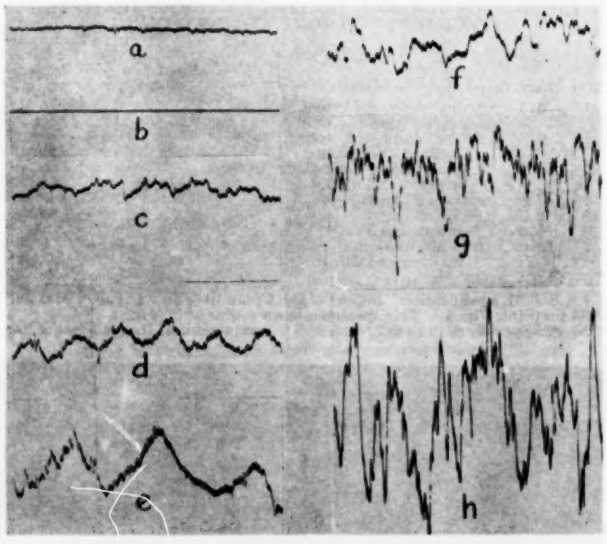


FIG. 15 PROFILOGRAPH RECORDS, FULL SIZE, FOR VARIOUS TYPES OF SURFACE CONDITIONS

(Rollers used were $^7/_{16}$-in. diam. with $1^1/_2$-in. contour radius. Scale used is vertical magnification 2000×, where $^1/_{32}$ in. = 15 millionths and horizontal magnification 30×, where $^1/_{32}$ in. = 0.001 in.)

Curve	Specimen no.	Normal rolling pressure per roller, lb	Equiv. roller feed across specimen in threads per inch
a	2–5	0	...
b	Optical flat for comparison purposes		
c	1–3	183	224
d	4–2	210	224
e	2–23	250	40
f	4–1	250	224
g	Cast-iron cylinder bore with one tool		
h	Cast-iron cylinder bore with another tool		

the data obtained to date on 1-in. specimens is given in Table 7. Referring to Fig. 10, it is seen that the $^7/_{16}$-in. × $^3/_{32}$-in. rollers and 0.3-in. specimen are geometrically similar to the 1.46-in. × 0.31-in. rollers and 1-in. specimen. It may then be expected that the percentage increase in endurance limit would be the same with both 0.3-in. and 1-in. specimens rolled at pressures in the ratio of the squares of the diameters of the two specimens. In this respect the test results shown in Table 8 and Fig. 12 indicate that the percentage of increase in fatigue strength for the 1-in. rolled specimens approaches that for the 0.3-in.

TABLE 9 EFFECT OF ROLLER FEED ON ENDURANCE LIMIT

Roller feed — Equiv. threads per in.	In.	Endurance limit — Lb per sq in.	Per cent	Ratio of roller contact width[a] to roller feed in.
0	0	39800	100	0
20	0.050	44500	112	1.01
40	0.025	45900	115	2.02
80	0.0125	46200	116	4.03
224	0.00446	46500	117	11.3

[a] Actual contact width between roller and length of specimen = 0.0505 in. A discussion of static indentation tests is given later in the paper under the heading "Effect of Roller Shape."
NOTE: The roller used had a diameter of $^7/_{16}$ in. and a contour radius of $1^1/_2$ in. The normal pressure per roller was 130 lb.

TABLE 10 EFFECT ON ENDURANCE LIMIT OF MACHINING SPECIMEN AFTER ROLLING

Amount removed from diam, in.	Endurance limit — Lb per sq in.	Per cent
0	46500	100
0.0056	43000	93
0.0152	42100	90
0.0214	41300	89
0.0322	40500	87
0.0402	40000	86

NOTE: Specimen rolled at a feed equivalent to 224 threads per in. with a roller having a diameter of $^7/_{16}$ in. and a contour radius of $1^1/_2$ in.

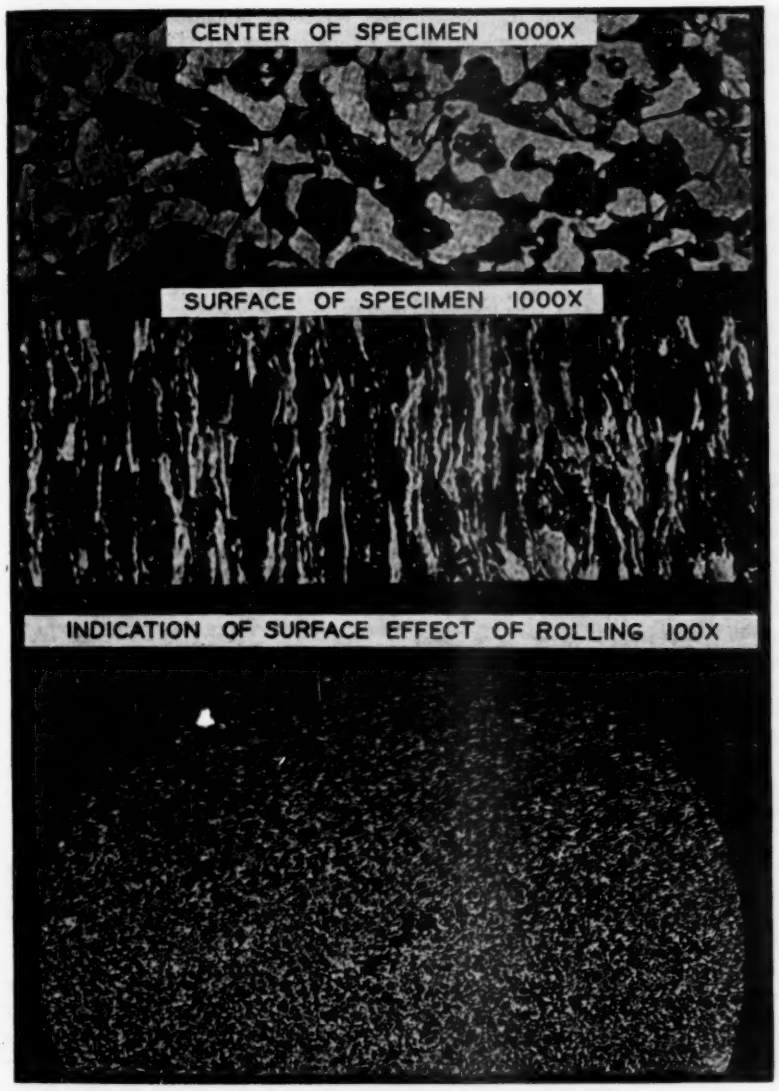


FIG. 16 PHOTOMICROGRAPHS OF SPECIMEN 4-1 THROUGH A TRANSVERSE SECTION
(The surface of the specimen was rolled with a normal pressure of 250 lb per roller with rollers of $^{7}/_{16}$-in. diameter and $1^{1}/_{2}$-in. contour radius fed at a speed equivalent to 224 threads per in.)

In this comparison, the 0.3-in. specimens were selected from the region of negligible bending stress of the drive-coupling end of a 1-in. specimen after it was tested in reversed bending. In this manner the same material and heat-treatment were used and there was no effect of prestressing. Also the 0.3-in. specimens were cut from the region of the larger specimen at $^{1}/_{2}$-in. radius so as to present material in the same surface layer.

TABLE 11 DEPTH OF PENETRATION OF COLD-WORKING ON 0.3-IN. SPECIMENS

Normal pressure per roller, lb	Depth of penetration, in. $^{7}/_{16} \times {}^{3}/_{32}$-in. roller	Depth of penetration, in. $^{7}/_{16} \times 1^{1}/_{2}$-in. roller
183	0.0075[a]	0.006
250	0.0150	0.011

[a] Specimen given a mild heat-treatment after rolling.

Effect of Roller Feed on Endurance Limit. A study of the effect of the roller feed on the endurance limit is given in Table 9. It may be concluded from the values given in this table that very little increase in fatigue strength results with feeds above 40 equivalent threads per inch for the particular shape of roller and pressure used. This is readily apparent from the endurance feed curve in Fig. 13.

The manner in which the roller feed affects the endurance limit is believed to be due to the degree of uniformity to which the surface of the specimen is cold-worked. As an expression of this uniformity we may use the ratio

$$\frac{\text{Actual contact width between roller and length of specimen}}{\text{Equivalent thread pitch of roller feed}}$$

It would be desirable to know the proper feed for other pressures using the same roller and to extend this still further to apply to rollers of different contours and other materials. This determination could be made by a large number of fatigue tests, but it is believed that this ratio equal to 2 would be suitable for calculating the proper feed.

Effect on Endurance Limit of Machining Specimen After Rolling. The rolled surface was machined off a number of 0.3-in. diameter specimens rolled under the same conditions with the results shown in Table 10 and Fig. 14. The conclusions to be derived from this work would indicate that the first 0.0056 in. removed from the diameter reduces the endurance limit by 7 per cent or by an amount equal to one-half of the total increase due to rolling. The removal of this surface layer on small parts would be important, but the decrease in endurance limit would become less as the parts become larger.[6]

Surface Profiles of Rolled Specimens. A comparison of surface irregularities of specimens which were surface-rolled with machined surfaces is given in Fig. 15. It will be observed that the rolled surfaces are more irregular than a 000-emery finish and yet the fatigue strength is higher as given by Table 4.

Effect of Mild Heat-Treatment After Rolling. A mild heat-treatment at 525 F for 1 hour with a furnace cool after rolling of the specimen effected an increase in the fatigue strength of 6 per cent over the specimen not heat-treated.

The effect of aging and mild heat-treatment in increasing the low proportional limit of materials subjected to cold-working has been known for a long time (21). The phenomena underlying this action are not known. Several theories have been advanced of which one indicates that mild heat-treatment causes a release of internal stress (21) while another indicates that the slip planes present in the cold-worked material are healed by the mild heat-treatment (22, 23).

The way in which this low proportional limit was effective was displayed by many cases of the bending of the rolled specimens, instead of resulting in fracture in the fatigue tests. The specimens and testing machines were checked for eccentricity and the load applied slowly after the machine had reached its operating speed. The vibration of the specimen would not begin immediately but after a period of operation at load. This period was from several minutes to several hours, depending on the magnitude of the stress above the endurance limit. It should be mentioned that this vibration has apparently no effect on the values of endurance limit established, since no vibration took

[6] This statement assumes that the larger parts will be rolled at proportional rolling pressures.

place at the endurance limit or below it. A difference of about 400 lb per sq in. bending stress exists between the fatigue strength of two specimens that will both run 10 million cycles, but with one specimen there is no perceptible vibration and with the other specimen the vibration builds up at the beginning of the test, but then does not increase as for specimens under higher stress.

Stress-strain curves were made from static-bending tests and hysteresis loops formed at 2000 lb per sq in. below the endurance limit for the rolled specimens. No hysteresis could be measured for either the specimens given a mild heat-treatment after rolling or specimens not rolled when both were tested at 2000 lb per sq in. above the endurance limit. This checks with the fatigue tests where practically no difficulty with vibration was experienced.

It is believed that the static-bending tests are good reasons for the vibration phenomenon encountered. Kimball (24, 25) has shown that internal friction, as for instance elastic hysteresis, may cause a shaft to whirl.

It should be mentioned that when the rolling process is used to strengthen local regions of a machine part instead of the whole this difficulty with vibration may not be much of a factor. In any event a mild heat-treatment will remedy the situation.

Microstructure of Surface-Rolled Specimens. Typical structural conditions after rolling, shown in Fig. 16, indicate that the stretching and elongation of the crystals with parallel fibers tend to improve the quality of the steel. The depth of penetration of cold-working was measured from photomicrographs at 1000 magnification. The results obtained are shown in Table 11. The depth given was measured at the point where the fibrous structure begins to merge into the normal structure. With this type of steel, the proportions of ferrite and pearlite make it difficult to determine the depth of penetration, and for this reason these measured depths are of a qualitative rather than a quantitative value.

Hardness of Surface-Rolled Specimens. The hardness of a rolled specimen as determined by the Bierbaum scratch test and Vickers pyramidal diamond test gives different results as shown in Fig. 17. The pyramidal diamond test indicates that the surface hardness is 25 per cent greater than the core, while the scratch test shows a difference of 100 per cent. This divergence may be attributed to the different nature of the two types of hardness tests.

It is interesting that cold-stretched and rolled specimens which gave pyramidal diamond measurements of about the same surface hardness also exhibited a fatigue strength of practically the same value.

Tensile Tests of Rolled and Cold-Stretched Specimens. From the summary data shown in Tables 2, 4, and 15, it may be concluded that the degree of cold-working used by surface-rolling, cold-stretching, and rolling in accordance with steel-mill practice, affects the physical properties of the $^1/_2$-in. diameter cold-rolled S.A.E. 1045 normalized-and-drawn steel as given in Table 12. In general, Table 12 indicates the desirable high fatigue strength of surface-rolled specimens which is obtained with small loss of ductility measured by per cent elongation, reduction in area, and area under tensile-test diagram. The mill-rolling and cold-stretching operations are more uniform in their action throughout the section and considerably reduce the ductility. A measure of this ductility is taken from the elongation, reduction in area, and area under the tensile-test diagram of Table 15.

Effect of Roller Shape.[7] It was thought that a measure of the pressure distribution over the roller indentations made on the fatigue specimens in the rolling process would give some relation to the fatigue strength through the strain hardening produced. Static indentation tests were therefore made using the rollers as an indentor. The indentations were made on 0.3-in. and 1-in. diameter specimens covered with a thin film of smoke. The size of the recovered indentation was determined with a micrometer microscope. There may be a very small difference between the recovered and unrecovered area of indentation but its determination would require considerable refinement of equipment. Table 16 summarizes the calculated and actual indentation characteristics.

TABLE 12 EFFECT OF COLD WORKING ON PHYSICAL PROPERTIES OF SPECIMENS[a]

	Surface rolling	Cold stretching	Mill rolling
Ultimate strength	+ 8 to 12	+22 to 25	+30
Yield point	.....	+82	..
Reduction in area	— 1 to 14	—10 to 14	—41
Elongation	—10 to 12	—45 to 47	—52
Area under tensile-test diagram	— 8 to 22	—45 to 62	..
Endurance limit	Up to +29	Up to +36	+30

[a] All values given in per cent.

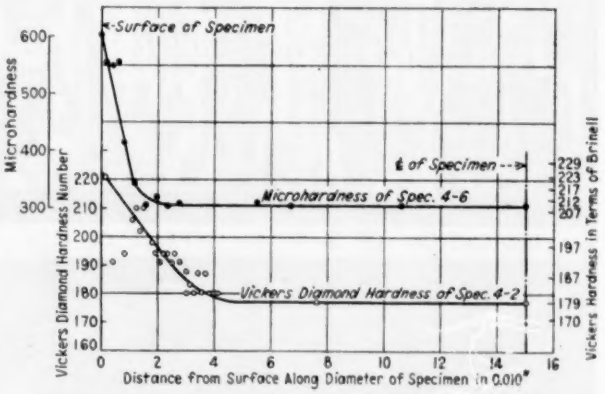


FIG. 17 COMPARISON ON SURFACE-ROLLED SPECIMENS OF HARDNESS OVER A CROSS SECTION BY TWO DIFFERENT METHODS

(Hardness comparisons made by the Bierbaum diamond-point scratch test using a 3-g load and by the Vickers pyramidal diamond test using a 30-kg load. The specimens were rolled with a normal pressure of 250 lb per roller with rollers of $^7/_{16}$-in. diameter and $1^1/_2$-in. contour radius fed at a speed equivalent to 224 threads per in. The specimen design is shown in Fig. 3*d*.)

The calculated values[8] assume elastic conditions and the actual values are based on the measured indentations. In both cases an ellipsoidal pressure distribution similar to the Hertz theory is used to determine q_o, the maximum pressure at the center line of ellipsoid of contact between the roller and specimen.

Fig. 18 shows the value of q_o determined from the static indentation tests plotted against per cent increase in endurance limit determined from the fatigue tests on rolled specimens. It may be noted that q_o reaches a limiting value and then drops off when the fatigue strength becomes a maximum. It would also seem as though the fatigue strength for the 1-in. diameter specimen would continue to increase with higher rolling pressures, but the increase would be small compared to the pressure.

Fatigue Strength of Cold-Stretched Specimens. The results of these tests are given in Table 13 with a comparison of similar tests made by Moore and Kommers (29) on a different steel. The

[7] The effect of roller shape on the endurance limit was considered by Thum and Bautz (8). They proposed a relation between roller pressure and favorable roller shape to produce "the desired increase in fatigue strength." The application of their relation to the tests outlined in this paper on 0.3-in. diameter specimens gives a minimum roller pressure of 600 lb as compared to 200 lb actually used to obtain maximum fatigue strength. Their relation is not satisfactory, and since they do not give the reason or derivation of their proposed relation, no further discussion can be made.

[8] Calculations showing how these values were derived are too lengthy to include in this paper but the procedure may be found in the literature (26, 27, 28).

TABLE 13 FATIGUE STRENGTH OF COLD-STRETCHED SPECIMENS

Brinell hardness[a] No.	Brinell hardness[a] Per cent	Stretched condition of specimen	Ultimate strength — Author[a] Lb per sq in.	Ultimate strength — Author[a] Per cent	Ultimate strength — M. and K.[b] Lb per sq in.	Ultimate strength — M. and K.[b] Per cent	Endurance limit — Author[a] Lb per sq in.	Endurance limit — Author[a] Per cent	Endurance limit — M. and K.[b] Lb per sq in.	Endurance limit — M. and K.[b] Per cent
183	100	No stretch	88600	100	61500	100	39800	100	28000	100
194	106	8% red. area	111500	125	67600	110	50000	126	35000	125
228	125	17–18% red. of area	107800	122	73400	119	54000	135	41000	146

[a] Results as obtained by the author on specimens made from 1/2-in. diameter cold-rolled 0.48-C steel, normalized and drawn.
[b] Results as given by Moore and Kommers (29) for 1/2-in. diameter hot-rolled 0.18-C reinforcing steel bars.

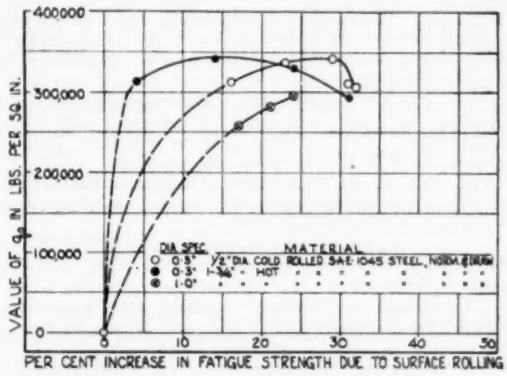


FIG. 18 VARIATION OF q_0 WITH INCREASE IN FATIGUE STRENGTH

data presented in Table 13 indicate that strain hardening[9] in the absence of residual stresses was directly responsible for the large increase in fatigue strength. In all these tests it has been found that an increase in hardness is associated with higher fatigue strength. While it is too early to state definitely, the results of these tests seem to indicate that the strain-hardening phenomena rather than the residual stresses are largely responsible for the increased fatigue strength found with rolled specimens.

[9] A discussion of strain hardening and residual stresses may be found in the literature (27).

TABLE 14 EFFECT OF SURFACE-ROLLING ON COLD-WORKED SPECIMENS

Condition of specimen	Without rolling Lb per sq in.	Without rolling Per cent	With rolling[a] Lb per sq in.	With rolling[a] Per cent
Normalized and drawn	39800	100	46500	117
Cold-rolled as received from mill	51600	100	54500	106
Cold-stretched, 8% red. area	50000	100	53500	107

[a] Rolling pressure of 130 lb per roller used with a 7/16-in. diameter × 1 1/2-in. contour-radius roller. Specimen rolled with a feed equivalent to 40 threads per in.

Effect of Surface Rolling of Previously Cold-Worked Specimens. In these tests, the surface rolling of specimens which had already been cold-worked showed a much smaller percentage of increase in endurance limit than found on the normalized-and-drawn steel. The results of these tests are summarized in Table 14. It has been shown by O'Neill (30) that an increase in indentation hard-

TABLE 15 SUMMARY OF TENSILE-TEST DATA OBTAINED ON SURFACE-ROLLED AND COLD-STRETCHED SPECIMENS[a]

Specimen Kind	Specimen No.	Surface rolling data — Norm. press. per roller, lb	Roller diam, in.	Contour radius, in.	Feed equiv. thds. in.	Ult. strength — Lb per sq in.	Ult. strength — Per cent	Yield point — Lb per sq in.	Yield point — Per cent	Red. of area — Per cent red. of area	Red. of area — Per cent	Elongation[c] — Per cent elongation	Elongation[c] — Per cent	Tensile-test diagram — Area, sq in.	Tensile-test diagram — Per cent	Tensile-test diagram — Fig. no.[e]
Surface-rolled	8–6	0	...	...	..	88600	100	58600	100	56.9	100	33.2	100	6.20	100	34a
	8–9	130	7/16	1 1/2	40	96600	109	See stress-	...	56.5	99		..	5.73	92	34b
	8–13[b]	130	7/16	1 1/2	40	99000	112	strain	...	54.5	96	29.2	88	5.41	87	34c
	8–11	130	7/16	3/32	96	99200	112	curves	...	49.1	86	29.1	88	4.84	78	34d
	8–10	210	7/16	1 1/2	40	95700	108	Fig. 37[e]	...	56.6	99	30.0	90	5.28	85	35a
Cold-tension-stretched	7–1	Stretched 7.96% reduction area				110500	125	106200	182	48.8	86	17.5	53	3.39	55	35b
	7–9[d]	Stretched 8.29% reduction area				111100	125	107200	183	51.2	90	18.3	55	2.98	48	35c
	8–8	Stretched 18.6% reduction area				107800	122	107200	183	50.4	89	18.3	55	2.34	38	35d

[a] Specimens machined to design shown in Fig. 3b from 1/2-in. diameter cold-rolled steel which had been normalized and drawn.
[b] Specimen given a mild heat-treatment after rolling for 1 hr at 525 F and left to cool in a closed furnace.
[c] Gage length of the specimen L used in these tests was 1.2 in., and is obtained from the relation $L = 4.51\sqrt{A}$, where A is the cross-sectional area of the specimen (2).
[d] Specimen given a mild heat-treatment after stretching for 1 hr at 525 F and left to cool in a closed furnace.
[e] Space does not permit inclusion of these references in this report.

TABLE 16 RESULTS OF STATIC-INDENTATION TESTS

Specimen diam	Pressure, lb per roller	2a,[a] in. Calcd.	2a,[a] in. Actual	2b,[b] in. Calcd.	2b,[b] in. Actual	ab, in. Calcd.	ab, in. Actual	q_0[c], lb per sq in. Calcd.	q_0[c], lb per sq in. Actual	Endurance limit, lb per sq in.	End. limit increase, Per cent
0.3 in., cut from 1/2-in. stock[d]	70.5	0.0135	0.0176	0.0132	0.0244	0.000140	0.000338	754000	313000	46400	16
	130.0	0.0166	0.0235	0.0162	0.0315	0.000211	0.000581	925000	336000	49000	23
	183.0	0.0186	0.0274	0.0182	0.0372	0.000266	0.000799	1036000	343000	51400	29
	210.0	0.0194	0.0308	0.0190	0.0417	0.000290	0.001010	1085000	312000	52200	31
	250.0	0.0206	0.0343	0.0202	0.0454	0.000327	0.001220	1150000	306000	52600	32
0.3 in., cut from 1 3/4-in. stock[e]	70.5	0.0135	0.0178	0.0132	0.0240	0.000140	0.000336	754000	314000	41500	4
	130.0	0.0166	0.0235	0.0162	0.0310	0.000211	0.000571	925000	342000	45200	14
	183.0	0.0186	0.0287	0.0182	0.0375	0.000266	0.000830	1036000	331000	49500	24
	210.0	0.0194	0.0313	0.0190	0.0435	0.000290	0.001072	1085000	294000	52000	31
	250.0	0.0206	0.0358	0.0202	0.0460	0.000327	0.001290	1150000	290000	...	..
1.0 in., cut from 1 3/4-in. stock[f]	780.0	0.0452	0.0776	0.0437	0.0745	0.0001544	0.004540	760000	258000	42100	17
	1440.0	0.0552	0.1004	0.0535	0.0973	0.002321	0.007670	930000	282000	43700	21
	2030.0	0.0619	0.1149	0.0600	0.1147	0.002917	0.010350	1043000	294000	44600	24
	2330.0	0.0648	0.1231	0.0628	0.1216	0.003200	0.011750	1093000	297000	...	..
	2770.0	0.0686	0.1319	0.0665	0.1313	0.003587	0.013600	1157000	307000	...	..

[a] $2a$ = the major axis of the area of contact.
[b] $2b$ = the minor axis of the area of contact.
[c] $q_0 = 3/2 \times ab$ = maximum pressure at the center line of ellipsoid of contact between the roller and specimen.
[d] This specimen machined from 1/2-in. diameter cold-rolled S.A.E. 1045 steel, normalized and drawn; endurance limit = 39800 lb per sq. in. The specimen was rolled with rollers of 7/16 in. diameter and 3/32-in. contour radius.
[e] This specimen machined from 1 3/4-in. hot-rolled S.A.E. 1045 steel, normalized and drawn; endurance limit = 39800 lb per sq in. The specimen was rolled with rollers of 7/16 in. diameter and 3/32-in. contour radius.
[f] This specimen was machined from 1 3/4-in. hot-rolled S.A.E. 1045 steel, normalized and drawn; endurance limit = 36000 lb per sq in. The specimen was rolled with rollers of 1.46 in. diameter and 0.31-in. contour radius.
NOTE: The 7/16-in. diameter by 3/32-in. contour-radius rollers were slightly worn at the contour radius due to therolling of a large number of specimens. At the time these indentation tests were made, the 3/32-in. contour radius measured about 1/8 in. The calculated values given in the table are based on the 3/32-in. radius.

ness due to cold-working is accompanied by a decrease in the work-hardening capacity. This would explain the results obtained in Table 14.

Conclusion

The conclusions to be derived from this work, based on the limitations imposed by the nature of the materials, tests, and size of specimens tested, may be summarized as follows:

1 Fatigue strength may be increased from 24 to 32 per cent as shown in Figs. 11 and 12.

2 Roller shape has little effect on the increased fatigue strength as shown by Tables 5 and 7.

3 The minimum roller feed for about maximum fatigue strength may be expressed by a ratio of roller contact width to roller feed equal to 2 or greater.

4 The irregularity of the surface finish obtained by rolling may be controlled by proper feed as shown by Fig. 14.

5 Mild heat-treatment after rolling gives 6 per cent additional fatigue strength.

6 Cold-stretching of fatigue specimens in tension increased fatigue strength as much as 35 per cent as shown in Table 13.

7 Surface rolling of previously cold-worked steel does not give as great an increase in fatigue strength as the annealed steel.

Acknowledgments

Much credit is due Prof. S. Timoshenko of the University of Michigan for his invaluable suggestions during the progress of this work. The interest of T. V. Buckwalter, vice-president of Timken Roller Bearing Company, has been of primary importance in furthering this research. The assistance of Dr. J. L. Maulbetsch has been of great value in these tests.

BIBLIOGRAPHY

1 W. Zander and K. Gunther, "Der Einfluss von Oberflächenbeschädigungen auf die Biegungsschwingungfestigkeit," Veröffentlichungen des Wöhler-Instituts, Braunschweig, heft 1 and 2, NEM-Verlag, Berlin, 1928.

2 O. Föppl, "Das Drücken der Oberfläche von Bauteilen aus Stahl," *Stahl und Eisen*, vol. 49, 1929, p. 575.

3 Paul Behrens, "Das Oberflächendrucken zur Erhöhung der Drehschwingungsfestigkeit," Veröffentlichungen des Wöhler-Instituts, Braunschweig, heft 6, NEM-Verlag, Berlin, 1930.

4 G. S. von Heydekampf, "Cold Rolling Raises Fatigue or Endurance Limit," *Iron Age*, Sept. 18, 1930, p. 775; also "Should Steel Machinery Parts Be Cold Rolled," *Iron Age*, Oct. 2, 1930. p. 928.

5 Hilmar Döring, "Das Drücken der Oberfläche und der Einfluss von Querbohrungen auf die Biegungsschwingsfestigkeit," Veröffentlichungen des Wöhler-Instituts, Braunschweig, heft 5, NEM-Verlag, Berlin, 1930.

6 R. Kühnel, "Achsbrüche von Eisenbauhfahrzengen und ihre Ursachen," *Glasers Annalen*, vol. 110, 1932, p. 39. Also *Stahl und Eisen*, vol. 52, Oct. 6, 1932, p. 965.

7 A. Thum and F. Wunderlich, "Einfluss von Einspann- und Kraftangriffsstellen auf die Dauerhaltbarkeit der Konstruktionen," *Zeit. V.D.I.*, vol. 77, no. 31, Aug. 5, 1933, p. 851.

8 A. Thum and W. Bautz, "Steigerung der Dauerhaltbarkeit gekerbter Konstrucktionsteile durch Eigenspannugen," *Zeit. V.D.I.*, vol. 78, no. 31, Aug., 1934.

9 A. Thum and H. Oschatz, "Steigerung der Dauerfestigkeit bei Rundstäben mit Querbohrungen," *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 3, no. 2, March–April, 1932, p. 87.

10 H. Oschatz, "Gesetzmässigkeiten des Dauerbruches und Wege zur Steigerung der Dauerhaltbarkeit," heft 2, *Mitteilungen der deutschen Materialprüfungsanstalten an d. T. H., Dormstadt*, Berlin, 1933.

11 Wilhelm Staedel, "Dauerfestigkeit von Schrauben," heft 4, *Mitteilungen der deutschen Materialprüfungsanstalten, an d. T. H., Dormstadt*, Berlin, 1933.

12 H. Koch, "Die Biegewechselfestigkeit einer Keilverbindung (Passfederanordnung) und die Erhöhung der Dauerhaltbarkeit durch das Oberflachendrücken," *Mitteilunjen des Wöhler-Instituts, Braunschweig*, heft 20, Berlin, 1934.

13 A. Thum and F. Wunderlich, "Dauerbegefestigkeit von Konstruktionsteilen an Einspannungen, Nabensitzen und ähnlichen Kraftangriffstellen," heft 5, *Mitteilungen der deutschen Materialprüfungsanstalten, an d. T. H., Darmstadt*, Berlin, 1934.

14 R. E. Peterson and A. M. Wahl, "Fatigue of Shafts at Fitted Members, With a Related Photoelastic Analysis," A.S.M.E. Journal of Applied Mechanics, March, 1935.

15 T. V. Buckwalter, "Locomotive Axle and Wheel Research," Engineers Society of Western Pennsylvania, Meeting in Pittsburgh, Pa., 1935.

16 O. Föppl and A. Thum, "Steigerung der Dauerhaltbarkeit durch Oberflächendrucken," *Zeit. V.D.I.*, vol. 77, no. 50, Dec., 1933, p. 1335.

17 E. J. Abbott and F. A. Firestone, "The Profilograph," *Automotive Industries*, Aug. 19, 1933, p. 204.

18 E. J. Abbott, "Surface Finish and How It Can Be Measured and Specified," *S.A.E. Journal*, vol. 36, no. 3, March, 1935, p. 112.

19 R. E. Peterson, "Stress Concentration Phenomena in Fatigue of Metals," Trans. A.S.M.E., vol. 55, 1933, paper APM-55-19, p. 157.

20 R. Faulhaber, "Über den Einfluss des Probestabdurchmessers auf die Biegeschwingungsfestigkeit von Stahl," *Mitteilungen aus dem Forschungs-Institut der Vereinigte Stahlwerke Aktiengesellschaft Dortmund*, vol. 3, no. 6, June, 1933.

21 S. Timoshenko and J. M. Lessells, "Applied Elasticity," Westinghouse Technical Night School Press, Pittsburgh, 1925.

22 L. Aitchison, "The Low Apparent Elastic Limit of Quenched or Work-Hardened Steels," Carnegie Scholarship Memoirs, Iron and Steel Institute, vol. 12, 1923.

23 W. Rosenhain and D. Ewen, "Intercrystalline Cohesion of Metals," *Journal Institute of Metals*, vol. 10, 1913, p. 119.

24 A. L. Kimball, "Internal Friction Theory of Shaft Whirling," *General Electric Review*, vol. 27, no. 4, 1924, p. 244.

25 A. L. Kimball, "Vibration Prevention in Engineering," John Wiley and Sons, Inc., New York, 1932, p. 124.

26 S. Timoshenko, "Theory of Elasticity," McGraw-Hill Book Company, New York, 1934.

27 H. Thomas and V. Hoersch, "Stresses Due to the Pressure of One Elastic Solid Upon Another," Engineering Experiment Station University of Illinois, Bulletin No. 212, July, 1930.

28 N. M. Belajef, Bull. Inst. Engineers of Ways of Communications, no. 102, Leningrad, 1929.

29 H. F. Moore and V. B. Kommers, "An Investigation of the Fatigue of Metals," Engineering Experiment Station, University of Illinois, Bulletin No. 124, Urbana, Ill., 1921.

30 Hugh O'Neill, "The Hardness of Metals and Its Measurements," Chapman and Hall, London, 1934.

31 E. Meyer, "Untersuchungen Über Härteprüfung und Härte," *Zeit. V.D.I.*, vol. 52, 1908, p. 645.

32 O. Schwarz, "Zugfestigkeit und Härte bei Metallen," *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 313, 1929.

33 Zay Jeffries and Robert Archer, "The Science of Metals," McGraw-Hill Book Co., New York, 1924

A Method of Harmonic Analysis

By FRANK M. LEWIS,[1] NEW YORK, N. Y.

The method of harmonic analysis described in this paper is of the selected ordinate type. It is similar to the Fischer Hinnen analysis in that a different spacing of ordinates is used for each harmonic component to be determined, and no multiplications are necessary. The basis of derivation is the assumption that the curve to be analyzed can be replaced by a parabola passing through certain ordinates in each half period of the harmonic component. A spacing of these ordinates is then found such that the sum of the ordinates is proportional to the harmonic component of the parabolas replacing the original curve.

Spacings have been determined for 3, 5, and 7 ordinates per half period, corresponding to parabolas of second, fourth, and sixth orders. In the five and seven ordinate rules it is necessary to use a multiplying factor of $^1/_2$ for the end ordinates.

In use a set of transparent charts with the ordinates for each component are constructed. Laid over the curve to be analyzed, the ordinates can be quickly summed on a strip of paper.

The accuracy of the method is relatively high and in use it is of the utmost simplicity, so that with a set of charts once prepared, correct results can be obtained by the most unskilled computer.

METHODS of harmonic analysis by selected ordinates are of two types: Analyses in which the identical spacing of ordinates is used to determine all the components; and analyses in which a different spacing of ordinates is used to determine each harmonic component.

Methods of harmonic analysis by identical ordinates have been devised by several writers.[2] In general, the procedure is to divide the period into a number of equidistant intervals, enter the ordinates at these points in tabular forms, multiply each ordinate by a set of constants, and by proper combinations and additions of the products the harmonic coefficients can be determined up to an order number which depends upon the number of intervals into which the period is divided. The tedious multiplications are an undesirable feature in this procedure, and the use of the tabular forms requires considerable skill and care on the part of the computer.

The Fischer Hinnen analysis and the method to be presented in this paper are examples of the second type.

The Fischer Hinnen analysis,[3] in which a different spacing of ordinates is used for each of the harmonic components avoids the multiplications of the identical ordinate methods. It may be stated as follows:

"If starting at $x = 0$, we measure $2n$ ordinates at intervals $l/2n$, where l is the periodic length, the average of these ordinates, taken alternately plus and minus, is equal to the sum of the amplitudes of the nth, $3n$th, $5n$th, etc., cosine components."

A similar rule holds for the sine components.

If the higher components are small or absent, or a high degree of accuracy is not necessary, this rule is particularly simple and rapid in application. It is desirable to lay down the curves to a standard length and construct a set of ruled transparent charts with the ordinate spacing, one for each of the components.

If higher components are present and accuracy is desired, it is first necessary to determine a number of the higher components, and use these to correct the lower, and the rule loses somewhat in simplicity.

The method of analysis to be described in this paper is similar to the Fischer Hinnen analysis in that a different spacing of ordinates is used for each of the harmonic components and multiplications are avoided, but it has an entirely different basis of derivation. It is particularly useful in those cases where higher components are present and accuracy is wanted. Any particular component is determined independently of the others.

The three-ordinate rule will be described first.

Referring to Fig. 1 the period AB is divided into $2n$ equal intervals, where n is the order number of the harmonic coefficient to be determined. For sine components the intervals will all be equal, while for cosine components there will be half intervals at the ends. Taking an origin at the center of any one of these intervals any harmonic coefficient, sine or cosine, will be given by

$$A_n = \Sigma \frac{1}{nh} \int_{-h/2}^{h/2} f(x) \cos \frac{\pi x}{h}\, dx \qquad [1]$$

Σ meaning the sum of the contributions from the separate intervals.

It is assumed that in each interval the arbitrary curve $y = f(x)$ can be replaced by a parabola of the form

$$y = a_0 + a_1 x + a_2 x^2 \qquad [2]$$

The spacing of three ordinates will be determined so that if in any interval, $f(x)$ is replaced by the parabola passing through these ordinates, the integral in Equation [1] will be proportional to the sum of the ordinates.

This procedure is suggested by Tchebycheff's rule for areas. In Tchebycheff's rule an ordinate spacing is found so that the area is proportional to the sum of the ordinates. In this paper a harmonic coefficient is made proportional.

It will be assumed that one of the ordinates is at the center of the interval and that the others are equidistant x_1 to left and right of the center.

Then

A_n (CD contribution)

$$= \frac{1}{nh} \int_{-h/2}^{h/2} (a_0 + a_1 x + a_2 x^2) \cos \frac{\pi x}{h}\, dx$$

$$= M \begin{bmatrix} a_0 \\ + a_0 + a_1 x_1 + a_2 x_1^2 \\ + a_0 - a_1 x_1 + a_2 x_1^2 \end{bmatrix}$$

$$= M(3a_0 + 2a_2 x_1^2)$$

---

[1] Professor of Engineering, Webb Institute of Naval Architecture. Mem. A.S.M.E.

[2] "Higher Mathematics for Engineers and Physicists," by I. S. and E. S. Sokolnikoff, McGraw-Hill Book Company, New York, N. Y., 1934. This text contains a discussion of several of these methods as well as a discussion of the Fischer Hinnen analysis.

[3] "Method zur schnellen Bestimmung harmonische Wellen," by J. Fischer Hinnen, *Elektrotechnische Zeitschrift*, vol. 22, no. 9, May 9, 1901, pp. 369–398.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1936, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

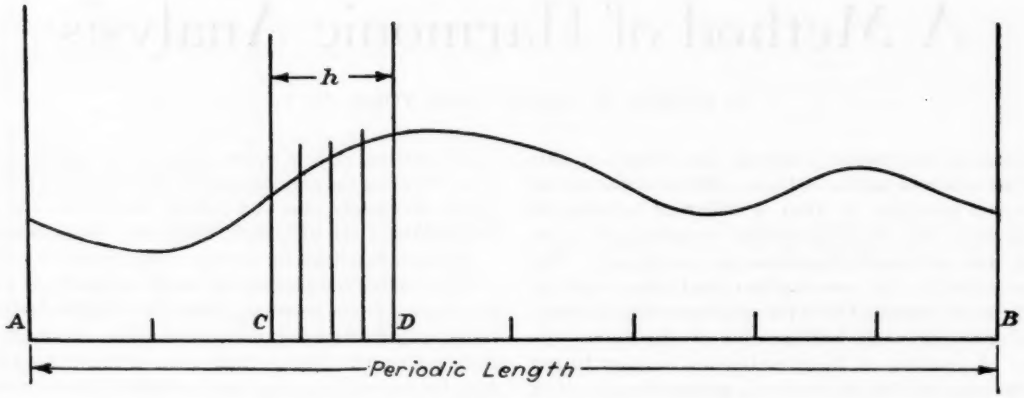


FIG. 1

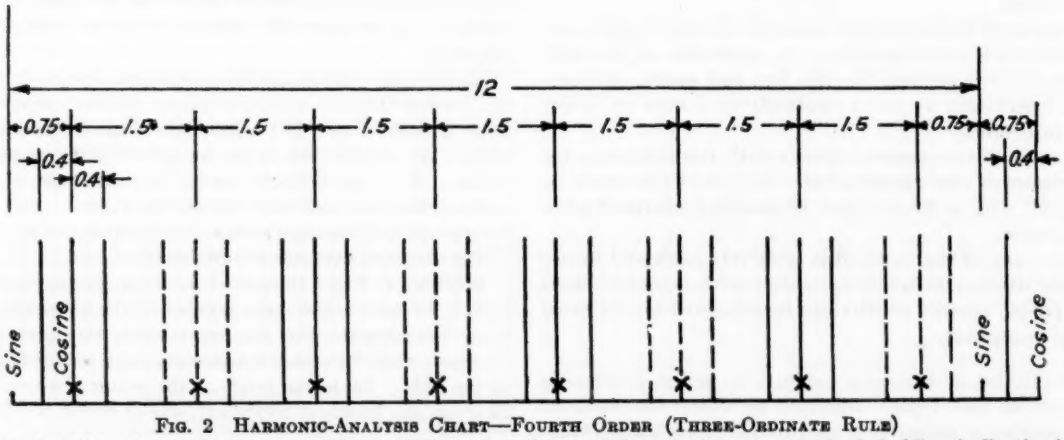


FIG. 2 HARMONIC-ANALYSIS CHART—FOURTH ORDER (THREE-ORDINATE RULE)

(a_4 or $\beta_4 = [1/8\pi]$[sum of the ordinates represented by solid lines minus the sum of the ordinates represented by dashed lines.] For the shorter Fischer Hinnen rule, a_4 or $\beta_4 = [1/8]$ [sum of the ordinates represented by the solid lines minus the sum of the ordinates represented by the dashed lines], taking only the ordinates marked x.)

Evaluating the integral and equating the coefficients of a_1 and a_2 there is obtained

$$M = (4\pi)(1/6n)$$

$$a_1 = (\sqrt{3}/2\pi)[\sqrt{(\pi^2/2) - 4}]h = 0.26652\,h$$

Fig. 2 illustrates the spacing of ordinates for a complete chart as prepared for the fourth-order components. The period is taken as 12 in. This is divided into eight intervals of $1^1/_2$ in. each. Ordinates are laid off to the right and left of the center of each of these a distance 0.26652×1.5 in. $= 0.399$ in.

The curve to be analyzed is laid down to a periodic length of 12 in. To determine the fourth-order sine coefficient, the fourth-order chart is laid over the curve so that the termini marked "sine" are at the termini of the curve, and the sums of the ordinates under the ruled lines determined, the full lines being taken as positive and the dashed lines as negative. The sum of these ordinates multiplied by $M = 1/6\pi$ will be an approximation to the fourth-order coefficient given by the integral in Equation [1]. How close an approximation is obtained depends upon how close the individual sections of the actual curve approximate to parabolas.

To obtain the cosine coefficient, the chart is shifted until the termini marked "cosine" terminate the period and exactly the same procedure is followed. The end ordinates must be added only once.

The charts should be laid over the curve so that the lowest ordinate of the curve is above the axis of the charts. This obviates any difficulty from sign of the ordinates.

The additions can be made very rapidly and accurately by marking off the positive ordinates in succession on one edge of a strip of paper and the negative ordinates on the opposite edge, and taking the difference of the lengths.

Charts for any other order can be prepared in exactly the same manner.

Where it is known that harmonics, which are an odd multiple of the order to be determined, are not present, or where high accuracy is not needed, the same charts may be used to determine the harmonic coefficients by the Fischer Hinnen method. In this case, only the central ordinates of each group are utilized and their sums (ordinates taken alternately plus and minus) are divided by the number of ordinates.

The method which has been described is designated as the three-ordinate rule, there being three ordinates per half period of the harmonic component. For low-order components, this number of ordinates may not fix the curve to be analyzed closely enough to determine the coefficients with accuracy. For this purpose a five-ordinate rule and a seven-ordinate rule have been devised.

For the five-ordinate rule it is assumed that the curve to be analyzed can be represented in each interval by an equation of the form

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$

It is found that the resultant ordinate spacings are $x_1 = 0.0881h$, and $x_2 = 0.3326h$. The three central ordinates are closely grouped and, as a result, the curve will be defined with

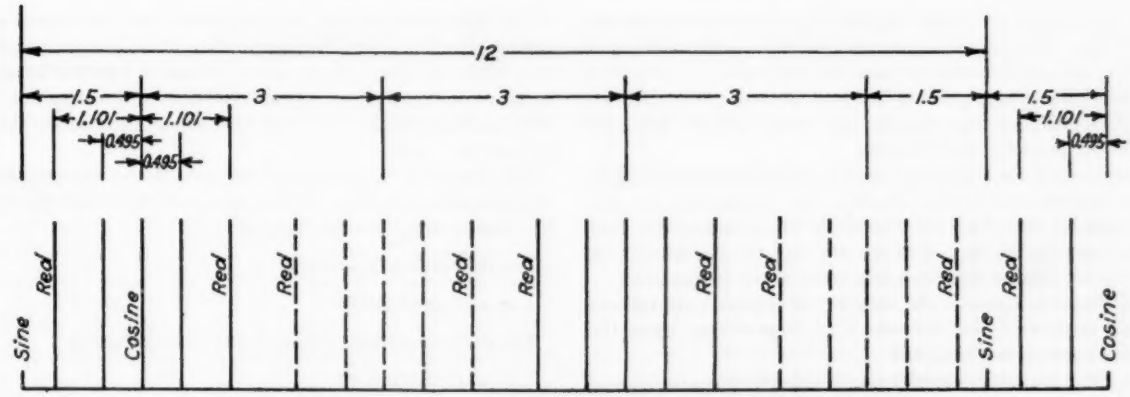


FIG. 3 HARMONIC-ANALYSIS CHART—SECOND ORDER (FIVE-ORDINATE RULE)
(a_2 or β_2 = [1/4π] [sum of the black ordinates plus one-half of the red ordinates]. The solid lines are taken as positive and the dashed lines are taken as negative.)

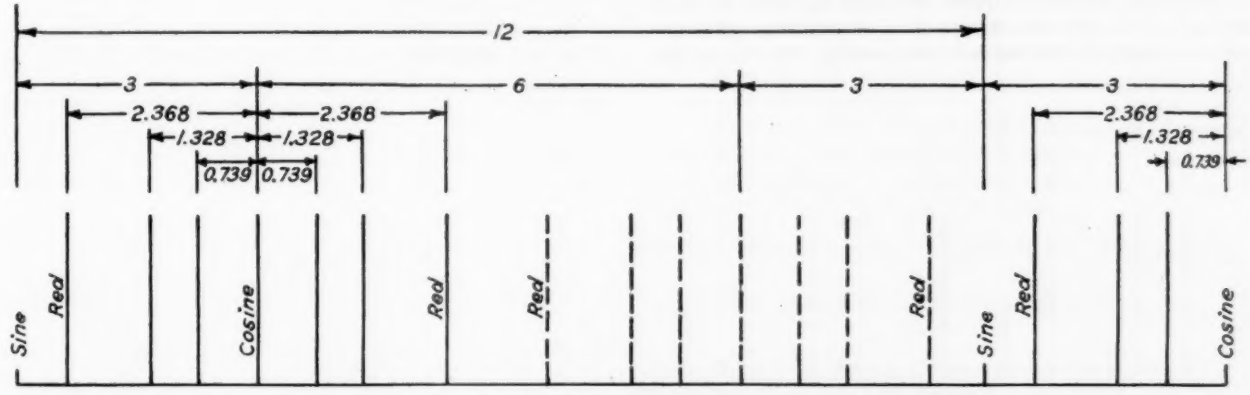


FIG. 4 HARMONIC-ANALYSIS CHART—FIRST ORDER (SEVEN-ORDINATE RULE)
(a_1 or β_1 = [1/2π] [sum of the black ordinates plus one-half of the red ordinates]. The solid lines are taken as positive and the dashed lines are taken as negative.)

but little greater accuracy than when the three-ordinate rule is used.

A five-ordinate rule of much higher accuracy can be obtained by taking the sum of the three central ordinates plus one half the sum of the two end ordinates. The conditions for this case for any order n lead to

$$\frac{1}{nh}\int_{-h/2}^{h/2}(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4)\cos\frac{\pi x}{h}\,dx$$

$$= M(4a_0 + 2a_2x_1^2 + a_2x_2^2 + 2a_4x_1^4 + a_4x_2^4)$$

from which by equating the coefficients of the a's

$$M = (4/\pi)(1/8n)$$

$$2x_1^2 + x_2^2 = (2h^2/\pi^2)[(\pi^2/2) - 4]$$

$$2x_1^4 + x_2^4 = (4h^4/\pi^4)[(\pi^4/16) - 3\pi^2 + 24]$$

solving

$$x_1 = 0.1652h$$

$$x_2 = 0.3673h$$

Fig. 3 shows a second-order chart spaced for the five-ordinate rule with half end ordinates. In making up an actual chart, the half ordinates should be ruled in red.

A seven-ordinate rule with full ordinates cannot be obtained as the equations for the ordinate spacing have imaginary roots. However, a seven-ordinate rule with half end ordinates can be obtained. The equations are

$$M = (4/\pi)(1/12n)$$

$$\frac{1}{nh}\int_{-h/2}^{h/2}(a_0 + a_1x + a_2x^2 \ldots\ldots a_4x^6)\cos\frac{\pi x}{h}\,dx$$

$$= M[6a_0 + a_2(2x_1^2 + 2x_2^2 + x_3^2) + a_4(2x_1^4 + 2x_2^4 + x_3^4) + a_6(2x_1^6 + 2x_2^6 + x_3^6)]$$

leading to

$$2x_1^2 + 2x_2^2 + x_3^2 = (6h^2/\pi^2)[(\pi^2/4) - 2]$$

$$2x_1^4 + 2x_2^4 + x_3^4 = (6h^4/\pi^4)[(\pi^4/16) - 3\pi^2 + 24]$$

$$2x_1^6 + 2x_2^6 + x_3^6 = \frac{6h^6}{\pi^6}\left[\frac{\pi^6}{64} - \frac{\pi^4 \times 6 \times 5}{16} + \frac{\pi^2 \times 6 \times 5 \times 4 \times 3}{4} - 6!\right]$$

solving

$$x_1 = 0.1232h$$
$$x_2 = 0.2214h$$
$$x_3 = 0.3946h$$

In Fig. 4 is shown the first-order chart using the seven-ordinate rule with half end ordinates. It will be seen that the distribution of the ordinates is such as to determine the curve very accurately.

It is evident that the foregoing rules by no means exhaust the possibilities. Rules might be devised having an even number of ordinates, use other multiples than one-half, rules of more than seven ordinates, etc., etc. It is believed, however, that the foregoing three-ordinate, five-ordinate, and seven-ordinate rules will cover all the practical requirements.

In practice, for the analysis of such curves as the turning-effort curve of internal-combustion engines, we customarily use the seven-ordinate rule for the first-order component, the five-ordinate rule for the second order, and the three-ordinate rule or the Fischer Hinnen rule for components of any higher order.

It is difficult to appraise the accuracy of approximate rules of harmonic analysis. This depends to a large extent upon the particular curve being analyzed.

The error in the foregoing rules may be expressed as a function of the higher components which are odd multiples of the order under consideration, but no clear idea of the accuracy of the rules would be given by such equations. In curves with high-order components of large amplitude (ripples) particular care must be taken, and any of the approximate methods of analysis may fail to give accurate results for the low-order components. It is noted that if the rules are applied to analyze a sine curve, the actual amplitude of which is unity, the three-ordinate rule gives an amplitude of 0.9925; the five-ordinate an amplitude of 1.00000723 and the seven-ordinate an amplitude of 1.0000231, which does not imply that in actual use the seven-ordinate rule is less accurate than the five-ordinate rule.

The author is indebted to Frederick Porter for assistance in the numerical solution of the equations given in this paper. More accurately the spacings are:

For the three-ordinate rule

$$x_1 = 0.266526638h$$

For the five-ordinate rule with half end-ordinates

$$x_1 = 0.165155076h$$
$$x_2 = 0.367257584h$$

For the seven-ordinate rule with half end-ordinates

$$x_1 = 0.123223004h$$
$$x_2 = 0.221410966h$$
$$x_3 = 0.394629380h$$

Thermal Stresses in Plates[1]

By JOHN L. MAULBETSCH,[2] CANTON, OHIO

The author presents solutions of the problem of thermal stresses in thin plates of uniform thickness with simply supported edges when the temperature varies only along the thickness of the plate. For the case of polygonal-shaped plates it is shown that the solution can be deduced from the problem of the torsion of prismatical bars, the cross section of which has the same shape as the plate. The exact solutions for cases of equilateral triangular and square plates are given. Although only the case of small deflections is considered, its limitations are treated in detail.

Introduction

IF A PLATE of uniform thickness has its two faces kept at different uniform temperatures, there will be a flow of heat from one face to the other, and for a homogeneous material, there will be a temperature distribution varying linearly over the thickness of the plate. If the plate is free of constraints, there will be, in general, no stresses resulting from such nonuniform heating. Stresses will be produced if displacements of the

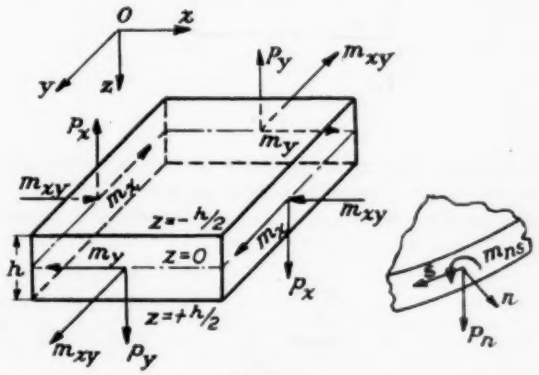


FIG. 1

plate are prevented, for instance, by some type of supports along the edges.[3] In this paper thin plates with simply supported edges are considered.

The application of the results is limited by the usual assumptions made in the theory of thin plates with small deflections. The assumptions are: (1) That the material is homogeneous, isotropic and follows Hooke's law, (2) that straight lines initially normal to the middle surface remain straight and normal to that surface after bending, and (3) that deflections are small compared to the thickness.

The following nomenclature will be used:

t_o = difference of temperature between the upper and lower faces of the plate. This temperature difference is positive if the temperature of the lower face is greater than the temperature of the upper face

x, y, z = coordinate axes. The middle plane of the initially flat plate is chosen as the x-y plane. The lower face of the plate is the plane $z = +h/2$, and the upper face $z = -h/2$

w = deflection of the plate. This deflection is positive when it occurs in the direction of the positive z-coordinate

α = coefficient of thermal expansion

E = modulus of elasticity

ν = Poisson's ratio

$D = Eh^3/12(1-\nu^2)$ = flexural rigidity of the plate

m_x, m_y = bending moments per unit length

m_{xy}, m_{ns} = twisting moment per unit length

p_x, p_y, p_n = shearing forces per unit length

The positive directions of moments and forces are shown in Fig. 1.

Thermal Stresses in Plates With Simply Supported Edges

Under nonuniform heating, a plate with free edges is bent to a spherical surface. This bending can be eliminated by applying uniformly distributed bending moments along the edges of the plate. When the plate is held flat, let it be fastened to simple supports along the edges, and then let the applied couples be removed. The removal of these couples will produce some bending in the plate, and the stresses due to this bending, together with the stresses produced initially by the uniformly distributed couples, will represent the thermal stresses which are produced in the plate due to nonuniform heating.

Thus, the problem of thermal stresses in plates with simply supported edges is divided into (1) the determination of stresses in a plate free from constraints produced by uniformly distributed couples, and (2) the determination of stresses in a simply supported plate submitted to couples uniformly distributed along the edges. The first problem can be solved readily, but the second requires, in the case of plates of polygonal shape, solution of a differential equation of the second order.[4] The same type of differential equation occurs also in the torsion problem of prismatical bars and in the one of the deflection of membranes. Solutions are known for these cases and can be used in the cases considered in this paper.

Thermal Stresses in an Equilateral Triangular Plate Simply Supported at the Edges. Let a be the height of the triangle and x and y be the coordinate axes as shown in Fig. 2. The deflection w due to nonuniform heating is found, as shown in Appendix 2, to be

$$w = -\frac{(1+\nu)}{2}\frac{\alpha t_o}{h}\left[\frac{(x^2+y^2)}{2} - \frac{(x^3-3xy^2)}{2a} - \frac{(2a^2)}{27}\right] \quad \ldots [1]$$

The bending and twisting moments are

---

[1] From a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of Michigan, February, 1935.

[2] Research Engineer, Timken Roller Bearing Company. Jun. A.S.M.E.

[3] "Strength of Materials," by S. Timoshenko, vol. 2, 1930, D. Van Nostrand Company, Inc., New York, N. Y., p. 487.

Presented at the National Applied Mechanics Meeting of The American Society of Mechanical Engineers held at the University of Michigan, Ann Arbor, Mich., June 18–19, 1935.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1936, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] See Appendix 1.

$$\left.\begin{aligned} m_x &= -(1/24)\ \alpha t_o Eh^2\ (1 + 3x/a) \\ m_y &= -(1/24)\ \alpha t_o Eh^2\ (1 - 3x/a) \\ m_{xy} &= (1/24)\ \alpha t_o Eh^2\ (3y/a) \end{aligned}\right\} \quad \ldots\ldots\ldots [2]$$

In the case of plates of polygonal shape, as is shown in Appendix 1, the shearing forces p_x and p_y vanish, and the sum of the bending moments m_x and m_y is constant at every point of the plate and is equal to $-Et_o h^2/12$.

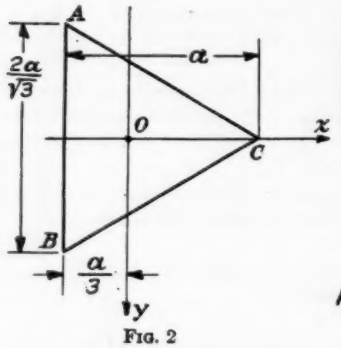


FIG. 2

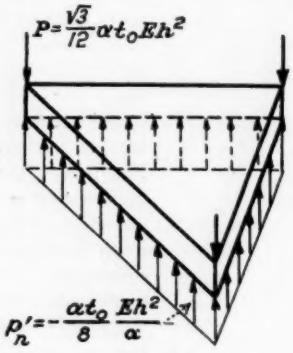
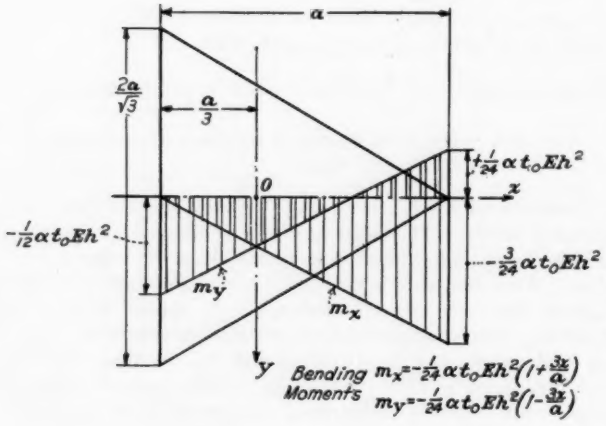


FIG. 3



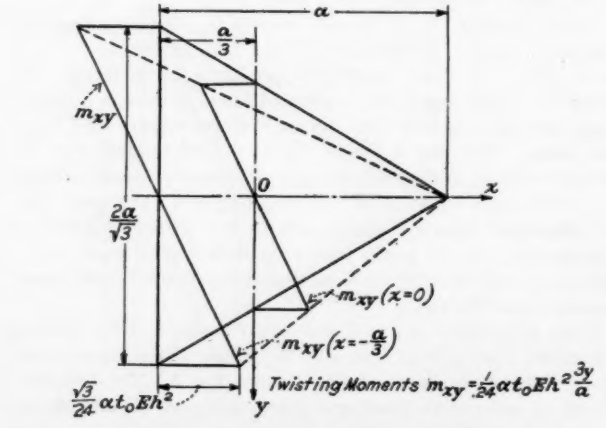


FIG. 4 DISTRIBUTION OF BENDING AND TWISTING MOMENTS IN EQUILATERAL TRIANGULAR PLATE

These expressions do not contain ν, that is, thermal stresses in such plates are independent of Poisson's ratio. The moments m_x, m_y, and m_{xy} depend only on the coefficient of linear expansion and on the modulus of elasticity E of the material, for a plate of definite dimensions a and h and for a certain temperature difference t_o.

Along the boundary of the equilateral triangular plate, there is a uniform distribution of reaction forces of intensity $(-\alpha t_o/8)(Eh^2/a)$ and at each corner there acts a concentrated reaction $(\sqrt{3}/12)\alpha t_o Eh^2$ as shown in Fig. 3. This system of boundary forces is in equilibrium.

The variations of m_x, m_y, and m_{xy} over the plate are shown in Fig. 4. The bending moments have the largest values at the corners. Therefore, since the stresses are proportional to the moments, the greatest tensile and compressive stresses are also at the corners. In the case of brittle materials such as glass, the failure of which can be predicted by means of the maximum-stress theory, cracks due to nonuniform heating would be expected to develop at these points. In the case of ductile materials such as steel, the maximum-shear theory applies, and it can be seen that the maximum shearing stress occurs also at the corners. Yielding of the material will therefore start at these points.

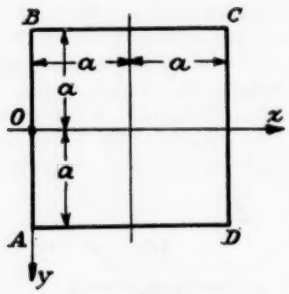


FIG. 5

Square Plate. Let $2a$ be the length of the sides of the square and let the system of coordinates be as shown in Fig. 5. The expressions for the deflection and moments, as derived in Appendix 2, are

$$w = \frac{16}{\pi^3} a^2 (1 + \nu) \frac{\alpha t_o}{h} \sum_{n = 1,3,5\ldots}^{\infty} \frac{1}{n^3} \left[1 - \frac{\cosh (n\pi y/2a)}{\cosh (n\pi/2)} \right] \times \sin \frac{n\pi x}{2a} \quad \ldots\ldots [3]$$

$$\left.\begin{aligned} m_x &= -(E/3\pi) h^3 \alpha t_o\ S_1 \\ m_y &= -(E/3\pi) h^3 \alpha t_o\ [(\pi/4) + S_1] \\ m_{xy} &= (E/3\pi) h^3 \alpha t_o\ S_2 \end{aligned}\right\} \quad \ldots\ldots [4]$$

In Equations [4]

$$S_1 = \sum_{n = 1,3,5\ldots}^{\infty} \frac{1}{n} \left[\frac{\cosh (n\pi y/2a)}{\cosh (n\pi/2)} - \frac{e^{n\pi y/2a}}{e^{n\pi/2}} \right] \cos \frac{n\pi x}{2a} + \frac{1}{4} \log \frac{\cosh [(\pi/2) - (\pi y/2a)] + \cos (\pi x/2a)}{\cosh [(\pi/2) - (\pi y/2a)] - \cos (\pi x/2a)}$$

$$S_2 = \sum_{1,3,5\ldots}^{\infty} \frac{1}{n} \left[\frac{\sinh (n\pi y/2a)}{\cosh (n\pi/2)} - \frac{e^{n\pi y/2a}}{e^{n\pi/2}} \right] \cos \frac{n\pi x}{2a} + \frac{1}{4} \log \frac{\cosh [(\pi/2) - (\pi y/2a)] + \cos (\pi x/2a)}{\cosh [(\pi/2) - (\pi y/2a)] - \cos (\pi x/2a)}$$

The distribution of m_x, m_y, and m_{xy} is shown in Figs. 6 and 7. It is seen that the maximum bending moments occur along the

edges of the plate and the largest twisting moments near the corners. The reactions of the supports along the edges of the plate are illustrated in Fig. 8. Due to symmetry the same conditions exist along all four edges of the square plate. Between points B and B_1 there will be a distribution of reaction forces of intensity

$$p'n = -\frac{E}{6}\alpha t_o \frac{h^2}{a}\left[\sum_{1,3,5\ldots}^{\infty}\left(\tanh\frac{n\pi}{2} - 1\right)\sin\frac{n\pi x}{2a} + \frac{1}{2\sin(\pi x/2a)}\right] \quad \ldots\ldots\ldots [5]$$

The distribution of forces near the corners is treated in Appendix 2. It is shown that over a distance c from the corners, their resultant is

$$P_{AB} = (m_{xy})_{\substack{x=c \\ y=a}} \quad \ldots\ldots\ldots [6]$$

and, relative to point B, their resultant moment is

$$M_{AB} = \frac{2}{3\pi^2} Eh^3\alpha t_o \sum_{1,3,5\ldots}^{\infty}\frac{1}{n^3}\tanh\frac{n\pi}{2}\sin\frac{n\pi c}{2a} \quad \ldots\ldots [7]$$

The distance c from the corner, along which the exact distribution of forces is unknown may be taken equal to the thickness of the plate, since it is usual to assume in the theory of thin

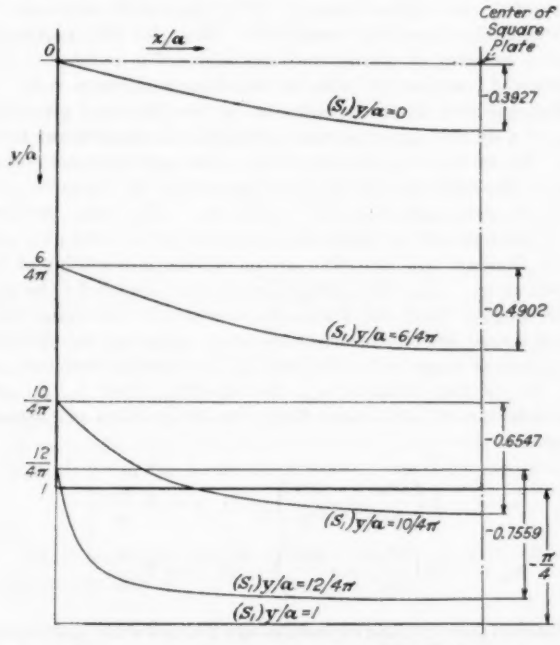


FIG. 6 DISTRIBUTION OF S_1 OVER ONE QUADRANGLE OF THE SQUARE PLATE

(Bending moments: $m_x = -[E/3\pi]h^3\alpha t_o S_1$ and $m_y = -[E/3\pi]\alpha t_o h^2[(\pi/4) + S_1]$.)

plates that the stress distribution is not given correctly by the mathematical theory for points situated at distances from the edges less than the thickness of the plate. If there is yielding of the material near the corners, then the distance c must be taken up to the point where yielding begins if this distance is larger than the thickness of the plate. The investigation of the danger of yielding of the material can be made only for specific dimensions, material, and temperatures.

CONCLUSIONS

The stresses σ_x, σ_y, and τ_{xy} vary linearly over the thickness of the plate and their maximum values occur at the upper and lower faces of the plate. They are determined by means of the known relations[5]

$$\sigma_x = \pm 6(m_x/h^2),\ \sigma_y = \pm 6(m_y/h^2),\ \tau_{xy} = \pm 6\,(m_{xy}/h^2)$$

In the cases of triangular and rectangular plate the moments

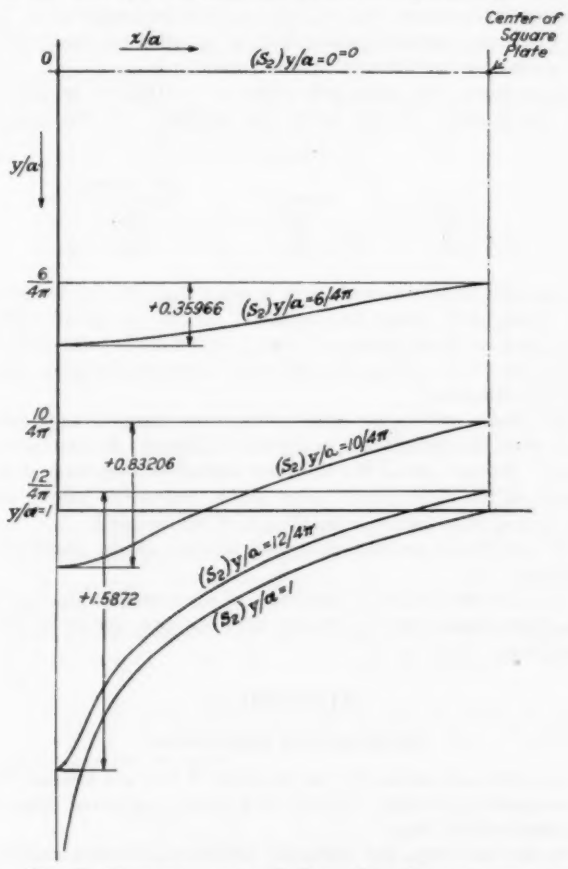


FIG. 7 DISTRIBUTION OF S_2 OVER ONE QUADRANGLE OF THE SQUARE PLATE

(Twisting moment $m_{xy} = [E/3\pi]h^3\alpha t_o S_2$.)

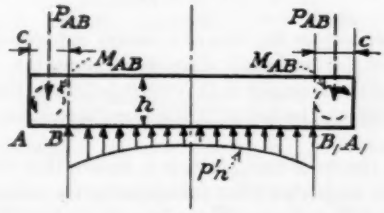


FIG. 8

are proportional to h^2, hence the thermal stresses will be independent of the thickness h. They are proportional to the coefficient of linear expansion α, the modulus of elasticity E and the difference of temperature t_o. However, the values of t_o are larger in thick plates than in thin plates for the same difference in gas temperatures,[6] so that the thicker the plate the greater the thermal stresses.

[5] "Die Elastische Platten," by A. Nadai, Julius Springer, Berlin, Germany, 1925.

[6] The difference of temperature t_o is not the difference of temperatures between the fluids or gases in contact with the faces of the

The results derived apply only to the case of deflections which are small compared to the thickness of the plate. In the case of the square plate, the maximum deflection occurs at the center of the plate, and from Equation [3]

$$w_{max}/h = (4/\pi^3)\,(2a/h)^2\,(1+\nu)\,\alpha t_o\,0.57107$$

It is seen that for a given material, the ratio w_{max}/h is proportional to $(2a/h)^2$ and to the difference of temperature t_o. Assuming, for instance, that w_{max}/h must not be greater than $^1/_2$, the maximum admissible value for t_o up to which the theory may be applied can be calculated in functions of $2a/h$.

Assuming a steel plate and taking $\alpha = 0.056/10^4$ in. per in. per deg F and $\nu = 0.3$, Table 1 is obtained. In the case of

TABLE 1

$2a/h$	t_o max, F	Max stress, lb per sq in.
50	375	31500
100	94	7900
200	23	1900

plates with small ratios of $2a/h$, smaller than 100 for instance, the theory will always be applicable because to obtain values of t_o such as those given in Table 1, it would be necessary to have very large temperature differences between the gases separated by the plate.[7]

For plates with larger ratios of $2a/h$ the theory is applicable only when the difference of temperature between the two faces is small. In such plates the thickness cannot be very large if the length of the plate is to be kept within reasonable values, and the drop of temperature in the plate itself will be small.

If concrete is considered instead of steel similar results are obtained.

It is seen then that the limitation of the theory to the case of small deflections does not greatly limit its applicability to practical cases.

Appendix 1

Mathematical Derivations

As explained previously the problem of thermal stresses due to nonuniform heating of a plate with simply supported edges is divided into two steps.

In the first step, the uniformly distributed couples, counteracting the thermal bending of the free plate produce at every point of the plate a bending moment[3]

$$M = -(1+\nu)\,D(\alpha t_o/h) \qquad [8]$$

The second step is the case of a simply supported plate submitted to couples uniformly distributed along the edges. The magnitude of these couples is $(1+\nu)\,D\,(\alpha t_o/h)$ so that when the second step follows the first step, the edges become free of bending moments.

From the theory of thin plates it is known that the deflection w of a simply supported plate submitted to the action of couples uniformly distributed along the edges must satisfy the differential equation[8]

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \qquad [9]$$

with the boundary condition $w = 0$

$$\frac{\partial^2 w}{\partial n^2} + \nu\,\frac{\partial^2 w}{\partial s^2} = -(1+\nu)\,\frac{\alpha t_o}{h}$$

where n indicates the direction of the outer normal to the boundary and s is the tangent to boundary of the plate, as shown in Fig. 1.

If the plate is of polygonal shape, the partial differential equation of the fourth order may be reduced to one of the second order.[8] It is shown that a polygonal boundary consists of straight lines along which $(\partial^2 w/\partial s^2) = 0$, since $w = 0$ at the boundary. Therefore, along a straight boundary, the second boundary conditions are

$$(\partial^2 w/\partial n^2) = -(1+\nu)\,\alpha t_o/h$$

but[9]

$$\frac{\partial^2 w}{\partial n^2} + \frac{\partial^2 w}{\partial s^2} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

and since $(\partial^2 w/\partial s^2) = 0$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -(1+\nu)\alpha t_o/h \qquad [10]$$

This equation needs to be satisfied along the boundary of the plate only. However, if w satisfies this equation over the whole region with the boundary condition $w = 0$, it is seen that w will also satisfy the original Equation [9] of the fourth order and its corresponding boundary conditions. Thus, for this particular case of bending of plates, the calculation of w reduces to the solution of Equation [10] with the boundary condition $w = 0$.

Superposition of the second step on the first step gives the case of a simply supported plate submitted to nonuniform heating. In the first step the plate remains flat, and the total deflection is the deflection obtained by integration of Equation [10] with the prescribed boundary conditions. The same problem of thermal stresses in plates was treated by N. Yamaguti and R. G. Olsson,[10] who derived a partial differential equation of the fourth order. The first author also gave a solution for the case of rectangular plate, but it contains very slowly converging series which do not lend themselves to accurate numerical calculations. By means of known relations[5] between the bending moments m_x, m_y, the twisting moment m_{xy}, the shearing forces p_x, p_y, and the deflection w, one obtains finally for the problem of thermal stresses

$$\left.\begin{aligned} m_x &= -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right) - (1+\nu)\,D\,\frac{\alpha t_o}{h} \\ m_y &= -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\,\frac{\partial^2 w}{\partial x^2}\right) - (1+\nu)\,D\,\frac{\alpha t_o}{h} \end{aligned}\right\} \qquad [11]$$

the second parts in these expressions are the uniformly distributed moments of the first step

$$m_{xy} = -D\,(1+\nu)\,\frac{\partial^2 w}{\partial x \partial y} \qquad [12]$$

---

plate, but only a fraction of it, due to the drop of temperature in the layers of fluids touching the faces. Assuming still air on both sides of a steel plate, one finds that t_o is 1/300 of the difference of temperatures between the gases if the plate is 1 in. thick, if the plate is 6 in. thick, then this ratio is 1/27.

[7] It is to be noted that the theory must be applied to cases where the maximum temperature of the plate is still low enough to have the material under conditions satisfying the assumptions of the theory of elasticity.

[8] "Berechnung der ringsum frei aufliegenden gleichseitigen Dreieckplatte," by S. Woinowsky-Krieger, *Ingenieur-Archiv*, vol. 4, 1933, page 254.

[9] The sum of the radii of curvature in two perpendicular planes is a constant.

[10] "Thermal Flexure of a Thin Plate Heated Uniformly on One Surface," by N. Yamaguti, *Journal of the Faculty of Engineering*, Tokyo Imperial University, vol. 18, no. 1, 1928.

"Die Biegung der rechteckigen Platte veränderlichen Dicke," by R. G. Olsson, Dissertation, Berlin, 1932.

$$\left.\begin{aligned} p_x &= -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \\ p_y &= -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [13]$$

It is to be noted that for a polygonal plate one has because of Equations [13] and [10]

$$p_x = p_y = 0$$

that is, the shearing forces are zero.

Also adding m_x and m_y and making use of Equation [10]

$$m_x + m_y = -E\,\alpha t_0 h^2/12$$

The sum of the bending moments m_x and m_y is constant at every point of the plate and is independent of Poisson's ratio.

The reactions of the supports along the edges should represent the twisting moments. These moments can be replaced by a distribution of shearing forces

$$p_n' = -\partial m_{ns}/\partial s \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

and by concentrated forces at the corners. If m_1 and m_2 are the twisting moments at a corner of the plate, the concentrated force P at this corner is

$$P = m_1 - m_2 \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

The rules for the sign of the moments and forces in Equations [14] and [15] are: The twisting moment m_{ns} is positive when the vector m_{ns} representing the twisting moment in a right-handed system coincides with the outer normal to the boundary, and the forces p'_n and P are positive when their direction coincides with the positive z-axis.

Appendix 2

Equilateral-Triangular Plate

Referring to Fig. 2, the solution of Equation [10] in the case of an equilateral triangle, from the problem of torsion, is

$$w = -\frac{(1+\nu)}{2}\frac{\alpha t_0}{h}\left[\frac{(x^2+y^2)}{2} - \frac{(x^3-3xy^2)}{2a} - \frac{2a^2}{27}\right]$$

By means of Equations [11] and [12], the expressions for the moments, Equation [2], are derived. The boundary conditions, shown in Fig. 3, are determined by Equations [14] and [15].

Square Plate

Again using the solution of Equation [10] as obtained in the torsion problem

$$w = \frac{16}{\pi^3}(1+\nu)a^2\frac{\alpha t_0}{h}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n^3}\left[1-\frac{\cosh(n\pi y/2a)}{\cosh(n\pi/2)}\right]\sin\frac{n\pi x}{2a}$$

and from Equations [11] and [12]

$$m_x = -\frac{E}{3\pi}h^2\alpha t_0\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\frac{\cosh(n\pi y/2a)}{\cosh(n\pi/2)}\sin\frac{n\pi x}{2a} \ldots [16]$$

$$m_y = +\frac{E}{3\pi}h^2\alpha t_0\sum_{n=1,3,5\ldots}^{\infty}\left[\frac{1}{n}\frac{\cosh(n\pi y/2a)}{\cosh(n\pi/2)}\sin\frac{n\pi x}{2a} - \frac{\pi}{4}\right] \ldots [17]$$

$$m_{xy} = \frac{E}{3\pi}h^2\alpha t_0\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\frac{\sinh(n\pi y/2a)}{\cosh(n\pi/2)}\cos\frac{n\pi x}{2a} \ldots\ldots [18]$$

As in the case of the triangular plate, these expressions are independent of Poisson's ratio.

N. Yamaguti[10] calculated the values of the moments by taking the first five terms of these series and obtained errors as large as 5 per cent in the numerical values. These series may be transformed so as to improve the accuracy of the computations. It is done as follows:

Bending Moments m_x and m_y. It is noted that with increasing values of n, the terms of the series m_x as given in Equation [16] differ less and less from the terms of[11]

$$\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\frac{e^{n\pi y/2a}}{e^{n\pi/2}}\sin\frac{n\pi x}{2a} = \frac{\tan^{-1}}{2}\frac{\sin(\pi x/2a)}{\sinh[(\pi y/2a)-(\pi/2)]} \ldots [19]$$

An accurate approximation of the sum of the series m_x can be found by taking the sum of the series given in Equation [19] and adding to it a correction due to the difference in value of the terms of the two series. This difference is[12]

$$\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\left[\frac{\cosh(n\pi y/2a)}{\cosh(n\pi/2)} - \frac{e^{n\pi y/2a}}{e^{n\pi/2}}\right]\sin\frac{n\pi x}{2a}$$

This new series is rapidly convergent and the first three terms are sufficient to give an accuracy of 0.01 per cent.

The expression for the bending moment m_x is then as given by Equations [4].

The values of the bending moment m_y follow from those of m_x since the sum of m_x and m_y is a constant at every point of the plate.

Twisting Moment m_{xy}. The series of m_{xy} as given in Equation [18] may be similarly transformed into a better converging series, if one makes use of[11]

$$\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\frac{e^{n\pi y/2a}}{e^{n\pi/2}}\cos\frac{n\pi x}{2a} = \frac{1}{4}\log\frac{\cosh[(\pi/2)-(\pi y/2a)]+\cos(\pi x/2a)}{\cosh[(\pi/2)-(\pi y/2a)]-\cos(\pi x/2a)}$$

One obtains then the expression given in Equations [4] for m_{xy}.

To find the boundary forces, consider the edge AD shown in Fig. 5 where $y = a$ and $0<x<a$. Due to symmetry the conditions along the other edges of the square plate will be identical.

From Equations [4], one obtains for the forces at the edge

$$p_n' = -\frac{E}{6}\alpha t_0\frac{h^2}{a}\left[\sum_{n=1,3,5\ldots}^{\infty}\left(\tanh\frac{n\pi}{2}-1\right)\sin\frac{n\pi x}{2a} + \frac{1}{2\sin(\pi x/2a)}\right]$$

[11] "Elementary Treatise on Fourier's Series, and Spherical, Cylindrical and Ellipsoidal Harmonics With Application to Problems in Mathematical Physics," by W. E. Byerly, Ginn & Company, Boston, Mass., 1893, p. 100.

[12] This procedure is justifiable since the two series are uniformly convergent for an interval $\delta \leq x \leq 2\pi - \delta$, where δ is a quantity satisfying the condition $0 < \delta < \pi$. See "Theorie und Anwendung der Unendlichen Reihen," by K. Knopp, Julius Springer, Berlin, Germany, 1930, p. 360.

The variation of $(p_n')_{y=a}$ as given by this equation is shown in Fig. 8. The series is uniformly convergent for an interval $\delta \leqq x \leqq 2\pi - \delta$ with $0 < \delta < \pi$. When δ is made very small, x approaches the value 0 or $2a$, and p_n' increases without limit.

The concentrated forces at the corners cannot be obtained from Equation [15], for when x approaches 0 or $2a$, the value of the

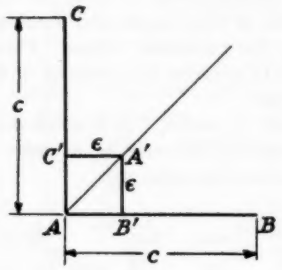


FIG. 9

twisting moment tends to approach infinity. The infinite forces given by the mathematical solution at the corners cannot exist in practical cases, but some indications of the conditions near these points can be derived from this solution.

Consider the element $AB'A'C'$ of side ϵ in Fig. 9 as being separated from the plate. If the stress conditions within the rest of the plate are not to be altered, the internal stresses existing along $B'A'$ and $A'C'$ prior to removal of the element must be considered as applied forces. The vertical forces resulting from the distribution of forces along $AB'A'C'C$ may be determined. When ϵ is made smaller and smaller this force will be the resultant of the forces acting along BA and AC. Because of symmetry, the resultant of the forces distributed along BA is equal to the resultant along AC so that the resulting force acting on BA may be determined. Hence

$$P_{AB} = (m_{xy})_{\substack{x=c \\ y=a}}$$

The moment M_{AB} of the resultant force P_{AB} relative to point B must be equal to the sum of the twisting moments m_{xy} distributed along AB, since the system of forces p_n' and P_{AB} must be equivalent to the system of twisting moments m_{xy}. Therefore

$$M_{AB} = \frac{2}{3\pi^2} Eh^3\alpha t_o \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n^2} \tanh \frac{n\pi}{2} \sin \frac{n\pi c}{2a}$$

ACKNOWLEDGMENTS

The proof given in Appendix 1 was originated by Prof. S. Timoshenko, and the author wishes to thank him for his helpful suggestions given during the preparation of this paper.

The Strength and Stiffness of Cylindrical Shells Under Concentrated Loading

By R. J. ROARK,[1] MADISON, WIS.

In certain structures and under certain conditions of use, cylindrical or nearly cylindrical shells are subjected to concentrated loading. This paper presents empirically derived formulas for the stresses and deflections produced by such loading, and describes the tests on which these formulas are based.

INTRODUCTION

IN CERTAIN structures and under certain conditions of use, cylindrical or nearly cylindrical shells are subjected to concentrated loading. Examples of this are found in standpipes or water tanks with internal spider bracing, in jacketed pressure containers and boilers, in airplane fuselages of the monocoque type, and in submarine hulls. Usually the magnitudes of the forces involved are more or less conjectural, and the degree of uncertainty as to form, area over which the load is distributed, and boundary conditions, is such as to make the development and application of a precise theoretical method for calculating stresses and deflections impracticable. It would seem more expedient to resort to empirical methods based on experiment.

In connection with tests to determine the bending stresses present in partly filled pipes of large diameter, the author made use of specimens and equipment at hand to investigate the effect of concentrated loading. From the data secured, empirical formulas have been developed which are believed to be of value in predicting the stress and deflection to be expected from the application of widely separated concentrated loads to a thin cylindrical shell whose length is great in comparison with its diameter. It is the purpose of this paper to describe these tests and to present the formulas based thereon.

Test Specimens

For investigating stresses and local deformations, two large steel cylinders were used. One of these was a specially constructed, longitudinally welded cylinder 0.129 in. thick, 23 in. diameter, and 10 ft long, with open ends. The other was a segment of seamless rolled pipe, 0.330 in. thick, 23.5 in. diameter, and 12 ft long. This second specimen was tested first with open ends and afterward with constrained ends. Constraint was secured by welding plates, $^1/_8$ in. thick and 3 ft square, against the end of the pipe and then cutting holes in these plates of nearly the same diameter as the pipe, leaving, in effect, wide flanges on the ends of the pipe sufficiently rigid to prevent appreciable deformation. Fig. 1 represents this test piece with flanges attached.

The first cylinder described was very nearly uniform in thickness; the second, though regular in outline, varied considerably in thickness, measurements ranging from 0.318 in. to 0.331 in. It was therefore necessary to cut holes and make local measurements at the point of loading to ascertain the thickness there, and the results for stress at points remote from the load are in some measure vitiated by uncertainty as to the exact thickness of metal at those points.

For further study of deformations, two long seamless brass tubes were used. One of these was 12 ft long, 2.5 in. diameter, and 0.029 in. thick; the other was 6 ft long, 3 in. diameter, and 0.065 in. thick. It is believed that because of the great length of these tubes in comparison with their diameters, end effects were eliminated, and conditions secured comparable to those in a cylinder of infinite length.

Method of Testing

Steel Cylinders. In order to eliminate the effect of variation in the supporting reactions and in order to secure symmetry it was desired to employ a loading that comprised two equal and opposite concentrated forces, acting at points diametrically opposite, and so applied as to produce no change in the forces

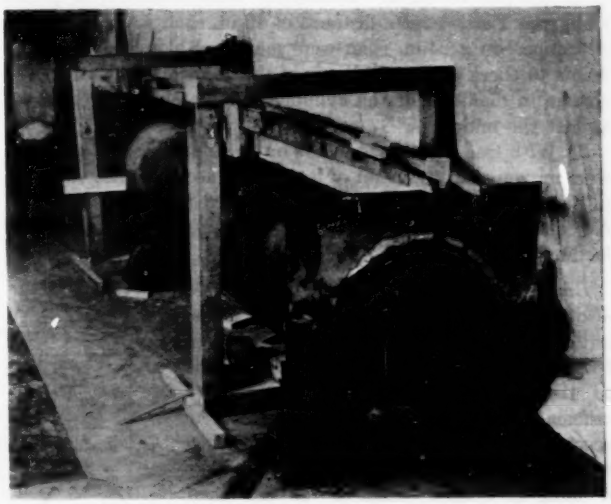


Fig. 1 Steel Cylinder With Flanges Used to Determine Effect of End Constraint

that supported the specimen. This was accomplished by means of the system of levers and deadweights shown in Fig. 2. The specimen was supported at the ends by light rods. The lever system, constructed of light planking, was balanced by means of counterweights W_1 and W_2 so that when separately supported at C it hung free in balance. A deadweight W, hung on the knife edge at D, tended to spread the levers, producing equal and opposite forces at A and B which were transmitted through round-ended pins to the inner surface of the cylinder. The apparatus was carefully calibrated in a testing machine of small capacity, and the load-multiplication factor was found to agree very closely with that calculated from the dimensions of the lever arms, showing that there was very little friction and that distortion and binding had little effect. The deadweight W was so chosen as to make each of the applied loads 120 lb for the thin cylinder and 620 lb for the thick cylinder, these loads

[1] Professor of Applied Mechanics, University of Wisconsin.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 30, 1935, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

being judged the greatest that could be used without danger of overstressing.

Strains were measured by means of Huggenberger extensometers, and a gage length of $^1/_2$ in. was used except at points remote from the load, where a gage length of 1 in. was used. For purposes of reference, the top element of the cylinder was taken as the Y-axis and the circumference at mid-length as the X-axis. In accordance with this, longitudinal stresses are

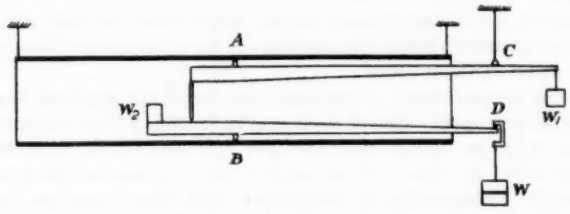


FIG. 2 METHOD OF LOADING USED IN TESTING STEEL CYLINDERS FOR STRESS AND DEFLECTION

herein denoted by the subscript y and circumferential stresses by the subscript x.

Tests and strain measurements were made as follows:

1 With load at $x = 0$, $y = 0$, x and y strains were measured at points along the Y-axis from the load to the end of the cylinder, and at points along the X-axis each way from the point of loading until bending vanished. Deflections were measured at points along the Y-axis over a distance of 10 in. each way, and center deflections on a 24-in. span were measured along adjacent elements to check the x coordinate of the point of inflection. The change in diameter of the cylinder was measured at each end and at the middle.

2 With load at various points on the Y-axis, x and y strains were measured at the point of loading, to ascertain the variation in maximum stress as the load approached the end of the pipe. No deflection measurements were taken for this off-center loading in the case of the steel pipes.

Brass Cylinders. The brass tubes were also subjected to equal and opposite loads, but on account of the small diameter of these specimens it was necessary to apply the loads to the outer surface. Fig. 3 represents schematically the method employed. The specimen was supported at the ends by counterweighted cords running over pulleys, so that it hung in neutral balance. It was then brought down until it contacted the round-pointed pin A, after which it was loaded by a dead load W placed on the yoke or frame Y, the pressure being transmitted to the specimen by the pin B, diametrically above A. This produced an equal and opposite reaction at A. Deflections at the point of loading were measured by the separately supported Ames dial C, the increment in the dial reading giving twice the actual deflection. Deflections at points away from the load were measured by two Ames dials mounted on a movable frame, one below and one above the specimen.

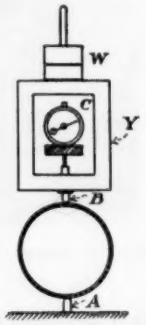


FIG. 3 METHOD OF LOADING USED IN TESTING BRASS TUBES FOR DEFLECTION

These specimens were used to study: (1) The deflection produced at various points along the loaded element when the load was applied at a point remote from the ends, and (2) the deflection produced at the load when the load was applied at various points, from the extreme end of the tube back to the point at which the deflection became practically the same as for loading at mid-length.

TEST RESULTS AND FORMULAS

STRESSES

The strain measurements, made over lengths of at least $^1/_2$ in., provided means of ascertaining only the average stresses over corresponding distances. Because of the rapid variation of stress near the load, it was necessary in some cases to estimate, from these average values, the probable value of the stress at definite points. This required certain assumptions as to the law of stress variation. After the assumptions are made, the method used for adjusting the average stresses, and the empirical formulas adopted, can best be considered jointly for each of the questions of stress intensity and distribution studied. These will now be enumerated in order.

(1) Variation of circumferential stress along the loaded circumference, or

$$s_x = f(x) \text{ at } y = 0, \text{ load at } x = 0, y = 0$$

(2) Variation of longitudinal stress along the loaded circumference, or

$$s_y = f(x) \text{ at } y = 0, \text{ load at } x = 0, y = 0$$

(3) Variation of circumferential stress along the loaded element, or

$$s_x = f(y) \text{ at } x = 0, \text{ load at } x = 0, y = 0$$

(4) Variation of longitudinal stress along loaded element, or

$$s_y = f(y) \text{ at } x = 0, \text{ load at } x = 0, y = 0$$

(5) Variation of circumferential and longitudinal stress at point of loading as load approaches end of cylinder, or

$$s_x \text{ max and } s_y \text{ max} = f(y) \text{ with load at } x = 0, y = y$$

(6) Effect of end constraint on stresses.

These six questions are now discussed in turn.

(1) $s_x = f(x)$ *at* $y = 0$. It appeared that the results obtained by the application of the theory of elasticity to the case of rectangular flat strips might be adapted to this case, since the tests showed that only a comparatively narrow strip along the loaded element of the cylinder was appreciably stressed. Nadai[2] and Timoshenko[3] discuss the problem of the flat strip, and Nadai gives formulas for finding the transverse stress s_x produced in a strip of infinite length by a concentrated load applied at mid-width which reduces to

$$M_x = [(1 + \theta)/4\pi]P \log (2a/\pi x) \quad [1]$$

$$\max M_x = [(1 + \theta)/4\pi]P \log(2a/\pi b) + (P/4\pi) \quad [2]$$

where M_x = transverse bending moment per linear inch, in-lb; θ = Poisson's ratio (here taken as 0.27 for steel); P = concentrated load, lb; a = width of strip, in.; x = distance from load, measured crosswise of the strip, and supposed to be very small, in.; b = radius of the circular area over which it is assumed that the nominally concentrated load is distributed, in.

Equation [1] was divided through by the section modulus of the unit strip, $^1/_6\ t^2$ (where t = thickness of the strip, in.) and written as

$$s_x = [KP \log(CR/x)]/t^2 \quad [3]$$

where R = radius of the cylinder, in., and K and C are constants to be determined empirically. It was found that both circum-

[2] "Über die Spannungsverteilung in einer durch eine Einzelkraft belasteten rechteckigen Platte," by A. Nadai, *Der Bauingenieur*, January 15, 1921, p. 11.

[3] "Über die Biegung der allseitig unterstutzen rechteckigen Platte unter wirking einer Einzellast," by S. Timoshenko, *Der Bauingenieur*, January 31, 1922, p. 51.

ferential and longitudinal strains practically vanished at $x = 2.5$ in., indicating for C a value of about 0.215. By setting Equation [3] equal to the measured stress, values of K for different values of x were determined, and the average of these was found to be 0.42. The values thus found for K and C inserted in Equation [3] give, as the adopted empirical formula for $s_x = f(x)$ at $y = 0$

$$s_x = [P/t^2][0.42 \log (0.215\, R/x)] \quad [4]$$

To obtain s_x at the load, the term $(1 + \theta)/4\pi$ in Equation [2] was replaced by 0.07 and the term $2a/\pi b$ by $0.215R/b$, to correspond with the empirically determined values used in Equation [4]. The resulting formula for maximum circumferential stress is

$$\max s_x = [P/t^2][0.42 \log (0.215R/b) + (6/4\pi)] \quad [5]$$

In Fig. 4 is plotted a graph obtained from Equations [4] and [5], evaluated for the load used in testing the thin cylinder and for the dimensions of the thin cylinder. The plotted points

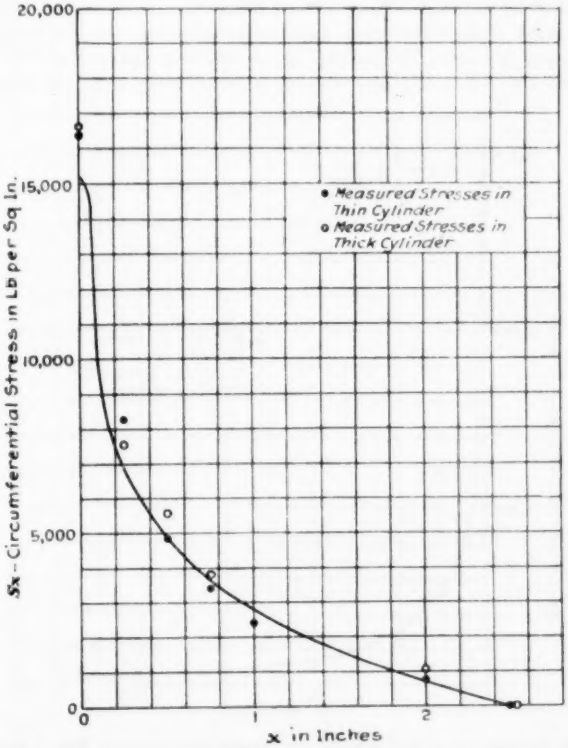


FIG. 4 CIRCUMFERENTIAL STRESS IN CYLINDRICAL SHELL RESULTING FROM CONCENTRATED LOAD. STRESS PLOTTED AS FUNCTION OF x, DISTANCE FROM LOAD MEASURED CIRCUMFERENTIALLY

indicate the measured stresses, the solid circles representing results secured with the thin cylinder, and the open circles results secured with the thick cylinder, these latter being the measured stresses multiplied by the factor $P't^2/Pt'^2$, where P and P' are the loads, and t and t' the thicknesses, for the thin and thick cylinders, respectively. For $x = 0$ the plotted point does not represent the average stress as measured over a gage length of $^1/_2$ in. centered on $x = 0$, $y = 0$; it represents the *estimated* stress at the point $x = 0$, $y = 0$, and was obtained from the average stress as follows: Equation [4] was plotted to a large scale for values of x up to 0.25 in. From this graph, it was found that the ratio of maximum stress (at $x = 0$) to the average stress (between $x = 0$ and $x = 0.25$) was 1.5. The measured stress over the $^1/_2$-in. gage length was therefore simply multiplied by 1.5 to give the value plotted in Fig. 4.

(2) $s_y = f(x)$ *at* $y = 0$. According to the analysis for flat strips previously referred to the values of s_x and s_y are equal at

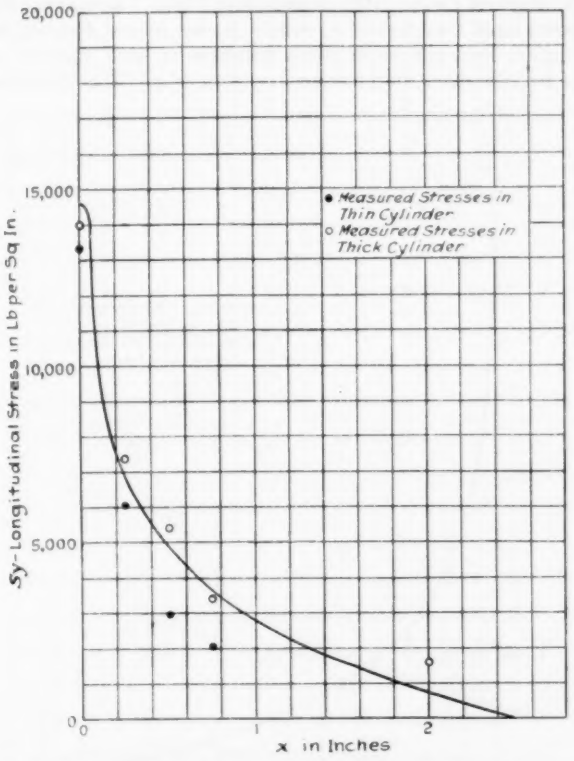


FIG. 5 LONGITUDINAL STRESS IN CYLINDRICAL SHELL RESULTING FROM CONCENTRATED LOAD. STRESS PLOTTED AS FUNCTION OF x, DISTANCE FROM LOAD MEASURED CIRCUMFERENTIALLY

any point except the point of loading, hence Equation [4] gives the longitudinal as well as the circumferential stress. For the point of loading, the formula for bending in the flat strip is

$$\max M_y = [(1 + \theta)/4\pi]P \log(2a/\pi b) + (\theta P/4\pi) \quad [6]$$

Making the same modifications as were made for the case of max M_x, we have

$$\max s_y = [P/t^2][0.42 \log(0.215R/b) + (6\theta/4\pi)] \quad [7]$$

When s_y was calculated by Equations [6] and [7] and plotted, the ratio of max s_y to the mean s_y over a gage length of $^1/_2$ in. was found to be 1.47. The curve of Fig. 5 is a graph of Equation [4] and Equation [7] and the plotted points represent experimentally determined values of s_y, the results obtained with the thin cylinder being indicated as before by solid circles and those obtained with the thick cylinder by open circles. The values at $x = 0$ were obtained by multiplying the mean stresses measured over a $^1/_2$-in. gage length there by the factor 1.47.

(3) $s_x = f(y)$ *at* $x = 0$. The analysis of the flat strip does not yield a formula that can be adapted for this case, and so, as in the cases subsequently discussed, an attempt was made to fit a purely empirical equation to the experimental results. It was considered necessary that the equation agree with Equation [4] at $y = b$ (that is, at the edge of the loaded area) and that it should represent with reasonable fidelity the stresses adjacent to the load; for points remote from the load the stresses are so small as to be of practically no consequence.

The equation adopted was

$$s_x = [P/t^2][0.42 \log(\beta R/y)] \quad [8]$$

where $\beta = 0.215 + 0.5\,[(y - b)/R]^{1/2}$

The graph of Equation [8] is plotted in Fig. 6. Because of the known rapid variation of s_x with x, it was judged desirable to estimate, from the mean stress measured at each value of y, the *probable* stress at the point $x = 0$, $y = y$. This was done

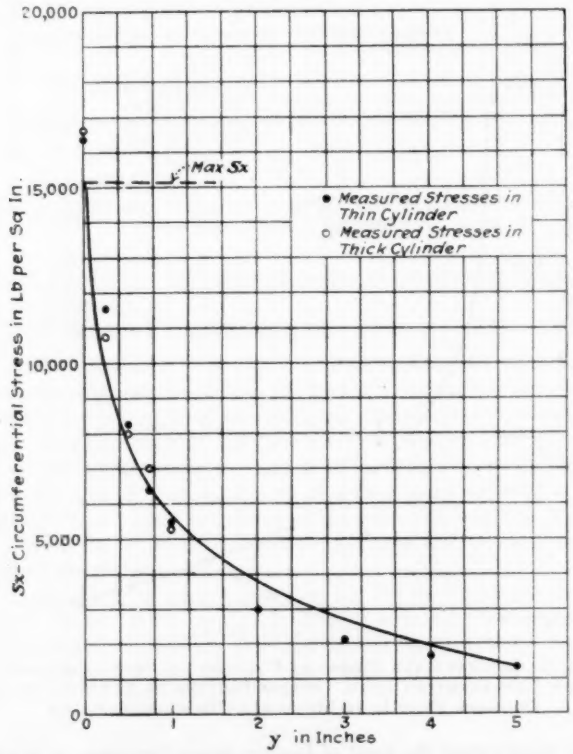


FIG. 6 CIRCUMFERENTIAL STRESS IN CYLINDRICAL SHELL RESULTING FROM CONCENTRATED LOAD. STRESS PLOTTED AS FUNCTION OF y, DISTANCE FROM LOAD MEASURED LONGITUDINALLY

by plotting the mean stresses s_x over $^1/_{\bar{4}}$-in. gage lines centered at $x = 0$, $^1/_4$, $^1/_2$, and $^3/_4$ in. The curve of estimated actual stress was then drawn in so as to be consistent with these mean values. This procedure was carried out for several different values of y and the ratio of s_x at $x = 0$ to mean s_x at $x = 0$, at any given place along the cylinder, was determined by interpolation from the several values thus obtained. The correction factors secured in this way ranged from 1.5 (at $y = 0$) down, approaching unity as y increased. The values of s_x thus arrived at are of course approximate, but it is believed that they represent the true stresses much better than do the mean stresses as measured.

(4) $s_y = f(y)$ *at* $x = 0$. The equation

$$s_y = [1 - 1.1(y/R)^{1/4}]s_x \quad [9]$$

was found to fit the test results fairly well. Fig. 7 shows a graph of this equation, and the plotted points represent the mean measured stresses, the same legend being followed as before. Because of the essentially straight-line character of the curve over short distances, the mean stresses are taken to represent the actual stresses with sufficient accuracy.

(5) s_x *max and* s_y *max* $= f(y')$ *with load at* $x = 0$, $y = y'$. Tests to determine the variation in maximum stresses as the load approached the end of the cylinder were made on the thin cylinder only. The measured mean stresses, corrected in each case by the proper factor (1.5 for s_x and 1.47 for s_y) are plotted on Fig. 8, and the equations adopted are graphed. These curves have the equations

$$\max s_x = \max s_{x0}[0.7\,R/(y' + b)]^{1/4} \quad [10]$$

$$\max s_y = \max s_{y0}(y'/0.2R)^{1/2} \quad [11]$$

where y' denotes the distance from the end of the cylinder to the load and max s_{x0} and max s_{y0} denote the maximum stresses produced by a load remote from the end of the cylinder, as given by Equations [5] and [7].

Equations [10] and [11] apply only for values of y' less than those which make max s_x = max s_{x0} and max s_y = max s_{y0} that is, Equation [10] applies only when $y' + b < 0.7R$ and Equation [11] applies only when $y' < 0.2R$.

(6) *Effect of End Constraint on Stresses.* The stresses at and near the load were found to be unaffected by end constraint except when the load was applied very near the ends. This would be expected from the results indicated in Fig. 8, which show that end effect vanishes within a short distance. Tests on the thick cylinder with constrained ends indicated that the maximum stresses produced by loading up to within a distance of 0.2 R from the end could be assumed approximately equal to those

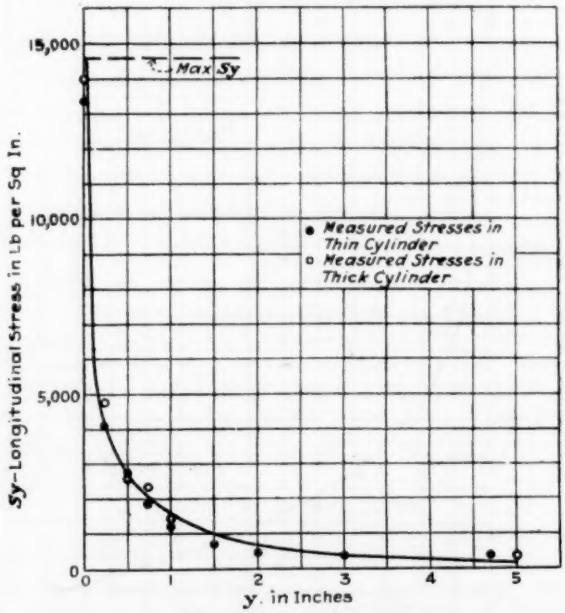


FIG. 7 LONGITUDINAL STRESS IN CYLINDRICAL SHELL RESULTING FROM CONCENTRATED LOAD. STRESS PLOTTED AS FUNCTION OF y, DISTANCE FROM LOAD MEASURED LONGITUDINALLY

produced by loading at or near mid-length, and that from that point to the end the stresses at the point of loading may be assumed to diminish linearly to zero.

DEFLECTIONS

The only laws of deflection for which formulas were sought were:

(*a*) The deflection d at any point (0, y) along the loaded element, with load remote from end of cylinder, and

(*b*) The deflection D at the point of loading, with the load applied at any point (0, y).

As has been explained, the steel cylinders were used to study the deflection adjacent to the point of loading, while the brass tubes were used to study the deflection at various distances

from the load, under conditions comparable to those in a cylinder of infinite length, and to study the influence of the position of the load on the maximum deflection.

(a) $d = f(y)$, *Load Remote From End of Cylinder.* It was thought that a strip along the loaded element of the cylinder might be regarded as a beam on an elastic foundation, the support being provided by the series of elastic hoops comprising successive segments of the shell. It was assumed that the deflection at the point of loading would be proportional to P and to the square of the radius, and inversely proportional to E and the cube of the thickness, that is, that $D = C(PR^2/Et^3)$, where C is an empirical constant. Comparison with test results gave 0.135 as an average value of C, and so the formula

$$D = 0.135PR^2/Et^3 \quad [12]$$

was adopted, as giving the deflection at the point of loading when the load is remote from the end of the cylinder.

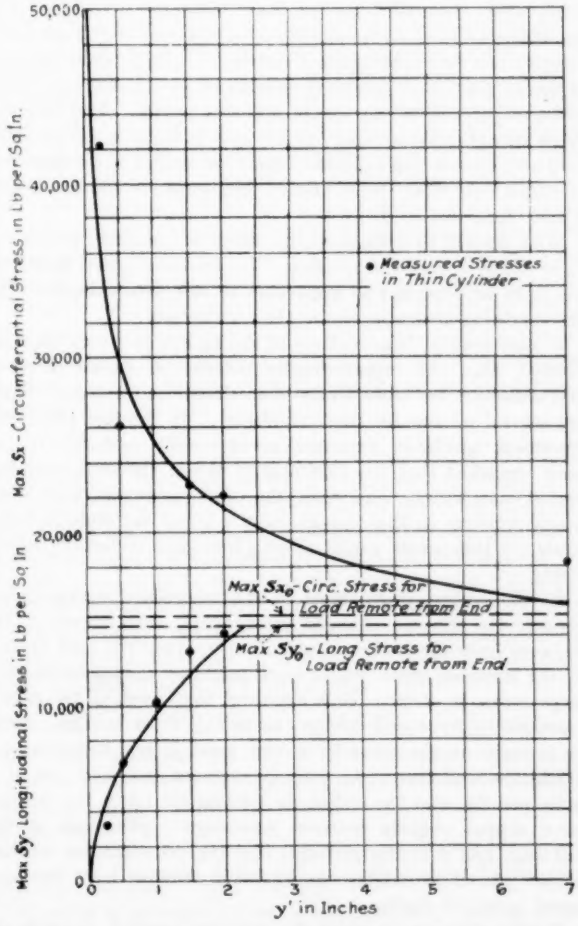


FIG. 8 MAXIMUM STRESSES IN CYLINDRICAL SHELL DUE TO CONCENTRATED LOAD. STRESSES PLOTTED AS FUNCTION OF y', DISTANCE OF LOAD FROM END OF CYLINDER

The deflection at any point of a beam on an elastic foundation may be expressed by the equation $d = De^{-\beta y}(\cos \beta y + \sin \beta y)$ where D = deflection at the load, d = deflection at a distance y therefrom, and β is equal to $(K/4EI)^{1/4}$, where K = elastic resistance of the foundation per linear inch of beam per unit of deflection. Since, in the case of the cylinder, K is measured by the stiffness of a unit ring or segment, it is proportional to Et^3/R^3. Substituting this for K in the expression for β, and noting that the width of the hypothetical strip that constitutes the beam may reasonably be assumed to be proportional to R and that therefore I is proportional to Rt^3, it is found that β is proportional to $1/R$. It was found, with the brass tubes, that the deflection practically vanished at $y = 8R$, and this condition gives $0.3125/R$ for the value of β. In the case of the elastic beam, the deflection is of course a periodic function and

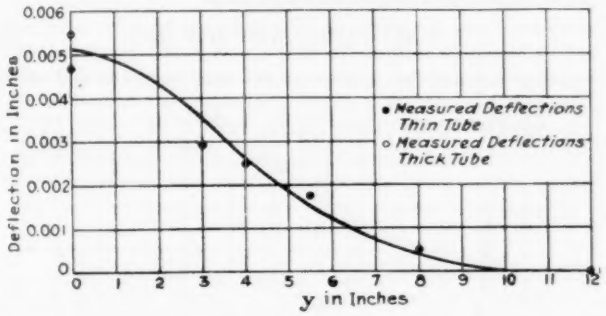


FIG. 9 DEFLECTION OF BRASS TUBE PRODUCED BY CONCENTRATED LOAD. DEFLECTION PLOTTED AS FUNCTION OF y, DISTANCE FROM LOAD MEASURED LONGITUDINALLY

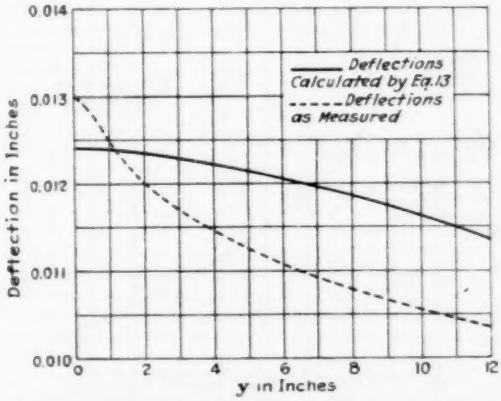


FIG. 10 DEFLECTION OF STEEL CYLINDER PRODUCED BY CONCENTRATED LOAD. DEFLECTION PLOTTED AS FUNCTION OF y, DISTANCE FROM LOAD MEASURED LONGITUDINALLY

negative deflections are obtained; this may very well be the case also with the cylinder, but such deflections, if present, are negligibly small, and in the specimens tested could not be detected with the apparatus used. It is necessary therefore only to consider the deflections between the point of loading and points distant $8R$ to either side thereof, and the formula

$$d = (0.135\ PR^2/Et^3)(e^{-\beta y})(\cos \beta y + \sin \beta y) \quad [13]$$

where $\beta = 0.3125/R$ was adopted as being in reasonable accord with experimental results. The curve of Fig. 9 represents Equation [13] plotted for the thin brass tube and the measured deflections for that specimen. The deflection at load as determined for the thick tube is also plotted, the actual measured deflection having been multiplied by the ratio of the quantities PR^2/Et^3 for the two specimens to make it comparable with the deflection of the thin tube.

In Fig. 10 are shown the graph of Equation [13], calculated for the thin steel cylinder and the actual elastic curve as determined by careful measurement made near the load. It is clear that Equation [13] does not represent closely the *form* of the deflection curve near the load, and that the analogy of the beam on an elastic support could not be expected to yield a satis-

factory formula for stresses. The sharp curvature under the load indicated by the experimentally determined curve is consistent with the high stresses found at that point.

(*b*) $D = f(y)$, *Load at* $(0, y)$. The variation of deflection at load with distance from the end of the cylinder was found to be as shown in Fig. 11, where the plotted points represent experimental results secured with the thin brass tube. The equation of the curve found to best fit the points is

$$[0.48 - 0.22(y'/R)^{0.35}][PR^2/Et^2] \quad [14]$$

where y' denotes the distance of the load from the end of the cylinder. When y' becomes equal to 4.4 R, the deflection is practically equal to that produced by a load indefinitely remote from the end.

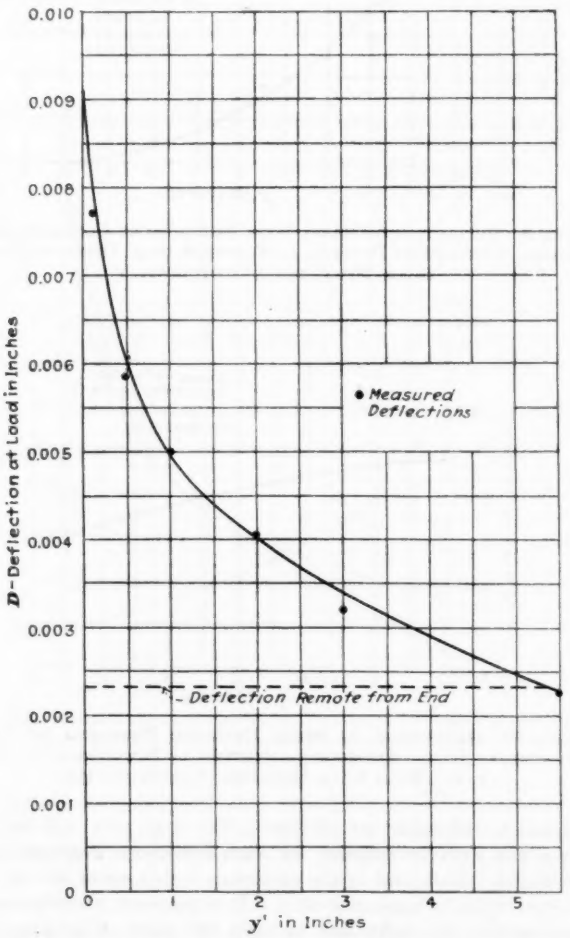


FIG. 11 DEFLECTION UNDER 5-LB CONCENTRATED LOAD OF A BRASS TUBE 2.5 IN. DIAM AND 0.029 IN. THICK. DEFLECTION PLOTTED AS FUNCTION OF y', DISTANCE FROM END OF TUBE TO THE LOAD

CHANGES IN DIAMETER

In addition to the measurements of deflection on which Equations [12], [13], and [14] are based, the changes in the vertical and horizontal diameters of the thick steel cylinder were determined at the loaded section and at the ends. Under loads each equal to 698 lb, the vertical diameter increased 0.0229 in. at the loaded section and 0.0039 in. at the ends, while the horizontal diameter decreased 0.0126 in. at the loaded section and 0.0036 in. at the ends.

It may be of interest to ascertain the length of a segment of the cylinder which, under the same loads uniformly distributed along opposite elements, would suffer the same change in vertical diameter as occurred in the cylinder at the loaded section. Using the ring formula: $\Delta D = 0.149\ PR^3/EI$, where ΔD denotes change in diameter, it is easy to show that the length of this segment is 6.9 times the radius. A similar computation for the brass tubes gives a length 7.3 times the radius. It appears, therefore, that the change in diameter produced in a long cylinder at the point of loading can be calculated on the assumption that the loads are uniformly distributed along opposite elements of a ring having a length equal to seven times the radius.

CONCLUSION

The results of the tests indicate that both the stiffness and strength of a thin cylinder under concentrated loading are greater than probably would be supposed. As is the case with flat plates, the stresses become negligibly small at a comparatively short distance from the load. Because of the fact that the higher stresses are so sharply localized, end effects are small except when the load is applied very near the ends, and it appears probable that other boundary conditions would also have little effect on stresses at any except adjacent points. The formulas given should therefore apply to any case in which a concentrated load acts upon a thin cylindrical plate, no matter how supported, provided only that the boundary and supports are sufficiently remote from the load.

With respect to deflections, the effect of boundary conditions is, as might be expected, greater. The formulas given, therefore, can only be regarded as applicable to complete cylinders that are very long in comparison to their diameters.

It may appear, from a study of the figures on which they are plotted, that the experimentally determined values of stress and deflection are more erratic than should be the case. There are several reasons for this irregularity. In the first place, the specimens available were not geometrically perfect. It has been remarked that the thickness of one of the steel cylinders varied considerably, and while the other specimens were more nearly uniform in this respect, even a small variation in these relatively thin shells would affect stress and deflection appreciably.

In the second place, most of the quantities measured, especially the stresses, changed very rapidly with respect to the distance from the load, and so failure to apply the load exactly at the intended point would have produced disproportionately large errors in stress. It is apparent that most of the points representing measured stresses, as in Fig. 6 for example, could be brought on the curve by a very small lateral displacement. While the load was never entirely removed during a test, it is quite possible that the end of the loading pin (A in Fig. 2) may have slipped slightly between successive applications of the full load, and it is also probable that the extensometer was not always placed at exactly the intended distance from the supposed point of loading.

Finally, the way in which the load was applied, through the placing by hand of a deadweight on a yielding support, made it very difficult to avoid some dynamic effect. Every effort was made to ease the load on so as to reduce this effect to a minimum, but the operation was a fatiguing one and it is possible that there was occasional overloading, too slight to be revealed by recovery or vibration and yet sufficient to introduce an appreciable error. It should, however, be remarked that every experimental value, whether representing a stress or a deflection, is the mean of several observations, the load being applied and removed until at least two successive readings were obtained that agreed as closely as the instrument could be read.

Recent Research Work in Lubrication

By GEORGE B. KARELITZ,[1] NEW YORK, N. Y.

I—GENERAL

THE increase in speed of machinery and the larger loads imposed on rubbing surfaces bring the matter of lubrication and the mechanism of bearings forcibly to the attention of mechanical engineers. Much has been done to disseminate and popularize the recent findings on the theory of lubrication. In this respect the review and bibliography given herein will be helpful as will reference to the work done under the auspices of the A.S.M.E. (1).[2]

II—HYDRODYNAMICAL THEORY AND DESIGN APPLICATIONS

In regard to hydrodynamical theory, Needs (2, 3) has shown the effect of a finite bearing length and the resulting end leakage upon the pressure distribution, carrying capacity, and friction of the oil film. In this work Needs employed the electric analogy of Kingsbury to study the conditions in a 120-deg bearing. The data obtained give a firmer basis for the application of the theory to the design of power bearings. Further tests by Baudry and Tichvinsky (4) on the performance of actual large-size bearings have been reported and the results compared with the theoretical values expected. An interesting review of standard practice in bearing design in the light of hydrodynamic theory has been presented by Howarth (5, 6). This review was based on statistical design data collected from the leading machinery manufacturers. Troublesome vibration which sometimes occurs in bearings due to the instability of the oil film has been discussed by Newkirk (7). In this work, means to prevent and cure the vibration were presented.

III—OILINESS

The question of oiliness, its physical meaning and relation to the lubricating quality of lubricants has received much attention both here and abroad. The mechanism of the extreme-pressure lubricants and the effect of sludging or oxidation on the lubricating qualities of the oil has been investigated and discussed by various authorities (8, 9, 10, 11). Among the several methods advocated for measuring the adhesive property of the lubricant or the thickness of the adhered layer a centrifugal device has been described by Sperry (12).

IV—INTERNAL-COMBUSTION ENGINE LUBRICATION

Particular attention has been paid to the necessary physical properties of lubricants for, and the behavior of such in internal-combustion engines both for automobile and aviation purposes (13, 14, 15).

V—VISCOSITY MEASUREMENT

Standardization of viscosities for industrial lubricating oils has been considered and charts for the practical classification of mineral lubricants have been published (6, 16). The advisability of the universal adoption of absolute-viscosity units instead of the commercial units now in common use, has been discussed and advocated. In this connection, further work has been done on instruments for measuring the absolute viscosity directly, both for laboratory and industrial uses (17, 18).

BIBLIOGRAPHY

1 "Notes on History of Lubrication," by M. D. Hersey, *Journal American Society of Naval Engineers*, vol. 45, no. 4, 1933, pp. 411–429 and vol. 46, no. 3, 1934, pp. 369–385.

2 "Effect of Side Leakage in 120-Deg Journal Central Bearings," by S. J. Needs, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-16, pp. 721–732.

3 "Side Oil Flow in a 120-Deg Bearing," by G. B. Karelitz, *Mechanical Engineering*, vol. 57, 1935, pp. 292–293.

4 "Performance of Large Journal Bearings," by R. Baudry and L. M. Tichvinsky. Paper presented at the A.S.M.E. Annual Meeting, December, 1934.

5 "Current Practice in Pressures, Speeds, Clearances, and Lubrication of Oil Film Bearings," by H. A. S. Howarth, Trans. A.S.M.E., vol. 56, 1934, paper MSP-56-2, pp. 891–911.

6 "The Loading and Friction of Thrust and Journal Bearings With Perfect Lubrication," by H. A. S. Howarth, Trans. A.S.M.E., vol. 57, 1935, paper MSP-57-2, pp. 169–187.

7 "Oil-Film Whirl—A Non-Whirling Bearing," by B. L. Newkirk and L. P. Grobel, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-10, pp. 607–615.

8 "Oxidation Lubrication and the Blending of Mineral Oils to Obtain Maximum Lubricating Value," by R. O. King, *Journal of the Institute of Petroleum Technology*, vol. 20, 1934, pp. 97–137.

9 "Friction Tests on Lubricating Oils," by J. W. Donaldson and D. R. Hutchinson, Trans. Society of Chemical Industry, vol. 52, 1933, pp. 424–429.

10 "Friction in the Greasy and Semi-Fluid Phase," by E. Bodart, *Revue Universelle des Mines*, vol. 9, nos. 11 and 12, 1933, pp. 285–297 and pp. 320–329.

11 "What Is Oiliness?" by O. C. Bridgeman, *Physics*, vol. 5, 1934, pp. 125–130.

12 "Researches in Oiliness Conducted on the Adheroscope," by E. A. Sperry, *Journal of the Society of Automotive Engineers*, vol. 35–36, February, 1935, p. 37.

13 "Causes and Effects of Sludge in Motor Oils," by D. P. Barnard, E. R. Barnard, T. H. Rogers, B. H. Shoemaker and R. E. Wilkin, *Journal of the Society of Automotive Engineers*, vol. 34, May, 1934, pp. 167–178.

14 "Behavior of Lubricating Oils in Automobile Engines," by R. Orel, *Petroleum*, vol. 30, no. 15, supplement, 1934, pp. 2–7.

15 "Change of Viscosity Curve of Oil in Service," by L. Steiner, *Petroleum*, vol. 30, no. 6, supplement, 1934, pp. 2–3.

16 "Classification of Lubricants According to Change in Viscosity Over Given Temperature Range," by C. M. Larson and W. C Schwaderer, *Oil and Gas Journal*, vol. 33, no. 5, 1934, p. 10.

17 "A Direct Reading Viscosimeter," by J. G. A. Rhodin, *Engineer*, vol. 158, 1934, p. 95.

18 "Absolute Method of Determining the Coefficient of Viscosity," by A. Sokolsky, Comptes Rendus, Académie de Sciences, U.S.S.R., 1934, pp. 341–347.

[1] Associate Professor, Department of Mechanical Engineering, Columbia University. Mem. A.S.M.E.

[2] Numbers in parentheses refer to bibliography at the end of the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expression of their authors, and not those of the Society.

Design Data

It is important that the data contained in technical papers be made readily available to designing engineers. In order to satisfy these needs of industry, this section of the Journal will include a concise presentation of data and information drawn chiefly from papers previously published by the Applied Mechanics Division of The American Society of Mechanical Engineers.

The data are prepared by a subcommittee of the Applied Mechanics Division on strength of materials (R. E. Peterson, chairman), under the guidance of S. Timoshenko.

Torsional Stresses in Shafts Having Grooves or Fillets

By L. S. JACOBSEN,[1] STANFORD UNIVERSITY, CALIF.

THE general problem of torsional stresses in shafts having grooves or fillets was first solved by Michell (1) in 1900. A little later Föppl (2) outlined a graphical method whereby solutions might be obtained in specific cases, and in 1907 Willers (3) actually determined the magnitudes of such stresses in two typical sections by a graphical method of his own. About fifteen years later, Liljeblad (4) found an analytical expression for the stress concentration near a fillet in a shaft and applied it to practical sections. Shortly after, the author of this article described an electrical analogy (5) of torsion by means of which experimental determinations of stresses might be readily made for any axial outline of a shaft of circular cross section. Finally, in 1929, Sonntag (6) published an investigation, using an approximate method, of the three most typical forms of axial changes used in practice, as shown in Fig. 1.

By means of the hydrodynamic analogy it can be shown that the torsional stress at the bottom of the semicircular groove, Fig. 1*a*, approaches a value of twice the maximum stress in the straight part of the shaft as the radius of the groove is diminished. Sonntag's results for this case apply to a groove of any size and are

$$\tau_{max} = \frac{16M}{\pi d^3}\left(\frac{2D}{D + 2r}\right)$$

The more general section, shown in Fig. 1*b*, has also been studied by Sonntag, and for this groove his results are

$$\tau_{max} = \frac{16M}{\pi d^3}\left[\frac{(D - d + 2r)(d + 2r)^2 + 4r^2(D - d - 2r)}{2rD(d + 4r)}\right]$$

The section shown in Fig. 1*c* is perhaps the one most commonly used in practice, inasmuch as very few shafts are of one diameter only. Willers considered this case graphically for the ratio of $D/d = 1.33$. He found that the maximum stress occurs at a point slightly to the right of the point of tangency of the fillet with the outline of the smaller shaft. The studies made by the author (5) using the electrical analogy, confirm this fact and are, moreover, in good agreement with Willers' results. Five ratios

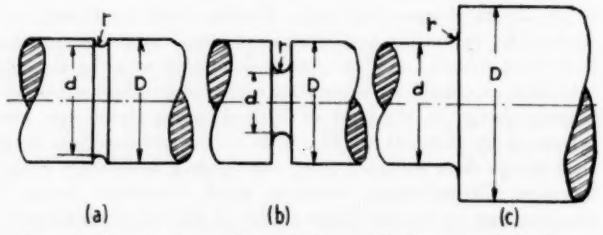


FIG. 1 NOTATION FOR SHAFTS HAVING GROOVES OR FILLETS

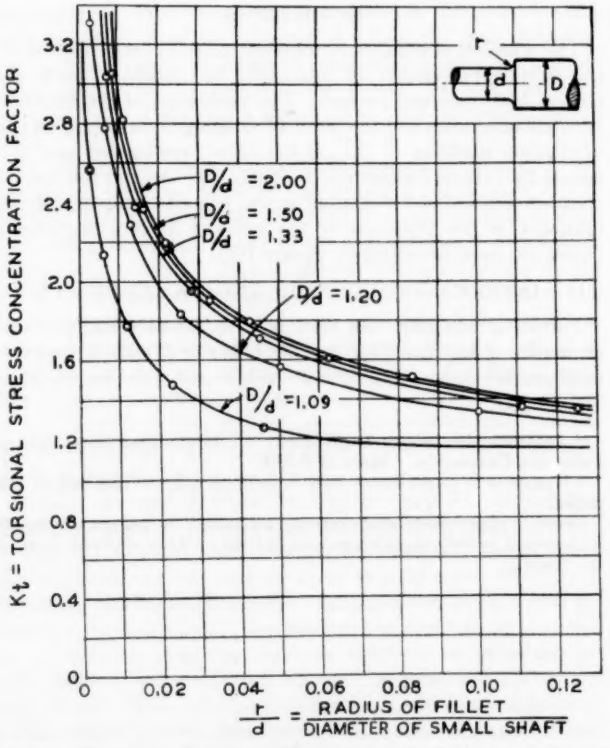


FIG. 2 TORSIONAL-STRENGTH-CONCENTRATION FACTORS IN SHAFTS WITH FILLETS

[1] Professor, Department of Mechanical Engineering, Stanford University, Calif.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of D/d were investigated, namely: 1.09, 1.20, 1.33, 1.50, and 2.00. Values of the stress-concentration factor K_t, i.e., the ratio between the peak stress in the fillet to the nominal computed torsional stress $\tau = \frac{16M}{\pi d^3}$ in the smaller shaft, as found by these tests are given in Fig. 2, for various values of the ratio D/d and r/d. For values of D/d not shown on the curves interpolation may be used with sufficient accuracy for most practical purposes.

Sonntag's approximate analytical formula for the case shown in Fig. 1*c* is

$$\tau_{max} = \frac{16M}{\pi d^3 D}\left[\frac{3}{2}\frac{(d+2r)(d+4r)}{(d+6r)} + \frac{(D-d-2r)(d+12r)}{12r}\right]$$

Results computed by this formula are in fairly good agreement with those obtained experimentally (5) for values of D/d near unity; however, for larger values of D/d there is considerable difference and for such cases the curves of Fig. 2 should be used.

If shafts of complicated axial outlines are to be studied for torsional stresses, the electrical analogy will give answers without the expenditure of much mental effort. All that is necessary for carrying out the analogy is a two-dimensional, or nearly two-dimensional, conductor, e.g., a plate. However, in order that the analogy may be established, the conductivity of the plate must be proportional to the cube of the distance from one edge. In the practical case, this is effected by making the thickness of the plate proportional to the cube of the distance from the edge; in other words, the cross section of the plate resembles that of a hollow-ground razor blade.

The plate is then cut to have the same boundary as the outline of the axial section of the shaft, the thin edge of the plate coinciding with the center line of the shaft, and the thickness of the plate at the boundary being proportional to the cube of the distance of the boundary from the thin edge.

Assuming that the ends of the plate are maintained at a constant electric potential, so that the current near the boundary of the plate follows the boundary, the potential gradient anywhere in the plate may be measured experimentally by a Wheatstone bridge and a galvanometer. The torsional stress at a point in the shaft is then proportional to the measured potential gradient in the plate at the corresponding point multiplied by the distance of the point from the thin edge of the plate. It is usually a matter of a few minutes of experimentation to find the point at which the maximum stress occurs, i.e., maximum gradient times distance from the thin edge.

In order that the accuracy of the analogy may be satisfactory, the thickness of the plate must be kept as small as possible, so as to approximate a two-dimensional current flow. But in order that the manufacture of the plate may not be too expensive, due to the accuracy needed, the thickness must be appreciable. These two demands are contradictory, and a compromise must be sought. In 1926 the author used an analytic bath of shallow proportions in which the depth increased as the cube of the distance. A 10 per cent copper-sulphate solution is satisfactory. Work of this type has been carried out in Germany recently (7).

BIBLIOGRAPHY

1 J. H. Michell, "The Uniform Torsion and Flexure of Incomplete Tores With Application to Helical Springs," Proceedings, London Mathematical Society, vol. 31, 1899, pp. 130–146.

2 A. Föppl, "Über die Torsion von runden Stäben mit veränderlichem Durchmesser," Sitzungsberichte der mathematisch-physikalischen, Klasse der K. B. Akademie der Wissenschaften zu München, vol. 35, 1905, pp. 247–262.

3 F. A. Willers, "Die Torsion eines Rotationskörpers um seine Achse," *Zeit. für Mathematik und Physik*, vol. 55, 1907, p. 225.

4 R. Liljeblad, "Torsionsspänningen i halkälar och kilspar a för vridning utsatta axlar," *Teknisk Tidskrift*, Mekanik, vol. 11, 1921.

5 L. S. Jacobsen, "Torsional Stress Concentrations in Shafts of Circular Cross-Section and Variable Diameter," Trans. A.S.M.E., vol. 47, 1925, p. 619.

6 R. Sonntag, "Zur Torsion von runden Wellen mit veränderlichen Durchmesser," *Zeit. für angewandte Mathematik und Mechanik*, vol. 9, 1929, p. 1.

7 A. Thum and W. Bautz, "Die Ermittlung von Spannungsspitzen in verdrehbeanspruchten Wellen durch ein elektrisches Modell," *Zeit. V.D.I.*, vol. 78, no. 1, 1934, p. 17.

Book Reviews

Anecdotal History of the Science of Sound

ANECDOTAL HISTORY OF THE SCIENCE OF SOUND. By Dayton Clarence Miller, D.Sc., L.L.D. Professor of Physics, Case School of Applied Science, Cleveland. The Macmillan Company, New York, N. Y., 1935. Cloth, 5¾ × 8½ in., 114 pp., 15 plates, $2.50.

REVIEWED BY A. L. KIMBALL[1]

NO ONE interested either in the theory or the application of sound should fail to possess this little volume written by the most distinguished exponent of this field we have in the country today. Doctor Miller with his fine physical perception, together with his long experience, and wide contacts, is eminently fitted for this task.

Although, as the name implies, much of the book is given to anecdotes, these are chosen in a way which gives an illuminating picture of the development of the science of acoustics in both its experimental and theoretical aspects, with many side lights on the parallel development of musical instruments and sound-measuring devices.

An attractive feature of the book is a number of plates showing portraits of many of the pioneers and illustrations of apparatus made by them.

Many facts of interest appear which are not generally known, some even to specialists, I venture to say. It appears, for example, that Galileo's researches give "the first enunciation of the principles of sound which are really scientific," and his own words, quoted in this connection on the vibration of strings and cords, show how far ahead of his time was this great pioneer of the inductive method of scientific research.

Not the least interesting of many personal side lights are those regarding more recent investigators, such as Tyndall, König, Rayleigh, Webster, Sabine, and Bell, with all of whom, except Tyndall, Dr. Miller was personally acquainted.

This book is written with an appreciation of pioneer work of a more analytical character as well as of experimental, which will be pleasing to those who incline toward the theoretical.

A bibliography of all works referred to is given at the end.

Aerodynamic Theory

AERODYNAMIC THEORY. Dr. William Frederick Durand, editor-in-chief. Vol. 2. Division E. Julius Springer, Berlin, 1935. Cloth, 6 × 9 in., xv plus 367 pp., 113 figs., and 4 plates. Special price only until the publication of the last volume, rm. 20.

REVIEWED BY J. C. HUNSAKER[2]

TO present a useful review of this, the second volume of the series of six, it seems necessary to consider its place in the whole work. Dr. Durand, under his precept as general editor has undertaken to organize and coordinate the work of the world's most eminent authorities into a comprehensive and permanent record of the present state of aerodynamic theory. That such a record is in effect a record of twenty-five years' progress is due to the fact that modern aerodynamics has been developed within these few years. Consequently, the contributors drafted by Dr. Durand are themselves men whose own work constitutes a large part of the body of this theory.

The six volumes, of which four have appeared in rapid succession, will contain a stock taking of general fluid mechanics; the mechanics of perfect fluids, of viscous and of compressible fluids, the application of theory and experimental technique to airfoils, bodies, propellers, etc., the dynamics of the airplane and airship, performance of aircraft, the hydrodynamics of sea planes and the aerodynamics of cooling. From this plan it can be seen that volume one, reviewed in the June, 1935, issue of this JOURNAL, is an introduction and a foundation for the entire structure. It set forth the mathematical and analytical methods which are to be later applied, as aerodynamics, to the design of aircraft.

Volume two, now before us, is evidently the first floor of this figurative structure. Here Professor Th. von Kármán of Pasadena and Professor J. M. Burgers of Delft, in happy collaboration, present a comprehensive and masterly treatment of the mechanics of nonviscous imcompressible fluids and in particular the aerodynamics of wings. This treatment is rigorous and elegant, and it is also clear, because the authors have taken pains to make it so. The editor states that the whole is the joint work of the two authors but that von Kármán primarily contributed the chapter on the theory of wings of infinite span and the chapter on the wake, while the other five chapters are by Burgers. The reader should be grateful to whichever of these able scientists proposed the wholesome innovation of commencing each chapter with a precise yet nonmathematical statement of the physical problem to be considered, a discussion of the methods of attack which might be employed in the solution, and finally a forecast of just what will be done, with reasons for the course to be taken. In consequence, each chapter has unity and purpose and a definite relation both to what precedes and to what follows.

Chapter 1 gives a general description of the nature of the flow of air around a wing and explains the process by which lift is created, develops the conceptions of circulation and induced drag, bound and trailing vortices, and establishes the approximate equivalence of an airfoil and a system of vortices.

Chapter 2 deals with the idealized case of two-dimensional flow around an airfoil of infinite span in an ideal fluid. This is the classical Lanchester-Prandtl wing theory. The general wing theory is postulated as the problem of calculating the velocity and pressure field around a wing of given cross section. The development proceeds from a first approximation with a vortex filament replacing the wing, then to a second approximation using a vortex sheet, and finally to the theory of the thin airfoil. The various methods of conformal transformation are next used to compute, from potential flow theory, the flow paterns of the well known families of thick "mathematical" airfoils (Joukowski, von Kármán, Trefftz, von Mises).

Chapters 3 and 4 treat the more nearly practical problem of flow about an airfoil of finite span. This is the most important section of the volume and is entirely new (to this reviewer) in that the starting point is the investigation of the influence the surface forces, on the airfoil, have on the motion of a fluid. The present treatment is logical and powerful. Chapter 3 begins with the Euler hydrodynamical equations of motion, completed by generalized force terms. By successive approximations, these

[1] Engineering General Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[2] Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

equations of motion are solved and the principal theorems regarding induced drag, wake, downwash, lift, vorticity, etc. are established as a consequence. This chapter is essentially a mathematical tool for application in chapter 4 to a series of important problems.

Chapter 4 is in three parts: Part *A* is devoted to the computation of the distribution of lift over the span and the corresponding induced drag and the effect of ailerons, sweep back, taper, twist, and yaw. Part *B* deals with the distribution of lift and drag over multiplanes. Part *C* establishes a thoretical basis for estimating the effect on airfoils of boundaries parallel to the direction of general flow, such as the ground, wind-tunnel walls, etc.

Chapter 5 deals with nonuniform and curvilinear motion and, due to the relatively undeveloped nature of this field, the discussion is more brief. Perhaps when flapping wings become of other than academic or ornithological interest this material may form a basis for further extension of the theory.

Chapters 6 and 7 discuss the development of the vortex system downstream of the airfoil and the theory of the wake. The latter is suggestive of the need for future progress as the whole phenomenon of resistance is involved. At a certain inclination to the direction of motion the flow of real fluids separates from the back of an airfoil giving rise to a turbulent wake, high drag, and reduced lift. In this short chapter von Kármán confines himself to the results obtained from the Helmholtz theory of the discontinuous potential, the von Kármán "vortex street" and Oseen's "vortex field." These assume an ideal fluid. The turbulence of real fluids will be a subject for a later volume.

There is a good index and for each chapter there is an adequate bibliography. It should be realized that the entire wing theory is "applied mathematics," but the presentation here is so clearly done that, with the aid of volume 1 of the series, a diligent reader requires no more than an engineer's equipment of mathematics.

The Hardness of Metals and Its Measurement

THE HARDNESS OF METALS AND ITS MEASUREMENT. By Hugh O'Neill, D.Sc., M.Met. Senior Lecturer in Metallurgy in the Victoria University of Manchester. Chapman & Hall, Ltd., London, 1934. Cloth, 6 × 9½ in., 292 pp., 119 figs., 123 tables, 25s.

REVIEWED BY J. R. TOWNSEND[3]

THIS book is an exhaustive study of the property of hardness of metals. Aside from the distinction of being the first and only book on this important subject it is a model of clear and balanced exposition of a well-known but not fully understood property of metals. This book should be of interest not only to testing engineers but to metallurgists and scientists as well. It is another volume added to the fast-growing library on physical metallurgy or the science and use of metals.

Dr. O'Neill has boldly faced the problem of hardness in his introductory essay by bringing to bear all of the existing physical, mechanical, and physical-chemical theory on the problem. He shows the present practices of hardness measurement are widespread regardless of the fact that they may not be on a good foundation of fundamental theory. On the other hand consideration of the available fundamental theory does not permit one to formulate a hardness test that is wholly satisfactory in the sense that the hardness of all materials may be referred to a fundamental concept in terms of length, mass, and time. The conclusion is drawn that for all practical purposes the hardness of metals may be expressed in terms of "mass hardness." This term takes into consideration the fact of the heterogeneity of metals; that metals as we have to deal with them are a conglomerate of crystals of different hardness, plasticity, orientation, and size. The mass hardness then of a metal at a given temperature is its reaction to local indentation and this reaction should be expressed in terms which recognize viscosity and elasticity in metals and which allow for the fact that the intensity or degree of indentation may affect the specific reaction.

In spite of the fact that there is not available means of making such a test for the hardness of metals the practice of hardness testing with a variety of machines has progressed to the point that practice has far outstripped theory. In fact there has been excellent standardization of hardness testing in the field of specifications. The fact that these methods are in general use and widely understood is no reason why we should be satisfied with the present state of our knowledge. Dr. O'Neill's book is very provocative of thought in this regard and stimulating in presenting the broad panorama of facts that must be taken into consideration in the solution of the problem.

Aside from the basic statement of the problem the book contains an exhaustive review of the various hardness tests that have been devised in Europe and America. Many comparisons are drawn between the hardness values determined by the various hardness-testing machines for commonly used metals. While more emphasis is placed on European practice this is hardly a defect to thoughtful readers familiar with American practices. The book does not cover adequately the subject of the hardness test as related to specification use or to calibration and methods of manipulation of specific hardness testers. This is a very important subject in the technology of metals. While data are given showing the relationship of hardness to per cent reduction, heat treatment, and correlations between tensile strength, fatigue endurance, and hardness there is little if anything on the subject of minimum thickness of metal for reliable hardness measurement, or the minimum width, or how curved or irregular surfaces should be treated. These are most important problems in the engineering of metals. However, in view of the complete review of fundamental knowledge that is given, the absence of such data does not impair the usefulness of the book in giving a thorough understanding of the background of knowledge on the subject.

Dr. O'Neill quite properly accents the work done by Meyer and his analysis of the Brinell test or ball-indentation test. Meyer's test consists in measuring the load to produce unit deformation of the metal. The difficulty of measuring the diameter of an indentation in a metal by a ball because of the rim of metal that is thrown up and the irregular shape of the indentation due to irregular recovery of the metal and its directional reaction caused by cold work is clearly brought out in the book. The solution of this problem is not adequately explored, however. Since the deformation of a metal by a ball combines such factors as elasticity, plasticity, elastic recovery, and the directional effect of this recovery in cold-rolled metals the solution is obviously in the direction of measuring the deformation in terms of penetration of the spherical indentor with the load on. The author quite properly states that depth measurements after the load is removed are more unreliable than diameter measurements, due to shallowing of the indentation. Furthermore, it is possible to measure better the indentation of a ball penetrator with the load on than the diameter of an irregular indentation since accurate means of mechanically measuring depth are available while width measurements would require a microscope and visual observation. Meyer's constants may be accurately determined by the load required to produce unit deformation in terms of depth of indentation. Such measurements have been reported by the reviewer in the Proceedings of the American Society for Testing Materials.

[3] General Development Laboratory, Bell Telephone Laboratories, Inc., New York City.

In conclusion therefore it may be said that Dr. O'Neill has produced a valuable summary of hardness testing of metals which should be found in the library of every engineer, metallurgist, and scientist interested in the important subject of hardness. The Proceedings of the American Society for Testing Materials cover exhaustive studies of hardness as affecting specifications and use and the work of Dr. O'Neill is a fitting companion to this work to present the broad picture of hardness.

Resistance of Materials

RESISTANCE OF MATERIALS. By Fred. B. Seely, Professor of Theoretical and Applied Mechanics, University of Illinois. Second edition. John Wiley & Sons, New York. Chapman & Hall, Ltd., London, 1935. Cloth, 6 × 9 in., 436 pp., 383 figs., $3.75.

REVIEWED BY T. MCLEAN JASPER[4]

THE proper use of materials is one of the most important tasks which confront the engineer of today, and Professor Seely in his new edition of "Resistance of Materials" has succeeded in presenting, in a concise and simple manner, many of the more recent methods of applying materials to structural design.

The method employed of clarifying involved principles by practical illustrative problems should prove most helpful to the student and practicing engineer.

One feature of the book is the profuse collection of illustrative diagrams which helps the engineer to grasp fully the problems with which he is confronted.

The language of the book is simple and direct which allows no misinterpretation of the information available within its covers.

Teachers and design engineers will do well to study this book for its treatment of current engineering problems.

Metallic Arc Welding

METALLIC ARC WELDING. By H. Harris, Ph.D., D.I.C., A.R.C.S. Longmans, Green & Company, New York, N. Y. Edward Arnold & Co., London, 1935. Cloth, 5¾ × 9 in., viii plus 199 pp., 99 illustrations, $6.

REVIEWED BY CHAS. H. JENNINGS[5]

DR. HARRIS' book on metallic arc welding is a comprehensive summary of the present knowledge on the metallurgy of arc welding and should be of great assistance to engineers in connection with the production of metallic arc welds of the quality required by present-day practice.

The major portion of the book is devoted to the analysis of welds made on low-carbon structural steels of the types used for general pressure vessels, machinery, and constructional work. Excellent discussions are given on the effect of welding on the base metal, crystallization in multiple-layer welds, peening, residual stresses and stress relief, and the physical properties of welds. Coated and bare electrodes are discussed from the standpoint of effect of coating composition on the soundness of the weld deposit and the chemical changes resulting from the transfer of metal through the arc.

The influence of oxygen and nitrogen in steel and weld metal is thoroughly discussed in accordance with present-day theories. Typical test data are given to illustrate the age-hardening phenomena attributed to these elements. Metallurgical problems encountered while welding (1) alloy structural steels, (2) austenitic stainless, (3) manganese steels, (4) nickel, and (5) monel metal are also considered.

The book is ably written and illustrated and its value is greatly enhanced by the inclusion of the large number of references used in compiling the data given.

Noise

NOISE. A COMPREHENSIVE SURVEY FROM EVERY POINT OF VIEW. By N. W. McLachlan, D.Sc. (Engineering) London, M.I.E.E. Oxford University Press, New York, 1935. Cloth, 4¾ × 7¼ in., 148 pp., 62 figs., $2.25.

REVIEWED BY A. L. KIMBALL[6]

THIS little book is among the best the writer has seen on this subject, in that it combines to an unusual degree a clear grasp of fundamentals and practical experience. To anyone who wishes to acquire an overall picture of the subject in a reasonable number of pages, including its technical physiological and psychological aspects, this book is recommended.

The first chapter on general considerations presents the difficult subjects of the relation between noise-meter readings, loudness sensation, and masking effects unusually clearly, although the author prefers to use the term "reference tone level," abbreviated "reftone level," rather than "loudness level," because the decibel scale is not such as to truly measure loudness.

The second and third chapters cover measurement of noise and frequency analysis.

The fourth chapter takes up noise in buildings in an interesting way with illustrations. The specific method used is the most direct and practical one which the writer has thus far seen, and he intends to make use of it in his own work.

The next three chapters cover briefly, traffic noise, train noise, and noise of airplanes, ships, propellers, and motor vehicles. An outline of what it has been possible to accomplish thus far in these fields is given with a few illustrations.

Chapters 8 and 9 go into the important subjects of vibration of machinery and noise from electrical apparatus, with a simple illustration of how to use elastic suspensions in preventing noise transmission. A brief résumé of the sources of noise in generators and in motors, and the cause of thermionic-valve noise also appears.

The final chapter goes into the physiological and psychological effects of noise.

It is, of course, impossible to treat such a broad subject in as much detail as would be required by a specialist in one particular field, but the author has succeeded in doing a very good piece of work considering the small size of the book.

Geometry

ANALYTICAL GEOMETRY OF THREE DIMENSIONS. By D. M. Y. Sommerville, M.A., D.Sc., Professor of Pure and Applied Mathematics, Victoria University College, Wellington, N. Z. The Cambridge University Press, Cambridge, Eng., and The Macmillan Company, New York, 1934. Cloth, 6 × 9 in., 416 pages, 49 figs., $4.75.

REVIEWED BY J. ORMONDROYD[7]

THE style in which this text on analytical geometry is written makes it difficult to understand unless studied under the guidance of an instructor. It can be recommended to engineers only as a reference work which is slightly more elementary, compact and modern than Salmon's classical treatise on the same subject.

---

[4] Director of Research, A. O. Smith Corporation, Milwaukee, Wis. Mem. A.S.M.E.

[5] Research Laboratories, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

[6] Engineering General Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[7] Experimental Division, Westinghouse Electric & Manufacturing Company, South Philadelphia, Pa. Mem. A.S.M.E.

Siphon Spillways

SIPHON SPILLWAYS. By A. H. Naylor, M.Sc., Assoc. M.Inst.C.E., M.Inst.W.E. Longmans, Green & Company, New York. Edward Arnold & Co., London, 1935. Cloth, 5¼ × 8½ in., 80 pp., 40 figs., 7 plates, $3.40.

REVIEWED BY M. P. O'BRIEN[8]

ALTHOUGH many papers have already been published on the design of siphon spillways, this small volume will prove useful because it presents in some detail the basic considerations underlying the design of an important installation. Following a review of the principal types of siphons and a discussion of the head necessary to cause priming, application is made to the Laggan Siphons of the North British Aluminum Company. The computations are given in detail and the computed discharge is compared with that shown by model tests. Useful data on the formation of intake vortices and the necessary size of air vents are included.

It is regrettable that the author did not expand his theoretical treatment of siphons, especially that portion dealing with the head necessary to prime and the mapping of streamlines. The latter problem appears to be one which is especially suitable for solution by the electric analogy but the method outlined by the author is probably sufficiently accurate. It is an example of the value of the mathematical approach in the solution of practical problems.

The author proposes as the measure of siphon efficiency the ratio of the actual discharge to that which would occur if a perfect vacuum existed at the highest point of the throat. This concept was originally advanced by J. C. Stevens but was abandoned by him in favor of a coefficient based upon the discharge area. The author's proposal does not give a direct comparison of different siphons because low-head siphons will show low efficiencies no matter how small their friction losses may be. The coefficient based on discharge area gives a direct indication of these losses and is believed to be preferable.

The many practical observations regarding siphon design and behavior are an important feature of this work. In general, the treatment is clear and concise and it is recommended for reference by engineers concerned with the design of this type of hydraulic structure.

Science and Social Needs

SCIENCE AND SOCIAL NEEDS. By Julian Huxley. Harper and Brothers, New York and London, 1935. Cloth, 5½ × 8½ in., 287 pp., illustrated, $2.75.

REVIEWED BY JOHN M. LESSELLS[9]

THE basis for "Science and Social Needs" was furnished by a "tour of British Science" which the author made and refers specifically to conditions of science in Great Britain. Some of the material was broadcast by the British Broadcasting Corporation but considerable amplification and additions have been made to the material to form the contents of this volume.

The first chapter is an introduction by Sir William Bragg, entitled "Science—Friend or Enemy." In this Sir William prepares the way for the author's presentation by taking us on an imaginary journey through the Royal Institution. The experiments of Rumford on heat are mentioned. These are well known but it is illuminating to learn that Rumford was interested in applying knowledge to relieve the common lot. Davy's work on the miner's lamp, the electrical experiments of Faraday, Tyndall's work on bacteriology, Rayleigh's on acoustics, and Dewar's on the liquefaction of gases are all cited as scientific discoveries which have later blossomed into useful industries. A warning note is struck, however, when he reminds his readers that while science has been shown by these and other examples to have been beneficial to the human race there are other cases such as the use of science in war where it is applied for mutual injury.

Four chapters take the form of a dialogue between the author and Professors Levy, Blackett, and Sir T. Barlow, while the remainder discuss the state of science in certain industries as the author found it during his tour.

While the book describes conditions as they exist in Great Britain, and for this reason is of more interest to British readers, it nevertheless possesses a certain attraction for American readers as indicating the relative status of research in the two countries, judged by expenditures per capita. Nevertheless, the reader gains the impression that while such material may be impressive when heard over the radio it is tedious to follow. The basic questions as to the proper control of scientific research and whether science is responsible for our present economic ills are left unanswered.

If the purpose of the book is to make the man in the street au fait with the present-day achievements and future possibilities of applied science then it has a worth-while objective. In this respect, although the book is sketchy and not at all conclusive, it paints a picture in simple language which may be absorbed by the non-technical reader.

Higher Mathematics for Engineers and Physicists

HIGHER MATHEMATICS FOR ENGINEERS AND PHYSICISTS. By Ivan S. Sokolnikoff, Assistant Professor of Mathematics, University of Wisconsin; and Elizabeth S. Sokolnikoff, formerly Instructor in Mathematics, University of Wisconsin. McGraw-Hill Book Company, New York, 1934. Cloth, 6 × 9 in., 482 pp., $4.

REVIEWED BY GEORGE B. KARELITZ[10]

IN the preface to this work the authors state: "Intimate contact with students in engineering and other applied sciences has convinced the authors that most of the students emerging from the usual course in calculus are not sufficiently mature mathematically to undertake independent reading of the more advanced works on mathematics." This deficiency also interferes with the reading of the more advanced works in engineering and the volume may be well recommended as a means for overcoming, in large measure, these shortcomings.

Chapter 1 deals with elliptic integrals. Chapter 2 gives the approximate solution of algebraic equations followed by chapter 3 on determinants and matrices. Chapter 4 deals with infinite series and tests for convergence, expansion in the power series, and gives a demonstration of its application to approximate formulas. Chapter 5 discusses partial derivatives and differentiation with illustrative problems in geometry and physics, together with a discussion of maxima and minima of functions of two or more variables. Chapter 6 discusses Fourier series, chapter 7 multiple integrals with suitable geometrical illustrations and problems. Chapter 8 is on line integrals and Green's theorem, while chapter 9 deals with improper intergrals. Chapter 10 on ordinary differential equations is well developed, both in its elementary form of presentation and in the manner in which the authors

[8] Associate Professor of Mechanical Engineering, College of Engineering, University of California, Berkeley, Calif. Assoc-Mem. A.S.M.E.

[9] Consulting Mechanical Engineer, Swarthmore, Pa. Mem. A.S.M.E.

[10] Associate Professor, Department of Mechanical Engineering, Columbia University, New York. Mem. A.S.M.E.

develop the significance of the methods. Applications by illustrations and problems are given in the various fields of physics. Partial-differential equations in chapter 11 are treated in the same manner. Chapter 12 gives a brief discussion of vector analysis with several pertinent examples of the application to physics. Chapter 13 discusses probability and contains a number of problems illustrating the practical application. Chapter 14 is devoted to curve fitting and empirical formulas, interpolation and, numerical integration. Chapter 15 discusses the elements of conformal representation.

The book presents an assembly of separate additional chapters of algebra and calculus which should be incorporated as the working tools of the engineer and physicist. The contents are rather disjointed and unsystematic but the reader obtains the impression that they are the results of careful study of the actual needs of the student andt hat the material has been chosen on the basis of the mentioned "intimate contact." It is to be regretted that the authors did not find it possible to include in the book paragraphs treating with approximate solutions of differential equations as a counterpart of the chapter on approximate solutions of algebraic equations. The book would probably be of more value to students had the authors included a larger number of physical applications, such as additional examples of setting up differential equations, or practical applications of Green's theorem. The authors may also be reproached for excessive sketchiness in parts, as for instance in the last chapter where a few paragraphs on complex numbers would be very useful to the reader.

Apart from these criticisms the volume covers a vast amount of material which has been chosen on the basis of experience collected at the University of Wisconsin where "engineering mathematics" has been taught for a number of years. The book is now introduced as a text in a corresponding course in mathematics at Columbia University and has been found to be very useful. It will also be adequate for any engineer who would wish to improve his mathematical equipment.

Thermodynamics

THE THERMODYNAMICS OF ELECTRICAL PHENOMENA IN METALS. By P. W. Bridgman. The Macmillan Company, New York, 1934. Cloth, 5¾ × 8½ in., 200 pp., 33 figs., $3.75.

REVIEWED BY JOHN A. GOFF[11]

IN THIS rather compact volume, Professor Bridgman has collected the substance of a number of papers which he has written during the last ten years on the interrelations of a thermodynamic character between various electrical properties of metals. The volume is written primarily for the physicist and discusses phenomena with which the engineer is perhaps unfamiliar. Nevertheless, there appear at various places applications of the laws of thermodynamics which cannot help but clarify their meaning and emphasize their limitations even to the engineer. Notable among these applications is the demonstration on page 30 that a fluid in motion must convect with it in addition to its intrinsic energy a quantity of energy equal to the product of pressure times volume. This demonstration is based on the broad proposition that it is impossible to add energy to any region of space without effecting some real physical and permanent changes within the region.

In chapter 1, Professor Bridgman examines the fundamental notions and experiments involved in the notion of the electrical field. He discusses a number of idealized experiments intended to show the nature of the fundamental ideas and to distinguish quantities having direct physical meaning from mathematical constructions devised to cover regions incapable of direct physical exploration but which satisfy thermodynamics and join on smoothly with directly measurable quantities. Into the list of already accepted constructional quantities he introduces the ideas of "driving emf" and "working emf" and shows that in complicated cases, such as a metal carrying currents derived from a thermal-electromotive force, the two are not at all equal. Of particular interest is the author's steadfast recognition of the reality of the Poynting vector which he invokes to show that no information whatever can be obtained from energy considerations as to the distribution of potential jumps making up the observed Volta difference of potential between two metals; and that energy of position in the electrostatic field cannot properly be assigned to electricity as it moves about.

In chapter 2, Professor Bridgman discusses thermoelectric phenomena and shows that no rigorous application of the Second Law of Thermodynamics comprehending the irreversible effects of Joulean heat and thermal conduction has yet been made which imposes any useful restriction on the magnitudes of the various quantities involved. He attempts to show, however, that the analysis of Kelvin which ignores these irreversible effects is all right by applying the Second Law to a thermocouple so proportioned as to make the irreversible effects a maximum. The author also attacks the conventional account of the action at an interface between two metals when there is a Peltier heat, or the action in the body of a metal with a temperature gradient where Thomson heat is absorbed, and offers an account in which, at the interface, there is no "driving emf" but there is "working emf" equal to the Peltier heat, while in the body of the metal where there is a temperature gradient there is a "driving emf" distributed according to the temperature. On the basis of this account, he deduces an expression for the quantity of energy convected by the current which is essentially different from that deduced from the conventional account.

In chapter 3, the Volta effect is subjected to a searching examination which shows from thermodynamic reasoning the reality of what Tonks and Langmuir have called "surface heat of charging" and suggests that the way toward experimental verification lies in the direction of improved vacuum technique to yield satisfactory reproducible values of the Volta difference of potential for comparison with the Peltier heat. In chapter 4, the subject is thermoionic emission and the laws of thermodynamics are applied to the electron gas surrounding every metal to draw important deductions regarding the surface heat of charging.

There follow in succeeding chapters similar discussions of the effect of surface charge on vapor pressure and electron emission, thermoelectric phenomena in crystals, transverse phenomena including the Hall effect, and connections with the electron theory of metals and photoelectric phenomena. To those equipped to read these later chapters, Professor Bridgman's philosophy must contribute an added interest by bringing into focus with his vigorous and penetrating mind many things which have heretofore evaded such attention.

---

[11] Professor of Thermodynamics, University of Illinois, Urbana, Ill. Assoc-Mem. A.S.M.E.

IN TWO SECTIONS—SECTION TWO

TRANSACTIONS

of The American Society of Mechanical Engineers

SOCIETY RECORDS—PART 1

Ralph E. Flanders, President of The American Society of Mechanical Engineers, 1934–1935 . . . . . RI-2

Council and Committee Personnel RI-5–RI-29

Officers and Council 5
Standing Committees 5
Special Technical Committees 6
Special Council Committees 7
Special Administrative Committee 8
Special Publications Committee 8
Professional Divisions 9
Local Sections 12
Student Branches 18
Special Research Committees 20
Standardization Technical Committees . . . 22
Power Test Codes Technical Committees . . 26
Safety Technical Committees 27
A.S.M.E. Representatives on Other Activities 29

Awards RI-30

Honorary Members RI-32

Past-Presidents RI-32

Index to Committees RI-33

FEBRUARY, 1935

VOL. 57, NO. 2

Published by The American Society of Mechanical Engineers

TRANSACTIONS

of The American Society of Mechanical Engineers

Published on the tenth of every month, except March, June, September, and December

Publication Office, 20th and Northampton Streets, Easton, Pa.
Editorial Department at the Headquarters of the Society, 29 West Thirty-Ninth Street, New York, N. Y.

Includes Aeronautical Engineering

Members of Council, 1934–1935

PRESIDENT

RALPH E. FLANDERS

VICE-PRESIDENTS

Terms expire December, 1935

WILLIAM L. BATT
H. L. DOOLITTLE
ELY C. HUTCHINSON
ELLIOTT H. WHITLOCK

PAST-PRESIDENTS

Terms expire December

CHARLES M. SCHWAB 1935
ROY V. WRIGHT 1936
CONRAD N. LAUER 1937
A. A. POTTER 1938
PAUL DOTY 1939

VICE-PRESIDENTS

Terms expire December, 1936

EUGENE W. O'BRIEN
JAMES H. HERRON
HARRY R. WESTCOTT

MANAGERS

Terms expire December, 1935

R. L. SACKETT
ALEX D. BAILEY
JOHN A. HUNTER

Terms expire December, 1936

JAMES A. HALL
ERNEST L. OHLE
JAMES M. TODD

Terms expire December, 1937

BENNETT M. BRIGMAN
JILES W. HANEY
ALFRED IDDLES

TREASURER

ERIK OBERG

SECRETARY

C. E. DAVIES

Chairmen of Standing Committees of Council

AWARDS, W. L. BATT
CONSTITUTION AND BY-LAWS, H. H. SNELLING
EDUCATION AND TRAINING FOR THE INDUSTRIES, GEORGE A. SEYLER
FINANCE, WALTER RAUTENSTRAUCH
LIBRARY, E. P. WORDEN
LOCAL SECTIONS, W. L. DUDLEY
MEETINGS AND PROGRAM, R. I. REES
MEMBERSHIP, H. A. LARDNER
POWER TEST CODES, F. R. LOW
PROFESSIONAL CONDUCT, C. G. SPENCER
PROFESSIONAL DIVISIONS, W. A. SHOUDY
PUBLICATIONS, S. W. DUDLEY
RELATIONS WITH COLLEGES, W. L. ABBOTT
RESEARCH, G. M. EATON
SAFETY, W. M. GRAFF
STANDARDIZATION, C. W. SPICER

Committee on Publications

S. W. DUDLEY, *Chairman*
S. F. VOORHEES W. F. RYAN, *Vice-Chairman*
G. F. BATEMAN M. H. ROBERTS
EDITOR: GEORGE A. STETSON

Advisory Members

E. L. OHLE, ST. LOUIS, MO.
E. B. NORRIS, BLACKSBURG, VA.
A. J. DICKIE, SAN FRANCISCO, CALIF.
O. B. SCHIER, 2D (JUNIOR MEMBER)

BY-LAW: The Society shall not be responsible for statements or opinions advanced in papers or . . . printed in its publications (B2, Par. 3).

Entered as second-class matter March 2, 1928, at the Post Office at Easton, Pa., under the act of August 24, 1912. Price $1.50 a copy, $12.00 a year; to members and affiliates, $1.00 a copy, $7.50 a year. Changes of address must be received two weeks before they are to be effective on our mailing list. Please send old, as well as new, address.

Copyrighted, 1935, by THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

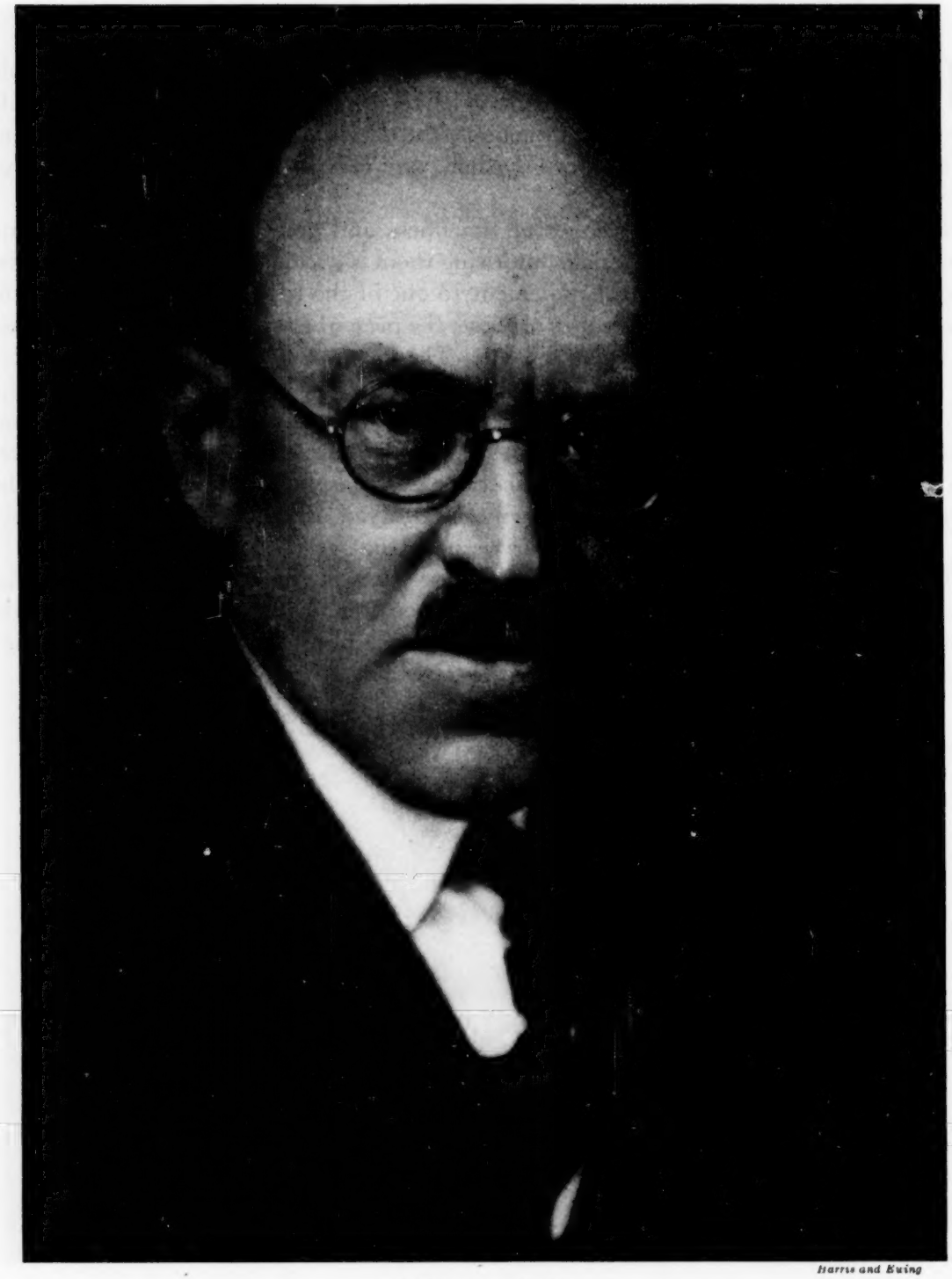
Foreword

THE Transactions of The American Society of Mechanical Engineers include selected technical papers and reports delivered at meetings of the Society, its Professional Divisions, and its Local Sections, the *Journal of Applied Mechanics* (contributions of the Applied Mechanics Division), and certain Records of the Society of permanent value.

In order to secure the advantages of timeliness and greater usefulness in issuing these Society Records, the material comprising them is divided into a number of parts, each one of which is mailed as a supplement to one of the regular monthly sections of the Transactions. For 1935, the first of these, the present issue, contains the personnel of the Council and committees for the year. The second, to be issued sometime later in the year, will contain the memorial notices of deceased members. Following the plan adopted in 1934, it is expected that the reports of Council and the Society's committees will appear as a supplement to the November issue. The presidential address and the indexes to miscellaneous publications, *Mechanical Engineering*, and to the Transactions themselves, must, necessarily, be delayed until 1936, and will probably be mailed as a supplement to the January issue of that year.

In binding the 1935 Transactions, all of these parts of the Society Records will be assembled at the back of the volume as has been customary for several years. To aid in locating references in the bound volumes, the page numbers of the sections containing the Society Records are preceded by the letter RI.

THE COMMITTEE ON PUBLICATIONS



Harris and Ewing

RALPH E. FLANDERS

PRESIDENT OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
1934–1935

Ralph E. Flanders

RALPH E. FLANDERS, president of The American Society of Mechanical Engineers for the term 1934–1935, is president of the Jones & Lamson Machine Company and of the Bryant Chucking Grinder Company, both of Springfield, Vt. He was born at Barnet, Vt., on September 28, 1880, and was educated at the Central Falls (R. I.) High School. His early experience in machine design and construction was received at Brown & Sharpe Manufacturing Company, Taft-Pierce Manufacturing Company, International Paper Box Machine Company, and General Electric Company. After four years as associate editor of *Machinery* and two years as sales engineer with the Fellows Gear Shaper Company, he joined the Jones & Lamson Machine Company in 1912 as manager of the Fay Lathe Department. In 1914 he was made general manager and in 1933 president of the company.

Mr. Flanders is regarded as a national authority on machine design and construction, especially on the engineering problems of screw threads and thread grinding. His 1924 A.S.M.E. paper on "The Design, Manufacture, and Production Control of a Standard Machine" brought a successful solution to the problem of building machinery economically at widely varying rates of demand.

Mr. Flanders has devoted a large amount of time and effort to the welfare of his profession, serving on many committees of The American Society of Mechanical Engineers. Since 1921 he has been a member and since 1930 chairman of the Sectional Committee on Standardization and Unification of Screw Threads. He represented the A.S.M.E. on the National Screw Thread Commission from 1919 to 1924. He has been a member since 1926 and chairman since 1930 of the Special Research Committee on Strength of Gear Teeth. He served on the Publications Committee from 1918 to 1924 and acted as chairman in 1925 through 1928. He was a manager of the Society from 1926 to 1929 and vice-president in 1930 and 1931.

He was a delegate of the A.S.M.E. to the American Engineering Council in 1924–1925. Since 1932 he has been active in the Public Works program of the Council, and was a member of its Committee on Government Reorganization. As chairman of its Committee on the Relation of Consumption, Production, and Distribution he has been in a position of strong leadership.

Serving as president of the National Machine Tool Builders Association in 1924, Mr. Flanders presented a presidential address that dealt with current economic problems. In this address he pointed out the importance of the transition from the economy of need to the economy of plenty, a contribution of great importance to economic thinking of the times. His address before The American Society of Mechanical Engineers in December, 1930, on "Engineering, Economics, and Social Well-Being" marks his entry into a course of economic study, discussion, and writing that has brought him to the forefront of leadership in his field and led to his selection as a member of the American Engineering Council's Committee on the Relation of Consumption, Production, and Distribution, already mentioned. He is also a director of the Social Science Research Council.

Upon the passage of the National Industrial Recovery Act in 1933 and the organization of the National Recovery Administration, Mr. Flanders was called into service as a member of the Industrial Advisory Board. He is also a member of the Business Advisory and Planning Council appointed by Secretary of Commerce, Daniel C.

Roper. At the conference of the industrial leaders engaged in the administration of various codes for industries held in Washington the first week in March, 1934, his grasp of the basic industrial and economic problems at the present time enabled him to step into a position of leadership in the discussion, which had a profound effect on the deliberations of that important occasion.

Mr. Flanders has written on a wide range of subjects. In his early career his papers and books on gears and gear machinery were highly regarded. Later his discussions of industrial and economic problems have appeared in a wide range of magazines and and his book, "Taming Our Machines," published in 1931, was very well received.

Mr. Flanders was granted the degree of Mechanical Engineer by Stevens Institute of Technology in June, 1932, and the degree of Master of Arts by Dartmouth College the same year. In June, 1934, he was honored with the degree of Doctor of Science, by Middlebury College, and Doctor of Engineering by Brooklyn Polytechnic Institute. He is lecturer at the Tuck School of Business Administration at Dartmouth College and has spoken before many groups of economists, engineers, and industrialists on subjects dealing with economics, engineering, and related problems.

The American Society of Mechanical Engineers

THE members of the Council and of its standing and special committees given on the following pages are those in office on February 1, 1935, serving for the official year 1934–1935. The terms of office of members of other committees are not fixed by the official calendar.

Officers and Council

PRESIDENT

RALPH E. FLANDERS

PAST-PRESIDENTS

Terms expire December

CHARLES M. SCHWAB (1935) — CONRAD N. LAUER (1937)
ROY V. WRIGHT (1936) — A. A. POTTER (1938)
PAUL DOTY (1939)

VICE-PRESIDENTS

Terms expire December, 1935	*Terms expire December, 1936*
WILLIAM L. BATT	EUGENE W. O'BRIEN
HAROLD L. DOOLITTLE	JAMES H. HERRON
ELY C. HUTCHINSON	HARRY R. WESTCOTT
ELLIOTT H. WHITLOCK	

MANAGERS

Terms expire December, 1935	*Terms expire December, 1936*
ROBERT L. SACKETT	JAMES A. HALL
ALEX D. BAILEY	ERNEST L. OHLE
JOHN A. HUNTER	JAMES M. TODD

Terms expire December, 1937

BENNETT M. BRIGMAN — JILES W. HANEY — ALFRED IDDLES

TREASURER

ERIK OBERG

SECRETARY

C. E. DAVIES

Executive Committee of Council

RALPH E. FLANDERS, *Chairman* — ALFRED IDDLES
WILLIAM L. BATT — HARRY R. WESTCOTT
JAMES A. HALL

Advisory Members: Chairmen of the Finance Committee, the Local Sections Committee, and the Professional Divisions Committee

Chairmen of Standing Committees

Representatives on Council but without vote

Finance, WALTER RAUTENSTRAUCH
Meetings and Program, R. I. REES
Publications, S. W. DUDLEY
Membership, H. A. LARDNER
Professional Divisions, W. A. SHOUDY
Local Sections, W. L. DUDLEY
Constitution and By-Laws, H. H. SNELLING
Awards, W. L. BATT
Relations With Colleges, W. L. ABBOTT
Education and Training for Industries, G. A. SEYLER
Library, E. P. WORDEN
Research, G. M. EATON
Standardization, C. W. SPICER
Power Test Codes, F. R. LOW
Safety, W. M. GRAFF
Professional Conduct, C. G. SPENCER

ASSISTANT SECRETARIES

ERNEST HARTFORD — C. B. LEPAGE

EDITOR

G. A. STETSON

Standing Committees

FINANCE

WALTER RAUTENSTRAUCH, *Chairman and Representative on Council* (1936)

W. T. CONLON (1935) — T. R. WEYMOUTH (1938)
W. D. ENNIS (1937) — K. M. IRWIN (1939)
Council Representatives { E. C. HUTCHINSON (1935); J. H. HERRON (1936)

MEETINGS AND PROGRAM

R. I. REES, *Chairman and Representative on Council* (1935)
E. C. HUTCHINSON (1936) — CLARKE FREEMAN (1938)
H. N. DAVIS (1937) — R. F. GAGG (1939)
Junior Adviser: To be appointed

PUBLICATIONS

S. W. DUDLEY, *Chairman and Representative on Council* (1936)
W. F. RYAN, *Vice-Chairman* (1937) — M. H. ROBERTS (1938)
S. F. VOORHEES (1935) — G. F. BATEMAN (1939)
O. B. SCHIER, 2ND, *Junior Adviser* (1935)

Advisory Members: A. J. DICKIE, E. B. NORRIS, E. L. OHLE
(Personnel of Special Committee, p. 8)

MEMBERSHIP

H. A. LARDNER, *Chairman and Representative on Council* (1935)
R. H. MCLAIN (1936) — L. R. FORD (1938)
C. L. DAVIDSON (1937) — F. C. SPENCER (1939)

Advisory Member: O. E. GOLDSCHMIDT (1935)

PROFESSIONAL DIVISIONS

W. A. SHOUDY, *Chairman and Representative on Council* (1935)
K. H. CONDIT (1936) — CROSBY FIELD (1938)
G. B. PEGRAM (1937) — L. K. SILLCOX (1939)

(Personnel of Professional Divisions' Executive Committees, p. 9)

LOCAL SECTIONS

W. L. DUDLEY, *Chairman and Representative on Council* (1935)
To be appointed (1936) — W. R. WOOLRICH (1938)
R. E. W. HARRISON (1937) — D. B. PRENTICE (1939)

Junior Advisers

JENSEN CLAUSEN — S. E. OREN
F. X. KROGMANN — J. M. TUCKER

(Personnel of Local Sections' Executive Committees, p. 12)

CONSTITUTION AND BY-LAWS

H. H. SNELLING, *Chairman and Representative on Council* (1935)
RICHARD KUTZLEB, JR. (1936) — W. H. KAVANAUGH (1938)
H. B. LEWIS (1937) — R. D. BRIZZOLARA (1939)

Junior Adviser: G. N. COLE (1935)

AWARDS

The members of this Committee serve as members also of the Special Council Committee on Honors

W. L. BATT, *Chairman and Representative on Council* (1935)
HERMAN DIEDERICHS (1936) — L. P. ALFORD (1938)
R. C. H. HECK (1937) — HARTE COOKE (1939)
Junior Adviser: C. B. ARNOLD (1935)

RELATIONS WITH COLLEGES

W. L. ABBOTT, *Chairman and Representative on Council* (1935)
E. W. BURBANK (1936) — W. A. HANLEY (1938)
R. V. WRIGHT (1937) — F. V. LARKIN (1939)
Junior Advisers { R. F. WARNER, JR. (1935); G. D. WILKINSON, JR. (1935)

(Student Branches and Officers, p. 18)

EDUCATION AND TRAINING FOR THE INDUSTRIES

G. A. SEYLER, *Chairman and Representative on Council* (1935)
C. J. FREUND (1936) — J. A. RANDALL (1938)
JOHN YOUNGER (1937) — P. E. BLISS (1939)

LIBRARY

E. P. WORDEN, *Chairman and Representative on Council* (1935)
G. F. FELKER (1936) — J. S. KERINS (1938)
L. K. SILLCOX (1937) — The Secretary, C. E. DAVIES, *Ex-Officio*

RESEARCH

Organized in 1909 to supervise all research activities of the Society, to cooperate with similar committees of kindred societies, to encourage research, and to disseminate knowledge of researches conducted in the United States and in other countries

G. M. EATON, *Chairman and Representative on Council* (1935)
D. B. BULLARD (1936) — H. A. JOHNSON (1938)
C. R. RICHARDS (1937) — N. E. FUNK (1939)

(*Personnel of Special Committees, p. 20*)

STANDARDIZATION

Organized in April, 1911, to supervise all standardization activities of the Society and to advise concerning the Society's participation in the activities of the American Standards Association

C. W. SPICER, *Chairman and Representative on Council* (1935)
ALFRED IDDLES (1936) — WALTER SAMANS (1938)
L. A. CORNELIUS (1937) — O. A. LEUTWILER (1939)

(*Personnel of Special Committees, p. 22*)

POWER TEST CODES

Organized December, 1918, to revise and extend the Power Test Codes of the Society. These codes had been formulated by various technical committees appointed to develop particular codes. This work began in 1886

F. R. LOW, *Chairman and Representative on Council* (1935)
R. H. FERNALD, *Acting Chairman*

Term expires 1935
A. T. BROWN
R. H. FERNALD
C. F. HIRSHFELD
F. R. LOW
R. J. S. PIGOTT

Term expires 1936
A. G. CHRISTIE
PAUL DISERENS
E. C. HUTCHINSON
G. A. ORROK
W. M. WHITE

Term expires 1937
HARTE COOKE
E. R. FISH
O. P. HOOD
H. B. OATLEY
W. J. WOHLENBERG

Term expires 1938
HANS DAHLSTRAND
LOUIS ELLIOTT
G. A. HORNE
H. B. REYNOLDS
E. N. TRUMP

Term expires 1939
C. H. BERRY
FRANCIS HODGKINSON
D. S. JACOBUS
L. F. MOODY
E. B. RICKETTS

(*Personnel of Technical Committees, p. 26*)

SAFETY

Appointed in October, 1921, to extend the knowledge of accident prevention, to promote cooperation in this field, and to supervise all safety code activities of the Society with the exception of those of the Boiler Code group of committees

W. M. GRAFF, *Chairman and Representative on Council* (1936)
M. H. CHRISTOPHERSON (1935) — H. L. MINER (1938)
H. H. JUDSON (1937) — To be appointed (1939)

(*Personnel of Special Committees, p. 27*)

PROFESSIONAL CONDUCT

C. G. SPENCER, *Chairman and Representative on Council* (1935)
E. R. FISH (1936) — E. F. SCOTT (1938)
J. H. HERRON (1937) — HUGO DIEMER (1939)

Special Technical Committees

BOILER CODE

F. R. LOW, *Chairman*
D. S. JACOBUS, *Vice-Chairman*
C. W. OBERT, *Honorary Secretary*
M. JURIST, *Acting Secretary*
C. A. ADAMS
H. E. ALDRICH
W. H. BOEHM
R. E. CECIL
F. S. CLARK
W. F. DURAND
A. J. ELY
E. R. FISH
V. M. FROST
C. E. GORTON
A. M. GREENE, JR.
F. B. HOWELL
J. O. LEECH
M. F. MOORE
I. E. MOULTROP
C. O. MYERS
H. B. OATLEY
JAMES PARTINGTON
WALTER SAMANS
C. L. WARWICK
A. C. WEIGEL
H. LEROY WHITNEY

Honorary Members
F. W. DEAN
T. E. DURBAN
C. L. HUSTON
W. F. KIESEL
H. H. VAUGHAN

Conference Committee to the Boiler Code Committee

T. R. ARCHER, Delaware
L. M. BARRINGER, Seattle, Wash.
H. D. BOMBECK, St. Joseph, Mo.
B. M. BOOK, Pennsylvania
J. C. BRYAN, St. Louis, Mo.
JOHN CAMPBELL, Pennsylvania
T. C. CANNON, Tulsa, Okla.
A. L. DANIELS, Parkersburg, W. Va.
JAMES DONOHUE, Indiana
L. F. DUGE, Maryland
M. A. EDGAR, Wisconsin
E. W. FARMER, Rhode Island
M. J. FLYZIK, Washington
C. W. FOSTER, Omaha, Nebraska
W. H. FURMAN, New York
GERALD GEARON, Chicago, Ill.
E. M. GOODMAN, Evanston, Ill.
C. H. GRAM, Oregon
P. M. GREENLAW, District of Columbia
F. A. HECKINGER, Memphis, Tenn.
J. M. LUKENS, Philadelphia, Pa.
J. M. LYNCH, Erie, Pa.
C. E. MCGINNIS, Los Angeles, Calif.
H. H. MILLS, Detroit, Mich.
A. C. MITCHELL, Scranton, Pa.
C. O. MYERS, Ohio
J. D. NEWCOMB, Arkansas
W. L. NEWTON, Oklahoma
DANIEL O'CONNOR, Michigan
F. A. PAGE, California
L. C. PEAL, Nashville, Tenn.
J. J. RYAN, Kansas City, Mo.
E. K. SAWYER, Maine
A. H. SCHLEMAN, Tampa, Florida
J. F. SCOTT, New Jersey
C. I. SMITH, Utah
WM. E. SMITH, Hawaii
GEORGE WILCOX, Minnesota

EXECUTIVE COMMITTEE

D. S. JACOBUS, *Chairman*
H. E. ALDRICH
W. H. BOEHM
E. R. FISH
V. M. FROST
C. E. GORTON
F. R. LOW
C. W. OBERT
JAMES PARTINGTON

SUBCOMMITTEE ON BOILERS OF LOCOMOTIVES

JAMES PARTINGTON, *Chairman*
F. H. CLARK
H. B. OATLEY
A. G. PACK, *Conference Member*

SUBCOMMITTEE ON CARE OF STEAM BOILERS AND OTHER PRESSURE VESSELS IN SERVICE

F. M. GIBSON, *Chairman*
V. M. FROST
J. R. GILL
J. A. HUNTER
H. J. KERR
W. H. LARKIN, JR.
S. T. POWELL
C. W. RICE
H. F. SCOTT
NICHOLAS STAHL
F. G. STRAUB

SUBCOMMITTEE ON HEATING BOILERS

F. B. HOWELL, *Chairman*
W. H. BOEHM
C. E. BRONSON
J. A. DARTS
C. E. GORTON
F. W. HERENDEEN
W. E. STARK
J. W. TURNER

SUBCOMMITTEE ON MATERIAL SPECIFICATIONS

Members of A.S.M.E. Boiler Code Committee
P. R. CASSIDY, *Chairman*
A. M. GREENE, JR.
J. O. LEECH
C. L. WARWICK

Members of Conference Committee of American Society for Testing Materials

C. L. WARWICK, *Chairman* E. J. EDWARDS
C. F. W. RYS

Members of Conference Committee of Association of American Steel Manufacturers, Technical Committees

J. O. LEECH, *Chairman* E. F. KENNEY
A. D. PEACE

SUBCOMMITTEE ON MINIATURE BOILERS

C. E. GORTON, *Chairman* W. H. FURMAN
JAMES PARTINGTON, *Vice-Chairman* C. O. MYERS
E. R. FISH C. W. OBERT
J. H. PLUNKETT

SUBCOMMITTEE ON RULES FOR INSPECTION

J. A. COLLINS, *Chairman* WILLIAM FERGUSON
S. H. BARNUM C. E. GORTON
L. E. CONNELLY F. W. HERENDEEN
JAMES PARTINGTON

SUBCOMMITTEE ON SPECIAL DESIGN

D. B. WESSTROM, *Chairman* H. E. ROCKEFELLER
W. L. BOWLER D. B. ROSSHEIM
R. E. CECIL W. H. ROWAND
PAUL DISERENS E. O. WATERS
F. S. G. WILLIAMS

SUBCOMMITTEE ON UNFIRED PRESSURE VESSELS

E. R. FISH, *Chairman* R. E. CECIL
C. A. ADAMS PAUL DISERENS
W. H. BOEHM A. W. LIMONT, JR.
C. E. BRONSON H. S. SMITH
D. B. WESSTROM

SUBCOMMITTEE ON WELDING

Members of A.S.M.E. Boiler Code Committee

JAMES PARTINGTON, *Chairman* E. R. FISH
C. A. ADAMS R. K. HOPKINS
A. M. CANDY T. MCLEAN JASPER
R. E. CECIL L. A. SHELDON
J. H. DEPPELER HENRY TORRANCE, JR.

Members of Conference Committee of American Welding Society

C. W. OBERT, *Chairman* F. C. FYKE
L. H. BURKHART J. W. OWENS
J. J. CROWE A. KIDD
E. H. EWERTZ H. E. ROCKEFELLER
L. H. ROLLER

API-ASME COMMITTEE ON UNFIRED PRESSURE VESSELS

A.S.M.E. Representatives

R. E. CECIL D. S. JACOBUS
E. R. FISH T. MCLEAN JASPER
JAMES PARTINGTON

A.P.I. Representatives

WALTER SAMANS, *Chairman* K. V. KING
A. J. ELY (P. D. MCELFISH, *Alternate*)
R. C. POWELL T. D. TIFFT

SHAFT COUPLINGS

Appointed in December, 1928

A.S.M.E. Members (Total personnel, 6)

D. J. MCCORMACK H. G. REIST
R. E. B. SHARP

DEVELOPMENT OF APPLICATIONS OF STATISTICS IN ENGINEERING AND MANUFACTURING

Joint sponsorship with the American Society for Testing Materials. Appointed in December, 1929

A.S.M.E. Members (Total personnel, 4)

W. H. FULWEILER †L. K. SILLCOX

STANDARD HEIGHT FOR LOADING PLATFORMS AT FREIGHT TERMINALS AND WAREHOUSES

Joint sponsorship with the Society of Terminal Engineers. Appointed in December, 1931

A.S.M.E. Members (Total personnel, 13)

†C. B. CROCKETT

Special Council Committees

ADVERTISING REVENUE

S. W. DUDLEY, Chairman of Committee on Publications
WALTER RAUTENSTRAUCH, Chairman of Finance Committee
H. R. WESTCOTT, Chairman of Committee on Policies and Budget

ADVISORY BOARD ON TECHNOLOGY

A. A. POTTER, *Chairman*

D. B. BULLARD, Research
K. H. CONDIT, Professional Divisions
S. W. DUDLEY, Publications
R. I. REES, Meetings and Program

ADVISORY BOARD ON STANDARDS AND CODES

ALEX D. BAILEY, *Chairman*

V. M. FROST, Boiler Code
W. M. GRAFF, Safety
FRANCIS HODGKINSON, Power Test Codes
L. A. CORNELIUS, Standardization

BOARD OF REVIEW (DELINQUENT MEMBERS)

A. D. BLAKE J. P. NEFF
H. B. OATLEY

BOND ISSUE (CERTIFICATES OF INDEBTEDNESS)

DEXTER S. KIMBALL, *Chairman* J. H. LAWRENCE
PAUL DOTY W. R. WEBSTER
W. A. HANLEY W. H. WINTERROWD
C. F. HIRSHFELD ERIK OBERG, *Ex-Officio*

Trustees for Issue of Certificates of Indebtedness:

D. S. KIMBALL ERIK OBERG R. V. WRIGHT

CAPITAL GOODS INDUSTRIES

L. P. ALFORD, *Chairman* L. W. W. MORROW
R. E. FLANDERS ERIK OBERG

CITIZENSHIP (MANUAL ON)

A. R. CULLIMORE, *Chairman* J. W. ROE
LILLIAN M. GILBRETH W. H. WINTERROWD
R. V. WRIGHT

COOPERATIVE RELATIONS

H. V. COES, *Chairman* R. I. REES
D. ROBERT YARNALL

ECONOMIC STATUS OF THE ENGINEER

C. F. HIRSHFELD, *Chairman* H. L. WHITTEMORE
D. S. KIMBALL W. E. WICKENDEN
C. N. LAUER W. L. ABBOTT } *Ex-Officio*
H. B. OATLEY W. L. DUDLEY }

EMPLOYMENT (MEMBER RELATIONS)

CROSBY FIELD J. N. LANDIS
W. A. SHOUDY

ENGINEERING HISTORY

(To be appointed)

FREEMAN SCHOLARSHIP

CHARLES T. MAIN, *Chairman* CLARKE FREEMAN
E. C. HUTCHINSON

† Official A.S.M.E. representatives serving on this committee.

HONORS

The members of the Standing Committee on Awards (see page 5) also serve as a Special Committee on Honors

JUNIOR PARTICIPATION

W. A. HANLEY
A. A. POTTER
D. B. PRENTICE
W. H. WINTERROWD
(Junior Member to be appointed)

MANUAL OF PRACTICE

W. A. SHOUDY, *Chairman*
ALFRED IDDLES
WYNN MEREDITH
J. M. TODD
B. F. WOOD
C. G. SPENCER, *Ex-Officio*
THEODORE BAUMEISTER, JR., *Acting Secretary*

Junior Adviser, PHILIP WERNER

POLICIES AND BUDGET

H. R. WESTCOTT, *Chairman*
L. P. ALFORD
B. M. BRIGMAN
H. M. BURKE
W. H. CARRIER
ALFRED IDDLES
A. C. JEWETT
J. N. LANDIS
J. H. LAWRENCE
R. G. MACY
A. L. MAILLARD
M. C. MAXWELL
ERIK OBERG
L. K. SILLCOX
W. H. WINTERROWD

PUBLIC AFFAIRS

R. V. WRIGHT, *Chairman*
L. P. ALFORD
PAUL DOTY
R. E. FLANDERS
A. A. POTTER
J. W. ROE
D. ROBERT YARNALL

REGISTRATION OF ENGINEERS

J. H. LAWRENCE, *Chairman*
PAUL DOTY
C. F. HIRSHFELD
D. S. KIMBALL
V. M. PALMER
J. M. TODD

CALVIN W. RICE MEMORIAL

H. N. DAVIS, *Chairman*
J. D. CUNNINGHAM
W. F. DURAND
C. E. FULLER
C. N. LAUER
E. W. O'BRIEN
J. W. PARKER
R. L. SACKETT

SPIRIT OF ST. LOUIS MEDAL BOARD OF AWARD

(*Dates in parentheses denote expiration of terms*)

V. J. AZBE, *Chairman* (1943)
J. H. DOOLITTLE, *Secretary* (1940)
H. I. CONE (1937)
W. B. MAYO (1937)
ORVILLE WRIGHT (1940)
C. B. MILLIKAN (1943)

GEORGE WESTINGHOUSE BUST

AMBROSE SWASEY, *Honorary Chairman*
D. S. KIMBALL, *Chairman*
L. B. STILLWELL, *Vice-Chairman*
W. W. ATTERBURY
K. T. COMPTON
S. W. DUDLEY
C. H. LAUER
L. A. OSBORNE
C. F. SCOTT

GEORGE WESTINGHOUSE MEMORIAL

(*Ninetieth Birthday in 1936*)

R. V. WRIGHT, *Chairman*
S. W. DUDLEY
J. H. MCGRAW
C. F. SCOTT
R. I. REES, *Ex-Officio*

Special Publications Committee

BIOGRAPHY ADVISORY COMMITTEE

R. V. WRIGHT, *Chairman*
L. P. ALFORD
R. E. FLANDERS
F. R. LOW
G. A. ORROK
J. W. ROE
W. H. WINTERROWD

Special Administrative Committee

REGULAR NOMINATING COMMITTEE (FOR 1935)

GROUP

I — W. K. SIMPSON, Waterbury, Conn.
PROF. C. H. BERRY, Cambridge, Mass., *Alternate*

II — A. D. BLAKE, New York, N. Y.
E. A. KERBEY, New York, N. Y., *Alternate*

III — PROF. A. G. CHRISTIE, Baltimore, Md.
A. R. STEVENSON, JR., Schenectady, N. Y., *1st Alternate*
PROF. F. V. LARKIN, Bethlehem, Pa., *2nd Alternate*

IV — DEAN B. R. VAN LEER, Gainesville, Fla.
R. P. KOLB, Raleigh, N. C., *Alternate*

V — A. N. GODDARD, Detroit, Mich.
PROF. F. W. MARQUIS, Columbus, Ohio, *Alternate*

VI — F. H. DORNER, Milwaukee, Wis.
A. A. LUEBS, Lincoln, Neb., *Alternate*

VII — PROF. R. L. DAUGHERTY, Pasadena, Calif.
H. P. VAIL, Los Angeles, Calif., *1st Alternate*
R. R. ROBERTSON, Los Angeles, Calif., *2nd Alternate*

LOCAL SECTIONS IN NOMINATING COMMITTEE GROUPS

GROUP I

BOSTON
BRIDGEPORT
GREEN MOUNTAIN
HARTFORD
MERIDEN
NEW BRITAIN
NEW HAVEN
NORWICH
PROVIDENCE
WATERBURY
WESTERN MASSACHUSETTS
WORCESTER

GROUP II

METROPOLITAN (N. Y.) AND FOREIGN MEMBERS

GROUP III

ANTHRACITE-LEHIGH VALLEY
BALTIMORE
CENTRAL PENNSYLVANIA
ONTARIO
PHILADELPHIA
PLAINFIELD
ROCHESTER
SCHENECTADY
SUSQUEHANNA
SYRACUSE
UTICA
WASHINGTON, D. C.

GROUP IV

ATLANTA
BIRMINGHAM
CHARLOTTE
CHATTANOOGA
FLORIDA
GREENVILLE
HOUSTON
KNOXVILLE
MEMPHIS
NEW ORLEANS
NORTH TEXAS
RALEIGH
SAVANNAH
VIRGINIA

GROUP V

AKRON-CANTON
BUFFALO
CINCINNATI
CLEVELAND
COLUMBUS
DAYTON
DETROIT
ERIE
INDIANAPOLIS
LOUISVILLE
PENINSULA
PITTSBURGH
TOLEDO
WEST VIRGINIA
YOUNGSTOWN

GROUP VI

CHICAGO
KANSAS CITY
MID-CONTINENT
MILWAUKEE
MINNESOTA
NEBRASKA
ROCK RIVER VALLEY
ST. JOSEPH VALLEY
ST. LOUIS
TRI-CITIES

GROUP VII

COLORADO
INLAND EMPIRE
LOS ANGELES
OREGON
SAN FRANCISCO
UTAH
WESTERN WASHINGTON

Professional Divisions

(*Personnel of Standing Committee, page 5*)

Aeronautic Division

Organized, 1920

EXECUTIVE COMMITTEE

J. H. Doolittle, *Chairman*
Alexander Klemin, *1st Vice-Chairman*
Jerome Lederer, *Secretary*
R. M. Mock
E. A. Sperry
B. M. Woods

COMMITTEE ON INDUSTRIAL AERODYNAMICS

Alexander Klemin, *Secretary*
Oliver Allen
W. H. Carrier
H. E. Davison
H. P. Frear
W. G. Grove
O. E. Hovey
A. L. Kimball
R. J. S. Pigott
L. K. Sillcox

REPRESENTATIVES ON OTHER ACTIVITIES

Aircraft Safety and Inspection, Jerome Lederer
Marking of Obstructions to Air Navigation, J. E. Whitbeck
Spirit of St. Louis Medal Board of Award, V. J. Azbe
Daniel Guggenheim Medal Fund, E. E. Aldrin

Applied Mechanics Division

Organized, 1927

EXECUTIVE COMMITTEE

J. A. Goff, *Chairman*
E. O. Waters, *Secretary*
Rupen Eksergian
J. C. Hunsaker
C. R. Soderberg

Associates

A. L. Kimball
J. M. Lessells
G. B. Pegram

JOURNAL OF APPLIED MECHANICS

J. M. Lessells, *Technical Editor*

SUBCOMMITTEE CHAIRMEN

Elasticity, Stephen Timoshenko
Hydromechanics, Theodor von Karman
Materials, R. E. Peterson
Plasticity, A. Nadai
Thermodynamics, J. A. Goff
Vibration, F. M. Lewis

Fuels Division

Organized, 1920

EXECUTIVE COMMITTEE

K. M. Irwin, *Chairman*
W. G. Christy, *Secretary*
L. C. Bosler
M. D. Engle
T. A. Marsh
R. A. Sherman

Associates

A. D. Blake
A. C. Stern
John Van Brunt
F. M. Van Deventer

SUBCOMMITTEE ON PROGRAMS, MEETINGS, AND AIMS

M. D. Engle, *Chairman*
Ollison Craig
R. M. Hardgrove
J. N. Landis
J. F. Muir
H. G. Thielscher

SUBCOMMITTEE ON FUELS ENGINEERING PROGRESS REPORT

K. M. Irwin, *Chairman*

SUBCOMMITTEE ON CINDERS AND FLY ASH

A. C. Stern, *Chairman*

COMMITTEE ON SAMPLING PULVERIZED FUEL IN A MOVING GAS STREAM

(Research Committee; see page 21)

K. M. Irwin, *Chairman and Representative of Fuels Division*

COMMITTEE ON REMOVAL OF ASH AS MOLTEN SLAG FROM POWDERED-COAL FURNACES

(Research Committee; see page 21)

K. M. Irwin, *Chairman and Representative of Fuels Division*

PURE AIR COMMITTEE

Organized, 1932

E. C. Hutchinson, *Chairman*
E. H. Whitlock, *Secretary*
J. W. Armour
M. D. Engle
O. P. Hood
W. F. Keenan

Representatives

Philip Drinker, American Society of Heating and Ventilating Engineers
H. C. Murphy, *Alternate*
K. M. Irwin, A.S.M.E. Fuels Division
H. B. Meller, Mellon Institute of Industrial Research
E. B. Ricketts, Edison Electric Institute

Hydraulic Division

Organized, 1926

EXECUTIVE COMMITTEE

C. F. Merriam, *Chairman*
S. Logan Kerr, *Secretary*
B. R. Van Leer
Paul Diserens
D. J. McCormack

MEETINGS AND PAPERS COMMITTEE

S. Logan Kerr, *Chairman*
R. W. Angus

COMMITTEE ON WATER HAMMER

Honorary Member, Lorenzo Allievi, Rome, Italy

S. Logan Kerr, *Chairman*
Eugene Halmos (A.S.C.E.)
N. R. Gibson
L. F. Moody
R. S. Quick
E. B. Strowger

COMMITTEE ON TRANSLATIONS AND TERMINOLOGY

B. R. Van Leer, *Chairman*

Iron and Steel Division

Organized, 1927

EXECUTIVE COMMITTEE

T. H. Wickenden, *Chairman*
J. H. Hitchcock, *Secretary*
W. A. James
S. M. Marshall
Willis McKee
W. R. Webster

Associates

F. C. Biggert, Jr.
A. J. Boynton
A. G. McKee
C. S. Robinson
G. T. Snyder
W. Trinks

S. M. Weckstein

Machine Shop Practice Division

Organized, 1921

EXECUTIVE COMMITTEE

R. E. W. Harrison, *Chairman*
G. F. Nordenholt, *Secretary*
B. P. Graves
E. R. Norris
B. G. Tang
Carlos de Zafra

SUBCOMMITTEE ON FOUNDRY PRACTICE

James Thomson, *Chairman*
R. E. Kennedy, *Secretary*

Machine Shop Practice Division (continued)

SUBCOMMITTEE ON MACHINE DESIGN

J. A. Hall, *Chairman*
J. B. Armitage
G. H. Ashman
G. F. Cosgrove
Frederick Franz
G. B. Karelitz
G. F. Nordenholt
Albert Palmer

SUBCOMMITTEE ON LUBRICATION ENGINEERING

W. F. Parish, *Chairman*
C. H. Bromley
G. B. Karelitz
C. M. Larson
H. J. Masson

SUBCOMMITTEE ON CUTTING METALS

Coleman Sellers, 3rd, *Chairman*

SUBCOMMITTEE ON WELDING

Everett Chapman, *Chairman*

Management Division

Organized, 1920

EXECUTIVE COMMITTEE

J. A. Piacitelli, *Chairman*
G. W. Kelsey, *Vice-Chairman*
W. H. Kushnick, *Secretary*
C. I. Day
Walter Rautenstrauch
J. R. Shea

Associates

C. W. Lytle
F. E. Raymond

Student Associates

F. Weaver, Stevens Institute of Technology
R. Brugger, Polytechnic Institute of Brooklyn
M. J. Massimo, New York University
V. S. Myers, Columbia University
R. H. Curtiss, Columbia University

SUBCOMMITTEE CHAIRMEN

Economics, Kerr Atkinson
Finance and Accounting, K. W. Jappe
Human Relations, R. Likert
Job Shop Management, A. J. Graf
Management Research, W. E. Freeland
Marketing, G. W. Kelsey
Metropolitan Section, A. F. Ernst
Quality Control, M. F. Skinker
Time and Motion Study, D. B. Porter
Waste Elimination, C. B. Auel

NATIONAL MANAGEMENT COUNCIL

(*See page 30*)

Materials Handling Division

Organized, 1920

EXECUTIVE COMMITTEE

J. B. Webb, *Chairman*
M. W Potts, *Secretary*
N. W. Elmer
G. E. Hagemann
F. E. Moore
G. L. Morehead

Associates

F. D. Campbell
J. A. Jackson
R. H. McLain
R. B. Renner
F. J. Shepard, Jr.
E. D. Smith

National Defense Division

Organized, 1920

EXECUTIVE COMMITTEE

H. I. Cone, *Chairman*
W. C. Dickerman
Ralph Earle
T. A. Morgan
J. L. Walsh

Oil and Gas Power Division

Organized, 1921

EXECUTIVE COMMITTEE

Ralph Miller, *Chairman*
M. J. Reed, *Secretary*
L. B. Jackson
E. J. Kates
Ernest Nibbs
Lee Schneitter

Associates

Harte Cooke
L. R. Ford
L. M. Goldsmith
H. C. Major
H. T. Moore
L. H. Morrison

SUBCOMMITTEE ON 1934 OIL ENGINE POWER COST REPORT

H. C. Major, *Chairman*
M. J. Reed, *Secretary*
L. R. Ford
W. G. G. Godron
K. M. Irwin
E. J. Kates
H. C. Lenfest
Howard McCurdy
A. B. Morgan
L. H. Morrison
H. A. Person
Lee Schneitter
P. H. Schweitzer
H. C. Thuerk
C. A. Trimmer

Petroleum Division

Organized, 1925

EXECUTIVE COMMITTEE

H. F. Brindel, *Chairman*
Harold Adkison, *Midwestern Secretary*
H. J. Masson, *Secretary*
W. H. Carson
F. H. Eberle
W. G. Heltzel
T. D. Tifft

Associates

E. H. Barlow
H. P. Porter
Walter Samans

SUBCOMMITTEE ON PRODUCTION

W. H. Carson, *Chairman*
R. M. Carr
H. W. Manley
R. R. Hawkins
H. Decker
J. R. Johnston

SUBCOMMITTEE ON OIL TRANSPORTATION

W. G. Heltzel, *Chairman*
A. N. Horne
H. N. Hunter
B. P. Sibole
H. M. Stevenson
F. A. Stivers
F. E. Warterfield
Oscar Wolfe

SUBCOMMITTEE ON GAS TRANSPORTATION

R. W. Hendee, *Chairman*

SUBCOMMITTEE ON PETROLEUM REFINING

T. D. Tifft, *Chairman*
T. H. Hamilton, *Vice-Chairman*

SUBCOMMITTEE ON LUBRICATION ENGINEERING

(Machine Shop Practice Division; see preceding column)

H. J. Masson, *Representative of Petroleum Division*

SURVEY COMMITTEES ON PETROLEUM PROBLEMS

(Now at work or in the process of formation in the Mid-Continent Section)

Unfired Pressure Vessels
Diesel Engine Driven Reciprocating Pumping Stations
Uniform Code of Economic Analysis of Different Types of Oil Pipe Line Pumping Stations

Power Division

Organized, 1920

EXECUTIVE COMMITTEE

J. M. Brentlinger, *Chairman*
W. A. Carter, *Secretary*
W. E. Caldwell
A. E. Grunert
J. C. Hobbs

Printing Industries Division

Organized, 1922

EXECUTIVE COMMITTEE

EDWARD EPSTEAN, *Chairman*
EDWARD P. HULSE, *Secretary*
J. M. FARRELL
R. G. MACDONALD
JOHN CLYDE OSWALD
BURT D. STEVENS

Associates

V. WINFIELD CHALLENGER
SUMMERFIELD ENEY, JR.
F. M. FLYNN
HARRY L. GAGE
A. E. GIEGENGACK
WM. C. GLASS
HARRY GROESBECK, JR.
WELLS F. HARVEY
T. R. JONES
HADAR ORTMAN
H. M. TILLINGHAST

CHAIRMEN OF DIVISIONAL COMMITTEES

Meetings and Programs, G. D. BEARCE
Paper and Pulp, W. R. MAULL
Progress Report, W. S. HUSON
Research and Survey, ARTHUR C. JEWETT

The Division sponsors the Conference of the Technical Experts in the Printing Industry, a forum for the discussion of the mechanical and process problems of the entire graphic arts field; also the Graphic Arts Research Bureau, formed to act as a clearing house for graphic arts research and for the collection, correlation, and distribution of research information pertaining to the industry and for the sponsorship of research work.

Process Industries Division

Organized, 1934

EXECUTIVE COMMITTEE

VICTOR WICHUM, *Chairman*
W. K. MCAFEE, *Vice-Chairman*
T. R. OLIVE, *Secretary*
C. E. HARRINGTON
H. D. MUNSON
C. W. THOMAS

SUBCOMMITTEE CHAIRMEN

Air Conditioning, G. B. BAILEY
Brewing, C. F. KAYAN
Ceramics, W. K. MCAFEE
Cottonseed Processing, W. R. WOOLRICH
Drying, C. W. THOMAS
Food Processing, G. L. MONTGOMERY
Pulp and Paper, H. D. FISHER
Pulverizing and Grinding, J. C. HARDIGG
Sanitation, WILLIAM RAISCH
Sugar, F. M. GIBSON
Unit Operation Costs, H. J. MASSON

HEAT TRANSFER COMMITTEE

J. H. SENGSTAKEN, *Chairman*
R. E. BIRCH
T. H. CHILTON
M. S. VAN DUSEN
F. H. EBERLE
H. C. HOTTEL
C. E. LUCKE

Railroad Division

Organized, 1920

EXECUTIVE COMMITTEE (RR1)

C. E. BARBA, *Chairman*
G. W. RINK, *1st Vice-Chairman*
W. H. WINTERROWD, *2nd Vice-Chairman*
C. T. RIPLEY, *3rd Vice-Chairman*
E. C. SCHMIDT, *4th Vice-Chairman*
M. B. RICHARDSON, *Secretary*

GENERAL COMMITTEE (RR2)

C. E. BARBA, *Chairman*
O. C. CROMWELL (1935)
J. R. JACKSON (1935)
M. H. ROBERTS (1935)
A. I. LIPETZ (1936)
F. E. LYFORD (1936)
ELIOT SUMNER (1936)
T. C. MCBRIDE (1937)
P. C. MORALES (1937)
K. F. NYSTROM (1937)
W. G. BLACK (1938)
W. H. CLEGG (1938)
G. A. YOUNG (1938)
L. H. FRY (1939)
F. E. RUSSELL (1939)
R. W. SALISBURY (1939)

PAST-CHAIRMEN (RR3)

E. B. KATTE* (1920–1922)
JAMES PARTINGTON (1923–1924)
C. E. CHAMBERS (1925)
H. B. OATLEY (1926–1927)
WM. ELMER (1928)
R. S. MCCONNELL (1929)
A. F. STEUBING (1930)
ELIOT SUMNER (1931)
T. C. MCBRIDE (1932)
L. K. SILLCOX (1933)
C. B. PECK (1934)

CHICAGO GROUP (RR4)

W. O. MOODY, *Chairman*
E. L. WOODWARD, *Secretary*
WALTER DUNHAM
PETER PARKE
C. T. RIPLEY
W. H. WINTERROWD

COMMITTEE ON MEETINGS AND PAPERS (RR5)

W. H. WINTERROWD, *Chairman*
W. G. BLACK, L. H. FRY } Spring Meeting, 1935

COMMITTEE ON SURVEY (RR6)

A. I. LIPETZ, *Chairman* (Locomotive and Foreign Developments)
W. H. CLEGG (Canadian and Automotive Equipment)
P. C. MORALES (Mexican and South American Developments)
K. F. NYSTROM (Cars)

COMMITTEE ON TECHNICAL CONTACTS (RR7)

E. C. SCHMIDT, *Chairman*
O. C. CROMWELL
J. R. JACKSON
F. E. LYFORD
F. E. RUSSELL
R. W. SALISBURY

COMMITTEE ON PROFESSIONAL SERVICE (RR8)

L. K. SILLCOX, *Chairman* (Membership)
H. B. OATLEY (Professional Survey)
M. B. RICHARDSON

COMMITTEE ON RESEARCH (RR9)

L. H. FRY, *Chairman*
T. C. MCBRIDE
JAMES PARTINGTON
G. A. YOUNG
G. W. RINK
A. F. STEUBING
ELIOT SUMNER

Textile Division

Organized, 1921

EXECUTIVE COMMITTEE

H. D. LEARNARD, *Chairman*
M. A. GOLRICK, JR., *Secretary*
A. W. BENOIT
WENDELL BROWN
W. L. CONRAD
C. H. RAMSEY

Associates

H. M. BURKE
PAUL MERRIAM
ALBERT PALMER
EARLE STALL

Wood Industries Division

Organized, 1921

EXECUTIVE COMMITTEE

G. ROLLO PETRIE, *Chairman*
H. B. CARPENTER, *Secretary*
A. D. SMITH, JR.
A. W. KEUFFEL
L. M. NICHOLS

Associates

C. L. BABCOCK
P. H. BILHUBER
F. P. CARTWRIGHT
A. S. KURKJIAN
SERN MADSEN
J. H. MANSFIELD
J. S. MATHEWSON
R. H. MCCARTHY
P. T. NORTON, JR.
T. D. PERRY

SAWS AND KNIVES COMMITTEE

C. L. BABCOCK, *Chairman*
G. E. FRENCH, *Secretary*

RESEARCH SURVEY COMMITTEE

G. R. PETRIE, *Chairman*
A. KOEHLER, *Secretary*

* Deceased.

Local Sections

(*Personnel of Standing Committee, p. 5*)

Midwest Office

R. R. LEONARD, Midwest Representative,
Room 1617, 205 West Wacker Drive, Chicago, Ill.

Mid-Continent Office

J. HAROLD ADKISON, Mid-Continent Petroleum Secretary,
213 Midco Bldg., Tulsa, Okla.

AKRON–CANTON

Organized: 1920
Territory: Counties of Richland, Ashland, Medina, Summit, Portage, Wayne, Stark, Holmes, Tuscarawas, Carroll, and Coshocton in Ohio
Number of Members: 108

EXECUTIVE COMMITTEE

U. A. WHITAKER, *Chairman*
H. E. WANER, *Vice-Chairman*
H. A. TRISHMAN, *Secretary-Treasurer*
R. F. BACHE
F. P. BROWN
C. H. BRUGGEMEIR
JAS. FORREST
A. J. KELLER
A. D. MACLACHLAN
C. W. TRAUT

ANTHRACITE–LEHIGH VALLEY

Organized: 1920, as Lehigh Valley; reorganized, 1928, as Anthracite–Lehigh Valley
Territory: Counties of Bradford, Susquehanna, Wayne, Sullivan, Wyoming, Lackawanna, Columbia, Luzerne, Monroe, Pike, Schuylkill, Carbon, Berks, Lehigh, Northampton in Pennsylvania, and Warren in New Jersey
Place of Meeting: One meeting annually at Allentown, Bethlehem, Easton, Hazleton, Pottsville, Reading, Scranton, and Wilkes-Barre
Local Organization: The Engineers' Club of Lehigh Valley
Number of Members: 205

EXECUTIVE COMMITTEE

F. H. DECHANT, *Chairman*
H. C. PICKEL, *Vice-Chairman*
M. C. STUART, *Vice-Chairman*
P. B. EATON, *Vice-Chairman*
C. W. BELL, *Secretary*
R. J. PRINCE, *Treasurer*
W. W. HAGERTY
A. W. LUCE
C. M. MERRICK
J. A. POWELL
CHRISTIAN SCHILLINGER
E. E. SMITH
P. A. WEAVER
D. G. WILLIAMS

ATLANTA

Organized: 1913
Territory: Radius of sixty miles from Atlanta, Ga.
Place of Meeting: Atlanta Athletic Club
Number of Members: 70

EXECUTIVE COMMITTEE

E. W. KLEIN, *Chairman*
W. C. WROE, *Vice-Chairman*
R. M. MATSON, *Secretary-Treasurer*
GEORGE BRAUNGART, JR.
J. W. PARKER, JR.
P. R. YOPP

BALTIMORE

Organized: 1916
Territory: Radius of thirty miles from Baltimore, Md.
Place of Meeting: Engineers' Club of Baltimore
Luncheon meeting every Wednesday at 12:00 noon at Engineers' Club
Number of Members: 158

EXECUTIVE COMMITTEE

J. C. STROTT, *Chairman*
F. W. KOUWENHOVEN, *Secretary-Treasurer*
J. R. BAKER
O. C. CROMWELL
A. W. TAYLOR
K. A. HAWLEY
N. B. HIGGINS
J. E. HOWARD
A. L. PENNIMAN, JR.

BIRMINGHAM

Organized: 1915
Territory: Radius of sixty miles from Birmingham, Ala.
Place of Meeting: Auditorium of the Alabama Power Company Building
Number of Members: 51

EXECUTIVE COMMITTEE

J. A. SIRNIT, *Chairman*
J. W. ESHELMAN, *Vice-Chairman*
R. A. POLGLAZE, *Secretary-Treasurer*
J. M. GILFILLAN
J. W. ORCUTT

BOSTON

Organized: 1909
Territory: Radius of thirty miles from Boston, Mass.
Place of Meeting: Rooms of the Engineering Societies of Boston
Local Organization: Engineering Societies of Boston
Number of Members: 501

EXECUTIVE COMMITTEE

A. W. BENOIT, *Chairman*
J. T. CROGHAN, *Vice-Chairman*
G. C. EATON, *Secretary-Treasurer*
E. L. ROOT
W. E. FARNHAM
W. F. RYAN

BRIDGEPORT

Organized: 1917, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: Fairfield County, Conn.
Place of Meeting: University Club
Number of Members: 107

EXECUTIVE COMMITTEE

H. E. WELLS, *Chairman*
H. P. HARRIS, *Vice-Chairman*
W. H. SNIFFEN, *Secretary*
C. N. HOAGLAND, *Treasurer*
T. H. BEARD
JULIUS BRENZINGER
ARTHUR BREWER
W. R. CLARK
A. H. EMERY, JR.
A. W. HAGAN
I. C. JENNINGS
R. C. MOODY

BUFFALO

Organized: 1915
Territory: Radius of thirty miles from Buffalo, N. Y.
Place of Meeting: Hotel Statler
Local Organization: Engineering Society of Buffalo
Number of Members: 162

EXECUTIVE COMMITTEE

J. L. YATES, *Chairman*
PAUL DUBOSCLARD, *Acting Vice-Chairman*
W. A. MILLER, *Secretary*
C. E. HARRINGTON, *Treasurer*
C. GALE KIPLINGER
J. ARTHUR FISH
H. D. MUNSON

CENTRAL PENNSYLVANIA

Organized: 1921
Territory: Radius of approximately sixty miles from State College, Pa.
Place of Meeting: Pennsylvania State College, State College, Pa.
Number of Members: 51

EXECUTIVE COMMITTEE

F. E. BURPEE, *Chairman*
C. E. BULLINGER, *Secretary-Treasurer*
G. F. ELLIOTT
H. A. EVERETT
S. K. HOFFMAN
C. H. KENT

CHARLOTTE

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Charlotte, N. C.
Number of Members: 23

EXECUTIVE COMMITTEE

W. S. LEE, JR., *Chairman*
H. M. MCKELVIE, *Vice-Chairman*
V. E. FULLER, *Secretary-Treasurer*
J. H. SADLER
ASA HOSMER, *Ex-Officio*

CHATTANOOGA

Organized: 1922
Territory: Radius of sixty miles from Chattanooga, Tenn.
Number of Members: 14

EXECUTIVE COMMITTEE

NEWELL SANDERS, *Chairman*
H. H. BAILEY, *Vice-Chairman*
F. WARD REILLY, *Secretary-Treasurer*
T. C. ERVIN
M. P. WALL

CHICAGO

Organized: 1913
Territory: Radius of fifty miles from Chicago, Ill.
Headquarters: Mid-West A.S.M.E. Office, 205 West Wacker Drive, Chicago, Ill.
Local Organization: Western Society of Engineers
Number of Members: 639

EXECUTIVE COMMITTEE

R. D. BRIZZOLARI, *Chairman*
A. E. GRUNERT, *Vice-Chairman*
F. B. ORR, *Secretary*
HUGO DIEMER, *Treasurer*
C. C. AUSTIN
C. B. COLE
L. D. GAYTON
W. W. WINTERROWD

CINCINNATI

Organized: 1912
Territory: Radius of thirty miles from Cincinnati, Ohio
Place of Meeting: Engineers' Club Rooms, Ninth & Race Sts.
Local Organization: Engineers' Club of Cincinnati
Number of Members: 199

EXECUTIVE COMMITTEE

J. T. FAIG, *Chairman*
G. A. SEYLER, *Vice-Chairman*
C. A. JOERGER, *Secretary-Treasurer*
C. H. ANDERSON
J. W. BUNTING
F. E. CARDULLO
F. S. DEWEY
W. E. M. FIELMAN
O. C. HILMER
C. L. KOEHLER
E. A. MULLER
R. S. PARKER
W. W. TANGEMAN
H. C. UIHLEIN

CLEVELAND

Organized: 1918
Territory: Counties of Lorain, Cuyahoga Lake, Geauga, and Ashtabula in Ohio
Place of Meeting: Statler Hotel
Local Organization: Cleveland Engineering Society
Number of Members: 238

EXECUTIVE COMMITTEE

L. E. JERMY, *Chairman*
A. J. STOCK, *Vice-Chairman*
H. M. HAMMOND, *Secretary*
FERDINAND JEHL, *Treasurer*
E. H. WHITLOCK
DAVID GAEHR
H. A. MACKENZIE
MCREA PARKER
G. L. TUVE

COLORADO

Organized: 1919
Territory: Entire State of Colorado
Place of Meeting: Parisienne Rotisserie Inn, Denver, Colo.
Local Organization: Colorado Engineering Council (Colorado Society of Engineers)
Number of Members: 84

EXECUTIVE COMMITTEE

F. A. LOCKWOOD, *Chairman*
G. A. RICHTER, *Secretary-Treasurer*
L. D. CRAIN
F. H. PROUTY
ARTHUR HALLIWELL
J. A. HUNTER
D. J. NEVILL

COLUMBUS

Organized: 1920
Territory: Counties of Union, Delaware, Licking, Madison, Franklin, Fayette, Pickaway, and Ross in Ohio
Place of Meeting: Batelle Memorial Institute and Ohio State University
Local Organization: Engineers' Club of Columbus
Number of Members: 79

EXECUTIVE COMMITTEE

F. W. MARQUIS, *Chairman*
H. S. DICKERSON, *Vice-Chairman*
E. M. SAMPSON, *Secretary-Treasurer*
S. R. BEITLER
T. H. KERR

DAYTON

Organized: 1926
Territory: Counties of Drake, Miami, Champaign, Preble, Montgomery, Greene, and northern part of Butler and Warren in Ohio
Place of Meeting: Engineers' Club of Dayton
Local Organization: Engineers' Club of Dayton
Number of Members: 73

EXECUTIVE COMMITTEE

J. Q. SALISBURY, *Chairman*
R. W. MARTIN, *Vice-Chairman*
W. E. BLANK, *Secretary*
B. E. TATE, *Treasurer*
G. A. BUVINGER
T. F. RATAICZAK
C. M. RIPSCH
H. C. WEINLAND

DETROIT

Organized: 1916
Territory: Radius of thirty miles from Detroit, Mich.
Place of Meeting: Place varies
Local Organization: Associated Technical Societies of Detroit
Number of Members: 356

EXECUTIVE COMMITTEE

L. J. SCHRENK, *Chairman*
SABIN CROCKER, *Secretary-Treasurer*
B. W. BEYER, JR.
J. A. CLAUSS
C. L. EKSERGIAN
L. T. KNOCKE
F. J. LINSENMEYER
R. W. SMITH
STEPHEN TIMOSHENKO
P. H. SMITH, *Ex-Officio*

ERIE

Organized: 1917
Territory: Radius of thirty miles from Erie, Pa.
Place of Meeting: Auditorium of Pennsylvania Telephone Company
Number of Members: 51

EXECUTIVE COMMITTEE

A. J. WOODWARD, *Chairman*
G. S. BREWER, *Vice-Chairman*
H. C. MITCHELL, *Vice-Chairman*
H. O. DAVIDSON, *Vice-Chairman*
H. L. KAUFFMAN, *Vice-Chairman*
N. A. NEWTON, *Vice-Chairman*
M. E. SMITH
W. J. BRENNER, *Secretary*
FRANK DERBY, *Treasurer*
GEORGE BACH
H. E. GOETZ
W. L. HUNTER
HERMAN MUELLER

FLORIDA

Organized: 1925
Territory: State of Florida
Place of Meeting: Place varies
Local Organization: Florida Engineering Society, Gainesville, Fla.
Number of Members: 55

EXECUTIVE COMMITTEE

H. S. RIDDLE, *Chairman*
BURDETT LOOMIS, JR., *1st Vice-Chairman*
ROBERT PEYINGHAUS, *2nd Vice-Chairman*
B. R. VAN LEER, *Secretary-Treasurer*
F. J. HOWE, *Student Chairman*
CHARLES BEENSEN
C. M. LOWRY
H. J. B. SCHARNBERG
G. H. SMITH
J. P. WARREN

GREEN MOUNTAIN

Organized: 1923
Territory: Entire State of Vermont and neighboring and closely related communities of Claremont and Hanover, N. H.
Place of Meeting: Springfield, Windsor, Vt., and Claremont, N. H.
Local Organization: Vermont Engineering Society
Number of Members: 37

EXECUTIVE COMMITTEE

J. B. JOHNSON, *Chairman*
M. H. ARMS, *Secretary-Treasurer*
C. H. ADAMS
O. F. ANDERSON
C. S. BEACH
F. E. CHEEVER
C. J. DEWELL
H. R. FINN
R. H. HACKEL
F. A. JOY
H. J. LOCKWOOD
E. L. SUSDORFF

GREENVILLE

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Greenville, S. C.
Place of Meeting: Meetings held at Greenville, Clemson College, S. C., Canton, Asheville, and Enka, N. C.
Number of Members: 26

EXECUTIVE COMMITTEE

J. R. GILL, *Chairman*
R. H. CHAPMAN, *Vice-Chairman*
J. B. MAYO, *Secretary-Treasurer*
C. D. BLACKWELDER
W. R. CRUTE
B. E. FERNOW
R. B. FULLER
W. H. TAYLOR
R. C. TRAMMELL

HARTFORD

Organized: 1917, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: Hartford County except that portion served by Meriden and New Britain Sections
Place of Meeting: Connecticut State Trade School
Number of Members: 101

EXECUTIVE COMMITTEE

E. R. FISH, *Chairman*
H. B. VAN ZELM, *Vice-Chairman*
E. S. WOLSTON, *Secretary-Treasurer*
W. C. BEEKLEY
E. P. HERRICK
L. W. STEVENS

HOUSTON

Organized: 1919
Territory: South Texas and the northern part of the State not included in the North Texas Section territory
Place of Meeting: Electric Bldg., Houston, Tex.
Number of Members: 112

EXECUTIVE COMMITTEE

J. H. DUBENDORF, *Chairman*
J. M. ROBERTSON, *Vice-Chairman*
J. E. MONTGOMERY, *Secretary*
W. T. ALLIGER
V. M. FAIRES
G. G. HARRINGTON
J. H. POUND
C. M. ROSEBRUGH
B. E. SHORT
J. K. SWINFORD
E. R. BREAKER

INDIANAPOLIS

Organized: 1916
Territory: Radius of eighty miles from Indianapolis, within Indiana
Place of Meeting: Place varies
Local Organization: Indiana Engineering Society
Number of Members: 115

EXECUTIVE COMMITTEE

D. B. PRENTICE, *Chairman*
J. H. MAGUIRE, *Vice-Chairman*
J. C. SIEGESMUND, *Secretary-Treasurer*
F. C. HOCKEMA
HOMER RUPARD

INLAND EMPIRE

Organized: 1921
Territory: East of Columbia River in State of Washington, and Counties of Okanogan and Benton, and portion of Northern Idaho
Place of Meeting: Davenport Hotel, Spokane
Luncheon Meetings every Wednesday at 12:00 noon, Davenport Hotel, Spokane
Local Organization: Associated Engineers of Spokane
Number of Members: 21

EXECUTIVE COMMITTEE

D. R. GRAY, *Chairman*
U. B. HOUGH, *Vice-Chairman*
C. I. CARPENTER, *Secretary-Treasurer*
H. F. GAUSE
H. J. MACCAMY
L. J. POSPISIL

KANSAS CITY

Organized: 1921
Territory: Radius of sixty miles from Kansas City, Mo.
Place of Meeting: Hotel President
Local Organization: Engineers' Club of Kansas City
Number of Members: 89

EXECUTIVE COMMITTEE

E. L. MCDONALD, *Chairman*
E. D. HAY, *Vice-Chairman*
H. A. SMITH, *Secretary*
P. A. WHITE, *Treasurer*
H. A. ATWATER
W. G. CRAMER
E. L. HENDRICKSON
F. J. [illegible]BAUR
A. C. KIRKWOOD
J. A. WALTER

KNOXVILLE

Organized: 1923
Territory: All the counties east of the west boundaries of the following, Morgan, Roane, Loudon, McMinn, Scott, and Polk, Tenn., and Bell County, Ky.
Place of Meeting: Andrew Johnson Hotel
Number of Members: 39

EXECUTIVE COMMITTEE

E. W. PALMER, *Chairman*
J. P. FERRIS, *Vice-Chairman*
E. L. CARPENTER, *Secretary-Treasurer*
WALTER CARSON, JR.
WENDELL KENNEDY
W. F. SEARLE, JR.

LOS ANGELES

Organized: 1915
Territory: South of southern boundaries of following counties, Monterey, Kings, Tulares, and Inyo, Calif.
Place of Meeting: Place varies
Local Organization: Technical Societies of Los Angeles
Number of Members: 318

EXECUTIVE COMMITTEE

D. P. VAIL, *Chairman*
J. R. HOFFMAN, *Vice-Chairman*
S. M. DUNN, *Secretary-Treasurer*
E. C. BARKSTROM
D. E. DICKEY
L. G. METCALF
W. ROY SHETTEL

LOUISVILLE

Organized: 1922
Territory: Radius of thirty miles from Louisville, Ky.
Place of Meeting: Engineers' and Architects Club of Louisville
Local Organization: Engineers' and Architects Club
Number of Members: 34

EXECUTIVE COMMITTEE

J. F. HURST, *Chairman*
L. S. VANCE, *Vice-Chairman*
H. H. FENWICK, *Secretary-Treasurer*
B. M. BRIGMAN
J. H. ROMANN

MEMPHIS

Organized: 1923
Territory: Radius of sixty miles from Memphis, Tenn.
Number of Members: 28

EXECUTIVE COMMITTEE

L. H. HUNGATE, JR., *Chairman*
R. V. DOWNS, *Vice-Chairman*
J. J. RYAN, *Secretary-Treasurer*
T. H. ALLEN
M. W. RICE
W. H. ROBERTS
J. S. ROBINSON

MERIDEN

Organized: 1917, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: Meriden, Middletown, Southington, Portland, Plantsville, and Wallingford, Conn.
Place of Meeting: State Trade School Auditorium
Number of Members: 25

EXECUTIVE COMMITTEE

C. A. NEWTON, *Chairman*
R. W. STETSON, *Secretary-Treasurer*
H. C. CASHEN
J. A. HUTCHINSON
L. B. MARCY
E. A. ROBINSON

METROPOLITAN

Organized: 1910
Territory: Metropolitan District, New York and New Jersey
Place of Meeting: Engineering Societies Building, New York, N. Y.
Number of Members: 3459

Executive Committee

J. N. Landis, *Chairman*
R. B. Purdy, *Secretary*
W. H. Armacost, *Treasurer*
W. W. Clinedinst
E. S. Crosby
W. C. Glass
John Hoffhine
T. E. Keating
W. L. Tann
G. B. Pegram, *Ex-Officio*

MID-CONTINENT

Organized: 1919
Territory: Entire States of Oklahoma and Arkansas, and a part of Louisiana. In Texas north of the southern boundaries of the counties of Gaines, Dawson, Bordon, Scurry, Fisher, Jones, and Shackelford
Place of Meeting: 213 Midco Building, Tulsa, Okla.
Number of Members: 148

Executive Committee

Hollis P. Porter, *Chairman*
J. M. McGregor, *Vice-Chairman*
J. F. Eaton, *Secretary*
H. W. Manley, *Treasurer*
F. S. Kelly, Jr.
V. L. Maleev
R. G. Paddock

MILWAUKEE

Organized: 1904
Territory: Radius of fifty miles from Milwaukee, Wis.
Place of Meeting: Milwaukee Athletic Club
Local Organization: Engineers' Society of Milwaukee
Number of Members: 196

Executive Committee

W. D. Bliss, *Chairman*
A. H. Luedicke, *Secretary-Treasurer*
C. A. Cahill
Hans Dahlstrand
F. H. Dorner
Arthur Simon

MINNESOTA

Organized: Minneapolis, 1913; St. Paul, 1913; merged the two Sections, 1934
Territory: Entire State of Minnesota
Place of Meeting: Minnesota Union
Local Organization: Minneapolis Engineers' Club, Minnesota Federation of Architectural and Engineering Societies
Number of Members: 92

Executive Committee

G. F. Endicott, *Chairman*
Melvin Ovestrud, *Vice-Chairman*
L. A. Cobb, *Secretary-Treasurer*
P. J. Frawley
R. E. Gibbs
C. A. Herrick
H. O. Washburn

NEBRASKA

Organized: 1922
Territory: State of Nebraska, and Council Bluffs, Iowa
Place of Meeting: Lincoln and Omaha
Local Organization: Engineers' Club of Lincoln and Omaha
Number of Members: 32

Executive Committee

W. L. DeBaufre, *Chairman*
R. B. Boals, *Vice-Chairman*
A. A. Luebs, *Secretary-Treasurer*
A. E. Bunting
C. F. Moulton
R. H. Park
R. J. Prohaska

NEW BRITAIN

Organized: 1921, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: New Britain, Plainville, Forestville, Bristol, Kensington, and Berlin, Conn.
Place of Meeting: Auditorium of the State Trade School
Number of Members: 35

Executive Committee

R. A. Grise, *Chairman*
P. W. Bauer, *Vice-Chairman*
C. W. Lund, *Secretary-Treasurer*
R. H. Barlow
B. S. Lewis
H. L. Spaunburg
C. C. Stevens
W. Clements Zinck

NEW HAVEN

Organized: 1912; reorganized, 1923
Territory: Portions of New Haven and Middlesex Counties, Conn.
Place of Meeting: Mason Laboratory, Yale University
Number of Members: 88

Executive Committee

C. W. Taylor, *Chairman*
H. R. Polleys, *Secretary-Treasurer*
A. L. Breckenridge
M. J. Radecki
P. H. English
G. E. Hulse
J. C. Menzies

NEW ORLEANS

Organized: 1916
Territory: All of Louisiana except the northern part allotted to Mid-Continent Section
Place of Meeting: Room 422, St. Charles Hotel
Local Organization: Louisiana Engineering Society
Number of Members: 83

Executive Committee

R. F. Muller, *Chairman*
C. A. Bender, Jr., *Secretary-Treasurer*
D. W. Stewart
J. S. Netherwood
A. D. Stancliff

NORTH TEXAS

Organized: 1922
Territory: Radius of one hundred and twenty-five miles from Dallas, in Texas
Place of Meeting: University Club, Dallas, Texas
Local Organization: Technical Club of Dallas
Number of Members: 50

Executive Committee

E. W. Burbank, *Chairman*
R. R. Crowdus, *Secretary-Treasurer*
R. W. Howe
P. M. Cordell
W. B. Gregory

NORWICH

Organized: 1930
Territory: Counties of Tolland, Windham, and New London in Connecticut, and Westerly District in Rhode Island
Place of Meeting: Arcanum Club, 150 Main St., Norwich
Number of Members: 30

Executive Committee

Lovelock Holm, *Chairman*
W. E. Beaney, *Secretary-Treasurer*
C. E. Barber
W. L. Edel
F. S. English
C. W. Phelps
L. E. Whiton

ONTARIO

Organized: 1917
Territory: Province of Ontario, Canada
Place of Meeting: Mining Building, University of Toronto
Number of Members: 86

Executive Committee

O. W. Ellis, *Chairman*
F. G. East, *Secretary-Treasurer*
E. A. Allcut
W. S. Ball
C. H. McL. Burns
P. G. Welford
F. H. Eland
S. L. Fear
W. G. McIntosh
W. A. Osbourne
W. A. Richards

OREGON

Organized: 1919
Territory: State of Oregon and that territory in Washington within a radius of thirty miles from Portland, Ore.

Place of Meeting: University Club, Portland, Ore.
Local Organization: Oregon Society of Engineers
Number of Members: 52

EXECUTIVE COMMITTEE

W. H. MARTIN, *Chairman*
J. E. DYER, *Secretary-Treasurer*
S. M. LISTER
G. F. MCDOUGALL
A. A. OSIPOVICH

PENINSULA

Organized: 1923
Territory: West of the east boundaries of the following counties: Emmet, Charlevoix, Antrim, Kalkaska, Missaukee, Clare, Isabella, Gratiot, Clinton, Eaton, Calhoun, and Branch, Mich.
Place of Meeting: Grand Rapids, Mich.
Local Organization: Engineers' Club of Grand Rapids
Number of Members: 50

EXECUTIVE COMMITTEE

LEROY L. BENEDICT, *Chairman*
B. A. PARKS, *Vice-Chairman*
F. H. MEYER, *Secretary-Treasurer*
S. H. DOWNS
C. H. GRINNELL
E. E. NORMAN

PHILADELPHIA

Organized: 1912
Territory: Counties of Bucks, Montgomery, Chester, Philadelphia, Delaware, Pa., and the State of Delaware
Place of Meeting: Philadelphia Engineers' Club
Local Organization: Philadelphia Engineers' Club
Number of Members: 771

EXECUTIVE COMMITTEE

G. E. CROFOOT, *Chairman*
W. F. OBERHUBER, *Vice-Chairman*
J. P. HARBESON, *Secretary-Treasurer*
E. R. GLENN
C. C. JONES
COLEMAN SELLERS, 3RD

PITTSBURGH

Organized: 1920
Territory: Counties bounded by and including Beaver, Butler, Venango, Forest, Jefferson, Indiana, Somerset, Fayette, Greene, and Washington, Pa.
Place of Meeting: Engineers' Society of Western Pennsylvania, William Penn Hotel
Local Organization: Engineers' Society of Western Pennsylvania
Number of Members: 365

EXECUTIVE COMMITTEE

L. E. HANKISON, *Chairman*
K. F. TRESCHOW, *Secretary-Treasurer*
R. B. AMBROSE
LOUIS ELLMAN
W. N. FLANAGAN
R. J. S. PIGOTT

PLAINFIELD

Organized: 1921
Territory: Plainfield and territory included between Elizabeth, Bound Brook, Metuchen, and Watchung, N. J.
Place of Meeting: Elks Club, Elizabeth, Park Hotel, Plainfield
Local Organization: Plainfield Engineers Club, Singer Engineering Society
Number of Members: 217

EXECUTIVE COMMITTEE

F. C. SPENCER, *Chairman*
RICHARD KAIER, *Vice-Chairman*
D. V. WATERS, *Secretary*
J. P. FABER, *Treasurer*
W. B. UPDEGRAFF

PROVIDENCE

Organized: 1920
Territory: Radius of thirty miles from Providence, R. I.
Place of Meeting: Providence Engineering Society Building, 195 Angell St., Providence, R. I.
Local Organization: Providence Engineering Society
Number of Members: 134

EXECUTIVE COMMITTEE

A. C. CHICK, *Chairman*
Z. R. BLISS, *Vice-Chairman*
A. WILLIAM MEYER, *Secretary-Treasurer*
F. A. CHIFFELLE
C. F. MILLER
S. A. VAULE

RALEIGH

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Raleigh, N. C.
Place of Meeting: N. C. State College, Raleigh, N. C.
Local Organization: N. C. Engineering Council, Raleigh Engineers Club
Number of Members: 31

EXECUTIVE COMMITTEE

R. P. KOLB, *Chairman*
R. S. WILBUR, *Vice-Chairman*
J. M. FOSTER, *Acting Secretary-Treasurer*
R. D. ANTHONY
B. B. GUY

ROCHESTER

Organized: 1919
Territory: Radius of thirty miles from Rochester, N. Y.
Place of Meeting: Rochester Engineering Society Rooms, Sagamore Hotel
Local Organization: Rochester Engineering Society, Sagamore Hotel
Number of Members: 99

EXECUTIVE COMMITTEE

A. E. SCHELL, *Chairman*
W. D. SEELEY, *Vice-Chairman*
I. G. MCCHESNEY, *Secretary-Treasurer*
O. V. SPRAGUE
J. F. ANCONA
K. C. CASTLE, JR.
C. C. ROSS

ROCK RIVER VALLEY

Organized: 1926
Territory: Radius of thirty miles from Rockford, Ill.
Local Organization: Rockford Engineering Society
Number of Members: 43

EXECUTIVE COMMITTEE

F. P. GRUTZNER, *Chairman*
M. B. MACNEILLE, *Vice-Chairman*
HERMAN HUGLE, *Secretary-Treasurer*
A. C. MATTISON
ROBERT VON ROTZ

ST. JOSEPH VALLEY

Organized: 1929
Territory: Counties of La Porte, Starke, Pulaski, St. Joseph, Marshall, Fulton, Elkhart, and Kosciusko in Indiana, and Cass and Berrien Counties in Michigan
Place of Meeting: Morningside Hotel, South Bend, Ind.
Local Organization: St. Joseph Valley Engineers' Club
Number of Members: 35

EXECUTIVE COMMITTEE

C. R. ADAMS, *Chairman*
C. C. WILCOX, *Vice-Chairman*
K. W. KNORR, *Secretary-Treasurer*

ST. LOUIS

Organized: 1909
Territory: Radius of thirty miles from St. Louis, Mo.
Place of Meeting: Place varies
Local Organization: Engineers' Club of St. Louis
Number of Members: 213

EXECUTIVE COMMITTEE

R. M. BOYLES, *Chairman*
G. L. SHANKS, *Vice-Chairman*
E. H. SAGER, *Secretary-Treasurer*
E. W. DAVIS
J. R. JACKSON
F. W. RABE

SAN FRANCISCO

Organized: 1910
Territory: All territory north of the northern boundaries of the counties of San Luis Obispo, Kern, and San Bernardino
Place of Meeting: Engineers' Club, 206 Sansome St.
Luncheon Meetings every Thursday at 12:15 Noon at the Engineers' Club
Local Organization: San Francisco Engineers' Club
Number of Members: 337

EXECUTIVE COMMITTEE

H. B. LANGILLE, *Chairman*
F. W. COLLINS, *Vice-Chairman*
R. L. GRUTZMACHER, *Secretary-Treasurer*
S. R. DOWS
W. M. MOODY
M. P. O'BRIEN
H. J. SMITH
LAWRENCE WASHINGTON

SAVANNAH

Organized: 1923
Territory: Radius of 125 miles from Savannah in Georgia
Place of Meeting: Savannah Hotel
Local Organization: Engineers' Council of Savannah Chamber of Commerce
Number of Members: 15

EXECUTIVE COMMITTEE

A. P. KEISKER, *Chairman*
BRUCE SAMS, *Vice-Chairman*
T. R. JONES, *Secretary*
FRANK EXLEY, *Treasurer*
W. H. ARTLEY
D. E. KEHOE
A. M. ORMOND

SCHENECTADY

Organized: As a Branch, 1919; as a Section, 1927
Territory: Radius of thirty miles from Schenectady, N. Y.
Place of Meeting: Edison Club Hall
Number of Members: 173

EXECUTIVE COMMITTEE

W. E. JOHNSON, *Chairman*
J. E. ANDERSON, *Secretary*
B. O. BUCKLAND, *Treasurer*
A. I. LIPETZ
A. J. NERAD
E. L. ROBINSON

SUSQUEHANNA

Organized: 1927
Territory: Counties of Cumberland, Dauphine, Lebanon, Adams, York, and Lancaster
Place of Meeting: Engineering Society of York
Local Organization: Engineering Society of York
Number of Members: 67

EXECUTIVE COMMITTEE

WILLIAM NOYES, *Chairman*
JACOB FISCH, *Vice-Chairman*
H. B. MARTIN, *Secretary-Treasurer*
WALTER KNAPP
R. M. SPENGLER

SYRACUSE

Organized: 1920
Territory: Radius of thirty miles from Syracuse, N. Y.
Place of Meeting: Ball Room of the Onondaga Hotel
Local Organization: The Technology Club of Syracuse
Number of Members: 115

EXECUTIVE COMMITTEE

H. B. CRAIG, *Chairman*
S. T. HART, *Vice-Chairman*
E. K. RHODES, *Secretary-Treasurer*
K. H. CASKEY
D. W. DIEFENDORF
A. G. FORSSELL
E. W. ZIMMERMAN

TOLEDO

Organized: 1920
Territory: Radius of thirty miles from Toledo, Ohio
Place of Meeting: University Club, Toledo, Ohio
Local Organization: Affiliated Technical Societies of Toledo
Number of Members: 58

EXECUTIVE COMMITTEE

E. P. RAYMOND, *Chairman*
HENRY KERR, *Vice-Chairman*
C. W. KIRSCH, *Secretary-Treasurer*
R. M. BATCH
D. M. PALMER, *Ex-Officio*
D. L. FELTHAM
J. R. MOSER
C. B. SCHAFER
I. F. ZAROBSKY

TRI-CITIES

Organized: 1920
Territory: Radius of thirty miles from Moline, Ill.
Place of Meeting: Rock Island, Ill., Moline, Ill., and Davenport, Iowa
Luncheon Meeting every Wednesday, Davenport Hotel, 12:00 Noon
Number of Members: 62

EXECUTIVE COMMITTEE

R. M. BARNES, *Chairman*
G. T. SCHOEMAKER, *Vice-Chairman*
C. A. CARLSON, *Secretary-Treasurer*
J. M. HARTMAN
W. P. HUNT
J. H. PLOEHN

UTAH

Organized: 1923
Territory: State of Utah
Place of Meeting: University Club, Salt Lake City
Local Organization: Utah Society of Engineers
Number of Members: 26

EXECUTIVE COMMITTEE

E. W. PACE, *Chairman*
W. J. COPE, *Vice-Chairman*
M. B. HOGAN, *Secretary-Treasurer*
JULIUS BILLETER
H. D. LANDES
F. W. MCENTIRE

UTICA

Organized: 1920
Territory: Radius of thirty miles from Utica, N. Y.
Local Organization: Mohawk Valley Technical Club
Number of Members: 26

EXECUTIVE COMMITTEE

E. G. MUNSON, *Chairman*
REX WITHERBEE, *Secretary-Treasurer*
W. J. CLEMENT

VIRGINIA

Organized: 1919
Territory: State of Virginia
Place of Meeting: Richmond, Norfolk, Charlottesville, Roanoke, University, Petersburg
Number of Members: 126

EXECUTIVE COMMITTEE

A. F. MACCONOCHIE, *Chairman*
F. T. MORSE, *Secretary*
H. C. HESSE, *Treasurer*
F. F. GROSECLOSE
J. B. WOODWARD
A. F. KEANE
E. F. KRONER
E. W. MILLER
E. J. F. WILSON

WASHINGTON, D. C.

Organized: 1919
Territory: District of Columbia
Place of Meeting: Auditorium, Potomac Electric Power Co., 10th & E Sts., Washington, D. C.
Number of Members: 162

EXECUTIVE COMMITTEE

H. N. EATON, *Chairman*
M. E. WESCHLER, *Vice-Chairman*
J. F. FOX, *Secretary-Treasurer*
M. X. WILBERDING
J. G. ADAIR
TAWSON PRICE
A. J. SCHWARTZ

WATERBURY

Organized: 1917, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: Litchfield County and a portion of New Haven County
Place of Meeting: Waterbury Club
Number of Members: 72

EXECUTIVE COMMITTEE

M. J. DEMPSEY, *Chairman*
A. L. DAVIS, *Vice-Chairman*
R. L. PALATINE, *Secretary-Treasurer*
S. G. BEAN
E. J. DALY
J. H. ROBERTS

WEST VIRGINIA

Organized: 1925
Territory: State of West Virginia
Place of Meeting: Charleston, W. Va.
Number of Members: 61

EXECUTIVE COMMITTEE

E. L. HUDSON, *Chairman*
J. F. MALLOY, *Secretary-Treasurer*
E. R. HABICHT
ANDREW HILL

WESTERN MASSACHUSETTS

Organized: 1922
Territory: Includes counties of Berkshire, Franklin, Hampden, and Hampshire
Place of Meeting: Highland Hotel, Springfield, Mass.
Local Organization: Engineering Society of Western Massachusetts
Number of Members: 95

EXECUTIVE COMMITTEE

P. W. BIDWELL, *Chairman*
R. L. BOSWORTH, *Vice-Chairman*
L. G. CARLTON, *Secretary-Treasurer*
C. E. MAYNARD
E. LOVELL SMITH
G. R. WHOLEAN

WESTERN WASHINGTON

Organized: 1919
Territory: State of Washington west of the Columbia River with the exception of the territory included in the thirty-mile radius of Portland, Ore.
Place of Meeting: Engineers' Club, Arctic Bldg., Seattle, Wash.
Local Organization: Seattle Engineers' Club
Number of Members: 77

EXECUTIVE COMMITTEE

F. B. LEE, *Chairman*
R. E. JOHNSON, *Vice-Chairman*
R. L. DYER, *Secretary-Treasurer*
G. S. WILSON
C. P. MANN
W. A. MCKENZIE
R. E. WALTER

WORCESTER

Organized: 1915
Territory: Radius of thirty miles from Worcester, Mass.
Place of Meeting: Rooms of the Worcester Engineering Society
Local Organization: Worcester Engineering Society
Number of Members: 138

EXECUTIVE COMMITTEE

H. C. KENDALL, *Chairman*
S. N. MCCASLIN, *Secretary-Treasurer*
H. W. H. BETH
A. W. BLACKBURN
E. H. CARROLL
ROBERT ERICKSON
V. C. KING
T. L. F. LARSSON
K. C. MONROE
F. W. ROYS

YOUNGSTOWN

Organized: 1928
Territory: Counties of Trumbull, Mahoning, and Columbiana in Ohio, and Mercer and Lawrence in Pennsylvania
Place of Meeting: Central Y. M. C. A., Youngstown, Ohio
Number of Members: 67

EXECUTIVE COMMITTEE

E. M. RICHARDS, *Chairman*
S. M. ADAMS, *Secretary-Treasurer*
GORDON ARMSTRONG
E. E. KENDALL
R. M. SMITH
CORNEL KMENTT
M. H. MAWHINNEY
H. OVESEN
F. R. SCHAEFER

Student Branches

(Personnel of Standing Committee on Relations With Colleges, page 5. Communicate with Student Branch through Honorary Chairman)

Name and Location	Year Authorized	Chairman	Secretary	Honorary Chairman
Akron, Univ. of, Akron, Ohio	1924	WILLIAM LEAVENWORTH	PAUL WAGNER	F. S. GRIFFIN
Alabama Polytechnic Inst., Auburn, Ala.	1920	F. R. BALL, JR.	D. O. NICHOLS, JR.	C. R. HIXON
Alabama, Univ. of, University, Ala.[1]	1931	L. E. MORRISON	HERMAN SCHWABE	J. M. GALLALEE
Arkansas, Univ. of, Fayetteville, Ark.	1910	J. W. CARTINHOUR	R. H. BLOOD	R. G. PADDOCK
Armour Inst. of Technology, Chicago, Ill.	1909	J. H. DEBOO	F. J. MEYER	DANIEL ROESCH
Brooklyn, Polytechnic Inst. of, Brooklyn, N. Y.	1909	DONALD MCCORMICK	J. R. BLIZARD	GERADO IMMEDIATO
Brown Univ., Providence, R. I.	1923	R. S. SHAW	R. F. HOPKINS	J. A. HALL
Bucknell Univ., Lewisburg, Pa.	1916	J. D. MORRIS	G. F. ZIMMERMAN	G. M. KUNKEL
California Inst. of Technology, Pasadena, Calif.	1914	ALLEN RAY	ROBERT HALLANGER	HOWARD CLAPP
California, Univ. of, Berkeley, Calif.	1912	CHARLES HARBAND	W. L. RODMAN[2]	N. F. WARD
Carnegie Inst. of Technology, Pittsburgh, Pa.	1913	W. H. STAFFORD	WALTER APPLEGATE	S. B. ELY
Case School of Applied Science, Cleveland, Ohio	1913	R. A. TABORSKY	R. M. SKIDMORE	E. S. AULT
Catholic Univ of America, Washington, D. C.	1922	C. J. DVORAK	R. J. GREEN	M. E. WESCHLER
Cincinnati, Univ. of, Cincinnati, Ohio[3]	1909	GEORGE GENTRY	A. P. MCARTHUR	C. A. JOERGER
Clarkson College of Technology, Potsdam, N. Y.	1930	S. O. CARPENTER	E. I. BRICKER	J. H. DAVIS
Clemson Agricultural College, Clemson College, S. C.	1921	H. A. PLOWDEN	J. B. COX, JR.	B. E FERNOW
Colorado Agricultural College, Fort Collins, Colo.	1914	CARL RITTER	JOHN SUDDUTH	JOSEPH PINSKY
Colorado, Univ. of, Boulder, Colo.	1914	W. T. LUCKING	W. B. SWAN	G. S. DOBBINS
Columbia Univ., New York, N. Y.	1909	WILLIAM GEOGHEGAN	M. A. SHRIRO	J. R. HICKS
Cooper Union, New York, N. Y.	1920	N. M. STEFANO	M. YESOWITZ	H. F. ROEMMELE
Cornell Univ., Ithaca, N. Y.	1908	H. A. MASON	J. S. LESLIE	F. O. ELLENWOOD
Delaware, Univ. of, Newark, Del.	1929	EUGENE MADEY	D. W. SELBY	W. FRANCIS LINDELL
Detroit, Univ. of, Detroit, Mich.	1930	W. B. OAKLEY	KARL SANTTI	H. E. MAYROSE
Drexel Inst., Philadelphia, Pa.	1920	J. W. REID	A. E. JURAM[4]	DAWSON DOWELL
Florida, Univ. of, Gainesville, Fla.	1926	FRED HOWE	E. W. MCKNIGHT	B. R. VAN LEER
George Washington Univ., Washington, D. C.	1924	E. J. THOMAS	C. H. SWANSON	B. C. CRUICKSHANKS
Georgia School of Technology, Atlanta, Ga.	1915	M. G. KEISER	L. J. DRUM	N. C. EBAUGH
Idaho, Univ. of, Moscow, Idaho	1925			H. E. GAUSS
Illinois, Univ. of, Urbana, Ill.	1909	W. A. JOHNSON	C. W. BRISSENDEN	C. H. CASBERG
Iowa State College, Ames, Iowa	1919	D. E. PFITZENMAIER	W. J. SCHLAGEL	N. P. BAILEY
Iowa, State Univ. of, Iowa City, Iowa	1913	WILLIAM BUSBY	D. E. NELSON	R. M. BARNES
Johns Hopkins Univ., Baltimore, Md.	1917	T. H. MARSHALL	J. R. CURTIS, JR.	A. G. CHRISTIE
Kansas State Agricultural College, Manhattan, Kan.	1914	T. G. BECKWITH	R. E. TORKELSON	A. J. MACK
Kansas, Univ. of, Lawrence, Kan.	1909	D. C. WILLIAMS	R. G. WARREN	J. A. KING
Kentucky, Univ. of, Lexington, Ky.	1911	STANFORD NEAL	ELIZABETH WARREN	C. C. JETT
Lafayette College, Easton, Pa.	1919	G. L. WILLIAMS	M. E. ACKERLY	C. M. MERRICK, 3RD
Lehigh Univ., Bethlehem, Pa.	1911	G. A. DORNIN, JR.	R. M. EICHNOR	A. W. LUCE
Lewis Inst., Chicago, Ill.	1933	CHARLES MACKIE	DAVID REDMAN	J. S. KOZACKA
Louisiana State Univ., Baton Rouge, La.	1916	I. J. KIRKMAN	LUKE HADNOT	HAMILTON JOHNSON
Louisville, Univ. of, Louisville, Ky.	1928	V. E. FURNAS, JR.	K. W. SCOTT	R. S. TROSPER
Lowell Textile Inst., Lowell, Mass.	1921	E. H. FAIRBANKS	J. RAYMOND KAISER	H. J. BALL

[1] Aeronautic Division, E. BONIFACE, *Chairman*, J. SMOLE, *Secretary*
[2] FREDERICK WOODWARD, *Corresponding Secretary*
[3] Section 2, HARRY PAINE, *Chairman*, R. J. THOMPSON, *Secretary*
[4] BAYARD QUINN, *Corresponding Secretary*

Student Branches (*continued*)

Name and Location	Year Authorized	Chairman	Secretary	Honorary Chairman
Maine, Univ. of, Orono, Maine	1910	S. T. Favor	R. L. Perkins, Jr.	I. H. Prageman
Marquette Univ., Milwaukee, Wis.	1923	J. W. Krueger	Joseph Kropka	J. E. Schoen
Massachusetts Inst. of Technology, Cambridge, Mass.	1909	P. P. Johnston	H. B. Kimball	James Holt
Michigan College of Mining and Technology, Houghton, Mich.	1930	H. C. Cuskie	R. J. Cosgrove	H. W. Risteen
Michigan State College, East Lansing, Mich.	1917	Paul DeKoning	H. G. English	W. E. Reuling
Michigan, Univ. of, Ann Arbor, Mich.	1914	J. F. Schmidt	L. V. Colwell	O. W. Boston
Minnesota, Univ. of, Minneapolis, Minn.	1913	E. S. Howard	M. L. Olson	J. J. Ryan
Mississippi State College, State College, Miss.	1926	W. W. Denton, Jr.	S. L. Foster, Jr.	O. D. M. Varnado
Missouri School of Mines and Metallurgy, Rolla, Mo.	1930	George Nolde	Rudolph Knoll	R. O. Jackson
Missouri, Univ. of, Columbia, Mo.	1909	B. Morris Baker	K. W. Miller	J. R. Wharton
Montana State College, Bozeman, Mont.	1920	Wayne Linthacum	G. B. Hill	Eric Therkelsen
Nebraska, Univ. of, Lincoln, Neb.	1909	H. E. Simonson	E. R. Dexter	W. F. Weiland
Nevada, Univ. of, Reno, Nev.	1923	Lawrence Kearney	James Crawford	F. H. Sibley
Newark College of Engineering, Newark, N. J.	1924	B. B. Reilly	A. B. Sweet	R. B. Rice
New Hampshire, Univ. of, Durham, N. H.	1926	Joseph VanderHoeff	H. H. Wilkins	T. J. Laton
New York, College of the City of, New York, N. Y.	1922	Bernard Maslow	Milton Dormont	Frederick Kuhle
New York Univ., New York, N. Y.[5]	1917	Bartholomew Antonucci	M. E. Mundel	A. C. Coonradt
New York Univ., Evening Division, New York, N. Y.	1933	E. A. Horner	H. W. Koster	A. C. Coonradt
North Carolina State College, Raleigh, N. C.	1920	J. L. Summers	W. F. Moody, Jr.	L. L. Vaughan
North Carolina, Univ. of, Chapel Hill, N. C.	1929	Calder Atkinson	J. D. Maynard	Colin Carmichael
North Dakota Agricultural College, Fargo, N. D.	1929	Ernest Hall	Charles Martin	R. M. Dolve
North Dakota, Univ. of, Grand Forks, N. D.	1923	C. F. MacKean	J. H. Reinertson	A. J. Diakoff
Northeastern Univ., Boston, Mass.	1922	M. C. Bennett (Div. A) C. Vachert (Div. B)	A. W. Dudley (Div. A) R. L. Allen (Div. B)	J. W. Zeller (Divs. A and B)
Notre Dame, Univ. of, Notre Dame, Ind.	1929	Paul Dougher	Louis Crystal	C. C. Wilcox
Ohio Northern Univ., Ada, Ohio	1922	Stanley Schear	Charles Bailey	J. A. Needy
Ohio State Univ., Columbus, Ohio	1911	H. E. Allspach	J. A. Lucas	F. W. Marquis
Oklahoma A. & M. College, Stillwater, Okla.	1921	Louis Sumpter	Sanford Kroeher	R. E. Venn
Oklahoma, Univ. of, Norman, Okla.	1917	A. C. Frampton	James Mills	H. V. Beck
Oregon State Agricultural College, Corvallis, Ore.	1909	Ross Roberts	R. H. Edson	J. C. Othus
Pennsylvania State College, State College, Pa.	1909	R. Lee Homsher	F. H. Light	C. L. Allen
Pennsylvania, Univ. of, Philadelphia, Pa.	1925	O. E. Jansson	C. M. Hoffner	W. A. Sloan
Pittsburgh, Univ. of, Pittsburgh, Pa.	1917	Joseph Schmieler	R. P. Haase	J. A. Dent
Porto Rico, Univ. of, Mayaguez, P. R.	1923	Rafael Pumarada	E. H. Ruiz	Luis Stefani
Pratt Inst., Brooklyn, N. Y.	1923	John Hanby	E. C. Smith	J. W. Hunter
Princeton Univ., Princeton, N. J.	1926	Irving Warner, Jr.	Robert Cheesman	L. F. Rahm
Purdue Univ., Lafayette, Ind.	1909	J. E. Pearson	P. F. Lilly	F. C. Hockema
Rensselaer Polytechnic Inst., Troy, N. Y.	1910	D. M. Rickey	W. W. Manville	T. F. Fitzgerald
Rhode Island State College, Kingston, R. I.	1930	Raymond Pearson	Fritz Sattler	C. D. Billmyer
Rice Inst., Houston, Tex.	1925	R. H. Nolley	B. R. Ramey	A. H. Burr
Rose Polytechnic Inst., Terre Haute, Ind.	1926			H. C. Gray
Rutgers Univ., New Brunswick, N. J.	1920	L. A. Benton	Jack Spachner	U. C. Holland
Santa Clara, Univ. of, Santa Clara, Calif.	1925	J. W. Conley	E. E. Polomik	G. L. Sullivan
Southern California, Univ. of, Los Angeles, Calif.	1929	Orrin Broberg	C. H. Collins	S. F. Duncan
Southern Methodist Univ., Dallas, Tex.	1933	N. Blankenship	J. F. Schenewerk	R. R. Slaymaker
Stanford Univ., Stanford University, Calif.	1909	J. W. Clyne	K. C. Burch	Lawrence Washington
Stevens Inst. of Technology, Hoboken, N. J.	1908	W. H. Molinari	A. E. Blirer	R. F. Deimel
Swarthmore College, Swarthmore, Pa.	1921			G. A. Bourdelais
Syracuse Univ., Syracuse, N. Y.	1912	H. Carlyle Dugan	H. F. Howe	A. R. Acheson
Tennessee, Univ. of, Knoxville, Tenn.	1923	R. S. Robinson	Virgil Condron	F. L. Wilkinson
Texas, A. & M. College of, College Station, Tex.	1921	Karl White	H. M. Long	J. A. Trail
Texas Technological College, Lubbock, Tex.	1930	Truman Green	Lyle Hardgrave	J. C. Hardgrave
Texas, Univ. of, Austin, Tex.	1921	Marvin Williams	E. H. Moss	M. L. Begeman
Toronto, Univ. of, Toronto, Ont., Can.	1933	L. P. Baker	M. Willer	R. W. Angus
Tufts College, Tufts College, Mass.	1917	Mark Goedecke	F. Robt. Hartin	E. E. Leavitt
Tulane Univ. of Louisiana, New Orleans, La.	1933	J. T. Barrow	D. W. B. Murphy	W. B. Gregory
U. S. Naval Academy, Post Graduate School, Annapolis, Md.	1925			P. J. Kiefer
Utah, Univ. of, Salt Lake City, Utah	1923	F. C. Allen	T. R. Mackay	M. B. Hogan
Vanderbilt Univ., Nashville, Tenn.	1928	B. M. Bayer	Frank Pittman	J. E. Boynton
Vermont, Univ. of, Burlington, Vt.	1922	J. M. Libby	W. H. Connor	E. L. Sussdorff
Villanova College, Villanova, Pa.	1925	M. A. Donnelly	G. O. Hazzard	J. S. Morehouse
Virginia Polytechnic Inst., Blacksburg, Va.	1915	A. N. Bodine	B. W. Bishop	F. F. Groseclose
Virginia, Univ. of, University, Va.	1923	C. T. Montgomery	R. C. Carrick	F. T. Morse
Washington, State College of, Pullman, Wash.	1920	Don Appel	Louis Ellis	H. H. Langdon
Washington Univ., St. Louis, Mo.	1911	Arnold Schainker	L. Gross	E. H. Sager
Washington, Univ. of, Seattle, Wash.	1917	Clarence Gerber	Sholto Salmon	R. H. G. Edmonds
West Virginia, Univ. of, Morgantown, W. Va.	1922	J. F. Millan	R. B. Creel	L. D. Hayes
Wisconsin, Univ. of, Madison, Wis.	1909	Harold Mittelstaedt	William Meade	B. G. Elliott
Worcester Polytechnic Inst., Worcester, Mass.	1914	E. J. Abendschein	F. M. Angevine	H. W. Dows
Wyoming, Univ. of, Laramie, Wyo.	1925	Gale Baker	Eldon Messersmith	R. S. Sink
Yale Univ., New Haven, Conn.	1910	J. F. G. Miller	Neil Macneale, Jr.	S. W. Dudley

[5] Aeronautic Division, Sidney Freedman, *Chairman*, Irving Berler, *Secretary*

Special Research Committees

(*Personnel of Standing Committees, p. 6*)

LUBRICATION

Appointed October, 1915, to investigate the fundamental problems of lubrication, to formulate results of investigations previously made, and to keep in touch with contemporary research in this field

(The Committee is now being reorganized. In December, 1934, the Standing Committee on Research appointed a special committee to formulate a statement of the lubrication problem in its broad, present-day aspects and to recommend a program of research under the auspices of the A.S.M.E. The personnel of this special committee is as follows: N. E. FUNK, *Chairman*, D. B. BULLARD, M. D. HERSEY, G. B. KARELITZ, A. L. KINGSBURY, and W. F. PARISH.)

FLUID METERS

Appointed 1916, to develop the theory of fluid meters of all kinds and to report on the best methods for their installation and use

(Reorganized July, 1929)

R. J. S. PIGOTT, *Chairman*
J. R. CARLTON, *Secretary*
H. S. BEAN
S. R. BEITLER
R. K. BLANCHARD
G. D. CONLEE
W. W. FRYMOYER
LOUIS GESS
T. H. KERR
W. S. PARDOE
E. S. SMITH, JR.
R. E. SPRENKLE
E. C. M. STAHL
T. R. WEYMOUTH

THERMAL PROPERTIES OF STEAM

Appointed in December, 1921, to direct research on the thermal properties of water-vapor and steam from 0 C to the upper limits of temperature and pressure

(Reorganized April, 1929)

W. L. ABBOTT, *Vice-Chairman*
G. L. BOURNE
H. N. DAVIS
H. C. DICKINSON
ALEX DOW
A. M. GREENE, JR.
R. C. H. HECK
D. S. JACOBUS
J. H. KEENAN
F. G. KEYES
L. S. MARKS
G. A. ORROK
R. J. S. PIGOTT
H. V. RASMUSSEN
E. L. ROBINSON

STRENGTH OF GEAR TEETH

Appointed in December, 1921. Is investigating factors affecting the strength and life of gear teeth

R. E. FLANDERS, *Chairman*
C. H. LOGUE, *Secretary*
C. G. BARTH
EARLE BUCKINGHAM
A. M. GREENE, JR.
C. W. HAM
F. E. MCMULLEN
E. W. MILLER
ERNEST WILDHABER

CUTTING OF METALS

Appointed in September, 1923. Is studying the problems of metal cutting, including tool materials, tool design, lubrication, cooling, and speeds and feeds

COLEMAN SELLERS, 3RD, *Chairman*
L. N. GULICK, *Secretary*
A. L. DELEEUW
KING HATHAWAY
L. P. ALFORD
C. J. OXFORD
R. D. PROSSER

ELEVATOR SAFETIES

Appointed June, 1924, as a subcommittee of the Sectional Committee on Safety Code for Elevators, to study the function and operation of elevator safeties and buffers and their associated mechanisms and to develop methods of test for the approval of elevator safety devices

M. H. CHRISTOPHERSON, *Chairman*
O. P. CUMMINGS, *Vice-Chairman*
J. A. DICKINSON, *Secretary*
BASSETT JONES
D. L. LINDQUIST
M. G. LLOYD
J. J. MATSON
M. B. MCLAUTHLIN
W. S. PAINE
S. F. VOORHEES
S. W. JONES

MECHANICAL SPRINGS

Appointed December, 1923, to determine the status of the mechanical-spring art, to promote and conduct necessary and adequate research, and to develop the art to the point of standardization

J. R. TOWNSEND, *Chairman*
C. T. EDGERTON, *Secretary*
C. E. BARBA
W. G. BROMBACHER
R. W. COOK
W. T. DONKIN
RUPEN EKSERGIAN
G. E. HANSEN
BENJAMIN LIEBOVITZ
DAVID LOFTS
(R. D. BRIZZOLARA, *Alternate*)
A. N. LUKENS
D. J. MCADAM, JR.
R. E. PETERSON
J. B. REYNOLDS
J. W. ROCKEFELLER, JR.
B. W. ST. CLAIR
M. F. SAYRE
KEITH WILLIAMS
J. KAYE WOOD
F. P. ZIMMERLI
O. B. ZIMMERMAN

EFFECT OF TEMPERATURE ON THE PROPERTIES OF METALS

Appointed December, 1924, as a joint research committee of the A.S.T.M. and the A.S.M.E. to encourage the investigation and accumulation of data on the properties of metals used in the mechanic arts at extremely high and low temperatures

H. J. FRENCH, *Chairman*
N. L. MOCHEL, *Secretary*
A. D. BAILEY
R. A. BULL
E. S. DIXON
H. W. GILLETT
LOUIS JORDAN
H. J. KERR
C. L. KINNEY, JR.
H. W. MAACK
C. E. MACQUIGG
J. A. MATHEWS
P. E. MCKINNEY
E. L. ROBINSON
A. E. WHITE

BOILER FEEDWATER STUDIES

Appointed March, 1925, as a Joint Research Committee of the American Boiler Manufacturers Association, American Railway Engineering Association, American Water Works Association, Edison Electric Institute, the American Society for Testing Materials, and the A.S.M.E., to study methods of analysis and treatment of boiler feedwater for stationary and railroad practice

W. L. ABBOTT
S. B. APPLEBAUM
A. D. BAILEY
A. G. CHRISTIE
H. C. DINGER
D. K. FRENCH
R. E. HALL
J. A. HOLMES
C. F. HIRSHFELD
C. R. KNOWLES
A. L. PENNIMAN, JR.
E. B. POWELL
T. E. PURCELL
C. W. RICE
F. N. SPELLER
KURT TOENSFELDT
A. E. WHITE
J. D. YODER

CONDENSER TUBES

Appointed May, 1925, to investigate and report on the causes of failure of tubes used in steam condensers and similar heat interchange apparatus

A. E. WHITE, *Chairman*
BERT HOUGHTON, *Vice-Chairman*
P. A. BANCEL
R. H. BARBER
D. K. CRAMPTON
H. M. CUSHING
H. F. EDDY
V. M. FROST
W. L. GREEN, JR.
C. F. HARWOOD
C. F. HIRSHFELD
H. W. LEITCH
E. F. MILLER
W. B. PRICE
H. A. STAPLES
W. R. WEBSTER
ROBERT WORTHINGTON
REPRESENTATIVE OF BUREAU OF ENGINEERING, U. S. NAVY DEPARTMENT

WORM GEARS

Appointed May, 1927, to investigate certain problems in connection with the action of worm gear drives and to recommend improvements in their design, manufacture, and use

EARLE BUCKINGHAM, *Chairman*
G. H. ACKER
L. R. BUCKENDALE
W. H. HIMES
D. L. LINDQUIST
A. A. ROSS
B. F. WATERMAN
REPRESENTATIVE OF BUREAU OF ENGINEERING, U. S. NAVY DEPARTMENT

BOILER FURNACE REFRACTORIES

Appointed June, 1925, to determine the principal factors governing the failure of refractories in various types of installations, to subject these factors to detailed experimental analysis, to undertake the formulation of suitability tests and, if necessary, to attempt to develop a suitable refractory to meet the needs of severe service

W. A. Carter, *Chairman*
S. H. Barnum
G. A. Bole
M. C. Booze
W. H. Fulweiler
C. B. Grady
R. A. Heindl
C. F. Hirshfeld
O. P. Hood
R. K. Hursh
N. E. Lewis
J. S. McDowell
(F. A. Harvey, *Alternate*)
S. J. McDowell
Percy Nicholls
S. M. Phelps
E. B. Powell
R. A. Sherman
R. B. Sosman
L. J. Trostel
G. B. Wilkes

VELOCITY MEASUREMENT OF FLUID FLOW

Appointed November, 1927, to sponsor the development of an absolute method for determining the velocity of the flow of fluids by means of the location of nodal points in wave systems

W. F. Durand, *Chairman*
T. R. Weymouth

MEASURES OF MANAGEMENT

Appointed March, 1928, to attempt the reconciliation of certain economic laws affecting production, to develop formulas for management, and to collect and report information on management research

W. E. Freeland, *Chairman*
F. E. Raymond, *Secretary*
J. H. Barber
T. H. Brown
R. C. Davis
G. E. Hagemann

ABSORPTION OF RADIANT HEAT IN BOILER FURNACES

Appointed April, 1928, to make a study of the absorption of radiant heat in boiler furnaces with the purpose of developing recommendations on improved furnace design

W. J. Wohlenberg, *Chairman*
E. G. Bailey
R. M. Gates
C. W. Gordon
E. L. Lindseth
G. A. Orrok
R. J. S. Pigott
John Van Brunt

STRENGTH OF VESSELS UNDER EXTERNAL PRESSURE

Appointed June, 1929, to develop reliable design data on the strength of cylindrical and spherical surfaces under external pressure, particularly with reference to jacketed vessels

W. D. Halsey, *Chairman*
A. J. Ely
F. V. Hartman
T. McL. Jasper
A. W. Limont, Jr.
Carl Rigdon
H. E. Saunders
E. E. Shanor
J. H. Taylor
(F. S. G. Williams, *Alternate*)
D. B. Wesstrom
D. F. Windenburg

WIRE ROPE

Authorized in October, 1928, to study the factors affecting the life of wire rope so that it may be better understood and more effectively used

W. H. Fulweiler, *Chairman*
W. M. Bager
H. LeR. Brink
D. L. Lindquist
G. W. Martin
C. A. McCune
A. H. McDougall
B. V. E. Nordberg, Jr.
W. S. Paine
W. J. Ryan
George Simpson
L. E. Young

HEAVY DUTY ANTI-FRICTION BEARINGS

Appointed March, 1929, to investigate the possibilities and limitations of anti-friction bearings when applied to roll necks of rolling mills

W. Trinks, *Chairman*
H. E. Brunner
W. R. Clark
G. C. Farkell
H. H. Talbot
S. M. Weckstein
H. A. Winne

REMOVAL OF ASH AS MOLTEN SLAG FROM POWDERED-COAL FURNACES

Appointed March, 1929, to investigate the adding of fluxes as a means of increasing the fluidity of slag in boiler furnaces and thus permit its removal at operating furnace temperatures

K. M. Irwin, *Chairman*
Andrew Carnegie
T. G. Coghlan
H. M. Cushing
C. F. Hirshfeld
Percy Nicholls
E. B. Powell
P. B. Rice

AUTOMATIC OIL PIPE LINE PUMPING STATIONS

Authorized March, 1930, to develop methods of automatic control for oil pipe line pumping stations

W. G. Heltzel, *Chairman*
J. N. Hunter, *Vice-Chairman*
T. D. Williamson, *Secretary*
W. S. Baugh
W. C. Dreyer
W. H. Elliot
J. B. Ford
L. T. Gibbs
F. A. Graham
C. F. Guinn
A. N. Horne
J. K. McGoldrick
J. M. McGregor
J. B. McMahon
R. L. Middleton
O. L. Olsen
William Parkerson
W. R. Reed
F. A. Stivers
W. H. Stueve
Fred Thilenius
J. B. Thomas
F. E. Warterfield
Oscar Wolf

PRIME MOVERS FOR ROTARY DRILLING OF OIL WELLS

Appointed June, 1930, to investigate existing types of prime movers used for rotary drilling of oil wells as to their relative efficiencies, costs of operation, and general satisfaction

D. L. Trax, *Chairman*
Raymond Carr
W. H. Carson
R. R. Hawkins
H. W. Manley

CRITICAL PRESSURE STEAM BOILERS

Appointed June, 1931, to study the characteristics of high-pressure forced-circulation steam-generating units

A. A. Potter, *Chairman*
A. D. Bailey
E. G. Bailey
A. G. Christie
F. S. Clark
A. M. Houser
H. J. Kerr
G. A. Orrok
H. L. Solberg
P. W. Thompson

SAMPLING PULVERIZED FUEL IN A MOVING GAS STREAM

Appointed November, 1932, to investigate the present methods of sampling pulverized fuel and to evolve a generally satisfactory method that may be adopted as a standard

K. M. Irwin, *Chairman*
F. M. Gibson, *Secretary*
John Blizard
Ollison Craig
M. D. Engle
C. S. Gladden
A. E. Grunert
R. M. Hardgrove
J. C. Hardigg
H. J. Klotz
Henry Kreisinger
J. W. McKenzie
W. S. Morrison
G. B. Randall
R. C. Roe
E. H. Tenney

COTTON SEED PROCESSING

Appointed December, 1932, to study the mechanical problems involved in storing, conditioning, and cooking cotton seed meats

W. R. Woolrich, *Chairman*
Homer Barnes
E. L. Carpenter
W. D. Edwards
C. E. Garner
B. J. Sams

A.S.M.E. Representatives on Other Research Committees

See also A.S.M.E. Representatives on Other Activities, page 29

CORROSION COMMITTEE

American Society of Refrigerating Engineers

A. D. Little

HIGHWAY RESEARCH

Advisory Board of National Research Council

J. G. Bergquist

NON-FERROUS METALS AND ALLOYS

Advisory to the National Bureau of Standards' Committee

C. H. BIERBAUM

PROPERTIES OF REFRACTORY MATERIALS

Advisory to the National Bureau of Standards' Committees

E. B. POWELL

NATIONAL COMMITTEE ON WOOD UTILIZATION

Department of Commerce, National Bureau of Standards

A. E. HALL

FATIGUE PHENOMENA OF METALS

American Society for Testing Materials

W. R. WEBSTER

Standardization Technical Committees

(*Personnel of Standing Committee, p. 6*)

SHAFTING

**Sole sponsorship. Organized October, 1918*

A.S.M.E Members (Total personnel, 17)

†C. M. CHAPMAN, *Chairman*
†A. A. ADLER
†A. H. BEYER
†J. E. BUSHNELL
E. E. GREVE
H. C. E. MEYER
L. C. MORROW
C. W. SPICER
†G. N. VANDERHOEF
†L. W. WILLIAMS

BALL AND ROLLER BEARINGS

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized December, 1920*

A.S.M.E. Members (Total personnel, 18)

†W. P. KENNEDY, *Vice-Chairman*
†G. R. BOTT
H. E. BRUNNER
F. H. BUHLMANN
L. A. CUMMINGS
F. G. HUGHES
†G. E. HULSE
†W. L. ILIFF
L. F. NENNINGER
†A. E. NORTON

GEARS

**Joint sponsorship with the American Gear Manufacturers Association. Sectional Committee organized June, 1921*

A.S.M.E. Members (Total personnel, 33)

B. F. WATERMAN, *Chairman*
†EARLE BUCKINGHAM, *Vice-Chairman*
G. H. ACKER
L. H. FRY
C. B. HAMILTON
D. T. HAMILTON
†AUGUST HOFFMAN
O. A. LEUTWILER
G. L. MARKLAND, JR.
H. E. VEHSLAGE

BOLT, NUT, AND RIVET PROPORTIONS

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized March, 1922*

A.S.M.E. Members (Total personnel, 52)

†A. E. NORTON, *Chairman*
F. C. BILLINGS
B. G. BRAINE
(D. L. BRAINE, *Alternate*)
†J. H. BUCKLEY
ELLWOOD BURDSALL
G S. CASE
T. G. CRAWFORD
M H. FLYNN
H. P. FREAR
HERMAN KOESTER
S. F. NEWMAN
E. W. REED
†F. O. WELLS
R. J. WHELAN
E. M. WHITING
V. R. WILLOUGHBY
(J. J. MCBRIDE, *Alternate*)
O. B. ZIMMERMAN

---

* *Note:* All of the Standards Committees for which the Society is Sponsor, Joint Sponsor, or on which it has representation are organized under the procedure of the American Standards Association.

† Official A.S.M.E. representatives serving on this committee.

PIPE FLANGES AND FITTINGS

**Joint sponsorship with the Heating, Piping, and Air Conditioning Contractors National Association and the Manufacturers Standardization Society of the Valve and Fittings Industry. Sectional Committee organized October, 1921*

A.S.M.E. Members (Total personnel, 53)

†C. P. BLISS, *Chairman*
J. J. HARMAN, *Secretary*
W. R. CONARD
SABIN CROCKER
FERDINAND FINK
H. E. HALLER
J. S. HESS
†N. S. HILL, JR.
FRANCIS HODGKINSON
†H. A. HOFFER
A. M. HOUSER
D. S. JACOBUS
L. H. JENKS
W. R. KREMER
JOHN KNICKERBACKER
C. R. KNOWLES
M. B. MACNEILLE
F. H. MOREHEAD
†E. L. MORELAND
†W. S. MORRISON
L. S. MORSE
C. W. MOWRY
(A. L. BROWN, *Alternate*)
A. L. PENNIMAN, JR.
WALTER SAMANS
†C. W. STEPHEN
†J. R. TANNER
J. H. TAYLOR
H. L. UNDERHILL
G. W. WATTS
J. H. WILLIAMS
J. H. ZINK

SCHEME FOR IDENTIFICATION OF PIPING SYSTEMS

**Joint sponsorship with the National Safety Council. Sectional Committee organized June, 1922*

A.S.M.E. Members (Total personnel, 30)

W. L. BUNKER
CROSBY FIELD
WILLIS LAWRENCE
†W. S. MORRISON
A. K. OHMES
†R. S. PECK
A. L. PENNIMAN, JR.
G. E. SANFORD
H. S. SMITH
F. N. SPELLER

SMALL TOOLS AND MACHINE TOOL ELEMENTS

**Joint sponsorship with the National Machine Tool Builders Association and the Society of Automotive Engineers. Sectional Committee organized September, 1922*

A.S.M.E. Members (Total personnel, 26)

C. W. SPICER, *Chairman*
F. O. HOAGLAND, *Vice-Chairman*
F. S. BLACKALL, JR.
E. J. BRYANT
EARLE BUCKINGHAM
†F. H. COLVIN
K. H. CONDIT
S. A. EINSTEIN
†H. E. HARRIS
J. E. LOVELY
†SIMON MACKAY
†W. C. MUELLER
†E. R. NORRIS
ERIK OBERG
C. J. OXFORD
D. M. PALMER

TECHNICAL COMMITTEE NO. 1 ON T-SLOTS

A.S.M.E. Members (Total personnel, 11)

ERIK OBERG, *Chairman*
E. P. BURRELL
HARRY CADWALLADER, JR.
HERMAN CASLER
S. A. EINSTEIN
R. T. HAZELTON
F. O. HOAGLAND
E. R. NORRIS

TECHNICAL COMMITTEE NO. 2 ON TOOL HOLDERS AND TOOL POST OPENINGS

A.S.M.E. Members (Total personnel, 13)

E. P. BURRELL
J. D. KARLE
P. M. MUELLER
R. R. WEDDELL

TECHNICAL COMMITTEE NO. 3 ON MACHINE TAPERS

A.S.M.E. Members (Total personnel, 21)

F. S. BLACKALL, JR., *Chairman*
E. J. BRYANT
EARLE BUCKINGHAM
B. P. GRAVES
F. H. COLVIN
J. B. DILLARD
H. E. HARRIS
F. O. HOAGLAND
J. H. HORIGAN
A. H. LYON
L. F. NENNINGER
C. W. SPICER
F. W. STEIN

Technical Committee No. 4 on Spindle Noses and Collets for Machine Tools

A.S.M.E. Members (Total personnel, 26)

J. E. Lovely, *Chairman*
B. P. Graves
F. O. Hoagland
A. M. Johnson
M. E. Lange
A. H. Lyon
J. H. Mansfield
L. F. Nenninger
H. W. Ruppel
L. D. Spence

Technical Committee No. 5 on Milling Cutters

A.S.M.E. Members (Total personnel, 21)

A. N. Goddard
J. H. Horigan
G. L. Markland, Jr.
E. K. Morgan
Erik Oberg
C. J. Oxford
E. D. Vancil

Technical Committee No. 6 on Designations and Working Ranges of Machine Tools

A.S.M.E. Members (Total personnel, 22)

K. H. Condit, *Chairman*
Earle Buckingham
T. H. Doan
B. P. Graves
J. J. McBride
Charles Schenck
E. R. Smith
W. E. Whipp

Technical Committee No. 7 on Twist Drill Sizes

A.S.M.E. Members (Total personnel, 9)

C. R. Alden, *Secretary*
J. H. Horigan
W. C. Mueller
C. J. Oxford

Technical Committee No. 8 on Drill Bushings

A.S.M.E. Members (Total personnel, 9)

C. R. Alden, *Secretary*
J. H. Horigan
W. C. Mueller
C. J. Oxford
H. E. Wells

Technical Committee No. 9 on Punch Press Tools

A.S.M.E. Members (Total personnel, 17)

D. M. Palmer, *Chairman*
D. H. Chason
A. J. Cummings
N. W. Dorman
H. E. Harris
W. C. Mueller
H. E. Wells

Technical Committee No. 10 on Circular Forming Tools and Holders

A.S.M.E. Members (Total personnel, 9)

W. C. Mueller
D. H. Montgomery
L. D. Spence
H. E. Wells

Technical Committee No. 11 on Chucks and Chuck Jaws

A.S.M.E. Member (Total personnel, 9)

J. E. Lovely, *Chairman*

Technical Committee No. 12 on Cut and Ground Thread Taps

A.S.M.E. Member (Total personnel, 7)

W. C. Mueller

Technical Committee No. 13 on Splines and Splined Shafts

A.S.M.E. Members (Total personnel, 11)

C. W. Spicer, *Chairman*
W. F. Groene
R. E. W. Harrison
F. O. Hoagland
B. F. Waterman

Technical Committee No. 14 on Electric Welding Dies and Electrode Holders

A.S.M.E. Members (Total personnel, 12)

Lewis Fine
N. McD. Loney

Technical Committee No. 15 on Milling Machine Tables

A.S.M.E. Members (Total personnel, 6)

B. P. Graves
L. F. Nenninger

Technical Committee No. 16 on Rotating Tool Shanks

A.S.M.E. Members (Total personnel, 19)

E. J. Bryant, *Chairman*
J. H. Horigan, *Secretary*
J. B. Dillard
E. W. Howe
W. C. Mueller
C. J. Oxford
H. E. Wells

Technical Committee No. 17 on Nomenclature

A.S.M.E. Members (Total personnel, 15)

H. E. Harris, *Chairman*
O. W. Boston, *Secretary*
F. S. Blackall, Jr.
F. H. Colvin
F. O. Hoagland

Ex-Officio Members

C. R. Alden
B. P. Graves
A. N. Goddard
C. J. Oxford
C. W. Spicer
H. E. Wells

Technical Committee No. 18 on Multiple Spindle Drilling Heads

A.S.M.E. Member (Total personnel, 10)

H. E. Wells

Technical Committee No. 19 on Single Point Cutting Tools

A.S.M.E. Members (Total personnel, 2)

F. H. Colvin, *Chairman*
O. W. Boston

SCIENTIFIC AND ENGINEERING SYMBOLS AND ABBREVIATIONS

**Joint sponsorship with the A.S.C.E., A.I.E.E., A.A.A.S., and S.P.E.E. Sectional Committee organized January, 1926*

A.S.M.E. Members (Total personnel, 33)

S. A. Moss, *Vice-Chairman*
R. M. Anderson
(E. P. Warner, *Alternate*)
K. H. Condit
†H. N. Davis
†(R. J. S. Pigott, *Alternate*)
G. W. Lewis

PLAIN AND LOCK WASHERS

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized March, 1926*

A.S.M.E. Members (Total personnel, 37)

†S. G. Baits
Eugene Caldwell
T. G. Crawford
†B. S. Lewis
C. H. Loutrel
J. J. McBride
H. C. E. Meyer
E. H. Waring
E. M. Whiting
O. B. Zimmerman

MACHINE PINS

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized March, 1926*

A.S.M.E. Members (Total personnel, 18)

†M. E. Steczynski, *Chairman*
†E. J. Bryant
J. J. McBride
H. C. E. Meyer
O. B. Zimmerman

DRAWINGS AND DRAFTING ROOM PRACTICE

**Joint sponsorship with the Society for Promotion of Engineering Education. Sectional Committee organized July, 1926*

A.S.M.E. Members (Total personnel, 52)

F. DeR. Furman, *Chairman*
C. W. Keuffel, *Secretary*
Douglas Baker
C. E. Coolidge
T. G. Crawford
H. C. Fletcher
T. E. French
A. C. Harper
A. M. Houser
Alfred Iddles
†Samuel Ketchum
F. R. Laney
H. B. Langille
W. L. McIntosh
F. W. Ming
E. B. Neil
†J. K. Olsen
J. W. Owens
F. C. Panuska
Carl Rossmassler
†E. S. Smith, Jr.

CODE FOR PRESSURE PIPING

**Sole sponsorship. Sectional Committee organized March, 1926*

A.S.M.E. Members (Total personnel, 72)

E. B. Ricketts, *Chairman*
G. S. Coffin
H. C. Cooper
Sabin Crocker
(J. A. Walker, *Alternate*)
H. D. Edwards
C. A. Ellis
Charles Fitzgerald
O. S. Hagerman
H. E. Haller
(G. J. Stuart, *Alternate*)
J. J. Harman
J. S. Haugh
(E. B. Severs, *Alternate*)
J. S. Hess
H. A. Hoffer
G. G. Hollins
†A. M. Houser
†Alfred Iddles
J. H. Lawrence
M. B. MacNeille
G. W. Martin
H. C. E. Meyer
J. W. Moore
F. H. Morehead
H. H. Morgan
W. S. Morrison
A. W. Moulder
Fred Nolde
E. W. Norris
G. A. Orrok
A. L. Penniman, Jr.
†C. S. Robinson
G. W. Saathoff
G. K. Saurwein
H. S. Smith
(H. H. Moss, *Alternate*)
F. N. Speller
C. G. Spencer
J. R. Tanner
J. H. Vance
F. H. Wagner
H. L. Whittemore
J. H. Williams
T. F. Wolfe
G. H. Woodroffe

GRAPHIC PRESENTATION

**Sole sponsorship. Sectional Committee organized November, 1926*

A.S.M.E. Members (Total personnel, 38)

†G. E. Hagemann, *Secretary*
M. F. Behar
C. N. Bigelow
Wallace Clark
H. G. Crockett
E. F. DuBrul
T. E. French
†D. B. Porter

TRANSMISSION CHAINS AND SPROCKETS

**Joint sponsorship with the Society of Automotive Engineers and the American Gear Manufacturers Association. Sectional Committee organized December, 1926*

A.S.M.E. Members (Total personnel, 16)

†F. V. Hetzel, *Chairman*
†G. M. Bartlett, *Secretary*
W. J. Belcher
Joseph Joy
†L. V. Ludy
†E. B. Nichols
F. L. Morse
G. A. Young
O. B. Zimmerman

WIRE AND SHEET METAL GAGES

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized November, 1927*

A.S.M.E. Members (Total personnel, 28)

J. F. Howe
†E. E. Rose

PIPE THREADS

**Joint sponsorship with the American Gas Association. Sectional Committee reorganized December, 1927. In August, 1908, the A.G.A. and the A.S.M.E. appointed a Joint Committee on this subject which was reorganized as a Sectional Committee under the procedure of the A.S.A. in the spring of 1919*

A.S.M.E. Members (Total personnel, 43)

B. H. Blood
A. F Breitenstein
E. J. Bryant
L. A. Cornelius
E. S. Cornell, Jr.
J. J. Crotty
J. J. Harman
†A. M. Houser
F. B. Howell
A. H. Jarecki
J. O. Johnson
W. R. Kremer
A. S. Miller
P. V. Miller
F. H. Morehead
W. C. Morris
S. F. Newman
E. S. Sanderson
L. N. Shannon
W. D. Sizer
J. H. Williams

ELECTRIC MOTOR FRAME DIMENSIONS

**Joint sponsorship with the National Electrical Manufacturers Association. Sectional Committee organized November, 1927*

A.S.M.E. Members (Total personnel, 31)

†W. F. Dixon, *Chairman*
C. A. Adams
S. A. Einstein
E. W. Ely
F. S. English
W. F. Jones
A. L. McHugh
P. G. Rhoads
†A. G. Trumbull
†E. H. Waring

WROUGHT IRON AND WROUGHT STEEL PIPE AND TUBING

**Joint sponsorship with the American Society for Testing Materials. Sectional Committee organized April, 1928*

A.S.M.E. Members (Total personnel, 39)

H. H. Morgan, *Chairman*
Sabin Crocker, *Secretary*
J. B. Aston
†A. M. Houser
†D. S. Jacobus
(F. S. Clark, *Alternate*)
W. R. Kremer
H. C. E. Meyer
F. H. Morehead
C. W. Mowry
†H. B. Oatley
A. L. Penniman, Jr.
(A. B. Morgan, *Alternate*)
F. N. Speller
J. R. Tanner
A. E. White
H. LeR. Whitney
G. H. Woodroffe

SPEEDS OF MACHINERY

**Sole sponsorship. Sectional Committee organized May, 1928*

A.S.M.E. Members (Total personnel, 29)

†C. M. Bigelow
R. C. Deale
Paul Diserens
F. S. English
A. E. Hall
D. C. Jackson
W. F. Jones
H. C. E. Meyer
John Reid
P. G. Rhoads
F. C. Spencer
O. B. Zimmerman

SCREW THREADS FOR HOSE COUPLINGS

**Sole sponsorship. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 24)

A. L. Brown, *Secretary*
†A. F. Breitenstein
W. L. Curtiss
†W. E. Dunham
J. J. Harman
(F. C. Ernst, *Alternate*)
A. M. Houser
H. C. E. Meyer
J. H. Williams

PLUMBING EQUIPMENT

**Joint sponsorship with the American Society of Sanitary Engineering. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 30)

†L. A. Cornelius
J. J. Crotty
O. E. Goldschmidt
G. W. Martin
(A. H. Morgan, *Alternate*)
†W. R. Webster

ROLLED THREADS FOR SCREW SHELLS OF ELECTRIC SOCKETS AND LAMP BASES

**Joint sponsorship with the National Electrical Manufacturers Association. Sectional Committee organized March, 1929*

A.S.M.E. Members (Total personnel, 18)

†E. J. Bryant
†Earle Buckingham
†E. S. Sanderson

STOCK SIZES, SHAPES, AND LENGTHS FOR HOT AND COLD FINISHED IRON AND STEEL BARS

**Sole sponsorship. Sectional Committee organized April, 1929*

A.S.M.E. Members (Total personnel, 24)

J. B. Aston
F. H. Dechant
†L. W. Williams
G. H. Woodroffe
O. B. Zimmerman

STANDARDIZATION AND UNIFICATION OF SCREW THREADS

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee originally organized in June, 1921. Reorganized in February, 1928*

A.S.M.E. Members (Total personnel, 37)

†R. E. Flanders, *Chairman*
†F. O. Wells, *Vice-Chairman*
Earle Buckingham, *Secretary*
H. E. Bilger
E. J. Bryant
G. S. Case
T. G. Crawford
†A. M. Houser (R. E. Perry, *Alternate*)
H. C. E. Meyer
†P. V. Miller
O. B. Zimmerman

PRESSURE AND VACUUM GAGES

**Sole sponsorship. Sectional Committee organized July, 1930*

A.S.M.E. Members (Total personnel, 45)

M. D. Engle, *Chairman* (A. B. Morgan, *Alternate*)
†A. W. Lenderoth, *Secretary*
E. J. Bryant
†J. P. Cavanaugh
Paul Diserens
W. F. Jones
R. J. Kehl
†J. C. McCune
A. H. Morgan
†S. A. Moss
H. B. Reynolds
C. Z. Rosencrans
W. C. Schoenfeldt

ALLOWANCES AND TOLERANCES FOR CYLINDRICAL PARTS AND LIMIT GAGES

**Sole sponsorship. Sectional Committee originally organized in June, 1920. Reorganized in September, 1930*

A.S.M.E. Members (Total personnel, 44)

F. E. Banfield (E. E. Blake, *Alternate*)
F. S. Blackall, Jr.
H. E. Brunner (William Jetter, *Alternate*)
E. J. Bryant
†Earle Buckingham
†F. H. Colvin
R. E. W. Harrison
F. O. Hoagland
N. E. Jacobi
H. C. E. Meyer
P. V. Miller
W. C. Mueller
†E. C. Peck
W. C. Schoenfeldt
F. W. Stein
C. C. Stevens
O. B. Zimmerman

FOUNDRY EQUIPMENT AND SUPPLIES

**Joint sponsorship with the American Foundrymen's Association. Sectional Committee organized February, 1931*

A.S.M.E. Members (Total personnel, 23)

E. S. Carman, *Chairman*
E. W. Ely
F. W. Herendeen (F. B. Howell, *Alternate*)
G. F. Jenks
A. S. Phelps
†H. P. Van Cleve
O. B. Zimmerman

SPECIFICATIONS FOR LEATHER BELTING

**Sole sponsorship. Sectional Committee organized February, 1931*

A.S.M.E. Members (Total personnel, 24)

R. W. Chandler
H. T. Coates
†R. W. Drake
King Hathaway
Fred Nolde
J. E. Rhoads
C. A. Schieren
†C. O. Streeter
O. B. Zimmerman

UNIFICATION OF RULES FOR THE DIMENSIONING OF FURNACES FOR BURNING SOLID FUEL

**Sole sponsorship. Sectional Committee organized June, 1933*

A.S.M.E. Members (Total personnel, 22)

C. E. Bronson, *Temporary Chairman*
W. G. Christy
John Hunter
†V. G. Leach
T. A. Marsh
W. L. Martwick
J. F. McIntire
†John Van Brunt

CLASSIFICATION AND DESIGNATION OF SURFACE QUALITIES

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized May, 1932*

A.S.M.E. Members (Total personnel, 56)

C. G. Mettler, *Temporary Chairman*
John Cetrule
J. S. Chafee
T. G. Crawford
†R. C. Deale
U. S. Eberhardt
S. A. Einstein
R. F. Gagg
J. J. Harman
†R. E. W. Harrison
F. V. Hartman
F. O. Hoagland
H. J. Holtzclaw
R. T. Kent
A. H. Lyon
M. J. Reed
F. C. Spencer
C. C. Stevens
J. S. Tawresey
C. H. Whitaker

A.S.M.E. Representatives on Miscellaneous Standardization Committees

See also A.S.M.E. Representaitves on Other Activities, page 29

SPECIFICATIONS FOR CAST IRON PIPE

**Sponsor bodies: American Gas Association, American Society for Testing Materials, American Water Works Association, and New England Water Works Association*

G. W. Biggs
J. E. Gibson

CLASSIFICATION OF COAL

**Sponsor body: American Society for Testing Materials*

F. R. Wadleigh

DRAINAGE OF COAL MINES

**Sponsor body: American Mining Congress*

M. J. Lide
O. M. Pruitt
W. M. White

FIRE TESTS OF BUILDING CONSTRUCTION AND MATERIALS

**Sponsor bodies: A.S.A. Fire Protection Group, National Bureau of Standards, and the American Society for Testing Materials*

E. C. Rack

MANHOLE FRAMES AND COVERS

**Sponsor bodies: A.S.A. Telephone Group and American Society of Civil Engineers*

Anton Hansen
Homer Rupard

SPECIFICATIONS FOR AND RECOMMENDED PRACTICE IN THE USE OF WIRE ROPE FOR MINES

**Sponsor body: American Mining Congress*

J. L. Harrington

ROTATING ELECTRICAL MACHINERY

**Sponsor bodies: American Institute of Electrical Engineers and the National Electrical Manufacturers Association*

H. R. Sewell (C. A. Booth, *Alternate*)

SPECIFICATIONS FOR FUEL OILS

**Sponsor body: The American Society for Testing Material*

W. H. Butler
Harte Cooke
H. W. Dow
L. H. Morrison
Lee Schneitter
Denistoun Wood

ELECTRIC WELDING

**Sponsor bodies: American Institute of Electrical Engineers and the National Electrical Manufacturers Association*

N. McD. Loney

SPECIFICATIONS FOR SIEVES FOR TESTING PURPOSES

**Sponsor bodies: American Society for Testing Materials and National Bureau of Standards*

D. McM. Blackburn

METHODS OF RATING RIVERS

**Sponsor body: U. S. Geological Survey*

D. W. Mead

METHODS OF TESTING WOOD

**Sponsor body: U. S. Forest Service and the American Society for Testing Materials*

C. M. Bigelow

METHODS OF TESTING PETROLEUM PRODUCTS AND LUBRICANTS

**Sponsor body: American Society for Testing Materials*

A. E. Flowers

MISCELLANEOUS OUTSIDE COAL-HANDLING EQUIPMENT

**Sponsor body: American Mining Congress*

J. H. Stratton

SPECIFICATIONS FOR MATERIALS FOR USE IN MANUFACTURE OF SPECIAL TRACKWORK

**Sponsor body: American Transit Association*

W. R. Hulbert

ELECTRICAL DEFINITIONS

**Sponsor body: American Institute of Electrical Engineers*

C. H. Berry J. V. B. Duer

ACOUSTICAL MEASUREMENTS AND TERMINOLOGY

**Sponsor body: Acoustical Society of America*

E. E. Free
(P. H. Bilhuber, *Alternate*)
R. V. Parsons
(J. S. Parkinson, *Alternate*)
W. B. White
(H. S. Read, *Alternate*)

SPECIFICATIONS FOR CLEAN BITUMINOUS COAL

**Sponsor body: American Institute of Mining and Metallurgical Engineers*

R. A. Sherman (E. L. Lindseth, *Alternate*)

PREFERRED NUMBERS

**Special Committee of A.S.A.*

K. H. Condit

FOREST FIRE PROTECTION

Committee of National Fire Protection Association

E. H. Davis

BUILDING CODE COMMITTEE

Advisory Committee of Department of Commerce

Thornton Lewis

SPECIFICATIONS FOR TOOL STEEL

Subcommittee No. XIV of A.S.T.M. Committee A1 on Steel

O. W. Boston C. M. Inman

PETROLEUM SPECIFICATIONS

Advisory Board of U. S. Bureau of Mines

H. A. S. Howarth

Power Test Codes Technical Committees

(*Personnel of Standing Committee, p. 6*)

(1) GENERAL INSTRUCTIONS

Appointed December, 1918

W. H. Kavanaugh, *Chairman* C. F. Hirshfeld
A. M. Greene, Jr. M. C. Stuart

(2) DEFINITIONS AND VALUES

Appointed December, 1918

F. R. Low L. S. Marks
A. C. Wood

(3) FUELS

Appointed December, 1918

W. J. Wohlenberg, *Chairman* F. G. Philo
E. G. Bailey G. S. Pope
B. L. Boye C. R. Richards
L. P. Breckenridge E. B. Ricketts
H. W. Brooks F. M. Rogers
S. B. Flagg E. X. Schmidt
D. M. Myers Nicholas Stahl
E. N. Trump

(4) STATIONARY STEAM-GENERATING UNITS

Appointed December, 1918

E. R. Fish, *Chairman* E. B. Powell
A. D. Bailey E. B. Ricketts
Alfred Iddles C. U. Savoye

(5) RECIPROCATING STEAM ENGINES

Appointed December, 1918

A. G. Christie, *Chairman* J. A. Hunter
Harte Cooke H. G. Mueller
K. S. M. Davidson B. V. E. Nordberg
Herman Diederichs J. F. M. Patitz
Henrik Greger A. V. Saharoff
Thomas Hall A. G. Witting
J. C. Workman

(6) STEAM TURBINES

Appointed December, 1918

C. H. Berry, *Chairman* V. M. Frost
I. E. Moultrop, *Secretary* A. E. Grunert
O. D. H. Bentley Francis Hodgkinson
W. E. Caldwell S. A. Moss
A. G. Christie R. O. Muller
Hans Dahlstrand T. E. Purcell
C. C. Thomas

(7) RECIPROCATING STEAM-DRIVEN DISPLACEMENT PUMPS

Appointed December, 1918

R. D. Hall, *Chairman* J. E. Gibson
C. H. Anderson G. L. Kollberg
E. H. Brown M. B. MacNeille
J. N. Chester D. W. Mead
L. A. Quayle

(8) CENTRIFUGAL AND ROTARY PUMPS

Appointed December, 1918

W. B. Gregroy, *Chairman*
Max Spillman, *Secretary*
W. C. Brown
M. B. MacNeille
L. F. Moody
F. H. Rogers
W. M. White

(9) DISPLACEMENT COMPRESSORS AND BLOWERS

Appointed December, 1918

Paul Diserens, *Chairman*
G. T. Felbeck
J. F. Huvane
R. M. Johnson
S. B. Redfield

(10) CENTRIFUGAL AND TURBO-COMPRESSORS AND BLOWERS

Appointed December, 1918

A. T. Brown, *Chairman*
E. L. Anderson
C. A. Booth
W. H. Carrier
L. E. Day
E. S. Dean
Z. G. Deutsch
S. H. Downs
P. E. Good
J. J. Grob
H. F. Hagen
Paul Hoffman
H. D. Kelsey
A. L. Kimball
L. S. Marks
Arvid Peterson
H. F. Schmidt
M. C. Stuart

(11) COMPLETE STEAM POWER PLANTS

Appointed December, 1918

F. M. Van Deventer, *Chairman*
W. F. Davidson
C. H. Fellows
R. A. Foresman
V. M. Frost
W. W. Johnson
E. W. Norris
C. U. Savoye
D. S. Wegg
H. S. Whiton

(12) CONDENSERS, WATER HEATING, AND COOLING EQUIPMENT

Appointed December, 1918

G. A. Orrok, *Chairman*
P. H. Hardie, *Secretary*
C. H. Baker, Jr.
R. N. Ehrhart
J. F. Grace
D. W. R. Morgan
J. J. Mullan
H. B. Reynolds
P. E. Reynolds

(13) REFRIGERATING SYSTEMS

Appointed December, 1918

R. M. Anderson, *Chairman*
G. B. Bright
N. H. Hiller
G. A. Horne
G. T. Voorhees

(14) EVAPORATING APPARATUS

Appointed December, 1918

E. N. Trump, *Chairman*
B. N. Bump
E. A. Newhall
H. L. Parr
L. C. Rogers

(15) STEAM LOCOMOTIVES

Appointed December, 1918

E. C. Schmidt, *Chairman*
W. F. Kiesel, Jr.
H. B. Oatley
G. E. Rhoads
L. K. Sillcox
W. E. Woodard

(16) GAS PRODUCERS

Appointed December, 1918

W. T. Magruder, *Chairman*
R. H. Fernald
C. D. Smith
H. F. Smith

(17) INTERNAL-COMBUSTION ENGINES

Appointed December, 1918

Max Rotter, *Chairman*
Harte Cooke
L. B. Jackson
B. V. E. Nordberg
J. H. Suter

(18) HYDRAULIC PRIME MOVERS

Appointed December, 1918

E. C. Hutchinson, *Chairman*
C. M. Allen
H. L. Doolittle
W. F. Durand
N. R. Gibson
J. P. Growdon
T. H. Hogg
D. J. McCormack
L. F. Moody
R. V. Terry
W. M. White

(19) INSTRUMENTS AND APPARATUS

Appointed December, 1918

C. F. Hirshfeld, *Chairman*
W. A. Carter, *Secretary*
C. M. Allen
E. G. Bailey
H. S. Bean
L. J. Briggs
C. R. Cary
J. D. Davis
R. E. Dillon
F. M. Farmer
J. B. Grumbein
W. H. Kenerson
E. S. Lee
E. L. Lindseth
Osborn Monnett
S. A. Moss
R. J. S. Pigott
E. B. Ricketts
W. A. Sloan
R. B. Smith

(20) SPEED-RESPONSIVE GOVERNORS

Appointed December, 1921

Francis Hodgkinson, *Chairman*
Harte Cooke
J. F. M. Patitz
F. H. Rogers

(21) DUST SEPARATING APPARATUS

Appointed October, 1934

M. D. Engle, *Chairman*
J. W. Armour, *Secretary*
E. L. Anderson
A. D. Bailey
H. M. Bubar
W. G. Christy
H. O. Croft
J. M. DallaValle
Philip Drinker
H. F. Hagen
P. H. Hardie
C. W. Hedberg
J. H. Leech
H. E. Macomber
B. F. Tillson

A.S.M.E. Representatives on Other Committees

See also A.S.M.E. Representatives on Other Activities, page 29

DEVELOPMENT OF DEFINITIONS FOR THE NET CALORIFIC VALUE AND GROSS CALORIFIC VALUE OF FUELS

Sponsor Body: American Society for Testing Materials

W. J. Wohlenberg

COMMITTEE ON REDEFINING SO-CALLED STANDARD TON OF REFRIGERATION

Sponsor Body: American Society of Refrigerating Engineers

G. B. Bright

Safety Technical Committees

(*Personnel of Standing Committee, p. 6*)

SAFETY CODE FOR MECHANICAL POWER-TRANSMISSION APPARATUS

**Joint sponsorship with the International Association of Industrial Accident Boards and Commissions, National Bureau of Casualty and Surety Underwriters. Sectional Committee organized February, 1921*

A.S.M.E. Members (Total personnel, 27)

†C. B. Auel, *Chairman*
†L. A. DeBlois
W. J. Hamilton
R. McA. Keown
†G. M. Naylor
W. W. Nichols
W. S. Paine
P. G. Rhoads
†D. C. Wright
(G. N. VanDerhoef, *Alternate*)

SAFETY CODE FOR ELEVATORS

**Joint sponsorship with the American Institute of Architects and the National Bureau of Standards. Sectional Committee organized November, 1922*

A.S.M.E. Members (Total personnel, 35)

O. P. Cummings, *Vice-Chairman*
C. R. Callaway
M. H. Christopherson
I. N. Haughton
†D. L. Holbrook
S. V. James
Bassett Jones
D. L. Lindquist
N. O. Lindstrom
J. J. Matson
J. C. McCabe
M. B. McLauthlin
W. S. Paine
S. F. Voorhees

SAFETY CODE ON MACHINERY FOR COMPRESSING AIR

**Joint sponsorship with the American Society of Safety Engineers—Engineering Section, National Safety Council. Sectional Committee organized May, 1923*

A.S.M.E. Members (Total personnel, 23)

Paul Diserens
H. D. Edwards
W. M. Graff
W. J. Graves
O. P. Hood
†H. H. Judson
C. E. Pettibone
D. L. Royer

SAFETY CODE FOR CONVEYORS AND CONVEYING MACHINERY

**Joint sponsorship with the National Bureau of Casualty and Surety Underwriters. Sectional Committee organized November, 1925*

A.S.M.E. Members (Total personnel, 45)

M. H. Christopherson
E. L. Gifford
W. J. Graves
F. V. Hetzel
†M. A. Kendall
R. McA. Keown
W. S. Paine
C. G. Pfeiffer
D. L. Royer
(W. M. Graff, *Alternate*)
†William Staniar
G. R. Wadleigh
J. G. Wheatley

SAFETY CODE FOR CRANES, DERRICKS, AND HOISTS

**Sole sponsorship. Sectional Committee organized November, 1925*

A.S.M.E. Members (Total personnel, 58)

B. F. Tillson, *Secretary*
H. LeR. Brink
J. F. Howe
W. D. Keller
Franklin Moeller
†Lewis Price
F. H. Schwerin
†R. H. White
H. L. Whittemore

A.S.M.E. Representatives on Other Safety Committees

See also A.S.M.E. Representatives on Other Activities, page 29

SAFETY CODE CORRELATING COMMITTEE

M. H. Christopherson (T. A. Walsh, Jr., *Alternate*)

SAFETY CODE FOR AMUSEMENT PARKS

**Sponsor bodies: National Association of Amusement Parks, and National Bureau of Casualty and Surety Underwriters*

G. P. Smith

SAFETY CODE FOR FLOOR OPENINGS, RAILINGS, AND TOE BOARDS

**Sponsor body: National Safety Council*

William Collins

SAFETY CODE FOR FORGING AND HOT METAL STAMPING

**Sponsor bodies: American Drop Forging Institute and the National Safety Council*

O. F. Luckenbach
C. F. Park

SAFETY CODE FOR LADDERS

**Sponsor body: National Safety Council*

W. T. Hatch

SAFETY CODE FOR LAUNDRY MACHINERY AND OPERATION

**Sponsor bodies: Laundry Owners National Association, Association of Governmental Officials in Industry of U. S. and Canada, and National Association of Mutual Casualty Companies*

E. J. Carroll

SAFETY CODE FOR LIGHTING FACTORIES, MILLS, AND OTHER WORKS PLACES

**Sponsor body: Illuminating Engineering Society*

L. A. Blackburn

SAFETY CODE FOR LOGGING AND SAWMILL MACHINERY

**Sponsor body: National Bureau of Standards*

J. H. Dickinson

SAFETY CODE FOR PAPER AND PULP MILLS

**Sponsor body: National Safety Council*

R. L. Weldon

SAFETY CODE FOR POWER PRESSES, AND FOOT AND HAND PRESSES

**Sponsor body: National Safety Council*

E. E. Barney

SAFETY CODE FOR TEXTILES

**Sponsor body: National Safety Council*

G. L. Warfield

SAFETY CODE FOR VENTILATION

**Sponsor body: American Society of Heating and Ventilating Engineers*

P. A. McKittrick (L. H. Eggert, *Alternate*)

SAFETY CODE FOR WALKWAY SURFACES

**Sponsor bodies: American Institute of Architects and National Safety Council*

W. M. Graff

SAFETY CODE FOR WINDOW WASHING

**Sponsor body: National Safety Council*

W. G. Boyle

SAFETY CODE ON COLORS FOR IDENTIFICATION OF GAS MASK CANISTERS

**Sponsor body: National Safety Council*

L. C. Lichty

SAFETY CODE FOR PROTECTION OF INDUSTRIAL WORKERS IN FOUNDRIES

**Sponsor bodies: American Foundrymen's Association and National Founders Association*

H. M. LANE

SAFETY CODE FOR PREVENTION OF DUST EXPLOSIONS

**Sponsor bodies: National Fire Protection Association and U. S. Department of Agriculture*

J. H. MORROW

SAFETY CODE FOR WORK IN COMPRESSED AIR

**Sponsor body: International Association of Industrial Accident Boards and Commissions*

L. J. EIBSEN

SAFETY CODE FOR EXHAUST SYSTEMS

**Sponsor body: International Association of Industrial Accident Boards and Commissions*

A.S.M.E. Representative

J. C. HARDIGG (E. H. DECONINGH, *Alternate*)

SPECIFICATIONS AND METHODS OF TEST FOR SAFETY GLASS

**Sponsor bodies: National Bureau of Casualty and Surety Underwriters and National Bureau of Standards*

T. A. WALSH, JR. (G. E. SANFORD, *Alternate*)

LOW VOLTAGE ELECTRICAL HAZARD

Special Committee of the American Society of Safety Engineers—Engineering Section, National Safety Council

J. P. JACKSON

A.S.M.E. Representatives on Other Activities

See also A.S.M.E. Representatives on Other Research Committees, etc., pages 21, 25, 27, and 28

(Dates in parentheses denote expiration of terms)

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

SECTION M, ENGINEERING

(To be appointed)

AMERICAN BUREAU OF WELDING

JAMES PARTINGTON

COMMITTEE ON STRUCTURAL STEEL WELDING

G. A. ORROK

AMERICAN ENGINEERING COUNCIL

(One year term)

R. E. FLANDERS, *Chairman* — PAUL DOTY
L. P. ALFORD — A. A. POTTER
D. ROBERT YARNALL
Alternates: H. V. COES, J. W. ROE

AMERICAN STANDARDS ASSOCIATION

C. W. SPICER (1937) — *Alternates:* V. R. WILLOUGHBY, C. B. LEPAGE

AMERICAN YEAR BOOK CORPORATION

C. E. DAVIES

THE ENGINEERING FOUNDATION

A. E. WHITE (1936) — W. H. FULWEILER (1937)
D. ROBERT YARNALL (1937)

EDUCATIONAL RESEARCH COMMITTEE

H. N. DAVIS

RESEARCH PROCEDURE COMMITTEE

W. H. FULWEILER

ENGINEERING SOCIETIES EMPLOYMENT SERVICE

NATIONAL BOARD

C. E. DAVIES

METROPOLITAN BOARD

ERNEST HARTFORD, *Chairman*

ENGINEERS' COUNCIL FOR PROFESSIONAL DEVELOPMENT

W. E. WICKENDEN (1935) — C. F. HIRSHFELD (1936)
To be appointed (1937)

ENGINEERS NATIONAL RELIEF FUND

W. A. SHOUDY

JOHN FRITZ MEDAL BOARD OF AWARD

R. V. WRIGHT (1935) — A. A. POTTER (1937)
C. N. LAUER (1936) — PAUL DOTY (1938)

GANTT MEDAL BOARD OF AWARD

J. W. ROE (1935) — D. B. PORTER (1936)
F. E. RAYMOND (1937)

DANIEL GUGGENHEIM MEDAL FUND, INC.

H. I. CONE (1935) — E. E. ALDRIN (1937)
P. H. ADAMS (1936) — B. M. WOODS (1938)

JOSEPH A. HOLMES SAFETY ASSOCIATION

O. P. HOOD

HOOVER MEDAL BOARD OF AWARD

AMBROSE SWASEY (1935) — To be appointed (1937)
C. N. LAUER (1939)

INTERNATIONAL CONGRESSES OF PHOTOGRAPHY, JULY, 1935

AMERICAN NATIONAL COMMITTEE

EDWARD EPSTEAN

INTERNATIONAL ELECTROTECHNICAL COMMISSION

U. S. NATIONAL COMMITTEE

H. N. DAVIS — FRANCIS HODGKINSON
E. C. HUTCHINSON
Alternates: C. HAROLD BERRY, PAUL DISERENS

MECHANICAL STANDARDS ADVISORY COUNCIL

W. S. MONROE

NATIONAL BUREAU OF ENGINEERING REGISTRATION

J. H. LAWRENCE

NATIONAL FIRE WASTE COUNCIL

H. O. LACOUNT — J. A. NEALE

NATIONAL MANAGEMENT COUNCIL

J. A. Piacitelli (1936)—G. W. Kelsey, *Alternate*
C. W. Lytle (1937)
H. V. Coes (1938)—J. W. Roe, *Alternate*

NATIONAL RESEARCH COUNCIL

DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH

W. R. Webster (1935) F. M. Farmer (1936)
W. D. Ennis (1935) D. S. Jacobus (1936)

C. E. Davies, *Secretary, Ex-Officio*

NEW YORK STATE JOINT LEGISLATION COMMITTEE

Sigmund Firestone J. H. Lawrence

ALFRED NOBEL PRIZE

A. M. Greene, Jr.

PROFESSIONAL ENGINEERS COMMITTEE ON UNEMPLOYMENT

W. T. Conlon J. N. Landis G. B. Pegram
H. N. Davis A. H. Meyer Walter Rautenstrauch

STUDY OF COAL

JOINT COMMITTEE WITH THE A.I.M.E.

A. D. Bailey H. Drake Harkins A. L. Penniman, Jr.
E. H. Barry J. C. Hobbs E. B. Ricketts
F. M. Gibson E. H. Tenney

UNITED ENGINEERING TRUSTEES, INC.

H. V. Coes (1936) D. Robert Yarnall (1937)
W. L. Batt (1938)

ENGINEERING SOCIETIES MONOGRAPHS COMMITTEE

S. W. Dudley W. A. Shoudy

WESTERN SOCIETY OF ENGINEERS

WASHINGTON AWARD

G. F. Gebhardt (1935) C. B. Nolte (1936)

Awards

The following paragraphs deal with the awards, scholarships, and loan funds which come within the jurisdiction of the A.S.M.E. The Society also participates with other engineering societies in a number of joint awards.

Honorary Membership, to which persons of acknowledged professional eminence are elected by unanimous vote of Council under the provisions of the By-Laws and Rules. A list of honorary members is given on page RI-32.

Life Membership, which may be conferred by the Council for distinguished service to the Society; or secured by a member by payment for an annuity in accordance with the provisions of the By-Laws.

A.S.M.E. Medal, established by the Society in 1920 to be presented for distinguished service in engineering and science. May be awarded for general service in science having possible application in engineering.

Holley Medal, instituted and endowed in 1924 by George I. Rockwood, Past Vice-President of the Society, to be bestowed for some great and unique act of genius of engineering nature that has accomplished a great and timely public benefit.

Worcester Reed Warner Medal, provision for which was made in the will of Worcester Reed Warner, Honorary Member of the Society, is a gold medal to be bestowed on the author of the most worthy paper received, dealing with progressive ideas in mechanical engineering or efficiency in management.

Melville Medal, established in 1914 by the bequest of Rear-Admiral George W. Melville, Honorary Member and Past-President of the Society, to be presented for an original paper or thesis of exceptional merit, presented to the Society for discussion and publication, to encourage excellence in papers. The medal may be presented annually.

Spirit of Saint Louis Medal, endowed by members of the Society and citizens residing in St. Louis, Mo., to be awarded for meritorious service in the field of aeronautical engineering. This medal will be awarded at the discretion of the Council of the Society at approximately three-year periods upon the recommendation of a Spirit of Saint Louis Medal Board of Award made up of six members, each appointed for a term of nine years and the terms of two members expiring at each three-year period. The St. Louis Section and the Aeronautic Division will each be responsible for the nomination of three members.

Junior Award, annual cash award of $50, established in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with an engraved certificate, for the best paper or thesis submitted by a Junior Member.

Student Awards, two annual cash awards of $25 each, established in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with engraved certificates, for the best papers or theses submitted by members of Student Branches.

Charles T. Main Award, annual cash award of $150, established in 1919 from a fund created by Charles T. Main, Past-President of the Society, to be awarded to a student of engineering, preferably a member of a Student Branch of the Society, for the best paper within the general subject of the "Influence of the Profession Upon Public Life." The exact subject is assigned by the Committee on Awards, subject to the approval of the Council, and is announced each year through the Honorary Chairmen of the Student Branches.

SCHOLARSHIPS AND LOAN FUNDS

Max Toltz: Loan Fund of $15,000 established by Major Max Toltz, former member of the Council of the Society, the income to be used for assistance to students.

John R. Freeman: Fund of $25,000 established in 1926 by John R. Freeman, Past-President of the Society, the income to be used for travel scholarships and research.

Woman's Auxiliary: Scholarship or Fellowship offered by the Woman's Auxiliary to the Society to assist sons and daughters of members or worthy students of mechanical engineering.

RECIPIENTS OF AWARDS

The names of the recipients of the different awards to date are given in the following lists, together with the dates of presentation, and the services or papers for which the awards were made. There were no awards for the years not listed.

A.S.M.E. Medal

1921 Hjalmar Gotfried Carlson, in recognition of the services rendered the Government because of his invention and part in the production of 20,000,000 Mark III drawn steel booster casings used principally as a component of 75-mm high-explosive shells, but also used extensively in gas shells and bombs

1923 Frederick Arthur Halsey, for his paper describing the premium system of wage payments presented before the Society at the Providence Meeting in 1891, as the adoption of the methods there proposed has had a profound effect toward harmonizing the relations of worker and employer

1923 John Ripley Freeman, for his eminent service in engineering and manufacturing by his meritorious work in fire prevention and the preservation of property

1926 R. A. Millikan, in recognition of his contributions to science and engineering

1927 Wilfred Lewis, for his contributions to the design and construction of gear teeth

1928 Julian Kennedy, for his services and contributions to the iron and steel industry

1929 William LeRoy Emmet, for his contributions in the development of the steam turbine, electric propulsion of ships, and other power-generating apparatus

1931 Albert Kingsbury, for his research and development work in the field of lubrication

1933 Ambrose Swasey, for his contributions to the advancement of the engineering profession and for his part in the development of the turret lathe and the astronomical telescope

1934 Willis H. Carrier, in recognition of his research and development work in air-conditioning.

Holley Medal

1924 Hjalmar Gotfried Carlson, for his inventions and processes which made possible the timely production of drawn steel booster casings for artillery ammunition, thereby aiding victory in the World War

1928 Elmer Ambrose Sperry, for achievements and inventions that have advanced the naval arts, including the gyroscope that has freed navigation from the dangers of the fluctuating magnetic compass

1929 Baron Chuzaburo Shiba, for his contributions to knowledge through fundamental research, including the field of aerodynamics, by the development of ultra-rapid kinematographic methods.

Worcester Reed Warner Medal

1933 Dexter S. Kimball, for his contributions to efficiency in management as exemplified by his recently revised "Principles of Industrial Organization" and by his many articles, engineering society papers, and public addresses

1934 Ralph E. Flanders, for his contributions to a better understanding of the relationship of the engineer to economic problems and social trends as exemplified by the many papers which he has presented.

Melville Medal

1927 Leon P. Alford, "Laws of Manufacturing Management"

1929 Joseph Wickham Roe, "Principles of Jig and Fixture Practice"

1930 Herman Diederichs and William D. Pomeroy, "The Occurrence and Elimination of Surge or Oscillating Pressure in Discharge Lines From Reciprocating Pumps"

1931 Arthur E. Grunert, "Comparative Performance of a Pulverized-Coal-Fired Boiler Using Bin System and Unit System of Firing"

1932 Alexey J. Stepanoff, "Leakage Loss and Axial Thrust in Centrifugal Pumps"

1933 William E. Caldwell, "Characteristics of Large Hell Gate Direct-Fired Boiler Units."

Spirit of Saint Louis Medal

1929 Daniel Guggenheim, founder of the Guggenheim Fund for the Promotion of Aeronautics

1932 Paul Litchfield, for his work in encouraging and sponsoring airship design and construction in this country.

Junior Award

1915 Ernest O. Hickstein, "Flow of Air Through Thin Plate Orifices"

1916 L. B. McMillan, "The Heat Insulating Properties of Commercial Steam-Pipe Coverings"

1919 E. D. Whalen, "Properties of Airplane Fabrics"

1921 S. Logan Kerr, "Moody Ejector Turbine"

1922 R. H. Heilman, "Heat Losses From Bare and Covered Wrought-Iron Pipe at Temperatures Up to 800 Degrees Fahrenheit"
F. L. Kallam, "Preliminary Report on the Investigation of the Thermal Conductivity of Liquids"

1923 S. S. Sanford and S. Crocker, "The Elasticity of Pipe Bends"

1924 R. H. Heilman, "Heat Losses Through Insulating Material"

1925 Gilbert S. Schaller, "An Investigation of Seattle as a Location for a Synthetic Foundry Industry"

1927 Wm. M. Frame, "Stresses Occurring in the Walls of an Elliptical Tank Subjected to Low Internal Pressure"

1928 M. D. Aisenstein, "A New Method of Separating the Hydraulic Losses in a Centrifugal Pump"

1929 Arthur M. Wahl, "Stresses in Heavy, Closely Coiled Helical Springs"

1930 Ed Sinclair Smith, Jr., "Quantity-Rate Fluid Meters"

1931 M. K. Drewry, "Radiant-Superheater Developments"

1932 Edmond M. Wagner, "Frictional Resistance of a Cylinder Rotating in a Viscous Fluid Within a Coaxial Cylinder"

1933 Townsend Tinker, "Surface Condenser Design and Operating Characteristics"

1934 John I. Yellott, Jr., "Supersaturated Steam."

Student Award

1916 Boynton M. Green, Stanford University, "Bearing Lubrication"
Howard Stevens, Rensselaer Polytechnic Institute, "An Investigation of the Dynamic Pressure on Submerged Flat Plates"
M. Adam, Louisiana State University, "The Adaptability of the Internal Combustion Engine to Sugar Factories and Estates"

1917 H. R. Hammond and C. W. Holmberg, Pennsylvania State College, "Study of Surface Resistance With Glass as the Transmission Medium"

1919 C. F. Leh and F. G. Hampton, Stanford University, "An Experimental Investigation of Steel Belting"
W. E. Helmick, Stanford University, "An Experimental Investigation of Steel Belting"

1920 Howard G. Allen, Cornell University, "Wire Stitching Through Paper"

1921 Karl H. White, University of Kansas, "Forces in Rotary Motors"
Richard H. Morris and Albert J. R. Houston, University of California, "A Report Upon an Investigation of the Herschel Type of Improved Weir"

1923 Charles F. Olmstead, University of Minnesota, "Oil Burning for Domestic Heating"
H. E. Doolittle, University of California, "The Integrating Gate: A Device for Gaging in Open Channels"

1924 George Stuart Clark, Stanford University, "Two Methods Used for the Determination of the Gasoline Content of Absorption Oils in Absorption Plants"
L. J. Franklin and Charles H. Smith, Stanford University, "The Effect of Inaccuracy of Spacing on the Strength of Gear Teeth"

1925 Harry Pease Cox, Jr., Rensselaer Polytechnic Institute, "A Study of the Effect of End Shape on the Towing Resistance of a Barge Model"
W. S. Montgomery, Jr., and E. Ray Enders, Jr., Pennsylvania State College, "Some Attempts to Measure the Drawing Properties of Metals"

1926 R. E. Peterson, University of Illinois, "An Investigation of Stress Concentration by Means of Plaster of Paris Specimens"
Cecil G. Heard, University of Toronto, "Pressure Distribution Over U. S. A. 27 Aerofoil With Square Wing Tips Model Tests"

1927 Alfred H. Marshall, Princeton, "Evaporative Cooling"
Roger Irwin Eby, University of Washington, "Measurement of the Angular Displacement of Flywheels"

1928 Clarence C. Franck, Johns Hopkins University, "Condition Curves and Reheat Factors for Steam Turbines"

1929 Frank Vernon Bistrom, University of Washington, "An Investigation of a Rotary Pump"
William Wallace White, University of Washington, "An Investigation of a Rotary Pump"

1930 Gerard Eden Claussen, Polytechnic Institute of Brooklyn, "High-Temperature Oxidation of Steel"
Harold L. Adams and Richard L. Stith, University of Washington, "A Wind Tunnel for Undergraduate Laboratory Experiments"

1931 Jules Podnossoff, Polytechnic Institute of Brooklyn, "Pressure and Energy Distribution in Multi-Stage Steam Turbines Operating Under Varying Conditions"

1932 H. E. Foster, Jr., University of Tennessee, "Factors Affecting Spray Pond Design"
William A. Mason, Stanford University, "An Experimental Investigation of the Flame Propagation in Internal-Combustion Engines"

1933 Hugo V. Cordiano, Polytechnic Institute of Brooklyn, "Thermal Analysis of Lithium-Magnesium System of Alloys"
James A. Ostrand, Jr., Princeton University, "Sudden Enlargement in the Open Channel"

1934 H. Reynolds Hudson, Georgia School of Technology, "Dynamic Balance and Functional Utility Applied to Automotive Design."

Charles T. Main Award

1925 Clement R. Brown, Catholic University of America. Subject: "The Influence of the Locomotive on the Unity of the United States"

1926 W. C. Saylor, Johns Hopkins University. Subject: "The Effect of the Cotton Gin Upon the History of the United States During Its First Seventy Years"

1927 No award. Subject: "Scientific Management and Its Effect Upon the Industries"

1928 Robert M. Meyer, Newark College of Engineering. Subject: "Scientific Management and Its Effect Upon Manufacturing"

1930 Jules Podnossoff, Polytechnic Institute of Brooklyn. Subject: "The Value of the Safety Movement in the Industries"

1931 Robert E. Klise, University of Michigan. Subject: "Interchangeability—Its Development and Significance in Industry"
1932 Marshall Anderson, University of Michigan. Subject: "Apprenticeship and Vocational Training"
1933 George D. Wilkinson, Jr., Newark College of Engineering. Subject: "Progress in the Prevention of Smoke and Atmospheric Pollution"
1934 Philip P. Self, Colorado State College, "Air Conditioning—Its Practicability and Relation to Public Welfare."

Freeman Travel Scholarship

1927 Herbert N. Eaton
1928 Blake R. Van Leer
1929 Robert T. Knapp
1931 Reginald Whitaker
1932 G. Ross Lord
1933, 1934 H. J. Casey.

Honorary Members

HONORARY MEMBERS IN PERPETUITY

Alexander Lyman Holley, Founder of the Society. Died 1882.
John Edson Sweet, Founder of the Society. Died 1916.
Henry Rossiter Worthington, Founder of the Society. Died 1880.

DECEASED HONORARY MEMBERS

	ELECTED	DIED
Horatio Allen	1880	1889
Sir William Arrol	1905	1913
Sir Benjamin Baker	1886	1907
Johann Bauschinger	1884	1893
Sir Henry Bessemer	1891	1898
Sir Frederick Joseph Bramwell	1884	1903
John Alfred Brashear	1908	1920
Gustave Canet	1900	1908
Andrew Carnegie	1907	1919
Daniel Kinnear Clark	1882	1896
Rudolph Julius Emmanuel Clausius	1882	1888
Sir John Coode	1889	1892
Peter Cooper	1882	1883
Carl Gustaf Patrick de Laval	1912	1913
Rudolph Diesel	1912	1913
James Dredge	1886	1906
Victor Dwelshauvers-Dery	1886	1913
Thomas Alva Edison	1904	1931
Alexandre Gustave Eiffel	1889	1923
Marshal Ferdinand Foch	1921	1929
Sir Charles Douglas Fox	1900	1921
John Ripley Freeman	1932	1932
John Fritz	1900	1913
Major-General George Washington Goethals	1917	1928
Franz Grashof	1884	1893
Rear-Admiral Robert Stanislaus Griffin	1920	1933
Otto Hallauer	1882	1883
Charles Haynes Haswell	1905	1907
Friedrich Gustav Herrmann	1884	1907
Gustav Adolph Hirn	1882	1890
Joseph Hirsch	1889	1901
Ira N. Hollis	1928	1930
Robert Woolston Hunt	1920	1923
Benjamin Franklin Isherwood	1894	1915
Henri Léauté	1891	1916
Erasmus Darwin Leavitt	1915	1916
Anatole Mallet	1912	1919
Charles H. Manning	1913	1919
Rear-Admiral George Wallace Melville	1910	1912
The Honorable Sir Charles Algernon Parsons	1920	1931
Charles Talbot Porter	1890	1910
Auguste C. E. Rateau	1919	1930
Sir Edward J. Reed	1882	1906
Franz Reuleaux	1882	1905
Calvin Winsor Rice	1931	1934
Palmer C. Ricketts	1931	1934
Henri Adolphe-Eugene Schneider	1882	1898
C. William Siemans	1882	1883
Viscount Ei-ichi Shibusawa	1929	1931
Henry Robinson Towne	1921	1924
Henri Tresca	1882	1885
William Cawthorne Unwin	1898	1933
Oskar von Miller	1912	1934
Francis A. Walker	1886	1897
Worcester Reed Warner	1925	1929
George Westinghouse	1897	1914
Sir William Henry White	1900	1913
Sir Alfred Fernandez Yarrow	1914	1932

LIVING HONORARY MEMBERS

	ELECTED
Sir John Audley Frederick Aspinall	1911
William Wallace Atterbury	1925
Mortimer Elwyn Cooley	1928
William Frederick Durand	1934
Charles de Fréminville	1919
Nathanael Greene Herreshoff	1921
Herbert Clark Hoover	1925
David Schenck Jacobus	1934
Masawo Kamo	1929
Henri Le Chatelier	1927
Grande Ufficiale Ing. Pio Perrone	1920
Charles M. Schwab	1918
Ambrose Swasey	1916
Elihu Thomson	1930
Samuel Matthews Vauclain	1920
Right Honorable Lord Weir	1920
Orville Wright	1918

Past-Presidents

A list of past vice-presidents, managers, treasurers, and secretaries will be found in the 1930 Record and Index, pages 10–12. Dates in parentheses denote year of death.

Alexander Lyman Holley, *Chairman of the Preliminary Meeting for Organization of The American Society of Mechanical Engineers* (1882)

1880–1882 Robert Henry Thurston (1903)
1883 Erasmus Darwin Leavitt (1916)
1884 John Edson Sweet (1916)
1885 Josephus Flavius Holloway (1896)
1886 Coleman Sellers (1907)
1887 George H. Babcock (1893)
1888 Horace See (1909)
1889 Henry Robinson Towne (1924)
1890 Oberlin Smith (1926)
1891 Robert Woolston Hunt (1923)
1892 Charles Harding Loring (1907)
1893–1894 Eckley Brixton Coxe (1895)
1895 Edward F. C. Davis (1895)
1895 Charles Ethan Billings (1920)
1896 John Fritz (1913)
1897 Worcester Reed Warner (1929)
1898 Charles Wallace Hunt (1911)
1899 George Wallace Melville (1912)
1900 Charles Hill Morgan (1911)
1901 Samuel T. Wellman (1919)
1902 Edwin Reynolds (1909)
1903 James Mapes Dodge (1915)
1904 Ambrose Swasey
1905 John Ripley Freeman (1932)
1906 Frederick Winslow Taylor (1915)
1907 Frederick Remsen Hutton (1918)
1908 Minard Lafever Holman (1925)
1909 Jesse Merrick Smith (1927)
1910 George Westinghouse (1914)
1911 Edward Daniel Meier (1914)
1912 Alexander Crombie Humphreys (1927)
1913 William Freeman Myrick Goss (1928)
1914 James Hartness (1934)
1915 John Alfred Brashear (1920)
1916 David Schenck Jacobus
1917 Ira Nelson Hollis (1930)
1918 Charles Thomas Main
1919 Mortimer Elwyn Cooley
1920 Fred J. Miller
1921 Edwin S. Carman
1922 Dexter Simpson Kimball
1923 John Lyle Harrington
1924 Frederick Rollins Low
1925 William Frederick Durand
1926 William Lamont Abbott
1927 Charles M. Schwab
1928 Alex Dow
1929 Elmer Ambrose Sperry (1930)
1930 Charles Piez (1933)
1931 Roy V. Wright
1932 Conrad N. Lauer
1933 A. A. Potter
1934 Paul Doty

Index to Committees

Abbreviations and Symbols....RI-23
Acoustical Measurements and Terminology.RI-26
Administrative, Special....RI-8
Advertising Revenue....RI-7
Advisory Board on Standards and Codes....RI-7
Advisory Board on Technology....RI-7
Aeronautic Division, Executive....RI-9
Alfred Nobel Prize....RI-30
Allowances and Tolerances for Cylindrical Parts and Limit Gages....RI-25
American Association for the Advancement of Science....RI-29
American Bureau of Welding....RI-29
American Engineering Council....RI-29
American Standards Association....RI-29
American Year Book Corporation....RI-29
Amusement Parks, Safety Code....RI-28
API-ASME on Unfired Pressure Vessels....RI-7
Applied Mechanics Division, Executive....RI-9
Ash Removal From Powdered-Coal Furnaces....RI-21
Automatic Oil Pipe Line Pumping Stations.RI-21
Awards....RI-5
Ball and Roller Bearings....RI-22
Bearings, Heavy-Duty Anti-Friction....RI-21
Biography Advisory....RI-8
Board of Review (Delinquent Members)....RI-7
Boiler Code....RI-6
Boiler Code Conference....RI-6
Boiler Code, Executive....RI-6
Boiler Feedwater Studies....RI-20
Boiler Furnace Refractories....RI-21
Boiler Furnaces, Absorption of Radiant Heat in....RI-21
Bolt, Nut, and Rivet Proportions....RI-22
Bond Issue....RI-7
Building Code....RI-26
Calvin W. Rice Memorial....RI-8
Capital Goods Industries....RI-7
Care of Boilers and Pressure Vessels in Service....RI-6
Cast Iron Pipe....RI-25
Certificates of Indebtedness....RI-7
Chucks and Chuck Jaws....RI-23
Cinders and Fly Ash....RI-9
Circular Forming Tools and Holders....RI-23
Citizenship (Manual on)....RI-7
Coal, Classification of....RI-25
Coal, Clean Bituminous....RI-26
Coal-Handling Equipment....RI-26
Coal Mines, Drainage of....RI-25
Coal, Study of....RI-30
Colleges, Relations With....RI-5
Compressed Air, Work in, Safety Code....RI-29
Compressors and Blowers, Centrifugal and Turbo....RI-27
Compressors and Blowers, Displacement....RI-27
Condensers, Water Heating, and Cooling Equipment....RI-27
Condenser Tubes....RI-20
Constitution and By-Laws....RI-5
Conveyors and Conveying Machinery, Safety Code....RI-28
Cooperative Relations....RI-7
Correlating Committee, Safety Code....RI-28
Corrosion....RI-21
Cotton Seed Processing....RI-21
Council, Executive....RI-5
Council, Special....RI-7
Cranes, Derricks, and Hoists, Safety Code..RI-28
Cut and Ground Thread Taps....RI-23
Cutting Metals....RI-10, 20
Cutting Tools, Single Point....RI-23
Daniel Guggenheim Medal Fund, Inc....RI-29
Definitions and Values, Power Test Codes..RI-26
Delinquent Members, Board of Reviews....RI-7
Designations and Working Ranges of Machine Tools....RI-23
Design of Boilers, Special....RI-7
Displacement Pumps, Reciprocating Steam-Driven....RI-26
Drawings and Drafting Room Practice....RI-23
Drill Bushings....RI-23
Dust Explosions, Safety Code....RI-29
Dust Separating Apparatus....RI-27
Economic Status of the Engineer....RI-7
Education and Training for the Industries..RI-6
Effect of Temperature on Properties of Metals....RI-20
Electrical Definitions....RI-26
Electric Motor Frame Dimensions....RI-24
Electric Sockets and Lamp Bases, Rolled Threads for Screw Shells of....RI-24
Electric Welding....RI-25
Electric Welding Dies and Electrode Holders RI-23
Elevator Safeties....RI-20
Elevators, Safety Code....RI-28
Employment (Member Relations)....RI-7
Engineering Foundation....RI-29
Engineering History....RI-7
Engineering Registration, National Bureau of....RI-29
Engineering Societies Employment Service..RI-29
Engineering Societies Monographs Committee....RI-30
Engineers' Council for Professional Development....RI-29
Engineers' National Relief Fund....RI-29
Evaporating Apparatus....RI-27
Exhaust Systems, Safety Code....RI-29
Feedwater Studies, Boiler....RI-20
Finance....RI-5
Fire Tests of Building Construction and Materials....RI-25
Floor Openings, Railings, and Toe Boards, Safety Code....RI-28
Fluid Flow, Velocity Measurement of....RI-21
Fluid Meters....RI-20
Forest Fire Protection....RI-26
Forging and Hot Metal Stamping, Safety Code....RI-28
Foundry Equipment and Supplies....RI-25
Foundry Practice....RI-9
Freeman Scholarship....RI-7
Fritz Medal Board of Award....RI-29
Fuel Oils....RI-25
Fuels Division, Executive....RI-9
Fuels, Gross and Net Calorific Values of....RI-27
Fuels, Power Test Codes....RI-26
Furnaces, Dimensioning for Burning Solid Fuel....RI-25
Gages, Limit, and Cylindrical Parts, Allowances and Tolerances for....RI-25
Gages, Pressure and Vacuum....RI-25
Gages, Wire and Sheet Metal....RI-24
Gantt Medal Board of Award....RI-29
Gas Mask Canisters, Colors for Identification of, Safety Code....RI-28
Gas Producers....RI-27
Gas Transportation....RI-10
Gears....RI-22
Gear Teeth, Strength of....RI-20
General Instructions, Power Test Codes....RI-26
George Westinghouse Bust....RI-8
George Westinghouse Memorial....RI-8
Graphic Presentation....RI-24
Guggenheim Medal Fund....RI-29
Heating Boilers....RI-6
Heat Transfer....RI-11
Highway Research....RI-21
Holmes Safety Association....RI-29
Honors....RI-8
Hoover Medal Board of Award....RI-29
Hose Couplings, Screw Threads for....RI-24
Hydraulic Division, Executive....RI-9
Hydraulic Prime Movers....RI-27
Identification of Piping Systems....RI-22
Industrial Aerodynamics....RI-9
Industrial Workers in Foundries, Protection of, Safety Code....RI-29
Industries, Education and Training for....RI-6
Inspection of Boilers, Rules for....RI-7
Instruments and Apparatus, Power Test Code....RI-27
Internal Combustion Engines....RI-27
International Congresses of Photography..RI-29
International Electrotechnical Commission..RI-29
Iron and Steel Bars, Stock Sizes, Shapes and Lengths for Hot and Cold Finished....RI-24
Iron and Steel Division, Executive....RI-9
John Fritz Medal Board of Award....RI-29
Joseph A. Holmes Safety Association....RI-29
Junior Participation....RI-8
Ladders, Safety Code....RI-28
Laundry Machinery, Safety Code....RI-28
Leather Belting....RI-25
Library....RI-6
Lighting Factories, Mills, etc., Safety Code..RI-28
Loading Platforms, Standard Height for....RI-7
Local Sections, Executive....RI-12
Local Sections, Standing....RI-5
Locomotives, Boilers of....RI-6
Logging and Sawmill Machinery, Safety Code....RI-28
Low Voltage Electrical Hazard....RI-29
Lubrication....RI-20
Lubrication Engineering....RI-10
Machine Design....RI-10
Machine Pins....RI-23
Machinery for Compressing Air, Safety Code....RI-28
Machine Shop Practice Division, Executive..RI-9
Machine Tapers....RI-22
Machine Tool Elements....RI-22
Management Division, Executive....RI-10
Manhole Frames and Covers....RI-25
Manual of Practice....RI-8
Manual on Citizenship....RI-7
Materials Handling Division, Executive....RI-10
Material Specifications....RI-6
Measures of Management....RI-21
Mechanical Power-Transmission Apparatus, Safety Code....RI-27
Mechanical Springs....RI-20
Mechanical Standards Advisory Council..RI-29
Meetings and Program....RI-5
Membership....RI-5
Metals, Cutting of....RI-10, 20
Metals, Effect of Temperature on Properties of....RI-10, 20
Metals, Fatigue Phenomena of....RI-22
Milling Cutters....RI-23
Milling Machine Tables....RI-23
Miniature Boilers....RI-7
Multiple Spindle Drilling Heads....RI-23
National Bureau of Engineering Registration....RI-29
National Defense Division, Executive....RI-10
National Fire Waste Council....RI-29
National Management Council....RI-30
National Research Council....RI-30
New York State Joint Legislation Committee....RI-30
Nobel Prize....RI-30
Nomenclature for Machine Tools....RI-23
Nominating....RI-8
Non-Ferrous Metals and Alloys....RI-22
Oil and Gas Power Division, Executive....RI-10
Oil Transportation....RI-10
Oil Wells, Rotary Drilling Prime Movers..RI-21
Paper and Pulp Mills, Safety Code....RI-28
Petroleum Division, Executive....RI-10
Petroleum Production....RI-10
Petroleum Refining....RI-10

Petroleum Specifications....RI-26
Photography, International Congresses....RI-29
Pipe and Tubing, Wrought Iron and Wrought Steel....RI-24
Pipe Flanges and Fittings....RI-22
Pipe Threads....RI-24
Piping Systems, Identification of....RI-22
Plain and Lock Washers....RI-23
Plumbing Equipment....RI-24
Policies and Budget....RI-8
Power Division, Executive....RI-10
Power Test Codes, Standing....RI-6
Power Test Codes, Technical....RI-26
Practice Manual....RI-8
Preferred Numbers....RI-26
Presses, Power, Foot, and Hand, Safety Code....RI-28
Pressure and Vacuum Gages....RI-25
Pressure Piping, Code for....RI-24
Pressure Vessels in Service, Care of....RI-6
Prime Movers for Rotary Drilling of Oil Wells....RI-21
Printing Industries Division, Executive....RI-11
Process Industries Division, Executive....RI-11
Professional Conduct....RI-6
Professional Divisions, Executive....RI-9
Professional Divisions, Standing....RI-5
Professional Engineers Committee on Unemployment....RI-30
Properties of Refractory Materials....RI-22
Public Affairs....RI-8
Publications, Special....RI-8
Publications, Standing....RI-5
Pulverized Fuel, Sampling in Moving Gas Stream....RI-21
Pumping Stations, Automatic Oil Pipe Line....RI-21
Pumps, Centrifugal and Rotary....RI-27
Punch Press Tools....RI-23
Pure Air....RI-9
Railroad Division, Executive....RI-11
Rating Rivers....RI-26
Refractories, Boiler Furnace....RI-21
Refractory Materials, Properties of....RI-22
Refrigerating Systems....RI-27
Registration of Engineers....RI-8
Relations With Colleges....RI-5
Representatives on Other Activities....RI-29
Research, Special....RI-20
Research, Standing....RI-6
Rice Memorial....RI-8
Rolled Threads for Screw Shells of Electric Sockets and Lamp Bases....RI-24
Rotating Electrical Machinery....RI-25
Rotating Tool Shanks....RI-23
Safety Glass, Specifications and Methods of, Test for....RI-29
Safety, Standing....RI-6
Safety, Technical....RI-27
St. Louis Medal Board of Award....RI-8
Sampling Pulverized Fuel in Moving Gas Stream....RI-21
Saws and Knives....RI-11
Scientific and Engineering Symbols and Abbreviations....RI-23
Screw Threads for Hose Couplings....RI-24
Screw Threads, Standardization and Unification of....RI-25
Shaft Couplings....RI-7
Shafting....RI-22
Sieves for Testing Purposes....RI-26
Small Tools and Machine Tool Elements....RI-22
Specifications, Material....RI-6
Speed-Responsive Governors....RI-27
Speeds of Machinery....RI-24
Spindle Noses and Collets for Machine Tools....RI-23
Spirit of St. Louis Medal Board of Award....RI-8
Splines and Splined Shafts....RI-23
Springs, Mechanical....RI-20
Standardization, Standing....RI-6
Standardization, Technical....RI-22
Standards and Codes, Advisory Board on....RI-7
Standard Ton of Refrigeration....RI-27
Standing....RI-5
Statistics in Engineering and Manufacturing....RI-7
Steam Boilers, Critical Pressure....RI-21
Steam Boilers in Service, Care of....RI-6
Steam Engines, Reciprocating....RI-26
Steam Generating Units, Stationary....RI-26
Steam Locomotives....RI-27
Steam Power Plants....RI-27
Steam, Thermal Properties of....RI-20
Steam Turbines....RI-26
Steel and Iron Bars, Stock Sizes, Shapes and Lengths for Hot and Cold Finished....RI-24
Strength of Vessels Under External Pressure....RI-21
Student Branches....RI-18
Study of Coal....RI-30
Surface Qualities....RI-25
Symbols and Abbreviations....RI-23
Technical, Special....RI-6, 20, 22, 26, 27
Technology, Advisory Board on....RI-7
Testing Petroleum Products and Lubricants....RI-26
Testing Wood....RI-26
Textile Division, Executive....RI-11
Textiles, Safety Code....RI-28
Tool Holders and Tool Post Openings....RI-22
Tool Steel....RI-26
Trackwork Manufacturing Materials....RI-26
Transmission Chains and Sprockets....RI-24
T-Slots....RI-22
Twist Drill Sizes....RI-23
Unfired Pressure Vessels, API-ASME....RI-7
Unfired Pressure Vessels, ASME Subcommittee on....RI-7
United Engineering Trustees....RI-30
Ventilation, Safety Code....RI-28
Walkway Surfaces, Safety Code....RI-28
Washers, Plain and Lock....RI-23
Water Hammer....RI-9
Welding (Boiler Code)....RI-7
Welding (Machine Shop Practice)....RI-10
Western Society of Engineers....RI-30
Westinghouse Bust....RI-8
Westinghouse Memorial....RI-8
Window Washing, Safety Code....RI-28
Wire and Sheet Metal Gages....RI-24
Wire Rope (Research)....RI-21
Wire Rope for Mines....RI-25
Wood Industries Division, Executive....RI-11
Wood Utilization....RI-22
Worm Gears....RI-20

IN TWO SECTIONS—SECTION TWO

TRANSACTIONS

of The American Society of Mechanical Engineers

SOCIETY RECORDS—Part II

(Part I of Society Records for the year 1935, containing Council and Committee Personnel and other general information, was issued as Section Two of the Transactions for February, 1935)

Calvin Winsor Rice (*Portrait*) RI-36

Life of Calvin Winsor Rice RI-37

Dr. Calvin W. Rice's Contribution to International Friendliness . . . . . *John H. Finley* RI-40

Memorial Notices RI-42

AUGUST, 1935

VOL. 57, NO. 6

Published by The American Society of Mechanical Engineers

TRANSACTIONS

of The American Society of Mechanical Engineers

Published on the tenth of every month, except March, June, September, and December

Publication Office, 20th and Northampton Streets, Easton, Pa.
Editorial Department at the Headquarters of the Society, 29 West Thirty-Ninth Street, New York, N. Y.

Includes Aeronautical Engineering

Members of Council, 1934–1935

PRESIDENT

RALPH E. FLANDERS

VICE-PRESIDENTS

Terms expire December, 1935

WILLIAM L. BATT
H. L. DOOLITTLE
ELY C. HUTCHINSON
ELLIOTT H. WHITLOCK

PAST-PRESIDENTS

Terms expire December

CHARLES M. SCHWAB 1935
ROY V. WRIGHT 1936
CONRAD N. LAUER 1937
A. A. POTTER 1938
PAUL DOTY 1939

VICE-PRESIDENTS

Terms expire December, 1936

EUGENE W. O'BRIEN
JAMES H. HERRON
HARRY R. WESTCOTT

MANAGERS

Terms expire December, 1935

R. L. SACKETT
ALEX D. BAILEY
JOHN A. HUNTER

Terms expire December, 1936

JAMES A. HALL
ERNEST L. OHLE
JAMES M. TODD

Terms expire December, 1937

BENNETT M. BRIGMAN
JULES W. HANEY
ALFRED IDDLES

TREASURER

ERIK OBERG

SECRETARY

C. E. DAVIES

Chairmen of Standing Committees of Council

AWARDS, W. L. BATT
CONSTITUTION AND BY-LAWS, H. H. SNELLING
EDUCATION AND TRAINING FOR THE INDUSTRIES, GEORGE A. SEYLER
FINANCE, WALTER RAUTENSTRAUCH
LIBRARY, E. P. WORDEN
LOCAL SECTIONS, W. L. DUDLEY
MEETINGS AND PROGRAM, R. I. REES
MEMBERSHIP, H. A. LARDNER
POWER TEST CODES, F. R. LOW
PROFESSIONAL CONDUCT, C. G. SPENCER
PROFESSIONAL DIVISIONS, W. A. SHOUDY
PUBLICATIONS, S. W. DUDLEY
RELATIONS WITH COLLEGES, W. L. ABBOTT
RESEARCH, G. M. EATON
SAFETY, W. M. GRAFF
STANDARDIZATION, C. W. SPICER

Committee on Publications

S. W. DUDLEY, *Chairman*
S. F. VOORHEES — W. F. RYAN, *Vice-Chairman*
G. F. BATEMAN — M. H. ROBERTS
EDITOR: GEORGE A. STETSON

Advisory Members

E. L. OHLE, ST. LOUIS, MO.
E. B. NORRIS, BLACKSBURG, VA.
A. J. DICKIE, SAN FRANCISCO, CALIF.
O. B. SCHIER, 2D (JUNIOR MEMBER)

BY-LAW: The Society shall not be responsible for statements or opinions advanced in papers or . . . printed in its publications (B2, Par. 3).

Entered as second-class matter March 2, 1928, at the Post Office at Easton, Pa., under the act of August 24, 1912. Price $1.50 a copy, $12.00 a year; to members and affiliates, $1.00 a copy, $7.50 a year. Changes of address must be received two weeks before they are to be effective on our mailing list. Please send old, as well as new, address.

Copyrighted, 1935, by THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

Memorial Notices

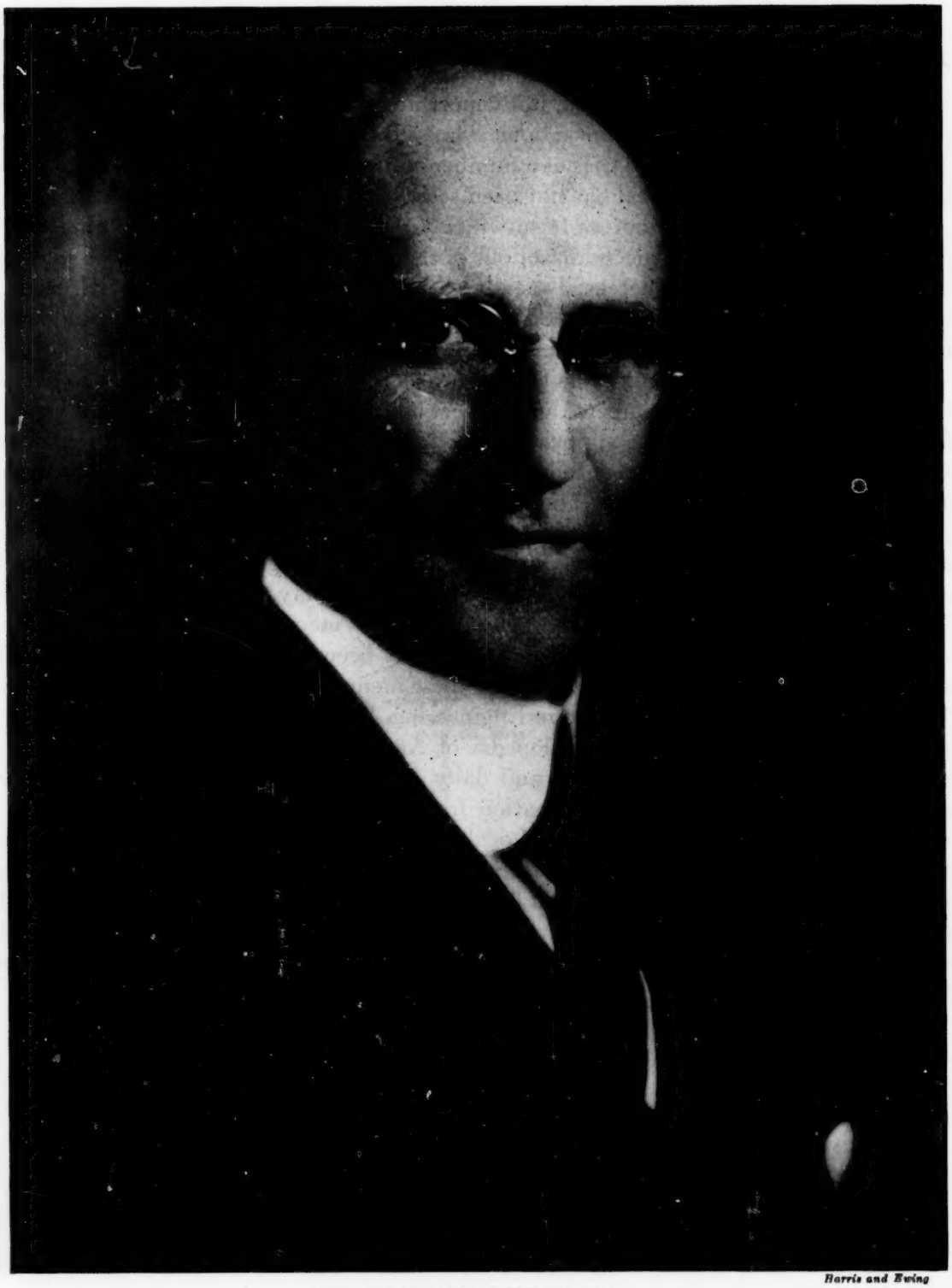
THIS group of memorial notices contains records of members of the Society whose deaths occurred prior to 1935, but concerning whom material was not ready for publication when the 1934 Transactions was issued. In fact, it is still necessary to delay the publication of obituaries of a number of members who died in 1933 and 1934 because sufficient information has not been secured.

There are a number of sources from which biographical material concerning engineers may be secured, but the Transactions of the A.S.M.E. is naturally the only place in which obituaries of all deceased members of the Society appear. Special care is therefore given to the preparation of these memorials and no obituary is published until as complete a record as possible has been secured.

The first source of information is the application file of the Society. In the case of those who have become members during recent years, the applications usually yield fairly complete records. The applications of those who became members in the early days of the Society are not as detailed, however, and these are the cases which often present considerable difficulty. The member may have been retired for some years prior to his death, so that business associates cannot be located, and in some cases members of his family cannot be found. Only by following up every slightest lead can even the main facts be assembled, and a long period may therefore elapse between the time of a member's death and the completion of his memorial.

The Society appreciates the assistance that has been given by relatives, business associates, and friends in the preparation of these memorials. It also acknowledges its debt to such sources as Who's Who in Engineering, Who's Who in America, and similar publications; the National Cyclopedia of American Biography and New International Year Book; the technical and daily press; and to engineering and other societies which have supplied information from their records.

Relatives, business associates, and Local Section and Student Branch officers are urged to notify the Society promptly of the deaths of members. Newspaper clippings or obituaries in any other form should be sent whenever available and the names and addresses of those who can supply further information should be furnished. A special form for supplying complete details will be forwarded by the Office of the Society upon request.



Harris and Ewing

CALVIN WINSOR RICE, 1868–1934

SECRETARY OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
1906–1934

Life of Calvin Winsor Rice

STRICKEN at his desk in the midst of his daily work, and in the building which his vision, enthusiasm, and energy had greatly helped to bring into being as a headquarters for the professional engineering societies of the United States, Calvin Winsor Rice, honorary member and for twenty-eight years secretary of The American Society of Mechanical Engineers died as a result of cerebral hemorrhage on October 2, 1934, a few hours after his removal to a nearby hospital.

Although he had won distinction as an electrical engineer in the practice of that profession prior to 1906, he will be most widely remembered for his services to The American Society of Mechanical Engineers as its secretary and for his vision and leadership in that office, which not only developed the Society greatly in numbers and in virility, but the entire engineering profession as well. For the spirit of cooperation with which he worked brought together the numerous individually organized groups of engineers in joint activities for common purposes. Of this spirit, the Engineering Societies Building in New York is a substantial and glorious monument, the physical embodiment of that vision and idealism that were Dr. Rice's most abundant gifts.

His Qualities in Professional-Society Organization

At the time Dr. Rice was made an honorary member of The American Society of Mechanical Engineers, in December, 1931, Dr. Karl T. Compton, president, Massachusetts Institute of Technology, delivered an address on his work in professional-society organization and paid a splendid tribute to this phase of his career. In the address, which was published in the January, 1932, issue of MECHANICAL ENGINEERING, Dr. Compton listed six essential qualifications for leadership in engineering-society organization which Dr. Rice possessed to a marked degree: (1) The ability and desire to cooperate with others; (2) the ability and judgment to recognize a good project when it was suggested; (3) initiative and drive to carry a project through to completion; (4) daring to undertake an audacious project, once convinced of its merit; (5) originality of thought; and (6) ability to organize, to delegate authority, and to spur others on to take active part in the affairs of the Society. Dr. Compton listed in addition four principles that were basic to Dr. Rice's philosophy of society organization and operation: (1) Unselfish cooperation with related societies; (2) planning actively for the future, so that development might not be haphazard, and so that opportunities for development in the desired direction might be quickly and firmly grasped when they presented themselves; (3) giving the most effective service to the members of the profession; and (4) leading the profession in rendering the most valuable possible service to society. The great number of engineers who knew Dr. Rice were conscious of these elements of his character and these principles of his philosophy.

His International Reputation

One characteristic feature of his career, resulting from these personal qualities, brought distinction to him, to the society he served, to his country, and to the profession of engineering. This was his relationship with engineers in this and other countries. In Great Britain, in Europe, in Mexico and South America, and in the Orient, Dr. Rice was well known in engineering circles. He traveled frequently in the discharge of his duties, and his office in New York was a focal point upon which converged the paths of engineers, eminent and obscure, who came to the United States. He was never too busy to give letters of introduction to engineers from abroad, or to engineers of this country planning to travel in Europe. Members of the Society frequently found that letters from Dr. Rice were more effective in gaining for them admission to some foreign plant or factory than similar letters from business men and engineers in the country they were visiting. The bread of hospitality and service he continually cast upon the waters wherever he happened to be returned in abundant measure to him by way of courtesies shown all over the world to those for whom he bespoke consideration. In students coming to this country Dr. Rice had great interest, going out of his way to have arranged for them itineraries of plant visits and advising them on educational and professional programs. These active international relationships brought Dr. Rice into high esteem, both here and abroad, and year by year added to the list of those beholden to him for help, advice, and innumerable lesser courtesies.

Significant Events in His Career

Dr. Rice was born at Winchester, Massachusetts, on November 4, 1868, the son of Edward Hyde and Lucy J. (Staples) Rice. After attending public schools in Boston, New Haven, and Winchester, he spent four years as a student in the Massachusetts Institute of Technology, from which he was graduated in 1890 with the degree of bachelor of science in electrical engineering. He then held, successively, positions as assistant engineer in the power and mining department of the Thomson-Houston Company in Lynn; as engineer in the General Electric Company in Schenectady; district engineer for that company in Cincinnati; engineer with the Silver Lake Mines in Colorado; consulting engineer for the Anaconda Copper Mining Company in Anaconda, Montana; electrical engineer of the Kings County Electric Light and Power Company, and later with the New York Edison Company and the Consolidated Subway Company; vice-president of the Nernst Lamp Company; consulting engineer with the General Electric Company in New York. It was from this rich and varied experience in electrical, hydraulic, and steam engineering, combined with managerial and administrative work, that Dr. Rice was called to the secretaryship of the A.S.M.E. in 1906.

The Engineering Societies Building in New York

Dr. Rice became a member of the A.I.E.E. in 1897, and was active in its affairs. In 1900 he joined the A.S.M.E. The story of his part in the bringing together of the engineering societies in a common headquarters building in New York and of the manner in which the funds were secured was told by Dr. Compton in his tribute to Dr. Rice previously referred to. Dr. Compton said:

> In 1902, as chairman of the Building Committee of the A.I.E.E., he called a dinner meeting of the committee, together with the president of the A.I.E.E., Prof. Charles F. Scott, and several others, to discuss plans for a modest building for the Institute, primarily to house the Latimer Clark Library, which had been presented to the Institute by Dr. S. S. Wheeler on the condition that a fireproof building be secured to house it. The Committee had, at that time, definite prospects of only about $250,000. When President Scott suggested that consideration be given to the possibility of a building for housing the four National Engineering Societies, with a common library and a common auditorium and individual rooms for the headquarters of each society, doubts were expressed as to whether the four societies could be brought into such a cooperative project.
>
> Strenuous efforts, which were at the last minute successful, were made to get Mr. Andrew Carnegie as a guest at the next annual dinner of the Institute. At this dinner President Scott outlined his ambitious plan and pointed out its fine features—including the library.
>
> The next day Mr. Carnegie asked Dr. Rice to come to his residence at five o'clock, and Dr. Rice, with characteristic thoughtfulness for others, as well as admiration for the lofty character of his president,

brought with him Professor Scott. There Mr. Carnegie asked them further about the work of the Institute, about the finances of the engineering societies, about the relation of the proposed building to the Engineers' Club (of which he was a member). Dr. Rice cleverly inferred that an obstacle in Mr. Carnegie's mind was the securing of the land, for the latter was not in the habit of buying the land on which the libraries which he donated were built. Dr. Rice then optimistically remarked that the Engineering Societies would be able to provide the land, whereupon Mr. Carnegie gave a cheerful smile and said, "If you can provide the land, I will put up the building." Dr. Rice was made chairman of the building fund.

Then the money had to be raised to buy the land; complications and difficulties in perfecting the organization and developing the plans had to be overcome. In the words of Professor Scott, "Mr. Rice's devotion to the idea of a building for the Institute and his skill in directing the early conference with Mr. Carnegie and his enthusiastic and faithful assistance in subsequent service to the Institute in carrying out the project were fundamental factors in the creation of the Engineering Societies Building and the separate building for the Engineers' Club."

The Kelvin Memorial

Among other incidents that illustrate Dr. Rice's faculty of initiating projects that bore fruitful results may be mentioned the Kelvin Memorial Window, in Westminster Abbey, told by Dr. Compton in the A.S.M.E. address, and the establishment of the Officers' Reserve of the United States Army, a brief account of which was contained in a letter by the late Gen. William Barclay Parsons published in *The Military Engineer* for March-April, 1931. Dr. Compton's account of the Kelvin Memorial Window is as follows:

> In 1910 Dr. Rice had a most unique experience, an account of which has never before been published. That year the A.S.M.E. made a return visit to the Institution of Mechanical Engineers at their Birmingham meeting. Remembering that in a modest way the A.S.M.E. had contributed to the memorial window in Westminster Abbey to Sir Benjamin Baker, honorary member, A.S.M.E., Dr. Rice wrote to the Dean asking if it would be permissible for the members of the A.S.M.E. when passing through London on a certain Sunday, to visit the Abbey and view the window. Not only was permission granted but a special service was arranged, with a sermon on engineering by the Bishop of Lewes, and on this occasion the "Hallelujah Chorus" was rendered by the full surpliced choir. The event was further made memorable by having the A.S.M.E. audience arranged in a semi-circle about the memorial window, the movable pulpit having been placed beside it.
>
> Dr. Rice noticed that every window in the entire Abbey, save one, was a memorial window. The unappropriated window was apparently an original plain-glass window and was very dull by comparison. The next day Dr. Rice called on the Dean to express gratitude, and in conversation commented on the unoccupied window. The Dean immediately responded that the Abbey would appreciate a gift of a memorial window. Dr. Rice thereupon sensed the situation and offered a window, knowing it would be an easy matter to collect from the entire English-speaking world an amount sufficient to install a window to an engineer.
>
> Dr. Rice proposed a window to his friend Lord Kelvin as one mutually desired by the Abbey and by engineers. Consistently he arranged that this memorial be provided through the cooperation of the engineering bodies of Great Britain and the United States. Having obtained instant approval of influential persons in England, he used the same method in the United States, and, when the undertaking was assured, placed the whole proposition in the hands of the Institution of Civil Engineers, the oldest and most important engineering organization in the world, for announcement of the popular subscription.
>
> The result was so successful that not only was the window provided but the Kelvin Medal was founded. This is probably the only joint undertaking of this nature by the English-speaking world.

The Officers' Reserve

In a letter to General Parsons, Dr. Rice said that he had commenced agitation for an Officers' Reserve in 1902, when he went to Washington to take the matter up with Gen. Nelson A. Miles, then Chief of Staff. He was unsuccessful, but he kept repeating his suggestion up to 1914. From General Parson's letter in *The Military Engineer* the following passages are quoted:

> By the end of the year 1914, there were some engineers who perceived that the world was in for a long period of war and that no matter what position of neutrality the United States might for a while assume, it would probably inevitably be drawn into the conflict. Mr. Calvin W. Rice, secretary of The American Society of Mechanical Engineers, had this view and became firmly impressed with the belief that the engineers should be organized for action.
>
> After conferring with Major-General Leonard Wood, then commanding the Department of the East, Mr. Rice organized a luncheon in February, 1915, at which were present, besides Mr. Rice, Dr. Henry S. Drinker, President of Lehigh University, Mr. Elmer E. Corthell, Mr. Ralph Mershon, Mr. Bradley Stoughton, and a few others, with General Wood as the guest of honor. On being asked for advice, General Wood pointed out that for many years there had been established, as part of the Army, a Medical Officers' Reserve Corps, who were duly commissioned in the Army and who were subject to call to duty in case of war and in times of peace with their consent. This Reserve had been found to be most beneficial, as officers were drawn from it at times of emergency and then could return to private life when the stress was passed. He showed that engineers, like doctors, were always mobilized for the practice of their profession and that an Engineer Officers' Reserve, parallel to the Medical Officers' Reserve, might be established.
>
> This suggestion of General Wood's was promptly taken up, and committees were appointed on behalf of the engineering societies. . . . These committees were immediately organized and began to work, but it was found more expedient to form a central executive and operating committee of the chairmen of the separate committees. . . . The Chairman of the Joint Committee [Wm. Barclay Parsons] and the Committee proceeded to Washington to lay the proposed plan before the War Department. The Committee went with some misgivings as to how such a radical suggestion, emanating from civilians, would be received. Their misgivings, however, were quickly dispelled. The Secretary of War, Mr. Lindley M. Garrison, received them cordially and sympathetically and presented them to Major-General Hugh L. Scott, then Chief of Staff. General Scott, after hearing what the chairman had to say, introduced him to Major-General Tasker H. Bliss, Assistant Chief of Staff. General Bliss listened attentively and with much interest, and at the conclusion of a long interview sent for Major W. D. Connor, now Major-General Connor, an officer of the Corps of Engineers attached to the General Staff. General Bliss instructed Major Connor to confer with Mr. Parsons and to prepare a joint report for submission. When this was done, General Bliss studied it carefully and said, "You have proved your case for engineers, but why limit it to them? It has always been my wish that there should be established a general reserve as part of the Army of the United States." He then returned the report, asking them to make a plan for such a general reserve. . . .
>
> The outcome was that the joint committee reported to the five societies under date of June 23, 1916, that "a bill, which recently passed the Congress, has been signed by the President and will become effective July 1, 1916. This bill, known as the Army Reorganization Act of 1916, contains provisions for the organization of an Officers' Reserve, including the engineers."
>
> Thus a movement initiated by Mr. Rice, put into concrete form by General Wood, and carried into execution by the Committee of the Engineering Societies, was authorized by law.

Dr. Rice, in his modest manner, was very proud of the part he played in the establishment of the Officers' Reserve. He was commissioned a major in the Officers' Reserve Corps in 1922 and a Lieutenant-Colonel in 1929.

His Educational Affiliations

Inspired by the work and character of Dr. Oskar von Miller, distinguished director of the Deutsches Museum, for whom he had a warm affection and high regard, Dr. Rice became interested in the educational possibilities of exhibits of the industrial arts. This brought him into active working contact with a group of men of enthusiasm and vision who established what is now the New York Museum of Science and Industry. As honorary secretary and a member of the board of this institution he gave freely of his energy and advice up to the time of his death.

To his Alma Mater he gave his services as member of the Corporation of the Massachusetts Institute of Technology and chairman of the visiting committee of the department of mechanical engineering of that institution. His broad acquaintanceship

among engineers brought him into close contact with other institutions, for his advice was frequently sought when a professorship, a deanship, or a presidency was to be filled.

Honors and Professional Contacts

In addition to membership in the A.S.M.E. and the A.I.E.E., of which he was a former vice-president, Dr. Rice was a member of many other engineering and professional societies. He was an honorary member of the association of members in Argentina of the National Engineering Societies; of the Koninklijk Instituut van Ingenieurs, of Holland; of the Club de Engenharia, of Rio de Janeiro; of the American Society of Safety Engineers; of the Masaryk Academy, Czechoslovakia; and of the Deutsches Museum, Munich, Germany. He was corresponding member of the Instituto de Engenieros de Chile and of the Technisches Museum, of Vienna. In 1915 he served as a member of the Jury of Award of the Panama-Pacific Exposition. In 1922 he received a gold medal at the Centennial Exposition of Brazil. On him were bestowed the Order of the White Lion of Czechoslovakia, and the Golden Ring of Honor of Bavaria. He was a fellow of the American Association for the Advancement of Science, and a member of the Institution of Electrical Engineers, of London, and of the New York Electrical Society. His well-known interest in research won for him membership in the division of engineering and industrial research of the National Research Council, and the position of National Counselor of the Purdue Research Foundation.

As a delegate of The American Society of Mechanical Engineers to the Seventy-Fifth Anniversary of the Verein deutscher Ingenieure, held in Cologne, in 1931, he was the recipient of a medal of honor "in appreciation of his services to technical-scientific achievement, particularly in promoting the mutual international interests of the engineers of the entire world." Among other honors in Germany he received the honorary degree of Doctor of Engineering (Dr.-Ing. E. h.), from the Technische Hochschule, of Darmstadt, Germany, in 1926.

Dr. Rice married Ellen M. Weibezahn, of Winchester, Mass., August 6, 1904, who, with his children, Edward Winslow and Marjorie Charlotte, survives him.

Tribute of the A.S.M.E. Council

A tribute to Dr. Rice was presented at the Business Meeting of the Society in December, 1934, in behalf of the committee of three past-presidents, Dexter S. Kimball, chairman, D. S. Jacobus, and Alex Dow, appointed for this purpose at the October 24 meeting of the Executive Committee of the Council of the Society. The tribute follows:

In the passing of Calvin Winsor Rice this Society has suffered an irreparable loss and many of its members have lost a kind friend and a wise advisor. To some of us who have been close to him for many years it does not seem possible that we shall never again see his cheerful smile, never again feel the hearty clasp of his hand, and never again see his bold signature at the end of a letter.

The accomplishments of his long and busy life are well known and will be recorded elsewhere, but it is very fitting that we should here be reminded of the great work that he performed in building up this Society. In season and out of season he labored wholeheartedly, and much of the success and reputation of The American Society of Mechanical Engineers is due to his foresight and faithfulness. Nor has this Society alone been benefited by his work. He had long been a source of information and inspiration to all who were concerned with the affairs of the engineering profession.

Especially should we remember that the existence of the Engineering Societies Building that has done so much to stimulate and consolidate the engineering fraternity is largely the result of his enthusiasm and diplomacy. As a consistent and energetic promoter of the life within the walls of this building, his work has been preeminent and will ever be remembered.

Mr. Rice was a man of remarkably broad and sympathetic nature. All good works claimed his attention and support and his range of activities was surprisingly great, both in this country and abroad. As a natural result he had a world-wide host of friends and acquaintances. He had indeed a peculiar ability to make friends of all manner of men. Few men in any walk of life have made so many personal contacts as he, and this in no small way redounded to the credit and reputation of the Society. The loss of these many connections can never be regained.

He was particularly considerate of young men and their problems. Many such sought him out for the advice and help that never failed and which was given freely and graciously. His unfailing good humor and his patience, diplomacy, and steadfastness, even when under criticism, fitted him admirably for the great work that he accomplished as secretary of the Society.

The engineering fraternity has lost a distinguished member, the Society has lost a great leader, and many of us have lost a friend and counselor. Because of such men the race makes progress, and the world is the better for his presence. And his work will endure. The labors of no man, who builds so intelligently and industriously as he, are ever lost but become a part of the inheritance of the race. "Though he were dead, yet shall he live." It is for those of us who remain to build upon the work he directed, so long and so well, the greater and more serviceable society that he always visualized, weaving into it not only technical excellence, but also the fine human qualities that so marked his personality.

"Build thee more stately mansion, O my Soul!
As the swift seasons roll!
Leave thy low-vaulted past!
Let each new temple, nobler than the last,
Shut thee from heaven with a dome more vast,
Till thou at length art free,
Leaving thine outgrown shell by life's unresting sea!"

Dr. Calvin W. Rice's Contribution to International Friendliness[1]

By JOHN H. FINLEY[2]

IT has been announced that I am to give a lecture, but what I shall say will not deserve such a formal or didactic description. It will, I hope, the rather be but as a preface to a series of lectures to bear the name of Calvin Rice—lectures devoted to the causes to which he gave himself, soul, mind, and body. It will so be that though dead he will yet speak on through the years and even centuries, in which we cannot doubt that the earth will be under a unified planetary physical control of engineers with Rice's humanizing and coordinating spirit, so far as man is progressively able to master the forces of nature.

Prometheus, the first of engineers, the demi-god as Shelley recounts his alleviation of man's estate,

> brought the fire to man, tortured to his will iron and gold, the slaves and signs of power, found what was hid beneath the mountains and the waves . . . gave science . . . taught the implicated orbits woven of the wide-wandering stars . . . taught to rule the tempest-winged chariots of the ocean and made the Celt to know the Indian . . . Cities were built and through their snow-like columns flowed the warm winds and the azure ether shone and the blue sea and shadowy hills were seen.

But these were only the beginnings of the ministry of the engineer. He was a Titan in the Greek myth, this half-god, half-man who secreted the fire in a rod of fennel and carried it to mortals, as later the modern engineer, whose name meant originally the man of genius, caught the fire of electricity and carried it in a wire to the inhabitants of all the earth—the mythical precursor of the engineer who has equated the unknown from the known by algebraic signs and evoked the seen from the unseen by divinations of intersecting lines and incantations of logarithms.

Prometheus is no longer bound. A Titan he is free to avail of all that his mind can bring forth to light. You remember into what a rapture Shelley rises at last in his "Prometheus Unbound,"

> To suffer woes which Hope thinks infinite;
> To forgive wrongs darker than death or night;
> To defy Power, which seems omnipotent;
> To love, and bear; to hope till Hope creates
> From its own wreck the thing it contemplates;
> Neither to change, nor falter, nor repent;
> This, like thy glory, Titan, is to be
> Good, great and joyous, beautiful and free;
> This is alone Life, Joy, Empire, and Victory.

It was a Promethean labor that Calvin Rice, Engineer, protagonist of engineers, friend of all mankind, performed, yet happily without Promethean suffering or "withering in destined pain." He went in the midst of his joyous planetary employment, conscious of the appreciation and the goodwill of the world which he was leaving.

I have often used the phrase "planetary consciousness" as something deeper than international-mindedness, importing a basic human relationship without regard to race or nationality. I think from what I have heard that few possessed that consciousness more highly developed than Calvin Rice—and not merely as an official acquaintanceship. Dr. A. B. Hart, Professor of Government Emeritus in Harvard University, has sent me a letter which illustrates that particular phase of the life of Calvin Rice:

> I have received the invitation to be present at your address in the Calvin W. Rice Lecture Series on "International Friendliness." Surely that term is applicable to Mr. Rice, who belonged to the large group of international scientific men; for whatever race, condition, language, or government, there is but one world group composed of those who adhere to the principles of that modern religion which is founded on the establishment, recognition, and application of truth. A more candid man than Calvin W. Rice could not be found, and he was a remarkable example of the international spirit of personal goodwill, of recognition of achievements, and of distribution of the results of research and generalization for the common welfare of mankind.
>
> It is that universality of research, that generous distribution of the results of research throughout the world which characterizes the science of the Twentieth Century.
>
> Added to that indelible search for and acceptance of the truth was the personal side of the man—the humor, the good temper, the forecasting, the belief in the universality of research and of learning Calvin W. Rice is a shining example of that true, indivisible, and universal science which is the pride of our age.

Here are particular instances of his "international friendliness" in which his planetary consciousness of the common fate of mankind expressed itself—first from Dr. L. S. Rowe, Director General of the Pan American Union:

> Mr. Rice was a constant and enthusiastic collaborator in the work of the Pan American Union. He was particularly helpful to us in bringing about closer relations between the engineering societies in Latin America and the national engineering societies of the United States. In the furtherance of this work he made a trip through the countries of South America and was warmly received everywhere. In devising plans for the development of closer cultural ties between the United States and Latin America I constantly turned to him for advice and suggestion. In a word, he was one of the most efficient collaborators in the furtherance of the plan for closer understanding between the nations of this continent.

Next a letter from the General Secretary of the Committee on Friendly Relations Among Foreign Students, Mr. Charles D. Hurrey:

> Dr. Calvin Rice was an esteemed member of our Committee on Friendly Relations Among Foreign Students for the past six years. He was never too busy to receive foreign student callers and to advise them with reference to obtaining practical experience in engineering before returning to their native lands. I have never known a more thoughtful man in writing letters of sympathy and appreciation to foreign student leaders; because of his world travels he was literally a citizen of the world and always manifested a profound respect for our visiting students and professors from abroad.
>
> Outstanding in his international services was his support of the plan by which several hundred German graduates of technical schools came to America after the war to obtain practical training in American industry. Dr. Rice was one of the moving spirits in initiating and carrying through this plan, which has established an abiding friendship between American industrial leaders and hundreds of the brightest younger business and professional men of Germany.
>
> I have in my files many letters from Dr. Rice, and in my memory is a happy recollection of many conversations in which he expressed a conviction that an engineer must be first a real, intelligent world citizen and secondarily an efficient technician.

One more only may I read—from Dr. S. P. Duggan, Director of the Institute of International Education:

> Dr. Calvin Rice was of inestimable aid to me in establishing one of the important activities which took place almost at the beginning of this Institute's career; namely, the exchange made with France

---

[1] Extracts from address delivered on December 5, 1934, during the Annual Meeting, New York, N. Y., of The American Society of Mechanical Engineers.

[2] Associate Editor of the *New York Times*.

of engineering professors. . . . His own efforts in . . . respect (to securing funds) and others in connection with the exchange were of great value to us.

He has since been a real friend to all our undertakings, particularly to young foreign exchange students of engineering who have come to study at American colleges and universities. His advice was eagerly sought by them and we have felt it a privilege to have such an authority place himself at our disposal.

These are only several instances of his friendship; but it is difficult to enumerate all the occasions when he was helpful. It is needless to say that the Institute and myself feel that the passing of Calvin Rice is an irreparable loss.

His passing would be an irreparable loss if his life were not a prophecy of better human relations as well as physical comforts. His humanizing and coordinating spirit will live on.

Memorial Notices

HAROLD FRANCIS ADAMS (1868–1934)

Harold Francis Adams, whose death occurred at Sayville, L. I., N. Y., on January 30, 1934, was born at Tansborough, N. J., on August 3, 1868, the son of Charles Edwin and Ellen Maria (French) Adams. His early education was secured in the schools of Atlantic City, N. J. After completing high school there he studied at Pierce's Business School and the Franklin Art Institute, both in Philadelphia, Pa., and then at the University of Pennsylvania, from which he secured a certificate of proficiency in 1897.

During the next five years Mr. Adams was a practicing architect in Atlantic City. From 1902 to 1904 he was associated with Herbert D. Hale and H. G. Morse, Jr., in charge of their Philadelphia office as chief draftsman and also in charge of the work of designing the interior fittings of the S.S. *Manchuria* and S.S. *Mongolia*, built by the New York Shipbuilding Company, Camden, N. J. During the following year he was employed by Wilson Eyre, of Philadelphia, directing the construction of a large private home in Jenkintown, Pa.

From 1905 to 1907 Mr. Adams was again connected with Hale and Morse, with whom James Gamble Rogers was also associated at that time. Mr. Adams held the position of assistant chief draftsman and had charge of exterior work on a number of large buildings in New York, including the McCreery store and Engineering Societies Building.

In 1907 and 1908 he was employed by the Purdy & Henderson Co., of New York and Havana, in charge of drawings for the Custom House and other buildings in Santiago de Cuba, and from 1917 to 1920 he was chief draftsman and head of the Havana office of that company and made designs for both concrete and structural steel buildings there. During the years 1909–1917 he worked successively for Mr. Rogers, directing construction on the estate of E. S. Harkness, New London, Conn.; for Goldwin Starrett & Van Vleck, New York, on plans for the Albany (N. Y.) high school; and for Willauer, Shape & Bready, New York, as chief draftsman, working on plans for a number of large buildings in that city.

While in Havana with Purdy and Henderson Mr. Adams attracted the attention of the Royal Bank of Canada, and in 1920 he became manager of its Bank Premise Department there. Later he was transferred to Montreal, Canada. He left the bank's employ in 1923 to open an office of his own in New York, where he engaged in architectural work until his retirement in 1931.

Mr. Adams became a member of the A.S.M.E. in 1921 and also belonged to the American Institute of Architects and the American Arbitration Association, as well as to the masonic fraternity. He had served as captain of the Morris Guards of Atlantic City. He invented a honing device, which was patented in 1918, and was interested in photography. He is survived by his widow, Minnie A. (Rocks) Adams, and two sons, Harold Francis, Jr., and Lewis Edwin, all of Sayville, and a daughter, Helen B. Adams, of Baltimore, Md.

MARION EUGENE BABIONE (1908–1934)

Marion Eugene Babione, whose death occurred on June 2, 1934, was born on May 6, 1908, at Luckey, Ohio, the son of Dr. Augustus A. and Anna (Bollini) Babione. He secured an A.B. degree at Adelbert College, Western Reserve University, Cleveland, Ohio, in 1930, and two years later a B.S. degree in mechanical engineering at the Case School of Applied Science. While at Adelbert College he was elected to the Pi Kappa Alpha fraternity, and he later became a member of Sigma Xi, honorary scientific fraternity. He became a junior member of the A.S.M.E. in 1932.

For eighteen months prior to his death Mr. Babione was employed as mechanical engineer for the Hanna Coal Company, Cleveland.

JOSEPH E. BACHELDER (1868–1932)

Joseph E. Bachelder, who died in Chicago, Ill., on December 12, 1932, following two months' illness, was born at Palmyra, Maine, on November 3, 1868. At an early age his family removed to Holyoke, Mass. Following his graduation from high school, he entered the employ of the Deane Steam Pump Company of Holyoke, progressing rapidly in the organization, and from 1897 to 1901 was assistant to the superintendent of the company. He then went to Chicago to become manager of the Pump Sales Department of Fairbanks, Morse & Co., and remained in the employ of this company, with the exception of a year as vice-president and manager of the Temple Pump Company, Chicago (1914–1915) until his retirement in 1930. From 1901 to 1906 he was manager of the Pump Sales Department. He then was transferred to the Pump Manufacturing Department at Beloit, Wis., where he remained until 1914. After the year with the Temple Company he was appointed engineering salesman and New York branch manager of the Fairbanks organization. He was transferred to Baltimore in 1922 and to St. Louis in 1923, serving as manager in each city.

Mr. Bachelder became a junior member of the A.S.M.E. in 1905 and an associate of the Society in 1921. He is survived by his widow, Grace Swift Bachelder, and by a son and two daughters.

HERBERT A. BARRE (1875–1934)

Herbert A. Barre, chief engineer of the Southern California Edison Company, Ltd., died suddenly of a cerebral hemorrhage at his home in Los Angeles, Calif., on June 28, 1934. He had been in the continuous service of the Southern California Edison Company and its predecessor companies for 23 years, holding successively the positions of electrical and mechanical engineer, executive engineer, and chief engineer.

Mr. Barre was born at Pictou, N. S., Canada, on January 26, 1875, the son of John R. and Elizabeth F. (Merriman) Barre. He went to San Francisco while still a child and received his education in the schools of that city. He secured a B.S. degree from the University of California in 1897 and was a member of Tau Beta Pi, honorary engineering fraternity. He was given an honorary degree by the California Institute of Technology in 1924.

Practically all of Mr. Barre's engineering work was done in the West in connection with electric utility systems. Following his graduation in 1897 he secured employment as electric station operator for the Redlands (Calif.) Electric Light & Power Co. and subsequently held similar positions with the Los Angeles Railway and the Mt. Whitney Power Company, Hammond, Calif., and was assistant electrical engineer with the traction company at Oakland Calif. From 1900 to 1902 he was connected with the Independent Electric Light & Power Co. and Independent Gas Company, San Francisco, as inspector of construction and assistant to the chief engineer of the former, and assistant superintendent of the latter. In 1902 he returned to Los Angeles as assistant to R. S. Masson, consulting engineer of the Henry P. Huntington properties, which included the Los Angeles Railway, the Pacific Electric Railway, and the San Joaquin Light & Power Co.

In 1906, Mr. Barre became associated with the Electric Bond & Share Co., of New York, N. Y., where he was engaged in engineering valuation and operating activities. Construction of the Glenwood and Boulder plants of the Denver Gas & Electric Co. took Mr. Barre West again to Denver in 1907. He became interested in the formation of the Electric Construction Company, which built the Fossil Creek plant of the Arizona Power Company, and was actively engaged in the building of that plant.

In 1911, when the Pacific Light & Power Corp. (a Huntington property, merged in 1917 with the Southern California Edison Company) was beginning the construction of the Big Creek-San Joaquin River hydroelectric development with its 240-mile transmission lines to Southern California, Mr. Barre was engaged as electrical and mechanical engineer. It was in connection with the large hydroelectric and steam system of the Southern California Edison Company that his most outstanding achievements were accomplished. He was possessed of high engineering ability coupled with a very sound economic understanding of the utility industry. His recommendations as engineer and economist are permanently written into the great hydroelectric and steam system of the company he served. In 1911, when Mr. Barre became associated with the properties now comprising the Southern California Edison Company, this system consisted of a development of some 180,000 horsepower, represented by a plant investment of about $50,000,000. During his active participation in the development of this system, the plant capacity was enlarged to more than 1,250,000 horsepower and the investment in properties increased to $350,000,000.

Probably the most outstanding achievement of Mr. Barre's work was the successful construction and operation of the first 150,000-volt long-distance electric transmission lines in the world, transmitting energy from the water power development on Big Creek and the San Joaquin River to Los Angeles, a distance of approximately 240 miles. Later, when the construction of additional power development at Big Creek brought the need for greater transmission capacity to Southern California, Mr. Barre met the requirement by reconstructing the 150,000-volt lines for operation at 220,000 volts. These lines so converted were the first commercial lines in the world to ac-

complish electric transmission at this high voltage. These attainments brought to Mr. Barre world recognition as an engineer and attracted attention from outstanding engineers in America and foreign countries who went to Los Angeles to learn the details of construction and operation. This contribution in the field of electric transmission has not only saved many millions of investment in transmission expense on the projects with which he was associated, but has also had its influence on the development of all the large hydro developments in the world that are distant from the market to which the energy is transmitted.

Mr. Barre became a member of the A.S.M.E. in 1912. He also belonged to the American Institute of Electrical Engineers. He married Annie E. McTear, in Los Angeles, in 1904, and is survived by her and by two children, Ben E. and Ruth E. Barre, and also by his father.

WILLIAM HASTINGS BASSETT (1868–1934)

William Hastings Bassett, a pioneer metallurgist in the non-ferrous metal industry, connected since 1902 with The American Brass Company, died at his home in Cheshire, Conn., on July 21, 1934. He was born at New Bedford, Mass., on March 7, 1868, the son of William A. and Almira D. (Mayhew) Bassett. With forebears who were seafaring men and a childhood spent in such an environment as New Bedford, he had a love for the water which always made boating his favorite recreation. Until within a year of his death he spent a month annually at Martha's Vineyard.

Not choosing to follow the sea as a profession, however, he entered the Massachusetts Institute of Technology, from which he was graduated in 1891 with a Bachelor of Science degree. He spent the next five years as chemist and superintendent for the Popes Island Manufacturing Company, in New Bedford, and then taught chemistry at the Swain Free School there until 1900. During the next two years he was chief chemist at the Newark Laboratory of The New Jersey Zinc Company.

In 1902 Mr. Bassett went to Torrington, Conn., as chemist for the Coe Brass Manufacturing Company, a branch of the newly formed American Brass Company of Waterbury. The following year he was made chief chemist and metallurgist of The American Brass Company, in 1912 its technical superintendent and metallurgist, and in 1930 metallurgical manager of the company. Of his position in the non-ferrous metal industry and the achievements which led to it, Dr. H. Foster Bain, formerly secretary of the American Institute of Mining and Metallurgical Engineers, wrote in an appreciation published in *Mining and Metallurgy* for September, 1934, as follows:

"After beginning as chemist he rose through various grades of laboratory and plant service with ever increasing responsibilities until he became metallurgical manager for the entire company, itself the largest producer of brass and copper goods in the world. He was the recognized unofficial dean of metallurgists in his chosen field long before he died.

"To him, more than any other single individual, though many had part in the movement, was due the introduction and development of technical control of production in copper-alloy manufacture. When he went to Torrington, brass making was an art handed down from father to son or master to apprentice. The manager of a plant was in the hands of his melters who kept close in their own control knowledge of composition, temperature, and time necessary to produce given quantities in an alloy. Results were irregular, unexplained, and often unsatisfactory. Bassett being a chemist at once began taking samples and making analyses. Also, as Holland and Pringle have related in a most interesting chapter in 'Explorers in Science,' he had a microscope. It was at least among the first of those instruments put into effective use in the industry. The result was that composition and quality were accurately related and the metallurgist came to give directions for what went into the pots and when. . . . A firm foundation was laid for large-scale manufacture of standard goods even from high-priced metals.

"Mr. Bassett's researches in alloys were numerous and wide ranging. A large number of patents were secured in his name or in that of members of his staff and still others were bought by the company on his recommendations. It was not only in chemical matters that he was an authority but in much besides. An important reinforced hollow copper cable is one of his inventions and his wide knowledge and sound judgment of methods of testing were recognized by his professional compeers in his election to the presidency of the American Society for Testing Materials which he held at the time of his death."

Mr. Bassett was president of the A.I.M.E. in 1930. Prior to that he had served as chairman of the Institute of Metals Division of the Institute and was instrumental in bring about the merging of the older Institute of Metals with the A.I.M.E. He was awarded the James Douglas Medal by the A.I.M.E. in 1925 "for constructive research in copper and brass and other non-ferrous metals and their alloys and for his contributions to the establishment of high standards of quality."

Mr. Bassett's service in the American Society for Testing Materials was also outstanding. C. L. Warwick, secretary-treasurer of the Society, has summarized it in the following paragraph:

"In June, 1934, after serving a two-year term as vice-president, he was elected president of the American Society for Testing Materials, with which he had been affiliated since 1903. As a personal member and the official representative of his company, he participated actively in the work of a number of A.S.T.M. committees. He took a leading part in the organization in 1909 of the first two A.S.T.M. standing committees in the non-ferrous field, B-1 on Copper Wire and B-2 on Non-Ferrous Metals and Alloys, and served continuously on these committees until his death. He served as a member of the society's executive committee in 1916–1918. He always took a keen interest in advancing the knowledge of the properties of non-ferrous metals and supported actively a number of research and standardization projects in the society."

Mr. Bassett also took part in the work of the National Research Council and during the World War was a member of the War Department's Advisory Committee on Materials for Airplane Construction. He had served on the Metallurgical Advisory Committee to the Bureau of Standards and on the Army Ordnance Advisory Committee, Watertown Arsenal. He was a member of the Army Ordnance Association.

Mr. Bassett became a member of the A.S.M.E. in 1924. He was a former director of the American Institute of Chemical Engineers, a Fellow of the American Association for the Advancement of Science, and held membership in many scientific and engineering societies in this country and abroad, including the American Chemical Society, Society of Automotive Engineers, American Electrochemical Society, Mining and Metallurgical Society of America, American Foundrymen's Association, Society of Naval Architects and Marine Engineers, American Welding Society, The Franklin Institute, American Geographical Society, Institute of Metals and Society of Chemical Industry, London, and Deutsche Gesellschaft für Metallkunde. He was a 32d degree Mason and a Shriner, trustee of the public library in Cheshire, and a deacon in the First Congregational Church there since 1914. He belonged to the Engineers' and Chemists' Clubs in New York, as well as to clubs in Waterbury and Torrington.

Mr. Bassett married Sarah Hedge Whiting in 1892 and is survived by her and by two children, Alice Whiting Bassett and William Hastings Bassett, Jr., the latter technical superintendent and metallurgist for the Anaconda Wire & Cable Co., Hastings-on-Hudson, N. Y.

HENRY HOWELL BAUMGARTNER (1893–1933)

Henry Howell Baumgartner died on November 16, 1933, at his home in Belleville, N. J., of injuries sustained in an automobile accident during the summer. He was president and treasurer of the Essex Engineering Company, organized at Belleville in 1928.

Mr. Baumgartner was born at Flemington, N. J., on July 10, 1893, the son of Frederick and Marguerite (Burkitt) Baumgartner. He attended the United States Steam School at Hoboken, N. J., for five months during the winter of 1917–1918 and served in the United States Navy until early in 1920. He then spent about a year as second engineer on the Standard Oil tanker, "S. V. Harkness." He was refrigerating engineer for Swift & Co. from September, 1921, to February, 1922, and spent the remainder of that year in his former position of second engineer of Standard Oil tankers.

From then until June, 1925, Mr. Baumgartner was connected with the Public Service Gas & Electric Co., of New Jersey, as assistant boiler room engineer and technical assistant. During the next three years, until he organized the Essex Engineering Company, he was refractory and combustion engineer for the Drake Non-Clinkering Furnace Block Company, of New York, N. Y. Several patents, dealing with furnaces and combustion methods, were issued to Mr. Baumgartner.

Mr. Baumgartner became an associate-member of the A.S.M.E. in 1928, and was past-president of the National Association of Power Engineers. He belonged to the Belleville post of the American Legion. His widow, Ida E. (Hughes) Baumgartner, whom he married in 1918, and two sons, William Darrow and Richard Balz Baumgartner, survive him.

JOSEPH ROBERT BLAINE (1884–1934)

Joseph Robert Blaine died at his home in Oak Park, Ill., on December 17, 1934. He had been connected since 1913 with the Miehle Printing Press & Manufacturing Co., Chicago, in charge of the design of offset printing presses.

Mr. Blaine was born in Chicago on November 14, 1884, the son of John Smith and Frances (Birkholz) Blaine. He received a B.S. degree at the University of Wisconsin in 1905, and an M.E. degree from that institution in 1911. For two years prior to entering college he had worked in the shop and toolroom of Edward P. Allis, Milwaukee, and for a year on the erecting floor of Pawling & Harnischfeger, in that city. He spent his vacations in the latter work, also, and after his graduation in 1905 continued with the company for two years as mechanical draftsman.

From 1907 to 1913 Mr. Blaine was engaged by the Berlin Machine Works, Beloit, Wis., as designer of band-sawing machinery, spending four months of the time in Europe in special work for the company. He held some patents on woodworking machinery but his most important work was in the graphic arts field, in which he held about fifty patents for inventions on typographic and lithographic presses and other printing equipment.

Mr. Blaine became a member of the A.S.M.E. in 1917. He is survived by his widow, Mrs. Attolle (Frost) Blaine, a daughter, Virginia P. Blaine, and a son, Robert F. Blaine.

DAVID REES BOWEN (1865–1934)

David Rees Bowen, vice-president of Farrel-Birmingham Company, Inc., Ansonia, Conn., and for many years its chief engineer, died at his home in Ansonia on December 29, 1934. He played an important part in the development of machinery for non-ferrous rolling mills, sugar-cane grinding, stone and ore crushing, and for the manufacture of paper, rubber, linoleum, and other products, and was widely known in the industries where such machinery is used.

Mr. Bowen was born in Cwmavon, Wales, on October 22, 1865. He received his early education in the British national schools and at Llandovery Collegiate School. At the age of 17 he came to the United States with his parents and in 1883 entered the employ of the Farrel Foundry & Machine Co. as a machinist's apprentice. After serving his apprenticeship for three years he worked as a machinist two years longer, then entered the drafting room of the company. His exceptional ability soon led to his appointment as superintendent, then chief engineer. He held the latter position until 1930, when he was elected vice-president, in charge of engineering. In July, 1933, when ill health compelled him to relinquish the active direction of engineering, he became consulting engineer for the company, continuing also as vice-president until his death.

Mr. Bowen had been a member of the A.S.M.E. since 1899. He was also a member of the masonic fraternity and the Elks. He took an interest in civic affairs, serving as alderman of Ansonia and as a member of the board of trustees of the public library there. He was also a vestryman of Christ Episcopal Church. He was very fond of books and through his reading had acquired a fund of knowledge on many subjects. He spoke the Welsh language fluently and was deeply interested in his native land, to which he returned for many visits.

He is survived by his widow, a son, a daughter, and two grandchildren.

HENRY K. BROOKS (1869–1934)

Henry K. Brooks, president of the Capital Iron Works Company and of the Steel Fixture Manufacturing Company, of Topeka, Kan., died in that city on August 12, 1934, of a cerebral hemorrhage.

Mr. Brooks was born at Kettering, Northamptonshire, England, on January 8, 1869, and was educated in that country. His early schooling was received through private instruction and at the Wisbech Schools, Cambridgeshire. After securing government certificates in drawing, he served an apprenticeship with the firm of Gimson & Co., general engineers, of Leicester, at the same time attending the Leicester Technical Evening College. Upon the completion of his apprenticeship he continued in the employ of Gimson & Co. for several years. He also worked for a short time for the Midland Railway Company, in England, before coming to the United States.

During his first years in this country Mr. Brooks was also engaged in railroad work, being employed by the Atchison, Topeka & Santa Fe, the Southern Kansas, and the Southern Pacific railway companies. In 1895 he accepted a position as instructor at the Kansas State Agricultural College, in charge of the foundry, blacksmith, and machine departments. The following year he returned to the industrial field, in the capacity of general manager for the Capital Iron Works. A short time afterward he and his brother, George W. Brooks, purchased the company and he became its president. He spent three years with the company, during this first period, and was responsible for the work under many federal, state, and railroad contracts for structural iron and machinery.

In 1899 Mr. Brooks became interested in the electrical field and took a position with the Electric Axle Light & Power Co., of New York, N. Y., in charge of its equipment in operation on railroads. The following year he became assistant chief engineer for the Consolidated Railway Light & Equipment Co., New York, and for several months had charge of its factories in New York and Connecticut. Later in 1900, having made some improvements in electric train-lighting apparatus, Mr. Brooks was offered a position as chief engineer of the United States Light & Heating Co., New York. He remained with this company until 1908 and was in complete charge of its factory and all mechanical apparatus.

In 1908 Mr. Brooks returned to Topeka to again take over the management of the Capital Iron Works, and he and his brother also formed the Steel Fixture Manufacturing Company for the manufacture of steel office furniture. These two fabricating companies are among the largest of their kind in Kansas and their products are widely distributed.

Mr. Brooks became a member of the A.S.M.E. in 1907. He was a charter member of the Kansas Day Club, New York, N. Y., a member of the board of directors of the Kansas State Historical Society, past-president of the Associated Industries of Kansas, past-president of the Lakeview Hunting and Fishing Club of Topeka, and a Scottish Rite Mason.

Mr. Brooks married Miss Edith Harrison, of Ottawa, Kansas, in 1895. She died in 1927. He is survived by a daughter, Mrs. R. E. Kreuger, of Topeka, as well as by a number of brothers and sisters.

JAMES PARK CALDERWOOD (1884–1934)

James Park Calderwood, head of the Department of Mechanical Engineering at Kansas State College of Agriculture and Applied Science, Manhattan, Kan., died at his home there on August 9, 1934. He is survived by his widow, Coral E. (Parker) Calderwood, whom he married in 1910.

Professor Calderwood was born at Middleport, Ohio, on January 1, 1884, the son of Andrew and Christina (Thompson) Calderwood. He was graduated from the Middleport High School in 1903 and from the Pomeroy (Ohio) Academy the following year, and in 1908 secured his M.E. degree at Ohio State University. The following year he held a fellowship in experimental engineering at the university.

In 1909 Professor Calderwood became instructor in mechanical engineering at Pennsylvania State College. He was made assistant professor in charge of all heat-engine subjects the following year and in 1911 took charge of the experimental engineering laboratory. In 1913 he was advanced to an associate professorship, with the administration of the experimental engineering department, and he continued in this post until 1917. In addition to his regular teaching and administrative work he designed and installed a heating and ventilating system for new liberal arts and mining buildings.

Special research work conducted by Professor Calderwood while at Pennsylvania State College included an investigation of the effect of the velocity and humidity of air on heat transmission through building material; a study of the pounding in gas engines when using kerosene as a fuel; the development of a method of determining the ultimate from the proximate analysis of coal; and a study of the purchase of coal by specification, which was primarily a study of the methods of coal sampling. The results of all of these investigations were published by the Engineering Experiment Station of the college.

Professor Calderwood served as a member of the Pennsylvania State Board for the examination of boiler and elevator inspectors from 1916 to 1917, and was offered, but declined, an appointment as mechanical engineer for the Department of Labor and Industry of the State of Pennsylvania. In 1916 the Pennsylvania State College conferred the degree of Master of Science upon him.

In 1917 Professor Calderwood accepted a position in the Engineering and Inspection Division of the Travelers Insurance Company at Hartford, Conn. He spent a year with the company, working on special engineering problems connected with liability and indemnity insurance, including the special inspection of industrial plants, studies on accident prevention, and the development of a method for the rating of chemical risks. During this year he also served as assistant engineer for the State of Connecticut in connection with the United States Fuel Administration.

Professor Calderwood had been a member of the faculty of Kansas State College since 1918, when he became professor of steam and gas engineering there. He was made head of the Department of Mechanical Engineering the following year. Reports on some of his work there have been published by the Engineering Experiment Station of the college. He had also made frequent contributions to the technical press and was co-author of several textbooks, including "Engineering Thermodynamics," in collaboration with James A. Moyer; "Elements of Steam and Gas Power," with A. A. Potter; and "Elements of Engineering Thermodynamics," with Moyer and Potter. He had been called upon by the public service commissions

of Kansas and Missouri and the Federal Court of Nebraska in cases involving controversy over natural gas.

Professor Calderwood became a junior member of the A.S.M.E. in 1913 and a member six years later. He had served two terms on the executive committee of the Mid-Continent Section of the Society, being vice-chairman in 1925–1926 and again a member of the committee in 1929–1930. He was honorary chairman of the Student Branch of the Society at Kansas State College from 1925 to 1932.

He was also a member of the Society for the Promotion of Engineering Education, was president of its Kansas-Nebraska Section in 1930, and a member of its membership committee in 1928. He belonged to the Kansas Engineering Society and served on its legislative committee in 1925, was faculty advisor of Sigma Tau, of which he was a member, and was advisory editor of the *Kansas State Engineer*. He was also a member of the masonic fraternity, Delta Upsilon, and the honorary fraternities, Phi Kappa Phi and Sigma Xi, as well as of a number of clubs.

ADAMS BURTON CHAMBERLAIN (1875–1934)

Adams Burton Chamberlain, assistant superintendent of the Station Engineering Department of The Edison Electric Illuminating Company of Boston, died at his home in Wollaston, Mass., on January 22, 1934.

Mr. Chamberlain was born in LaCrosse, Wis., on April 1, 1875, the son of Eli and Mary L. (Porter) Chamberlain. At an early age he went to Newport, Vt., to live and was graduated from the St. Johnsbury Academy. After that he was successively employed by the Tremont and Suffolk Mills, Lowell, Mass., on cost and inspection work, by C. R. Makepeace, mill engineer, Providence, R. I., as a draftsman, and by the New England Telephone & Telegraph Co. on construction work.

In January, 1901, he entered the employ of The Edison Electric Illuminating Company of Boston in the Station Engineering Department, where he rose through various grades to the position he held at the time of his death.

Mr. Chamberlain became a member of the A.S.M.E. in 1911. He was also a member of the Engineers Club of Boston and a 32d degree Mason. He is survived by his widow, L. Josephine Eaton, whom he married in 1903, and by a daughter, Kathryn.

WALTON CLARK (1856–1934)

Walton Clark, consulting engineer and former vice-president of the United Gas Improvement Company, Philadelphia, Pa., died in that city on July 30, 1934. He was the inventor of a process for the complete gasification of coal, and contributed to the development of processes for operating water-gas sets. He was the author of numerous scientific articles.

From 1907 to 1924 he was president of The Franklin Institute and in 1926 the institute honored him with its first award of the Walton Clark Medal, for his "distinguished and outstanding contributions" to the gas industry. At the time of the presentation of the award Dr. W. C. L. Eglin, president of the institute, said that Mr. Clark was "a pioneer in the association of physical and chemical laboratories with the development work of gas companies."

Mr. Clark was born in Utica, N. Y., on April 15, 1856, the son of Erastus and Frances (Beardsley) Clark. He attended the public schools of Utica until his seventeenth year, when he entered the service of the New Orleans Gas Company, where he served in various capacities until 1886. In 1883 he was temporarily assigned to the United Gas Improvement Company to complete the installation of a water-gas plant in New Orleans, work on which had been interrupted by a yellow fever epidemic. Mr. Clark was immune to the disease, and so was able to finish the installation, which would have been impossible for an unacclimated engineer.

He served successively in gas companies in Jefferson City, La., and in Kansas City, and following the death of the assistant general superintendent of the United Gas Improvement Company, Mr. Clark was appointed, in August, 1888, to take his place. Six years later he was made general superintendent, a position which covered the duties of the chief engineer of the company and supervisor of the many enterprises under its control. He became a vice-president of the company in 1904 and held that office until he resigned from active duties in 1923. He had been consulting engineer to the company since then.

In 1904 the honorary degree of Master of Engineering was conferred upon Mr. Clark by Stevens Institute of Technology, and in 1911 he received the degree of Doctor of Science from the University of Pennsylvania.

He was past-president (1906) of the American Gas Institute, of which he was one of the founders; became a member of the A.S.M.E. in 1890; and also belonged to the American Institute of Electrical Engineers and the American Society of Mining and Metallurgical Engineers. He was chairman of the board of trustees of the Free Correspondence School for Gas Works Employees.

He also was a member of the Society of the Cincinnati, the Society of the Founders and Patriots of America, and the Sigma Psi fraternity. His clubs included the Engineers' Club, New York, and in Philadelphia the Engineers, Union League, University, and a number of country and athletic clubs. His home was at Chestnut Hill, Pa.

Mr. Clark in 1880 married Miss Alice M. Shaw, of Natchez, Miss., who died in 1882. In 1885 he married Miss Louise Beauvais, of New Orleans. Surviving him, in addition to Mrs. Clark, are Frank Shaw Clark, a son by his first marriage; and three sons and one daughter by his second marriage, Walton, Jr., Theobald Forstall, Beauvais, and Darthela Clark.

WILLIAM JAY COFFIN (1875–1934)

William Jay Coffin, connected with the New York State Department of Public Works since 1922, died at Wethersfield, Connecticut, on June 14, 1934, after several months' illness.

Mr. Coffin was born in Albany, N. Y., on December 20, 1875, the son of William Latham and Anna McHarg Coffin. He received his M.E.E. degree from Cornell University in 1898 and immediately began work as electrical repairman in the Signal Department of the New York Central Railroad, Albany. He was connected with this railroad, in various capacities, until 1914. He served an apprenticeship in the Motive Power Department in 1900 and 1901 and worked in that department successively as stationary engineer, roundhouse foreman, and mechanical foreman at Albany until 1909. He was then sent to the general manager's office in New York, N. Y., to take charge of locomotive tests and tonnage rating. After several months in that work he was made assistant to the superintendent of motive power of the Western Division, investigating mechanical problems in shops and on the road. In April, 1910, he was appointed assistant engineer in the president's office in New York, where he remained until 1914, making technical investigations dealing with locomotive operation and tonnage rating for the entire New York Central system.

In January, 1914, Mr. Coffin was appointed first deputy commissioner of Public Works and he served the City of Albany in that capacity until January, 1922, having charge of the adaptation of motor apparatus to municipal work and establishing repair facilities. He entered the employ of the New York State Department of Public Works as Fleet Commander in March, 1922, in charge of floating equipment on the state barge canal, including tugs and dredges; he also supervised dredging work and wrecking operations. He was made assistant civil engineer in February, 1923, and continued in that capacity until his death. He was assigned to the division of highways of District No. 1, and had charge of several highway construction projects.

Mr. Coffin became a member of the A.S.M.E. in 1923. He also belonged to the masonic fraternity. He is survived by a sister, Helen Coffin, of Hartford, Conn., and by a daughter, Lois Lilian Coffin, of Brooklyn, N. Y. His wife, Selora (Gaskill) Coffin, died in 1922.

SAMUEL DUNLAP COLLETT (1868–1933)

Samuel Dunlap Collett, whose death occurred on December 26, 1933, was born on October 25, 1868, at Newport, Ind. He attended the public schools there and secured his B.S. and M.S. degrees at Rose Polytechnic Institute, where he studied mechanical, electrical, and civil engineering.

Before completing his college work Mr. Collett worked for a short period as a surveyor on the Chicago, Evansville & Chattanooga Ry., took a training course in the factory of the Thomson-Houston Electric Company, and worked in the Boston and Pittsburgh offices of that company.

In the fall of 1895 Mr. Collett entered the employ of the Metropolitan Telephone & Telegraph Co. (now the New York Telephone Company) as engineer in the construction department and for two years was in charge of the interior block department and underground cable work. He then took the position of eastern manager of the Elevator Supply & Repair Co., New York, of which he later became vice-president. He was associated with the company until 1920, and from then until his retirement in 1925 was with the Elevator Supplies Co., Inc., Hoboken, N. J.

Mr. Collett had been a member of the A.S.M.E. since 1902. He was also a member of the American Institute of Electrical Engineers.

ERNEST HARRY CORNELIUS (1890–1934)

Ernest Harry Cornelius, president of the Oklahoma Steel Castings Company, Tulsa, Okla., died in Cleveland, Ohio, on May 8, 1934,

from complications following an operation for appendicitis. The son of Harry B. and Georgiana (Phillips) Cornelius, he was born on October 28, 1890, at Hastings, Neb., and received his early education there. He entered Northwestern University, at Evanston, Ill., at the age of 17, but because of an injury sustained in a fall during his junior year, he was obliged to leave college. After spending about a year and a half in California, he returned to Nebraska and entered the state university at Lincoln, from which he was graduated with a B.S. degree in 1913.

Following his graduation Mr. Cornelius engaged for a time in sales work for the Jacques Steel Company, in northern Texas. During the first part of 1914 he assisted in appraisal work in Kansas City and then took a position with the Metropolitan Street Railway in that city inspecting construction of new trackage and rehabilitation of existing lines. He also made special designs for cars and did other drafting work, remaining with the company for about two years. He spent part of 1916 with the Kansas City Southern Railway Company, as assistant to the bridge engineer. He then moved to Tulsa, and during the next two years was connected with the Oklahoma Structural Steel Company, subsidiary of the Oklahoma Iron Works, as chief engineer and sales manager the greater part of the time.

In January, 1918, Mr. Cornelius began private practice in the design and construction of steel structures, chiefly for the oil industry. This business was incorporated as the Industrial Construction Company two years later, with Mr. Cornelius as president. In 1922 he also organized the Oklahoma Steel Castings Company, which he had since served as president, greatly developing the size of the plant and extending the territory it serves.

Mr. Cornelius took a keen interest in civic affairs and the development of industry in Oklahoma. He was a past-president and for some years had been a director of the Associated Industries of Oklahoma. He had also served as president of the Public Welfare Society of Tulsa, director of the Tulsa Community Fund and the Tulsa Chamber of Commerce, member of the Board of Trustees of the University of Tulsa, president of the Rotary Club in Tulsa, and Member of the Board of Trustees and chairman of the Financial Committee of the Boston Avenue Methodist Episcopal Church. Just previous to his illness he had been appointed to the NRA Code Authority for the steel-castings industry.

Mr. Cornelius became a member of the A.S.M.E. in 1927 and was a former director of the American Iron and Steel Institute. He belonged to Sigma Nu and the masonic fraternities, and to the Tulsa Club and Tulsa Country Club.

In addition to all these activities he found time to take up flying, first as a hobby and later as a means of transportation for business trips.

Mr. Cornelius is survived by three children, Marjorie Ann and Ernest H. Cornelius, Jr., and Mrs. Virginia (Cornelius) Eby. His wife, Virginia Moseley, whom he married in Lincoln, Neb., in 1914, died in 1923.

ALBERT NELSON CRAMER (1883–1934)

Albert Nelson Cramer, whose death occurred on June 21, 1934, was born in Philadelphia, Pa., on March 27, 1883, the son of Harry Howell and Fannie H. (Slack) Cramer. After attending the Manual Training School in Philadelphia for three years, he entered the employ of the Baldwin Locomotive Works there, and for five years worked on detail drawings for the company. He subsequently spent a year and a half each in similar work for the Link-Belt Engineering Company, Nicetown, Pa., and W. E. Hamilton, Columbus, Ohio, and about six months with the Schoen Steel Wheel Company, Philadelphia. In 1908 and 1909 he was engaged in field engineering for P. A. Kley, Philadelphia, and layout work for the Jeffrey Manufacturing Company, Columbus.

During the next ten years Mr. Cramer was designer in charge of experimental development and construction for the Federal Glass Company, Columbus. He then went to Toledo, Ohio, to take the position of assistant engineer, in charge of special engineering development, with the Owens Bottle Company. He left there in the spring of 1929 and spent the next two years as assistant engineer in the research department of the American Optical Company, Southbridge, Mass.

Since July, 1931, Mr. Cramer had devoted all his time to developing inventions of his own. Prior to that date he had already taken out more than twenty patents on glass-making machinery, including blowing, molding, and cutting machines, feeders, and handling apparatus. Subsequently he developed additional equipment for the glass industry, including a number of machines for forming tumblers and hollow glass articles, patents on some of which were pending at the time of his death.

Mr. Cramer became a member of the A.S.M.E. in 1929 and was active in the masonic fraternity in both Columbus and Toledo. He is survived by his widow, Mrs. Alice P. (Bietsch) Cramer, and by a daughter, Elizabeth B. Cramer.

RALPH CROOKER (1854–1934)

Ralph Crooker, a member of the A.S.M.E. since 1890, died at his home in Acton, Mass., on June 2, 1934. He is survived by his widow, Louise C. (Little) Crooker, whom he married in 1905.

Ralph Crooker (3d) was born in South Boston on December 25, 1854, the son of Ralph and Ann A. (Bailey) Crooker. He was graduated from the English High School of Boston with the class of 1872, and had always kept in touch with the members of that class through its annual reunion dinners in Boston.

At the age of eighteen he entered training as a draftsman at the Bay State Iron Works, Boston, under the direction of his grandfather, who was a member of the firm and superintendent of the works. He remained with that company until 1879, when he went to Springfield, Ill., where he spent about six months with the Springfield Iron Company. From 1880 to 1884 he served as engineer and superintendent with the Colorado Coal & Iron Co.

In 1885 Mr. Crooker entered the employ of the Jones & Laughlin Steel Co., Pittsburgh, Pa., and between then and 1907 he spent the greater part of his time with that company, the last period being from about 1903 to the latter part of 1907. His other connections were with the Bessemer Works of the Springfield Iron Company in 1887 and 1888, with the Johnson Company, Lorain, Ohio, during part of 1894, and with the Lorain Steel Company, about 1900.

He had made his home in Acton since 1907 and had not been active in business except for a little consulting work and a few months in 1919 making alterations in piping for T.N.T. for Mackintosh-Hemphill & Co., Pittsburgh. His chief pastime during his latter years was the study of maps and of plans for ocean liners and boats. He also liked to study the rock formations and wild life in the woods near his home.

Mr. Crooker was a Legion of Honor member of the American Institute of Mining and Metallurgical Engineers and also belonged to the Engineers' Society of Western Pennsylvania.

FRANK GARFIELD CUTLER (1881–1934)

Frank Garfield Cutler, of Birmingham, Ala., died on June 17, 1934, at the Manhattan Eye, Ear and Throat Hospital, New York, N. Y., of complications following a mastoid operation.

Mr. Cutler was born on May 13, 1881, at Fort Scott, Kan., the son of Samuel and Ella B. (Dickerson) Cutler. He studied manual training in Louisville, Ky., and then entered the University of Kentucky, from which he received his Bachelor's degree in mechanical engineering in 1901, and the degree of Mechanical Engineer three years later. He was graduated with highest honors and in 1902, when the honorary engineering fraternity, Tau Beta Pi, was organized there, he was made a member.

Immediately upon leaving college in 1901 Mr. Cutler went to Chicago to become assistant in the steam engineering department at the South Works of the Illinois Steel Company. He remained in that position until the fall of 1906, then took charge of the steam engineering department of the Ensley Division of the Tennessee Coal, Iron & Railroad Co. He was made chief of the Bureau of Steam Engineering of the company in 1914 and held that position until his death.

Mr. Cutler was considered an authority on steam power-plant design and construction and especially its application to steel mills. In 1928 he completed studies for and wrote a paper on "Combination Firing of Blast-Furnace Gas and Pulverized Coal," which was presented at the Semi-Annual Meeting of the A.S.M.E. in Pittsburgh in May of that year.[1] These studies were carried on as new boilers and equipment were put in at the mill, and continued practically up to his death.

Mr. Cutler became a member of the A.S.M.E. in 1914 and had served several years on the Executive Committee of the Birmingham Section of the Society, being chairman in 1921. He was secretary in 1917–1918 and president in 1921 of the Alabama Technical Association, and in 1922 was vice-chairman of the Association of Iron and Steel Electrical Engineers. He had also served as chairman of the Jefferson County Orphans Home Association and as a director of the Ensley Rotary Club and a member of the board of governors of the Woodward Gold Club. His hobbies were golf and contract bridge, and especially long automobile trips in both the North and South. He is survived by his widow, the former Blanche B. Duffy, of Chicago, whom he married in 1909, and by three children, Mary E., Samuel M., and Grace L. Cutler.

[1] Trans. A.S.M.E., vol. 49–50, 1927–1928, FSP-50-35, pp. 102–106.

ORTON GOODWIN DALE (1870–1934)

Orton Goodwin Dale was drowned in Barnegat Bay on February 18, 1934, when ice broke beneath him as he was walking on the bay near his summer home at Mantoloking, N. J. Mr. Dale was consulting engineer to the Texas Gulf Sulphur Company, of New York, and a director of the New York & New Jersey Steamboat Company. He is survived by his widow, the former Miss Amy Slade, of Trenton, N. J., whom he married in 1896, and by a daughter, Mrs. D. R. Abbes, of Burlingame, Calif., and two sons, F. Slade Dale, yachtsman, Bay Head, N. J., and Orton G. Dale, Jr., vice-president of Bowne & Co., financial printers, New York.

Mr. Dale was born in Helensburgh, Scotland, on November 8, 1870, but had lived in the United States since he was ten years old. He was graduated from the Stevens Institute of Technology in 1893 and spent the following year in the drawing room of the National Sugar Refining Company, Yonkers, N. Y. He then became New York manager of the Mead-Morrison Manufacturing Company. In 1918 he was connected with the Singer Manufacturing Company, Elizabeth, N. J., and for the next two years was assistant chief engineer of the American International Shipbuilding Corporation, Philadelphia, Pa. He had been with the Texas Gulf Sulphur Company since that time.

Mr. Dale resided in Plainfield, N. J., for some years, but moved to New York in 1916. It was his custom to spend the weekends at his Mantoloking home and he had recently been elected to the Mantoloking Borough Council. He became a junior member of the A.S.M.E. in 1894 and was promoted to the grade of member in 1909. He was appointed a member of the Committee on Publications of the Society in 1922 and served as its chairman in 1925 and vice-chairman the following year.

CYRUS EATON DAVISON (1890–1934)

Cyrus Eaton Davison, formerly chief research engineer for Martin Motors, Inc., New York, N. Y., died on March 8, 1934, after a year's illness. He is survived by his widow, Anna R. (Tong) Davison, whom he married in 1918, and by a son, Robert C., and daughter, Ruth B. Davison, residing at Stapleton, S. I., N. Y.

Mr. Davison was born at Huntsport, Nova Scotia, Canada, on August 26, 1890, the son of Cyrus C. and Laura (Eaton) Davison. He attended the Huntsport High School and at the age of twenty-one entered the employ of the Southern Pacific Company. He served as assistant engineer and chief engineer on various ships of that company until December, 1917. He then entered the United States Navy as an engineer officer, with the grade of lieutenant, serving, during the next year and a half, aboard the U.S.S. *Lakewood* and U.S.S. *Montclair*. During the summer of 1919 he was connected with the United States Shipping Board in the capacity of assistant machinery inspector and chief machinery inspector, supervising the installation of boilers, turbines, and reciprocating engines, and all other mechanical equipment on ships being built by the Federal Shipbuilding Company for the Emergency Fleet Corporation. He was graduated from the Naval Turbine School at Carnegie Institute of Technology, Pittsburgh, Pa., in 1919.

In the fall of 1919 Mr. Davison took charge of the Educational Department of the Ocean Association of Marine Engineers, of which he was a member, and was instructor of its engineering classes for three years. During the following year he was assistant engineer for the French Company, directing the operation and repair of turbo-generators and electric motors.

Mr. Davison again became an instructor in marine engineering in July, 1923, when he took the position of chief engineer and senior engineering instructor at the New York State Nautical School, New York, N. Y. He continued in that work until January, 1926, then spent two years as assistant superintendent of buildings and grounds at Columbia University. He had been with Martin Motors, Inc., since that time, located at the research laboratory in East Rutherford, N. J., at first. He was engaged in the design and construction of internal-combustion engines for test purposes, and in running power and efficiency tests.

Mr. Davison became an associate-member of the A.S.M.E. in 1926 and a member three years later. He also belonged to the masonic fraternity.

WILLIAM EDWARD DEAN, JR. (1888–1933)

William Edward Dean, Jr., who died of pneumonia at Hamilton Ontario, Canada, on December 11, 1933, was a native of Minneapolis Minn., where he was born on June 12, 1888. His parents were William Edward and Elmina (Clapp) Dean. He attended grammar and high school at Los Angeles, Calif., and then served a four-year apprenticeship with the Southern Pacific Company, of that city. In 1910 he entered the University of California, from which he was graduated with a B.S. degree in mechanical engineering in 1914. During his summer vacations he did drafting-room and testing-laboratory work.

Immediately following his graduation Mr. Dean took a position in the Engineering Department of the Westinghouse Air Brake Company, Wilmerding, Pa., and in the fall of that year was made personal assistant to the manager of engineering, to work on the design, manufacture, and performance of air brakes and related devices, such as automatic train control.

In August, 1917, Mr. Dean entered the Second Officers Training Camp at Fort Oglethorpe, Ga. He was commissioned second lieutenant in the Signal Reserve Corps, Aviation Section, and served five months at Park Field, Tenn., and five at Langley Field, Va., as engineer officer in charge of installation and operation of machine, motor-repair, and blacksmith shops. He was then promoted to first lieutenant in the Air Service and until January, 1919, was stationed at Washington, D. C., in the engine and plane maintenance branch of the service.

Upon his return to civilian life, Mr. Dean was made assistant engineer of tests by the Westinghouse Air Brake Company, continuing to carry on research on air brakes. Early in 1921 he was sent to Kingston, Jamaica, on special consulting work for the Jamaica Government Railways. Upon his return he was appointed engineer of tests, and at the beginning of 1926 assistant chief engineer in charge of tests and inspection.

At the close of 1928 Mr. Dean was appointed chief engineer of the Westinghouse Brake Company of Australasia Ltd., and for several years was located in New South Wales, Australia. In August, 1932, he was transferred to the position of chief air-brake engineer of the Canadian Westinghouse Co. Ltd., at Hamilton, Canada, the position he held at the time of his death.

Mr. Dean became a junior member of the A.S.M.E. in 1921 and a member four years later. He also belonged to the Air Brake Association and to the honorary fraternities, Tau Beta Pi and Sigma Xi. He had patented many air-brake system devices.

Surviving Mr. Dean are his widow, Doris (Brown) Dean, whom he married in 1923, and three sons, Frederick Brown, William Corner, and Robert Stanley Dean.

GEORGE PASS DeHAVEN (1886–1934)

George Pass DeHaven, son of William Henry and Margaret M. (Pass) DeHaven, both descendants of old Pennsylvania families, was born at Harrisburg, Pa., on September 3, 1886. His early education was secured in the schools of that city. After his graduation from high school there he studied for two years at the Stevens Institute of Technology, Hoboken, N. J., and two years at Drexel Institute, in Philadelphia. He then entered the employ of the Pennsylvania Steel Company, at Steelton, where he spent a year each in the pattern and machine shops, and about six months each in the iron and steel foundries.

In January, 1914, he took a position in the engineering department of the Bethlehem Steel Corporation, Steelton, where he remained for four years, working on the design of rolling mill machinery. From then until the summer of 1923 he was connected with the Harrisburg Foundry and Machine Works, spending four years on the design, testing, and preparing specifications and estimates for steam engines, then serving successively as assistant purchasing agent, assistant works manager, and production manager.

He was next employed for about a year as designer and checker for the Koppers Company, Pittsburgh, working on gas producers and machinery for operating them, and during the winter of 1924–1925 designed rolling-mill machinery for the Birdsboro Steel Foundry & Machine Co., Birdsboro, Pa. He then became assistant chief engineer for the Taylor-Wilson Manufacturing Company, McKees Rocks, Pa., with which he remained until he was appointed chief engineer, General and Maintenance Employees, of the Bureau of Public Grounds and Buildings of the Commonwealth of Pennsylvania. He subsequently was made superintendent of the Bureau of Maintenance of Public Grounds and Buildings, the position he held at the time of his death, which occurred on February 19, 1934, at his home in Harrisburg.

Mr. DeHaven became an associate-member of the A.S.M.E. in 1926 and in that year he also took some courses in the shaping of steel and roll pass design at the Carnegie Institute of Technology. He had served as corporal and quartermaster sergeant on the Governor's Troop of the Pennsylvania National Guard and belonged to the masonic fraternity and the Elks. He is survived by his widow, Margaret M. (Edwards) DeHaven, whom he married in 1906, and by their son, Charles A. DeHaven.

EDWARD H. J. DILLON (1887–1934)

Edward H. J. Dillon, who died in Philadelphia, Pa., on March 28, 1934, was born in Brooklyn, N. Y., on April 15, 1887, the son of John and Anna (Mohan) Dillon. He attended the public and high schools there, and subsequently studied for two years at Brooklyn Polytechnic Institute and St. John's University, Brooklyn.

At the age of seventeen Mr. Dillon entered the employ of the Griscom-Russell Company, of Philadelphia, marine and stationary power plant engineers and manufacturers, and had spent his entire professional life with that company, rising to the position of manager of the Philadelphia office.

Mr. Dillon became an associate of the A.S.M.E. in 1920 and belonged to the Knights of Columbus. He is survived by his widow, Margaret Dillon, whom he married in 1915.

FRED DOEPKE (1862–1934)

Fred Doepke, founder and president of the Wrought Washer Manufacturing Company, Milwaukee, Wis., died in Los Angeles, Calif., on April 23, 1934, after having been in failing health for several years. Mr. Doepke became a member of the A.S.M.E. in 1922 and since 1926 had served as secretary of the Sectional Committee on the Standardization of Plain and Lock Washers, for which the A.S.M.E. and the Society of Automotive Engineers are joint sponsors. He was also a past vice-president of the Engineering Society of Milwaukee and belonged to a number of social clubs in Milwaukee and vicinity.

Mr. Doepke was born at Rockford, Ill., on March 28, 1862, the son of Gotlieb and Bertha (Smith) Doepke. The family later moved to Milwaukee and he attended school there from 1879 to 1883, at the same time serving an apprenticeship in the machine shops of the old Filer & Stowell Co. After working for a time in 1883 in machine shops in Cleveland, Ohio, he went to New York, where he studied at Cooper Union nights and was employed by E. W. Bliss Co., the North River Iron Works, and the Freeland Tool Works. Subsequently he went to Philadelphia, where he likewise studied evenings, taking courses offered by The Franklin Institute, and worked in the daytime.

During these years Mr. Doepke became interested in automatic machinery and conceived the idea of automatic washer-making machinery. He returned to Milwaukee in 1885 and started in business with A. J. Read. The Wrought Washer Manufacturing Company was founded about 1887 and he served as vice-president of the company until 1910, when he became its president. He not only designed and built the plant itself, but also practically all of its special machinery. Under his guidance the plant became one of the largest in the world devoted to the production of washers.

Mr. Doepke had traveled extensively, visiting many foreign countries, and was widely known in engineering circles and throughout the metal industries. For some years prior to his death he had made his home in St. Petersburg, Fla. He is survived by his widow, Anna M. (Disch) Doepke, whom he married in 1892, and by one son, Fred C. Doepke, of Milwaukee.

ROSCOE IRVING DUNTEN (1888–1934)

Roscoe Irving Dunten, whose death occurred on June 17, 1934, was born at Calcium, N. Y., on April 13, 1888, the son of Milton Thomas and Anna Elizabeth Dunten. To supplement his high-school education he took courses in steam and electrical engineering through the International Correspondence School, completing these studies in 1907. He then secured work as assistant at the steam power plant of the St. Regis Paper Co. at Deferiet, N. Y., where he remained for two years, part of the time being chief engineer.

From 1909 to 1912 Mr. Dunten was in charge of the steam-electric power plant of the Delaware & Hudson R. R. Co. at Oneonta, N. Y., and during the next year directed the operation and maintenance of an 80-ton ice-making plant of the Tampa Ice Company, at Palmetto, Fla.

In 1913 the Southern Utilities Company, of Palatka, Fla., engaged Mr. Dunten as engineer in its electric plant at Bradentown, Fla. After three years there he was transferred to the combined ice and electric plant at West Palm Beach as chief engineer. Something over two years later he was made supervising engineer of all of the company's properties, consisting of more than twenty plants located at different points in Florida and Georgia. He remained with the company until the latter part of 1921, his work including not only the supervision of existing plants but also the design, layout, and erection of new plants and research on burning oil fuel under power boilers.

During the next seven years Mr. Dunten was vice-president and general manager of the San Juan Ice and Refrigeration Company, in Porto Rico. He directed the reconstruction and enlargement of the company's plant at San Juan, greatly increasing the output and reducing fuel consumption. He returned to Florida in 1928 to become manager of the Caribbean Division of Pan-American Airways, Inc., with headquarters at Miami, and held that position at the time of his death.

Mr. Dunten joined the A.S.M.E. with the grade of associate-member in 1921 and was promoted to full membership three years later. He was Rear Commodore of the Biscayne Yacht Club and a member of the masonic fraternity. He is survived by his widow, Luvenia M. (Wood) Dunten, whom he married in 1909, and by a daughter, Florence A. (Dunten) Foster.

GEORGE C. FARKELL (1872–1934)

George C. Farkell, superintendent of the rail and blooming mills, at the Lorain, Ohio, works of the National Tube Company, died at St. Joseph's Hospital in that city of pneumonia on October 22, 1934. He had been connected with the Lorain Works since early in 1905, when he became assistant superintendent of the skelp mills. The following year he was advanced to the position of superintendent of the skelp mills, and in 1907 was made superintendent of the rail and mechanical departments of the works. He held this position for ten years, then was superintendent of the rolling mills until April, 1934, when he was further promoted to the position he held at the time of his death.

Mr. Farkell was born on January 12, 1872, at Canajoharie, Montgomery County, N. Y., the son of John and Catherine Farkell. He was graduated in 1887 from the local high school and entered Cornell University from which he was graduated in 1892 with an M.E. degree in electrical engineering. While at Cornell he was elected to membership in the honorary scientific fraternity, Sigma Xi.

For two years after leaving college Mr. Farkell was engaged in electrical construction work for H. Ward Leonard & Co., New York, and from then until the end of 1895 was on the staff of the Physical Testing Laboratory of the Lukens Iron & Steel Co., Coatesville, Pa. During the next five years he was assistant engineer with the United States Revenue Cutter Service, for three years of that period assistant to the engineer-in-chief detailed for inspection duty and for engineering work in connection with the construction of steam machinery for new revenue cutters.

Mr. Farkell became assistant chief inspector of the Homestead Steel Works of the Carnegie Steel Company in January, 1901. In October of that year he was made assistant to the superintendent of the 128-in. plate mill, the 42-in. universal, and the 30-in. slabbing mill of the Homestead works, where he remained until May, 1904. Between then and his connection with the Lorain Works, he held the position of assistant superintendent of the slabbing, plate, rail, and structural mills of the Lackawanna Steel Company, Buffalo, N. Y.

Mr. Farkell contributed greatly to the development of rolling-mill practice and held patents on a manipulator used in handling rolled-metal shapes; a speed-controlling apparatus for electrically driven rolling mills (joint patent with Edwin S. Lammers); a method of rolling T-shaped rails; and a device for tightening nuts on screw-threaded ends of bolts.

Mr. Farkell had been a member of the A.S.M.E. since 1913. He also belonged to the American Society for the Advancement of Science, the American Iron and Steel Institute, and the masonic fraternity. He was very active in community affairs in Lorain. He was one of the founders of the local Red Cross chapter and served on its executive committee from 1917 to the time of his death, part of the time as chairman. He had been a member of the board of directors of the Lorain Y.M.C.A. since 1925 and was chairman of the committee which had charge of erecting a new Y.M.C.A. building. He was a member of the board of directors of the Lorain Community Fund during the last five years of his life. In recognition of these services he was selected in 1933 for the *Lorain Journal* Achievement Award, as the citizen who during the year had performed the most noteworthy service to the community.

Radio construction was his special hobby and golf his favorite pastime.

His widow, the former Miss Ouida E. Pease, whom he married in 1909, survives him.

THOMAS FARMER (1852–1934)

Thomas Farmer, a member of the A.S.M.E. since 1891, died of pneumonia at the United Hospital in Portchester, N. Y., on November 6, 1934. He is survived by his widow, formerly Miss Maud H. White, whom he married in 1879, and by a daughter, Elizabeth G. (Farmer) Kelsey, and a son, Thomas, the fourth in line to bear that name.

Mr. Farmer was born in Boston, Mass., on October 12, 1852, his

mother being Henrietta B. (Cobb) Farmer. After his graduation from high school in Roxbury in 1869 he learned the machinist's trade at the shops of A. Leitelt Co. in Grand Rapids, Mich. He spent three and a half years there, then went to Aurora, Ill., where he worked in the locomotive shops of the Chicago, Burlington & Quincy R.R. He next returned East, working for a time on erecting for the Providence Machine Company. During these years he studied drawing and other engineering subjects.

In 1874 he returned to Grand Rapids, where he obtained the position of superintendent of the city water works, which he held for five years, resigning to go into business for himself. Owing to lack of capital his venture was not successful and he went to Chicago, where he was employed for a time as tool saleman. He did not find this work congenial, however, and took a position as general superintendent of the Somersworth Western Manufacturing Company of Bloomington, Ill. During two years spent with this company he built new shops and designed special tools. He resigned that position to become superintendent of the Detroit Radiator Company (which later became a part of the American Radiator Corporation). He directed the laying out and building of a new plant and had charge of the design of new tools and the operation of all departments of the plant.

Mr. Farmer's next connection was with the Russel Wheel & Foundry Co. of Detroit, for which he built a new plant and served as its superintendent for a time. About 1894 he became chief engineer of the Detroit Citizens Railway, with which he remained for nine years. From Detroit he went to Cleveland, Ohio, where he was associated for ten months with the Kuhlman Car Company. He then opened an office for consulting work in the street-railway field. He gave this up to rebuild and electrify the plant of the United States Heater Company (subsequently known as the United States Radiator Corporation) in Detroit, after which he became sales manager of the Detroit office of the Warner & Swasey Co. He spent nine years in that capacity, prior to becoming manager of the Detroit office of Manning, Maxwell & Moore, Inc. He had not been active in business since 1925.

EDWIN E. A. FISHER (1870–1934)

Edwin E. A. Fisher was born in Providence, R. I., on August 31, 1870, the son of James Potter and Caroline Martha (Butts) Fisher. He attended Cooper Union, New York, then entered Cornell University, from which he was graduated in 1891 with an M.E. degree. He specialized in electrical engineering during his senior year and his first position after graduation was with the Field Engineering Company, working on the installation of an electric railway between Paterson and Passaic, N. J. After a year in that connection he went to Lynn, Mass., to take a course at the General Electric Company works there. In 1893–1894 he engaged in electric-railway installation for the Union Railway Company, Providence, R. I., and the following year was instructor in physics and electrical engineering at Lehigh University.

From 1895 to 1912 Mr. Fisher was located the greater part of the time outside the United States. He studied physics, chemistry, and mathematics for a year at the University at Göttingen, Germany, after which he was located in England and Canada. For ten years beginning in 1901 he was connected with the Bureau of Education in the Philippine Islands, as division superintendent of schools, superintendent of the Philippine School of Arts and Trades, and acting assistant director of education in charge of schoolhouse construction in the Philippines, and devised the unit system for schoolhouses there. He next spent a year as assistant commissioner of the interior of Porto Rico.

Since his return from Porto Rico Mr. Fisher had been engaged in technical and statistical research in relation to life insurance. He was in the statistician's department of the Prudential Insurance Company, Newark, N. J., from 1912 to 1921 and since then, until his death on March 3, 1934, had been in the risk classification department of the John Hancock Mutual Life Insurance Company, Boston, Mass.

Mr. Fisher became a member of the A.S.M.E. in 1922 and also belonged to the American Statistical Association, American Association for the Advancement of Science, and Boston Life Underwriters Association. He was vice-president and director of Fisher & Thompson, Inc., New York. In Dover, Mass., where he made his home, he was treasurer of the First Parish of Dover and vice-president of the Dover Historical and Natural History Society. He married Sophie L. Armington, of Providence, in 1907, and is survived by her and by three sons and a daughter.

STANLEY GRISWOLD FLAGG (1860–1934)

Stanley Griswold Flagg, president of Stanley G. Flagg & Co., Philadelpha, Pa., died of a heart attack on March 14, 1934, just a week after the death of his wife, Elizabeth (Windrim) Flagg. He is survived by his son, Stanley Griswold Flagg, 3rd, of Philadelphia, and his daughter, Mrs. Edward S. Nugent-Head, of London.

Mr. Flagg was born in Philadelphia on January 21, 1860. His mother was Adelaide (Gordon) Flagg and his father Stanley Griswold Flagg, founder of the company, which manufactures malleable-iron, cast-iron, and brass fittings, brass valves, and specialties. He learned the business under his father and succeeded him to the presidency of the company.

Mr. Flagg was one of the early advocates of standardization in the valve and fittings industry. As a member of the A.S.M.E., which he joined in 1891 and of which he was a manager for the term 1910–1913, he was appointed a representative on the American Engineering Standards Committee soon after its formation in 1918. He continued to serve on that body, later reorganized as the American Standards Association, until 1930.

For some years prior to the formation of the A.E.S.C. he served on special A.S.M.E. committees on pipe and screw threads. He represented the Society on the Sectional Committee on Standardization and Unification of Screw Threads from the time of its organization in 1921 until 1930, and was also the Society's representative on the Sectional Committee on Pipe Threads. He also represented the Manufacturers Standardization Society of the Valve and Fittings Industry on the Sectional Committee on Pipe Flanges and Fittings.

He was a most active worker on these and other committees to which he was appointed by the various technical organizations to which he belonged, losing no opportunity to cooperate in the sound development of the industry as a whole. He made frequent trips abroad, promoting standardization at international conferences and through his many personal contacts.

Mr. Flagg was a past-president of the American Foundrymen's Association and a member of the American Institute of Mining and Metallurgical Engineers, the American Society for Testing Materials, and the Iron and Steel Institute. He had also served as president of the Pennsylvania Society of Sons of the Revolution, vice-president of the General Society of the Sons of the Revolution, and treasurer of the Society of Colonial Wars in the Commonwealth of Pennsylvania. One of his pastimes was the collecting of rare books.

WILLIAM FRAY (1865–1934)

William Fray, who died at the Bridgeport (Conn.) Hospital, on July 22, 1934, after a two years' illness, was born at Cornwall, England, on July 2, 1865, the son of Samuel and Emma (Sweet) Fray. He came to the United States as a boy and attended the Bridgeport public schools. At the age of sixteen he was apprenticed to the machinist and toolmaking trade and he worked up in that field, studying mechanical drawing, mathematics, and other subjects evenings, until he became assistant foreman at the Farrel Foundry & Machine Co., Waterbury, Conn., in 1890, and foreman two years later.

He resigned in 1900 to take the position of superintendent of the wire mill of the Benedict & Burnham Manufacturing Co., Waterbury, with which he remained four years. He then became production superintendent for John S. Fray & Co., Bridgeport, manufacturers of small tools for mechanics. He was there until 1910 when he became superintendent of the Standard Brass & Copper Tube Co., New London, Conn. After two years he was made secretary of the company, as well as superintendent, and two years later became its president and general manager. He greatly enlarged the plant during his connection with it. After the sale of the business in 1917 Mr. Fray retired from active business, but from time to time acted as a consulting engineer on brass and copper-tube manufacture, serving, among others, the Bridgeport Brass Company, Wheeler Condenser & Engine Co., Detroit Brass & Copper Co., Scovill Manufacturing Company, Dominion Copper Products Company, and Philadelphia Roll & Machine Co.

Mr. Fray became a member of the A.S.M.E. in 1920 and was treasurer of the Bridgeport Section of the Society from 1926 to 1931. He was also a member of the Bridgeport Engineers Club and the masonic fraternity. After his retirement he spent some time in travel. He is survived by his widow, Minnie J. (Steiber) Fray, whom he married in 1883, and by a son, George H. Fray, and daughter, Florence (Fray) Clark.

DAVID WILLIS FRENCH (1860–1934)

David Willis French, retired superintendent of the Hackensack (N. J.) Water Company, died at his home in Englewood, N. J., on August 9, 1934, following a few days' illness. He is survived by his widow, Wilburta F. French, and by a sister, Mrs. Laura E. Libby of Boston. His first wife, Amelia (Eddy) French, whom he married in 1884, died in 1929.

Mr. French was born at North Weymouth, Mass., on March 24, 1860, his father's name being John French. At the age of sixteen he entered upon a three-year apprenticeship in the shops of the Whittier Machine Company, Boston, during which period he also attended night school four evenings a week. The following year he took a special course at the Massachusetts Institute of Technology. In 1881 he became associated with the Henry R. Worthington hydraulic works in South Brooklyn, N. Y., as erecting engineer. In 1882 the firm contracted to install a pump for the Hackensack Water Company at the New Milford plant. Mr. French supervised this installation and upon its completion was retained by the Hackensack Water Company. In 1883 he had charge of installing the plant for the high service system at Weehawken, and subsequently was engaged to operate it for a year under Charles B. Brush, chief engineer and superintendent. He was engineer in charge of the high service system for eight years, then was made deputy assistant superintendent of the company, and after the death of Mr. Brush in 1896 was made superintendent of the company, the position he held until his retirement in 1928.

Some of his major projects were the reconstruction of the New Milford pumping plant, the installation of the present great filter system, construction of the sub-pumping station at North Bergen, and the extension of the water system to supply about sixty towns in Hudson and Bergen counties. In 1900 he recommended to the board of directors of the company the purchase of a subsidiary water company located at Spring Valley, N. Y., and in 1912 he reconstructed this plant to supply ten towns in Rockland County.

In 1913 Mr. French perfected a machine which makes it possible to install a line gate without the interruption of service, a machine now widely used throughout the United States.

Mr. French became a member of the A.S.M.E. in 1916. He was also a member of the American Water Works Association, which he served as president in 1909, was a director of the Hoboken Savings Bank, a member of the Union City Masonic Lodge, member of the board of the First Presbyterian Church, Englewood, director of the New Jersey Utilities Association for twenty years, superintendent of the Grove Church Sunday School for a number of years, member of the Executive committee of the Boy Scouts, North Hudson County Council, and a former member of the Board of Education of Union City.

A. O. FRICK (1852–1934)

A. O. Frick, chairman of the board of the Frick Company, Waynesboro, Pa., and active in the affairs of the firm for more than sixty years, died at a hospital in Baltimore, Md., on April 20, 1934.

Mr. Frick was born on June 16, 1852, at Ridgeville (now Ringgold), Md., where his father, George Frick, a native of Switzerland with a genius for mechanical things, then had his engine shop. The family having moved to Waynesboro in 1861, Mr. Frick's education was completed in the public schools of that town. At the age of fifteen he began work as an apprentice in the Frick shops under his father, later becoming foreman, then draftsman, then mechanical engineer.

In the late 60's and the 70's Mr. Frick, with an engineering insight years ahead of his time, improved the designs for the Frick portable steam engines and boilers. Some of his early drawings for these engines, bearing the date 1869, are still preserved, and illustrate with their fine, accurate lines, the craftsmanship he displayed in all his work.

Between 1876 and the early 80's he developed the Frick steam traction engine, doing the ground work which led through such steps as steering by a team of horses, chain drive from the engine shaft to the gearing, the use of a train of small gears to replace the chain, devices for springing the rear axle, feedwater heaters, etc. He also patented an improved balanced slide valve, some of his early drawings showing the adaptability of this valve to locomotive practice.

In 1882 and 1883 Mr. Frick made the drawings for the original refrigerating machines with which the firm pioneered in this field, and he subsequently assisted in enlarging the line of ice-making equipment. About 1889 he spent a year in Springfield, Mo., building and operating an ice plant. Following this he devoted himself particularly to managing the engineering work of the company. He was made vice-president in 1896 and elected to the presidency of the company in 1904, but continued to serve as consulting engineer on all of its work.

Mr. Frick had been chairman of the board since 1924, and though in recent years he had retired from active work, he always took the keenest interest and pride in the company's progress. When he was past the age of 75 he took over the duties of chief engineer for several months, during his absence. He was the last living member of a group of thirteen men who banded together in 1873 to save the firm from the effects of the panic of that year.

In addition to his connection with the Frick Company, Mr. Frick was a director of the Citizens National Bank & Trust Co., a director of the Chamber of Commerce and of the Beneficial Fund Association of Waynesboro, a member of the board of managers of the Waynesboro Hospital, a member of the Waynesboro Country Club, and charter member of the local masonic lodge. He became a member of the A.S.M.E. in 1885.

He is survived by his widow, the former Miss Margaret Mehaffey, a sister, and two brothers, Amos and Ezra Frick.

ALFRED BROOKS FRY (1860–1933)

Alfred Brooks Fry, Commodore, U.S.N.R. (Retired), died in Coronado, Calif., on December 4, 1933. Mr. Fry entered Government service in 1886 as inspection engineer of public buildings under the Supervising Architect's Office of the Treasury Department and held many important posts in civil and naval service prior to his retirement in 1927.

Mr. Fry was born in New York, N. Y., on March 3, 1860, a son of Major Thomas William G. Fry (U.S.V.) and Frances (Olney) Fry, and a great-grandson of Captain Benjamin Fry, who served in the Continental Army. He attended private schools and Morse's School in New York and studied engineering at Columbia University. He spent several years at sea as an oiler and assistant engineer and also had shop experience with the Corliss Engine Company, Providence, R. I., and at Portland, Me., and Boston, Mass., with the Portland Company and Globe Mill Company. He was captain of the Boston Fire Department from 1882 to 1886.

In civil service in the Treasury Department Mr. Fry was advanced from inspection engineer to chief engineer and supervising chief engineer. He had charge of the construction, engineering operation, and repairs of United States public buildings and their mechanical and electrical plants. Much of his time was spent on work at New York but he also had assignments in Boston, Chicago, Washington, New Orleans, and Galveston and on the Pacific Coast and in connection with quarantine stations and marine hospitals on the Atlantic, Gulf, and Pacific Coasts. From 1916 to 1926 he was consulting engineer for the U. S. Immigration Service, in charge of the construction and maintenance of immigration buildings. He served on a Congressional Commission on the mechanical transmission of mails in 1913.

By order of the President Mr. Fry served as a member of the Board of Consulting Engineers for the Improvement of State Canals in New York from 1904 to 1912, and he also studied and reported on European and Egyptian canal, river, and port improvements in 1909 and 1918. He was an engineer member of the Special Panama Canal Commission in 1921.

Mr. Fry was consulting engineer for the Association for the Protection of the Adirondacks, from 1908 to 1920, and for the Department of Water Supply, Gas, and Electricity of the City of New York, from 1914 to 1917. He was a member of the Committee on Water Storage and Waterways of the Merchants' Association of New York from 1918 to 1926.

In 1892 and 1893 he organized the first Engineer Division of the United States Naval Militia at Boston and New York. He served as engineer lieutenant and engineer lieutenant-commander from 1892 to 1910 and as commander and chief of staff of the Naval Militia, New York, from then until 1923, when he was made commodore. He was retired in 1924 with the rank of rear-admiral.

In the U.S.N.R. he was advanced through the ranks from lieutenant to commodore, retiring in 1927. He was acting chief engineer in the U.S.N. during the Spanish-American War, 1898, and engineer-aide to Rear-Admiral Burd, industrial manager of the Third Naval District and New York Navy Yard, during the World War. He was engineer-observer on the first run of the U.S.S. *Leviathan* in 1917 and also saw duty at sea and at English and French ports. He was consulting marine engineer for the U.S. Army Transport Service in 1922. He was a member of the International Congresses of Navigation.

Since his retirement Mr. Fry had served on the Harbor Committee of the San Diego Chamber of Commerce in 1929 and on the City Council of Coronado in 1929–1930 and was mayor pro-tem of Coronado in 1930–1931, and mayor from then until the time of his death. He served on the employment committee of Southern California in 1931.

Mr. Fry became a member of the A.S.M.E. in 1892. He was also a member of the American Society of Civil Engineers, American Society of Naval Engineers, Society of Constructors of Federal Buildings, of which he had been president. He was a member of a committee of engineers representing the National Engineering Societies at the New York Constitutional Convention in 1915. He belonged to the Society of Colonial Wars, The Society of the Cincinnati of the Providence Plantations, and the Military Order of the Spanish-

American War, and his clubs included the Columbia University, New York, and Army and Navy, Washington. He was an honorary member of the Yachting Department of the New York Athletic Club.

Surviving Mr. Fry are his widow, Emma V. (Sheridan) Fry, whom he married in 1890, and their son, Sheridan Brooks Fry.

GEORGE WARREN FULLER (1868–1934)

George Warren Fuller, who became internationally known as a sanitary engineer and expert on water supply and sewage disposal died at his home in New York, N. Y., on June 15, 1934. He was born at Franklin, Mass., on December 21, 1868, and was the son of George Newell and Harriet (Craig) Fuller. After his graduation from the Massachusetts Institute of Technology in 1890 with an S.B. degree, he studied for about a year at the University of Berlin and in the private office of the engineer of the Berlin water works. He then devoted about nine years to research work in water purification, chiefly at the Lawrence Experiment Station of the Massachusetts State Board of Health, and at Louisville, Ky., and Cincinnati, Ohio, where his work on coagulation and rapid sand filtration laid the basis for American water purification practice.

Since 1899 Mr. Fuller had engaged in consulting work, with his headquarters in New York. From 1901 to 1911 he was in partnership with Rudolph Hering under the firm name of Hering & Fuller. During the next few years he continued practice under his own name. In 1916 he formed a partnership with James R. McClintock, under the firm name of Fuller & McClintock, of which he was senior member at the time of his death.

Mr. Fuller served as consulting engineer for major water works and sewerage improvements in more than 150 large cities in this country and abroad. He had been consultant to the New York Board of Water Supply since 1906. In connection with the development of the Catskill reservoirs and supply systems he made an intensive study of the principal systems in Wales and Scotland which led to the installation of aerating stations below the Ashokan and Kensico reservoirs. He also served as consultant to the Metropolitan Sewage Commission and in 1928 and 1929 reported to the city upon the Wards Island sewage-treatment project, plans for which were subsequently drawn by his firm.

In 1917 and 1918, during the World War, Mr. Fuller was a member of a central committee at Washington having to do with the engineering, planning, and sanitation of various army camps, and he was also consulting engineer to the Construction Division of the Army. Directly after the Armistice he served as a member of the Franco-American Engineering Congress which convened at Paris to consider reconstruction and economic problems in France.

One of the many important posts held by Mr. Fuller since the World War was that of chairman of the Engineering Board of Review for the Sanitary District of Chicago, which was formed in 1924 to consider a new sewage-disposal system in that city. The recommendation of the board to draw sufficient water from Lake Michigan to carry sewage in satisfactory dilution through the Illinois Canal to the Mississippi River was adopted and successfully carried out. In 1934 he was recalled to advise a special board of review in regard to additions to this system.

Mr. Fuller had always taken a keen interest in public-health work. He was consulting engineer to the United States Public Health Service in connection with municipal improvements necessary to public health, contributed largely to the development and adoption of the Standard Methods of Analysis for Water and Sewage sponsored by the American Public Health Association, and as chairman of its Council on Standardization, helped the American Water Works Association in the publication of its Manual of Water Works Practice.

Mr. Fuller was a member and chairman of an engineering advisory committee of the Reconstruction Commission of the State of New York, and advisor to the International Joint Commission on Boundary Waters between the United States and Canada. Among the projects on which the advice of his firm was sought during recent years were water works for Shanghai, China; the Wanaque aqueduct, for the New Jersey Water Policy Commission and the North Jersey District Water Supply Commission; new water supply and purification systems for Kansas City, Mo., Memphis, Tenn., and Wilmington, Del.; sewage-treatment works at New Haven, Conn., and Lexington, Ky.; a comprehensive system of intercepting sewers and sewage treatment for the Hackensack Valley Sewerage Commission of New Jersey involving some fifty communities bordering on the Hackensack River and its tributaries; the collection and treatment of sewage and industrial wastes reaching Narragansett Bay from the city of Providence and neighboring municipalities, a study of which was made for the Metropolitan Sewerage Commission of Rhode Island; and the future water supply of the Minneapolis-St. Paul district, a report on which was completed just prior to his death.

Mr. Fuller's work was not entirely in the hydraulic and sanitary fields, however. He was an expert in the valuation of public utilities and in adjusting disputes on rates and charges by utilities supplying water and other services. During the World War he was a member of the Pittsburgh valuation board, appraising the street railways in that city. He had also been a member of the valuation committee handling street-railway adjustments in Philadelphia and was consulting expert for the International Railways Company, owner of the street-car system in Buffalo, N. Y., and its suburbs on both sides of the Niagara Valley.

In addition to his consulting work Mr. Fuller found time to write many papers in his field, and three books: "Water Purification at Louisville," published in 1898; "Sewage Disposal," 1912; and "Solving Sewage Problems," 1926, of which his partner, James R. McClintock, was co-author.

At the time of his death Mr. Fuller was chairman of the board of The Engineering Foundation. He was appointed a member of the board by the American Society of Civil Engineers in 1931, served as vice-chairman the following year, and was elected chairman in 1933. He was a past-president and director of the American Society of Civil Engineers and a past-president of the American Water Works Association and the American Public Health Association. He became a member of the A.S.M.E. in 1910 and belonged to many other professional and scientific societies, including the Institution of Civil Engineers, Verein Deutscher Ingenieure, Association Generale des Hygienistes et Techniciens Municipaux, of France, Engineering Institute of Canada, The Franklin Institute, American Chemical Society, American Institute of Consulting Engineers, and the American Society of Bacteriologists. He was chairman of the Committee on Promotion and Attendance for the World Engineering Congress at Tokio, Japan, in 1929.

Among Mr. Fuller's clubs were the Engineers' and Machinery, New York; University, Chicago; Cosmos, Washington; Seaview Golf, Absecon, N. J.; Everglades, Palm Beach; Canoe Brook Country, Summit, N. J.; and the Pleiades Club, New York, of which he was president at the time of his death.

He is survived by his widow, Eleanor Todd Fuller; three sons by previous marriages: Myron E., Kemp G., and Asa W. Fuller; and three step-sons, Kenneth B., Gordon B., and George B. Fuller; and a sister, Mrs. Carl W. DeVoe, of Jerome, Idaho.

THAYER PRESCOTT GATES (1879–1934)

Thayer Prescott Gates, manager of the Finishing Department of the Riverside and Dan River Cotton Mills in Danville, Va., died suddenly of angina pectoris on the evening of January 1, 1934, while in Richmond, Va.

Mr. Gates was born in Lowell, Mass., on May 14, 1879, the son of Prescott Coburn and Ellen (Kittridge) Gates. He prepared for college at Phillips Andover Academy and took a mechanical engineering course at the Massachusetts Institute of Technology. He was vice-president of his class there one year and a member of Sigma Chi fraternity.

Textile manufacturing had been Mr. Gate's work ever since he left the Institute. After some early experience under his father at the mill of Josiah Gates & Sons, in Lowell, he was employed on the construction of mills and layout and erection of machinery at the Lawrence, Mass., mills of the American Woolen Company in 1902–1903. During the next year he secured drafting-room and shop experience with the Jeffrey Manufacturing Company, Columbus, Ohio.

In February, 1904, Mr. Gates became master mechanic for the Wilkes-Barre (Pa.) Lace Manufacturing Company, where he remained for two years. He then took the position of foreman of the J. and W. Lyall Machine Works and master mechanic of the Brighton Mills at Passaic, N. J. In 1907 he became efficiency engineer for the Sayles Bleacheries at Saylesville, R. I., and continued with that mill and the Glenlyon Dye Works, reorganizing them along the lines of scientific management and handling all engineering problems in connection with construction, power supply, maintenance, and operation until 1917. The following year he was connected with Collins and Aikman, of Philadelphia, manufacturers of plushes.

In the fall of 1918 he opened an office in Providence, R. I., as a consulting engineer and textile specialist. He transferred his office to New York in 1930, and continued in practice until 1932, when he took the position he held at the time of his death.

Mr. Gates became a junior member of the A.S.M.E. in 1906 and a member in 1913, and was a Knight Templar in the masonic fraternity. He was a contributor to the textile press and had worked on an emulsifier for use in textile manufacture.

Surviving Mr. Gates are his widow, Cecile (Lovell) Gates, whom he married in 1904, and a daughter, Marjorie (Gates) Alley.

FREDERICK TAYLOR GAUSE (1866–1934)

Frederick Taylor Gause, for nearly forty years connected with the Standard Oil Company of New York, died in San Diego, Calif., on October 17, 1934.

Mr. Gause was born at Kennett Square, Chester County, Pa., on March 16, 1866, son of S. Irwin and Edith (Taylor) Gause. After leaving public school he served an apprenticeship with the Harlin & Hollingsworth Co., and secured drafting and shop experience with that company and at the Trenton Iron Works. He then entered Stevens Institute of Technology, from which he was graduated in 1891.

From the time of his graduation until his retirement in 1930, Mr. Gause was in the employ of the Standard Oil Company. The first few years were spent mainly on experiments to determine the merits of pure petroleum oil for lubricating reciprocating marine engines. In 1894 he was sent to the Far East to report on the possible demand for heavy petroleum products in the principal commercial centers there, particularly at the seven centers in India, China, and Japan where the company had opened offices the previous year to sell kerosene oil. As a result of his investigation the company added to these offices organizations to market heavy petroleum products, and Mr. Gause was assigned to supervise their establishment. Later he made similar trips to South America, South Africa, Australia, New Zealand, etc., establishing marketing branches at many points and otherwise assisting in the conduct of the Export Marketing Department of the company.

Mr. Gause became a member of the A.S.M.E. in 1893, belonged to the Engineers' Club, New York, and was a former member of the American Association for the Advancemnt of Science. He is survived by his widow, Gertrude (Grier) Gause.

FREDERICK AUGUST GEIER (1866–1934)

Frederick August Geier, president of the Cincinnati Milling Machine Company, Cincinnati, Ohio, died suddenly of a heart attack at his home in that city on March 27, 1934.

Mr. Geier was born in Cincinnati on June 23, 1866, the son of Philip and Sophia L. (Otten) Geier. He was graduated from the Woodward High School there in 1884, and for a time worked in a bank in Newton, Kansas. At the age of 21 he became associated with the Cincinnati Screw & Tap Co., which in 1889 became the Cincinnati Milling Machine Company. He served as manager of the company for a time, was made secretary and treasurer, and finally became president.

Within a few years after Mr. Geier's affiliation with the company, increased business necessitated the removal of the plant from Second and Plum Streets, where it had been established in 1884, to new quarters at Spring Grove Avenue and Alfred Street. By this time the company had entered the foreign trade field and exhibited a revolutionary milling machine at the Paris World Fair in 1900.

After 19 years at Spring Grove Avenue and Alfred Street, the company was housed in a plant at Oakley where Mr. Geier was a moving spirit in organizing a factory colony that did much to make the suburb one of the leading machine-tool centers of the world.

Mr. Geier was also president of the Factory Power Company, Factory Colony Company, Cincinnati Grinders Incorporated, Cincinnati Rubber Manufacturing Company, and Cincinnati Morris Plan Bank. He was a director of a number of other companies and institutions, including the Cincinnati Bickford Tool Company, Central Trust Company, Lincoln National Bank, and the Children's Home, all of Cincinnati. He was a leader in promoting vocational education in Cincinnati and aided in the establishment of the cooperative engineering system at the University of Cincinnati when he was a trustee and president of the board of the university. He also established a loan fund for cooperative engineering students. At the time of his death he was president of the Ohio Mechanics Institute.

From 1913 to 1915 Mr. Geier was chairman of the Council of Social Agencies for Cincinnati, from 1915 to 1917 chairman of the Central Budget Committee, and from 1917 until his death, a member of the Executive Committee of the Cincinnati Community Chest.

Other organizations in which Mr. Geier was actively interested included the Cincinnati Bureau of Governmental Research, the National Crime Commission, and the Ohio Valley Improvement Association.

Mr. Geier became an associate of the A.S.M.E. in 1900. He served as chairman of the Cincinnati Section in 1917 and was a manager of the Society from 1917 to 1920. He was interested in the standardize tion and research work of the Society and contributed to the support of the work of the Special Research Committee on Gears.

Mr. Geier was an early advocate of the eight-hour day and helped to organize the National Metal Trades Association. In 1910 and 1911 he was president of the National Machine Tool Builders' Association and in 1933 he helped to organize the Machinery and Allied Products Institute. He was also a member of the American Academy of Political and Social Science, American Institute of Banking, National Municipal League, National Society for Promotion of Industrial Education, Chamber of Commerce of the United States, Cincinnati Chamber of Commerce, and many other such organizations. His clubs included the Engineers, Commercial, Queen City, Camargo, and others in Cincinnati and vicinity.

Surviving Mr. Geier are his widow, Juliet (Esselborn) Geier, formerly of Portsmouth, Ohio, whom he married in 1904, and two sons and two daughters.

HAROLD LOTHROP GOODWIN (1897–1933)

Harold Lothrop Goodwin was born in Roxbury, Mass., on November 14, 1897, son of Homer and Mabel A. (Bates) Goodwin. He was graduated from the Roxbury Latin School in 1916 and received an S.B. degree in mechanical engineering from the Massachusetts Institute of Technology in 1921. Following his graduation he engaged in construction work in Boston with L. P. Soule, soon securing a position with Stone & Webster, Inc. In 1924 he was sent to Philadelphia, Pa., in connection with the construction of a building for the Insurance Company of North America, and in the following year worked on a power-plant building for the Savannah Light & Power Co., and on construction at Davis Islands, Tampa, Fla.

He returned to Boston in 1928 and was associated with L. P. Soule again in constructing an addition to the John Hancock Life Insurance Company building there. In 1930 he took a position in the Generating Office Department of the Edison Electric Illuminating Company of Boston, where he was employed at the time of his death on December 4, 1933.

Mr. Goodwin became a junior member of the A.S.M.E. in 1921. He is survived by his widow, the former Miss Martha L. Dewey, of Great Barrington, Mass., whom he married in 1925.

SAMUEL MARTIN GREEN (1864–1934)

Samuel Martin Green was born at Benton Harbor, Mich., on April 13, 1864, son of Martin Green, a civil engineer well-known for his connection with some of the greatest engineering projects in the United States. His mother was Mary Frances Stuart (Hathaway) Green.

Mr. Green was graduated from the course in mechanical engineering at Worcester Polytechnic Institute in 1885. Following his graduation he was draftsman for about a year with each of the following companies: F. E. Reed Company, Worcester, Mass., machine-tool manufacturers; the John T. Noye Manufacturing Co., Buffalo, N. Y.; the Deane Steam Pump Company, Holyoke, Mass.; and the Merrick Thread Company, Holyoke. He then became master mechanic of the last-named company, which permitted him to do outside consulting work. About 1898 the American Thread Company was formed and the Merrick Thread Company merged with it. He served as consulting engineer to the company until he organized his own business in the fall of 1908 at Springfield, Mass., under the firm name of Samuel M. Green Company.

Among the numerous companies which Mr. Green served as consulting engineer prior to the establishment of his own company were the Union Metallic Cartridge Company, Bridgeport, Conn., and the United States Envelope Company. For the latter company he designed and supervised the construction of a three-story factory building at Waukegan, Ill., and a five-story structure at Springfield in connection with which he designed and supervised the installation of a steam boiler and power plant and electrified the entire plant.

In connection with Mr. Green's work for the Union Metallic Cartridge Company, the following is taken from a memoir prepared by Harry B. Hopson, of Springfield, for the American Society of Civil Engineers, of which Mr. Green was also a member:

"In 1906, Mr. Green designed a steam boiler and turbine plant for the Union Metallic Cartridge Company. When this work was completed, the company requested him to make an examination of its primer loading plants after a disastrous explosion. He revised the method of loading primers entirely; built a new primer-loading plant; and devised an entirely new method of handling and mixing fulminate, which, to that time, had been an exceedingly dangerous operation. Since the revision of the method by Mr. Green the company has never had an explosion in its plant.

"About that time, the company had a disastrous explosion in its powder-storage farm where many tons of powder had been stored. Mr. Green was given the problem of designing a powder-storage farm that would be safe. He designed roads, bunkers, and small storage houses for powder, and supervised the installation of this equipment.

The Union Metallic Cartridge Company was the owner of the Remington Arms Company, at Ilion, N. Y. This latter company was not making a profit, and Mr. Green was given the commission of investigating the entire process of manufacturing arms. He spent two or three years at the Remington plant, designing new equipment and re-arranging machinery, until, finally, it was operating on a profitable basis."

Concerning Mr. Green's work after 1908, Mr. Hopson writes: "Among his more important clients was the Farr Alpaca Company, of Holyoke, which spent under his direct supervision and charge from $12,000,000 to $15,000,000.

"In 1911, he was given a commission by Max Henkels, who had come to the United States from Germany to engage in the lace-manufacturing business, at Bridgeport. Mr. Green designed and supervised the construction of a weave shed; a two-story and basement brick mill building; and in 1915, a three-story building of reinforced concrete. The design of this latter building was rather extraordinary because the posts had to be located so that Nottingham-lace looms could be installed in the building. The floors of the building were of the mushroom type of construction.

"In 1912 and 1913, Mr. Green designed and supervised the construction of a paper mill for the Strathmore Paper Company, at Woronoco, Mass., together with its power plant, filter plant, etc. For this same company, in 1923, he designed a reinforced-concrete bridge across the Westfield River, a span of approximately 400 ft, supported by three high arches.

"In 1915, he designed and supervised the construction of the plant of the Warner-Klipstein Chemical Company, at South Charleston, W. Va. This was a caustic-chlorine plant consisting of many buildings, including a power plant of about 2000-kw capacity, a plant for the production of carbon tetrachloride, carbon bi-sulphide, chlorine, and caustic soda.

"In 1918, Mr. Green was awarded the contract by the United States Government to design and supervise the construction of the large chlorine plant at the Edgewood (Md.) Arsenal. This chlorine plant was the largest of this character that had ever been built to that time. Its capacity was 100 tons of chlorine gas and caustic soda per 24 hr. Mr. Green's work consisted in designing all the buildings which were of brick and wood construction. This plant consisted of a building for chlorine cells; a salt-treating plant; a boiler plant, of approximately 2000 boiler-hp; an evaporator plant for salt caustic evaporation, containing eight double-effect evaporators; and a caustic fusion plant. He designed all the machinery and supervised its installation, and was under contract with the Government to supervise the installation of all the equipment and the starting of its operation. This plant was successfully constructed, and operation was started before the end of the World War, or in approximately six months.

"In 1926, Mr. Green designed a new boiler plant for the Fitchburg Paper Company, of Fitchburg, Mass. This work consisted of a complete revision of an old boiler-house and the installation of one 1000-hp, 250-lb pressure boiler, with complete pulverized-coal equipment, superheaters, and piping system. In connection with this work he also designed a high-pressure steam line, with all the supports, about 1200 ft long, running from this boiler plant to one of the other mills over a bridge, about 100 ft long, also designed by Mr. Green. This work was entirely under his supervision; he let all the contracts and supervised the completion of the work.

"In 1927, he designed a new boiler installation for Joseph Bancroft and Sons Company, at Wilmington, Del. This work consisted in the removal of two old boilers and the installation of one 1000-hp, 250-lb pressure boiler, together with all its auxiliary equipment, stokers, etc., and a revision of the boiler-house for this installation.

"During 1927 and 1928, Mr. Green also engineered and supervised the construction of the Integrity Trust Building in Philadelphia, Pa."

Mr. Green had been a member of the A.S.M.E. since 1890, and in addition to his membership in the A.S.C.E., belonged to the Engineering Institute of Canada, Engineering Society of Western Massachusetts, and the American Geographical Society. He was a member of the Chamber of Commerce and Taxpayers' Association of Springfield, and president of the latter at the time of his death. He also belonged to the Constitutional Liberty League of Massachusetts, National Economy League, and Navy League of the United States. He was a member of all the branches of the York Rite and of the Scottish Rite of the masonic fraternity, being a Knight Templar, a Thirty-second Degree Mason, and a member of the Shrine.

Mr. Green's death occurred at Springfield on March 22, 1934. He is survived by his widow, formerly Miss Ida McKown, of Holyoke, and by their two married daughters, Mrs. Mildred (Green) Page, of Longmeadow, Mass., and Mrs. Lydia (Green) Meadows, of New York, N. Y.

JOSEPH RUDD GREENWOOD (1883–1934)

Joseph Rudd Greenwood, who retired in 1930 as president and treasurer of the New York Vitreous Enamel Products Corporation, Flushing, L. I., N. Y., died in New York on March 2, 1934.

Mr. Greenwood was born in New York on May 27, 1883, the son of Isaac J. and Mary A. (Rudd) Greenwood. He attended the Collegiate and Halsey schools in New York and the Hill School at Pottstown, Pa., and was graduated as a civil engineer from Princeton University in 1905. After serving as an instructor in civil engineering at Princeton in 1905–1906, he took a position as assistant engineer with The Ballwood Company in New York, manufacturers of single- and four-valve engines, high- and low-pressure and mixed-flow turbines, and high-pressure pipe joints and bends. He was made superintendent of the pipe shop in 1908, superintendent of construction in the field in 1910, general superintendent in 1911, and general manager in 1914.

In 1916 Mr. Greenwood left The Ballwood Company to be mechanical engineer in the office of Charles H. Higgins, but the following year entered the World War as a driver and section leader in the American Ambulance Field Service in France. He rose to the rank of captain in the service.

After the close of the War he returned to the employment of Mr. Higgins, with whom he remained for several years before becoming connected with the New York Vitreous Enamel Products Corporation.

Mr. Greenwood became a member of the A.S.M.E. in 1920, and belonged to the Sons of the Revolution, and the University and Princeton clubs. He is survived by his widow, Ruth M. (Dayton) Greenwood, whom he married in 1919.

ELWOOD GRISSINGER (1869–1934)

Elwood Grissinger, of Buffalo, N. Y., whose death occurred on October 8, 1934, was born at Mechanicsburg, Pa., on March 3, 1869. After completing his high school course there he studied telegraphy in the ticket office of the Cumberland Valley Railroad at Mechanicsburg. At the age of 17 he secured an appointment as a telegraph operator on the Norfolk & Western R.R. Subsequently he studied stenography at Chaffee Institute, Oswego, N. Y., and worked in the general passenger office of the New York, Ontario & Western R.R. and for the Pennsylvania Railroad, at Harrisburg, Pa.

In 1890 Mr. Grissinger entered Lehigh University, from which he was graduated four years later with a degree in electrical engineering. His scholastic standing was such that he was appointed orator for Founder's Day and commencement. He was elected to membership in Tau Beta Pi, honorary engineering fraternity, as well as in Delta Upsilon. In 1916 the university conferred an honorary M.S. degree upon him.

After his graduation from Lehigh University Mr. Grissinger worked for two years in the shops of the Westinghouse Electric & Manufacturing Co., and for three years was district engineer and salesman for the company at Syracuse and Buffalo. From 1899 to 1906 he was commercial engineer for the Buffalo General Electric Company and the Cataract Power & Conduit Co. The following year he was general agent for the Niagara Falls Power Company and the Canadian Niagara Power Company.

Since 1906 Mr. Grissinger had engaged in consulting engineering in the mechanical and electrical fields, and in research work. He patented a complete system of long-distance telephony, including telephonic speech amplifiers and the only known operable method of associating them with the speech transmission circuits. This system has been adopted for use on transcontinental lines and all classes of long-distance circuits, the United States patents having been purchased by the American Telephone & Telegraph Co., and was used in Europe by the United States Army and its Allies during the World War. He also patented a method of sound amplification as exemplified by phonograph horns.

Mr. Grissinger became a member of the A.S.M.E. in 1912. He was a Fellow of the American Institute of Electrical Engineers and a member of The Franklin Institute. He had contributed special articles on electric railroading to the journals of the Brotherhood of Locomotive Engineers and Firemen.

Mr. Grissinger married Lucy Martin Ash, of Oil City, Pa., in 1897, and is survived by her and a son, Elwood.

CONRAD VELDER HAHN (1890–1933)

Conrad Velder Hahn, associate professor of mechanical engineering at Drexel Institute, Philadelphia, Pa., and head of the mechanical engineering department of the Drexel Evening Diploma School, died on December 3, 1933, at the Temple University Hospital.

Dr. Hahn was born in Philadelphia on May 9, 1890, the son of John G. and Elizabeth (Velder) Hahn. He attended the Central Manual Training High School in Philadelphia and was graduated from the University of Pennsylvania with a B.S. degree in engineering in 1908. An M.E. degree was conferred upon him by the same university four years later and San Jose University gave him the degrees of Doctor of Engineering and Doctor of Philosophy.

Dr. Hahn was a member of the faculty of the civil engineering department of Temple University Evening School from 1910 to 1912 and of the mechanical engineering department of the University of Pennsylvania during the next two years. He had been connected with Drexel Institute since 1919, when he became director of the mechanical engineering department of the evening school.

He had also carried on a consulting practice since 1910 as managing director of the Hahn Company, Philadelphia. He was also a director and associate engineer of the International Development Company, Washington, D. C., consulting mechanical engineer for the Turbo Motor Company, East Radford, Va., and a member of N. T. Whitaker & Co., Washington.

During the World War he served in the Ordnance Department, as first lieutenant and captain, and received special mention for technical service at the Army Ordnance Base Depot, France, and the Aberdeen Proving Ground, Scotland. He held the rank of major in the Ordnance Reserve Corps.

Dr. Hahn became a junior member of the A.S.M.E. in 1915. He also belonged to The Franklin Institute, National Philosophical Society, masonic fraternity, and a number of fraternal orders and clubs, including the honorary engineering fraternities, Sigma Xi, Tau Beta Pi, and Pi Tau Sigma.

He is survived by his widow, Lillian (Roth) Hahn, and by his mother, a brother, and a sister.

THEODORE COMMODORE HAILES, JR. (1883–1934)

Theodore Commodore Hailes, Jr., field engineer for the New York State Temporary Emergency Relief Administration, Binghamton, Division Office, died in that city on October 29, 1934.

Mr. Hailes was born in Albany, N. Y., on September 15, 1883, the son of Theodore Commodore and Elizabeth (Brooks) Hailes. After his graduation from the Albany High School he entered Rensselaer Polytechnic Institute, from which he received a C.E. degree in 1907. He took part in practically all extra-curricula activities while at the Institute and was one of the founders of the Rensselaer Technical Society, which later became Pi Kappa Phi fraternity.

From the time of his graduation until the United States entered the World War, Mr. Hailes engaged in the general practice of engineering and contracting, with an office in Albany. His work included examinations and reports on a number of hydroelectric-development projects; the design and installation of signal equipment on the Erie Canal; construction of several telephone lines; destruction by dynamite of ice gorges in the Hudson River several years; supervision of construction of state highways, and quarrying and crushing of stone for them; and investigation and reports as expert witness in many cases involving engineering.

In 1917 he accepted a position as superintendent of the Neemes Brothers Iron Foundry, at Troy, N. Y., which was engaged in war work. He continued with that company for several years after the close of the war, then became manager for the Tool & Device Corp., of Troy. More recently he had been secretary and treasurer of the Engineering Service Corporation, Albany.

Mr. Hailes became a member of the A.S.M.E. in 1922 and was a 32d degree Mason. He is survived by his widow, Charlotte S. (Welch) Hailes, whom he married in 1909, two sons, Theodore C. Hailes, 3d, and Gardner M. Hailes, and three sisters.

FRANCIS EDWARD HALLORAN (1882–1932)

Francis Edward Halloran was born on June 5, 1882, in Brooklyn, N. Y., the son of Patrick and Anne (Burke) Halloran. He was graduated in 1900 from the Boys High School in Brooklyn and later continued his studies through the International Correspondence Schools.

The greater part of Mr. Halloran's engineering work was for the E. W. Bliss Co., Brooklyn. He was associated with this company from early in 1902 until after the close of the World War with the exception of about a year during which he worked for C. J. Regan, Maspeth, L. I., designing dies for sheet-metal ware and attachments for spinning lathes. His first assignments for the Bliss company were in the general drafting room. He was then transferred, upon his request, to the drafting room of the die department. After the year at Maspeth he began his most important work for the company, in connection with torpedoes. In the capacity of mechanical draftsman in the torpedo drafting room at the main works of the company he worked from 1906 to 1910 laying out the mechanisms in the Bliss-Leavitt torpedo; helping to develop the outside superheater system; experimenting with gyroscopic steering, hydrostatic depth regulating, and reducing-valve gears; redesigning dynamometers; and developing new tools and fixtures for the manufacture of torpedo parts. He was made chief draftsman in 1910 and during the next three years was engaged in the design and construction of an "angle-fire" gyroscope, depth mechanism, steering engines, launching tubes, and high-pressure air accumulators for the company's proving steamer *Emblane*.

In 1913 the torpedo department was relocated at the 53d Street works of the company and for about ten years longer Mr. Halloran devoted himself to supervising all the experimental and research work incident to further increasing the speeds, range, and reliability of torpedoes.

After leaving the Bliss company Mr. Halloran was mechanical designer first at the Edison Lamp Works of the General Electric Company at Harrison, N. J., and then with the Trans-Lux Daylight Picture Screen Corporation, Brooklyn. He helped to perfect the machinery for manufacturing frosted bulbs, in the former position, and in the latter worked on the Trans-Lux stock quotation projection machine and motion picture projection. Poor health forced his retirement from the Trans-Lux company, and his death from heart disease occurred on March 20, 1932, at the home of his sister, Mrs. M. Gertrude Quinn, Corona, L. I., N. Y.

Mr. Halloran became a member of the A.S.M.E. in 1920.

JOHN HANKIN (1865–1933)

John Hankin, for nearly forty years president of John Hankin & Brother, consulting engineers, New York, N. Y., died at his home in Passaic, N. J., on November 30, 1933.

Mr. Hankin was born on September 21, 1865, in Farnworth, Lancashire, England, the son of William and Susannah Hankin. He came to the United States with his parents in 1874 and secured his early education in the public schools in Rome and Watertown, N. Y. Subsequently he attended the Brooklyn Evening High School for one year and took mechanical and scientific courses at Cooper Institute.

Mr. Hankin's first employment was in Watertown, N. Y., in the shops of the Davis Sewing Machine Company, in 1880. He then served a four-year apprenticeship at the Steam Engineering Works there, after which he was employed as a journeyman machinist at the works and by the Eames Vacuum Brake Company, Watertown, the James Brady Manufacturing Company, Brooklyn, and Arnoux & Hochausen Electric Co., New York. For two years, beginning in March, 1888, he was draftsman for the Buffalo Forge Company, and for the next three years engaged in sales engineering for the Akron Heating & Ventilating Co., Akron, Ohio, and Cook & Co. and Jas. Beggs & Co., New York.

From August, 1893, to a year from that time Mr. Hankin was a partner in the Armstrong Engineering Company, New York. He then went into the general engineering business for himself. The firm of John Hankin & Brother was established in 1900.

Mr. Hankin became a member of the A.S.M.E. in 1913 and belonged to the National Association of Heating and Piping Contractors, American Society of Professional Engineers, American Institute of the City of New York, and Building Trades Employers' Association of New York. He was also a member of the masonic fraternity, the Blizzard Men of 1888 Club, and the Transportation Club, New York.

Surviving Mr. Hankin are his daughter, Mrs. E. Hans Jacob, of New York; a brother, Dr. Walter Hankin, of Bisbee, Ariz.; a stepson, Dr. John Townsend, of Passaic, a son of his deceased wife, the former Mrs. Ida Dieter Townsend, of Buffalo; and two grandsons, Robert Earl and John Edward Jacob.

JAMES HARTNESS (1861–1934)

The American Society of Mechanical Engineers lost one of its most honored members, and the profession one of its most distinguished engineers, in the death of Ex-Governor James Hartness, of Springfield, Vermont, after a long illness, on February 2, 1934. His loss will be deeply felt by all who have ever known him.

James Hartness was born in Schenectady, N. Y., on September 3, 1861. His father, John W. Hartness, a man of dry, salty humor and keen understanding of human nature, moved to Cleveland in 1863, where he was a foreman in various machine shops. James Hartness was educated in the Cleveland public schools and began learning his trade at sixteen, at a wage of 45 cents a day, first with Younglove, Massey, and Company, of Cleveland, where his father was superintendent, and then in the machine shop of the Union Steel Works. In the latter shop he first came into contact with close, accurate work

under Jason A. Bidwell, one of the fine New England mechanics of the older generation. Later, Mr. Hartness went to the Lake Erie Iron Works as a toolmaker. In 1882 he secured a position, through correspondence, as foreman of the Thomson, Stacker Bolt Company in Winsted, Conn. When he reached there the superintendent exploded at the youth of the man whom he had hired. Hartness waited until the storm was over and then offered to release the firm from its contract but to stay until his successor was installed. He remained three years. In 1885 he went to the Union Hardware Company, of Torrington, Conn., manufacturers of gun implements, first as toolmaker, then foreman, and later as inventor. During the year 1888 he worked for a few months each at the Pratt & Whitney Company in Hartford, at Scottdale, Pa., and with the Eaton, Cole, and Burnham Company in Bridgeport.

Connection With Jones & Lamson Machine Company Established in 1889

He went to the Jones & Lamson Machine Company in March, 1889, the year in which the company moved from Windsor to its present location at Springfield, Vermont. He was at first superintendent until 1893, then manager until 1900, and president from then until his retirement last year. Mr. Hartness' going to the Jones & Lamson Machine Company added another to the list of great mechanics who made the history of this company, or rather the succession of companies of which it is a part. It began about 1838 at Windsor, Vermont, as the Robbins and Lawrence Company, and through the middle of the century, under Richard S. Lawrence, Frederick W. Howe, and Henry D. Stone, this small shop made engineering history. The ever-widening influence of their work is felt to this day. Here was developed and first manufactured the hand-operated turret lathe, the beginnings of the milling machine later known as the Lincoln miller, drop hammers, and many tools adapted to arms manufacture and other types of accurate work. Through various changes the firm became Lamson, Goodnow, and Yale, E. G. Lamson and Company, the Windsor Manufacturing Company, and in 1879 the Jones & Lamson Machine Company. Mr. Hartness brought with him the fundamental ideas of the Hartness flat turret lathe which he had had in his mind for a number of years. He undertook, as superintendent, to develop and manufacture this machine, and in so doing opened up a new period in the history of the company. Like other shops of the time, the successive companies at Windsor had scattered their energies over a wide line. An old "poster" of the Windsor Manufacturing Company shows them offering engine lathes, turret lathes, planers, saw-mill and quarrying machinery, engines and boilers, trip-hammers, sewing machines, rifles, "etc., etc." Things had improved somewhat in this respect, but with the removal of the company to Springfield and the coming of Mr. Hartness there was a radical change. He concentrated its activities on a single size of the flat turret lathe, and the tools for it, dispensed with agents and sold direct, and instituted a new type of sales literature. Under his leadership the company rapidly acquired a world-wide recognition. This is all the more remarkable for its being located in a small town in a Vermont valley six miles off a minor railway, a location, by the way, which he always defended and said he would in no wise change.

Services to A.S.M.E. and the Public

Mr. Hartness joined The American Society of Mechanical Engineers in 1891. He was a manager from 1909 to 1912, and vice-president in 1912 and 1931. On the German trip of the Society in 1913, President Goss was unable to go and Mr. Hartness, as senior vice-president, acted as head of the party on all the formal occasions. He did this so acceptably to all that he was elected president of the Society in the following year, 1914. He was president of the American Engineering Council from 1924 to 1926, and as a past-president, served as a member of its Assembly for the six years following his presidential term. He therefore held the highest elective positions in the gift of his profession.

In 1912 he was appointed a member of the A.S.M.E. Special Committee on Tolerances in Screw-Thread Fits, and he served for many years on this and other committees on screw-thread standardization.

His public services were as distinguished as those for the engineering profession. He served his state as chairman of the State Board of Education for six years (1914–1920), was chairman of the Committee of Public Safety during the War, and also Federal Food Administrator, and, in 1921 to 1923, served as the sixty-first governor of Vermont. In national affairs he was a member of the commission which represented the United States at the Inter-Allied Air Craft Standardization Conferences in London and Paris; and was vice-chairman of the Congressional Screw-Thread Standardization Committee from 1918 to 1920.

He was a member of the Society of Automotive Engineers, the Aero Club of America, the American Astronomical Society, a Fellow of the American Association for the Advancement of Science, the Royal Astronomical Society, the Royal Society for Encouragement of Arts, the Royal Societies Club of London, and the Institution of Mechanical Engineers in London. In 1921 he was awarded the John Scott Medal by the Board of Directors of the City Trusts of Philadelphia, and the Edward Longstreth Medal by The Franklin Institute of Philadelphia.

He is the author of a number of papers before the A.S.M.E., including his presidential address on "The Human Element, The Key to Economic Problems." He also wrote two books, "The Human Factor in Works Management" which was widely distributed, and translated abroad, and "Machine Building for Profit."

He was given the honorary degree of M.E. by the University of Vermont in 1910, honorary M.A. by Yale University in 1914, and LL.D. by the University of Vermont in 1921.

In 1885 he married Lena Sanford Pond, of Winsted, Conn., who died in March, 1933. He is survived by two daughters, Anna Jackson, who married Dr. William H. Beardsley, and Helen Edith, who married Mr. Ralph E. Flanders.

Mr. Hartness a Prolific Inventor

Mr. Hartness was a prolific inventor and he held patents in this country and abroad for more than 100 inventions. Among these was the flat turret lathe which he began developing from the time of his first coming to Springfield. In the old type of high turret lathes, developed originally by the Robbins and Lawrence Company, the turret revolved about a large vertical pin and all subsequent development had followed that design. Mr. Hartness felt that a stiffer and more accurate construction was possible and investigated the disk turret, the barrel turret, and the flat turret. He settled finally upon the flat turret which appealed to him as offering all the advantages of the old type with possibilities of greater stiffness and precision. It was low, could be clamped at points far apart, and permitted the location of the locking pin directly under the cutting tool. Following out this general design, he developed a lathe which has found wide use here and abroad for the large output of accurate work. Among the improvements were the cross-sliding head in 1903, the double-spindle type in 1910, and the Hartness automatic lathe.

The Lo-Swing and Fay automatic lathes are two more machines invented by him, or greatly influenced by his work. In connection with lathes, he developed many special tools to be used in connection with them, and among these were the well-known Hartness threading dies, which became an important element in the company's business. Associated with these was the Hartness thread comparator, a refined method for the gaging of screw threads by optical means, which not only tells whether the threads are off standard, but precisely wherein they depart from it. He also invented the Hartometer, another type of gage for screw-thread inspection.

His Interest in Astronomy

Mr. Hartness' avocations were as interesting as his vocation. He spent four summers on a yacht following down the coast as far as Chesapeake Bay. In 1916 he learned to fly and won an amateur pilot's license. His interest in aviation continued throughout his whole life and he was largely influential in establishing a landing field at Springfield as a memorial to the soldiers and sailors of the World War. It was at this field that Lindbergh landed for his stop in Vermont during the tour of the country after his Paris flight.

For years astronomy claimed his attention. After suffering from a cold acquired in using the old-fashioned type of telescope, he devised the turret equatorial telescope, which was an adaptation of the fundamental idea of the flat turret lathe, wherein a large ring was substituted for one of the axes in the usual equatorial type. This permitted the observer to work inside, under warm and comfortable conditions at all times. One of the happy incidents of the A.S.M.E. trip in 1913 was to run into a model of this type of telescope in the astronomical section of the Deutsches Museum at Munich. This invention was the subject of a valuable paper before the Society at its Annual Meeting in 1911, and gained for him membership in both English and American astronomical societies. It was mounted over an underground room, one of a long series, running under the lawn in front of his home, connected by a heated and air-conditioned tunnel, and containing workrooms, library, and lounging room. Here he did much of his later work, and it formed a unique feature of the home whose charming hospitality will long be remembered.

His interest in astronomy aroused many others, so that there is probably no community of size similar to Springfield where there is such widespread interest in astronomy and in the building of amateur telescopes.

His Influence on Other Men

An outstanding characteristic of Mr. Hartness' life was his influence on and encouragement of young men. A number of impor-

tant machines have been developed by men connected with his company and are now being manufactured by other firms. He discovered and helped in various ways many men now well known in the engineering profession, among whom are George O. Gridley, inventor of the Gridley automatic, and vice-president of the New Britain-Gridley Machine Company; Edwin R. Fellows, inventor of the gear shaper, and president of the Fellows Gear Shaper Company; the late William L. Bryant, who was president of the Bryant Chucking Grinder Company; Frederick P. Lovejoy, president of the Lovejoy Tool Company; and Ralph E. Flanders, who succeeded Mr. Hartness as president of the Jones & Lamson Machine Company. Mr. Hartness' concentration on his own machines was so great that he would not be diverted in his own work by the inventions and ideas of these younger men, but as they brought their ideas to him he freely gave them help and encouragement and was a factor in their success.

Though Springfield is a small country town, there has grown up there a large group of important tool-building plants, largely through his influence.

Governor Hartness was one of the foremost mechanical engineers of this country and generation, and a splendid example of the engineer in public life. The Society holds him in affection and honor as an engineer and as a citizen.—J. W. Roe.[1]

WILLIAM JOSEPH HENNESSY (1873–1934)

William Joseph Hennessy, a veteran of the Spanish-American War, died in Washington, D. C., on November 26, 1934, and was buried in Arlington Cemetery with military honors. When war was declared with Spain Mr. Hennessy was employed by the American Line Steamship Company on the *St. Paul*, which was taken over by the Navy Department and placed under the command of Captain Charles D. Sigsbee, who had commanded the battleship *Maine*. Mr. Hennessy enlisted as a chief machinist and served on the *St. Paul* until the close of the war.

He was born at Jersey City, N. J., on September 27, 1873, son of Michael J. and Ellen T. (Clancy) Hennessy. His father was chief engineer in the United States Navy. After he had completed his high-school course in Jersey City he became an apprentice machinist with the William Cramp & Sons Ship & Engine Building Co. and during his apprenticeship also took evening courses in mechanical engineering at The Franklin Institute, and correspondence courses in marine engineering through the International Correspondence School.

He entered the employ of the American Line in 1895, and with the exception of his six months in the Spanish-American War, was connected with the company for five years, serving from junior to second engineer on vessels between the United States and England. In 1901 and 1902 he was a machinist in the Construction and Repair Department at the United States Navy Yard, New York, engaged for the most part in experimental work. During the next two years he was first assistant engineer for the Mallory Steamship Company, on boats running between New York and Georgia, Florida, and Texas cities.

He returned to Government service in November, 1905, as chief engineer in the United States Coast and Geodetic Survey. During about eight of the twelve years he spent in this service he was connected with important surveying expeditions in the Philippines, Alaska, and the West Indies, as well as work on the Atlantic Coast. Four years were spent in shore duty in the office of the Survey at Washington, and in different parts of the United States. One of his assignments was the inspection of the steam yacht *Isis* in 1915, which was taken over by the Navy upon his recommendation. He designed, wrote the specifications,and supervised the construction of the boilers, engines, and auxiliaries of the steamer *Surveyor*, built in 1917 for the Survey at the works of the Manitowoc Ship Building Company in Wisconsin, and served as chief engineer on its maiden trip to Washington and Norfolk.

During the World War Mr. Hennessy was inspector of engines and boilers for the Emergency Fleet Corporation of the United States Shipping Board, in connection with work being done in the Central States area.

Subsequently he was superintendent, for a time, of marine engine construction for the Dodge Manufacturing Company, in Indiana, and more recently had served as chief engineer in charge of the plants at Woodstock College, Gonzaga College, and St. Aloysius Church, Washington. He was in poor health the last few years of his life.

Mr. Hennessy received medals from the United States Navy and Shipping Board, and from the State of New Jersey for his war service.

Mr. Hennessy was not married. He is survived by a brother, the Reverend Charles J. Hennessy, of Woodstock, Md., and by two sisters, Miss Ella Hennessy and Mrs. Lydia Bahlman, both of Washington. He became an associate-member of the A.S.M.E. in 1918.

[1] Professor of Industrial Engineering, New York University, New York, N. Y., and author of "English and American Tool Builders," McGraw-Hill Book Co., 1926. Mem. A.S.M.E.

WILLIAM THOMAS HENRY (1845–1933)

William Thomas Henry, of Fall River, Mass., died in that city on July 22, 1933. He was born there on February 27, 1845, son of James and Martha (Whitaker) Henry, and received his early education in local schools. He took courses in civil and mechanical engineering with the class of 1870 at the Massachusetts Institute of Technology. He then entered the employ of Josiah Brown, civil engineer, Fall River, and upon his death five years later took over the business, which he continued until his retirement from business. He was connected with the design and construction and engineering problems of many of the textile mills in that part of the country and the inventor of a belt-driving mechanism patented in 1910.

Mr. Henry became a member of the A.S.M.E. in 1887 and belonged to the American Historical Society in Fall River and the Methodist Episcopal church there. His first wife, S. Louise Hadcock, whom he married in 1884, died nine years later. He is survived by his widow, Carrie S. Henry, whom he married in 1898.

FREDERICK WILLIAM HESPE (1889–1934)

Frederick William Hespe, whose death occurred on April 16, 1934, was born in Jersey City, N. J., on July 27, 1889. He attended the Stevens School and Stevens Institute of Technology, Hoboken, N. J. His first position was with the Public Service Corporation of New Jersey, Newark, where he worked on machine design and in the claims department from 1911 to 1918. The next year was spent as chief inspector of metal parts for the Standard Aircraft Corporation, Elizabeth, N. J. In 1919–1920 he was machine designer for the American Sawmill Machinery Company, Hackettstown, N. J., and from 1920 to 1923 held a similar position at the Picatinny Arsenal, Dover, N. J. He became co-partner of the Wharton (N. J.) Foundry in 1923 and owner in 1926, and engaged in the manufacture of gray iron castings until 1928. He then went to the Fokker Aircraft Corporation of America, Passaic, N. J., where he worked first as machine designer and subsequently as supervisor of the shop of the research department. More recently he had been supervisor of research at the General Aviation Manufacturing Corporation, Passiac.

Mr. Hespe became a member of the A.S.M.E. in 1931. He is survived by his widow, Rita Hespe.

FRANK JOSEPH HESS (1896–1934)

Frank Joseph Hess, instructor at Brooklyn (N. Y.) Technical High School from 1926 until the time of his death on December 26, 1934, was born in Buffalo, N. Y., on August 17, 1896. His parents were Frank William and Mary H. (Schroeder) Hess. He secured his early education in the schools of Clifton, N. J., then attended the Stevens Institute of Technology for two years. He secured a B.S. degree from New York University in 1920 and his M.E. degree two years later. Columbia University conferred a posthumous M.A. degree upon him in February, 1935.

Mr. Hess was an assistant superintendent with the Tidewater Oil Company in Bayonne, N. J., from 1920 to 1924. He was an engineer at Columbia University between then and the time he joined the faculty of the Brooklyn Technical High School.

Mr. Hess became a junior member of the A.S.M.E. in 1921 and also belonged to the masonic fraternity. He was an ensign in the United States Naval Reserve in 1918 and a sergeant in field artillery in the United States National Guards, in New York, from 1926 to 1930. He is survived by his widow, Edna M. (Flewelling) Hess, whom he married in 1924.

EDWARD MARRIOTT HEWLETT (1866–1934)

Edward Marriott Hewlett, an electrical engineer and inventor for many years associated with the General Electric Company and known for his connection with important engineering projects, died at his home in Schenectady, N. Y., on May 24, 1934, of a cerebral hemorrhage.

Mr. Hewlett was born at Cold Spring Harbor, L. I., N. Y., on September 14, 1866, a son of Edward T. and Eliza (Marriott) Hewlett. He was educated in private schools and by private study in civil and mechanical engineering and physics. From 1884 to 1890 he was general manager and civil engineer of the Cascade Town Improvement Company, Cascade, Colo. In 1888 he surveyed and laid out the first road to the top of Pike's Peak.

Mr. Hewlett took up electrical engineering in April, 1890, when he

went to work for the Thomson-Houston Electric Company, at Lynn, Mass., where he assembled, tested, and repaired electrical machinery. After about a year there he was sent to South River, N. J., where he worked until March, 1892, on the design and construction of embroidery machinery. He was then transferred to the New York office of the Thomson-Houston company and assisted in the installation of a focusing lamp and strings of incandescent lamps on the Statute of Liberty for the Columbian Exposition. After this work was completed he returned to Lynn, where he assisted in designing electrical switchboards for railways.

In 1894 Mr. Hewlett went to Schenectady as assistant design engineer for the General Electric Company, which had taken over the Lynn plant. Two years later he was advanced to the position of design engineer in the railway department, and when the switchgear department was organized in 1898 he was appointed its head. He was active in the work of this department until 1927, when he became consulting engineer for it. He retired from service in 1931.

Mr. Hewlett designed switchboards for the control of the locks of the Panama Canal and for the Metropolitan Street Railway Company, New York Edison Company, Manhattan Elevated Railway, Interborough Rapid Transit Company, and Grand Central Terminal, all in New York. He also handled the electric lighting of Tiffany's store in New York and designed special electrical effects for outdoor displays.

Between 1905 and 1907 Mr. Hewlett, in association with H. B. Buck, then engineer of the Niagara Falls Power Company, evolved the Hewlett suspension insulator, which made possible the transmission of voltages between 100,000 and 220,000 and higher, as against about 60,000 volts which could be sent with the earlier pin-type insulator. In 1906 he received a silver medal at the St. Louis Exposition for the development of the oil switch, from which sprang the entire modern line of oil circuit breakers, operating at high interrupting frequencies. He also originated the electrical relay, an indispensable factor in electrical control systems. He received the Coffin Award of the General Electric Company for special marine apparatus. He proposed and designed the modern fire-control system of the United States Navy, and during the World War was chief engineer of the Navy Experimental Station at New London, Conn. Altogether, Mr. Hewlett held about 160 patents for his inventions.

Mr. Hewlett became a junior member of the A.S.M.E. in 1897 and a member three years later. He was a Fellow of the American Institute of Electrical Engineers, and belonged to the masonic fraternity as well as to a number of clubs in New York and Schenectady, including the Engineers' in New York and the Mohawk and Electrical in Schenectady.

Surviving Mr. Hewlett are his widow, Mrs. Susan (Mauger) Hewlett, who resides with her son, John Mauger Hewlett, of Teaneck, N. J.; and two other sons, Edward P. Hewlett, of Schenectady, and Paul Marriott Hewlett, of Albany, N. Y.

LOUIS JOSEPH HIRT (1854–1933)

Louis Joseph Hirt, retired consulting engineer, New York, N. Y., died in that city on September 20, 1933. He was born in Paris, France, on September 26, 1854, and was educated in that country. He served as an apprentice in the shop of his father, Joseph Hirt, for two years and secured further experience in the marine shop of Esher Wyss & Co., Zurich, Switzerland, and with the Northern Railroad of France. He went to London in 1876 and was employed successively by Leger & Co., manufacturers of mathematical and optical instruments, and by J. Moore as general workman on watchmaker's tools. In 1878 and 1879 he was foreman, first at the Keystone Valve Company in Wolverhampton, England, and then at the steam-pump works of J. Evans & Son there.

Mr. Hirt came to the United States in July, 1880, and until the fall of 1882 was employed as foreman at the machine shop of Hanan & Lewes, New York. From then until 1885 he was superintendent of the machine shop of Merrill Bros., Brooklyn. He went to Rochester, N. Y., in 1885 and for two years was master mechanic of the Standard Nail Company there. During the next year he was chief draftsman at the Walker Manufacturing Company, Cleveland, Ohio, and from 1888 to 1890 traveling engineer for the Hill Clutch Works of that city.

From 1890 to 1896 Mr. Hirt was connected with the West End Street Railway, Boston, Mass., as master mechanic and chief engineer. He then returned to New York, taking the position of mechanical engineer for the New York Cable Railroad (later known as the Metropolitan). After two years with that company he became chief engineer for the New England Gas & Coke Co., Everett, Mass.

In 1900 Mr. Hirt took a position as mechanical engineer for the Pearson Engineering Corporation, New York, later becoming its vice-president and continuing in that capacity until 1920. In 1906 he took up consulting work for the Toronto Power Company, Toronto, Canada, and the Winnipeg Electric & Manitoba Power Companies, Winnipeg, Canada, which he served until 1926. In 1910 he also became consulting engineer for the Brazilian Traction, Light & Power Co., Ltd., Mexican Light & Power Co., Ltd., and Barcelona Traction, Light & Power Co., Ltd., the head offices of all of which were located in Toronto. His service to these companies continued until his retirement in 1925, and he therefore played an important part in the inception and development of their hydroelectric properties. On the occasion of the golden wedding anniversary of Mr. and Mrs. Hirt in 1925 they were presented with a gold bowl from these three companies, the inscription on which included a tribute to the skill and technical ability of Mr. Hirt.

Mr. Hirt held many patents in the street-railway field and on gas engines. He had been a member of the A.S.M.E. since 1894, and had belonged to the New York Athletic Club and Lawyers Club prior to his retirement. He is survived by a son, Edward L. Hirt, of Toronto, and by two daughters, Mrs. Sidonia (Hirt) Jackson, of South Orange, N. J., and Mrs. Florence (Hirt) Ellis, of Longmeadow, Mass. His wife, Alice M. (Flammand) Hirt, whom he married in 1875, died in 1934.

PLINY EASTMAN HOLT (1872–1934)

Pliny Eastman Holt, director of the Caterpillar Tractor Company, San Leandro, Calif., died of heart disease at his home in Stockton, Calif., on November 18, 1934. He will be remembered not only for his inventions and improvements in tractors, harvesters, and other farm equipment, and for his service to the city of Stockton as chairman of its Port Commission; but more widely for his developments in the field of ordnance.

In this field Mr. Holt's achievements were threefold. During 1917 he assisted the Ordnance Department of the United States Army in the development of $2^1/_2$-, 5-, and 10-ton artillery tractors, at the Peoria, Ill., plant of the Caterpillar Tractor Company. He was appointed chairman of the Motor Artillery Board and during the remainder of the War was located in Washington, declining a commission and serving without a salary. He completed the designs for a one-man tank, supervised the design of a 150-ton tank, and had begun work on a much larger unit when the War ended. Perhaps his most outstanding achievement was the caterpillar gun mount, the design for which he completed shortly before the close of the War. Subsequently he supervised the construction and testing of a working model. The design provided for the mounting of light- and medium-weight field guns directly on a caterpillar chassis, armed and equipped for military service, and capable of traversing terrain utterly impassable to anything except a track-laying tractor. The design was accepted and the mount has become a standard part of army equipment.

Mr. Holt was born at Loudon, near Concord, N. H., on August 27, 1872. He was the son of William Harrison Holt, who, with his brothers, Charles and Benjamin Holt, started the firm of Holt Brothers in San Francisco in 1869. His mother was Clara Susan (Cate) Holt. His education, secured in the schools of Minneapolis, Minnesota, included three years' special work in mechanical and electrical engineering at the University of Minnesota.

At the age of 23 Mr. Holt took a position with the Carter Brake Company, in Chicago, Ill., as superintendent, and for about a year was engaged in the manufacture of a special street-car brake. In 1896 he became a draftsman for the Holt Manufacturing Company at the Stockton plant. He was made superintendent in 1903, vice-president in 1906. He was also president of the Aurora Engine Company and treasurer of the Houser & Haines Manufacturing Co. He resigned these positions in 1909 to establish the Holt Caterpillar Company at Peoria, but after serving the new company as president and general manager for about a year he was forced to resign because of ill health. In 1912 he became vice-president and general manager of the Holt Manufacturing Company, at Stockton, with which had been consolidated the Holt Caterpillar Company, Aurora Engine Company, and Houser & Haines Manufacturing Co. Ill health again necessitated his retirement as manager in 1916, but he retained the office of vice-president, and since the close of the War had served the company as director.

The inventive ability of Mr. Holt accounted in a large measure for the success of the Holt company and the industrial growth of his community. Nearly all of the agricultural machinery manufactured by the company was designed by him or had met improvement at his hands.

In recent years Mr. Holt had given much of his time to civic activities, and particularly to the Stockton Port Commission, of which he had been chairman since its formation. His careful, first-hand study of the industrial and technical details of the development of the port

were invaluable. He was a member of the Statewide Water Commission and had sponsored the movement for the construction of the Central Valley Water Project. He belonged to the San Francisco and Stockton Chambers of Commerce, was a director of the Stockton Land, Loan and Building Association, and a member of the advisory board of the Bank of America.

Mr. Holt became a member of the A.S.M.E. in 1915 and also belonged to the Society of Automotive Engineers, Society of Agricultural Engineers, Institute of Radio Engineers, American Association for the Advancement of Science, and other engineering and scientific organizations, as well as to the masonic fraternity and to many clubs. He was a director of the Army Ordnance Association and president of the San Francisco post of the Association.

In addition to his widow, Mrs. Florence (Guernsey) Holt, whom he married in 1909, he is survived by three sons and a daughter, Pliny Guernsey Holt, Frank Harrison Holt, Richard Eastman Holt, and Miss Harriet Holt.

BRONSON LOVETT HUESTIS (1890–1934)

Bronson Lovett Huestis, who died on May 29, 1934, was born in Long Island City, N. Y., on June 15, 1890, the son of Robert Sidney and Hanna M. (Isberg) Huestis. He attended the Erasmus Hall High School and Brooklyn Polytechnic Institute, receiving his mechanical engineering degree in 1913. Prior to his graduation, in the summer of 1912, he worked as a patent draftsman.

During the latter part of 1913 and early 1914, Mr. Huestis taught evening classes in machine design at Brooklyn Polytechnic Institute and was employed daytimes by the Stresau Engineering Company, Brooklyn, on the design of special machinery. He was next employed by the International Callophone Corporation, New York, N. Y., first as draftsman on new types of telephone apparatus and later in charge of a small machine shop for developing this apparatus. Later in the year he assisted his father in connection with plumbing and wiring for new residences. After a period of illness he again became connected with Brooklyn Polytechnic Institute, as instructor in machine-shop practice and toolmaker for laboratory apparatus.

At the close of the school year in 1915, Mr. Huestis entered the employ of Westinghouse Church Kerr & Co., New York. He was connected with that company until November, 1917, working as draftsman on sanitation, heating, and ventilation layouts for plants, and testing materials of construction, principally concrete and steel.

In November, 1917, Mr. Huestis became assistant inspector of ordnance matériel for the Ordnance Department of the United States Army, and until April, 1919, was stationed in Connecticut, his duties being at the New Britain Machine Company and the Winchester Repeating Arms Company, New Haven. He was in charge of a division of ballistic tests of small-arms ammunition, and the selection of all Government test specimens at the Winchester company.

In the fall of 1919 he returned to civilian work with the Pratt Engineering & Machine Co., New York, in charge of the mechanical design department, working particularly on chemical plants and equipment. This led to his connection two years later with the Dorr Company, New York, as designer of chemical plants and equipment and the following year as designer of sugar-mill equipment and plants for the Francisco Sugar Company, New York.

Beginning in December, 1922, Mr. Huestis was employed by the American Railway Association, New York, until his death. He was first connected with the Freight Container Bureau, inspecting methods of fruit packing all over the United States, and also carrying on independent research work on freight handling and preparing articles for the technical press on this subject. His book on "Shipping Containers" was published in 1925. In 1923 he became supervisor of the Accident Department of the Bureau of Explosives and subsequently was editor of the "Accident Bulletin" published by the Bureau.

Mr. Huestis became a junior member of the A.S.M.E. in 1913, an associate-member in 1917, and a member in 1923. He was particularly interested in photography, including motion-picture photography. He was unmarried.

EDWARD WALTER JONES (1889–1934)

Edward Walter Jones, vice-president and manager of the Laclede Stoker Company, St. Louis, Mo., died in that city on March 24, 1934. He is survived by his widow, Marguerite (Rickert) Jones, whom he married in 1920, and by two children, Edward R. and Marion F. Jones.

Mr. Jones was born in Chicago, Ill., on March 10, 1889, son of Lewis F. and Margaret (Goss) Jones. After securing a B.S. degree at the University of Illinois in 1911 he became testing engineer for the Commonwealth Edison Company, Chicago, and remained in that position until 1916. He then worked for the American Tar Products Company, Chicago, as fuel and power engineer for about three years. In 1919 he became chief engineer of the Stoker Department of Laclede-Christy Clay Products Company in St. Louis. He supervised the design and development of the Laclede chain-grate, forced-draft stoker. He became vice-president and manager of the Laclede Stoker Company in 1924. He was also active in smoke-abatement work in St. Louis.

Mr. Jones became a member of the A.S.M.E. in 1930 and belonged to the Prairie Club in Chicago.

ALFRED JOHN JUPP (1875–1934)

Alfred John Jupp, a vice-president of The Lunkenheimer Company, Cincinnati, Ohio, died of a cerebral hemorrhage at the Roosevelt Hospital, New York, N. Y., on April 10, 1934, while in the city on business.

Mr. Jupp was born in Cincinnati on June 25, 1875, son of Basil John and Rosina (Kolker) Jupp. He entered the employ of The Lunkenheimer Company at the age of fifteen. In 1896 he was made New York representative and continued in that position until 1913. He then returned to the home office, becoming a vice-president and director of the company.

Mr. Jupp was active in the Manufacturers Standardization Society of the Valve and Fittings Industry and in the Valve and Fittings Institute, and for a number of months prior to his death had contributed much of his time to code work on behalf of the Institute, under the National Industrial Recovery Act. He was a member of the American Committee to the World Power Conference in 1930.

He became an associate of the A.S.M.E. in 1917 and also belonged to the American Society for Testing Materials, Society of Naval Architects and Marine Engineers, American Petroleum Institute, Zoological Society of Cincinnati, American Red Cross Society, masonic fraternity and Elks, and the Engineers', Cincinnati, Queen City, and Cuvier Press Clubs in Cincinnati. He was unmarried and made his home with his sister, Mrs. Sarah J. Rice.

HERBERT EUGENE KAIGHN (1875–1934)

Herbert Eugene Kaighn, consulting engineer, Wilmington, Del., died on March 18, 1934. He was born at Washington, D. C., on August 19, 1875. During his high-school vacations he secured practical experience in the ordnance drafting room at the Navy Yard at Washington, and subsequently took instruction in ordnance construction and electrical engineering there and at the Naval Torpedo Station, Newport, R. I. He also took private lessons in electrical and mechanical engineering.

From 1894 to 1896 Mr. Kaighn was in charge of an electrical course for officers at the Naval Torpedo Station at Newport, and the next year was assistant engineer for the Newport Illuminating Company. In 1898–1899 he was in charge of electrical equipment, marine district, for the New York, New Haven, and Hartford Railroad. From 1900 to 1904 he was again at the Naval Torpedo Station, as ballistic engineer in charge of the powder factory and naval magazine, and the following two years he held a similar position at the Naval Proving Ground, Indian Head, Md. During the early part of 1907 he was at Philadelphia, Pa., in charge of manufacture of powder for the army and navy and the design of machinery for that purpose, and was also inspector of ordnance at the Midvale Steel Works.

Later in 1907 Mr. Kaighn entered the employ of the E. I. du Pont de Nemours Powder Co. at Wilmington as ballistic and experimental engineer in charge of all ballistic and physical tests of powder and the design and improvement of methods and apparatus used in this connection. He remained with the company until 1917, when he established a consulting practice, specializing in designing wiring systems for hospitals and other institutions. Since 1926 he had specialized in radio and acoustic and amplifying systems. He was also consultant for the American Car & Foundry Co. on marine installations.

Mr. Kaighn became a member of the A.S.M.E. in 1913. He also belonged to the American Institute of Electrical Engineers and the United States Naval Institute.

FREDERICK KAREL (1882–1934)

Frederick Karel, appointed car designer by the Board of Transportation of the City of New York in 1927, died on December 8, 1934, after a short illness.

Mr. Karel was born in New York on February 26, 1882, the son of William and Antoinette (Pekarek) Karel. He attended Cooper Insti-

tute from 1902 to 1905 and later took a course in mechanical engineering through the International Correspondence School. Between 1905 and 1920 he was employed nearly all the time by the Central Railroad of New Jersey as locomotive and car designer. Between 1909 and 1916, while serving as senior mechanical draftsman in the office of the superintendent of motive power he instructed apprentices in mechanical drawing and elementary mechanics. He was boiler and construction engineer for the company from 1916 to 1920.

During the next four years Mr. Karel was inspector of locomotives and equipment for the United States Railroad Administration and the Interstate Commerce Commission, and from then until his appointment by the Board of Transportation was chief draftsman in the office of the superintendent of motive power of the Long Island Railroad Company. He did important design work for the Board of Transportation on the 207th Street Shops, Concourse, and Jamaica Yards and in 1911 patented a coupling carry-iron.

Mr. Karel became an associate-member of the A.S.M.E. in 1921 and belonged to the masonic fraternity. He is survived by his widow, Louise (Vanek) Karel, whom he married in 1908, and by a daughter, Mrs. Frances (Karel) Connolly, and two sons, Frederick and Arthur Karel.

EDWARD J. KEARNEY (1868–1934)

Edward J. Kearney, secretary and treasurer of the Kearney & Trecker Corporation, Milwaukee, Wis., died in that city on January 12, 1934, of pneumonia, after having been in failing health for several months.

Mr. Kearney was born at Little Cedar, Mitchell County, Iowa, on April 7, 1868, the son of James Hamilton and Emeline (Smith) Kearney. Left an orphan at the age of 12, he began work as an engine wiper in a roundhouse on the Chicago, Milwaukee & St. Paul R.R. at Egan, S. D. He continued in the employ of the road, working in various capacities, until a year after his graduation, in 1893, from Iowa State College, from which he received the degree of Bachelor of Mechanical Engineering. During the year following his graduation he was located in the West Milwaukee locomotive shops.

In June, 1894, Mr. Kearney took a position as draftsman with the Kempsmith Machine Tool Company, Milwaukee, manufacturers of milling machines. Here he met Theodore Trecker and in 1898, during the Spanish-American war, the two friends decided that if Admiral Dewey were victorious they would start in business together. On May 17, 1898, therefore, a partnership was formed under the name of Kearney & Trecker for the manufacture of milling machines. The business was incorporated under its present name in 1906.

Mr. Kearney became a member of the A.S.M.E. in 1913. He served a five-year term on the Society's standing committee on standardization, terminating as chairman of the committee during 1930. He then became chairman of the Sectional Committee on Allowances and Tolerances for Cylindrical Parts and Limit Gages. After serving the National Machine Tool Builders' Association as vice-president for four years he was elected its president in 1922. He was also secretary and treasurer of the Kearney & Trecker Investment Co., a director of the LeRoi Company, a trustee of Milwaukee Downer College, and one of the original members of the Wisconsin State Board of Vocational Education. During the World War he acted as Director of Sales for the Liberty Loan drives in the Seventh Federal District. He belonged to the Delta Tau Delta fraternity and to a number of clubs in Milwaukee and vicinity.

Mr. Kearney married Ella B. Norton in 1895, and is survived by her and by two daughters, Katherine (Kearney) Carpenter and Alice (Kearney) Fuldner.

WILLIAM THOMAS KEOUGH (1866–1933)

William Thomas Keough, whose death occurred on January 20, 1933, was born in Boston, Mass., on December 22, 1866. He was graduated from the special course in marine engineering and naval architecture at the Massachusetts Institute of Technology in 1888, and after taking post-graduate work and lecturing at the Institute he entered the employ of the Atlantic Works in East Boston. He remained there nine years, the last five as chief engineer.

For the next ten years he carried on work in his own private office as consulting engineer in marine engineering and naval architecture. Then he was elected a member of the Boston School Committee and from 1907 to 1924 held the position of business agent for the school board, establishing modern and efficient business methods. He was appointed a member of the Boston Finance Commission by Governor Allen in 1930 and held that position until his death.

Mr. Keough became a member of the A.S.M.E. in 1900 and belonged also to the Society of Naval Architects and Marine Engineers and the American Society of Marine Engineers. He was a trustee and member of the board of investment of the Home Savings Bank and a director and vice-president of the Enterprise Cooperative Bank. He was a member of the Charitable Irish Society and president of the Catholic Alumni Sodality. He is survived by his widow, who was Rose Butler, of Boston, and by two sons, Paul and John, and a daughter, Katherine.

JAMES WALTER LAWRENCE (1858–1933)

James Walter Lawrence, emeritus professor of the Colorado Agricultural College, Fort Collins, since his retirement in 1917, died on April 3, 1933.

He was born on December 14, 1858, at Toronto, Ontario, Canada, the son of George and Honora (Naughton) Lawrence. His early education was secured in the public schools of Boston, and he subsequently attended the Massachusetts Institute of Technology. After working for several years at the Pacific Mills, Lawrence, Mass., he became a member of the faculty of the Colorado Agricultural College, teaching mechanical engineering and directing the shops and laboratory. He was acting president in 1892 and dean of the faculty from 1907 to 1913. The college conferred an M. E. degree upon him in 1902.

Professor Lawrence also served as president of the Fort Collins School Board from 1902 to 1912. He belonged to the Fort Collins Lodge of Masons and the Colorado School Masters Club, and was treasurer of the Fort Collins Pioneer Society and a member of the St. Lukes Episcopal Church of Fort Collins. He became a member of the A.S.M.E. in 1892.

Professor Lawrence is survived by his widow, Elizabeth (Coy) Lawrence, whom he married in 1890, and by a son, George Coy Lawrence, of Rochester, N. Y.

RUSSELL ELLSWORTH LAWRENCE (1889–1934)

Russell Ellsworth Lawrence, founder of the Lawrence Institute of Technology, located at Highland Park, Mich., died at his home in Redford, Mich., on June 15, 1934. He founded the Institute in 1932 and headed it as president and dean. It was his wish that the school, which in 1933–1934 had an enrolment of more than eight hundred students, be continued as an institution where any student, regardless of financial status, religion, or nationality, may secure engineering training.

He was born in Terre Haute, Ind., on November 4, 1889, a son of Ellsworth Lyman and Mary C. (Holmes) Lawrence. He received a B.S. degree in mechanical engineering from Rose Polytechnic Institute in 1913 and during the next three years was employed in steam-turbine research for the General Electric Company at Lynn, Mass. He then took graduate work at Rose Polytechnic Institute for his master's degree, which he received in 1916. Subsequent work toward his doctor's degree at the University of Cincinnati was interrupted by the World War, and he served in 1918–1919 as a sergeant in the Signal Corps, Aviation Section, United States Army.

From 1919 to 1922 he taught mechanical engineering at the University of Detroit and was acting dean of the College of Engineering. He became dean in 1922 and held that position until he founded the Lawrence Institute of Technology in 1932. He also did a great deal of consulting work while at the University of Detroit.

He became a member of the A.S.M.E. in 1929 and also belonged to the Society of Automotive Engineers and The Franklin Institute, and was a member of the masonic order and Phi Kappa Upsilon fraternity. He is survived by his widow, Mrs. Frieda Lawrence, and by two daughters.

WILLIAM STATES LEE (1872–1934)

William States Lee, inventor of the "Lee pin," an insulator pin used to support large insulators in the transmission of power, and a noted figure in the industrial development of the Carolinas, died at at his home in Charlotte, N. C., on March 24, 1934, of a cerebral hemorrhage. The American Institute of Electrical Engineers, of which he was a past-president, published a tribute in the July, 1934, issue of *Electrical Engineering*, which read, in part:

"Throughout his career, his ability as an engineer, his keen perception of the controlling factors in any situation encountered, and his constant attention to the human values involved made him an outstandingly effective and a highly respected leader of men.

"As president of the American Engineering Council, during the years 1932 and 1933, he worked intensively and enthusiastically to make the engineering profession as helpful as possible in all problems involving engineering and affecting the public welfare."

William States Lee was born at Lancaster, S. C., on January 28, 1872, one of nine children of W. S. and Jenny Lind (Williamson) Lee.

He attended public school at Anderson, S. C., and was graduated as a civil engineer in 1894 from the South Carolina Military Academy at Charleston, now The Citadel. He had held a scholarship in return for which he was obligated to teach for two years in the public schools of the state. In February, 1896, he secured a position with the Greenwood-Anderson & Western Railway. He spent five months in the drafting room, three months as transitman on location, and then was resident engineer on construction until March, 1897. He next became resident engineer with the Anderson Water, Light & Power Co., on a hydroelectric development on the Seneca River, about ten miles from Anderson. His first assignment was to string a transmission line across the river, a task in which others had failed. He was successful and before the close of the year had completed the plant and put it into operation. Following this he served for several months as resident engineer on construction for the Pickens Railway.

From March to November, 1898, Mr. Lee was connected with the United States Engineering Department as assistant engineer on coast defense. He helped with work on forts on Charleston Harbor and assisted in mining the harbor. He was next associated with the Columbus (Ga.) Power Company. From November, 1898, to September, 1901, he was resident engineer for the company, and from then until March, 1903, was its chief engineer. He directed a 8000-hp hydroelectric development and also the construction of a 27,000-spindle fine-yarn mill.

In 1903 Mr. Lee became chief engineer for the Catawba Power Company, Charlotte, a post which brought him into contact with James B. Duke, the tobacco manufacturer. His first task, to dam a river at a plant near Rock Hill, S. C., after three previous attempts to do so had been unsuccessful, was finished in seven months. This was the beginning of the Carolinas superpower system, financially backed by Mr. Duke and designed and built under the direction of Mr. Lee. He served as vice-president and chief engineer of the Southern Power Company, which took over the Catawba Power Company, and later became vice-president and chief engineer of the Duke Power Company, holding this position until his death.

In addition to the huge Carolina system, Mr. Lee designed and built the Isle Maligne station of the Duke-Price Power Company on the Saguenay River, in Quebec, Canada, and was associated with other projects on the Saguenay, part of the Duke system in that section.

So high was Mr. Duke's regard for him that his will provided that Mr. Lee be a trustee of the great Duke foundation of some $80,000,000.

Mr. Lee was president of the W. S. Lee Engineering Corp., of Charlotte, and was retained as consulting engineer on many hydroelectric projects throughout the East and South. He designed and built and for years had operated the Piedmont & Northern Railway, a high-speed, 1,500-volt, direct-current railroad, consisting of about 160 miles of track located in North and South Carolina, an accepted model of electric-railway design, carrying both freight and passengers.

During the World War his advice was sought in connection with plans for naval fortifications along the Atlantic seaboard. He was a director of the American Cyanamid Company, and vice-president and chief engineer of the Great Falls Power Company, Western Carolina Power Company, Wateree Power Company, Catawba Manufacturing & Electric Power Co., Duke-Price Power Company, and the Quebec Development Company, Ltd.

Mr. Lee became a member of the A.S.M.E. in 1907. He also belonged to the Engineering Institute of Canada, the American Society of Civil Engineers, the American Electrochemical Society, the masonic fraternity, and various clubs. In 1929 he was given an honorary degree of Doctor of Science by Davidson College, and he was a trustee of North Carolina State College of Agriculture and Engineering.

He married Miss Mary Martin, of Columbus, Ga., in 1901, and is survived by her and their three children, William States Lee, Jr., Mrs. William H. Williamson, and Martin Lee.

WILLIAM HENRY LEFFINGWELL (1876–1934)

William Henry Leffingwell, president of W. H. Leffingwell, Inc., New York, N. Y., died at his home in Westfield, N. J., on December 19, 1934, of pneumonia.

Mr. Leffingwell was born on June 14, 1876, in Woodstock, Ontario, Canada, the son of American parents, Wendell Phillips and Mary Catherine (Edwards) Leffingwell. After attending high school in Grand Rapids, Mich., he spent ten years as stenographer, clerk, and office manager for different companies. From 1904 to 1910 he was general manager of several large mail-order publications in New York, no longer published. From then until the outbreak of the World War he was in Europe acting as professional office organizer for various firms.

With this varied and practical background Mr. Leffingwell entered more seriously upon his career as a business efficiency expert. He was organization and management engineer at the Chicago Ferrotype Company in 1914–1915, and from then until 1918 was manager of the office-efficiency department of L. V. Estes, Inc., industrial engineers of Chicago. By 1918 he had founded the W. H. Leffingwell Company in Chicago. Two years later the name was changed to The Leffingwell-Ream Company, management engineers, and subsequently the firm was incorporated in New Jersey under its present name.

Mr. Leffingwell believed that for efficient management of an office a highly specialized knowledge was necessary. He applied scientific methods to his task by dividing office management into 93 parts, thus permitting a test rating to be given to each. Then suggestions for improvement were made on the resulting diagnosis. He considered office management inefficient unless each of the 93 parts had the best known practice.

Mr. Leffingwell was the author of a number of books and pamphlets, as well as of articles for scientific societies to which he belonged. His book on "Scientific Office Management," published in 1917, was one of the first on the application of engineering principles to the office. His "Office Management Principles and Practice," published in 1925, is used as a textbook in many schools and colleges. He contributed articles on office management and office appliances to the Encyclopedia Britannica and in 1928 was associate editor of the magazine *System*. He read papers at the Third International Congress of Scientific Management in Rome, 1927, the fourth congress in Paris, 1929, when he was official delegate from the United States, and the fifth congress in Amsterdam, in 1932. He had prepared a paper for the sixth congress to be held in London in 1935.

As aids in carrying out his principles Mr. Leffingwell invented a typewriter measuring scale, a desk and a posture chair for clerks, and a lighting-demonstration device. He enjoyed amateur photography, sculpture, handicraft, and gardening.

Mr. Leffingwell became an associate of the A.S.M.E. in 1921. He was a past-president of the National Office Management Association and was president of the Taylor Society at the time of his death. He was a member of the Society of Industrial Engineers and chairman of its New York Chapter, and belonged to the American Management Association, the Association of Consulting Management Engineers, the Engineers' and City Clubs, New York, the Union League Club, Chicago, and the Sons of the American Revolution.

Mr. Leffingwell married Miss Anna Short, of Chicago, in 1900, and is survived by her and by two daughters, Evelyn E. (Mrs. Joseph W.) Lewis and Dorothy A. (Mrs. Harold F.) Scribner, both of Westfield. His father, two sisters, and a brother, also survive him.

SYLVESTER MAURICE LEONARD (1896–1934)

Sylvester Maurice Leonard, born in Northampton, Mass., on August 16, 1896, died in the Bridgeport (Conn.) Hospital on December 31, 1934. Mr. Leonard, whose parents were James Sylvester and Felicie (Gillet) Leonard, supplemented his grammar school education with study through correspondence schools and by evening work at the New Haven College. He served an apprenticeship in drafting at the Baird Machine Company, Stratford, Conn., and worked as a special machine designer there between 1916 and 1921. Although not continuously employed during the next two and a half years he gained experience in shoe and paper machinery and miscellaneous tool work. During the summer of 1923 he was draftsman at the Continental Paper & Bag Mills, and the following winter worked on wire-drawing and wire machinery for the Waterbury Machine Company. From then until his death he was employed by the Crawford Oven Company, New Haven. He was senior draftsman for about three years and had been chief draftsman since then.

Mr. Leonard became an associate-member of the A.S.M.E. in 1930 and belonged to the masonic fraternity and Odd Fellows. He married Beatrice Hudson in 1925, and is survived by her and their son, Raymond Frederick Leonard.

EDWIN S. LORSCH (1869–1934)

Edwin S. Lorsch, whose death occurred on October 4, 1934, was born in New York, N. Y., on November 19, 1869, a son of Sigmund and Jenny (Simon) Lorsch. He was graduated from the Stevens Institute of Technology in 1891 as a mechanical engineer and during the next two years worked in the drafting, testing, and erecting departments of the Geo. F. Blake Manufacturing Co., manufacturers of pumping machinery, East Cambridge, Mass. He then returned to New York, where he was employed by the Electrical & Mechanical Engineering Co. and in the New York office of the Fort Wayne Electric Company until he became a member of the firm of Sussfeld, Lorsch & Co., in 1896.

Mr. Lorsch was responsible for the organization of the export department of the company in 1899 and for many years was in charge of this department. The firm was reorganized to Sussfeld, Lorsch, and Schimmel on January 1, 1920, and he remained a member of the firm until it was dissolved in 1929.

Mr. Lorsch became a member of the A.S.M.E. in 1903 and belonged to the North Shore Country Club and Harmonie Club of New York. He had made several trips around the world, recorded in a collection of photographs which he took. He was unmarried and is survived by his brother, Henry Lorsch, of New York.

JAMES LYMAN (1862–1934)

James Lyman, consulting engineer, retired, for the firm of Sargent & Lundy, Chicago, Ill., died on March 28, 1934, at Del Monte, Calif., where he had been spending the winter months with his son, Oliver B. Lyman.

Mr. Lyman was born at Middlefield, Conn., on September 1, 1862, the son of David and Catherine (Hart) Lyman. He was graduated from the Sheffield Scientific School of Yale University in 1883 with the degree of Ph.B. and during the summer and fall of that year was foreman of the machine shops of the Metropolitan Manufacturing Company, at Middlefield.

In December, 1883, Mr. Lyman became construction electrician for the Edison Construction Company, with which he remained about six months, assisting in the installation of a number of lighting plants in Pennsylvania and Ohio. During the next two years he was assistant superintendent of the Marr Construction Company, which was organized to take over the construction work for the Edison company. He had entire charge of the installation of plants at a number of places in Pennsylvania, including Johnstown, Wayne, and DuBois, and also gave exhibitions of incandescent lighting.

Resigning from the Marr company in 1886, Mr. Lyman returned to Middlefield as superintendent of the shops and foundry of the Metropolitan Manufacturing Company, in which his family held an interest. During the seven years he spent there the company became part of the American Wringer Company.

In 1893 Mr. Lyman took up post-graduate engineering work at Sibley College, Cornell University, and received an M.E. degree in 1894 and M.M.E. degree in 1895. He then entered the employ of the General Electric Company at Schenectady and as assistant to Dr. Charles P. Steinmetz conducted special experimental testing and designed electric apparatus. Later he was transferred to the power and mining department and in 1899 was appointed assistant engineer of the Chicago branch of the company. Three years later he became district engineer and continued in that capacity until 1911.

From 1911 until his death Mr. Lyman was associated with the firm of Sargent and Lundy, Inc., as electrical engineer, member of the firm, vice-president (1926–1930), and as consultant retired since 1930.

Mr. Lyman became a member of the A.S.M.E. in 1899, was a Fellow of the American Institute of Electrical Engineers, and belonged also to the American Electrochemical Society, Western Society of Engineers, and the Institution of Electrical Engineers. He was a member of the University and Union League Clubs, Chicago, and several clubs in Evanston, where he made his home for more than thirty years.

Mr. Lyman married Miss Anna J. Bridgman, of Boston, Mass., in 1891.

WALTER ARNOLD LYNCH (1902–1934)

Walter Arnold Lynch, who died of typhoid fever and meningitis at Carrizo Springs, Texas, on September 23, 1934, was born in Philadelphia, Pa., on August 16, 1902, the son of Edmund Wright and Alice (Clayton) Lynch. He attended high school in Darby, Pa., and received a B.S. degree in mechanical engineering at Drexel Institute, Philadelphia, in 1926.

Between the time of his graduation and 1929, Mr. Lynch engaged in farming in Carrizo Springs and was employed at different periods by the Atlantic Refining Company, Philadelphia, in the conversion of a ship from steam to Diesel-electric drive; by the Mayhue Lumber Company in the installation of deep-well pumps; and by the Central Power & Light Co. in the installation of refrigeration machinery.

In 1929 he secured employment as a junior hydraulic engineer with the Texas State Board of Water Engineers in an underground water survey, and in 1933 began similar work with the United States Geological Survey, with which he was connected at the time of his death.

Mr. Lynch had devised a portable device for testing for salt water in deep wells and for determining the position of the salt-water leaks. He also designed and constructed a wind-driven generator for charging a storage battery on his farm, and had constructed many models for airplanes. Radio was perhaps his principal hobby; he had constructed many sets and was a licensed amateur sender.

He served as chairman of the A.S.M.E. Student Branch at Drexel Institute in 1926 and became a junior member of the Society upon his graduation. He was also president of the Phoenix Club at the Institute in 1926, and served as a sergeant in the Headquarters Battery of the 108th Field Artillery of the Pennsylvania National Guard from 1921 to 1925. He was unmarried.

NELSON THEODORE MANN (1899–1934)

Nelson Theodore Mann, who had been employed as a research analyst since 1927 by Scudder, Stevens & Clark, New York, N. Y., died on February 17, 1934, of pneumonia. He is survived by his widow, Beatrice Smith Mann, whom he married in 1928, and by a son, Timothy Charles Nelson Mann.

Mr. Mann was born at Hawley, Mass., on November 10, 1899, attended the Medford (Mass.) High School, and secured an S.B. degree in Mechanical Engineering at the Massachusetts Institute of Technology in 1923. He spent one vacation as rodman for the Scully Construction Company, Boston, and several with the New York Central Railroad, working as pipefitter's helper and in other capacities in the Electrical Department. He served nine months in the United States Marine Corps during 1918.

After his graduation Mr. Mann served a special apprenticeship with the Westinghouse Air Brake Company, Wilmerding, Pa., and worked for that company as assistant to the mechanical engineer and as commercial engineer for a time prior to his connection with Scudder, Stevens & Clark. His work for the latter company had been mainly on machinery, tin containers, railroad equipment, and nonferrous metal mining, analyzing their investment possibilities.

Mr. Mann became a junior member of the A.S.M.E. in 1924, and belonged to the New Rochelle Tennis Club and the Huguenot Yacht Club.

NORMAN ISAAC MARSHALL (1865–1933)

Norman Isaac Marshall, who died in New York, N. Y., on March 28, 1933, was born at Hampstead, N. H., on December 6, 1865, the son of Isaac Hull and Julia (Bement) Marshall. He prepared for college at the Bromfield Academy, Harvard, Mass., and was graduated from Worcester Polytechnic Institute in 1886 with a B.S. degree in mechanical engineering.

For several years following his graduation Mr. Marshall was employed by the Marr Construction Company, assisting in the erection of central-station light and power plants in Pennsylvania, Ohio, and Wisconsin. He next had charge of similar work for the Westinghouse Electric & Manufacturing Co. in Texas, later becoming eastern manager for the company, with headquarters in Boston. From 1901 to 1903 he was designer of machinery and fittings and superintendent of shops for the Star Electric Company, Philadelphia, Pa., and during the next two years manager of the Iona Manufacturing Company and Anchor Electric Company, Boston.

In 1905 Mr. Marshall went into business for himself as owner and manager of the Marshall Electric Company, Boston. Subsequently he sold this business and formed the Elastoid Fibre Company, in West Newton, Mass., later located in Waltham, Mass. He was president and treasurer of the company until its sale in 1927, since when he had not been active in business except for consulting work.

Mr. Marshall held many patents in the electrical and mechanical fields. He was one of the first to manufacture sockets, switches, fuses, etc., and patented the first automatic machinery for the manufacture of incandescent bulbs. From 1916 to 1918 he was connected with the science and research department of the United States War Department, and jointly with Dr. Louis Bell invented and perfected the ultra-violet signalling apparatus for use in the Signal Corps during the World War.

Mr. Marshall served as a lieutenant in the naval militia during the Spanish-American War.

Since 1927 Mr. Marshall had spent a great deal of his time in travel. He was an excellent shot with the rifle and had hunted big game in Alaska, China, and India. He wrote several articles on hunting for publication, and had also written some short stories. He greatly enjoyed walking and mountain climbing and was an able botanist.

Mr. Marshall became a member of the A.S.M.E. in 1910 and also belonged to the American Institute of Electrical Engineers, Boston University Club, Lake Placid Club, and Appalachian Mountain Club.

He married Miss Hortense Carver in 1898 and is survived by her and by three children, Thomas C. Marshall, of Fairfield, Conn., Mrs. M. H. Hull, Still River, Mass., and Mrs. C. V. Russell, Boston.

WILLIAM CROSBY MARSHALL (1870–1934)

William Crosby Marshall, consulting mechanical and civil engineer, and former professor of machine design at the Sheffield Scientific School, Yale University, died at the Gaylord Farm Sanitorium, Wallingiord, Conn., on February 1, 1934. Mr. Marshall, a descendant of Robert Treat, an early governor of Connecticut and founder of Newark, N. J., retired in 1929 and since that time had made his home at Milford, Conn. He was born at Avon, Conn., on September 21, 1870, the son of Henry G. and Marietta (Crosby) Marshall.

After being graduated from Yale University in 1890, with a Ph.B. degree, Mr. Marshall worked for three years as a draftsman and structural engineer for the Berlin Iron Bridge Company, Berlin, Conn. He then returned to Yale to study for a mechanical engineering degree, which was conferred upon him by Sheffield Scientific School in 1894. He remained at the school, first as instructor and later as professor in machine design, until 1913. He was given a civil engineering degree there in 1900. He studied at the École des Beaux Arts and École du Genie Maritime, in Paris, in 1907–1908.

For a year after he left Yale in 1913 Mr. Marshall was chief engineer for the Federal Sugar Refining Company, New York, N. Y. He then became research engineer at the Union Metallic Cartridge Company, Bridgeport Works of the Remington Arms Company. During the early part of 1917 he was connected with the Holt Manufacturing Company, Peoria, Ill., as mechanical engineer, then served during the World War as captain in the Ordnance Department, U. S. A., in charge of twenty-five plants. He also served as trade commissioner to Italy for the U. S. Bureau of Foreign and Domestic Commerce in 1919.

Returning to civil life, Mr. Marshall was engaged as mechanical engineer for the National Spun Silk Company, New York, and carried on consulting work in that city on machine designing, testing, graphical charting, and materials handling and traveled in England inspecting various plants for new and comparative material. In 1921–1922 he was chief enginer for the U. S. Hoffman Machinery Corp., New York; 1922–1923, mechanical engineer for The Trexler Co. of America, Wilmington, Del.; 1923–1924, maintenance and plant engineer, Richard Hellman, Inc., Long Island City, N. Y.; 1924–1925, director, Yale Graduate Placement Bureau, Yale Club, New York; 1925–1926, instructor in machine drawing, Pratt Institute, Brooklyn, N. Y. From then until his retirement he was an associate editor of The Engineering Index.

While living in Milford, Mr. Marshall served as town-planning and building inspector there. He was the author of a textbook on "Descriptive Geometry," published in 1910, "Machine Drawing and Design," 1912, and "Graphical Methods," 1921, as well as of many magazine articles.

Mr. Marshall became a junior member of the A.S.M.E. in 1901 and a member in 1909. He also belonged to the Sigma Xi fraternity, Yale Engineering Association, and Yale Club, New York. He is survived by his widow, Mrs. Genevieve Holbrook Marshall, and by a son, John.

ROYAL MATTICE (1878–1934)

Royal Mattice, president of the Mattice Engineering Company, Philadelphia, Pa., died at his home in that city on February 18, 1934, and was buried with military honors in the Arlington National Cemetery. He was born at Birmingham, Ala., on July 27, 1878. At the age of 13 he began work as a water boy at the Carnegie Steel Company. He served an apprenticeship as a patternmaker and for six years took evening courses, specializing in metallurgy. In 1897 he was employed as assistant manager and metallurgical engineer for the Kokomo (Ind.) Nail & Brad Co., and after serving in the Spanish-American War in 1898, returned to that company in the position of general manager in charge of sales and production.

From 1900 to 1920, with the exception of periods of service on the Mexican Border and in the World War, Colonel Mattice was metallurgical engineer for the American Steel & Wire Co. He was division sales manager for the company at Chicago, Ill., Oklahoma City, Okla., Wichita, Kan., and Cincinnati, Ohio. Although his connection with this company, as well as with the Kokomo Nail & Brad Co., was officially in the sales department, he had practically nothing to do with selling except in a technical way. He was chiefly concerned with improvements in the firm's products and production methods.

After the World War, Colonel Mattice established the Mattice Engineering Company, devoted almost entirely to electric welding of cast iron.

Colonel Mattice became a member of the A.S.M.E. in 1924, and also belonged to The Franklin Institute, the Military Order of Foreign Wars, the Engineers Club in Philadelphia, and the masonic order. He was president of the Welding Contractors' Association of Greater Philadelphia and Camden.

He is survived by a son, Royal Mattice, Jr., and a daughter, Nancy Ellen.

DABNEY HERNDON MAURY (1863–1933)

Dabney Herndon Maury, retired consulting engineer, chiefly in the water-supply field, died at his home in Chevy Chase, Md., on May 11, 1933. Mr. Maury was born in Vicksburg, Miss., on March 9, 1863, the son of Dabney Herndon and Nannie Rose (Mason) Maury. His early education was secured in private schools in New Orleans, La., and Richmond, Va. He then attended the Virginia Military Institute, at Lexington, Va., from which he was graduated in 1882. Two years later he secured his M.E. degree from Stevens Institute of Technology, Hoboken, N. J.

Mr. Maury worked during his summer vacations while attending college. In 1881 he was rodman on surveys for a railroad bridge across the Ohio River at Point Pleasant, W. Va. The following summer he was chief of a party making preliminary surveys for the Brighthope Railway, near Richmond, Va., and draftsman at Colt's Armory, Hartford, Conn. During the summer of 1883 he was draftsman for locomotive works in Richmond and in 1884 assistant to Prof. Robert H. Thurston in the mechanical laboratory at Stevens Institute.

In October, 1884, Mr. Maury went to Texas as engineer for the Grand Belt Copper Company, of New York, a connection he retained until the summer of the following year. He then became principal assistant engineer on location and construction for the Ft. Worth and New Orleans Railway. From March to November, 1886, he was engaged in making land surveys for the Southern Pacific Railway Company in western Texas.

From Texas Mr. Maury went to Colombia, S.A., where his father, General Dabney H. Maury, was stationed as Envoy Extraordinary and Minister Plenipotentiary of the United States to Colombia. He served as private secretary to his father until March, 1887, when he became general engineer for Green & Garcia, mine agents in Colombia. He remained in their employ until May, 1890, serving part of the time as general manager of the Tolima, Organoz, Socorro, and other gold and silver mines, erecting machinery and building dams and ditches for the Silencio, Tetuan, and Colon mines, and making surveys, plans, and estimates for other mines in Colombia.

During the remainder of that year Mr. Maury traveled in Great Britain and the United States. He returned to Colombia as general manager for the Saldana Syndicate, Ltd., of Liverpool, England, and until the winter of 1892 had charge of all of its interests there, designing and constructing dams and ditches and directing the operation of mines. He was also general manager for the Anape Syndicate, Ltd.

After his return to the United States in 1892 Mr. Maury took postgraduate work at Stevens Institute through the winter. He then became engineer and superintendent for the Peoria (Ill.) Water Company and later continued in the same capacities with the reorganized Peoria Water Works Company, remaining with the company until August, 1912. He designed and constructed a number of new pumping plants, developed improvements in operation, and designed sewer systems for suburban districts.

During the latter part of this period he had also conducted a private practice as a consulting engineer, and in August, 1912, he opened his own office in Chicago. He was located there until the United States entered the World War in 1917, and again from July, 1919, until his retirement in 1927, during the last four years as senior partner in the firm of Maury and Gordon. His advice on water supplies was sought by municipalities in many parts of the country.

In May, 1917, Mr. Maury put aside his private practice to serve as advisory engineer on water supply for the Construction Division of the United States Army. He was in charge of the design of water systems for all projects of the Division in the United States and possessions. He was also borrowed by the Marine Corps for its water supplies at Paris Island and Quantico, and by the Navy for supplies at Pearl Harbor and other points in Hawaii. He was specially assigned by the War Industries Board to develop water supplies for the army, navy, shipping board, and housing corporation in the Hampton Roads district. He was given the rank of commanding major in the Engineers Reserve Corps in 1917 and of lieutenant-colonel in the Quartermaster Corps in 1918. He was honorably discharged in May, 1919.

In 1920 Mr. Maury won the James Laurie Prize of the American Society of Civil Engineers for a paper on "Water Supplies for the Cantonments, Camps, and Other Activities Built by the Construction Division of the Army." He contributed other papers to the technical press and to the societies to which he belonged. He held several patents on pumping machinery and wells.

Mr. Maury became a member of the A.S.M.E. in 1890. He was past-president of the American Water Works Association, Illinois

Society of Engineers, and Illinois Water Supply Association. He was also a member of the American Society of Civil Engineers, American Institute of Consulting Engineers, Western Society of Engineers, and New England Water Works Association, as well as of Kappa Alpha (Southern) and Tau Beta Pi, honorary engineering fraternity, and a number of clubs.

Mr. Maury married Mary McCaw, of Richmond, in 1893, and is survived by her and their son, Dabney Herndon Maury, Jr.

RICHARD JUSTIN McCARTY (1851–1934)

Richard Justin McCarty was born on March 12, 1851, at Clarksburg, W. Va., the son of Joseph and Ann (McCally) McCarty. He attended Soulé University, Chapel Hill, Texas, from 1862 to 1869 and then studied for two years at the University of Virginia, being graduated in pure mathematics in 1871. Subsequently he took courses in applied mathematics and civil engineering there, completing his work in 1875.

Following a short period as foreman for the erection of a rolling mill at Rosedale, Kan., he was employed by the Kansas Rolling Mill Company in various capacities until 1879. He then worked in the engineering, accounting, and operating departments of steam railroads until 1888, when he became chief engineer of the Metropolitan Street Railway Company at Kansas City, Mo. He held this position until 1895, having charge of designing and constructing extensions to the cable lines and power plants and the maintenance and operation of all of the company's property. After he left the regular service of the company in 1895 he was recalled to make changes in another of its lines and was also employed in 1895 and 1896 by the Kansas City Stock Yards Company to design a pumping and lighting plant and to handle construction work.

In 1897 Mr. McCarty became auditor for the Kansas City, Pittsburg & Gulf R.R., which later became the Kansas City Southern Railway. In 1903 he resigned to devote two years to scientific study, then returned to his former position. Later he was made vice-president of the company and chairman of its valuation committee. He retired permanently in 1918 and spent his remaining years in scientific and philosophical study and writings. His work on "Valuation of Railroad Property" was published in 1915; "Elements of Plane Trigonometry, 1920; Wisdom and Purpose, 1922; Work and Play: An Autobiography, 1925; "Essays in Poetry," written jointly with his wife, 1930; and Vital Evolution, 1933.

He died in Kansas City of heart disease on June 16, 1934. He is survived by his widow, the former Miss Mary Louise Allen, whom he married in 1877, and by three sons, Allen, Richard J., and Charles E. McCarty.

Mr. McCarty became a member of the A.S.M.E. in 1901 and also belonged to the American Society of Civil Engineers, Sons of the American Revolution, Royal Societies Club, London, and the Zeta Psi fraternity.

JOHN McGEORGE (1852–1933)

John McGeorge, honorary member of the Cleveland Engineering Society, was instantly killed on the evening of February 27, 1933, when struck by an automobile while walking near his home in Cleveland. Mr. McGeorge had been a member of the A.S.M.E. since 1891.

He was born in Manchester, England, on May 2, 1852. Leaving school when about fourteen years of age, he served a seven-year apprenticeship with Emmerson Murgatroyd & Co., Stockport, England, working in both the shops and drawing office. He also studied evenings at the Mechanic's Institution of Stockport and later, while employed in Manchester, took evening courses at the Owens College there.

After completing his apprenticeship, Mr. McGeorge worked successively as draftsman with Galloway & Sons, engineer with the Boiler Insurance Company, and draftsman and manager of Deakin Parker & Co., all of Manchester. He was connected with the Deakin company for about seven years and during this period invented a drum or shaft governor. Later he engaged in drafting for companies in Nottingham, Grantham, and Guilford, England.

In 1884 Mr. McGeorge came to the United States. During his early years here he constructed special machinery for the Bellaire Stamping Company, Bellaire, Ohio, including machinery of his own design for making the Mason fruit jar cap. Subsequently he was employed by the Pittsburgh Iron & Steel Engineering Co., Pittsburgh, Pa., as chief draftsman.

In 1892 he became associated with Samuel T. Wellman, of the Wellman-Seaver Engineering Company (now the Wellman Engineering Company), and served as chief engineer of the company from 1896 to 1903. With Mr. Wellman he developed the rolling open-hearth furnace, the open-hearth charging machine, and other equipment which meant much to the development of the steel industry. Since 1903 his work had been largely in the consulting field. He also invented an electric factory truck in wide use as a labor-saving device in the handling of materials.

Mr. McGeorge was elected to membership in the Cleveland Engineering Society in 1896 and honorary membership was conferred upon him in 1931. He was also a life member of the Society of Automotive Engineers.

DANIEL GEORGE McMILLAN (1871–1934)

Daniel George McMillan, engineer for the Jersey Central Power & Light Co., South Amboy and Sayreville, N. J., died in Elizabeth, N. J., on May 30, 1934. He is survived by his widow, Eleanor T. (Christie) McMillan, whom he married in 1907, and by a daughter, Mildred Eleanor.

Mr. McMillan was born at St. Louis de Gonzague, Quebec, Canada, on May 14, 1871, the son of Neil and Flora (Menish) McMillan. He secured his early education in Montreal schools, served an apprenticeship at the Royal Electric Company there, and spent three years in the mechanical and electrical laboratories of McGill University. For five years he served as chief engineer of the Canadian Singer Factory at St. Johns, Quebec, then came to the United States as assistant chief engineer of the Singer Manufacturing Company at Elizabethport, N. J. He remained with this company for nearly ten years, being its chief engineer the latter part of the time. Subsequently he was superintendent of power for the Willys Corporation, Elizabeth, and maintenance engineer of the Elizabeth General Hospital, prior to his connection with the Jersey Central Power & Light Co.

Mr. McMillan became an associate-member of the A.S.M.E. in 1917. He was very active in the masonic fraternity, in which he had held a number of important posts, and was a member of Everymans Bible Class at the Third Presbyterian Church in Elizabeth.

HIRAM LEES MELLOR (1865–1933)

Hiram Lees Mellor, owner of the Lawrence Pump & Engine Co., Lawrence, Mass., a member of the A.S.M.E. since 1908, died on January 5, 1933. He was born at Sandwich, N. H., on December 2, 1865, but moved with his parents to Lawrence at an early age and was educated in the public schools there. He served an apprenticeship as a machinist and draftsman with S. S. & W. O. Webber, Lawrence, manufacturers of textile machinery, printing presses, shop tools and jigs, and attended night school, thus fitting himself for a position as draftsman with the Lawrence Machine Company. He spent four years in that work, then became superintendent of the company, which manufactured centrifugal pumping machinery, steam engines, and related equipment. About 1904, he started in business for himself, founding the Lawrence Pump & Engine Co., which he continued to manage until his death.

He had served as treasurer and deacon of the United Congregational Church in Lawrence. He was married, and is survived by his widow and a daughter, Mrs. J. L. Dean, whose husband now manages the business.

EDGAR HAMILTON MERRICK (1886–1934)

Edgar Hamilton Merrick, whose death occurred in Cleveland, Ohio, on January 21, 1934, was born at Gananoque, Ontario, Canada, on May 4, 1886, son of Dr. Edgar H. and Sara E. (Carpenter) Merrick. He was graduated from Sibley College, Cornell University, in 1908, with an M.E. degree, and during the next six years was machinery salesman for Hill, Clarke & Co., Chicago, Ill. He then became secretary and sales manager for the Perkins Windmill & Engine Co., Mishawaka, Ind., where he remained for three years.

From 1917 to 1927 Mr. Merrick was connected with Manning, Maxwell & Moore, Inc., New York, N. Y., as district manager of their Cincinnati office for seven years and of the Cleveland office for three years. He engaged in machinery business for himself during the next year, then became sales engineer for Dravo-Doyle Company, of Pittsburgh, Pa., handling the products of the Shepard-Niles Crane & Hoist Corp. Since 1931 he had been in business for himself as sales engineer, in Cleveland.

Mr. Merrick became an associate of the A.S.M.E. in 1932, and was a 32d degree Mason. He is survived by his widow, Florence C. (Earle) Merrick, whom he married in 1917, and by two children, Frances Chapin and Richard Henry Merrick.

VLADIMIR VICTOR MESSER (1877–1932)

Vladimir Victor Messer, who died in 1932, on or about May 2, was born in St. Petersburg, Russia, on December 2, 1877. He attended

the naval academy of the Imperial Russian Navy and was commissioned a lieutenant in 1896. His first work in the United States was as chief draftsman for the Los Angeles Gas & Electric Co., in 1906. The following year he became consulting engineer for the Sweetland Filter Press Company and Brubaker Bros., in San Dimas, Calif., and subsequently served the Commercial Gas Engine Company, of Los Angeles, in connection with irrigation equipment. In 1912 he engaged in the design of the steel structure for the Canadian Pacific Railway terminal at Vancouver, as an employee of Coughlan & Sons, of that city. He then returned to Los Angeles, where he became structural engineer for Fitzhugh & Fitzhugh, architects. He subsequently held a similar position with Lescher & Kibbey, architects, in Phoenix, Ariz.

About the year 1915 Mr. Messer went to New York, where he was associated with Percival R. Moses and Frederick Pope as consulting engineer. During the World War, in 1918-1919, he was chief engineer for the Newport Chemical Works and Newport Hydro-Carbon Company, Milwaukee, Wis., and from then until 1924 again associated with Moses and Pope. At that time he formed the V. V. Messer Manufacturing Co., located first in Long Island City, and later in New York.

Much of his time during the latter part of his life was spent in work on inventions and on special missions to other countries, including the building of a steel plant and railroad in Russia, and zinc plant in South America, and in connection with the cork industry in Europe. Other special connections were with the Goodyear Tire & Rubber Co., in Akron, Ohio, the National Analine Company, for which he built an indigo plant in Buffalo, N. Y., and the Miller Rubber Company, Inc., New York.

Mr. Messer became a member of the A.S.M.E. in 1916 and belonged to the Chemists' Club, New York. He is survived by his widow, Edithe Maxham Messer, whom he married in 1917. Their only son, Paul, died in October, 1933.

HAROLD R. MILLER (1906–1934)

Harold R. Miller was born at Norwalk, Ohio, on July 3, 1906. He secured his early education there and was graduated from the Ohio State University in 1928 with a B.S. degree in mechanical engineering. He immediately entered the employ of the Philip Carey Manufacturing Company, at Lockland, Ohio, and at the time of his death on March 31, 1934, was assistant superintendent of power.

Mr. Miller became a junior member of the A.S.M.E. in 1933 and belonged to the Theta Xi and Tau Beta Pi fraternities. He is survived by his widow, Mary (Ross) Miller, whom he married in 1926, and by a daughter, Marjorie Anne Miller.

JAMES MILNE (1865–1934)

James Milne, chief electrical and mechanical engineer in the Department of Works of the City of Toronto, Ontario, Canada, in charge of water supply, for about twenty years, died in that city on May 21, 1934.

Mr. Milne was born on January 29, 1865, at Woodside, Aberdeen, Scotland, the son of William and Jessie (Hird) Milne. He secured his technical training at Robert Gordon's College, Aberdeen, and as an apprentice with C. Davidson & Sons and Abernethy & Co., Aberdeen. He then went to Montreal, Canada, where he worked for a time in a machine shop and later secured employment with the Edison General Electric Company. He spent three years there and during the next six years was superintendent of the Incandescent Electric Light Company at Toronto. For two years he taught machine construction and drawing, applied mechanics, and steam engineering at the Toronto Technical School, then spent eight years as general manager and chief engineer of the Underfeed Stoker Company, Toronto, at the same time carrying on a consulting engineering practice. Early in February, 1906, he became general superintendent of the British Columbia Electric Railway Company, Vancouver, operating street railways, light, and power in Vancouver, Victoria, and New Westminister, B.C. From 1910 to 1913 he was assistant manager of the Windsor Salt Works, Windsor, Ontario, and engaged in consulting work. He began his long connection with the City of Toronto in 1913.

Mr. Milne became a member of the A.S.M.E. in 1907. He also belonged to the American Institute of Electrical Engineers, Engineering Institute of Canada, and the Association of Professional Engineers of Ontario, and was a member of the Engineers and Parkdale Canoe Clubs in Toronto and of the masonic fraternity. He was a contributor to technical publications and a member of the Toronto Symphony Orchestra, in which he played the contra-bassoon.

Mr. Milne had been twice married. His first wife died in 1899 and his second in 1929. He is survived by a son, James C. R. Milne, and by a married daughter.

GUY KÖCHLING MITCHELL (1878–1934)

Guy Kochling Mitchell, formerly president and proprietor of the Standard Electric Machinery Company and partner in the Standard Appraisal Company, both of Baltimore, Md., died of pneumonia at his home in that city on January 15, 1934.

Mr. Mitchell was born in Baltimore on July 5, 1878. He attended the Baltimore Polytechnic Institute and in 1899 began work for the United Electric Light & Power Co. as draftsman for the department handling equipment and operation. He spent six years in that position, then in 1902, after a few months at Embreeville, Tenn., installing an electric plant and transmission line for the Embree Iron Company, he returned to Baltimore to do estimating and layout work in the distribution department of the United Electric Light & Power Co.

In 1906 Mr. Mitchell became manager of the electrical department of Crook Horner Company, Baltimore. When this company dissolved the following year he purchased the electrical and elevator departments and established the Standard Electric & Elevator Co. operating it as a manufacturing and contracting concern. The name of the company was changed to the Standard Electric Machinery Company in 1921.

During the early days of the company it specialized in replacing steam drives by electric-motor drives, and under the direction of Mr. Mitchell installations varying from 50 hp to 1000 hp were made in plants in Baltimore and vicinity. At the same time he carried on the development of electric elevators, especially the single-phase, and a new line of dumbwaiters, hoists, and cranes. During the World War he devised an automatic system for reconnecting and rerating motors and generators of all makes and types.

Mr. Mitchell became an associate of the A.S.M.E. in 1913 and a member six years later. He was also a member of the American Institute of Electrical Engineers. He is survived by his widow, Katherine V. Mitchell.

ROBERT LEVIS MITCHELL, JR. (1911–1934)

Robert Levis Mitchell, Jr., son of Doctor and Mrs. Robert L. Mitchell, of Baltimore, Md., died at Perth Amboy, N. J., on December 28, 1934. He had been a member of the engineering staff of the National Lead Company at Perth Amboy since early in July.

Mr. Mitchell was born in Baltimore on September 21, 1911. He attended the Friends School there and secured his degree in mechanical engineering from Johns Hopkins University with the class of 1934. He was a student member of the A.S.M.E. at the university and became a junior member of the Society upon graduation. He was also a member of the university amateur theatrical group, the Barnstormers, and of Alpha Tau Omega fraternity.

Surviving Mr. Mitchell, in addition to his parents, are a sister, Miss Nancy Mitchell, and a brother, William.

LESTER A. MORELAND (1874–1934)

Lester A. Moreland, former production engineer for the Worthington Pump & Machinery Corp., Harrison, N. J., died on March 21, 1934, after an illness of more than a year. Mr. Moreland was born at Skaneateles, N. Y., on December 11, 1874, the son of Lewis P. and Harriet (Daniels) Moreland. After leaving public school he began to learn the machinist's trade with the Straight Line Engine Company, of Syracuse, N. Y., but soon decided that he would prefer toolmaking. He therefore transferred to the E. E. Stearns Co., Syracuse, and there and in other shops learned the trade.

In 1898 he entered the employ of the Oliver Typewriter Company, at Woodstock, Ill., as a toolmaker, and soon was placed in charge of of the department. As the company had no drafting or designing department, he made what drawings were required. After six years with the company, he became foreman of the tool designing, experimental, and toolmaking departments of the Illinois Sewing Machine Company, at Rockford. Later he was made assistant superintendent in full charge of all departments, including the foundry and patternmaking.

In 1908 Mr. Moreland went to Providence, R. I., taking charge of the machine department of the American Locomotive Automobile Company, and later of its toolroom and grinding department. The following year he became tool supervisor of the Warner Gear Company, Muncie, Ind., with which he was connected until 1915. One of the special machines which he designed for the company was an automatic machine for inserting corks into clutch plates for automobile clutches.

After leaving the Warner company, Mr. Moreland spent a year with the Remington Arms Company, at Eddystone, Pa., as assistant superintendent of the tool department, and two years with the British Munitions Company, of Montreal, Canada, as master mechanic in the fuse loading and assembling plant.

In 1918, when the World War was nearly over, Mr. Moreland went to Trenton, N. J., to become chief engineer and designer of the John E. Thropp's Sons Co. Among the designs made for the company by him were a special machine for engraving non-skid designs in tire molds, a fabric-spooling machine for taking the fabric from the bias cutter, a special banding machine, and a collapsible core and chuck. He had been with the company about ten years when he became production engineer for the Worthington Pump & Machinery Corp.

Mr. Moreland became a member of the A.S.M.E. in 1923, and belonged to the Engineers Club in Trenton and to the masonic fraternity. He is survived by his widow, Emma J. (Chappel) Morelan i, whom he married in 1899, and by a son, Lester D. Moreland.

GEORGE WILLIAM MORETON (1860–1934)

George William Moreton, who died in Wilmington, Del., on March 2, 1934, after a year's illness, was English by birth. The son of John and Annie (Smith) Moreton, he was born on July 23, 1860, at Minshull Vernon, Cheshire, England. He secured his early education at public schools in Crewe and Birmingham and later was a science and art student at the Crewe Mechanics Institute, and winner of a Whitworth Scholarship in 1885.

Mr. Moreton began work as an office boy for the London and Northwestern Railway in Birmingham at the age of fourteen, and three years later was transferred to its shops at Crewe as an apprentice. He completed his training in 1881 and continued to work for the company as machinist and draftsman until he came to the United States.

Mr. Moreton's first employment in this country was with the Pond Machine Tool Company, Plainfield, N. J., with which he was connected as draftsman from 1888 to 1892. He then engaged in similar work for the Betts Machine Company, of Wilmington, of which he became general superintendent in 1898, and later served also as secretary and vice-president. When the company was sold, during the World War, becoming the Betts Shops of E. I. duPont de Nemours & Co., Mr. Moreton was retained as superintendent and consulting engineer for several years. Since then he had been associated with the Consolidated Machine Tool Corporation of Rochester, N. Y., as consulting engineer and manager of the Hilles & Jones plant in Wilmington.

Mr. Moreton became a member of the A.S.M.E. in 1909. He also belonged to the Wilmington Country and Wilmington Whist Clubs and the Whitworth Society of England. He was a director of the Artisans Saving Fund, Wilmington, and a member of Trinity Church there. He is survived by his widow, Hilda W. (Brotherhood) Moreton, whom he married in 1906, and by two children, John B. and Winnifred Moreton.

THOMAS MORRIN (1853–1934)

Thomas Morrin, son of John and Mary Morrin, was born at Waterloo, N. Y., on August 6, 1853, and died at his home in San Francisco, Calif., on August 1, 1934. He attended public school in Waterloo and at the age of fifteen began an apprenticeship at the Island Works in Seneca Falls, N. Y., where steam fire engines were manufactured. After the completion of his apprenticeship in 1871 he worked for fifteen years as a machinist, being employed in Bay City, Mich., and San Francisco and Marysville, Calif.

In 1886 Mr. Morrin became master mechanic for the Carson & Tahoe Lumber & Co., at Glenbrook, Nev., where he had full charge of railroad, steamboat, and sawmill machinery. Two years later he accepted a similar position with the Western Beet Sugar Company in Watsonville, Calif., where he remained until July, 1890. During the next two years he was foreman in charge of the construction of a machine shop, then became master mechanic and chief engineer for the D. O. Mills properties in and near San Francisco, a position which he held until 1908.

Mr. Morrin also practiced independently as a consulting mechanical engineer from 1904 until 1928, when he practically retired, though he continued to act as supervising engineer for the Phelan estate, San Francisco, until the date of his death.

Mr. Morrin became a member of the A.S.M.E. in 1897 and belonged to the Engineers Club in San Francisco. He was especially interested in fine cabinet work in mahogany and walnut, and took walking trips into the Sierra Nevada Mountains. He is survived by a daughter, Miss Mary Irene Morrin. His wife, Irene (Hoyt) Morrin, died in 1916.

VICTOR EMANUEL MUNCY (1860–1934)

Victor Emanuel Muncy, whose death occurred at Ashland, Ky., on October 26, 1934, was born at Louisa, Ky., on December 1, 1860, the son of Samuel K. and Teresa (Carter) Muncy. After teaching at the Academy of the State (Agricultural and Mechanical) College of Kentucky for a time, he became interested in engineering. He received a B.S. degree from the college in 1891 and secured a position there as teacher in elementary mathematics and physics. He continued in that position until 1905, except during 1900–1901, when he served as business agent for the college.

He joined the faculty of the Ohio Mechanics Institute in 1905 as instructor in applied electricity and physics. He was given an M.S. degree by the State College of Kentucky in 1907, and during the scholastic year 1907–1908, on leave of absence from Ohio Mechanics Institute, he took a course in electricity and power plant engineering at the State University of Kentucy which brought him a B.M.E. degree in 1908. He then returned to his duties at Ohio Mechanics Institute, where in 1911 he was appointed dean of the School of Mechanics and Electricity, later becoming head of the Department of Physics and Electricity. He resigned in 1930 after a severe illness. The State University of Kentucky conferred an M.E. degree upon him in 1913.

Professor Muncy became a member of the A.S.M.E. in 1914. He also belonged to the American Institute of Electrical Engineers, the Illuminating Engineering Society, the Royal Society of Arts, American Association for the Advancement of Science, the Engineers Club of Cincinnati, and Tau Beta Pi, honorary engineering fraternity.

Professor Muncy's first wife, Emma (Garred) Muncy, died in 1893, two years after their marriage. He was separated by divorce in 1913 from Mary O. (Hodges), whom he married in 1908.

HENRY HOTCHKISS MURRAY (1871–1934)

Henry Hotchkiss Murray, mechanical engineer and manager for the Mechanical Improvements Corporation, Camden, N. J., died at the Burlington County Hospital, Mount Holly, N. J., on February 28, 1934. Mr. Murray was born at Viola, Kent County, Del., on March 16, 1871, the son of George Henry and Olive I. (Purinton) Murray. He prepared for college at the Hillhouse High School, New Haven, Conn., and was graduated from the Sheffield Scientific School of Yale University with a Ph.B. degree in 1893. The following year he did post-graduate work there in mechanical engineering and was an assistant instructor in shop visiting.

From 1894 to 1906 Mr. Murray was draftsman for McIntosh, Seymour & Co., Auburn, N. Y., with the exception of the first six months of 1896, when he served as acting professor of mechanical and electrical engineering at Delaware College, Newark, Del. From 1906 to 1930 he was connected with the Victor Talking Machine Company, Camden, serving in the capacities of designer, chief draftsman, mechanical engineer, chief engineer, consulting engineer, and manager of the field technical service. He had been with the Mechanical Improvements Corporation since 1930.

Mr. Murray held many patents pertaining to the improvement of the talking machine and a number for camera cases for underwater photography. He was very active in the community of Riverton, N. J., where he made his home. From 1915 to 1923 he was a member of the borough council; from 1916 to 1923, chairman of the Memorial Park Committee; from 1917 to 1934, director of the Cinnaminson Building and Loan Association; from 1926 to 1932, director of the Cinnaminson Bank & Trust Co.; and in 1932–1933, member of the Riverton Board of Education. He served as chairman of the committee on the location of the Philadelphia-Camden bridge of the Camden Chamber of Commerce in 1921, and during the World War was active in Riverton in connection with the sale of liberty bonds and other war work. At that time he was in charge of airplane parts, gun stocks, etc., manufactured by the Victor Talking Machine Company.

Mr. Murray became a junior member of the A.S.M.E. in 1899 and a member in 1905. He also belonged to The Franklin Institute, Yale Engineering Society, Union League of Philadelphia, Society of Colonial Wars, Riverton Country Club, and Yale Club, New York. He married Miss Georgiana Groot in 1898 and is survived by her and their daughter, Cornelia Groot Murray.

ALFRED NATHAN (1866–1933)

Alfred Nathan, president of the Nathan Manufacturing Company, New York, N. Y., died at the Presbyterian Hospital in that city on May 22, 1933, following an operation. He was a native of New York, where he was born on November 21, 1866, the son of Max and Rosalie Nathan.

Following his graduation from Stevens Institute of Technology with an M.E. degree in 1890 he became assistant mechanical engineer for The Geo. F. Blake Manufacturing Co., a position which he held until 1897. He had been associated with the Nathan Manufacturing Company since then, as assistant secretary until 1903, and vice-

president from that time until he became president of the company in 1909.

Mr. Nathan had been a junior member of the A.S.M.E. since 1891. He belonged to the Beta Theta Pi and Sigma Alpha Epsilon fraternities, and to a number of clubs in New York and vicinity, including the Uptown and Criterion clubs, and the Deal and Hollywood golf clubs. He is survived by his widow, Mabel E. (Lauer) Nathan, whom he married in 1892, and by a son, Alfred Nathan, Jr.

HAROLD VAN HOUTEN NEEFUS (1883–1934)

Harold Van Houten Neefus, for many years associated with Francisco and Jacobus, engineers and architects, New York, N. Y., died in Greenwich, Conn., on August 23, 1934, of pneumonia.

Mr. Neefus was born in Newark, N. J., on August 21, 1883, the son of Henry Freeman and Olive (Van Houten) Neefus. After being graduated from the Stevens Institute of Technology with an M.E. degree in 1904, he was employed for three years as a draftsman by the Public Service Corporation of New Jersey. During the next two years he engaged in experimental work, including drafting and shop work, for the Blevney Machine Company, Newark. In 1909 he took a position as assistant to the secretary and general manager of the Foos Gas Engine Company, with which he remained until 1911. He then went to Buffalo to take charge of manufacturing and sales for the Bogart Gas Power Company. In 1914 he became mechanical engineer for the Northern Division, including six plants, of The Texas Company, New York.

During the World War, from 1916 to 1918, Mr. Neefus was engaged in the production of munitions at the Canada Car & Foundry Co. Subsequently he served for two years as special engineer for the City of Chicago in connection with the electrification of the Illinois Central Railroad and the development of the Union Station. He became connected with Francisco and Jacobus in sales and promotion work in 1920 and later was put in charge of the Chicago office and mid-west territory. At the time of his death he was serving as manager for the Eastern District, New Jersey Department, of the Home Owners Loan Corporation, with headquarters at Newark.

Mr. Neefus became a member of the A.S.M.E. in 1915 and belonged to the Delta Tau Delta fraternity. His wife, Grace (Berger) Neefus, died the year following their marriage in 1910. He is survived by a son, Van Houten Neefus.

VICTOR ROBERT NELSON (1898–1934)

Victor Robert Nelson, whose death occurred on October 3, 1934, was born in Cleveland, Ohio, on June 3, 1898, the son of Niels and Annie (Madsen) Nelson. During the summer after his graduation from the Central High School in Cleveland he was employed by the Linde Air Products Company, and he spent subsequent vacations, while attending the Case School of Applied Science, working in various shops for practical experience. He was a member of the Reserve Officers Training Corps at the School in 1917–1918.

In the summer of 1920, followng his graduation with a B.S. degree in mechanical engineering, Mr. Nelson was employed by Van Dorn & Dutton Co., Cleveland, in connection with production routing and time records. In the fall he secured a position in the U.S. Ordnance Engineer's Office in Cleveland, working under the supervision of the resident engineer on the design and specifications for an artillery tractor and other equipment. After he had been there about a year the Cleveland office was discontinued and he was transferred to the Rock Island Arsenal, where he continued in similar work until March, 1922.

He then returned to the Van Dorn & Dutton Co., which in 1928 became a division of Gears & Forgings, Inc. His early work for the company was in the sales and estimating departments. He then became production supervisor and at the time of the merger was put in charge of the organization of a department combining and coordinating the work of the order and production departments, and certain phases of the work of the engineering department. He continued to serve as production engineer for the company until his death, increasing his knowledge in this field by a course in industrial management through the LaSalle Extension University.

Mr. Nelson became an associate-member of the A.S.M.E. in 1931. He had served as a member of the local Committee on Apprenticeship of the National Metal Trades Association, the Committee on Standards of the Cleveland Engineering Society, the Production Standards Committee (as chairman) of the Associated Industries of Cleveland, and the Production Executive Committee of the Cleveland Chamber of Commerce. He belonged to the masonic fraternity.

Mr. Nelson married Miss Ruth Ellen Heininger in 1926 and is survived by her and by two daughters, Margaret Ellen and Karen Elizabeth Nelson.

FRED NOLDE (1874–1934)

Fred Nolde, secretary and treasurer of the Refrigerating Machinery Association, Philadelphia, Pa., died at his home in that city on December 19, 1934. He was born at Norristown, Pa., on January 16, 1874, the son of Henry and Caroline Nolde. He attended the Central High School in Philadelphia and then the Drexel Institute there, from which he was graduated with a B.A. degree in 1891.

Following his graduation Mr. Nolde took a position as draftsman for the Pennsylvania Iron Works Company in Philadelphia. He remained with this company until 1903, being advanced to the position of chief draftsman and sales engineer, and engaging in the design, construction, operation, and sales of cable railroad machinery, steam engines, hydraulic machinery, ice making and refrigerating machinery, and gasoline engines. During the next year he was connected with James Beggs & Co., selling power machinery in New York, and after that held a similar position with the Allis-Chalmers Manufacturing Company, in the Philadelphia district.

From 1905 to 1927 Mr. Nolde was associated with the De La Vergne Machine Company, as sales engineer, chief engineer, and manager in New York until 1926, when he became consulting engineer to the company, with headquarters in Philadelphia. After about two years in that position he became eastern sales agent for the Vilter Manufacturing Company, of Philadelphia, a position which he held until he took the office of secretary and treasurer of the Refrigerating Machinery Association in October, 1927, first on a part-time basis and latterly as a full-time occupation.

Mr. Nolde became a junior member of the A.S.M.E. in 1901. He resigned in 1910 and was reinstated in 1918 as a full member. He was also a member of the American Society of Refrigerating Engineers and belonged to the masonic fraternity and to a number of clubs, including the Engineers' and Bankers in New York. He married Miss Hedwig P. Haenchen in 1913 and is survived by her and their son, Fred Nolde, Jr.

ARVID NORDIN (1881–1933)

Arvid Nordin, whose death occurred on July 19, 1933, had been associated with the M. H. Treadwell Co., New York, N. Y., for nearly twenty years.

Mr. Nordin was born on July 30, 1881, at Westkinde, Sweden, and was educated in that country. After being graduated from the college at Venersborg he studied mechanical engineering for three years at the Polytechnic Institute at Gothenburg. During his college vacations he worked as an apprentice machinist and boilermaker at the Eriksbergs Mechanical Works, Gothenburg, and Nydqvist & Holm Locomotive Works, Trollhattan.

Mr. Nordin came to the United States in 1901 and during the next four years worked as draftsman and designer-checker for the Niles-Bement-Pond Company, Philadelphia, Pa., Westinghouse Electric & Manufacturing Co., Pittsburgh, Pa., Carnegie Steel Company, Sharon, Pa., Allis-Chalmers Company, Milwaukee, Wis., Illinois Steel Company, South Chicago, and Sargent & Lundy, Chicago, Ill. From 1905 to 1912 he was connected with the Tennessee Copper Company, Copperhill, Tenn., in the positions of designer-checker, chief draftsman, and mechanical engineer. From then until he took up his work with the Treadwell organization he was engineer for the Link-Belt Company, Chicago.

During his association with the M. H. Treadwell Company, Mr. Nordin devoted a large measure of his ability and energy to the development of the various items of hydraulic equipment which they manufacture. He was the inventor of many improvements in the design and details of spillway and intake gates, hoists and accessory equipment, and held many valuable patents on these improvements. As contracting engineer for the company, he enjoyed an unusually wide acquaintance among hydraulic engineers and in the hydroelectric power industry in general. In recognition of his ability and accumulated experience with respect to hydraulic equipment, his advice and opinions were in constant demand.

Mr. Nordin became a member of the A.S.M.E. in 1913. He is survived by his widow, Ruth L. Nordin, of Allegan, Mich., and by a son, John Arvid Nordin.

EDWARD WILLIAM O'BRIEN (1883–1934)

Edward William O'Brien, who died at St. Joseph's Hospital in Providence, R. I., on August 4, 1934, was born in that city on November 29, 1883. His parents were Edward and Mary (Lindsay) O'Brien. Mr. O'Brien attended the Technical High School in Providence and was graduated from the evening course at the Rhode Island School of Design in 1903. Later he took special courses at Franklin Union, Boston, and the Boston Y.M.C.A. in applied mechanics, hydraulics, and other subjects.

From 1903 to 1907 Mr. O'Brien was employed by Knight C. Richmond, engineer and architect of Providence, as a draftsman and designer, working on power plants and industrial buildings. He spent the next two years in Fitchburg, Mass., at the D. M. Dillon Steam Boiler Works, engaged in special work for the New England Boiler Manufacturers Association on boiler design standardization, and compiling a catalog for the company.

During the years 1910–1917, with the exception of about a year, he was associated with Charles H. Fish, consulting engineer of Boston, working part of the time in the office on the design of industrial buildings, and making appraisals and reports, and part of the time in the field, supervising construction. In 1913–1914 he was engaged by the General Electric Company at Lynn to design and serve as inspector during the construction of a reinforced-concrete storehouse.

The Harry M. Hope Engineering Co., Boston, employed Mr. O'Brien from 1919 to 1921, keeping him in the field the greater part of the time. He spent about eight months as general inspector during the construction of a large power plant in Connecticut, and more than a year in St. Louis, Mo., in connection with the construction and equipment of a power house and three eight-story reinforced-concrete buildings for the United Drug Company.

He was next connected with the American Printing Company, Fall River, Mass., as assistant to the plant engineer. His most important work for this company, with which he remained until 1930, was the supervision of the design, construction, and equipment of a large cotton mill in Tennessee, including the mill itself, and a storehouse, power plant, and pumping station, as well as some two hundred dwellings for mill operatives, with water and sewer systems.

Since 1930 Mr. O'Brien had been connected with the Public Buildings Department of the City of Providence, being second deputy to the inspector of public buildings, in charge of zoning, at the time of his death.

Mr. O'Brien became a member of the A.S.M.E. in 1927. He is survived by his widow, Gertrude (Curran) O'Brien, whom he married in 1926, and by two children, Margaret J. and Robert E. O'Brien.

SPENCER OTIS (1858–1933)

Spencer Otis, whose death occurred on June 10, 1933, was born in Orange, N. J., on February 26, 1858, the son of Daniel Carmichael and Clarissa Otis. He entered Amherst College in 1876 but was obliged to leave because of illness. The following year he secured work at a brick and terra cotta factory in Louisville, Ky., and in 1877 he began an apprenticeship with the Union Pacific Railroad at its Omaha, Neb., shops. Upon the completion of his training he worked for that road for some years, advancing to the position of master mechanic.

In 1888 he became general manager of the Kansas City Switch & Frog Co., and two years later president of the Phoenix Foundry Company. He engaged in consulting work, beginning in 1891, and five years later organized the Spencer Otis Company, of Chicago, manufacturers and agents for railway supplies. He served as president of this company until its sale some twelve or fifteen years later. For a number of years, beginning in 1892, he was also president of the National Dump Car Company, Chicago, which in 1904 was combined with the Rodger Ballast Car Company, with Mr. Otis as vice-president. He held many patents pertaining to railway cars and equipment.

About 1902 Mr. Otis became interested in large-scale farming, and much of the latter part of his life was devoted to it, especially to the development of pastures in the Middle West. His chief engineering connection in recent years was with the National Boiler Washing Company, of which he became president in 1919, and the Locomotive Terminal Improvement Company, which he formed in 1925 to manufacture plants for the former company, and to develop and build plants for locomotive terminals. At the time of his death he had retired and was residing in Waco, Texas, with a married daughter.

Mr. Otis became a member of the A.S.M.E. in 1890 and also belonged to the Western Railway Club, the Union League Club, Chicago, and Barrington Hills Country Club.

He was twice married, his first wife, Eleanor G. Beard, dying in 1885 and his second, Julia Melchers, in 1902. He is survived by one son, Spencer Otis, Jr., of Chicago, and by five daughters.

GEORGE BAILEY PAGE (1887–1934)

George Bailey Page, Philadelphia district manager for The Terry Steam Turbine Company, of Hartford, Conn., died at his office from a hemorrhage on September 10, 1934. His widow, Helen (Ferguson) Page, whom he married in 1919, and two children, George F. and Jane Helen Page, survive him.

Mr. Page was born at Williamsport, Pa., on April 27, 1887, the son of Earnest A. and Sarah (Bailey) Page. He attended the Dickinson Seminary in Williamsport and was graduated from Cornell University with a mechanical engineering degree in 1911. He then served an apprenticeship at the DeLaval Steam Turbine Company, Trenton, N. J., and worked for the company in different capacities until early in 1916. During the next five years he was district engineer for the Rush Machinery Company, Pittsburgh, Pa., holding the honorary title of vice-president. In 1921-1922 he was Pittsburgh district sales engineer for the Underfeed Stoker Company of America and then held a similar position with the Sanford Riley Stoker Company for nearly two years.

Between then and 1926 Mr. Page was a member of the firm of Geo. B. Page & Co., in Pittsburgh, leaving there to become associated with B. F. Sturtevant Co., New York. He had been with The Terry Steam Turbine Company since January 1, 1932. He was well versed in both sales and installation work in his field.

Mr. Page became an associate-member of the A.S.M.E. in 1924, and had belonged to the Engineering Society of Western Pennsylvania and the Association of Iron and Steel Electrical Engineers, and to Alpha Chi Rho fraternity.

GEORGE EDWARD PERKINS (1877–1934)

George Edward Perkins, who died at the Brooklyn Hospital on May 4, 1934, was born in Newburyport, Mass., on November 23, 1877, the son of George Collyer and Sarah (Hinckley) Perkins. He attended high school in Amesbury, Mass., for three years, and then found work in the boiler and dynamo rooms of the electric light company there. From June, 1900, to November, 1902, he secured varied experience as operating engineer and electrician in resort hotels in New England and Florida. He then became operating engineer in charge of the steam plant of W. W. Cross & Co., Inc., Brockton, Mass., with which he remained until 1909, and subsequently held a similar position with the T. D. Barry Co., also in Brockton.

Mr. Perkins entered the employ of the Fidelity & Casualty Co. of New York, in November, 1911, as a boiler inspector. About a year later he was transferred to the home office of the company as assistant to the chief boiler inspector. He remained with the company until the end of January, 1920, his work in connection with the inspectors and the insured requiring a knowledge of the design, construction, and operation of steam boilers and engines. Since then he had been assistant engineer, in the Engineering Department, of the Royal Indemnity Company, New York, his duties being similar to those of his previous position, but involving more responsibility.

Mr. Perkins became a member of the A.S.M.E. in 1925, and was active in the masonic fraternity and Odd Fellows, holding several offices. He is survived by his widow, Alice L. (Duston) Perkins, whom he married in 1904, and by a daughter, Helen M. Perkins.

GEORGE SIMEON PERKINS (1868–1933)

George Simeon Perkins, who had been engaged in contracting and superintending construction in and about San Francisco, Calif., for some years, died suddenly on June 21, 1933, at his home in Oakland, Calif. He had been in failing health for two years.

Mr. Perkins was born at Bridgeport, Conn., on August 14, 1868, the son of George B. and Maryetta (Blood) Perkins. After completing his high school course in that city he attended the Stevens Institute of Technology, Hoboken, N. J., securing a degree in mechanical engineering in 1891. During the next six years he worked for the Dow Typesetting & Composing Co., New York, the Philadelphia Traction Company, and the East Jersey Water Company, in connection with the installation of pumping engines at Little Falls, N. J. Subsequently he was employed by the Ingersoll-Sargeant Company, on central station air plants, and by several portland cement plants no longer in existence, including the Pacific, Hecla, and Midland.

Mr. Perkins became a member of the A.S.M.E. in 1926. He is survived by his widow, Adela B. Gayer, and by three children, Daniel Carter, George Stephen, and Mary Gayer Perkins.

JOHN McCLARY PERKINS (1878–1934)

John McClary Perkins, treasurer of the Wittenmeier Machinery Co. of N. Y., New York, N. Y., died in the New York Hospital on August 10, 1934, of pneumonia.

Mr. Perkins was born in Washington, D. C., on September 4, 1878, the son of John McClary and Lucy (Flagg) Perkins. He attended high school at Arlington, Mass., and was graduated from the Massachusetts Institute of Technology in 1901. During the next three years he worked in the steel foundries of the Everett, Mass., branch of the General Electric Company, becoming a foreman molder. He

then took a position as foundry superintendent at the Saco-Lowell Shops, where he installed a complete foundry accounting and cost system and put the foundry on a piece-work basis. He left there in 1909 to build, equip, and put into operation a foundry for the Chalmers Motor Company.

From 1911 to 1916 he was connected with Turner & Seymour, Torrington, Conn., first as foundry manager, and for the last three years as sales manager. He then became works manager of the Gilbert & Barker Manufacturing Co., a subsidiary of the Standard Oil Company, and put into successful operation there the conference committee plan later adopted by all branches of the Standard Oil. In 1919 he went to the Kewanee Works of the Walworth Manufacturing Company, where he settled a strike and formed a conference committee. He became manager of the McNab & Harlin Manufacturing Co., Paterson, N. J., in 1921, and remained there until about 1923, when he became treasurer of the Lapham-Perkins Engineering Company, of New York. He had been treasurer of the Wittenmeier Machinery Company since 1926.

Mr. Perkins became a member of the A.S.M.E. in 1923, and was a former president of the Refrigeration Association of New York. He also belonged to the masonic fraternity, Elks, and New York Rotary Club. He is survived by his widow, Floye (Dinwiddie) Perkins, whom he married in 1906, and by a son, Wilkes D. Perkins.

ARTHUR FRENCH POOLE (1872–1934)

Arthur French Poole, who had been granted many patents for his inventions in connection with electric clocks and adding machines, died at his home in Ithaca, N. Y., on April 26, 1934, after a short illness.

Mr. Poole was born in Cumberland, Md., on August 21, 1872, son of Arthur A. and Virginia (French) Poole. He received his early education there. Later the family moved to Washington, Pa., where he learned the watchmaker's trade under his father, then entered Washington and Jefferson College, with the class of 1891, and studied there for three years. Having become interested in the newly founded Leland Stanford Jr. University, he went there in 1894, taking an A.B. degree in astronomy in 1896. At Stanford he made use of his watchmaking training to work his way through college. He also spent considerable time at Lick Observatory.

Mr. Poole returned to Pennsylvania after his graduation and went to work as a draftsman for John Brashear, noted maker of telescope lenses and scientific instruments, at Alleghany. He left in 1897 to work on a clock system of his own invention and in 1899 formed the Poole Clock Company, Wheeling, W. Va., to manufacture the clock. The company was dissolved in 1902, because of business conditions. His interest turning to electrical work, he joined the Mountain State Electric Company, of Wheeling, to do development work in connection with automatic telephones, and party line systems. Many of his early patents reflect his work here.

In 1908 Mr. Poole went to Chicago, Ill., where for several years he engaged in special development and research work. He worked for Jackson and Jackson on a workmen's time register, for the Holtzer Cabot Company, of Boston, on a frequency meter, and independently on a speedometer. In 1911 he took charge of the patent and development department of the Wahl Adding Machine Company there, and some of his most brilliant work was done in the development of adding machine attachments for typewriters. In 1918 the company acquired patents on automatic pencils, which enabled it to develop and market the Eversharp pencil, and Mr. Poole's inventive genius assisted materially in this work. He was made a director and vice-president of the company in 1920, and two years later, when the Remington Accounting Machine Corporation, a subsidiary of the Remington Typewriter Company, was formed to take over the Wahl adding machine business, he went to New York as vice-president and general manager of the new organization. He remained active as a director of the Wahl company, however, until his death.

Perhaps due to his early training, he was always deeply interested in clocks, especially electric clocks and electric clock systems. His first patent, granted in 1899, was for an electric clock. He was a pioneer in the development of the synchronous motor clock, and methods of frequency control for them, and many of the earliest patents in this field were granted to him. In 1924 he formed the Poole Manufacturing Company for the purpose of making a battery clock of his invention. He established a factory at Westport, Conn., and manufactured these clocks there until 1926, when the business was affiliated with the Morse Chain Company, of Ithaca, N. Y., and moved to Ithaca. His work in inventing and designing this clock led The Franklin Institute to award him a Certificate of Merit in 1933.

Mr. Poole retired from active business in 1931, but continued in development work. At the time of his death he had many patents pending in the patent office. Since his first patent was granted in 1899, he had received more than one hundred, chiefly in the adding machine and electric clock fields.

Like many scientists, Mr. Poole was a lover of music. He had a large collection of symphonic scores, played the 'cello and piano, and while in Chicago was a member of a string quartet.

Mr. Poole became a member of the A.S.M.E. in 1921 and also belonged to The Franklin Institute.

Surviving Mr. Poole are his widow, Maryline (Barnard) Poole, whom he married in 1904; two daughters, Mrs. Martin D. Hardin, Jr., of New York, N. Y., and Miss Elizabeth Poole, of Ithaca; and three sons, Frank and John Poole of Ithaca, and Arthur B. Poole, of Bristol, Conn.

WILLIAM BANCROFT POTTER (1863–1934)

William Bancroft Potter, for many years engineer for the railway department of the General Electric Company, Schenectady, N. Y., died at his home in that city on January 15, 1934. Mr. Potter was one of the best known electric traction engineers in the United States. His career went back prior to 1890, when electric street cars were still struggling to overcome technical difficulties and public prejudice. His contributions to the electric traction art were substantial and long continuing. Many of them were covered by the 130 patents which had been issued in his name.

Mr. Potter was born on February 19, 1863, at Thomaston, Conn., the son of Horace and Charlotte (Pierce) Potter. He gave early evidence of his mechanical aptitude and during the latter years of his school life spent his vacations working in the Seth Thomas clock factory. After completing the Thomaston high-school course he served an apprenticeship as a machinist with Sawtell and Judd, in Hartford, Conn. He continued with that company for two years after the completion of his apprenticeship, securing further training in machine shop and steam engineering work. While repairing stationary engines at the plant of the Hartford Electric Light Company he became interested in electrical apparatus. He foresaw that the development of the electric dynamo would have a marked influence upon the future of the steam engine, and this trend in his thinking led him to study dynamos and electric generating stations. In the summer of 1887 he entered the employ of the Thomson-Houston Electric Company, Lynn, Mass. After a period of shop training he was sent into the field to adjust trouble in electric light plants which the company had installed, and served for a time as superintendent of the electric light company at Raleigh, N. C.

In the fall of 1889, having seen his first electric street car, he returned to Lynn and became a shop student in electric traction work. This was again supplemented by practical experience in the field, particularly on the West End Street Railway Company's system in Boston, and others of less importance.

Later, returning to the Lynn plant, he aided in the development of the Thomson-Houston single-reduction motor for street cars, and when Walter H. Knight was appointed engineer of the railway department he became Mr. Knight's principal assistant. He was active in working out railway circuit breakers, fuse blocks, lightning arresters, and switches. The most serious problem in electric railway work at that period was the matter of an efficient controller for electric cars. Knight and Potter both worked on this, and ultimately they solved the problem by the invention of the series-parallel controller, which became the basic method in use all over the country from that time forward. Mr. Potter's part in the development of the controller led to his being placed in charge of all control work for street railways.

In 1894, when the Thomson-Houston Electric Company became part of the General Electric Company, Mr. Potter was transferred to Schenectady, and in 1895 he was appointed engineer of the railway department, succeeding Mr. Knight. He continued in this position until his retirement in 1930. In that long interval he witnessed and aided materially the tremendous expansion of electric traction in the street railway and interurban field, and also participated in the earliest installations of electrical equipment upon the lines of steam railroads.

The major projects in which he was associated were the electrification of the Manhattan Elevated Railway of New York, the electrification of the Grand Central Terminal, and the Paris and Orleans railroad in France. Other projects with which he was prominently identified included the West Jersey and Seashore, Detroit tunnel, Great Northern Railroad, Southern Pacific, Butte, Anaconda and Pacific and Milwaukee, Chicago, St. Paul and Pacific railroads.

A suggestion made by Mr. Potter in the spring of 1896 resulted in the establishment of the old General Electric test railroad, or test track, along the berm bank of the Erie canal, stretching outward from the local General Electric plant about a mile and a half. There all tests of electric railway equipment were conducted until 1919, when the berm bank track was discontinued.

On this track were tested early types of gas-electric rail cars, some of which later went into commercial service on eastern railroad lines. Mr. Potter participated in many of these test trips and had an active part in directing the development of the propulsion equipment for these cars.

Possibly his most valuable work within the General Electric Company during latter years had been his organization and development of a great engineering staff for electric railway and electric railroad work. His supervision of the department of which he was the head was practically an unbroken stretch of harmonious relations.

Mr. Potter became a member of the A.S.M.E. in 1897. He also belonged to the American Society of Civil Engineers, American Institute of Electrical Engineers, and Society of Naval Architects and Marine Engineers. He served as a member of the A.I.E.E. traction and transportation committee during 1916–1917, 1919–1921, and 1923–1924; its transportation committee, 1925–1932; and its education committee the year before his death. He had contributed many articles to the technical societies.

Mr. Potter had been interested for many years in community life and the development of Schenectady. He particularly followed the fortunes of the Schenectady Civic Players. He was a man of various hobbies, which gave him a means on many occasions of delighting his friends. He was a consistent radio enthusiast from the early days of the art, and had done independent research and development work on a type of radio receiver.

Mr. Potter's club memberships included the Engineers', Transportation, and Railroad, New York; Mohawk, Mohawk Golf, and Edison, Schenectady; and Griswold, Erie. He was twice married, his first wife being Loretta Harward, of Raleigh, N. C., whom he married in 1890, and his second, who survives him, Rose Hubbard, formerly of Sandusky, Ohio, whom he married in 1912. A daughter, Dorothy (Potter) Bird, also survives him.

THOMAS PROSSER (1854–1934)

Thomas Prosser, senior member of the firm of Thomas Prosser & Son, metal dealers, New York, N. Y., died on December 23, 1934, at his home in Garden City, L. I., N. Y., after a long illness. A daughter, Mrs. John Good, and a son, Thomas H. Prosser, both of Garden City, survive him, as do three sisters and four brothers.

Mr. Prosser, who was the eldest of eleven children, was born in Brooklyn, N. Y., on August 24, 1854. He attended the Lafayette and Brooklyn Polytechnic Institutes and at the age of twenty-one became a member of the firm, which was established by his grandfather. He made frequent trips abroad in connection with business for the Krupp Steel Works, of Essen, Germany, for which the firm became American representative in 1848, and also traveled extensively in the United States.

Mr. Prosser became a member of the A.S.M.E. in 1893 and also belonged to the Railroad and Engineers Clubs and the Chamber of Commerce of New York.

FREDERICK GEORGE PROUTT (1870–1934)

Frederick George Proutt, consulting engineering and chairman of the City Board of Water Commissioners of Memphis, Tenn., died in that city on May 27, 1934, after an illness of six months. Mr. Proutt was born at Bowmanville, Ont., Canada, on November 27, 1870, the son of Mark James and Martha (Burke) Proutt. His mother died at his birth. He secured his early education in Canadian public schools and at the age of seventeen entered the employ of the Canadian Bell Telephone Company. He worked there until 1890, when he took a position as superintendent of the Bowmanville Electric Light Company.

In 1892 Mr. Proutt came to the United States and for two years studied electrical engineering at the works of the General Electric Company at Lynn, Mass. Upon completion of the course in 1894 he became chief electrician and assistant to the general manager of the Malden Electric Company, Malden, Mass. He left New England in 1897, accepting the position of chief electrician of the Memphis Consolidated Gas & Electric Co., of which he later became general superintendent. During the nine years he spent with this company he designed plans for and supervised the installation of the first extensive underground conduit system for distribution of electricity in the city.

In 1906–1907 Mr. Proutt was general manager for the Electric Railway Light & Power Co. at Jackson, Miss., for which he designed and installed electric power and gas plants. He returned to Memphis at the beginning of 1908 and opened an office for consulting engineering practice, specializing in the design and management of public utilities. He designed a number of plants in Arkansas, Mississippi, and Tennessee, including light and heat plants for the University of Mississippi, Light and Power for the Brownsville Cotton Oil & Ice Co., water works and sewage disposal systems for the city of Brownsville, and other public utility plants at Friar Point, Laurel, and Cleveland, Miss., and at Blytheville, Osceola, and Luxora, Ark., as well as private installations in Memphis and vicinity.

In May, 1917, during the World War, Mr. Proutt entered the First Officers Training Camp at Chickamauga Park, Ga. He was commissioned a captain of engineers but subsequently discharged because of defective eyesight. He was then appointed consulting engineer for the War Finance Corporation at Washington, D. C., and devoted two years to Government work in this civil capacity, part of the time being spent in Washington and the remainder in Memphis. His work included a survey of Tennessee industries for the Naval Consulting Board.

Mr. Proutt became chairman of the Memphis City Board of Water Commissioners in 1920, and held that office until his death. His able representation of the city in 1920 in the appraisal of the plants of the Memphis Consolidated Gas & Electric Co., which were subsequently acquired by the Memphis Power & Light Co., led also to his selection in 1921 as consulting engineer for the State Railroad and Public Utilities Commission of Tennessee; this connection also continued throughout the remainder of his life.

Mr. Proutt had also served as president of the Consumers Water Company, at Blytheville, and as treasurer of the Home Light & Ice Co., at Cleveland. At the time of his death he was president of the Peoples Finance & Thrift Co., at Memphis, and a trustee of the William R. Moore School of Technology, and for five years he was a trustee of the Memphis and Shelby County Tubercular Hospital.

In 1914 Mr Proutt aided in organizing and was the first president of the Engineers' Club of Memphis. To all club activities he was generous with his time and resources, the Technical Education Fund, which since his death has been given his name, being his chief interest. He served as chairman of this fund from the time of its inception.

He was also a charter member of the Memphis Section of the American Institute of Electrical Engineers. He became a member of the A.S.M.E. in 1915. He was a vestryman at St. Luke's Episcopal Church in Memphis and had attained the highest degrees in Masonry.

Surviving Mr. Proutt are his widow, the former Miss Laura J. Yarnold, of Whitby, Ontario, Canada, whom he married in 1895, and a daughter, Miss Jean Proutt. A younger daughter, Marjorie, died in 1925.

MARK ANTHONY REPLOGLE (1861–1934)

Mark Anthony Replogle, president of the Replogle Heater Company, Akron, Ohio, died at his home in that city on May 25, 1934, after a year of poor health. He was born at Martinsburg, Blair County, Pa., on September 8, 1861, son of Samuel B. and Margaret A. (Hanawalt) Replogle. His father was the inventor of an automatic buffer and coupling widely used on railroads. His childhood was spent in Mifflin County, Pennsylvania, where he attended the public schools. Subsequently he taught in the schools there for a time, then went to Iowa, where, after a period on a farm, he worked in machine shops in Hampton and Ackley.

In 1885 Mr. Replogle entered the Iowa State Normal School at Cedar Falls, where he took a three-year mechanical course, spending his vacations as representative of the Akron, Ohio, firm of Aultman Miller & Co. in the harvest fields around Cedar Falls. During the next two years he again taught school in Pennsylvania, then became associated with H. E. Olbrich and H. H. Clay, of Cedar Falls, in the manufacture and sale of water-wheel governors. In 1895 the manufacturing rights were purchased by the Selle Gear Company, of Akron, and the Replogle Governor Works was organized, with Mr. Replogle as chief engineer. The Selle Gear Company was later absorbed by the Webster, Camp & Lane Co., for which he became hydraulic engineer in 1899. In 1906 he organized the Lombard-Replogle Engineering Company, specializing in water-wheel governors. The Replogle Heater Company was formed in 1925.

Mr. Replogle's interest in water wheels went back to his childhood, when he made wheels which he tried out in neighborhood streams. From 1889 on he devoted himself to design in this and related fields. He held patents in the United States and other countries for his original inventions, which include a discharge gate for turbine wheels, an electrical governor, a mechanical speed governor, a sliding toggle, mechanical movements, and a smokeless home heater. He was associated with the design and construction of a number of water-power plants, including one at Sault Ste. Marie, Mich., while he was with the Webster, Camp & Lane Co., which was the largest erected in the United States up to that time. He furnished the governors and turned on the water that started the first turbines of the Niagara Falls Paper Company.

In 1894 he published a treatise entitled, "Electricity and Water-Power and Their Inter-relations," republished by the Electrical Re-

view Publishing Company, New York, in 1896, and his paper on "Speed Regulation in Water-Power Plants," prepared at the request of The Franklin Institute, was one of the earliest works in this country on that subject. Other papers by him have appeared from time to time in the technical press in this and other countries.

Mr. Replogle became a member of the A.S.M.E. in 1898. He was a 32d degree Mason. He is survived by a son, George B. Replogle, and by two daughters, Mrs. Edith (Replogle) Harley and Mrs. Wilhelmina (Replogle) Groff, all by his first wife, Emma H. (Steeley) Replogle. His second wife, Carrie (Pardee) Replogle, died in December, 1933.

MYRON B. REYNOLDS (1880–1934)

Myron B. Reynolds, city engineer of Chicago, Ill., died at the Edward Hines Hospital in that city on January 27, 1934, of cancer of the throat. He had been in the service of the city, in various capacities, since 1907, although inactive because of his illness for some time prior to his death.

Mr. Reynolds was born at Pana, Ill., on December 12, 1880. He entered college at Rose Polytechnic Institute in 1900 and spent two years there. He then worked for a year as draftsman on elevated railway rolling stock and structures in Chicago and a year in St. Paul, part of the time on bridge design for the Great Northern Railway and the remainder as draftsman in the Public Works Department of St. Paul. He reentered college as a junior at Armour Institute in 1904 and was graduated two years later.

Immediately following his graduation Mr. Reynolds entered the employ of the City of Chicago. During the next three years he worked as engineer in the Bureau of Engineering, in connection with tunnel construction, and as office assistant to the assistant city engineer. He then spent two years as superintendent of concrete bridge construction for a contracting engineer in Chicago. He returned to the employ of the city in 1911 in the position of assistant engineer of the Bureau of Engineering. After two years he was made engineer of water-works design and held that position until 1923, when he became assistant city engineer. In 1917 he was granted leave of absence for military duty and served as a major in the Corps of Engineers for about fifteen months, being stationed at several different camps in the United States.

In 1927, after the death of John Ericson, under whom he had been serving as assistant city engineer, Mr. Reynolds was called upon to fill the position of acting city engineer for several months and in 1931 he was appointed city engineer. Many of his recommendations have been adopted in modernizing the Chicago water-works system, particularly in connection with metering and filtration projects. He was senior author, with Arthur E. Gorman, city filtration engineer, of a paper on "Chicago's Water Pollution Problem, Past, Present, and Future," which brought him the award, just a few days before his death, of the medal of the Illinois Society of Engineers for the best paper published by one of its members in 1933.

Mr. Reynolds became a member of the A.S.M.E. in 1924. He also belonged to the American Water Works Association and was commander of a post of professional engineers of the American Legion in Chicago. He is survived by his widow, Irene Reynolds, and by a son, Myron, Jr.

PALMER CHAMBERLAINE RICKETTS (1856–1934)

Palmer Chamberlaine Ricketts, president and director of the Rensselaer Polytechnic Institute, which he had served as instructor to president during sixty successive years, and honorary member of the A.S.M.E., died at the Johns Hopkins Hospital, Baltimore, Md., on December 10, 1934.

From an institution of less than two hundred students he had seen Rensselaer grow to seventeen hundred students and from two buildings to twenty or more. For most of this growth he was largely responsible, owing to his scholarship, foresight, and administrative ability. Of the present buildings used by the Institute, only three were erected before his presidency. At present there are twelve distinct courses offered by the Institute, of which only one was offered at the time he undertook the direction of the Institute. This expansion has been the result of his conception of the field of endeavor of a true polytechnic institute.

Palmer Ricketts, the son of Palmer C. and Eliza (Getty) Ricketts, was born at Elkton, Cecil County, Maryland, on January 17, 1856. His early boyhood was spent in Maryland, but in 1865 after the death of his father the family moved to Princeton, N. J., and here he was prepared for college by Mr. John Howard O'Brien, a tutor in Latin of Princeton College, and entered the Rensselaer Polytechnic Institute in September, 1871, at the early age of fifteen.

His high scholastic record was remarkable and was gained while he participated in the student activities of that day. In 1875, at the age of nineteen, he received his degree of Civil Engineer with twenty-three other young men.

In 1874, during the student days of Palmer Ricketts, the Institute celebrated the Fiftieth Anniversary of its founding and from the student body of less than 200 members, eleven men received the degree of Civil Engineer in that year. The C.E. degree was the only degree conferred by the Institute at that time.

On account of his high scholarship Palmer Ricketts was offered an instructorship in mathematics and astronomy, which so appealed to him that he accepted this rather than entering at once into the practice of engineering in the activities of that day, although he carried on practical work in connection with his teaching, as assistant engineer of the Troy and Boston Railroad in 1876.

In 1882 he was made an assistant professor of mathematics and astronomy, and in 1884 he was appointed the William Howard Hart Professor of Rational and Technical Mechanics, which had previously been held by the late William H. Burr. The success of his teaching in structures and hydraulics was so outstanding that he was a consultant in bridge engineering of the Rome, Watertown, and Ogdensburg Railroad from 1887 to 1891, the engineer of the Public Improvement Commission of Troy from 1891 to 1892, the River Commission of Corning from 1897 to 1898, and the designer and supervisor of erection of a number of bridges and hydraulic structures.

His work at the Institute prepared him as an expert in patent cases and he presented testimony in important litigations between the years 1886 to 1897.

In 1892, after the resignation of Director D. M. Green, Palmer Ricketts, at the age of thirty-six, became the director of the Institute under the presidency of John Hudson Peck, a lawyer of the City of Troy, N. Y. The separation of the directorship from the presidency caused much difficulty in the management and development of the Institute, owing to the lack of understanding of the educational needs of the Institute by the presiding officer of the Board, so that the early days of Director Ricketts were filled with trips from the "Hill" to the city with requests and arguments. Fortunately, in 1901 the two offices were combined and Director Ricketts became president and director. Although president for the last thirty years he was always affectionately called "the Director," a title which his loyal friends used, and, we believe, he liked.

On November 12, 1902, Director Ricketts married Miss Vjera Conine Renshaw, of Baltimore, Md., and this wedding was marked by demonstrations of affection and admiration for the Director by the faculty and student body. This affection and admiration for him has been extended by the faculty and students to his wife, who entered into and shared all of his plans and activities during the full term of his presidency.

In 1904 the burning of the main building was the beginning of a new Institute, the monument to the ability, wisdom, and perseverance of Director Ricketts. Such a catastrophe as the fire is often a blessing in disguise. It proved so in this case. With the major part of the plant destroyed, an offer was made to transfer the Institute to a large university with the retention of the name of the Rensselaer Polytechnic Institute, but President Ricketts with his faculty determined to remain at Troy and build again on the foundations established three-quarters of a century before.

After a conference with the late Capt. Robert W. Hunt, Director Ricketts went to Andrew Carnegie and persuaded him to give $125,000 for a new building. Then money was raised to purchase the Warren House and grounds on Eighth Street and money was given by Mr. J. J. Albright for part of the cost of the William Weightman Walker Laboratory, named in memory of a graduate of the class of 1886.

Director Ricketts then persuaded Mrs. Russell Sage to give a sum of one million dollars to found a course in mechanical engineering as a memorial to her husband. This was in the year 1907 and at this time the Institute, under the guidance of the Director, inaugurated the courses in mechanical engineering and electrical engineering.

From here on the years have brought to the Institute gifts for buildings and endowment from alumni, friends, and citizens of Troy and an enlargement of its field of education. A recital of this progress would show the regard and trust which these donors have had in the man who indicated the needs of the institution which he had loved, served, and directed for more than a generation.

Following the introduction of courses in mechanical and in electrical engineering, the course in chemical engineering, and, subsequent to 1922, courses, in arts, science, and business administration, in physics, in chemistry, in biology, in architecture, in metallurgical engineering, in industrial engineering, and in aeronautical engineering have been added. A new building which will house the aeronautical, industrial, and metallurgical departments, will be called the Ricketts Building, in his honor.

In all of this development Director Ricketts was guided by the

thought of service. His theory of admissions and his desire to increase the work offered by the Institute was inspired by his desire to give to those who wanted an education in some field of technology a chance to prove their worth. He always thought that offering a chance to those who might make good was more important than the insistence upon formal admission examinations.

In the administration of the Institute, he was ever the champion of those who did not receive the "fair deal." Although believing in student activities and student life, he was not willing to have hazing or freshman rules interfere with the free life of every member of the student body. His stand against hazing came only when this form of student life went beyond bounds of decency, fair play, and even morality. He was ever willing to remain firm to his convictions, even under adverse criticism from students or alumni. His ability to handle student discipline was remarkable and effective. In its administration he had his fixed conviction, but his great sense of humor was a wonderful aid in this work.

He was interested in the possibility of aid, financial and scholastic, for the students needing such. To him personally many owe their graduation, for without his aid it would have been impossible to continue.

Director Ricketts stood for the highest scholarship from students and faculty, and never did he deviate from the motto of the Institute, "Knowledge and Thoroughness," in his administration of Institute educational policies. He insisted on the traditions of daily recitations, and topic work and even in the "scholastic amusements," as Amos Eaton called the work in the laboratories, the work had to be tested by final examination.

In 1924 the Institute celebrated the Centennial of its founding and Director Ricketts was most happy in the recognition which came to the Institute from governments, prominent engineers, and educational institutions of the whole world through the greetings of those in attendance at these extensive ceremonies.

Director Ricketts was active in civic affairs as shown by the positions held at the time of his death. He was a trustee and vice-president of the Troy Public Library, a trustee of Dudley Observatory of Albany, N. Y., of the Albany Academy, of the Albany Medical College, and the New York State College for Teachers. He was a director of the Samaritan Hospital of Troy and vice-president of the Rensselaer County Tuberculosis and Public Health Association. He was also a Director of the National City Bank of Troy.

In 1905 Stevens Institute of Technology conferred upon Director Ricketts the degree of Doctor of Engineering and in six years later New York University conferred the degree of Doctor of Laws upon him.

Director Ricketts was a member of the American Society of Civil Engineers for many years, a director in 1899–1901, and a vice-president in 1916–1917. His connection with The American Society of Mechanical Engineers extended over forty-four years, and in 1932 he was elected to Honorary Membership. Following this election by our Society he was also elected an honorary member of the American Society of Civil Engineers. He was a commander of the Order of the Crown of Italy and a Commander of the Legion of Honor (France). He was a fellow of the American Association for the Advancement of Science and a member of the American Philosophical Society. He was also a member of the American Institute of Mining and Metallurgical Engineers, the Institution of Civil Engineers of Great Britain, and Sigma Xi.

Director Ricketts has written two histories of the Rensselaer Polytechnic Institute and a number of papers have been contributed by him to the State Commission on Railroads and on Education, to the technical and daily press, and to the Proceedings of technical societies. In all of these he was a clear and forceful writer, free from all sham and uncertain expressions.

Not only was Director Ricketts a man of great foresight, administrative ability, and perseverance but he was a most helpful leader under whom to work. He would listen carefully to your plans and suggestions and then give you the authority to carry out plans which he approved. He never withheld praise for good work and in many cases where adverse criticism might have been offered by others he would take the responsibility himself and assume any blame attached to the transaction. His faculty and his trustees were always ready to follow his lead after a decision had been reached by discussion. His force of character, his human interests, and his devotion to Rensselaer remain as a cherished memory and guiding influence for our work in the future.

As was written of Wren in St. Paul's so of Ricketts at Rensselaer we may write in clear letters: "If you ask for my monument look about you."—ARTHUR M. GREENE, JR.[1]

[1] Dean, School of Engineering, Princeton University, Princeton, N. J. Past Vice-President, A.S.M.E.

WILLIAM ROBINSON (1876–1934)

William Robinson, master mechanic for the Eberhard Faber Pencil Company, Brooklyn, N. Y., died in that city on December 31, 1934, after an illness of several months. He is survived by his widow, Elizabeth (Thomson) Robinson, whom he married in 1906, and by two daughters, Mrs. Helen (Robinson) Fairchild and Miss Dorothy T. Robinson.

Mr. Robinson was born in Calderbank, Scotland, on March 3, 1876, son of Andrew and Margaret (Watson) Robinson. He was educated in the United States, attending grammar school and evening high school in Brooklyn, and taking a mechanical engineering course through the Alexander Hamilton Institute. He then served an apprenticeship as a machinist with Ball & Jewell, Brooklyn, shortly afterward (in 1898) becoming a naturalized citizen of the United States.

After working as a machinist for R. Hoe & Co., the Progressive Iron Works, Chelsea Fibre Mills, and a few small shops, he entered the employ of the Eberhard Faber Pencil Company in 1907. He was a machinist for the company for about three years and foreman for two and a half years prior to becoming master mechanic. He did much to develop the products of the company, and held patents on several inventions of his own, including a mechanical pencil and a pencil lengthener.

Mr. Robinson became an associate-member of the A.S.M.E. in 1915, was president of the Bayview Park Association, Port Washington, L. I., at the time of his death, and belonged to the masonic fraternity.

VERNON ROYLE (1846–1934)

Vernon Royle, president and treasurer since 1885 of John Royle & Sons, machinery manufacturers of Paterson, N. J., died of pneumonia at his home in that city on December 17, 1934. Four distinct lines of industry bear the impression of his inventive and manufacturing ability—photo-engraving, textile and fire-hose weaving, rubber goods, and electrical insulation.

Mr. Royle was born in Paterson on June 9, 1846, the eldest son of John Royle, Sr., of English ancestry, who founded the Royle machinery business in Paterson in 1855. His mother was Agnes Houston Royle, a native of Scotland. His early education was acquired in the "pay schools" of Paterson, public schools not having been established there until about 1858.

He served an apprenticeship in patternmaking with Wm. G. & J. Watson, machinists of Paterson, from 1864 to 1868. He then secured a position in the engravers' joining business of Heber Wells, later Vanderburg & Wells, in New York, N. Y. This contact with the boxwood engraving trade and with the early experiments in photo-engraving led to his later improvements in machinery for that industry.

In 1877 Mr. Royle decided to join the Paterson machine business. He became president when his father retired eight years later and under his guidance and subsisting almost wholly upon his own inventions, the Royle machine shop of the seventies grew to a modern manufacturing plant doing a business that was international in its scope, and the leader in each of its numerous lines. It remained a partnership until 1898, when it was incorporated under the same name of John Royle & Sons. Mr. Royle continued active in the management of the business until his death.

In photo-engraving and related processes, Vernon Royle's machinery for routing, lining, beveling, trimming, and blocking plates for the printing press has materially furthered the development of the world's illustrative arts during the past fifty years. Substantially all commercial weaving of figured textile goods is done today through the medium of jacquard pattern cards made on Royle card-cutting machines. Woven fire hose and gasoline service station hose are productions of Royle circular looms.

In the rubber industry, automobile tire treads and inner tubes are made in vast quantities on Royle tubing machines. The same is true of miscellaneous tubing, garden hose, pencil tips, rubber heels, and hundreds of other articles in soft and hard rubber and celluloid. Royle machinery for applying rubber insulation upon electric wire and cable is likewise the standard process of manufacture.

Mr. Royle was keenly interested in his community. He was secretary of the Paterson Board of Education from 1872 to 1889 and for several terms was a member of the board. He had been a director of the Hamilton Trust Company of Paterson for 25 years, was a director of the Cedar Lawn Cemetery Association, and a member of the local Taxpayers Association. He was a former member of the Passaic Valley Sewage Commission.

Mr. Royle became a member of the A.S.M.E. in 1907 and also belonged to the New Jersey Historical Society, the masonic fraternity, and the Hamilton Club, of Paterson.

He was married in 1872 to Miss Jeannie Malcolm, who died in 1908. He is survived by a son, Vernon E. Royle, now president of the company, and by three grandchildren and two great grandchildren.

GEORGE NICHOLAS SAEGMULLER (1847–1934)

George Nicholas Saegmuller, formerly a member of the firm of Bausch & Lomb Optical Co., Rochester, N. Y., died at his home in Clarendon, Va., on February 12, 1934. He was born just 87 years before, on February 12, 1847, at Neustadt-an-der-Aisch, Bavaria, the son of J. L. and Babette (Bertholdt) Saegmuller.

After four years' training at the polytechnic school at Nuremberg, Mr. Saegmuller went to England, where he was employed as draftsman for four years, 1865–1869, by Cooke & Sons, York, manufacturers of astronomical instruments. The following year he served in the German Army as a one-year volunteer and from 1870 until 1905 was a partner in the firm of Fauth & Co., Washington, D. C., manufacturers of astronomical and engineering instruments. This firm combined with the Bausch & Lomb Optical Co. in 1905 and Mr. Saegmuller held the office of vice-president of the latter until he retired from business in 1925.

In Who's Who in Engineering for 1931 will be found the following statement regarding Mr. Saegmuller's achievements: "Among engineers (he was) best known as the inventor of the solar attachment by means of which the true meridian is found by a single observation of the sun and which is largely used in the surveys of public lands. In astronomical circles he is known as the inventor of the Star Dials which make circles on equatorially mounted telescopes unnecessary and by means of which a star may be found without computing the hour angle; also for accurate graduations produced on automatic dividing engine. For the last thirty years his most important work was in connection with the developments of fire control instruments for the American navy. He, with the then Captain Samson, originated for our navy the first telescopic gunsights, boresights, and in fact all fire control instruments used in our navy and most important of all the development of our large rangefinders which are found in every turret of our battleships."

Mr. Saegmuller became a member of the A.S.M.E. in 1911. He also belonged to The Franklin Institute, the American Association for the Advancement of Science, and several clubs.

Mr. Saegmuller married Maria J. Vandenbergh, in Washington, D. C., in 1874, and is survived by her and by their three sons, J. L., Fred B., and George M. Saegmuller.

EDWIN JOEL SANDERSON (1877–1934)

Edwin Joel Sanderson, manager of the A. C. Ramsey Co., an insurance firm of Terrell, Texas, since 1934, die in Dallas, Texas, on December 27, 1934. He was born at New Vienna, Ohio, on February 18, 1877, the son of Francis Wyatt and Frances B. (West) Sanderson. After a year at the Ohio State University he entered the employ of James Leffel and Company, Springfield, Ohio, manufacturers of water wheel turbines. He remained with this company until 1931, working as draftsman for five years, the latter part of the time in charge of the department, then eight years in the engineering department, four years as sales engineer, twelve as sales manager, and the remainder of the time as general manager of the turbine department. He was manager of the Clark Screw Machine Company, Springfield, for a short time before going to Texas.

Mr. Sanderson became a member of the A.S.M.E. in 1930. He had been secretary-treasurer of the Terrell Building and Loan Association since 1932, had served as a director of the Rotary Club in both Springfield and Terrell and was chairman of the club's Crippled Children's Committee. He was a 33d degree Mason and had been a director of the Chamber of Commerce in Springfield and Terrell. He is survived by his widow, Carrie D. (Van Loan) Sanderson, whom he married in 1932.

NORMAN ROBERT SEIDLE (1877–1934)

Norman Robert Seidle, whose death occurred in Chicago, Ill., on May 14, 1934, was born in Lebanon, Pa., on January 25, 1877, the son of Christian and Jane (Mull) Seidle. He studied at Franklin and Marshall College for two years and at Swarthmore College for one year, then entered the employ of the Standard Boiler Works at Lebanon. With the exception of about a year (1900–1901) spent with the North Penn Iron Company, Philadelphia, he was connected with the Standard Boiler Works until 1904, rising to the position of superintendent and assistant manager, in charge of steel-plate construction.

From 1904 to 1919 Mr. Seidle was manager of the Plate Construction Department of Joseph T. Ryerson & Son, Chicago. He had charge of the design and manufacture of boiler specialties, and of steel pipe lines for hydroelectric developments and water supply, as well as of general plate construction and erection and pressed steel work.

After leaving the Ryerson company Mr. Seidle spent three years as general manager of James G. Heggie & Sons Co., Joliet, Ill.; about a year in the same capacity with the General Boiler Company, Waukegan, Ill.; a similar period as manager of the boiler department of the Harrisburg (Pa.) Manufacturing & Boiler Co.; seven years as treasurer and general manager of the McAleenan Bros. Co., and vice-president of its successor, the McAleenan Corporation, Pittsburgh, Pa.; and two years as vice-president and general manager of the Youngstown Boiler & Tank Co. In December, 1933, he became executive director of the National Steel Tank Association and secretary-treasurer of the Code Authority of the metal tank industry, the position he held at the time of his death.

During the World War Mr. Seidle was a civilian member of the Procurement Division of the United States Army, and a "Minute Man." He became an associate of the A.S.M.E. in 1917, was a past-secretary of the Joliet Rotary Club, and belonged to the Delta Tau Delta fraternity and the Odd Fellows. He was deeply interested in religion and for many years had served as a vestryman in the Episcopal Church.

Mr. Seidle is survived by two sons, N. Robert and Charles A. Seidle. His wife, Florence (Wedekind) Seidle, whom he married in 1905, died in 1926.

GEORGE HOTCHKISS SMITH (1851–1934)

George Hotchkiss Smith, whose death occurred in Providence, R. I., on June 7, 1934, had been a member of the A.S.M.E. since 1881. The fifty-year medal of the Society was presented to him in 1931.

Mr. Smith was born in New Britain, Conn., on December 22, 1851. He supplemented his public schooling with study at Cooper Union, New York, and served an apprenticeship in the machine shop of Smith & Garvin, at Bricksburg (now Lakewood), N. Y. In 1878 he went to Providence, to work as a draftsman for the Brown & Sharpe Manufacturing Co. He was made chief draftsman the following year, and remained in that position until 1885, when he entered business with Elmer A. Beaman under the firm name of the Beaman & Smith Co. He retired from business about the year 1915 because of ill health.

Mr. Smith was a past-president of the Providence Engineering Society and a member of the masonic fraternity. He is survived by a son, Walter J. B. Smith, of Providence. His wife, Louise J. (Luther) Smith, whom he married in 1882, died in 1916.

NATHANIEL AUSTIN SMITHWICK (1864–1934)

Nathaniel Austin Smithwick died on July 6, 1934, at his summer home, Waites Landing, Falmouth Foreside, near Portland, Maine. He had made his home in Maine since his retirement in 1912.

Mr. Smithwick was born at Damariscotta, Me., on March 25, 1864, the son of Captain Frank and Caroline (Austin) Smithwick, and as a boy he sailed with his father to many ports. He attended Lincoln Academy, Newcastle, Me., and Phillips Exeter Academy, Exeter, N. H., spent six months as an apprentice machinist with Barrett Bros., Boston, and was graduated from Worcester Polytechnic Institute, with a B.S. degree, in 1886.

After his graduation Mr. Smithwick spent a year as machinist at the Thomson-Houston Electric Company, at Lynn, Mass., and two years as draftsman for Erastus Woodward, mechanical engineer of Boston. The remainder of his engineering work was done for the Goulds Manufacturing Company, of Seneca Falls, N. Y. He was draftsman for five years, manager of the designing and experimental department for eleven years, and after that chief engineer of the company. He was largely instrumental in introducing triplex power pumps into mining, oil refining, water works, and other fields. He took part in a series of efficiency tests of motor-driven triplex pumps during his early days with the company, and later designed and supervised the installation, at Austin, Texas, of the first electrically operated water works pumping plant in the United States. In 1910 he went to South Africa to further the use of electric power pumps in mining operations. He held patents for governor valves for automatic pumps and for an improvement in vertical suction pumps.

Mr. Smithwick became a member of the A.S.M.E. in 1916 and was always keenly interested in scientific and engineering developments. Yachting and yacht designing were naturally of special interest to him, in view of his early trips with his father.

Surviving Mr. Smithwick are his widow, Mabel (Kenney) Smithwick, whom he married in 1901, and their only son, Austin K. Smithwick.

THOMAS WADE STONE (1877–1933)

Thomas Wade Stone, vice-president and general manager of The Western Gas Construction Company, Ft. Wayne, Ind., died on September 28, 1933, at the Methodist Hospital in that city, a few hours after suffering a stroke of apoplexy.

Mr. Stone was born on October 4, 1877, at New Corydon, Ind., a son of Dr. Michael and Mary (Elzey) Stone. The family moved to New Bremen, Ohio, when he was a small child and he attended public schools there and was graduated from the New Bremen High School in 1894. He then taught school for two years at Kettlersville, Ohio, a small town near New Bremen, and for one year at New Bremen. He attended Armour Institute at Chicago for a year and then entered Ohio State University at Columbus, from which he was graduated in 1902 with a degree in mechanical engineering. He won honors in mathematics and was elected to membership in Sigma Xi, honorary scientific fraternity.

Following his graduation from the university Mr. Stone went to Fort Wayne to become draftsman for The Western Gas Construction Company. He was advanced to the position of chief draftsman the following year, became chief engineer in 1916, and was made vice-president of the firm in 1928. In 1931 he also became general manager of the plant. Patent office records dating back to 1914 show the result of Mr. Stone's inventive ability in the gas manufacturing industry. The inventions credited to him include various mechanisms for use in the manufacture of gas, purifying devices, and other apparatus for the industry.

Mr. Stone became a junior member of the A.S.M.E. in 1903 and a member in 1916. He also belonged to the American, Canadian, and Indiana Gas Associations and to a number of clubs in Ft. Wayne, in many of whose civic activities he was actively interested. He belonged to several of the higher orders in the masonic fraternity. During the World War he aided in the manufacture of phenol at the plant of The Western Gas Construction Company for use in making explosives. He participated in numerous drives for the sale of Liberty Loan bonds, raising money for the Red Cross, and other war activities.

Mr. Stone was married to Miss Dorothy M. Greiwe, of New Bremen, in 1905, and is survived by her and by two sisters.

EDWIN STRUCKMANN (1872–1934)

Edwin Struckmann, since 1928 connected with the Lone Star Portland Cement Company of Kansas, died in Kansas City, Mo., on March 15, 1934. He was born at Aalborg, Denmark, on April 24, 1872, the son of a Danish sea captain. He served a four-year apprenticeship at the Nielsen & Winther Tool & Machinery Co., Copenhagen, and worked for some months as a mechanic at Kofoed & Hauberg's Machine Co. in that city. He also passed examinations for engineering service in the Danish Mercantile Marine and was a reserve assistant engineer in the Danish Royal Navy for about seven months.

His desire to become a navigating officer, however, was not to be fulfilled. On the return from a trip before the mast to South America his eyesight proved defective. He turned to engineering with the hope of becoming a ship's engineer, but after his graduation in 1896 from the Technical Institute in Copenhagen, he became interested in brick plants, and entered the employ of F. L. Smidth & Co., of Copenhagen. He was connected with this company for ten years, working in various capacities from draftsman to engineer in charge of construction and operation of brick and cement plants. One of his assignments was the construction of a brick plant in the northern part of Sweden, where the region was so desolate that a railroad and town had to be built before work on the plant itself could be started. Later he was sent to Russia to reconstruct and operate a brick plant and was there during the Russo-Japanese War, suffering many of the hardships of the times. A daughter, born to Mr. and Mrs. Struckmann in Sweden, died while they were in Russia. His final assignment with the company was the construction of a plant for the Canadian Portland Cement Company at Port Colborne, Ontario, Canada.

Since 1907 Mr. Struckmann had been connected with a number of portland cement companies in the United States. For something over three years he was superintendent of the Kansas City Portland Cement Company's plant and from then until the fall of 1911 held a similar position with the Continental Portland Cement Company of St. Louis, Mo. He then went to Des Moines, Iowa, where he was superintendent of the Iowa Portland Cement Company's plant until 1920 and then was connected with the Pyramid Portland Cement Company, first as vice-president and general manager, and from 1922 to 1925 as president and general manager.

From 1925 to 1929 he was employed by the Knickerbocker Portland Cement Company, Hudson, N. Y., first as superintendent, then as general superintendent. In 1929 he became general superintendent of the Western Division of the Lone Star Cement Company of Kansas, at Kansas City, Mo. At the time of his death he was stationed at Bonner Springs, Kan.

Mr. Struckmann became a member of the A.S.M.E. in 1913, and belonged also to the American Society of Danish Engineers. He is survived by his widow, Marie V. (Olsen) Struckmann, whom he married in 1900, and by a son, Holger Struckmann, as well as by a sister, Mrs. Holden Knudsen, of Copenhagen, Denmark.

CHARLES ANTHONY SUPIOT (1906–1933)

Charles Anthony Supiot, mechanical engineer for Hires, Castner & Harris, Inc., Philadelphia, Pa., died of pneumonia on May 21, 1933.

Mt. Supiot was born at Westtown, Pa., on October 6, 1906, the son of Charles Aime and Cecilia Supiot. He was graduated from the University of Pennsylvania in 1929 with a B.S. degree in mechanical engineering. He had been connected with Hires, Castner & Harris, Inc., since his graduation, beginning as a draftsman and subsequently becoming mechanical engineer for the company.

Mr. Supiot was a member of the A.S.M.E. Student Branch at the University of Pennsylvania and became a junior member of the Society upon graduation. He is survived by his mother; his father died in November, 1933.

CARROLL D. THOMAS (1863–1934)

Carroll D. Thomas, for nearly fifteen years chief boiler inspector for the State of Oregon, died at St. Vincent's Hospital in Portland, Ore., on December 24, 1934. He was born on November 13, 1863, at Hollansburg, Ohio, where he spent his early life. He attended the Central Normal College at Danville, Ind., for two years, and taught school and was principal of public schools in Ohio and Indiana for a number of years. He went to Oregon in 1884 and taught school in that state the following two years. He joined the gold rush to Alaska during 1887 and became operating engineer for the Treadwell Gold Mining Company, of Juneau. He returned to the States in 1888, engaging in mining operations in Colorado. In 1890 he went to Unionville, Mont., where he worked for a year on erection for the McCann Gold Mining Company, and from then through 1893 as designing, erecting, and operating engineer for the Whitlatch Union Gold Mining Company. For ten years beginning in 1894 he was engaged in erection work in Colorado mines, part of the time in the Cripple Creek district and from 1898 on as chief operating engineer of the Portland Gold Mining Company at Victor, Colo.

Mr. Thomas entered the power field in 1905 as operating engineer for the Pacific Power & Light Co. at Astoria, Ore. Two years later he became boiler inspector for the Hartford (Conn.) Steam Boiler Inspection and Insurance Company, in the Portland, Oregon, district. He continued in this position until he became state boiler inspector in 1921.

Mr. Thomas became an associate-member of the A.S.M.E. in 1925, and was a member of the Conference Committee to the Boiler Code Committee of the Society. He had served as chairman of the National Board of Boiler and Pressure Vessel Inspectors and as technical adviser to the Industrial Accident Commission of the State of Oregon.

Surviving Mr. Thomas are his widow, Mrs. Beatrice Thomas, of Salem, and a son, Carl F. Thomas, of Portland, as well as three brothers and two grandchildren.

OSKAR VON MILLER (1855–1934)

Oskar von Miller, pioneer in electrical generation and distribution in Germany, founder of the world-famous Deutsches Museum, Munich, and honorary member of the A.S.M.E., died in Munich on April 9, 1934. His long and honorable career was closely identified in Germany with the growth of the electrical industry which has wrought far-reaching changes in industrial and domestic life and in western culture. He labored with vision, intelligence, and energy, keenly conscious of these changes and astute enough to realize the necessity of impressing upon the youth of Germany the significance of the powerful technological influences that determine the environment in which their lives are lived.

Oskar von Miller was born at Munich, May 7, 1855, the tenth son in a family of fourteen children. His grandfather had been director in the Academy of Art in Munich, and his father, Ferdinand von Miller, the head of a bronze foundry, was an artizan of considerable ability and won government awards for his statues. His mother, Anna von Pösl, the daughter of a family of high social and governmental standing, whose marriage with Ferdinand von Miller was

contrary to the wishes of her parents, found time, in spite of the large family, to keep the books for the foundry.

The children were strictly brought up and encouraged in self-support at an early age. Oskar von Miller attended the Technische Hochschule at Munich, and, after having been graduated from it at the head of his class, entered the civil service, holding several transportation posts during the next few years. Among his early engineering tasks was the building of a bridge over the River Main in the winter of 1879 when the weather was so cold that no one else could be found to do the work.

But von Miller's major engineering interest, that of the electrical industry, began in 1881 when he asked for and received a furlough and journeyed to Paris to see the First Electrical Exposition. Here he became intensely interested in electricity and its applications with the result that he organized an electrical exhibit in Munich in 1882, at which the French engineer, Deprez, successfully transmitted direct current from Miesbach to the Glaspalast in Munich, a distance of 57 km.

Following the successful Munich exhibit, von Miller was sent abroad to France, England, and America to study the technology of the infant electrical industries. It was on this journey that he met Edison for the first time, and while in Vienna, after his return home, he was invited by Emil Rathenau to join in the introduction of Mr. Edison's electric lighting plants in Germany. He refused this unexpected and flattering offer because of his desire to spend his talents and energy in the service of the Bavarian state. He envisioned the development of the water-power resources of Bavaria, but his superior considered the plans premature. Perhaps it was the spirit of the times, combined with his own enthusiasm and the experiences of his travels that inspired young von Miller in the vision he was to live to see a reality many years later. The young German empire, politically powerful after the crushing defeats of Austria and France, and guided by the forceful Bismark, was emerging from an agricultural into an industrial nation of first rank, rich in resources of materials and human energy, and possessed of the scientific spirit which provides the basis for technological growth. Progress and technological development were in the air.

In Berlin, Emil Rathenau had formed the German Edison Company, now known as the A.E.G., and in 1883 von Miller joined him as a director of the company. By 1884 they had opened the first central station in Berlin. Here the boilers were set above the engine room where Edison bipolar dynamos were driven by belt, three to every engine. For the next five years they were busy developing electrical power plants throughout Germany.

Yielding to his desire to return to his native Munich, von Miller left the A.E.G. in 1889. Life was not easy, but the director of the Portland Cement Works at Lauffen sought him out with a proposition to supply the city of Heilbronn with power from Lauffen. This he accomplished, completing the work just prior to the electrical exposition at Frankfurt, in 1891, of which he became the president.

While power had been transmitted at the Munich exposition, electrical generation was there the major interest. Electrical transmission over considerable distances appealed to von Miller and the idea of transmitting electricity from his Lauffen plant to Frankfurt seemed worth trying. Consultation with Charles E. L. Brown convinced him that an alternating current could be so transmitted, as indeed it was, at 25,000 volts over a three-phase system 178 km in length. Of the 234 horsepower at Lauffen, 181 reached Frankfurt, a loss of only 22.6 per cent. In the same year, at Kassel, von Miller installed the first alternating-direct-current system, in which electricity was transmitted as alternating current and converted to direct current for use.

Following the Frankfurt exhibition, von Miller was firmly established as an electrical engineer and his genius was providing electric lighting and power plants throughout Germany. Space will not permit even a list of these projects. His interests spread to railway electrification and to the use of electricity in the chemical industry and for cooking and domestic appliances. In 1911 there were initiated the plans for the comprehensive hydroelectric power development of Bavaria, totaling 600,000 kva and comprising the famous 168,000-hp project at Walchensee, opened in 1924, with its 1000 km, 100,000-volt transmission system, and a yearly output of 180 million kilowatthours. Thus came to fruition the dream of his young manhood. In 1930, an extensive plan for the electrification of all of Germany engaged his attention.

But important as these engineering achievements of von Miller proved to be, he is best known throughout Germany and the world at large for the Deutsches Museum in Munich. His interest in the museum idea is said to have originated in a visit to Kensington Museum in 1878 when he saw the famous "Puffing Billy," built by William Hedley in 1813, and Stephenson's "Rocket." Lying fallow in his mind for a quarter of a century, the idea of a museum for his native city in which laymen and young people could see the industrial foundations on which our present civilization is based was presented by him to a group of government representatives, fellow citizens, scholars, and technologists in 1903, and he became the founder, president, and director of the institution. To this project he gave the best of his energy and enthusiasm and drove it through to an amazingly popular success in spite of tremendous difficulties.

When it outgrew its original quarters a few years after its opening, the city of Munich provided a commanding site for new buildings on a beautifully located island in the River Isar. Here, in 1925, after the long delay of the War, the present museum was opened to the public. Thousands of visitors to Munich pass through its 400 separate rooms and halls and from a city of 700,000, more than a million pass every year along the nine miles of its exhibits. It has provided inspiration for the founding of similar museums in this country. Space will not permit an estimate of what this great contribution means to the world's understanding of the conditions under which men live and how technology is constantly undergoing development.

Mrs. von Miller, whose death in the fall of 1933 broke a family group unusual in its solidarity, was Marie Seitz, the daughter of Franz Seitz, a professor at the University of Munich. Of her marriage to Dr. von Miller in 1884 seven children were born, three sons and four daughters, two of whom died at an early age. The family was accustomed to meet for a day or an evening together every two weeks, and each year gathered for a religious service at the little church in Neuhausen where Dr. and Mrs. von Miller were married.

Von Miller possessed to a rare degree human qualities that endeared him to every one. His tireless energy permitted him to accomplish great deeds and his infectious enthusiasm carried others along with him in the pursuit of his objectives. Breadth of vision provided these objectives in heroic measure, and a keen sense of his obligation to his country and its culture led him to give his services to such projects of social significance as the electrical exhibitions of his early manhood and the museum and the electrical development of Bavaria which occupied his later years. When the great von Miller saw a need, he did not stop until he had satisfied it, true to his family motto, "If I rest, I rust."

JOSEPH ADDISON WADDELL, JR. (1877–1934)

Joseph Addison Waddell, Jr., western district manager of the Spencer Heater Company, a subsidiary of the Cord Corporation, with offices in Chicago, Ill., died suddenly of coronary thrombosis on April 28, 1934, while in St. Louis, Mo.

The son of Legh Richmond Waddell and nephew of Senator Joseph Addison Waddell, of Virginia, he was born at Staunton, Va., on November 5, 1877. He received a B.S. degree in mechanical engineering in 1900 and M.E. degree two years later from Virginia Polytechnic Institute. While working for his Master's degree he taught mathematics and experimental engineering at the Institute, and during the next year was employed in the erecting and layout department of the Deane Steam Pump Company, Holyoke, Mass.

Mr. Waddell first became associated with the Spencer Heater Company, then located in Scranton, Pa., in 1903 as engineer in charge of sales and mechanical engineering. Two years later he was made general superintendent and in 1913 works manager, with entire charge of production. He held this post until 1917, when he was released to serve in the Production Division of the Emergency Fleet Corporation, United States Shipping Board. He had charge of the Boiler Section and was responsible for the allocation of material to the various manufacturing companies throughout the United States which were building boilers for the Corporation. After the armistice he adjusted contracts and claims arising from cancelations of orders, remaining with the Corporation until the end of 1919.

During the next four years Mr. Waddell was connected with Morris and Company, of Chicago, as manager of its Phosphate Mining Department at Bartow, Fla. He directed the construction of a hydraulic plant, including the power plant, washing and separating plants, and drying and storage plants, repair shop, and other buildings, and installed the machinery and other equipment. After the completion of the plant he directed the operation of the mine and the production of pebble phosphate rock until the company was purchased by Armour and Company, Chicago. A model of the Bartow phosphate mine has been placed in the Smithsonian Institution in Washington, D. C.

From then until 1927 he was not engaged in engineering work, and in 1924 he resigned his membership in the A.S.M.E., which he had held since 1913. He applied for reinstatement in 1930.

When Mr. Waddell again took up engineering duties in 1927 it was with the Spencer Heater Company, located at Williamsport, Pa., in the capacity of chief engineer. He held patents on Spencer heaters dating back to as early as 1910 and after his return to the company

developed the present line of products and was granted patents in both the United States and Canada on magazine stoking boilers for heating purposes. Since 1933 he had been stationed at Chicago as western district manager, but had also continued as chief engineer of the company.

Surviving Mr. Waddell are his widow; a daughter, Martha; and a son, Joseph Addison Waddell, 3rd, also with the Spencer Heater Company.

VINCENT L. WALTERS (1868–1934)

Vincent L. Walters, who became an associate-member of the A.S.M.E. in 1920, died on April 30, 1934. He was born at Bilston, England, on February 6, 1868, and was educated in that country. His father was master mechanic in a large steel mill and the son trained for erecting and mill work. He had not worked long in England, however, when his brother, in the United States, advised him to come to this country. He wished for a position as draftsman, but there were so many applicants for such work in New York that for a number of years he found erecting and mill work offered better pay.

Later, however, he obtained work in which he was more interested as designer and experimentalist for the Ideal Cash Register Company, Bound Brook, N. J. He was connected with this company until 1901, designing, among other things, a spring winding machine and special dies, and having charge of the tool and gage room.

From 1901 to 1912 Mr. Walters was employed by the Weston Electrical Instrument Company, Newark, N. J. He designed intricate mechanisms for a variety of instruments and machines and for three years was in charge of the tool department of the company.

After a few months in 1912 and 1913 as experimentalist and model maker for Sloan and Chase, Newark, Mr. Walters went to Orange, N. J., for the manufacture of the Monroe calculating machine, on the experimental stages of which he had been working for some time. He designed all the tools necessary for its manufacture and was assistant superintendent of the Monroe Calculating Machine Company for four years.

Mr. Walters next spent several months with the Mehl Machine Tool & Die Co., Roselle, N. J., then became superintendent of A. Rimelspacher & Co., designing a special thread rolling machine and directing the manufacture of machinery and tools. In the summer of 1917 his aid was solicited by Joseph Gough, of Newark, and during the following winter he worked on the perfection of a universal joint gage for a Government assignment. Subsequently he became foreman for Jackson Waddell, Newark, where he remained until the latter part of 1919. He then became general foreman for the Pierce Accounting Machine Company, Inc., New York, for whom he carried on experimental as well as manufacturing work until 1924.

FRANK ROSE WHEELER (1878–1934)

Frank Rose Wheeler, a specialist in condensing equipment, died of a cerebral hemorrhage at his home in Chicago, Ill., on March 1, 1934.

Mr. Wheeler was born at Wheeler, N. Y., on April 1, 1878, a son of Don D. and Mary (Rose) Wheeler. He was graduated from high school in Tacoma, Wash., and subsequently was a special student at Leland Stanford Jr. University in 1898 and 1899 and from 1901 to 1903. During these years he also worked as a draftsman for the City of Tacoma and the Northern Pacific Railway, and following his graduation, for C. C. Moore & Co., of Los Angeles and San Francisco, and as engineer and vice-president of the Tracy Engineering Company, Los Angeles, working upon the design of power plants.

In 1904 Mr. Wheeler became Pacific Coast manager for the C. H. Wheeler Mfg. Co., of Philadelphia, in charge of the sale of condensing apparatus. He took over the Chicago district in 1916 and remained with the company until 1929, when he became manager of the Condenser Heater Department of the Elliott Company, Jeannette, Pa. He spent three years in this position, and during the remainder of his life was connected with the Utilities Power & Light Co. in Chicago.

Mr. Wheeler became a member of the A.S.M.E. in 1909 and also belonged to the Western Society of Engineers, the Beta Theta Pi fraternity, and to several clubs in Chicago and San Francisco. He was a contributor to the technical press.

Mr. Wheeler is survived by his widow, Blanche (Harper) Wheeler, whom he married in Pittsburgh in 1912.

GEORGE E. WHIPPLE (1890–1934)

George E. Whipple, solicitor for United Drydocks, Inc., New York, N. Y., died of a heart attack at his home in Teaneck, N. J., on May 3, 1934. He was born in Brooklyn, N. Y., on January 3, 1890, son of George E. and Nellie T. (Doody) Whipple. He attended public schools in Brooklyn and studied art in New York. The most of his engineering experience was with Theo. A. Crane's Sons Co., of Brooklyn, with which he became associated in 1917. From then until 1928 he was located in the New York office of the company, as draftsman and designer, and assistant executive. He was also assistant superintendent of the shipyard, supervising all repairs on boilers, engines, and hulls. He became solicitor for ship repairs for the company in 1929 and the following year took the position of solicitor with United Drydocks, Inc. He was a licensed ship engineer, first class.

Mr. Whipple became an associate-member of the A.S.M.E. in 1927. He was a 32d degree Mason and belonged to the Sons of the American Revolution, Odd Fellows, and Rosicrucians. He was a deputy warden of the New Jersey Fish and Game Commission. In addition painting in oils and sketching in pen and ink, he was interested in photography and in floral horticulture.

Mr. Whipple is survived by his widow, Marie B. (Arden) Whipple, whom he married in 1920.

WILLIAM HIGH WILLISTON (1879–1934)

William High Williston was born on October 10, 1879, at Somerville, Mass., where he secured his grammar and high-school education. He was graduated from the Massachusetts Institute of Technology in 1902 with a B.S. degree, and was employed continuously from December of that year until his death on June 7, 1934, by Manning, Maxwell & Moore, Inc. He spent his early years with the company in drafting and designing at the Hancock Inspirator Company and Hayden & Derby Manufacturing Co., rising to the position of chief designer. He became mechanical assistant to the vice-president of the parent company in 1913 and was made assistant works manager of the Boston plant three years later. Subsequently he was promoted to the position of works manager of that plant, then railroad sales manager. He was appointed vice-president in charge of railroad sales in 1927, and at the time of his death was director and vice-president in charge of railroad sales of the subsidiary company, Consolidated Ashcroft Hancock Co., Inc., as well as works manager of the Boston plant.

Mr. Williston became a member of the A.S.M.E. in 1925. He was an enthusiastic philatelist. He is survived by his widow, a son and a daughter, and his father, Belvin T. Williston.

WALTER WOOD (1849–1934)

Walter Wood, of R. D. Wood & Co., Philadelphia, Pa., manufacturers of cast-iron water and gas pipe, died on April 20, 1934, in Washington, D. C., while there on a business trip. He was born in Philadelphia on December 6, 1849, a son of Julianna (Randolph) Wood and of Richard Davis Wood, who was a partner in the firm of R. D. Wood & Co., and one of the organizers of the Pennsylvania Railroad.

Mr. Wood was graduated from Haverford College in 1867 with the degree of Bachelor of Arts and took post-graduate work at Harvard University, securing his A.B. degree there in 1868. He then began work in the office of his father, becoming managing partner when his father died in 1869. He was also president of the Florence (N. J.) Pipe Foundry & Machine Co., owner of The Millville (N. J.) Daily Republican, and treasurer of the Cumberland County Gas Company, the Millville Electric Light Company, the Millville Water Company, and the High Pressure Supply Company.

Mr. Wood also interested himself in civic affairs. He became a member of the Committee of One Hundred (Philadelphia) in 1880, and served for four years with this group, which investigated and attempted to improve local governmental conditions. From 1887 to 1895 he served as a Civil Service examiner and was a trustee of Haverford College. In recent years he had been an active proponent of the plan to build a ship canal across New Jersey, linking the Delaware River at Bordentown with Raritan Bay. He advocated the project before the House Rivers and Harbors Committee a few weeks prior to his death.

Mr. Wood became an associate of the A.S.M.E. in 1880 and a member ten years later. He was one of the organizers and charter members of the American Foundrymen's Association and a member of the American Institute of Mining and Metallurgical Engineers, American Society for Testing Materials, American Waterworks Association, and of a number of clubs in Philadelphia, New York, Washington, D. C., and Boston. He was a director of the Philadelphia Bourse and of the Burlington City Loan & Trust Co. He was unmarried and is survived by six nieces, and by two nephews, Richard and Grahame Wood, all of Philadelphia.

EDMUND WILHELM ZEH (1867–1934)

Edmund Wilhelm Zeh, president of the Zeh & Hahnemann Co., Newark, N. J., manufacturers of presses, dies, and automatic machinery for working sheet metals, died on March 17, 1934.

Mr. Zeh was a native of Germany, having been born at Frankfort-on-Main on September 12, 1867, the son of Carl Wilhelm and Catharina (Emmel) Zeh. At the age of 16 he entered the shops of Collet & Engelhard, at Offenbach-on-Main and after two years there spent four more in securing drafting-room practice with the company. He then entered the technical high school at Darmstadt, from which he was graduated in 1891 with an M.E. degree. He returned to the employ of Collet & Engelhard as a constructor and continued in that work until 1893.

Mr. Zeh came to the United States in 1893 to see the World's Fair at Chicago and remained to work as a designer for the Niagara Machine & Tool Works, in Buffalo, N. Y. He was made superintendent of the works the following year, and held that position until 1904, when he helped to found the Zeh & Hahnemann Co. He held patents on percussion power presses, details for double crank presses, double action presses, stacking devices, guards, button machinery, and can body machinery.

Mr. Zeh became a member of the A.S.M.E. in 1915. He was president of the Newark Technical Society and a member of the Verein deutscher Ingenieure. He is survived by his widow, Anna M. (Mantz) Zeh, whom he married in 1889, and by one son, now president of the Zeh & Hahnemann Co.

New Pioneers on a New Frontier

By RALPH E. FLANDERS

THE greatest fact in American history has been the existence of the frontier.

For centuries the centers of industry, commerce, and settled agriculture lay in the East, expanding decade by decade in a westward progress that was uninterrupted and successful. The border line of that movement was held by the pioneers. Before them lay the unknown regions which the imagination pictured as teeming with wealth of forest, mine, and soil. Behind them came the settlers, who found and developed the underlying realities of the pioneer's dream.

This physical, geographical frontier for many generations served our nation in three important ways; it gave unlimited opportunity to youth, it furnished a seemingly boundless field for the profitable investment of capital, and it generated a favorable psychology throughout the whole structure of American society.

The opportunity for youth was a visible actuality at so recent a date as to be remembered by many in this gathering tonight. In the eastern farming regions the process was almost automatic. With the large families of a generation or two ago it was obviously impossible for all to gain a good living on the restricted home acres. The boys in due time moved outward, some to the cities (themselves growing with the growing commerce to and from the frontier) and others directly to the virgin soil and forest of the newly opened lands, where they could find the opportunity and the room to make new lives for themselves on a more ample scale.

But so widespreading and rich was the western domain that the expanding population of the eastern seaboard was insufficient to occupy and develop its resources. Their numbers were swelled by the millions from the crowded countries of Europe who descried the opportunities from afar, swarmed into the transatlantic shipping, and crowded the immigrant trains into our virgin territory. The numbers of these immigrants were numbered by the millions and tens of millions. Our own population growth did not suffice. There was room for all.

Money was needed as well as men. There were farms to be equipped, mines to be opened, roads and railroads to be built, cities to be erected. For generations the savings of the settled East poured across the Alleghanies, across the Mississippi, across the Continental Divide, and found profitable investment in these new and rich territories. Fortunes were lost, but more fortunes were made. There was never a word of "overproduction" or "overinvestment." The thought was so ridiculous that the words were unneeded and uninvented.

As with men, so with money. The savings of the East were inadequate to the needs and opportunities of the West. In consequence, millions of dollars came pouring out of the coffers of Europe to finance the settlement of western America, and the expansion of eastern commerce and industry which western development demanded. The millions grew into billions before the demand was satisfied and the stream slacked its flow.

All of this produced a national psychology of a fortunate sort. We were convinced that the future held more for us than the present and the past. We were willing to risk time and money in productive enterprise. The future, we were sure, would not betray us—nor did it. Even as our physical surrounding made our psychology, so our psychology made our future. The process was natural, healthful, and fortunate beyond all accounting.

But—the frontier is gone!

No doubt we will continue, now and then, to find new oil fields. New areas of fertile soil will be irrigated and added to our inheritance of arable land. New mineral deposits will be found and exploited. All this will go on continuously for many years, yet the total effect will be small. It will not constitute the outstanding, controlling fact of our nation's history, as it did for nearly three hundred years. Sometime in the period since the turn of the century—not on any given day or in any given year—the physical frontier in the grand sense disappeared and disappeared forever.

This was the greatest event of our times. If we are looking for some effective cause for the evils which have been afflicting us, why look further? We build up our false or inadequate theories of "overproduction" when we have never had a decent general standard of living in this country; of "technological unemployment" when on technology alone can we found our hopes for the higher standards of the future; of "overinvestment" when an adequate provision of goods and services to the mass of our fellow citizens requires larger and more efficient productive facilities than we have ever dreamed of. All of these explanations and many more are shallow and inadequate.

The outstanding historical event of our generation is the disappearance of the frontier, and the consequent disappearance of the old, trusted, and tried opportunities for youthful opportunity and the profitable investment of capital; and with these gone there is beginning to go that favorable, constructive type of national psychology which has energized our progress hitherto.

What shall we do in the face of an emergency so fundamental?

Two courses are open to us. We can readjust our social, political, and industrial institutions to a future of limited opportunities for men and for money, with a national outlook keyed down to safety as our objective, instead of the old purposeful advance toward a better future. The alternative is that we find an equivalent for the lost frontier—an equivalent which will as effectively open new opportunities for men and for capital, and make it possible for us to retain our constructive national psychology.

For the time being, we seem to be making the first and poorer choice. In some directions deliberately and in others by indirection we are slowing down our production of goods and services. In place of a confident advance toward business risk and personal adventure, we are all playing safe—business man, worker, and farmer. We are at this moment organizing a new world of restricted production, restricted human enjoyment, and restricted opportunity for the coming generation. We have chosen to play the lesser part.

We need not do this and we must not do it. For the effective equivalent of the old physical frontier lies ready at our hands for our occupation, development, and enjoyment. That frontier lies so close at hand that it escapes our notice. Its separate features are so familiar to us that we have missed its significance as a whole. For the new frontier is not a distant physical region, it is an ever-present social possibility. This new social frontier is *a greatly raised standard of living for the mass of our fellow citizens.*

At first sight this proposal for a new frontier may seem trite and inadequate. But let us examine it before passing hasty

Eighth Henry Robinson Towne Lecture on Engineering and Economics, delivered as the 1935 Presidential Address at the Annual Meeting, New York, N. Y., December 2–6, 1935, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

judgment. Let us consider first and principally its material characteristics. Of what are they composed?

A higher material standard of living does not mean higher wages, higher salaries, or higher dividends. It does not mean higher prices for goods or services—least of all does it mean restriction of output. It does mean more and more goods and services at lower and lower prices relative to incomes.

Let us first consider whether a higher standard of living, defined in these terms, is physically possible. Next, let us see whether, if attained, it can take the place of the lost frontier in furnishing opportunity for youth and for capital, and in providing a favorable national psychology. Finally, if these inquiries lead to favorable conclusions, let us see what it is that prevents our occupation of this new frontier, so that we may remove the hindrances and proceed with our new social advance.

There can surely be no question as to the physical possibility of a rising standard of living. We have the material resources required. The very abundance of our coal fields, oil wells, and fertile soil have embarrassed our clumsy institutions until we have imagined abundance to be a curse instead of a blessing. For the materials we need and do not have there are ample stores of native products for exchange in the markets of the world.

We are favored as well in the character of our population. Drawn from a diversity of races, they bring to our land every useful type of acquired skill and native ability that could be asked for; and their energies are heightened by climates that are for the most part particularly favorable for effective work of hand or mind.

Finally, years of experience have produced in this country a body of business and technical experience directly adapted to the production and distribution on a large scale, at low cost, and with minimum human effort, of a rich variety of desirable goods and services.

The rising standard of living is physically possible.

Quite obviously, when we consider the matter, this new social frontier of the higher standard of living offers the same advantages as did the old physical frontier if we can but enter into and occupy it.

We seek more and better goods and services at less relative cost. This can only be effected to the desired degree by both expansion and refinement of our productive machinery and processes. The expansion of industry will call for more and more workers. The needed expansion and improvement of production will call for more and more investment. Only by this expansion of operations and revival of investment can we simultaneously produce more and better goods and services, distribute them more widely, do so at a less relative cost, and employ more men in the process at a number of working hours per week which tends to decrease rather than increase. All other programs fall short of this complete solution.

Our progress toward a higher standard will thus definitely provide the opportunities for men and for money that were furnished by the old frontier; and as this fact becomes obvious, it is plain that our old psychology of firmly based optimism will revive, and our newly injected defeatism will wither and disappear—a consummation devoutly to be wished for.

If this new social frontier is physically attainable, and offers a continuation of the services which were rendered to us for so many generations by the old frontiers, we must be in earnest in our determination to discover and surmount the obstacles which deter us from entering into it.

We have made good progress toward an ever-rising standard of living in the past, but in recent years our forward movement has been checked, and we have even been forced backward from our goal. The causes of our trouble have not been single, but many. But, of the many, two are of such overshadowing importance that it is folly to disturb ourselves about the minor difficulties until we have found means of surmounting these major ones. In fact, we cannot even see the other problems clearly until these two have been solved. Thus attacked, the whole problem will more readily yield to solution.

The first obstacle is a false idea, and it is the more dangerous from being held by a great majority of the population of the country. Business, labor, and agriculture account for almost the whole body of persons engaged in providing a high standard of living and all three of these groups are imbued with this false idea and strive to make it effective. That false idea is that the interests of each separate group are best served by control or restriction of output and by maintenance or raising of prices.

Industry has long been fascinated by the possibility of understandings between competitors which lead directly or indirectly to price and production control. So evidently profitable does this appear that the movement toward agreement and combination has been almost irresistible. As early as 1890 Congress recognized the rapid growth of the movement and passed the Sherman anti-trust act forbidding "conspiracies in restraint of trade." But in spite of that legislation and of the succeeding Clayton act and the establishment of the Federal Trade Commission, business has, openly or secretly, consciously or unconsciously, moved in that direction. Some industries have, indeed, been so "successful" in coming to agreements that they have encouraged outside competition and quite evidently defeated their purposes.

Labor-union policy has been identical with business policy. But since the commodity dealt in is labor and skill, the fallacy is expressed in somewhat different terms. Labor-union policy seeks "shorter hours and higher wages" without any expressed or discernible limit. And some unions, like some businesses, have been so "successful" in this policy that their members are idle for months and years when other workers are profitably employed.

Agriculture has long looked with envy on the "success" of business and labor in carrying out this false idea and at last it too has its chance, under the AAA, to limit production and raise prices; and it too is meeting with the "success" of lost foreign markets, stimulated foreign competition, and actual importation of products which our farmers are well able to supply.

When business, labor, and agriculture hold the same beliefs, be they true or false, we must not be too surprised if the administration hastens to embody those beliefs in the law of the land. This was done, and as a result we had AAA and NRA. But legislation cannot purge a fallacy of its folly. These two institutions together lead to a universal condition of less goods at higher prices. But our goal of a higher standard of living requires more and more goods at relatively less prices. The governmental policy is surely false. It leads away from our goal. There is no magic algebra which can prove the truth of the equation, AAA + NRA = Prosperity. It does not equal prosperity. It equals a lowered scale of living for the people of this country, and no other answer to this equation can be found.

This first obstacle, then, is in the nature of a drag or brake or a heavy load to be carried. The second has more the nature of a recurring catastrophe which, when it occurs, not merely retards our advance but stops it completely. It not merely stops it, but throws us backward and destroys the very mechanism of progress, so that we must spend weary months and years in repairing it before we can regain our lost ground and resume our advance.

This second obstacle is the periodic onslaught of speculative frenzy to which our social organization is subjected.

We should clearly understand what we mean by "speculative." In the sense in which we are using the word it has no connection with business enterprise or business risk. A man may risk his time and all the funds at his command in the development of a new machine or a new product, or simply in an attractive business opportunity of the conventional sort. The outcome may be doubtful and may be called "speculative." But the proceeding is essentially healthy if pursued with judgment and in the light of experience. On the willingness of the business community to engage in such risks are based our hopes for expanding employment and a rising living standard. Speculation is quite another thing and is most unhealthy when carried out on a large scale. It will best be understood if we describe the conditions under which it flourishes.

Periodically business men become discouraged with the slow and meager profits of normal business, which is the business of producing and distributing goods and services. Normal business profits are small. If on the one hand we admire the returns from certain well-managed or especially fortunate organizations, on the other we observe the thousands of businesses which struggle along for years without profit, finally sinking into bankruptcy with losses which run to hundreds of millions in the aggregate. Between the two extremes lies the general average of business success which, with infinite work and worry, returns to its owners little more than the normal interest on the capital invested.

From time to time the business world, discouraged with this meager return, looks about and discovers lying ready at hand a marvelous mechanism which promises to work to a most attractive end. Instead of finding in his hands at the end of the year only the small profits of this year's business, this mechanism promises that the business man can hold in his hands to use or spend and enjoy *now*, not only the profits of this year but next year's profits as well—the profits of five years ahead—the imagined profits of future generations. This mechanism, ready at hand for such delectable use, is the speculative market for securities, real estate, and commodities, with the financial mechanism to which it is geared. The endeavor to realize far future profits by the use of this mechanism is "speculation" in the sense in which we are using the term.

When the professional speculators are joined by thousands of business men, and finally by the general public in the operation of this mechanism, it does seem to work. For many months or for a term of years we do find new, spendable wealth in our hands—new millions, new billions. Apparently we can reach far into the future, seize its profits and enjoy them now.

But it is in appearance only. There is no mechanism which will reach into the future and seize the profits of years not yet born. The reality is ugly and sordid. If there are new billions of "wealth," those billions came directly from bank borrowing by purchasers of the securities, real estate, or commodities, to finance their purchases above cash margins which were comparatively narrow. Bank-credit money, from its very nature, is generated by the process of borrowing and disappears when the debt is paid. In consequence, for every billion of new wealth there is somewhere a billion of new indebtedness, and if that billion of indebtedness becomes due and must be paid, then somewhere a billion of "wealth" must disappear.

That volume of indebtedness—that source of fallacious wealth—is an incredibly flimsy and fantastic structure when we reach the end of a protracted speculative boom. It finally becomes evident to every one that the future profits on which present prices are based can never be realized. When that does become evident prices fall and debts become due. And then, for long months and years every cent of savings, every mite that can be spared from the bare expenses of existence, must be devoted to the liquidation of that indebtedness. These sums are lost to purchasing power, and these months are months of destruction and defeat. The damage must be painfully repaired before we can again resume our progress toward the new social frontier, the raised standard of living.

These two obstacles, the false idea and the destructive calamity, are not light matters. We have struggled with them for generations, and they have grown stronger with the years. What hope is there that we will ever control or overcome them?

I am glad to stand before you tonight and to say that to me the prospect for sanity and intelligence in these matters looks brighter than at any time in the past. We have done much thinking. Experience has brought to us bitter lessons. Some of them we have learned.

We have, for instance, one example before our eyes of an industry which has never for a moment worshipped the false gods of production control and raised prices. The automotive industry has always been willing to submit to severe competition, and in consequence it has year by year built more and more and better and better cars and sold them for lower and lower prices. In further consequence it has made the greatest increase in employment and payroll dollars and is the most prosperous of any of our great industries. It has made the greatest contribution to industrial recovery. Other industries are taking notice and considering their ways.

Furthermore, there is a growing realization that if industries agree on price and production instead of competing, they open the door to government regulation. Indeed under those conditions, government control becomes an inescapable duty. On the other hand, for an industry which submits to the hazards of competition, and governs its prices and output thereby, governmental control is an unwarranted impertinence and a social calamity. Viewed in this light, our antitrust legislation is the strongest protection available to business. These things are becoming clear, and if there is any drift of business opinion it is away from the fallacious search for safety and toward intelligent courage.

Industry must lead the way. We cannot expect labor or agriculture to do so. The prosperity which will come to industry as a whole in so doing will so directly increase the demand for labor and for agricultural products that the artificial methods hitherto applied by these two groups will be clearly seen as the hopeless proceedings which in reality they are.

As to the control of speculative frenzy, we can point to much more definite items of achievement. Beginning with the Glass-Steagall act of the last months of President Hoover's administration, and ending with certain useful elements of the banking act passed last summer, we have provisions which by no means insure us against a recurrence of this calamity, but which do provide us with tools which, if wisely used, will greatly diminish the severity of any future attack.

Hitherto it has not been possible to restrain the expansion of speculative credit without throttling the flow of credit for normal business in the process. Now, for the first time, we can restrain the evil while supporting the good. We need have no repetition of the period from 1926 to 1929 with its disastrous consequences. We can, if we will, prepare a future freed of the worst follies of the past, in which we can move steadily and purposefully toward our social objective of a raised standard of living for our people as a whole.

It must be clear to all that engineers have a particular interest and responsibility in these matters. We cannot sit idly by and see the choice made for us.

If, faced with the disappearance of the physical frontier, we cravenly resign ourselves to accepting that fact as final, and key our national policy and our national hopes down to that dismal

decision, there will be little future for the engineer. There will be fewer of us, and our services will be of a subordinate order.

If, on the contrary, we make the more courageous decision to replace the lost physical frontier with the new social frontier of the raised standard of living, then the engineer will come into his own.

For the raised standard of living demands more goods and services, of better quality and at lower cost. This is the service which the engineer is qualified to render. To this was he born; for this was he educated; this is his life work. In a society dedicated to this end he will find profitable employment, satisfying recognition, and useful essential service of the general good. With the scientist, the inventor, and the broadly visioned business executive he will pioneer in the founding of a new order as far surpassing our present one as that does the barbarism of earlier ages.

We are faced with these alternatives—to retreat or to advance. Is there any doubt as to which the American people should choose? Nay, rather, is there any doubt as to which they *will* choose?

IN TWO SECTIONS—SECTION TWO

TRANSACTIONS

of The American Society of Mechanical Engineers

SOCIETY RECORDS—Part III

(Including Indexes to Publications)

[*Part I of Society Records for the year 1935 (containing Council and Committee Personnel and other general information) was issued as Section Two of the Transactions for February, 1935, and Part II (Memorial Notices), August, 1935.*]

Annual Report of the Council for 1934–1935 ..... RI-81

Reports of Standing and Special Committees RI-83

Finance	83	Advisory Board on Technology	106
Meetings and Program	89	Advisory Board on Standards and Codes	107
Publications	89	Board of Review	107
Local Sections	91	Boiler Code	108
Professional Divisions	94	Bond Issue	108
Membership	96	Capital-Goods Industries	109
Constitution and By-Laws	96	Citizenship Manual	109
Relations With Colleges	96	Cooperative Relations	109
Awards	97	The Economic Status of the Engineer	109
Education and Training for the Industries	98	Engineering History	109
Library	98	Junior Participation	109
Research	98	Manual of Practice	110
Standardization	102	Policies and Budget	110
Power Test Codes	104	Public Affairs	110
Professional Conduct	106	Calvin W. Rice Memorial	110
Safety	106	George Westinghouse Bust	110

Reports of Activities Carried on Jointly With Other Organizations RI-110

Service Rendered by the Office of the Society RI-116

Depositories for Transactions RI-117

Indexes to Papers and Publications RI-121

Regular Society Publications, 1935 ..... RI-121

Special Publications, 1935 ..... RI-121

Miscellaneous Papers, 1935 ..... RI-121

Publications Developed by the Technical Committees RI-123

Index to *Mechanical Engineering*, 1935 ..... RI-125

Index to A.S.M.E. Transactions, 1935 (including Society Records Supplements) RI-134

JANUARY, 1936

VOL. 58, NO. 1

Published by The American Society of Mechanical Engineers

TRANSACTIONS

of The American Society of Mechanical Engineers

Published on the tenth of every month, except March, June, September, and December

Publication Office, 20th and Northampton Streets, Easton, Pa.
Editorial Department at the Headquarters of the Society, 29 West Thirty-Ninth Street, New York, N. Y.

Includes Aeronautical Engineering

Members of Council, 1936

PRESIDENT

WILLIAM L. BATT

VICE-PRESIDENTS

Terms expire December, 1936

EUGENE W. O'BRIEN
JAMES H. HERRON
HARRY R. WESTCOTT

PAST-PRESIDENTS

Terms expire December

ROY V. WRIGHT	1936
CONRAD N. LAUER	1937
A. A. POTTER	1938
PAUL DOTY	1939
RALPH E. FLANDERS	1940

VICE-PRESIDENTS

Terms expire December, 1937

ALEX D. BAILEY
JOHN A. HUNTER
R. L. SACKETT
WILLIAM A. SHOUDY

MANAGERS

Terms expire December, 1936

JAMES A. HALL
ERNEST L. OHLE
JAMES M. TODD

Terms expire December, 1937

BENNETT M. BRIGMAN
JILES W. HANEY
ALFRED IDDLES

Terms expire December, 1938

WILLIAM L. DUDLEY
WALTER C. LINDEMANN
JAMES W. PARKER

TREASURER
W. D. ENNIS

SECRETARY
C. E. DAVIES

Chairmen of Standing Committees of Council

ADMISSIONS, R. H. McLAIN
HONORS AND AWARDS, H. DIEDERICHS
CONSTITUTION AND BY-LAWS, H. H. SNELLING
EDUCATION AND TRAINING FOR THE INDUSTRIES, C. J. FREUND
FINANCE, WALTER RAUTENSTRAUCH
LIBRARY, E. P. WORDEN
LOCAL SECTIONS, W. L. DUDLEY
MEETINGS AND PROGRAM, E. C. HUTCHINSON (Acting Chairman, R. I. REES)
POWER TEST CODES, F. R. LOW
PROFESSIONAL CONDUCT, E. R. FISH
PROFESSIONAL DIVISIONS, K. H. CONDIT
PUBLICATIONS, S. W. DUDLEY
RELATIONS WITH COLLEGES, E. W. BURBANK
RESEARCH, N. E. FUNK
SAFETY, W. M. GRAFF
STANDARDIZATION, A. IDDLES

Committee on Publications

S. W. DUDLEY, *Chairman*

G. F. BATEMAN
C. B. PECK
M. H. ROBERTS
W. F. RYAN, *Vice-Chairman*

EDITOR: GEORGE A. STETSON

Advisory Members

W. L. DUDLEY, SEATTLE, WASH.
J. M. TODD, NEW ORLEANS, LA.
A. N. GODDARD, DETROIT, MICH.
L. S. MARKS, CAMBRIDGE, MASS.
L. N. ROWLEY, JR. (JUNIOR MEMBER)
O. B. SCHIER, 2D (JUNIOR MEMBER)

BY-LAW: The Society shall not be responsible for statements or opinions advanced in papers or . . . printed in its publications (B2, Par. 3).

Entered as second-class matter March 2, 1928, at the Post Office at Easton, Pa., under the act of August 24, 1912. Price $1.50 a copy, $12.00 a year; to members and affiliates, $1.00 a copy, $7.50 a year. Changes of address must be received two weeks before they are to be effective on our mailing list. Please send old, as well as new, address.

Copyrighted, 1936, by THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

Annual Report of the Council

DURING the past year Society activities were on a substantially better basis than in 1933–1934. Rigorous economy has been observed and it is believed real progress has been made. For the first time in three years the Society income at the end of the first six-months' period was greater than estimated and it was possible to make some increase in appropriations at that time. The financial report for the year will show a modest addition to surplus and a reduction in the debt.

Information About Members

The improved financial status of the Society enabled the Council and its Executive Committee to give more thought to the interests of the individual members. As a first step, a professional-data form was circulated during the spring, on which members were asked to record their experience and special qualifications. The analysis of these forms is about 60 per cent complete at the close of the fiscal year. Approximately 7000 members returned data and the information will be valuable in formulating programs for publications and activities to meet the general interests of the members. We have up-to-date records of 7000 members that will enable us to give aid to those employers seeking men with unusual experience and qualifications. We also have an accurate record of 860 consulting engineers, showing their detailed specialties, from which to answer inquiries for men qualified to handle specific problems. About 750 members have volunteered for service on committees.

Improving the Profession

The enhancement of the status of the engineer is the stated purpose of the Engineer's Council for Professional Development in which the A.S.M.E. is participating. The Engineers' Council has made splendid progress in its program of accrediting engineering schools and, as the fiscal year draws to a close, plans are under way to examine some twenty schools in the New England and Middle Atlantic areas. These two areas were chosen for the first trial of accrediting and the development of the scheme will depend upon its success in this experimental area. The Engineers' Council has also promulgated two programs in which the Society is called upon to participate actively through its sections.

(1) The guidance of young men planning to enter engineering as a career.

(2) Stimulation of a self-development program for young men who have been graduated from engineering schools and are preparing themselves to become full-fledged engineers and for those young men who have not had the advantage of a formal education.

Plans for both of these programs have been prepared and communicated to the Sections. The successful execution of these plans provides a direct problem for each Section.

Registration

During the past year the Council has given a great deal of consideration to the movement looking to the establishment of registration laws for engineers throughout the United States. As the year closes, thirty-five states have such laws.

In June the Council adopted an important policy in this matter as follows:

(1) The A.S.M.E. will present the broad aspects of registration to its members in those states not having registration and stimulate its consideration by them.

(2) The A.S.M.E. will not publicly urge registration in those states where the movement has not taken definite shape.

(3) In those states where a registration movement has been initiated which has the support of representative members, the A.S.M.E., through its members in those states, will offer to support the movement by helping to organize it further, by assisting to formulate a proper legislative bill, by soliciting the support of other engineering societies and outstanding members, by advising appropriate strategy, and by other suitable and proper means.

(4) In those states where registration laws have been enacted, the A.S.M.E., through its members, will encourage (a) amendment of the registration laws for the purpose of improving and making them more uniform, and (b) stricter enforcement of those laws.

At the close of the year Council was giving consideration to the steps necessary to carry out this policy.

American Engineering Council

The American Engineering Council has completed a good year. Its report appears in detail on another page and is well worth painstaking study. The American Engineering Council has made substantial progress in relating itself to government agencies on engineering matters and has initiated a series of contacts looking to the placement of engineers in government services. It has been instrumental in initiating a special census of engineers, with particular reference to educational background, occupation, income, and present status of employment. Over 50,000 records of individuals are in the hands of the Bureau of Labor Statistics of the U. S. Department of Labor and it is expected that the report will be available about the end of the calendar year.

Junior Activity

One of the important problems pressing for early solution is the assimilation into the Society activities of the large number of graduates of engineering schools. The program of post-graduate training planned by the E.C.P.D. previously mentioned offers an excellent opportunity for the Society to take an important position of leadership. During the past year a special committee on junior participation was appointed, which has made some valuable suggestions for junior activity. Many sections have adopted active programs for junior participation, but there is still a great opportunity for future developments in this program.

Changes in Membership Grades

An important change in the membership structure was initiated during the year. As a result of discussions in the E.C.P.D. and in the Committee on Policies and Budget, a revised plan of membership grades was advanced and, after acceptance by the members in meeting at Cincinnati in June, 1935, was submitted to letter ballot. The plan abolishes the present Associate-Member grade, makes some changes in the requirements for a new grade of Member, and adds the Student Member, and the Fellow, the latter being the engineer of distinction.

General Activities

The meetings of the Society during 1934–1935 were generally satisfactory. The plan of having annually a Calvin W. Rice Memorial Lecture was initiated at the Cincinnati meeting by Dr. Adolph Meyer, of Zurich, Switzerland.

The increase in income from advertising enabled the Council to provide more money for publications during the latter part of the fiscal year. The *Journal of Applied Mechanics* was issued with some assistance in financing from universities, companies,

CHANGES IN MEMBERSHIP, OCTOBER 1, 1934, TO SEPTEMBER 30, 1935

	Membership		
	Oct. 1, 1935	Oct. 1, 1934	Net Changes
Honorary Members	16	17	−1
Life Members	67	67	0
Members	6375	6548	−173
Associates	268	293	−25
Associate-Members	2604	2658	−54
Juniors (10)	1448	1497	−49
Juniors (20)	3239	3147	+92
	14,017	14,227	−210

and individuals. The publication problem is, however, a major one and at the close of the fiscal year is being given intensive study by the Publication Committee, the Professional Divisions, the Advisory Board on Technology, and the Council.

With appropriations less than customary two years ago, the operation of Sections was carried on with reasonable satisfaction. Three hundred and one meetings were reported. A scale of measurement of Section operation has been developed and put into use. By this it is hoped that the work of a Section during any period can be more accurately evaluated. Also it aids the Section officers to gage the results of their administration.

In addition to cooperating in the preparation of programs for sessions at annual and semi-annual meetings, the Professional Divisions held five successful national meetings during the year.

This year marked the completion of the installation of the new plan of student branch operation. The number of members in student branches reached a total of 3250. Ten student meetings were held with splendid success.

The Committee on Awards has completed a new scheme for administering the honors and awards for the Society, by which a Board of Honor and a Medal Committee will be established. Revisions of the By Laws are under consideration by the Council at the close of this fiscal year.

The technical committee work of the Society was carried on with unexpected activity in view of the condition of the times. Nineteen of the twenty-four research committees showed substantial progress. Ten standards were completed and presented to the American Standards Association for approval. One new Power Test Code and one revised code were published and two new sections and one revised section of one of the codes were completed and approved by the Council.

During the year progress was made in improving contact between the members of the Council and the members of the Society at large. In addition, the President and Secretary visited 48 Sections and 34 Student Branches.

The financial report indicates that the Society has operated well within its income, and has reduced its indebtedness materially.

Advisory Boards

This year saw the initiation of three advisory boards, formed to coordinate the functioning of certain related committees: The Advisory Board on Technology is concerned with meetings, divisions, research, and publications; the Advisory Board on Standards and Codes is concerned with the boiler code, standardization, power test codes, and safety; the Advisory Board on Professional Status is concerned with membership, registration, and the E.C.P.D. These boards made up of representatives of committees with a chairman designated from the Council and set up on an experimental basis for two years, have started to function and have made substantial progress.

Reports of Committees

The complete reports of the standing and special committees follow. They are worthy of painstaking study.

The Parker Case

Shortly before the opening of the fiscal year, Mr. John C. Parker brought action in the Supreme Court looking to an investigation of Society activities, particularly in connection with the Engineering Index and the Boiler Code. The investigation was started in April and continued intermittently through the rest of the year. It is expected that the report of the investigation will be completed before the Annual Meeting. Upon its completion, the Council will render a complete report to the membership.

C. E. Davies, *Secretary*

Reports of Standing and Special Committees

FINANCE

For the second consecutive year since the effects of the depression in the fall of 1931 began to be reflected in its financial affairs, the Society has lived within its income.

(*A*) Yearly Operating Results based upon inclusion of initiation fees:

	1931–32	1932–33	1933–34	1934–35
Total Income.....	$576,590.14	$409,729.73	$395,642.53	$383,830.05
Total Expense and Charges.....	644,588.52	441,281.86	354,936.56	362,905.06
Net Income or Deficit for the Year.	$ 67,998.38 (deficit)	$ 31,552.13 (deficit)	$ 40,705.97 (net income)	$20,924.99 (net income)

(*B*) Yearly Operating Results based upon exclusion of initiation fees:

	1931–32	1932–33	1933–34	1934–35
Total Income	$560,301.81	$396,934.29	$380,287.93	$368,029.82
Total Expense and Charges.....	644,588.52	441,281.86	354,936.56	$362,905.06
Net Income or Deficit for the Year.	$ 84,286.71 (deficit)	$ 44,347.57 (deficit)	$ 25,351.37 (net income)	$ 5,124.76 (net income)

The Finance Committee considers this accomplishment an encouraging indication that the financial condition of the Society is improving. The actual total income during the past year is 6 per cent greater than the budgeted estimate. The situation with respect to dues is considerably better than it was a year ago. For the past two years there has been a slow, but steady increase in the revenue received from advertising and sale of publications. The earnings from investments have continued to hold at an average of approximately 4 per cent. The financial position of the Society, as reflected by its balance sheet, is increasingly liquid. Questionable assets have been written down to amounts believed to be conservative; the current liabilities have been reduced. If these improvements may be considered as indicative, it would seem that a critical period in the history of the Society has been successfully passed.

Statements

In accordance with the practice of recent years, this annual report of the Finance Committee, for the fiscal year ended September 30, 1935, includes detailed comparative statements of the Society's operations for the past two fiscal years. A brief explanation of the statements may assist the membership and committees in a clearer understanding of what the Finance Committee has set forth.

The summary of expanses given in the income and expense statement in Exhibit A is supported by detailed statements, viz.: direct appropriations to Standing Committees of Council in Schedule 1; printing and distribution expenses of publications in Schedule 2; and the itemized expenses of operating the office of the Society in Schedule 3. Additional schedules are included for those members who prefer to have the expenses shown differently, as follows: the departmental expenses of the office are shown in Schedule 4, with the explanation of the activities carried on by each department; in Schedule 5 is given the total cost of the many activities of the Society, obtained by adding to the direct appropriation for each activity its proportionate share of the office expense; finally, the total cost of Society activities is summarized into major groups of services in Schedule 6. The accompanying chart of income and expenditures per member is based upon the total income, including initiation fees, as given in the first column of Exhibit A and upon total expenses and charges as given in the first column of Schedule 6.

The amount collected from initiation fees was not used for operating expenses, but was put directly into surplus as shown in Exhibit A. The net income for the year shown at the bottom of Exhibit A was, likewise, added to surplus. It represents the difference between all income, except that from initiation fees, and total expenses and charges. These additions to surplus, together with other entries directly affecting surplus, are shown in Exhibit B. The balance in the surplus account on September 30, 1935, is the difference between the total assets and the total liabilities of the Society on that date and is so shown in the balance sheet, Exhibit C, before providing for decline from the purchase price to the present quoted market value of the Society's investments, exclusive of trust-fund investments.

The annual audit of the Society's accounts for the fiscal year ended September 30, 1935, has again been made by Messrs. Haskins & Sells, Certified Public Accountants. As explained in the notation following this report, Exhibits A, B, and C and Schedules 1 and 3 were prepared from the auditor's report, whereas, the statements Schedules 2, 4, 5, and 6 were prepared from the Society's records. The statements from the two sources are in entire agreement.

Financial Management

Evidence of the efforts made by the Council to improve the financial position of the Society is apparent from the following: (*a*) insistence upon strict adherence to the budget, (*b*) write-offs of uncertain assets, (*c*) wipe-off of deferred charges, (*d*) facing the facts about present market values of securities, (*e*) reduction of the bank debt, (*f*) advantageous disposal of marketable securities, (*g*) constant attention to the Society's investments, (*h*) unremitting study of trust-fund and other fund requirements, and (*i*) judicious formulation of future policies.

Budget Adherence

Some of the major budget policies formulated by Council in June, 1934, which have been strictly adhered to are: (*a*) the amount available for budget appropriations for Society expenses not to exceed 95 per cent of the estimated gross income, (*b*) an amount equal to the income received from initiation fees to be appropriated from surplus for the reduction of the bank debt, (*c*) efforts to be made to reduce the bank debt by the advantageous sale of securities as opportunity arose, (*d*) no new activity to be undertaken unless definitely known that the funds required would be available without decreasing appropriations or existing essential activities.

Recent Write-Offs

Upon recommendation of the Finance Committee, after careful consideration, Council has approved many drastic write-offs of uncertain assets and deferred charges during the past four years, in order to present more accurately the Society's condition. An examination of the surplus statements and balance sheets for September 30, 1932, 1933, 1934, and 1935 shows the following major write-offs or creation of reserves:

1932, Dues.....	$ 55,000.00	
Deferred deficits of Engineering Index from Oct. 1, 1927, to Sept. 30, 1931.....	81,740.99	$136,740.99
1933, Dues.....	$164,317.89	
Advertising account.....	20,384.69	
Obsolete and inactive items in inventory.....	78,660.48	$263,363.06
1934, Dues.....	$120,448.65	
Revaluation of Engineering Index inventory and transfer of receivables to Engineering Index, Inc.	34,217.78	
Deferred charge for fiftieth anniversary.....	24,399.66	
Adjustment in publication inventory	10,098.19	$189,164.28
1935, Dues.....		$ 10,000.00

Dues Situation

After a three-year adherence to a lenient dues-payment policy, the Council deemed it necessary in 1934 to begin clarifying the membership status by dropping or suspending, on September 30, 1934, those members who on that date owed dues for two full years or more. Further analysis of the dues accounts in 1935 resulted in the dropping of additional members on June 18, 1935. Although the actual write-offs of the unpaid dues of these members did not take place until the current fiscal year, provision therefore was made out of income in the years shown below:

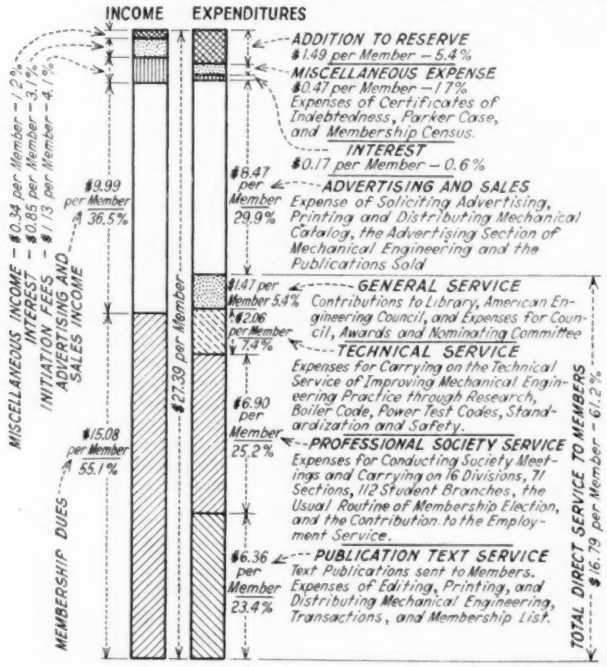
Year ended September 30, 1932.....	$ 55,000.00
Year ended September 30, 1933.....	164,317.89
Year ended September 30, 1934.....	120,448.65

The sum of these three charges when added to the corresponding expense of $10,000.00 for the present fiscal year, makes the total provision for uncollectible dues for the entire four-year period ended September 30, 1935, $349,766.54.

As the result of the foregoing write-offs and creation of reserves, the amount of dues receivable shown on the balance sheet, Exhibit C, is considered to be collectible in its entirety.

Furthermore, a most favorable dues situation has resulted from the efforts of Council to clarify the membership status. This is

A.S.M.E. INCOME AND EXPENDITURES PER MEMBER, 1934-35



apparent from the fact that, despite a 30-per cent shrinkage in the membership during the past year, the cash collections of dues were improved, as shown:

Year ended September 30, 1934.....$199,868.73
Year ended September 30, 1935..... 201,967.21

Current Condition

On the basis of quoted market values of investments, the current condition of the Society is more liquid than it was a year ago, as shown by the ratio of current assets to current liabilities:

At September 30, 1934.....1.3
At September 30, 1935.....1.9

This corresponds with an increase in the net working capital, or excess of current assets over current liabilities of:

	Current Assets	Current Liabilities	Net Working Capital
At September 30, 1934.....	$154,741.85	$114,562.88	$40,178.97
At September 30, 1935.....	135,289.88	74,771.93	60,517.93

The foregoing ratios and statement of net working capital are computed on the conservative basis used in the Society's balance sheets, and do not take into consideration real estate mortgage bonds and certificates both unpledged and pledged as collateral to bank notes payable having a quoted market value of $71,350.00 at September 30, 1934, and $75,685.00 at September 30, 1935. The reason for the exclusion of these items is their less ready marketability as explained later under the heading "Investments."

Debt Reduction

The demand note held by the bank has been steadily reduced as shown below:

	Amount of Note	Yearly Reductions
September 30, 1933.....	$164,000.00	
September 30, 1934.....	57,500.00	$106,500.00
September 30, 1935.....	17,000.00	40,500.00

The reduction has been accomplished by the application of funds received from three sources:

	1933–34	1934–35
Current income.....	$ 8,897.36	$13,951.93
Sale of securities.....	61,287.42	17,713.29
Sale of certificates of indebtedness to members.....	36,315.22	8,834.78
Total reduction in bank debt.....	$106,500.00	$40,500.00

Sale of Securities

By close observation of the daily market quotations of the issues of railroad and public-utility bonds owned by the Society, the Finance Committee, upon authority of Council, was able to sell certain securities from the Society's portfolio at a profit and thereby to reduce the bank loan. The face value, cost, and net selling price of the securities sold during the past two years are given in the accompanying Table 1.

TABLE 1

	Face Value	Cost	Net Selling Price
(A) Securities sold during 1933–34			
Binghamton Light, Heat and Power Co. 5%, 1946.....	$ 5,000.00	$ 5,000.00	$ 4,931.62
Central Maine Power Co. 5%, Series D, 1955.....	10,000.00	10,100.00	9,974.37
Commonwealth Edison Co. 5%, 1935	5,000.00	5,118.75	5,194.50
Cumberland County Power & Light Co. 4½%, 1959.....	2,000.00	1,890.00	1,877.95
Dallas Power & Light Co. 5%, Series C, 1952.....	5,000.00	5,000.00	5,184.87
Metropolitan Edison Co. 5%, Series C, 1953.....	5,000.00	5,012.50	5,036.12
New York Central and Hudson River Railroad Co. debenture 4%, 1942.	25,000.00	23,062.50	23,943.12
San Diego Consolidated Gas and Electric Co. 5%, Series B, 1947.....	5,000.00	5,075.00	5,144.87
Total during 1933–34.....	$62,000.00	$60,258.75	$61,287.42
(B) Securities sold during 1934–35			
Alabama Power Co. 5%, 1951.....	$ 5,000.00	$ 5,000.00	$ 4,831.90
Cumberland County Power & Light Co. 4½%, 1959.....	3,000.00	2,835.00	2,816.86
Public Service Company of Northern Illinois 5%, 1966.....	5,000.00	4,962.50	5,107.26
Texas Power & Light Co. 5%, 1956...	5,000.00	4,850.00	4,957.27
Total during 1934–35.....	$18,000.00	$17,647.50	$17,713.29

Sale of Certificates of Indebtedness

On March 21, 1934, the Council authorized an aggregate issue of $125,000.00 of four per cent Certificates of Indebtedness, due January 1, 1936, to January 1, 1944, to be sold to the membership of the Society. Between that date and September 30, 1934, certificates amounting to $36,450.00 were purchased by members. In the current year additional certificates amounting to $8,700.00 have been purchased by members, thus bringing the amount issued to $45,150.00 on September 30, 1935. Of this amount $600.00 face value of certificates have been donated to the Society by members and are held in the treasury. Those members who purchased Certificates of Indebtedness are to be commended for their willingness to assist the Society. By their loyal support they have not only enabled the Society to reduce its indebtedness to the bank and to lower the annual interest rate by one per cent from the former level; but, in instances where purchased certificates have been donated to the Society, those members have beneficently opened a new channel of revenue through which the Society's financial condition may now be continuingly strengthened.

Investments

The status of each real-estate mortgage bond and certificate has been analyzed and studied. With the exception of two mortgage bonds with a combined face value of $54,000.00, all mortgage bonds and certificates owned by the Society, whether as investments or as trust-fund securities, are issues of the Lawyers Mortgage Company. Practically all mortgage bonds of the Society are secured by real-estate properties in New York City. The Department of Insurance and the Mortgage Commission of the State of New York have endeavored to rehabilitate the Lawyers' Mortgage Company by supervising its operations during the depression. The quoted market value for real-estate mortgages in New York City is low, inasmuch as investors are not buying that form of security; but the majority of the properties

in which the Society has a participating interest are being well managed. This is evidenced by the average yield on real-estate mortgage bonds and certificates owned by the Society which was:

	Trust-Fund Investments	Other Investments	Average
For Fiscal year: 1933-34	4.46%	3.37%	3.64%
1934-35	4.13%	3.59%	3.75%

Although the average yield is exceptionally good for this class of investment at the present time, the mortgages are still in a "distress market." In most cases the market value of these investments is expected ultimately to return to nearly the original purchase price.

The complete schedule of securities owned by the Society appeared in the annual report of Council for 1932–1933 on page 25 of the Record and Index Section of Transactions for 1933. A schedule of securities as of September 30, 1935, has not been included in this report inasmuch as the only changes are those previously noted under the heading "Sale of Securities."

Respectfully submitted,

WALTER RAUTENSTRAUCH, *Chairman* — WM. H. GESELL
WM. T. CONLON — K. M. IRWIN
W. D. ENNIS — ERIK OBERG, *Treasurer*
ELY C. HUTCHINSON, JAMES H. HERRON — *Council Representatives*

ACCOUNTANTS' CERTIFICATE

The American Society of Mechanical Engineers:

We have made an examination of your accounts for the year ended September 30, 1935. Under the terms of our engagement, the accounts receivable, other than advertising accounts, at September 30, 1935, and collections during the year were not verified by confirmations obtained from the debtors, nor were the quantities of the inventories verified by us.

In accordance with recommendations made by us in previous years, the Society during the year under review has compiled detailed records of dues receivable from members. This compilation shows balances receivable at September 30, 1935, aggregating approximately $21,000.00 in excess of the amount shown by the general records. No effect has been given in the accompanying statements to any part of such excess, and therefore the income of the Society for the year under review is conservatively stated. A study now being made by us in conjunction with the Society's staff indicates that a substantial part of the above-mentioned excess has its origin in actions taken by the Society to continue in membership certain individuals whom it had previously expected to drop from the rolls. A supplemental report will be issued setting forth the extent to which the surplus will be adjusted during the fiscal year beginning October 1, 1935, on account of the foregoing.

In conformity with the practice followed by the Society, there have not been included in its accounts nor in the accompanying statements any accruals of interest receivable or payable other than on outstanding certificates of indebtedness.

In our opinion, subject to the foregoing and to the notations on the accompanying statements, the attached balance sheet and statements of income and expenses and of surplus set forth, respectively, your financial condition at September 30, 1935, and the results of your operations for the year ended that date.

(Signed) HASKINS & SELLS

New York
November 30, 1935

NOTE: The following Exhibits A, B, and C and Schedules 1 and 3 have been prepared from the report of Messrs. Haskins & Sells; Schedules 2, 4, 5, and 6 have been prepared by the Finance Committee from the Society's records.

EXHIBIT A

COMPARATIVE STATEMENT OF INCOME AND EXPENSES

For Two Fiscal Years Ended September 30, 1935

INCOME	1934–35	1933–34
Initiation fees (carried to Surplus)—See Exhibit B	$ 15,800.23	$ 15,354.60
Membership dues (Less provision for dues considered uncollectible at September 30, 1935, $10,000.00; September 30, 1934, $120,448.65)	$201,660.34	$198,731.15
Student dues	9,738.50	6,781.35
Interest and discount	11,871.06	13,883.15
Mechanical Engineering advertising	53,503.64	49,195.37
Mechanical Catalog advertising	42,258.15	41,156.63
Publication sales	47,712.06	39,522.83
Miscellaneous sales	1,440.25	1,486.59
Journal of Applied Mechanics contributions	1,531.00	
Engineering Index		25,715.18
Contributions unrestricted	23.00	982.06
Technical Committee contributions	400.00	1,577.28
	$370,138.00	$379,031.59
Less: Provision for and write-offs of accounts receivable other than dues, and adjustment of controlling account, less recoveries	3,466.97	3,972.33
	$366,671.03	$375,059.26
Authorized transfer from Trust Funds		4,200.00
Net profit on securities sold	65.79	1,028.67
Sales of equipment	1,293.00	
TOTAL INCOME	$368,029.82	$380,287.93
EXPENSES AND CHARGES		
Committee expense—See Schedule 1	$ 59,687.04	$ 50,893.96
Publications expense—See Schedule 2	103,535.63	85,625.30
Office expense—See Schedules 3 and 4	190,703.57	188,129.88
Engineering-Index expense		21,235.20
Certificates-of-Indebtedness—interest and issuing expense	1,895.18	1,047.64
Journal of Applied Mechanics—Editorial Board expense	285.43	
Parker Case expenses paid to date	2,326.37	
Membership census	2,054.66	
TOTAL EXPENSES—See Schedules 5 and 6	$360,487.88	$346,931.98
Interest and discount charges	2,417.18	8,004.58
Total expenses and charges	$362,905.06	$354,936.56
Net Income for the year (carried to Surplus)—See Exhibit B	$ 5,124.76	$ 25,351.37

SCHEDULE 1. COMMITTEE EXPENSE

(Direct Appropriation Only)

	1934–35	1933–34
Council	$ 2,560.98	$ 3,521.23
Engineering Societies Library	8,509.53	8,193.71
American Engineering Council	8,778.00	6,055.00
Sections Committee—mileage	853.10	552.18
Sections Delegates—mileage	1,716.74	3,214.69
Sections operation	13,690.55	9,761.43
Awards	364.38	394.03
Nominating Committee	416.06	740.11
Relations with Colleges—mileage	74.95	110.47
Research (Grants to Initiate Projects)		208.00
American Standards Association	500.04	625.01
Divisions Meetings	2,781.52	2,444.63
Society's Meetings	4,822.90	4,333.47
Engineering Societies Employment Service	8,240.27	6,923.58
Student-Branches Operation	6,378.02	3,816.42
TOTAL	$ 59,687.04	$ 50,893.96

SCHEDULE 2. PUBLICATIONS EXPENSE

(Printing and Distribution Only)

	1934–35	1933–34
Mechanical Engineering Text Pages	$ 24,950.48	$ 22,769.94
Membership List	3,869.29	
Transactions	22,849.96	21,358.69
Journal of Applied Mechanics	3,929.75	
Mechanical Engineering Advertising Pages	11,205.49	9,616.57
Mechanical Catalog	16,927.92	15,709.09
Other Publications Sold	19,802.74	16,171.01
TOTAL	$103,535.63	$ 85,625.30

SCHEDULE 3. OFFICE EXPENSE

(Classified by Items)

	1934–35	1933–34
Salaries	$143,201.78	$146,424.05
Commissions	2,323.99	1,583.30
Postage	8,040.75	7,650.45
Printed matter	5,827.53	5,223.79
Publicity	541.55	
Traveling	5,967.34	3,679.83
Collections	197.10	145.42
Telephone and telegraph	4,187.40	4,151.03
Insurance	991.16	1,149.27
Office Supplies	4,446.91	3,399.17
Secretary's Emergency Fund	257.82	50.00
Rent	7,270.58	8,509.83
Stationery	1,658.81	1,151.27
Maintenance and repair	706.78	455.10
Cost of Miscellaneous sales	955.39	935.19
Depreciation of furniture and equipment	1,300.00	1,300.00
Unclassified	3,058.93	2,588.37
	$190,933.82	$188,396.07
Less Receipts	230.25	266.19
TOTAL	$190,703.57	$188,129.88

SCHEDULE 4. OFFICE EXPENSE

(Classified by Departments)

	Salary expense	Other expense[a]	Total Expense 1934–35	Total Expense 1933–34
General Department: General supervision, staff service to Council, Executive Committee, Special Committees of Council, Committees on Finance and Publications, central accounting, purchasing, stores, addressograph, duplicating, files, clerical and stenographic	$50,448.02	$15,445.49	$65,893.51	$65,823.02
Field Department: Staff service for Sections, Branches, Meetings, Divisions, Membership, Awards, Employment Service, Chicago and Tulsa Offices	28,293.33	6,868.40	35,161.73	32,969.24
Technical Department: Staff service for Committees on Boiler Code, Power Test Codes, Research, Safety and Standardization	14,782.32	2,107.46	16,889.78	18,896.05
Editorial Department	19,210.41	1,036.69	20,247.10	20,913.77
Advertising Department	24,888.38	9,116.76	34,005.14	31,436.90
Publication Sales Department	5,579.32	3,365.25	8,944.57	7,131.80
Rent, Insurance, Depreciation	.....	.....	9,561.74	10,959.10
TOTAL	$143,201.78	$37,940.05	$190,703.57	$188,129.88

[a] Other expense includes postage, printed matter, travel, telephone and telegraph, office supplies, stationery, maintenance and repair, and unclassified.

SCHEDULE 5. DETAILED COST OF A.S.M.E. ACTIVITIES

Activities	Committee Expense	Printing and Distribution Expense	Office Expense	Total Cost 1934–35	Total Cost 1933–34
Council (President; Executive Committee)	$ 2,560.98	.....	.....	$ 2,560.98	$ 3,521.23
Standing Committees of Council					
Awards	364.38	.....	.....	364.38	394.03
Library	8,509.53	.....	.....	8,509.53	8,193.71
Local Sections	16,260.39	.....	$18,418.14	34,678.53	31,022.89
Operation	(13,690.55)	.....	.....	.....	.....
Delegates Conference	(1,716.74)	.....	.....	.....	.....
Committee Mileage	(853.10)	.....	.....	.....	.....
Meetings and Program	4,822.90	.....	7,619.52	12,442.42	11,188.20
Membership					
Applications	.....	.....	5,936.54	5,936.54	5,218.74
Development	.....	.....	5,936.55	5,936.55	5,218.74
Power Test Codes	.....	.....	4,274.00	4,274.00	7,356.22
Professional Divisions	2,781.52	.....	7,619.51	10,401.03	9,299.36
Publications					
Engineering Index	.....	.....	.....	.....	21,235.20
Journal of Applied Mechanics	(285.43)	$ 3,929.75	2,215.78	6,430.96	.....
Mechanical Catalog	.....	16,927.92	25,203.91	42,131.83	39,372.40
Mechanical Engineering Advertising Pages	.....	11,205.49	30,951.23	42,156.72	39,030.16
Mechanical Engineering Text Pages	.....	24,950.48	18,119.95	43,070.43	42,943.80
Membership List	.....	3,869.29	667.75	4,537.04	.....
Publications for sale	.....	19,802.74	14,749.82	34,552.56	28,214.93
Transactions	.....	22,849.96	12,443.62	35,293.58	36,498.60
Relations With Colleges (Student Branches)	6,452.97	.....	11,076.45	17,529.42	13,647.01
Operation	(6,378.02)	.....	.....	.....	.....
Committee Mileage	(74.95)	.....	.....	.....	.....
Research	.....	.....	7,772.09	7,772.09	7,929.76
Technical Inquiries	.....	.....	877.51	877.51	843.75
Safety	.....	.....	2,676.09	2,676.09	2,641.05
Standardization	.....	.....	6,674.09	6,674.09	9,290.88
Special Committees of Council					
Boiler Code	.....	.....	6,593.51	6,593.51	5,086.14
Certificates of Indebtedness	.....	.....	.....	1,895.18	1,047.64
Membership Census	.....	.....	.....	2,054.66	.....
Nominating Committee	416.06	.....	.....	416.06	740.11
Parker Case	.....	.....	.....	2,326.37	.....
Joint Activities					
American Engineering Council	8,778.00	.....	.....	8,778.00	6,055.00
American Standards Association	500.04	.....	.....	500.04	625.01
Engineering Societies Employment Service	8,240.27	.....	877.51	9,117.78	10,317.42
GRAND TOTAL	$59,687.04	$103,535.63	$190,703.57	$360,487.88	$346,931.98

SCHEDULE 6. TOTAL COST OF A.S.M.E. ACTIVITIES

	1934–35	1933–34
General Service—Contributions to Library, American Engineering Council, and expense of Council, Awards, and Nominating Committees	$ 20,628.95	$ 18,904.08
Society Activities—Expenses for Society Meetings, Professional Divisions, Local Sections, Student Branches, Membership Election, and contributions to Employment Service	96,919.78	86,756.11
Technical Service—Expenses of Committees on Boiler Code, Power Test Codes, Research, Safety, and Standardization	28,489.82	32,929.06
Publication Text Service—Expenses of editing, printing, and distributing text section of *Mechanical Engineering*, Transactions, *Journal of Applied Mechanics*, and Membership List	89,332.01	79,442.40[a]
Advertising and Sales—Expenses of soliciting advertising, printing, and distributing Mechanical Catalog, Advertising Section of *Mechanical Engineering*, and publications sold	118,841.11	106,617.49
Engineering-Index—Expenses of editing, printing, soliciting, and distributing Card Service and Annual Volumes		21,235.20
Certificates of Indebtedness—Expenses of soliciting sales, printing certificates, legal service and interest	1,895.18	1,047.64
Cost of Parker Case—Expenses of legal counsel, minutes of hearings, and printing papers on appeal	2,326.37	
Membership Census—Expenses of printing, distributing, and analyzing biographical data sheets of members	2,054.66	
TOTAL	$360,487.88	$346,931.98

[a] No expense of *Journal of Applied Mechanics* nor Membership List in 1933–34.

EXHIBIT B

STATEMENT OF SURPLUS

For Fiscal Year Ended September 30, 1935

BALANCE, OCTOBER 1, 1934, PER SOCIETY ANNUAL REPORT		$222,596.59
CREDITS:		
Initiation Fees	$15,800.23[a]	
Adjustment of Prior Years' Interest—Retirement-Fund Reserve	1,810.46	
Reduction in Capital Investment	557.56	
Adjustment of Prior Year's Inventory	115.76	
Net Income for the Year, per Exhibit A	5,124.76	23,408.77
TOTAL		$246,005.36
CHARGES:		
Write-Off of Loan to Committee on Thermal Properties of Steam	$ 4,000.00	
Assessment by United Engineering Trustees, Inc., for construction of Library stacks in a prior year	2,500.00	
Reduction of Brashear Biography Inventory value	457.92	
Adjustment of Custodian-Fund Interest Applicable to prior year	197.99	7,155.91
BALANCE, SEPTEMBER 30, 1935 (representing excess of income, including initiation fees, over operating expenses and other charges, and subject to provision not having been made for decline of $163,200.64, as of September 30, 1935, in the quoted market value of the Society's investments, exclusive of trust-fund investments)		$238,849.45

[a] It being the practice of the Society to take up initiation fees only as and when collected, the above statement does not include such fees receivable at September 30, 1935, and October 1, 1934.

EXHIBIT C

BALANCE SHEET, SEPTEMBER 30, 1935

ASSETS

CURRENT ASSETS:			
Cash		$ 17,421.80	
Accounts receivable:			
Dues (less reserve for uncollectible items, $32,253.17)	$10,295.34		
Publications and advertising (less reserve for uncollectible items, $4,000.00)	44,840.61		
Miscellaneous	1,232.16	56,368.11	
Inventories:			
Publications for sale	$35,378.54		
Publications in process	8,092.49		
Supplies	4,741.94	48,212.97	
Securities—At cost (quoted market value, $13,287.00)—$23,177.64 deposited as collateral to bank note payable		23,277.64	
TOTAL CURRENT ASSETS			$ 145,280.52
REAL-ESTATE MORTGAGE BONDS AND CERTIFICATES—AT COST (quoted market value, $110,540.00):			
Pledged as collateral to bank note payable		$ 86,000.00	
Pledged as collateral to Certificates of Indebtedness		68,750.00	
Unpledged		109,000.00	
TOTAL REAL-ESTATE MORTGAGE BONDS AND CERTIFICATES			263,750.00
TRUST-FUND ASSETS:			
Corporate stocks and real-estate mortgage certificates (quoted market value, $66,370.52)		$118,127.16	
Cash		2,640.44	
TOTAL TRUST-FUND ASSETS			120,767.60
PROPERTY-FUND INVESTMENTS:			
One-fourth interest in real-estate and other assets of United Engineering Trustees, Inc., exclusive of trust funds		$496,948.48	
Office furniture and fixtures (depreciated value)		23,719.60	
Library books		1.00	
Engineering Index—Title and good-will		1.00	
TOTAL PROPERTY-FUND INVESTMENTS			520,670.08
DEFERRED CHARGES—Society Activities Applicable to Subsequent Years			141.46
TOTAL			$1,050,609.66

LIABILITIES

CURRENT LIABILITIES:			
Note payable—Bank (See securities pledged, contra)		$ 17,000.00	
Accounts payable		21,382.00	
Unfilled commitments—Estimated liabilities relating to "Mechanical Catalog" for 1935–1936, etc.		13,700.20	
Assessment payable—United Engineering Trustees, Inc.		2,000.00	
Custodian funds		16,243.23	
Reserve for prepaid subscriptions		4,000.00	
Accrued interest on certificates of indebtedness		446.50	
TOTAL CURRENT LIABILITIES (exclusive of Certificates of Indebtedness to be redeemed January 1, 1936)			$ 74,771.93
FOUR PER CENT CERTIFICATES OF INDEBTEDNESS, Due January 1, 1936, to January 1, 1944 (10% of the total amount issued to be redeemed January 1, 1936)—See investments pledged, contra:			
Authorized	$125,000.00		
Issued		$ 45,150.00	
Less in treasury		600.00	44,550.00
TRUST-FUND RESERVES:			
Principal		$111,120.02	
Income		9,647.58	120,767.60
PROPERTY FUND RESERVE			520,670.08
RESERVE FOR EMPLOYEES' RETIREMENT ALLOWANCES			15,921.61
DEFERRED CREDIT—DUES RECEIVED IN ADVANCE			35,078.99
SURPLUS (representing excess of income, including initiation fees, over operating expenses and other charges, and subject to provision not having been made for decline of $163,200.64 as of September 30, 1935, in the quoted market value of the Society's investments, exclusive of trust-fund investments), per Exhibit B			238,849.45
TOTAL			$1,050,609.66

NOTATION:

In accordance with the Society's practice, initiation fees receivable are not included in the above statement as they are taken up by the Society as assets only as and when collected.

MEETINGS AND PROGRAM

With each year there are changes in the aspects of the work of the Committee on Meetings and Program and its problems are constantly shifting. Through it all there remain, however, certain fundamental requirements upon which it is periodically necessary to place pressure that the Society as a whole may continue its progress and provide for the service and welfare of the members.

In the year just concluded the Committee's slogan has been "Better meetings and bigger attendance, within the budget." Necessarily the latter presented limitations. Nevertheless, much has been accomplished and reactions to the work of the Committee from the members in general have been encouraging.

For the purpose of developing greater uniformity of action, a suggested policy for the Society with respect to exhibitions held in conjunction with meetings was prepared by the Committee and submitted to the Board of Technology for consideration.

The Annual Meeting which was held December 3 to 7, 1934, was generally satisfactory. Dr. John H. Finley, editor of the *New York Times* delivered the Calvin W. Rice Memorial Lecture, given to commemorate Dr. Rice's contribution to the development of international friendliness in the engineering profession. Because Dr. Rice's services to the Society were outstanding in international relationships in that field. This memorial should be repeated as occasion is offered, as it was when Dr. Adolph Meyer of Switzerland was the lecturer at the Semi-Annual Meeting at Cincinnati.

At the annual banquet the plan of securing as speaker a well-known and outstanding lecturer upon world affairs was extremely well received. To make it possible for members not attending the banquet to listen to the speaker, it was arranged that for a small entrance fee such members would be admitted to the banquet room. The plan worked moderately well, and it is hoped to develop it further.

At the Annual Meeting a small registration fee was charged for nonmembers who were not invited to attend the Meeting. The plan serves to emphasize the value of the Society to the individual member, and it is hoped to develop it further.

The meeting in Cincinnati was eminently successful in every way. Programs for both meetings were published in the publications of the Society and are to be considered as a part of this report by reference.

The revised pamphlet, "Suggestions to Authors" has resulted in increased excellence of presentation of papers. The policy of printing papers in advance of the meetings has been continued with growing popularity.

Council has allotted for all activities conducted under the supervision of the Meetings and Program Committee for the year 1935–1936 the sum of $5700. This amount will be disbursed by the Committee in such manner as to provide the greatest number of meetings. There are now scheduled the 1935 Annual Meeting to be held in December, 1935, the 1936 Semi-Annual Meeting at Dallas, Texas, as well as regional meetings in San Francisco and Niagara Falls also to be held during 1936.

To enable a larger number of members to enjoy the ceremonies and addresses which have been scheduled heretofore on Tuesday evening of Annual Meeting week, known as "President's Night" it has been decided to transfer some of these events this year to the Banquet night on Wednesday. The President's reception and the Presidential Address will be combined with the Banquet and followed by dancing. Tuesday evening will be designated as "Honors Night" at which time the medals and honors will be presented, to be supplemented by one or two addresses or lectures of importance and current interest.

The committee has studied the general programs of the individual Local Sections and has offered to the officers of the Sections suggestions for improving their programs and aid where suitable speakers could not be found locally. The Committee fostered a policy which will insure publication of Local Section papers that meet the standards required of national meeting papers.

Respectfully submitted,

ROBERT I. REES, *Chairman* CLARKE FREEMAN
ELY C. HUTCHINSON R. F. GAGG
HARVEY N. DAVIS D. C. CORY, *Junior Representative*

PUBLICATIONS

At the beginning of the fiscal year the Committee on Publications faced a further decrease in funds below the insufficient amount allotted for publications last year. This forced further curtailment in the services of the committee during the early part of the year, although there was no decrease in the number of papers being presented and recommended for publication. The committee solicited the aid of the Professional Divisions in recommending for publication only papers of outstanding quality and timeliness. The Advisory Board on Technology assisted materially in assenting to a policy for allocating available funds and space among the several technical-paper production agencies of the Society, a plan yet to be worked out in detail, and in urging Council to increase the allotment of funds for publication.

Fortunately an increase in income, both from dues and advertising, enabled the Council to provide more money for publications during the latter part of the fiscal year. Members of the Society expect and are entitled to this direct evidence of an increase in available income. The Committee is continuing to urge the importance of all means for improving the quantity, quality, and timeliness of the technical publications of the Society and for enlarging and improving "Mechanical Engineering."

SUGGESTIONS OF LOCAL SECTIONS DELEGATES

From the Conference of Local Sections Delegates held in connection with the 1934 Annual Meeting the Committee received several suggestions and points of view, all of which were discussed and acted upon.

The confusion existing regarding the name "Record and Index," applied to a publication issued in several sections, only one of which was an index, was cleared up by adopting the name "Society Record" for the sections which are records and "Index" for the final section which is made up of indexes to the Transactions, *Mechanical Engineering*, and miscellaneous publications.

The suggestion that a handbook of codes and standards be distributed free to members was referred to the Advisory Board on Technology.

A suggestion of an assistant editor representing, and resident on, the Pacific Coast was noted but action was withheld for financial reasons.

Comments and suggestions relating to publication policy, requested by the Committee and reported from the individual group conferences were noted and discussed. No serious objections to these policies were registered, but early in 1935 several individuals protested vigorously against the policy of including in *Mechanical Engineering* papers on non-technical subjects, particularly in economic, sociological, and management fields. The Committee attempted to explain its point of view to these critics, and pointed out that in practically every instance the papers on these subjects had been submitted by some official group within the Society, and that the Management Division, whose contributions were objected to on the ground "that they were not engineering," was one of the largest groups in the Society.

For several years the Committee has been alive to criticism of this sort and from time to time has attempted to secure opinions from widely distributed individuals and groups. In the spring of 1935 a "check list" was circulated among selected groups of junior members. One of the objects of this check list was to determine the distribution of reader interest. In all of these attempts to learn what readers want, results have been generally negative. That is, approximately as many do not care for non-technical articles as do. There are very positive statements of preference by some individuals on both sides of this question. It is hoped that the recent census of biographical and professional data will reveal patterns of member interest to guide in selecting material for *Mechanical Engineering*.

TRANSACTIONS

During the fiscal year 1934–1935 the Transactions contained 76 papers, a total of 730 pages; these figures include three issues during 1935 of the *Journal of Applied Mechanics*.

In a memorandum to the Council dated July 2, 1934, it was pointed out that with the proposed budget of $18,000 it would be possible to publish only 64 papers. The Council, in noting the memorandum, expressed the opinion that a minimum of 100 papers per year was desirable and put itself on record as wishing to increase the budget for Transactions as soon as conditions would permit so that this number could be attained. While the figure of 64 papers represented a calendar year, the number 76 actually published in the 12 months of the fiscal year shows how closely that desired number may be realized.

During the summer of 1935 eight papers on hydraulic subjects, whose publication had been postponed because of lack of funds, appeared in the Transactions.

Improvement in the Committee's ability to publish papers resulting from meetings is shown in the record of the Cincinnati meeting. Either in *Mechanical Engineering* or the Transactions 32 of the 38 papers for which manuscripts recommended by the sponsoring groups were received either were published or are shortly to be published.

"Journal of Applied Mechanics"

Under the auspices of the Applied Mechanics Division, a board of editors, headed by John M. Lessells, has been providing material for the section of the Transactions known as the *Journal of Applied Mechanics*. Three issues of the Journal, each of 40 pages, have appeared. The *Journal*, which goes to all members of the Society in good standing, to all subscribers of the Transactions, and to a small list of subscribers to the *Journal* only, has been departmentalized. The departments are: Technical papers; reviews of subject matter within the purview of the Division (to date largely sections of the Division's 1934 Progress Report); design data, a section of material arranged for the immediate use of designers; book reviews; and discussions of papers published in previous issues.

The wealth of available material pressing for publication has proved embarrassing for the editors. At a meeting held in Ann Arbor, Mich., in June, it was decided to limit papers to six pages in order to make possible the publication of a greater number of papers.

Sponsors of the *Journal* are of the opinion that the *Journal* is distributed much more widely than the interest of those who receive it in the subject matter of the papers warrants. They would like to see the circulation reduced to include only those actually desirous of receiving papers of this type. However, a promise was made at the 1933 Annual Meeting to the members of the Society that every member would receive all Transactions papers. This implies that all members shall also receive the *Journal*. The Committee on Publications believes that no change in this policy should be made. In holding this opinion the Committee has in mind not only the 1933 pledge, but the belief that applied mechanics is fundamental to most mechanical-engineering practice. It feels that the wide distribution of all Transactions papers tends to hold the Society's technical interests together, and it recognizes that the great number of younger members, whose careers are still in the formative stage, are better served by "exposure" to a wide range of technical literature than by a too early specialization of interest in what may be, for the individual, a temporary connection with a given field of practice. While the Committee would welcome the voluntary relinquishment of the privilege of receiving the *Journal* by members who have no use for such a publication, it does not wish to be placed in the position of requesting members to give up the *Journal*. Much less would it wish to make every member ask to have the *Journal* sent to him, a request that would not be in keeping with the 1933 pledge.

In order to assist in financing the Journal, its board of editors has solicited funds from interested individuals and organizations. A summary of contributions is as follows:

Universities	$1000.00
Companies	379.00
Individuals	157.00
Total	$1536.00

The Committee has no way of estimating how much money can be collected from these sources during the coming year.

The Committee views the *Journal* as being still in the experimental stage, and withholds its comment on the success of the venture. No criticism of the quality of the technical papers is implied nor intended by this statement, and the Committee wishes to record its deep appreciation of the enthusiasm and services to the Society of the Applied Mechanics Division, the board of editors of the Journal, and John M. Lessells, technical editor of the Journal, on whom the burden of providing the material and soliciting the funds has fallen.

Society Record

Formerly known as the "Record and Index," the present Society Record has been issued from time to time as a supplement to the Transactions. With the February, 1935, issue of Transactions the Society Record contained a complete list of the Society's committee personnel and other reference material as of the administration year that opened in December, 1934. There was thus provided at the earliest practicable moment as complete and up-to-date information on the Society's committees as could be assembled. Heretofore these lists have been of historical interest only as they represented, not the current committee personnel, but that of the previous year.

The scheme of issuing the Society Record in supplements to the Transactions makes it possible to distribute the information contained in the Record as soon as it is available, and also places a more uniform load on the editorial staff. The committee personnel, as already explained, is published as soon as committee appointments have been made; reports of committees are published immediately following the end of the fiscal year and before the Annual Meeting; the indexes to publications appear as soon as the December issues are off the press. Memorial notices of deceased members are issued during the summer.

Membership List

For the first time since 1931 a membership list was published in December, 1934. It was distributed as a supplement to Transactions for December in order to save postage, and the format was consequently changed.

The Committee feels that the effect on the members of the Society of resuming publication of the Membership List was distinctly favorable and valuable. It was therefore with extreme regret that the Committee noted the vote of the Council at its Cincinnati meeting not to publish the Membership List in 1936. It sustains the hope that early resumption of the membership list as a regular annual publication will be ordered by the Council.

"Mechanical Engineering"

Plans for the development and improvement of *Mechanical Engineering* still await increase in Society income from dues and advertising, which will permit additions to an overburdened editorial staff. As already noted, the budget for *Mechanical Engineering* was increased during the summer, permitting the continuance of an average of 67 pages per issue, which otherwise would have been cut materially.

The revenue from advertising this year shows an increase over last year's income. It has been understood that any gain should be applied to the publication, and additional funds were made available for publications as already noted. The Committee's financial statement indicates what has been accomplished.

The Committee has received from several sources queries on the relative proportions of subject matter in *Mechanical Engineering*. This led to an analysis covering about 17 issues which showed the following percentages: Technical papers, 54.3 per cent; professional matter, 14.7 per cent; general, 15.5 per cent; and departmental, 15.5 per cent. It also showed the following distribution as to sources from which the material was derived: Meetings and Program Committee, 29 per cent; Professional Divisions, 26 per cent; Local Sections, 11 per cent; Society Committees, 10 per cent; joint activities with other societies, 2 per cent; miscellaneous contributions and staff articles, including "Engineering Progress," 22 per cent.

Student Branch Bulletin

For eight months during the school year the Student Branch Bulletin, a four-page publication, is distributed with *Mechanical Engineering* to the 3250 members of the Society's 112 student branches.

A hopeful experiment with this publication was inaugurated in September, 1935. A board of editors made up of junior members in the Metropolitan district has taken over the preparation of material for this publication. The following, from the initial issue, explains the objective:

> "The task of editing the 'Student Branch Bulletin' has been taken over by a committee of Junior members of the Society partly to relieve the publications staff, partly to provide an outlet for the energies of younger men, and partly with the hope that the result would be a publication more nearly tuned to the needs and desires of the student engineer."

The Committee wishes to acknowledge its gratitude to this committee and to commend the spirit in which the young men have responded to the opportunity to serve the Society in building up interest among student members.

Mechanical Catalog

The 1935–1936 edition of the Mechanical Catalog marks the twenty-fifth year of reference service to industry. Improved in makeup, the issue for this year has over 10 per cent more pages of product descriptions than the preceding edition and a new index feature with the title, "Manufacturers' List." The "Classified Index" in the volume has some 4000 product headings, carefully checked as to technical nomenclature.

Biennial Report

In 1932, under the chairmanship of W. H. Winterrowd, the Committee on Publications made an extensive study of its functions and problems and presented the results to President Lauer in the form of a comprehensive report. Among other recommendations of this report it was suggested that a similar report be prepared every two years. During the spring of 1935 a new report was prepared and presented to the Council at the Cincinnati meeting. At the suggestion of President Flanders the report was assigned for study and discussion at its October meeting.

Further Study of Publications Policy

Following the presentation of the Biennial Report to the Council the Committee on Publications continued its study of policies at a

meeting on September 6 as a result of more recent developments, among which was a suggestion that the *A.S.M.E. News*, suspended January 1, 1933, be resumed in some form. The Committee's recommendations were presented to the Council at its October meeting.

Conclusions and Recommendations

As is true of all Society committees, the work of the Committee on Publications has been seriously curtailed during the last few years. It wishes to emphasize the following points:

1 Serious curtailment in funds for publication has had a cumulative and adverse effect on Society prestige and on membership morale. Instances have been cited in annual reports of the committee where other agencies have published papers that should be a part of the Society's literature; of committees and divisions that are discouraged and dissatisfied because papers and reports considered by them to be valuable have not been published by the Society, and of groups that have financed their own publications in impermanent form with the result that they are quickly lost to future researchers in these fields. At the base of most publications problems lies lack of funds.

2 Failure to provide adequate and timely publicity on the Society's activities has resulted in growing disinterest in the Society, ignorance and misunderstanding of what is being accomplished, and a consequent weakening of the morale of members, committees, and divisions. An effective house organ or publicity medium is urgently needed. Whether this should be an independent publication, as was the *A.S.M.E. News*, abandoned for reasons of economy a few years ago, or a development of the news function of *Mechanical Engineering* depends on other questions of publication policy. It is the opinion of the Committee on Publications that the Council should, as soon as possible, make provision for an effective and adequate method of giving publicity to the Society's affairs.

3 The Committee is eager to undertake its long deferred plans for developing and improving *Mechanical Engineering*, and earnestly solicits additional funds for this purpose.

4 Assistance to the Committee's continued study of its problems looking to long-time plans for development and improvement of all its publications is solicited.

Respectfully submitted,

S. W. Dudley, *Chairman*
W. F. Ryan, *Vice-Chairman*
S. F. Voorhees
M. H. Roberts
G. F. Bateman
O. B. Schier, 2d, *Junior Adviser*
A. J. Dickie, E. B. Norris, E. H. Ohle } *Advisory Members*

LOCAL SECTIONS

The administrative responsibility of the Committee on Local Sections includes matters of finance as well as the correlation and general direction of Local Sections. It acts in a liaison capacity between Sections and Council, Professional Divisions, Research and Standardization Committees, etc., to promote the interest of all. It directs and plans the activities of the Sections and maintains knowledge of these activities by rating, by report, and by personal contact whenever possible.

General Conditions

Considering the industrial conditions of the country the Local Sections may be considered to be in a satisfactory condition, but they have suffered both from lack of sufficient funds and from loss of membership and leaders.

There is a certain unrest difficult to explain, partially due to inadequate financing in maintaining desired and planned activities, with a consequent lag in interest; partially due to lack of intelligent leadership or initiative on the part of the Section Executive Committees. This, fortunately, is not the case in all Sections. Unrest is fostered by lack of knowledge of Society structure, affairs, and purposes. The members of the Committee on Local Sections have tried by personal contact to advise and educate members in these matters.

The attendance of Sections Delegates both at Group Conferences, and at the Group Delegates' Conference has been a matter of extreme concern to the Committee. The attendance has approximated 60 per cent of that necessary, and nonattendance is due simply to inability of members designated as delegates to attend without serious financial loss to them under the present expense allotment. This is a serious matter to those traveling long distances, and a careful survey indicated that, on the average, every dollar supplied by the Society was matched by a dollar from the pocket of the delegate. This has never been expected nor required, and attendance will not be sufficiently large to permit these conferences to function properly or to voice adequately the advice, desires, and suggestions of the membership unless some method is found to finance completely the delegate's trip.

Budget, 1935–1936

The Committee regrets that the estimate of Society income was insufficient to allow the amount it requested and considered the minimum necessary for adequate Section operation. Requests for additional funds are frequent. This is a continuing indication of the insufficiency of funds necessary for constructive operation. Dissatisfaction is expressed as to the disbursement of Society funds and is made the excuse for non-payment of dues.

Group Conferences

Last fall only 53 out of 71 Sections were represented at the Group Conferences. Of these 53, six attended Conferences held in their home cities, therefore only 47 delegates actually traveled to attend. This was the cause of the low expenditure of delegates' mileage, not a commendable saving. Every section is entitled to representation, but that result is secured only when delegates' expenses are fully met.

The superiority and advantages of these Conferences is increasingly evident in better developed agenda, and constructive suggestions to Council. The Committee feels that the work accomplished by these Conferences is most essential.

The Committee approved the following method of electing delegates: "That at each Group Conference Meeting in 1935 two delegates be elected to attend the Delegates' Meeting in New York in December, one delegate to be elected for one year and the other delegate to be elected for two years; and thereafter one delegate shall be elected each year for a term of two years, and be it further provided:

That the Senior Delegate in each Group shall automatically become Chairman of his Group Conference; and it shall be mandatory upon a Local Section to elect such Senior Delegate as its representative to the Group Conference. In such instances the Section will be entitled to another delegate at the Group Conference."

Redistricting Local Sections

At the suggestion of the Committee on Policies and Budget, the Executive Committee of Council requested the Committee on Local Sections to make a study of the principle of redistricting Local Sections along geographical lines perhaps better suited to the purposes and administration of the Society. The Committee made a complete study of the geographical significance, railroad fares, time of travel, and community of Section interest. Careful inquiry was made of Sections' interests and desires, and several tentative groupings were considered and investigated prior to the Sections Delegates' Conference at Cincinnati.

It was decided from this study by the Committee on Local Sections and the Group Delegates that it would be difficult to develop a better arrangement than that of the present seven groups. Any increase in the number of groups would tend to increase expense, correspondence, postage, travel, exchange of information, details of operation, delay in obtaining action or approval. These are a few of the many factors involved.

Student Meetings

The Student Meetings held in 1934–1935 were unqualified successes from the standpoints of interest, college and Section cooperation, attendance, and technical papers. It is apparent that the students appreciate the attendance at these Conferences of members of Council, staff, and standing committees. Fortunately it was possible for President Ralph Flanders to attend and address some of the Conferences. This interest shown by the officers of the Society is most impressive to the student. Such contacts are an inspiration to both students and members, and will bear fruitful results. The Committee wishes it were possible for such contacts to be made a permanent policy.

Schedule and Place of Group Conferences in 1935

Place	Tentative date	Committeeman to attend
Group I, Providence	October 26–27	Ernest Hartford
Group II, New York	November 1–2	Ernest Hartford
Group III, Ithaca	October 26–27	R. E. W. Harrison
Group IV, Chattanooga	October 26–27	W. R. Woolrich
Group V, Columbus	October 27–28	D. B. Prentice
Group VI, St. Louis	November 2–3	D. B. Prentice
Group VII, Spokane	November 8–9	W. L. Dudley

Semi-Annual Meeting in 1936

Upon the invitation of E. W. Burbank, Chairman of the North Texas Section, Council scheduled the 1936 Semi-Annual Meeting at Dallas, Texas, to be held during the third week of June.

TABLE 1 SUMMARIZED LOCAL SECTIONS' ACTIVITIES 1934–1935

(Based Upon Estimates Made According to the Scale of Measurements)

	(a) Service to Membership					(b) Service to Society							
	A	B	C	D		E	F	G	H	J	K		
Section	No. of meetings	Exec. comm. meetings	Assist. to Junior Mem.	Aid to Student Mem.	Total (a)	Ratio unpaid total Mem.	Ratio new Mem. to total Mem.	Cooperation with headquarters	Performance of delegate	Functioning of individual Mem.	General Sect. activity	Total (b)	Section result, points or per cent
Akron-Canton	135	95	95	95	420	65	20	20	100	65	150	420	840
Anthracite Leh. Valley	150	100	25	50	325	32	17	20	120	50	130	369	694
Atlanta	150	100	20	100	370	20	22	20	100	55	150	367	737
Baltimore	...	...	...	...	...	...	...	...	...	...	...	...	...
Birmingham	150	100	50	70	370	65	20	25	120	60	200	490	860
Boston	...	...	...	...	...	...	...	...	...	...	...	...	...
Bridgeport	75	100	0	50	225	50	50	15	50	50	200	415	640
Buffalo	...	...	...	...	...	...	...	...	...	...	...	...	...
Central Pa.	...	...	...	...	...	...	...	...	...	...	...	...	...
Charlotte	38	50	25	0	113	10	18	10	0	25	50	113	226
Chattanooga	...	...	...	...	...	...	...	...	...	...	...	...	...
Chicago	150	100	25	25	300	50	20	20	120	50	150	410	710
Cincinnati	150	100	100	50	400	60	13	25	120	75	150	443	843
Cleveland	150	100	100	100	450	40	20	25	120	75	160	440	890
Colorado	150	100	100	50	400	55	22	25	120	50	200	472	872
Columbus	...	...	...	...	...	...	...	...	...	...	...	...	...
Dayton	150	50	0	0	200	40	8	15	0	25	100	188	388
Detroit	150	100	100	75	425	100	30	25	120	75	180	530	955
Erie	75	100	50	0	225	50	40	25	120	25	100	360	585
Florida	120	75	60	100	355	41	23	20	100	60	150	394	749
Green Mt.	75	25	0	0	100	78	15	5	75	75	75	323	423
Greenville	150	100	20	75	345	33	30	20	120	50	125	378	723
Hartford	150	75	25	25	275	55	21	15	50	50	120	311	586
Houston	...	...	...	...	...	...	...	...	...	...	...	...	...
Indianapolis	90	80	50	80	300	40	10	15	96	25	100	286	586
Inland Empire	...	...	...	...	...	...	...	...	...	...	...	...	...
Kansas City	150	100	75	75	400	50	30	25	120	50	110	385	785
Knoxville	150	100	20	100	370	40	28	25	120	70	110	393	763
Los Angeles	150	100	75	50	375	100	10	25	120	75	180	510	5
Louisville	150	100	0	75	325	72	18	20	100	50	175	435	60
Memphis	...	...	...	...	...	...	...	...	...	...	...	...	...
Meriden	...	...	...	...	...	...	...	...	...	...	...	...	...
Metropolitan	150	100	100	80	410	39	18	23	120	80	180	460	870
Mid-Continent	150	100	90	55	395	67	13	20	120	50	150	420	815
Milwaukee	...	...	...	...	...	...	...	...	...	...	...	...	...
Minnesota	...	...	...	...	...	...	...	...	...	...	...	...	...
Nebraska	110	75	30	50	265	70	25	15	100	50	100	360	625
New Britain	150	100	0	0	250	75	20	25	120	60	175	475	725
New Haven	150	100	30	100	380	50	30	25	120	65	175	465	845
New Orleans	150	100	0	75	325	62	21	15	120	50	125	393	718
North Texas	150	100	25	75	350	40	20	20	120	50	105	355	705
Norwich	...	...	...	...	...	...	...	...	...	...	...	...	...
Ontario	150	100	50	80	380	40	12	20	100	50	150	372	752
Oregon	...	...	...	...	...	...	...	...	...	...	...	...	...
Peninsular	...	...	...	...	...	...	...	...	...	...	...	...	...
Philadelphia	150	100	85	70	405	55	30	22	118	70	185	480	885
Pittsburgh	...	...	...	...	...	...	...	...	...	...	...	...	...
Plainfield	150	100	50	0	300	50	30	25	120	75	130	430	730
Providence	150	100	75	75	400	75	15	25	120	75	200	510	910
Raleigh	150	75	50	50	325	80	11	25	120	25	80	341	666
Rock River Valley	150	100	50	30	330	70	18	25	0	40	200	353	683
Rochester	150	100	50	50	350	75	50	25	80	75	120	425	775
St. Joseph Valley	...	...	...	...	...	...	...	...	...	...	...	...	...
St. Louis	...	...	...	...	...	...	...	...	...	...	...	...	...
San Francisco	150	100	100	100	450	65	17	25	120	75	180	482	932
Savannah	...	...	...	...	...	...	...	...	...	...	...	...	...
Schenectady	150	100	90	35	375	50	30	25	120	75	135	435	810
Susquehanna	...	...	...	...	...	...	...	...	...	...	...	...	...
Syracuse	150	100	0	0	250	25	18	20	120	20	65	268	518
Toledo	...	...	...	...	...	...	...	...	...	...	...	...	...
Tri-Cities	150	100	50	75	375	50	20	25	120	75	175	465	840
Utah	...	...	...	...	...	...	...	...	...	...	...	...	...
Utica	...	...	...	...	...	...	...	...	...	...	...	...	...
Virginia	...	...	...	...	...	...	...	...	...	...	...	...	...
Washington, D. C.	...	...	...	...	...	...	...	...	...	...	...	...	...
Waterbury	...	...	...	...	...	...	...	...	...	...	...	...	...
Western Mass.	150	100	0	0	250	20	22	25	60	50	100	277	527
Western Wash.	...	...	...	...	...	...	...	...	...	...	...	...	...
West Virginia	...	...	...	...	...	...	...	...	...	...	...	...	...
Worcester	150	100	50	50	350	50	22	20	120	75	200	487	837
Youngstown	150	75	10	0	235	60	30	18	80	50	130	368	603

Regional Meetings, 1935–1936

The Committee on Local Sections has received an invitation from the Buffalo Section to hold a Regional Meeting at Niagara Falls, now scheduled tentatively for September, 1936; also an invitation from San Francisco to hold a Regional Meeting there in March or April, 1936. This will be accomplished by expanding plans for a western meeting of the Machine Shop Practice Division to include other Divisions.

Manual for the Operation of a Section

The "Manual for the Operation of a Local Section" has been revised and distributed to officers of Local Sections.

"Scale-of-Measurement" Revision

The method of rating and recording the activities of a Local Section as regards its service to the membership, and its service to the Society, which has been in use by the Committee since 1933, has proved extremely useful both to the Sections themselves in evaluating activities and personnel, and to the Committee in guiding and planning the work for any particular Local Section.

The "Scale-of-Measurement" has been revised and simplified. It is believed the change has improved its utility, and that it will be of more positive value.

Junior Program

The Committee has been vitally interested in the useful participation of the Junior Member in the activities of the Local Sections, and concerned for his reception by the members of the profession and the members of the Society. An intelligent solution for the adequate reception and participation of the Junior must be found, and the Committee on Local Sections is interested in planning intelligently toward this end.

President D. B. Prentice of Rose Polytechnic Institute, a member of the Committee on Local Sections, and Chairman of the Committee on Junior Members which cooperates in this work, presented from this committee eight recommendations for stimulating and maintaining the interest and participation of the Juniors in the Society.

TABLE 2 LOCAL SECTIONS VISITED BY MEMBERS OF THE COMMITTEE ON LOCAL SECTIONS, 1934–1935

Section	W. L. Dudley *Chairman*	R. E. W. Harrison	W. R. Woolrich	D. B. Prentice	E. Hartford *Secretary*
Atlanta	.....	.....	.....	.....	Apr. 1935
Baltimore	.....	.....	.....	.....	Mar. 1935
Birmingham	.....	.....	.....	.....	Apr. 1935
Charlotte	.....	.....	.....	.....	Apr. 1935
Chattanooga	.....	.....	.....	.....	Mar. 1935
Chicago	Mar. 1935	Mar. 1935	.....	.....	Sept. 1934
Cincinnati	June 1935	.....	June 1935	June 1935	Mar. & June 1935
Colorado	Apr. 1935	.....	.....	.....	Oct. 1934
Florida	.....	.....	.....	.....	Apr. 1935
Greenville	.....	.....	.....	.....	Apr. 1935
Houston	.....	.....	.....	.....	Oct. 1934
Indianapolis	.....	.....	.....	(Resident)	.....
Inland Empire	.....	.....	.....	.....	Oct. 1934
Kansas City	.....	.....	.....	.....	Oct. 1934
Knoxville	Mar. 1935	Mar. 1935	(Resident)	Mar. 1935	Mar. 1935
Los Angeles	Apr. 1935	.....	.....	.....	Oct. 1934
Metropolitan	Dec. 1934	Dec. 1934	Dec. 1934	Dec. 1934	(Resident)
Mid-Continent	.....	.....	.....	.....	Oct. 1934
Milwaukee	.....	.....	.....	.....	Sept. 1934
Minnesota	.....	.....	.....	.....	Sept. 1934
Nebraska	Mar. 1935	.....	.....	.....	.....
North Texas	.....	.....	.....	.....	Oct. 1934
Oregon	.....	.....	.....	.....	Oct. 1934
Raleigh	.....	.....	.....	.....	Apr. 1935
St. Louis	.....	.....	.....	.....	Oct. 1934
San Francisco	Apr. 1935	.....	.....	.....	Oct. 1934
Savannah	.....	.....	.....	.....	Apr. 1935
Utah	.....	.....	.....	.....	Oct. 1934
Virginia	.....	.....	.....	.....	Apr. 1935
Washington, D. C.	Apr. 1935	(Resident)	.....	.....	Jan. 1935
Western Wash.	(Resident) Apr. 1935	.....	.....	.....	Oct. 1934

Note: Mr. W. W. Macon attended the Annual Meeting in December, 1934, in New York City. He died on January 1, 1935.

Their accomplishment is a planning responsibility of which the Committee is keenly aware. It appreciates both the work and suggestions of the Junior Committee and promises its utmost in cooperation.

Cooperation With Student Branches

The Committee recognizes its obligation to Student Members and Branches through staff contacts such as were made by its secretary, Ernest Hartford in his recent trip. The fine cooperation of the Local Sections whose members have arranged for attendance at Student Branch Meetings, the attendance of students at Local Sections Meetings, and the participation of students in inspection trips arranged by Sections all show the spirit of cooperation. Many Local Sections offer prizes to win the active interest of undergraduates.

Joint Meetings of Local Sections

Sections are encouraged to hold joint meetings with the local engineering organizations and clubs, or sections of other national engineering societies. Through these joint meetings a community of interest is fostered and developed. Much of the investigation and development of the problem of engineering registration has resulted from them.

Employment Service

The Society undertakes to assist members to secure positions, maintaining offices in New York, Chicago, and San Francisco, with the cooperation and help of other National Societies. The Local Sections have cooperated to the fullest in extending the service to engineers and employers.

Cooperation With Other A.S.M.E. Activities

During the last few years the Standing and other Committees of the Society have felt keenly the need and perceived the necessity for cooperation to correlate their efforts and activities, that the greatest good might be obtained from their respective efforts. An effort is being made to realize this aim.

In most cases the Local Sections are the outlet for putting into effect many of the activities of the various Committees in their planned programs and activities.

The work of the Professional Divisions of the Society is invaluable to the Local Sections, as they are equipped to supply planned technical programs to be presented before Local Sections, or before meetings sponsored by Professional Divisions and held with the approval and cooperation of the Local Sections. Such meetings have been successful, but there still remains a large opportunity for the Professional Divisions to submit developed programs of a strictly technical nature which can be utilized in planning for comprehensive Sections programs.

This procedure was suggested to the Professional Divisions by the Advisory Board on Technology on January 12, 1935. Such programs when submitted will be distributed to those Sections within whose sphere of interest the subjects lie.

Inventory and Planning

Local Sections are encouraged to devote some of their meetings to the branches of industry dominant in their locality, and to matters of public interest where the mechanical-engineering profession can make a contribution.

A most effective way in which the members of the Society can attack the problems confronting them is by adopting a program for Section activities in which those well-informed persons available to a Section may analyze the problem, thus providing suitable papers for discussion.

The Committee offers some thirty topics for which qualified speakers are available. These topics are intended to supplement programs of the Divisions and include such general topics as "American Manufacturing Technique in Competition With the World," "Engineering Functions in a War-Time Procurement Program," "The Effect of the Business Cycle on Engineering Activity," "Railroads From the Lay Engineers' Viewpoint," "Competition Between Industries," and many others of a similar trend.

The Committee has, after considerable study, outlined and put into action a planned program for the Local Sections. This planning is an effort to chart the course of the activities related to Local Sections and includes the following items:

Junior program and Section program; Section administration; Group conferences; membership development; Section policies; employment; joint relationships; field offices; cooperation with engineering faculties; representation on A.S.M.E. Council by geographical groups of Sections; services and development program for personnel; work of Engineers Council for Professional Development; cooperation with American Engineering Council; Student Branches; Meetings and Program; Professional Divisions; local engineering societies; registration of engineers; and legislation affecting engineers activities.

This program is to be fully developed in the Group, Annual, and Semi-Annual Meetings by the delegates, and put into effect by them.

The chairmen and executive committees of all Local Sections have had submitted to them a preamble of objectives for Local Section development to be supported by (*a*) The work of inventory; (*b*) Evolving a planning procedure; (*c*) The execution or realization of this plan into a working program.

The planning procedure and the hoped-for execution has been outlined above. The work of inventory is a careful summation of the possibilities inherent in the local Section that can be intelligently utilized, each possibility to be developed by a committee of the Section. This has a multiple purpose. It is to acquaint the members by Committee service of the functions of their Section, and their application to Society and public service. This in itself provides an activity of extreme importance to the members, the Section, and the Society and gives a knowledge of the work upon which the Society is engaged.

Local Sections Papers

The Advisory Board on Technology states that worth-while papers presented at meetings of Local Sections will continue to receive

TABLE 3 MEMBERSHIP LOCAL SECTIONS, APRIL 1, 1935

Section	Total membership Apr. 1, '35	Oct. 1, '34	Members in good standing	Meetings 1934–35
Akron-Canton	119	108	80	3
Anthracite Leh. Valley	201	205	133	3
Atlanta	78	70	47	2
Baltimore	164	158	118	7
Birmingham	55	51	45	3
Boston	551	501	394	5
Bridgeport	105	107	86	2
Buffalo	166	162	121	3
Central Pa.	61	51	47	1
Charlotte	33	23	18	1
Chattanooga	18	14	13	1
Chicago	711	639	509	3
Cincinnati	193	199	151	5
Cleveland	255	238	178	14
Colorado	94	84	71	7
Columbus	75	79	60	5
Dayton	63	73	45	3
Detroit	388	356	282	4
Erie	48	51	35	1
Florida	76	55	54	3
Green Mountain	37	37	28	1
Greenville	44	26	28	1
Hartford	111	101	85	4
Houston	113	112	74	2
Indianapolis	105	115	73	1
Inland Empire	16	21	10	8
Kansas City	111	89	75	7
Knoxville	49	39	34	1
Los Angeles	297	318	248	10
Louisville	35	34	24	4
Memphis	24	28	15	0
Meriden	25	25	14	1
Metropolitan	3,707	3,459	2,518	26*
Mid-Continent	138	148	91	3
Milwaukee	173	196	151	5
Minnesota	96	92	63	3
Nebraska	54	32	44	4
New Britain	47	35	24	7
New Haven	86	88	54	3
New Orleans	93	83	75	6
North Texas	54	50	38	2
Norwich	38	30	26	3
Ontario	82	86	58	3
Oregon	53	52	27	1
Peninsula	42	50	30	1
Philadelphia	884	771	636	7
Pittsburgh	395	365	294	4
Plainfield	211	217	148	4
Providence	159	134	124	7
Raleigh	20	31	17	3
Rock River Valley	49	43	32	9
Rochester	98	99	69	5
St. Joseph Valley	32	35	21	1
St. Louis	194	213	127	2
San Francisco	337	337	244	6
Savannah	16	15	13	1
Schenectady	167	173	121	19
Susquehanna	66	67	50	1
Syracuse	99	115	62	4
Toledo	57	58	34	7
Tri-Cities	65	62	48	4
Utah	27	26	21	4
Utica	14	26	8	0
Virginia	124	126	96	0
Washington, D. C.	194	162	149	6
Waterbury	81	72	63	3
Western Mass.	98	95	60	4
Western Wash.	87	77	54	6
West Virginia	64	61	47	0
Worcester	148	138	111	8
Youngstown	49	67	42	4
	12,819	12,155	9,085	301

Membership in Society, April 1, 1935, 14,650
Membership in Sections, April 1, 1935, 12,819
Membership in Society, October 1, 1934, 14,227
Membership in Sections, October 1, 1934, 12,155

* In addition to 64 separate Junior activities.

special consideration by the Committee on Publications, and the Editor of "*Mechanical Engineering.*"

Midwest Office

The Committee on Local Sections is glad to express the appreciation of the Sections and Student Branches in the Midwest of the fact that Council has continued the Midwest Office. Many activities and members have found the services of that office of value.

Death of W. W. Macon

Last January Mr. Macon, whose years of effort in behalf of the Society had culminated in his service on the Committee on Local Sections, passed away suddenly and unexpectedly. Your Committee expresses its deepest regret at his loss which it and the Society feel very keenly. All will miss his analytical ability and kindly humor, both given so freely in the service of the Society.

In Conclusion

Your Committee expresses its gratitude to the many sincere and conscientious officers and executive committees of the Local Sections in the many centers of the country for their generous cooperation, and also to the officers, staff, and Council of the Society, together with the Standing Committees for their help in, and their sympathetic understanding of, the many problems with which the membership is confronted.

Respectfully submitted,

William Lyle Dudley, *Chairman* — W. R. Woolrich
R. E. W. Harrison — D. B. Prentice

PROFESSIONAL DIVISIONS

The Professional Divisions have had a successful year during 1934–1935. The outstanding accomplishment was the publication quarterly of the *Journal of Applied Mechanics* and the financial contributions secured by that Division from outside sources to increase the number of technical papers to be published.

During the year five national division meetings were successfully held by the following Divisions: Oil and Gas Power, May, Tulsa; Applied Mechanics, June, Ann Arbor; Graphic Arts, June, Cincinnati; Machine Shop Practice, September, Cleveland; Aeronautic, October, St. Louis. In addition the Textile Division assisted the Greenville, S. C., Section in holding a Southern textile meeting. The programs of these meetings were effectively organized with the close cooperation of the Sections and the Divisions. For the Aeronautic meeting special credit is due to the St. Louis Section for arranging preprints of the meeting papers in pamphlet form.

The Professional Divisions were particularly active during the year in organizing large technical programs for the Semi-Annual and 1935 Annual Meetings. There was a noticeable improvement in cooperative activities between the Divisions, more sessions having been jointly organized. At the 1934 Annual Meeting the Standing Committee successfully arranged the presentation of a joint progress report of the power divisions. This was presented at the close of one of the power sessions by C. F. Hirshfeld, who was assisted in the preparation of his report by the following Divisions: Fuels, Hydraulic, Oil and Gas Power, and Steam Power. It is planned to increase this type of cooperation.

An effort was made by the Divisions during the year to assist the Local Sections in securing speakers for local meetings. The Railroad Division took the lead in this line of endeavor by organizing a special committee whose members were well distributed over the country and by notifying the Sections that these members were available as speakers for local railroad meetings. The Management Division was also active in assisting in coordinating local management meetings, particularly in New York and Boston. The Machine Shop Practice Division encouraged the Sections to hold meetings on the subjects of welding and casting.

Division Activities

Aeronautic Division. The Aeronautic Division held its National Meeting in October at St. Louis, Mo., under the auspices of the St. Louis Section. Over thirty-five papers were presented at the meeting and they were published by the Section in a photo-offset pamphlet, which was sold for $1.50. The Spirit of St. Louis Medal was awarded posthumously at the meeting to Will Rogers for his splendid work in advancing aeronautics.

Other activities consisted of holding a session at both the Semi-Annual and the 1935 Annual Meetings and in assisting several Local Sections in holding local aeronautic meetings. The Division has a special Subcommittee on Industrial Aerodynamics and is actively assisting both the A.S.M.E. Committee on Aircraft Safety and Inspection and the Aviation Committee of the American Engineering Council.

Applied Mechanics Division. The Applied Mechanics Division held its National Meeting on June 18 and 19 at the University of Michigan, Ann Arbor, Mich., under the auspices of the Detroit Section. Thirteen papers were presented at the meeting. The Division is sponsoring or participating in sessions at the 1935 Annual Meeting. It has six special subcommittees: elasticity, strength of materials, plasticity, dynamics, mechanics of liquids and gases, and thermodynamics.

This year saw the publication of the *Journal of Applied Mechanics* as a separate quarterly under the auspices of the Society. The Division secured nearly $2000 in special donations to assist in publishing more technical papers. This, however, does not as yet take care of the quantity of valuable papers that are presented, and the Division

has recommended that the *Journal* be sent only to those members requesting it so as to conserve funds for publication of additional material.

The Division is also undertaking a study of symbols and abbreviations used in applied mechanics, in cooperation with other interested groups, as a preliminary step in the preparation of a list which will be acceptable to a majority and form the basis of a recommended standard.

Fuels Division. The Fuels Division arranged two sessions at the Semi-Annual Meeting and two sessions at the 1935 Annual Meeting. During the year the Special Subcommittee on Fly-Ash and Cinders continued its survey to secure data that industry had accumulated. The Pure Air Committee has continued its activities. The Fuels Division is also cooperating with the A.I.M.E. in the work of the Committee on Fuel Values. This Committee has three special subcommittees on effect of ash on Btu, effect of fusion temperature of ash on fuel value, effect of sulphur with special reference to corrosive effect.

Graphic Arts Division. The Graphic Arts Division is the new name adopted this year for the former Printing Industries Division. The Division held its National Meeting as part of the Semi-Annual Meeting. Ten papers and lectures were presented at the National Meeting, which was held jointly with the Graphic Arts Research Bureau. The Bureau was organized at the Division's National Meeting in Philadelphia in October, 1934. The object of the Bureau is to coordinate, stimulate, and sponsor research in the Graphic Arts field. The Bureau at its Cincinnati meeting voted to request affiliation with the Society. Membership in the Bureau consists of about 400 individual members and three affiliated organizations. The Bureau and the Division are planning a National Meeting to be held in Washington, D. C., in March or April, 1936.

Hydraulic Division. The Hydraulic Division held a session on Hydraulic Turbine developments at the Semi-Annual Meeting in Cincinnati. At the 1935 annual meeting, two sessions will be devoted to the study of Cavitation and have been arranged in cooperation with the Power Division of the A.S.C.E. and the Applied Mechanics Division of the A.S.M.E.

The Division is continuing the activities of the Water Hammer Committee and has secured the cooperation of many prominent engineers in other societies and in various foreign countries. Plans are being completed for a second symposium on Water Hammer in December, 1936.

Iron and Steel Division. The Iron and Steel Division held one session at the Semi-Annual Meeting and two sessions at the 1935 Annual Meeting, one of which was held jointly with the Applied Mechanics Division.

Machine Shop Practice Division. The Machine Shop Practice Division held four sessions and cooperated with the Railroad Division in a fifth at the Semi-Annual Meeting. For the 1935 Annual Meeting two sessions are planned and a joint luncheon with the Job Shop Committee of the Management Division. The Division also cooperated in the Machine-Tool Congress held in Cleveland, Ohio, and jointly with the Cleveland Section, held two evening meetings on September 10 and 11. At the beginning of the year the Lubrication Engineering Subcommittee of the Petroleum Division was transferred to the Machine Shop Practice Division and reorganized. Other subcommittees are: foundry, welding, and machine design. The Division also has liaison connection with the Research Committee on Cutting Metals, Cutting Oils, and Gears. The Foundry Committee together with the Birmingham Local Section and the American Foundrymen's Association held a two-day meeting on February 28, and March 1, 1935. Several Local Sections held meetings on the subjects of welding and casting as a result of the Division's suggestions and cooperation. These meetings were well attended and created considerable interest.

Management Division. The Management Division sponsored three sessions at the Semi-Annual Meeting and is sponsoring four sessions at the Annual Meeting as well as cooperating in a luncheon and one other session. The Division has started work on a Supplement to the Bibliography of Management Literature to cover literature published from January, 1931, to date.

The Management Division assisted the National Management Council in securing papers for the Sixth International Congress for Scientific Management held in London, England, during July, 1935, and on behalf of the National Management Council planned and organized a series of nine joint meetings to be held in the New York Metropolitan area during 1935–1936 in which local sections of ten societies and associations interested in managerial affairs will participate.

During the past year the Waste Elimination Committee has been active in sponsoring sessions at the Semi-Annual and Annual Meeting.

The Job Shop Management Committee is to hold a joint meeting with the Machine Shop Practice Division at the coming Annual Meeting and it has developed considerable interest in Job Shop Management problems by sending out several questionnaires during the year.

The Marketing Committee has undertaken a national survey to determine the present status of marketing research in industry and has also in the course of preparation a Bibliography of Literature on Marketing Research.

The Division has the following subcommittees: Economics, human relations, job shop management, management research, marketing, quality control, time and motion study, waste elimination, and statistics in industry. It has representation in the National Management Council.

Materials Handling Division. The Materials Handling Division was active this year for the first time in several years and sponsored three sessions at the Semi-Annual Meeting in Cincinnati.

Oil and Gas Power Division. The Oil and Gas Power Division held its Eighth National Meeting on May 8 to 11, 1935, in Tulsa, Okla., in cooperation with the Mid-Continent Section. There were presented nine technical papers which were preprinted in the *Oil and Gas Journal* and *Diesel Power*. The division will have a session at the 1935 Annual Meeting.

The special Subcommittee on Oil-Engine Cost will present its seventh yearly report at the 1935 Annual Meeting. This report for the year 1934 covers over one hundred and fifty plants. The report is available in pamphlet form and may be purchased from the Society.

Petroleum Division. The Petroleum Division is holding a session at the 1935 Annual Meeting. The subcommittees were active in the Mid-Continent area and a series of local meetings was held in Tulsa. The Subcommittee on Lubrication Engineering was transferred this year from the Petroleum Division to the Machine Shop Practice Division. The Division is planning a national meeting next year to be held at the time of the Petroleum Exposition in Tulsa, Okla., in May, 1936.

Power Division. The Power Division held two sessions at the Semi-Annual Meeting and has planned three sessions at the 1935 Annual Meeting besides a cooperative session with the Applied Mechanics Division. The Industrial power group has planned to hold, at the 1935 Annual Meeting, its usual Wednesday luncheon conference. Among the special research committees in the power field the Boiler Feedwater and Steam Research committees are planning to hold sessions at the 1935 Annual Meeting.

Process Division. The Process Industries Division continued its active role during its second year. At the Annual Meeting the Subcommittee on Sugar will hold a session, its Subcommittee on Heat Transfer will hold two sessions on radiant heat and cooperate in a third session. The Subcommittee on Drying presented at the Semi-Annual Meeting its recommended practice on Testing Drying Equipment and is sponsoring a session on Dust Separation at the 1935 Annual Meeting. The Air Conditioning Subcommittee held a Metropolitan Section meeting in October. The Heat Transfer Committee has issued in pamphlet form the previous papers presented at its sessions. The Division has developed relations with the American Institute of Chemical Engineers and the American Ceramic Society both of which have appointed liaison representatives.

Railroad Division. The Railroad Division held a joint session with the Machine Shop Practice Division at the Cincinnati Semi-Annual Meeting and has planned to hold at the 1935 Annual Meeting two sessions, including a symposium on Railroad Lubrication. Besides this the Division will hold a joint session with the Applied Mechanics Division. During the year the Division organized in order to assist certain Local Sections that might appropriately hold local railroad meetings.

Textile Division. The Textile Division plans to hold its session at the 1935 Annual Meeting in cooperation with the Graphic Arts and the Management Divisions. There will also be the usual luncheon conference after the Annual Meeting session. The Division assisted the Greenville Section of the Society in holding a meeting there on April 10 during the Southern Textile Exposition. This meeting was to have been held in October, 1934, but was postponed until the spring of 1935.

Wood Industries Division. The Wood Industries Division held a two-session symposium on Wood Housing at the Cincinnati Semi-Annual Meeting. The Division is holding a session at the 1935 Annual Meeting on Modern Timber Construction.

Respectfully submitted,

W. A. Shoudy, *Chairman*
K. H. Condit — Crosby Field
G. B. Pegram — L. K. Sillcox

MEMBERSHIP

The Committee on Membership submits the following statistical report covering the recommendations of the Committee to Council during the fiscal year October 1, 1934, to October 1, 1935:

REPORT

The Committee on Membership held twelve regular monthly meetings during the fiscal year 1934–1935.

The following are the applications considered in the transaction of its work and a summary showing the action taken:

Applications pending October 1, 1934	137
Special cases before Council	57
Applications received during fiscal year 1934–1935	2206
Total applications handled during the year 1934–1935	2400
Recommended for membership	2222
Transfers denied	3
Deferred	4
Withdrawn, incomplete, and canceled	40
Applications pending October 1, 1935	131
Total applications handled during year 1934–1935	2400

The 2222 recommended for membership were divided into the following grades:

Members	104
Transfers to Member	89
Transfers from Student Member to Member	1
Associates	3
Associate-Members	79
Transfers to Associate-Member	107
Transfers from Student Member to Associate-Member	2
Juniors	165
Transfers from Student Associate to Junior	76
Transfers from Student Member to Junior	1596
Total recommended	2222
Transfers	196
Total new members recommended	2026

During the fiscal year 1934–1935 the Committee on Membership made the following recommendations:

Transfers from Student Member to Junior declared void	735
Elections to other grades declared void	28
Total elections declared void	763

During the year closing, the Committee has devoted a portion of several meetings in addition to special meetings to joint discussion with the Secretary and also representatives of other Committees interested in effecting the proposed changes in the Constitution and By-Laws[1] pertaining to Membership, as the Amendments now appear on page 532 of the August, 1935, issue of *Mechanical Engineering*.

Certain changes in the By-Laws have been suggested by the Membership Committee, including the recommendation that the title of the Standing Committee on Membership be changed to the Standing Committee on Admissions.

A canvass of all Local Sections was initiated through a letter of April 5, 1935, from the chairman of the Committee on Membership to the chairmen of the Local Sections, requesting a report from the Executive Committees stating whether or not they wish, in the future, to receive a copy of the professional record of every applicant for membership within their respective territories.

The following response resulted: Thirteen replies were received in which four wish full copy of record, and nine wish only special cases referred to them.

A new reference form for new members and transfers has been adopted.

The committee now has under consideration the policies which will govern the effective administration of its work of passing on applications not only for the proposed grade of Fellow but for all other grades in order to maintain the highest standard obtainable.

Respectfully submitted,

H. A. LARDNER, *Chairman*
R. H. MCLAIN
C. L. DAVIDSON
L. R. FORD
F. C. SPENCER
O. E. GOLDSCHMIDT, *Advisory Member*

CONSTITUTION AND BY-LAWS

For some little time it has been appreciated that there should be somewhat more uniformity in the admission requirements for the several grades in the national engineering societies and recommendations have been made from time to time that our Society and others should add the grade of Fellow to give recognition to the senior engineers who have achieved distinction in the engineering field.

To meet these needs the Committees on Policies and Budget, and Membership, at the request of Council, submitted to the Committee on Constitution and By-Laws a draft of suggested amendments along these lines, which amendments as revised were submitted to the membership in the usual printed ballot form in August, 1935, for vote to be announced at the Annual Meeting in December.

The new articles of the Constitution divide the membership into two groups, the first, including all of the voting members, is to be entitled the "Corporate Membership" which consists of the new grade of Fellows, the Members, and the Junior Members. The second group will also include three grades, namely, Honorary Members, Associates, and Student Members.

The requirements for Fellow are such as to insure that the grade shall have the proper distinction. It is not necessary that one should be able to meet the full requirements if exceptionally qualified because provision has been added that the Council may, by unanimous vote, elect a Fellow who has distinguished scientific attainments. In order to form the nucleus of Fellow a temporary provision permits any past or present member of Council to apply for the grade of Fellow and if properly qualified to become a Fellow automatically. This section was a controversial one but in the form submitted to the membership is thought to be in accord with the wishes of the membership at large. The Associate Members have been advanced to the grade of Member if over thirty years of age and will be automatically advanced upon reaching that age so that in a few years the present grade of Associate Member will be abolished.

The new amendments to the By-Laws which have been reviewed and proposed for approval by the Council include a change in the title and duties of the Standing Committee on Membership and in a rewording of Paragraph 3 of Article B14 providing that the income from initiation fees shall not be used for current expenses but shall be added to the surplus at the close of each fiscal year. This change makes no real alteration in the method of handling funds but does simplify the bookkeeping and will aid the auditors in their annual report.

Respectfully submitted,

H. H. SNELLING, *Chairman*
RICHARD KUTZLEB, JR.
HERBERT B. LEWIS
WM. H. KAVANAUGH
R. D. BRIZZOLARA
G. N. COLE, *Junior Adviser*

RELATIONS WITH COLLEGES

The past year was notable in the history of the Committee because of the fact that it saw the completion of the new plan and of changing from the old system of operating Student Branches to the new. This was accomplished through the introduction of the plan in the early part of the college year when a visit was made by a representative of the Committee to each of the colleges which comprise Groups VI and VII.

Naturally the number of members in Student Branches enrolled under the new plan increased considerably and reached a total membership of 3250. This further development of the plan was sufficient to encourage the presentation at the Semi-Annual Meeting, Cincinnati, of an amendment to the Constitution of the Society providing for the grade of Student Member, and also to suspend the initiation fee of these Student Members who transfer to the grade of Junior within a limited time after graduation. In the 112 colleges a total of 336 meetings were held.

The economic conditions of the country, while improved, have not created a sufficient demand for recent graduate engineers to show any marked increase in the number of Student Members transferred to the grade of Junior upon graduation. Of 1318 Student Members graduated with the class of 1934, 530 completed their transfer to Junior membership by the payment of the initiation fee. The loss of the remaining 800 can be accounted for largely because of the eco-

TABLE 1 COMPARISON OF BRANCH ACTIVITIES BY GROUPS

Group	No. of branches	Avg no. meetings per branch	Avg attendance per meeting
I	15	7	42
II	18	4	41
III	51	8	37
IV	16	5	39
V	17	6	54
VI	15	6	36
VII	15	5	24
Puerto Rico	1	–	—

[1] See Report of the Committee on Constitution and By-Laws, page 8.

TABLE 2 GROUP MEETINGS, 1935

GROUP I, NEW ENGLAND STUDENT MEETING, CAMBRIDGE, MASS.

Attendance: 175 Papers presented: 11

Prize	Recipient	Title of Paper	College
First	H. T. Sawyer	Development and Investigation of an Indicator for High-Speed Engines	Rensselaer Poly. Inst.
Second	R. F. Hopkins	Effect of Transverse Slots and Holes on the Torsional Properties of Shafting	Brown University
Third	H. B. Kimball	Problems in Miniature Design	Mass. Inst. of Tech.
Fourth	W. R. Steur	Water Channel Method of Flow Visualization	Worcester Poly. Inst.

GROUP II, EASTERN STUDENT MEETING, NEW YORK, N. Y.

Attendance: 225 Papers presented: 10

First	Benjamin O. Delaney	Engine Indicators	Princeton Univ.
Second	John A. Sauer	A New Apparatus for Testing Materials in Combined Bending and Torsion	Rutgers Univ.
Third	Alan S. Compton	Maintenance Welding	Drexel Inst.
Fourth	John R. Blizard	Air Resistance of Tube Bundles	Poly. Inst. of Brooklyn

GROUP III, ALLEGHENIES STUDENT MEETING, STATE COLLEGE, PA.

Attendance: 175 Papers presented: 10

First	Philip S. Criblet	The Precipitation of Dust From Flue Gases	Johns Hopkins Univ.
Second	R. L. Homsher	Governing Hydraulic Turbines for Best Efficiency	Penn State College
Third	Wm. Leavenworth	The What and Why of a Whirling Arm	Univ. of Akron
Fourth	J. J. Prendergast	Tests on a Streamlined Power Craft	Case School of Applied Science

GROUP IV, SOUTHERN STUDENT MEETING, KNOXVILLE, TENN.

Attendance: 180 Papers presented: 13

First	A. C. Todd	Economic Aspects of Stream Lining for Automobiles	Georgia School of Tech.
Second	H. B. Edwards	Uses of Glass in Mechanical Engineering	Univ. of Virginia
Third	Luke R. Hadnot	A Dryer for the Artificial Dehydration of Forage Crops	Louisiana State University
Fourth	John R. Kilgore	A New Kind of Mirror	Vanderbilt Univ.

GROUP V, MIDWEST STUDENT MEETING, CHICAGO, ILL.

Attendance: 210 Papers presented: 15

First	Edw. J. Wellauer	A Mathematical Determination of the Contact Length of Helical and Herringbone Gearing	Marquette University
Second	Einar W. Jensen	Stresses in an Automobile Engine Crankshaft	Univ. of Iowa
Third	Janus DeHamer	The Development of a Test for Set in Split-Bamboo Fishing Rods	Mich. College of Mining & Tech.
Fourth	Walter Schlagel	Employment Conditions Among Recent Engineering Graduates	Iowa State College

GROUP VI, NORTHERN UNIT STUDENT MEETING, LINCOLN, NEB.

Attendance: 75 Papers presented: 9

First	E. D. Beachler	Diesel Boilers	University of Nebraska
Second	Harold Grasse	Artistic Industrial Design	Univ. of Kansas
Third	H. C. Bates	A Study of Causes of Failures of Tubes in Refinery Cracking Stills	Kansas State Agri. College
Fourth	H. E. Simonson	The Regenerative Vapor Cycle	Univ. of Nebraska

GROUP VI, SOUTHERN UNIT STUDENT MEETING, DALLAS, TEXAS

Attendance: 116 Papers presented: 13

First	Glenn W. King	Combining Motion Study and Machine Design for Efficient Production	Rice Institute
Second	Saviour Perrone	Steam Jet Vacuum Refrigeration	Univ. of Texas
Third	Alex W. Francis	Some Aspects of Increasing Ultimate Recovery of Crude Oil From Natural Reservoirs	Univ. of Oklahoma
Fourth	W. C. Rodgers	Application of Refrigerating Cycle to the Heating and Cooling of Buildings	Texas A. & M. College

GROUP VII, NORTHWEST STUDENT MEETING, SEATTLE, WASH.

Attendance: 110 Papers presented: 10

First	Robt. E. Laughlin	The Diesel Field	Univ. of Washington
Second	James Kratzer	The Photoelectric Cell	Wash. State College
Third	Robert D. Watt	Design of a Universal Air Conditioner	Univ. of Washington
Fourth	Edward Wegman	Air Conditioning at High Altitudes	Montana State College

GROUP VII, CENTRAL UNIT STUDENT MEETING, BOULDER, COLORADO

Attendance: 95 Papers presented: 7

First	Carl L. Ritter	Cooling of Boulder Dam	Colorado State College
Second	Thomas R. Mackay	San Gabriel River Flood Control	University of Utah
Third	Alvin Welton	Brake Equipment on Modern High-Speed Trains	Colorado State College
Fourth	Harry L. Hoffman	Discussion of Flotation	Univ. of Colorado

GROUP VII, SOUTHWESTERN STUDENT MEETING, BERKELEY, CALIF.

Attendance: 106 Papers presented: 17

First	Bradley H. Young	Design of Oleo-Pneumatic Shock Absorbers for Aircraft	California Inst. of Technology
Second	Byron Masterson	Spot Welding in Thin Sheet Structures	University of California
Third	A. L. Buonaccorsi	Seismogram Wave Analysis	Santa Clara Univ.
Fourth	Clyde C. Chivens	The Pump-Testing Laboratory of the Metropolitan Water District of Southern California	California Inst. of Tech.

nomic conditions which made it impossible for them to obtain engineering employment or perhaps any employment.

Another important development during the past year was the extension of Student Meetings from five in number to ten, made necessary by the addition of Groups VI and VII. The geographical area covered by these Groups is so enormous that it is impractical to schedule a single meeting for each Group with the expectation of any reasonable attendance from all of the colleges within the Group area. Therefore, the Committee deemed it wise to arrange two meetings for Group VI and three meetings for Group VII. The other Groups comprise practically all colleges east of the Mississippi and have but one meeting for all colleges in each Group.

Only twice in the history of the Student Branches has any Branch been discontinued. Last year the University of South Dakota discontinued its course in mechanical engineering, and this year upon request of the Student Branch at Harvard University it was discontinued. One new Student Branch was authorized at the Northwestern University, Evanston, Illinois.

The Committee is pleased to report that it was able to authorize five loans to Student Members from the Max Toltz Fund amounting to $850, thus enabling these students to continue their engineering studies.

Eugene W. O'Brien resigned from the Committee but was persuaded to serve as advisory member. Edward W. Burbank was appointed to fill his unexpired term.

Respectfully submitted,

WILLIAM L. ABBOTT, *Chairman* WILLIAM A. HANLEY
EDWARD W. BURBANK F. V. LARKIN
ROY V. WRIGHT EUGENE W. O'BRIEN, *Advisory Member*

AWARDS

The Committee on Awards made the following recommendations to the Council for awards which were to be presented during the fiscal year 1934–1935:

A.S.M.E. Medal to Willis H. Carrier, for his pioneer work in air conditioning.

Holley Medal to Irving Langmuir, for his contributions to science and engineering, including the development of gas-filled incandescent lamps, thoriated filament for thermionic emission, atomic hydrogen welding, phase-control operation of the thyratron tube, and fundamental research in oil films.

Worcester Reed Warner Medal to Ralph E. Flanders, for his contributions to a better understanding of the relationship of the engineer to

economic problems and social trends as exemplified by the many papers which he has presented.

Junior Award to John I. Yellott, Jr., for his paper, "Supersaturated Steam."

Charles T. Main Award to Philip P. Self, for his paper on "Air Conditioning—Its Practicability and Relation to Public Welfare."

Student Award, Undergraduate, to H. Reynolds Hudson, for his paper on "Dynamic Balance and Functional Utility Applied to Automatic Design."

No recommendations were made for the Melville Medal and Postgraduate Student Award.

The topic for the 1936 Charles T. Main Award is "Development in the Generation and Distribution of Power and Their Effect Upon the Consumer."

Upon recommendation of the Committee on Policies and Budget, the Council approved the extension of the work as heretofore carried on whereby a Board of Honors would be established. Council named the same personnel on the Board of Honors as comprises the Committee on Awards. It was understood that the Board of Honors would operate in an advisory or trial capacity for a period of two years. Acting in this capacity they proceeded to study and make recommendations regarding changes in the By-Laws and Rules of the Society and methods of procedure.

The first recommendations made to Council were as follows:

Resolved that the By-Laws be duly amended to change Par. 23 of Article B-8 of the By-Laws which now reads: "The Standing Committee on Awards shall, under the direction of the Council, have supervision of the awards of the Society as detailed in the Rules or prescribed by Council. The Committee shall consist of five (5) members and the term of one member shall expire at the close of each Annual Meeting," to the following:

"The Board of Honors and Awards shall, under the direction of the Council, have supervision of the awards of the Society as detailed in the Rules or prescribed by Council. Recommendations for representatives on joint bodies of award shall be made to Council by this Board. The Board shall consist of five (5) members, and the term of one member shall expire at the close of each Annual Meeting". . . .

Further resolved that the Board of Honors and Awards shall be authorized to constitute a Medal Committee to be comprised of twenty (20) persons, five of whom shall be the members of the Board of Honors and Awards, and in addition fifteen persons, three appointed each year for a period of five years.

It is anticipated that this Medal Committee will suitably organize itself into a functioning body which shall be charged with the duty of recommending to the Council nominations for the A.S.M.E. Medal, the Holley Medal, the Warner Medal, and in addition thereto nominations for representatives of the Society on joint Boards of Award.

The Board desires to provide better methods for the review of papers submitted for awards and intends to experiment with various means to this end. For this year the Publications Committee has been asked to submit its recommendations regarding papers that should be considered for the Melville Medal and the Junior Award. These papers will be reviewed by qualified persons or agencies and through them a recommendation made to the Board. It is expected that student papers may be reviewed by the faculties of the various universities under a plan to be definitely formulated and the assistance of the Committee on Relations With Colleges will be invited to this end.

The objective behind these recommendations is, first, that there should be a single responsibility for all awards in which the Society is interested; second, that there should be a body of highly representative members of the Society who may give to the consideration of awards, the attention which the subject deserves; and third, that the creation of the Medal Committee would afford a body which should be particularly charged with forward-looking plans and policies, not alone for those honors which are now available but for others which may later be available for various types of recognition.

It is the belief of the present Standing Committee on Awards that these suggested steps will contribute to the elevation of the whole awards situation and will enable them to be used in a larger manner to the good of the Society and the profession.

The nominations which the Awards Committee offered to the Council for the Medal Committee cover the fields of education, technical journalism, consultation, and industry, and represent a geographical cross section of the membership as well. The names are: E. O. Eastwood, A. M. Greene, Jr., D. C. Jackson, R. H. Fernald, J. H. Herron, F. M. Gunby, J. L. Harrington, H. C. Meyer, Jr., K. H. Condit, R. V. Wright, Robert Sibley, J. W. Parker, C. L. Bausch, E. R. Fish, and L. W. Wallace.

Respectfully submitted, W. L. Batt, *Chairman*

R. C. H. Heck — Herman Diederichs
Harte Cooke — L. P. Alford

EDUCATION AND TRAINING FOR THE INDUSTRIES

Due to several changes in the personnel the Committee on Education and Training for the Industries was reorganized during the year.

One of its important activities was the evolving of a statement of its objectives as it is felt that the opportunities for this Committee have broadened and become considerably more important because of the changed economic conditions during the past five years.

The Committee sponsored a session at the Annual Meeting of the Society and at its Semi-Annual Meeting. The session was well attended and caused much favorable comment.

Respectfully submitted,

George A. Seyler, *Chairman*
Clement J. Freund — John A. Randall
John Younger — Warner Seely

LIBRARY

The work of the Engineering Societies Library has been carried on steadily during the year with marked appreciation by its users. These numbered over forty thousand, of whom over ten thousand were members living at a distance who were assisted by correspondence.

Sixty-five hundred books were added in 1934 to the collection, which now contains about 147,000 volumes and maps.

The conditions portrayed so graphically in our report of last year still continue to hamper the growth of this useful activity.

The Library is called upon for a constantly increasing variety of services. More and more it is becoming a general information bureau for the profession, as well as its outstanding depot for the literature. An increasing demand for assistance constantly calls for more books, increased staff, and larger quarters. With a budget much lower than it was five years ago, it is impossible to provide these facilities. We trust that the time is near when more generous provision may be made for this important activity, which at some time or another is used by almost every member of the Society.

Respectfully submitted,

E. P. Worden, *Chairman*
G. F. Felker — J. S. Kerins
L. K. Sillcox — C. E. Davies, *Ex-Officio*

RESEARCH

A review of the annual reports of the Society's special research committees reveals an unexpected amount of activity and strength in organization. Only five of the twenty-four committees reported no activity during the year and a considerable number of them have functioned in a normal way.

As in previous years, a number of these committees have sponsored sessions at the general meetings of the Society or have provided the papers for the programs of other meetings set by the A.S.M.E. and similar organizations. For example, during the 1934 Annual Meeting the following five special research committees sponsored and conducted regular technical sessions; Fluid Meters, Thermal Properties of Steam, Cutting of Metals, Mechanical Springs, and Boiler-Feedwater Studies. In addition, three committees, Fluid Meters, Wire Rope, and Cottonseed Processing, cooperated with certain A.S.M.E. professional divisions in the preparation of the programs for three other sessions.

At the Society's Semi-Annual Meeting in Cincinnati in June and the Machine-Tool Congress in Cleveland in September the committees on the Cutting of Metals and Mechanical Springs supplied papers for certain of the technical sessions sponsored by the professional divisions.

The financial condition of the committees has remained much the same this year as it was last. Approximately $3000 was secured from industry. The Engineering Foundation has been both considerate and generous in its responses to the requests for assistance addressed to it by several of the committees. In all $7500 was given as grants and an added $1500 was pledged as a loan to the Subcommittee on Metal-Cutting Data. This latter sum is intended to assure prompt publication of the first of the series of handbooks to be issued by this committee.

Sometimes the members of administrative committees such as the Research Committee are tempted to be skeptical as to the value of the results accomplished by an activity for which they are responsible. At such times it is reassuring to find, in the reports of a special committee, statements such as the following which was taken from the report of the Committee on Lubrication:

"An interesting letter was received a few days ago from Mr. F. P. D. of the M. C. Company. After describing the work of his company in installing oil-film roll-neck bearings to withstand shock loads exceeding 1,500,000 pounds, he says: 'The operating ZN/P for these bearings is low. But, according to experiments made by the members of the lubrication committee, with this factor as a criterion, these bearings should and do perform satisfactorily. I know of no better way to pay tribute to the efforts of the committee than to outline this example of a very practical use of the reports of the Special Research Committee on Lubrication.' "

Brief reports of the special and joint research committees which were organized and are sponsored by the Society will be found in the following sections of this report.

Special and Joint Committee Reports

Lubrication, A. E. Flowers, Chairman. A successful meeting of the A.S.M.E. Special Research Committee on Lubrication was held in New York on December 5, 1934. Reports were received on the Committee's activities from the members present. The chairman made a statement concerning the committee's funds which covered an expenditure of approximately $500 and a balance on hand of $524. Mr. Hersey reported on the progress he had made in the preparation of abstracts of the 249 papers listed in the committee's selected bibliography. Approximately 100 have been completed. He presented specimens of these abstracts and after discussion it was decided to reduce further the number of titles in the selected bibliography to those papers of prime scientific importance and to include in each of the abstracts the usable data and the summary or conclusions given in the paper. A general discussion of the work of the committee followed during which the members reported a total of 12 papers prepared during the year or in course of preparation by members of the committee or their immediate associates.

A reorganization of this committee is under way in order that a broad program of work may be undertaken that will cover both the physics and chemistry of lubrication and practical service problems. To this end the Society's Research Committee at its meeting in December, 1934, appointed a special Survey Committee on Lubrication to investigate the possibilities and to formulate a program.

This committee under the chairmanship of N. E. Funk held two meetings in March and May, 1935, and subsequently presented its report to the Research Committee. It was unanimous in its recommendation that the A.S.M.E. should continue to sponsor a special Research Committee on Lubrication. A consideration of the scope of the project in the light of a review of the 19 years of activity of this special research committee led to the recommendation that the scope of the reorganized committee should be broad enough to give it freedom in the selection of problems but narrow enough to insure the interest of industry. The proposed scope is to include the following: (1) Investigations of the fundamental physical and mechanical problems of lubrication, (2) interpretation of the results of research in this field for practical application, and (3) coordination of data from contemporary research in lubrication.

Fluid Meters, R. J. S. Pigott, Chairman. The Special Research Committee on Fluid Meters has had as its principal work for the year the completion of the revision of Part 1, "Fluid Meters—Their Theory and Application." This revision is divided into three sections: Section *A* on the classification and nomenclature of fluid meters, Section *B* on the theory of fluid measurement, and Section *C* which is to contain figures and tables for use in solving practical fluid-measurement problems and illustrative examples. The Committee expects that the completed revision will be ready for publication early in 1936.

The joint research on orifice coefficients for the measurement of air and water was completed by the A.G.A.-A.S.M.E. Joint Committee on Orifice Coefficients in the fall of 1934 and the final report on these studies is now practically completed.

At present the special committee is perfecting its plans for research at the National Bureau of Standards on flow nozzles. This program is being developed in cooperation with the A.S.M.E. Power Test Codes Committee and a campaign to raise the necessary funds is under way. H. S. Bean will be in direct charge.

Though no reports have been received this year from Prof. W. H. Carson the committee understands that he is continuing his experimental work on meter coefficients for use in measuring viscous fluids at the University of Oklahoma.

Thermal Properties of Steam, W. L. Abbott, Vice-Chairman. Following the Third International Steam Tables Conference held in the United States in September, 1934, there has been a natural pause in the work assigned to the special research committee. Prof. J. H. Keenan undertook for the committee the preparation of the summary of the results of the conference. A revised skeleton table is its principal part. This report will be published in the November, 1935, issue of *Mechanical Engineering*. With the completion of the committee's present program Alex Dow asked to be relieved of his duties as chairman. His resignation was accepted with much regret but he was urged to continue to serve as a member of the committee.

Strength of Gear Teeth, R. E. Flanders, Chairman. The financial assistance given by the Engineering Foundation has made it possible for the Special Research Committee on the Strength of Gear Teeth to keep a research assistant engaged since the beginning of 1935 on tests of the surface fatigue of materials in rolling contact under the direction of Prof. Earle Buckingham. Tests have been made on rolls one inch wide with diameters up to four inches. In all of these tests, one roll was of hardened and ground steel. The mating rolls were of several varieties of cast iron, leaded brass, chilled phosphor bronze, and phenolic laminated material. A progress report on these tests is now in preparation.

With these materials, the tests indicate a tendency of an outer lamina of the material to travel by plastic flow in the direction of rolling, and also for this lamina to start to shear under the surface. On some of the cast-iron alloys, it is not yet certain whether the initial failure starts below the surface, or starts at the surface. It is hoped that investigations may be continued with other combinations of materials, and that it will be possible also to make still further tests with the same combinations of materials under combined rolling and sliding contact. Thus far, the tests have shown a very consistent relationship between load and number of repetitions of stress to cause surface failure, giving hopes that some definite relationships may eventually be established between surface fatigue and flexural or bending fatigue. Such tests take much time, but the information to be obtained from them will be invaluable to the designer in the selection of combinations of materials to avoid or minimize wear.

Cutting of Metals, Coleman Sellers, 3rd, Chairman. The work of this Committee during the past year consisted mainly of coordinating and encouraging the work of its four subcommittees. The personnel of the special committee has been enlarged and changed to include only those taking an active part in the work.

Metal Cutting Data. The Subcommittee on Metal Cutting Data formed to carry on the work so ably started by Frederick W. Taylor has been engaged for the past three years in examining all available experimental data on the cutting of metals, supplementing this with research of its own, and preparing the data thus obtained for publication in the form of handbooks for the use of industry. This should be a very valuable contribution to metal manufacturers as there is nothing of its kind in existence although the need for it has long been felt. The subcommittee has been generously supported by the Engineering Foundation. In June, 1935, the Foundation agreed in addition to underwrite the preparation of the first publication to the extent of $1500. During the coming year, the subcommittee expects to bring this first handbook on the Cutting of Metals with Single-Pointed Paring Tools to publication. It is also continuing to encourage experimental work already under way in various parts of the country in an effort to bring them to successful conclusions. L. P. Alford is chairman of this special research committee and R. C. Deale is its secretary.

Cutting Fluids. During the past year the Subcommittee on Cutting Fluids has been reorganized and enlarged under the chairmanship of Prof. O. W. Boston, of the University of Michigan. The sixth progress report of the committee was presented at the semi-annual meeting of the Society in Cincinnati in June in the form of a paper entitled "The Influence of Cutting Fluids on Tool Life in Turning Steel," by Messrs. Boston, Gilbert, and Kraus. The committee has prepared a research program of a comprehensive nature and is now attempting to raise funds from industry which, with the $1000 grant from the Engineering Foundation already appropriated, will be used in a research on cutting fluids with particular reference to the influence of cutting fluids on the tool life of single-point tools. Various physical and thermal properties of the oils also will be determined, and it is hoped that a correlation between all known properties may be obtained by which specifications may be set forth for use in selecting the proper cutting fluid to give tool life, low power, or good finish as desired on any particular material.

Bibliography. The Subcommittee on Bibliography on the Cutting of Metals issued its second report during the past year. This new portion of the bibliography is a continuation of Part I, published in 1930, and consists of 1257 new references with abstracts.

Metal Cutting Materials. The accomplishment of the Subcommittee on Metal Cutting Materials during the year will be its report on the "Questionnaire on the use of New Cutting Materials." This questionnaire was sent out to approximately 750 concerns in April, 1935. A detailed report on the data and information received was made at the Machine-Tool Congress held in Cleveland, Ohio, in September, 1935, during one of the sessions sponsored by the A.S.M.E. Machine Shop Practice Division. In addition this subcommittee has

discussed the important development of the diamond-impregnated wheels for the shaping of the cemented carbides, and expects to sponsor a paper regarding the same in the near future.

The special Research Committee on the Cutting of Metals has felt severely the lack of funds for publication by the Society. This has made it increasingly difficult to obtain research papers. Some means should be found to preserve for reference, papers of this type which, although not of wide and popular interest, are nevertheless valuable as contributions to the extremely large subject of the cutting of metals.

Mechanical Springs, J. R. Townsend, Chairman. The Special Research Committee on Mechanical Springs held one meeting during the past year in December, 1934. Two research programs have been practically completed during this period. The research at Wright Field directed by J. B. Johnson, chief of the Materials Branch of the Air Corps, has progressed to the point where the report is in preparation. This work covers fatigue tests on large-size helical springs. The second study by E. E. Weibel on the correlation of spring-wire torsion and fatigue data for small wire springs under the sponsorship of Prof. F. P. Zimmerli and his subcommittee at the University of Michigan has been completed and the report presented. This work was supported by funds subscribed by industry. Dr. D. J. McAdam's work on a monograph entitled "A Résumé of the Physical Properties of Spring Materials" has been carried forward and a portion of this report was presented at the Semi-Annual Meeting of the Society in June, 1935. At the present time the committee is reviewing several papers for presentation at the Annual Meeting of the Society in December, 1935. These papers are on the correlation of spring-wire bending, torsion-fatigue tests, and maximum shearing stress in an eccentrically loaded helical spring of circular wire. In February, 1935, Dr. W. G. Brombacher at the National Bureau of Standards completed and distributed the tenth supplement to the Committee's Bibliography on Mechanical Springs.

Effect of Temperature on Properties of Metals, H. J. French, Chairman (Joint Committee with the American Society for Testing Materials). Two meetings of the joint committee were held during the year in December and June and meetings of the subcommittees as required.

During the calendar year 1934, an additional contribution of $3000 to the Joint Committee's funds was received from Engineering Foundation, which made possible continuation of the sponsorship of researches at the Battelle Memorial Institute during the year 1935. Procurement of additiona' funds for the sponsored researches of the Joint Committee is in the hands of a newly organized Committee II on Finances, the chairmanship of which has been accepted by R. A. Bull.

Work has been continued on the study of embrittlement of austenitic nickel-chromium steels as a result of prolonged exposure to high temperatures and a summary has been prepared. The work in question relates to the progressive changes in toughness of an austenitic steel when exposed for a long period to high temperatures, both with and without stress. Work was initiated on a second project, relating to the interpretation of creep tests, the results of which are now broadly utilized by engineers in determining allowable stresses for high-temperature service.

So many logical questions are still unanswered respecting the behavior of steels under creep, especially in very long periods of time, that the Joint Committee decided to sponsor a program of research at Battelle Institute, in which steels of known creep characteristics (as determined by the customary test of 1000 hr or more) would be used and these steels further tested to durations of two or more years. For one of the materials, K-19 was selected to represent the austenitic type because creep values by the usual orthodox test have already been reported on this identical steel. A pure ferritic steel (K-20) was also selected. The latter is a 0.40 per cent carbon commercial steel of exceptional physical uniformity, furnished gratis by the Bethlehem Steel Company through the good offices of P. E. McKinney. The first progress report on this steel, dated June 2, 1935, was presented to the Joint Committee at the Detroit meeting by Messrs. A. C. Cross and F. B. Dahle. At that time it was voted to continue the 8000 lb per sq in. load at 850 F and also to maintain a sample at 7500 lb per sq in. also at 850 F. The K-19 steel is under test at the load and temperature decided upon as the creep limit under the former tests, i.e., 8345 lb per sq in. and 1200 F.

During the year a number of subcommittees have been at work on other parts of the program and in some cases they have enlisted the cooperation of company and university laboratories. Space available will permit only a list of the committees and the names of their chairmen. "High-Temperature Test Methods," N. L. Mochel, chairman; Subcommittee D, H. J. Kerr, chairman, checking Method E21–34; Subcommittee E, M. S. Northrup, chairman, checking Method E22–34T; Subcommittee F, H. W. Gillett, chairman, work on fatigue suspended for the present; Subcommittee N, J. W. Bolton, chairman, has prepared a report summarizing published data on methods for the determination of wear and seizure at high temperatures; Subcommittee O, Jerome Strauss, chairman, on corrosion of austenitic nickel-chromium steels at elevated temperatures, has prepared a report during the year; and Subcommittee P, H. W. Russell, chairman, organized in March, 1935, to consider the effects of low temperatures on metals.

Committee III on Projects, C. E. MacQuigg, chairman, made a comprehensive report at the June, 1935, meeting of the Joint Committee and pointed out the importance of raising $10,000 from industry in the next two years.

It has been impracticable during the past year to formulate a program acceptable to all interests represented on Committee V on Oil Refinery Problems, but such a program is considered to be necessary before funds can be secured for support of research in this field.

The Joint Committee has been in close touch with the interested committees of the A.S.T.M. and the A.S.M.E. on questions relating to the effect of temperature on the properties of metals and a considerable amount of time has been devoted to these semi-official cooperative projects during the past year.

Boiler-Feedwater Studies, C. H. Fellows, Chairman (Joint Committee with the American Boiler Manufacturers Association, American Railway Engineering Association, American Society for Testing Materials, American Water Works Association, and Edison Electric Institute). During the period covered by this 1934–1935 annual report the activities of the Joint Research Committee on Boiler Feedwater Studies have been limited to a continuation of the work in which its subcommittees were engaged during the preceding fiscal year.

An important change in the organization of the committee has occurred as the result of the resignation of S. T. Powell as chairman. Early in the past decade Mr. Powell perceived the importance of extending the knowledge of the industry in the reactions between water and steel under the conditions obtaining as the temperature and pressure of boiler operation increased and organized the joint research committee. With the aid of leaders in the industries vitally interested in these studies, with whom he associated himself, a comprehensive program of research was outlined. Not all phases of the general problem were studied at once; but emphasis was laid first on methods of water analysis and on a study to determine the cause of foaming and priming. The interest stimulated by Mr. Powell's leadership, however, has in the past ten years, advanced in a remarkable manner our knowledge of the reactions occurring within industrial water itself and between it and boiler metal under the conditions of association in modern steam power plants. It was, accordingly, with considerable regret that the members of the Executive Committee acceded to Mr. Powell's wishes and allowed him to relinquish the chairmanship.

The following officers were elected by the executive committee at its meeting in Cincinnati in May of this year to carry on the work of the joint research committee: C. H. Fellows of The Detroit Edison Company, chairman; R. C. Bardwell of The Chesapeake & Ohio Railway Company, vice-chairman; and J. B. Romer of The Babcock & Wilcox Company, secretary. These officers are planning a reorganization of the joint research committee in an effort to stimulate increased financial interest in its work and, at the same time, to further the objectives originally drawn up.

During the past year the work of the Subcommittee on Methods of Boiler Water Analysis was continued at the University of Michigan. A progress report on the incompleted research program concerning methods for determining dissolved oxygen was presented before a session on Boiler Water at the Annual Meeting of the American Water Works Association at Cincinnati in June, 1935, by C. H. Fellows, then chairman of the subcommittee. The research work on the determination of dissolved oxygen undertaken by this technical committee has served to stimulate a number of other investigators in this field. As a result M. C. Schwartz and W. B. Gurney, J. D. Yoder and his associates, D. O. Lima, and others have published results of investigations in this field. The future work of this committee will be under the direction of Prof. A. H. White of the University of Michigan. Professor White has accepted the invitation of the Executive Committee to undertake the chairmanship of this group, which was made vacant when its former chairman was elected to the chairmanship of the joint committee.

The work of the Subcommittee on Zeolite Softeners, Internal Treatment, Priming and Foaming, Electrolytic Scale Prevention under Prof. C. W. Foulk dealing primarily with the causes and prevention of foaming and priming, has been temporarily discontinued. It is planned, however, that this work will be resumed during the coming year.

The work of the Subcommittee on Alkalinity and Sulphate Relations in Boiler-Water Salines, under the direction of J. H. Walker as chairman, has progressed to an important extent during the past

year. W. C. Schroeder has been actively engaged on this project since its inception under the direction of E. P. Partridge at the New Brunswick Station of the U. S. Bureau of Mines. The original objectives have been reached and a report covering this research will be presented at the coming Annual Meeting of the A.S.M.E.

In addition to this completed study, Mr. Walker's committee has initiated a more basic research on the phenomenon of caustic embrittlement. Through the continued prosecution of these studies it is anticipated that many new and important data will be disclosed regarding this subject, permitting a clearer understanding of its development and prevention.

Condenser Tubes, A. E. White, Chairman. The results of the work of the Special Research Committee during the year 1934 were coordinated and presented at a meeting of the Committee held during the Annual Meeting of the Society in December, 1934. This report included the progress of short-time methods of corrosion testing of condenser-tube alloys, particularly, by methods of impingement attack and by the use of experimental condensers operating under controlled conditions. This latter work included a comparison of the effect of circulating-water velocity on the corrosion rate of the tubes. Research work on a modified form of apparatus for impingement testing was presented for further investigation and calibration with comparative tube life in service.

Some rather well-defined cases of corrosion in condenser tubes caused by the presence of foreign matter within the tube were shown. Conjecture regarding the effect of foreign matter on tube life is a matter of long standing, however, very few concrete cases taken from service have been available.

A rather comprehensive survey of the experience in service of a number of the newer tube alloys were made. The use of the newer alloys has been increasing steadily. They apparently have a very definite field, at least in certain places, where the tube life of the older alloys has been short.

During 1935 the committee has continued its development of standard methods for corrosion testing and will include in its December, 1935, report a discussion of causes of end corrosion and improved methods for prolonging tube life in condensers where end corrosion is a serious problem.

Boiler-Furnace Refractories, W. A. Carter, Chairman. During the past year there has been little progress because of lack of funds. However, some laboratory work on the equilibrium diagram for the alumina-lime-silica-iron oxide system has been continued at the National Bureau of Standards. In addition, the chairman has endeavored to get some help from co-members of the A.S.T.M. Committee C-8 on Refractories to carry on the laboratory slag-test program that the committee was forced to discontinue at the University of Illinois because of lack of funds. The prospects of getting this project under way in the near future are, however, encouraging.

Absorption of Radiant Heat in Boiler Furnaces, W. J. Wohlenberg, Chairman. The Society authorized the appointment of a special research committee on absorption of radiant heat in boiler furnaces early in 1928. In 1930 a fellowship was established under the direction of Professor Wohlenberg and financed by The Superheater Company in order to study the problem from both the experimental and theoretical points of view. The experimental work was to consist of radiation and temperature measurements in boiler furnaces together with other data necessary for establishing the energy distribution, the particular furnaces in which the measurements were to be taken to be selected after a study as to type and convenience of arrangements for obtaining the information desired. These tests were carried out on ten boilers in different power plants in the United States and involve stokers, pulverized-fuel, oil, and gas firing with various combinations of refractory protected and water-cooled furnaces.

When the results of the investigations were presented to the Committee at its December 5, 1934, meeting, it was decided to present this information to the Society in the form of one or more papers and to further recommend that one session of the 1935 Annual Meeting be devoted exclusively to this subject. Such a session is now being sponsored by the Fuels Division and the Subcommittee on Heat Transfer of the Process Division. The papers to be read at this session have been written by W. J. Wohlenberg, W. H. Armacost, C. W. Gordon, and H. F. Mullikin.

Velocity Measurement of Fluid Flow, W. F. Durand, Chairman. A variant method for determining air velocity in a pipe using sound impulses rather than wave trains of definite frequency was investigated. The method consists in starting a sound pulse at a point in the pipe carrying the air and adjusting a length measured upstream against a fixed length measured downstream so that the pulse arrives at the two terminations simultaneously. The method employed for determining simultaneity consisted in converting the arrived sound pulses to electrical pulses by means of telephone receivers and applying the electrical pulses to the grids of two gas-type electronic tubes so coupled that the tube receiving the impulse registers and thereby prevents the other tube from registering. By manipulating the length of one of the sound paths, either one or the other electronic tubes can be made to register for a steady state of fluid flow. This enables the path length of simultaneous sound-pulse arrival to be determined. The difference in path length of the adjustable arm over that determined when the fluid is at rest, is a measure of the fluid velocity.

The method was devised to avoid the effect of sound reflections in the pipe which hitherto have been found extremely troublesome. This scheme depends for its operation only on the first pulse arriving, the apparatus thereafter being rendered automatically inoperative for a time sufficient to allow sound disturbances to subside.

The scheme was found to work successfully over the fluid-velocity range possible with the apparatus at hand, namely velocities from 1 to 22 ft per sec. The fluid velocities were checked by means of a gasometer and stopwatch. The reproducibility of velocity readings obtained with the acoustic method was superior to that obtained with the gasometer and the average velocities determined by the two methods agreed to within one per cent.

Measurements were made of air velocity in two different pipes, the first an iron conduit of 1.37 in. internal diameter, the second a brass tube with an internal diameter of 0.293 in. Fluid velocities from 1 to 7 ft per sec could be produced in the large pipe, and velocities up to 22 ft per sec in the small tube.

Fluid turbulence apparently does not affect the operation of the device in a significant way, at least for the velocities and pipe diameters investigated. Also it was found that it was not necessary to use an arbitrary correction factor to reconcile the velocity values obtained by the acoustic method with those obtained with the gasometer standard. Accordingly it appears that the acoustic method may be regarded as an absolute method yielding correct velocity values from simple length measurements and the velocity of acoustic propagation in the fluid medium.

Strength of Vessels Under External Pressure, W. D. Halsey, Chairman. The special research committee is at present investigating the possibilities of furnishing simple means for computing the dimensions and stresses of vessels under external pressure constructed of materials other than steel and also of steel vessels but at elevated temperatures. Whereas it would be possible to provide charts for the calculation of such vessels, the preparation of such charts would entail an enormous amount of work and it is hoped that a more simple means may be developed.

The committee expects to furnish also within the near future a discussion of the underlying principles of the report it has already published and which has been incorporated as a section of the A.S.M.E. Code for Unfired Pressure Vessels. In the meantime, it is undertaking to answer questions raised by the membership regarding the application of the rules given in its first report.

Wire Rope, W. H. Fulweiler, Chairman. The National Bureau of Standards has completed the report of the tests on some 200 samples of discarded wire rope. This report shows the close relationship between wear and broken wires in a rope and its remaining strength. Publication of the report is being delayed until after its findings have been checked by wire-rope inspectors in the field.

Heavy-Duty Anti-Friction Bearings, W. Trinks, Chairman. During the year the committee has continued its search through published and privately obtained data for additional findings relating to the various phases of its research program. While some additional information has been obtained, no developments have been brought to light which require revision or immediate amplification of the committee's previous reports. The report summarizing the results of the committee's experimental investigation has just been published as a separate pamphlet under the title "Roll-Neck Bearings."

In so far as the committee's plans are concerned, the immediate future may be characterized as a period of watchful waiting, to be continued until foreign or private investigators unearth new data, or until demand arises for resumption of an active program by the committee.

Rotary Drilling of Oil Wells, D. D. Trax, Chairman. The activity of the Special Research Committee on Rotary Drilling of Oil Wells has been limited this year to a recasting of the report on performance and efficiency of gas-electric prime movers for rotary drilling which was presented at the Petroleum Division meeting held in Tulsa in May, 1934. When the report is completed the committee proposes to recommend its publication as a separate pamphlet.

Critical-Pressure Steam Boilers, A. A. Potter, Chairman. The major portion of the year was devoted to a study of the viscosity of water and of steam. There are no data available for the viscosity of water at temperatures above 320 F and in the case of steam the values for viscosity above atmospheric pressure are not known with sufficient accuracy. Up-to-date data are available on the viscosity of water from 212 F up to and including the critical temperature.

Interesting results have also been secured on the effect of pressure on the viscosity of subcooled water.

The viscosity of steam has been investigated since June, 1935, at pressures up to 3500 lb per sq in. and at temperatures up to 1000 F. The results of the investigations of the viscosity of water and of steam under the previously mentioned conditions will be presented in a paper at the coming Annual Meeting. The plans for the next year include a study of dissociation at high pressures and temperatures.

Sampling of Pulverized Fuel in Moving Gas Streams, K. M. Irwin, Chairman. At its meeting in New York in December, 1934, the committee decided not to solicit funds to cover further laboratory tests but to limit the committee's activities to an inquiry from the present users of pulverized-fuel equipment relative to the methods they are now using to obtain pulverized-fuel samples and also to attempt to secure an expression of opinion from them as to the relative accuracy of the methods which are being employed.

A questionnaire is in preparation for this purpose which has been reviewed by a few members of the special research committee. It is hoped that before the end of 1935, final approval on the form of the questionnaire will be reached and that during the next year the committee will be successful in collecting a considerable amount of information.

Cottonseed Processing, W. R. Woolrich, Chairman. The University of Tennessee, the Tri-State Cotton Oil Mill Superintendents Association, the Tennessee Valley Authority, the Engineering Foundation and the A.S.M.E. Special Research Committee are cooperating in this research program. Through laboratory investigation at the University, fundamental and basic facts relative to the processing of cottonseed are uncovered and proved. When necessary, experimental equipment is built and trial runs made to determine the value of new processes or products. A progress report was presented to the Society during the year at the 1934 Annual Meeting, by Chairman Woolrich.

Future plans involve the finding of more new uses for the cottonseed bi-products and the extension of the research program into the demonstration stage, especially that of a continuous cooker of the type which has been developed within the laboratory.

Inactive Committees. Five of the twenty-four special research committees on Elevator Safeties, Worm Gears, Measures of Management, Removal of Ash as Molten Slag from Powdered-Coal Furnaces, and Automatic Oil Pipe Line Pumping Stations have been inactive during the past year for financial and other reasons.

Respectfully submitted,

G. M. Eaton, *Chairman*
D. B. Bullard — H. A. Johnson
N. E. Funk — C. R. Richards

STANDARDIZATION

A review of the year's activity among the standardization committees sponsored by the Society reveals a commendable amount of progress and the Standardization Committee takes considerable pleasure and pride in reporting the results accomplished. Within the fiscal year just closed (October 1, 1934, to October 1, 1935) eight standards were completed and presented to the American Standards Association for approval. These standards are as follows:

Shafting and Stock Keys
Screw Threads
Adjusted Pressure Ratings for Steel Flanges and Flanged Fittings
Jig Bushings
Drawings and Drafting-Room Practice
Graphical Symbols
Code for Pressure Piping
Hose Coupling Screw Threads
Wrought-Iron and Wrought-Steel Pipe
Cast-Iron Soil Pipe and Fittings

In the paragraphs which follow will be found brief statements covering the year's progress of every committee that has been active during that period. For the convenience of reference the present status of the several projects for which the Society holds sponsorship or joint sponsorship under the A.S.A. procedure is indicated also by classifying them under the following six heads that represent definite steps in that procedure but in reverse order according to time: (1) Before the A.S.A. for final approval and released for publication; (2) proposed standard before sponsor bodies; (3) revised proposal before sectional committee; (4) completion of report by subcommittee; (5) review of comments by subcommittee; and (6) development of proposal by subcommittee.

(1) Before the A.S.A. for Final Approval and Released for Publication

Shafting and Stock Keys, C. M. Chapman, Chairman. Four standards formerly approved by the A.S.A., B17a—1924, B17b—1925, B17d—1927, and B17e—1927 were revised with additions and republished in one pamphlet after approval in December, 1934.

Screw Threads, R. E. Flanders, Chairman. The two sponsor bodies, the Society of Automotive Engineers and The American Society of Mechanical Engineers approved the first revision of the American Standard for Screw Threads, originally published in 1924, in February, 1935, and November, 1934, respectively. Accordingly, in March, the proposal was transmitted to the American Standards Association for final approval and designation as an American Standard. This designation was given in April, 1935, and the standard known as B1.1—1935 is now available in pamphlet form.

Adjusted Pressure Ratings for Steel Flanges and Flanged Fittings, A. M. Houser, Chairman. The letter-ballot vote of the members of the Sectional Committee on the Standardization of Pipe Flanges and Fittings was completed in March, 1935, and resulted in a favorable recommendation in regard to the revised tables for pressure-temperature ratings that were prepared by Subcommittee No. 4 on Materials and Stresses to replace Table 1 in the American Standard for Steel-Flanged Fittings and Companion Flanges (B16e—1932). The Heating, Piping, and Air Conditioning Contractors National Association, one of the sponsors, approved this proposal in April, 1935, while the other two sponsors, the Manufacturers Standardization Society of the Valve and Fittings Industry and The American Society of Mechanical Engineers gave their approval in May, 1935. The proposal was placed before the A.S.A. for approval in May, 1935. This was granted in September.

Jig Bushings, F. S. Walters, Chairman. The National Machine-Tool Builders Association, the Society of Automotive Engineers, and The American Society of Mechanical Engineers, as sponsors, approved the proposed American Standard for Jig Bushings during the past year, and it was presented to the American Standards Association for final approval in March, 1935. This approval was given in April and this new American Standard (B5.6—1935) is now available.

Drawings and Drafting-Room Practice, F. DeR. Furman, Chairman. The American Society of Mechanical Engineers as one of the sponsors for the proposed American Standard for Drawings and Drafting-Room Practice signified its approval of this proposal in February, 1935. Approval of the other sponsor, the Society for the Promotion of Engineering Education, was given the previous June. It was transmitted to the American Standards Association in March and its approval and designation as an American Standard was given in May. This standard therefore, is available now in pamphlet form with the serial designation 214.1—1935.

Graphical Symbols, T. E. French, Chairman. The proposed American Standard for Graphical Symbols was approved by The American Society of Mechanical Engineers as sponsor in May, 1935. The other joint sponsor, the Society for the Promotion of Engineering Education gave its approval in June, 1934, contingent on approval by the sectional committee and the A.S.M.E. However, several suggestions made by certain members of the Sectional Committee on the Standardization of Drawings and Drafting-Room Practice at the time of the letter-ballot vote made necessary a further review of a number of the proposed standard symbols. Accordingly, in June, 1935, a summary of these suggestions was sent to the entire membership of the sectional committee for vote by letter ballot. The changes which were approved by this ballot were included in the proposed standard submitted to the A.S.A. in September, 1935.

Code for Pressure Piping, E. B. Ricketts, Chairman. The letter-ballot vote of the members of the Sectional Committee on Code for Pressure Piping was completed in December, 1934. The Code was then submitted to The American Society of Mechanical Engineers for approval in its capacity as sole sponsor. This approval was given in February, 1935, and in March the proposed American Tentative Standard Code for Pressure Piping was presented to the American Standards Association for final approval. Favorable action was announced in June, and the code was made available immediately in pamphlet form with the number B31—1935.

Wrought-Iron and Wrought-Steel Pipe, H. H. Morgan, Chairman. In November, 1934, the American Society of Mechanical Engineers, as one of the two joint sponsors, gave its approval to the proposed American Tentative Standard for Wrought-Iron and Wrought-Steel Pipe, and The American Society for Testing Materials, the other sponsor for this project, gave its approval in February, 1935. While the proposal was presented to the A.S.A. in March, 1935, some delay in securing its approval has resulted from a disagreement concerning slight changes which had been made by the editing committee in the wording of Paragraph 3 of the introductory notes.

Hose Coupling Screw Threads, H. W. Bearce, Chairman. Objec-

tions raised during A.S.A. vote necessitated another meeting of Sectional committee. Standard finally approved and published in July, 1935.

Cast-Iron Soil Pipe and Fittings, J. J. Crotty, Chairman. The proposed American Standard for Cast-Iron Soil Pipe was completed in April, 1935; it was submitted to the members of the Sectional Committee on Plumbing Equipment for approval by letter ballot in May and was presented to the two sponsor societies in July, 1935. The American Society of Sanitary Engineering and The American Society of Mechanical Engineers, gave their approval to the proposed standard in August, and in the same month it was transmitted to the A.S.A. for final approval and designation as an American Standard. W. C. Groeniger is chairman of the Sectional Committee.

(2) Proposed Standard Before Sponsor Bodies

Brass Fittings for Flared Copper Tubes, F. L. Riggin, Chairman. The development of a standard for brass fittings for flared copper water tubes was initiated by a subgroup composed principally of manufacturers and in 1930 a tentative draft was submitted for review to the members of Subcommittee No. 7 of the Sectional Committee on Plumbing Equipment. After some revision, the proposed standard was distributed broadly to industry for criticism in February, 1931. Then followed considerable correspondence between the members of the subcommittee and groups in the industry. Finally in March, 1934, it was approved in revised form by Subcommittee No. 7 and submitted to the sectional committee for vote on approval by letter ballot. This vote was favorable and in September, 1935, the proposed standard was submitted to the sponsor bodies, the American Society of Sanitary Engineering and The American Society of Mechanical Engineers for approval and transmission to the A.S.A.

Socket Set Screws and Socket-Head Cap Screws, H. Koester, Chairman. In April, 1935, the final draft of the proposed American Standard for Socket Set Screws and Socket-Head Cap Screws was released by Subcommittee No. 9 to the Sectional Committee on the Standardization of Bolt, Nut, and Rivet Proportions for vote on approval by letter ballot. This vote was favorable and the proposed standard was submitted to the two sponsors, the Society of Automotive Engineers and The American Society of Mechanical Engineers in August, 1935.

(3) Revised Proposal Before Sectional Committee

Ammonia Flanged Fittings and Companion Flanges, W. R. Kremer, Chairman. The letter-ballot vote of the members of the Sectional Committee on the Standardization of Pipe Flanges and Fittings on the proposed American Tentative Standard for Ammonia Flanged Fittings and Companion Flanges was completed in October, 1934. The results of this ballot indicated, however, that certain extensive changes and additions were desired by the members of the sectional committee. The officers of the committee, after considerable correspondence prepared a revised draft of the proposal and resubmitted it to the members of the sectional committee in September, 1935, for vote on approval by letter ballot.

(4) Completion of Report by Subcommittee

Taper Pipe Threads, S. B. Terry, Chairman. Subcommittee No. 2 completed its revision of the American Standard for Pipe Threads (B2—1919) in September, 1935. As the next step in the development of the proposed revision, it will be submitted to the members of the Sectional Committee on Pipe Threads for approval by letter-ballot vote.

Machine Tapers, F. S. Blackall, Jr., Chairman. The final draft of the proposed American Standard for Machine Tapers with certain minor corrections and changes was approved by Technical Committee No. 3 of the Sectional Committee on Small Tools and Machine Tool Elements at its meeting in December, 1934. After final checking of all details, the proposed standard will be mailed to the members of the sectional committee for vote by letter ballot.

Spindle Noses and Collets, J. E. Lovely, Chairman. Technical Committee No. 4 on Spindle Noses and Collets of the Sectional Committee on Small Tools and Machine-Tool Elements was organized in December, 1928. It cooperated with Technical Committee No. 11 in the preparation of a draft standard on Spindle Noses and Chucks distributed to industry for comment in May, 1930.

However, as a result of suggestions made at numerous conferences with members of the industry, Technical Committee No. 4 in March, 1931, drew up a revised draft of this proposal dealing only with lathe-spindle noses. In October, 1932, this draft was approved by the lathe groups of the National Machine Tool Builders Association.

During 1933, the committee carried on tests on new forms of spindle noses. Subsequently sufficient experience with the proposed standard type of spindle nose has been had and recorded, and the new designs have met with such universal approval by industry that in October, 1934, copies of the proposal in its final form covering spindle noses for turret lathes and automatic lathes were sent to the members of the Technical Committee. In March, 1935, the Technical Committee released this proposed standard to the Nomenclature Committee for review. Following this review it will be submitted to the members of the Sectional Committee for vote by letter ballot.

Chucks and Chuck Jaws, J. E. Lovely, Chairman. Work on the preparation of a proposed standard for Chucks and Chuck Jaws was begun in December, 1928, with the organization of Technical Committee No. 11. Subsequently, Technical Committee No. 11 collaborated with Technical Committee No. 4 on Spindle Noses and Collets in preparing a draft standard on Spindle Noses and Chucks. In May, 1930, these committees distributed to industry for comment a joint proposal entitled proposed American Standard for Spindle Noses and Chucks. The next year a revised draft of this proposal dealing only with chucks and chuck jaws and based on the comments received as a result of the previous canvass of industry was circulated to industry for further comment.

After extensive correspondence relating to the adjustment of various changes and corrections suggested by the manufacturers of chucks, Technical Committee No. 11 completed the final draft of the proposed standard and released it to the Sectional Committee in March, 1935. Following review by Technical Committee No. 17 on Nomenclature, it will be submitted to the members of the Sectional Committee for vote by letter ballot.

Circular Forming Tools and Holders, W. C. Mueller, Chairman. Work on a proposed standard for circular forming tools and holders for automatic screw machines was begun in 1931 by Technical Committee No. 10. During the course of the next three years, four questionnaires were distributed to industry for criticism and comment and the suggestions received were, incorporated as far as possible, in successive revisions of the proposed standard. In August, 1934, a final draft of the proposal was sent to the members of the Technical Committee, and in November, 1934, it was released to the Sectional Committee.

Technical Committee No. 17 was asked to develop the nomenclature for this standard and when this is completed the proposed standard will be sent to the members of the Sectional Committee for vote on approval by letter ballot.

(5) Review of Comments by Subcommittee

Inspection of Gears, F. W. England, Chairman. In May, 1935, the Recommended Practice of the American Gear Manufacturers' Association for Inspection of Gears was mailed to a selected list of users of gears with a request for their criticism and comment. Subcommittee No. 9 of the Sectional Committee on the Standardization of Gears proposes that these rules form the basis of the American Standard for the Inspection of Gears. The replies which were received are now in the hands of the subcommittee for study.

Large Rivets, R. N. S. Baker, Chairman. As a result of the meeting of Subcommittee No. 1 on Large and Small Rivets held in New York in December, 1934, the proposed American Standard for Large Rivets was broadly distributed in June, 1935, for criticism and constructive suggestions. The replies received are now in the hands of the subcommittee.

Plain Washers, C. W. Squier, Chairman. During the past year a tentative draft of the proposed American Standard for Plain Washers for use with American Standard Bolt Heads and Nuts was completed by Subcommittee No. 1 and distributed broadly for the general criticism and comment of industry. Consideration is now being given to the replies received.

Speeds of Machinery, A. E. Hall, Chairman. The special subgroup appointed to study the comments received from industry following the distribution of copies of a tentative draft of the proposed American Standard for Machine Speeds held a meeting in New York, in April, 1935. A revised draft of the proposal based on the subgroup's recommendations is now being prepared.

Traps, A. R. McGonegal, Chairman. In February, 1935, a draft of the proposed American Standard for Traps for use with plumbing fixtures was distributed to industry for review and comment. Subcommittee No. 5 of the Sectional Committee on Plumbing Equipment is now reviewing the replies which were transmitted to the chairman late in April. These suggestions will form the basis of a further revision of this proposal.

Pressure and Vacuum Gages, M. D. Engle, Chairman. Subcommittee No. 3 on Gage Sizes and Mounting Dimensions, H. B. Reynolds, chairman, and Subcommittee No. 4 on Accuracy and Test Methods, O. J. Hodge, chairman, are reviewing letters received as a result of the general distribution of their parts of the proposed standard to industry for criticism and comment. Section 3 was mailed in November, 1934, and Section 4 was distributed in April, 1935. There is a possibility that all of the three sections of this proposed

standard will be ready early this fall for the editing committee, which is to mold them into a unified whole.

(6) DEVELOPMENT OF PROPOSAL BY SUBCOMMITTEE

Round Unslotted Head Bolts. At the December, 1934, Meeting of Subcommittee No. 5, a proposed revision of the American Tentative Standard for Round Unslotted Head Bolts published in 1928 (B18e—1928) was approved. Draft copies of this proposed revision had been distributed previously to the committee members and others interested for their consideration. It was found necessary to prepare a new table for thread lengths, copies of which were distributed to the members of the committee in July, 1935. If satisfactory, this table will be incorporated in the proposed revision which then will be circulated to industry at large for criticism and comment.

T-Slots, Erik Oberg, Chairman. Technical Committee No. 1 of the Sectional Committee on Small Tools and Machine-Tool Elements completed its report on the "Nomenclature and Glossary of Terms for T-Slots," in May, 1935. This report is now in the hands of Technical Committee No. 17 on Nomenclature after which this supplement to B5a—1927 will be submitted to the members of the sectional committee for approval.

Punch-Press Tools, D. M. Palmer, Chairman. The reorganized Technical Committee No. 9 on Punch-Press Tools held a meeting in Detroit in February, 1935, at which substantial progress was made toward the completion of a tentative draft of the committee's proposal intended to cover certain parts of all classes of die sets considered suitable subjects for standardization.

Single-Point Cutting Tools, F. H. Colvin, Chairman. Technical Committee No. 19 which was organized in December, 1933, completed its report on the nomenclature for single-point tools in May, 1935. This report will be submitted shortly to the members of the sectional committee for approval.

Preferred Practice in Graphic Presentation, A. H. Richardson, Chairman. Subcommittee No. 3 of the Sectional Committee on Standards for Graphic Presentation at its meeting held in New York in June, 1935, adopted, with a few minor changes, a tentative draft of a code for preferred practice in graphic presentation which had been in course of preparation for a number of years. The subcommittee plans to have about five hundred copies of this proposed code printed for preliminary review, the expense to be covered by sales to the public. This would be in the nature of a trial edition which thus could be tested in practice and, if found to be satisfactory, presented to The American Society of Mechanical Engineers as sponsor society for approval and transmission to the American Standards Association.

Engineering and Scientific Graphs, W. A. Shewhart, Chairman. The special subgroup of Subcommittee No. 4 appointed in April, 1934, to prepare a standard nomenclature for the shapes of curves and plots, under the chairmanship of E. T. Cope, completed its report in May, 1935, and distributed copies of it to the members of the subcommittee in June of this year for examination and comment.

The special subgroup under the chairmanship of H. F. Dodge appointed also in April, 1934, held a meeting in December of that year to consider a preliminary draft of a brochure on "Engineering and Scientific Graphs for Publications." Consideration was given also at this meeting to an outline of the subgroup's report submitted by Messrs. Stone and Hanscom. Three subsequent meetings were held in January, February, and April and the suggestions developed at these meetings have been incorporated in a redraft of the proposal.

Tolerance Systems, R. E. W. Harrison, Chairman. Subcommittee No. 1 of the Sectional Committee on Allowances and Tolerances for Cylindrical Parts and Limit Gages held a meeting in New York in December, 1934. At this meeting a full discussion of the differences between the present standard (B4.1—1925) and the I.S.A. Standard with special reference to ranges and classes of fit took place. D. R. Miller, a member of the committee, then was requested to prepare a set of ranges and tolerances following the general trend of B4.1—1925 and the I.S.A. standard but with the fewest possible and widest practical ranges. This task was completed and copies of the report were distributed to the members of the subcommittee in June, 1935.

Cut and Ground Thread Taps, C. M. Pond, Chairman. At its meeting in December, 1934, Technical Committee No. 12 discussed and approved revisions of the American Standard for Cut and Ground Thread Taps (B5e—1930). This revision is based on comments and suggestions received as a result of the distribution of a tentatively revised draft to industry at large in May, 1934. After approval by the members of the Sectional Committee on Small Tools and Machine-Tool Elements this revision will be submitted to the sponsor societies and the American Standards Association.

Gasoline, Oil, and Grease Separators, J. J. Crotty, Chairman. Meetings of Subcommittee No. 9 were held in November, 1934, and March, 1935, and good progress was made toward the completion of the first draft of a proposed American Standard for Gasoline, Oil, and Grease Separators. At the November, 1934, meeting, a report on the "Recommended Use for Oil Separators and Their Construction" was presented and discussed. There is now before the subcommittee for its consideration a draft of a proposed standard ordinance to regulate the handling, storage, transportation, and disposition of waste oils and grease resulting from the use and operation, cleaning, repairing, or servicing of vehicles, machinery, or appliances of any kind. This project is part of the program of the Sectional Committee on Plumbing Equipment.

Iron and Steel Bars, F. H. Frankland, Chairman. The Sectional Committee, on the Standardization of Stock Sizes, Shapes, and Lengths for Hot-Rolled and Cold-Finished Iron and Steel Bars held a meeting in New York on April 23, 1935. The committee discussed seriously the need for this standard and the form which it should take. It was finally decided that the gathering together of the several present trade practices within the scope of the committee's work and the placing them in form for review and approval by the American Standards Association would render a useful service.

Accordingly, certain members of the committee agreed to supply a list of shapes, sizes, lengths, and tolerances of hot-rolled bars together with similar data for hot-rolled reinforcing bars. Another group offered to supply the same data for cold-finished steel bars. It was then decided that, when this material had been assembled and worked up into the form of a standard, it should be distributed to the members of the Sectional Committee as a tentative proposal.

Classification and Designation of Surface Qualities, C. G. Mettler, Chairman. Subcommittee No. 1 on Standardization of Surfaces Produced by Tools and Abrasives, J. Cetrule, chairman, held a meeting in December, 1934, at which a comprehensive report of Subgroup No. 2 on Machine Surfaces Produced by Tools, of which J. S. Chafee is chairman, was read and discussed. As a result of this discussion the subgroup was assigned further duties and from subsequent correspondence with Mr. Chafee it is expected that a report will be forthcoming shortly.

In December, 1934, a meeting was held also by Subcommittee No. 5 on Ways, Means, and Apparatus for Measuring Quality of Surface, J. R. Weaver, Chairman, at which Prof. F. A. Firestone's report on definitions and notation for measurement of surface quality was read and discussed. At this meeting consideration was given also to a program of experimental work which had been outlined in a letter sent out by the secretary of the subcommittee the previous spring and, as a result of the discussion, a subgroup headed by G. A. Bouvier, was appointed to carry out that part of the program relating to the measurement, by various methods, of samples of representative surfaces.

Furnaces, C. E. Bronson, Temporary Chairman. Upon the recommendation of the A.S.M.E. Pure Air Committee together with a large number of representative and interested organizations, a Sectional Committee on the Unification of Rules for Dimensioning of Furnaces for Burning Solid Fuel was organized in June, 1933, in Chicago under the procedure of the American Standards Association. The present economic conditions have operated to delay the completion of the personnel of this sectional committee and the five subcommittees which were authorized by the sectional committee at its first meeting. Mr. Bronson gives assurance, however, that this task is now nearly completed.

As a matter of record, the following statistics are given for the purpose of indicating the extent of this activity and the number of persons having a part in it. Number of Sectional Committees 31; number of subcommittees and subgroups 273; number of committee members, members of A.S.M.E. 371, non-members 812; number of committee meetings held in year, 24; number of cooperating organizations, 206.

Respectfully submitted,

C. W. SPICER, *Chairman*
ALFRED IDDLES — WALTER SAMANS
L. A. CORNELIUS — O. A. LEUTWILER

POWER TEST CODES

The Committee on Power Test Codes takes pleasure in reporting that during the fiscal year 1934–1935 one new test code and one revised code were completed, approved, and published while two new sections of Instruments and Apparatus and one revised section were completed and approved by the Council. A more detailed statement covering this activity is given in the following paragraphs.

ITEMS OF GENERAL INTEREST

Election of Acting Chairman of Power Test Codes Committee. In recognition of the service which Dr. Robert H. Fernald had so willingly rendered the main Committee during the period of Dr. Low's illness, the committee requested him at its December, 1934,

meeting to serve as its acting chairman until such a time as Chairman Low will be able again to take his place.

Btu per Hour Equivalents of Horsepower and Kilowatt. In a comment on the November, 1934, draft of the proposed revision of the Test Code for Steam-Condensing Apparatus attention was called to the disagreement between the values there given for the Btu per hour equivalents for "horsepower" and the "kilowatt," and the similar values given in the Keenan Steam Tables. The constants 2543 and 3413 Btu per hr respectively in the code were taken from the Code on Definitions and Values and those in the Keenan Steam Tables, 2543.1 and 3411.5, are based on the International calorie which was defined at the 1929 International Steam Tables Conference (London) as 1/860 of the International watt-hour.

Since the publication of the Keenan Steam Tables, the National Bureau of Standards has adjusted the basic values of its electrical standards to agree with the International values and the American Standards Association has accepted similar modifications.

In view of these developments which have taken place since the Code on Definitions and Values was issued in June, 1931, Committee No. 2 on Definitions and Values has been requested to revise its Btu-per-hour equivalents for horsepower and the kilowatt for use in new or revised power-test codes.

A.S.T.M. Committee on Gaseous Fuels. Because of the direct interest in this subject of the Power Test Codes Committee No. 3 on Fuels, E. X. Schmidt, a member of that committee was named as its representative on the A.S.T.M. committee, which now has been organized to standardize the nomenclature and the methods of sampling and testing gaseous fuels in so far as they apply to purchases and sales and the requirements of regulatory bodies. Mr. Schmidt has been active in the development of the A.S.M.E. Power Test Code for Gaseous Fuels.

Committee Meetings. As a matter of record it should be stated that the following committees held regularly called meetings during the past twelve months: Power Test Codes Committee, three meetings, October, December, and April; Committee No. 6 on Steam Turbines; Committee No. 10 Subcommittee on Fans; Committee No. 11 on Complete Steam Power Plants; Committee No. 18 on Hydraulic Prime Movers; special Subcommittee on Measurement of Fluid Flow of Committee No. 19 on Instruments and Apparatus; and Committee No. 21 on Dust-Separating Apparatus.

Committee Progress

Fuels. Committee No. 3 on Fuels, W. J. Wohlenberg, Chairman, completed its review of the preliminary draft of the Test Code for Gaseous Fuels in November, 1934. The code is now in shape for distribution to a selected list of individuals and firms for criticism and comment.

Reciprocating Steam Engines. Committee No. 5 on Reciprocating Steam Engines, A. G. Christie, Chairman, completed an extensive revision of the Test Code for Reciprocating Engines and on February 6, 1935, it was approved by the Council and subsequently published in pamphlet form.

Steam Turbines. Committee No. 6 on Steam Turbines, C. H. Berry, Chairman, met during the 1934 Annual Meeting of the Society to discuss the voluminous material that had been distributed to the members dealing with the proposed revision of the Test Code for Steam Turbines. A subcommittee of three was appointed at that time to develop the revision of the code. This subcommittee is working effectively and definite progress is being made toward the completion of the revised draft of this test code.

Centrifugal and Rotary Pumps. Committee No. 8 on Centrifugal and Rotary Pumps, W. B. Gregory, Chairman, is now considering the advisability of an immediate revision of this code which was approved and published in February, 1928. This step was urged in a communication received from the Hydraulic Institute. In the use of this code as its standard of testing practice, the Institute has realized more and more that it is not altogether adequate and, through the efforts of one of its committees, it has been endeavoring to agree upon certain desirable revisions, and additions, and the standardization of certain of the testing methods and equipment. As the principal purpose of these codes is to serve the practical needs of industry, the Committee, when it has acquainted itself with the exact nature and extent of the changes desired, will begin work on a revised draft.

Displacement Compressors and Blowers. Committee No. 9 on Displacement Compressors and Blowers, Paul Diserens, Chairman, is now laying plans for a complete revision of the Test Code for Displacement Compressors and Blowers. Before undertaking this work the committee is awaiting the report of the Instruments and Apparatus Special Subcommittee on the Measurement of Fluid Flow covering rules for the measurement of air flow from a gaging tank into the atmosphere and from the atmosphere into a gaging tank.

Centrifugal and Turbo-Compressors and Blowers. Chairman, A. T. Brown, Committee No. 10, reports the publication in pamphlet form in March, 1935, of Part 1 on Compressors and Blowers of the Test Code for Centrifugal Compressors, Exhausters, and Fans.

The Subcommittee on Fans, M. C. Stuart, Chairman, held two meetings within the year, during which the first tentative draft of the Test Code for Fans was outlined. The development of the code has been delayed due to the lack of adequate information and agreement on the standard methods of measuring air. Accordingly, several subgroups have been organized to formulate standard methods for the measurement of air under the conditions to be found in the testing of fans. To provide the necessary data and information investigations are under way at Lehigh University under the direction of Professor Stuart; at Harvard University, under the direction of Professor Marks; and in the laboratories of several of the fan manufacturers. They cover (1) measurement of flow by pitot tube in inlet or discharge ducts; (2) measurement of flow by nozzle in inlet or discharge ducts; and (3) measurement of flow with large chamber by nozzle or orifice. The subcommittee hopes that before the end of another year it will have completed the first tentative draft of the Test Code for Fans.

In order to bring about closer cooperation between the A.S.M.E. committee and the committees of the American Society of Heating and Ventilating Engineers interested in fan testing, Thomas Chester, consulting engineer of Detroit, has been appointed to membership on the committee. Mr. Chester is chairman of the Special Council Committee of the American Society of Heating and Ventilating Engineers, organized to review that Society's Standard Code for Disk and Propeller Fans, Centrifugal Fans and Blowers.

During the two years since the organization of the Subcommittee on Fans it has been consolidating data and information bearing on its work. To develop further some of the ideas which have been advanced it applied for permission to hold a technical session during the 1934 Annual Meeting. This was granted and on Tuesday, December 4, a session was presided over by Willis H. Carrier and papers were presented by L. S. Marks, N. P. Bailey, W. E. Somers, N. C. Ebaugh and R. Whitfield, and M. C. Stuart. So great was the interest in this session that another is being planned for the 1935 Annual Meeting.

Steam-Condensing Apparatus. Geo. A. Orrok, Chairman, of Committee No. 12 on Condensers, Water-Heating and Cooling Equipment, reported in December, 1934, the completion of the proposed revision of the Test Code for Steam-Condensing Apparatus. The present revision of the code includes a tentative method for determining the cleanliness factor of condenser tubes. Copies of a tentative draft have been distributed to an extensive selected list of firms and individuals for criticism and comment. As a result of this mailing a number of constructive criticisms of this proposed revision of the code have been received. The duplication of this material was generously arranged for by the Brooklyn Edison Company through P. H. Hardie, secretary and member of Committee No. 12.

Hydraulic Prime Movers. E. C. Hutchinson, Chairman of Committee No. 18 on Hydraulic Prime Movers, reports that the committee has completed the drafting of the proposed revision of the Test Code for Hydraulic Prime Movers. The committee held meetings during the annual meetings of the A.S.M.E. and A.S.C.E. which made it possible to reach agreement on all of the controversial points which had been raised in the development of the earlier drafts. Its plans call for the distribution of copies of the code to a selected list of firms and individuals for criticism and comment before it is presented for final approval.

Instruments and Apparatus. C. F. Hirshfeld, Chairman, Committee No. 19 on Instruments and Apparatus, reports that the work completed by the committee during the year just closed includes the following sections which were approved for publication, namely: Part 1 on General Considerations (second edition); chapter 2 on Radiation Pyrometers of Part 3 on Temperature Measurement, and Part 15 on Measurement of Surface Areas.

In addition to the above, Committee No. 19 has made good progress toward the completion of Part 2, Pressure Measurement, chapter 4 on Bourdon, Bellows, Diaphragm and Deadweight Gages and chapter 5 on Manometer or Liquid Column Gages; Part 3, Temperature Measurement, chapter 3 on Thermocouple Thermometers; and chapter 4 on Resistance Thermometers.

Special Subcommittee on Measurement of Fluid Flow. Inasmuch as there had been some misunderstanding concerning the scope of the work of this special Instruments and Apparatus subcommittee, it was deemed advisable to change its name from Instruments and Apparatus Subcommittee on Flow Nozzles to Instruments and Apparatus Special Subcommittee on Measurement of Fluid Flow.

Due to the urgent need for its reports, the subcommittee is preparing as quickly as possible recommended rules for the measurement of fluid flow which can be made mandatory in the codes prepared by the several individual committees. Preliminary drafts of sections

covering nozzles, orifices, and venturi tubes are being written by the members of the subcommittee. The preliminary draft of the orifice section has been completed and it is expected that similar drafts of the sections on flow nozzles and venturi tubes will be completed within the next few months.

Industry is now being canvassed for funds for the research on flow nozzles which is to be made at the National Bureau of Standards under the auspices of the A.S.M.E. Special Research Committee on Fluid Meters.

As a representative of the American Standards Association, W. A. Carter, Chairman of this special subcommittee, attended the meeting of the International Standards Association Committee No. 30 on the Measurement of Fluid Flow held in Stockholm, Sweden, in September, 1934. He presented the American comments on the I.S.A. Proposal for Measurement of Fluid Flow and urged the inclusion of certain other rules covering the use of nozzles.

A meeting of this special subcommittee was held in New York during the A.S.M.E. Annual Meeting in December, 1934.

Dust-Separating Apparatus. The organization meeting of Committee No. 21 on Dust-Separating Apparatus was held on Wednesday, December 5, 1934. At this meeting the members of Committee No. 21 expressed an appreciation of the magnitude of the project and a feeling that several years would elapse before a final report would be forthcoming. M. D. Engle was elected chairman of the committee. The following subcommittees have been appointed to carry forward the program in the development of the Test Code for Dust-Separating Apparatus: (1) on plan and scope; (2) on definitions and terms, bibliography; (3) on collection of data on test methods employed in past and results obtained, equipment available; (4) on the obtaining of representative samples from gas streams carrying dust in suspension; and (5) on separation of the solid matter from the gas of the sample obtained.

International Cooperation

Through membership on the U. S. National Committee of the International Electrotechnical Commission,[1] the Society, and for it the main Committee on Power Test Codes has been actively engaged in assisting to bring about international agreements on acceptance tests for steam turbines and internal-combustion engines. The Secretariat for the Advisory Committees on Steam Turbines, Internal-Combustion Engines and Hydraulic Turbines is held by the U. S. National Committee of the I.E.C. The Society was represented on this committee this year by Harvey N. Davis, Francis Hodgkinson, and Ely C. Hutchinson, with C. Harold Berry and Paul Diserens serving as alternates.

Respectfully submitted,

F. R. Low, *Chairman*
R. H. Fernald, *Acting Chairman*
C. H. Berry
A. T. Brown
A. G. Christie
H. Cooke
H. Dahlstrand
P. Diserens
L. Elliott
E. R. Fish
C. F. Hirshfeld
F. Hodgkinson
O. P. Hood
G. A. Horne
E. C. Hutchinson
D. S. Jacobus
L. F. Moody
H. B. Oatley
G. A. Orrok
R. J. S. Pigott
H. B. Reynolds
E. B. Ricketts
E. N. Trump
W. M. White
W. J. Wohlenberg

PROFESSIONAL CONDUCT

During this past year this Committee has had but one case referred to it. Investigation developed that no charge of unprofessional conduct on the part of a member of the Society was made. Accordingly no action could be taken by this Committee. This case was referred to the Secretary of the Society.

Respectfully submitted,

C. G. Spencer, *Chairman*
Edwards R. Fish
J. H. Herron
E. F. Scott
Hugo Diemer

SAFETY

The Standing Committee on Safety has continued during the past year to promote industrial safety through the activities of the Society and, in reporting the progress made during that period, desires to record first the technical session in the form of a symposium on the engineering aspects of occupational-disease disability, which it sponsored during the 1934 Annual Meeting of the Society. Five papers dealing with different phases of the subject of industrial-disease hazards were read by authorities in each field and were freely discussed. This session drew such a large and interested audience that the committee was encouraged to plan for a similar session at the 1935 Annual Meeting. Good progress has already been made toward the selection of speakers and the subjects of the papers which are to be presented. At the Committee meeting on June 14 it tentatively agreed to the following three divisions of the subject as an outline for the meeting: (*a*) occupational-disease legislation, (*b*) engineering control of occupational-disease hazards, and (*c*) limitations of protective devices and equipment available.

The committee has agreed to act as co-sponsor with the Management Division of the Society for a session on compensation laws which is also to be held during the 1935 Annual Meeting. There are many aspects of this subject in which the committee is directly interested since it holds that there is a close relation between the compensation laws of a state and its laws bearing on disease disability.

Safety Code for Machinery for Compressing Air, D. L. Royer, Chairman. During the past year the Sectional Committee on a Safety Code for Machinery for Compressing Air released for criticism and comment two successive drafts of its proposed code which indicates that good progress is being made in its work.

Safety Code for Cranes, Derricks, and Hoists, J. C. Wheat, Chairman. No meetings of the Sectional Committee or any of its subcommittees have been held during the past year but from correspondence which this committee has had with Mr. Wheat it has assurances that by the first of the next fiscal year more active work will be resumed toward the completion of this important code which is now in the hands of the editing subcommittee.

Safety Code for Elevators, Sullivan W. Jones, Chairman. The Sectional Committee on a Safety Code for Elevators held a meeting on April 26, 1935. This meeting was called to consider certain changes in the American Standard "Safety Code for Elevators" published in 1931 which had been proposed by the Subcommittee on Elevator Research, Interpretations and Recommendations. The subcommittee had held meetings on October 30, 1934, and April 24, 1935.

The proposed American Recommended Practice in the form of an Inspector's Handbook was approved by letter-ballot vote of the Sectional Committee. The committee plans to delay its submittal to the sponsor societies, however, until the new references to the revised Safety Code for Elevators are known.

Junior Advisory Member. Upon the recommendation of the Committee Carl Endlein, mechanical engineer connected with the C. J. Tagliabue Manufacturing Company, Brooklyn, N. Y., was appointed by President Flanders as the Junior Adviser to the Safety Committee.

It is with deep regret that the committee reports the death of one of its members, M. H. Christopherson, on September 9, 1935.

Respectfully submitted,

W. M. Graff, *Chairman*
H. H. Judson
H. L. Miner
J. B. Chalmers

Special Committees

ADVISORY BOARD ON TECHNOLOGY

On December 7, 1934, the Council of The American Society of Mechanical Engineers took the following action:

Voted: That the Advisory Board on Technology be established for two years, under the direction of the Council, with functions as stated below:

(1) Assume the responsibility of coordinating the activities of the general meetings and congresses, divisions, research, and publications of the Society.

(2) Assume the responsibility of leadership and initiative with respect to coordination.

(3) Interpret the provisions of the Society's By-Laws concerning the jurisdictions and policies of the several committees represented on the Board.

(4) Review and recommend policies proposed by the several committees for the control of their activities.

(5) Hold at least two meetings each year.

(Note: This Board is to consist of five, a Chairman from the Council, named by the President, and one member of each of the following Standing Committees: Meetings and Program, Professional Divisions, Research, and Publications.)

[1] For Report of A.S.M.E. representatives see page 26.

In the discussion, it was emphasized that the responsibilities of coordination, leadership, and initiative gave the Board a tremendous opportunity to advance the Society's purposes which are embodied in the new By-Laws adopted by Council on December 3, 1934, as follows:

(*a*) Encouraging engineering research, tests and other original work.

(*b*) Encouraging the preparation of original papers on engineering topics.

(*c*) Holding meetings for the presentation and discussion of original papers and participating in international engineering congresses.

(*d*) Publishing papers and reports and disseminating knowledge and experience of value to engineers.

Personnel

The members of the Board appointed by President Flanders (for a term of two years) and the committees they represent are: Robert I. Rees, Meetings and Program; K. H. Condit, Professional Divisions; S. W. Dudley, Publications; D. B. Bullard, Research; and A. A. Potter, Council, chairman.

Meetings and Actions

Two meetings of the Board have been held, January 25 and April 18, 1935. Discussions at both meetings centered around problems relating to the coordination of the activities of the committees represented on the Board and to publications. At the second meeting the principle of planning the work of the Society in the technical and educational development of the profession and of members was also discussed.

The view was expressed that every encouragement be given to local sections to send in for consideration by the Committee on Publications the manuscripts of papers presented at their meetings. The committee and the editor were represented as being willing to consider, as in the past, original papers of permanent value for publication in the Transactions, and papers of current interest for publication in *Mechanical Engineering*, which is a record of advancement.

The problems of the Committee on Publications in adequately serving the committees and agencies that provide papers and reports were discussed. Every effort is to be expended in raising the quality of papers and assuring that the papers of permanent value are received sufficiently in advance of meetings to insure adequate review and preprinting so that full discussion will result when the papers are discussed. The principle of budgeting space in the publications to these committees and agencies, as a first step in coordination and planning, was approved.

Relations with the technical press were clarified.

The relation of the professional divisions to the programs of local sections was discussed.

The importance of research as related to other activities of the Society was emphasized, including the opportunity for the research committees and professional divisions to cooperate in preparing bibliographies.

Emphasis upon educational quality should be the major guide and it was considered important to keep the individual member engineering-minded.

A coordinated budget for the committees represented on the Board of Technology was approved.

Respectfully submitted,

A. A. Potter, *Chairman*
D. B. Bullard — S. W. Dudley
K. H. Condit — R. I. Rees

ADVISORY BOARD ON STANDARDS AND CODES

One of the recommendations of the Special Committee on Policies and Budget of which Harry R. Westcott is chairman, was the establishment of an Advisory Board on Standards and Codes. It is expected that this Board will further coordinate the activities of the four committees in charge of the standardization and code-making activities of the Society. Several conferences of representatives of these activities with members of the Committee on Policies and Budget were held last fall and the statement of purpose of this board was finally developed at a meeting of these representatives on December 6, 1934. It was approved by the Council on December 7.

The Advisory Board on Standards and Codes is established for two years, under the direction of the Council, with the following organization and functions. This Board is to consist of five members, a chairman appointed by the President from the membership of the Council and one member named by each of the following four committees, Boiler Code, Standardization, Power Test Codes, and Safety. This Board shall be advisory to the Council and shall assist in (1) establishing general policies pertaining to standards and codes, (2) initiating and developing standards and codes, and (3) coordinating the activities of the several committees which are represented in its membership. It plans also to cooperate with the Board of Technology.

The first meeting of the Board was held on May 9, 1935, at which the time was spent principally in discussing general policies and the methods of work of the constituent groups. It considered also the request of the Committee on Policies and Budget to the effect that the Advisory Board on Standards and Codes cooperate with it in determining the broad purposes and long-range policies which should control the Society's standards and codes activities.

Respectfully submitted,

A. D. Bailey, *Chairman*
L. A. Cornelius (Standardization) — W. M. Graff (Safety)
V. M. Frost (Boiler Code) — Francis Hodgkinson (Power Test Codes)

BOARD OF REVIEW

The Board of Review held nine meetings during 1935 to consider the requests of members for dues cancelation and inactive status, provided in the policy which was adopted by the Council on December 2, 1934.

The number of requests received for consideration total 378, and in response, recommendations were made to the Executive Committee of Council, as follows:

Approved (in part or in full)	345
Denied	30
Tabled	3
	378

Dues canceled in connection with the 345 requests approved and the fiscal years to which such dues applied are given below. The cancelation of this amount in reality does not represent a real financial loss in Society revenues since the Board in each instance received satisfactory evidence of the impossibility of collecting such arrears. Furthermore, in some cases, the cancelation of part-dues arrears was made contingent on and resulted in the immediate payment of the dues-arrears balance or dues for the current year.

Fiscal year dues ending	Amount canceled
1930	$5.00
1931	60.00
1932	350.00
1933	1,597.07
1934	4,782.58
1935	4,418.33
	$11,212.98

In considering each request to cancel dues, the Board carefully weighed the all-important matter of loss of income to the Society. In several instances the unsolicited pledge of the member was voluntarily given that, if temporarily relieved of previous dues incurred and for which no publications or other services were rendered by the Society, he would immediately assume current dues obligations or those payable after October 1, 1935. In recommending favorable action in such cases, the continued interest of the member in the Society's welfare was retained with a reasonable certainty that active status with regular dues payments would follow in the near future. Further, the grateful acknowledgments received after notification was sent of favorable action leads the Board to believe that the Society will continue to hold indefinitely the good will of those members who have suffered serious misfortunes during the past few years. Where circumstances deemed it advisable, the Board canceled only that part of the dues arrears that would permit retaining the delinquent member on the Society roster. In such cases the member was asked to endeavor to pay the balance of arrears and current dues.

With each request for dues cancelation a detailed confidential statement was submitted by the member and individual attention given by the Board to the facts presented in such application. Where complete information was omitted, a communication was dispatched to the member and a further opportunity given to submit additional details. The recommendations of Local Sections received in certain instances were most helpful in assisting the Board.

In acting for the Council, the Board was most sympathetic in its treatment of cases where there had been a long period of faithful membership. On the other hand, the employment difficulties of the

recently graduated Junior members received equally sympathetic consideration.

In the majority of cases where dues cancelation were recommended to Council, two or more of the following conditions existed:

(*a*) Continued period of unemployment
(*b*) At present temporarily employed in relief projects
(*c*) At present unemployed
(*d*) Drastic salary readjustment
(*e*) Loss of income, due to bank failures, bankruptcies, etc.
(*f*) Loss in income due to ill health or old age.

The Board is firmly convinced of the necessity of handling promptly the membership status of dues-delinquent members and of the return to the enforcement of the Society's By-Laws canceling membership after a lapse of 15 months in membership dues. Continuation of such a policy in the future insures elimination of the suspension and dropping of several thousand delinquent members at one time as resulted in September, 1934, following a most liberal dues policy.

In conclusion, the Board offers the following recommendations for further consideration by Council:

1 That the Council continue to enforce the provisions of Par. 4, Article B5, Fees and Dues, with a liberal policy toward those members anxious to retain their membership but unable to pay annual dues for the reasons enumerated above.

2 That the provision for inactive status without membership privileges be extended one year from October 1, 1935.

3 That no member of the Board of Review serve longer than a three-year term.

Respectfully submitted,

HENRY B. OATLEY, *Chairman*
A. D. BLAKE
JOHN P. NEFF

BOILER CODE

The Boiler Code Committee submits the following report for the fiscal year ending September 30, 1935, covering activities other than its regular routine work.

During this period the Committee held nine regular meetings and several executive committee meetings which were devoted to interpretations of the various sections of the Code and to the formulation of revisions and addenda thereto.

To keep abreast of the progress being made in the development of pressure vessels and of the materials of which they are constructed, the Committee has made necessary revisions and additions to the various sections of its Code. These changes are to a great extent the result of suggestions from manufacturers, inspectors, users, and others. These revisions were formally adopted by the Council on July 23, 1935, and distributed in pink-colored addenda sheets.

The Committee has under consideration the adoption of more comprehensive rules for the use of cast iron in the construction of cast-iron unfired pressure vessels under limited conditions. It has been brought out that there appears to be a continued demand for permission to use cast iron in Code vessels. It also appears that there are a great many unfired pressure vessels in use with some portion made of iron castings and, therefore, it appears desirable that some recognition be given in the Code to the use of cast iron which will enable manufacturers to take advantage of the extensive use of this material.

Various non-ferrous alloys as well as ferrous alloys are being developed which appear to have considerable merit for use in the construction of special pressure vessels for varying types of service. As a result, a special committee is investigating these materials with the expectation of introducing suitable provisions for Code specifications and working stresses for such materials.

Consideration was given to a request for provisions in the Code for acceptance of malleable iron for pressure parts of pipes, fittings, valves, and their bonnets. A similar provision has appeared in the Code for some time for the use of cast iron and the feeling was expressed that malleable iron should also be provided for. As a result a rule has been incorporated in the Code to allow for the use of high-grade malleable iron, which complies with the material specifications in the Code for malleable iron, for boiler and superheater connections under pressure.

Three A.S.T.M. specifications for condenser tubes and ferrule stock for unfired pressure vessels have been added by the Committee for incorporation in the Material Specifications Section of the Code. These specifications are as follows: Specifications S-29 for Seamless 70-30 Brass Condenser Tubes and Ferrule Stock (A.S.T.M. Designation B55-33); Specifications S-30 for Seamless Muntz Metal Condenser Tubes and Ferrule Stock (A.S.T.M. Designation B56-33); Specifications S-31 for Seamless Admiralty Condenser Tubes and Ferrule Stock (A.S.T.M. Designation B44-33).

Since the adoption in 1934 of a new set of rules for flat heads, the desirability of providing for additional types of flat heads was reported on by the Subcommittee on Special Design. As a result, additional rules have been added to cover the designs of crimped flat heads, of screwed-in flat heads, and of flat heads with mechanical locks.

Further study has been given to the Rules for Bolted Flanged Connections which required qualification of some of its requirements. As a result, revisions have been made of some of its requirements to eliminate the inconsistencies.

Consideration has been given to a proposal for coordination of the A.S.M.E. and the A.P.I.-A.S.M.E. Codes for Unfired Pressure Vessels. As a result, a special committee has been appointed to revise Section VIII of the A.S.M.E. Boiler Code which will embody the best features of the A.S.M.E. and the A.P.I.-A.S.M.E. Codes for Unfired Pressure Vessels.

As a result of data submitted to substantiate the reliability and safety of the flame-cutting method of preparing the edges of plate for pressure vessels to be welded under the requirements of the Code, the Code has been revised to permit the use of this method.

New rules have been adopted and incorporated in the Unfired Pressure Vessel Code to cover the use of electric-resistance butt welding for unfired pressure vessels. The requirements are similar to those in the Power Boiler Code.

After considerable experimentation and work, the Special Committee on X-Ray Requirements has selected a master set of radiographs, reproductions of which are now available.

The Committee has cooperated with the U. S. Naval Experiment Station in submitting comments on its specifications for boiler-water gages, including glasses, frames, fittings, illuminators, etc.

New rules for the construction of marine boilers and pressure vessels adopted by the U. S. Bureau of Navigation and Steamboat Inspection and based upon a report of the Special Committee to Coordinate the Marine Boiler Rules were finally adopted and issued. The part taken by the A.S.M.E. in preparing these rules was outlined in a statement published in the September, 1935, issue of *Mechanical Engineering*.

The Committee appointed V. M. Frost as its representative on the Advisory Board on Standards and Codes which was formed by the Council to establish general policies pertaining to standards and codes.

C. A. Adams, A. J. Ely, and Walter Samans were appointed members of the Main Committee of the Boiler Code Committee. John A. Darts was appointed a member of the Subcommittee on Heating Boilers. C. E. Bronson was appointed to replace M. F. Moore as a member of the Subcommittee on Heating Boilers. E. O. Waters was appointed a member of the Subcommittee on Special Design. Commander W. P. Portz replaced Commander R. F. Frellsen as the representative of the Bureau of Engineering of the U. S. Navy Department.

The Committee reports with regret the death of C. D. Thomas, chief boiler inspector of the State of Oregon and a member of the Conference Committee to the Boiler Code Committee.

The States of Maine and North Carolina have adopted the A.S.M.E. Boiler Code as their standard for the construction of power boilers, which makes a total of 21 states in which the Code is operative.

Respectfully submitted,

The Boiler Code Committee

F. R. LOW, *Chairman*
D. S. JACOBUS, *Vice-Chairman*
C. W. OBERT, *Honorary Secretary*
C. A. ADAMS
H. E. ALDRICH
W. H. BOEHM
R. E. CECIL
F. S. CLARK
W. F. DURAND
A. J. ELY
E. R. FISH
V. M. FROST
C. E. GORTON
A. M. GREENE, JR.
F. B. HOWELL
J. O. LEECH
M. F. MOORE
I. E. MOULTROP
C. O. MYERS
H. B. OATLEY
JAMES PARTINGTON
WALTER SAMANS
C. L. WARWICK
A. C. WEIGEL
H. LEROY WHITNEY

BOND ISSUE

During the current fiscal year, the Bond Issue Committee has received subscriptions for $8200 for the purchase of Certificates of

Indebtedness, bringing the total up to $44,650, of which $600 in certificates are held in the treasury of the Society, being gifts of the purchasers.

Collateral of the face value of $68,750 is being held by the Trustees of the Certificates of Indebtedness. The interest due on the outstanding certificates of indebtedness was paid in full on January 1 and July 1, 1935, and for the coming fiscal year there is provision in the budget for the payment of interest when due and for the retirement of 10 per cent of the face value of the certificates outstanding.

It is expected that further activity of this Committee will not be required and it therefore requests its discharge.

Respectfully submitted,

DEXTER S. KIMBALL, *Chairman*
PAUL DOTY
W. A. HANLEY
C. F. HIRSHFELD
JOHN H. LAWRENCE
W. R. WEBSTER
W. H. WINTERROWD
ERIK OBERG, *Ex-Officio*

CAPITAL-GOODS INDUSTRIES

The situation in the captial-goods industries in the United States has been so confused and uncertain during the current year, largely because of the impact of economic forces without the industries, that the committee has found little that it could do effectively.

One research project was carried to completion, however. The January, 1935, issue of *Mechanical Engineering* published a paper on "Code Restrictions on Machinery and Production," by W. E. Hopkins and J. F. Nelson. It is a privilege to express appreciation of the diligent work of these two Junior Members of The American Society of Mechanical Engineers, and through them to acknowledge the cooperation of the Junior group in Society affairs.

Respectfully submitted,

L. P. ALFORD, *Chairman*
R. E. FLANDERS
L. W. W. MORROW
ERIK OBERG

CITIZENSHIP MANUAL

The committee during the course of the year has printed and distributed approximately sixty-five hundred copies of a pamphlet entitled "The Engineer's Duty as a Citizen," by Roy V. Wright. This pamphlet was the revision of one used in a discussion course given by Dr. Wright at the Newark College of Engineering. A copy of the pamphlet was mailed to every Student and Junior Member of the Society, and in the letter of transmittal the committee directed attention to the fact that:

"It is the object of this pamphlet to stimulate, help, and direct young men toward a larger participation in community activities, both political and otherwise; and if there is any matter which you think should be added or modified or deleted, the committee would value your suggestions very highly. Particularly in the matter of suggested questions, we are anxious to secure the reaction of the younger members of the Society.

"We are asking for this contribution on your part because we know that the success of this movement depends upon the younger men, and it is absolutely necessary that we have your suggestions and criticisms if this material is to be really helpful."

The committee received a very considerable number of criticisms and suggestions widely scattered over the United States. Most of the criticisms were from student members and came from suggestions arising from discussion in the student branches of the Society. The criticisms in the main were constructive and pointed out the need of some group discussion in connection with the distribution of the pamphlet.

The great need for something of this kind and the particular necessity for focusing the attention of the younger man on this phase of the fuller life of an engineer was generally accepted, and it was felt that the pamphlet, supplemented by some discussion conducted by a local engineer of prominence, would be particularly fruitful.

Dr. Wright has during the past few months made a further revision, and this is now ready for mailing. The committee contemplates the distribution of this pamphlet to the student branch advisers in the various colleges in the United States with the particular hope that it will furnish the basis for discussion on the part of the student members of the Society.

Respectfully submitted,

ALLAN R. CULLIMORE, *Chairman*
ROY V. WRIGHT
W. H. WINTERROWD
LILLIAN M. GILBRETH
JOS. W. ROE

COOPERATIVE RELATIONS

This Committee was asked to study the problem of the relations of the Society to the various bodies in engineering and the relations between the functions of the joint bodies. Its report was presented to the Council in June, 1935, and the recommendations are under consideration by the Council.

Respectfully submitted,

HAROLD V. COES, *Chairman*
ROBERT I. REES
D. ROBERT YARNALL

THE ECONOMIC STATUS OF THE ENGINEER

The general economic status of the engineering profession has improved somewhat during the past year. The improvement in the manufacturing industries has been reflected in the slight improvement in engineering employment. Durable-goods industries are still operating at only partial capacity, and although statistics available show that losses were incurred in these industries during the past year the earnings of the group as a whole have increased.

In cooperation with the American Engineering Council the United States Department of Labor conducted a questionnaire study of the earnings of engineers. About seventy thousand responses were received by the Department. These are being tabulated and it is expected that a report will be available shortly after the first of October. This Committee is holding itself in readiness to interpret the findings of the study in so far as they affect mechanical engineers and present them for publication and discussion if such discussion will prove helpful.

Respectfully submitted,

C. F. HIRSHFELD, *Chairman*
D. S. KIMBALL
N. B. OATLEY
H. L. WHITTEMORE
W. E. WICKENDEN
C. N. LAUER
W. L. DUDLEY, *Ex-Officio*
W. L. ABBOTT, *Ex-Officio*

ENGINEERING HISTORY

At the 1934 Annual Meeting of The American Society of Mechanical Engineers a session on Engineering History was held, at which a resolution was adopted asking the A.S.M.E. to take the lead in organizing a joint movement for the study of Engineering History.

A committee has been organized with representatives on it from the A.S.C.E., A.I.M.E., and the A.I.Ch.E.

Progress is being made.

Respectfully submitted,

GEO. A. ORROK
JOS. W. ROE

JUNIOR PARTICIPATION

The Committee on Junior Participation approved the following methods for stimulating interest among the 3500 junior members of the Society in mechanical engineering as a profession and in the work of the Society:

1 All Local Sections should arrange dinner meetings at which at least two members should be assigned to each Junior Member with the idea of bringing the Juniors in contact with the older members of the profession in their own localities, and with the hope that personal friendships would thereby be stimulated.

2 Meetings should be arranged at which the program would be entirely in the hands of the Juniors, or, if this is not possible, that Juniors should be urged to present papers at Section Meetings.

3 Juniors should be encouraged to take part in the discussion of papers at Section Meetings, by appointment, and by sending them advance copies of the papers if possible.

4 Juniors should be urged to prepare articles for publication in *Mechanical Engineering*, and the Publications Committee should be asked to give special consideration to these papers.

5 Members of the Society should be designated as special advisers to Junior Members in their Sections.

6 The Committee should offer its assistance to General Rees of the E.C.P.D. in carrying out his program for recent graduate engineers with reference particularly to developing the characteristics which the proposed examination for certification demand.

7 In every Local Section one or more of the older and more promi-

nent members should be requested to bring about the proper mental attitude of the members in their locality with reference to the younger entrants to our profession. Invitations to Juniors to the homes of the older members should be the more general custom.

8 The Secretary of the A.S.M.E. should send every year a list of Junior Members in each Local Section to the chairman or secretary of that Section. Upon the receipt of this list the chairman should send a communication to each Junior Member inviting him to visit him personally and to attend the meetings of the Section. In the case of large Sections the chairman may wish to divide the Junior list among a number of prominent members who will be willing to correspond and to show special courtesies to a small group of Juniors.

This program is being carried out in cooperation with the Committee on Local Sections.

Respectfully submitted,

D. B. Prentice, *Chairman*
W. A. Hanley
A. A. Potter
W. H. Winterrowd

MANUAL OF PRACTICE

The procedure adopted by the Committee looking to the ultimate preparation of a manual for the Society has been that of citing specific instances or classes of problems with which the manual should be concerned. At the Council Meeting of December, 1934, there was presented a statement concerning the engineering duties and practices to be expected of the engineer and the manufacturer, respectively. This statement was approved by Council and published in *Mechanical Engineering* for February, 1935. Subsequent to the appearance of this statement the Committee has had numerous interviews with consulting engineers and manufacturers in various commercial fields. There has been a unanimity of approval in all cases for the principles contained in the statement.

Since the promulgation of the Committee's first statement several other specific matters have been brought to our attention but it is unfortunate that because of their limited scope no action is considered advisable at this time.

On the other hand the Committee feels that the general membership of the Society should be kept constantly aware of our problems, particularly through the issues of *Mechanical Engineering*, and the Committee again urges that a definite program be initiated to ask the Society membership for cases, items, and matters which might be the subject of Committee discussion and action.

The Committee regrets the retirement of W. A. Shoudy as his leadership was of inestimable worth.

Respectfully submitted,

B. F. Wood, *Chairman*
Alfred Iddles
Wynn Meredith
J. M. Todd
C. G. Spencer, *Ex-Officio*
Theodore Baumeister, Jr., *Acting Secretary*
Philip Werner, *Junior Adviser*

POLICIES AND BUDGET

The first phase of this Committee's work, completed in 1933, was a study of Society operation and finances used as a basis for a reduced budget for 1933–1934. The second phase of its work, which is still under way, was initiated with a new statement of Society purposes, approved for inclusion in the By-Laws by Council in December, 1934, the suggestion of changes in membership grades which are to be voted by the membership at the end of the fiscal year, and certain modifications in Society organization to simplify it and make it more responsive to the needs of the membership.

Respectfully submitted,

Harry R. Westcott, *Chairman*
L. P. Alford
B. M. Brigman
H. M. Burke
W. H. Carrier
Alfred Iddles
A. C. Jewett
J. N. Landis
John H. Lawrence
R. G. Macy
A. L. Maillard
M. C. Maxwell
Erik Oberg (*Ex-Officio*)
J. W. Parker
L. K. Sillcox
W. H. Winterrowd

PUBLIC AFFAIRS

The Committee on Public Affairs was authorized by Council in July, 1934, to recommend to the Society policies in regard to participation in national, state, and local affairs in accord with a modification of the By-Laws made in December, 1934, which included the following statement:

"Increasing the usefulness of the organized engineering profession by ..... (*C*) encouraging engineers to participate in public affairs."

To insure complete cooperation with American Engineering Council, this Committee was constituted of present and past representatives of this Society on the Council.

This Committee has held one meeting at which a number of procedures for carrying out the purposes of the Committee were canvassed. Further activity depends in some measure on the program of American Engineering Council.

Respectfully submitted,

Roy V. Wright, *Chairman*
L. P. Alford
Paul Doty
R. E. Flanders
A. A. Potter
J. W. Roe
D. Robert Yarnall

CALVIN W. RICE MEMORIAL

The Committee on Calvin W. Rice Memorial was appointed in October, 1934, to consider suggestions for a suitable memorial to Dr. Calvin W. Rice.

As guides for the purposes of suitable memorials, the Committee has selected Dr. Rice's keen interests in international friendliness, in young men, and in the portion of the membership not able to attend the great annual gatherings of the Society in New York.

The Calvin W. Rice Memorial Lecture has been established. This lecture is to be given at the Semi-Annual Meeting of the Society by a distinguished foreign engineer or other person with international interest. In the record book of the lectureship are the names of Dr. John H. Finley of New York, who delivered the tribute to Doctor Rice at the 1934 Annual Meeting, and of Dr. Adolph Meyer of Switzerland, lecturer at Cincinnati in June, 1935.

Further plans are under consideration.

Respectfully submitted,

H. N. Davis, *Chairman*
W. F. Durand
C. E. Fuller
J. W. Parker
J. D. Cunningham
R. L. Sackett
C. N. Lauer
E. W. O'Brien

GEORGE WESTINGHOUSE BUST

This committee is charged with the responsibility of advising the Council as to the custody of a Bust of George Westinghouse, Honorary Member and Past-President of the A.S.M.E.

At a meeting of the Committee, December 5, 1934, Ambrose Swasey was elected honorary chairman of the committee, Dean Dexter S. Kimball chairman, L. B. Stillwell, vice-chairman, and C. E. Davies, secretary. On nomination of the Committee Messrs. L. A. Osborne and Karl T. Compton were added to its personnel.

Respectfully submitted,

D. S. Kimball, *Chairman*
Ambrose Swasey, *Honorary Chairman*
L. B. Stillwell, *Vice-Chairman*
C. E. Davies, *Secretary*
W. W. Atterbury
K. T. Compton
S. W. Dudley
C. N. Lauer
L. A. Osborne
C. F. Scott

Joint Activities

Reports for the year 1933–1934 were also presented to the Council by representatives of the Society on a number of joint activities. Reference to some of these will be found in the Report of the Council and others are presented or summarized in the following pages.

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

Section M

During the period between October 1, 1934, and September 30, 1935, one meeting of Section M, Engineering, of the American Association for the Advancement of Science was held in connection with the winter meeting of the Association, in Pittsburgh, December 27,

1934, to January 2, 1935. The principal feature of the meeting was the address of the retiring vice-president, Dr. C. F. Kettering on "Some Future Problems of Science and Engineering." The program included also problems of stress distribution and plastic deformation in metals and joint sessions on Cost and Cost Theory and History of Interesting Technological Developments, with the Section on Social and Economic Sciences and the Section on Historical and Philological Sciences, respectively.

No session of Section M was held during the summer meeting of the A.A.A.S. in Minneapolis in June, 1935.

The coming winter meeting of Section M will be held at St. Louis and active steps are under way to develop a strong engineering program.

Respectfully submitted,

SUMNER B. ELY, JOHN R. DUPRIEST } *A.S.M.E. Representatives*

AMERICAN BUREAU OF WELDING

The relations between the A.S.M.E. Boiler Code Committee and the American Bureau of Welding continue to be on the most cordial basis and all items of welding which are referred jointly to the American Welding Society Committee and our Subcommittee on Welding are given most careful attention.

Cooperation of the American Welding Society is perfect, and our relations most satisfactory.

Respectfully submitted,

JAMES PARTINGTON, *A.S.M.E. Representative*

AMERICAN ENGINEERING COUNCIL

The membership of The American Society of Mechanical Engineers participates in national public affairs of engineering significance through the American Engineering Council, with offices in Washington, D. C. In keeping with its long-established policy of seeking to serve the common interests of all engineers as well as those of its own members, the A.S.M.E. has actively supported the work of the A.E.C. from the beginning.

The membership of American Society of Mechanical Engineers is officially represented in these common activities of the American Engineering Council through five delegates. Both nationally and locally, many other members participate in the work. Their delegates, on the occasion of the annual meeting of the Council, held in January, in Washington, and through the year, through their service on committees of the Council and through coordination between the staff of the A.S.M.E. and the staff of the Council forward many matters of common interest to all engineers and especially to the members of our society.

With the great increase in the legislative and administrative activities of the Federal Government as they have affected industry and engineering during the past three years, there has been ample opportunity for mechanical engineers to relate themselves to this work. Contacts in Washington, long established, understanding of the procedures of the Federal Government, and the development of staff and committees to relate the American Engineering Council to these rapidly growing government activities have enabled the profession to participate intimately through what has come to be called the Washington Embassy of American engineers and engineering.

This rapid development of Government agencies relating in one way or another to the development of our natural resources, our public utilities, our capital goods, including machinery and construction, and our transportation and communication industries, has made the task of the American Engineering Council very much greater than ever before.

The following of legislation, the interpretation of Executive Orders, and the aiding in the staffing of government bureaus and consultations on policies and methods of fulfilling much of the new legislation has related American Engineering Council in new ways to our national program. It is literally true that more legislation has been passed directly affecting the welfare of engineers during the last two years than through the ten years preceding. To the original purpose of the founders of the American Engineering Council to provide an organization to aid in molding legislation on engineering matters of public interest has been added the past year or two the practical task of molding methods in the application of legislation.

The first organized approach to the study of the present widely expanded government program with regard to furnishing a means of interpreting to the profession as a whole the details of the many legislative programs took place at the annual meeting of the Council in January, 1935. In line with the need, instead of devoting the entire time to business sessions, a symposium of Federal activities was held and a first hand account was given by members of the Federal Government announcing their plans and deeper purposes as related to:

(1) Planning and the development of natural resources
(2) Construction
(3) Durable-goods industries.

Among the prominent Federal officials who addressed the several sessions were: Hon. Marriner S. Eccles, Governor, Federal Reserve Board; Hon. M. L. Wilson, Assistant Secretary of Agriculture; Dr. Harlow S. Person, of the National Resources Board; Capt. R. S. Patton, director, U. S. Coast and Geodetic Survey; Dr. Isador Lubin, commissioner of labor statistics; Col. D. H. Sawyer, director, Federal employment stabilization office; Clarence McDonough, chief of the engineering division, Public Works Administration; Charles C. Anthony, industrial adviser, and Thomas Hibben, chief engineer, Federal Housing Administration; Wm. P. Witherow, vice-chairman, industrial advisory board, National Recovery Administration; and R. E. W. Harrison, chief, machinery division, U. S. Bureau of Foreign and Domestic Commerce.

Dr. Harold G. Moulton, president, The Brookings Institution, addressed the largest group ever assembled to attend the Annual Dinner. He discussed the economic trends leading to the depression.

The opportunity to hear these Government officials was afforded not only to members of the assembly and to engineers resident in Washington and environs, but also to secretaries of engineering societies who had assembled from all parts of the United States to discuss their own problems at a conference held under the auspices of American Engineering Council and in conjunction with its annual meeting.

In line with this aggressive policy of coordination, the annual assembly voted to provide a method for further correlation of activities of engineers in public affairs through a plan for local membership whereby local organizations as well as national organizations could participate in the work of Council without duplicating financial structures. To this end a committee, consisting of Messrs. Bickelhaupt, Alford, Trullinger, and Wendt, with Dr. McClellan as adviser, proposed a plan of membership to open the way to affiliation with the Council to a large number of state and local engineering organizations. This new plan permitted such an organization to become a member for the nominal fee of $25 per thousand members per year, the expenses of the delegate at the annual meeting to be paid by the member-organization.

Under this plan the following engineering organizations have joined Council since the first of the year:

Arkansas Engineers Club
Cleveland Engineering Society
Engineers Club of Baltimore
Engineers Club of Columbus
Engineers Club of Philadelphia
Engineers Club of St. Louis
Engineering Societies of New England
Engineering Society of Western Pennsylvania
Florida Engineering Society
Louisiana Engineering Society
Michigan Engineering Society
Providence Engineering Society
South Carolina Society of Engineers

A third policy established during the year was to strengthen and further cooperation between member-organizations and the Council through the dissemination of information as widely as possible. With the cooperation of the editors of the journals of the Founder Societies and of the editors of the other technical journals, regular monthly reports of Council's activities are published for the information of the members.

Beginning July 1, 1935, the information service has been further strengthened by the establishment of a plan of placing on file in the offices of member-organizations the basic laws of the administrative and executive orders dealing with engineering matters. This additional "Washington Embassy Service" has been developed from the viewpoint of increasing not only the general knowledge of members of the societies but of making available locally information concerning government expenditures which would constitute the basis for employment of many individuals.

A mimeographed monthly news letter is mailed to delegates, committee members, and officers of local sections, for their personal information.

The fourth policy in course of development is the establishment in each state of public-affairs committees. In accordance with a plan approved at the annual meeting in January, these committees are

to be coordinated with the national Committee on Public Affairs. This plan of coordination of state committees on public affairs rests for its success upon the cooperation of local sections and of local engineering societies and can only come about gradually through the active participation of local organizations in districts not now represented in national affairs. The general purpose of this program is to take advantage of present organizations to provide the machinery for securing the organized discussion of public problems by engineers instead of setting up any new organization or duplicating the activities of others.

In the fifth place, to provide coordination within the committee activities of American Engineering Council itself. The Executive Committee of Council approved the development of three broad classes of committees:

(1) Public-affairs committees
(2) Engineering and economic research and inquiry committees
(3) American Engineering Council operating committees.

In order to simplify procedure, in so far as possible, special committees have been made subcommittees of standing committees.

In the first group is included the Committee on Public affairs and related to it as subcommittees are the committees on administration of public works, aeronautics, competition of Government with engineers in private practice, engineers water power policy, flood control, patents, rural electrification, and water resources, also representation on the Advisory Council of the Federal Board of Surveys and Maps.

In the second group are included the committee on engineering and allied technical professions, the relation of consumption, production and distribution, and the naval towing tank; also representation on the board of directors of the National Bureau of Economic Research and the Board of National Councilors of Purdue Research Foundation.

In the third group, are included the committee on constitution and by-laws, to which the committee of tellers serves as a subcommittee, the finance committee, committee on membership and representation, publicity and publications, and regional activities.

The activities of four committees of the Council will have special interest for members of The American Society of Mechanical Engineers.

(1) The Committee on Aeronautics, under the chairmanship of Grover C. Loening, is proceeding actively in carrying out the program by Council. The major features of the program are:

(*a*) That a sum of $50,000 be appropriated each year as an addition to the budget of the National Advisory Committee for Aeronautics and subsequently earmarked for research work to be done by universities and other institutions of public learning.
(*b*) That none of such appropriation should be employed for educational purposes, construction of buildings or basic equipment.
(*c*) That a committee be appointed entitled "Committee on Cooperation between the N.A.C.A. and Universities Interested in Aeronautical Engineering;" further that this committee should contain representatives of the National Advisory Committee for Aeronautics, of the Army, of the Navy, of the Department of Commerce, of the industry selected by the Aeronautical Chamber of Commerce, and of the universities engaged in aeronautical work.
(*d*) That scientific workers at the universities or technical institutes desiring to conduct research would submit to the N.A.C.A. a formal plan for such research, stating the object of the investigation, and the personnel available for execution for a given piece of research. The allotment of funds would be entirely within the discretion of these committees.
(*e*) Publications of worth-while research would be in the form of N.A.C.A. technical reports or technical notes.

Dr. Alexander Klemin, secretary of the committee, has made several contacts in Washington in the interest of forwarding the program.

(2) Committee on Engineering and Allied Technical Professions.

The work of this committee is described in a later part of this statement.

(3) The Committee on Patents, under the chairmanship of Dean A. A. Potter, is studying ten bills relating to patents and to copyright laws which are pending before the Patents Committee of the House of Representatives. The legislation deals with such subjects as provision of counsel for the defense and prosecution of rights of indigent patentees, recording of patent-pooling agreements and contracts with the Commissioner of Patents, permitting single signature in patent applications and validating joint patent for sole invention; limiting the life of a patent to a term commencing with the date of application, compensation of owner for infringement of patents by U. S. Government, prevention of fraud, deception, and other improper practice in connection with business before the U. S. Patent Office, providing protection for registration of designs for textiles and other materials, making effective in the United States provisions of the International Convention for the Protection of Industrial Property relating to time limitations in which a foreigner may make application for a design patent, or a trade-mark application.

A.S.M.E. members of the committee are Dean A. A. Potter, chairman, W. C. Lindemann, Edwin J. Prindle, and E. N. Trump.

(4) The Committee on Water Resources, under the chairmanship of Wm. S. Conant, a former representative of the A.S.M.E. on the assembly of the Council, has submitted a report which the Council approved. The report points out that two fundamental needs for a National Water Resources Policy are: (1) complete and correlated data; (2) comprehensive study of water-control legislation. A federal Bureau of Water Resources is endorsed in principle and an interdepartmental Board of Water Resources Investigations is recommended to correlate investigational functions of federal units. Extension of the work of the Water Planning Committee is recommended through a National Advisory Water Planning Agency for comprehensive, integrated drainage-basin planning.

The annual report of the Council will include an analysis of the work of all the committees.

Procedure as to Legislation

With so much of engineering interest at stake in the session of Congress just passed, it seems timely to summarize the policies and methods followed by the Council in legislative matters. The staff has had for its guidance:

(1) Policies adopted by the Assembly and Administrative Board at Annual Meetings.
(2) Advice on new matters from special and standing committees.
(3) Precedent of several years on matters which do not require new rulings.
(4) The viewpoint of public interest as a fundamental reason for participating in legislative activities.
(5) The corollary viewpoint of advancing the economic status of the profession.

The factors of public interest and the status of the profession are closely related in problems constantly arising in connection with the Federal program, due to the extraordinary relation of government to business at present. *Competent engineers must be employed where engineers belong if the public interest is to be protected* against faulty planning and wasteful execution.

Although the "Washington Embassy" of engineers may not jump into drastically new lines of action in the name of the profession without due consultation and approval, it is not hemmed in by ponderous procedure. The precedent set on past work, recognized as in the interest of the profession, gives sufficient latitude for quick action on immediate, practical steps and leaves only the long-range phases for more deliberate action.

Legislation which Council follows for engineers includes the following general categories:

(1) *Construction:* Federal appropriations and administrative machinery for construction. (The work-relief bill.)
(2) *Development of Industries:* Amendments to the National Industrial Recovery Act, new patent legislation, etc.
(3) *Development of Natural Resources:* Bills relating to water power, navigation, reclamation, soil-erosion control, surveys and maps, and similar public activities wherein the engineer serves in the development of "our national plant."
(4) *Engineers' Welfare:* General legislation in the field of unemployment insurance, old-age pensions, etc; bills such as civil-service measures, engineers' compensation on federal and relief work, etc., more directly affecting the engineer.

Scores of bills under each of the above headings have been under consideration. With its limited staff the Council has found itself obliged to concentrate on those of the greatest importance to the profession. Where needed, information and arguments have been presented to Congressional Committees in open hearings or otherwise. The Public Affairs Committee and its subcommittees are sent copies of important bills and documents and kept generally informed.

Cooperation With Government and Placement of Engineers

As indicated above, so active have been the developments of Government agencies that what might be called a by-product in normal Council activities has become very important.

From time to time the Council has been called upon to recommend engineers for positions and works in cooperation with the Engineering Societies Employment Service in which the A.S.M.E. participates. At the request of the Government, names of engineers technically qualified for positions, have been submitted to various agencies. Some 74 Government divisions now employ engineers.

In addition to working from day to day on placement problems, the Council has made a number of analyses of the number and distribution of engineers. These analyses led to a further activity, namely, the census of engineers which is being made with the cooperation of the national, state, and local societies. Under the direction of a standing committee of Council on engineering and allied technical professions the Bureau of Labor Statistics of the U. S. Department of Labor

conducted a special census of engineers with particular reference to educational background, occupation, income, and present status of employment. The results of the questionnaire will be presented in a report by the Bureau of Labor Statistics and it is expected to have a far-reaching influence on policies concerning engineering education and percentage distribution of engineers both by profession and by industry, the basis for professional consideration or compensation, and other policies of direct interest to the individual engineeer.

Officers and Delegates

President J. F. Coleman, past-president of the American Society of Civil Engineers, and consulting engineer of New Orleans, La., is serving the second of his two-year term as president. C. O. Bickelhaupt, of New York, Alonzo J. Hammond of Chicago, Paul Doty of St. Paul, and W. H. Woodbury of Duluth are vice-presidents. C. E. Stephens of New York is treasurer.

Work of the Council in Washington is essentially a staff activity. The personnel of the organization is headed by Frederick M. Feiker, as executive secretary, serving his second year. During the year, with the approval of the executive committee, Lemuel V. Reese joined the organization as assistant secretary. Mr. Reese, joining the staff from the F.E.R.A. organization, brought a first-hand knowledge of the present government personnel, as well as a background of experience in engineering and industry. The secretaries of Council and of the national member organizations meet each month to assure coordination of effort.

The last three years have seen a constantly increasing opportunity for the engineering profession to contribute its viewpoint and constructive suggestion in public affairs. The delegates of The American Society of Mechanical Engineers believe it fortunate that there has been available in Washington an organization and headquarters for service, the American Engineering Council. The organization has been pointed to by societies of the other professions as an example to be followed.

The American Society of Mechanical Engineers, from the beginning has supported this coordinated center for common action and its representatives believe it has possibilities of even wider and greater usefulness to the public and indirectly to the profession.

Respectfully submitted,

R. E. Flanders, *Chairman*
L. P. Alford
Paul Doty
A. A. Potter
D. Robert Yarnall
H. V. Coes } *Alternates*
J. W. Roe }

AMERICAN STANDARDS ASSOCIATION

On April 25, 1935, the Standards Council of the American Standards Association voted to reorganize parts of the Association's structure to cope with the widening range of standards work. The chairman of the Standards Council, J. C. Irwin, was authorized to organize a mechanical committee, a textile committee, and a building-code correlating committee. Other intra-industry committees will be formed from time to time as the need develops. Because of the increased interest in standards for consumer goods, an advisory committee on ultimate consumer goods was authorized. This body will advise the Standards Council on the problems in this field.

During the year the Federal Housing Administration became a member body of the American Standards Association, and the American Automobile Association, Manufacturing Chemists Association, and the Motor Truck Association of America have become associate members. A total of 36 member bodies represents 40 national organizations. Of these nine are governmental bodies, eight are technical societies, and 19 are trade associations. The 13 associate members consist of six technical societies and seven trade associations. More than 600 national groups are now cooperating in developing and revising standards and safety codes under the procedure of the American Standards Association.

Officers of the Association are: Howard Coonley, representing the A.S.M.E., president; F. E. Moskovics, representing the Society of Automotive Engineers, vice-president; P. G. Agnew, secretary, and Cyril Ainsworth, assistant secretary. J. C. Irwin is chairman of the Standards Council and F. M. Farmer is vice-chairman.

As of September 10, 1935, the Association has approved 301 standards, 65 of which are in the mechanical field. Since October 1, 1934, 36 standards have been approved, six important projects being in the mechanical field. During the past 12 months nine requests for the initiation and approval of existing standards were received.

Respectfully submitted,

C. W. Spicer, *A.S.M.E. Representative*

THE ENGINEERING FOUNDATION

The most important feature of The Engineering Foundation's year of activity was the celebration of the twentieth anniversary of its founding. For the celebration the regular meeting, October 18, 1934, was selected. Dr. Swasey accepted the Foundation's invitation to be the guest of honor. Invitations to a subscription dinner drew acceptance from 78 persons: past and present members of the Foundation and United Engineering Trustees, Inc., officers of the Founder Societies and cooperating organizations of engineers and scientists, and a few personal friends of the Founder. Chairman Charlesworth presided and at the close of the program presented Dr. Swasey an engrossed testimonial signed by all the persons present. Addresses were made by Harry P. Charlesworth, Frank B. Jewett, a former vice-chairman of the Foundation, Karl T. Compton, president of Massachusetts Institute of Technology, and representing Harold V. Coes, president, United Engineering Trustees, Inc., vice-president George L. Knight.

Total book value of endowments, December 31, 1934, was $882,000; and the E. H. McHenry bequest in the hands of executors until decease of two life beneficiaries, appraisal at probate of will in 1931, was approximately $400,000. These capital funds are held and administered by United Engineering Trustees, Inc. The net income from endowment was $5000 in 1915 and $40,000 in 1934. The Foundation Board has discretionary power in the use of income. For enterprises which the Foundation has aided, large contributions of money, services, and materials have often been obtained from organizations and firms in industry.

The activities in 1934 included investigations of concrete and reinforced-concrete arches, earths and foundations, and plastic properties of concrete in the civil-engineering field; critical review of the world's literature on alloy irons and alloy steels since 1890, and barodynamic research, both sponsored by the American Institute of Mining and Metallurgical Engineers; in cooperation with committees of The American Society of Mechanical Engineers, studies of effect of temperature on properties of metals, cutting of metals, thermal properties of steam, mechanical springs, riveted joints, wire rope, boiler-feedwater studies, fluid meters, and strength of gear teeth; under sponsorship of the American Institute of Electrical Engineers and electric-welding research on pure-iron electrodes.

Assistance was given also to the Engineers' Council for Professional Development, Personnel Research Federation, and for a survey by The Engineering Index.

The present officers of the Engineering Foundation are: Chairman, Harry P. Charlesworth; first vice-chairman, D. Robert Yarnall; second vice-chairman, Edwards R. Fish; members of executive committee, Otis, E. Hovey, John V. N. Dorr, and Edwards R. Fish; representative on executive board of National Research Council, Harry P. Charlesworth; director and secretary, Alfred D. Flinn.

Revised Rules of Administration have been drafted in accordance with the revised by-laws of the United Engineering Trustees, Inc. Copies of these rules are now available in pamphlet form. They govern applications for grants and the other procedures of the Foundation.

Welding Research. On the invitation of A.I.E.E. a conference was held on April 22 of 18 representatives of technical organizations and the industry. This conference discussed plans and possibilities for a comprehensive welding research project, and nominated eight members of a Committee to be appointed by the Foundation. This Committee was appointed on April 25 and held a meeting on May 6 attended by all members, the chairman of the conference and the director of the Foundation. This meeting stated the principal elements of a program, appointed a Subcommittee on Literature and a Subcommittee on Research Projects, and directed that a canvass be made of contributions to be expected from industrial concerns, especially of services and materials for the research.

The members of the Welding Research Committee are: Comfort A. Adams, chairman, professor of electrical engineering, Harvard Engineering School; David S. Jacobus, advisory engineer, Babcock and Wilcox Company; Henry M. Hobart, consulting engineer, General Electric Company; James H. Critchett, vice-president, Union Carbide and Carbon Research Laboratories; Glen F. Jenks, commanding officer, Watertown Arsenal; Frederick T. Llewellyn, research engineer, U. S. Steel Corporation; John J. Crowe, engineer in charge of apparatus research and development for the Air Reduction Company; and William Spraragen, secretary, consulting engineer.

Respectfully submitted,

W. H. Fulweiler (1936)
D. Robert Yarnall (1936)
A. E. White (1939)
} *A.S.M.E. Representatives*

ENGINEERING SOCIETIES EMPLOYMENT SERVICE

The Engineering Societies Employment Service, a joint activity conducted by the four National Societies, maintains offices in the Engineering Societies Building in New York, at Chicago, and San Francisco. Each of these offices is in charge of a manager who is supervised by a local Advisory Board. Each Advisory Board is made up of a member of each of the Four National Societies.

In New York, the Society of Naval Architects and Marine Engineers participates; in Chicago, the Western Society of Engineers; and in San Francisco, the Engineers' Club and the California Section of The American Chemical Society.

It has been found necessary during the last few years to restrict the service to members of the participating organizations, unless no member can be found who has the necessary qualifications. In New Yorl , non-members of the Societies have been aided through a special activity, known as the Professional Engineers' Committee on Unemployment, which was organized and conducted by the New York Local Sections of the four National Societies. In Chicago and San Francisco, where there was no activity comparable to that carried on by the P.E.C.U., the Employment Service served a reasonably large number of non-members. This was in cases of openings for stationary engineers, chemical engineers and special classifications generally found within the membership of the participating societies. The fees collected through these placements reduce the expense of operation on the part of the societies to underwrite any deficit that occurs from year to year.

Tables showing the registrations and placements for each of the offices follow directly.

REGISTRATIONS, BY SOCIETIES

For New York, Chicago, and San Francisco offices from August 1, 1934, to July 31, 1935, inclusive

	A.S.C.E.	A.I.M.E.	A.S.M.E.	A.I.E.E.	W.S.E.	S.N.A.	A.C.S.	E.C.	N.M.	Total
New York.....	341	110	405	313	4	9	..	..	74	1256
Chicago.....	56	32	83	88	62	..	..	..	281	602
San Francisco....	92	97	92	62	..	..	17	..	252	612
Total.....	489	239	580	463	66	9	17	..	607	2470

PLACEMENTS, BY SOCIETIES

For New York, Chicago, and San Francisco offices from August 1, 1934, to July 31, 1935, inclusive

	A.S.C.E.	A.I.M.E.	A.S.M.E.	A.I.E.E.	W.S.E.	S.N.A.	A.C.S.	E.C.	N.M.	Total
New York.....	116	27	242	112	1	1	..	..	116	615
Chicago.....	14	1	32	18	52	..	..	..	130	247
San Francisco....	18	27	36	9	1	1	5	1	80	178
Total.....	148	55	310	139	54	2	5	1	326	1040

Respectfully submitted,

C. E. DAVIES, *A.S.M.E. Representative*

ENGINEERS' NATIONAL RELIEF FUND

From August 15, 1934, to September 4, 1935, the Board of Direction of the Engineers' National Relief Fund considered and acted upon seven applications for loans. Of these cases two were those of the A.S.C.E., one, A.S.M.E., and four were of non-members. A total of $570 was paid out of which $345 was returned during this time.

Except in extreme and exceptionally worthy cases, the Loan Fund was available only to members of the four National Societies or to members of local societies with which the Local Sections of the four National Societies are affiliated. It was understood that the loan was to be on the basis of a revolving fund, made for the briefest possible period consistent with providing an emergency service. The basis for this policy was to have the fund circulating to the greatest extent. Assistance to engineers in dire distress was generally determined by a study of answers to a group of questions, which answers wherever possible were furnished as a result of investigation by local organizations.

Respectfully submitted,

WILLIAM G. ATWOOD (Am.Soc.ofC.E.), *Chairman*
LANDON F. STROBEL (A.I.M.E.)
E. B. MEYER (A.I.E.E.)
WILLIAM A. SHOUDY (A.S.M.E.)[1]

[1] W. W. Macon, A.S.M.E. representative, died January 1, 1935. Mr. Shoudy was appointed to succeed him on the Board of Direction.

INTERNATIONAL ELECTROTECHNICAL COMMISSION

The eighth Plenary Meeting of the International Electrotechnical Commission convened on June 18, 1935, at Scheveningen, Holland, for the first week of meetings and continued during the following week in Brussels, Belgium.

There were over four hundred and fifty delegates present, of whom 14 represented the United States National Committee of the I.E.C. The American delegation was led by Dr. C. H. Sharp, President of the U.S.N.C., and the delegates were selected in the greater part from the sectional committees of the A.S.A. and the A.S.M.E. Committees on Power Test Codes. American industry, therefore, participated directly in the deliberations of the I.E.C.

Twenty advisory-committee meetings were held during the two weeks, including those on steam turbines and internal-combustion engines. The committees which held sessions at Scheveningen dealt with nomenclature, aluminum, insulating oils, rules and regulations for overhead lines, radio communication, electrical installations on ships, electric cables, electronic devices and steam turbines, while at Brussels meetings were held on symbols, lamp caps and holders, standard voltages and currents and high-voltage insulators, electric-traction equipment, measuring instruments, terminal markings, switchgear, accumulator batteries, and internal-combustion engines.

The International Electrotechnical Commission embraces practically every branch of the electrical industry and from the point of view of standardization in its broadest sense is dealing with matters of the highest commercial importance. The I.E.C. is engaged in preparing standard specifications which can serve as models for the use of the several member nations in international trade. In its 30 years of existence it has accomplished more than is generally recognized. And, although the I.E.C. recommendations are not so much in the public eye as they might be, they are constantly permeating the national standards as well as industry as a whole, where their identity or origin is often lost.

The U. S. National Committee of the I.E.C. functions as Secretariat for Advisory Committees No. 1 on Nomenclature; No. 4 on Hydraulic Turbines; No. 5 on Steam Turbines; No. 14 on Rating of Rivers; and No. 19 on Internal-Combustion Engines. During the year preparations had been made for the meetings of Advisory Committees No. 1, No. 5, and No. 19.

Advisory Committee No. 5 on Steam Turbines held a week of meetings at Scheveningen while No. 19 on Internal-Combustion Engines met for a week in Brussels. At both meetings a considerable degree of international agreement was attained, which was due, in part, to the fact that all business coming before the committees had been the subject of extended correspondence between the various national committees and the Secretariat over a period of four years. The written opinions of the various national committees were condensed into a number of printed Secretariat reports which were submitted at the meetings.

At the meetings of Advisory Committees No. 5 and No. 19, the U. S. National Committee was represented by Francis Hodgkinson, Acting Director of the Secretariat on Steam Turbines and Director of that on Internal-Combustion Engines, by K. McH. Irwin, delegate and by C. B. LePage, Assistant Director of both of the Secretariats.

Respectfully submitted,

H. N. DAVIS
E. C. HUTCHINSON
FRANCIS HODGKINSON } *A.S.M.E. Representatives*

ENGINEERS' COUNCIL FOR PROFESSIONAL DEVELOPMENT

The program of Engineers Council for Professional Development looking toward the enhancement of the status of the engineer is divided into four parts:

(1) Educational and vocational orientation of young men
(2) Cooperation between the engineering profession and engineering schools
(3) Further personal and professional development of young engineering graduates
(4) Establishment of suitable standards for professional recognition.

During the year substantial progress has been made in all elements of this program.

The most tangible progress has been made in the matter of co-operation between the profession and education. In this field the preliminary work on the program for accrediting curricula in engineering schools has been completed and as the year drew to a close steps were being taken to accredit some twenty institutions in the New England and Middle Atlantic areas.

In the establishment of suitable standards for professional recognition, two Societies, the American Society of Civil Engineers and our own, have initiated changes in their Constitutions to provide for modification of the requirements for the grade of Member and the addition of a grade of Fellow.

In the vocational orientation of young men, groups have been organized in over a score of centers throughout the United States to provide this guidance for prospective candidates for engineering schools.

In the post-college development work a leading list and a personal self-analysis blank have been perfected and distributed.

The complete report of the work of this important body may be secured for the sum of ten cents by writing to the Secretary at 29 West 39th Street, New York City.

Respectfully,

C. F. HIRSHFELD
W. E. WICKENDEN
WILLIAM L. BATT
A.S.M.E. Representatives

JOINT AWARDS

The A.S.M.E. participates in a number of joint awards. These follow together with brief statements giving the purpose of the awards, the participating bodies, the names of the A.S.M.E. representatives for 1934–1935, and the awards made between October 1, 1934, and September 30, 1935.

John Fritz Medal, for notable scientific or industrial achievement; Am.Soc.C.E., A.I.M.E., A.S.M.E., A.I.E.E.; R. V. Wright, C. N. Lauer, A. A. Potter, Paul Doty. Award for 1935 to Frank Julian Sprague.

Gantt Gold Medal, for distinguished achievement in industrial management as a service to the community; Institute of Management Division of A.S.M.E.; Jos. W. Roe, David B. Porter, F. E. Raymond. Award for 1935 to Arthur Young.

Daniel Guggenheim Medal, for notable achievement in advancement of aeronautics; A.S.M.E., S.A.E., Canada, England, France, Germany, Holland, Italy, Japan; H. I. Cone, Porter H. Adams, E. E. Aldrin, B. M. Woods. Award for 1935 to William F. Durand.

Herbert Hoover Medal, for distinguished public service; Am.Soc. C.E., A.I.M.E., A.S.M.E., A.I.E.E.; Ambrose Swasey, C. N. Lauer. No award.

Alfred Nobel Prize, for most meritorious technical paper published by participating societies; Am.Soc.C.E., A.I.M.E., A.S.M.E., A.I.E.E., Western Society of Engineers; Arthur M. Greene, Jr. Award for 1935 pending.

Washington Award, for advancing human progress through engineering; Am.Soc.C.E., A.I.M.E., A.S.M.E., A.I.E.E., Western Society of Engineers; Geo. F. Gebhardt, C. B. Nolte. Award for 1935 to Ambrose Swasey.

NATIONAL MANAGEMENT COUNCIL

During the past year the National Management Council has been engaged primarily in plans for American participation in the Sixth International Congress for Scientific Management, held in London, England, July, 1935. The A.S.M.E. Management Division cooperated in this effort. The Congress was reported as being highly successful with an attendance of over 2000. The American management bodies were represented by five, all members of the A.S.M.E. The next International Management Congress will be held in the United States in 1938.

The informal suggestion from the National Management Council resulted in the planning and organizing of a series of nine meetings in the Metropolitan area under the joint auspices of the Sections of societies and associations concerned with management problems.

Respectfully submitted,

J. A. PIACITELLI,
C. W. LYTLE
H. V. COES
A.S.M.E. Representatives

NATIONAL RESEARCH COUNCIL

DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH

In the Division of Engineering and Industrial Research there are four main research projects in which steady progress is being made: highway research; electrical insulation; heat transmission; and welding research.

Highway Research. The Highway Research Board held the largest meeting of a series of 14 annual meetings on December 6 and 7, 1934. The registered attendance was 383 against 314 for last year and 282 for 1932. From the program of this meeting 20 papers have been selected for publication in the Proceedings of the Board which will be issued later (in an edition of 2000 copies). In addition the Board issues Highway Research Abstracts (mimeographed) ten times during the year for the presentation of information concerning important unpublished research work.

Among its active projects are:

(*a*) Investigations on the use of high-elastic-limit steel for concrete reinforcement
(*b*) The warping of concrete pavement slabs
(*c*) The stabilization of the surface of low-cost-type roads
(*d*) Studies on highway costs in relation to economic planning.

Electrical Insulation. The Committee on Electrical Insulation held its seventh annual meeting on October 25 and 26, 1934, at the University of Illinois with an attendance of about 90. Nineteen technical papers were presented dealing with current investigations on the theory and practice of electrical insulation. These papers have been published (in planographed form) for the committee with the assistance of the Brooklyn Edison Company.

Annual meetings of this Committee are now recognized as very useful occasions for the exchange of information in regard to current research in this country on dielectrics. In addition to publishing the papers at these meetings, the Committee has sponsored the preparation of a series of four monographs (published commercially) the last of which will be issued next spring. These monographs deal with "The Nature of a Gas," "Liquid Properties of Glass," "Liquid Dielectrics," and "Impregnated Paper Insulation."

Heat Transmission. The Committee on Heat Transmission has sponsored the preparation of a second treatise on heat insulation which will be prepared by E. C. Rack, of the Johns Manville Corporation, and is expected to be issued early in 1936. This new volume, together with the book on "Heat Transmission" of Prof. W. H. McAdams, of the Massachusetts Institute of Technology, issued in 1933, will provide a complete review of present knowledge of the transmission and conservation of heat.

Welding Research. The Committee on Fundamental Research in Welding held its fourth annual conference in New York on October 2, 1934, with an attendance of 42. The object of this Committee is to place welding engineers in touch with university physicists and metallurgists who can undertake research on fundamental problems encountered in the fusion of metals in welding, particularly iron and steel. Ten special reports were completed during the year, by collaborators with this committee and the committee is in touch altogether with some 66 research projects now in progress in university, governmental, and industrial laboratories.

Cooperation With the Science Advisory Board. The Division of Engineering and Industrial Research has also materially assisted the Science Advisory Board of the National Research Council throughout the year by maintaining contacts with government officials in the Department of Commerce. The Director of the Division has made frequent trips to Washington and has been closely in touch with a number of governmental agencies which are concerned with progress of industrial research.

Respectfully submitted,

DAVID S. JACOBUS
F. MALCOLM FARMER
BERT HOUGHTON
A.S.M.E. Representatives

UNITED ENGINEERING TRUSTEES, INC.

The United Engineering Trustees, Inc., has three departments, The Engineering Foundation, the Engineering Societies Library, and the Administrative.

Under By-Laws revised in 1934, operations have been greatly simplified and clarified. The newly created Real Estate Committee has functioned efficiently in matters outside the technical operation of the Engineering Societies Building. It has cooperated in many ways with the Finance Committee, thereby distributing the heavy responsibilities placed upon a few men already overloaded by their

professional interests. Distinct progress has been made toward the initiation of another permanent committee for the purpose of raising funds by gift, deed, bequest, or other method, "for the furtherance of research in science and engineering or for the advancement in any other manner of the profession of engineering and the good of mankind."

The corporation continues as treasurer for the Professional Engineers' Committee on Unemployment and has recently accepted appointment as treasurer for Engineers' Council for Professional Development, thus further serving the profession and its interrelated organizations, and fulfilling the purpose of its creation by the Founder Societies.

Members of the governing bodies of the Founder Societies and officers of their joint functional organizations met on the evening of May 20 and heard progress reports of the many projects which are operated jointly by the Founder Societies. The benefit of social contact by members of these groups was felt to be an important factor in tending to knit together more firmly the entire profession, through better understanding of the work and objectives of the different groups of joint activities of the Founder Societies.

Important improvements are being made in the public-address system in the Engineering Auditorium in order that it shall represent the ultimate in service to its users. This new system, coupled with our motion-picture projectors will make possible sound and color pictures in the auditorium, and should prove attractive to Society users and patrons and bring additional engagements. Seats have been added on the main floor of the auditorium thus increasing comfort by the more judicious use of space. Windows throughout the Engineering Societies Building have been repaired to prevent drafts and discomfort to the occupants of the offices, at the same time conserving steam, and the physical property.

The several departments of United Engineering Trustees, Inc., have remained within their budgets which were made on a most conservative basis. The income from the Engineering Societies Building activities was at a minimum owing to the continued reduction on a most conservative basis.

Full statistical and detailed reports on the activities of the United Engineering Trustees, Inc., is presented to the Societies in the Annual Reports of the President and the General Manager in October of this year.

Respectfully submitted,

D. Robert Yarnall
Walter Rautenstrauch
Harold V. Coes
A.S.M.E. Representatives

Service Rendered by the Office of the Society

The function of the office of the Society is to perform the regular duties connected with the many committee activities of the Society, to serve as a service bureau to the members and on behalf of the members as a service bureau to the public, in so far as facilities and resources permit. In general, the office staff keeps the books, maintains an accurate address record, edits the publications, secures the advertising, and maintains contacts with the Local Sections, Professional Divisions, Student Branches, and Technical Committees, all under the policy direction of the Council. In detail, however, the activities of the office are complicated. To aid in understanding them the following brief statement is given as an indication of the volume of work being performed in each group.

Secretary's Department (*3*): Secretary and two assistants.

Supervision. Coordination. Contacts with joint bodies. Direct service to the Council, Executive Committee, and Special Committees.

Field Department (*12*): Assistant Secretary and eleven assistants nine in New York, one in Chicago, one in Tulsa.

Seventy-one Local Sections (visited 38 in 1935), correspondence, speakers lists, appropriations.

Seventeen Professional Divisions (with about 60 subcommittees), six National Meetings.

Annual Meeting, 50 sessions, 105 authors, 60 committee meetings.

Semi-Annual Meeting, 20 sessions, 42 authors.

One hundred and thirteen Student Branches (visited 48 in 1935), 3300 applications for student membership; 10 district conferences.

Admissions, 2000 applications requiring 6000 communications, 340 resignations.

Employment, Engineer's National Relief Fund, Professional Engineers' Committee on Unemployment, Engineering Societies Employment Service.

Special Duties, technical inquiries, foreign visitors, awards, Max Toltz Fund, Freeman Fund, handling cases of 6240 dropped or suspended members.

Editorial Department (*6*): Editor, five full-time assistants, two part-time. Twenty-one hundred pages in *Mechanical Engineering* text and Transactions (including *Journal of Applied Mechanics* and Student Branch Bulletin), edited, styled, proofread, and published. Reports of Committees, Codes, etc., and special-meeting printing.

Advertising and Sales Department (*15*): Advertising manager, three sales representatives, seven on production of *Mechanical Engineering* advertising and Catalog including promotion, special service, layout, proof checking, publishing, and billing. Separate distribution list of 14,000 for catalog.

Publications Sales (**4**): Sales of codes, reports, reprints, etc.

Technical Committee Department (*6*): Assistant Secretary and five assistants.

Four hundred and thirty-nine Committees and subcommittees of Boiler Code, Power Test Codes, Standardization, Safety, and Research. Follow-up and publication of reports. Committee meetings, etc.

Relations with American Standards Associations, International Electrotechnical Commission, etc.

General Office (*22*): Office manager in charge.

Accounting (3): 15,000 individual accounts, 200 commercial accounts, 30,000 individual cash receipts annually, 5000 checks written annually.

Shipping, stores, and mail (4): 250,000 pieces of mail received, 8400 orders for publications, badges, etc. handled.

Addressograph and mimeograph (4): 16,000 address plates, 8000 changed each year; 30,000–40,000 address imprints per month; 50,000 copies from 250 stencils each month.

Filing (2).

General clerical and stenographic (8): master address record, 14,000 names, reception (800 visitors per month), stencil cutting, mail opening, etc.

C. E. Davies, *Secretary*

Depositories for A.S.M.E. Transactions in the United States

BOUND copies of the complete Transactions of The American Society of Mechanical Engineers will be found in the libraries in the United States and other countries which are listed on the following pages.

Alabama

Auburn — Engineering Library, Alabama Poly. Inst.
Birmingham — Public Library

Arkansas

Fayetteville — Engineering Library, Arkansas University

California

Berkeley — Library, University of California
Long Beach — Public Library
Los Angeles — Public Library
University of Southern California
Oakland — Oakland City Library
Teachers' Professional Library
Pasadena — Library, California Institute of Technology
Santa Clara — Library, University of Santa Clara
San Diego — Public Library
San Francisco — Public Library (Civic Center)
Engineers Club of San Francisco
Mechanics Institute
Stanford Univ — Library, Stanford University

Colorado

Boulder — Library, University of Colorado
Denver — Public Library
Fort Collins — Colorado State Agricultural College

Connecticut

Bridgeport — Public Library
Hartford — Public Library
New Haven — Public Library and Yale University
Waterbury — Silas Bronson Library

Delaware

Newark — University of Delaware
Wilmington — Wilmington Free Institute

District of Columbia

Washington — Scientific Library, U. S. Patent Office
Library of Congress
Bureau of Standards Library
George Washington University
Catholic University

Florida

Gainesville — University of Florida
Jacksonville — Free Public Library
Miami — Public Library
Tampa — Public Library

Georgia

Atlanta — Carnegie Public Library
Georgia School of Technology
Savannah — Public Library

Idaho

Moscow — University of Idaho

Illinois

Chicago — John Crerar Library
Western Society of Engineers
Lewis Institute of Technology
Library, Armour Institute of Technology
Public Library of Chicago
Moline — Public Library
Urbana — University of Illinois

Indiana

Evansville — Public Library
Fort Wayne — Public Library
Indianapolis — Public Library and Indiana State Library
Notre Dame — Library, University of Notre Dame
Terre Haute — Rose Polytechnic Institute
West Lafayette — Library, Purdue University

Iowa

Ames — Iowa State College
Des Moines — Public Library
Iowa City — State University of Iowa

Kansas

Kansas City — Public Library, Huron Park
Lawrence — Library, University of Kansas
Manhattan — Kansas State Agricultural College
Wichita — Wichita City Library

Kentucky

Lexington — University of Kentucky
Louisville — Speed Scientific School
University of Louisville

Louisiana

Baton Rouge — Louisana State University
New Orleans — Howard Memorial Library
Louisiana Engineering Society
Public Library
Tulane University

Maine

Orono — University of Maine

Maryland

Annapolis — United States Naval Academy
Baltimore — Johns Hopkins University
Engineers Club of Baltimore
Public Library

Massachusetts

Boston — Engineering Societies of New England
Northeastern University
Boston Public Library
Cambridge — Harvard University (Engineering Library)
Massachusetts Institute of Technology
Fall River — Public Library
Lowell — Lowell Textile Institute
Lynn — Free Public Library
New Bedford — Free Public Library
Springfield — Springfield City Library
Tufts College — Tufts College
Worcester — Worcester Polytechnic Institute
Free Public Library

Michigan

Ann Arbor — University of Michigan
Detroit — Public Library
Cass Technical High School
Highland Park Public Library
University of Detroit
East Lansing — Michigan State College
Flint — Public Library
Grand Rapids — Public Library
Houghton — Michigan College of Mining & Technology
Jackson — Public Library

Minnesota

Duluth — Public Library
Minneapolis — University of Minnesota
Minneapolis Public Library (Engineering and Circulating Libraries)
St. Paul — James Jerome Hill Reference Library

Mississippi

State College — Mississippi State College

Missouri

Columbia — University of Missouri
Kansas City — Public Library
Rolla — Missouri School of Mines and Metallurgy
St. Louis — Engineers Club of St. Louis
Public Library
Washington University
Mercantile Library

Montana

Bozeman — Montana State College

Nebraska

Lincoln — University of Nebraska
Omaha — Public Library

Nevada

Reno — University of Nevada Library

New Hampshire

Durham — University of New Hampshire

New Jersey

Bayonne — Free Public Library
Camden — Free Public Library
Elizabeth — Free Public Library
Hoboken — Stevens Institute of Technology
Jersey City — Free Public Library
Newark — Newark College of Engineering
Free Public Library
New Brunswick — Rutgers University
Paterson — Free Public Library
Princeton — Princeton University
Trenton — Free Public Library

New York

Albany — New York State Library
Brooklyn — Polytechnic Institute
Pratt Institute
Brooklyn Public Library
Buffalo — The Grosvenor Library
Engineering Society of Buffalo
Buffalo Public Library
Ithaca — Cornell University
Jamaica, L. I. — Queens Borough Public Library
New York — Engineering Societies Library
Public Library
College of the City of New York
Cooper Union
Columbia University
New York Museum of Science and Industry
New York University Library
Potsdam — Clarkson College of Technology
Rochester — Rochester Engineering Society
Schenectady — Union College
Syracuse — Syracuse University
Public Library
Troy — Rensselaer Polytechnic Institute
Utica — Public Library
Yonkers — Public Library

North Carolina

Chapel Hill — University of North Carolina (Engineering Library)
Raleigh — North Carolina State College

North Dakota

Fargo — North Dakota State Agricultural College
Grand Forks — University of North Dakota

Ohio

Ada — Ohio Northern University
Akron — Public Library
University of Akron
Canton — Public Library
Cincinnati — University of Cincinnati
Public Library
Engineers Club of Cincinnati
Cleveland — Public Library
Case School of Applied Science
Cleveland Engineering Society
Columbus — State of Ohio Library
Public Library
Ohio State University
Dayton — Engineers Club of Dayton
Toledo — Public Library
University of Toledo
Youngstown — Public Library

Oklahoma

Norman — Oklahoma University
Oklahoma City — Public Library
Stillwater — Oklahoma Agricultural and Mechanical College
Tulsa — Public Library

Oregon

Corvallis — Oregon State Agricultural College
Portland — Portland Library Association

Pennsylvania

Allentown — Free Library
Bethlehem — Lehigh University
Easton — Public Library
Lafayette College
Erie — Public Library
Lewisburg — Bucknell University
Philadelphia — Engineers Club
Drexel Institute
The Free Library
University of Pennsylvania
Franklin Institute
Pittsburgh — University of Pittsburgh
Engineers' Society of Western Pennsylvania
Carnegie Institute of Technology
Carnegie Library (Schenley Park)
Carnegie Free Library of Allegheny
Reading — Public Library
Scranton — Public Library
State College — Pennsylvania State College
Swarthmore — Swarthmore College
Villanova — Villanova College
Wilkes-Barre — Public Library

Rhode Island

Kingston — Rhode Island State College
Providence — Brown University
Providence Engineering Society
Public Library

South Carolina

Clemson College — Library, Clemson College

Tennessee

Kingsport — Public Library
Knoxville — University of Tennessee
Memphis — Goodwin Institute
Nashville — Vanderbilt University

Texas

Austin — University of Texas
College Station — Agricultural & Mechanical College of Texas
Dallas — Public Library
Southern Methodist University
El Paso — Public Library
Fort Worth — Carnegie Public Library
Houston — Rice Institute
Public Library
Lubbock — Texas Technological College (School of Engineering)
San Antonio — Carnegie Library

Utah

Salt Lake City — University of Utah
Public Library

Vermont

Burlington — University of Vermont

Virginia

Blacksburg — Virginia Polytechnic Institute
Charlottesville — University of Virginia
Norfolk — Public Library
Richmond — Virginia State Library

Washington

Pullman — State College of Washington
Seattle — Public Library
Engineers Club
University of Washington
Spokane — Public Library
Tacoma — Public Library

West Virginia

Morgantown — West Virginia University

Wisconsin

Madison — Library, University of Wisconsin
Milwaukee — Public Library
Board of Industrial Education, Vocational School Library
Marquette University

Wyoming

Laramie — Wyoming University

Depositories for A.S.M.E. Transactions Outside the United States

Argentine

Buenos Aires — Biblioteca de la Sociedad Cientifica

Australia

Adelaide — Public Library of Adelaide
Melbourne — Public Library of Victoria
Perth — University of Western Australia Library
Sydney — Public Library, N. S. W., Sydney

Belgium

Louvain — University of Louvain

Brazil

Rio de Janeiro — Bibliotheca da Escola Polytechnica
Bibliotheca Nacional
Sao Paulo — Bibliotheca da Escola Polytechnica

Canada

Montreal — McGill University
Engineering Institute of Canada
Toronto — University of Toronto, Library

Chile

Santiago — Universidad de Chile, Facultad de Ciencias Fisicas y Matematicas (Engrg. School)

China

Peiping — College of Technology of Peiping University

Cuba

Havana — Cuban Society of Engineers

Czechoslovakia

Prague — Masarykova Akademie Prace
Society of Czechoslovak Engineers

Danzig Free City — Bibliothek der Technischen Hochschule

Denmark

Copenhagen — The Royal Technical College

England

Birmingham — Birmingham Public Libraries
Bristol — University of Bristol
Cambridge — University of Cambridge
Leeds — University of Leeds
Liverpool — Public Library of Liverpool
Liverpool Engineering Society
London — City & Guild Engineering College
Institution of Automobile Engineers
Institution of Mechanical Engineers
Institution of Civil Engineers
Institution of Electrical Engineers
The Junior Institution of Engineers
The Royal Aeronautical Society
Manchester — Manchester Public Libraries (Reference Library)
Oxford — University of Oxford
Newcastle-upon-Tyne — The North East Coast Institution of Engineers and Shipbuilders
Sheffield — Sheffield Public Libraries

Wales

Cardiff — Cardiff Public Library

France

Lyons — University of Lyons
Paris — École Nationale des Arts et Metiers
École Nationale Supérieure de L'Aeronautique
École Centrale des Arts et Manufactures de Paris
Société des Ingénieurs Civils de France

Germany

Berlin — Verein deutscher Ingenieure
Bibliothek der Technischen Hochschule
Breslau — Bibliothek der Technischen Hochschule
Cologne (Köln) — Universitäts- und Stadtbibliothek

Germany (Continued)

Dresden — Bibliothek der Technischen Hochschule
Düsseldorf — Bücherei des Vereines deutscher Eisenhüttenleute
Frankfort — Technische Zentralbibliothek
Hamburg — Bibliothek der Technischen Staatslehranstalten
Hanover — Bibliothek der Technischen Hochschule
Karlsruhe — Bibliothek der Technischen Hochschule
Leipsic — Stadtbibliothek
Munich — Bibliothek der Technischen Hochschule
Bibliothek des Deutschen Museums
Stuttgart — Bibliothek der Technischen Hochschule

Hawaii

Honolulu — University of Hawaii Library

Holland

Amsterdam — Koninklijke Akademie von Wetenschappen
Delft — Bibliotheek der Technische Hoogeschool
The Hague — Koninklijk Instituut van Ingenieurs
Rotterdam — Nationaal Technisch Scheepvaartkundig Institut

India

Bangalore — Mysore Engineers Association
Calcutta — Bengal Engineering College
Poona — Poona College of Engineering
Rangoon — University of Rangoon

Ireland

Belfast — Queen's University of Belfast

Italy

Milan — Biblioteca della R. Scuola d'Ingegneria
Comitato Autonomo per l'Esame della Invenzioni
Naples — Biblioteca della R. Scuola d'Ingegneria
Rome — Biblioteca della R. Scuola d'Ingegneria
Consiglio Nazionale delle Ricerche presso il Ministero della Educazione Nazionale
Turin — Biblioteca della R. Scuola d'Ingegneria

Japan

Kobe — Kobe Technical College
Tokyo — Imperial University Library
The Society of Mechanical Engineers
Yokohama — Library of Yokohama

Mexico

Mexico City — Asociacion de Ingenieros y Arquitectos de Mexico
Library of the Escuela de Ingenieros Mecanicos y Electricistas

Norway

Oslo — Den Polytekniske Forening

Poland

Warsaw — Bibljoteka Publicazna

Porto Rico

Mayaguez — University of Porto Rico

Portugal

Lisbon — Institute Superior Technico

Roumania

Bucharest — Scoala Polytechnica din Bucharest

Scotland

Glasgow — Royal Technical College
Mitchell Library

South Africa

Cape Town — University of Cape Town
Johannesburg — South African Institute of Engineers

Sweden

Stockholm	Kungl. Tekniska Hogskolan
	Svenska Teknologföreninger
Gothenburg	Chalmers Tekniska Institut

Switzerland

Zurich	Bibliothek der Eidg. Technischen Hochschule

Turkey

Istanbul	Robert College

U.S.S.R.

Kharkov	Supreme Economic Council of Ukraine
Leningrad	Leningrad Polytechnic Institute
Moscow	Supreme Council of National Economy
Tomsk	Tomsk Polytechnic Institute

Index photographed at the
beginning for the convenience
of the microfilm user.